TAGGART


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HANDBOOK
OF
ORE DRESSING

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BY

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TO MY FRIEND

JAMES FARLEY McCLELLAND

TO WHOM

MORE THAN ANY OTHER

THE INSPIRATION FOR THIS BOOK IS DUE

PREFACE

It is hoped that this book may serve as a reference handbook for engineers practicing or investigating ore-dressing processes and also as a text-book for students of the subject. With these ends in view the text has been set in type of two sizes, the thread of description and principle being carried along in the larger type while matter of reference character, numerical data, mill performances, supplementary argument, *etc.*, are set in the smaller type.

No reference handbook in ore dressing has been published since the appearance of the third and fourth volumes of Richards' monumental work in 1909. Since that time practice in the art has been revolutionized by froth-flotation concentration of sulphide-ore slimes and by the use of ball mills and the like for fine grinding.

The subject matter of the book is arranged on the principle that the metal or mineral as used in the arts is the proper focus of interest at every mine; that the operations converging on this point are mining and treatment of the ore, refining of the treatment-plant product, and sale of the refined product; and that the unit in ore treatment is the whole mill. Pursuant to this idea the book starts off, after a short introduction, with alphabetical presentation of the metals and minerals whose ores require dressing as part of their preparation. As is, of course, natural in a book on ore treatment, dressing is the principal topic under each substance heading, but discussion of treatment is, in each case, preceded by a thorough summary of the economic elements, occurrence, uses, production, price range, *etc.* In subsequent chapters individual milling operations, such as coarse and intermediate crushing, fine grinding, screening, classification, various methods of concentration, dewatering, *etc.*, are treated separately. Sections on sampling, handling of ores and treatment products, designs of milling plants, hydrometallurgy, and on the engineering fundamentals, physics, mathematics and mechanics follow and have been contributed by associate editors who are experts in the various subjects. The sections on mathematics and mechanics were written in an endeavor so to present these engineering tools that they may be most easily picked up by an engineer who has grown rusty in their use; the presentation is, therefore, in the form of a series of rules for the solution of typical problems, with numerous examples of methods of use; the theory underlying derivation of the rules must be sought in more mathematical texts.

It follows from the statement of plan that treatment of ores of non-metallic minerals has not been segregated from treatment of metallic ores, except in so far as the alphabetical arrangement of Section 2 so serves. Such segregation is not justified on physical grounds and both metallic and non-metallic practice have suffered in the past from a wholly arbitrary separation which has resulted in ignorance among the practicioners in each field of the developments in the other.

The handbook of an art is necessarily somewhat encyclopedic in character. Notwithstanding that ore crushing, gravity concentration, dewatering, *etc.*, are merely practical applications of well understood physical phenomena, yet each operation introduces so many diverse factors, complexly interrelated and practically incapable of segregation, that it is not possible to set down quantitative rules for operation nor formulas for predicting results, and estimates of future behavior must be based principally on study of past performances. In the present book, therefore, performances of machines and processes, critically culled from the replies to an elaborate questionnaire and a large number of letters, as well as from thorough study of periodical literature, make up a large part of the text.

If exception is taken to the space allotted to such apparatus as the gravity stamp, Chilean mill, grinding pan, film sizers and to the older flotation processes, the reply is that as regards the first three the battle of extinction is only recently decided, if yet finally decisive, and that the picture of the battlefield is useful to deter others from renewing the fight; that the film sizers, imperfect as they are, remain the only tools for treating slimes not amenable to froth flotation or hydrometallurgy; and that thorough understanding of the physical principles of flotation and the no less important legal pitfalls is not to be had without acquaintance with the "prior art."

Costs as given in the text are specifically dated or the date indicated by the context. They may usually be related to present time by a multiplier representing the ratio of combined labor and commodity index figures for the published date and the present.

Textual reference to the literature is a perplexing problem in all writing, and when the number of such references runs into the thousands, as in a handbook, the difficulty is aggravated, and no solution is entirely satisfactory either to author or reader. The plan adopted herein, in all but one section, is to run the reference in the text, set off in parenthesis and abbreviated in the form adopted in legal writing where, perhaps more than in any other writing, textual references abound. The method has the distinct advantage over number reference to a bibliography that it makes immediately apparent to the reader acquainted with the literature the general source of the authority cited and eliminates the irritation involved in search for the page of bibliography, while the interruption to thought caused by the parenthetical insertion need be no greater where the parenthesis contains six or eight characters than where it contains one or two. The abbreviations used are listed on page xi.

The editor hereby makes grateful acknowledgment to his associates and publisher for their forbearance with his delay in publication, occasioned both by the magnitude of the work of writing and editing and by the stress of professional engagements; to the men and mining companies listed on the following page, for answering and permitting answer to the elaborate questionnaire that formed the foundation upon which much of the book was built; to all of the mining-machinery manufacturers who freely offered and furnished information relative to their products; and to many friends who gave uncomplainingly of their time to help.

ARTHUR F. TAGGART

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ABBREVIATIONS

References to articles in periodicals are written in the order: volume number, abbreviation for periodical name, page number; thus *100 J 110* is to be read, Engineering and Mining Journal, vol. 100, page 110. References to books are made by giving the surname of the author in italics. An alphabetical list of the periodical abbreviations and book references follows.

A Transactions American Institute of Mining [and Metallurgical] Engineers.
Aa Australasian Institute of Mining Engineers.
ACS Journal of the American Chemical Society.
AES Transactions of the American Electrochemical Society.
AGLJ American Gas Light Journal.
AIM American Institute of Metals.
BANCROFT. Applied colloid chemistry, W. D. Bancroft, McGraw-Hill Book Co., Inc., 1921.
BOWIE. A practical treatise on hydraulic mining in California, A. J. Bowie, Jr., D. Van Nostrand, 1905.
Bul. Bulletin.
CA Coal Age.
CI Coal Industry.
CIT Carnegie Institute of Technology.
CMI Transactions Canadian Mining Institute.
CMIA Proceedings Coal Mining Institute of America.
CMJ Canadian Mining Journal.
CAMB. PHIL. Transactions Cambridge Philosophical Society.
DHH Transactions of the Delaware and Hudson and Hudson Coal Co. Mining Institute, May, 1915.
DANA. System of mineralogy, J. N. Dana, John Wiley and Sons, Inc., 1903.
Text book of mineralogy, Dana and Ford (W. E.), John Wiley and Sons, Inc., 1922.
DAVY AND FARNHAM. Microscopic examination of the ore minerals, W. M. Davy and C. M. Farnham, McGraw-Hill Book Co., Inc., 1920.
EL Engineer, London.
EN Engineering News; Engineering News-Record.
ER Electrical Review.
EW Electrical World.
FULTON. A manual of fire assaying, C. H. Fulton, McGraw-Hill Book Co., 1911.
GILLETTE. Handbook of cost data for contractors and engineers, H. P. Gillette, Myron C. Clark Publishing Co., 1910.
HETZEL. Belt conveyors and belt elevators, F. V. Hetzel, John Wiley and Sons, Inc., 1922.
HOOVER. Concentrating ores by flotation, T. J. Hoover, Mining Magazine, London, 1916.
IEC Journal of Industrial and Engineering Chemistry.
IME Transactions of the Institution of Mining Engineers.
IMM Transactions Institution of Mining and Metallurgy.
J Engineering and Mining Journal; Engineering and Mining Journal-Press.
JCM Journal of the Chemical, Metallurgical and Mining Society of South Africa.
JOHANNSEN. Manual of petrographic methods, A. V. Johannsen, McGraw-Hill Book Co., Inc., 1918.

JULIAN, SMART and ALLEN. Cyaniding gold and silver ores, H. Forbes Julian and Edgar Smart, revised by A. W. Allen; J. B. Lippincott, 1921.
KETCHUM. The design of walls, bins and grain elevators, Milo S. Ketchum, McGraw-Hill Book Co., 1919.
KIDDER. The architects' and builders pocket-book, F. E. Kidder and Thos. Nolan, John Wiley and Sons, Inc.
LSMI Proceedings Lake Superior Mining Institute.
LADOO. Non-metallic minerals, R. B. Ladoo, McGraw-Hill Book Co., Inc., 1925.
LIDDELL. Handbook of chemical engineering, Donald M. Liddell, McGraw-Hill Book Co., 1922.
LOUIS. The dressing of minerals, Henry Louis, Arnold, London, 1909.
MCJ Mining Congress Journal.
MEW Mining and Engineering World.
MH Mines Handbook, W. H. Weed, The Mines Handbook Co., Annual.
MIMM Mexican Institution of Mining and Metallurgy.
MIS Transactions of the Mining Institute of Scotland.
MM Mining Magazine.
MMt Mining and Metallurgy.
MR Mineral Resources of the U. S.; U. S. Geological Survey, Annual.
MSM Minnesota School of Mines, Engineering Experiment Station.
M & M Mines and Minerals.
MACLEOD and WALKER. Metallurgical analysis and assaying, W. A. MacLeod and C. Walker, Griffin and Co., Ltd., 1903.
MARKS. Mechanical engineers' handbook, L. S. Marks, McGraw-Hill Book Co., 1916.
MERRIMAN. Mechanics of materials, Mansfield Merriman, John Wiley and Sons, Inc., 1906.
MIN. and ENG. REV. Mining and Engineering Review, Australia.
MURDOCH. Microscopical determination of the opaque minerals, Jos. Murdoch, John Wiley and Sons, Inc., 1916.
OD Ore dressing, Robert H. Richards, McGraw-Hill Book Co., 1908.
P Mining and Scientific Press.
PC Private communication.
PP Pre-print.
PEELE. Mining engineers' handbook, Robert Peele, John Wiley and Sons, Inc., 1918.
PHIL. MAG. London, Edinburgh and Dublin Philosophical Magazine.
PRO. ENG. SOC. W. PA. Proceedings Engineering Society of Western Pennsylvania.
PRO. INST. CIV. ENGRS. Proceedings of the Institute of Civil Engineers.
Q Questionnaire.
RMP A textbook of Rand metallurgical practice, Chas., Griffin & Co., Ltd., 1913.
RICHARDS. Ore dressing, R. H. Richards, McGraw-Hill Book Co., 1908; Text book of ore dressing, 1909; Revision, with C. E. Locke, 1925.
RITTINGER. Lehrbuch der aufbereitungskunde, P. ritter von Rittinger, Ernst and Korn, Berlin, 1867.
ROGERS. Study of minerals, Austin Flint Rogers, McGraw-Hill Book Co., 1912.
SCI Journal of the Society of Chemical Industry.
SMQ School of Mines Quarterly.
SEAMON. A manual for assayers and chemists, W. H. Seamon, John Wiley and Sons, Inc., 1910.
SO. AF. JOUR. IND. South African Journal of Industry.
SPURR and WORMSER. The marketing of metals and minerals, J. E. Spurr and F. E. Wormser, McGraw-Hill Book Co., Inc., 1925.
TAIEE Transactions American Institute of Electrical Engineers.
TB Textbook of ore dressing, Robert H. Richards, McGraw-Hill Book Co., 1909.
TP Technical paper.
TRAUTWINE. Civil engineers' pocket-book, J. C. Trautwine, Trautwine Pub. Co., 1919.
TRUSCOTT. A textbook of ore dressing, S. J. Truscott, Macmillan, 1923.
UI University of Illinois, Engineering Experiment Station.
UId University of Idaho, Pamphlets of the U. S. Bureau of Mines and the Idaho Department of Mines and Geology.
USGS United States Geological Survey.
UU University of Utah, Engineering Experiment Station.
UW University of Washington, Engineering Experiment Station.
VAN WAGENEN. Manual of hydraulic mining, T. F. Van Wagenen, Van Nostrand, 1913.
WIARD. The theory and practice of ore dressing, E. S. Wiard, McGraw-Hill Book Co., 1915.
WILSON. Hydraulic and placer mining, E. B. Wilson, John Wiley and Sons, Inc., 1918.

Reference dates. Table 1 gives dates corresponding to volume numbers for the periodicals cited most frequently. When citations are made to earlier volumes or to periodicals not in the table, dates are bracketed in the reference, if the date is important.

Volume numbers of periodicals

Reference letter	A	Aa	CA	CMI	CMJ	EN	IMM
1913	45, 46, 47	9,10,11,12	3, 4	16	34	69, 70	22
1914	48, 49, 50	13,14,15,16	5, 6	17	35	71, 72	23
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HANDBOOK OF ORE DRESSING

SECTION 1

INTRODUCTION

A large proportion of the inorganic and many of the organic substances that form so important a part of the structure of present-day civilization have their source in the earth's crust in such admixture, chemically or mechanically, with other substances that they are substantially useless. Rendering them into form fit for use is, very largely, a process of separation from start to finish, mining to separate them as an impure mass from the surrounding material in the earth, ore dressing to effect mechanical separation of the value-bearing constituents in this impure mass from the associated barren impurities, and finally, in most cases, chemical treatment of various kinds to break the bonds that hold the substance sought in undesirable association. Lead is typical. It occurs at scattered localities in the earth's crust, principally as a chemical compound with sulphur (galena), usually associated with other metallic sulphides and with a large preponderance of one or more of the ordinary rock-forming minerals. Its principal uses in the arts are in the form of the metal, as in pipe, tank linings, storage batteries and the like, and as a carbonate of high purity in paints. A few tons per year would satisfy the world's demand for galena as such. Chemical treatment is necessary to break the bond between lead and sulphur and collect the lead in useful form.

In the exceptional cases where galena is found in such large masses that it can be mined in a state of comparative purity, the requisite chemical treatment can be applied directly to the material as mined. But in most cases the galena occurs in small masses and grains intimately admixed with large quantities of worthless substances such as quartz or limestone, and the cost of chemical treatment to break up the galena and collect the lead is increased enormously by the presence of these diluents. Under such circumstances the material mined is first subjected to a process of mechanical separation by means of which the galena is segregated from the associated materials at a relatively small cost. The economic advantages of this treatment are made apparent in the following example.

Assume an ore consisting of galena scattered as grains and small veinlets through a body of dolomite. Assume that the mass as mined assays 5 per cent. lead, that the market price for lead is $0.07 per lb., that mining and delivery at the surface costs $3 per ton, that freight to the nearest smelter costs $5 per ton, that the smelter charge for treating material containing 5 per cent. lead is $10 per ton, that 97 per cent. of the lead in the ore is recover-

able by smelting and that the smelter pays 95 per cent. of the market price for the lead recovered. Assume further that by a process of ore dressing galena from the ore as mined can be separated from the dolomite in the form of a concentrate assaying 60 per cent. lead, that in the process 5 per cent. of the lead will be lost, that the process costs $0.75 per ton, that the freight charge on concentrate is $10 per ton, the smelter charge $2 per ton, and that 97 per cent. of the lead in the concentrate is recovered and paid for at 95 per cent. of the market price. The comparative receipts and expenditures on 100 tons of ore are shown in Table 1. In this case, which is typical, concentration changes a deficit of $11.55 per ton into a profit of $1.43 per ton.

Table 1. Comparison of profits from mining and treating a 5-per cent. lead ore, with and without concentration.

WITHOUT CONCENTRATION

Expenditures		*Receipts*	
Mining, 100 tons @ $3	$300.00	100 tons of ore produce 9700 lbs. of lead, which at 95 per cent. of 7 cents per lb. is	$645.05
Freight, 100 tons @ $5	500.00	Deficit	1154.95
Smelting, 100 tons @ $10	1000.00		
	$1800.00		$1800.00

WITH CONCENTRATION

Mining, 100 tons @ $3	$300.00	100 tons of ore produce 7.92 tons of concentrate containing 9500 lbs. of lead. This, when smelted, produces 9215 lbs. of lead which, at 95 per cent. of 7 cents per lb. is	$612.80
Concentrating, 100 tons @ $0.75	75.00		
Freight, 7.92 tons (*a*) @ $10	79.20		
Smelting, 7.92 tons @ $2	15.84		
Profit	142.76		
	$612.80		$612.80

a 100 tons of ore produce 7.92 tons of 60-per cent. concentrate under the conditions stated.

The problem of ore treatment confronts every mine manager. Mining should be looked upon as a manufacturing operation involving the production and sale of ore and ore products. If thus viewed, solution of the problem of ore treatment involves not only the technical knowledge necessary to decide upon the method or methods of treatment to which the ore is amenable, but also knowledge of the costs and possibilities of recovery by the various methods, and, more important yet, acquaintance with the possibilities of the market for the finished product. Such acquaintance must be founded on knowledge of the ultimate uses of the product, its production and price history, the ability of the market to absorb new production, the possibility and availability of substitutes, and the economic significance of differences in grade of the product marketed. The technical part of the problem is the simpler; if a relative weight can be assessed, the economic phase is the more important. The mining regions of the world are dotted with monumental failures, not the least of which are those in which every technical problem of production has been satisfactorily solved, but the problem of profitable disposal of product yet awaits solution.

Definitions. An ORE is any rock containing a valuable constituent in such amounts as to make mining and treatment of the rock to extract the valuable part a commercially profitable operation. The valuable constituent is ordinarily spoken of as VALUABLE MINERAL, often just MINERAL. The worthless part of ore is called GANGUE. In some ores the valuable mineral

is in the chemical state in which it is to be used in manufacture and the arts, for example, coal, graphite, sulphur and asbestos; in others, as for instance most ores of metals, the valuable mineral contains a worthless element in chemical combination with the metal.

The method of treatment depends upon the proportion of valuable mineral in the ore and its mode of distribution, both of which vary widely; and upon the chemical state of the valuable constituent. High-grade ores of metals are usually smelted without any preliminary treatment. For discussion of smelting see, among others, Schnabel, *Handbook of metallurgy;* Hofman, *General metallurgy, Metallurgy of lead;* Hofman and Hayward, *Metallurgy of copper;* Peters, *Principles of copper smelting, Practice of copper smelting;* Ingalls, *Metallurgy of zinc and cadmium;* Certain low-grade ores of metals are treated by methods that consist essentially in dissolving the metals from the ore by means of some solvent that does not dissolve the gangue, separating the solution from the solid residue, and separating the metal from the solution. Such processes are described in Sec. 15. All other metallic ores, if treated at all, are subjected to some mechanical process that results in separating the ore into a CONCENTRATE, containing the bulk of the valuable mineral in relatively pure form; and TAILING, consisting of gangue with but a small residue of valuable mineral. ORE DRESSING is the art of mechanical treatment of ores to produce concentrate and tailing.

Crushing is the most expensive part of ore dressing in the case of a large majority of ores. The economic minerals of gold, silver, copper, tin, lead and zinc (excluding gold- and tin-placer deposits) almost invariably occur in such a fine state of dissemination in the gangue that crushing must be carried down to a fraction of a millimeter in order to free them for separation. This work must be done in steps and there are certain machines best suited to each step in the reduction process. The crushers that first take the ore as it comes from the mine (some of them can take 6-ft. cubes) are called COARSE CRUSHERS; those that take the product of the coarse crushers and break it to a size suitable to feed to the final crushers are called INTERMEDIATE CRUSHERS. These machines take material in the range from 2½- to 6-in. and deliver it at from 1½- or 2-in. down to ½-in. size. FINE CRUSHERS or GRINDERS take the product of the intermediate crushers and deliver a product at any desired size down to about 0.05-mm. maximum.

Concentration involves subjecting the crushed product to one or more operations of various character that utilize differences in physical properties of the mineral and the gangue. The most important physical properties thus utilized are specific gravity, surface energy, color, luster and permeability. When differences in specific gravity form the basis of the concentrating process, it is called GRAVITY CONCENTRATION; the processes that utilize differences in surface energies are grouped under the name of FLOTATION; differences in color and luster are the basis for HAND PICKING; and differences in permeability are utilized in MAGNETIC CONCENTRATION.

Gravity concentration. Whenever two particles of different specific gravities are allowed to fall under the influence of gravity alone through a fluid that offers resistance to their passage, differences in the falling rate develop and these differences are greater the greater the difference in specific gravity and the greater the resistance offered by the fluid. Impractical extremes are the equal fall of a lead shot and a feather in a vacuum and the sinking of platinum and flotation of iron in a bath of mercury. Practical instances are the difference in rate of fall of gold and quartz particles of equal

size in water and the sinking of slate and flotation of coal in a semi-fluid mass composed of quartz sand mechanically maintained in suspension in water. When to this difference in falling rate is added difference in response to impulses such as the transporting effect of a substantially horizontal stream of water, the result is to spread out the different particles along the bottom of the water channel with the particles of highest specific gravity nearest the source and the lightest farthest down stream. This is the method by which gold has become concentrated in placer deposits and also that by which it is won from the accompanying gravels in sluice boxes.

The rate of fall of particles in a resistant fluid is proportional to their effective weights in the fluid. Hence large particles of a given specific gravity fall more rapidly than small particles, and of equal-sized particles of different specific gravities the denser falls more rapidly. It is apparent, then, that in a mixture of grains of different specific gravities and sizes there will be equal-falling grains of different specific gravities and that in all such equal-falling heterogenous groups the heavier will be the smaller. The process of treatment that involves separating a mixture of grains into equal-falling groups is called SORTING or CLASSIFICATION. A process that groups equal-sized grains together is called SIZING. All gravity-concentration processes that utilize as the separating medium a fluid less dense than any of the minerals in the ore consist of sorting and sizing as supplementary operations. Either may precede. If the separating medium is water, as is usual, the process is called WATER-GRAVITY CONCENTRATION, or, frequently, GRAVITY CONCENTRATION; if air is the medium, PNEUMATIC SEPARATION or AIR CLEANING. To be effective, gravity concentration requires a difference in specific gravity of at least 0.5 to 1.0, depending upon the absolute specific gravity of the lighter mineral; the smaller difference is sufficient when the lighter mineral has a density of 1.3 to 1.5 (coal), the greater is desirable when the lighter mineral is, say, quartz (sp. gr. = 2.6). The principal water-gravity concentrating machines are washers, jigs and shaking tables.

When the density of the separating medium is greater than that of the lighter mineral, sizing is unnecessary, since the buoyancy of any substance in a liquid heavier than itself is independent of the size of particle. The processes of this class are called HEAVY-FLUID PROCESSES. They have the advantage of being able to work with a smaller specific-gravity difference than water-gravity concentration or pneumatic separation. The only one of practical value at the present time is one that uses a suspension of sand in water as the separating fluid.

Flotation concentration consists in causing attachment of certain minerals, in a finely-ground state admixed with water, to gas bubbles of such size with respect to the mineral particles attached to them that the gas-mineral aggregates are buoyant in the aqueous mixture (PULP) and float away from their former associates. This differential gas attachment depends upon differences in the surface energy (connected with the luster) of the different minerals. In general, minerals of metallic luster will float while those of non-metallic luster are unaffected, but by suitable means differences in floatability of the minerals of metallic luster can be so accented that one can be floated away from another (DIFFERENTIAL FLOTATION), and, by other suitable means, certain minerals of non-metallic luster can be floated.

There are a number of distinctly different flotation processes, but only two, viz.: the AGITATION-FROTH PROCESS and the BUBBLE-COLUMN PROCESS, have been widely adopted.

Magnetic concentration utilizes differences in permeability to effect separation. Minerals such as magnetite, franklinite and pyrrhotite are relatively so highly permeable as to be readily separable from all other minerals by bringing the mixture into a weak magnetic field. Certain other minerals, notably hematite and pyrite, are rendered highly magnetic by roasting. A number of other minerals, *e.g.*, ferriferous blende, iron garnet, etc., may be separated readily from quartz, feldspar, calcite and the like by subjection to a powerful magnet. Magnetic separation may be practiced at any size, theoretically. Practically the range is below 2-in. for highly-magnetic particles and below 2-mm. for feebly-magnetic. At larger sizes the magnets must be made so heavy and powerful, in order to overcome the inertia of the particles and the necessarily large air gaps, that the machines become structurally impractical.

Hand-picking is possible whenever the valuable mineral and gangue differ distinctly in color and/or luster and when they occur as large individuals and break clean. This method of concentration is indicated when clean individuals of either gangue or mineral, 3-in. or larger in size, occur in sufficient quantity to permit one or more operators to keep busy in removing them from a traveling surface. Under such circumstances it will usually work out to be cheaper to recover valuable mineral or to discard waste in this fashion than to break further and make the separation mechanically. Hand-picking is particularly useful in coal cleaning.

Miscellaneous methods of concentration. Differences in electrical conductivity of minerals may be utilized for separation of finely-crushed, closely-sized grades. The process is called ELECTROSTATIC SEPARATION. In general the minerals of metallic luster are good conductors and those of non-metallic luster poor conductors. Blende is an exception to the general rule for metallic minerals, and this fact has been utilized (see p. 177) to effect separation from galena.

A few minerals, *e.g.*, fluorspar, DECREPITATE (disintegrate spontaneously on heating). Separation of these from others that have not this property may be made by first sizing the mixture of grains within small limits, then heating to cause decrepitation, and finally screening out the broken fragments. The process is of limited application.

Separation by oil and grease in bulk utilizes differences in the same property of minerals, *viz.*: surface energy or surface tension, that is utilized in flotation. The process is not universally applicable but two notable examples of its utility are: (a) the separation of diamonds from associated minerals, beryl, tourmaline, quartz, etc., by passing them in semi-suspension in water over a surface covered with stiff grease (GREASED TABLES), and (b) separation of coal from slate by violent agitation of the finely-ground mixture in water with a quantity of fuel oil (GRANULATION).

Recovery. Ratio of concentration. Technical success in concentration is measured by the ratio of the weight of valuable mineral in the concentrate to the weight in the original material (usually called FEED or HEADS), due regard being had also to the richness (GRADE) of the concentrate, *i.e.*, the percentage of metal contained. This ratio is known as RECOVERY or EXTRACTION. The ratio of the weight of feed to the weight of concentrate is called the RATIO OF CONCENTRATION. (For calculation and discussion of these terms see Sec. 22, Art. 14.)

The scheme of concentration (FLOW-SHEET) depends on the mineralogical character of the ore and on the size of the individual particles of the economic

mineral and of the barren or low-grade constituent. If either the economic mineral or the low-grade constituent occurs in aggregates upwards of three inches in size and in any considerable bulk at these sizes, hand-picking will probably prove desirable as the initial step in concentration, provided, as is almost invariably the case, there is sufficient difference in appearance to allow pickers to choose with speed and certainty. If the economic mineral has high unit value, it may be picked out at sizes down to 1-inch. Crushing prior to picking should be planned to do no more breaking than is necessary to sever the material that is to be picked from the balance of the ore. If, following hand-picking, the remaining ore is amenable to separation by both gravity concentration and flotation, the flow-sheet will usually show these two processes in the sequence named and the MIDDLING (partial concentrate consisting largely of unsevered grains of mineral and gangue) from treatment at coarse sizes will be re-ground to free the locked valuable mineral. If gravity concentration or magnetic separation only is applicable, it is usually applied to successively finer grades with re-grinding of the middling produced at each step. When both gravity concentration and flotation are applicable but the valuable mineral is so finely disseminated that gravity concentration at relatively coarse sizes is not possible, it becomes a serious question whether gravity concentration should be employed at all, or whether it is not best to grind immediately to flotation size and treat all the ore by this process alone. Gravity concentration is cheaper, but it ordinarily cannot reject sufficiently low-grade tailing; flotation must, therefore, be applied eventually and the saving in cost on the tonnage of concentrate removed by gravity concentration must pay for the cost of the gravity operation. This it cannot ordinarily do. The simplicity and relatively small size of the all-flotation plant would ordinarily dictate its selection even in the face of an estimated small saving in operating cost with the gravity plant ahead. If, however, the gravity plant increased the saving or raised the grade of concentrate, it would usually justify itself economically.

If the economic mineral is very finely disseminated and amenable to flotation, gravity concentration and, of course, hand-picking are out of the question entirely.

When low-intensity magnetic separation can be used, it is economically preferable to gravity concentration.

High-intensity magnetic separation, electrostatic separation and decrepitation are expensive operations, justified only when the simpler and cheaper processes fail.

The unit value of the economic mineral in an ore sets the limit to which the ultimate pursuit of that mineral in the mill can be carried profitably. It is apparent that the pursuit of gold, with a market value of \$20 per oz., can be carried further and prosecuted at greater expense per unit weight recovered than that of silver at \$0.60 per oz.; that silver can be sought more carefully and to a greater extent than copper; copper than lead, and lead than iron. It follows also that minerals of highest unit value may be profitably recovered from ores in which their dissemination is most scanty, and these ores are frequently the ones that require the greatest number of machines, *i.e.*, the greatest complexity of flow-sheet, for successful treatment.

Unit value has another phase, involving transportation and smelting charges. From the point of view of the mill operator the important unit value of his product is that in concentrate as delivered from his plant. The value of lead per pound in high-grade concentrate near a Missouri smelter

may easily be higher than that of copper in a low-grade concentrate several hundred miles from a smelter in Arizona. Similarly the net value of an ounce of gold in low-grade cyanide-process bullion, salable at the mint, may well be so much greater than that of the same ounce when contained in pyritic flotation concentrate with high freight and smelter charges yet to be paid that cyaniding will be the cheaper mill process notwithstanding probable greater costs and possible greater losses.

Mill tonnage. The size of the ore body and daily tonnage are closely related, especially if one accepts the Hoover theory of the economic life of mines (*H. C. Hoover, Principles of mining, McGraw-Hill, 1909*). Apart from this theory, a large ore body usually means a large daily tonnage and a long life, both of which elements justify large expenditures for plant and permit a multiplicity of treatment and re-treatment operations that would not be economically possible at small, short-lived mines. Consequently, all other things being equal, the flow-sheet at a large mine has more steps and is more complicated than that at a small mine.

Grade of ore. High-grade ore yields a higher recovery with a given grade of tailing than low-grade ore of the same metal, and, of course, more money per ton treated. A mill treating high-grade ore may find it economy to discharge a tailing assaying higher than the feed to a low-grade mill. In such case the flow-sheet of the high-grade mill may be and usually is simpler than that of the low-grade mill. This is particularly the case as between high-grade mills in isolated locations and low-grade mills near lines of transportation. Ordinarily, however, high-grade mills have the longer and more complex treatment schemes in proportion to the extent of treatment necessary to reduce the mill pulp to the metal content of the low-grade ore; both seek to make the same grade of tailing; both must give substantially the same final treatment, therefore, before discharge; but in the high-grade mill the process of impoverishment must be gradual, if it is to be efficient, hence in this mill a certain amount of preliminary concentration is necessary that is unnecessary in the low-grade mill.

Location of plant. (See also Sec. 23.) The principal angles from which plant location affects the flow-sheet are with respect to water, transportation and metal-buying centers. If water in excess of requirements for power and domestic use is lacking, flotation and water-gravity concentration are debarred and the ore must be transported to water (or, of course, water to the ore), or some method of dry concentration such as hand-sorting, magnetic or electrostatic separation, or air-cleaning must be employed. If ordinary transportation facilities are lacking, the flow-sheet must be one employing only machines of such size and weight (and, therefore, capacity) as can be brought in by means of the transportation available, and the process should be one that requires the minimum of outside supplies and provides a finished plant product of minimum bulk.

When transportation facilities are the poorest, economic necessity may dictate the location of a crude smelter at the milling plant. This is particularly true, if such crude smelting treatment as is possible results in considerable diminution in bulk of the plant product without too great loss of values. Ordinarily, however, milling-plant product must be transported to smelters, which are so located as to draw ores and concentrates from a considerable radius, and to be close to a source of fluxes and to a source of fuel or a direct and cheap fuel-transport line. In such case the proximity of the milling plant to the smelter is of considerable moment in determining the

character of the plant product and, therefore, of the plant flow-sheet. If the plant is near the smelter, then, all other things being equal, a lower grade of concentrate containing more moisture is permissible than when concentrate must be hauled a considerable distance. This makes for a simpler flow-sheet. In general, the greater the distance to the smelter, the smaller the bulk of concentrate must be and the greater the complexity of the plant required to produce it.

SECTION 2

METALS AND MINERALS

PROPERTIES, USES, ORES, PRODUCTION, SELLING, TREATMENT

1. Introduction

A mill is essentially a manufacturing plant for which the ore constitutes the raw material and the concentrate the finished product for market. Hence it is necessary for the designer and operator to know all that is possible about the kind and source of raw material available and the demand for and destination of the finished product. The fact that the mill owner is also usually the mine owner is not sufficient insurance as to the adequacy or kind of raw-material supply, nor is the further fact that one or more other manufacturing processes usually intervene between the mill and the ultimate consumer sufficient to excuse ignorance as to the general character of the final use of the mill product. Most mill managers are well acquainted with the demands of the immediate purchasers of their products, but these demands are sometimes based as much on the ignorance of the managers as upon the requirements of the consumers, and may be favorably modified in proportion to increase in the manager's knowledge. Omniscience is not expected nor required, but the further the operator advances from the attitude that his responsibility starts with the ore in the mill bins and ends with the concentrate in the railroad cars, the greater his value.

The elements determining the administrative problems of mill design and operation are: (*1*) The characteristics of the particular ore; (*2*) statistical facts as to world distribution and production of similar and dissimilar ores of the same metal or mineral; (*3*) uses of the finished product; (*4*) prices, current and for a sufficient time in the past to form a basis for judgment as to future trend; (*5*) mill-treatment schemes on the same and similar ores; (*6*) costs of milling similar and related ores.

Ore. The quantity and quality of ore available, either actually blocked out in the mine or positively indicated by geologic data, determine whether a mill should be built, and, if so, are also factors in the problems of capacity and type of construction. The mineralogical character of the ore determines the treatment scheme. Mill construction should never start until the existence of an amount of ore of paying quality, sufficient to repay the mill investment with interest, is established. Mill capacity is determined by potential mine capacity, first cost of mill construction, and demand for the mill product. Type of mill construction is determined by kind and size of machinery, probable life of mine, and relative costs of different materials. See also Sec. 23.

Production. Prices. Uses. District, state, national and world production are obtainable from publications of the U. S. Geol. Survey, the U. S. Department of Commerce, and various technical publications (see p. 14). Study of these together with average prices over a period of years, obtainable from the same sources, furnishes a basis for judgment as to the market for mill products, possible competition, etc. Knowledge of uses will tell something as to the probable stability of demand, wide and varied use tending toward stable, increasing demand.

The increase in the use of closed bodies for automobiles resulted in a great increase in the demand for aluminum, while the present tendency toward the use of fiber bodies threatens a decrease in this demand. Continued increase in centralization of power production together with the almost certain electrification of long stretches of railroad will result in increased demand for copper and aluminum. On the other hand, the enormous and relatively high-grade copper deposits of Central Africa are a potential damper on any great increase in copper prices for a considerable time.

In the case of certain substances, such, for instance, as zinc, graphite, molybdenite, and garnet, the grade of concentrate or its physical character, as dictated by the use, is highly important in determining the price of the mill product and even the existence of any market therefor.

2. Flow-sheets

The flow-sheet best suited to a given ore depends upon the physical character of the economic mineral and of the gangue with which it is associated, the ruling size of the particles of economic mineral, the size of the ore body, the daily tonnage, the unit value of the valuable constituent, the grade of the ore and the location of the plant with respect to markets, water, tailing-disposal space, etc. In simplest fundamentals, all concentration flow-sheets are alike in that all contain machines for severing valuable from waste mineral followed by other machines for separating the severed constituents. Further than this, however, identity ceases and cursory examination of a number of mill schemes taken at random shows a bewildering complexity. There is, nevertheless, a distinct order, which becomes apparent when the flow-sheets are analyzed in the light of the controlling factors above enumerated, and certain kinds of machines and definite sequences of these machines are found to occur with considerable regularity, coincident with similarities in fundamental conditions. This fact is not, of course, any chance or statistical agreement, but the result of recognition by flow-sheet designers of certain basic principles of design. Numerous variations in minor details are observable, which are to be attributed to local ideas or traditions, special availability of particular pieces of apparatus, or individual predilections of certain designers, but in the matter of fundamentals, arbitrary differences disappear, and the elements of the flow-sheets can be referred to a few definite principles

The important determinative characteristics of the ore are: kind of valuable mineral, kind of associated gangue minerals, relative quantities of valuable mineral and gangue, and size of valuable-mineral particles. The most important mineral properties, from the viewpoint of flow-sheet design, are specific gravity, surface energy, magnetic permeability and fracture.

Specific gravity. The important consideration is the ratio of specific gravities, each decreased by one, of the minerals to be separated. If this ratio is greater than two (the heavier mineral in the numerator) reasonable separation can be effected without too great difficulty; if the ratio is near one, it is difficult to get a high-grade concentrate, low-grade tailing is substantially impossible to obtain, and the tonnage of middling is very large. With a ratio of 2.5 it is possible to get clean concentrate, but difficult to get low-grade tailing and the tonnage of middling is large. With a ratio of 3 or more, gravity concentration is easy in all sizes down to the finest sands. No difference in specific gravity that exists between minerals is sufficient to make clean gravity separation of slime possible for the reason that with such fine particles the relation of surface to volume (and weight) is so large that the surface forces resisting settlement of the particles in fluids are sufficient to mask or completely nullify the effect of specific gravity.

Luster. If there is a marked difference in luster between valuable mineral and gangue, such as, for instance, exists between the metallic sulphides and the rock-forming silicates, flotation concentration can be used. Flotation, however, labors under a disadvantage converse to that suffered by gravity concentration in that it cannot be practiced on particles greater than 0.2-mm. diameter, and better not on particles larger than 0.15- or even 0.1-mm. size. Hence when the valuable mineral occurs in coarse aggregates and sufficient difference in specific gravity of the ore constituents exists to permit gravity concentration, flotation should ordinarily be an adjunct to gravity separation, unless the ratio of concentration is high and no gravity-concentration tailing can be made economically.

Magnetic concentration is employed under two general conditions, viz.: (*1*) when the valuable mineral is magnetite, (*2*) when the valuable and waste minerals are close together in specific gravity, not amenable to sharp separation by flotation and one is sensibly permeable to magnetic lines of force and differently permeable from the others. Magnetic concentration of highly permeable minerals can be practiced at all sizes below 2-in.; if the minerals are of low permeability, fine crushing is necessary.

Fracture. Concentration dependent upon difference in fracture of the ore constituents has limited application in treatment of metallic ores but is important in separating slate from coal.

Ratio of concentration. If the ratio of concentration (Sec. 22, Art. 15) of an ore is high, the flow-sheet should be of such character that concentration is effected at one size, in order to lessen the variety of concentrating machines and to permit use of large units with correspondingly low capital and operating charges. Under such circumstances the crushing plant may be greatly simplified, a minimum number of screens and classifiers is necessary, and expensive elevation and horizontal transport of pulp are greatly lessened. The all-flotation porphyry-copper mills are typical of this type of flow-sheet. If coarse tailing can be discarded and ultimate fine grinding is necessary, as for instance, is the case at the MESABI IRON CO. (Fig. 88) and ALASKA GASTINEAU (Fig. 75) some elaboration of flow-sheet is justified in order to save the cost of grinding low-grade material. If the ratio of concentration is low and, as usually follows, the valuable mineral occurs in coarse aggregates, a complicated flow-sheet with a multiplicity of screens, classifiers and concentrating machines of varied types may be justified on the grounds that coarse concentrate is usually of higher grade than fine, is more cheaply and efficiently smelted, that an appreciable tonnage is diverted from the grinding machines and over-all recovery is bettered. The basis of the final statement is that a particle of, say, a size that would go into coarse-jig concentrate will, if broken, form some slime particles, and since the recovery of slime particles is never perfect, a part of this broken particle is discarded as tailing, with resultant lowering of recovery. If slime recovery is efficient, this argument has but

little weight and a coarse concentrator must justify its inclusion on the first three grounds mentioned. Flotation is so efficient, and the size that can be handled on shaking tables has increased so greatly in recent years, that jigs have been eliminated from many mills. Such elimination, in addition to simplification of flow-sheet, permits removal of several screens and saves water, labor, considerable maintenance, floor space, head room and may even lessen the total power consumption. Tables are almost always used with ores of low concentrating ratio, if gravity concentration is feasible, even when flotation is most efficient. The flotation machines can make a lower-grade tailing with low-grade than with high-grade feed, some burden is taken off the grinding machines, and the cost of dewatering granular table concentrate is much less than that for flotation concentrate.

Rich ores both require and can stand the cost of more elaborate treatment than poor ores; large ore bodies justify not only larger but more elaborate mills than small; and valuable substances such as gold, silver and tin can, economically, be pursued further, *i.e.*, according to a more elaborate flow-sheet, than lead, zinc or iron. When freight rates are high or smelter penalties for impurities great, extensive (and expensive) treatment designed to raise the grade of concentrate is justified, or high-grade concentrate may be made at the expense of a low recovery, while with low concentrate-treatment and transportation charges, the flow-sheet should be designed for high recovery with less attention to the grade of the final product.

Tonnage is an important factor in flow-sheet design. If tonnage is small, say less than 500 tons per 24 hr., the simplest type of flow-sheet, only, should be considered, even when a low ratio of concentration might indicate an elaborate mill. Multiplication of machines under such circumstances makes for small and relatively inefficient units and increases capital and operating costs out of proportion to the savings otherwise effected. When tonnage is large, a mill is designed in independent sections, the flow-sheets of which are substantially the same, the tonnage going to each section being determined usually as that of the largest-tonnage unit or convenient group of units. The advantages of such design are: (*1*) that it permits one or more sections to be shut down for repairs, curtailment or the like without affecting the efficient operation of the balance of the plant, and (*2*) that responsibility for efficient operation of machines can be localized and a competitive spirit engendered between the workmen. Capacity is readily increased by adding sections without materially affecting current operations.

Water supply. Scarcity of water may dictate dry treatment when otherwise wet treatment was indicated, as *e.g.*, at some coal-cleaning plants; it may make it necessary to plan for a smaller daily tonnage than otherwise; and it usually requires inclusion in the flow-sheet of a more or less elaborate water-reclamation system.

Essential elements of a flow-sheet are crushing, grinding, concentrating, concentrate handling, sampling and weighing, storage and transportation of pulp. The first three of these elements, in any flow-sheet, are primarily dependent upon the ore as delivered from the mine; concentrate handling depends upon the method of concentration and upon the after-treatment of the concentrate; sampling and weighing are matters of efficient mill control; and storage and pulp transport are solely mechanical features, substantially independent of the character of the ore and of the process of concentration, but of supreme importance from the standpoint of smooth and efficient operation.

Element	Specific gravity	Relative hardness (diamond, 10)	Melting point, deg. C.	Boiling point, deg. C.	Modulus of elasticity, tension, million lb. per sq. in.	Coefficient of linear expansion		Specific heat		Thermal conductivity		Latent heat (cal. per gm.)		Electrical resistivity	
						Coefficient (multiply by 10^{-5})	Temperature	Coefficient	Temperature	C. G. S. units	Temperature	Fusion	Vaporization	Microhms per cu. cm.	Temperature
Aluminum	2.60	2.9	657	1800	10	2.313	40	0.210 0.223	0 100	0.3435 0.3619	0 100	76.8	......	2.8	0
Antimony	6.62	3.0	630	1500 to 1700	....	1.692	40	0.050	0–100	0.042	0–30	......	......	40.5	0–30
Arsenic	5.73	3.5	sublimes from solid state at 450		....	0.559	40	0.083	.21–68	....	......	......	......	35.0	0
Bismuth	9.80	2.5	268	1300 ±	4.5	1.316	0–100	0.030	9–102	0.0194 0.0161	18 100	....	......	113.0	19
Cadmium	8.64	2.0	322	770	10	3.159	0–100	0.055	0–100	0.2216 0.2149	18 100	13.7	......	6.9	0
Chromium	6.50	9.0	1515 ±	....	....	......	......	0.121	0–100	....	......	......	......	......	......
Cobalt	8.6	125f	1467 ±	Nickel	....	1.236	40	0.103	15–100	....	......	......	......	12.0	100
Copper	8.93	3.0	1084	2100	15 19	1.718	0–100	0.093	0–100	0.8915 0.8771	18 100	5.86	......	1.5	0–30
Gold	19.32	2.5	1064	2530	12	1.470	0–100	0.032	0–100	0.7003 0.7027	18 100	....	......	2.4	18
Iridium	22.42	6.5	2200 ±	....	....	0.700	40	0.032	0–100	....	......	......	......	......	......
Iron	7.86	4.5	1700 ±	....	16.5 to 30	1.182	0–100	0.116	23–100	0.1665 0.1627	0 100	....	......	10.6	0–30
Lead	11.37	1.5	327	1700 ±	2.5	2.799	0–100	0.031	18–100	0.0836 0.0764	0 100	5.86	......	19.8	0–30
Manganese	7.39	5.0	1260	1900	....	......	......	0.122a	14–97	....	......	......	......	......	......
Mercury	13.55	b	−39	357	....	18.2c	0–100	0.033	20–50	0.0197	0–34	2.85	62	94.0	0
Molybdenum	8.6	.....	2500 ±	....	....	......	......	0.072	15–91	....	......	......	......	......	......
Nickel	8.9	65f	1435	2325	29 34	1.279	40	0.109	18–100	0.1420d 0.1384	18 100	4.64	......	6.9	0
Osmium	22.48	7.0	2500 +	....	....	0.657	40	0.031	19–98	....	......	......	......	9.5	20
Palladium	11.4	4.8	1585	....	13.8	1.10	0–100	0.059	0–100	0.1683 0.1817	18 100	....	......	10.2	0
Platinum	21.50	4.3	1780	2450	22 24	0.880	0–100	0.032	0–100	0.1664 0.1733	18 100	....	......	11.0	0
Silver	10.50	2.7	955	1955	10	1.921	40	0.056	0–100	1.006e 0.992	18 100	22	......	1.5	0
Tin	7.29	1.8	233	2200	2.4 5.9	2.234	40	0.056	0–100	0.1528 0.1423	0 100	14	......	13.0	0
Titanium	3.54	.....	1800 ±	....	....	......	......	0.112	0–100	....	......	......	......	......	......
Tungsten	19.6	.....	3267	....	60	0.35	18	0.034	0–100	....	......	......	......	4.42	......
Uranium	18.7	.....	....	....	....	......	......	0.017	11–98	....	......	......	......	......	......
Vanadium	5.5	.....	Elec. furn.	....	....	......	......	0.115	0–100	....	......	......	......	......	......
Zinc	7.1	2.5	419	929	12.4	2.914	40	0.094	0–100	0.2653 0.2619	18 100	28	......	5.4	0

a Containing some Si. *b* Liquid at ordinary temperatures. *c* Cubical, not linear, expansion. *d* Contains 3 per cent. impurities. *e* 999.8 fine. *f* Brinell scale.

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3. The marketing of metals and minerals, J. E. Spurr and F. E. Wormser, McGraw-Hill Book Co., N. Y., 1925.

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5. Engineering and Mining Journal, annual review numbers and weekly price lists.

3. Aluminum, Al.

Properties. Metal; bluish-white, lustrous, highly malleable and ductile; TENSILE STRENGTH, pure cast aluminum, 18,000 lb. per sq. in.; rolled sheet and wire up to 40,000 lb. per sq. in.; ELASTIC LIMIT from 8500 lb. per sq. in. for castings to 33,000 lb. per sq. in. for fine wire. (See also Table 1.) AT. WGT., 27.0. Practically unchanged in air due to rapid formation of a thin, firmly-adhering layer of aluminum oxide. Substantially without action on water at any temperature. Reacts with strong bases to form aluminates and with hydrochloric and sulphuric acids to form corresponding salts. With nitric acid oxidation takes place and the film of oxide protects the metal from further action. Organic acids attack pure aluminum only slightly. Aluminum ion is tri-valent. Aluminum alloys readily with copper, zinc, tin, nickel, magnesium and tungsten.

Uses. Aluminum metal is used for structural shapes and castings where lightness is essential, principally in airplane and automobile construction; electrical transmission, cooking utensils, scientific instruments, alloys, lithographic work as a substitute for stone or zinc, de-oxidation of steel in casting, thermit welding, explosives, etc. Bauxite, beside its use as a source of metallic aluminum, is used for manufacturing aluminum chemicals, for making abrasives such as alundum, and for refractories.

Ores. Aluminum is widely distributed as a large constituent of many different minerals, but extraction is commercially possible only from bauxite and cryolite. BAUXITE occurs as pisolites and clay-like masses in pockets or lenses in residual clays. CRYOLITE occurs as lenses or veins in metamorphic rocks. At present bauxite ores form practically the only commercial source of aluminum. Principal economic occurrences of bauxite in the United States are in Georgia, Alabama, Tennessee and Arkansas. Principal foreign localities are France, Italy, Dalmatia and British Guiana. Analyses range from 50 to 60 per cent. Al_2O_3, 2 to 20 per cent. SiO_2, 1 to 25 per cent. Fe_2O_3 and 1 to 3 per cent. TiO_2.

Production statistics are given in Tables 2, 3 and 4.

Table 2. World's production of aluminum (thousands of metric tons). (*After Mineral Industry.*)

Year	Austria	Canada	France	Germany	Great Britain	Italy	Norway	Switzerland	U. S.	Total
1913	5	6	13.5	0.8	10	0.9	2.5	10	29.5	78
1914	4	6.8	10	0.8	8	0.9	2.5	10	40.6	83
1915	2.5	8.5	6	2	6	0.9	3.5	12.5	45	87
1916	5	8.8	9.6	8	4	1.1	6	15	63	120
1917	5	11.8	11	15	6	1.7	8	15	90.7	164
1918	8	15	12	25	14	1.7	7.5	15	102	200
1919	5	15	10.3	15	10	1.7	4	15	90	166
1920	2	10	12.3	10	8	1.2	5	12	90	151
1921	2	6	10	10	5	0.7	4	10	28.8	76
1922	4	9	12	12	9.5	0.6	6	12	52	117
1923	4	16.5	12	13	9	1.5	14	12	97	179

Table 3. World production of bauxite (in thousands of metric tons). *(After Mineral Industry.)*

Country	1917	1918	1919	1920	1921	1922	1923
Austria:							
Dalmatia.....	135	94.5	*b*	*c*	*c*	*c*	*c*
Istria........	28	70	*b*	*d*	*d*	*d*	*d*
Lower Austria	....	....	....	0.4	2.6	4.1	2.7
British Guiana	2.8	4.3	2	32	20	*f*	112
British India....	1.4	1.2	1.7	6.4	6.8	5	7*e*
Dutch Guiana		...	...	.		18.8	20*e*
France..........	121	145	153	267	95	139	314
Germany.......	10.8	14.4	9.4	13.4	2*e*	12*e*	6*e*
Hungary .	28.8	*b*	*b*	*b*	*b*	*b*	*b*
Roumania......	....	....	....	....	....	12*e*	12*e*
Italy..........	7.8	7.8	3	13.1	49	67	98
Yugoslavia:							
Dalmatia.....	*a*	*a*	*a*	28	10	31	50*e*
Spain.	...	0.5	1.8	0.5	0.2	...	...
United Kingdom	15	9.7	9.4	11.2	2.3	6	4*e*
United States	578	615	333	530	142	315	553
Total......	928	963	569	902	330	610	1180

a Reported under Austria. *b* Figures not available. *c* Reported under Yugoslavia. *d* Reported with Italy. *e* Estimate. *f* Mines down.

Table 4. Production of bauxite in U. S. (thousands of long tons). *(USGS)*

Year	Ga., Ala., Tenn.	Ark.	Total
1914	24	195	219
1915	28	269	297
1916	49	376	425
1917	62	507	569
1918	43	563	606
1919	43	333	376
1920	40	481	521
1921	15	125	140
1922	43	267	310
1923	29	494	523
1924	35*a*	345*a*	380*a*

a Estimate *(119 J 95)*.

Selling. *(118 J 851.)* Prices for 99-per cent. aluminum metal since 1913 are given in Table 5; 98-per cent. metal is quoted about one cent less per pound. Metal prices are not necessarily an index to prices for mine products for the reason that extraction of metal from bauxite in the United States is a virtual monopoly, protected by a high tariff, and intrenched against competition by the extreme costliness of plant. Average prices for bauxite at the mines have ranged, since 1913, from \$4.75 per long ton in that year to \$6.04 in 1923, with a high of \$6.38 in 1921. Open-market quotations on domestic bauxite in 1924 ranged from \$5.50 to \$8.75 per long ton, depending on purity, and imported at \$5 to \$7.50 per ton at eastern ports. These prices also must be used with caution, since the Aluminum Co. of America, which is the principal user, is also the largest mine owner. Requirements as to composition of bauxite vary according to the use. Alumina content should be 50 to 52 per cent. Al_2O_3 as a minimum. Low silica (less than 7 per cent.) is required

for aluminum manufacture and Fe_2O_3 should not exceed 6.5 per cent.; low iron and titanium (2.5 to 3 per cent. maximum) are preferred in the manufacture of aluminum chemicals, but silica may be high (15 to 20 per cent.); for manufacture of aluminum abrasives silica and ferric oxide should be less than 5 per cent. each and titanium oxide less than 4 per cent. For refractories, iron and titanium must be low but silica may be high.

Table 5. Price range of aluminum at New York (99-per cent. grade; cents per pound) (a)

Year	Average
1913	23.6
1914	18.6
1915	34.0
1916	60.7
1917	51.6
1918	33.5
1919	32.2
1920	32.7
1921	21.2
1922	20.2
1923	25.4
1924	28*b*

a Eng. and Min. Jour.
b Approximate.

Treatment. Practically all the bauxite now mined is of sufficiently high grade so that no mechanical separation from associated minerals is necessary. In the past loose mixtures of bauxite and clay have been concentrated in log washers, but the low-grade deposits cannot compete at present. Treatment at one large plant consists of coarse breaking, the hard rock in gyratories, the soft in Fairmount crushers; followed by drying in coal-burning rotary kilns, about 7 × 50 ft., at a temperature near 1100° F.; thence to a cooling conveyor and shipping bins. Residual mechanically-held moisture is from 0.5 to 1 per cent. The bauxite also contains from 15 to 33 per cent. combined water, which, if the ore is to be used for abrasives and refractories, is driven out by heating at about 2000° F. in 6 × 60-ft. kilns, leaving a total moisture content of about 0.5 per cent. Simpler methods of the same general character are followed at smaller plants. Some mines ship the ore as mined, but this is extremely uneconomical, if the haul is long, on account of the high water content.

Aluminum is obtained from bauxite by electrolysis in a fused bath of cryolite. The best grades of metal contain 99.5 to 99.9 per cent. Al; poorer grades, 94 to 96 per cent. Al. Impurities are Fe and Si. Grade of finished metal is largely dependent on purity of bauxite.

4. Antimony, Sb

Properties. Metal, silver-white, lustrous, hard and brittle. (See also Table 1.) Impurities dull the luster and make the metal less crystalline. AT. WGT., 121.8. Unacted upon by dry air at ordinary temperatures but burns in air, if sufficiently heated, forming dense fumes of white oxide. Slightly tarnished by moist air at ordinary temperatures. Decomposes water at red heat. Oxidized by nitric acid. Dissolves in aqua regia and hot concentrated sulphuric acid. Antimony ion is tri-valent and quinque-valent, acid, and both base- and acid-forming. Antimony alloys freely with other metals.

Uses. Antimony metal has little or no use as such. The principal use is in alloys such as Babbitt metal, Britannia metal, shrapnel lead, hard lead for plumbing and similar purposes, type metal, white metal, pewter, storage-battery plates, solder and various other soft metals used in making foil, metal tubes, etc. Antimony persulphide is used in vulcanizing rubber, the trisulphide for safety matches, and the powdered metal, trioxide and trisulphide are all used for pigments.

Ores. The economic minerals are stibnite, senarmontite, valentinite and cervantite. Ores are found in most countries, but the Chinese deposits around Changsha yield 80 to 90 per cent. of the world supply. These

deposits consist of stibnite of remarkable purity occurring as pockets and bunches in dolomitic limestone.

Production. See Table 6.

Table 6. World's production of antimony (metric tons). (*USGS and MI*)

Country	1913	1914	1915	1916	1917	1918	1919	1920	1921	1922
United States	*a*	*a*	1,760	1,420	310	45	......	......	......	......
Algeria......	186	320	2,740	8,940	4,550	2,218	605	1,000	103	530
Bolivia......	30	102	9,859	15,077	12,860	4,770	105	588	336	......
China.......	13,032	19,647	23,357	42,800	31,000	18,112*b*	8,923	13,109	14,752	14,316
France......	5,170	540	893	2,430	2,354	1,329	976	1,413	1,118	814
Mexico......	2,340	1,570	200*b*	829	2,647	3,269	628	1,572	......	457
Victoria.....	960	890	1,300	1,320	......	576	580	375	150	730
Others......	2,798	490	3,071	8,818	3,445	440	138	1,691	488	......
Total...	24,516	23,559	43,180	81,634	57,166	30,759	11,955	19,748	16,947	......

a No antimony metal produced from ores but considerable as a constituent of hard lead from lead ores containing antimony. *b* Data incomplete; probably larger.

Selling. Market demand is for antimony metal or REGULUS containing at least 99 per cent. antimony, as little arsenic as possible and of uniform quality. Because of the fact that the trade has become accustomed to a radiating crystalline structure, caused by doing the final refining beneath a layer of flux, it is practically necessary to market the metal in this "star" condition, although equal purity may be obtained in a different physical state. Average New York price since 1913 is shown in Table 7. Supply is far in excess of demand under normal conditions.

Table 7. Yearly price range of antimony metal, cents per pound. (*J*)

Year	Price
1913	7.5
1914	8.8
1915	29.5
1916	25.3
1917	20.7
1918	12.6
1919	8.2
1920	8.5
1921	5.0
1922	5.5
1923	7.9
1924	10.8

Treatment. (*8 AIM 196*). Only high-grade ore is mined, most of which is sufficiently concentrated by crude hand picking. If further concentration is necessary, it may be readily effected by liquation at a low temperature. Both methods are extremely wasteful (liquation losses may run as high as 30 per cent. antimony) but only high-grade material can be shipped from isolated districts to the smelters and even in the period of high costs during the war it was considered more economical to bear the losses attendant upon these crude methods than to practice more refined (and hence more costly) concentration methods. Three methods are used to recover metal from concentrate. (*1*) Roast to the volatile trioxide or non-volatile tetroxide and reduce to metal in furnaces or crucibles under a flux of sodium chloride, soda ash or sodium sulphide. (*2*) Fuse the sulphide in crucibles with metallic iron and suitable fluxes, producing iron sulphide and crude metallic antimony, and refine the latter by fusion in a reverberatory furnace or crucible with potash, antimony sulphide and other fluxes. (*3*) Electrolysis of a solution of stibnite in sodium sulphide. This produces substantially pure metallic antimony in powdered form and this must be melted down under fluxes to

produce the "Star-brand" ingots. Antimonial or HARD LEAD containing 10 to 20 per cent. Sb is obtained as a by-product in smelting and refining antimonial-lead concentrate.

5. Arsenic, As

Properties. Metal; steel-gray, lustrous and crystalline, or black, dull and amorphous; brittle. (See also Table 1.) AT. WGT., 75.0. Unaltered in dry air, becomes tarnished in moist air, burns at 180° C. in air or oxygen with bluish flame forming arsenic trioxide. Not attacked by water. Does not dissolve in hydrochloric or sulphuric acid or bases, but is slowly oxidized and dissolved by nitric acid. Combines directly with most elements. Arsenic is tri- and quinque-valent and both base- and acid-forming. It alloys with many metals, almost invariably forming a brittle alloy.

Uses. The principal uses are for insecticides such as Paris green and lead arsenate for plant sprays and calcium arsenate for boll-weevil extermination. Sodium arsenite is used in considerable quantities on railroads and highways for a liquid weed killer. The white oxide is used in glass making to mask the coloring effect of metallic oxides and impart brilliancy. It is also used in making sheep and cattle dips. The red and yellow sulphides are used for pigments in paints, calico printing, dyeing and tanning. Small amounts of the element are used for certain alloys, principally lead shot.

Ores. Arsenic occurs as a constituent of 30 or more minerals (Hess, *MR, 1914 p. 957*) but the principal sources are arsenopyrite, arsenides and sulpharsenides associated with ores of lead, copper and the precious metals, the arsenic being a by-product. Arsenopyrite is mined for the arsenic content in a few places. Realgar and orpiment occur as vein minerals associated with barite, stibnite, quartz, pyrite, and precious metals, but there is little or no mining of such deposits for arsenic content.

Production. See Table 8.

Table 8. World's production of white arsenic (in metric tons). (*MR and MI*)

Country	1913	1914	1915	1916	1917	1918	1919	1920	1921	1922	1923
United States...	2280	4237	4988	5430	5580	5736	5469	10,434	4342	9096	12,947
Canada........	1535	1576	2174	1983	2663	3230	3075	2,231	1353	2337	3,307
France.........	.....	132	323	874	677	993	735	606	......	......	......
Germany.......	1892	1637	1456	1280	2081	3592	1475	2,077	2000	2000	......
Japan..........	21	15	15	.....	.....	245	835	933	1395	2041	......
Mexico.........	.....	.....	.....	.....	1285	1881	2246	2,183	785	271	......
Portugal.......	925	960	859	.....	.....	.....	536	653	268	......	......
United Kingdom	1722	2020	2536	2583	2668	2387	2568	2,029	1049	994	......
Others (*a*)......	600	235	140	560	820	425	1120	2,160	1910	2865	......

a Includes Belgium, China, Greece, Queensland, Rhodesia, Spain, Union of South Africa.

Table 9. Price range of arsenic trioxide. (*MR, MI, J*)

Year	Cents per lb.
1916	3.5 to 8
1917	8.2 to 15
1918	9
1919	8 to 10.5
1920	9.1 to 14.5
1921	7 to 12
1922	6.9 to 14.5
1923	9 to 15.5
1924	6 to 13.5

Selling. The commercial form is the trioxide, 99 per cent. or better purity, white and finely powdered. A small amount of lower-grade off-color material is sold. The price has increased in recent years due to the growing use of calcium arsenate in controlling boll-weevil damage to cotton. Production facilities have increased greatly as a result until now they are probably in excess of normal demand. Range of 5-yearly average prices of trioxide

from 1901 to 1915 was 2.74 to 3.15 cents per lb. Range since 1915 is given in Table 9.

Treatment. Arsenical fumes and dust from bag houses and Cottrell precipitators in lead and copper smelters are calcined or roasted in reverberatory or special furnaces and the fume condensed in an ordinary flue system. This fume is known as black dust and contains about 90 per cent. As_2O_3. It is refined in a coke-fired furnace connected to a special flue system in which the refined (+ 99 per cent.) As_2O_3 collects.

6. Asbestos

Properties. Asbestos is a name describing a fibrous form of several different minerals rather than the name of one definite mineral. The usual minerals that occur in fibrous form and are classed as asbestos are chrysotile, $3\ MgO \cdot 2\ SiO_2 \cdot 2\ H_2O$, a variety of serpentine; tremolite, $CaO \cdot 3\ MgO \cdot 4\ SiO_2$; actinolite, $CaO \cdot 3(Mg,Fe)O \cdot 4\ SiO_2$; crocidolite, $NaFe(SiO_3)_2 \cdot FeSiO_3$; and anthophyllite $(MgFe)O \cdot SiO_2$, all varieties of amphibole. Another variety of amphibole recently discovered in South Africa and there mined is called amosite. Its ANALYSIS, compared with average analyses of chrysotile, crocidolite and anthophyllite is given in Table 10. SP. GR. of chrysotile is 2.2 to 2.65 and of the amphibole

Table 10. Analyses of asbestos minerals. (*After Ru Keyser, 113 J 619*)

	Chrysotile *a*	Crocidolite *b*	Amosite *c*	Anthophyllite *e*
SiO_2	40.49	51.64	50.24	57.12
Al_2O_3	1.27	....	....	0.75
FeO	....	....	7.80	6.36
Fe_2O_3	2.53	34.38	32.00	
MgO	41.41	2.64	3.96	29.44
CaO	....	0.05	Tr.	....
Na_2O	....	7.11	2.12	....
Water	14.06	4.01*d*	3.00	5.47

a Cirkel, F., Chrysotile asbestos, its occurrence, exploitation, milling and uses, 2nd ed., *Mines Branch, Canada Dept. of Mines, 1910.* *b* Dana, System of Mineralogy. *c* Private analysis. *d* Ignition. *e* Asbestos, 1913–1919, *Imperial Mineral Resources Bureau (Great Britain), 1921, p. 6.*

varieties, 2.9 to 3.2. LENGTH OF FIBER ranges ordinarily from ⅛ in. or less to 1½ or 2 in.; exceptionally fiber as long as 24 in. has been found. Amosite fiber is longer than the average. Fiber strength and flexibility, fineness of fiber, and resistance to heat and acids are important elements in judging the value of asbestos. Chrysotile has high flexibility and medium strength and resistance to heat and acids. Crocidolite has higher tensile strength than chrysotile but less resistance to heat. Anthophyllite has low strength and flexibility but high resistance to heat and acids. Amosite has less strength and flexibility than chrysotile, but is similar in its resistance to heat and acids.

Uses. Asbestos is used widely for purposes involving resistance to acids and to relatively low degrees of heat. Long-fiber material is consumed principally in making automobile brake bands; other uses are for fireproof curtains and articles of clothing, packings and gaskets, thread and wicking, electrical insulation, chemical-laboratory apparatus and supplies, etc. Short-fiber grades are used in making shingles and asphaltic roofings, pipe coverings, paper and millboard, and asbestos cements that are used for heat insulation on hot and cold pipes, stoves and furnaces, fireproof safes and the like.

Ores. Asbestos fiber occurs in three forms in the country rock, known respectively as cross-fiber veins, slip-fiber veins and mass fiber. In cross-fiber veins the filaments are oriented at right angles to the vein walls. Filaments in slip-fiber veins are not so regularly oriented as the cross-fiber, but such orientation as exists is generally parallel to the vein walls. Mass fiber occurs in unoriented masses disseminated through the country rock. Cross-

fiber is much the best from the point of view of fiber length. The veins of asbestos occur in association with serpentine in limestone and dolomite and in altered peridotites.

Production. The United States is the principal consumer of asbestos but a negligible producer. Imports of unmanufactured asbestos have increased from 64,000 long tons in 1914 to nearly 190,000 in 1923, while production since 1918 has ranged between 67 tons minimum in 1922 and 1650 tons maximum in 1920. Canada is the principal world producer, with a record volume of 233,000 short tons in 1923. In the same year Rhodesia produced about 20,000 tons; Union of South Africa, 8400 tons; and seattered small amounts were mined in Australia, Corsica, Cyprus, Madagascar, New Zealand and Russia. The latter country exported about 11,000 tons annually before the revolution.

Selling. Fiber is graded differently by different producers, but in general, the grading corresponds approximately to that shown in Table 11 (*114 J 277*). The average price of all grades sold in Canada ranged between $10 and $20 per ton from 1912 until 1916, when it began to climb, reaching $80 in 1919. The peak was reached in 1921. In that year No. 1 crude sold at $3000 per ton compared to $350 in 1913.

Table 11. Grades and prices of Canadian asbestos.

Trade name	Fiber length	Price (Oct., 1925), dollars per short ton
No. 1 crude	+ ¾-in.	400–450
No. 2 crude	− ¾, + ⅜-in.	250–300
Long spinning fiber	2–8–4–2*a*	110–175
Magnesia and compressed-sheet fiber	0–8–6–2*a*	75–110
Pipe-covering fibers	0–5–8–3*a*	100
Shingle stock	0–1½–9½–5*a*	50–75
Paper and mill-board stock	0–0–10–6–*a*	35–45
Cement stock	0–0–5–11*a*	8–12
Floats or shorts (*b*)	....	8–12
Sand	....	6–8

a Mill fiber is tested in a standard machine consisting of three nested shaking screens, respectively 2-mesh 11-gage, 4-mesh 17-gage and 10-mesh 20-gage, and a bottom box. Screen frames and box are of wood, rectangular, 16 × 26 in. × 5 in. deep. A 1-lb. sample is placed on the top (coarsest) screen and the nest is mechanically shaken for two minutes at 300 r.p.m. Oversizes and final undersize are weighed in ounces and the weights in order from coarse to fine are recorded, as in the table. Thus 2–8–4–2 means 2 oz. on 2-mesh, 8 oz. on 4-mesh, 4 oz. on 10-mesh and 2 oz. through 10-mesh. *b* This is material finer than any of the preceding and containing much dust and dirt.

Treatment of asbestos-bearing rock involves many problems not present in ordinary concentrating practice. In the first place, as may be seen by reference to Table 11, the value of long fiber is much greater than the value of the same material in shorter lengths, hence every endeavor is made to prevent fiber breakage. The valuable and gangue minerals are not of sufficiently different specific gravities to make them separable by gravity-concentration processes without close sizing, and the nature of the asbestos aggregates prevents such preparation. As a result of these facts a system of treatment has evolved that consists of coarse crushing, drying, screening, separating asbestos from screen oversize by suction, and subsequently separating rock dust from the sucked-up product by screening. This final screening is arranged to divide the asbestos into market grades (see Table 11). Crushing

is carried down as far as possible in jaw and gyratory crushers because these machines do the least injury to fiber. Rolls are unsuitable as they mat together the fiber and associated clayey material. In the fine-crushing section free fiber is separated ahead of each crusher. Treatment differs somewhat according to whether the ore is of the cross-fiber variety or slip or mass-fiber rock. Ru Keyser (*113 J 672*) gives the generalized flow-sheets shown in Figs. 1 and 3 to illustrate typical Canadian practice.

Recent experiment has indicated a good possibility that coarse fiber may be separated by jigging and fine fiber by tabling, the fiber discharging, irrespective of specific gravity, from the tailing outlets of the respective machines. The principle involved is the same as that which acts in the present screening practice, *i.e.*, the fiber works to the top (more properly, the granular material works to the bottom) of a mass of rock and asbestos when it is sufficiently agitated to give the particles partial relative mobility, due to the fact that the elongated and somewhat fluffy asbestos bridges over the interstices in the mass while the granular rock particles enter these interstices and work toward the bottom. If further experiment proves wet separation effective, milling plants should be much improved. Dust with its evil mechanical and physiological effects will be largely eliminated, drying will be confined to a small proportion of the mill tonnage, and it should be possible to substitute cylinder mills for the relatively inefficient jumbos, cyclones, and the like.

Typical slip-fiber asbestos mill. Fig. 1. (*113 J 672*).

Location: East Broughton, Que., Canada.

Ore: Chrysotile in serpentine associated with basic igneous rocks. Slip-fiber occurrence.

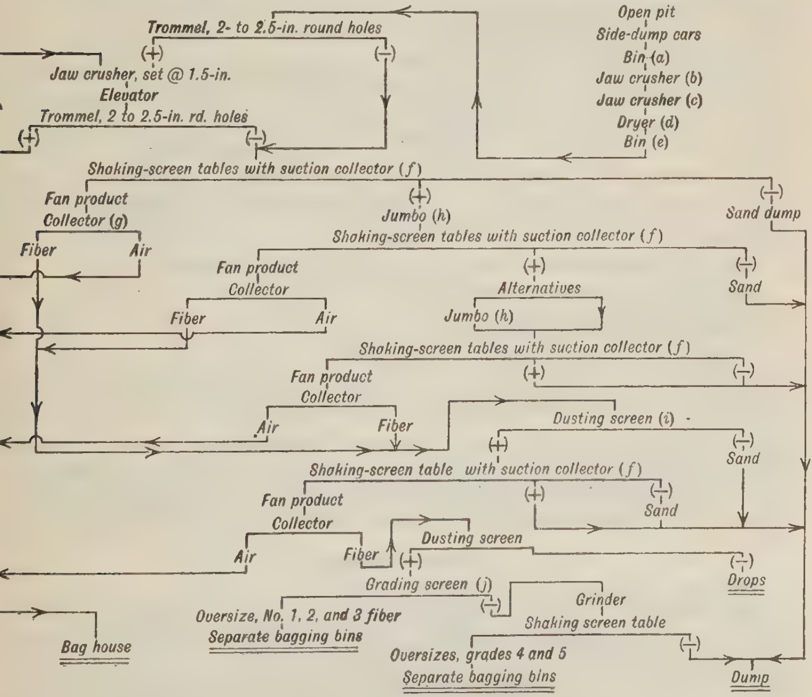


FIG. 1.—Typical slip-fiber asbestos mill.

a, Usually not more than 1-day capacity. Great difficulty is encountered in moving material through this bin in winter on account of freezing. At the JOHNS-MANVILLE

plant the cars dump directly into the primary crusher (see note *b*). *b*, At KING mine, 2 @ 32 × 72-in. At JOHNS-MANVILLE 1 @ 60 × 84-in. set 10 in. Capacity 3000 tons per 8 hr. Nip angle 18° on account of slippery nature of rock. *c*, At KING mine, 3 @ 15 × 20-in. set 2.5-in. Usual practice is 2 or 3 stages of jaw or gyratory crushing from run-of-mine to 2- or 2.5-in. *d*, In all but one mill rotary cylindrical dryers of either the direct- or indirect-heat type are used, 4½ to 6 ft. diam. × 30 to 50 ft. long. One mill uses a tower dryer 40 ft. high. Extent of drying is important. Too little makes subsequent separation difficult, too much causes partial dehydration and injury of fiber. *e*, Bulk of storage capacity is usually placed at this point. At two of the mines dry-rock bin capacity is 25,000 tons. With such storage it is possible to keep the mill running and shut down the quarry and coarse crushers in the worst weather. *f*, These average about 6 ft. wide × 20 ft. long, frequently stepped into 2 or 3 decks, and are covered with screen having about 1/16-in. openings. Speed @ 300 r.p.m. The suction pipe of a rotary fan, suitably shaped (see Fig. 2), is suspended above the screen near the end of each deck. *g*, Usual conical type. *h*, Or other type of disintegrator, *e.g.*, cyclone, swing-hammer pulverizer or the like. (See Sec. 4, Art. 21.) *i*, Usually a rotary or hexagonal trommel with about 1/16-in. screen. Vibrating screens are now coming into use. *j*, Screen apertures correspond to those on asbestos-testing sieves, *viz.*: 2-mesh, 4-mesh and 10-mesh. (See note *a*, Table 11.)

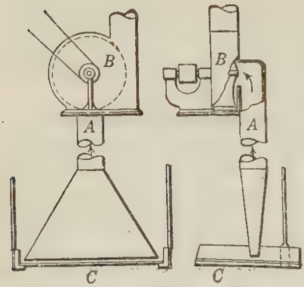


FIG. 2.—Suction fan for asbestos table.

Capacity: 800 to 1000 tons per 24 hr.

Assays: Feed and products are not capable of ordinary chemical assay, and an actual run is necessary in order to determine the amount of recoverable fiber present.

Recovery is stated as the percentage of total rock mined (or milled) that is shipped as asbestos. This method, of course, gives no measure of the efficiency of the concentrating process. Average per cent. of fiber extraction at the Quebec mines has been between 6 and 7.

Cost: Ru Keyser gives the following as typical of an 800-ton plant in 1919: Total expense per ton mined and milled, $1.65; value of ore per ton milled, $2.60; recovery of fiber, 4.81 per cent.; average selling price of fiber, $60 per ton; average operating cost of producing fiber, $38 per ton.

Typical cross-fiber asbestos treatment (*113 J 673*). Fig. 3.

Location: Thetford district, Que., Canada.

Ore: Chrysotile in serpentine.

Capacity: 800 to 1000 tons per 24 hr.

Assays: See corresponding note on slip-fiber mills.

Recovery: See corresponding note on slip-fiber mills. Recovery of No. 1 crude ranged downward from 0.1 per cent. in 1910 to 0.033 per cent. in 1920. Recovery of No. 2 crude has increased in the same time but total recovery of crude is not, in general, over 0.75 per cent.

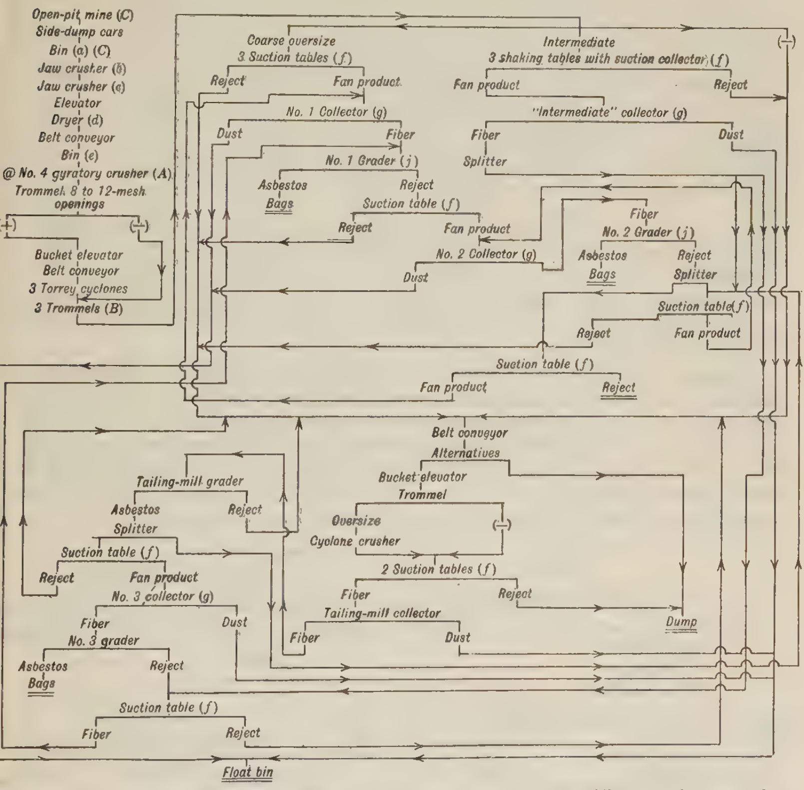
Cost: Ru Keyser gives the following as typical of 1921 operation. Value of fiber per ton of rock mined, $2.51; value of crude and fiber per ton mined, $3.43; total expense per ton mined and milled, $1.48; average value of fiber, $329.75 per ton; average total expense per ton of fiber, $147.25. Cost of milling, bagging, loading and of mine (and mill) overhead is about 45 to 50 cents per ton milled. About two-thirds of the rock mined goes to the mill.

7. Barite, $BaSO_4$

Properties. Crystalline, concretionary or earthy. COLOR: white when pure; impure varieties are red, yellow, gray, blue or brown. LUSTER: vitreous to resinous or pearly. SP. GR., 4.3 to 4.6. HARDNESS, 2.5 to 3.5. Insoluble in water and acids. Decrepitates and loses SO_3 on heating; fuses at high temperatures.

Uses. The principal uses are in the manufacture of white pigments and in making barium chemicals for various uses. Ground barite itself is used as a pigment, as a filler for paper, cardboard, rubber, linoleum, artificial ivory and in many other places where a heavy, inert filler is desired. LITHOPONE, which is the principal barium pigment, is composed of about 70 per cent. precipitated barium sulphate and 30 per cent of a mixture of 1 to 3 per cent. zinc oxide and the balance zinc sulphide (see *Ladoo, Non-metallic minerals,*

p. 78, for outline of method of manufacture). Various barium chemicals, including the precipitated sulphate, the nitrate, chloride, sulphide, oxides and hydroxide, chlorate and chromate are used in the manufacture of white-rubber goods, asbestos cement, ceramics, enamels, boiler compounds, insecticides, fertilizers, hydrogen peroxide, special glass, etc. (For detailed extensive list of uses see *Bul. 35, Geol. Surv. of Ga.*) Forty to 50 per cent. of the barite produced goes into lithopone manufacture, 25 per cent. into manufacture of barium chemicals; the balance is ground to fine powder and used thus.



a, b, c, etc., For lower-case reference numbers read notes to Fig. 1. *A*, Probably better practice than the jaw crusher in corresponding position in slip-fiber flow-sheet. *B*, Two screens on each, finer 8- to 12-mesh. *C*, Ore is usually cobbed both at the mine and ahead of or following the primary breaker for the recovery of "crude" fiber.

Fig. 3.—Typical cross-fiber asbestos mill.

Ores. Barite ($BaSO_4$) and witherite ($BaCO_3$) are the principal economic minerals. United States deposits are barite in the form of particles ranging from small grains to masses containing 1 or 2 cu. ft. together with grains and masses of chert in residual, unconsolidated clays and soils. The barite particles may be either hard and crystalline or soft and granular. The latter is preferred, as it is easier to grind. Some foreign barite deposits are of vein type.

Production. World production of barite minerals for the years 1919 to 1923 compared to that for 1913 is shown in Table 12. The principal producing regions in the United States are in Missouri and Georgia. These two states accounted for 80 per cent. of the total domestic production in 1923, with Tennessee third with 11 per cent.

Table 12. World production of barite, metric tons. (*Mineral Resources, 1923*)

Country	1913	1919	1920	1921	1922	1923
United States....	41,093	189,900	206,940	60,209	140,649	194,303
Australia........	550	2,827	3,874	1,494	2,292	*a*
Austria..........	*a*	*a*	*a*	2,080	2,140	2,377
Belgium.........	12,000	....	15,240	17,050	1,480	*a*
France..........	12,236	11,861	18,782	11,297	12,634	*a*
Germany........	78,394	63,188*b*	111,864*b*	108,759*b*	16,139*b*	*a*
India...........	....	2,632	689	1,480	2,430	*a*
Italy...........	12,970	15,920	15,380	13,500	21,300	27,592
Spain...........	3,049	11,459	13,773	910	4,500*c*	11,764
United Kingdom.	50,848	61,051	65,180	25,065	41,606	44,195
Others (*d*, *b*).....	582	425	1,009	245	1,462	899

a Data not available. *b* Complete data not available. *c* Estimated. *d* Algeria, Canada, Rhodesia, Russia.

Selling. (*114 J 109*) Crude barite should contain upward of 90 per cent. $BaSO_4$. The usual washed product contains 95 to 98 per cent. The average price of crude per long ton, f.o.b. mines ranged from $5.66 in 1917 through a peak of $9.08 in 1920 to $7.77 in 1923. *Eng. and Min. Jour.* quotation, Oct. 3, 1925, was $7 @ $8.50 per long ton of crude, f.o.b. mines; ground off-color, $14 @ $17 per short ton, f.o.b. So. Carolina mills; water-ground, bleached and floated, $23 to $24 f.o.b. works, St. Louis.

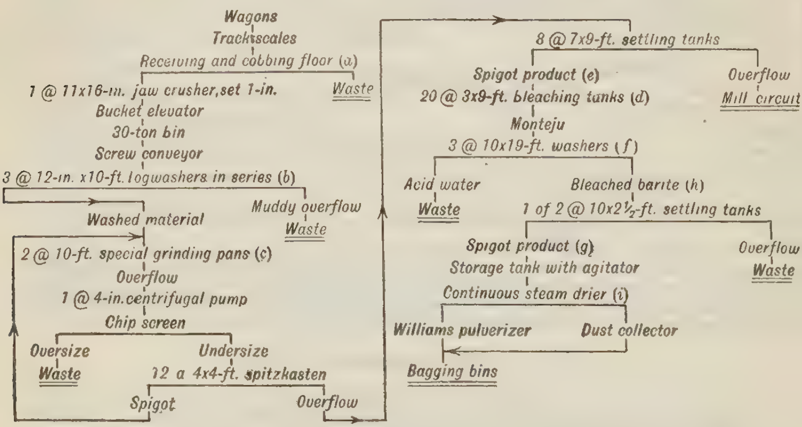


Fig. 4.—Point Milling and Manufacturing Co.

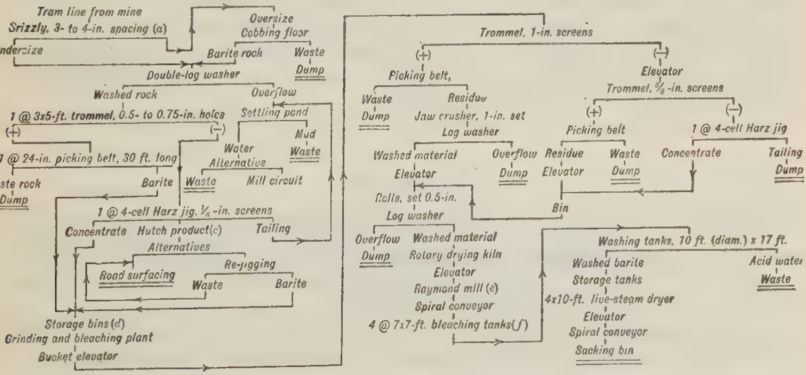
a, Pneumatic chisel used for cleaning lumps. *b*, Logs made of screw-conveyor spiral on 20° slope. Conveyor spiral has V-notches, 1.5 in. deep at 4-in. intervals. Spiral submerged for two-thirds of its length. Unsubmerged end is sprayed with 25 @ ⅛-in. jets under 45-lb. pressure. Each machine requires 40 gal. water per min. Capacity of the three is 120 tons washed sand per 24 hr. *c*, Steel sides 3 ft. high. Bottom paved with cut

blocks of local granite. 20 r.p.m. Drag stones roughly triangular, 36 in. on a side and 18 in. thick when new, arranged so that the leading angle can be changed as necessary to produce even wear. Life of stones about 3 yr.; life of bottom about 18 mo. to 2 yr. Should be about 8 in. of barite on the bottom for the best operation. Capacity of each mill is 12 to 18 tons per 24 hr. and each requires about 15 hp. ***d,*** Tanks are made of vitreous tile surrounded by hard lead which is in turn surrounded by oak staves and iron hoops. Charge is 2 tons of barite containing 25 per cent. water and a minimum of 240 lb. @ 66 per cent. H_2SO_4. About 45 min. is required to heat the mixture nearly to boiling with steam and 6 to 7 hr. minimum to bleach. If bleaching is not complete in 12 hr. more acid is added. ***e,*** About 25 per cent. moisture. ***f,*** Arranged for washing by continuous decantation. ***g,*** 15 per cent. moisture. ***h,*** Acid must not be completely removed or product turns yellow. Leave just enough so that the iron sulphate also present gives a slight blue color with potassium ferrocyanide. ***i,*** 3 × 8-ft. wrought-iron pipe, driven 1.5 r.p.m. and heated inside with live steam. Wet material is fed onto the outside at the top by means of a shaking feeder 8 ft. wide (200 r.p.m.), which in turn is fed in fine streams 4 in. apart by pin valves. Thus the feed comes onto the dryer in drops and adheres. Dried material is scraped off on the rising side near the feed point. The cylinder is mounted over a hopper lined with pipes carrying exhaust water from the cylinder. Spills are caught and dried therein.

Treatment varies with the character of the ore and the scale of mining. At the usual small-scale Missouri plant the ore is roughly picked, dried on floors and then "rattled" in a box screen with 1-in. holes. This treatment removes clay and sand as undersize which may be hand-jigged or rejected. Oversize is hand picked. Larger plants wash the crude ore in log-washers, with or without preliminary crushing, screen the washer sand, hand-pick the oversize and jig undersize. Fig. 4 shows the flow-sheet of a typical Missouri washer and Fig. 5 the rather more complicated scheme of a Georgia washer. Cost of the Georgia washing plant, down to and including the washed-ore storage bin was \$5000 to \$6000 in 1919.

Point Milling and Manufacturing Co. (*40 A 711*). Fig. 4.
Location: Mineral Point, Mo.
Ore: Barite and chert in residual clay.
Capacity: 35 tons cleaned barite per 24 hr., 500 to 600 tons feed.
Ratio of concentration: 15 or 20 to 1.
Product is said to all pass a 300-mesh sieve.

Thompson-Weinman Co. (*Bul. 36, Geol. Survey of Ga.*). Fig. 5.
Location: Cartersville, Ga.
Ore: Barite and chert in loose clay and soil.
Capacity: 60 tons white barite per day.



a, 7 steel rails, 12 ft. long. ***d,*** Better than9 2 per cent. $BaSO_4$. ***e,*** Product, 200-mesh. ***f,*** Lead- and brick-lined.

Fig. 5.—Thompson-Weinman Co., barite plant.

8. Bismuth, Bi

Properties. Metal; reddish-white, lustrous, crystalline; soft, brittle. Bismuth is the most highly diamagnetic substance known. (See also Table 1.) AT. WGT., 209.0. Remains unchanged in air and water at ordinary temperatures. At red heat burns in air and slowly decomposes steam. Not affected by dilute acids nor by concentrated hydrochloric acid. Concentrated sulphuric acid and concentrated nitric acid dissolve it, with the formation of the corresponding salts. Bismuth ion is tri-valent and quinque-valent and, so far as known, base-forming only. Bismuth alloys readily with other metals.

Uses. Metallic bismuth is used almost exclusively in making low melting-point alloys with tin, lead, cadmium, and mercury. Bismuth compounds are used extensively in medicine. They find some use also in porcelain and glass making, and in pigments.

Ores. The principal minerals are the native metal and bismuthinite, bismite and bismutite. The principal deposit of the world is in Bolivia, where bismuthinite occurs in veins associated with tin and silver minerals. Native bismuth and bismuthinite commonly occur in veins associated with pyrrhotite, pyrite, molybdenite, wolframite and precious metals. Native bismuth is sometimes found in sluice boxes. Carbonates and oxides of bismuth occur as residual deposits from leaching of rocks containing primary bismuth minerals. In such occurrences the bismuth is often associated with clays.

Production. Domestic production amounts to 100 to 150 tons of metal per year and this satisfies the domestic demand. The principal foreign producers are Bolivia and Australia, the product of each country ranging between 100 and 250 tons of metal per year. Scattered production in Argentina, Peru, Spain, Japan and Germany probably totals less than 100 tons per year.

Selling. Metal for pharmaceutical preparations must be more than 99.9 per cent. pure and contain no arsenic. That used for alloys is not so rigidly specified but in general must contain at least 99.9 per cent. Bi. Price is substantially fixed by Johnson, Matthey & Co., London, and fluctuates but little. Pre-war price was \$1.50 @ \$2.00 per lb. It rose to \$4.00 during the war. *Eng. & Min. Jour.-Press* (Oct. 3, 1925) quotation was \$2.65 @ \$2.70 in ton lots. These prices could not be maintained in the face of large increase in production. The market form is 25-lb. ingots.

Treatment. Domestic sources are lead- and tin-refinery slimes. At the S. AND M. tin-tungsten mine in Tasmania and the BURMA QUEENSLAND CORP. (tungsten-molybdenum mine) in Australia (see "Tungsten") bismuth is an important part of the concentrate.

9. Cadmium, Cd

Properties. Metal; bluish-white, lustrous; soft, malleable, ductile (see also Table 1). AT. WGT., 112.4. Very slightly affected by air at ordinary temperature but burns when strongly heated. Decomposes water at red heat. Attacked slowly by hydrochloric and sulphuric acids, vigorously by nitric, forming corresponding salts. Ion is bi-valent, base-forming. Cadmium alloys freely with other metals.

Uses. Principal uses are in making alloys of low-fusing temperature; in electro-plating hardware, automobile parts and the like by the udylite process to resist rusting; in the standard Weston cell; and, in the form of sulphide, as a yellow or orange pigment.

Ores. Cadmium is obtained both in this country and abroad exclusively as a by-product from zinc smelting and refining, or from bag-house dust at

lead smelters where zinc is a constituent of the ores treated. Siebenthal (*MR, 1921*) states that cadmium occurs in all zinc ores in the proportion of about one of cadmium to 200 of zinc.

Production (*MR*). United States and Upper Silesia are the principal producers. Domestic production ranged from 54,000 lb. of metal and 17,000 lb. of sulphide in 1913 to a peak of 207,000 lb. of metal and 50,000 lb. of sulphide in 1917. Following a slump, the metal production in 1922 was 132,000 lb. and the sulphide 135,000 lb. Upper Silesian production of metal in 1913 was 82,000 lb. and in 1922, 55,000 lb.

Selling. The commercial metal is in the form of pencils of 99.5 per cent. standard purity but most of the metal marketed runs higher, up to 99.9 per cent. The sulphide ranges from 98 to 99.9 per cent. pure. Prices of metal and sulphide usually keep within 5 or 10¢ per lb. of each other. Metal price in 1913 was $0.77 per lb.; the peak in 1916 was $1.56; quotation (*Eng. & Min. Jour.-Press*) Oct. 3, 1925, was $0.60 per lb. Demand is growing rapidly but at present the potential supply is in excess of the demand, the market is highly competitive and prices are generally falling.

Treatment. Dry method: Crude metal is obtained by fractional distillation from blue powder and this metal is refined by further fractional distillation. Wet methods consist in solution of cadmium oxides in acids and precipitation of the cadmium either chemically or electrolytically. (For full discussion see Hofman, *Metallurgy of zinc and cadmium, McGraw-Hill Book Co., N. Y.*)

10. Chromium, Cr

Properties. Metal; gray-white, lustrous, crystalline structure, very hard. (See also Table 1.) AT. WGT., 52.0. Unchanged in air at ordinary temperatures; at red heat becomes slowly coated with a thin layer of oxide. Dissolved by dilute hydrochloric and sulphuric acids with formation of corresponding salts. Nitric acid does not attack it. Chromium ion is bi-, tri- and quadri-valent, base- and acid-forming. Chromium alloys freely with other metals.

Uses. Pure metallic chromium is not used commercially. Ferrochrome, an alloy composed chiefly of iron and chromium with small amounts of carbon, silicon, and other impurities, notably manganese, is widely used for hardening steel for railway-wheel tires, wearing parts incrushing machinery, and other steels where toughness and hardness are essential. Chromite bricks are highly refractory and are used for furnace linings. Chromates and bichromates are largely used as red, yellow, and green pigments for dyeing, calico and wall-paper printing, painting, and pottery. Chromium compounds are largely used in tanning and to some extent in medicine. Recently "stainless" steel and "rustless" iron, both containing 12 to 14 per cent. chromium, have come onto the market and will, with increasing use, increase chrome consumption. Chromizing by a process similar to zinc sherardizing and aluminum calorizing has been accomplished and chromium plating has also been effected. Both chromized and chrome-plated iron and steel resist atmospheric corrosion and the latter resists acid and ammonia fumes but not electrolytic corrosion in mineral acids. (*32 MI 118.*)

Ores. Chromite ($FeO \cdot Cr_2O_3$) in highly basic igneous rocks or in serpentine is the only ore. The natural mineral is rarely pure chromite, Cr_2O_3 being replaced by Al_2O_3 or Fe_2O_3. As a consequence the highest grade of ore contains only about 55 per cent. Cr_2O_3 and the Cr_2O_3 content in salable ores runs down from this figure to 38 to 40 per cent.

Production by countries is given in Table 13. Domestic production during wartime was all from small, not particularly high-grade deposits, mostly located in the west, many of them remote from transportation facilities. Under normal conditions of world trade, with consumption within a few hundred miles of the Atlantic seaboard, domestic production from the

West cannot compete with ore from Rhodesia and New Caledonia and there are no eastern deposits of sufficient size and purity to supply the demand.

Table 13. World production of chromite, metric tons. (*MR*)

Country	1913	1919	1920	1921	1922	1923
United States	259	5,161	2,542	287	361	231
Brazil	....	4,877	3,506	....	....	....
Canada	....	7,748.	9,993	2,538	696	2,654
Cuba	....	14,693	721	610	....	10,587
Greece	6,930	4,070	12,492	5,919	9,927	13,350
India	5,767	37,024	27,231	35,320	23,143	*b*
Japan	1,326	6,012	3,969	3,368	3,757	*b*
New Caledonia	63,370	23,548	91,534	29,458	19,374	23,226
Rhodesia	57,500	32,007	54,674	45,529	84,799	87,702
Russia	15,000	*b*	2,965	4,013	308	*b*
Turkey in Asia	14,000	*b*	25,000	*b*	2,540	....
Others (*a*)	1,106	2,552	3,911	1,552	1,244	1,211*c*
Total	165,258	137,692	238,538	128,594	146,149	*c*

a Includes Austria-Hungary, Australia, Great Britain, Guatemala, Norway, Yugoslavia, Union of South Africa. *b* Figures not available. *c* Not complete.

Selling. Chromite is usually sold on the basis of the long-ton unit (1 per cent.) of Cr_2O_3 and the price per unit increases with the Cr_2O_3 content, but ore for refractories is sold at a flat price with a guaranteed minimum chromic-acid content. *Eng. and Min. Jour.-Press* quotation, Oct. 3, 1925, was: "Crude, 45 to 50 per cent. Cr_2O_3, \$20.50 @ \$23.50 per net ton, f.o.b. shipping point. . . . Ground, in bags, \$29. New Caledonian, 52-54 per cent. Cr_2O_3, \$24; nominal." Average yearly price per long ton for domestic sales was \$11.19 in 1913, reached a peak of \$47.99 in 1918, and has since receded, with some fluctuation, to a range between \$18 and \$24.

Treatment. The foreign ores are mostly from high-grade deposits that require no concentration other than hand sorting. Chromium ores respond to simple gravity concentration and some such concentration is necessary with many of the domestic ores. A 50 per cent. Cr_2O_3 concentrate can readily be obtained unless the chromite itself is of too low grade.

11. Clay

Properties. Clay of commerce is not a definite mineral, but rather a mixture of minerals in which one with unctuous feel and plastic consistency predominates and imparts its physical aspect to the mixture. The predominating mineral is ordinarily kaolinite, a hydrous aluminum silicate ($2\ H_2O \cdot Al_2O_3 \cdot 2\ SiO_2$) formed by decomposition of feldspars, but it may be some other closely related hydrous silicate of aluminum such as halloysite or bentonite. The usual associated minerals are quartz and feldspar, mica, calcite, gypsum, limonite, and some of the more resistant minerals of the original feldspar-bearing rock, such as garnet, beryl, tourmaline, hornblende, titanite and ilmenite. Kaolinite is normally white but may be tinged gray, yellow, brown, blue or red by impurities. SPECIFIC GRAVITY is about 2.6; HARDNESS, 2 to 2.5. *Ladoo* gives the INDEX OF REFRACTION as 1.561; MELTING POINT, 1850° C; and says that it is soluble in hot sulphuric acid. Important physical properties depend upon use. Clays for fillers should be very fine-grained, free from grit, white in color, with low oil-absorption. For ceramic use plasticity, cohesion, shrinkage and behavior as to cracking when dried, tensile strength, porosity, and disintegration in water are important. Behavior on heating, particularly as regards shrinkage and cracking, and the character of the burnt clay, its tensile strength, porosity, absorptive capacity, color, translucency, hardness and toughness are important in ceramic use. The requirements of clay-using industries are so varied and the requirements of any particular industry so difficult, if not impossible, of quantification, that the only safe way to determine the value of the clay in a given deposit is to submit samples to prospective users.

Uses. High-grade clays are used for porcelain, china, pottery and high-grade tile; as a filler in paper, paint, linoleum, oilcloth, rubber, soap, asbestos products, plaster, cloth and a large variety of other materials and as a fire refractory for metallurgical and other furnace linings. Bentonite has a special field in de-inking newsprint. Lower-grade clays and clayey shales are used for terra-cotta ware, vitrified pipe and tile and brick of all kinds.

Ores. Deposits of low-grade clays are found in substantially all parts of the world. Deposits of high-grade clays are less abundant and extensive and are much less widely scattered. Those of the United States are almost all in the eastern and southern states. There are also important deposits in England, France, Germany and Austria.

Production of high-grade clays in the United States ranges between two and three million tons per year. Price range is from $1 per short ton for some high-grade brick clays to $12 or $15 average for kaolin. The production of low-grade clay is not available but is far in excess of the above figures.

Selling. There are no standard specifications. Clays are ordinarily sold on the basis of samples submitted to the purchaser. There is, consequently, no open market and price is a matter of individual arrangement between the producer and consumer.

Treatment (*Bul. 53, USBM*) consists in disintegrating, settling out the impurities in water, dewatering and drying. Refined lump clay that is to be used for fillers and like purposes must be again pulverized. A flow-sheet of a typical plant is shown in Fig. 6. Rake and bowl classifiers and continuous thickeners would undoubtedly give cheaper and better service than the crude settling and dewatering devices used.

a, See Fig. 7. Approximately 3 × 3 × 10 ft., 200 r.p.m. Usually two are placed in series. ***b***, See Fig. 8. Tank about 10 ft. long and 6 ft. high. Sand scooped up by shovel arms is drained in rising and discharged over the side of the box. Frequently two are placed in series or the flow is from the primary washer to a sand wheel, thence to a secondary washer and sand wheel. ***c***, There are two types: (1) The broad trough, about 2 ft. wide by 20 to 30 ft. long, set level, with a discharge baffle 4 to 8 in. high. (2) The square tank, 6 ft. long, 5 ft. wide, 1 ft. deep with outlet 3 in. below the inlet and located either at one side or at the inlet end. ***d***, U-shaped troughs about 1 ft. wide × 1 ft. deep × 40 to 50 ft. long, arranged either in series (U. S. practice) or radiating from a central basin and flared toward the discharge end (British practice). In the series arrangement the troughs are either set level with a drop of 1 in. from trough to trough, or each trough is sloped 1 in. in its length. Operation is periodic. Usual procedure is to run pulp for about 5 hr., then cut off feed, run clear water for 15 or 20 min., then close regular outlet, open waste outlet, and scrape and flush the sediment therethrough. About 1 hr. is required for this cleaning.

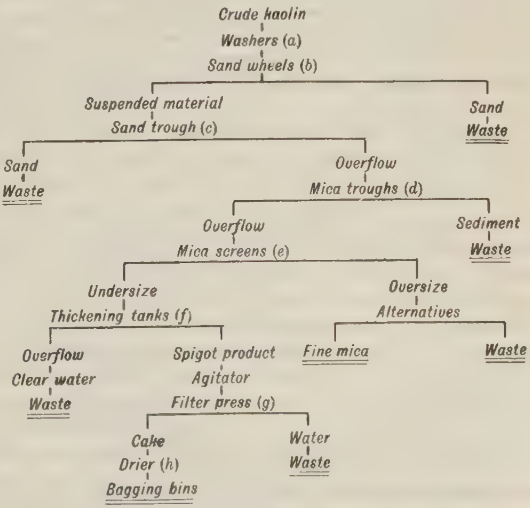


Fig. 6.—Typical kaolin-washing plant.

e, Stationary, vibrating, sliding and revolving screens are used. Usually 3 or more of the stationary screens are used in series, set at about 10° slope with 6-in. drop between screens. Screens are covered with fine brass cloth. *f*, Called locally "concentrating tanks." Lump alum, suspended in a bag in the feed launder, is commonly used as a flocculator to aid settling. The tanks are ordinarily intermittent and clear water is siphoned off. *g*, 100 to 120 lb. per sq. in. pressure. Cake contains 8 to 20 per cent. water. *h*, Atmospheric drying floors, unheated or steam-heated, or a drying tunnel through which air heated by passage over steam pipes is drawn, are used.

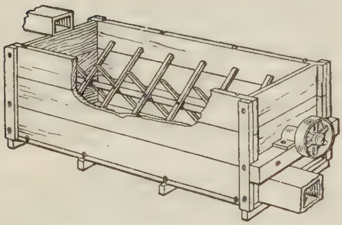


FIG. 7.—Kaolin washer.

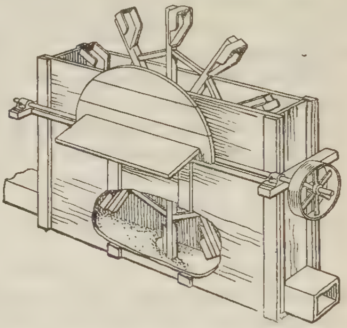


FIG. 8.—Sand wheel for clay washing.

12. Coal

Definition. In the present state of knowledge it is impossible to give an accurate, comprehensive definition of coal. What is known is that coal is a black or brown, rock-like, combustible substance, composed principally of carbon in various solid, liquid and gaseous compounds with hydrogen, oxygen and nitrogen and that any given mass represents a stage in a long series of complex geochemical transformations that start with dying and decaying vegetable matter and may end with graphite as the only residual solid product. The identities of the carbon compounds in different coals probably differ, certainly the proportions of the different compounds differ, but there is little or no exact knowledge of the composition of these compounds. In addition to the carbon compounds, coal also contains sulphur in various combinations, *e.g.*, pyrite, gypsum and unknown carbon compounds; water, and non-combustible "ash," this latter being principally silica or various silicates, carbonates of calcium, magnesium and iron, and minor amounts of various other mineral salts such as alkaline chlorides and calcium phosphate.

Rank. As above stated, coal has been derived by geologic processes from vegetable matter that has suffered slow decay and consolidation at or below water level, followed by further consolidation and metamorphosis due to the pressure and heat incident upon deposition of overlying strata and subsequent elevation and folding. These processes have been continuous and of varying intensity, through many geologic ages up to the present time, so that coal is found in all stages from the growing plant to graphitic carbon. The coal material in these various stages has different compositions and physical properties and widely different degrees of usefulness. As a result a rough classification of coals has developed, founded principally on use, but approximately expressible in terms of chemical analysis. The place of a given coal in this classification is called its RANK. There is no generally accepted classification according to rank.

Table 14 gives the classification in common use in U. S. Geol. Survey publications (*100–A USGS 3*). A somewhat more complete classification proposed in *The coal resources of the world* (*1913*) is summarized in Table 15. A classification proposed by Ashley (*1923 Pro. CMIA 29*), which further subdivides the bituminous rank on the basis of use, is given in Table 16. Several other classifications are summarized and discussed by Ashley in the last-named publication. Campbell (*Prof. Paper 48, USGS, Part I, p. 156*) discusses several possible bases of classification. The range in chemical analysis of coals of various ranks, as given by Moore (*Coal, John Wiley & Sons, 1922*), is shown in Table 17.

Properties of coal differ considerably with its rank.

Anthracite is brittle, grayish-black to black with dull to sub-metallic luster. HARDNESS varies from 2 to 2.5. It breaks with distinctly conchoidal FRACTURE, with sharp edges. SPECIFIC GRAVITY of low-ash particles ranges from 1.3 to 1.7 and rarely to 2.2 (Rhode Island) (*Bul. 615, USGS*). It ignites with difficulty and burns slowly without much disintegration, with a short hot flame and little smoke or odor. It does not coke.

Semi-anthracite is similar in appearance to anthracite but softer, and the FRACTURE is less distinctly conchoidal. SPECIFIC GRAVITY is about 1.4. It ignites more readily than anthracite, crumbles to some extent in burning and hence burns more rapidly, with a short, yellow flame. It does not coke.

Bituminous coals vary in COLOR from dark brown to black and in LUSTER from dull to resinous or brilliant. HARDNESS is less than anthracite, but on account of the brittle and

Table 14. Classification of coal. (*U. S. Geol. Survey*)

Rank	Fuel ratio	Approximate moisture, per cent. (*a*)	Physical characteristics
Anthracite	10–60	2–3	*b*
Semi-anthracite	6–10	3–6	*b*
Semi-bituminous	3–7	3–6	*b*
Bituminous	3	3–15	*b*
Sub-bituminous	....	18–30	*c*
Lignite	....	30–45	*d*

a Average run-of-mine coal. *b* But slightly affected by exposure to weather. *c* Black; no distinct woody texture; disintegrates and loses moisture on exposure to weather, but less rapidly than lignite. *d* Distinctly brown; either markedly clay-like or woody in appearance; falls into pieces on exposure to weather.

Table 15. Classification of coal. (*Coal Resources of the World, 1913*)

Rank	Fuel ratio, (*a*)	B.t.u. per pound	Carbon, per cent.	Volatile combustible matter, per cent.	Moisture, per cent.	$\frac{C+\frac{1}{2}V}{M+\frac{1}{2}V}$ (*b*)	Remarks
Anthracite	12+	14,500–15,000	93–95	3–5	.....	....	
Semi-anthracite	7–12	15,000	90–93	7–12	.....	....	
High-carbon bituminous	4–7	15,200–16,000	80–90	12–15	.....	....	Does not coke readily.
Bituminous	1.2–7	14,000–16,000	75–90	12–26	.....	....	Generally cokes.
Low-carbon bituminous	.....	12,000–14,000	70–80	35–	6–	2.5–3.3	Makes porous, tender coke.
Cannel	.....	12,000–16,000	.....	30–40*c*	.....	....	Very porous coke.
Lignitic or sub-bituminous	.....	10,000–13,000	60–75	......	6+*d*	1.8–2.5	
Lignite	.....	7,000–11,000	.....	45–65	20+	....	

a Percentage of fixed carbon ÷ percentage of volatile matter. *b* (*Fixed carbon + ½ volatile*) ÷ (*Hydroscopic moisture + ½ volatile*). *c* On distillation. *d* Runs up to 20 per cent. when freshly mined.

Table 16. Classification of coal. (*After Ashley*)

Usual name	Fuel ratio	Average analysis, per cent.			
		Moisture	Volatile matter	Fixed carbon	Ash
Anthracite	12+	3	2	88	7
Semi-anthracite	12–8	3	7	83	7
Semi-bituminous:					
"Admiralty"	8–5	3	13	77	7
"Low-volatile," "Smokeless," "Bunker"	5–3	3	20	70	7
Bituminous:					
"Medium-volatile," "Coking," "By-product"	3–2	3	27	63	7
"High-volatile," "Gas"	2–	3	34	56	7
"High-volatile"	2–	6	38	49	7
Sub-bituminous:					
"High-volatile"	2–	15	36	42	7
"High-moisture"	2–	25	33	35	7
Lignite	2–	40	25	28	7

Table 17. Analyses of various ranks of coal. (*After Moore*)

Rank	Percentages			
	Anthracite	Semi-anthracite	Semi-bituminous	Bituminous
Moisture	0.42– 5.61	1.97– 7.94	0.78– 8.99	0.04–34.33
Volatile matter	1.72–10.75	6.81–32.46	7.40–23.84	8.63–64.31
Fixed carbon	73.71–90.90	58.24–82.00	57.11–80.89	26.49–80.60
Ash	3.20–30.09	4.33–14.50	1.80–34.15	0.28–45.00
Sulphur	0.17– 2.60	0.57– 4.05	0.44– 6.47	0.0012–10.5
Hydrogen	1.89– 5.61	3.69– 4.81	3.34– 5.17	1.00– 8.80
Carbon	78.41–83.89	72.43–80.00	51.23–85.54	44.00–85.30
Nitrogen	0.63– 1.57	0.51– 1.45	0.81– 1.82	1.00– 9.20
Oxygen	3.80–11.54	5.46–10.02	3.38–13.70	0.95–46.90
Calorific value, B.t.u.	9,230–13,298	12,460–14,184	8,386–14,814	6,840–15,169
Air-drying loss	....	....	....	....

Rank	Percentages			
	Sub-bituminous	Lignite	Peat	Wood
Moisture	1.94–40.58	0.75–43.00	63 –90*b*	....
Volatile matter	7.50–70.86	27–53	43.38–73.60	....
Fixed carbon	18.00–83.00	16–51	10.39–33.91	....
Ash	2.06–55.40	2.60–42	1.05–32.95	....
Sulphur	0.15– 8.65	0.16–9	1*c*	....
Hydrogen	1.76– 6.98	5*a*	4.08–10.39	6.25
Carbon	30.68–86.85	58*a*	37.15–66.65	49.50
Nitrogen	0.49– 2.13	1*a*	0.77– 3.10	1.10
Oxygen	2.80–52.18	25*a*	18.59–42.63	43.15
Calorific value, B.t.u.	6205–14,843	*d*	5500–10,000	5800
Air-drying loss	0.80–28.00	....	....	....

a Average. *b* As cut. *c* Usually. *d* 5500 to 7000 un-dried; 10,000 to 12,000 on moisture-and-ash-free basis.

crumbly nature of many varieties, is difficult to test. The FRACTURE is usually along planes of weakness in two or three directions at substantially 90° to each other, resulting generally in the formation of cubical and rounded lumps. SPECIFIC GRAVITY of low-ash particles ranges from about 1.15 to 1.35.

Semi-bituminous coal is only slightly lower in the scale than semi-anthracite and is similar to it in appearance and structure but on account of the higher content of volatile matter, ignites more readily and burns more rapidly. It may coke but the coke is small and weak. Mixed in proper proportions with a good coking coal it may, however, strengthen the resulting coke. U. S. Navy specifications for Pocahontas coal are for the following maxima: moisture (as received), 2.5 per cent.; ash (dry basis), 6.5 per cent.; sulphur, 0.75 per cent.; volatile matter, 22 per cent.

Coking coal is a bituminous coal, usually brilliant black, of crumbling structure, containing from 20 per cent. to upwards of 40 per cent. volatile matter. It has the property of softening, when gradually heated in a closed container to the point of incipient decomposition (300 to 400°C), and forming a pasty mass which swells, with evolution of gas, and before 450°C is reached becomes a rigid, porous, combustible, carbonaceous solid. The phenomenon is called COKING and the solid product is COKE.

The greatest USE for coke is as a fuel in iron blast furnaces. For such use the coke should be strong, porous and in the form of large lumps and should contain, in general, less than 1.5 per cent. sulphur and less than 12 or 13 per cent. ash. Usually 1.0 per cent. is the maximum sulphur allowance and, if the coke is to be used for making acid open-hearth or Bessemer iron, the phosphorous content should not exceed 0.010 per cent. In coke for making basic iron and foundry iron the phosphorous content is relatively unimportant. Coke for non-ferrous smelting may contain more sulphur.

A good coking coal yields from 65 to 75 per cent. of its weight in the form of coke. This coke contains all of the ash that was present in the coal and, frequently, from 50 to 60 per cent. of the sulphur, hence there is a concentration of ash and, usually, some elimination of sulphur in the process of coking, and the allowable sulphur and ash contents of the coals are to be calculated accordingly. The specifications for the best inland slack trade for coal for by-product coking are: Moisture (as received), 5 per cent. max.; ash (dry basis), 7 per cent. max.; sulphur, 1 per cent. max.; volatile matter, 20 per cent. min. (*MCJ., Jan., 1926.*) If the removable ash content of the coal is in the form of shale particles, these form centers for radiating cracks in the resultant coke, and thus decrease its compressive strength.

It is generally figured that, on the basis of pre-war prices, each unit of ash in the coke adds approximately $0.20 per ton to the cost of pig iron obtained therewith, this increased cost being due to (*a*) the increased amount of limestone flux required to slag off the ash, (*b*) the greater volume of coke, limestone and slag to be handled, and (*c*) decreased furnace output (*18 CA 1130*). Sulphur adds similarly to the cost of pig iron. Sweetser stated that with pig iron costing $20 per ton, each reduction of 1 per cent. in the ash content of coke reduced the cost of the pig iron $0.30 per ton.

In general, the only sure method of determining whether a coal is suitable for coking is actual trial (Rose, *A*, *Feb., 1926*). White (*Bul. 29, USBM*) has called attention to the fact that in the best coking coals the ratio H/O is upward of 58. Some coals with a ratio of 55 to 58 make satisfactory cokes, and many with a ratio less than 55 coke, but the coke is weak and unsatisfactory. Pishel (*1908 AGLJ 445*) states that when bituminous coal is pulverized in an agate mortar, the pulverized material adheres to the mortar strongly and persistently if the coal is a good coking coal, otherwise the powder can be readily collected in the bottom by tapping the mortar.

Gas coal is a variety of coking coal containing usually from 33 to 38 per cent. volatile matter (dry basis) and yielding, on coking, a gas of high calorific value. The sulphur content should be low (less than 1.25 or, frequently, less than 1 per cent.), in order to keep down the amount of hydrogen sulphide in the oven gases, since this must be scrubbed out completely before the gas is put into the mains.

Classification of coking coals. Figs. 9 and 10, presented by Rose (*A*, *Feb., 1926*), permit tentative placing of a bituminous coal, knowing either the ultimate analysis (Fig. 9) or the calorific value and volatile matter (Fig. 10), it being remembered in all cases that the final proof is the full-scale oven test.

For thorough discussion of "The selection of coals for the manufacture of coke" see H. J. Rose (*A*, *Feb, 1926*).

Non-coking coal may be similar in appearance to the coking variety, although the term includes also varieties of bituminous rank that have a splintery fracture (SPLINT COAL), certain dull varieties, etc. Non-coking coals are used principally for steam

generation and for this purpose the important characteristics are the heating value per unit weight of ash-free coal and the kind and amount of impurities. The ash should be of such composition that it will slag sufficiently to permit a bed of ashes to be maintained on a grate, but not so readily fusible that clinkers (fused ash) form in sufficient size and quantity to make removal difficult. A higher sulphur content is permitted than in coking coal and it is the pyritic sulphur that is of most importance. The principal disadvantage of high pyritic sulphur is the fact that the iron oxide formed by its decomposition may lower the fusing temperature of the ash and thus cause too much clinkering. For analyses, see Table 17.

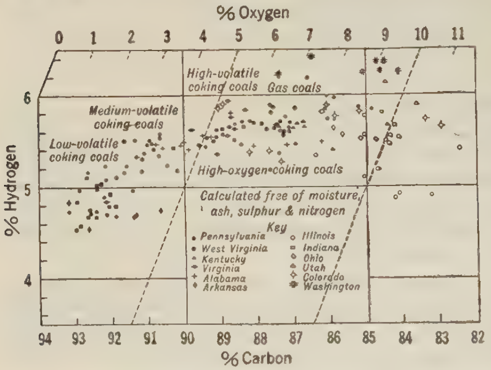


FIG. 9.—Ultimate analyses of 150 typical coking coals of the United States.

Sub-bituminous coal is the transition rank between lignite and bituminous coal. The FRACTURE is usually along one plane or, sometimes, conchoidal, so that it tends to break into slabs rather than into the cubical forms characteristic of the bituminous rank. The color is black and the luster glossy to resinous. (See also Table 17.)

Lignite is a brown, amorphous or woody form of coal representing the least change from peat. The luster is generally dull. SPECIFIC GRAVITY varies from 0.5 to about 1.3, according to the porosity (woodiness) and the amount of impurity. For analyses see Table 17.

Impurities in coal occur in several different varieties and forms. The impurities are classified as sulphur and ash.

Sulphur ranges between 0.5 and 8 to 10 per cent. of the coal as mined (*20 CA 3*). It occurs in three forms, *viz.*: as pyrite, as a sulphate, usually calcium sulphate, and in an organic combination. There is no constant relation between the amounts occurring in the different forms. Table 18 shows the results of sulphur analyses on typical coals from various domestic fields.

Ash content varies from 1 to 30 per cent. (*20 CA 3*). The ash occurs in several different forms: (*a*) As INHERENT ASH, which represents various inorganic minerals, principally silicates, so intimately associated with the coal as to be substantially inseparable therefrom by any mechanical means; X-ray examinations indicate that this ash consists of submicroscopic laminae of inorganic material alternating with thicker laminae of pure coal. (*b*) As RASH OR MOTHER-OF-COAL, which is a dull amorphous carbonaceous material of low specific gravity, but high inorganic content, found in bituminous coal. (*c*) In masses composed of fine inter-laminated layers of good coal and shale or slate so closely associated that fine crushing is required to separate them; these are known as BONE OR BONEY COAL. SULPHUR BONE is a similarly intimate mixture of pyrite and good coal. (*d*) As distinct masses of shale (with bituminous coals) or slate (with anthracite), clay, or other rocks which break

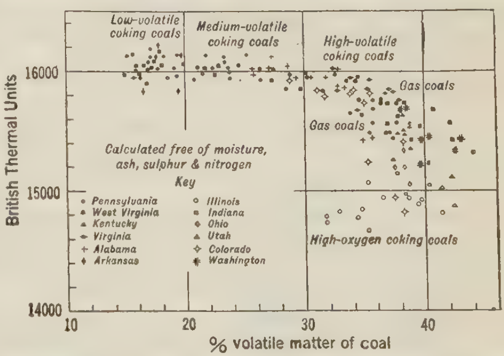


FIG. 10.—Volatile matter and calorific value of 150 typical coking coals of the United States.

cleanly away from the coal. (*e*) As crusts and veinlets of certain salts, of which the most important are sodium chloride and calcium phosphate.

Table 18. Forms of sulphur in raw coal. (*Bul. 11, UI*)

Coal from	Percentages of sulphur			
	Total	Pyritic	Sulphate	Organic
Tennessee	4.87	3.59	0.11	1.17
Kentucky	0.46	0.13	....	0.33
Illinois	1.83	1.04	Tr.	0.79
Illinois	3.51	1.84	....	1.67
Indiana	1.66	0.89	....	0.77

The important property of ash, apart from the fact that its presence lessens the heating value of the coal, is its fusibility. (For complete discussion see Fieldner and Selvig, *PP 1529-F A*)

Uses of coal. The principal uses are for fuel and as the source for a large number of important products derived by distillation. Anthracite is used principally for domestic fuel, but some large-size anthracite is used in gas producers and locomotives and the small sizes (buckwheat, rice and barley) are used for steam production in power plants. Table 19 shows the approximate distribution, according to use, of the soft coal consumed in the United States and exported in the year 1917. Fig. 11 shows some of the products obtained by distillation of bituminous coal.

Table 19. Consumption of soft coal in the United States, classified according to use (1917). (*After Tryon and McKenney, 21 CA 85*)

Use	Short tons	Per cent of total
Railroads	153,700,000	27.7
Industrials other than steel and coke	139,100,000	25.1
Steel plants	35,500,000	6.4
Coke:		
Beehive	36,000,000	6.5
By-product	47,740,000	8.7
Public utilities:		
Electric	31,700,000	5.7
Coal gas (*a*)	4,960,000	0.9
Domestic consumers	57,100,000	10.3
Coal-mine fuel	12,100,000	2.2
Exports	26,000,000	4.7
Bunkers	10,300,000	1.8
Totals	554,200,000	100.0

a Excluding by-product ovens.

Grades of coal as mined vary all the way from those sufficiently pure to be used with no treatment other than that required to bring them to suitable size, to grades so impure as to be economically impossible of exploitation. This statement holds for all of the different ranks. (See also **Treatment.**)

Production of coal in the United States in recent years is given in Table 20. The domestic production in 1924 was about 38.5 per cent of the world production (1,350,000,000 metric tons). The other principal producing countries in 1924 were Great Britain, 20.3 per cent.; Germany, 8.8 per cent.; France, 4.3 per cent.; Poland, 2.4 per cent.; and Belgium, 1.7 per cent. These countries stood in the same order in 1913. The remaining 24 per cent. of world production was from 44 countries scattered on all of the continents.

Selling. Notwithstanding the huge demand for coal, as evidenced by the consumption figures, the price at domestic mines for ordinary grades is kept

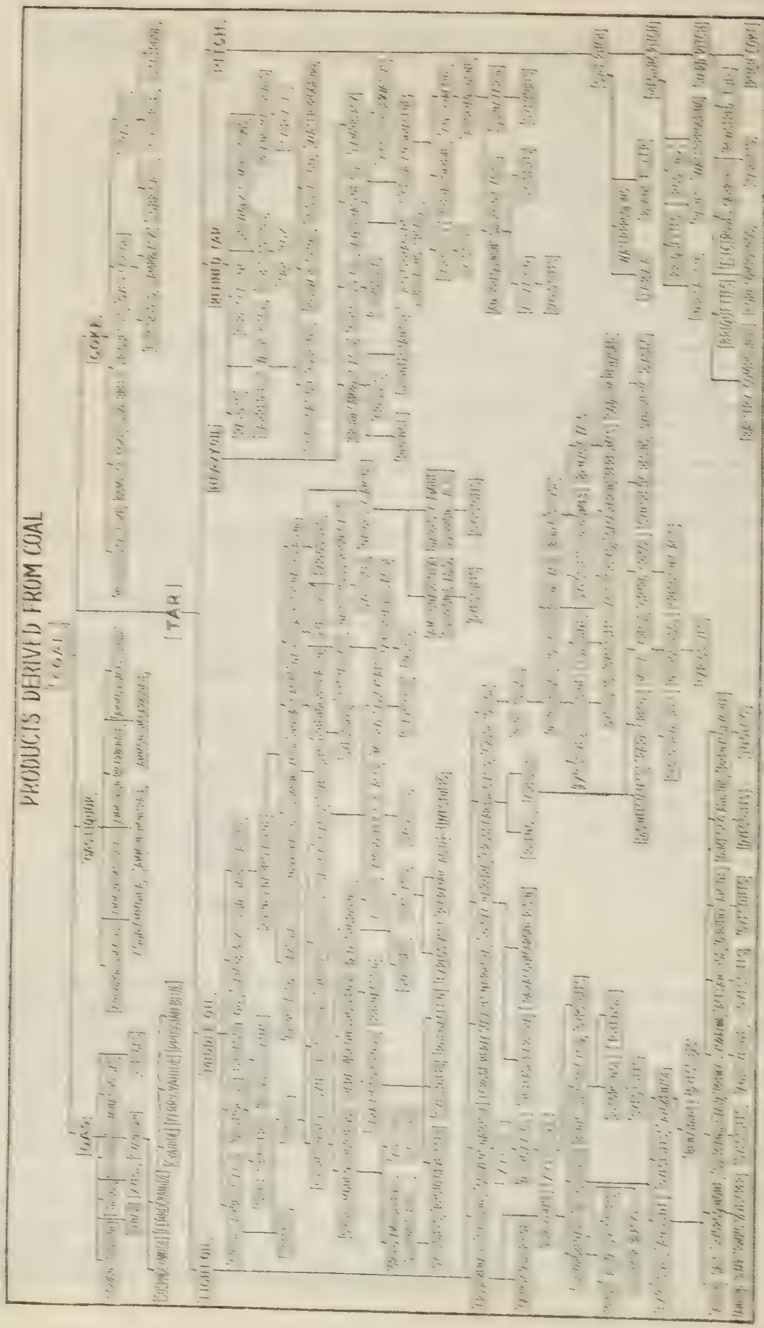


Fig. 11.—Products from the distillation of coal (after Barrett Mfg. Co.).

down by the excessive potential production capacity, close competition, and the high freight charge when distant shipments are necessary. These conditions apply less to anthracite than to bituminous coals, on account of the superiority of anthracite for domestic fuel, but in regions where a good grade of bituminous coal is locally available for domestic fuel, even anthracite suffers from the competition. Certain types of bituminous coal occupy a favored position because of their unique suitability for exacting uses. High-volatile coking coals (upwards of 30 per cent. volatile matter) which will yield upward of 10,000 cu. ft. of gas per short ton and contain less than 1 to

Table 20. Production of coal in the United States, short tons. (*MR, 1913 · MI, 1924*)

State	1913	1920	1921	1922	1923	1924 (*e*)
Alabama	17,678,522	16,140,099	12,568,899	18,324,740	20,457,649	19,490,000
Alaska	*c*	61,111	76,285	....	119,826	100,000
Arkansas	2,234,107	2,050,596	1,227,777	1,110,046	1,296,892	1,300,000
Colorado	9,232,510	12,274,225	9,122,760	10,019,597	10,346,218	9,840,000
Illinois	61,618,744	88,630,893	69,602,763	58,467,736	79,310,075	67,880,000
Indiana	17,165,671	29,090,585	20,319,509	19,132,889	26,229,009	22,340,000
Iowa	7,525,936	7,774,916	4,531,392	4,335,161	5,710,735	5,100,000
Kansas	7,202,210	5,838,408	3,466,641	2,955,170	4,035,404	4,150,000
Kentucky	19,616,600	35,528,762	31,588,270	42,134,175	44,777,317	45,000,000
Maryland	4,779,839	4,030,239	1,827,740	1,222,707	2,285,926	1,720,000
Michigan	1,231,786	1,487,765	1,141,715	929,390	1,172,075	820,000
Missouri	4,318,125	5,266,565	3,551,621	2,924,750	3,403,151	3,140,000
Montana	3,240,973	4,403,866	2,733,958	2,572,221	3,147,678	2,700,000
New Mexico	3,708,806	3,683,440	2,453,482	3,147,173	2,915,173	2,550,000
North Dakota	495,320	907,625	864,903	1,327,564	1,385,400	1,090,000
Ohio	36,200,527	45,032,653	31,942,776	26,953,791	40,546,443	29,200,000
Oklahoma	4,165,770	4,830,288	3,362,623	2,802,511	2,885,038	2,800,000
Penn. (*a*)	173,781,217	168,083,847	116,013,942	113,148,308	171,879,913	123,530,000
Tennessee	6,903,784	6,585,628	4,460,326	4,876,774	6,040,268	4,800,000
Texas	2,429,144	1,615,015	972,839	1,106,007	1,187,329	1,075,000
Utah	3,254,828	6,005,199	4,078,784	4,992,008	4,720,217	4,460,000
Virginia	8,828,068	11,244,106	7,492,378	10,491,174	11,761,643	10,900,000
Washington	3,877,891	3,753,093	2,428,722	2,851,165	2,926,392	2,400,000
West Virginia	71,308,982	89,450,707	72,786,996	80,488,192	107,899,941	110,000,000
Wyoming	7,393,066	9,623,271	7,200,666	5,971,724	7,575,031	6,850,000
Others	(*b*)330,777	(*d*)97,943	(*d*)104,183	253,126	142,084	45,000
Total bituminous	478,523,203	563,490,845	415,921,950	422,268,099	564,156,917	483,280,000
Pennsylvania anthracite	91,524,922	89,598,249	90,473,451	54,683,022	93,339,009	87,926,862
Grand total	570,048,125	653,089,094	506,395,401	476,951,121	657,495,926	571,207,000

a Bituminous only. *b* Georgia, Oregon, California, Alaska, Idaho, Nevada. *c* Included in "Others." *d* California, Idaho, Georgia, North Carolina, Oregon, South Dakota. *e* Estimated.

1.25 per cent. sulphur are in demand for gas works. High-volatile non-coking coals are demanded for locomotives pulling heavy, fast trains. Semi-bituminous coal (SMOKELESS COAL) makes the best kind of steam coal. This and semi-anthracite compete with anthracite for domestic use. Coals suitable for iron-furnace coke enjoy a marked preference over other bituminous grades. Other users with special requirements are malleable-iron furnaces, brick yards, and blacksmiths. (For a general analysis of markets see *Spurr and Wormser.*)

Prices of domestic sizes of anthracite (Table 21) are reasonably stable, due to the constancy of demand and close control of the major part of the total production. With the exception of a seasonal drop of about \$0.50 per ton in April, which, prior to 1916 was customary but has occurred only twice since, there has been a steady increase in average price since 1915 from \$3.75 per gross ton at the mines in that year to \$9.00 at the end of 1924 (N. Y. quotations). The so-called INDEPENDENT MINES (meaning small individual mines or groups which in all produce about 30 per cent. of the total output and sell on day-to-day quotation rather than on contract, as do the large COMPANY MINES or groups) have averaged considerably higher in their returns than the company mines, and for short periods have received much higher prices. For example, in the fall of 1916, the price of independent coal rose above \$8.00 against \$4.35 for company coal, and in the fall of 1920 the corresponding prices were \$14.25 and \$8.00. On the other hand, since Jan., 1916, the only time that the independent price has fallen below the company price was in Apr., 1919, when there was a spread of about \$0.25 per ton.

Table 21. Approximate sizes of market grades of anthracite

Name of size		Size of round opening, inches	
		Passing	Retained on
Domestic sizes	Lump	....	6
	Steamboat	6	4½
	Broken (grate)	4½	2¾–3¼
	Egg	2¾–3¼	2¼–2⁵⁄₁₆
	Stove	2¼–2⁵⁄₁₆	1½–1⅝
	Chestnut	1½–1⅝	⅞–1
Steam sizes	Pea	⅞–1	½–⅝
	No. 1 Buckwheat (*a*)	½–⅝	¼–⁵⁄₁₆
	No. 2 Buckwheat (Rice) (*a*)	¼–⁵⁄₁₆	⅛–³⁄₁₆
	No. 3 Buckwheat (Barley) (*a*)	⅛–³⁄₁₆	⁵⁄₆₄–⅛
	Culm	³⁄₃₂–⅛	....

a These three sizes are sometimes mixed under the name "Birdseye."

Prices. Steam sizes of anthracite (Table 21) compete with bituminous coal and are subject to the same fluctuations. For four years prior to 1917, the company price was substantially constant at close to \$1.60 per long ton at the mines. Since then it has risen to a maximum of about \$4.10 in the fall of 1920 and again in the fall of 1922. During the summer of 1924 the price was about \$3.05. Independent prices have averaged slightly higher with spreads of about \$2 per ton in their favor in the fall of 1916 and 1920.

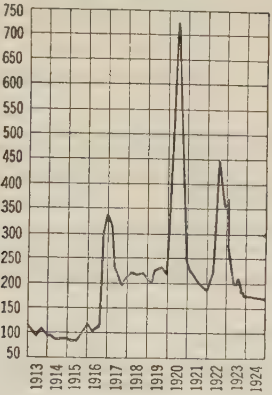
The average value per gross ton of anthracite in 1923, with due weighting for value and amount of domestic and steam sizes, local trade and mine fuel was \$5.43 in 1920 and \$6.08 in 1923. (*33 MI 137*).

Bituminous-coal prices fluctuate widely with seasonal demand and with variation in the industrial activity of the country. Furthermore, as set forth above, the price varies widely according to the grade. Fig. 12 shows the extent of the fluctuation in the years 1913 to 1924 incl.

Preparation

The treatment required to prepare coal for the market depends on the character of the material delivered from the mine and the demand of the particular market supplied. All anthracite requires some prepa-

ration, if only crushing and sizing, and much of it, as now mined, requires separation of impurities such as slate and bone. Much bituminous coal, on the other hand, is sold as mined or is given only the crudest kind of screening. Only between 5 and 10 per cent. of the total bituminous production is subjected to a concentrating treatment.



	1924 Dec. 29	1924 Dec. 22	1924 Dec. 15	1923 Dec. 31
Index	171	170	169	170
Weighted average price	$2.07	$2.06	$2.04	$2.17

This diagram shows the relative, not the actual, prices on fourteen coals, representative of nearly 90 per cent. of the bituminous output of the United States, weighted first with respect to the proportions each of slack, prepared and run-of-mine normally shipped, and second, with respect to the tonnage of each normally produced. The average thus obtained was compared with the average for the twelve months ended June, 1914, as 100, after the manner adopted in the report on "Prices of coal and coke; 1913–1918," published by the Geological Survey and the War Industries Board.

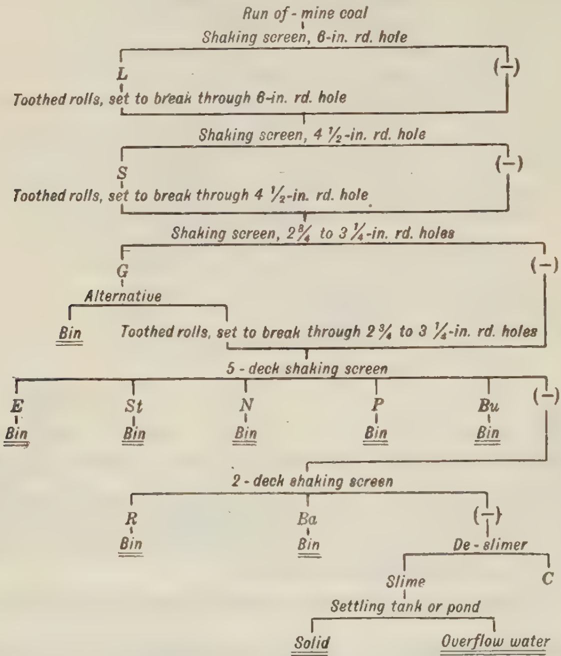
Fig. 12.—Coal Age index and spot prices of bituminous coal f.o.b. mines.

Preparation of anthracite. The principal impurity is shale or slate, which occurs generally in flat and elongated slabs and plates, is dull gray to black in color, and has a specific gravity between 2.5 and 2.7. Anthracite breaks into generally rounded lumps, its color is lustrous black and its specific gravity ranges between 1.3 and 1.7. The bone or middling particles consist of mixtures of coal and slate in all proportions. When the amount of bone is small, picking and gravity concentration are both relatively easy to apply. In hand picking the pickers are guided by the distinctive shapes, and, if the feed on the picking surface has been properly washed, by a distinctive color difference. If bone also is picked, the separation is more difficult, as both shape and color range between the extremes of rock and slate. However, since the bone is to be crushed and further treated, doubtful cases may be consistently resolved in favor of the bone circuit, with consequent increase in picking rate.

Separation of slate from anthracite by gravity concentration is relatively simple. The free-settling ratio (Sec. 6, Art. 1) of coal and slate is $D_{\text{coal}}/D_{\text{slate}} = (2.5 - 1)/(1.5 - 1) = 3.0$. Since market requirements demand sizing to a considerably closer sieve ratio than 3 : 1, (see Table 21), if the sizing is done prior to jigging, subsequent separation is simpler than that met with in most metal-concentrating mills. On the other hand, when there is

much bone present, the free-settling ratios become much smaller and separation is correspondingly more difficult.

For high-grade coal such as may be obtained from thick, horizontal or slightly inclined seams, treatment may consist of nothing more than crushing and screening to prepare the market sizes. In a breaker for such service the flow-sheet is essentially as shown in Fig. 13.



Ba, Barley size. ***Bu***, Buckwheat size. ***E***, Egg size. ***G***, Grate size. ***L***, Lump size. ***N***, Nut size. ***P***, Pea size. ***R***, Rice size. ***S***, Steamboat size. ***St***, Stove size.

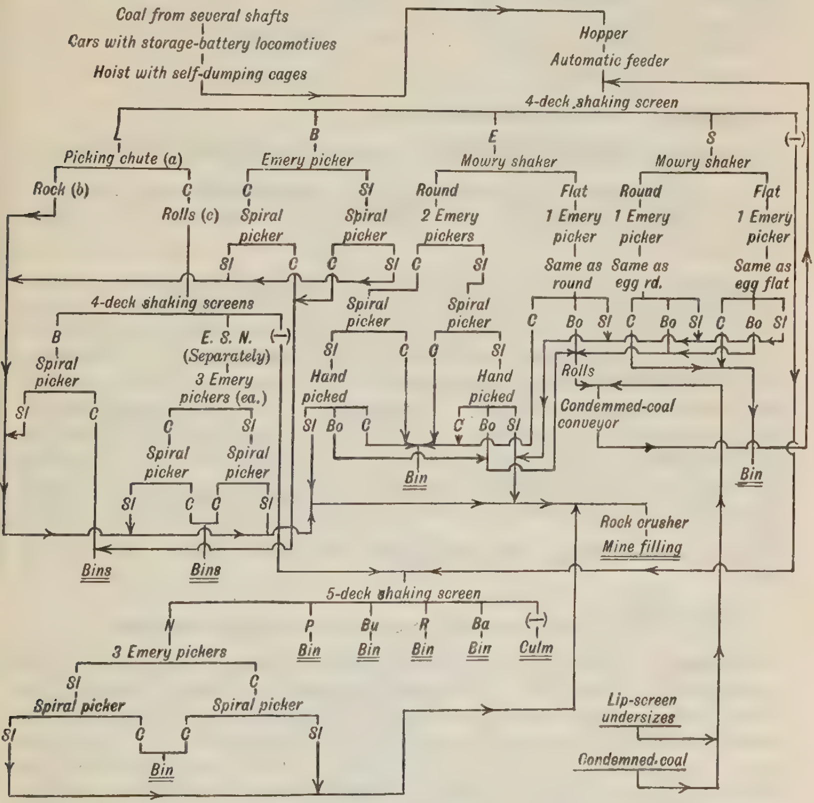
FIG. 13.—Typical pure-coal anthracite breaker.

Such breakers are substantially non-existent at the present day, but the figure illustrates the fundamental principle of arrangement of the apparatus for graded crushing and screening so as to produce the maximum amount of DOMESTIC-SIZE COAL (egg to chestnut sizes, inclusive) and to deliver all products with as little fine undersize as possible. Thus the coarse sizes are crushed separately one size-stage at a time, shaking screens are used in preference to other types, on account of the small amount of breakage that they effect, and special retarding chutes (Sec. 20, Art. 9) should be used throughout to lower the screened products into the bins. The coal passing the barley screen is suitable, after dewatering, for briquetting or for mixing back with the STEAM SIZES (pea and finer).

When, as is ordinarily the case, the coal as mined contains varying amounts of impurities, (see p. 34), the breaker must contain more or less concentrating equipment in addition to the breaking and sizing equipment. The

usual concentrating operations are picking, either hand or mechanical; jigging, and, of recent years, tabling and heavy-fluid separation. Froth flotation and granulation are also practical from the technical standpoint and only await the perfection of methods of utilization of fine coal thus cleaned to come into common use.

The type of flow-sheet employed when concentration is necessary depends upon the character of coal coming to the breaker. If this contains a relatively



a, An extension of the upper deck of the shaker. *b*, Picked. *c*, Set to produce broken size and smaller. *B*, Broken size. *Ba*, Barley size. *Bo*, bone. *Bu*, Buckwheat. *C*, Coal. *E*, Egg size. *L*, Lump size. *N*, Nut size. *P*, Pea size. *R*, Rice size. *S*, Stove size. *Sl*, slate.

FIG. 14.—Kingston Coal Co.

small amount of slate that breaks sharply and readily from the coal, the flow-sheet will be substantially the same as that shown in Fig. 13 except that each of the sizes down to and including chestnut will be hand-picked. Fig. 14 shows such a flow-sheet. If the amount of slate is large, water concentration is employed in addition to picking, usually jigs for the sizes down to and including buckwheat and, if necessary, shaking tables for the finer sizes. Figs. 15 to 22 inclusive illustrate several different variants of such flow-sheets.

When there is much clean coal in the large sizes a primary split of run-of-mine material is commonly made at steamboat size (4½-in.), slate is hand picked from the oversize and the coal is then run down separately in a so-called pure-coal section similar to the flow-sheet in Fig. 13. When the coal contains considerable bone, this is frequently separated on the coarse-picking tables and coarse jigs and crushed down so that the combustible material finally goes into the steam sizes. All of these contingencies are illustrated in the following flow-sheets.

Kingston Coal Co., Breaker No. 4 (*18 CA 935*). **Fig. 14.**

Location: Edwardsville, Pa.

Summary: Screening and picking only.

Lehigh and Wilkes-Barre Coal Co., Wanamie Breaker (*66 A 422*). Fig. 15.

Capacity: 700 tons per hr. Two sections. For tonnage distribution see numbers in parenthesis on the flow-sheet. See also Table 22 for distribution of products at Seneca breaker of Lehigh Valley Coal Co.
Building: Steel.

Summary. This flow-sheet is typical of the arrangement for treating exceptionally clean run-of-mine coal. It is perhaps not so simple to operate as the Kingston breaker (Fig. 14), but is more flexible and will yield a cleaner product with higher recovery on all except the highest grade of run-of-mine coal. The distinguishing characteristics are: (*a*) treatment of pure coal and boney coal in separate sections; (*b*) production of all sizes, from broken to No. 3 buckwheat, from picked lump in the pure-coal section, with no cleaning after crushing; (*c*) separation of bone and waste at egg and stove sizes from the steamboat- and broken-size bone by spiral pickers alone; (*d*) use of jigs to clean egg, stove and nut sizes in the boney-coal section; (*e*) no cleaning of sizes below nut.

Phila. and Reading Coal and Iron Co., Brookside breaker (*66 A 422*). Fig. 16.

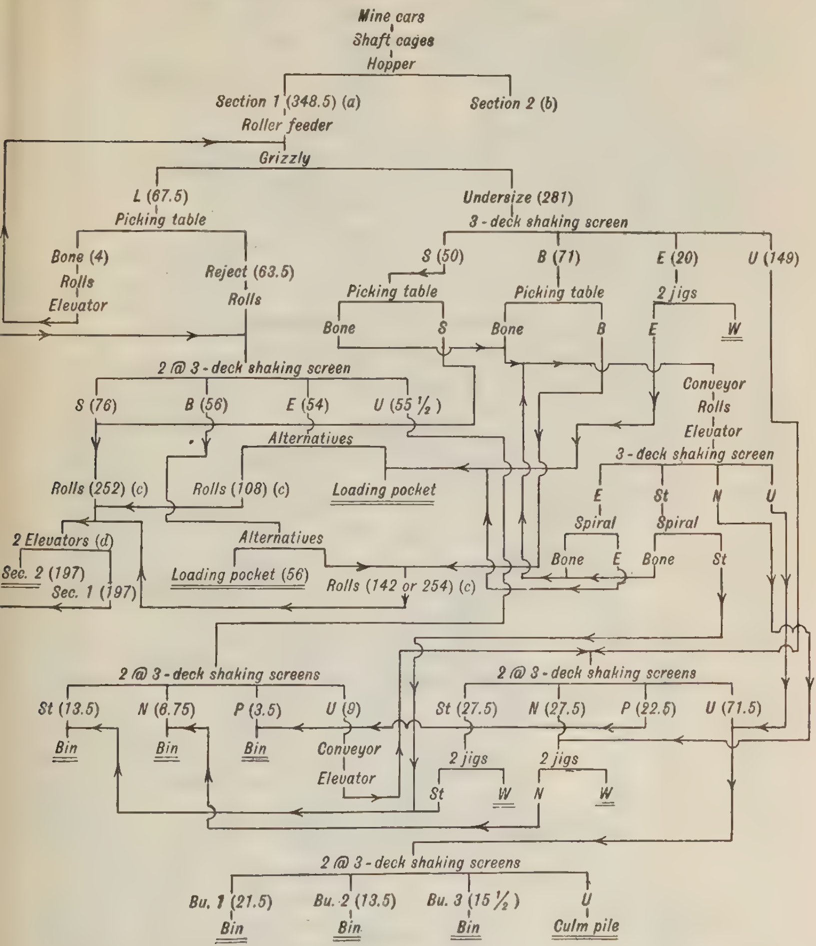
Location: Tower City, Pa.
Building: This is a hill-side breaker. A sectional drawing is shown in Fig. 17.

Summary. The coal fed to this breaker is relatively high grade. The characteristic features are: (*a*) separate pure-coal and boney-coal sections; (*b*) no cleaning in the pure-coal section following the lump-picking platform, except that the undersize of the buckwheat shakers goes to the boney-coal section; (*c*) use of jigs in the boney-coal section for all sizes from egg to rice, inclusive; (*d*) use of shaking tables for cleaning undersize of the rice shaker; (*e*) separation of egg, stove and nut sizes into flat and round grades by means of a mechanical picker prior to jigging, and separate jig treatment of the grades; (*f*) mechanical picking of the coal discharge from the egg- and stove-size coal jigs; (*g*) re-crushing of bone from the domestic-size bone jigs and pickers and return to the head of the boney-coal section.

Pennsylvania Coal Co. No. 1 (*66 A 422; 22 CA 786*). Fig. 18.

Location: Dunmore, Pa.
Building: Steel and concrete. This is a typical modern level-site breaker. See Fig. 19.
Capacity: 4000 tons per 8 hr.
Labor: 66 men per shift, as follows: 7 jig runners, 6 screen men, 34 pickers, 11 loaders, 1 machinist, 2 carpenters, 1 oiler, 3 sweepers, 1 foreman.
Estimated cost of breaker (1921–22) between $800,000 and $900,000.

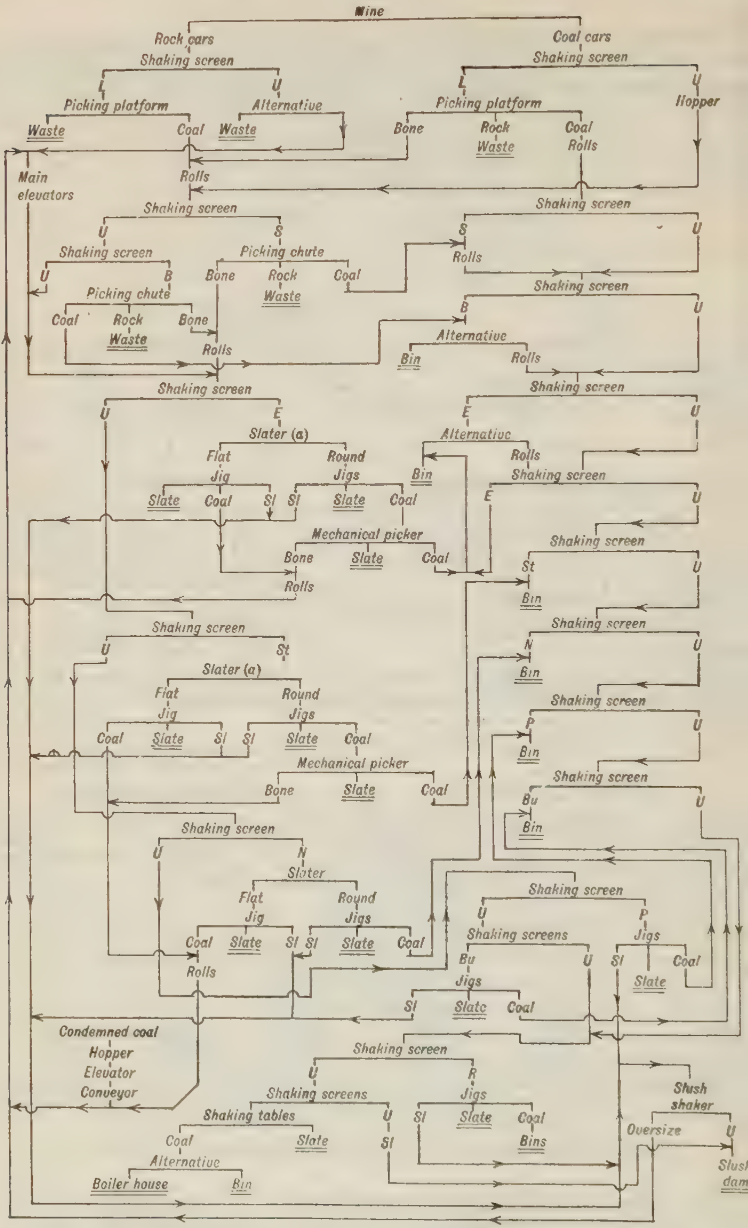
Summary. This breaker is suitable for treating a lower-grade run-of-mine coal than the preceding. There is no pure-coal section. The characteristic features are: (*a*) jigging of primary sizes from grate to No. 2 nut,



a, Numbers in parenthesis are tons per hr. *b*, Duplicate of Section 1. *c*, Common to both sections. *d*, One for each section. *B*, Broken. *Bu*, Buckwheat. *E*, Egg. *L*, Lump. *N*, Nut. *P*, Pea. *S*, Steamboat. *St*, Stove. *U*, Undersize. *W*, Waste.

Fig. 15.—Wanamie breaker, Lehigh and Wilkes-Barre Coal Co.

inclusive, making a 3-product separation at the two coarsest sizes; (*b*) hand picking of jigged coal with rejection of bone; (*c*) no cleaning of sizes below No. 2 nut; (*d*) crushing and return of bone to the main system. This is the simplest type of flow-sheet for this character of feed.



a, Mechanical device to separate flat and round particles. *B*, Broken size. *Bu*, Buckwheat size. *E*, Egg size. *L*, Lump size. *N*, Nut size. *P*, Pea size. *R*, Rice size. *S*, Steamboat size. *Sl*, Slush. *St*, Stove size. *U*, Undersize.

FIG. 16.—Brookside breaker, Philadelphia and Reading Coal and Iron Co.

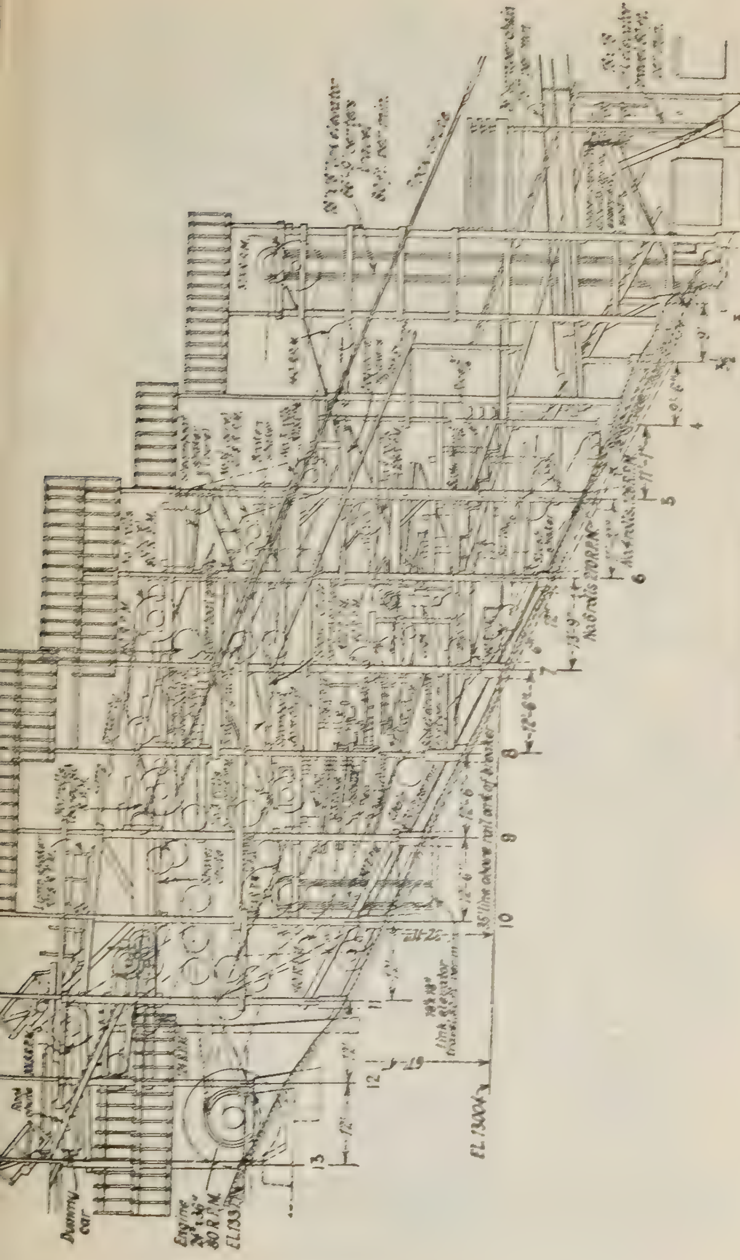
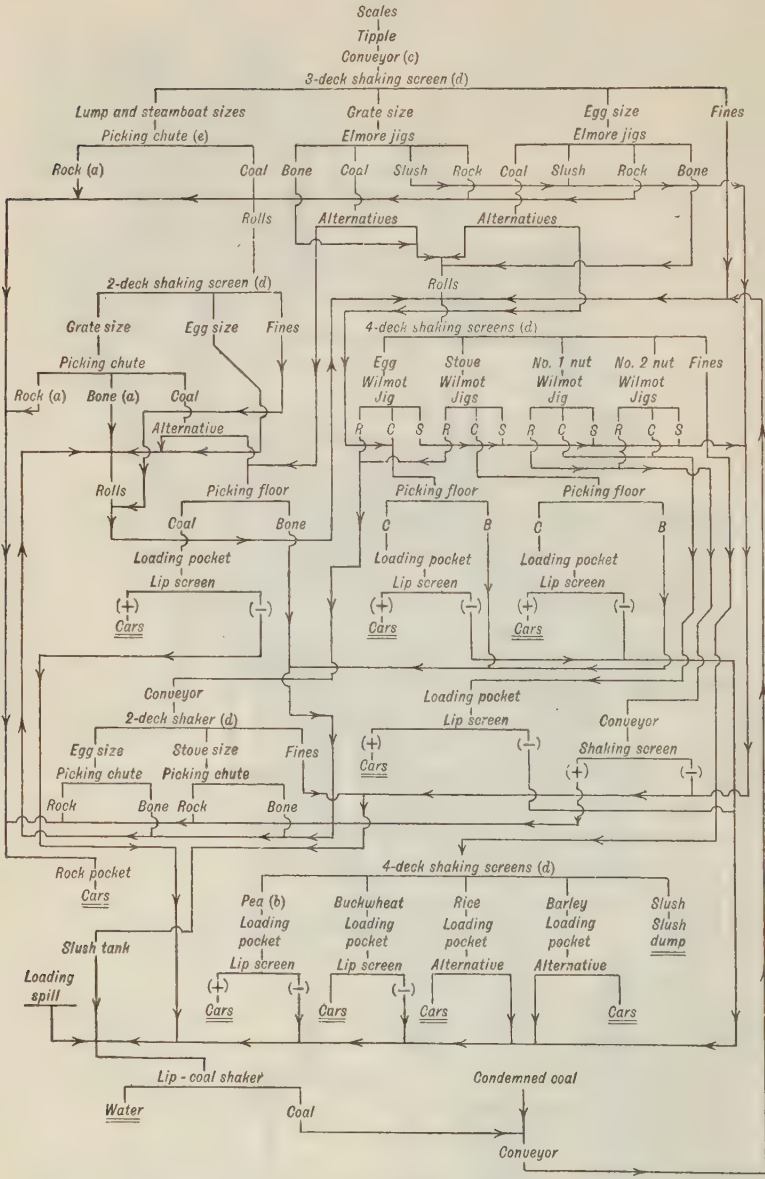


Fig. 17.—Brookside breaker.



a, Picked. *b*, There is an alternative arrangement to jig this size. *c*, Wilmot chai
conveyor, 385 ft. long, flights 4 ft. wide, 5-ft. spacing. *d*, See Sec. 5, Table 33. *e*, 16 me
B, Bone. *C*, Coal. *R*, Rock. *S*, Slush.

FIG. 18.—Pennsylvania Coal Co.

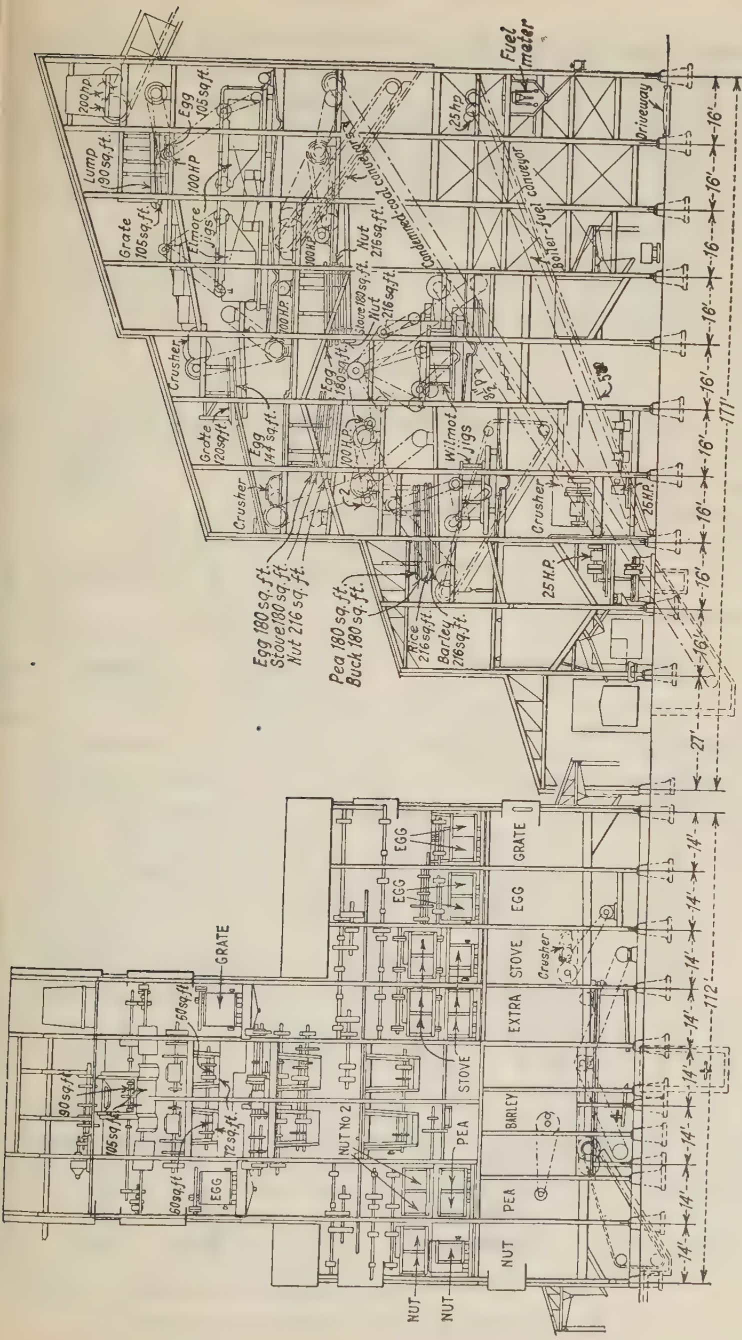


FIG. 19.—No. 1 breaker, Pennsylvania Coal Co.

Lehigh Valley Coal Co., Drifton No. 2 breaker (*59 A 335*). Fig. 20.

Location: Drifton, Pa.

Capacity: 2500 tons per 8 hr.

Labor: 1 breaker boss, 1 jig boss, 1 picking-table boss, 1 timekeeper, 2 car dumpers, 4 pickers on platform, 3 pickers on broken coal, 2 pickers on jig refuse, 2 jig runners, 4 screen and roll tenders, 1 breaker engineer, 1 oiler, 1 ropeman, 1 roustabout, 7 men loading coal.

Power distribution (approximate): Reciprocating feeder, 2 hp.; shaking screens, per deck, 2 hp.; No. 1 roll, 12 hp.; No. 3 rolls, 9 hp. ea.; 2 picking conveyors, 8 hp.; 2 elevators, 65 hp.; jigs, 4 hp. ea. Flight conveyors, 374 ft., 48 hp. Total, 272 hp. All figures include belting and line-shaft losses back to the mill engine.

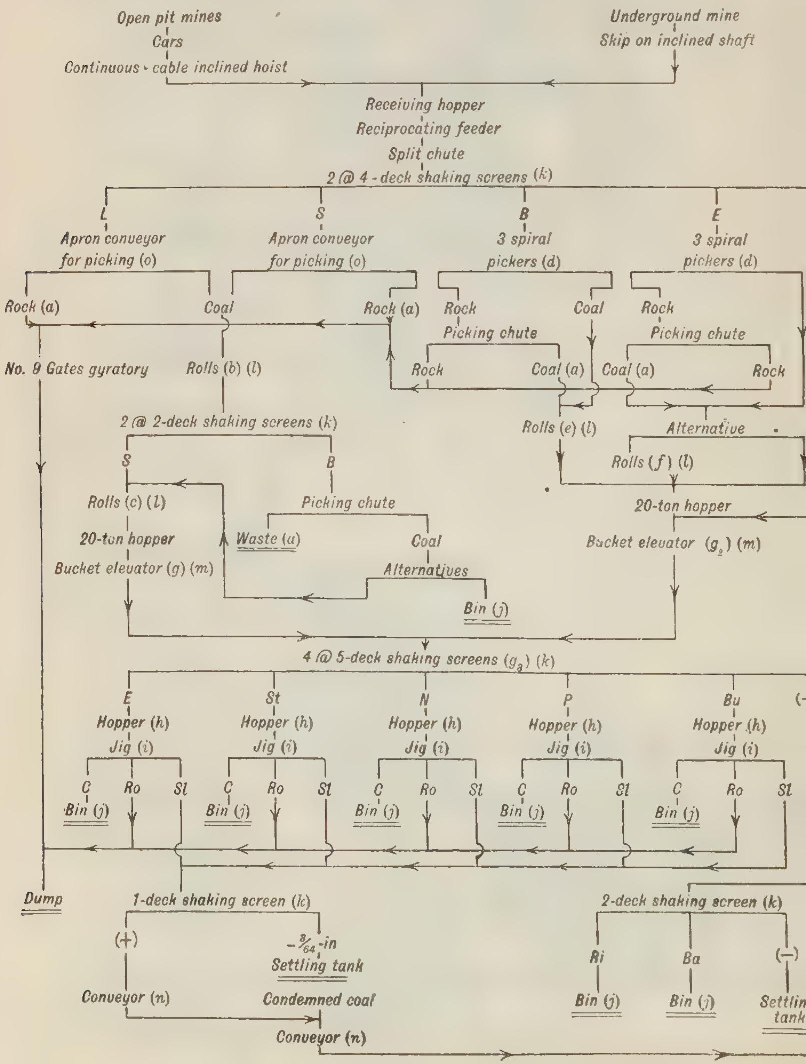


FIG. 20.—Drifton breaker, Lehigh Valley Coal Co.

a, Picked. *b*, No. 1 size. Slow speed. Set to break to broken size with a small percentage of steamboat. *c*, No. 3 size. Slow speed. Set to break to egg size. *d*, Adjusted to remove a maximum amount of rock. *e*, No. 3 size. Set to break to egg size. *f*, No. 3 size. Set to break to stove size. g, g_2, g_3, In 1917, when this breaker was first put into operation, the coal fed was very dirty and the breaker was operated as shown. It is so designed, however, that elevator g can feed two of the 5-deck shakers, which will send oversize products directly to the proper bins, thus acting as a pure-coal section, while elevator g_2 and the other two banks of shaking screens prepare impure coal for jigging. When handling 1500 to 1800 tons of feed per 8 hr., according to the flow-sheet shown, only two of the 5-deck shakers were used. *h*, With automatic discharge chute. Fed by vertical stepped telegraph (see Sec. 20, Art. 9). *i*, Simplex, 137 r.p.m. 20 jigs in all, arranged to allow diversion of feed as one or the other size of coal predominates. Tons per hour per jig on different sizes are shown in Sec. 9, Table 28. Fitted with automatic belt shifters actuated by the weight of coal in the feed chute. *j*, Fed with vertical stepped telegraph chutes. *k*, All shakers have 3 × 6-in. wood sides, 3½ × 3½ × 3/8-in. cross-frame angles, and are suspended by 1 × 6-in. hanger boards. Eccentrics have 3-in. throw. 3 × 6-in Parrish connecting arms. Decks are 4 ft. and 4 ft. 6 in. wide by 15 to 24 ft long. *l*, All rolls 36 × 36-in., compound geared, chilled cast-iron teeth. Peripheral speed 300 ft. per min. Capacity about 300 tons per hr. *m*, 22 × 25 × 12-in. buckets on two strands of 9-in. pitch, heavy-pattern chain. Bucket spacing, 18 in. Speed, 90 ft. per min. Capacity, 299 tons per hr. *n*, 6 × 18-in flights with 9-in. pitch chain. *o*, 4 ft. 6 in. wide, 30 ft. per min. Friction clutch. ***B***, Broken size. ***Ba***, Barley. ***Bu***, Buckwheat size. ***C***, Coal. ***E***, Egg size. ***L***, Lump size. ***N***, Nut size. ***P***, Pea size. ***Ro***, Rock. ***S***, Steamboat size. ***Sl***, **Slush.** ***St***, **Stove size.**

Table 22. Distribution of coal and refuse in Seneca breaker of Lehigh Valley Coal Co. (*After Ashmead, 22 CA 5*)

No.	Material or operation	Feed, tons per 8 hr.	Distribution					
			Coal			Refuse		
			Tons per 8 hr.	Per cent. of machine feed	Per cent. of breaker feed	Tons per 8 hr.	Per cent. of machine feed	Per cent. of breaker feed
1	Breaker feed	1800	......	....	....	......	....	....
2	Lump and steamboat sizes from bull shaker	216	......	....	....	26	12.0	1.4
3	Broken size, hand picked	105	70	66.7	3.9	35	33.3	1.9
4	Egg size, jigged	279	232	83.2	12.9	47	16.8	2.6
5	Stove size, jigged	377	317	84.1	17.7	60	15.9	3.3
6	Nut size, jigged	518	435	84.0	24.2	83	16.0	4.6
7	Pea size, no cleaning	117	117	....	6.5	......	....	....
8	Buckwheat size, no cleaning	138	138	....	7.7	......	....	....
9	Rice size, no cleaning	96	96	....	5.3	......	....	....
10	Barley size, no cleaning	99	99	....	5.5	......	....	....
11	Slush, to waste	45				45		2.5
12	Totals	1800	1504	....	83.7	296	....	16.3

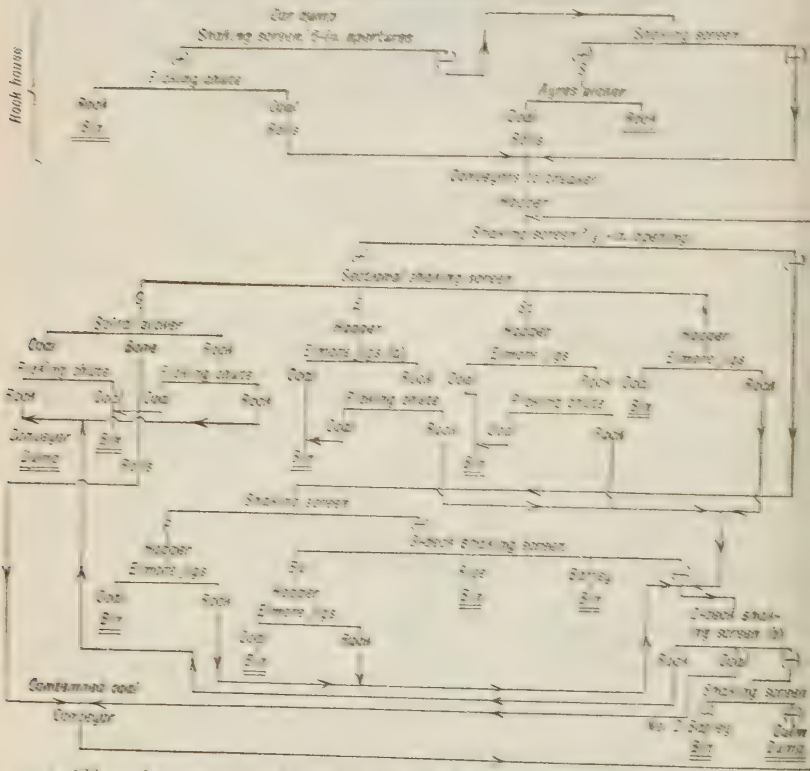
Summary. There is no pure-coal section. (*a*) Primary sizes from lump to egg are picked to reject rock, then crushed to egg size and joined with primary undersize; (*b*) all sizes from egg to buckwheat are jigged separately making finished coal and waste rock; (*c*) rice and barley sizes are not cleaned.

Lehigh Coal and Navigation Co., Rahn breaker (*66 A 422*). Fig. 21.

Capacity: 2000 tons per 8 hr.

Distribution of products.

Products	Weight, per cent	Slate in coal, per cent	Coal in slate, per cent
Grate	3.[illegible]		[illegible].0
Egg	[illegible]	2.[illegible]	[illegible].0
Stove	9.7	3.5	[illegible].8
Nut	[illegible].2	5.0	[illegible].1
Pea	10.6	8.0	[illegible].5
Buckwheat	[illegible]	[illegible]	4.1
Rice	15.3		
Barley	[illegible]		
Barley No. 2	[illegible]		



[illegible] strokes per min. [illegible] openings [illegible] Rock from the jigs [illegible] the first bank and [illegible] to the second bank. Bw, Buckwheat size. E, Egg size. G, Grate size. N, Nut size. P, Pea size. S, Steamboat size. St, Stove size.

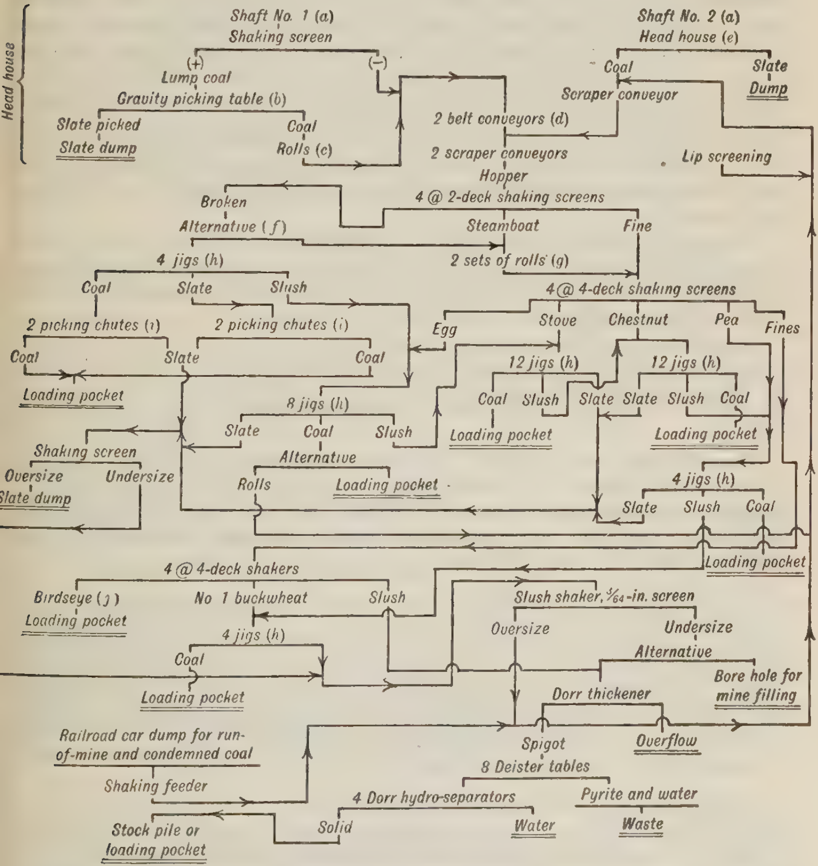
FIG. [illegible].—[illegible] breaker, Lehigh Coal and Navigation Co.

Summary. There is no [illegible] section. The characteristic features are: (a) mechanical picking at steamboat size; (b) separation of pea size

prior to separation into domestic sizes for subsequent cleaning; (c) cleaning of grate size by spiral pickers followed by hand-picking of coal from the spiral; (d) jigging of egg to buckwheat sizes; (e) hand-picking of the waste from the grate spiral and the egg and stove jigs; (f) non-cleaning of rice and barley sizes; (g) screening of the waste discharge from all jigs in order to reclaim the coal broken in jigging. The percentage of domestic sizes recovered is unusually low on account of the low grade of the breaker feed.

Hudson Coal Co., Marvine breaker (*66 A 422*). Fig 22.

Location: Marvine, Pa.
Capacity: 5000 tons per 8 hr.
Building: Steel and concrete.



a, About 2000 ft. apart. Identical equipment. *b*, Two men pick slate. *c*, Set to crush to steamboat size and smaller. *d*, 1100 ft. long. *e*, Duplicate of No. 1. *f*, Dependent upon market demand. *g*, Set to produce egg size. *h*, Delaware, piston type. (See Sec. 9, Fig. 15.) *i*, One boy on each chute. These boys are necessary in order to get clean coal and to keep coal out of the slate product. *j*, Nos. 2, 3 and 4 buckwheat mixed.

Fig. 22.—Marvine breaker, Hudson Coal Co.

Summary. There is no pure-coal section. The distinctive characteristics of the flow-sheet are: (*a*) jigging at broken size; (*b*) sizes from egg to No. 1 buckwheat, inclusive, are jigged; (*c*) material finer than No. 1 buckwheat is tabled to remove pyrite.

General summary of anthracite flow-sheets.

The characteristics common to all breakers are sizing before concentration and concentration at all sizes from +6-in. (lump) down to +1-in. (nut). With very clean run-of-mine coal, clean coal and waste slate are the only products made in each concentrating step (see the Kingston breaker). With less-clean breaker feed, bone (middling) may be taken out at the coarser sizes, but waste is rejected at all sizes from lump down and clean coal is made at all sizes from broken or egg down. (See Brookside, Pennsylvania, Drifton, Rahn and Marvine breakers.) Occasionally, as at the Wanamie breaker, rejection of waste does not start until the egg-size jigs are reached, although clean coal is taken at broken size. Concentration below pea size is the exception rather than the rule, but many recent breakers jig the buckwheat size (see Marvine, Rahn and Drifton); rice size is jigged at Brookside; and material finer than buckwheat is tabled at a number of breakers (see Marvine and Brookside).

Control of product. According to Ashmead (*21 C.A. 695*) a 50-lb. grab sample is taken from just below the surface of each carload of domestic-size anthracite. This is blown with a jet of live steam, then spread on a table where it quickly dries and is picked.

Allowances of slate and bone at HUDSON COAL CO. were: 2 per cent. slate and 4 per cent. bone in egg size; 4 per cent. bone and 4 per cent. slate in stove size, 6 per cent. slate in nut size and 10 per cent. slate in pea size. Cars failing these tests were 9.45 per cent. in March, 1922. These were dumped on the condemned-coal conveyor and returned to the plant.

Handling the coal *en route* is almost as important a part of the design of anthracite breakers as concentration, on account of the great spread in price between the domestic and steam sizes and the consequent necessity to prevent breakage. All apparatus is chosen with the view to eliminate tumbling, free fall, and rapid travel or sharp turns in chutes.

The importance of eliminating drops is shown by a test made at the breaker of the CRANBERRY CREEK CO. near Hazleton, Pa. (*18 C.A. 540*) to determine the effect of dropping run-of-mine coal 4 ft. into a chute at the head of the breaker. Results of sizing tests are shown in Table 23, indicating a reduction in tonnage of the prepared sizes of 8.4 per cent. due to the dump. At this breaker this amounted to 29,906 tons of coal out of 549,071 tons of breaker feed in 1918. During this period the average selling price of prepared sizes was $6.80 per ton against $4.18 for the smaller sizes, so that the breakage represented a loss of $2.62 per ton broken or $78,000 odd for the year. The increase in culm (slush) was 3.0 per cent., according to the table. If this is reckoned as derived from all sizes it represents 18,000 odd tons at $5.49 average price for all sizes or $99,000.

Special toothed rolls are used that break with a minimum production of fines (see Sec. 3, Art. 14). Shaking screens are preferred to revolving or fixed-inclined screens. Elevation is confined, as far as possible, to the initial entry into the breaker, and if intermediate elevation is necessary slow-speed drag conveyors are used, if possible. Bucket elevators should be employed only as a last resource, and should be arranged to load and discharge with minimum drop of material and be run at slow speeds. Special chutes are almost invariably used for lowering material from machine to

machine, whenever any considerable vertical drop is necessary, and for charging finished sizes into the loading bins. (See Sec. 20, Art. 9.)

Labor required in anthracite preparation (operating labor), as given by Ashmead (*66 A 422*) is shown in Table 24. The wet breakers employ the least men, the wet-and-dry the most.

Table 23. Breakage in dumping run-of-mine anthracite. (*After Ashmead*)

Size	Per cent. weight	
	Before dumping	After dumping
Lump	34.5	25.5
Steamboat	20.4	20.3
Broken	12.1	12.5
Egg	6.8	7.8
Stove	5.1	5.3
Nut	5.5	6.6
Pea, No. 1 Buckwheat, Rice, Barley	11.5	14.6
Slush	4.1	7.1
Total prepared sizes	84.4	78.0

Table 24. Operating labor required in anthracite breakers. (*After Ashmead*)

Wyoming Valley Field

	Preparation method employed		
	Dry	Wet-and-dry	Wet
Number of collieries reporting	14	22	30
Tons of coal produced	5,674,010	10,845,542	17,120,602
Tons of coal prepared per man employed on preparation	7,020	6,420	7,120
Percentage relation of preparation men to outside employees	38.7	34.4	34.3
Percentage relation of outside employees to total employees	20.8	24.1	22.2
Percentage relation of outside employees to inside employees	24.8	31.6	28.6

Lehigh Field

	Wet-and-dry	Wet
Number of collieries reporting	5	4
Tons of coal produced	2,241,783	1,597,216
Tons of coal prepared per man employed in preparation	4,930	6,000
Percentage relation of preparation men to outside employees	27.9	25.6
Percentage relation of outside employees to total employees	38.6	36.4
Percentage relation of outside employees to inside employees	68.0	57.2

Southern Field

	Wet-and-dry	Wet
Number of collieries reporting	7	64
Tons of coal produced	2,655,615	20,352,232
Tons of coal prepared per man employed in preparation	5,290	5,730
Percentage relation of preparation men to outside employees	33.5	26.0
Percentage relation of outside employees to total employees	31.2	34.0
Percentage relation of outside employees to inside employees	44.7	50.9

Variation in labor requirement with tonnage treated is shown in Fig. 23. At BUCK RUN breaker, treating 900 tons per day, wet, with no pure-coal section, 35 men are employed making 25.7 tons per man per 8-hr. day.

Water consumption in anthracite breakers ranges from 0.63 gal. per min. per daily (8-hr.) ton treated in the so-called dry breakers, which use water only for the fine sizes, to 1.54 gal. in the wet breakers in the lower (low-grade) field. (*66 A 422.*) Griffen (*66 A 514*) averages the water consumption at, roughly, 1 gal. per min. per daily ton of production.

Power. An idea of power consumption may be obtained from the description of the Drifton breaker (p. 48).

Bituminous-coal preparation.

From the viewpoint of preparation, bituminous coals, as mined, may be divided into two groups, the first, comprising the bulk of the total production, requiring no cleaning, while of the rest, which requires removal of some impurity, only a part, amounting to between 5 and 10 per cent. of the total bituminous production, requires WASHING (gravity concentration).

The important removable impurities in bituminous coal are shale, clay, pyritic sulphur and bone. When the run-of-mine product is coarse, hand picking is the easiest and cheapest method of removing shale and bone, but when, as is frequently the case, upwards of 50 per cent. of the product delivered at the surface is $-1\frac{1}{2}$-in. slack, separation by picking is uneconomical. With very soft coal the Bradford breaker takes advantage of the relative toughness of the shale to deliver it as oversize separated from the coal and fine shale, which discharge as undersize. Separation of pure coal from free shale and from free pyrite is easy, since the free-settling ratios are about five and twelve respectively, but if there is much bone (sp. gr. from 1.35 to 1.70) the problem becomes more difficult, particularly as regards the finer sizes, and the fact that crushing is rarely carried far enough to free either shale or pyrite with any pretense to completeness makes high elimination of either ash or sulphur unusual. While no general rules can be set down, experience shows that reduction in sulphur content ranges in general between 25 and 40 per cent., rising to 50 per cent. on some coals and even higher in a few exceptional cases. The organic sulphur is practically unaffected by washing, as is shown in Table 25. According to Campbell (*63 A 683*) a reduction of 50 to 60 per cent. in ash content is a fair expectation.

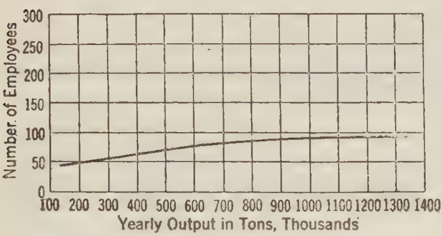


FIG. 23.—Labor in anthracite preparation.

Steam-coal tipple. Much coal is shipped directly as brought out of the mine. Most of the clean-coal mines, however, are provided with at least a simple screening plant (TIPPLE) in which crude separation is made on a screen of from 1- to 3-in. aperture (commonly $1\frac{1}{2}$-in.) between SLACK (undersize) and LUMP (oversize).

Fig. 24 shows a well-arranged tipple for handling steam coal. At the left is a dump-house for receiving run-of-mine coal that is to be shipped as such, a feeder and an elevating conveyor which delivers by way of an adjustable chute to railroad cars. At the right side are another hopper, feeder and conveyor delivering to the screening plant. This plant is flexibly arranged to permit of several dispositions of the coal, *e.g.*, (*1*) delivery to

screen (*a*) with aperture of from ¾-in. to 1½-in., the undersize going to the slack bin for shipment and the oversize to wagons or trucks for domestic consumption; (*2*) delivery to screens (*b*) fitted with, say, 4- to 6-in. screen, and screen (*c*) fitted with 1½- to 3-in. screen, thus producing lump coal on track No. 2, egg size on track No. 3, and slack through the bin onto track No. 4. (*3*) By putting blank sheets into screen (*c*), run-of-mine coal may be delivered onto track No. 2. (*4*) By putting a slack screen on screen (*b*) and delivering directly thereto from the conveyor, fine lump may be delivered on track No. 3 and slack on No. 4. (*5*) Finally, by putting blank plate on screen (*b*) and delivering directly thereto, run-of-mine coal may be delivered to track No. 3. Provision is also made to run slack to storage by means of a conveyor under the slack bin. The screens shown are of the fixed-inclined type (Sec. 5, Art. 4), but may also be either rotary or shaking types, which reduce the required headroom. They are built to allow ready change of plate, in order to attain the flexibility described.

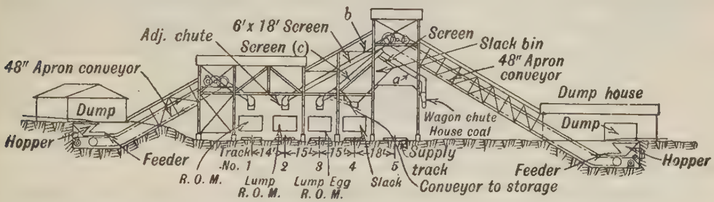


FIG. 24.—Typical steam-coal preparation plant (*after Read, 6 CA 255*).

When a coal requires cleaning, the treatment demanded depends upon the character of the coal. In some coals the removable impurity (FREE ASH) occurs in the coarser sizes and the slack is of better grade than the run-of-mine coal.

Example: At POCAHONTAS COAL CO., Palmer, Wash. (*28 Bul. UW 143*), the +⅜-in. raw coal assays 41 per cent. ash, the −⅜-in. 27.8 per cent., and the total 35.3 per cent.

In a converse case the impurity concentrates in the slack.

Example: At RENTON COAL CO., Renton, Wash. (*ibid.*), the entire coal assayed 18.7 per cent. ash, the + 3-in. size 13.3 per cent., and the −⅜-in. 25.6 per cent.

In another class of coal the impurity is of substantially uniform distribution and two cases arise: (*a*) when the coal is non-salable without cleaning and, (*b*) when cleaning improves the selling price in an amount greater than the cost of the cleaning operation. With a coal such as the latter it may be that the slack will not stand the cost of further treatment while the coarser sizes will. This is frequently the case when the coarse coal is sold for domestic use.

Table 25. Effect of washing on sulphur reduction in an Illinois washery. (*After Fraser & Yancey, 63 A 773*)

Material	Percentages of sulphur			
	Total	Pyritic	Sulphate	Organic
Raw coal..................	1.83	1.04	Trace	0.79
Washed coal, No. 1 size......	1.81	1.05	....	0.76
Washed coal, No. 2 size......	1.56	0.78	....	0.78
Washed coal, No. 3 size......	1.57	0.82	....	0.75
Washed coal, No. 4 size......	1.57	0.81	....	0.76
Washed coal, No. 5 size......	2.33	1.57	....	0.76

NOTE: In another case sulphate sulphur was reduced from 0.11 per cent. to less than 0.01 per cent. by washing.

The greater value of cleaned coal is due to its greater heating value per unit of weight, the lower freight charge per unit of heat, the greater absolute heating value due to the removal of non-combustible heat-consuming material, reduction in clinkering and in the weight of ash to be handled, reduction in the amount of smoke and soot, and reduction in corrosive effect of the gases and soot. When coal is to be used for making metallurgical coke, the requirements of high compressive strength and low-sulphur content in the coke bar high-ash high-sulphur coals although the coals themselves may possess the essential coking characteristic. Frequently washing will remove the objectionable ash and sulphur sufficiently so that the residue is suitable for coking, in which case the value of the coal is considerably enhanced.

Cleaning plants vary in complexity from relatively simple screening and hand-picking tipples, in which the coarse material only is treated, to relatively complicated gravity-concentration plants with close sizing preceding jigs or pneumatic tables. Shaking tables are used in some cases to clean fine sizes. Flotation and granulation are both applicable to cleaning very fine sizes, although both processes are relatively expensive and economically useful, at the present, only in a few special cases.

The flow-sheets for bituminous preparation are simpler than those employed for anthracite, the apparatus is generally fed at a higher rate, and the results, from the standpoint of concentration, are not so good.

The requirements of bituminous users are not, in general, so strict as those of anthracite consumers, and the selling price of bituminous coal will not permit more elaborate cleaning.

Table 26. Trade sizes of bituminous coal (35 Illinois washeries). (*After Lincoln*)

Name of size	Upper limiting screen, inches	Lower limiting screen, inches
Nut.........	2½ to 1¼	1¼ to ¾
Slack.........	2 to ¾	3/16 to 0
No. 1 extra..	3¾ to 3	3 to 2½
No. 1.......	3½ to 1½	2¼ to 1⅜
No. 2 extra..	2¼ to 1¾	1¾ to 1⅜
No. 2.......	2¼ to 1⅜	1½ to ⅞
No. 3.......	1½ to 1	1 to 3/32
No. 4.......	⅞ to ⅝	⅜ to 3/16
No. 5.......	⅜ to ¼	3/16 to 0

Sizing. When bituminous coal is prepared for domestic use, it is graded into a number of sizes as is domestic anthracite, but ordinarily not so many sizes are made and the trade sizes are not so closely standardized. Lincoln (*11 Bul. UI No. 9*) gives the data summarized in Table 26 showing Illinois practice.

Hand-picking plants are used for coals that have the impurity concentrated in the coarse sizes, or when the slack will not stand the cost of washing. Fig. 25 shows the flow-sheet of a relatively small but well-arranged plant. The apron feeder (*g*) and clutch drive on conveyor (*k*) permit individual carloads to be spaced on the picking table and the waste therefrom to be separately collected and weighed and, if desirable, set aside in cars for the loader's inspection.

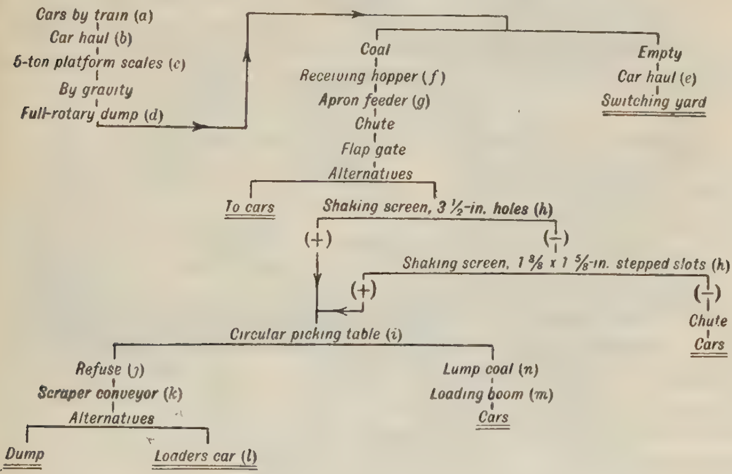
Imperial Coal Corporation, Diamond mine (*19 CA 1111*). Fig. 25.

Location: Johnstown, Pa.
Capacity: 225 tons per hr.

Summary. Screening into two sizes of lump coal for hand picking. Subsequent separate shipment or re-mixing as desired.

Picking and screening plants. Fig. 26 shows plan and elevation of an unusually elaborate plant for screening and picking bituminous coal, the slack being suitable for coking without further treatment other than crushing. The figure is self-explanatory, except that it is to be noted that the nut size

is re-screened in a revolving screen between the nut shaker and the picking table. Maximum CAPACITY of this plant is 750 tons per hr. Loading booms with adjustable finger chutes at the discharge end permit loading of sized products with a minimum of breakage. The flow-sheets (Figs. 27 and 28) show plants similar to a simple anthracite breaker for preparing coal coarser than slack for domestic use, the slack being sold without cleaning.



a, Upwards of 50 cars to a train, electric locomotive. *b*, 12 ft. 8 in. center to center of sprockets. Chain bars, $2\frac{1}{4} \times \frac{5}{8}$-in.; pins, 1-in. steel. Forged-steel hooks, engaging car axle or special plates, are placed at 10-ft. 8-in. intervals. Speed, 35 ft. per min. Head shaft, $3\frac{15}{16}$ in.; tail shaft, $2\frac{7}{16}$ in. Average carload is 4200 lb. and haul will move the entire train. 10-hp. motor drives through belt, cut-steel spur gears and roller chain. 14-in. friction clutch on drive pulley. 0.6-per cent. adverse grade. *c*, Quick-weighing type, with reading dial. *d*, Push-button control. Locking, turning and discharging empty cars, automatic. *e*, Capacity is 50 empty cars on adverse 0.6-per cent. grade. Empties weigh 1700 lb. each; 60 ft. per min.; 44 ft. center to center. Hooks at 5-ft. 4-in. centers. Chain and shafts as in (*b*). Rachet-and-pawl safety device on head pulley. *f*, Air-controlled flap gate permits discharge through either of two feeders. *g*, 17 ft. long, 6 ft. 6 in. wide, $\frac{1}{4}$-in. double-beaded pans. *h*, 6 ft. wide × 16 ft. long. $\frac{3}{16}$-in. side plates and $\frac{1}{4}$-in. bottom plates. *i*, 10 ft. inside diameter, 20 ft. outside diameter, divided by circular partition. Lump on outer deck and coal on inner. 50 ft. per min. mean speed. $\frac{1}{4}$-in. removable cover plate. 25-hp. motor for two tables. *j*, Picked. *k*, Conveyor chutes suspended from 1.5-ton scale so that waste from any car can be read. *l*, To be held for loader's inspection. *m*, Jointed 36-in. pan conveyor, outer 21.5 ft. hinged, and raised and lowered by 2-ton electric hoist. Belt speed, 75 ft. per min. Raising speed, 17 ft. per min. *n*, Sizes may be loaded separately.

FIG. 25.—Imperial Coal Corporation.

Economy Domestic Coal Co. (*19 CA 1151*). Fig 27.

Location: Riddlesburg, Pa.
Feed: Hard, semi-bituminous coal.

O'Gara Coal Co. (*25 CA 557*). Fig. 28.

Location: Harrisburg, Ill.
Capacity: 3000 tons per 8 hr.

Washing is employed when the slack requires cleaning. In some cases the coarse sizes are hand picked and the slack washed, in other cases both

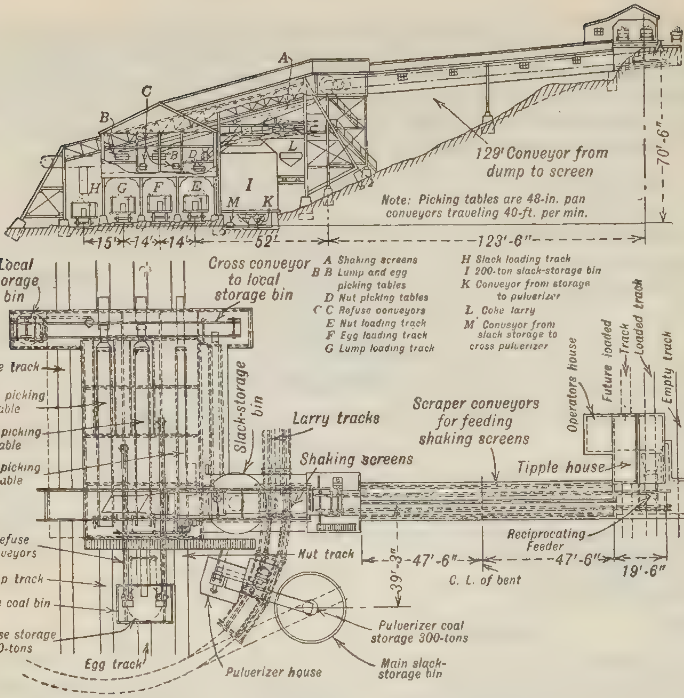
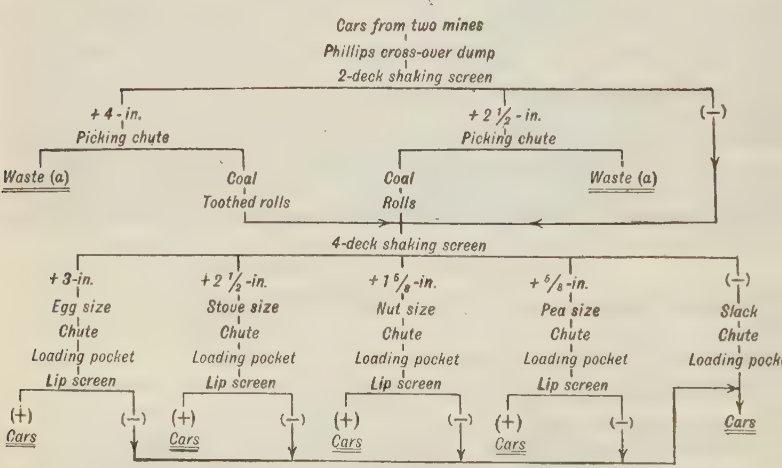


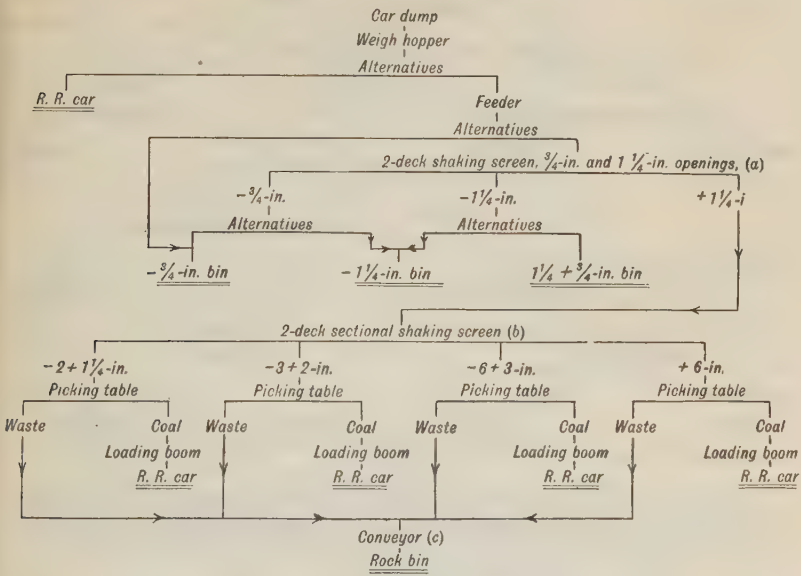
FIG. 26.—Screening and picking plant for preparing sized bituminous coal (*after Read, 6 CA 255*).



a. Picked.

FIG. 27.—Economy Domestic Coal Co.

coarse and fine sizes are washed, either together or separately, and in yet other cases the coarse material is crushed and the whole coal washed at −3-in. size or smaller.



a, Screening surface on each deck 29 ft. long. *b*, Upper deck has 18 ft. of 3-in. and 8 ft. of 6-in. screen followed by a blank section; lower deck has 18 ft. of 2-in. screen.

Fig. 28.—O'Gara Coal Co.

The devices used for washing range from crude trough sluices to fairly elaborate jigs. Recently, also, shaking tables have been used to some extent and several plants have been built that employ pneumatic concentration.

Washeries are frequently named from the particular type of concentrating device used, *e.g.*, a Stewart washery or a Luhrig washery, indicating that Stewart or Luhrig jigs are employed.

Trough washeries are the simplest and crudest form of washing plant. These plants have been used rather extensively at English and Scottish mines. Typical flow-sheet is given in Fig. 29. For data concerning size and performance of coal-washing troughs, see Sec. 8, Art. 12.

Elliott washery. See Sec. 8, Art. 12.

Rheolaveur washery is a modern development of the trough washery. (See Sec. 8, Art. 5, for description of the Rheolaveur apparatus.) Fig. 30 shows the arrangement of a typical plant. This type of washery has made considerable advance in England and has given promising

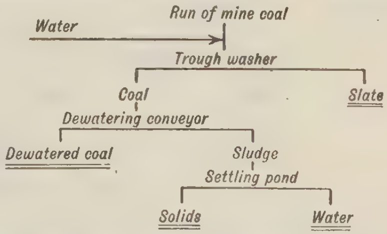
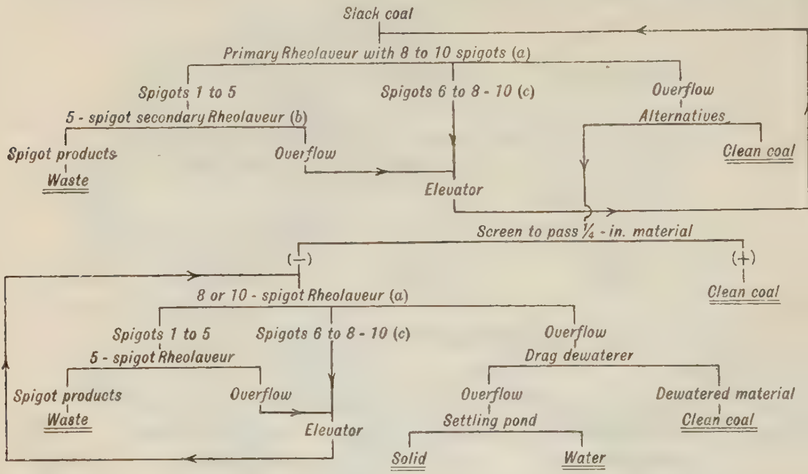


Fig. 29.—Flow-sheet of a trough washery.

results in the United States. The cost at a British washery (1923) treating 100 tons per hr. was $0.055 per ton (*67 IME 501*).



a, Run with weak rising currents. *b*, Run with strong rising currents. *c*, If a second-grade coal is desired spigot products 8 to 10, *e.g.*, may be diverted directly to this supply or sent to another, secondary Rheolaveur which will remove more impurity and insure a relatively constant grade.

Fig. 30.—Rheolaveur washery.

Tube washeries employ some form of tubular classifier as the separating device. They treat relatively fine, sized feeds. Fig. 31 shows a typical flow-sheet. (See also Draper washer, Sec. 8, Art. 5.)

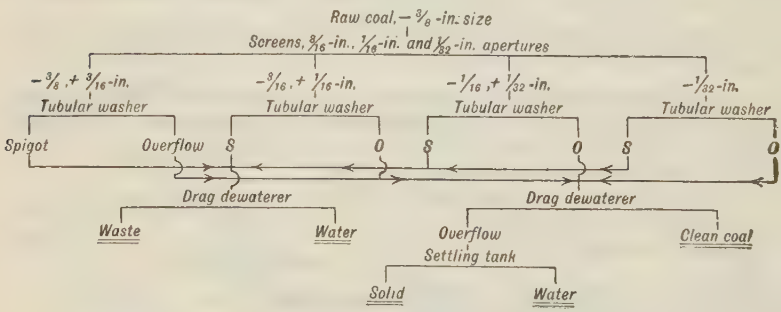
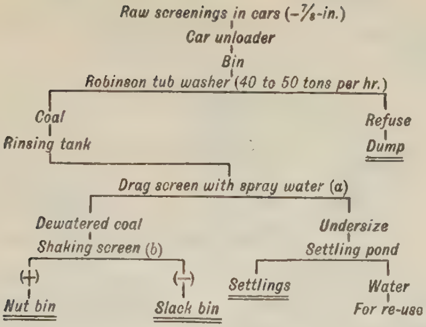


Fig. 31.—Tube washery.

Tank washeries are those that employ tanks like the Robinson, Howe, Chance or Conklin washers as the principal separating machines. Figs. 32 and 33 show a typical Robinson washery.



a, 4 × 20-ft., 3/32-in. round holes; slope, 40°; 100 ft. per min. *b*, Parrish type, supported, 4 × 10-ft., 15/16-in. round holes; slope, 4°; 132 @ 6-in. strokes per min. Feed, 34 tons per hr., −7/8-in. slot.

Fig. 32.—Robinson-tub washery at B. F. Berry Coal Co., Granville, Ill. (*after Lincoln*).

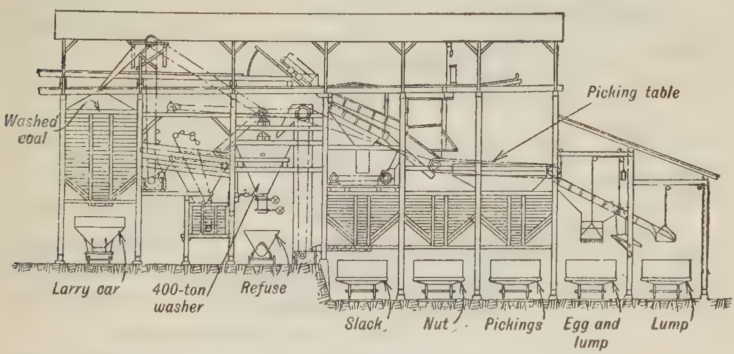


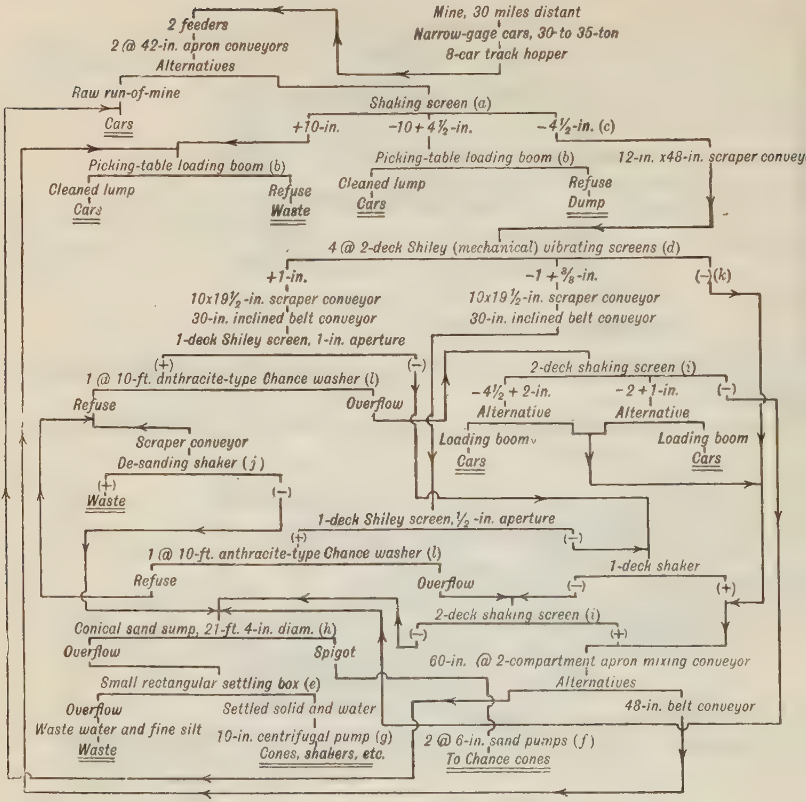
Fig. 33.—Robinson washery.

East Broadtop Railroad and Coal Co. (*PP 1535-F A.*) Fig. 34.

Location: Mt. Union, Pa.
Capacity: 500 tons per hr.
Coal: Low-volatile, slow-coking; ash of high-fushion temperature.
Power: See Table 27
Labor: 24 men per shift.
Analyses:

	Ash, per cent.	S, per cent.
Washed coal, −4½ +1-in.	7.5	1.10
Washed coal, −1 +3/8-in.	7.1	1.22
Raw coal, −3/8-in.	7.9	1.10
Shipped mixture, −1-in.	7.4	1.20

Total refuse = 4 to 5 per cent. of feed, indicating about 2½ per cent. ash reduction.
Estimated cost of plant, based on complete new tipple, $125,000 to $150,000.
Cost of operation: $0.0964 per ton of coal shipped, based on 24 days per month at 400 tons per day and average labor cost $0.80 per hr.



a, 6 ft. wide × 25 ft. long, 4½-in. rd. holes followed by bars with 10-in. spaces. *b*, Apron-conveyor type. *c*, Exceeds 400 tons per hr. *d*, Slope about 30 per cent., 1200 to 1500 vibrations per min. 1¼- to 1½-in. screens on upper deck and ½- to ⅝-in. screens on lower deck. *e*, Acting as feed box for centrifugal pump. *f*, Actual lift about 14 ft. Capacity about 1000 gal. per min. 25-hp. motors. *g*, 2500 gal. per min. capacity, 50-hp. motor. *h*, 200 to 300 gal. per min. of make-up water added here. *i*, Top deck 5 × 26 ft.; bottom, 5 × 23 ft. with a 4 × 17-ft. shaking-chute extension. Upper deck carries a 5 × 9-ft. de-sanding section with 3/32-in. round-hole bronze plate. Upper and lower decks driven by separate eccentrics set 180° apart and both screens driven from a common shaft with 48-in. flywheel to partially overcome lack of balance between upper and lower decks. 160 r.p.m. *j*, 3-ft. 6-in. × 11-ft. 6-in. top deck (3/32-in. screen) and 4-ft. × 11-ft. 6-in. bottom deck with blank plate, 160 r.p.m. *k*, About 70 tons per hr. *l*, Classifier columns are 22-in. diameter to permit handling large sizes. Hydraulic thrust valves, manually operated are used for trapping out slate. Sand loss was between 1.0 and 1.5 lb. per ton of coal washed. Sand cost was $1.59 per ton f. o. b. plant. Specific gravity in coarse cone, 1.55; fine cone, 1.45. Performance of coarse cone:

	Ash, per cent.	S, per cent.	B.t.u.
Raw coal, −4½ +1-in.	17.75	1.58	13,680
Washed coal	7.42	1.10	14,580
Refuse (including that from fine cone)	58.60	3.59	5,602

Density is tested four times per day.

FIG. 34.—East Broadtop Railroad and Coal Co.

Table 27. Motor distribution at the Mt. Union plant.

Use	Type	Drive	Hp.
No. 1 feeder	Squirrel cage	Unit belt	15
No. 2 feeder	Squirrel cage	Unit belt	15
No. 1 apron conveyor	Squirrel cage	Unit belt	30
No. 2 apron conveyor	Slip ring	Unit worm	30
Lump shaker	Squirrel cage	Unit belt	20
All transfer scrapers and apron conveyors	Squirrel cage	Group chain	40
48-in. belt conveyor	Squirrel cage	Unit worm	5
6 vibratory screens	Squirrel cage	6 @ 3-hp. V-bolt	18
Picking-table and loading booms	Squirre cage	Group gear	35
Boom hoists	Squirrel cage	Unit hoist	20
Total feeding, primary screening, conveying and loading	....	....	228 hp.
Cones, 30-in. belt	....	....	....
Conveyors, refuse	....	....	....
Shakers and conveyor	Squirrel cage	Group belt	75
Clean-coal shakers	F.T.R.	Unit belt	30
No. 1 sand pump	Squirrel cage	Direct connected	25
No. 2 sand pump	Squirrel cage	Direct connected	25
10-in. circulating pump	Squirre cage	Direct connected	50
Slate-valve pump	Squirrel cage	Direct connected	20
Make-up	Squirrel cage	Direct connected	15
Total cleaning and sizing	....	....	240 hp.
Total connected load for entire tipple	....	....	460 hp.

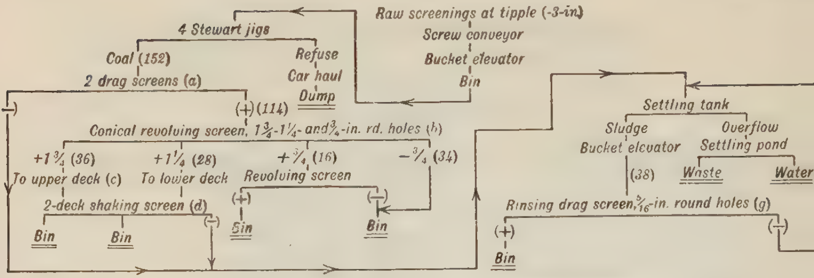
1.09 tons per hp.-hr. (installed) for entire plant; 1.37 tons per hp.-hr. (installed) for sizing and washing.

Summary. Feed divided at $4\frac{1}{2}$-in., oversize hand picked, fines screened from undersize and shipped without treatment, $-4\frac{1}{2} + 1$-in. washed by Chance process in two sizes.

Jig washeries

In these plants the principal separating machine is a jig. Two general types of flow-sheet have developed, depending partly on the character of the coal being treated and partly on whether a pan jig or a fixed-sieve jig is employed. With coals that are easy to clean and, in general, in pan-jig washeries, unsized slack as coarse as 3-in. maximum is fed directly to the jigs and any sizing desired is performed on the washed coal. Many plants treating difficult coals size closely before jigging and for treating the sized products use piston jigs. The Stewart washery (Fig. 35) is typical of the use of pan jigs. The Tennessee Coal, Iron and Railroad Co. (Fig. 36) shows the use of a piston jig in similar service, but taking a much finer feed. The plant at Woodward Iron Co. (Fig. 37) is similar to the preceding but uses a second piston jig to re-treat hutch product and crushed tailing from the first. At Republic Iron and Steel Co. (Fig. 38) shaking tables are used instead of the second jig for treating crushed bone from the primary jig. The Luhrig washery (Fig. 39) shows a plant for treating a difficult coal, with close sizing before jigging.

Chicago and Carterville Coal Co., Herrin, Ill. (*11 Bul. UI No. 9*). Fig. 35.

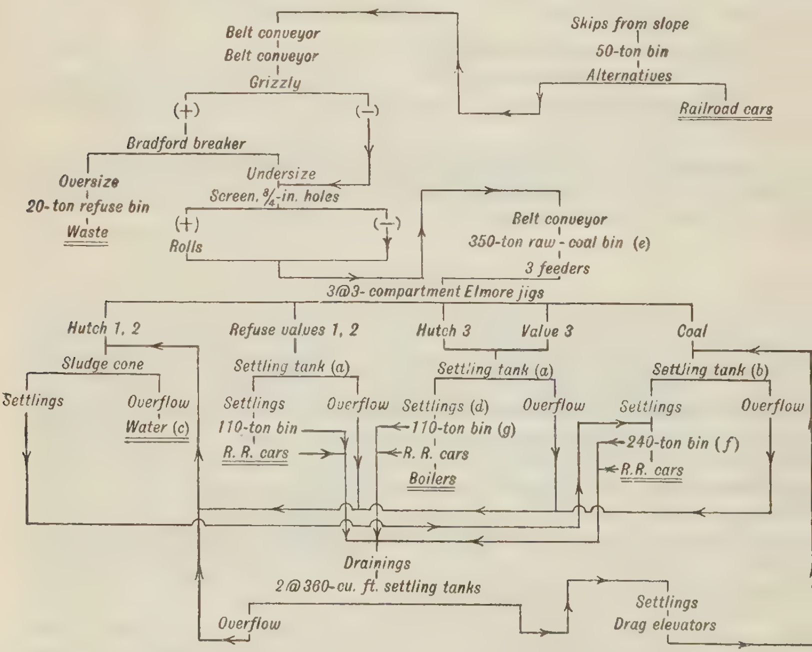


( ) Numbers in parenthesis = tons per hr. *a,* 3 × 28 ft.; lower, 7 ft. with 7/16-in. round holes; upper, 21 ft. with 3/8-in. round holes; slope, 16°; drag speed, 120 ft. per min.; feed, 76 tons per hr. each; 25 per cent. passes through the screen. *b,* Triple-jacketed, compound; inner jacket, 4 and 7½ × 17 ft.; intermediate jacket, 6½ and 10 × 16 ft.; outer jacket, 8 and 12 × 15 ft.; 12 r.p.m. Tons feed per sq. ft. per 24 hr. per mm. aperture: on coarse screen, 0.22; on intermediate screen, 0.14; on fine screen, 0.12. *c,* 3 × 10 ft., 1 1/16-in. round holes, 15° slope. *d,* 130 strokes per min. *g,* 3½ × 12 ft., set level; drag speed, 90 ft. per min.

FIG. 35.—Stewart washery at C. and C. Coal Co.

Tennessee Coal, Iron and Railroad Co. *(71 A 1100).* Fig. 36.

Location: Bayview, Ala. *Capacity:* 100 tons per hr.



a, 900 cu. ft. *b,* 10,600 cu. ft. *c,* 8.7 lb. of solid per 1000 tons of raw coal. *d,* 6.5 per cent. of total raw coal. *e,* 9 per cent. + ¾ in. (*j*). *f,* 19 per cent. + ½ in. (*j*). *g,* 20 per cent. + ¼ in. (*j*). *h,* 14 per cent. + ⅛ in. *i,* 42 per cent. – ⅛ in. *j,* These results are extremely variable.

FIG. 36.—Tennessee Coal, Iron and R.R. Co.

	Weight, per cent.	Proximate analyses			
		Volatile, per cent.	Fixed carbon, per cent.	Ash, per cent.	S, per cent.
Raw coal	100.00	26.42	61.14	12.44	1.76
Washed coal, −1.37 sp. gr.	82.05	28.18	67.06	4.76	1.30
Boiler coal, 1.37 –1.56 sp. gr.	6.53	25.45	59.73	14.82	2.25
Refuse, +1.56 sp. gr.	10.81 (*i*)	.....	.....	62.81	4.74
Refuse, −1.56 sp. gr.	0.61 (*h*)	.....	.....	5.19	1.49

Water circulated, 600 gal. per ton of coal washed. Water lost, 27 gal. per ton of coal washed.

Power: 1.24 kw.-hr. per ton of washed coal.

Summary. Crushing to −¾-in. in Bradford breaker and rolls. Jigging unsized feed. Recovery of fine coal by settling in elevated cone.

Woodward Iron Co. (*71 A 1094*). Fig. 37.

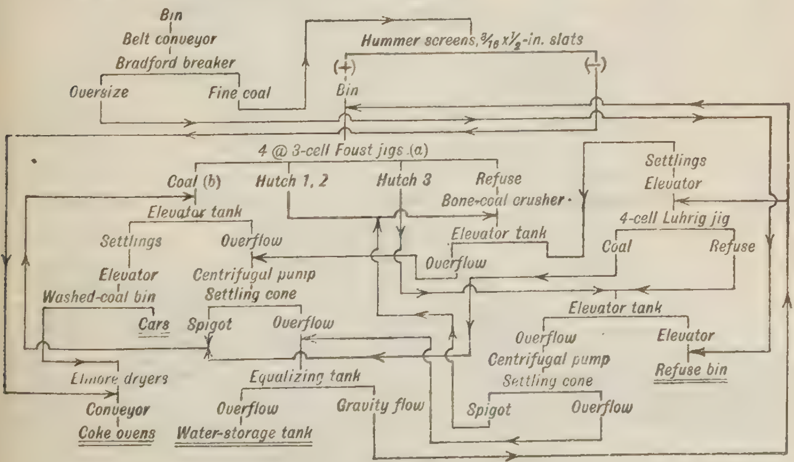
Location: Woodward, Ala.

Assays:

	Weight, per cent.			
	Volatile	Fixed carbon	Ash	Float (−1.37)
Raw coal	26.77	63.52	9.91	.. ..
Fine coal (−3⁄16-in.)	28.03	66.43	5.54	......
Washed coal from primary jigs	.....	.....	5.52	......
Re-washed coal from secondary jigs	.....	.....	9.46	......
Average washed coal	27.95	66.05	6.00	......
Refuse	.....	.....	....	3.21 (*a*)

(*a*) Contained 6.58 per cent. ash.

Water in circulation, 950 gal. per ton of coal washed. Water added, 41 gal. per ton of coal washed.



a, 11.89 per cent. ash. *b*, 5.52 per cent. ash.

Fig. 37.—Woodward Iron Co.

Summary. Screening to separate fine low-ash coal from run-of-mine. Bradford breaker to crush oversize and eliminate coarse refuse. Crushed coal jigged, primary-jig refuse re-crushed and re-jigged together with primary-jig hutch products. Sizing test of jigged coal follows: +0.75-in., 5.9 per cent.; +0.52-in., 2.4 per cent.; +0.26-in., 16.2 per cent.; +0.09-in., 47.4 per cent.; −0.09-in.; 28.1 per cent.

Republic Iron and Steel Co. (*71 A 1096; 19 CA 807*). Fig. 38.

Location: Risco, Ala. *Feed:* Coking coal. *Capacity:* 250 tons per hr.

Assays:

	Weight, per cent.			
	Volatile matter	Fixed carbon	Ash	Sulphur
Raw coal	27.75	55.50	16.75	0.90
Washed coal	28.50	62.25	9.25	0.80

Refuse is 8.5 to 9.5 per cent. by weight. Coal of less than 1.40 sp. gr. in refuse amounts to 1.4 per cent. of such coal fed to the washer.

Water: Consumption is about 25 gal. per ton of coal washed when draining conveyor is not working and coal is loaded out with 10 per cent. moisture. When conveyor is working, coal is loaded out at about 8 per cent. moisture, which is substantially the moisture content of the raw coal, and water loss is very low.

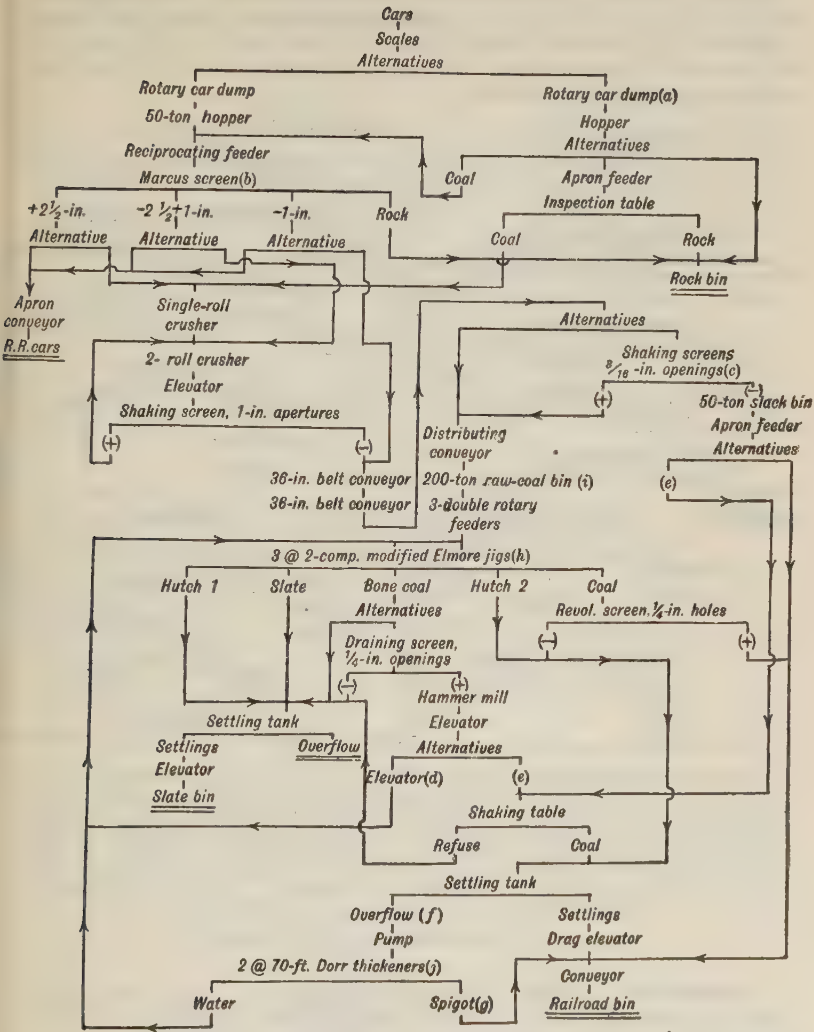
Summary. Crushing through 1-in. round hole in two steps, jigging unsized feed, and recovering fine coal from circulating water in Dorr thickener. The contemplated flow-sheet will separate raw coal at $^3/_{16}$ in. and table the undersize, if necessary, sending oversize only to the jigs.

Table 28. Sink-and-float test on raw coal. Republic Iron and Steel Co., Risco washery. (*After Geismer*)

Sp. gr.	Weight, per cent.	Ash content of grade, per cent.
−1.35	78.7	7.28
1.35–1.40	4.7	14.19
1.40–1.45	4.3	18.24
1.45–1.50	2.4	22.13
1.50–1.55	1.2	28.51
1.55–1.75	1.5	35.85
+1.75	7.2	72.14

Table 29. Sizing-assay (sink-and-float) test of raw coal, Republic Iron and Steel Co., Risco washery. (*After Geismer*)

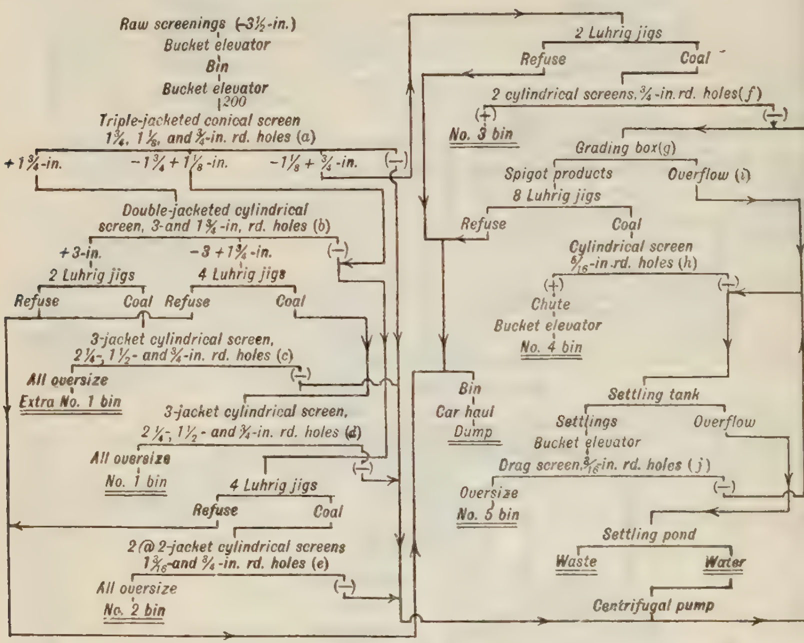
Size	Per cent.	Per cent. float at				Per cent. sink at 1.75 specific gravity
		1.35 specific gravity	1.35–1.45 specific gravity	1.45–1.55 specific gravity	1.55–1.75 specific gravity	
−1-in. +¾-in.	18.8	60.7	12.6	3.9	3.3	18.9
−¾-in. +3/16-in.	45.1	79.0	9.9	3.9	2.6	4.6
−3/16-in. +20-mesh	27.0	89.5	3.5	1.9	1.6	3.5
−20-mesh	9.1	89.6	....	....	10.4	....
Whole sample	100.0	79.7	7.8	3.0	3.2	6.3



a, When it is desired to inspect a carload. *b*, 1-in. and 2½-in. apertures. *c*, To be used when fines do not require washing and, after the addition of shaking tables, as contemplated, to separate shaking-table feed. *d*, At present. *e*, Proposed. *f*, −60-mesh. *g*, 50 per cent. solids. *h*, Capacity, 70 tons per hr. each. Circulating water, 1300 gal. per min. each jig. *i*, For sink-and-float test, see Table 28; for sizing-assay test (sink-and-float) see Table 29. *j*, With 4 to 6 per cent. solids in the feed the overflow carried 0.15 per cent. and the spigot product about 50 per cent. solids.

Fig. 38.—Republic Iron and Steel Co.

Taylor Coal Co., Carterville, Ill. (LUHRIG WASHERY) is diagrammed in Fig. 39 (*11 Bul. UI No. 9*). It differs fundamentally from the Stewart and other pan-jig washeries in that the feed to the jigs is carefully prepared by screen and hydraulic sizing. The result is a more complicated flow-sheet and a more expensive installation.



Feed rate, 200 tons per hr. *a*, Inside jacket, 5½ × 10 × 16 ft.; intermediate, 7¾ × 12¼ × 15 ft.; outside, 10 × 14½ × 14 ft. Axis level, 8 r.p.m. *b*, Inner jacket, 2½ × 5½ ft.; outer, 3½ × 5 ft. Slope, 4½°. 18 r.p.m. *c*, Inside jacket, 2½ × 7 ft.; intermediate, 3½ × 6 ft.; outside, 4½ × 5 ft. Slope, 3½°. 24 r.p.m. *d*, Inside jacket, 3½ × 7 ft.; intermediate, 4½ × 6 ft.; outside, 5½ × 5 ft. Slope, 3½°. 24 r.p.m. *e*, Each, 2¼ × 7 and 3⅙ × 6 ft. Slope, 3½°. 24 r.p.m. *f*, Each, 3⅙ × 7 ft. Slope, 3½°. 24 r.p.m. *g*, Horizontal-current multiple-spigot classifier. *h*, 5 × 8 ft. Slope, 1°. 12 r.p.m. *i*, Not shown by Lincoln. *j*, 4 × 16 ft. Level. 40 ft. per min.

FIG. 39.—Luhrig washery (*after Lincoln*).

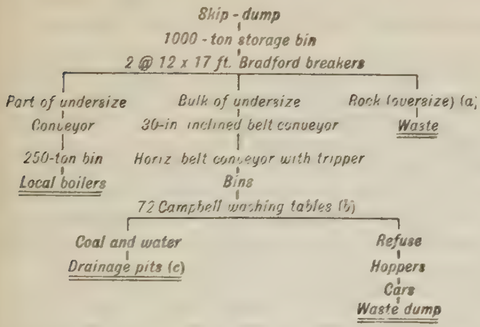
Bumping-table washery is illustrated by Fig. 40. (For description of bumping tables for coal see Campbell bumping table, Sec. 10, Art. 13.)

Cambria Steel Co., Rosedale plant (*23 CA 285*). Fig. 40.

Location: Johnstown, Pa.
Capacity: 4000 gross tons per 10 hr.

Air-cleaning plants is the name given to plants using pneumatic separators as the principal separating machines. Pneumatic machines require closely-sized granular feeds, not too coarse, hence screens are a large part of the plant requirement; all of the feed should be crushed to pass a 1-in. or 1.5-in. screen, or some other method should be provided for handling the

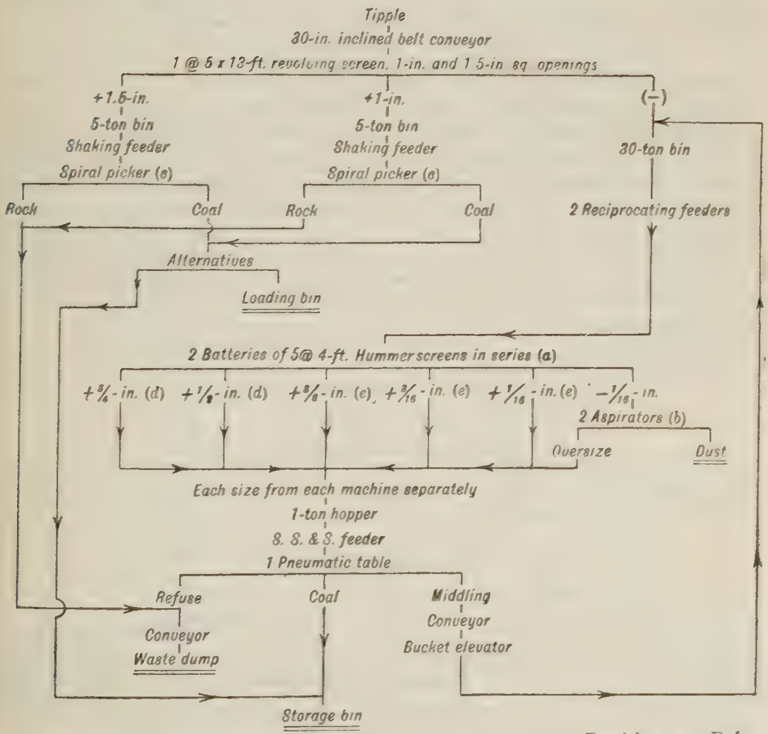
coarser material; and provision must be made for dust separation, collection and disposal. Fig. 41 shows a typical air-cleaning plant. See also Sec. 14, Art. 2.



a, Cobble loss of 1 per cent. *b*, For performance see Sec. 10, Table 35. *c*, 10 shallow rectangular concrete pits, capacity of each 1500 tons. Drainage period ordinarily four days. Drained coal contains 6 to 8 per cent. moisture. Pits are served by two unloading cranes with a chain-bucket digging boom delivering to a 36-in. belt conveyor serving all pits.

FIG. 40.—Cambria Steel Co. coal washery.

St. Louis, Rocky Mountain and Pacific Co., Brilliant mine (*23 CA 791*). Fig. 41.



a, Spaced 8 ft. 3 in. center to center. *b*, See Sec. 14, Art. 2. *c*, Parrish type. Refuse contains about 25 per cent. coal but this is as good as done by the ¾-in. (coarsest) pneumatic table. *d*, Square mesh. *e*, Ton-cap.

FIG. 41.—St. Louis, Rocky Mountain and Pacific Co.

Location: Raton, N. M.
Capacity: 800 tons per 8 hr.
Labor: One foreman and three laborers.
Performance: See Sec. 14, Table 3.
Cost: Young (*loc. cit.*) says that the first cost of a dry plant such as this is about 20 per cent. more than a wet-process plant; that the cost of power is a little higher and of labor about the same.

Flotation is not, at present (1926), practiced in any domestic washeries. There are a few plants in Great Britain. A flow-sheet of one of these is shown in Fig. 42 (*67 IME 510*). Ordinarily the feed is − 0.1-in. material, but this flow-sheet shows an application to a coarser feed.

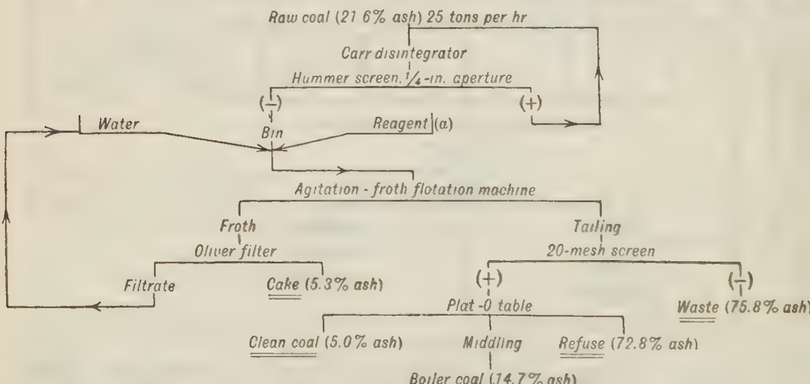
Crushing in coal preparation is never carried to the extent required to free a percentage of impurity equivalent to the percentage of metal freed in metal concentration. As a result, coal washing must be looked upon as a process of concentrating rich middling from poor middling and be judged accordingly.

In a test by Messmore, quoted by McMillan and Bird (*28 Bul. UW 198*), a sample of raw Washington coal, sized between ¾- and ⅜-in., was separated into various specific-gravity fractions, each was crushed to pass 60-mesh and again separated by a sink-and-float test. The lightest fraction of the original lot (− 1.35 sp. gr.), which analyzed 5.4 per cent. ash when crushed, yielded a small amount of material containing 66.7 per cent. ash and the heaviest fraction (+ 1.70 sp. gr.), which analyzed 69.7 per cent. ash yielded a small amount of material containing 5.5 per cent. ash and 3.3 per cent. with less than 9 per cent. The fraction 1.40 to 1.45 sp. gr., which contained 18 per cent. ash, yielded 62.3 per cent. containing 9.8 per cent. ash and 37.7 per cent. assaying 31.6 per cent.

Water required in bituminous-coal washeries varies with the type of washer. According to Campbell (*63 A 683*) the average water consumption of two-compartment jigs is three to four tons of water per ton of coal washed. O'Toole (*20 CA 3*) sets the water consumption in jig plants at 6 tons per ton washed. Shaking tables require a total of one to two tons of water per ton of feed, of which 75 per cent. is feed water. The water lost by leakage, evaporation and moisture in the coal and waste is about 50 to 60 gal. per ton of feed.

Moisture in washed coal is an important disadvantage of washing. O'Toole (*20 CA 3*) points out that when ash reduction is slight and the washed coal is to be shipped long distances the freight on moisture added in washing may counterbalance the enhanced value due to ash reduction.

He cites as an example a washed coal shipped from Indiana to Illinois that ranged



***a*, 0.77 lb. cresylic acid and 0.37 lb. of gas oil per ton.**

FIG. 42.—Flow-sheet for treatment of − ¼-in. slack coal by froth flotation at a British colliery (*after Sinnatt and Mitton*).

between 12.7 and 18.5 per cent. moisture as against a range in moisture content of unwashed Illinois coal received at the same plant of 6.5 to 11 per cent. and an average in Pocahontas coal of 2.3 per cent. High moisture content adds greatly to the cost of unloading in freezing weather. When coal is to be pulverized, it must first be dried, and here again washing is disadvantageous.

Breakage is by no means so important a consideration in bituminous-coal preparation as in anthracite, unless domestic sizes are being made and sold at a considerable advance over screenings. Drakeley (*54 IME 449*) states that the experience in English washeries is that there is much breakage in Robinson washers, less in trough washers and least in jigs. He quotes an experiment from *53 Black Diamond 165* to the effect that the breakage caused by dropping coal 10 to 15 ft. onto an iron plate was three to four times as great as when the coal dropped onto a wooden board.

Performance of bituminous-coal washeries. Tables 30 to 34 (*Lincoln, 11 Bul. UI No. 9*) indicate that so far as Illinois coals are concerned, the lump

Table 30. Dry-ash content of Illinois raw and washed coals. (*After Lincoln*)

Coal field	Raw coal: Face, per cent.	Raw coal: Mine run, per cent.	Raw coal: Lump, per cent.	Raw coal: Screenings, per cent.	Washed coal (*a*): Screenings, per cent.	Washed coal (*a*): No. 1, per cent.	Washed coal (*a*): No. 2, per cent.	Washed coal (*a*): No. 3, per cent.	Washed coal (*a*): No. 4, per cent.	Washed coal (*a*): No. 5, per cent.
Northern	8.89	16.95	11.40	28.58	11.06	9.74	11.43			
Northern	7.33	18.91	6.20	29.30	9.05	6.77	4.93	5.13	11.83	
Northern	5.64	15.45	6.80	27.10	6.69	5.8[illegible]	6.60	6.29	6.85	7.67
Central	11.01	13.62	11.72	16.48	10.29	9.61	9.99	10.79	10.29	12.92
Central	10.51	18.00	10.00	18.43	12.52	10.30	9.70	10.40	10.90	18.31
Central	12.75	15.15	12.93	25.76	10.83	11.57	6.04	10.60	8.80	14.25
Southern	12.43	17.03	10.58	20.51	9.07	5.63	8.88	8.78	7.44	14.11
Southern	7.94	11.73	10.68	12.43	10.08	9.12	9.60	9.28	10.76	11.[illegible]
Southern	9.73	11.42	8.35	13.47	9.15	9.09	8.16	8.78	9.46	11.60
Southern	9.80	12.50	7.90	16.00	9.39	7.65	8.41	8.10	8.25	13.45
Average	9.60	14.58	9.66	20.81	9.81	8.53	8.31	8.96	9.60	12.65

a For significance of size numbers, see Table 26.

Table 31. Dry-sulphur content of Illinois raw and washed coals. (*After Lincoln*)

Coal field	Raw coal: Face, per cent.	Raw coal: Mine run, per cent.	Raw coal: Lump, per cent.	Raw coal: Screenings, per cent.	Washed coal (*a*): Screenings, per cent.	Washed coal (*a*): No. 1, per cent.	Washed coal (*a*): No. 2, per cent.	Washed coal (*a*): No. 3, per cent.	Washed coal (*a*): No. 4, per cent.	Washed coal (*a*): No. 5, per cent.
Northern	3.23	3.38	3.12	3.93	3.76	3.72	3.77			
Northern	3.28	2.84	1.80	3.69	2.98	2.72	2.55	2.78	3.18	
Northern	2.59	2.41	2.30	2.90	2.77	2.64	2.80	2.81	2.78	2.74
Central	4.75	3.63	3.1[illegible]	4.39	3.90	4.05	3.98	3.49	3.33	3.68
Central	5.23	4.90	4.00	4.70	4.60	4.50	4.40	4.40	4.40	4.77
Central	5.51	4.73	5.34	5.89	4.16	3.75	3.55	4.14	4.45	4.60
Southern	3.51	4.31	2.69	5.51	2.92	2.[illegible]7	2.45	2.83	2.57	2.68
Southern	1.51	1.69	1.41	1.87	1.47	1.40	1.55	1.52	1.57	1.38
Southern	1.06	2.16	1.49	2.51	2.41	2.65	1.93	2.64	2.31	2.72
Southern	2.35	2.89	2.00	3.11	1.95	......	......	......	1.53	1.77
Average	3.30	3.29	2.67	3.84	3.03	3.11	2.98	3.12	3.00	3.13

a For significance of size numbers, see Table 26.

Table 32. British thermal units on dry coal in Illinois raw and washed coals. (*After Lincoln*)

Coal field	Unit coal (free from ash and sulphur)	Raw coal: Face	Raw coal: Mine run	Raw coal: Lump	Raw coal: Screenings	Washed coal: Screenings	Washed coal: No. 1	Washed coal: No. 2	Washed coal: No. 3	Washed coal: No. 4	Washed coal: No. 5
Northern	14,814	13,329	11,895	12,750	10,103	12,903	13,116	12,845 (No. 2 to No. 5)			
Northern	14,463	13,221	11,538	13,571	9,874	12,996	13,372	13,655	13,617	12,554 (No. 4 and No. 5)	
Northern	14,579	13,613	12,909	13,495	10,239	13,269	13,402	13,281	13,330	13,243	13,209
Central..	14,390	12,572	12,069	12,417	11,546	12,668	12,704	12,663	12,500	12,655	12,159
Central..	14,365	12,583	11,452	12,669	11,369	12,289	12,636	12,512	12,627	12,327	11,268
Central..	14,365	12,227	11,820	12,649	10,202	13,089	12,870	13,649	13,338	13,182	12,558
Southern.	14,912	12,907	11,752	12,829	11,172	13,074	13,486	13,465	13,070	13,163	12,140
Southern.	14,671	13,366	12,918	13,095	12,651	13,029	13,189	13,196	13,161	12,925	12,799
Southern.	14,763	13,229	12,939	13,753	12,397	13,170	13,086	13,253	13,134	13,037	12,687
Southern.	14,660	13,030	12,700	13,450	12,103	13,101	13,391	13,280	13,325	13,417	12,233
Average	14,598	13,007	12,199	13,068	11,166	12,959	13,125	13,180	13,095	12,935	12,445

Table 33. Percentages of washed coals and refuse produced at Illinois washeries. (*After Lincoln*)

Field	Washery No.	Nut	Slack	No. 1 extra	No. 1	No. 2 extra	No 2	No. 3	No 4	No. 5	Refuse
Northern.............	1	10.00	90.00	.....	.....	.....	.....	.....	.....	.....	32
	2	10.67	89.33	.....	.....	.....	.....	.....	.....	.....	36
	3	.....	100.00	.....	2.44	.....	21.98	75.58	.....	.....	32
	4	.....	.....	.....	.....	.....	.....	.....	.....	.....	33
	5	21.30	78.70	.....	.....	.....	.....	.....	.....	.....	24
	6	20.00	80.00	.....	.....	.....	.....	.....	.....	.....	25
	7	20.00	80.00	.....	.....	.....	.....	.....			25
	8	.....	.....	.....	8.00	.....	12.00	23.00	57.00 (No 4 and No. 5)		26
	9	.....	.....	.....	3.10	.....	5.90	33.50	25.70	31.80	19
(*a*) Average for 5 washeries making 2 sizes	......	16.40	83.60								
Central..............	15	.....	.....	.....	.....	.....	.....			12.00 (No. 3 to No. 5)	10
	17	.....	.....	.....	.....	.....	.....	60.70 (No. 3 and No 4)		39.30	11
	18	.....	.....	.....	14.26	.....	18.31	11.82	28.84	26.77	22
	19	.....	.....	.....	10.12	.....	22.98	23.27	21.72	21.91	11
	20	.....	.....	.....	17.11	.....	17.74	13.56	28.77	22.82	21
	21	.....	.....	.....	15.00	.....	18.00	23.00	26.00	18.00	....
	22	.....	.....	.....	1.24	.....	18.75	19.47	22.53	38.01	19
	23	.....	.....	.....	17.20	.....	12.30	20.30	32.00	18.20	10
	24	.....	.....	.....	17.09	.....	17.90	15.10	23.57	26.34	20
	25	.....	.....	.....	2.80	.....	17.40	21.20	30.30	28.30	13
(*b*) Average for 8 washeries making 5 sizes	......	.....	.....	.....	11.85	.....	17.92	18.47	26.72	25.04	
Southern.............	27	.....	36.10	.....	16.90	.....	21.60	25.40	.....	.....	...
	28	.....	.....	.....	9.80	.....	28.00	8.70	34.20	19.30	19
	31	.....	.....	.....	23.37	.....	18.09	10.78	22.40	25.36	5
	32	.....	.....	9.00	15.80	12.40	15.60	.....	27.20	20.00	9
	33	.....	.....	.....	24.00	.....	21.00	15.00	22.00	18.00	15
	35	.....	.....	.....	26.80	.....	18.60	18.50	27.90	8.20	5
(*c*) Average for 4 washeries making 5 sizes	......	.....	.....	.....	20.99	.....	21.42	13.25	26.62	17.72	

Table 34. Dry ash in products of Illinois washeries, one dry-cleaning plant and one re-screening plant. (*After Lincoln*)

Type of plant	Raw screenings, per cent.	Treated coal: No. 1, per cent.	No. 2, per cent.	No. 3, per cent.	No. 4, per cent.	No. 5, per cent.	All sizes, per cent.	Refuse, per cent.	Suspended matter, per cent.	Decrease in ash treated coal under raw screenings: Amount, per cent.	Proportion, per cent.	Increase in ash refuse over raw screenings: Amount, per cent.	Proportion, per cent.
Northern field													
Robinson washery...	29.96	not made		8.36			8.36	69.13	51.98	21.60	72.10	39.17	130.7
Forrester washery....	28.58	9.74	......	11.43			11.06	73.18	38.77	17.52	61.30	44.60	164.1
Forrester washery....	....	....	....		(14.85)*a*								
Pan-jig washeries.....	24.21	6.79	6.30	7.55	8.71	8.75	8.34	58.90	28.92	16.56	65.54	33.49	145.1
Danville field													
Forrester washery....	(19.30)*a*	....	......	(10.80)	(11.30)	(17.50)	......	(66.80)	....	......	....	47.50	246.1
New Century washery.	....	....	....	....	(9.66)	(16.39)	......	....	....			....	......
Central field													
Robinson washery....	....	....	9.63	....	....	13.95	..	60.14	46.23	....	....	....	......
Luhrig washeries.....	23.00	8.70	8.80	11.60	9.70	15.91	11.41	68.89	53.20	11.59	50.21	43.29	169.1
Pan-jig washeries.....	22.04	9.98	9.76	10.33	10.99	15.65	12.03	63.42	22.30	9.61	42.86	42.15	205.4
Springfield field													
Pan-jig washery......	(16.50)	(11.60)	(12.00)	(12.30)	(12.20)	(12.20)	......	55.00	....	....	......	43.40	374.1
Southern field													
Luhrig washeries.....	16.10	7.67	8.16	8.84	8.22	10.69	8.99	Coarse 47.30; Fine 43.42	47.12	5.07	36.30	....	......
Foust jig..........	18.90	......	......	......	9.46	11.60	11.31	74.13	16.07	7.59	40.16	55.23	292.2
Pan-jig washeries....	13.15	9.29	8.76	8.59	8.38	12.59	9.81	53.22	19.02	3.04	23.53	40.78	333.7
Campbell washery....	....	9.33	8.71	8.10	7.53	11.77	......	57.44	13.98	....	....	....	......
Dry cleaner..........	....	9.25	8.43	13.19	13.37	13.55	11.64	46.81	20.35*b*	....	....	....	......
Re-screener.	....	11.83	11.39	11.28	10.10		11.00	56.79*c*	......	....	....	....	......

a Results in parentheses were not obtained by Lincoln but are from sources believed by him to be reliable. *b* "Bone." *c* Hand picked.

product screened out of the run-of-mine coal and picked contains just about the same amount of ash and less sulphur than the unmined coal in the face and that the raw-coal screenings contain about twice as much ash and somewhat more sulphur than the unmined coal. Washing the screenings cuts the ash content just about in half, bringing it back to that of the unmined coal, and cuts off about 25 per cent. from the sulphur content, lowering it to slightly below that of unmined coal. The calorific values, of course, follow the ash content. Here then, except for the slight sulphur reduction, washing merely serves to remove the waste introduced in mining, but, since washing is a cheap process and mining expensive, it is more economical to remove mined waste in the washer than to mine so as to exclude the waste. Table 33 gives an idea of the size distribution of the washed coal. Table 34 compares average performances of the various types of washeries treating Illinois coals.

Cost of washing per ton of raw coal washed in Illinois in 1912 (*11 Bul. UI No. 9*) including power, labor, supplies, repairs and renewals averaged $0.105 for 15 washeries with a range from $0.03 to $0.18. Lincoln notes in giving these costs that where mine and washery are operated from the some power plant the power charge to the washery is frequently too low or omitted entirely, with the result that the above average is probably low. Average cost of Illinois washeries per hourly ton of rated capacity was $351 with a range of from $130 to $583. Average for 7 washeries with hourly capacities of 100 tons and under was $448, while that for 8 washeries of more than 100 tons hourly capacity was $266. Three Luhrig-jig washeries averaged $393, six Stewart-jig washeries $341, and two Robinson-tub washeries $322.

13. Cobalt, Co.

Properties. Metal; pinkish white, lustrous, malleable, ductile, very tenacious, readily polished, markedly magnetic. (See also Table 1) AT. WGT., 58.9. In compact form practically unchanged in air; in powder may ignite spontaneously. Dissolves slowly in most acids, readily in nitric, forming corresponding salts. Cobalt ion is both bi-valent and tri-valent, base- and acid-forming. Cobalt alloys readily with other metals, notably nickel and iron.

Uses. The principal use is in the ceramic and enamel industries, to impart a deep, intense blue. For this purpose the oxide is used. Alloys of cobalt with iron are used in making magnet steels and "stainless" steel, and an alloy with chromium and tungsten, STELLITE, is a substitute for high-speed tool-steel, although at present rather expensive. Cobalt salts are used and are very effective as catalyzers in drying oils and varnishes (*16 IEC 957; Circ. 186, Paint Mfr's Assoc. of U. S., 1*).

Ores. The economic minerals are: cobaltite, erythrite, smaltite and linnaeite, usually associated with nickel and iron. The main source of supply is Sudbury, Ontario, where the ore is a nickeliferous pyrrhotite which occurs in enormous masses at the contact of quartzite and diorite. There is also an important deposit in the Belgian-African copper field.

Production. To date the Canadian production only is of importance. This has ranged from a maximum of roughly 1,200,000 lb. of cobalt oxide (70 per cent. Co) and 475,000 lb. of cobalt metal in 1918 through a minimum of 217,000 lb. of oxide and 22,000 lb. of metal in 1921 to 750,000 lb. of metal in the form of oxide, metal, salts and treatment residues in 1924 (*33 MI 165*).

Selling. Canadian ores and treatment products containing from 5 to about 12 per cent. Co are sold to refiners in the United States on complicated schedules (see *Spurr and Wormser*), which, of course, depend on the prices obtainable for the metal and oxide. These prices have varied from $1.50 to $1.65 per lb. for the oxide and $2.25 to $2.50 for the metal in 1918 through

\$2.00 to \$4.00 for oxide and \$3.00 to \$6.00 for metal in 1920 and 1921 back to \$2.10 to \$2.25 and \$2.50 to \$3.25 for oxide and metal respectively in 1924. The oxide quoted is the black oxide containing 70 to 71 per cent. Co and not over 1 per cent. Ni. The metal is about 97 per cent. Co with Ni and C the principal impurities.

Treatment. Cobalt minerals are concentrated, usually with the silver and other metallic sulphides present in the ore, and the concentrate, either before or after cyanidation, is roasted to drive off sulphur and arsenic; the resulting oxides are then dissolved and separated by fractional precipitation of their salts.

14. Copper, Cu.

Properties. Metal: red, lustrous, soft, highly ductile, markedly low electrical resistivity (See also Table 1.) At. wt., 63.6. Unchanged by dry air at ordinary temperature. In moist air, after considerable time, a very thin layer of oxidation compounds forms which protects the body of the metal. At red heat it combines rather rapidly with oxygen. Dilute hydrochloric and sulphuric acids do not attack copper in the absence of oxidizing agents, but reaction takes place slowly in the presence of air. Hot concentrated sulphuric acid attacks it. Nitric acid in any dilution dissolves it, the activity of the reaction increasing with the concentration of acid. Copper ion is uni- and bi-valent, usually base-forming, but in some compounds forming part of a complex anion. Copper alloys freely with many other metals.

Uses of copper are multitudinous. It is, under most circumstances, the most economical conductor of electricity and hence is used widely for electrical machinery and transmission; its resistance to corrosion by air, water, and weak acids makes it valuable for certain parts of structures such as roofing, gutters, window screens, ship bottoms, for cooking utensils, and for pipes and containers in chemical and manufacturing plants. It is the main constituent of many alloys, notably brass and bronze which, like copper, can be drawn, and unlike copper, can be successfully cast and, being harder than copper, can be worked in a lathe. Copper chemicals such as the sulphate, carbonate, cyanide, etc., find wide use in the arts, and as germicides and fungicides.

Ores. The economic minerals are atacamite, azurite, bornite, bournonite, brochantite, chalcanthite, chalcocite, chalcopyrite, chrysocolla, native copper, covellite, enargite, cuprite, malachite, melaconite (tenorite) and tetrahedrite. Copper is found in practically every type of ore deposit and associated, in one place or another, with practically every metallic and rock-forming mineral. The largest and best-known deposits are the sulphide-vein deposits of Montana, the native-copper deposits of the Lake Superior region, and the "oxidized" and "porphyry" deposits of the southwestern United States. At Butte, Montana, the ore-bearing veins occur in granite, the chief copper minerals are chalcocite, enargite, bornite, and chalcopyrite, and the principal associated vein minerals are quartz and pyrite; minor minerals are covellite, tetrahedrite, tennantite, sphalerite, argentite, gold, and minerals containing bismuth, tellurium, selenium, nickel and manganese. In the Lake Superior region the native copper, associated with native silver, occurs as part of the cementing matter in a conglomerate, as a cavity filling in lava beds, and in veins cutting both igneous and sedimentary rocks. The usual gangue minerals are calcite and zeolites. The "oxidized" deposits of the southwest carry large amounts of the oxidized copper minerals malachite, azurite, and cuprite, and directly below or closely associated with these, bonanzas of massive secondary chalcocite. Both classes of minerals are derived from the primary copper sulphides, largely chalcopyrite and cupriferous pyrite, which are found unaltered in greater depths. The deposits occur in limestone or at limestone-porphyry contacts. There are also numerous granitic intrusions associated with the ore-bodies which have caused the formation of characteristic contact minerals. These deposits are char-

[illegible] of the Bisbee and Clifton districts. The deposits of the Jerome district [illegible]. [illegible] occur in fissures and as impregnations in the country rock near intruded granite dikes. The "porphyry coppers" are mineralized zones in granite porphyries and schists. The principal copper mineral is chalcocite, which occurs as grains and veinlets in the country rock. Pyrite, chalcopyrite, and magnetite are the principal metallic associates; quartz and the rock-forming silicates form the non-metallic gangue. An enormous disseminated deposit containing soluble sulphates of copper occurs at Chuquicamata, Chile, and in the Katanga province of Belgian Congo are a number of high-grade deposits in which the principal copper mineral is malachite.

Production statistics are shown in Tables 35 and 36.

Table 35. Copper production in the United States (millions of pounds) (a)

State	1913	1918	1919	1920	1921	1922	1923	1924
Alaska	23.4	67.1	56.3	69.1	76.1	39.9	65.6	77.1
Arizona	404.3	769.5	536.5	553.0	135.1	438.2	625.3	672.4
California	32.5	44.1	23.5	11.8	13.9	20.4	27.0	42.4
Colorado	9.1	7.6	4.9	4.3	6.3	3.6	4.3	4.2
Idaho	8.7	5.5	4.0	1.9	2.0	3.5	3.2	3.1
Michigan	155.7	231.1	177.6	153.5	100.9	122.3	157.7	145.5
Missouri	0.6	0.2	0.6	0.5	0.1	1.0		
Montana	283.7	326.4	176.5	177.7	49.5	135.3	224.4	246.[illegible]
Nevada	85.2	106.3	64.7	55.6	13.1	20.4	63.5	77.[illegible]
New Mexico	50.2	98.6	62.4	32.2	18.1	23.1	[illegible]	75.[illegible]
Tennessee	19.5	15.1	15.6	15.7	13.1	14.2	[illegible]	[illegible]
Utah	148.1	231.0	143.8	110.4	45.8	79.7	[illegible]	[illegible]
Washington	0.7	2.3	2.6	2.1	0.6	0.4	[illegible]	[illegible]
Others	0.8	5.4	19.4	3.3	4.7	2.1	[illegible]	[illegible]
Total	[illegible]	[illegible]	[illegible]	[illegible]	505.6	950.[illegible]	[illegible]	[illegible]

a U. S. Geol. Surv.

Table 36. World's production of copper [illegible] (a)

Country	1913	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	54,137	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	27,070	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
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[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
United Kingdom	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
United States	[illegible]	[illegible]	[illegible]	[illegible]	452,380	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Total	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

a Mineral Industry.

Selling. Electrolytic copper, 99.90 to 99.95 per cent. combined copper and silver; Lake copper, the melted and furnace-refined product from the Lake Superior mills, of the same purity; arsenical Lake, analyzing upwards of 99.88 per cent. copper plus silver plus arsenic; and re-melted scrap are the principal grades in which copper is sold in the United States. Some small amounts of high-grade blister copper and cathode copper are also dealt in. The metal is sold in the form of wire bars, ingots and ingot bars, and slabs and cakes. Selling is largely in the hands of a few agencies in New York City. (For details of selling methods see *Spurr and Wormser* and the bibliography therein.) Average prices in recent years are given in Table 37.

Table 37. Average prices of electrolytic copper at New York. (*Eng. and Min. Jour.*)

Year	Cents per pound
1913	15.27
1919	18.69
1920	17.46
1921	12.50
1922	13.38
1923	14.42
1924	13.02
1925	14.04

Most copper, as it leaves the mills, is in the form of a sulphide concentrate and the contained metal is subject to charges for freight, smelting, refining and selling. For typical schedules, see Art. 33.

Treatment of copper ores depends upon the nature of the ore itself. There are three general methods of treatment, viz.: *(1)* Direct smelting for high-grade ores, containing upwards of 6 per cent. copper. *(2)* Leaching followed by electrolytic precipitation or precipitation by iron, for oxidized ores. *(3)* Concentration followed by smelting of concentrate, for low-grade sulphide and native-metal ores. SMELTER TREATMENT of ores and concentrates consists in melting down in an oxidizing atmosphere, as a result of which an artificial copper-iron sulphide (MATTE) carrying precious metals is formed, rock-forming impurities are slagged off, and certain substances, *e.g.*, zinc and arsenic, pass off as fume. Coarse ore is treated in blast furnaces and fine in reverberatories. The molten matte is poured into a converter where air is blown through to drive off the sulphur as oxide and the iron is converted into an oxide which combines with silica to form slag. The copper product is a crude "BLISTER COPPER." This crude copper, in the absence of gold and silver, may be refined by further oxidation and subsequent reduction in a furnace similar to the "converter." If gold and silver are present the blister copper is refined electrolytically. Native-copper concentrate is melted down directly and the product refined in a furnace or, if desired, electrolytically. (See *Principles of copper smelting, E. D. Peters*, McGraw-Hill, 1907; *Practice of copper smelting, E. D. Peters*, McGraw-Hill, 1911; *Metallurgy of copper, Hofman and Hayward*, McGraw-Hill, 1924.)

Copper-ore Concentrators

The copper ores amenable to simple concentration are those containing native copper or sulphides of copper, to which both water-gravity and flotation concentration are applicable, and oxidized ores containing cuprite, which responds to gravity concentration from any of the usual gangues. Oxidized ores in which malachite, azurite and chrysocolla are the economic minerals do not respond to either gravity concentration or simple flotation. Sulphide-filming of the carbonates or solution of both carbonates and silicate with subsequent precipitation and flotation have been successful experimentally.

(Sec. 12, Art. 25.) Hydrometallurgical treatment of such ores is the commonest method at present (1925).

Native copper, coarse dissemination. The Champion mill of Copper Range Consolidated Mining Co. is typical of mills treating this class of ore. Hand-sorting and water-gravity concentration only are employed. Tailing is rejected at −0.25-in. and only middling products are re-ground.

Champion Copper Co. Fig. 43. (*Q*)

Location: Freda, Mich.

Ore: Amygdaloid containing an average of 37.5 lb. native copper per ton in all sizes from large masses to very small flakes.

Capacity: 2100 tons per 24 hr.

Assays, per cent. Cu: Feed, 1.87; concentrate, 65; tailing, 0.45–0.5.

Recovery: 88 per cent.

Ratio of concentration: 40 : 1.

Percentage possible running time: 95.8.

Labor: 13 tons per man-shift, total.

Power: 24 hp.-hr. per ton milled.

Water: 30 tons per ton of ore milled. No reclamation.

Distances: Mine to mill, 14 mi.; mill to smelter, 19 mi.; water transport, 1600 ft. from Lake Superior; electric power transmitted 14 mi. at 30,000 volts.

General: Sloping mill site.

Summary. Crushing: Jaw crusher, steam stamps and rolls in series from −24-in. to −0.25-in. Concentration: Hand-picking at 12-in. or larger down to 2- to 3-in.; steam-stamp classifier, 3-in. to 0.5-in.; jigs on −0.62 +0.25-in. sized feed; hydraulic classifiers followed by roughing (hutch-making) jigs and finishing tables on primary −0.25-in. feed. Fine-jig tailing discarded. Finishing-table tailing and middling re-ground in pebble mills and tabled without classification.

This relatively simple sorting and all-gravity flow-sheet is still considered best by the management for this particular ore, notwithstanding years of experimentation. Flotation, which is practiced by Calumet and Hecla, is not applicable on account of the comparative coarseness of the finest copper and the difficulty in grinding it to flotation size. The steam stamp is peculiar to Michigan native-copper mills and serves a purpose in separating the tough, irregular copper particles from the enclosing rock that no other crusher does as well.

With the exception of the elevator handling bull-jig tailing all elevation of pulps is accomplished by means of centrifugal sand pumps.

Native copper, fine dissemination. The conglomerate ores of the Lake Superior region are typical of this occurrence and the conglomerate sections of Calumet and Hecla mill illustrate an efficient treatment method. Hand picking, gravity concentration by jigs and tables and agitation-froth flotation are successively employed and sands from the re-grinding mills are leached.

Calumet and Hecla Mining Co. Conglomerate mill. Fig. 44 (*Q, 100 J 7; 117 J 277*).

Location: Torch Lake, Mich.

Ore: Native copper with a little native silver in an extremely hard rhyolitic conglomerate.

Capacity: 11,000 tons per 24 hr.

Assays, per cent. Cu: Feed (original mine ore), 1.75; concentrate, 60 to 62.

Recovery: 85 per cent (on mine rock).

Ratio of concentration: 35 : 1.

Percentage possible running time: 98, approx.

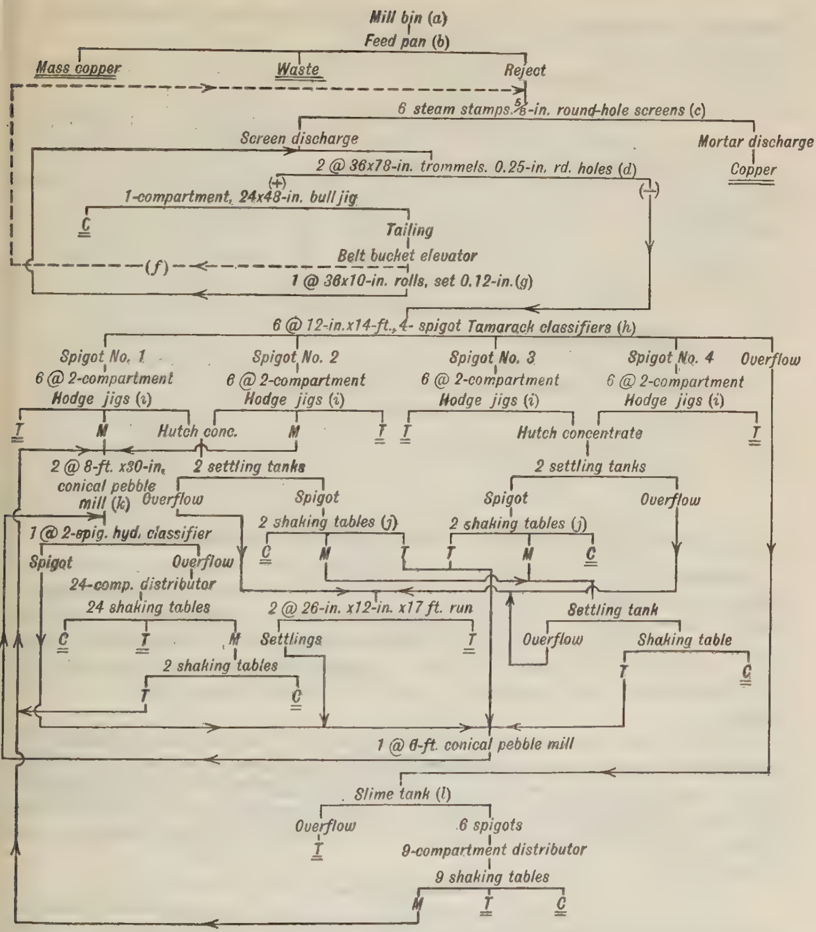
Labor: 19.5 tons per man-shift, total.

Power: 30 to 36 hp.-hr. per ton milled.

Water: 28 tons per ton milled. No reclamation.

Distances: Mines to mill: 5 mi. max.; mill to smelter, 0.5 mi.; water at mill; power transmitted 0.5 mi. at 13,500 volts.

Costs: See flow-sheet notes (*j*), (*k*), and (*s*) and Table 38.



a, 500-ton, 20 (diam.) × 44 ft. Contains −3.5-in. product of rock-house jaw crusher. (See Fig. 44 for a typical rock-house flow-sheet.) *b*, Inclined (−27°) converging chute with means for controlling flow. Operator picks waste and mass copper and regulates flow according to performance of stamp. Mass removed ranges from 6- to 12-in. size. One man picks about 110 lb. per hr. *c*, Simple (20 × 24-in. cylinder) and steeple-compound (see Sec. 3, Art. 16). *d*, 18 r.p.m. Slope, 1 in. per ft. Open-hearth high-carbon steel plate; life, 71 da. Total load, 363 tons per 24 hr.; 12,500 gal. water per hr. About 35 per cent. undersize in oversize. *f*, Alternative emergency flow. *g*, See Sec. 3, Table 22. *h*, Feed contains 80 per cent. water; spigot products, in order, 80, 83, 87 and 85 per cent., and overflow 96 per cent. water. Spigot apertures: No. 1, ¾-in. pipe, worn; No. 2, ¾-in. pipe, new; No. 3, ½-in. pipe, worn; No. 4, ½-in. pipe, new. Attendance, for 6 machines, one man part time. *i*, 24 × 36-in. compartments. (See Sec. 9, Art. 5.) *j*, See Sec. 10, Art. 11. *k*, See Sec. 4, Art. 16. *l*, 45 ft. long, 7 ft. deep, 9 ft. wide. *C*, Finished concentrate, to smelter. Moisture, 7.5 per cent. *T*, Finished tailing, combined and laundered to lake.

Fig. 43.—Champion Copper Co.

Table 38. Performance of flotation machines in Calumet and Hecla reclamation plant. (*After Benedict*)

Representative sample

	Feed		Tailing, Cu, per cent.	Recovery, per cent.
	Weight, per cent.	Cu, per cent.		
+200-mesh	6.0	0.289	0.243	15.9
−200-mesh	94.0	0.516	0.157	69.6
Total	100.0	0.502	0.162	67.8

Averages and totals, 1923

Feed, per cent. Cu	0.453
Tailing, per cent. Cu	0.164
Concentrate, per cent Cu	28.58
Recovery, per cent	63.89

Costs, 1923

General expense	$0.0278
Slime conveying and distribution	0.0280
Flotation	0.0547
Royalty	0.0449
Total	$0.1554

Cost per pound of copper produced, excluding smelting and selling, $0.0270.

Summary. No receiving bin. Crushing: Jaw crusher, 24- to 3-in.; steam stamp, 4- to 3/16-in.; pebble mills, 3/16-in. to 35-mesh. Concentration: Hand-picking on sizes to −3-in.; jigging in stamp mortar and on natural product through 3/16-in. screen with cleaning of jig-hutch product on shaking tables. Fine sands tabled without classification. Gravity-concentration residues divided into sands and slimes, the former leached and the latter floated.

Compare with the Copper Range flow-sheet. The copper in conglomerate ores is more finely disseminated than that in amygdaloid ores and is, therefore, less amenable to table concentration and more readily recoverable by flotation. Leaching eliminates the necessity of re-grinding all gravity-concentration tailing to flotation size and makes better extraction of coarse copper than flotation does.

Notes to Fig. 44.

a, Tilting pan, 9 ft. wide at upper end, 4 ft. at lower, with double-curved bottom. Head end is lowered below sliding angle of rock to receive grizzly oversize from dumped skip and raised by means of chains from friction-clutch drum on main line shaft after material has been picked. Pan bottom of ¾-in. plate, sides of ½-in. plate, all lined with ⅜-in. plate. 2 men required per shift, one picking and one operating winch. *b,* See Sec. 3, Table 10. *c,* 400-ton live capacity. One for each steam stamp. *d,* 28 of these. Capacity on conglomerate, 335 to 350 tons each per 24 hr. 3 tons water per ton of ore. (See Sec. 3, Table 28.) *e,* Four per stamp. 4 × 12-in. sieve compartments, 1-in. screens, 190–200 @ 2-in. strokes per min. *f,* See Sec. 9, Art. 5. First sieve 36 × 48-in., others 30 × 48-in. *g,* For 17 mill units. *h,* Capacity, 2640 tons per 24 hr. *i,* See Sec. 4, Table 77. *j,* Old mill tailing. 0.5 to 1.0 per cent. Cu. Total cost (1923) per ton of solid treated, including proportion of mine and mill administration:

General administration and miscellaneous	$0.044
Dredge (note *k*)	0.063
Shore plant (note *s*)	0.025
Re-grinding (note *s*)	0.164
Leaching	0.251
Flotation	0.034
Total	$0.581

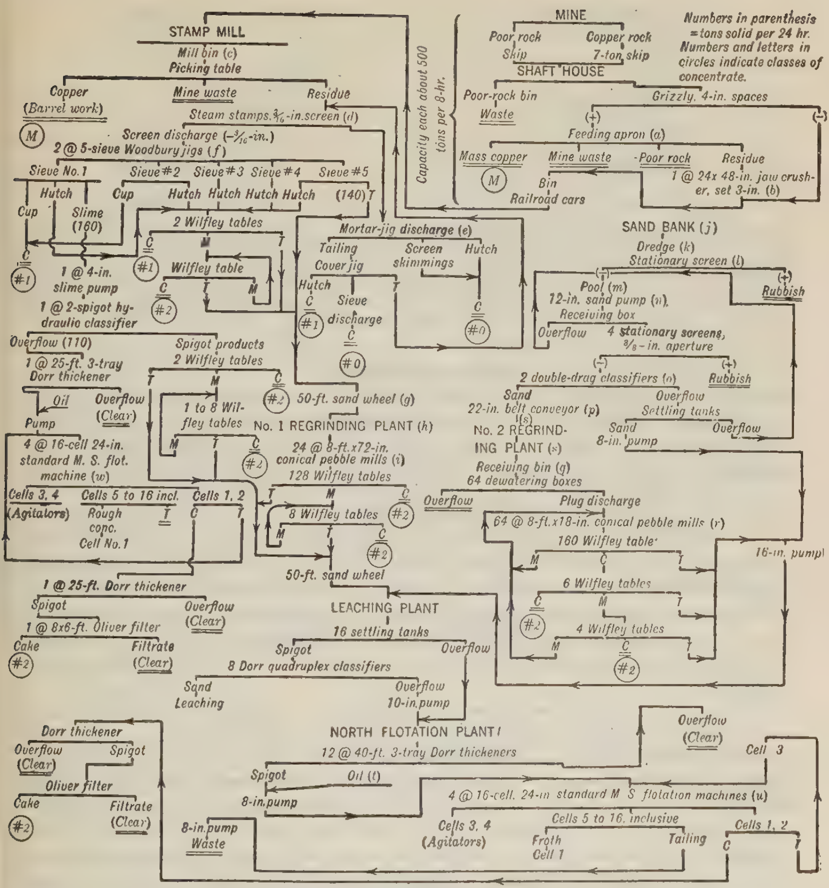


FIG. 44.—Calumet and Hecla, conglomerate mill.

Tons treated, 1,743,100. Assays, per cent. Cu: feed, 0.608; tailing, 0.124. Recovery, 80 per cent. *k,* Suction type. 10,000-ton daily capacity. 20-in. pump with 1250-hp. motor discharges through 3000 ft. of 20-in. pipe on pontoons to receiving pool on shore. Can dredge to 110 ft. below water level. COST (1923) for dredge operation with discharge through average length of 2000 ft. was $0.063 per ton of solid dredged. Distribution was: labor, 32 per cent.; power, 40 per cent.; pump renewals, 14 per cent.; other supplies, 14 per cent. *l,* On shore. 16 × 20 ft., 1-in. round holes. *m,* Capacity, 20,000 tons sand. *n,* Swinging suction of constant length giving uniform feed to re-claiming plant. Casing is split horizontally and lined throughout. Impeller 40-in. diam., direct-connected to 200-hp. motor. *o,* Each with 2 @ 20-in. belts carrying 6 × 4 × 24-in. angles. *p,* 275 ft. long; slope, 2½ in. per ft.; speed, 500 ft. per min. *q,* About 30-min. storage capacity. Discharged through gates by means of water jets. *r,* Capacity of 64 mills, 3000 tons per 24 hr. (See Sec. 4, Table 77.) 40-hp. motor each with flexible coupling and herringbone gears. Mills served by 15-ton crane that can pick up a full mill; time to change about 1 hr. *s,* Cost (1923) from pool to re-grinding plant was $0.0247 per ton of sand, of which cost 45 per cent. was labor, 30 per cent. power and 25 per cent. supplies. Cost per ton in re-grinding plant was:

General expense		$0.0182
Sand conveying and distribution		0.0186
Grinding:		
Labor	$0.0137	
Power	0.1363	
Pebbles and lining	0.0938	
Other supplies	0.0089	
		0.2527
Table treatment		0.0400
Total		$0.3295

Metallurgical results (1923): tons treated, 866,524. Assays, per cent Cu: feed, 0.723; tailing, 0.468, concentrate, 60. Recovery, 35.5 per cent. Cost per pound of copper, excluding smelting and selling, $0.064 (1923). About 1.5 lb. per ton of a mixture of coal tar from a local gas plant, Barrett coal-tar creosote, Semet Solvay residual coal-tar oil, Cleveland Cliffs wood creosote, and a little pine oil as required for frothing. No acid nor heat. u. Feed contains 25 per cent solids, enters 3rd cell. See also Table 38. a. See Table 38. Pulp fed to cell No. 3.

Copper-sulphide ores, coarse dissemination. The ANACONDA mill is typical of the old-style graded-crushing and gravity-concentration plant, with flotation added later. The antithesis is the Nacozari mill (MOCTEZUMA COPPER CO.), treating a somewhat similar ore and formerly making a similarly low-grade pyritic concentrate but now, with gravity concentration discontinued, eliminating the pyrite by selective flotation and making high-grade concentrate with substantially no loss in recovery and a new financial gain. The CHINO mill has been completely remodeled since the introduction of flotation, but gravity concentration had to be retained to scalp out oxidized copper minerals not amenable to flotation. At the ARIZONA HERCULES mill a similar concentrating problem is solved in a unique manner, by installing Harz jigs to treat the sand in the ball mill-classifier circuit. The BRITANNIA M. AND M. mill uses Hancock jigs to remove 5 to 8 per cent. of the feed at −0.25-in.

In general, with coarsely disseminated copper ores gravity concentration should precede flotation. Its use will probably increase recovery to some extent, even with non-oxidized ores, and will surely do so, if native copper and copper oxides are present; it will take some of the burden off the flotation machines, thus probably lessening the number required, and will simplify their operation; the granular concentrate from the gravity machines will decrease the amount of concentrate-dewatering equipment required and lower the moisture content of the concentrate as a whole; finally the product shipped will be easier to smelt. The disadvantages of including gravity concentration are increased complexity of flow-sheet and the addition of water on the gravity-concentrating machines, which water must be eliminated before flotation. These disadvantages are, in most cases, outweighed by the advantages listed. At the Nacozari mill the improvement in grade of concentrate by selective flotation over that obtainable by gravity concentration was an added advantage to the all-flotation flow-sheet which is not ordinarily present.

Anaconda Copper Co. Fig. 45. (PC.) Section 2. Nov. 1, 1923.

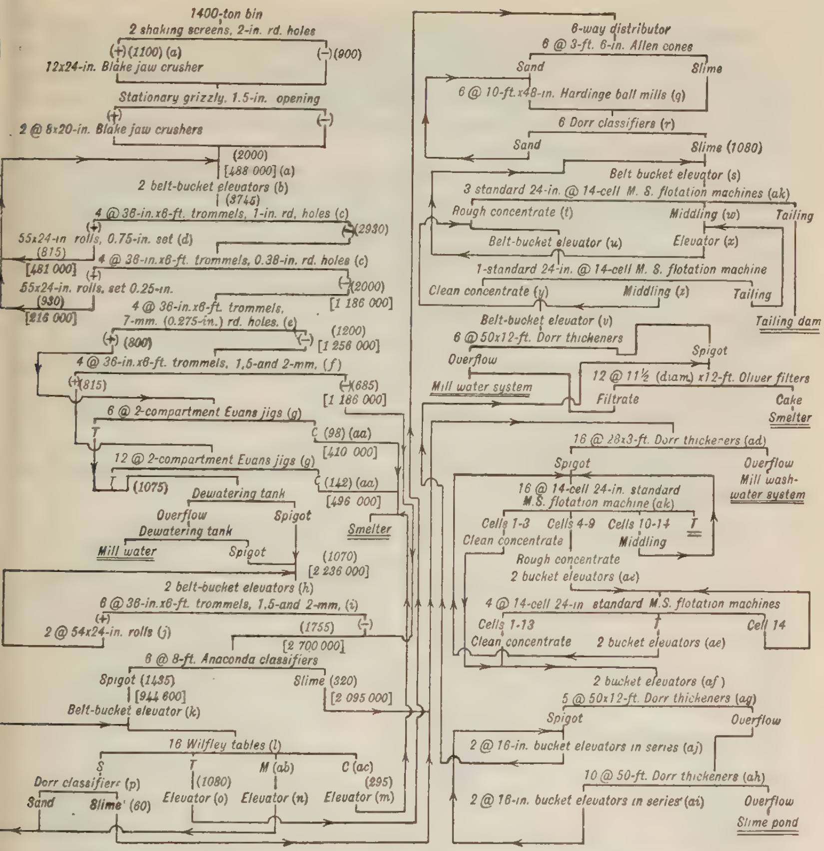
Location: Anaconda, Mont.

Ore: Bornite, chalcocite, chalcopyrite and pyrite coarsely disseminated in granite gangue.

Capacity: 2000 tons per section per 24 hr. 8 sections, [illegible]. Average daily mill tonnage, 15,000.

Assays, per cent Cu: Feed, 3.0 to 3.2; concentrate, 12 to 17; tailing, [illegible]

General: Ore from a group of underground mines at Butte, Mont., is hauled 32 miles in 50- and 60-ton standard-gage gondola cars to the mill at Anaconda, Mont.



a, Numbers in parenthesis are tons solid per 24 hr.; numbers in brackets are gallons of water per 24 hr. *b*, 461 ft. per min. 144 ft. of 18-in. @ 10-ply belt; 123 @ 8 × 16-in. buckets. *c*, 30 r.p.m. *d*, 85 r.p.m. *e*, 20 r.p.m. *f*, 2 × 12- and 1.5 × 12-mm. apertures. 20 r.p.m. *g*, 20 × 40-in. sieves, 8-mesh. *h*, 550 ft. per min. Each, 80 ft. of 24-in. @ 10-ply belt, 69 @ 10 × 22-in. buckets. *i*, As (*f*) except 30 r.p.m. *j*, Faces touching, 85 r.p.m. *k*, 450 ft. per min. 50 ft. of 20-in. @ 10-ply belt, 43 @ 10 × 18-in. buckets. *l*, 240 @ 1-in. strokes per min. Butchart riffling. *m*, Belt-bucket, 450 ft. per min. 84 ft. of 18-in. @ 10-ply belt, 72 @8 × 16-in. buckets. *n*, Belt-bucket, 450 ft. per min. 77 ft. of 18-in. @ 10-ply belt, 66 @ 8 × 16-in. buckets. *o*, Belt bucket, 440 ft. per min. 72 ft. of 20-in. @ 10-ply belt, 66 @ 10 × 18-in. buckets. *p*, 2 × 14-ft. simplex. *q*, 15 r.p.m. *r*, 2 × 14-ft. simplex, 23 r.p.m. *s*, 600 ft. per min. 47 ft. of 20-in. @ 10-ply belt; 43 @ 10 × 18-in. buckets. *t*, First 4 to 6 cells. *u*, 440 ft. per min. 50 ft. of 18-in. @ 10-ply belt; 46 @ 8 × 16-in. buckets. *v*, 440 ft. per min. 84 ft. of 18-in. @ 10-ply belt; 78 @ 8 × 16-in. buckets. *w*, Last 8 to 10 cells. *x*, 440 ft. per min. 96 ft. of 18-in. @ 10-ply belt; 89 @ 8 × 16-in. buckets. *y*, Cells 1 to 12, incl. *z*, Cells 13 and 14. *aa*, Assays @ 8 per cent. Cu, 15 per cent. insol. Drained in tanks to about 7 per cent. moisture. *ab*, Assay @ 0.9 per cent Cu. *ac*, Contains about 25 per cent insol. *ad*, 1 rev. in 12.25 min. *ae*, 440 ft. per min. 121 ft. of 18-in. @ 10-ply belt, 118 @ 8 × 16-in. buckets. *af*, 121 ft. of 20-in. @ 10-ply belt, 112 @ 10 × 18-in. buckets. *ag*, 1 rev. in 9.7 min. *ah*, 1 rev. in 21 min. *ai*, 1 @ 165 ft. of 18-in. @ 10-ply belt; 1 @ 102 ft. of the same. *aj*, 440 ft. per min. 1 @ 75 ft. of 18-in. @ 10-ply belt with 69 @ 8 × 16-in. buckets; 1 @ 129 ft. of 18-in. @ 10-ply belt. *ak*, 225 r.p.m. Reagents: Alkaline xanthate, pine oil and lime.

FIG. 45.—Anaconda Copper Co.

Summary. Receiving bin. Jaw crusher, 12- to 3-in.; jaw crusher, 3- to 1.5-in.; rolls, 1.5- to 0.75-in.; rolls 0.75- to 0.25-in.; rolls, 0.25- to 0.08-in.; one-stage ball milling, 2-mm. to 48-mesh. Screening between crushing steps and to prepare for jigging; hydraulic classification ahead of shaking tables. All sand tailing re-ground for flotation. Primary slime sent to separate flotation cells.

This plant is typical of large-tonnage, gravity-concentration plants with careful graded crushing and step concentration and flotation added at the end to recover sulphide mineral from the slimes. Compare with MOCTEZUMA COPPER CO. Concentrate is low-grade (about 10 to 12 per cent. Cu) on account of the large amount of pyrite present, but as the Anaconda Company also owns and operates the smelter and manufactures sulphuric acid both for its own hydrometallurgical uses and for sale, the pyrite is not objectionable. It is probable that were this mill to be built anew to-day jigs would be eliminated, the table feed would not be classified, and pneumatic flotation machines would be used.

Moctezuma Copper Co. (Phelps Dodge Corp.) Fig. 46. (*Q, 118 J 445.*)

Location: Nacozari, Sonora, Mex.

Ore: See Table 39. Chalcopyrite and copper-free pyrite in hard latite, andesite, monzonite, diorite and quartz porphyry.

Table 39. Analyses of feed, concentrate and tailing, Moctezuma Copper Co.

	Feed (*a*)	Feed (*b*)	General mill tailing (*b*)	Flotation concentrate (*b*)
SiO_2	56.3	70 (SiO_2, Al_2O_3, CaO, MgO)	54.00	4.84
Al_2O_3	16.5		16.30	1.97
CaO	....		2.10	0.46
MgO	....		1.99	0.22
FeO	1.8	....	....	....
Cu (Chalcopyrite)	3.30	....	....	....
Fe (Chalcopyrite)	2.92	....	....	....
S (Chalcopyrite)	3.95			
Fe (Pyrite)	5.08	....	....	....
S (Pyrite)	5.79	....	....	....
Cu, total	....	2.5–3	0.21	29.00
Cu, sulphide	....	....	0.15	27.96
Fe, total	....	11	10.24	28.50
Zn	....	....	0.49	0.60
S	....	....	6.30	33.74
Au	0.006 oz.	....	....	....
Ag	0.7 oz.	....	....	....
Remainder, incl. Na and K	....	....	8.37	0.67
Insoluble	....	....	....	7.50

a Questionnaire, 1919. *b* Composites, May, 1924. (*118 J 452.*)

Capacity: 3000 tons per 24 hr.

Assays: See Table 39.

Recovery: 92 to 94 per cent.

Ratio of concentration: 10 or 12 to 1.

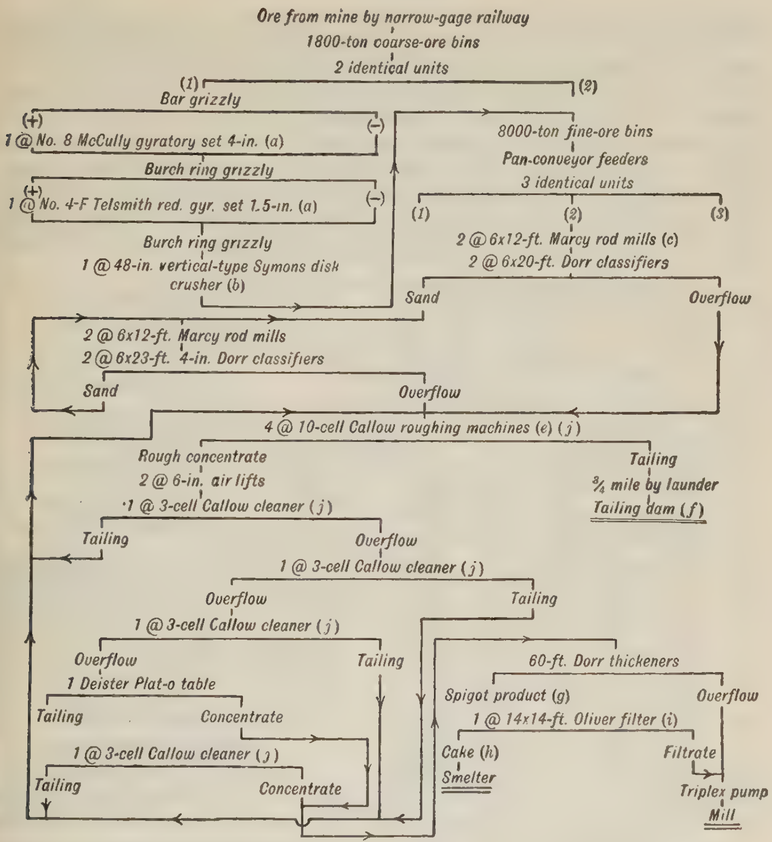
Percentage of possible running time: 97 per cent.

Labor: 31 tons per man-shift on operation; 75 per man-shift on repairs; 22 tons per man-shift, total.

Power consumption, hp.-hr. per ton: Coarse crushing, 1.1; all conveying ahead of rod mills, 0.7; two-stage grinding in rod mills, 12.5; flotation, including air lifts, 3.2, total, 23.2.

Water: 900,000,000-gal. reservoir with gravity flow to mill. Consumption: total, 1200 gal. per ton; reclaimed, 750 gal. per ton.

Distances: Mine to mill, 5 mi.; mill to smelter, 78 mi.; water, 3½ mi.; power, 1 mi.



a, See Sec. 3, Table 15. *b*, Product, 35 per cent. on 0.5 in. *c*, Product, −10-mesh. *e*, Feed − 48-mesh. Each bottom 42 × 42-in. canvas. Depth, 17 in. at head end to 26 in. at lower end. Flotation agents: 0.05 lb. crystalline potassium xanthate, 0.19 lb. steam-distilled pine oil; 5.5 to 6.5 lb. lime per dry ton of original mill feed. Best results are obtained when the tailing shows a lime equivalent of 0.15 to 0.35 lb. CaO per ton of clear solution and this is maintained by the operators by frequent titration. Soda ash, sodium silicate and other alkalis proved inferior. *f*, Now 105 ft. high. Sand placed by an overhead launder with de-sanding cones. *g*, 60 per cent. solids. *h*, 13 per cent. moisture. *i*, Change to xanthate and selective flotation has reduced filter requirement from three. *j*, The gradual elimination of iron and insoluble in a closely similar flow-sheet is shown in Fig. 47.

Fig. 46.—Moctezuma Copper Co.

Summary. Crushing by jaw and gyratory breakers and two sets of rolls in series from steam-shovel size to −8-mesh. Roughing-cleaning system of shaking-table concentration on −8-mesh primary feed and in re-grinding mill circuit. No gravity-concentration tailing made. Flotation by roughing-cleaning system with re-cleaning of concentrate.

This mill has been extensively remodeled since the introduction of flotation, but gravity concentration has been retained both on account of the

granular character of some of the sulphide and because of the presence of native copper, copper oxides and copper carbonates. The mill as originally built provided for graded-crushing, jigging the coarse sizes, with step hydraulic classification and treatment of classified products on shaking tables and vanners. Recovery by this method (1913) from 2-per cent. ore was 67.3 per cent.; ratio of concentration, 10.6 : 1; concentrate assay, 14.52 per cent. Cu. Iron content of ore was exceptionally high. In 1914 recovery was 73.4 per cent. and concentrate assayed 27.7 per cent. Cu. Capacity was about 6000 to 7000 tons per 24 hours. Capacity of the remodeled plant is about double this (12,000 to 14,000 tons per 24 hours), and recovery on feed containing only 1.5 to 1.6 per cent. Cu is between 75 and 80 per cent.

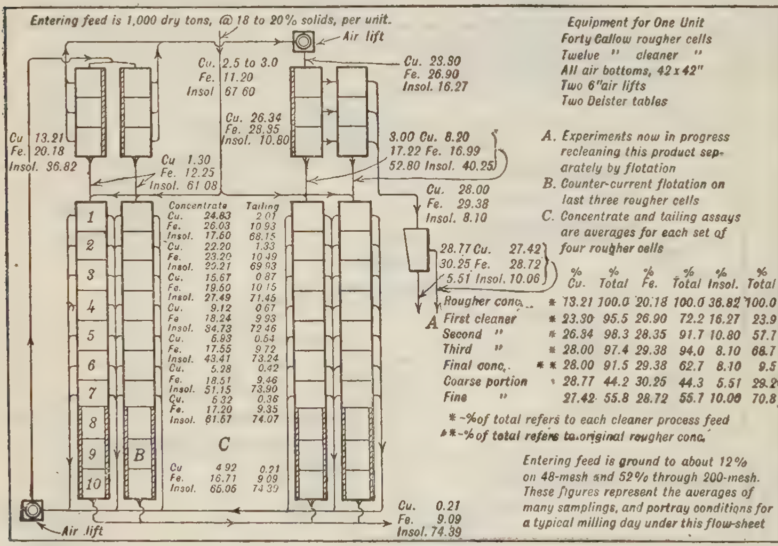


FIG. 47.—Interim assays in the flotation plant at MOCTEZUMA COPPER CO. (*after McDonald*).

Chino Consolidated Copper Co. Fig. 48. (*117 J 13, Q.*)

Location: Hurley, N. M.
Ore: Chalcocite and pyrite with some native copper and oxidized copper minerals in quartzite, diorite, porphyry and various metamorphic rocks (*116 J 984*)
Capacity: 12,000 tons per 24 hr.
Assays: Feed, 1.65 per cent. Cu; concentrate, 15.46 per cent; tailing, 0.554 per cent.
Recovery: 79; ratio of concentration, 13.6.

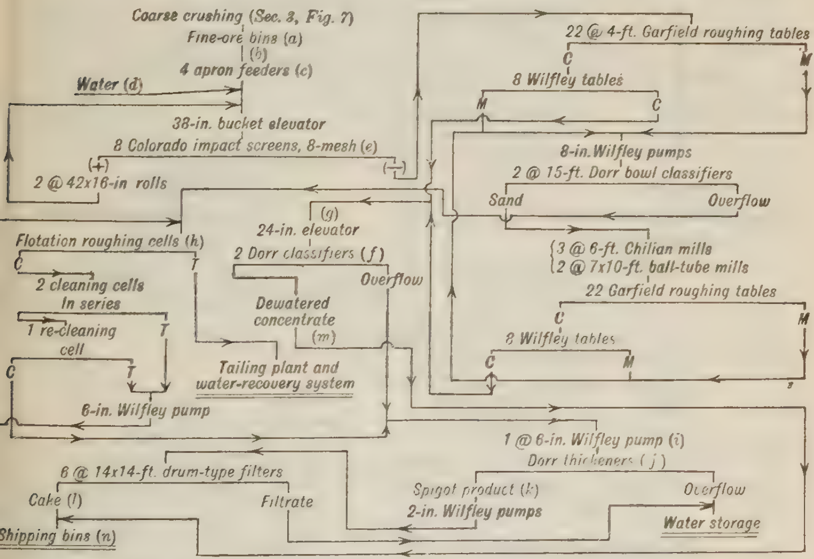
Britannia M. and M. Co. Fig. 49. (*28 MM 316; 115 J 375; 113 P 693.*)

Location: Howe Sound, Brit. Col.
Ore: Chalcopyrite and pyrite with a little sphalerite and galena in chloritic schist.
Capacity: 2500 tons per 24 hr.
General: Steeply sloping mill, six floors, 191 ft. high (216 ft. to the top of the receiving bins).

Summary. Underground crushing to −2.5-in.; 2-step reduction in rolls from 2.5- to 0.25-in.; closed-circuit tube milling to flotation size. CONCEN-

TRATION: Jigging at −0.25-in., flotation of primary slimes and re-ground jig tailing. Concentrate-middling routing.

Copper-sulphide ores, fine dissemination. Four general types of flow-sheets are employed in treating this class of ores, viz: *(1)* all-flotation; *(2)* gravity concentration followed by flotation; *(3)* flotation followed by gravity concentration; *(4)* flotation preceded and followed by gravity concentration. The first class is illustrated by the Magna mill of UTAH COPPER CO., the Superior mill of the ENGELS COPPER MINING CO., the No. 1 concentrator of the MOUNTAIN COPPER CO., the MIAMI COPPER CO. plant, and the MT. LYELL mill. The UTAH LEASING CO. mill, treating a dump tailing which is, artifi-



a, Cylindrical steel, 24 ft. 6 in. (diam.) × 40 ft. Two to a section, 14 in all. Live capacity 1000 tons each per 24 hr. *b,* One section. *c,* Two to a bin. *d,* 1.2 tons per ton of ore. *e,* 0.089-in. wire, 0.087-in. aperture. *f,* 4-ft. 6-in. duplex with vacuum trays and air jets. *g,* When magnetite is excessive in amount or is wanted for sponge iron, this concentrate is run to 3 drum-type magnetic separators. Sec. 13, Art. 13. *h,* Eight rows of Janney mechanical-air machines, each row with six agitating compartments (two ahead of the first air basket) and five @ 2 ft. 6 in. × 10-ft. air-basket compartments. *i,* Serves 1½ units. *j,* 5 @ 75 × 20 ft. and one @ 48 × 20 ft. *k,* 50 per cent. moisture. *l,* 20 per cent. moisture. *m,* 8 per cent. moisture. *n,* 17 per cent. moisture.

FIG. 48.—Chino Consolidated Copper Co.

cially, a finely disseminated ore, is of this same type. The mill at BRADEN COPPER CO. and the remodeled mill at the Morenci branch of PHELPS DODGE CORP. illustrate variants of the second type. INSPIRATION and CONSOLIDATED COPPERMINES are typical of the third class and the new Copper Queen concentrator of the PHELPS DODGE CORP. the fourth.

Utah Copper Co., Magna plant. Fig. 50. *(6 MMt 414; PC.)*

Location: Garfield, Utah.
Ore: Chalcocite and pyrite with a small amount of chalcopyrite in granitic porphyry.
Capacity: 25,000 tons per 24 hr.

Distances: Mine to mill, 18 miles; mill to smelter, 2 miles.
Cost of crushing: see note *v*.
For cross-section of crushing plant see Fig. 51.

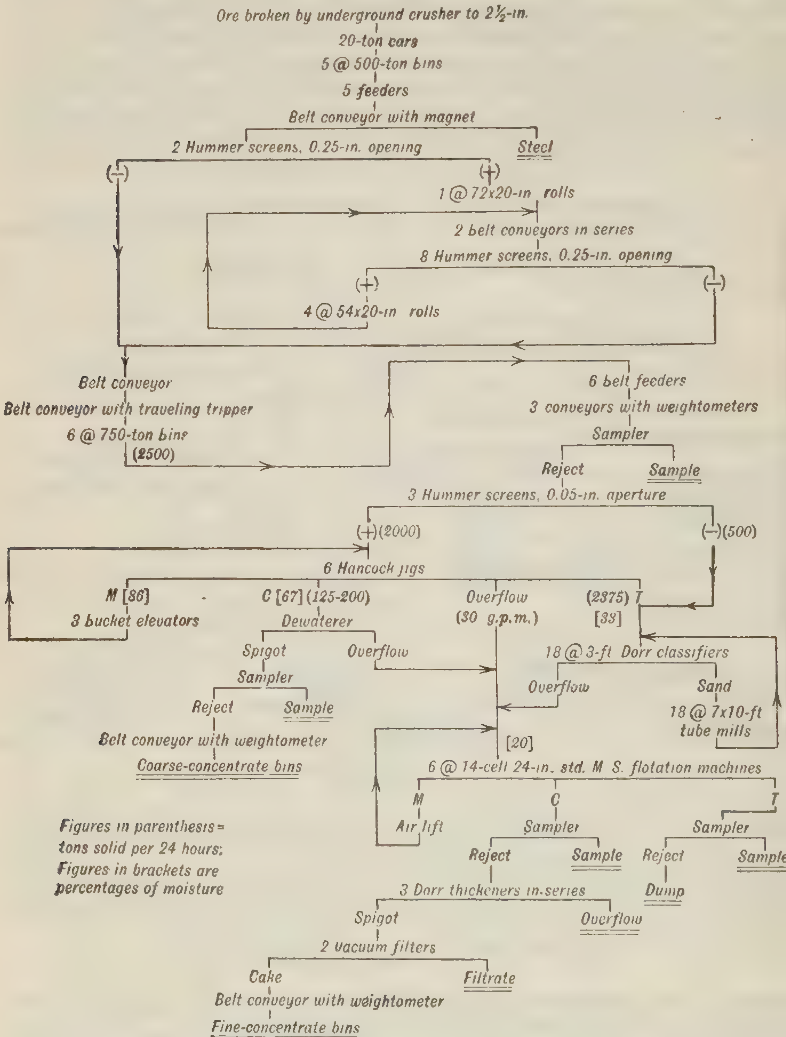


Fig. 49.—Britannia M. and M. Co.

Summary. No receiving bin. CRUSHING: gyratory, 54- to 6.5-in.; gyratory, 6.5- to 2.5-in.; rolls, 2 sets in series, 2.5- to 0.75-in. Circuit is closed on second rolls with impact screens. Rolls from 0.75- to 0.25- or 0.37-in. in closed circuit with impact screens. One-stage ball milling. All transport and

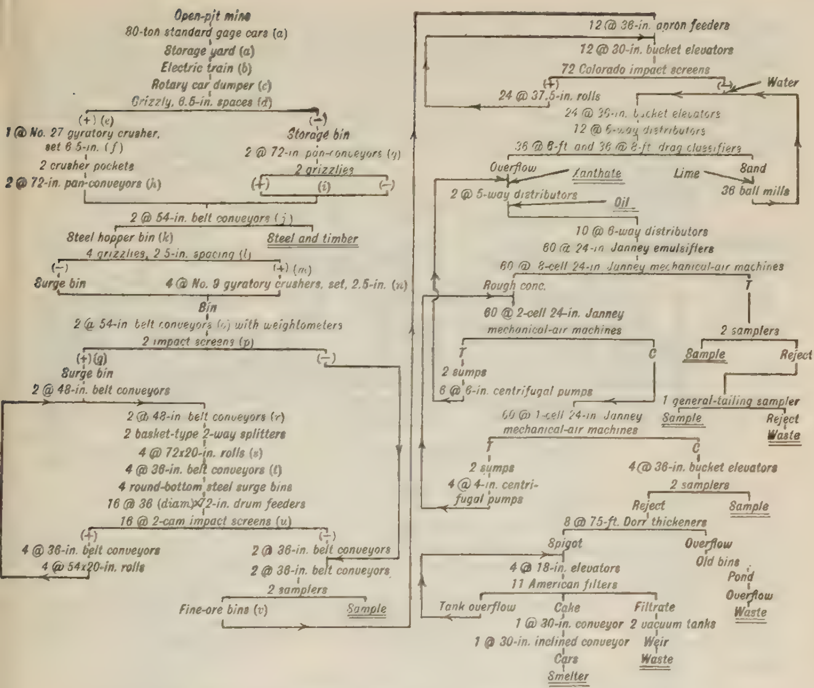


FIG. 50.—Utah Copper Co., Magna plant.

a, 45-car trains from mine to a 3-track loaded-storage yard with overhead trolley system. Storage capacity, 140 cars permanent and about 200 between train arrivals. Usual amount of ore in storage varies from 25 to 150 cars (2000 to 12,000 tons). ***b,*** 10- to 12-car trains hauled by electric locomotive and spotted two at a time in car dumper. Empties go by gravity to a 200-car, 3-track electrified empty-storage yard. Switching crews of 4 men. Two crews needed in summer and three in winter for medium tonnage. ***c,*** Two one-car units; may be operated separately or together. 50-hp. motor to each unit. 165° revolution. Dumper cycle is 2 min., but average time per car including spotting is 5 min. ***d,*** Dumped ore received on a pile of ore (see Fig. 51) which feeds two grizzlies side by side, each 1 car-length in width, 20 ft. long, sloping 42° from the horizontal. Each bar is a 20-in. T-beam capped with manganese-steel wearing plates. ***e,*** Estimated, 25 per cent. ***f,*** 54-in. gape, 2-way discharge, 250-hp. @ 300 r.p.m.; motor coupled direct to pinion shaft by flexible coupling. ***g,*** Driven by 2-speed motors. Controls tied in with controls of following belt conveyors, but feeder motors can be cut out independently by a push button. ***h,*** Driven by roller chain and sprocket through friction clutch from 54-in. belt-conveyor drive. ***i,*** Fines first onto belt. Crusher-pocket products come on after grizzly products. Object is to prevent cutting by large lumps. ***j,*** Each belt conveyor is fed by two pan conveyors, one each from a crusher pocket and the storage bin. Conveyors have ball-bearing troughing and return idlers. Slope, +19°. Each conveyor is driven through double-reduction gears by a 125-hp. 2-speed motor, fitted with automatic solenoid brake on a drum attached to the rotor shaft, in order to prevent the conveyor from running backward when stopped under load. Push buttons for starting and stopping are located near the head pulley and a button for stopping near the tail pulley. Speeds, 175 and 350 ft. per min., which allows one conveyor to take the full load when necessary. Steel and timber picked near head pulleys. ***k,*** For distributing purposes only. ***l,*** Standard manganese-steel T-bars. Each grizzly 6 ft. wide × 12 ft. long with bars spread slightly at the bottom. Slope, 42°. Spacing in summer, with dry ore, 1.25 in.; in spring and winter with wet and frozen ore, 2.5 in. ***m,*** About 30 per cent. of total. ***n,*** Short-head type. Direct-driven through flexible coupling by 125-hp. @ 450-r.p.m. motor. Normal control equipment at each motor and

cut-out buttons on the feed floor. Forced-feed lubrication with water-cooling for the circulating oil. An external oil-pumping and filtering system is provided for treating oil from all crushers. *o,* Ball-bearing idlers 6 sets of side idlers, equally spaced Head-pulley drive 50-hp motor each, 2-speed, double-reduction gears, flexible coupling, push-button control with stop buttons spaced at regular intervals along belts. Recording weightometers near head pulley. Moisture sample taken at head pulley. *p,* Opening varies from 1 in. square on dry ore to 1.5 × 3 in. on wet ore. Specially designed 4-cam

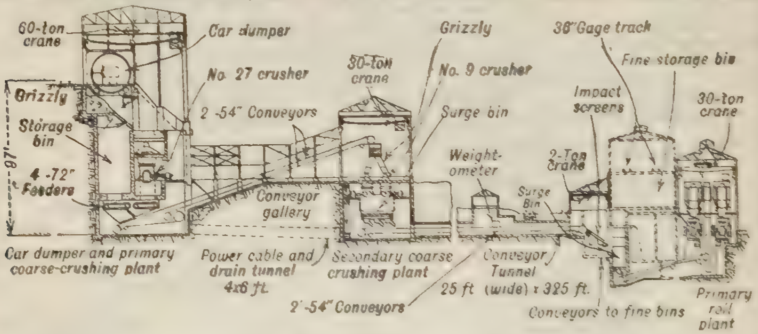


FIG. 51.—Coarse-crushing plant, Utah Copper Co., Magna mill.

screens. *q,* About 60 per cent. *r,* Incline, +21°. 420 ft. per min. Ball-bearing troughing and return idlers. Tandem drive. 100-hp. motor each with flexible coupling and reduction gears. *s,* Belt driven from line shaft, which is direct-driven through flexible coupling by a 250-hp. @ 300-r.p.m. motor. *t,* Slope, +21°. 370 ft. per min. Drive similar to 48-in. (note *r*) except 50-hp. @ 670-r.p.m. motors used. *u,* Aperture varies between 0.75 and 1 in. sq., as ore is dry or wet respectively. 1 @ 5-hp. motor to four screens. *v,* Cost (excluding overhead) for crushing to this size on average tonnage in 1925 was as follows:

	Per ton
Operating labor	$0.01053
Power	0.00828
Maintenance, labor and supplies:	
Car dumper	0.00072
Gyratory crushers	0.00294
Feeders and conveyors	0.00747
Screens	0.00173
Rolls	0.00737
Total	$0.03904

elevation in coarse-crushing plant by belt conveyors. CONCENTRATION: All-flotation. Rougher-cleaner routing. Concentrate cleaned twice.

This plant, built about five years later than the Arthur plant of the same company, is of interest in showing the retention of gyratories and rolls for crushing and of all-conveyor transport, but substitution of impact screens for 0.75-in. screening in place of the vibrating screens (Mitchell type) in the earlier plant. The impact type is the more rugged and the amplitude of vibration is greater. The later plant also contains more bins or hoppers in the course of the flow, which insures a more regular feed to all machines following the initial crusher and, therefore, better operation. The new plant uses more rolls and those for final crushing are of smaller diameter than in the old plant. This is in line with usual practice.

Engels Copper Mining Co., Superior mill. Fig. 52. (*Q; 123 P 183; 111 J 904*)

Location: **Engelmine, Calif.**

Ore: Very hard and tough. Altered diorite with bornite, chalcopyrite, chalcocite and covellite; some malachite and chrysocolla near surface.

Capacity: Have treated 1200 tons per 24 hr. on flow-sheet given. Average, 30,000 tons per month.

Assays: Feed, 2.25 per cent. Cu; concentrate, 30 per cent. Cu, 8 oz. Ag, 0.1 oz. Au; tailing, 0.45 per cent. Cu.

Recovery: 81 per cent.

Ratio of concentration: 15 : 1.

Labor: 30 tons per man-shift, operating; 130 tons per man-shift, repairs.

Water: 3.5 tons per ton of ore milled, none re-used. Transported 4½ miles by flume.

Percentage possible running time: 90.

General: Two mines, one at mill, the other 2½ miles away, deliver ore by aerial tram. Concentrate with 12 per cent. water shipped 600 miles to Garfield, Utah.

Arrangement: Steep mill site. See Fig. 53 for section of mill.

Flotation agents: 26 per cent. fuel oil, 53 per cent. Diesel oil (24° Bé. California fuel oil), 21 per cent. No. 14 Georgia pine; 80 per cent. added at ball mills; balance to tube mills and head flotation cells.

Metallurgical data, 1920: Tons milled, 239,612; lb. concentrate, 28,799,769; assays as above.

Costs: 1920: See Table 40.

Table 40. Milling costs at Engels Copper Mining Co., 1920

Item	Dollars per ton, milled		
	Operation	Repairs	Total
Coarse crushing	$0.0467	$0.0489	$0.0956
Classifying	0.0023	0.0042	0.0065
Roll crushing	0.0001	0.0058	0.0059
Grinding	0.1621	0.0856	0.2477
Re-grinding	0.0882	0.0214	0.1096
Conveying	0.0092	0.0070	0.0163
Pumping	0.0044	0.0153	0.0197
Concentrating	0.0923	0.0132	0.1055
Filtering	0.0262	0.0104	0.0366
Dewatering	0.0714	0.0985	0.1699
Power	0.2909	....	0.2909
Assaying	0.0086	....	0.0086
General	0.0146	....	0.0146
Heating and lighting	0.0816	....	0.0816
Boarding house	0.0317	....	0.0317
Total	$0.9303	$0.3103	$1.2407

Summary. CRUSHING: Jaw crusher, 16- to 5-in.; gyratory, 12- to 3-in.; gyratory, 3.5- to 1.5-in.; rolls, 2- to 0.75-in.; 2-stage ball milling from −2-in. and −1-in. (in parallel) to 65-mesh. CONCENTRATION: Flotation by a complex roughing-cleaning routing using both agitation-froth and pneumatic machines for roughing and pneumatic for cleaning and re-cleaning.

This plant is illustrative of the most favorable type of sloping mill site; the hill is steep and there is solid bed rock practically at the surface so that foundations are small and few retaining walls are necessary. With the exception of the inclined conveyor from the secondary gyratories to the trommel, the elevator closing the circuit on the rolls, and the pumps for circulating flotation pulps, the products flow by gravity from feed bin to concentrate bin and tailing dam. The mill is set higher on the hillside than otherwise necessary in order to provide head-room for tailing disposal one mile distant. As a result concentrate must be lowered from filters to the shipping bin at considerable inconvenience on account of spill. A better design would have placed the filters just above the shipping bins with thickened filter feed flowed down hill in pipes or launders. At MIAMI both thickeners and filters are placed at the bottom of the hill and unthickened concentrate is flowed to

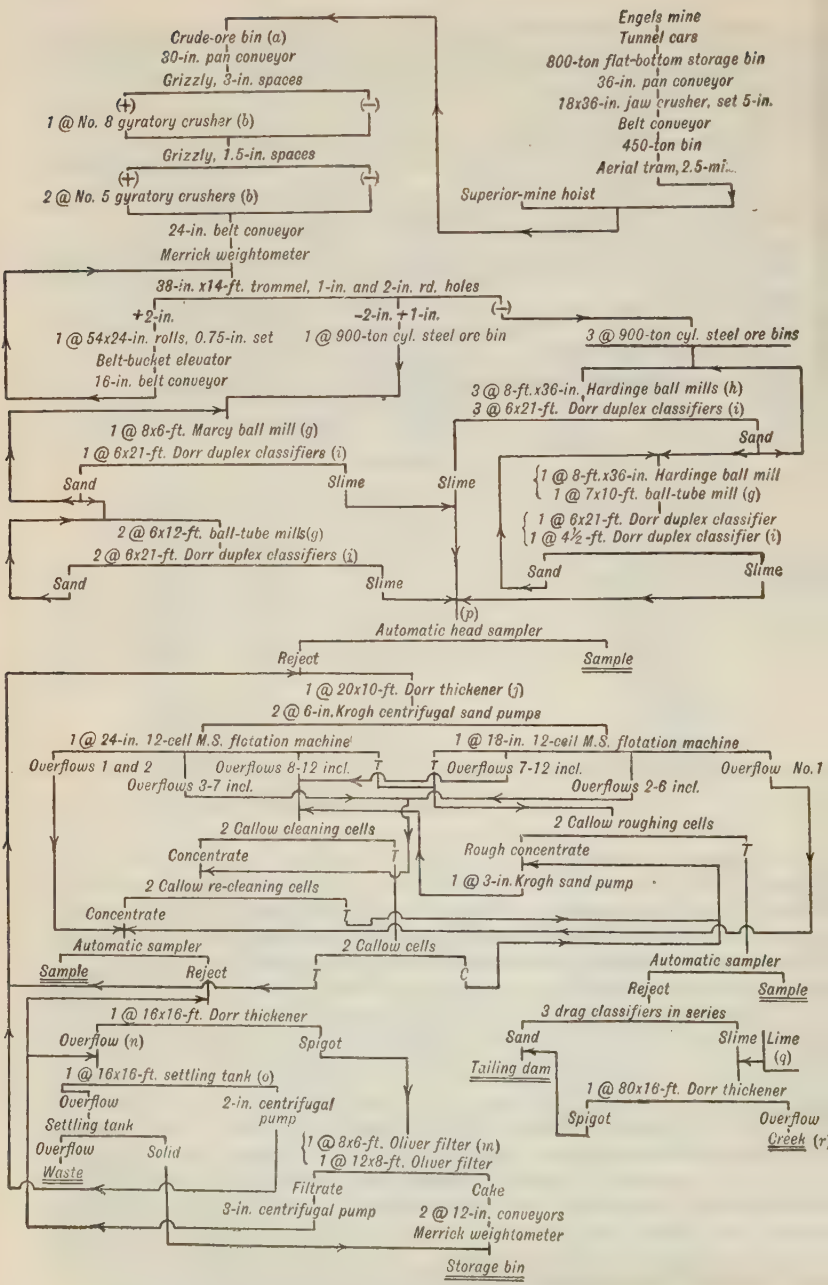


FIG. 52.—Engels Copper Mining Co., Superior mill.

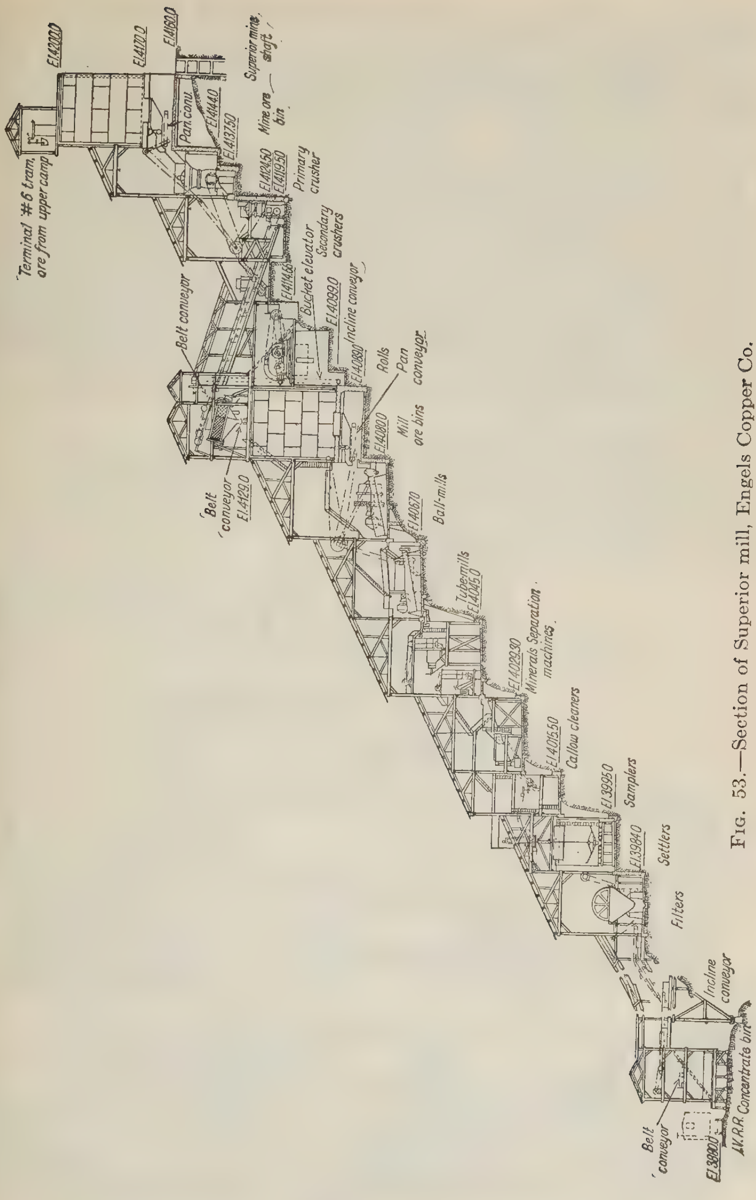


FIG. 53.—Section of Superior mill, Engels Copper Co.

the former. Outside inclined tramways, one each side of the mill, serve the different floors, at considerable expense.

Careful experimental work has proved that by crushing all ball-mill feed to pass 1-in. ring one ball mill followed by two tubes could grind 866 tons per day to 10 per cent. + 100-mesh at an expenditure of 10.7 kw.-hr. per ton ground against 444 tons for one ball and one tube mill, consuming 15.1 kw.-hr. per ton ground, according to the flow-sheet shown.

Notes to Fig. 52.

a, Cylindrical, steel, 24 ft. 6 in. (diam.) × 30 ft., flat-bottom, 837 tons live capacity. ***b,*** See Sec. 3, Table 15. These crushers have suffered undue breakage, probably due to iron. Cobbing magnets have now been put in ahead of the crushers. ***g,*** See Sec. 4, Table 5. ***h,*** See Sec. 4, Table 11. ***i,*** See Sec. 6, Table 40. ***j,*** Feed contains 33 per cent. solids; no overflow, acts as equalizer; 1.03 r.p.m.; draws 2.6 hp. This surge tank is a marked aid to good work in the flotation cells. ***m,*** The larger filter is sufficient. The smaller one is a reserve. ***n,*** Purposely cloudy in order to remove semi-colloidal silicious material that clogs filters. ***o,*** Has no rakes. Discharged intermittently. ***p,*** 10 per cent. + 100-mesh most economical. Finer grinding gives higher recovery but causes trouble in filtering, yields a wet cake, and in the final analysis costs more than the increased saving. ***q,*** Added as emulsion. 1.75 lb. per ton of original feed. ***r,*** Dewatering and clarification compulsory. Cost \$0.17 per ton of ore milled in 1920.

Mountain Copper Co., No. 1 Concentrator. Fig. 54. *(119 P 331.)*
Location: Shasta Co., Calif.
Ore: Chalcopyrite and pyrite in alaskite-porphyry gangue.
Capacity: 550 tons per 24 hr.
Assays: Feed, 2 per cent. Cu and 8 per cent. Fe; concentrate, 15 per cent. Cu.
Recovery; 92 per cent.
Ratio of concentration: 7 : 1.
Labor: 27.5 tons per man-shift (*e*) (*k*).
Power: 25.8 hp.-hr. per ton, installed.

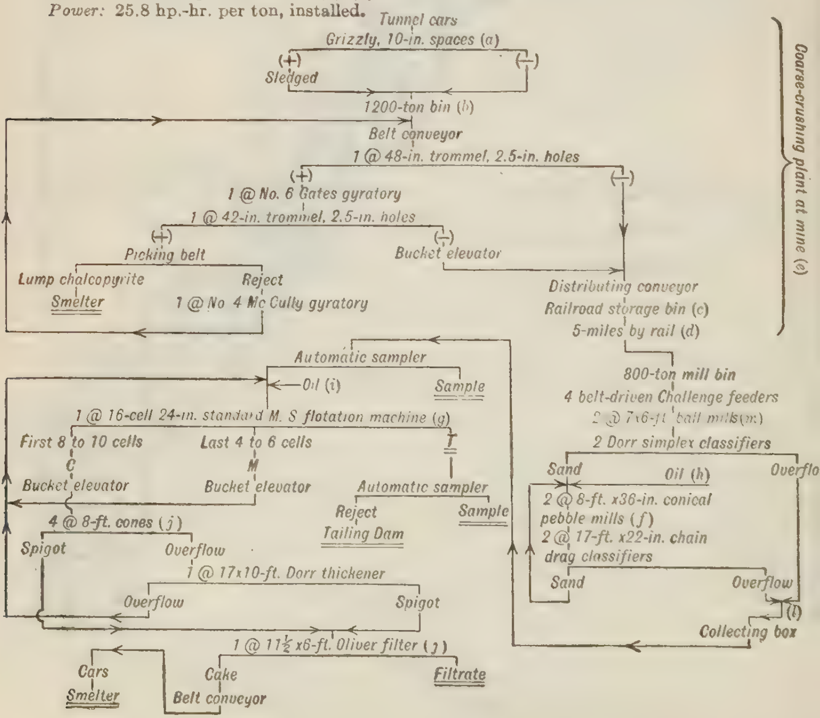


Fig. 54.—No. 1 concentrator, Mountain Copper Co.

a, 30-lb. rails over bin. One man sledges oversize through. *b*, 6 feed gates. One feeder man. *c*, 700-ton capacity. *d*, Narrow-gage track. Train: 1 Shay engine and 7 @ 20-ton cars. *e*, Capacity, 690 tons in 8 hr. Crew: 1 foreman, 1 oiler, 1 feeder, 1 roustabout, 1 grizzly man, 3 pickers. 100-hp. motor drives entire plant. *f*, Wedge-bar liners last 10 mo. in cylindrical and adjoining conical portion. Liners near trunnions last much longer. Mills not unloaded for liner renewal. Herringbone pinions last 22 to 26 mo. Consumption of California pebbles, 6 to 7 lb. per ton. *g*, Repairs for 2-years' operation total two sets of impellers, 2 sets of impeller-box liners and 1 set of regulating valves. Feed about 30 per cent. solid. *h*, Average 0.8 lb. per ton of ore: about one-half kerosene-acid sludge, balance a mixture of crude turpentine, tar oil and light fuel oil. *i*, Added to impeller boxes when needed. *j*, Entire flow to one cone until filled, then diverted to others in rotation. One purpose is to provide storage so that filter is run only half time. Principal purpose is to separate coarser concentrate, amounting to 75 per cent., which, when filtered separately, forms a 2- or 3-in. cake with 9 per cent. moisture. Dorr-thickener discharge forms ¾-in. cake with 14 per cent. moisture. If entire concentrate is sent to filter, cake carries 13 per cent. moisture. Settled concentrate in cones is loosened slightly with high-pressure water and then "boiled" with compressed air for 5 or 10 min. before filtering. It then draws readily. *k*, Mill crew: 1 flotation operator, 1 oiler, 1 man full time on concentrate handling and 1 man half time, the balance on sampling, repairs, etc.; 3 roustabouts on day shift, 1 foreman: total crew for 3 shifts, 12 men. *l*, Not over 4 per cent. −60 mesh. *m*, 4- and 5-in. chrome-steel balls, consumption, 1.8 to 2 lb. per ton. Chrome-steel liners last 4 mo.

Summary. Crushing: 2 gyratories in series, 10- to 2.5-in. in closed circuit with a trommel; 2-stage grinding in ball and pebble mills to 4 per cent. −60-mesh, ball mills in open circuit. Concentration: All-flotation, concentrate-middling routing, one agitation-froth machine.

Miami Copper Co. Fig. 55.

Location: Miami, Ariz.
Capacity: 7000 tons per 24 hr.
Ore: Chalcocite and pyrite finely disseminated in granitic schist.
Assay, per cent. Cu: Feed, 1.60 to 1.90; concentrate, 35 to 45; tailing, 0.1 to 0.15 sulphide; 0.25 to 0.30 total.
Recovery: 88 per cent. average.
Ratio of concentration: 20 or 21:1.
Labor: 46.2 tons per man per shift, operating; 241 tons per man per shift on repairs.
Water consumption: 2.8 tons per ton of ore milled; 66.5 per cent. re-used.
Power consumption: 15.8 hp.-hr. per ton milled.
General: Mill at mine. Water transported from 2 to 7 miles from wells. Railroad, ½ mile from mill. Power transmitted ½ mile. Concentrate shipped 1 mile. Gently sloping mill site.

Summary. Crushing: Gyratory from 12- in. to 2.5-in.; 2-stage roll crushing with second roll in closed circuit, 2.5- to 1.5-in. and 1.5- to 0.62-in.; 3-stage ball milling to 48-mesh. Concentration: All-flotation, rougher-cleaner routing. Primary flotation at 10 to 15 per cent. +48-mesh; slime from primary tailing re-floated in secondary machines, sands re-ground and returned to the primary machines.

Notes to Fig. 55.

Numbers in parentheses indicate tons solid per 24 hr., those in brackets, tons water per 24 hr. *a*, There is some difference between the different sections. *b*, See Sec. 23, Art. 3. *c*, Because of small storage capacity the coarse-crushing plant must be operated 24 hr. Hence two complete units like the preceding are maintained, although only one is in use at any time. *d*, Throw at discharge, 1-in.; 50-hp. motor, 22 hp. consumed. Spindle speed, 118 r.p.m. Oil consumption, 0.35 qt. per shift. Life of eccentric bushings, 30 days. Manganese-steel concaves last 100 days; about 8 hr. required for a change. *e*, 50-hp. motor, 125 r.p.m. Shells, Midvale steel, 5 in. thick when new and 2 to 3 in. when discarded. Bearings re-babbitted every 45 days, average. *f*, 200-hp. motor, 126 r.p.m. Shells, Midvale steel, 6½ in. thick new, wear to 2 in. in 45 days, when they are discarded. Manganese-steel cheek plates last 7 days. Bearings re-babbitted every 45 days, average. Eight

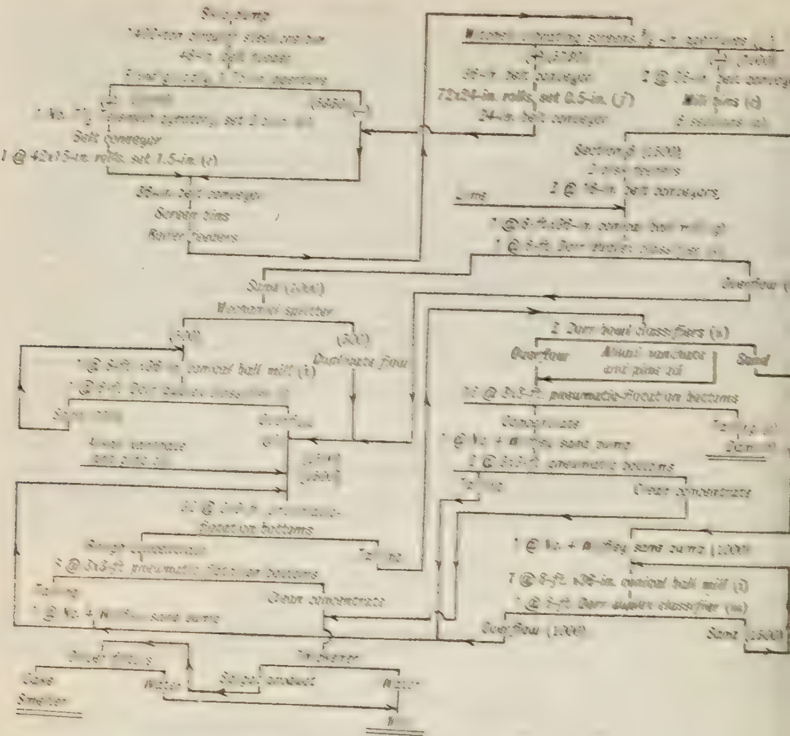


FIG. 35.—Mount Copper Co.

hours required to change shells. [illegible] 150-hp. motor, 21 r.p.m. 30 per cent. moisture in feed. Chrome-steel balls, consumption [illegible] per ton. Six to eight hours required to re-line, [illegible] men. *h*. See also note (*g*). Manganoid balls, consumption 1.5 lb. per ton. [illegible] manganoid balls, consumption 0.9 lb. per ton. (See also note *g*.) [illegible] 7 to 8 days. [illegible] per cent. undersize in oversize. *k*. 5-hp. motor. 28 @ 15-in. strokes per min. Slope, 2.8 in. per ft. 25 per cent. solids in overflow, 20 per cent. moisture in sand discharge. *l*. [illegible] motor. 28 @ 15-in. strokes per min. Slope, [illegible] in. per ft. 25 per cent. solids in overflow, 30 per cent. moisture in sand discharge. *m*. [illegible] motor, [illegible] @ 15-in. strokes per min. Slope, 2.5 in. per ft. Moistures as in note (*k*). *o*. 0.3 to 0.5 per cent. +48-mesh. *p*. 10 to 12 per cent. +48-mesh. *n*. 6-ft. [illegible] motor. 26 @ 15-in. strokes per min. Slope, [illegible] in. per ft. [illegible] per cent. moisture sand, [illegible] per cent. moisture overflow, [illegible] per cent. solids

Mount Lyell M. and M. Co. Ltd. Fig. 36. [illegible]

Location: Mount Lyell, Australia.

[illegible] chalcopyrite and pyrite [illegible] in schistose gangue with stringers of sulphide-bearing quartzite. [illegible] chalcopyrite, and pyrite in quartzite with some schist. For analysis see Table [illegible].

[illegible]

	Cu, per cent.	Ag, oz. per ton	Au, oz. per ton
Feed	3.43	0.54	0.018
Concentrate	10.95	1.47	0.047
Tailing	[illegible]		

Capacity [illegible] *tons per 24 hr.*
Recovery, per cent.: Cu, 90.4; Ag, 77 [illegible] Au, 72 [illegible]
Ratio of concentration [illegible]

Table [illegible] [illegible] Lyell ore. *(After Waterhouse)*

	[illegible] Comstock	[illegible] Lyell
Cu, per cent.	2.6	3.5
Ag, oz.	0.23	0.7
Au, oz.	0.02	0.01
SiO_2, per cent.	46.0	66.0
Fe, per cent.	11.5	8.0
Barite	1.4	2.0
Al_2O_3	15.5	7.7

Summary. CRUSHING: Jaw crusher, 10- to 2-in.; [illegible] slow-speed geared rolls in closed circuit with shaking screens, 2- to 0.12-in.; tube mills, open-circuit, from 0.12-in. to 60-mesh. CONCENTRATING: All-flotation, com[illegible]nation roughing. Concentrate handling is peculiar in that no thickener is used ahead of the filter. Bucket elevators are used throughout for lifting pulp. This is old-style and less satisfactory practice on fine sands and slimes than the usual American practice with centrifugal pumps.

Table 32. Sizing analysis of flotation feed and products [illegible] *(After Waterhouse)*

Screen, [illegible] mesh	Feed: Weight, per cent.	Feed: Cu, per cent.	Feed: Fe, per cent.	Concentrate: Weight, per cent.	Concentrate: Cu, per cent.	Concentrate: Fe, per cent.	Concentrate: Insoluble, per cent.	Tailing: Weight, per cent.	Tailing: Cu, per cent.	Tailing: Fe, per cent.	Recovery, per cent.
40	4.7	1.80	6.4	1.2	8.25	28.6	37.3	6.4	0.90	3.5	56.1
60	10.1	2.17	8.3	6.5	7.50	25.2	36.8	10.6	0.68	2.5	75.9
80	15.4	2.35	9.2	11.7	8.10	26.6	32.2	14.8	0.45	1.6	81.4
120	9.2	2.45	9.8	8.4	8.60	28.6	28.6	4.5	0.30	1.2	91.1
150	4.1	2.90	10.1	5.1	9.30	29.8	24.1	4.0	0.28	1.2	96.8
−150	56.5	3.12	8.7	67.1	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Total	100.0	2.77	[illegible]	100.0	10.01	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

Notes to Fig. 56.

a, [illegible] 16-ton hopper cars with [illegible] locomotive. *b*, [illegible] lined with [illegible] manganese-steel jaw plates. [illegible] *c*, Water spray [illegible] ft. long, [illegible] 275 ft. per min. [illegible] is scraped off into a launder at start of return run. [illegible] round-hole punched plate, 14 or 16 gage. [illegible] of ¼-in. plate. [illegible] @ 1½-in. strokes per min. *f*, 25 per cent. moisture. *g*, 24 r.p.m. [illegible] ged roll belt-driven, plain roll gear-driven. Life of chrome-steel [illegible] tons. *h*, [illegible] annular opening. *i*, [illegible] per cent. moisture. *j*, −[illegible]-mesh. *k*, [illegible] type. [illegible] r.p.m. Each gear-driven by [illegible]-hp. [illegible] rings on both rotor and counter shaft. [illegible] mechanical performance. Mills are run in [illegible] and liners. Life of ribbed cast-iron shell liners 25,000 tons [illegible]

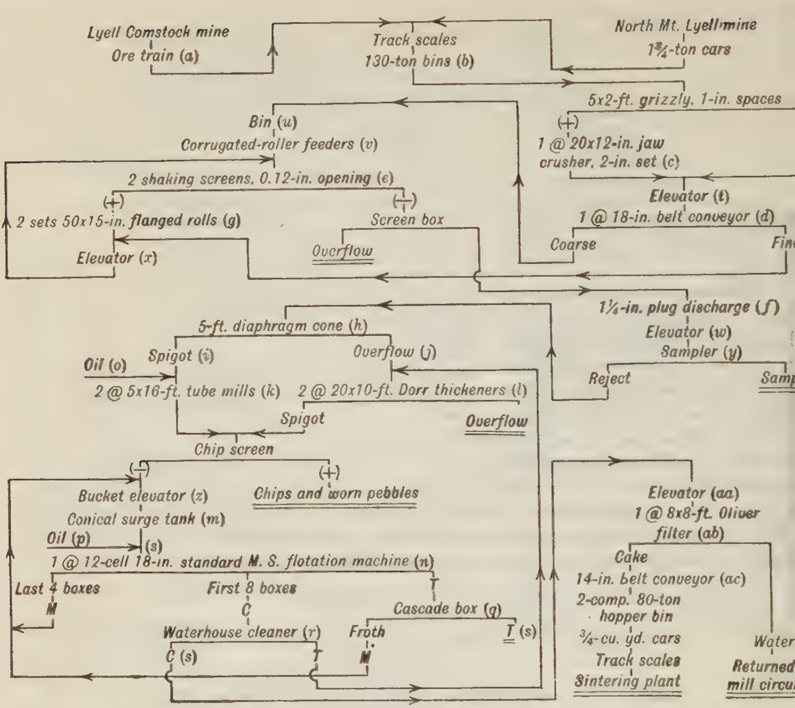


FIG. 56.—Mt. Lyell M. and M. Co.

sumption Tasmanian quartzite pebbles (2½- to 4-in. diam.). 11 lb. per ton. *l*, 8 min per rev. Spigot product, 40 per cent. solid. Tanks, 3/16-in. plate on sides and ¼-in. on bottom. Gravity discharge through 4-in. gate valve. *m*, 2½-ft. diam. × 3½ ft. deep. *n*, Cast-iron liners. 4-bladed impellers, 19-in. diam., 320 r.p.m., ¼-turn belt drive, 2-in impeller shafts, S. K. F. radial and thrust ball races. 4-in. ball-bearing line shaft. 75-hp motor, 14-in. camel-hair belt. 2-bladed froth skimmers, 19 r.p.m. Air added under first impeller. 18 per cent. solids in feed. Temperature varies from 47° F. in winter to 74° in summer. Operations are best at the higher temperatures. *o*, Mixture: one part eucalyptus oil and two parts coal tar by disk feeder; eucalyptus oil separately by pet cock. *p*, As needed. *q*, See Fig. 57. *r*, Not used regularly on account of excessive water consumption. Consists of box containing concentrate, into which high-pressure (80 lb.) water is passed by a jet ejector (3/16-in.) *s*, For sizing analysis see Table 42. *t*, 33¼-ft centers, 75° slope; 12-in., 8-ply balata belt; 12 × 6 × 5-in. buckets spaced 15 in. 380 ft per min. *u*, 2 @ 150 tons each. 45° bottom slope. *v*, 20 × 20 in. 2 r.p.m. *w*, 35½-ft centers, 80° slope, 12-in. @ 8-ply balata belt; 12 × 6 × 5-in. buckets spaced 9 in. 41 ft. per min. *x*, 25¾-ft centers, 80° slope, 12-in. @ 8-ply balata belt; 12 × 6 × 5-in. buckets spaced 9 in. 367 ft. per min. *y*, Double-cone type. Cuts 1 part in 3280. *z*, 27½-ft centers, 80° slope, 12-in. @ 8-ply balata belt; 12 × 6 × 5-in. buckets spaced 9 in. 35 ft. per min. *aa*, 40¼-ft. centers, 80° slope, 12-in. @ 8-ply balata belt; 12 × 6 × 5-in buckets spaced 24 in. 367 ft. per min. *ab*, 1 rev. in 8 min. Dry-vacuum pump, 3½ (diam.) × 8-in. 250 r.p.m. 22-in. vacuum. Feed contains 52 per cent. moisture and cake 11.5 per cent. Cake thickness, ¼ to 2 in. Capacity about 600 lb. solid per sq. ft. per 24 hr. Drum winding, 14-gage copper-clad steel wire, turns spaced ⅝ in. Life of canvas about 8 mo., 16 hr. required to re-cover and re-wind. Pneumatic agitation unsatisfactory. Filter replaced 3 @ 30 × 8-ft. steel draining tanks with filter bottom, connected with wet-vacuum pump. Concentrate drained therein to 15 per cent. moisture in one week. **ac, 50½-ft. centers, +26° slope, 137 ft. per min. 4-ply belt.**

Utah Leasing Co. Fig. 58. *(Q; 20 S. L. Min. Rev. 21; 105 J 535.)*

Location: Newhouse, Utah.

Ore: Dump tailing from Cactus mill of So. Utah Mines and Smelters. Chalcopyrite in quartzite gangue. Average copper content, 0.70 per cent., of which 0.10 per cent. is oxidized in sandy material and as much as 0.30 per cent. in slimes. Oxidation is greatest in dry, shallow parts of dump. For sizing tests of sand and slime portions see Table 43.

Capacity: 700 tons per 24 hr.

Assays, per cent. Cu (Sept., 1918): Feed, 0.584; concentrate, 16.85; tailing, 0.202. Concentrate contains about 2.5 oz. silver and 0.09 oz. gold per ton.

Recovery: (Aver. 1918) 62 to 65 per cent.

Ratio of concentration: Aver. 40 : 1.

Percentage possible running time: 92 (aver. 3½ years).

Labor: 104 tons per man-shift, operating; 310 tons per man-shift, repairs.

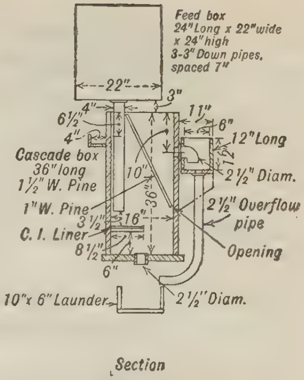


Fig. 57.—Cascade-type tailing-retreatment box, Mt. Lyell mill.

Power: 17 hp.-hr. per ton milled. Motors 5- to 20-hp., 440-volt; over 20-hp., 2200-volt.

Water: 750 gal. per ton of ore milled.

General: Dump material excavated with 30-ton steam shovel (a drag-line scraper failed) and hauled an average distance of 600 ft. to mill by dinky train. Cost per ton (1917), $0.12. Concentrate shipped 220 miles.

Summary. One-stage ball and pebble milling (mills in parallel) followed by agitation-froth flotation, concentrate-middling routing.

This is a typical, cheap dump-tailing flotation plant. Cost of such plants per ton of daily capacity is extremely low. (See Sec. 23, Table 37.)

Table 43. Sizing tests of feed to Utah Leasing Co. mill

Screen, mesh	Weight, per cent.		
	Sand	Slime	Mixed
4	0.5	....	....
8	5.5	....	5
14	30	....	25
28	29	....	28
35	9	....	....
48	7	....	18
65	5	1.2	6
100	5	2.0	5
150	4	6.5	....
200	2	3.5	....
− last screen	4	86.8	13

Notes to Fig. 58.

b, Belt-driven. Satisfactory. *c,* See Sec. 4, Table 11. *d,* 24 @ 12-in. strokes per min. Slope, 2⅞ in. per ft. 5-hp. motor. Overflow, 27 per cent. solid. Total load, 600 tons per 24 hr. Life of submerged rakes, 18 mo.; balance, 3 yr. *e,* For sizing test see Sec. 4, Table 11*a*. *g,* 23 @ 10-in. strokes per min. Slope, 2½ in. per ft. 3-hp. motors. Overflow, 27 per cent. solid. Total load, 400 tons per 24 hr. Life of rakes as above (*d*). *i,* 2 @ 22 × 10 ft., 1 @ 50 × 10 ft. Spigot products held at 55 to 60 per cent. moisture because elevator will not handle thicker material. *j,* Feed, 15 to 20 tons per 24 hr. all −65-mesh. Cake contains 18 to 22 per cent. water; as high as 26 per cent. in winter. Gage reading, 18 to 20 in. mercury. Speed, 1 rev. in 18 min. *k,* Cost (1917), $0.0805 per ton. Combined grinding cost (1917), $0.205 per ton. *m,* Cost of concentrate handling (1917), $1.26 per ton of concentrate or $0.0347 per ton of ore milled.

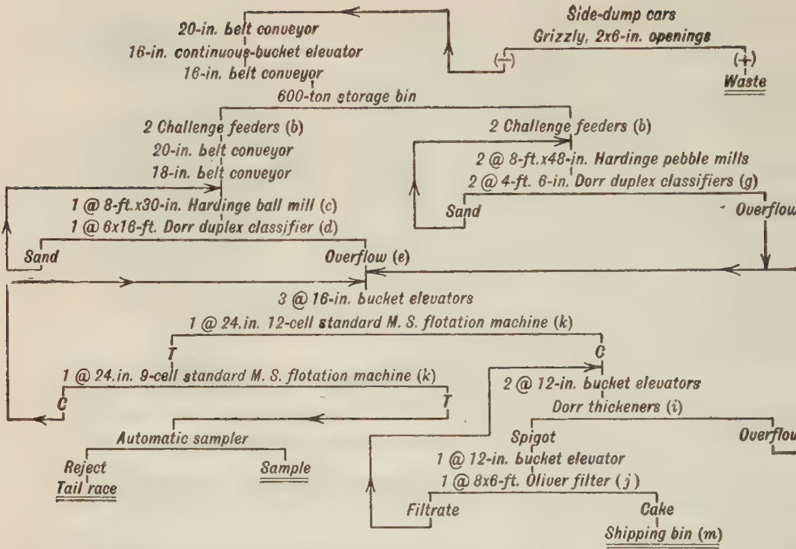


FIG. 58.—Utah Leasing Co.

Braden Copper Co. Fig. 59. (*Q; 101 J 315; 28 IMM 236.*)

Location: Sewell, Chile, S. A.

Ore: Chalcopyrite (75 per cent. of total copper), bornite and chalcocite in shattere andesite. Variable proportions of copper oxides, carbonates and silicates.

Capacity: 10,000 tons per 24 hr.

Assays, per cent. Cu: Feed, 2.0 to 2.3; concentrate, 18 to 20; tailing, 0.4 to 0.45.

Recovery: 80 to 82 per cent.

Ratio of concentration: 10 : 1.

Labor: 12 tons milled per man-day.

Power: 24.7 kw.-hr. per ton milled.

Water: 3.2 tons net per ton of ore.

General: Mill is about 1¼ miles from mine. Smelter on property. Power transmitte 18 miles at 33,000 volts. Ore hauled by electric railroad. Fall from top of coarse-ore bir to bottom of filter plant is 400 ft.

Summary. CRUSHING: Gyratory from 12- or 14- to 3-in.; disk crushers an rolls (in parallel) from 3- to 1-in.; ball mills from 1- to 0.16-in.; 2-stage ba milling with intermediate sizing, classification and table concentration t 40-mesh flotation size. CONCENTRATION: Shaking tables followed by flota tion of primary slimes and re-ground table tailing. Roughing-cleanin routing.

This mill replaced a smaller up-to-date gravity-concentration plan having jigs, tables and buddles, in which average recovery was only abou 50 per cent.

Notes to Fig. 59.

a, Numerator = tons solid per 24 hr.; denominator = tons water per 24 hr. *b,* 10-h motors. *c,* Length, 57 ft. 6-ply belt. 150 ft. per min. 15-hp. motors. 36 × 36-i Dings magnetic head pulleys. *d,* Length, 210 ft. 6-ply belt. 150 ft. per min. 10-h motor. 30 × 26-in. Dings magnetic head pulley. *e,* 30-in. × 18-ft. manganese stee *f,* McCully. 75-hp. motor. *g,* Length, 34 ft. 6-ply belt. 150 ft. per min. 5-hp. moto 30 × 30-in. magnetic head pulley. *h,* 88 r.p.m. 150-hp. motor each. *i,* 4 @ No. 7 Telsmith, 1 @ No. 7½ McCully. 75-hp. motor each. *j,* Lengths: 1 @ 24 ft., 4 @ 26 f

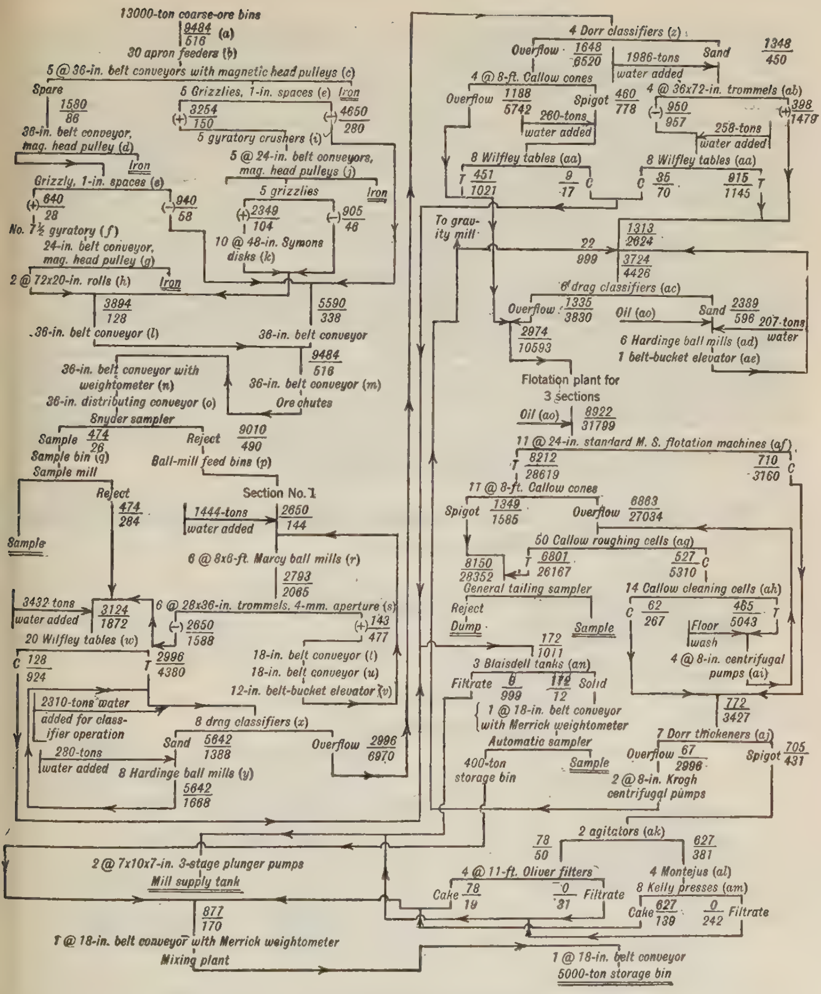


Fig. 59.—Braden Copper Co.

6-ply belt. 150 ft. per min. 5-hp. motor each. 30 × 26-in. magnetic pulleys. *k*, Horizontal-spindle type. Spindle, 100 r.p.m., 40-hp. motor. Eccentric, 30-hp. motor. See Sec. 3, Table 19*a*, for screen tests of feed and product. *l*, Length, 246 ft. 6-ply belt. 300 ft. per min. 20-hp. motor. *m*, Length, 55 ft. 6-ply belt. 190 ft. per min. 20-hp. motor. *n*, 8-ply belt. 350 ft. per min. 35-hp. motor. Merrick weightometer. *o*, Over ball-mill feed bins. Length, 365 ft. 6-ply belt. 350 ft. per min. 35-hp. motor. *p*, 7000-ton. 18 steel apron feeders each with 5-hp. motor. *q*, 500-ton. *r*, 17,500 lb. balls. 23 r.p.m. 225-hp. motors direct connected to mill countershaft. Herringbone gears, *s*, Attached to mill. Diagonal-slot plate screen. *t*, Runs in front of the 6 mills. Length, 54 ft. 5-ply belt. 140 ft. per min. 3-hp. motor. *u*, Length, 19 ft. 5-ply belt. 140 ft. per min. 3-hp. motor. *v*, Length, 45 ft. 8-ply belt. 300 ft. per min. Buckets, 6 × 6 × 10-in., spaced 18 in. 5-hp. motor. *w*, Butchart riffling. 240 r.p.m. 1 @ 50-hp. motor. Purpose is to remove free oxides and such oxide and carbonate as are adherent

to coarse sulphide and would be lost if slimed. *x*, Settling areas, 42 × 84 in. Chains 39 ft. long; blades, 3 × 32 × ¾ in., spaced 18 in. Chain speed, 40 ft. per min. 15-hp motor on line shaft for 8 drags. *y*, 2 @ 8-ft. × 48-in. and 6 @ 8-ft. × 36-in. Cascade liners. Cast-iron balls; average charge, 6600 lb. 27 r.p.m. 150-hp. motors. *z*, Duplex Model C. Slope, 2½ in. per ft. 15-hp. motor for four. *aa*, Butchart riffling. 240 r.p.m. 16 tables to 1 @ 15-hp. motor. *ab*, 2.5-mm. round-punched plate. 18 r.p.m. Slope, 1 in 10. 1 @ 7.5-hp. motor. *ac*, 42 × 90-in. settling areas. 24-ft. chains with 3 × 34 × ¾-in. blades spaced 18 in. Chain speed, 40 ft. per min. 15-hp. motor for 6 machines. *ad*, 5 @ 8-ft. × 30-in.; 1 @ 8-ft. × 22-in. Cast-iron cascade liners, aver ball load, 6600 lb. 27 r.p.m. 150-hp. motor each. *ae*, 18-in. @ 10-ply belt, 7 × 7 × 18-in. buckets spaced 16 in., 519 ft. per min. 25-hp. motor. *af*, Double-unit, Howard stirrers. 227 r.p.m. Each unit driven by direct-connected 150-hp. (390-r.p.m.) motor *ag*, 48 × 114-in. Slope, 4 in 10. Removable air baskets. Diaphragms of 3 thicknesses "Kelly" (filter) cloth and 1 of 16 oz. canvas duck, stitched. Air at 5 to 7 lb. 3 @ 20,000 cu. ft. multi-stage direct-connected turbo blowers for these and 14 cleaner cells. *ah*, Same size and type as roughing cells. *ai*, 36-ft. lift, 900 r.p.m. 2 pumps direct-connected to one motor. 1 @ 100-hp. and 1 @ 85-hp. motor. *aj*, 4 @ 60 × 13 ft., 1 @ 34 ft. 9 in. × 8 ft 7½ in., 1 @ 34 ft. 9 in. × 10 ft. 3 in., 1 @ 30 ft. × 10 ft. 3 in. 60-ft. tanks have 5-hp motors. *ak*, 12 (diam.) × 9 ft. 22 r.p.m. 3-hp. motor each. *al*, Each 225 cu. ft. capacity capable of 100 lb. per sq. in. *am*, Capacity, 175 cu. ft. each. 40- to 50-lb. pressure. *an* 30 (diam.) × 12 ft. 10-hp. motor each. Blaisdell "A" excavator, capacity 100 tons per hr. *ao*, Swedish or American pine-tar oil, fuel oil thinned with a small amount of kerosene. Some of the tar oil is added to the ball mills, the balance with the mineral oils and sulphuric acid is added at the primary flotation machine.

Phelps Dodge Corp., Morenci branch. Fig. 60. (*Q*; *Arthur Crowfoot PC*; *69 A 176*; *109 J 1349*.)

Location: Morenci, Ariz.
Ore: Chalcocite in monzonite porphyry, granite porphyry and quartzite.
Capacity: 4500 tons per 24 hr.
Assays: See Table 44.

Table 44. Phelps Dodge Corp., Morenci plant, August, 1925

Material	Percentages		
	Feed	Concentrate	Tailing
Copper, total	2.227	22.069	0.292
Copper, acid-soluble	0.176	0.623	0.132
Copper, sulphide	2.051	21.446	0.160*a*
Iron	3.2	26.20	1.0
Sulphur	3.1	32.8	0.2
Silica	63.0	11.3	68.0
Alumina	18.0	3.0	19.5
Lime	0.4	0.4	0.4
Insoluble	....	15.44	....

a With finer grinding this can be brought down to 0.12 per cent.

Recovery: 88.1 per cent. of total Cu; 92.9 per cent. of sulphide Cu.
Ratio of concentration: 11.25 : 1.
Water consumption: 282 gal. fresh water, 847 gal. reclaimed water, 1129 gal. total water per ton of ore.
Labor: Tons per man-shift, operating, 38.9; repair, 76.7; total, 25.8.
Power consumption: See Table 45.

General: Mine to mill, ¾ mile, max. Water pumped 6½ miles. Tailings run 12,000 ft through a flume on 2.85-per cent. grade. Railroad at mill. Sloping millsite. Power transmitted ½ mile at 2200 volts.

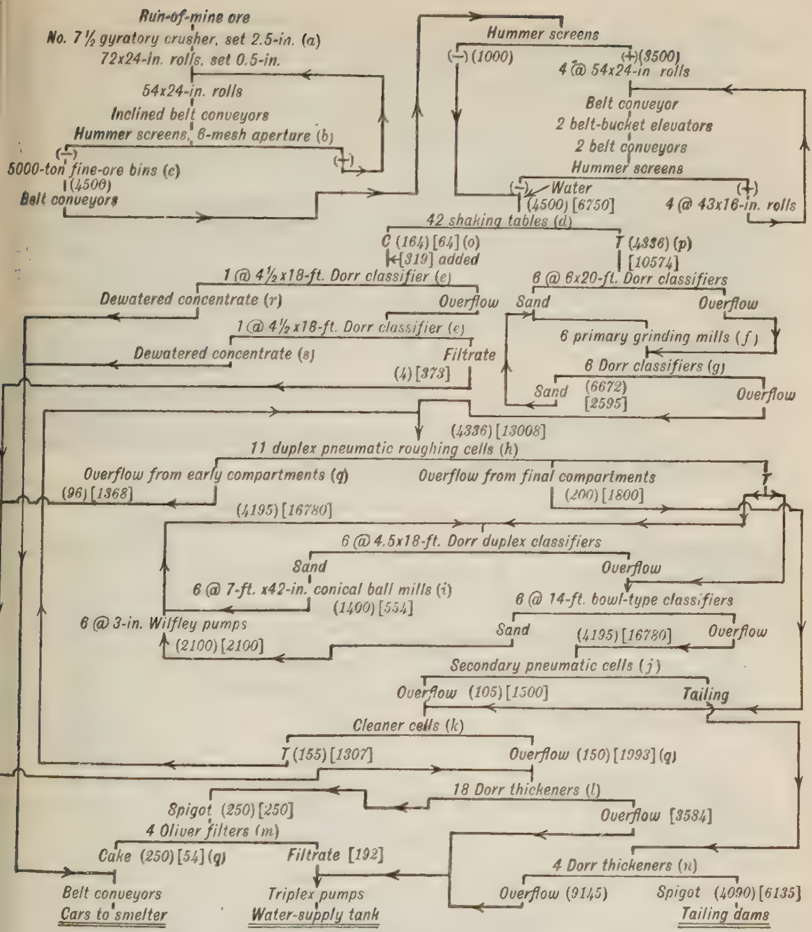


FIG. 60.—Phelps Dodge Corp., Morenci Branch.

Numbers in parentheses represent tons solid per 24 hr.; in brackets, tons water. *a*, See Sec. 3, Table 15. *b*, Screen area calculated to pass through 1.5 tons per sq. ft. per hr. *c*, Piped with compressed air to prevent hang-up of fine ore. *d*, 3886 tons wash water added per 24 hr. = 15.4 gal. per min. per table. *e*, With vacuum connection near sand-discharge end. *f*, Three 7 × 12-ft. cylindrical grate mills, two 8 × 6-ft. cylindrical grate mills, one 8-ft. × 48-in. conical ball mill. *g*, Four 8 × 26-ft. 8-in and two 6 × 25-ft. Dorr duplex classifiers. 2434 tons water added per 24 hr. *h*, 154 @ 33 × 42-in. bottoms, 1482 sq. ft. Flotation reagents: Alkali xanthate, 0.080 lb.; pine oil, 0.051 lb.; lime, 3.31 lb. per ton of ore. *i*, Lagged down from 8 ft. × 36 in. *j*, Three duplex cells, 60 @ 33 × 42-in. bottoms; one duplex cell, 28 @ 33 × 48-in. bottoms; 48 standard Callow cells (13.43 sq. ft. each); total porous bottom, 1537 sq. ft. *k*, For primary concentrate, 30 standard Callow cells (13.43 sq. ft. each) = 403 sq. ft. For secondary concentrate, nine 33 × 48-in. bottoms, 16 @ 33 × 42-in. bottoms and 12 standard Callow cells, total 416 sq. ft. *l*, 13 @ 30-ft., 2 @ 45-ft., 2 @ 50-ft., 1 @ 60-ft.; total, 19,124 sq. ft. = 76.5 sq. ft. per ton of solid per 24 hr.; 5.1 sq. ft. per ton of water per 24 hr. All roughed in one of the 30-ft. tanks with rakes at 40°; spigot product is granular, 60 per cent. solids; overflow goes to the other tanks. *m*, Two

11.5 (diam.) × 12-ft., one 11.5 × 8-ft., one 14 × 14-ft.; total area, 1338 sq. ft. = 375 lb solid per sq. ft. per 24 hr. *n,* One 200-ft., one 130-ft., two 60 ft. square; total area, 51,88 sq. ft.; = 12.7 sq. ft. per ton of soli per 24 hr. or 3.4 sq. ft. per ton c water. *o,* 19 per cent. Cu. *p,* 1.5 per cent. Cu. *q,* Combined flota tion concentrate, 23 to 24 per cent Cu. *r,* 12 per cent. moisture. *s,* to 8 per cent. moisture.

Table 45. Power consumption at Phelps Dodge Corp., Morenci mill

Operation	Kw.-hr. per ton
Primary crushing	0.70
Secondary crushing	1.79
Primary grinding	5.10
Secondary grinding	1.80
Flotation	2.21
Concentrator water (pumps)	1.17
Concentrator lights	0.25
Filters	0.46
Shops	0.02
Sampling and assaying (*a*)	0.02
Magnets and cranes (*a*)	0.02
Total	13.54

a Direct current.

Summary. CRUSHING Gyratory from 12- to 2.5-in. two sets of rolls in closed cir cuit to 6-mesh with Hum-me screen; two sets of rolls i closed circuit to table size one-stage primary ball-millin to primary flotation size; one stage re-grinding of sand tailin in ball mills. CONCENTRATION Tabling followed by primary and secondary flotation, rougher-cleaner routing.

Inspiration Cons. Copper Co. Fig. 61. (*55 A 576, 707; G. H. Ruggles, PC.*)

Location: Miami, Ariz.

Ore: Chalcocite and pyrite in decomposed schist and granite.

Capacity: 18,350 tons per 24 hr.

Assays: See Table 47.

Recovery: See Table 47.

Ratio of concentration: 41 : 1.

Labor: Tons per man-day; operating, 59.0; total, 42.0.

Power: See Table 46.

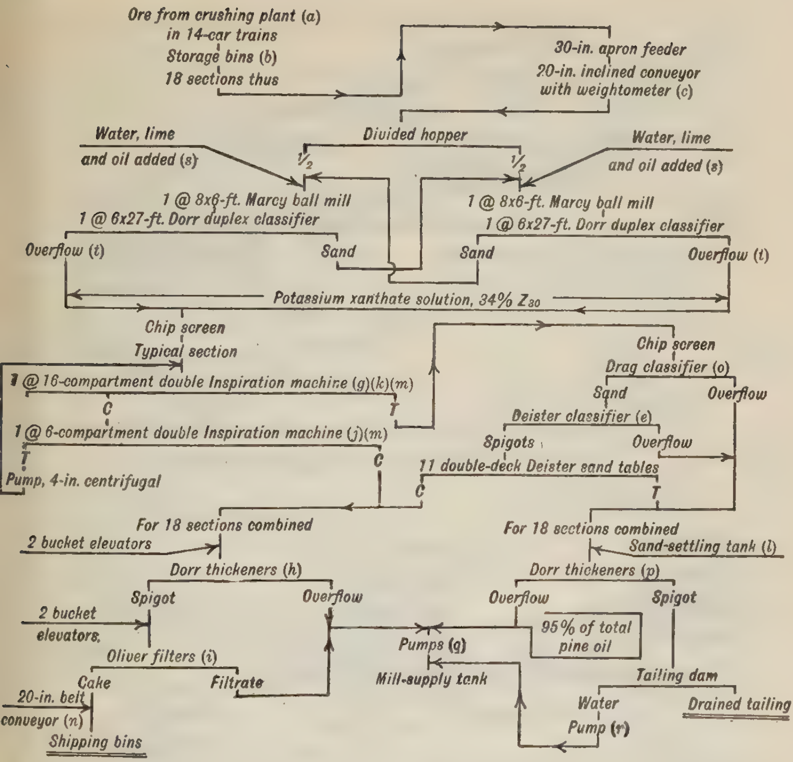
Water: See Table 48.

Table 46. Power consumption at Inspiration (May, 1925)

Operation	Kw.-hr. per ton of ore
Coarse crushing	0.44
Fine grinding	10.19
Flotation, pumps	0.13
Flotation, blowers	1.69
Drag classifiers	0.03
Tables	0.13
Filters	0.01
Elevators	0.01
Vacuum	0.04
Compressors	0.05
Reclaimed water	0.76
Fresh water	0.95
General, lights, etc.	0.23
Total	14.66

Table 47. Performance at Inspiration concentrator (May, 1925)

	Percentages		
	Feed	General concentrate	General Tailing
Total copper	1.114	36.238	0.235
Oxide copper	0.187	....	0.178
Sulphide copper	0.927	....	0.057
Iron	2.80	19.02	....
Sulphur	0.78	26.10	....
Insoluble	....	15.82	....
Lime	....	0.40	....
Moisture	....	8.50	....
Recovery, total calculated	....	79.42	....
Recovery, sulphide	....	94 00	....



a, See Sec. 23, Table 1, and Sec. 3, Fig. 13. *b*, 12,000-ton, 875 tons per section. Suspension-bunker type, two-tracked; length, 300 ft. Material, −2-in. disk-crusher product. *c*, 150 ft. per min, *d*, 3 ft. 3½ in. × 10 ft. 2 in. 16 cells in some sections. *e*, 22 in two parallel series of 11. *g*, Individual compartments, 3 × 4 ft. 3 in. For details see Sec. 12, Fig. 37. *h*, 2 @ 80-ft. Dorr tanks. 17 sq. ft. per daily ton of concentrate. When oil was used 5 @ 60-ft. and 3 @ 80-ft. tanks = 49 sq. ft. per ton were necessary. Spigot product contains 62 per cent. solids. *i*, 4 @ 11 ft. 6 in. × 12 ft. = 150 tons solid per filter per day. Cake contains 8.5 per cent. water, average. Moisture increases with increase in insoluble. With coal-tar as the selecting agent five or six filters were required to produce cake averaging 17 per cent. moisture, with the day-to-day performance highly variable. *j*, 3 × 3-ft. compartments. *k*, Water sample taken at the head of the cells and alkalinity determined. *l*, Consists of a series of Caldecott cones in a rectangular tank, 17 × 109 ft. Overflow to 3 @ 60-ft. Dorr thickeners. *m*, 4 single-stage centrifugal air compressors delivering 23,000 cu. ft. of free air per min. at 5.75 lb. per sq. in. 720 hp. installed. Average air consumption is 11.4 cu. ft. per sq. ft. of blanket surface per min. *n*, 100 ft. per min. *o*, See Sec. 6, Art. 6. *p*, 5 @ 60-ft., 1 @ 80-ft., 3 @ 100-ft., and 1 @ 200-ft. *q*, 4 vertical triplex pumps each geared to a 100-hp. synchronous motor. Capacity each pump, 2000 gal. per min. Lift, 113 ft. *r*, 1 @ 3000-gal. 2-stage centrifugal. *s*, Lime added in the form of a 4 per cent. lime emulsion. 5 per cent. of the total pine oil is also added to the lime water in a junction box and thence to the ore. *t*, 3 per cent on 48-mesh.

FIG. 61.—Inspiration Consolidated Copper Co.

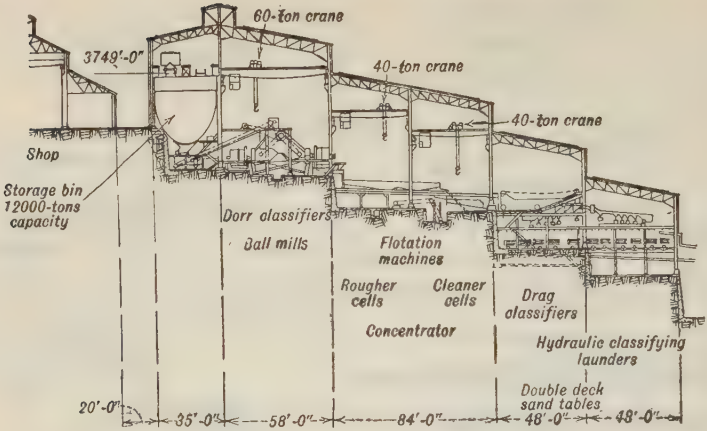


FIG. 62.—Section of Inspiration concentrator.

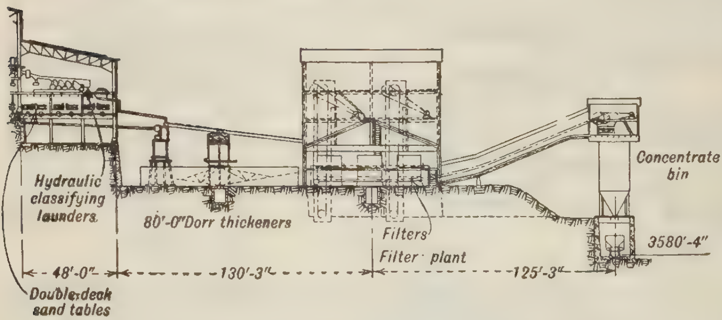


FIG. 63.—Section of Inspiration concentrate-handling plant.

Table 48. Water supply at Inspiration (May, 1925)

	Gallons per ton of ore	Tons per ton of ore	Per cent. of reclaimed water	Per cent. of total water
Reclaimed water				
Tanks	494.3	2.061	78.8	54.5
Ponds	132.8	0.554	21.2	14.6
Total	627.1	2.615	100.0	69.1
Fresh water	279.4	1.165	....	30.9
Total water	906.5	3.780	....	100.0

General: Mill has 20 units. For arrangement of machines see Figs. 62 and 63.

Flotation agents: Potassium xanthate (Z–30), 0.2435 lb; pine oil, 0.1513 lb.; lime, 1.216 lb. per ton.

Summary. One-stage ball milling from 3-in. to −35-mesh. Flotation by rougher-cleaner routing followed by tabling of de-slimed flotation tailing

Consolidated Coppermines Co., Fig. 64. *(64 A 816.)*

Location: Kimberley, Nev.

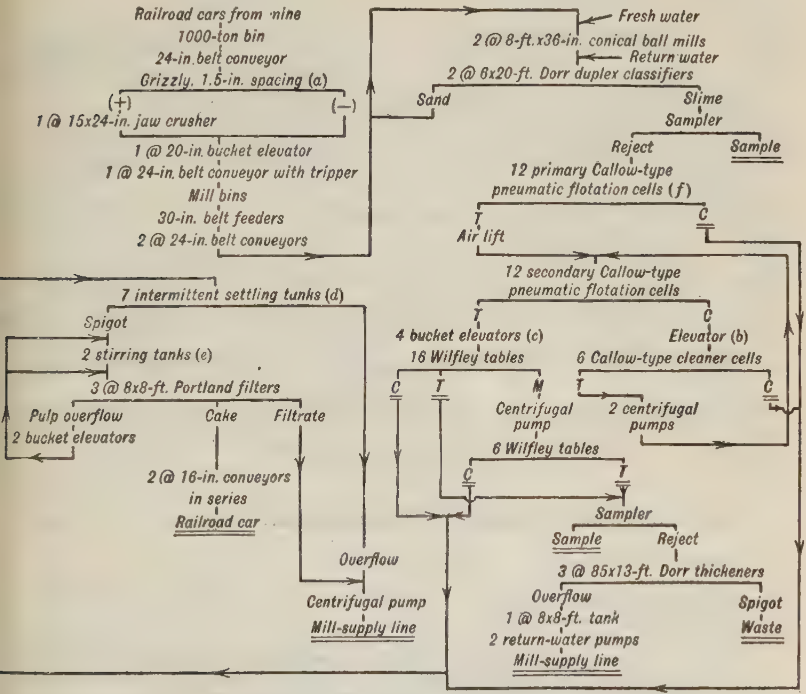
Ore: Typical porphyry copper ore containing chalcocite and some pyrite. Analysis: Cu, 1.4 per cent.; SiO_2, 64 to 85 per cent.; Al_2O_3, 4 to 12 per cent.; Fe, 4 to 5 per cent.; CaO, 0.5 to 2.5 per cent.; S, 2.7 to 3.2 per cent.; Au, 0.01 to 0.02 oz.; Ag, 0.05 to 0.08 oz.

Capacity: 1000 tons per 24 hr.

Assays: Feed, see above; concentrate, see Table 49.

Recovery: See Table 49.

Ratio of concentration: See Table 49.



a, 2 × 12 ft. *b*, Diaphragm pump in one section, bucket elevator in the other. *c*, In sets of 2. *d*, 6 @ 20 × 8-ft. Goldfield tanks and 1 @ 30 × 6-ft. 6-in. settling tank, in parallel. *e*, 20 × 8-ft., Goldfield type. *f*, Flotation agents were various mixtures of coal tar and pine oil from the start (May, 1917) to Aug., 1918; thereafter X–Y mixture (alpha- and beta-naphthylamine and xylidin), all in alkaline circuit with lime. For comparison of results see Table 49. Note the increase in recovery and grade of concentrate caused by the X–Y reagent. There was a corresponding reduction from 17.2 to 9.4 per cent. in the moisture content of filter cake. Savings were: smaller tonnage of concentrate to be filtered, freighted and smelted; quicker filtration (one shift with 1 or 2 filters against 3 shifts with 3 filters).

Fig. 64.—Consolidated Coppermines mill.

Summary. Crushing: Jaw crusher, 12- to 2-in.; one-stage closed-circuit ball milling to flotation size. Concentration: Pneumatic flotation, combination routing, with shaking-table treatment of flotation tailing without classification. Concentrate thickened in intermittent settling tanks.

Table 49. Monthly results of mill operation, Consolidated Coppermines Co. (*After Linton*)

Month	Reagent, kind and pound per ton							Pulp density	Per cent. on 48-mesh	Mill concentrate, per cent. Cu	Mill ratio of concentration	Mill recovery
	Coal tar	Coal-tar creosote	Pine oil(*a*)	Barrett No. 4	X-cake	Xylidin	Lime					
1917												
May......	1.39	0.09	0.03	0.12	......	......	2.39	2.76	5.2	13.80	12.4	60.7
June......	1.36	0.16	0.17	0.06	......	......	3.40	3.18	4.6	12.32	10.0	70.1
July......	1.49	0.28	0.21	0.03	......	......	2.36	3.22	4.2	13.52	10.2	71.0
August....	1.62	0.24	0.21	.....	......	......	2.59	3.29	4.9	15.50	10.0	78.7
September.	1.65	0.28	0.26	.....	......	......	3.36	3.07	4.3	18.42	10.4	83.9
October...	1.44	0.21	0.18	.....	......	......	3.77	3.00	4.5	15.40	11.4	82.2
November.	1.25	0.16	0.16	.....	......	......	3.16	3.10	3.6	15.20	13.4	75.6
December.	1.26	0.15	0.08	.....	......	......	3.58	3.00	3.3	15.10	12.1	77.4
1918												
March....	1.18	0.13	0.10	.....	......	......	3.26	2.9	3.3	15.20	15.0	73.5
April......	1.33	0.15	0.15	.....	......	......	3.52	3.0	3.8	11.30	13.2	65.8
May......	0.95	0.18	0.05	.....	......	......	2.58	3.0	4.7	13.10	17.7	72.8
June......	1.09	0.35	0.03	.....	......	......	1.38	3.3	5.7	17.00	13.7	84.8
July......	1.10	0.27	.....	.....	......	......	1.85	3.0	7.4	17.30	21.6	75.3
August....	1.17	0.28	.....	.....	......	......	1.63	3.15	6.1	20.18	20.4	75.0
September.	.....	.....	.....	.....	0.225	0.115	1.58	3.37	5.5	18.00	21.2	79.3
October...	.....	.....	.....	.....	0.129	0.080	1.87	3.42	5.3	20.65	21.1	83.3
November.	.....	.....	.....	.....	0.121	0.082	2.52	3.40	5.4	20.06	19.6	81.5
December.	.....	.....	.....	.....	0.130	0.066	2.99	3.40	7.3	18.85	17.5	78.0
1919												
January...	.....	.....	.....	.....	0.155	0.099	2.82	3.55	6.8	21.00	18.7	86.2

a Pensacola No. 80.

Phelps Dodge Corp. Copper Queen concentrator. Fig. 65. (*111 J 1079.*

Location: Bisbee, Ariz.
Ore: Porphyry.
Capacity: 4800 tons per 24 hr.
Assay: Feed, 1.5 per cent. Cu.
Recovery: 90 per cent.
Power: From smelter at Douglas. 6000 hp. Diesel auxiliary plant at mill.
Water: Total re uirement, 1516 gal. per ton. Of this 1140 gal. per ton is reclaimed the Dorr tanks and filters. Fresh water from mine, about ⅔ mile distant.
General: Sloping millsite. Building, steel with corrugated-iron siding and compositio roofing. Dorr tanks and flotation cells of concrete.

Summary. Crushing: Jaw crusher, 60- to 8-in.; gyratory, 8- to 4-in disk crushers, 4- to 1.25-in.; 2-stage rod-milling to 14- and 48-mesh, respectively, first stage open-circuit. Concentration: Shaking tables preceding and following pneumatic flotation, rougher-cleaner routing.

Notes to Fig. 65.

a, 335 ft. long. *b,* 150-hp. motor each. Rod charge, 36,000 lb. 3-in. high-carbon steel rods. 17½ r.p.m. Crushing to 14-mesh, single pass. *c,* Concrete, 22-compartment, 231 sq. ft. blanket area, each machine. *d,* Concrete, 6-compartment, 83 sq. ft. blanket area, each machine. *e,* Feed, 38.5 tons each. 1-hp. individual motors. *f,* F the 4 units. *g,* All conveyors have ball-bearing troughing and return idlers. Automat electrical interlock with crusher drives to prevent damage from stoppage of any machin

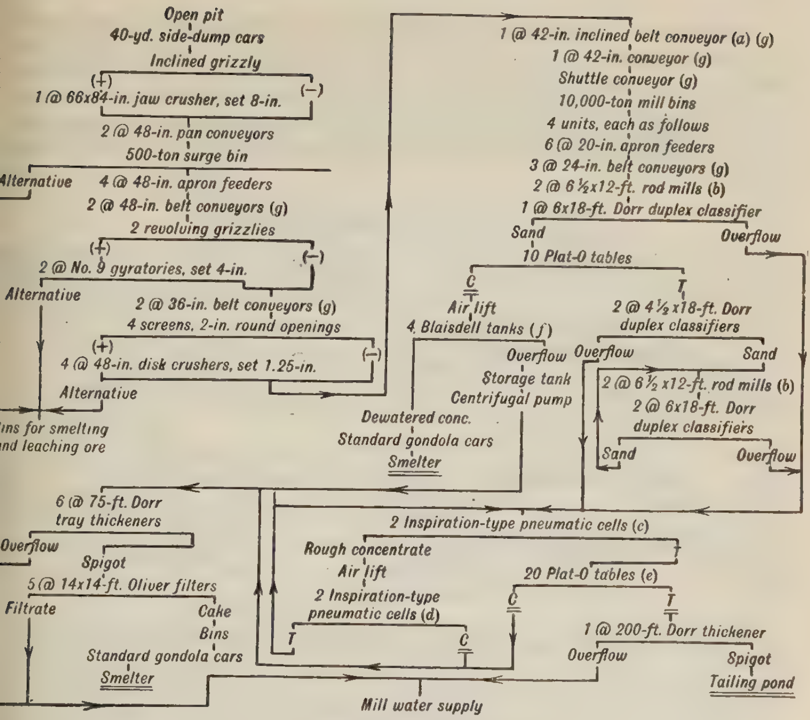


FIG. 65.—Copper Queen concentrator.

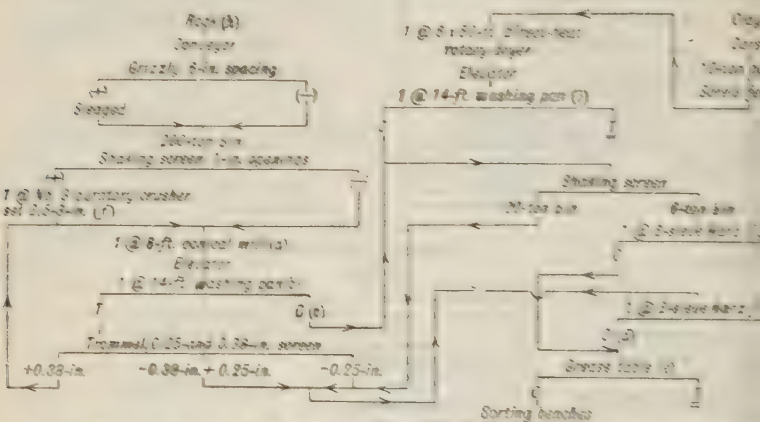
15. Diamond (C)

Properties. Diamonds are a crystalline form of carbon. They occur in two forms, *viz.*: the crystalline or gem variety and the amorphous-appearing variety, known as CARBON or CARBONADO. SPECIFIC GRAVITY of gem diamonds is 3.50 to 3.56; LUSTER of cleavage faces is adamantine, while the natural faces have a more greasy luster. SPECIFIC GRAVITY of carbonado ranges from 2.75 to 3.42 and the LUSTER is dull and earthy.

Uses. The principal use of the crystalline variety is, of course, as a gem stone. Chips and dust from cutting the gem varieties are used as polishing powder for gem diamonds and for other gems. BORT, a low-grade crystalline form, and carbonados are used as abrasive cutters, dies, and pivot bearings in delicate apparatus. The best carbons are used principally for setting diamond-drill bits. Poorer grades are used to point cutting tools in turning machines, glass cutters, and the like.

Ores. Until about 1870, the sources of gem diamonds were placer deposits, the principal ones being in India and Brazil. Since 1870, the South African mines (KIMBERLEY, JAGERSFONTEIN, PREMIER, etc.) have supplied the bulk of the world production. Here the diamonds occur scattered through an altered peridotite (blue ground). A somewhat similar occurrence is found near Murfreesboro, in Pike County, Ark., but while a considerable number of diamonds has been found in this deposit, it has not been proved up or worked on a commercial scale. Carbonados are found in only one locality, *viz.*: the highlands of the state of Bahia, Brazil. The deposits are in patches in quartzites and in river gravels derived from these rocks.

Production. The potential production of gem diamonds in South Afri[ca] exceeds the world demand, and, in order to maintain prices, the actual pr[o-]duction is closely limited by government officials working with the min[ing] companies. The demand for carbonados, on the other hand, is in exce[ss] of the supply with the result that prices (1924) ranged from \$90 or \$100 p[er] carat for the poorer and smaller stones to \$150 or even higher for choi[ce] stones.



a. No grinding medium. The larger lumps of hard rock are depended upon to eff[ect] disintegration. Fine grinding is not desired. b. See Sec. 8, Art. [illegible] c. 1 to 2 per ce[nt] of the original tonnage. d. [illegible] per cent of jig feed [illegible] per cent [illegible] This material consists of diamonds together with all heavy minerals from the rock. [In] South Africa these are pyrope, zircon, [illegible] chrome [illegible] pyroxene, [illegible] ilmenite, magnetite, chromite, [illegible] barite, [illegible] and pyrite. [illegible] Sec. [illegible] e. Will recover stones as small as [illegible] to the carat. Originally gravity concentrates were hand-picked to recover the diamonds. [illegible] weathering has been displaced on account of the cost of handling, [illegible] large amount of capital tied up in the weathering material. h. The following [illegible] may be taken as illustrative of South African practice. i. Frequently not [illegible] South African mills.

Drying and subsequent wetting as practiced on the clayey rock cause the clay to sl[ake] down to a thin mud, while without drying it is extremely difficult to disintegrate. T[his] treatment is a local development, not typical of South African practice.

Fig. 66.—Arkansas Diamond Corporation.

Treatment depends upon the kind of ore and the locality. The carbona[do] gravel deposits are worked by the Brazilian natives in bateas or, in the m[ore] elaborate plants, in sluices, sluice concentrate being worked up in the bat[eas.] A native with a batea can work about 1 cu. yd. of gravel per day when [it] is easily accessible, uncemented and near water [illegible]. The So[uth] African blue-ground deposits are treated in concentrating mills in which [the] general method of treatment is careful disintegration (formerly by weather[ing] for 3 or 4 months to two years, but recently by mechanical means), conce[n-]tration by washing in rotary pans, sizing of the concentrate, jigging, treatme[nt] of jig concentrate on greased shaking tables or greased-belt vanners, clean[ing] the concentrate by melting and pouring off the grease and washing the s[...] residue with hot caustic solutions and with hydrofluoric acid, and fin[ally] hand sorting and grading the ultimate concentrate [illegible]. The flow-sh[eet] of the Arkansas Diamond Corp. shows a recent mill scheme.

Arkansas Diamond Corp. Fig. 66. (*109 J 983.*)

Location: Murfreesboro, Ark.
Ore: Peridotite in place and residual clays resulting from weathering.
Capacity: 20 to 25 tons per hr.

Summary. Careful disintegration to free the diamonds without breaking them, followed by gravity concentration and re-treatment of gravity concentrate on greased tables.

16. Emery and Corundum

Properties. Corundum (Al_2O_3) occurs as the gem varieties, SAPPHIRE and RUBY; as a dull to dark, non-transparent crystalline variety, called CORUNDUM; and as a black or grayish-black granular variety, intimately associated with hematite and magnetite, called EMERY. HARDNESS, 9; SP. GR., 3.9 to 4.1.

Uses. The massive crystalline corundum and the granular emery are used in the form of wheels, bricks, coated papers, grains and powders as abrasives in metal and stone polishing, and as knife-edges and jewel bearings in delicate apparatus. As abrasives they come into competition with the artificial abrasives (carborundum and alundum) and, due to the fact that the latter are more uniform and reliable they have largely displaced the natural minerals.

Ores. Corundum occurs as loose fragments and crystals in residual soils; as placer deposits, and as a constituent of crystalline rocks, *e.g.*, pegmatites, gneisses, granites, schists and the like and also in crystalline limestones. Emery occurs in similar crystalline rocks. The principal corundum deposits of the world are in a region in the Transvaal where known deposits of all three varieties are scattered over 2000 square miles. (*So. Afr. Jour. Ind., Dec., 1924.*) There are also large deposits of both gem and crystalline corundum in India. Canada has important deposits. The most important emery deposits are in Greece. There are workable deposits in New York, Massachusetts and Virginia.

Production of corundum in Canada was about 1200 tons in 1913 and 400 tons in 1921; in India 400 tons in 1913, 2100 in 1917 and 200 in 1920; in South Africa, 13 tons in 1913, 3900 in 1918, and 600 in 1921. Production of emery in Greece in 1912 was 7900 tons and in 1921, 11,000 tons. Domestic production of emery was 1000 tons in 1913; 17,000 tons in 1917; 300 tons in 1921, and 2000 in 1924. The great expansion in 1917 was due to interruption with the Greek supply.

Selling. The value of corundum and emery depends on their hardness and toughness. The best grades are those tough enough not to crumble readily, but which crumble sufficiently at the surface, under ordinary grinding pressures, to present ever new, rough abrasive surfaces. There are no standards. Chemical analysis is indicative, but appearance and the behavior of sample lots are the final test. Both corundum and emery are marketed in crude lump form and finished by crushing and extremely close sizing in the plants of the abrasives manufacturers. The South African corundum output has ranged in value, from 1919 to 1924, inclusive, between \$30 and \$40 per ton. Exports of two tons of Canadian corundum in 1924 were valued at \$125 per ton. Domestic emery production since 1919 has ranged in value from about \$7.50 per ton in 1921 to \$12.75 per ton in 1923. (*33 MI 1.*) Best-grade imported (Greek and Turkish) emery was quoted (*J, Feb. 6, 1926*) at 6½ cents per lb.

Treatment. Residual and placer ores are treated by simple washing processes. The crystalline ores are treated by gravity concentration. THE MANUFACTURERS CORUNDUM Co. mill (Fig. 67), burned in 1913, was probably the most elaborate of such plants.

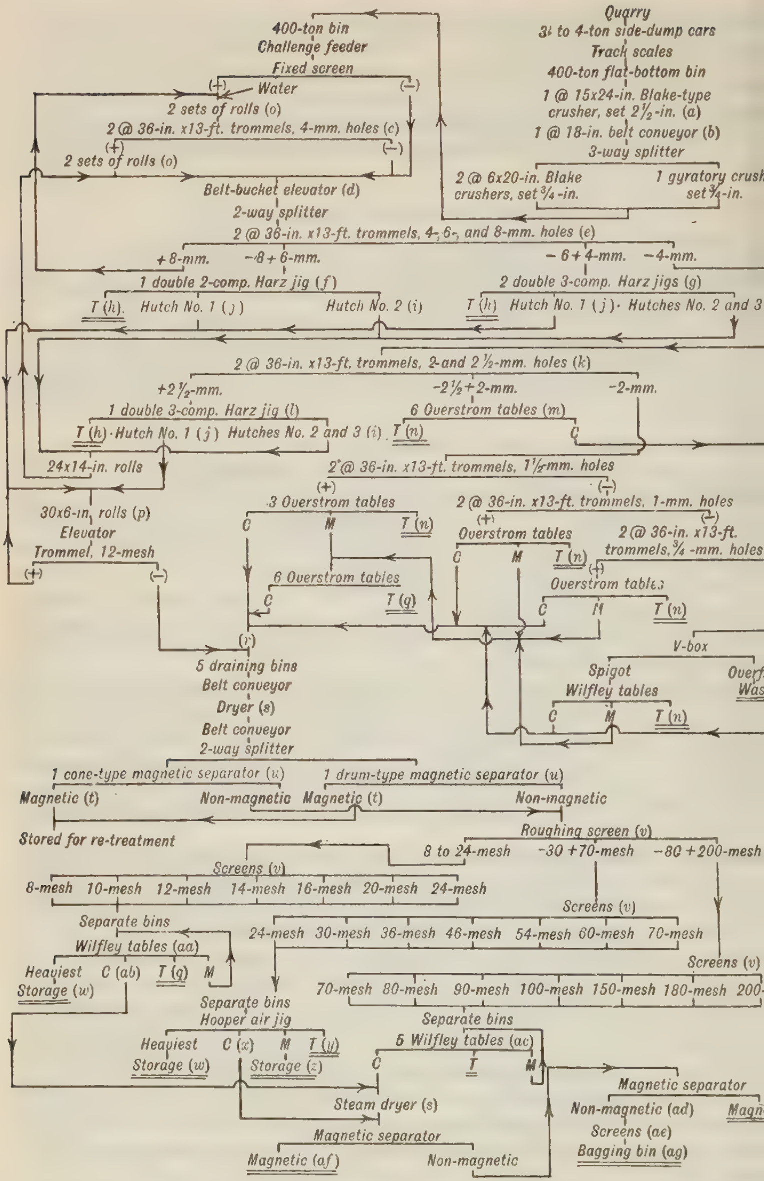


FIG. 67.—Manufacturer's Corundum Co.

a, 250 r.p.m. *b,* 85 ft. long, 20 per cent. grade. 300 ft. per min. *c,* 20 r.p.m.; sl 1 in. per ft. *d,* 18 × 6 × 6-in. buckets; 350 ft. per min. *e,* 20 r.p.m.; slope, 1 in. per rope drive. 6 ft. of 4-mm. screen, 4 ft. of 6-mm. screen, 1½ ft. of 8-mm. screen. *f,* 2

36-in. sieves with 6-mm. holes. 170 @ 1-in. strokes per min. Bedded with oversize corundum. *g*, 24 × 36-in. sieves with 8-mm. holes, 200 @ ⅞-in. strokes per min. Bedded with coarse corundum. *h*, About 3 per cent. corundum. Jigs overloaded. *i*, 35 to 45 per cent. corundum. *j*, About 50 per cent. corundum. *k*, 6 ft. of 2-mm. screen, 5 ft. of 2½-mm. screen. Speed and slope as in note (*e*). *l*, 24 × 36-in. sieves. 4-mm. holes. Bedded with coarse corundum. *m*, Feed is too coarse. *n*, 2 per cent. corundum. *o*, 40 × 14 in. *p*, Set to crush to 12-mesh. *q*, 5 per cent. corundum. *r*, 50 per cent. corundum. *s*, Two-deck, steam, 1¼-in. pipes under 4-mesh screen. Dry material drops through the screen and between the pipes onto a conveyor belt. *t*, 4 to 5 per cent. corundum. *u*, Feed contains 12 to 15 per cent. magnetic iron. *v*, Steel-wire screen from 8- to 30-mesh; silk cloth for finer sizes. *w*, Magnetite and pyrite. Stored outside for possible future treatment. *x*, 90 to 95 per cent. corundum. *y*, 4 to 6 per cent. corundum. *z*, Held until enough accumulates for re-treatment on the air jig. *aa*, 215 @ ¾-in. strokes per min. Different sizes treated separately. *ab*, 88 to 90 per cent. corundum. *ac*, 250 @ ⅜-in. strokes per min. Different sizes treated separately. *ad*, Contains 1 to 2½ per cent. combined iron. *ae*, Finishing screens to guard the preceding sizing. *af*, Coarse sizes, 7 per cent. corundum; fine, 3 per cent. *ag*, Sacked in 100-lb. bags. Three grades made for wheel-making; highest 90 to 95 per cent. pure.

Manufacturers Corundum Co. Fig. 67. (*Mem. 57, Canada Dept. of Mines, 300.*)

Location: Craigmont, Ont., Canada.
Ore: Corundum syenite.
Capacity: 150 tons per 24 hr.
Assays, per cent. corundum: Feed, 10 to 12; concentrate, 90 to 95 ; tailing, 5.
Recovery: 58 per cent.
Cost (1913) including mining, selling and all overhead was about $40 per ton shipped.

Summary. Graded CRUSHING from 10- or 12-in. to 8-mm. by two steps in crushers and two steps in rolls. Preliminary CONCENTRATION by jigs and tables, following close sizing. Final concentration of very closely-sized fine products by pneumatic jigs, shaking tables and magnetic separators. Note that the extremely close sizing preceding this final concentration is on account of the sizing demanded in the finished product; it is certainly unjustifiable from a concentrating standpoint.

17. Fluorspar

Properties. Fluorite, CaF_2, is a cleavable, light-colored, transparent to translucent mineral; SP. GR., 3 to 3.2; HARDNESS, 4. It decrepitates when heated to about 1200° F.

Uses. The principal consumption (80 to 85 per cent.) is as a flux in the basic open-hearth process of steel making. The balance is used in the ceramic industries, as a flux to increase fluidity of slags in copper blast furnaces, and in iron- and brass-melting furnaces; as a source of hydrofluoric acid and sodium fluoride; in making emery wheels and carbon electrodes; and in the extraction of potash from feldspar and cement-mill flue dusts. A small amount is used for optical lenses (*32 MI 247*).

Ores. Fluorspar occurs frequently as a gangue in lead-zinc ores, associated with calcite. It also occurs in veins in gneiss, slate, sandstone and limestone.

Production in the United States has ranged from 138,000 tons in 1919, 187,000 in 1920, and 35,000 in 1921 to 125,000 in 1924 (*USGS*); about 60 per cent. comes from Illinois and 35 per cent. from Kentucky. Imports have ranged in the same time from 6000 tons to 42,000 tons, Great Britain supplying about half of the tonnage.

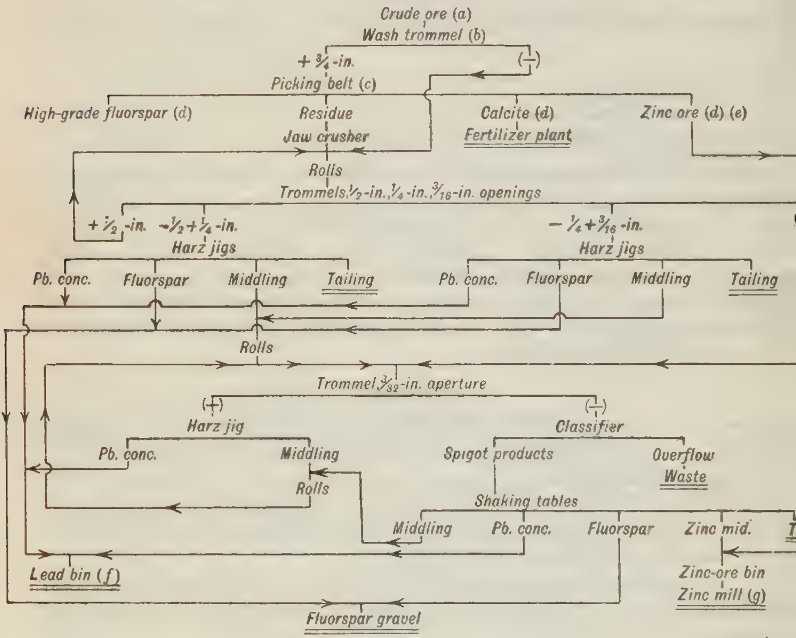
Selling. The market grades are lump, gravel and ground. The general silica limit for lump and gravel is 6 per cent; for the ceramic industry, 3 to 5 per cent.; for hydrofluoric acid, 1 per cent. For metallurgical purposes the standard specifications set 85 per cent. minimum CaF_2 and 5 per cent. maximum silica. Lead and zinc are objectionable in ceramic manufacture and calcium carbonate in acid manufacture. PRICES ranged from $25.50 per ton average

in 1919 to \$19.60 in 1924 with a low of \$17.80 in 1922. The quotations f.o.b. Kentucky and Illinois mines (*J, Feb. 6, 1926*) were \$17.50 per ton for gravel not less than 85 per cent. CaF_2 nor more than 5 per cent. SiO_2; \$20 for lump of the same purity; \$32.50 for material ground to substantially 100-mesh, 95 to 98 per cent. CaF_2 and not over 2½ per cent. SiO_2. Ground acid stock in bags, \$40 to \$45.

Treatment varies according to the character of the ore. In the Kentucky ores the waste is principally clay and sand and concentration is done by log washers. In the Illinois field the two largest mines are vein deposits with galena and sphalerite in a gangue of fluorspar and calcite.

Fairview Fluorspar and Lead Co. (Fig. 68) at Rosiview, Ill., is typical (*111 J 222.*)

Summary. Picking, graded crushing, sizing and classification followed by jigs and shaking tables.



a, Contains 10 to 30 per cent. waste. *b*, Part blank plate, part ¾-in. round-hole plate. *c*, About 50 ft. long, 24 in. wide, 25 to 40 ft. per min. *d*, Picked. *e*, Finely disseminated. *f*, Concentrate, 75 to 80 per cent. Pb and 7 to 8 oz. Ag per ton, amounting to 1 to 2 per cent. of the crude ore hoisted. *g*, Flotation is probably the best method of treatment, although electrostatic concentration has been tried.

Fig. 68.—Fairview Fluorspar and Lead Co.

At the Rock Candy mill, at Trail, B. C., the fluorite occurs in a silicious gangue and separation is made by crushing to about ½-in., sizing on ¼-in., 8-mesh and 15-mesh screens, heating the oversize products to about 1200° F. to cause decrepitation of the fluorite and then concentrating by re-screening. (*Ladoo.*)

18. Garnet

Properties. Garnet is a generic term covering several species of tri-silicates, viz.: grossularite, $Ca_3Al_2(SiO_4)_3$; pyrope, $Mg_3Al_2(SiO_4)_3$; almandite, $Fe_3Al_2(SiO_4)_3$; spessartite, $Mn_3Al_2(SiO_4)_3$, all known as aluminum garnets; iron garnet, andradite, $Ca_3Fe_2(SiO_4)_3$, and chrome garnet, uvarovite, $Ca_3Cr_2(SiO_4)_3$. All species have similar physical properties and crystal form. LUSTER is vitreous. SP. GR., 3.5 to 4.3. PERMEABILITY, see Sec. 13, Table 2. The important properties from an economic standpoint are the HARDNESS, varying from 6 to 7.5 in the different varieties, and TOUGHNESS.

Uses. Garnet has been used as a gem for a long time, but production for this purpose is small and relatively unimportant. A small amount of foreign garnet is used for watch jewels. The principal use is as an abrasive, both in loose form and mounted on paper and on cloth backing.

Ores. The most important garnet is almandite, which has a sp. gr. from 3.9 to 4.2, and hardness from 7 to 7.5. The color varies from red to black. The usual occurrence is in granitic and metamorphic rocks, such as schists, gneisses and crystalline limestones, associated with quartz, micas, feldspars, pyroxenes and hornblendes. Corundum, rutile, magnetite, pyrite, pyrrhotite and chalcopyrite are also common accessory minerals. For economical operation, the ore should ordinarily contain upwards of 6 per cent. garnet; some ores contain as high as 60 per cent., but 10 to 15 per cent. is the usual maximum. While garnet deposits are common in all parts of the world, yet with the above limitations, and the further requirement as to suitable hardness and toughness for abrasive service, the number of deposits of sufficient richness and so located as to repay exploitation are relatively few. This fact, coupled with the difficulty of concentration and the relatively small demand for the finished product has resulted in a virtual monopoly of the domestic industry in the hands of a few companies.

Production of abrasive garnet in the United States was 5000 tons in 1919, 3000 in 1921, 9000 in 1923 and 8000 in 1924. (*U.S.G.S.*) About half of this comes from the NORTH RIVER GARNET CO. and about 1000 tons from the BARTON MINES CORP. Canada is the most important foreign producer (1250 tons in 1924). Small amounts come from Japan, Spain and India (*33 MI 6*).

Selling. Unsized garnet concentrate (—2-in., assaying about 90 per cent. garnet) is shipped in 100-lb. sacks to the abrasive manufacturers who crush and grade it closely into 12 to 16 sizes, as per Table 56.

PRICES of domestic grades since 1920 have ranged from \$75 to \$85 per ton of unsized concentrate. Canadian is quoted \$5 to \$15 per ton cheaper f.o.b. mines and Spanish \$20 to \$25 cheaper, c.i.f. port of entry.

Treatment. The large mills treat the crude ore by close sizing and classification followed by jigging and tabling, and at the plant of the NORTH RIVER GARNET CO. (Fig. 69) the fine concentrate is re-cleaned on pneumatic jigs. Some small plants use pneumatic concentration throughout (Fig. 70). Electrostatic and magnetic concentration have both been tried, but have not been successful.

North River Garnet Co. Fig. 69. (*118 J 525.*)

Location: Near North Creek, N. Y.

Ore: Almandite in gneiss composed principally of hornblende and feldspar. The garnet crystals range in size from about 4-in. diameter downward, but average about 0.5-in.

Capacity: 400 to 500 tons per 20 hr.

Ratio of concentration about 25 : 1.

Concentrate assay about 90 per cent. garnet.

Labor: 25 to 30 tons per man-shift, operating.

Table 50. Comparative sizes of abrasive grains (*a*). (*After Ladoo, U.S.B.M., Ser. 2347*)
[Figures in columns represent grit numbers commonly used]

Standard screen mesh	Standard size of opening (inches)	Flint paper and cloth	Garnet paper and cloth	Silicon carbide and artificial corundum	Emery and artificial corundum paper and cloth	Silicon carbide and artificial corundum paper disks
			• 7/0			
200	.0029					
		• 4/0	• 6/0	• 220 • 200		
180	.0033					
		• 3/0	• 5/0		• 3/0	
160	.0038					
		• 2/0	• 4/0	• 180	• 2/0	
140	.0042					
			• 3/0	• 150 • 120	• 0	
120	.0046					
		• 0	• 2/0	• 100	• 100	• 120 • 100
100	.0055					
		• ½		• 90	• ½	• 90
90	.0059					
			• 0		• 1	
80	.0068					
		• 1		• 80 • 70	• 1½	• 80
70	.0073					
			• ½	• 60	• 2	• 70
60	.0097					
		• 1½	• 1	• 50		• 60
50	.0110					
		• 2	• 1½	• 40	• 2½	• 46
40	.0140					
		• 2½	• 2	• 36 • 30	• 3	• 36
30	.0198					
		• 3 • 3½	• 2½ • 3 • 3½	• 24	• 3½ • 4	• 24
20	.0340					
			• 4 • 5			
15	.0468					
						• 16 • 12
10	.0650					

a In this table the size of grain as related to screen mesh is indicated by the position of the dots between the horizontal lines. Thus, No. 2½ garnet is about 38 mesh.

Summary. Closely-graded crushing to 0.25-in. followed by rough jigging of unzised feed with rejection of tailing and step re-treatment of middling on other water jigs and pneumatic jigs after close sizing.

The mill is an old one and has developed slowly, under great difficulty in an isolated district, which facts go to explain the crude and inefficient handling methods and high labor requirement.

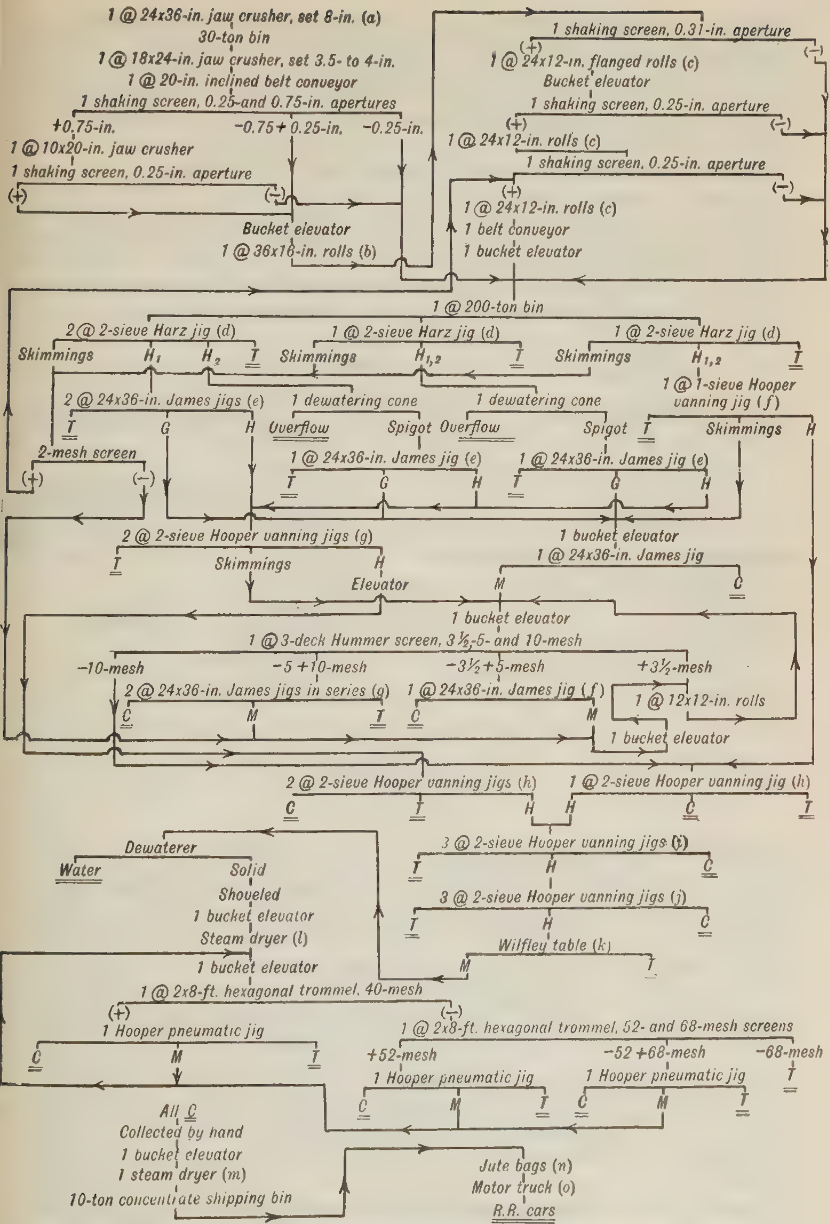


Fig. 69.—North River Garnet Co.

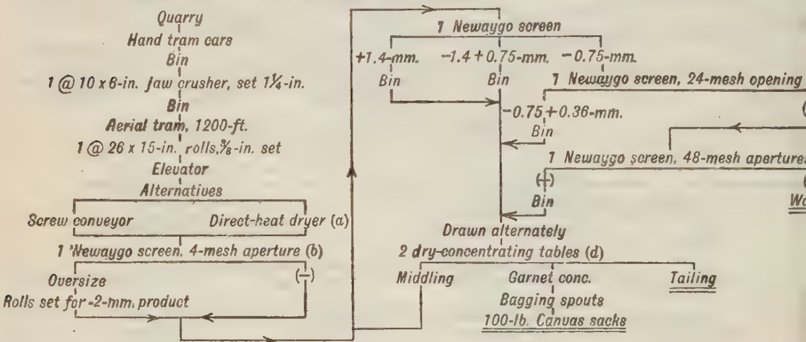
a, Hand-fed. *b*, Life of chrome-steel shells about 15 mo. *c*, Life of chrome-steel shells, 6 to 8 mo. *d*, 24 × 36-in. sieves, 0.25-in. and 0.19-in. screen on first and second

compartments, respectively. 200 @ 2-in. strokes per min. Jigs hand-skimmed at about 1-hr. intervals. *e*, 6-mesh screen, 225 @ ⅜- to ½-in. strokes per min. *f*, 6-mesh screen. *g*, 12-mesh screen. *h*, 16-mesh screen. *i*, 22-mesh screen. *j*, 30-mesh screen. *k*, 250 @ 1¼-in. strokes per min. *l*, Grid of 10 @ 10-ft. lengths of steam pipe inclined 50°, over which material flows. *m*, 7 ft. (wide) × 30 ft., inclined slightly more than angle of repose of sand. Bottom is 2 sets of 10 steam pipes. *n*, 130 to 135 lb. gross. Returned by consignee. *o*, 10 miles to North Creek.

Wausau Abrasive Co. Fig. 70. *(256 Bul. USBM 23.)*

Location: North Wilmot, N. Y.
Capacity: 50 tons of concentrate per week when running one 10-hr. shift per day.
Ore: Almandite crystals ¼- to ⅜-in. diameter constituting 40 to 60 per cent. of the rock. Feldspar and biotite the principal associates.

Summary. Graded crushing in jaw crushers and rolls; close sizing followed by pneumatic concentration. Fines (−48-mesh) wasted.



a Coal fired. *b*, −2-mm. undersize. *d*, Sutton, Steele and Steele.

Fig. 70.—Wausau Abrasive Co.

19. Gold and Silver

GOLD, Au

Properties. Metal; yellow, lustrous, soft, extremely malleable and ductile. See also Table 1. At. wgt., 197.2. Unchanged by air, water, dilute and concentrated acids at any temperature. Dissolves in the presence of free chlorine and of the soluble cyanides. Easily precipitated from its solutions by reducing agents. Uni- and tri-valent, both base- and acid-forming. Alloys freely with many metals, notably silver, copper, mercury and platinum.

Uses. The principal use is for coinage, for which purpose an alloy with copper is used. The composition of the standard coinage alloy in most countries is 900 parts gold and 100 parts copper (900 fine); Great Britain and Portugal, with standards 916.6 fine, are the principal exceptions. Jewelry is commonly manufactured from gold-copper alloys. The fineness of jewelry gold is expressed in carats. Pure gold, 1000 fine, is 24 carat. With this basic relation any other fineness or carat rating can be computed by simple proportion. Gold-silver alloys are used to some extent in jewelry manufacture, having a paler color than the usual alloys. An alloy of 750 parts gold and 250 parts iron is sometimes used by jewelers to make blue gold. A small amount of gold as leaf is used for gilding and gold lettering. Ruby-gold glass is a solid solution of colloidal gold in glass. Potassium aurocyanide is used in gold-plating by electrolysis. Certain gold salts are used in photography.

Ores. The economic minerals are native gold, calaverite and sylvanite. The principal ores are those containing native gold mechanically mixed with pyrite, less frequently with chalcopyrite, arsenopyrite, stibnite, sphalerite

and pyrrhotite, or with the oxidation products of these minerals or in the zones from which they have been leached. Native gold also occurs mechanically mixed with loose, water-sorted material (PLACERS). The metallic gold is almost invariably alloyed with more or less silver. The tellurides (sylvanite and calaverite) ordinarily occur associated with pyrite and with one or more of the other sulphides named above. Gold ores occur usually in fissure veins in sedimentary, igneous or metamorphic rocks or at contacts between any two of them. Hence the associated gangue minerals may be anything. The commonest gangue mineral is, undoubtedly, quartz.

Production. Gold in greater or less quantities is produced in more than 40 countries. World production for the years 1921 to 1924, inclusive (*USGS*) has ranged between 15,500,000 and 18,500,000 fine ounces per year. Of this more than one-half has come from the Transvaal, about 25 per cent. from North America, and about 15 per cent. of the total from the United States. California and Colorado account for about 50 per cent. of the domestic production.

Selling. Gold bullion containing less than 800 thousandths of base metal can be sold to United States assay offices in New York; Helena, Mont.; Seattle, Wash.; Deadwood, S. D., and Salt Lake City, Utah, and bullion of all degrees of fineness is bought by smelters. The amount paid by the government assay offices is the value of the fine gold at $20.67 per troy ounce less a charge to cover melting, parting, refining, etc. For a copy of the schedule and discussion of the sale of foreign gold see *Spurr and Wormser.*

SILVER, Ag

Properties. Metal; white, lustrous, soft, ductile. (See also Table 1.) AT. WGT., 107.88. Not attacked by air or water but becomes coated with a dark film of sulphide in air containing sulphuretted hydrogen. Not attacked by dilute acids, except nitric acid. Dissolves in hot concentrated sulphuric acid. Attacked slowly by hot concentrated hydrochloric acid, but a protecting layer of insoluble silver chloride quickly forms. Very resistant to basic substances even when these are in a fused state. Ion always basic, mono-valent. Alloys freely with most metals.

Uses. The principal use is as a component of an alloy for coins, jewelry and tableware. The chief alloys are those with aluminum, copper, zinc, nickel, and combinations of the same. Salts of silver are used to some extent in medicine and to a great extent in photography.

Ores. The economic minerals are metallic silver, argentite, argentiferous galena, cerargyrite, prousite, pyrargyrite, stephanite, tetrahedrite, polybasite. The most important ores are the silver-bearing lead ores in which the heavy mineral is principally argentiferous galena, usually associated with pyrite, sphalerite and rich silver-bearing minerals. The usual gangue minerals are quartz, calcite, barite and chert. The copper ores of Colorado, Utah, Montana and Arizona produce considerable silver. Silver is also usually associated with gold in both quartz-vein and placer types of deposit. Native silver and sulphides in quartz associated with a complex mixture of sulphides, arsenides, antimonides, etc., are important at Cobalt, Ont.

Production. World production is a fairly constant quantity, ranging, since 1893, between 5,000,000 and 7,900,000 kilograms (161,000,000 and 254,000,000 troy ounces) and averaging 5,900,000 kg. (190,000,000 oz.). Of this total, America produces 85 to 88 per cent.; Mexico, 37 to 39 per cent., and the United States, 25 to 30 per cent. The principal producing states, in order (1924), are, Utah, Montana, Nevada, Idaho, Arizona, California and Colorado.

Selling. Except in the case of the relatively small amounts of silver in gold bullion, most silver is sold by the mine in the form of a base-metal con-

trate and hence goes to the smelter. The net price paid by the smelter varies according to the particular contract, but usually 95 per cent. of the silver content of the concentrate is paid for at the New York price for bar silver at some particular time with respect to the date of delivery of the concentrate. The average price of silver varies considerably, as shown by Table 51. Much

Table 51. Average annual price of silver at New York (cents per troy ounce) (*a*)

1910	53.49	1918	96.77
1911	53.30	1919	111.11
1912	60.83	1920	100.90
1913	59.79	1921	62.65
1914	54.81	1922	67.52
1915	49.68	1923	64.87
1916	65.66	1924	66.78
1917	81.42	1925*b*	69.08

a Eng. and Min. Jour.-Press. *b* First 11 months.

of the apparent stability since 1920 has been due to large United States government purchases.

TREATMENT

The ores of gold and silver constitute an endless variety, but, from the point of view of milling they may be roughly classed as (1) placer deposits, (2) simple free-milling vein deposits, (3) simple sulphide ores, (4) complex or refractory ores in which the precious metals are the principal economic constituent, (5) complex ores in which the precious metals are important secondary economic constituents, (6) ores in which the precious metals are by-products of treatment.

Gold placers. Methods of treatment differ principally because of mining problems, the requirements of tailing disposal, and the water supply rather than because of the concentrating problem.

Panning and rocking. Very small high-grade placers where water is scarce are worked by panning or rocking (Sec. 8, Arts. 8 and 9).

Shoveling-in. When the deposits are larger, low-lying and not cemented, and more water is available, the long tom or a simple sluice line is used (Sec. 8, Art. 11). With more water, a HYDRAULIC ELEVATOR may be used for this type of deposit, or various mechanical means may be used to elevate the feed to the sluice line.

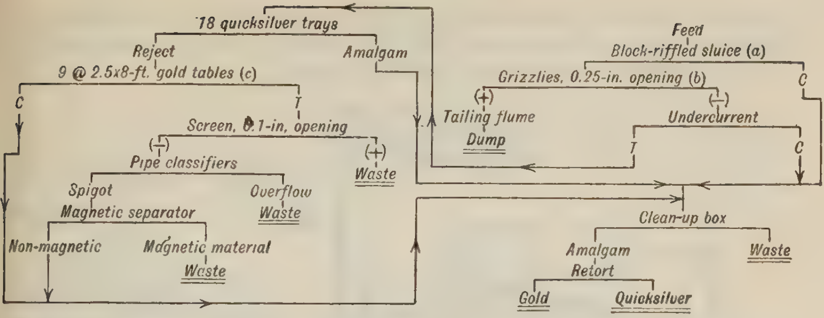
Hydraulic mining. When the deposits are large, lie well above the point of tailing disposal, and there is an abundance of water available under sufficient pressure, the gravel is broken down by hydraulic giants, transported to the concentrating plant by the water used for excavation, and flowed through sluices, usually fitted with undercurrents (Sec. 8, Art. 11) to catch the fine gold that cannot be held in the main sluice line on account of the strong current necessary to move the coarse gravel.

Camp Carson Mining and Power Co. Fig. 71. (*100 J 472.*)

Location: Union County, Ore.
Ore: Fine gold in loose gravel. Material as low as 8 to 9 cents per cu. yd. can be treated.
Cost, including hydraulicking, 5¢ per cu. yd.

Dredges are used when the deposit is low-lying, of large lateral extent, and not too great depth. The flow-sheet of the gold-saving devices depends on the character of the gravel and the requirements of tailing disposal.

Sluice dredge (Fig. 72) is best adapted for saving coarse gold and can be used when the percentage of large boulders is not too great. In a typical



a, The coarser gold is recovered in two lengths. (See Sec. 8, Art. 11.) *b*, In bottom of sluice. Made of round bars, about ¾-in. diameter, mounted in comb fashion and set in sluice with free ends downstream and overlapping the following grizzly. 8 sets used = about 16 ft. along bottom. *c*, Slope, 2 in. per ft.

Fig. 71.—Camp Carson Mining and Power Co.

sluice dredge the gravel passes first over a wash trommel with 3- to 4-in. openings, oversize discharges over the side of the boat into the pond, and undersize goes through a string of sluices, usually with an undercurrent and gold tables for the fine material. Fine tailing is discharged on top of the coarse from previous working.

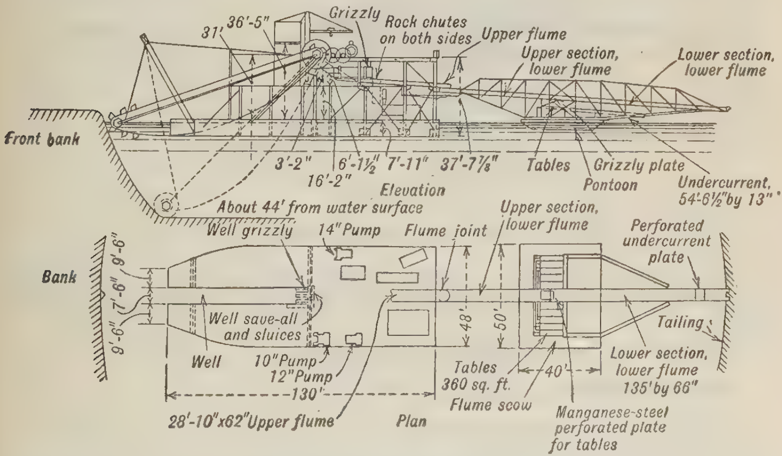


Fig. 72.—Sluice dredge.

Stacker dredge (Fig. 73) receives the bucket-line discharge in a large wash trommel with ½- to ¾-in. apertures; the oversize is discharged onto a stacking conveyor while the undersize goes first to a series of tables in parallel, thence through short sluices, frequently a number in parallel, to the pond. The stacker dredge is a better saver of fine gold than the sluice dredge, on account of the smaller depth and smaller velocity of the streams passing over the riffles, but it loses very coarse gold that will not pass the wash-trommel

perforations and also fine gold that balls up with clay and goes to the stacker. It also has the disadvantage that the coarse tailing is stacked on top of the fine, which ruins the land for any agricultural use.

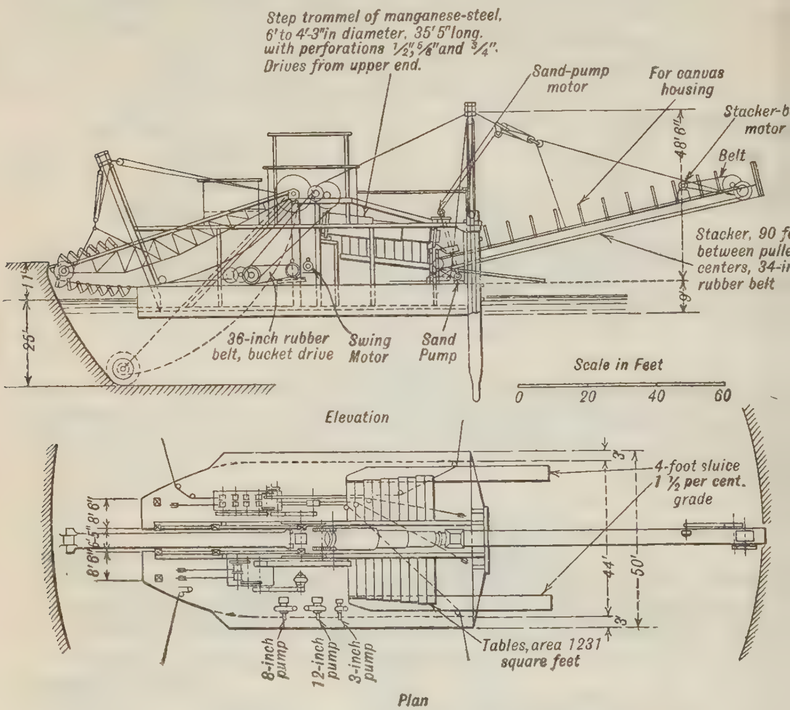


Fig. 73.—Stacker dredge.

Details of the gold-saving devices on four Montana sluices are given in Table 52. For further detail see *Bul. 121* and *127 USBM* and *Bul. 263 USGS.*

Clayey gravel requires thorough disintegration.

Siberian gravel mine. Fig. 74. (*105 J 859.*)

Ore: Clayey gravel containing free gold.

Capacity: 250 cu. yd. per 24 hr.

Water: 250 miner's inches (= @ 6 sec. ft.). This is a washing duty of 1 cu. yd. per 24 hr. per miner's inch. Hutchins calls attention to the fact that the corresponding figure for gold dredging is frequently 10 cu. yd.; for hydraulicking, 5 cu. yd.; for cemented gravel in California drift mines, 6 to 12 cu. yd.; and that even in hydraulic elevating the duty is greater than 1. cu. yd.

Recovery: About 67 per cent. Much of the loss was in clay balls from the first disintegrator.

Simple free-milling ores comprise the great bulk of the gold ores now mined. Substantially all of them are treated by amalgamation and/or cyanidation. (See Sec. 15.)

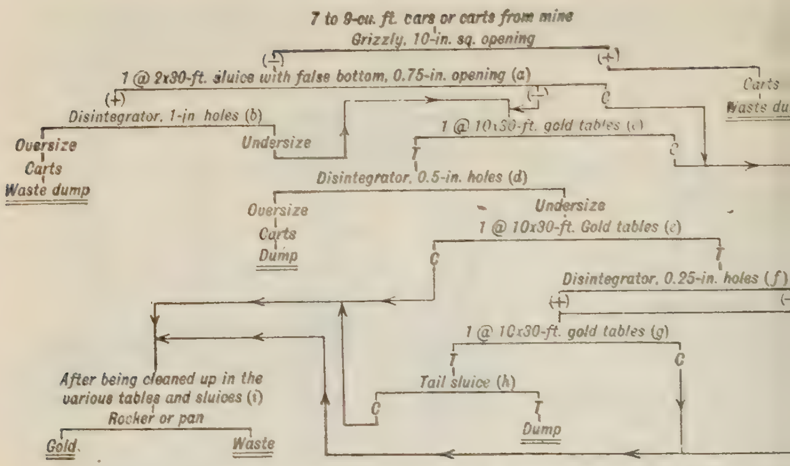
Table 52. Gold-saving devices on Montana dredges. (*After Jennings, Bul. 121, USBM 12*)

	Sluice dredge	Stacker dredge		
	No. 3	No. 1	No. 2	No. 4
Power	Elec.	Elec.	Elec.	Elec.
Cu. yd. per mo.	82,000	96,000	63,000	300,000
Wash trommel:				
Length, ft.	18	35	35	48½
Diam., in.	61	51 to 72*a*	51 to 72*a*	98
Plates, material	Mn	Mn	Mn	Mn
Plates, thickness, in.	⅝	⅝	⅝	1
Plates, aperture, in.	4¼×6	½–¾	½–¾	½–¾
Gold tables, number	10	20	20	24
Grade, per cent.	12	12½	12½	12½
Width, in.×length, ft.	30×12	30×18	30×18	30×11to29¾
Riffles, angle, in.	1¼	1¼	1¼	1¼
Perforated plate	⅜-in. hole	....	....	....
Water, gal. per min.	12,500	6700	6700	12,000
Tail sluices, number	None	2	2	12
Grade, per cent.	....	12½	12½	12½
Width, in.×length, ft.	....	48×40	48×40	48×17 to 44
Riffles, angle, in.	....	1¼	1¼	1¼
Undercurrents, number	2	None	None	12
Grade, per cent.	10	....	....	12½
Width, in.×length, ft.	13×54	....	....	34×7½
Riffles, material	Wood	....	....	Angle
Size, in.	1×1½	....	....	1¼
Perforated plates, slots, in.	1/16	....	....	¼*b*
Well save-all, (*c*) number	1	1	1	1
Grade, per cent.	8	8	8	8
Width, in.×length, ft.	18×26	18×7½	18×7½	18×18½
Upper flume, grade, per cent.	6	None	None	None
Riffles, material	Angles	None	None	None
Size, in.	2×2½	None	None	None
Width, in.×length, ft.	62×29	None	None	None
Lower flume, grade, per cent.	6	None	None	None
Riffles, material	Angles	None	None	None
Riffles, size, in.	2×2½	None	None	None
Width, in.×length, ft.	66×135	None	None	None
Total gold-saving area, sq. ft.	1264	1231	1231	3000
Sq. ft. per cu. yd. per day	0.46	0.38	0.59	0.30
Recovery, per cent. of total: (*d*)				
Upper flume	55.55	....	....	....
Lower flume, 1st section	26.81	....	....	....
Second section	7.14	....	....	....
Below undercurrent	0.53	....	....	....
Undercurrent	1.38	....	....	0.55
Well	5.05	4.22	2.83	2.59
Tables	2.93	92.76	93.34	87.92
Tail sluices	0.28	3.02	3.83	8.94
Miscellaneous	0.33	....	....	....

a Stepped, cylindrical. *b* Round hole. *c* A gold-saving table to treat the spill around the head of the digging ladder. *d* Actual recovery not ascertainable on account of practical impossibility of sampling feed.

Simple sulphide gold-silver ores

These ores contain gold and small amounts of silver intimately associated with sulphides in a simple rocky gangue. The usual method of treatment is concentration of the gold-bearing sulphides into small bulk, followed by smelting or hydro-metallurgical treatment of the concentrate. If the tailing



a, Slope, 18 in. in 12 ft. *b*, Locally called *chasha*. Shallow pan, 10 to 12 ft. diamete cast-iron bottom with perforations. 4 to 8 radial arms on a central spindle. Iron-sho projections from the arms rake over the material. Undersize works through the botto perforations and oversize is discharged intermittently by a bottom door. 15 to 20 r.p.n *c*, Slope, 27 in. in 12 ft. Occasional riffles only. *d*, Similar to first except that oversi is discharged over periphery by revolution of scrapers. 24 to 32 r.p.m. *e*, Slope, 24 in. i 12 ft. Occasional riffles only. *f*, 15 to 20 r.p.m. *g*, Slope, 18 in. in 12 ft. *h*, 4 ft. wid Slope, 14 in. in 12 ft. *i*, See Sec. 8, Art. 11.

Fig. 74.—Siberian gravel mine.

is of sufficiently high grade, it is cyanided separately. Alaska Gastineau typical of the first and the Mother Lode mills, California, making a gravit concentrate on tables and vanners, with grinding, amalgamation and cyanid treatment of concentrate and cyanidation of tailing are typical of the secon method of treatment.

Alaska-Gastineau Mining Co. Fig. 75. (Q. 63 A 488.)

Location: Thane, Alaska.

Ore: Gold and silver associated with pyrrhotite, galena, arsenical pyrite and sphale in quartz lenses occurring in slate, schist and meta-gabbro, which latter also carry mine values. Analysis, per cent: SiO_2, 59.7; Fe, 5.2; Al_2O_3, 21.4; CaO, 5.0; MgO, 0.4; S, 1 Pb, 0.1; Zn, 0.2.

Capacity: 10,000 tons per 24 hr.

Assays, Au: Feed, $1.25; concentrate, containing about 44 per cent. of the tot gold recovered, $400; dead bullion contains balance of gold recovered; tailing, $0.2 (1920, $0.17.)

Recovery: 75 to 80 per cent.

Ratio of concentration: 1000 or 1200 : 1.

Power: 13.3 hp.-hr. per ton milled distributed as follows: Coarse crushing, 0.52; f crushing, 6.02; concentrating, 4.78; re-treatment plant, 0.47; pumping, 1.17; lighting, 0.3 motor generator, 0.93.

Water: 5.2 tons per ton milled.

Labor: 78.9 tons per man-shift, operating.

Distances: Mine to mill, 3½ miles; mill to smelter, 2000 miles; water, a part comes mile by flume, the balance is pumped 1000 ft. with a consumption of 700 hp.; powe transmitted 8 miles at 22,000 volts.

Costs in 1917, before the period of full tonnage, but also before the war increase labor and supply prices were, in dollars per ton: Coarse crushing, 0.02804; fine crushi 0.09432; concentrating, 0.05136; general mill, 0.02778; power and light, 0.01467; gene

overhead, 0.02340; total, 0.23957. Of this total cost labor was 40.6 per cent.; supplies, including power, 22.8 per cent.; repairs, 15.8 per cent.; sundries, 20.8 per cent. In 1920 the total milling cost was $0.28 per ton.

General: Steeply sloping millsite. Sizing-assay tests of feed to primary Garfield tables (concentrating-mill feed), re-treatment-plant tailing and general tailing are given in Table 53.

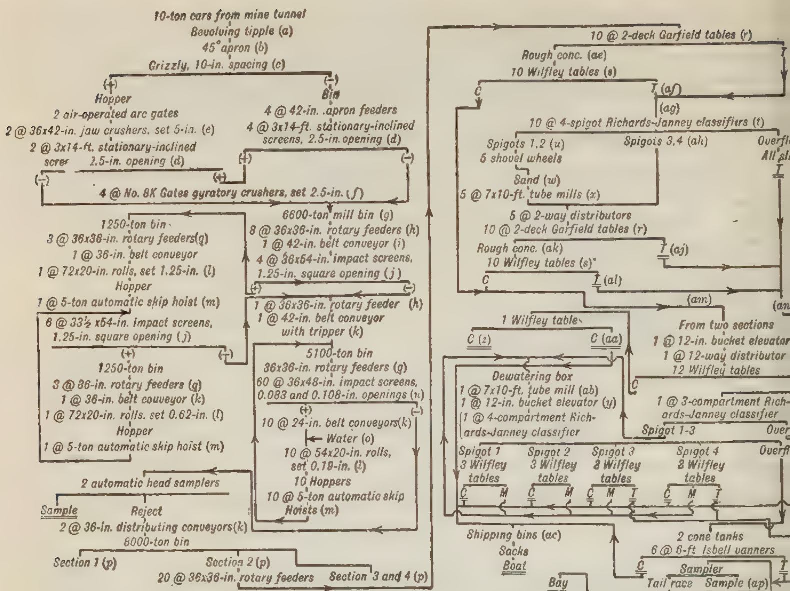
Table 53. Sizing-assay tests, Alaska Gastineau mill

Screen, Tyler mesh	Concentrating-mill feed			Re-treatment-plant tailing			General tailing		
	Weight, per cent.	Assay, $Au per ton	Value, per cent.	Weight, per cent.	Assay, $Au per ton	Value, per cent.	Weight, per cent.	Assay, $Au per ton	Value, per cent.
10	2.5	0.83	1.5	....	....	....	....	....	....
20	27.7	1.24	24.6	....	....	....	1.5	0.15	1.2
28	21.0	1.45	21.8	....	....	....	9.1	0.15	7.2
48	5.7	3.10	12.7	....	....	....	10.4	0.21	11.6
65	5.6	3.72	15.0	2.7	0.41	1.3	12.6	0.26	17.4
80	3.2	1.24	2.8	1.4	0.52	0.9	8.1	0.26	11.1
100	3.7	1.86	4.9	9.6	0.62	6.9	8.9	0.26	12.2
150	5.9	0.83	3.5	32.9	0.72	27.6	7.9	0.26	6.3
200	1.7	1.03	1.3	4.4	0.93	4.7	7.2	0.15	5.7
−200	23.0	0.72	11.9	49.0	1.03	58.6	34.3	0.15	27.3
Total	100.0	1.39	100.0	100.0	0.86	100.0	100.0	0.19	100.0

Summary. Table concentration by roughing-cleaning system. CRUSHING: Jaw crusher from 30-in. to 5-in.; gyratory from 10-in. to 2.5-in.; 2-stage roll crushing from 2.5- to 1-in.; one-stage choke crushing in rolls from 1-in. to 0.10-in.; tube mills from 0.1- to 0.04-in. CONCENTRATION: Roughing on primary Garfield tables with re-treatment of rough concentrate on Wilfley tables and re-grinding of coarse tailing followed by a similar roughing-cleaning step with rejection of tailing from both roughing and cleaning tables. Concentrate from the above operations was re-treated in two stages. The first stage was a roughing-cleaning operation on Wilfley tables making finished concentrate and tailing, the coarse part of which was re-ground, classified, and the spigot products re-treated in one pass over Wilfley tables that made finished concentrate and tailing and a middling returned to the re-grinding mill. It is an interesting feature of this flow-sheet that the slime is low-grade and that, consequently, the hydraulic classifiers are used as concentrators in so far as they reject overflow to the tail race.

This mill ceased operations early in 1921 after a life of some six years, during the last three of which operations were conducted at a loss. The causes of failure were incorrect sampling of the ore body as a result of which it was estimated that the grade of the feed would be between $1.50 and $1.75 while actual mining produced ore running nearer $1.25 per ton; failure to foresee that considerable waste would have to be mined with the ore, resulting in further reduction in grade of mill feed; and increases in labor and supply prices. The latter difficulty was nearly altogether overcome, in so far as the mill was concerned, as is evidenced by the fact that in 1920 the cost of milling was only $0.04 per ton higher than in 1917, but the 1920 mill heads carried only $0.85 gold, and while the yield was $0.68 per ton (80 per cent. recovery) this was insufficient to cover mining and milling costs.

Notwithstanding the failure of the enterprise, this mill was eminently well fitted to the treatment of a low-grade ore containing relatively coarse gold intimately associated with sulphides. The use of automatic skips for elevating in the roll circuits was unique in concentrating-mill practice, although they have been used in other lines.



a, Takes 4 cars at a time. Actuated through friction clutch and fitted with ba brake. 50-hp. motor installed is sufficient for two tipples. *b,* Covered with discarded r shells, 1 in. thick, straightened at red heat under steam hammer. *c,* 52 ft. (wide) × 21 Bars are 10-in. I-beams with 4-in. flanges. Upper flange covered with 6-in. channel, whi is, in turn, capped with manganese-steel shoes, 6 to the bar. Slope, 9½ in. per ft. *d,* Slo 45°. Crimped-wire cloth. *e,* See Sec. 3, Table 10. *f,* See Sec. 3, Table 15. *g,* Cut rock. *h,* 4.5 r.p.m. *i,* 600 ft. long, slight down grade. 8-ply belt. *j,* See Sec. 5, Tab 34. Specially reinforced all-steel screen for heavy service. *l,* See Sec. 3, Table 22. *m,* S Sec. 23, Art. 6. *n,* See Sec. 5, Table 34. *o,* Feed sprayed to lay dust. *p,* All sectio alike. *r,* See Sec. 10, Table 26. *s,* See Sec. 10, Table 1. *t,* See Sec. 6, Table 13. 2 tons solid per classifier. *u,* Contains 96 per cent. of the +48-mesh material. Assa 1st spigot, \$0.60 per ton; 2nd spigot, \$0.32. *w,* 90 to 93 per cent. +48-mesh; abo 33 per cent. moisture. Assay, \$0.49 per ton. *x,* See Sec. 4, Table 65. *z,* Conta practically all of the free gold with some coarse lead. Melted down at plant to high-gra bullion and matte. *aa,* Shipping concentrate. Assays about 50 per cent. Pb and \$800 A *ab,* Grinds to 100-mesh. *ac,* Shipping concentrate assays \$400 Au, 50 oz. Ag, 40 per ce Pb. *ad,* \$0.347 per ton. *ae,* \$13.15 per ton. *af,* \$0.844 per ton. *ag,* \$0.523 per t *ah,* Assays: 3rd spigot, \$0.26 per ton; 4th spigot, \$0.26. *ai,* \$0.16 per ton. *aj,* \$0.17 p ton. *ak,* \$5.45 per ton. *al,* \$0.46 per ton. *am,* \$70 per ton. *an,* \$0.177 per t *ao,* \$1.00 per ton. *ap,* \$0.20 per ton.

FIG. 75.—Alaska-Gastineau Mining Co.

Complex and refractory gold and silver ores

This class includes both ores in which the precious metals constitute t principal economic value and others in which the precious metals are secondary importance. The common characteristic is insusceptibility treatment by amalgamation and cyanidation. This may be due to the fa that the precious metals will not amalgamate and occur in association wi substances that consume undue amounts of cyanide, or that they occur particles of such size that solution requires an unduly long time, or in su small quantity that without the recovery of the associated metals treatme

[illegible]

Coniagas Mines, Ltd. [illegible]

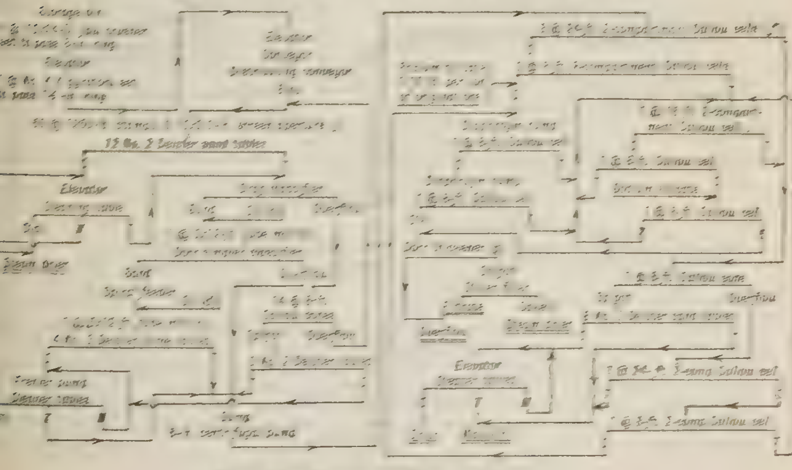


Fig. [illegible].—Coniagas Mines, Ltd.

[illegible]

coal tar. *e*, 600 oz. Ag per ton. *f*, Feed 20 per cent. solids, 3.4 oz. Ag per ton; concentrate, @ 50 oz. Ag per ton; tailing, 0.79 oz. per ton; 65 to 70 per cent. of this value is in the −200-mesh. *g*, 20 × 10-ft. *h*, 1.23 oz. Ag per ton. *i*, Analysis, per cent.: Insol., 42.3; Pb, 0.2; Cu, 3.6; Sb, 1.3; As, 6.0; Fe, 14.8; Al_2O_3, 8.8; Ni, 0.6; Co, 2.0; Zn, 0.1; CaO, 5.2; MgO, 2.2; S, 8.4; Ag, 1.7 (= 504 oz.). *j*, Shoes, chrome steel, wear 0.26 lb. per ton crushed; dies, hard iron, wear 0.14 lb. per ton. Tons per stamp per 24 hr. = 6.4. Sizing test of product, per cent.: +6-mesh, 36.0; +8, 9.9; +10, 8.2; +14, 8.5; +20, 4.4; +28, 6.6; +35, 3.9; +48, 3.9; +65, 3.1; +100, 2.5; +150, 2.2; +200, 1.1; −200, 9.6.

Location: Cobalt, Ont.

Ore: Native silver with niccolite, smaltite, chalcopyrite and some argentite, galena and pyrite in quartz and calcite veinlets in diabase.

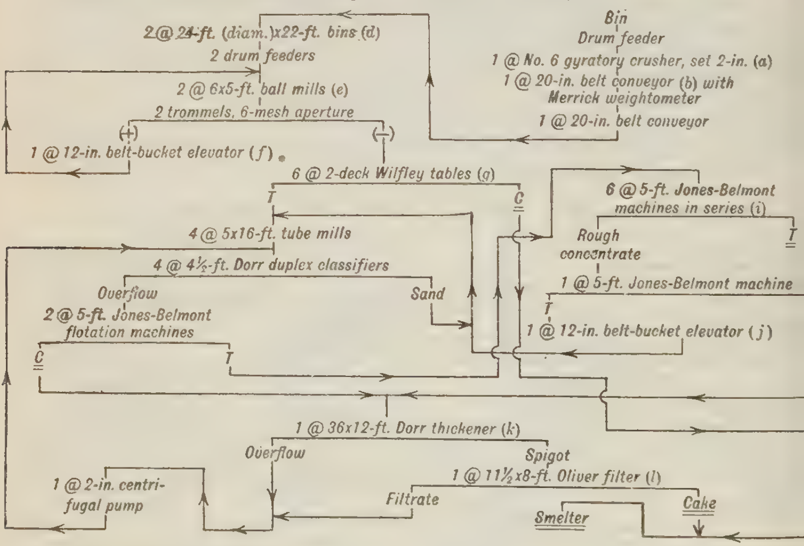
Capacity: 360 tons per 24 hr.

Assays: See notes *a*, *e*, *f*, *h*, *i*, Fig. 76.

Cost: $1.80 per ton milled.

Summary. Gravity concentration and flotation. CRUSHING: Jaw crusher to 3-in., gyratory to 1½-in., stamps to ½-in., 2-stage open-circuit tube milling to −60-mesh. CONCENTRATION: Roughing and cleaning of sands on shaking tables at −½-in. to 100-mesh; pneumatic flotation in three circuits, *viz.:* primary slimes, table and flotation middling, and primary-flotation tailing; concentrate in each circuit cleaned twice.

Belmont Surf Inlet Mine. Fig. 77. (*Q, 105 J 720.*)



a, 50-hp. motor. Run 8 hr. per day. *b*, 419 ft. long. Slope, −3.2 in. per ft. 280 ft. per min. *d*, Built of logs. Discharge gates in side near bottom. *e*, See Sec. 4, Art. 5. *f*, 12 × 6 × 6-in. buckets spaced 18 in., 20-ft. lift. 36-in. head pulley and 30-in. boot pulley. Installed in duplicate. *g*, 260 @ 0.75-in. strokes per min. *i*, See Sec. 12, Art. 15. *j*, 12 × 6 × 6-in. buckets, spaced 18 in.; 23-ft. lift. 36- and 30-in. head and tail pulleys, respectively. *k*, 1 rev. in 8 min. 86 per cent. moisture in feed, 30 to 40 per cent. in spigot. *l*, See Sec. 17, Art. 3.

FIG. 77.—Belmont-Surf Inlet mill.

Location: Surf Inlet, B. C.

Ore: Gold with a small amount of silver and copper and iron sulphide in hard white quartz.

Capacity: 350 to 400 tons per 24 hr.

Assays, Au, oz. per ton: Mill feed, 0.6; table concentrate, 5.36; table tailing, 0.33; flotation concentrate, 8.2 oz. Combined insoluble, 14 per cent.

Recovery: Gravity, 47.8 per cent.; total, 92.2 per cent.

Ratio of concentration ranges from 8 : 1 to 18 : 1, depending on pyritic content of feed.

Running time: 95 per cent. of possible.

Power: 42 hp.-hr. per ton milled.

Labor: 40 tons per man-shift, operating; 60 tons per man-shift on repairs.

Water: 3 to 4 tons per ton milled.

Distances: Mine at mill; mill to smelter, 500 miles; power transmitted, 5 miles at 23,000 volts; water pumped ⅓ mile from lake with consumption of 30 hp.

General: 130 miles from railroad. Steeply sloping millsite.

Summary. Tabling and flotation. CRUSHING: Gyratory from 12- to 2-in.; ball mills from 2.5- to 0.12-in.; tube mills from 0.12-in. to 56-mesh (70 per cent. −200-mesh). CONCENTRATION: Tabling at −0.12-in. to remove coarse gold; flotation by combination routing for balance of treatment.

Le Roi No. 2. Fig. 78. *(114 J 1118.)*

Location: Rossland, B. C.

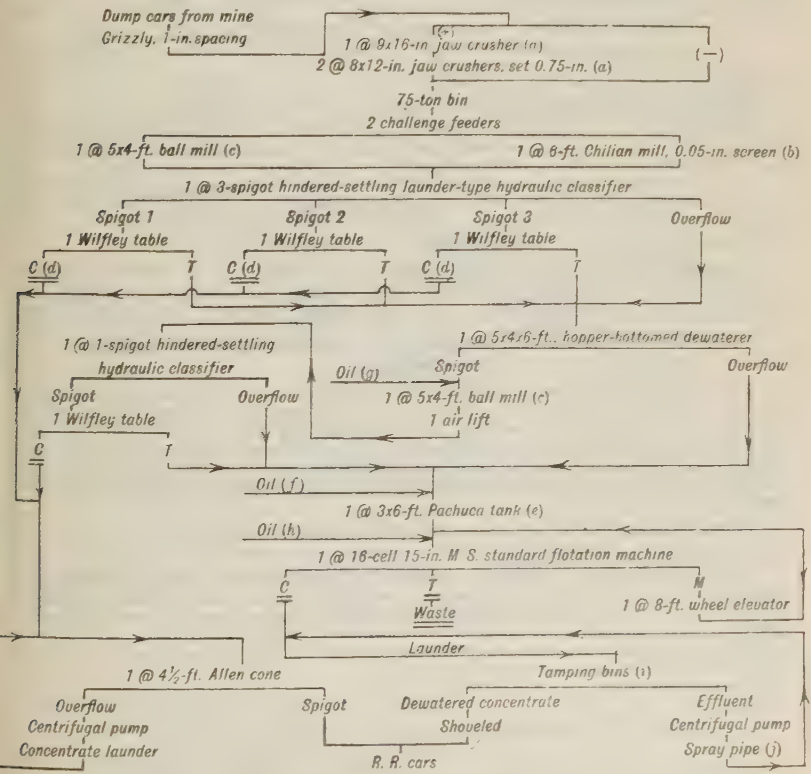


FIG. 78.—Le Roi No. 2 mill.

a, Repairs on the 3 crushers amount to $0.025 per ton crushed, indicating extreme hardness of ore. *b,* See Sec. 4, Table 88. *c,* See Sec. 4, Table 4. *d,* Principally pyrrhotite: assay, 1.0 oz. Au, 1.5 per cent. Cu. *e,* 2 air-lifts, one for circulation and one for discharge. *f,* Pine oil, @ 0.25 lb. per ton. *g,* Water-gas tar, @ 1 lb. per ton. *h,* Less than 0.25 lb.

per ton of the above oils added to middling cells as needed. *i,* Tank 1 ft. deep, 2 ft. wide, 22 ft. long, fed at one end, burlap screen at opposite end, effluent to a similar tank. Transverse baffles 6 in. high in bottom of tanks. Settled concentrate tamped for 2 or 3 min. every 2 or 3 hr. Compacted concentrate shoveled out as it accumulates. Product, 10 to 15 per cent. water. *j,* Over concentrate launder.

Ore: Gold-bearing chalcopyrite and barren pyrrhotite in augite porphyrite. Chalcopyrite finely disseminated in coarse pyrrhotite and in country rock. Gold is about 80 per cent. of total value. It occurs free in both sulphide and gangue rock.

Capacity: 100 to 150 tons per 24 hr., dependent upon water supply.

Assays: Concentrating ore, 0.15 to 0.50 oz. Au; 0 to 0.6 oz. Ag; 0.5 to 1.0 per cent Cu. Table 54 gives representative performance over a considerable period.

Table 54. Milling results at Le Roi No. 2. (*After Lay*)

	Au, oz. per ton	Cu, per cent.	Fe as sulphide, per cent.
Mill feed	0.155	0.42	8.1
Flotation feed	0.074	0.30	6.3
Table concentrate	0.93	1.40	29.6
Flotation concentrate	0.91	4.66	28.5
Flotation tailing	0.043	0.10	5.2

Recovery: Au, 75 per cent.; Cu, 80 per cent.

Ratio of concentration: 10 : 1.

Water: Scarcity prohibits erection of a large concentrator at the mine.

General: Gold recovery is more important than copper on account of the fact that the smelter pays for 95 per cent. of the gold in concentrate while only 65 per cent. of the copper is paid for, after deductions for treatment, etc.

Costs, dollars per ton of mill feed: coarse crushing, steel, 0.025; primary grinding, steel, 0.345; secondary grinding, steel, 0.165; power for crushing and tabling, 0.120; labor for crushing and tabling, 0.375; sundries, crushing and tabling, 0.065; flotation: oil, 0.072; labor, 0.148; power, 0.040; royalty and sundry, 0.050; total, 1.405.

Summary. Tabling followed by differential flotation. CRUSHING: 2-stage jaw crushing from 8- to 1-in.; 2-stage ball milling from −1-in. to 48-mesh. CONCENTRATION: Tabling after hydraulic classification in two stages with intermediate re-grinding of primary-table tailing; differential flotation of primary slime and secondary-table tailing.

Lay points out that higher recovery of both gold and copper could be made by collective flotation, but that the resulting higher recovery of iron would lower the financial return because of a combined freight and smelting charge of $6.40 per ton of concentrate. He also cites an experiment with heap-oxidized ore, *i.e.*, ore exposed to the weather for several months, that yielded a flotation concentrate containing 8 per cent. Cu, with copper and gold recoveries equal o or better than the present, and suggests the possibility of such treatment.

Seoul Mining Co. Fig. 79. (*33 IMM 3; 119 P 805.*)

Location: Tul Mi Chung, Korea.

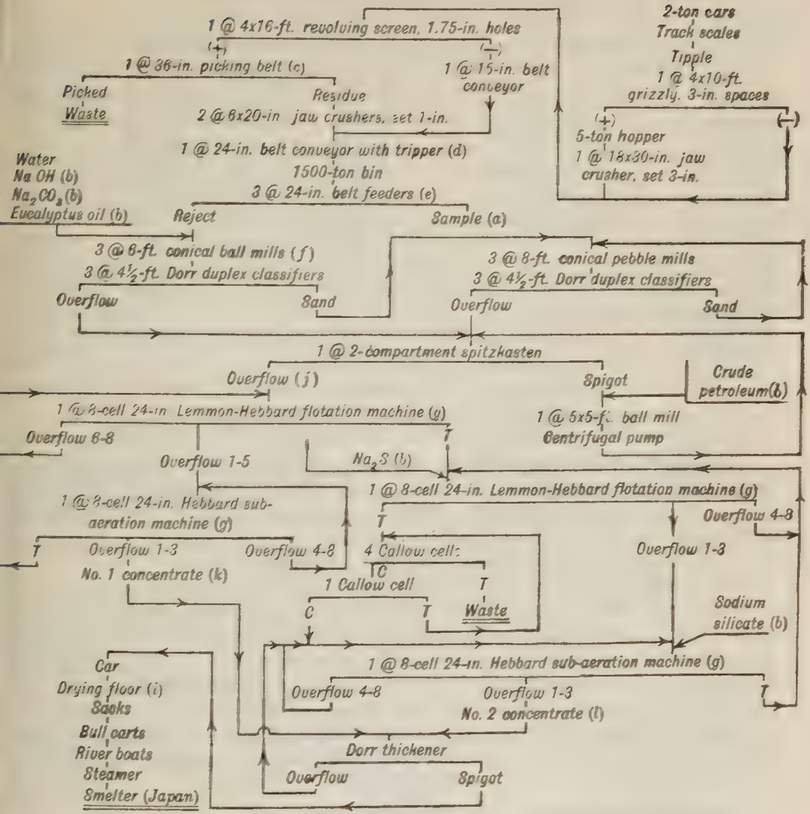
Ore: Highly complex. Gold and some silver with chalcopyrite, pyrite, mispickel and small amounts of lollingite (Fe_2As_3), galena, blende, bismuth and molybdenum minerals in a contact-metamorphic gangue containing much garnet, calcite, diopside, serpentine, mica, epidote, zoisite, actinolite, etc.

Analysis, per cent: Cu, 1.18; Fe, 7.45; As, 0.37; S, 1.76; Pb, 0.05; Zn, 0.30; Bi, 0.11; SiO_2, 67.07; Al_2O_3, 0.69; CaO, 10.76; MgO, 1.43.

Capacity: 450 tons per 24 hr.

Assays:

	Au, oz. per ton	Ag, oz. per ton	Cu, per cent.
Feed, aver.	0.3	0.5	1.0
Concentrate, total	5.0–5.4	....	20–24.5
Tailing	0.09	....	0.1



a, Hourly moisture sample taken by hand. *b,* NaOH, 1 lb. per ton; Na_2CO_3, 1 lb.; Na_2S, 0.5; Na_2SiO_3, 1 lb.; eucalyptus oil, 0.3 lb.; crude petroleum, 0.25 lb. *c,* 60 ft. long. About 2 per cent removed, principally clean limestone averaging $0.50 Au and a trace of Cu. *d,* 106 ft. long. *e,* Fed by 18 × 24-in. rack-and-pinion gate. *f,* See Sec. 4, Table 11. *g,* 1500 ft. per min. peripheral speed. *i,* See Sec. 18, Art. 2. *j,* Aver., 12 to 14 per cent. + 100-mesh. *k,* 88.8 per cent. of total concentrate, assaying 4.90 oz. Au and 25.23 per cent. Cu. *l,* 11.2 per cent. of total concentrate, assaying 6.17 oz. Au and 18.32 per cent. Cu.

FIG. 79.—Seoul Mining Co., Tul-Mi-Chung mill.

Recovery (1923): Au, 70.1 per cent.; Cu, 90 per cent.

Ratio of concentration: 30 to 35 : 1.

Power consumption (1923), hp.-hr. per ton milled: Crushing, 2.77; grinding, 19.41; flotation, 5.66; re-grinding, 3.26; water, 1.25; total, 32.35.

Water: 3.8 tons per ton of ore.

General: Steeply sloping millsite. Mine to mill, 3300 ft. Mill to railroad, 40 to 50 miles.

Cost (1923), $1.24 per ton milled.

Cost per ton milled, 1918; 154,300 tons milled

Department	Supplies	Power (*a*)	Labor (*b*)	Total
Crushing	$0.0643	$0.0322	$0.0177	$0.1142
Sorting	....	....	0.0033	0.0033
Grinding	0.1784	0.2541	0.0153	0.4478
Flotation	0.2588	0.0474	0.0074	0.3137
Tables	0.0074	0.0039	0.0016	0.0129
Re-grinding	0.0247	0.0390	0.0016	0.0654
Tailing elevating	0.0011	0.0137	....	0.0148
Slime plant	0.0046	0.0050	0.0064	0.0160
Tailing dam	0.0019	....	0.0188	0.0390
Water supply	0.0034	0.0354	0.0002	0.0390
Mill heating and lighting	0.0316	....	0.0007	0.0323
Building repairs	0.0009	....	....	0.0009
Fire protection	....	....	....	0.0004
Supervision	0.0188	....	0.0580	0.0768
Assaying	0.0226	....	....	0.0226
Total	$0.6185	$0.4307	$0.1310	$1.1818

a Electricians' labor included with power. *b* Mill labor, 20 to 35 cents per 8-hr. shift.

Summary. Flotation, combination routing. CRUSHING: Jaw crusher from 18- to 3-in.; jaw crusher from 4- to-1 in.; ball, pebble and ball mill in series from 1-in. to 10 per cent. +100-mesh. CONCENTRATION: 3-stage roughing with one cleaning of the concentrate from the first and second roughers and cleaning and re-cleaning of concentrate from the scavenger rougher.

The difficult problems at this plant are to save the gold and to keep down the tonnage of concentrate. The gold is finely divided and, while some occurs free, much is very closely associated with the chalcopyrite and the lollingite. The latter mineral does not float readily, and in order to bring it up, it is necessary to float considerable pyrite. Even thus, considerable gold is lost. Concentrate handling, freight and smelting cost $30 to $40 per ton of concentrate or upward or $1 per ton milled, hence it is not economically possible to increase gold recovery at the expense of any decided increase in tonnage of concentrate produced.

Mount Morgan mine. Fig. 80. (*34 IMM, 448.*)

Location: Mount Morgan, Queensland, Australia.

Ore: Gold with a trace of silver, pyrite and chalcopyrite in quartz. Gold is very finely disseminated in the quartz.

Capacity: 800 tons per 24 hr.

Assays:

	Au, oz., per ton	Cu, per cent.	Fe, per cent.	SiO_2, per cent.
Feed	0.293	2.00	15.00	64.74
Jig concentrate	0.318	1.84	38.67	15.65
Table concentrate	0.562	3.47	39.98	12.43
Flotation concentrate	1.113	11.99	26.06	29.75
Tailing	0.0104	0.19	3.54	....

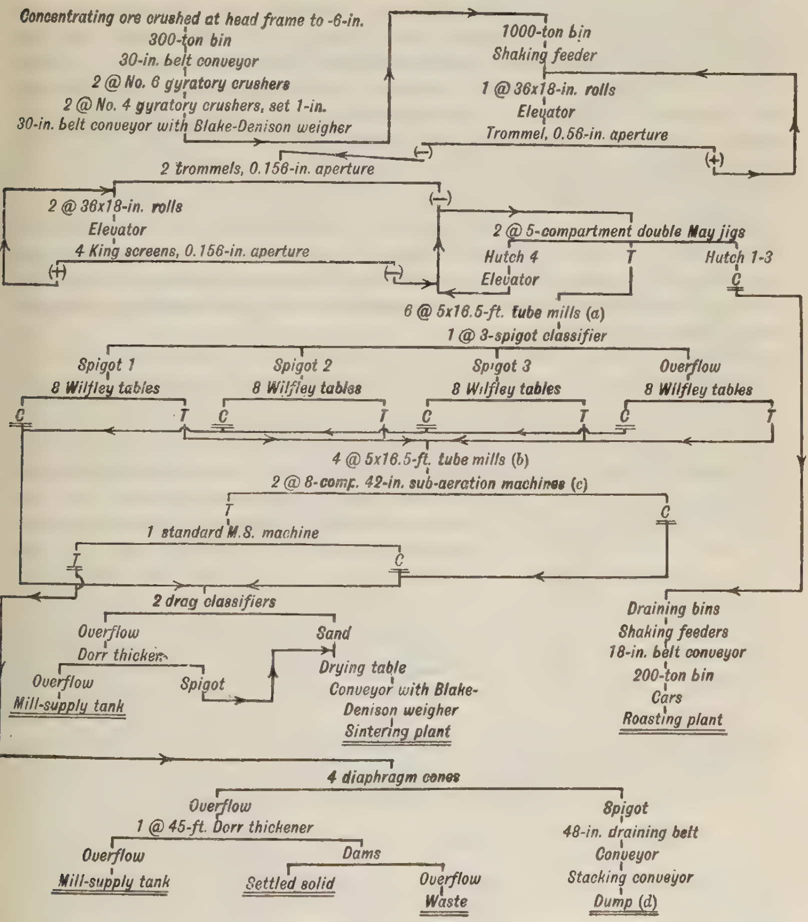
Recovery: 77.08 per cent. Au; 93.74 per cent. Cu.

Ratio of concentration: 2.81 : 1.

Water: new, 1 ton per ton milled.

Summary. Jigging, tabling, flotation. CRUSHING: Coarse breaking to 6-in.; 2-stage crushing in gyratories from 6- to 1-in.; 2-stage roll crushing from

1- to 0.156-in.; 2-stage tube milling from 0.156- to 0.0083-in. with intermediate tabling. Concentration: Jigging at −0.156-in.; tabling at 20-mesh; floating at 60-mesh, making pyritic concentrate in the gravity steps and a tailing for re-treatment; a copper concentrate and final tailing by flotation.



a, Grinding to 13 per cent. +40-mesh (0.0125-in.). *b*, Grinding to 4 per cent. +60-mesh (0.0083-in.). *c*, 18-in. shrouded impellers. *d*, Drawn as desired through tunnel and sent by aerial tram to mine for filling.

Fig. 80.—Mount Morgan mill.

20. Graphite (C)

Properties. Soft, amorphous or flaky, resists heat and chemical action, conducts electricity, and has good lubricating qualities.

Uses. Principal use is in the manufacture of crucibles. This industry accounts for 85 to 90 per cent. of domestic consumption of crystalline graphite. About 80 per cent. of the graphite in crucible mixes is Ceylon graphite, and, due to the belief of crucible manufacturers in the superiority of this variety, not more than 20 per cent. of the crucible demand

may be considered as a market for domestic production. Other uses for the flaky variety are as a lubricant, a filler for asbestos and metal packings, oil-less bearing compounds, pipe-joint compounds, paint, and shot polishing. AMORPHOUS variety is used for foundry stove polish and lead pencils.

Ores. Graphite is a mineral form of the element carbon (C); SP. GR. 2.1 to 2.9, the lower figure corresponding to greater purity. The usual impurity is silicious material occurring between adjacent laminæ of graphite. LUSTER is usually metallic. Occurs in laminated or sooty masses in vein deposits and as disseminated flakes or grains in metamorphic rocks. Flakes usually range in size from 0.5-in. down. Mica and chlorite are the most troublesome gangue minerals. The best-known deposits are the veins of flake graphite in Ceylon and of the amorphous variety in Mexico and Korea; the disseminated deposits of flake in Central Europe, eastern Canada and eastern United States.

Production. Average yearly post-war world production has been about 100,000 tons, of which the United States has contributed between 3000 and 10,000 tons. Austria, Czecho-Slovakia and Germany combined accounted for from 50 to 60 per cent. of the world production and Ceylon and Madagascar from 15 to 25 per cent.

Selling. Ceylon graphite is hand sorted into four grades, *viz.:* LUMP, from about 1- in. to 1/16-in.; CHIP, about 1/16-in.; DUST, the granular material finer than chip; and FLYING DUST, substantially impalpable powder. Flake graphite is sold in three grades, *viz.:* No. 1 FLAKE, passing 16-mesh and remaining on 90- or 100-mesh screen and having a graphitic carbon content of about 90 per cent.; No. 2 FLAKE, with slightly lower carbon content and lying between 60- and 120- to 140-mesh; and DUST with as low as 30 to 40 per cent. graphitic-carbon content and passing the fine screens. A sizing test of No. 1 Alabama flake is given in Table 55. Good-grade AMORPHOUS GRAPHITE should contain at least 80 per cent. graphitic carbon and should be brilliantly black and unctuous. Analyses of first-grade crystalline graphite from various producing sections is given in Table 56. Pre-war PRICE of best Ceylon graphite was 8 to 10¢ per lb. There was a slump in 1924 to 6¢. Sept.

Table 55. Sizing test of No. 1 Alabama flake graphite. (*After Dub*)

Screen, mesh	Weight, per cent.
+35	6.5
65	45.6
100	38.2
−100	9.7

Table 56. Analyses of first-grade crystalline graphite. (*After Brumell. 22 CMI 403 and Dub, Preparation of crucible graphite, USBM, 1918*)

Constituent	Alabama	New York	Pennsylvania	Ceylon	Canada
Volatile C	1.89	1.30	1.53	1.68	....
Graphitic C	87.03	88.97	88.80	85.06	93.87
SiO_2	5.85	4.34	5.24	7.81	3.61
Al_2O_3	4.17	2.40	2.05	2.82	1.33
Fe_2O_3	0.38	1.08	1.75	1.61	1.35
TiO_2	0.15	0.38	0.05	0.13	....
CaO	....	0.07	....	0.19	0.44
MgO	0.13	0.76	0.09	0.21	
K_2O	0.21	0.55	0.08	0.25	
Na_2O	0.04	0.12	0.12	0.11	
SO_3	0.04	....	0.21	0.005	....
P_2O_5	0.02	0.02	0.05	0.05	....
MnO	....	....	0.07	0.04	....
ZnO	....	....	....	0.03	....

1925, prices for Ceylon were 8½ @ 9¢ for lump, 6½ @ 7¢ for chip, 3½ @ 5¢ for dust; flake was quoted at 10 @ 30¢ per lb.

Treatment. Graphite from disseminated deposits must be concentrated before marketing and concentrate is usually further refined (*i.e.*, concentrated) by the larger purchasers before use. Large flake and high-grade concentrate are more important considerations in milling than high recovery. Substantially all kinds of concentrating processes have been tried in graphite-ore treatment and practically all failed prior to the introduction of froth flotation, Brumell (*22 CMI 378*), in a comprehensive summary of the American graphite industry, classifies the concentrating equipment in 38 Alabama mills in 1919 as follows: skin-flotation plants, 14½; froth-flotation, 15½; wet-gravity concentration, 5; electrostatic, 2; dry concentration, 1. He says that the mills using SKIN FLOTATION rarely made recoveries greater than 50 per cent. and usually about 35 per cent. The ELECTROSTATIC MILL was said to be highly profitable, but no operating data are available. The flow-sheet consisted of crushers and rolls in series to free the flake, grading and rough concentration by screens and air classifiers, and finishing on a Huff electrostatic machine. LOG-WASHER MILLS, classed above as wet-gravity concentration, treat the crushed material in log washers to which a small amount of kerosene is added. Overflow is concentrate. It is screened, undersize rejected, oversize washed on a cement floor, the residue being wasted, while the light washings are drained, dried and sent to the finishing mill. GRAVITY-CONCENTRATING MILLS using classifiers, shaking tables and film-sizing devices (buddles and the like) have been tried and failed many times. PNEUMATIC MILLS were probably the most successful from a concentrating standpoint, prior to froth flotation, but operating and mechanical difficulties were great on account of excessive dust. The flow-sheet of the American Graphite Co. is typical of a modern FROTH-FLOTATION MILL.

Graphite finishing mills treat the dried concentrate from the concentrating mills. The essential features of finishing practice are grinding in smooth-surfaced polishing rolls, buhr stones or pebble mills, de-dusting in crude air classifiers, usually of horizontal-current type, and sizing the de-dusted product into merchantable grades. Brumell (*loc. cit.*) cites two cases of such finishing treatment; in one a product containing 71.8 per cent. graphitic carbon was raised to 85.7 per cent.; in the other, 51-per cent. material was raised to 82-per cent.

The action of the grinders in finishing is to pulverize the more or less brittle gangue material while the soft, tough flake is flattened out but not broken. Hence subsequent separation into coarse and fine results in concentration of graphite in the coarser grades.

American Graphite Co. Fig. 81. (*120 P 567.*)

Location: Ticonderoga, N. Y.
Ore: Flake graphite in quartzitic schist.
Capacity: 100 tons per 24 hr.
Assays, per cent. graphitic carbon: Feed, 5.5 aver.; total conc., 86.5; coarse flake, 91.
Recovery (1919): 87 per cent.

Summary. Pneumatic flotation; repeated cleaning of concentrate with intermediate re-grinding in two stages.

Notes to Fig. 81.

a, Hand fed. *b,* Large circulating load is carried in order to grind flake as little as possible. *c,* 28-mesh. Much flake graphite overflows on account of presence of oil. *d,* Introduction of Deister cone classifiers at this point increased overflow of flake graphite from classifier system. The hydraulic-classifier sands went to the Dorr classifier, the overflow joined the Dorr overflow. *e,* Rectangular, one side canvas. Material drains in 8 hr. to about 40 per cent. moisture. Shoveled out by hand. *f,* Sides of all cleaners raised about 12 in.; 3 cross baffles, the lowest, at the discharge end, about the height of the original overflow, the next raised 3 in. and the feed-end baffle 6 in. Froth cascades over these and overflows at the discharge end; this aids in raising the grade of concentrate.

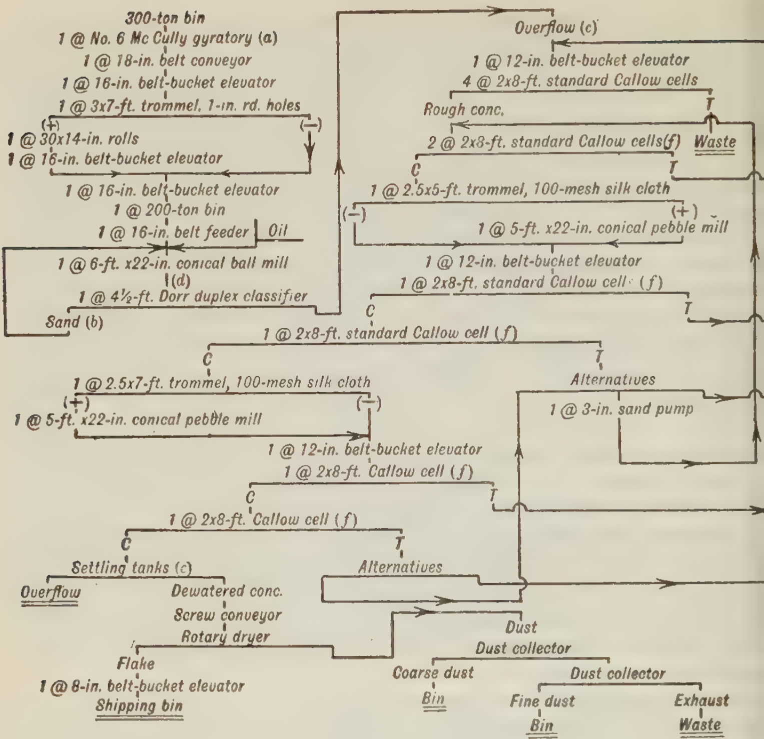


FIG. 81.—American Graphite Co.

21. Iron, Fe

Properties. Metal; gray-white, lustrous, tenacious, comparatively soft and ducti when pure, magnetic (see also Table 1). AT. WGT., 55.8. In compact form not attacke by dry air, but when very finely divided, as when produced by reduction with hydrogen, burns in dry air when gently warmed. It is attacked by moist air, with the formation RUST, a complex mixture of oxide and hydroxide of iron. Water is decomposed by iron all temperatures. Iron is dissolved by even the weakest acids, forming the correspondir bi-valent salts with iron as the base. These pass easily by oxidation into salts in whi the iron ion is tri-valent. Iron also enters into combination as a part of the acid constit ent of certain salts. Alloys of iron are numerous and extremely important commerciall

Uses of iron are multitudinous; it is used almost entirely as cast iron or steel, both which are alloys or mixtures of iron with carbon and other elements such as phosphoru manganese, chromium, nickel, tungsten, vanadium, molybdenum, cobalt, silicon, titaniu

Ores. The economic minerals are hematite, magnetite, limonite, siderit and very occasionally pyrite. For a deposit of one of the above minerals constitute an ore it must be favorably located with respect to fluxes, fuel ar markets; it must be of high grade; must occur in large quantity; and must l so that mining will be possible on a large scale at comparatively low cos Furthermore, it should be relatively free of titanium, phosphorus and sulphu which are harmful in the finished iron and steel products and difficult impossible to eliminate. Analyses of typical domestic ores are shown Table 57.

Table [illegible]. [illegible]

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	Magnetite	
					[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	0-0.008	0-0.01	0.04-0.11	0.01-0.06	0.05-0.42	[illegible]
[illegible]		20-40	8-17	11-15	1-20	[illegible]
[illegible]	....	....	3-6	2-6	0.7 to 4.5	[illegible]
[illegible]		....	1-25	[illegible]	[illegible]	0-3
[illegible]	....	....	0.4-3.0	0-0.4	0.2-4.2	0-6
[illegible]	....	....	0-0.3	0-3.0	0.04-1.4	0-0.3
[illegible]	....	....	....	....	0-Tr.	12-18
[illegible]	....	....	....	....	....	0-2.5
[illegible]						[illegible]
[illegible]	5-12	1-4	0-2	5-12	0-1.6	0-0.04

a Appalachians from N. Y. to Ala. b Ala., Pa., Texas, Iowa.

Production [illegible] Table [illegible] [illegible] production of pig iron is given in Table 59.

Table 58. [illegible]

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	43,260	33,660	30,458	17,811	28,760	41,348	31,602
[illegible]	14,460	15,440	17,510	7,283	10,454	14,174	12,351
[illegible]	5,550	5,050	5,804	2,878	5,235	6,783	6,963
[illegible]	1,633	870	926	470	444	542	256
[illegible]	1,140	1,060	881	257	577	871	460
[illegible]	642	410	457	235	332	379	383
[illegible]	565	627	734	147	781	963	357
[illegible]	542	316	321	74	31	156	90
[illegible]	414	284	278	26	156	267	159
[illegible]	364	494	432	50	90	365	65
[illegible]	184	225	274	111	118	265	130
[illegible]	427	286	302	141	130	325	351
Total	69,444	60,085	67,604	29,490	47,129	69,351	54,267

a [illegible] Tex.

Selling. [illegible] Table 60.

Table 59. World production of pig iron (thousands of gross tons). (*33 MI 389*)

Country	1913	1919	1920	1921	1922	1923	1924
United States...	30,966	31,015	36,926	16,688	27,220	40,361	31,40
France.........	5,126	2,373	3,380	3,308	5,147	5,347	7,57
United Kingdom.	10,260	7,398	8,035	2,616	4,902	7,439	7,31
Germany.......	16,499	5,566	6,931	7,725	9,249	4,859	7,07
Belgium........	2,446	247	1,099	863	1,588	2,116	2,76
Luxemburg.....	2,508	608	682	955	1,653	1,385	2,13
Russia.........	4,486	110	113	100	200	294	42
Others (*a*)......	5,245	3,395	3,474	2,968	2,448	4,187	3,94
Total......	77,536	50,712	60,640	35,223	52,407	65,988	62,64

a Sweden, Austria, Hungary, Czecho-Slovakia, Spain, Italy, Japan, India, Canada Australia.

Table 60. Average prices of iron ore per long ton at the mines

Year	Lake Superior district	South-eastern district	North-eastern district
1920	$4.25	$2.88	$4.90
1921	3.58	1.83	3.75
1922	3.33	1.74	2.33
1923	3.62	2.31	3.35
1924	3.06	2.13	2.59

Treatment. Most of the iron ore now mined and mined in the past ha been sufficiently high grade to smelt without concentration, but the end of th known high-grade deposits in the United States is in sight and an increasin tonnage of lower-grade ores is concentrated every year.

The simplest form of beneficiation is typified by that at the SUSQUEHANNA mine a Hibbing, Minn. (*102 J 787*) where masses of taconite are scattered through the hematite In earlier operations the taconite was picked out ahead of the steam shovels and haule away in dump cars. This expensive operation was replaced by screening. The ore i loaded by steam shovel into 12-cu. yd. air-operated side-dump cars, taken to the screenin plant and dumped onto a flat grizzly with 30-in. spacing. Oversize is sledged through into bin. From the bin the ore is drawn into a 5 × 24-ft. revolving stone screen with 2-in round holes. Oversize is waste, which discharges into a car-loading bin whence it is haule to a rock dump. Undersize goes to the shipping bins. Capacity of this plant is abou 250 tons per hr. and it requires only about 30 hp.

When, as is the case in many of the Lake Superior mills and the souther mines, the impurity in the ore is fine sand and clay the usual plant consist of screens and a log washer, supplemented in some cases by shaking tables fo treating the log-washer overflow. See Fig. 82.

In 1912 a complicated experimental plant was built for the AMERICAN-BOSTON MININ Co. (*95 J 1016*). It involved stage crushing through 10-mm. in gyratory crushers and roll 2-stage jigging of closely sized feeds (−10 +7-mm., −7 +4-mm., −4 +2-mm., an −2-mm.) and treatment of −20-mesh material by classification and tabling. This practic has not been followed by other companies, however, and is undoubtedly too expensive t allow the products to compete in the present markets.

MAGNETIC-IRON ORES carrying 25 to 45 per cent. Fe occur in enormou deposits underlying large areas in northern New Jersey and eastern New York High-grade deposits in these regions have been mined for over 100 years The coarsely crystalline, lower-grade deposits are concentrated by dry magneti methods typified by the flow-sheets of Witherbee, Sherman Co., mills Nos. and 5 (Figs. 85 and 86).

The finely disseminated deposits have not yet been worked profitably. At one of these, the BENSON mine, a plant was built in which the ore was crushed in stages from run-of-mine size to 18-mesh by 72 × 60-in. Edison giant rolls, 40 × 42-in. jaw crusher, 2 sets of 36 × 36-in. Edison rolls in series and 8 × 6-ft. ball mills, then concentrated in 2 stages on Gröndal wet-drum separators and the concentrate nodulized. At REPLOGLE STEEL Co. the economic minerals are magnetite and the non-metallic martite (Fe_2O_3); concentration is effected by dry magnetic separation followed by tabling (Fig. 87). At PENNSYLVANIA STEEL Co., Lebanon, Pa., magnetite occurs with pyrite and chalcopyrite (Fig. 90). The most recent plant is that of the MESABI IRON Co. (Fig. 88), treating a very finely disseminated low-grade magnetite ore. This plant is probably prophetic, in its essentials, of future practice in magnetic iron-ore concentration.

Alabama iron-ore washer. Fig. 82. *(108 P 458.)*

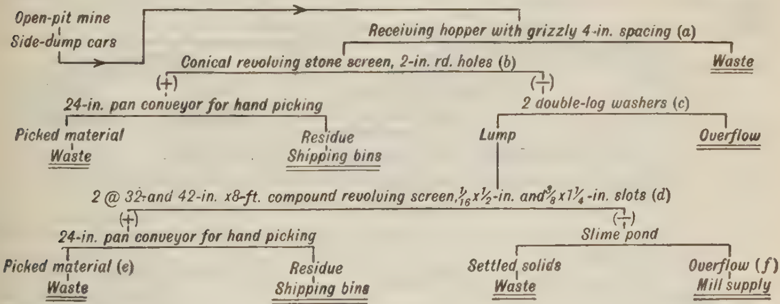
Ore: Limonite in clay and gravel.

Capacity: 100 to 150 cu. yd. per hr.

Ratio of concentration varies in this field from 3 : 1 to 12 : 1, average about 5 : 1, making a concentrate carrying 42 to 50 per cent. Fe, less than 1 per cent. Mn and 0.5 to 1 per cent. P.

Water: About 2000 gal. per ton of ore.

Cost, with a ratio of concentration not greater than 6 : 1, was about \$0.50 to \$0.75 per ton in 1914.



a, Hopper V-shaped, 5 to 6 ft. wide at top, 50 to 60 ft. long, 2 to 3 ft. deep at the upper end; bottom a flume 1½ ft. wide and sloping 2½ in. per ft. Grizzly of heavy rails set on 1½- to 2-in. per ft. slope and spaced 4 in. forms a false bottom. Oversize is sledged and picked on grizzly. Undersize is flushed down flume with water. *b,* With strong washing spray. *c,* Logs 18-in. octagonal and 20 to 30 ft. long, set 38 in. center to center. Blades 9 to 10 in. long, 5¼ in. wide and 1¼ in. thick. Logs slope 1 in. per ft. Trough flat-bottomed, 7 ft. 4 in. wide, about 4 ft. deep at lower end and 2 ft. deep at upper. Speed, 12 to 15 r.p.m. *d,* Fitted with strong internal wash sprays. *e,* Clay balls and gravel. *f,* About 50 per cent.

FIG. 82.—Typical Alabama iron-ore washer.

Oliver Iron Mining Co. Fig. 83. *(Q; 35 MEW 949; 107 J 683.)*

Location: Coleraine, Minn.

Ore: Loose mixture of hematite and taconite sand.

Capacity: 400 tons per unit per hr. (48,000 tons per 24 hr. for 5 units).

Assays, aver.:

	Fe, per cent.	SiO_2, per cent.	Mn, per cent.	P, per cent.	H_2O, per cent.
Feed	43.5	32.1	0.24	0.054	9.75
Concentrate	56.00	12.5	0.29	0.061	9.25
Tailing	18.65	70.6	0.14	0.041	....

See also notes *j* to *t*, Fig. 83.

Recovery: 80 to 85 per cent.

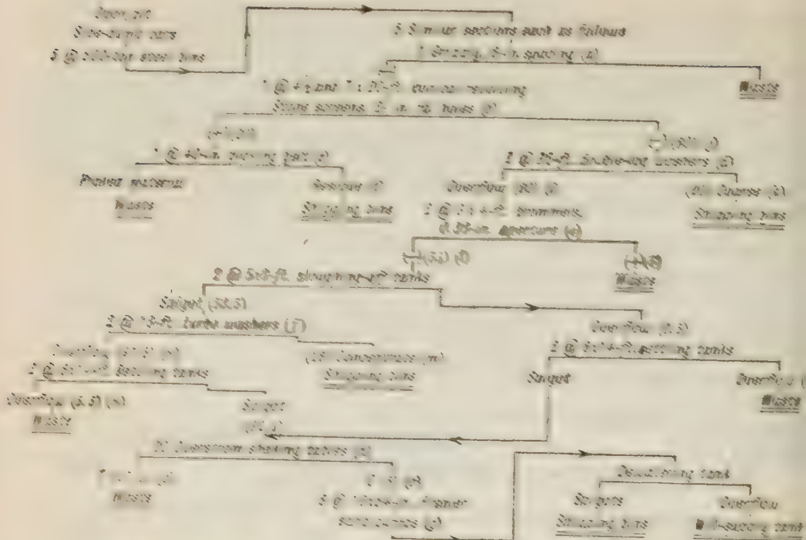


FIG. 83.—Oliver Iron Mining Co. Coleraine mill.
(Numbers in parentheses are average percentages of weight of original feed.)

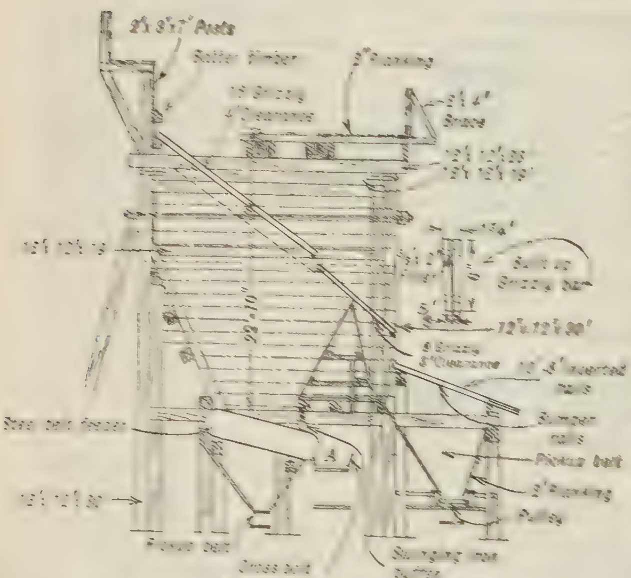


FIG. 84.—Receiving hopper and grizzlies in a Lake Superior iron ore washer.

a. Ore from b
sluiced over griz
with high-pressure j
Oversize ore sluic
through, waste pick
off. In later plants
other mines in the d
trict the arrangeme
shown in Fig. 84
used. b. [illegible] pla
holes [illegible] cente
Washing spray. c. 1
ft. per min. d. Tur
type, 14 r.p.m. Tan
6 ft. 8 in. wide; slop
1 in. per ft. e. C
[illegible]
9 r.p.m. [illegible]
[illegible]
About 350 lb. concen-
trate per table [illegible]
[illegible] per cent [illegible]
41.5; SiO_2 36. k. F
56; SiO_2 10. l. Fe. 3
SiO_2 44.5. m. Fe. 5
SiO_2 10. n. Fe. 2
SiO_2 61. o. Fe. 5
SiO_2 9. p. Fe. 21.
SiO_2 63. t. Fe. 5
SiO_2 12.5.

Ratio of concentration: 1.6 : 1.
Labor: Tons concentrate per man per day, based on total labor: 75 to 90.
Power: 2.8 hp.-hr. per ton (installed), exclusive of water supply.
Water consumption: 300 to 350 gal. per ton of crude ore.
Distance mine to mill: 2 to 5 miles.
Cost: (aver. for district) $0.06 to $0.10 per ton of feed.

Summary. Hand picking, 2-stage concentration in log washers, single-stage treatment of washer fines on shaking tables. No crushing except a small amount of sledging on the grizzlies.

Witherbee, Sherman Co., Mill No. 5 (high-grade ore). Fig. 85. *(Q)*

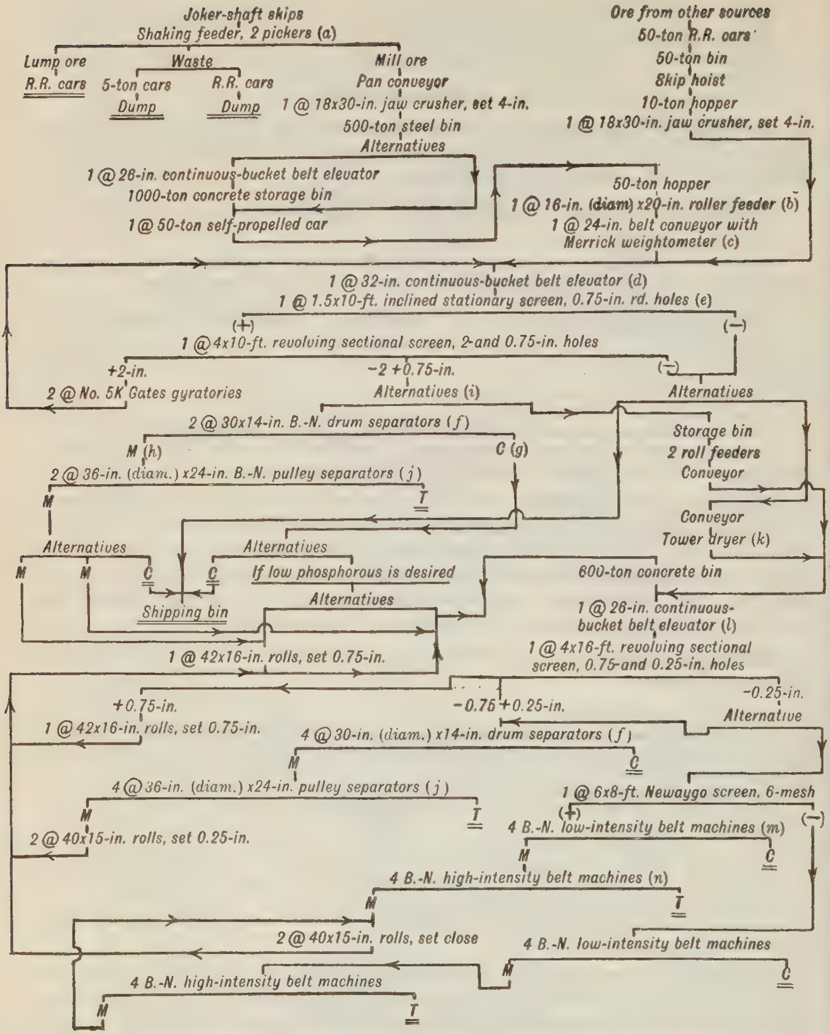
Location: Mineville, N. Y.
Ore: Banded magnetite in gneiss. Analysis: Fe, 49 per cent.; SiO_2, 15.75 per cent; Al_2O_3, 4 per cent; CaO, 8.75 per cent.; MgO 5 per cent.; P, 1 per cent; S, trace.
Capacity: 2000 tons per 24 hr.
Assays: Feed (aver.), 45 per cent. Fe; concentrate, 63 per cent Fe, 0.5 per cent. P; tailing, 6 per cent. Fe.
Recovery: 96 per cent.
Ratio of concentration: 1.4 : 1.
Running time: 90 per cent. of possible.
Power: 5.4 hp.-hr. per ton milled.
Labor: 39 tons per man-shift, operating; 390 tons per man-shift on repairs.
Distances: Mill 500 ft. from shaft mouth; concentrate sold and shipped to various eastern blast furnaces; power transmitted 0.5 mile at 3300 volts.

Summary. Hand sorting and dry magnetic concentration, making concentrate, middling and tailing at all sizes from run-of-mine down. (Compare Mill No. 4.) The magnetic separators for each size are arranged in two stages, low-intensity machines for the first stage, making concentrate and middling for re-treatment on high-intensity machines (second stage) that deliver finished tailing and a middling for re-grinding.

Witherbee, Sherman Co., Mill No. 4 (low-grade ore). Fig. 86. *(Q; 97 J 549; 56 A 899.)*

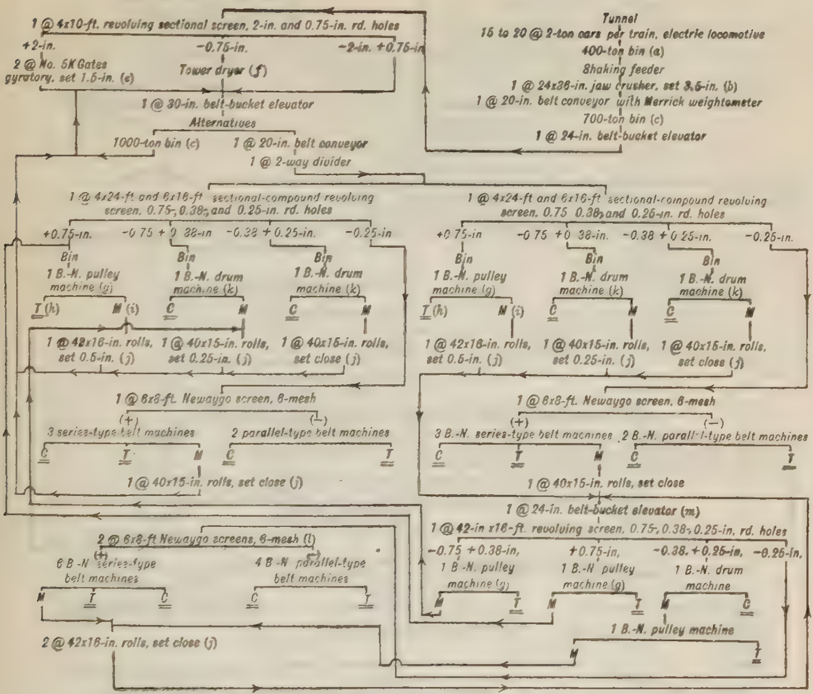
Location: Mineville, N. Y.
Ore: Banded and disseminated magnetite in gneiss, coarse crystallization. Analysis: Fe, 28 per cent.; SiO_2, 50.5 per cent.; Al_2O_3, 4.75 per cent.; CaO, 3.0 per cent.; MgO, 1.0 per cent.; P, 0.08 per cent.; S, *nil*.
Capacity: 2000 tons per 24 hr.
Assays, per cent. Fe: Feed, 28; concentrate, 64; tailing, 6.8.
Recovery: 84 per cent.
Ratio of concentration: 2.7 : 1.
Power: 8.7 hp.-hr. per ton milled.
Labor: 34 tons per man-shift, operating; 340 tons per man-shift on repairs.
Running time: 80 per cent. of possible.
Distances: Mill at tunnel mouth. Concentrate sold to eastern blast furnaces; power transmitted, 0.5 mile at 3300 volts.

Summary. Dry magnetic concentration, rejecting tailing at −2-in. and making concentrate at −0.75-in. CRUSHING: Jaw crusher from 18- to 3.5-in.; gyratory from 4- to 1.5-in.; 4- and 5-stage roll crushing of middling from 2-in. to 0.12-in. CONCENTRATION: Coarsest material divided into middling and tailing on high-intensity pulley machines; medium-coarse material treated on low-intensity drum machines making concentrate and middling; medium-fine material treated on series-type belt machines (low- and high-intensity) making concentrate, tailing and middling; finest material concentrated on parallel-type (low-intensity) belt machines making concentrate and tailing. All middling re-ground before re-concentrating, except that the middling of one low-intensity drum machine in the middling section is divided into tailing and a middling for re-grinding on a high-intensity pulley machine.



a, One man picks lump ore only; the other, lump ore and waste. *b*, Corrugated face, 10 r.p.m. *c*, 250 ft. long; slope, +2 in. per ft.; 330 ft. per min. *d*, 80-ft. lift, 10-ply belt, 350 ft. per min. 30 × 12 × 16-in. buckets. *e*, ⅜-in. steel plate, 40° slope. *f*, 16 poles. 8 amp. @ 125 volts. *g*, 63 per cent. Fe. *h*, 25 per cent. Fe. *i*, Depending upon character of feed. *j*, 24 magnets. 25 amp. @ 125 volts. *k*, See Sec. 18, Art. 3. *l*, Lift, 80 ft. 10-ply belt, 350 ft. per min. 24 × 10 × 14-in buckets. *m*, 12 magnets. 6 amp. @ 125 volts. *n*, 12 magnets. 9 amp. @ 125 volts.

FIG. 85.—Witherbee, Sherman Co., Mill No. 5.



a, Flat-bottom, wood. Air gate. All −18-in. rock. *b*, See Sec. 3, Table 5. *c*, Concrete. *e*, See Sec. 3, Table 15. *f*, See Sec. 18, Art. 3. *g*, See Sec. 13, Art. 4. *h*, 6 per cent. Fe. *i*, 18 per cent. Fe. *j*, See Sec. 3, Table 22. *k*, See Sec. 13, Art. 5. *m*, 12-in. buckets in 2 rows, staggered.

Fig. 86.—Witherbee, Sherman Co., Mill No. 4.

Replogle Steel Co. Fig. 87. (*Q*)

Location: Dover, N. J.
Ore: Magnetite and martite (Fe_2O_3) in gneiss. Fine crystallization.
Capacity: 3360 tons per 24 hr. aver.; 4800 tons maximum.
Assays, per cent. Fe: Feed, 31; concentrate, 60; tailing, 7.5.
Recovery: 87 per cent.
Ratio of concentration: 2.2 : 1.
General: Sloping mill site. Material dry during most of the milling. Note the large number of elevators and conveyors.

Summary. Dry magnetic separation of closely sized grades through 0.12-in. followed by table concentration at −14-mesh of magnetic tailing and re-ground magnetic middling. Crushing: Jaw crusher from 30-in. to 9-in.; jaw crusher from 9- to 3-in; disk crusher from 3- to 2-in.; rolls from 2- to 0.75-in.; disk crushers from 1- to 0.38-in.; rolls from 0.38- to 0.12-in.; rolls from 0.12- to 0.08-in. Concentration: Belt- and drum-type low-intensity separators making concentrate, tailing for gravity concentration and coarse middling for re-grinding and further magnetic treatment. Gravity concentration on sand and slime tables with re-concentration of primary-table middling without re-grinding. Sand tailing is de-slimed and sold for building sand.

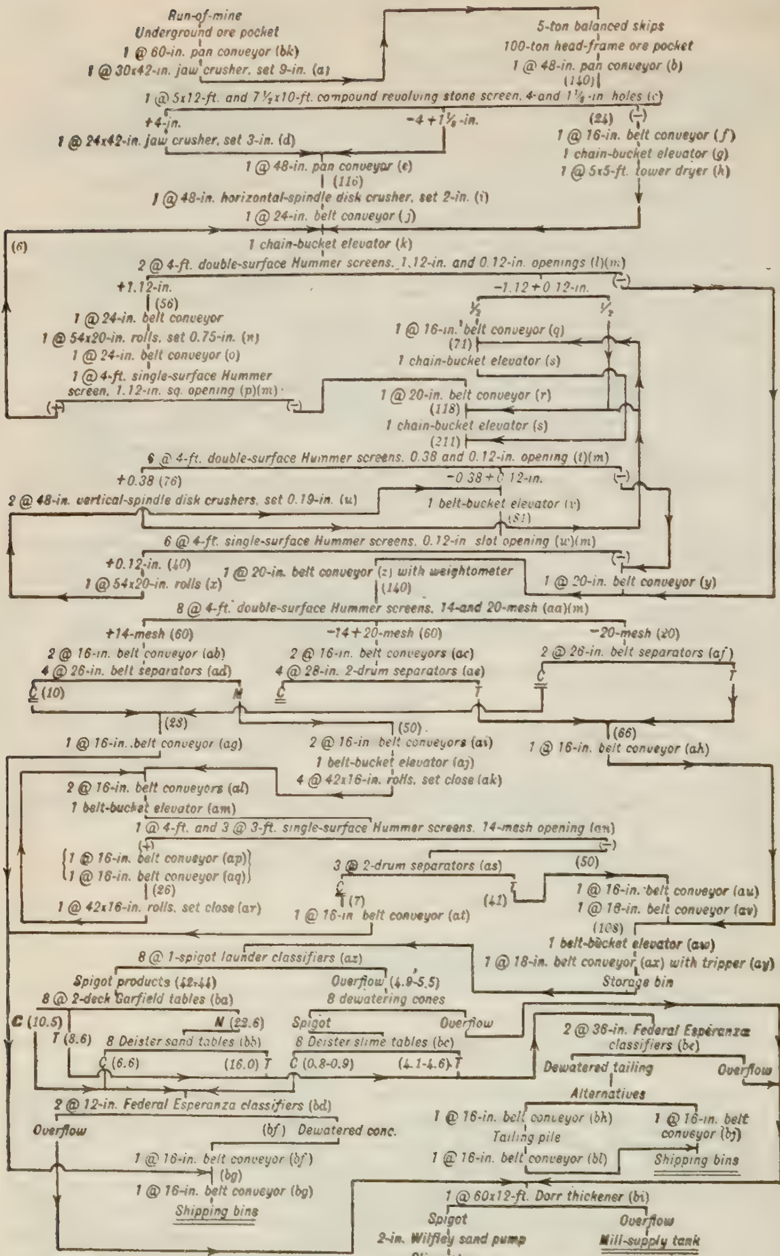


FIG. 87.—Replogle Steel Co.
( ) Numbers in parenthesis are tons per hour.

a, Underground. 200 r.p.m. 442 tons per hr. 100-hp. motor. *b*, 31.5 ft. long, +20° slope, beaded pans, 12.3 ft. per min. *c*, Inner screen, 4-in. round holes; outer, 1⅛ × 2½-in. slots; both punched-steel plate. 12 r.p.m. Slope, 1¼ in. per ft. *d*, 218 r.p.m. 100-hp. induction motor. Power draft ranges from 10 to 90 kw.; average, 18 kw. *e*, 30 ft. long, 35 ft. per min. +16° slope, single-beaded pans. *f*, See Table 61. *g*, Single-

Table 61. Belt conveyors in Replogle mill

Reference letter	Length, c. to c. of pulleys, feet	Slope, degrees	Speed, feet per minute	Number of belt plies	Pulley diameter, inches	
					Head	Tail
f	29.6	+4	138	4	20	18
j	9.9	+6	172	5	18	18
o	25.5	+22	198	5	14	18
q	36.3	+2 to +18	123	4	15½	17
r	13.1	+22	190	5	18	18
y	57.1	0	214	5	24	20
z	157.6	+16	245	5	24	20
ab	39.9	+11	154	4	18	18
ac	23.0	+11	151	4	18	18
ag	80.9	0	194	4	24	18
ah	77.4	0	186	4	24	18
ai	41.3	0	165	4	18	24
al	35.1	0	159	4	18	18
ap	40.2	+16	215	4	18	14
aq	38.7	+16	207	4	18	14
at	61.6	+2, 10, 15	158	4	18	24
au	64.9	+2, 10, 15	158	4	18	24
av	72.6	0	123	4	18	16
ax	65	0	314	4	30	24
bf	104.5	0, 4, 10	192	4	18	18
bg	177.2	0, 15	188	4	18	24
bh	97.7	0, 4, 10	192	4	18	18
bj	268.4	0,15	109	4	18	24
bl	288	....	120	4	18	18

chain type. 65 ft. long, slope +84°, 200 ft. per min.; 14 × 8 × 12-in. 8-gage steel buckets spaced 12 in.; manganese-steel chain. *h*, 45 ft. high. 14 per cent. moisture in feed. See Table 63. *i*, Main pulley, 100 r.p.m.; eccentric pulley, 250 r.p.m. A duplicate crusher is in place as a stand-by. See Table 63. *j*, See Table 61. *k*, Double-chain type. 61 ft. long, vertical, 150 ft. per min., 8-gage steel buckets, 24 × 9 × 12-in., spaced 2 ft. *l*, Type 37; upper screen square mesh, lower rectangular mesh. Slope, 38°. See Table 64. *m*, One motor-generator set for 23 screens. Generator: G. E., single-phase, 110-volt, 15-cycle, 900-r.p.m., 109-amp. Motor: Western Electric, 3-phase, 60-cycle, 440-volt, 12.5-amp., 10-hp. *n*, See Sec. 3, Table 22. *o*, See Table 61. *p*, See Table 64. *q*, See Table 61. *r*, See Table 61. *s*, Double chain. Length, 61 ft. Slope, 78°. 10-gage steel buckets, 24 × 9 × 11¾-in. spaced 2 ft. Manganese-steel chain. *t*, See Table 64. *u*, 50-hp. induction motor each. Power draft, 7.5 to 25 kw., aver. 15 kw. See Sec. 3, Table 20. *v*, See Table 62. *w*, See Table 64. *x*, 2 sets installed, one held as stand-by. 104 r.p.m.

Table 62. Belt-bucket elevators at Replogle Steel Co.

Reference letter	Width of belt, inches	Buckets, inches			Speed, feet per minute	Buckets		Belt plies	Slope, degrees	Length, center-to-center of pulleys, feet	Diameter of pulleys	
		Length	Width	Depth		Weight, gage number	Spacing, in.				Head	Tail
v	26	30	9	11¾	150	10	13	8	78	65.3	37	29½
aj	16	14	8	12	200	8	13	8	83½	46	32	24
am	16	14	8	12	200	8	12	8	76	54.2	32	24
aw	15	14	8	12	200	8	12	8	70	57.4	32	24

Table 63. Screen analyses of products in Replogle mill

	(*h*)*	(*i*)	(*ad*) Concentrate	(*ad*) Middling	(*ae*) Concentrate	(*ae*) Tailing	(*af*) Concentrate	(*af*) Tailing	(*ag*) Primary concentrate
+2.0 in.	...	16.8	...	...	...	...	...	...	...
+1.25 in.	7.0	36.2	...	...	...	...	...	...	...
0.75 in.	33.7	26.5	...	...	...	...	...	...	...
0.31 in.	33.4	10.6	...	...	...	...	...	...	...
0.12 in.	6.1	2.6	...	...	...	...	...	...	...
10-mesh	0.7	0.6	25.6	33.4	...	...	...	...	...
20-mesh	1.8	1.6	31.8	44.9	15.5	10.2	...	...	15.5
40-mesh	6.2	2.3	28.0	11.8	50.1	37.4	26.6	32.9	32.9
60-mesh	4.7	2.4	6.8	5.5	14.5	23.7	48.2	40.0	26.0
80-mesh	1.3	...	2.0	1.1	3.8	5.1	4.4	8.1	7.0
100-mesh	0.6	...	1.5	0.5	1.4	3.4	4.4	5.8	3.7
Through last screen	4.6	0.5	4.4	2.7	14.8	20.1	16.4	13.2	14.9

	(*ah*) Primary tailing	(*ai*) Primary middling	(*as*) Re-treatment drum separators F	(*as*) C	(*as*) T	(*az*) Launder classifiers F	(*az*) S	(*az*) O
+2.0 in.	...	...	...	...	...	...	...	...
+1.25 in.	...	...	...	...	...	...	...	...
0.75 in.	...	...	...	...	...	...	...	...
0.31 in.	...	...	...	...	...	...	...	...
0.12 in.	...	...	...	...	...	...	...	...
10-mesh	...	33.4	...	...	...	...	...	...
20-mesh	14.6	44.9	36.7	22.3	28.7	29.2	25.2	8.3
40-mesh	44.8	11.8	29.6	34.0	31.9	32.2	35.3	15.0
60-mesh	31.4	5.5	26.6	21.4	15.8	16.1	21.2	1.7
80-mesh	3.4	1.1	4.4	7.9	6.2	5.2	7.1	...
100-mesh	2.0	0.5	0.4	4.3	3.2	3.3	3.7	3.3
Through last screen	3.8	2.7	2.4	10.0	14.3	14.0	7.5	71.7

	(*ba*) Garfield tables F	(*ba*) C	(*ba*) M	(*ba*) T	(*bb*) Deister sand tables F	(*bb*) C	(*bb*) T	(*bf*) Wet-mill concentrate	(*bg*) Total mill concentrate	(*bh*) Wet-mill sand tailing	(*bi*) Wet-mill fine tailing
+2.0 in.	..	...	...	...	..	...	...	...	...	...	...
+1.25 in.	.	...	...	...	.	...	...	...	...	...	...
0.75 in.	..	...	...	...	..	...	...	...	...	...	...
0.31 in.	..	...	...	...	..	...	...	...	...	...	...
0.12 in.	.	...	...	...	.	...	...	...	...	...	...
10-mesh	.	...	...	...	..	...	...	...	3.9	...	...
20-mesh		12.8	22.0	53.8		32.5	17.4	19.4	20.2	32.6	...
40-mesh	See *az*, S	33.3	34.3	40.4	See *ba*, M	35.9	42.0	35.3	34.8	47.6	...
60-mesh		27.6	23.8	4.2		9.0	27.2	28.6	23.2	9.4	1.7
80-mesh		10.8	8.2	0.5		4.5	7.2	7.4	6.5	3.4	1.0
100-mesh		4.9	3.3	0.1		3.0	2.8	4.6	5.3	6.0	2.6
Through last screen		10.4	8.5	1.0		15.1	3.4	4.7	6.0	1.0	94.7

* For letters in column headings, see notes to flow-sheet.

1 @ 300-hp. induction motor for 2 rolls. Power draft 35 to 70 kw. (of which 25 kw. is for shafting). See Sec. 3, Table 22. *y,* See Table 61. *z,* See Table 61. *aa,* Slope, 35°. See Table 64. *ab,* In parallel. See Table 61. *ac,* In parallel. See Table 61. *ad,* Double-deck, double-belt; parallel type, 10 magnets per deck. Magnet belts, 335 ft. per min.; feed belts, 209 ft. per min. Feed, 31.37 per cent. Fe. Concentrate, 61.09 per cent. Fe. Middling, 27.01 per cent Fe. See Table 63. *ae,* Drums, 36 in. (diam.) × 28 in. 49 r.p.m. 14 magnets per drum. Feed, 33.35 per cent. Fe; concentrate, 61.98 per cent. Fe; tailing, 22.43 per cent. Fe. See Table 63. *af,* For mechanical detail see *ad.* Feed, 32.69

Table 64. Screens analyses of feed and products of Hum-mer screens, Replogle mill

Testing screens	Nos. 1 and 2 (*l*)				No. 3 (*p*)			Nos. 4 to 9 (*t*)			
	F	O_1	O_2	U	F	O	U	F	O_1	O_2	U
+2-in. rd.	16.4	35.1	.....	.....	.....	.....	.....	.....	.....	.....	.....
1.25-in. rd.	54.8	53.1	.....	.....	10.0	43.7	.....	.....	4.6	.....	.....
0.75-in. rd.	22.2	11.4	61.4	.....	27.8	49.4	33.8	21.6	32.4	.....	.....
0.31-in. rd.	3.4	0.3	30.4	.....	27.0	5.7	41.8	22.2	53.9	.....	.....
0.12-in. rd.	0.7	.....	5.9	2.9	9.0	0.4	10.2	13.4	7.0	44.5	17.4
10-mesh	0.1	.....	0.3	6.2	1.9	.....	1.8	5.2	0.2	23.0	9.8
20-mesh	.....	.....	.....	23.1	5.2	.....	3.2	11.8	0.4	21.5	23.2
40-mesh	.....	.....	.....	41.3	7.1	.....	3.8	12.6	.....	5.4	24.6
60-mesh	.....	.....	.....	23.7	4.8	.....	1.9	10.2	.....	2.2	20.0
80-mesh	.....	.....	.....	2.1	1.7	.....	0.9	1.7	.....	0.8	3.2
100-mesh	.....	.....	.....	0.4	1.2	.....	0.5	0.5	.....	0.4	0.8
Through last screen	2.4	0.2	2.0	0.2	4.2	0.8	2.1	0.8	1.4	2.2	1.0
Screen aperture	.....	1.12-in.	0.12-in.	.....	.....	1.12-in.	.....	.....	0.38-in.	0.12-in.	.....

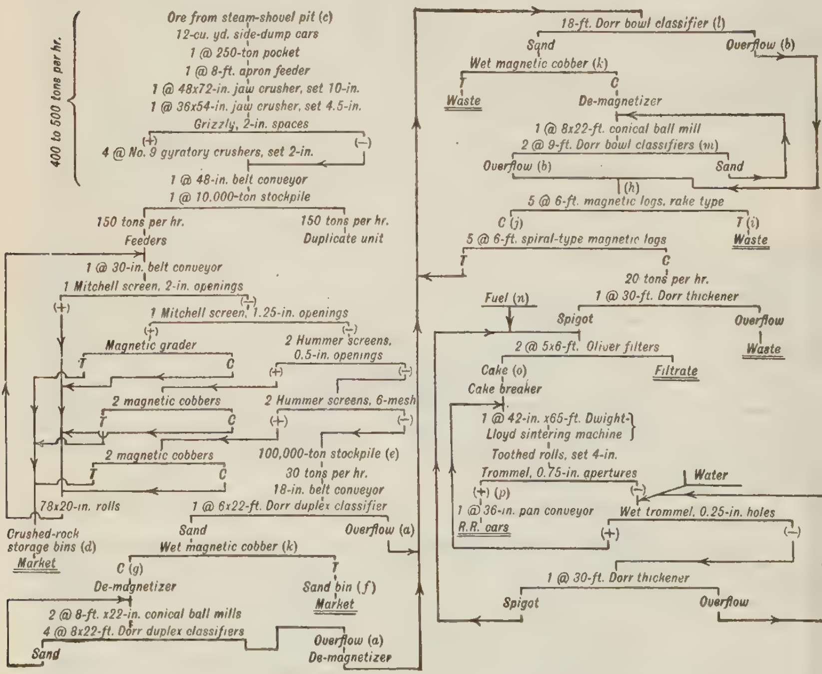
	Nos. 10–15 (*w*)			Nos. 16–23 (*aa*)				Nos. 24–27 (*an*)		
	F	O	U	F	O_1	O_2	U	F	O	U
+2in rd.	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
1.25-in. rd.	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
0.75-in. rd.	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
0.31-in. rd.	.....	9.9	.....	.....	.....	.....	.....	.....	.....	.....
0.12-in. rd.	.....	62.8	.....	.....	10.3	.....	.....	.....	.....	.....
10-mesh	See O_2, screens 4 to 9	17.8	0.2	13.7	8.0	.....	.....	19.5	43.5	.....
20-mesh		8.8	30.4	21.0	42.4	14.6	.....	41.6	51.0	36.7
40-mesh		0.5	28.7	27.5	20.4	37.5	33.0	18.2	3.6	29.6
60-mesh		0.1	18.3	17.0	8.1	30.4	27.8	8.1	0.6	26.6
80-mesh		.....	6.1	5.0	3.3	7.6	10.7	2.6	0.3	4.4
100-mesh		.....	2.5	4.6	1.8	3.1	5.9	1.4	.....	0.4
Through last screen		0.2	13.7	11.2	5.8	6.8	22.6	8.5	1.1	2.4
Screen aperture	.....	0.12-in.	.....	.....	14-mesh	20-mesh	.....	.....	14-mesh	.....

per cent. Fe. See Table 63. Concentrate, 59.12 per cent. Fe; tailing, 19.90 per cent. Fe. *ag,* See Tables 61 and 63. *ah,* See Tables 61 and 63. *ai,* See Tables 61 and 63. *aj,* See Table 62. 50-hp. motor. *ak,* 120 r.p.m. 1 @ 150- and 1 @ 100-hp. induction motor. Average power draft, 37.5 and 30 kw., respectively. See Sec. 3, Table 22. Two extra sets of rolls as stand-bys. *al,* See Table 61. *am,* See Table 62. 25-hp. motor. *an,* 2 @ 32° slope, 2 @ 35°. See Table 64. *ap,* See Table 61. In parallel with *aq.* *aq,* See Table 61. In parallel with (*ap*). *ar,* 120 r.p.m. See Sec. 3, Table 22. One extra set for stand-by. *as,* 2 with 36-in. (diam.) × 28-in. drums, 45 r.p.m. 14 magnets per drum; one with 30-in. (diam.) × 42-in. drum, 34 r.p.m., 27 magnets top drum, 45 magnets in bottom. Feed, 26.77 per cent. Fe; concentrate, 57.35 per cent. Fe; tailing, 20.64 per cent. Fe. See Table 63. *at,* See Table 61. *au,* See Table 61. *av,* See Table 61. *aw,* See Table 62. *ax,* See Table 61. *ay,* Self-propelling, self-reversing. *az,* Feed, 20.75 per cent. Fe. See Table 63. *ba,* 240 r.p.m. Feed, 20.75 per cent. Fe; concentrate, 58.96 per cent. Fe; middling, 11.03 per cent. Fe; tailing, 7.67 per cent. Fe. See Table 63. *bb,* 240 r.p.m. Assays, per cent. Fe: Feed, 11.03; concentrate, 58.01; tailing, 7.32. See Table 63. *bc,* 240 r.p.m. Assays, per cent. Fe: Feed, 12.55; concentrate, 55.37; tailing, 3.86. Tons per table, 0.62 to 0.68 per hr. *bd,* Single-chain type. 11.7 ft. c. to c. of sprockets, 22° slope, 6 × 12-in. blades

spaced 14 in. 13 ft. per min. On test, overflow from 2 machines was 165 gal. per min. carrying 38.64 tons solid assaying 33.27 per cent. Fe. *be,* Double-chain type, 11.7 ft. c. to c. of sprockets, 22° slope, 6 × 36-in. blades spaced 14 in. 35 ft. per min. *bf,* See Tables 61 and 63. *bg,* See Tables 61 and 63. *bh,* See Tables 61 and 63. *bi,* 5 min. per rev. Aver. tons solid feed per hr., 3.25. Feed pulp, 1.55 per cent. solids; spigot product, 33.02 per cent. water; overflow, clear water. Assay of solids, 11.21 per cent. Fe. See Table 63. *bj,* See Table 61, and *bh,* Table 63. *bk,* 12½ ft. long, +15°, 8 ft. per min. *bl,* See Table 61. In tunnel under tailing pile. For reclaiming sand tailing for shipment.

Mesabi Iron Co. Fig. 88. (*Q; 23 LSMI 111.*)

Location: Babbitt, Minn.
Ore: Magnetite in ferruginous chert (taconite).
Capacity: See flow-sheet. Ultimate planned, 75,000 tons per 24 hr.
Assays, per cent. magnetic iron: Feed, 26.5; concentrate, 61.75; tailing, from 3.1 to 10.8, average 8.2.
Recovery, magnetic iron: 80 per cent.
Ratio of concentration: 2.9 : 1.
Distances: Mine to mill, 3 miles.



a, −48-mesh. *b,* −150-mesh. *Assays,* per cent.: *c,* Mag. Fe, 26.50; weight, 100. *d,* Mag. Fe, 10.81; weight, 28.4. *e,* Mag. Fe, 32.72; weight, 71.6. *f,* Mag. Fe, 8.60; weight, 19.3. *g,* Mag. Fe, 41.61; weight, 52.3. *h,* Mag. Fe, 45.21; weight, 47.6. *i,* Mag. Fe, 3.10; weight, 12.8. *j,* Mag. Fe, 61.75; weight, 34.8. *k,* Rake-type, duplex, 4.5 × 14.7 ft. *l,* 6 × 36-ft. raking compartment. *m,* 8 × 32-ft. raking compartment. *n,* About 5 per cent. by weight of anthracite dust, coke breeze or other fine fuel. *o,* 11 per cent. moisture. *p,* Fe, 63 to 65; SiO_2, 8 to 10; P, 0.025 to 0.032 and a little Mn, Ca and Mg.

Fig. 88.—Mesabi Iron Co.

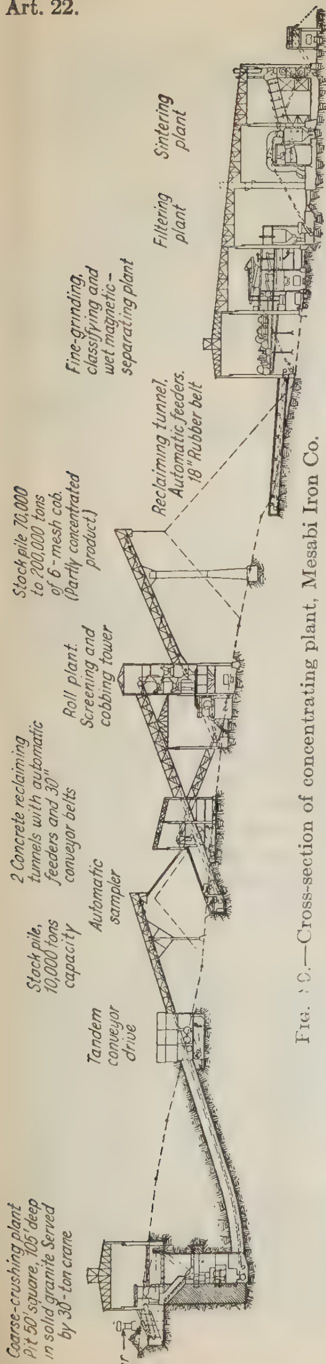


FIG. 89.—Cross-section of concentrating plant, Mesabi Iron Co.

Summary. Step magnetic concentration with rejection of tailing at each step and re-grinding of rough concentrate before cleaning. CRUSHING: Jaw crusher from 48- to 10-in.; jaw crusher from 10- to 5-in.; gyratory from 5- to 2-in.; 2-stage ball-milling, the first stage from −2-in. to 48-mesh, the second from 48- to 150-mesh. Primary mills in closed circuit with 8-ft. duplex Dorr classifiers, the secondary mills in closed circuit with 9-ft. bowl classifiers. CONCENTRATION begins at −2-in. and is carried forward in 3 stages, dry, down to 6-mesh. Minus 6-mesh material is concentrated wet in 3 stages, *viz.:* −6 +48-mesh, −48 +150-mesh and −150-mesh.

Arrangement of machines is shown in Fig. 89. The features of particular interest are the arrangement of the breakers in a rock pit, 50 ft. square by 105 ft. deep; the use of conveyors for all elevation of ore; and the use of stockpiles instead of bins for storage.

Pennsylvania Steel Co. Fig. 90. (*Q*) (1917).

Location: Lebanon, Pa.
Ore: Magnetite with some chalcopyrite and pyrite in silicious gangue.
Capacity: 600 tons per 24 hr.
Assays:

	Fe per cent.	Cu, per cent.	S, per cent.
Feed........	43.7	0.40	1.86
Iron conc....	59.0	0.24	0.9
Copper conc.	36.93	3.66	33.40
Tailing......	14.3	0.36	0.23

Recovery: Fe, 87 per cent.; Cu, 32 per cent.
Ratio of concentration: Fe, 1.55 : 1; Cu, 29 : 1.

Summary. Wet magnetic separation for magnetic iron followed by tabling and flotation for copper.

22. Lead and Zinc

LEAD, Pb

Properties. Metal; bluish-white, soft, plastic, somewhat ductile, not very tenacious. (See also Table 1.) AT. WGT., 207.2. Oxidizes rapidly in moist air, the oxide forming a superficial coating which protects the bulk of the metal. Oxidizes rapidly in dry air, when heated strongly. Pure water containing oxygen reacts with lead to form a slightly soluble hydroxide which thus does not form a protective coating. Impure waters, especially those carry-

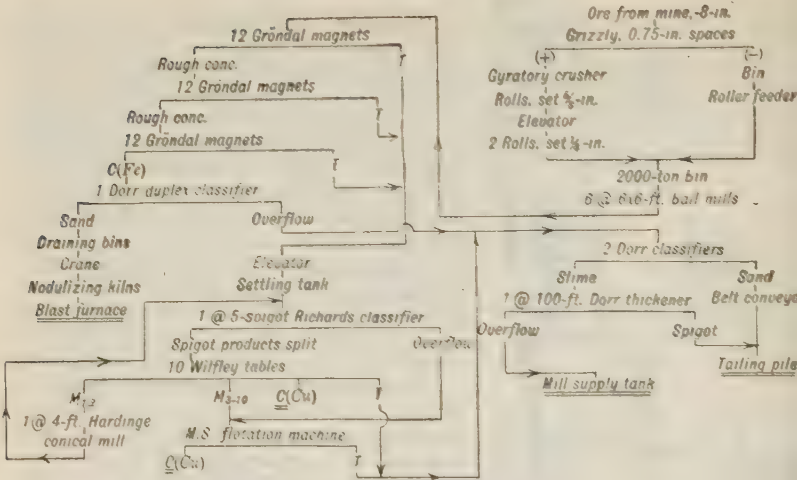


Fig. 90.—Pennsylvania Steel Co.

ing sulphates and carbonates, react superficially, but the corresponding salts, being insoluble, coat the lead and protect it from further action. Hot concentrated hydrochloric, sulphuric and nitric acids attack lead slowly, it combines directly with the halogens and with sulphur. The ion is bi- and quadri-valent, base- and acid-forming. Lead alloys freely with other metals.

Uses. The principal consumption of lead is as the metal and peroxide in storage batteries, as metal in cable coverings and as white lead ($xPbCO_3 \cdot yPb(OH)_2$) in paint. The metal, on account of its non-corrosive properties, also finds wide use for pipes, flashing, etc., in buildings. Considerable metallic lead is used for ammunition making. Important lead alloys are solder, pewter, type-metal, bearing-metal, and various low-fusing metals. The oxides, PbO and Pb_3O_4 are used in glass, rubber and paint manufacture. Lead arsenate is used as an insecticide, but calcium arsenate is more common.

Ores. The economic minerals are galena, cerussite, anglesite and pyromorphite. Galena ores comprise the great majority. There are three general classes: (a) those containing lead alone as an economic metal; (b) lead-zinc

Table 65. Production of lead in the United States (short tons) (b)

State	1913	1919	1920	1921	1922	1923	1924
Missouri	152,430	150,341	172,000	151,028	202,245	169,323	191,501
Idaho	137,892	89,091	117,191	99,707	91,487	127,797	123,709
Utah	71,069	65,102	64,006	51,872	63,130	104,678	119,318
Colorado	42,840	18,867	17,752	12,104	11,108	23,885	25,491
Arizona	4,901	5,407	5,987	3,313	7,218	8,828	9,372
Nevada	6,142	5,958	8,650	3,553	4,264	8,044	8,070
Oklahoma	3,214	49,984	68,494	46,902	67,436	59,602	56,017
Montana	3,256	17,513	13,231	11,565	14,551	18,345	21,226
California	3,294	2,004	2,260	614	3,018	5,168	2,305
New Mexico	1,821	1,418	1,123	384	1,230	1,638	2,263
Wisconsin	2,639	3,975	3,841	1,079	1,323	601	1,973
Kansas	1,504	7,951	8,421	10,939	10,900	20,207	12,895
Others (a)	5,518	10,214	11,391	2,227	3,779	5,920	7,860
Total	436,430	427,825	494,347	395,287	481,689	554,036	582,000

a Va., Ill., Alaska, Wash., Ark., Ky., Tex., S. D., Ore., Ia., N. C., Tenn. b USGS.

ores; (*c*) lead-silver ores. Calcite, dolomite and pyrite are the common gangue minerals of the first two classes, quartz of the third class.

Production of lead in the United States, by states, is given in Table 65. World production is shown in Table 66.

Table 66. World production of lead from smelters (metric tons). (*33 MI 437*)

Country	1913	1919	1920	1921	1922	1923	1924
United States	396,000	412,700	428,000	369,100	432,600	506,200	532,700
Mexico	109,700	78,600	85,200	60,500	120,800	167,100	161,300
Spain	198,800	125,700	175,200	135,900	119,200	127,500	137,100
Australia	113,700	84,000	6,100	57,200	109,000	124,000	130,500
Canada	17,100	19,900	16,300	30,200	42,300	50,500	80,600
Germany	181,100	51,500	59,000	75,000	73,600	51,200	61,200
Belgium	53,600	4,200	16,000	29,800	43,600	51,100	53,700
Burma	6,000	19,400	24,200	34,300	39,800	46,500	52,600
Italy	21,700	16,500	16,000	12,500	10,800	17,100	22,000
France	28,800	10,900	15,100	15,500	13,900	17,400	21,000
Others (*a*)	36,400	61,000	66,800	59,400	73,000	63,000	117,400
Total	1,162,900	884,400	907,900	879,400	1,078,600	1,221,600	1,370,100

a Austria, Greece, Japan, Rhodesia, Sweden, United Kingdom, etc.

Selling. For discussion of the sale of concentrate see p. 223. For marketing of metallic lead see *Spurr and Wormser*. Average yearly prices of lead at New York (*Eng. and Min. Jour.* quotations) are given in Table 67.

Table 67. Average yearly selling price of lead at New York (cents per pound).

Year	Price
1913	4.370
1919	5.759
1920	7.957
1921	4.545
1922	5.734
1923	7.267
1924	8.097
1925	7.7–10.575*a*

a Range.

ZINC, Zn

Properties. Metal; bluish-white, lustrous. Cast zinc is coarsely crystalline and brittle. When heated to somewhat over 100° C. it becomes soft, tenacious and malleable; it can then be hammered and rolled. Upon cooling, it retains its malleability. This malleable form, when heated to 200 to 300° C. becomes so brittle that it can be crushed to a powder. After such heating the metal retains much of its brittleness on cooling. (See also Table 1.) AT. WGT., 65.4. Oxidizes quickly in air or water at ordinary temperatures. The oxidation products form a protective coating, however, which resists further corrosion. Heated sufficiently in air, zinc burns with a bluish flame. Powdered zinc decomposes water slowly at ordinary temperatures. At red heat zinc decomposes water rapidly. The impure metal of commerce dissolves rapidly in dilute mineral acids. Pure zinc appears insoluble in pure acids, but a trace of the salts of heavy metals starts solution at once. Strong bases attack zinc with the formation of zincates. Zinc ion is always bivalent; it is both acid- and base-forming.

Uses. The important uses are in coating iron to protect it against corrosion (GALVANIZING); BRASS making (zinc, 20–50 per cent.; copper, 80–50 per cent. ± small amounts of tin, lead, and iron); as a constituent of other alloys such as GERMAN SILVER (copper, nickel, zinc) and WHITE METAL (zinc and copper, zinc predominating); in zinc-white pigment; as the positive pole or plate in electrical batteries; zinc shavings and dust in cyanide precipitation; gutters, household utensils, etc., where resistance to corrosion by air and water are desirable.

Ores. The economic minerals are sphalerite, smithsonite, calamine, franklinite, willemite and zincite. There are several distinct types of ores. Argentiferous and auriferous zinc sulphides with or without some lead, copper and iron sulphides in quartzose gangue are typical of the Rocky Mountain deposits. Sphalerite alone or with galena and, usually, with some pyrite

in limestone are typical of the Mississippi Valley deposits. Zinc as franklinite, willemite and zincite in a white, crystalline limestone is the characteristic occurrence in New Jersey. Sulphide ore bodies may be overlain by deposits of smithzonite, calamine and hydrozincite. Such deposits are often more valuable than the primary sulphides, both for the reason that they are more concentrated and that their metallurgical treatment is simpler.

Production in the United States is shown in Table 68. The United States production varies from 40 to 60 per cent. of the world production. Other important sources are Belgium, Great Britain, Germany, Poland, Tasmania, and Canada.

Table 68. Zinc production in the United States (short tons). (*USGS*)

State	1920	1921	1922	1923	1924
Oklahoma	219,188	121,372	209,682	242,421	269,137
Kansas	61,069	36,994	56,225	100,969	105,392
New Jersey	77,371	56,447	73,657	75,227	84,370
Montana	92,168	11,638	59,535	70,730	61,500
Colorado	24,395	1,180	11,477	26,780	26,000
Tennessee	19,217	9,692	15,568	15,900	14,376
Wisconsin	27,286	3,390	10,952	13,211	14,027
Missouri	24,422	10,845	16,171	18,265	12,920
New Mexico	5,300	114	2,250	8,248	10,000
Idaho	13,966	17	2,055	13,976	7,670
Nevada	5,349	35	1,309	7,084	5,501
New York	5,654	1,572	4,816	8,463	4,664
Others (*a*)	9,387	3,450	8,487	9,121	5,263
Total	584,772	256,746	472,184	610,395	620,820

a Ark., Ariz., Calif., Ky., Ill., Utah, Wash.

Selling. For discussion of the sale of concentrate see p. 225. For marketing of metallic zinc, see *Spurr and Wormser.* Average yearly prices at St. Louis are given in Table 69.

Table 69. Average yearly prices of slab zinc at St. Louis (cents per pound) (*a*)

Year	Price
1919	7.3
1920	8.1
1921	4.7
1922	5.7
1923	6.6
1924	6.3
1925	7.1 8.5*b*

a Eng. and Min. Jour.-Press.
b Range.

Treatment of lead and zinc ores. From the standpoint of concentration there is a great variety of lead and zinc ores, different in the amount of metal present, the relative proportions of the metals, the character of gangue and fineness of dissemination. Each of these differences has its effect on the type of flow-sheet employed in concentration. Table 70 is a rough classification, embracing the principal ore varieties and naming corresponding concentrating plants whose flow-sheets illustrate methods of treatment. The simple ores, *i.e.*, the sulphide ores of lead, zinc, lead-zinc or lead-silver, coarsely disseminated and with gangues of relatively low specific gravity, are easy to treat. The flow-sheets involve gravity concentration with jigs and tables to recover the bulk of the metallic minerals, followed by collective or differential flotation for the slimes. The complex sulphide ores consist primarily of intergrown lead and zinc sulphides in roughly equal amounts, usually accompanied by pyrite and small amounts of chalcopyrite and other copper sulphides, gold

and silver, in quartz or quartz-calcite gangues. Rhodonite, rhodochrosite, barite and siderite are frequently present. The concentrating problem varies according to the size of the individual grains of galena and sphalerite, and the specific gravity of the gangue minerals. When the sulphides are coarsely disseminated and there are no heavy gangue minerals, treatment consists primarily of selective gravity concentration to collect the bulk of the lead with flotation to catch and in some plants separate the sulphides in the gravity-concentration tailing. In the past iron sulphide has been most successfully separated from blende by magnetic concentration (see NATIONAL ZINC SEPARATING Co. and NORTHERN ORE Co.). At present differential flotation is

Table 70. Classification of lead and zinc ores, based on characteristics important in determining concentration

Mineralogical characteristic	Principal metal or metals	Relative character of dissemination	Relative specific gravity of gangue minerals	Plant
Sulphide ores	Lead	Coarse	Low	Federal Lead Co.
	Zinc	Coarse	Low	American Zinc Co.
	Zinc	Coarse	High (*a*)	Gennamari
	Lead, zinc	Coarse	Low	Eagle-Picher Lead Co.
	Lead, zinc, iron	Coarse	Low	Wisconsin zinc
	Lead, zinc, iron	Medium	Low	Timber Butte
	Lead, zinc, iron	Fine	Low	Cons. Mg. & Sm. Co.
	Zinc, iron	Fine	Low	Northern Ore Co.
	Lead, silver	Coarse	Low (*a*)	Bunker Hill & Sullivan
	Lead, silver	Coarse	Low (*a*)	Silver King Coalition
	Lead, zinc, silver	Coarse	High (*a*)	Federal M. & S. Co.
	Lead, zinc, silver	Coarse	High	Central Mine
	Lead, zinc, silver	Fine	High	Roseberry
	Lead, zinc, iron, silver, gold	Fine	Low	U. S. S. R. and M. Co.
	Lead, zinc, iron, silver, gold	Fine	Low	Sunnyside
Oxidized ores	Zinc	Coarse	Low	Wisconsin and Sardinia
	Zinc, iron, manganese	Coarse	High	N. J. Zinc Co.
	Lead, silver	Medium	Low	Amer. Sm. Sec. Co.
	Lead, silver	Medium	Low	Shattuck-Arizona
Mixed sulphide and oxide	Zinc, lead, iron	Medium	Low	Royal Asturiana
	Lead, silver, gold	Fine	Low	Chief Consolidated

a Quartz and siderite. Siderite gangue occurring with blende in the Gennamari and Federal M. and S. ores is classified as having relatively high specific gravity, but compared with galena the specific gravity is low.

highly successful. When there is a heavy gangue mineral such as barite, siderite or rhodonite, it is not possible to make a zinc concentrate by gravity methods. The usual procedure under such circumstances is to take out most of the lead (usually accompanied by the bulk of the silver) by gravity concentration and then either recover the zinc by collective flotation, tabling the flotation concentrate to separate a further amount of lead or, more recently, to use differential flotation. When lead and zinc sulphides occur in very small grains, so that fine grinding is necessary initially to sever them from the gangue and from each other, two general schemes are employed, *viz.:* (1) collective flotation of the sulphides followed by subsequent separation by gravity concentration; (2) selective flotation. Magnetic separation after roasting is practiced to separate iron sulphide from blende, electrostatic separation was

used for many years on fine granular zinc-iron gravity concentrate at the MIDVALE plant of the U. S. S. R. and M. Co., but differential flotation is now almost entirely successful in separating lead, zinc and iron, and will displace the older methods. Oxidized ores are difficult for the reason that differences in specific gravity between valuable and waste minerals are usually less than when the valuable metal is in the sulphide state and simple froth flotation cannot be applied. Mixed, *i.e.*, partially oxidized, ores present the concentrating problems of both sulphide and oxide ores. Normally the method of treatment is to concentrate in stages, first removing unaltered sulphide and throwing oxide into the tailing, then treating this tailing to collect the oxidized values.

Table 71. Typical analyses of southeastern-Missouri lead ores. (*After Watt*)

	Usual ore, per cent.	Rich ore, per cent.
Pb	4.32	6.06
SiO_2	4.83	7.40
FeO	6.64	6.10
Al_2O_3	1.16	5.50
CaO	30.80	26.70
MgO	17.96	13.60
S	0.97	1.70
Zn	0.50	Tr.
Ni+Co	Tr.	Tr.
Cu	0.03	0.50
CO_2, by difference	32.79	33.57

Federal Lead Co., Mill No. 4. Fig. 91. (*Q*; *57 A 322*)

Location: Elvins, Mo.

Ore: Essentially galena, rather coarsely disseminated in dolomite. There is some pyrite and a very small amount of sphalerite. See Table 71. Galena particles are generally less than ¼-in.; much is less than ⅛-in., some very small. 3-mm. crushing will free 90 to 95 per cent. of the total galena, 2-mm. crushing is practically the economic limit (with 4-cent lead), and at 0.21-mm. (65-mesh) substantially 100 per cent. is free. 10-mm. jig tailing assays about 0.6 to 0.8 per cent. Pb.

Capacity: 3000 tons per 24 hr.

Assays per cent. Pb: Feed, 4.75; concentrate, gravity, 75; flotation, 55; tailing, 0.75.

Recovery: 85 per cent.

Ratio of concentration: 18.5 : 1.

Labor: 37 tons per man-shift, total.

Power: 13.9 hp.-hr. per ton milled. See motor list, Table 72.

Water: 7 tons per milled. 85 per cent. of this re-used.

Running time: about 90 per cent. of possible.

Distances: Mill at mine; mill to smelter, 100 miles; water pumped, 1000 ft.; electric power transmitted 1¼ mile at 6600 volts.

General: Gently sloping mill site.

Summary. CRUSHING: Gyratory from 12- to 3-in.; disk crusher from 3- to 0.4-in.; rolls in closed circuit with screens from 0.88- to 0.4-in.; rolls, rod mills and ball mills in parallel, in closed circuit with screens from 0.4- to 0.08-in. CONCENTRATION: Jigging and tabling of primary ore, making tailing and concentrate; jigging and tabling of re-ground middling; flotation (combination routing) of primary and secondary slimes.

The principle underlying this flow-sheet is to crush the galena particles as little as is consonant with the requirement to free them from the gangue. This because gravity concentrate is 10 to 20 per cent. higher grade than flotation concentrate, is cheaper to handle and dry and cheaper to smelt. On account of the large difference in specific gravity between galena and the dolomite gangue, the range in allowable size of jig feed is large, hydraulic classification is unnecessary prior to tabling and only the slime need be removed. The variety of machines in re-grinding, tabling, and in secondary-flotation service is significant only of a progressive experimental turn on the part of the management.

This plant is typical of south-east Missouri lead mills. Minor variations in machines and arrangement are, of course, met, and one or two considerable

Table 72. Motor list, Federal Lead Co., Mill No. 4

Machines driven			Motors			
Number	Name	Reference letter on flow-sheet	Number	Speed, r.p.m. Synchronous	Speed, r.p.m. Full-load	Horsepower
1	Pan conveyor	*a*	1	900	865	50
2	Apron feeders	*c*	2	900	860	10
2	Gyratory crushers	....	2	900	865	50
1	Belt conveyor	*d*	1	900	860	20
4	Disk crushers	*f*	4	900	865	50
2	Belt conveyors	*g*	1	900	860	10
1	Belt conveyor	[illegible]	1	900	865	50
1	Sampling plant		1	900	860	25
1	Belt conveyor	[illegible]	1	900	860	20
4	Shaking feeders					
4	Drag dewaterers	*k, r*	2	900	860	20
4	Elevators					
12	Trommels	*l, m, o*	2	900	865	50
6	Hancock jigs	*q*	2	900	865	35
2	48×24-in. rolls	*n*	2	....	....	50
2	36×16-in. rolls	....	2	900	865	50
1	Conical ball mill	*s*	1	514	495	125
1	Rod mill	*t*	1	720	690	75
40	Shaking tables	[illegible]	2	900	860	20
2	Centrifugal pumps	*y*	2	900	860	10
2	Dewatering wheels	*z*	1	900	860	20
2	Belt conveyors	*aa, ab*	2	900	865	20
1	Drag dewaterer and bucket elevator	*ac, ad*	1	900	860	20
1	Centrifugal pump	[illegible]	1	900	860	20
1	40-ft. Dorr thickener		1	900	850	5
4	50-ft. Dorr thickeners		4	900	850	5
1	Centrifugal pump	*ae*	1	900	860	25
2	2-stage centrifugal pumps	*am*	2	900	860	250
18	Janney cells	*af, ah*	18	600	575	10
1	Centrifugal pump, cleaner-tailing		1	900	860	10
1	Blower for pneumatic cells	...	1	900	865	50
1	50-ft. Dorr thickener		1	1200	1150	5
1	2-stage centrifugal pump, slime-tailing	....	1	900	865	50
1	Bucket elevator	*ai*	1	900	860	10
1	Oliver filter	*aj*	1	900	860	10
1	Dry vacuum pump	*aj*	1	600	575	25
1	Centrifugal solution pump	*aj*	1	1200	1140	3
1	Air compressor	*aj*	1	900	860	20
1	Lowden dryer	*ak*	1	900	860	10
1	Coal elevator (for dryer)	....	1	1200	1150	5
1	Machine shop	....	1	900	860	10
1	A.C.-D.C. motor generator		1	1800	1730	7½
1	Janney concentrate unloader		1	1200	1155	3
1	24-in. tailing conveyor		1	900	865	50
1	Slime-tailing pump		1	900	865	50
2	Coarse trommels		1	900	860	20
1	Fresh-water pump	....	1	900	860	75

differences, such as large jaw crushers ahead of the gyratories at one or two of the plants, and relatively extensive hand picking at MINE LAMOTTE. The primary jaw crushers, where used, are expedients to increase capacity of the crushing plants coincidently with the increase in concentrating capacity effected by flotation. At MINE LAMOTTE the galena is deposited along fractures in a shattered dolomite and is, therefore, less finely disseminated

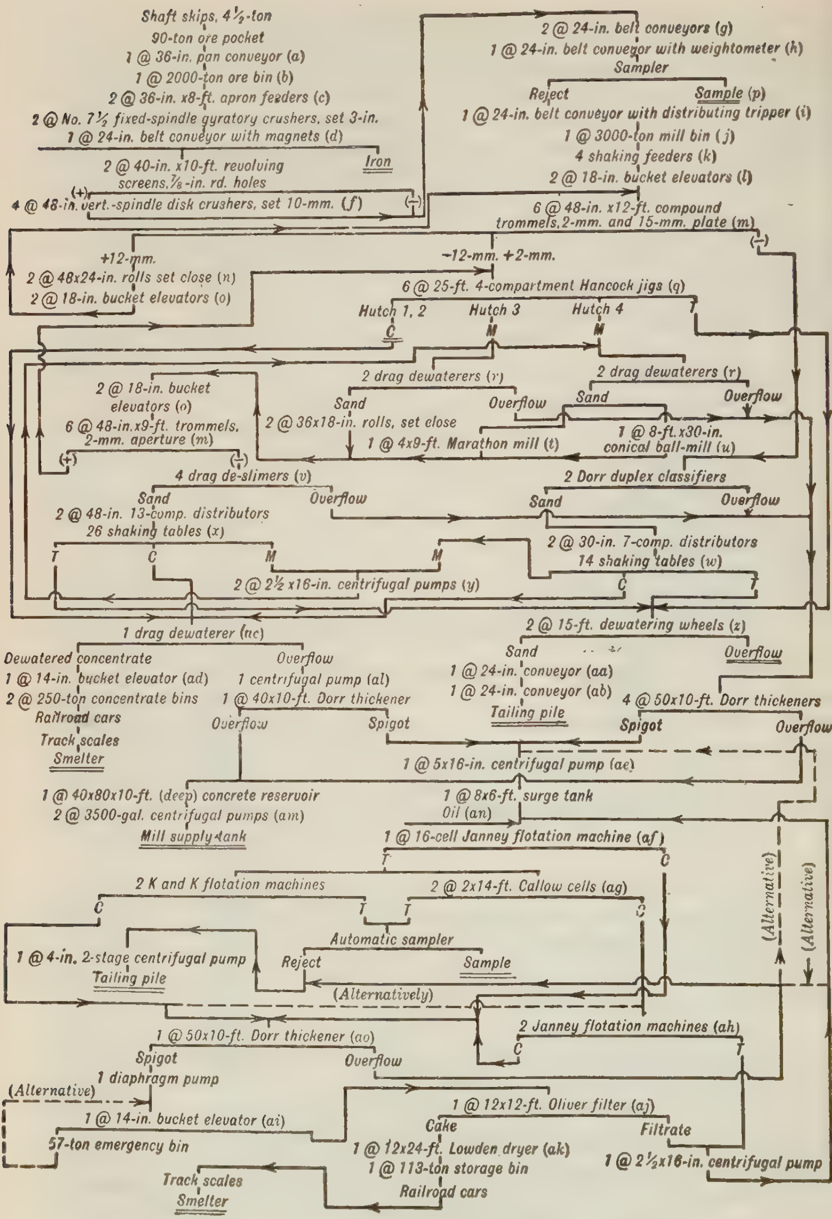


Fig. 91.—Federal Lead Co., Mill No. 4.

a, 243 ft. long; rise, 5 in 12, 101 ft. total; speed, 50 ft. per min. *b*, Flat-bottom, concrete, 30 ft. wide × 40 ft. long × 30 ft. deep. *c*, Aver. speed, 6 ft. per min. *d*, 6-ply

belt, 300 ft. per min.; length, 203 ft.; rise, 4 in 12, 68 ft. total. 1 mushroom magnet over belt; 1 @ 48 × 26-in. magnetic head pulley. *f*, 390 r.p.m. *g*, 64 ft. long, 4-ply belt, 200 ft. per min. *h*, 383 ft. long, 6-ply belt, 300 ft. per min. Merrick weightometer. *i*, 120 ft. long, 6-ply belt, 300 ft. per min. *j*, Flat-bottom, concrete, 95 ft. long × 28 ft. wide × 23 ft. high. *k*, 150 @ 2½-in. strokes per min. *l*, 385 ft. per min.; 72½-ft. lift. *m*, 20 r.p.m.; slope, 2 in 12. *n*, See Sec. 3, Table 22. *o*, 385 ft. per min.; 68½-ft. lift. *p*, Vezin-type, 1/500th cut in 3 steps without intermediate crushing. *q*, 65 r.p.m. See Sec. 9, Art. 7. *r*, Slope, 6 in 12; 30 ft. per min. *t*, 26 r.p.m. See Sec. 4, Table 50. *u*, 27 r.p.m. See Sec. 4, Table 11. *v*, 5 ft. 4 in. wide; slope, 4 in 12; 18 ft. per min. *w*, 10 Wilfley and 4 Deister-Overstrom. 275 r.p.m. See Sec. 10, Table 1. *x*, 20 Butchart tables and 6 Deister-Overstrom. 275 r.p.m. See Sec. 10, Table 6. *y*, 860 r.p.m. *z*, 24-in. face. 1.5 r.p.m. *aa*, 400 ft. long, 350 ft. per min.; rise ¾ in. per ft. *ab*, 200 ft. long, 350 ft. per min.; rise 3 in. per ft. *ac*, 6 ft. 5 in. wide; slope, 4 in 12. *ad*, 37-ft. lift; 350 ft. per min. *ae*, 425 gal. per min., 25 per cent. solids; 50-ft. head. 865 r.p.m. *af*, Mechanical, single-spitzkasten type. 575 r.p.m. 1 emulsifier (without spitzkasten) and 15 cells making finished concentrate. Typical feed analyses are given in Table 73. *ag*, Slope, 3¾ in 12. *ah*, Mechanical single-spitzkasten type. *ai*, 350 ft. per min.; 34½-ft. lift. *aj*, 1 @ 100-cu. ft. compressor; 1 @ 36 × 72-in. compressed-air receiver; 1 @ 4 × 8-in. duplex vacuum pump (215 r.p.m.); 1 @ 3 × 8-ft. vacuum receiver; 1 @ 1½ × 8-in. centrifugal solution pump (1130 r.p.m.); 1 @ 16 × 60-in. moisture trap. 23-in. vacuum. 7 to 8 rev. per hr. 50 tons concentrate per 24 hr. from 35 per cent. moisture to 15 per cent. moisture. 140° F. *ak*, Coal-fired, 2¼ strokes per min. Product 4 to 6 per cent. moisture. 2 tons of coal to 50 tons of concentrate per 24 hr. *al*, 5 × 20 in., 800 g.p.m., 43 ft. total head, 665 r.p.m. *am*, 12-in., 2-stage. *an*, Cleveland Cliffs No. 1, refined wood creosote, 0.5 lb. per ton. Disk feeder. Wood creosote ranging in amount from 0.5 to 1.5 lb. per ton is the usual reagent in the district. *ao*, Intermittent discharge dependent on dryer cycle of 1 to 2 hr.

Table 73. Sizing-assay tests of typical Lead-belt flotation feeds.
(*After Watt*).

Screen, mesh	Cumulative percentages			
	Sample No. 1		Sample No. 2	
	Weight	Lead	Weight	Lead
+100	2.5	Trace	2.2	0.7
150	7.0	Trace	1.7	1.1
200	14.0	Trace	16.2	1.6
−200	86.0	100 *a*	83.8	98.4*a*

a 95 per cent. of the lead is −300 mesh.

Chemical analysis, per cent: Pb, 3.0 (usual range is 2.5 to 6 per cent. 5 per cent. at Federal No. 4); Fe, 4.6; Zn, 0.2 (may run to 0.5 per cent. or even higher); Cu, 0.05; SiO_2, 9.0; CaO, 26.0.; MgO, 14.0; Al_2O_3, 4.0; Co, Ni, Trace.

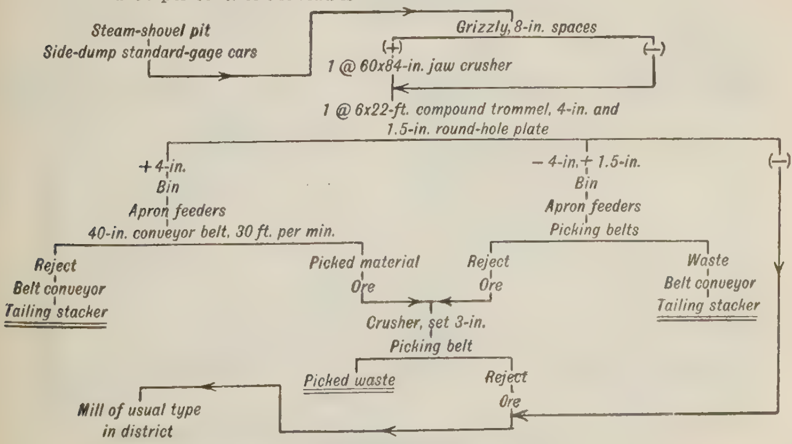


FIG. 92.—Crushing and hand-picking plant at Mine La Motte.

than at the other mines in the district. Further, the deposit lies near enough to the surface to permit steam-shovel mining. A sketch of the coarse-crushing and picking flow-sheet is shown in Fig. 92.

American Zinc Co. of Tennessee. Fig. 93. (*Q*; *118 J 407*.)

Location: Mascot, Tenn.

Ore: Sphalerite in brecciated dolomite. Analysis: calcite, 48.11 per cent.; dolomite, 35.36 per cent.; silica, 8.73 per cent.; alumina, 1.04 per cent.; Fe_2O_3, 1.33 per cent.; sphalerite, 5.43 per cent.

Capacity: 2400 tons per 24 hr.

Assays, per cent. Zn: Feed, 3.4 to 3.8; jig concentrate (about 25 per cent. of total concentrate), 60.8; flotation concentrate, 60.6; jig tailing, 0.7 to 0.85; flotation tailing, 0.09.

Recovery: In flotation, 98.1 per cent. Total recovery on above ore, 86.3 per cent. Total recovery on low-grade ore, about 50 per cent.

Ratio of concentration: 20 : 1.

Distances: Mill at No. 1 (low-grade) mine, 2000 ft. from No. 2 (principal) mine. Mill to smelter, 550 miles; water pumped about 350 ft. from creek; power transmitted 150 miles at 66,000 volts.

Costs, per ton milled: Crushing and jigging, \$0.32; flotation, \$0.08 (\$0.25 per ton floated); drying concentrate, \$0.04 (\$0.80 per ton of concentrate). (1925).

Summary. Jigging and flotation. CRUSHING: gyratory from 14- to 4-in.; disk crusher from 4- to 1.3-in.; rolls in closed circuit with screens from 1.3- to 0.62-in.; rolls in closed circuit with screens from 0.62- to 0.25-in.; ball and tube mills, 1-stage closed-circuit to 65-mesh. CONCENTRATION: Roughing and cleaning of $-\frac{5}{8}$-in. unsized primary feed on hutch-making jigs, both concentrate and tailing made; re-crushed ($1\frac{1}{4}$-in.) middling similarly roughed and cleaned, making concentrate and a tailing that is re-ground and floated in mechanical machines with combination routing. Coarse, sized tailing sold for ballast and concrete aggregate; fine tailing ground for agricultural limestone.

This flow-sheet is influenced by the commercial value of the coarse tailing and the fact that jigging equipment was already operating in a gravity-concentration mill when flotation became successful. Originally concentration was done by jigs and tables only, then flotation was added as an accessory. Within the last few years the hydraulic classifiers and tables (60 to 70 in all) have been dropped and it is a close question whether jigging also would not be eliminated were it not for the fact that the market value of the coarse tailing is greater than the value of the zinc (14 to 16 lb. per ton) at present therein.

Table 74. Elevators at American Zinc Co. mill

Reference letter *	Length of bucket, inches	Belt plies	Bucket spacing, inches	Pulley diameter, inches: Head pulley	Pulley diameter, inches: Tail pulley	Height, feet	Speed, feet per minute
g	22	....	16	....	....	55	446
n	22	7	16	54	24	63	425
w	22	7	16	30	24	25	425
aa	22	7	16	30	24	68	450
ab	22	7	16	36	24	40	350
ad	22	7	16	30	24	35	450
ae	22	7	16	30	24	45	425
ah	22	7	16	....	....	35	336
aj	22	7	16	30	24	65	358
ak	22	7	16	30	24	42	350
bh	18	7	32	30	24	....	240
cd	16	7	16	30	24	26	322
cg	14	7	16	30	24	36	322

* From flow-sheet.

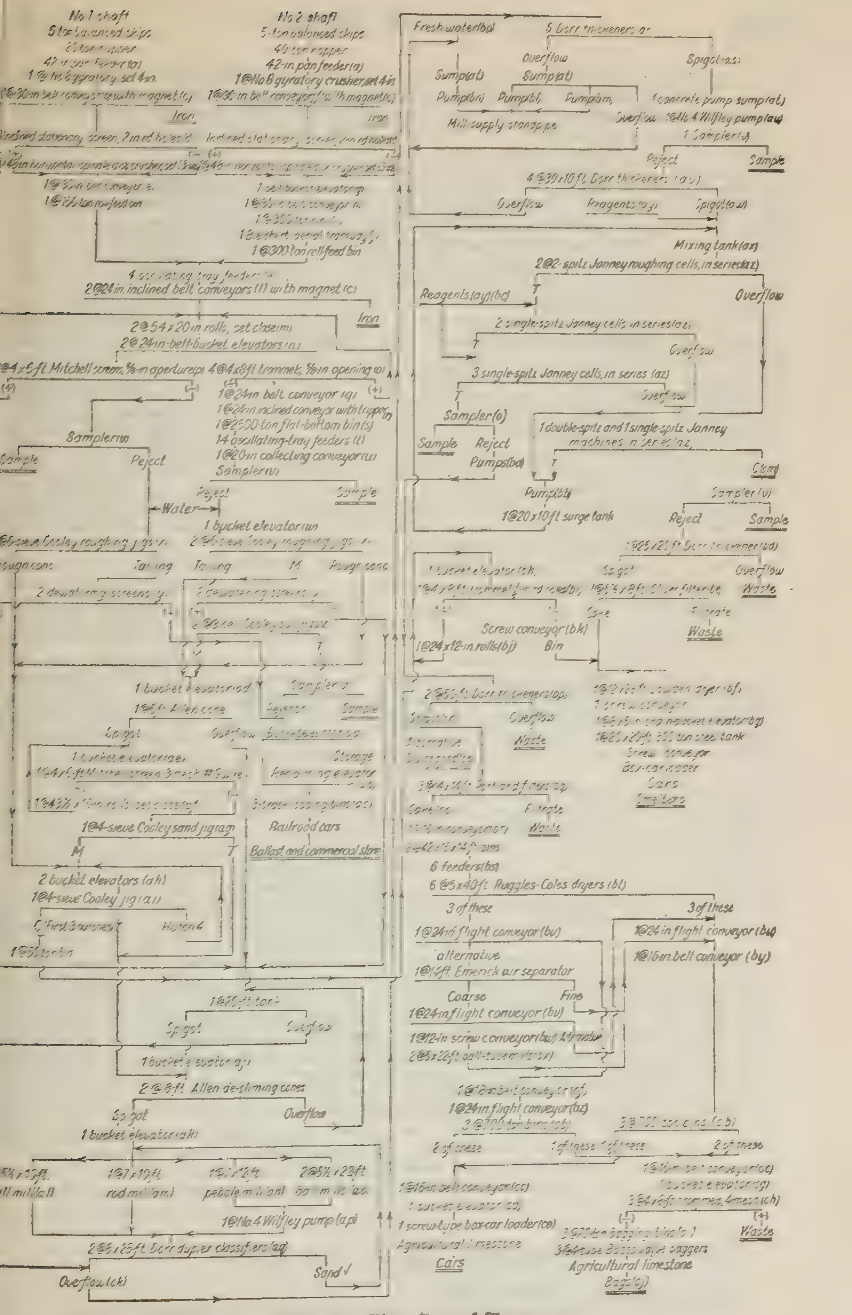


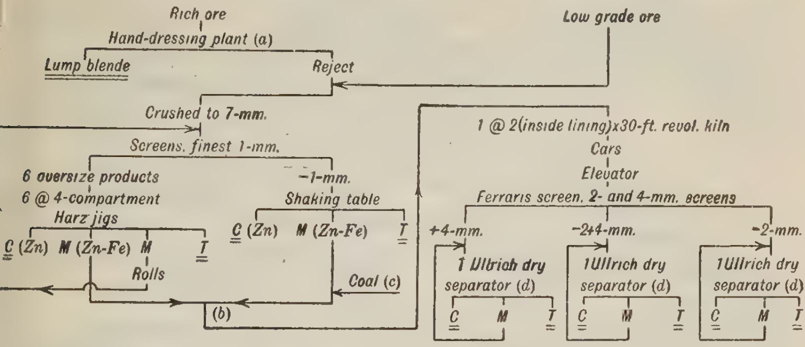
FIG. 93.—American Zinc Co., of Tennessee.

a, Ratchet-and-pawl drive, 9 ft. per min. *b*, 20 ft. long, 5½-ft. rise, 346 ft. per min., 5-ply. *c*, 54½-in. No. 4 Stephens Adamson lifting magnet, 27 amp., 220 volt. *d*, Manganese steel, ½ in. thick, 3 × 8-ft., 45° slope. *e*, 6-ply, 310 ft. per min. Rise, 13 ft. in first 52 ft., then 29 ft. in 107 ft. *f*, 28 ft. long, 9½-ft. rise, 336 ft. per min., 6-ply. *g*, See Table 74. *h*, 14 ft. long, 6-ply, 330 ft. per min. delivers to center of bin. *i*, 28 × 30, (rectangular) × 30 ft. deep, 4 arc gates each side. *j*, 1850 ft. long, 80 ft. net rise, 18 @ 1600-lb. buckets, 128 per hr. Speed, 506 ft. per min. Cast crucible-steel cables, 1½-in. load side, 1-in. return, both locked-coil; ⅝-in. traction cable, lang-lay. Works two-and-a-fraction shifts per day and requires 2 men per shift each end. *k*, 30 strokes per min. *l*, 50 ft. long, 11-ft. rise, 314 ft. per min., 6-ply belt. *m*, See Sec. 3, Table 22. *n*, See Table 74. *o*, See Sec. 5, Table 26. *p*, Replacing 4 trommels. *q*, 68 ft. long, 314 ft. per min., 6-ply belt. *r*, 20-ft. rise in first 55 ft. then 54 ft. horizontal; 320 ft. per min. 6-ply belt. Hand-propelled tripper. *s*, 30 × 60 × 24-ft. (deep). *t*, 28 strokes per min. *u*, 6-ply belt. 308 ft. per min. First 60 ft. horizontal then rises 11 ft. in 61. Merrick weigher. *v*, Automatic, air-driven, 7-min. cuts. *w*, See Table 74. *x*, 32 × 48-in. sieves. No. 10 slotted-steel plate. ⅛ × 1-in. openings. See Sec. 9, Art. 4. *y*, 30 × 42-in., stationary, 45° slope, ⅛ × 1-in. slotted No. 14 steel plate. *z*, 36 × 48-in. punched-plate ½-in. rd. holes; 82 r.p.m. See Sec. 9, Art. 4. *aa*, Tandem (2 belts in same housing). See Table 74. *ab*, See Table 74. *ac*, 5 × 9 × 6½-ft. each. These bins are also fed by an elevator loading washed gravel. *ad*, See Table 74. *ae*, See Table 74. *af*, See Sec. 3, Table 22. *ag*, 28 × 42-in. sieves, punched-plate, 14-gage, ⅛ × 1-in. slots. 200 strokes per min. See Sec. 9, Art. 4. *ah*, See Table 74. *ai*, 28 × 42-in. sieves, punched-plate, 14-gage, ⅛ × 1-in. slots. 200 strokes per min. See Sec. 9, Art. 4. *aj*, See Table 74. *ak*, See Table 74. *al*, 7 × 10-ft. pebble mill lagged down. 16,000 lb. balls. Local hard-iron liner. 23 r.p.m. 90-hp. motor, direct-connected, herringbone gear. *am*, Converted 7 × 10-ft. pebble mill. 40,000 lb. @ 2- and 3-in. rods. Local hard-iron liner. 16 r.p.m. 150-hp. motor. Belt drive, spur gear. *an*, 22,000 lb. pebbles. Forbes manganese-steel liner. 90-hp. motor, direct-connected. Herringbone gear. *ao*, Converted tube mills. 40,000 lb. @ 2-in. balls each. Local hard-iron liners. Belt-driven. Spur gears. 12 r.p.m. 1 @ 120-hp. and 1 @ 138-hp. motors. *ap*, 1080 r.p.m. *aq*, Model D. Slope 3½ in. per ft. 26 strokes per min. *ar*, 4 @ 30-ft., 3 min. per rev. 1 @ 50-ft. 138 sec. per rev. *as*, 20 per cent. solids. *at*, With chip screen. *au*, 1200 r.p.m. *av*, 225 sec. per rev. *aw*, 50 per cent. solids; 17 per cent. +100-mesh, 54 per cent. −200-mesh. *ax*, Rectangular tank with 3 Janney impellers. *ay*, Thiocarbanilid, 0.1 lb. per ton; pine oil, 0.25 lb.; copper sulphate, 0.3 lb. *az*, 570 r.p.m. *ba*, 2 @ No. 4 Wilfley pumps, 1200 r.p.m. *bb*, No. 4 Wilfley, 960 r.p.m. *bc*, As needed. Total average reagent consumption (1923): pine oil, 0.365 lb. per ton, thiocarbanilid 0.176 lb., copper sulphate, 0.363 lb. *bd*, 2 min. per rev. *be*, Oscillating agitators. No. 3 "0" canvas. 14 × 8-in. Type MO Oliver vacuum pump. ½-in. cake. 8 to 9 per cent. moisture. *bf*, 2⅓ @ 10-in. strokes per min. Oil-fired, gases to waste. *bg*, 60 ft. high, 270 ft. per min. *bh*, See Table 74. *bi*, 17 r.p.m. Slope, 2 in. per ft. *bj*, 80 r.p.m. *bk*, 12-in. 45 r.p.m. *bl*, 13 × 24-in. horizontal duplex double-acting Mesabi-type plunger pump. 2500 gal. per min. *bm*, 14 × 14-in. vertical triplex. 1000 gal. per min. *bn*, 13½ × 18-in. horizontal duplex double-acting plunger pump. 1700 gal. per min. *bo*, From river. *bp*, 1 r.p.m. *bq*, 7 min. per rev. 16- to 18-in. vacuum. One held as spare. 1 @ 17 × 10-in. IR duplex vacuum pump. Pump and filters driven by 50-hp. motor. *br*, 5-ply belt. 250 ft. per min. 65 ft. horizontal, then 29 ft. rise in 128 ft., then 226 ft. horizontal. *bs*, Flight type, ratchet-and-pawl drives. *bt*, 7 r.p.m. Unlined, direct-heat. Coal-fired. Coal pulverized to −20-mesh in a plant consisting of 1 @ 24-in. Jeffrey swing-hammer machine crushing to ¼-in. and 6 Type A Aero pulverizers. *bu*, 80 ft. per min. *bv*, 40 ft. long. 18-ft. rise. 80 ft. per min. *bw*, 18 ft. long. 100 r.p.m. *bx*, 40,000-lb. ball loads. Chilled-iron liners. 21 r.p.m. *by*, 5-ply high-temperature belt. 550 ft. long. 60-ft. rise. 228 ft. per min. *bz*, 50 ft. long. 80 ft. per min. *cb*, Reinforced concrete, 20 ft. diam. × 54 ft. high. *cc*, 65 ft. long. 240 ft. per min. *cd*, See Table 74. *ce*, 30 tons per hr. *cf*, 5-ply high-temperature belt. 485 ft. long. 55-ft. rise. 320 ft. per min. *cg*, See Table 74. *ch*, Square holes, 1½ in. per ft. *ci*, 10 × 14 × 9 ft. *cj*, 100-lb. paper or 200-lb. burlap, at will. *ck*, 20 per cent. +100-mesh. Normally 4 mills operate in closed circuit with one classifier. *cl*, 93 per cent. + ⅛-in. *cm*, 60.6 per cent. Zn, 75 per cent. of total conc.; 6 per cent. +100-mesh, 65 per cent. −200-mesh. *cn*, 60 per cent. solid. *co*, 12 per cent. moisture. *cp*, For storage during the dull season from April to August.

Gennamari-Ingurtosu mine. Fig. 94. *(100 J 911.)*

Location: Ingurtosu, Sardinia, Italy.
Ore: Blende in siderite, quartz, calcite, barite and schist. Coarse dissemination.

Assays: Feed, 10 to 15 per cent. Zn; magnetic concentrate, 53 per cent. Zn and 5 per cent. Fe; magnetic tailing, 2.5 per cent. Zn and 48 per cent. Fe.
Recovery: Magnetic plant, 96 per cent. Zn.



a, Cobbing, hand sorting, hand jigging. *b,* 25 to 35 per cent. Zn and 30 to 40 per cent. Fe. *c,* 10 per cent. by weight. *d,* 1.5 hp. for driving and 10 amp. at 125 volts for energizing.

FIG. 94.—Gennamari-Ingurtosu mill.

Eagle-Picher Lead Co. Netta mill. Fig. 95. (*105 J 727.*) This mill is typical of a large, modern Joplin plant.

Location: Picher, Okla.
Ore: Blende with varying amounts of galena and small amounts of marcasite and chalcopyrite in a chert gangue with varying amounts of limestone, shale and clay. The ores of the district vary considerably in lead content and in the fineness of dissemination of the sulphide minerals, but in most of the ores galena is distinctly less in quantity than blende and both minerals are coarsely disseminated.
Capacity: 1500 tons per 24 hr.
Assays: Feed ranges from 1.5 to 3.5 per cent. Zn with lead content varying from 10 to 50 per cent. of the zinc content. Average analysis of zinc concentrate (*107 J 558*) is Zn, 58.6 per cent; Fe, 1.6 per cent.; Pb, 1.0 per cent.; CaO, 0.9 per cent.; SiO_2, 5.2 per cent; tailing, 0.5 per cent. Zn; 0.1 per cent. Pb.
Costs (pre-war): about $0.25 per ton milled.
At the 1200-ton mill of the AMERICAN ZINC Co. at Joplin (*108 P 840*) in 1914, using jigs and tables only, the recovery was 68 to 69 per cent. and the cost $0.18 per ton, including $0.0275 for elevating and disposing of tailing.

Summary. Jigging, tabling, flotation. CRUSHING: Jaw crusher from 12- to 1.5-in.; 2-stage roll crushing from 2- to 0.38-in.; 2-stage roll crushing of middling with intermediate jigging from 0.38- to 0.08-in.; re-grinding −2-mm. material to 65-mesh in ball mill in closed circuit with Allen cones. CONCENTRATION: Roughing and cleaning jigs on −0.38-in. +0.06-in. feed, roughing and cleaning jigs on −0.25-in. re-ground middling, tabling of classified sands down to 65-mesh and of −65-mesh slimes, separate rougher-cleaner flotation of primary slimes and re-ground middling.

Notes to Fig. 95.

a, 1 James and 6 Butchart. *b,* Usual grade, 10 to 25 per cent. Zn. Some bed product shoveled from first two or three sieves is sent with this. *c,* This screen is omitted in many mills and unsized feed sent to the roughing jig. *d,* Use of a middling jig is unusual in the district; re-ground middling is ordinarily returned to the roughing jig, but as this material is about twice as rich as the original feed, the middling jig is good practice. *e,* Ferraris-type wet shaking screen (*93 J 461*). *f,* Hardwood creosote. *g,* At mills that cannot make substantially lead-free flotation feed, flotation concentrate is tabled to separate lead and zinc.

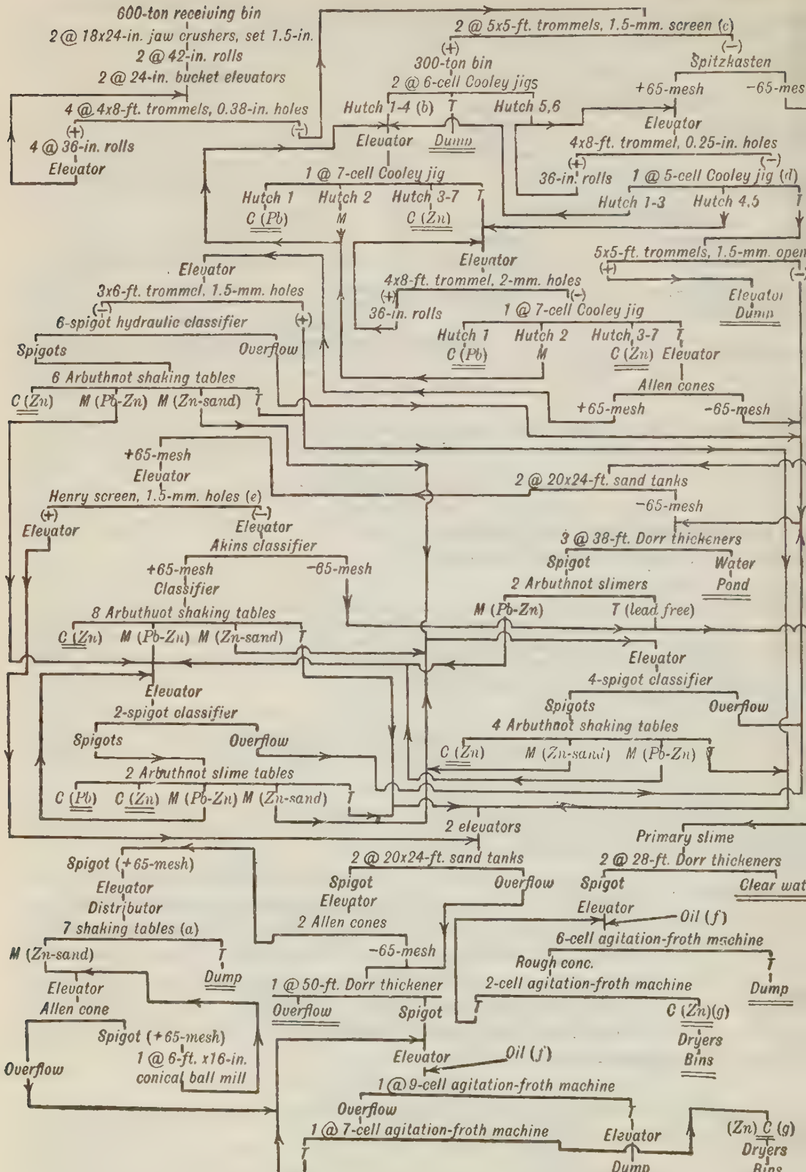
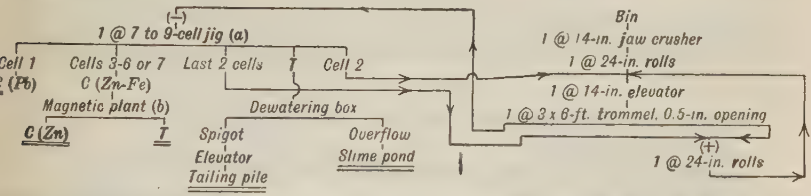


FIG. 95.—Eagle-Picher Lead Co., Netta mill.

Wisconsin zinc district, one-jig mills. Fig. 96. (*95 J 785; 59 A 117.*)

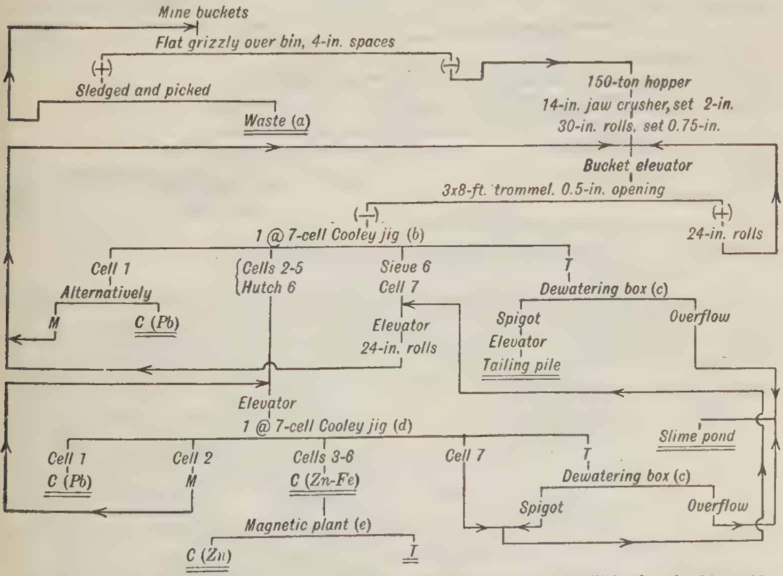
Ore: Galena, sphalerite, marcasite and pyrite in gangue of dolomite, calcite and barite.
Capacity: 100 to 150 tons per 10 hr.
General: This type of mill is applicable only to low-grade coarsely-disseminated ores having a high iron-zinc ratio and not much lead. With such ores much of the lead is recovered in a salable concentrate, and from 60 to 80 per cent. of the zinc in a zinc-iron concentrate assaying 20 to 40 or 45 per cent. Zn. This product is sold to the plants with magnetic separators. (See NATIONAL ZINC SEPARATING Co., p. 164.) If lead content of feed is relatively high, too much lead goes into the zinc concentrate and a 2-jig mill (*q.v.*) must be used.



a, 30 × 36-in. sieves. 200 r.p.m. Concentrate drawn continuously from hutches and shoveled from sieves at the discretion of the operator. *b,* See NATIONAL ZINC SEPARATING Co.

FIG. 96.—Typical Wisconsin one-jig mill.

Wisconsin zinc district, 2-jig mill. Fig. 97. (*59 A 117; 111 J 1065; TP 95 USBM.*)



a, Assay about 0.5 per cent. Zn. Amounts to 5 to 25 per cent. of mill feed. *b,* 30 × 42-in. sieves. *c,* On end of jig. *d,* 28 × 42-in. sieves. *e,* See following flow-sheet.

FIG. 97.—Typical Wisconsin two-jig zinc mill.

Ore: Galena, sphalerite, marcasite and pyrite in dolomite, calcite and barite. Coarse dissemination.
Capacity: 100 to 150 tons per 10-hr. shift.
Assays: Feed, 6 to 8 per cent. Zn; concentrate, unroasted, see Table 75; roasted, 59 to 61.5 per cent. Zn; tailing, 1.5 to 2 per cent. Zn.

Table 75. Analyses of mill (unroasted) concentrate, Wisconsin zinc district. (*After Deutman, 107 J 1108*)

Mine	Per cent. weight				
	Zn	Fe	Pb	CaO	S
Yewdall	21.60	27.40	0.82	2.15	41.60
Martin	39.95	17.12	0.20	1.67	38.30
Jefferson	45.90	12.85	0.22	1.50	35.73

Recovery: 65 to 80 per cent.

General: Tables are not used on account of the high iron and lime contents in table concentrate and the low recovery.

National Zinc Separating Co. Fig. 98. (*107 J 1107.*)

Location: Cuba City, Wis.

Ore: Zinc-iron concentrate from Wisconsin-district zinc mills.

Capacity: 275 tons per 24 hr.

Assays: Feed, 20 to 45 per cent. Zn; concentrate, 59 to 61.5 per cent. Zn; tailing, 4 to 5 per cent. Zn.

Costs: Roasting and separating, $1.28; Cottrell precipitator, $0.18; receiving and shipping, $0.91; general, $0.63; total, $3 per ton of roaster feed.

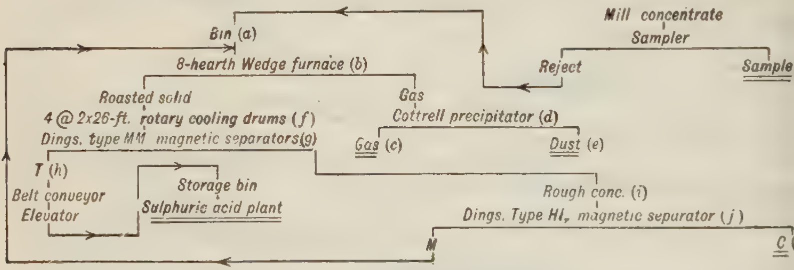


Fig. 98.—National Zinc Separating Co.

a, Separate compartments for concentrates from different mines. *b,* 22½-ft. diameter. 1 drying hearth and 7 for roasting. No fuel added. Maximum temperature at seventh hearth, 900° to 1000° F. Surface of marcasite oxidized to Fe_3O_4. Decrepitation during roasting (see Table 76) causes a decrease in CaO content through dusting. Linden Zinc Co., Linden, Wis., and Wisconsin Zinc Co., Cuba City, Wis., use an oil-fired rotary kiln on the ground that the degree of roasting can be better regulated. *c,* 4 to 5 per cent. SO_2. *d,* 2 units, can be operated together, but ordinarily operated separately and alternately for 2-week periods. Feed about 9000 cu. ft. of gas per min. at 290° F.; velocity, 6 ft. per sec. 36 collecting electrodes in each unit, made of 12-in. 14-gage steel pipe 15 ft. long. *e,* About 95 to 97 per cent. of total roasting loss. Comprises about 1.6 per cent. of solid feed to furnace. *f,* 6 r.p.m. Shells 5/16-in. riveted steel plate. Water-cooled by outside spray. 30 gal. water per min. per machine. Ore not completely cooled, as it is not then so magnetic as when slightly warm to the hand. *g,* See Sec. 13, Art. 8. Campbell, Wetherill and Knowles magnets used in other plants

Table 76. Screen analyses of feed and product of zinc-concentrate roaster

Size	Weight, per cent.	
	Feed	Product
+0.25-in.	0.20	0.10
0.12-in.	21.70	9.70
10-mesh	17.90	7.80
20-mesh	28.10	23.30
40-mesh	22.90	29.10
−40-mesh	9.20	30.00

in district. First poles draw 2 amp. at 225 volts; second, 3 amps. at 225 volts. Air gap of secondary magnets to shaking tray, ⅜ to ½ in. Capacity, 60 tons per 24 hr. *h,* Magnetic product. Averages 25 per cent. S and 4 to 5 per cent. Zn. *i,* 56 to 58 per cent. Zn and 4 to 5 per cent. Fe. *j,* 2 @ 12-in. feed belts, 70 ft. per min. 4 magnets in series drawing 7½ to 15 amp. Capacity, 40 to 45 tons per 24 hr. *k,* 61.5 per cent. Zn.

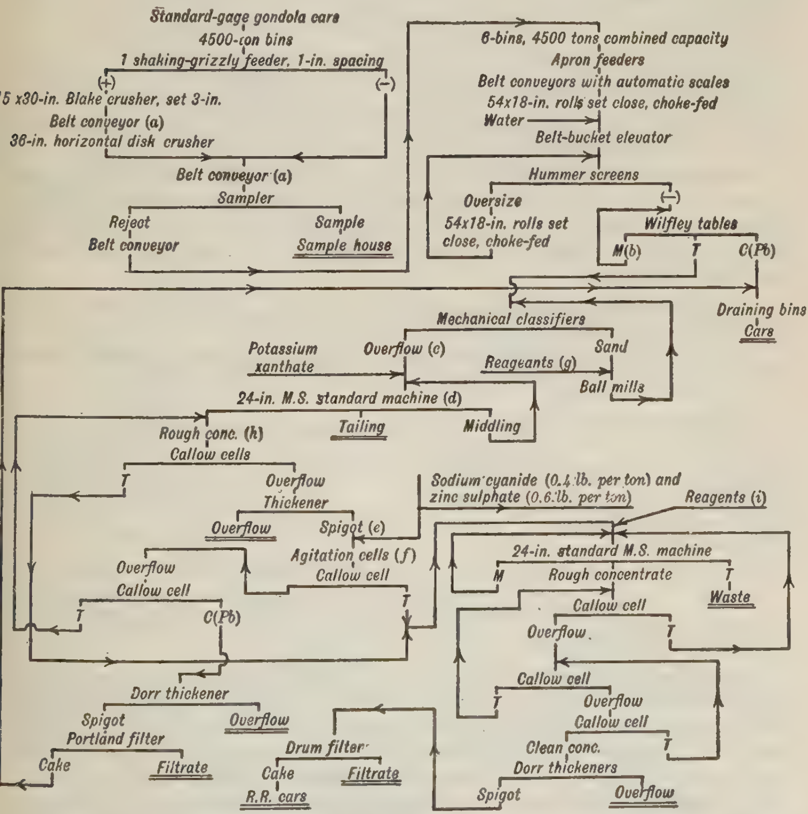
Summary. Roasting and magnetic separation.

Timber Butte Milling Co. Fig. 99. (*Q; 52 A 915; 120 J 685.*)

Location: Butte, Mont.

Ore: Sphalerite, galena, pyrite, bornite, chalcocite, tennantite, tetrahedrite, silver and some gold in quartzitic gangue containing also barite, fluorite, rhodonite and rhodochrosite. Galena is intimately mixed with blende. Considerable iron is combined with the zinc.

Capacity: 800 to 1200 tons per 24 hr.



a, Magnetic head pulley and suspended magnet. *b,* Large circulating load maintained. *c,* 60- to 100-mesh, according to the ore. *d,* Arranged for quick removal of froth. *e,* 50 per cent. moisture or less. *f,* Two in series. Pulp heated to 140° F. and held about one hour. *g,* Lime, 3 lb. per ton; Barrett No. 4, 0.20 lb. per ton. *h,* Pb, 25 per cent.; Zn, 36 per cent.; Fe, 4.0 per cent.; insol., 15 per cent. *i,* Copper sulphate, 1.3 lb. per ton; NaOH, 1.0 lb.; pine oil, 0.06 lb.; Barrett No. 634, 1.5 lb. per ton.

Fig. 99.—Timber Butte mill.

Assays: See Table 77.
Recovery: See Table 77.

Table 77. Performance at Timber Butte mill on Elm Orlu ore with differential flotation. (*After Robie*)

Material	Weight, per cent.	Assays					Recovery, per cent.			
		Ag, oz.	Per cent.				Ag	Fe	Zn	Pb
			Insol.	Fe	Zn	Pb				
Feed	100.0	3.8	.....	2.8	11.5	1.2	.....	.....	.....	.....
Table lead conc.	0.4	12.5	3.0	25.0	12.5	25.0	1.3	3.6	0.4	8.3
Flotation lead conc.	1.5	65.3	1.8	3.4	18.0	50.0	25.8	1.8	2.3	62.5
Flotation zinc conc.	18.3	12.1	5.8	2.7	56.7	1.5	58.2	17.8	90.3	22.5
Tailing	79.8	0.7	.....	2.7	1.0	0.1	.....	.....	.....	.....

General: Mine to mill, 13 miles. Copper smelter, Anaconda, Mont.; zinc smelter, Bartlesville, Okla.; lead smelter, Midvale, Utah. Water pumped 2 miles. Power transmitted 175 miles at 100,000 volts. Steep sloping mill site. An inclined tramway, 10-ton capacity, 37-hp. hoist, runs alongside the mill, serving all levels. Coarse-crushing plant and concentrate-handling plants are on the lowest level.

Summary. Four-step crushing in jaw crushers, disk crusher and rolls to pass 1/8-in. screen. Table lead concentrate made at this size. Balance ground in one step to 60- or 100-mesh, collective lead-zinc froth made and separated into lead and zinc concentrate by differential flotation with cyanide and zinc sulphate.

Consolidated Mining and Smelting Co. of Canada. Fig. 100. (*116 J 453.*)

Location: Kimberly, B. C.
Ore: Principally galena, marmatite, pyrite and pyrrhotite, with a small amount of quartz and calcite. Brittle, grinds readily. Sulphides intimately associated.
Capacity: 3000 tons per 24 hr.
Assays.

	Pb, per cent.	Zn, per cent.	Ag, oz.	Fe, per cent.	S, per cent.	Insol., per cent.
Feed	10–11	12–13	3	32–33	30–32	4–6
Lead concentrate	60+	7.5+	....	7+	....	....
Zinc concentrate	4	40+	....	....	....	....
Tailing *	1.8	2.7	....	....	....	....

* These are the tailing assays in the pilot plant.

Distance: mine to mill, 3.5 miles.

Summary. Differential flotation with tabling of the lead-zinc froth from the early cells of the zinc-flotation machines. CRUSHING: Jaw crusher from 36- to 8-in.; gyratory from 8- to 2.5-in.; rolls from 2.5- to 1-in.; rolls from 1- in. to 0.5-in.; 2-stage ball milling from 0.5-in. to 95 per cent. −200-mesh. CONCENTRATION: Primary lead flotation with 3-stage cleaning of lead concentrate: zinc flotation of lead-machine tailing making finished zinc concentrate; zinc-lead middling, which is tabled for zinc making a lead-iron concentrate that is returned to lead flotation and zinc middling; zinc middling which joins the table-zinc middling and is re-ground and re-floated in a secondary zinc-flotation machine; and a tailing which is classified into sand and slime, the sand being re-ground with the zinc middling while the overflow goes to the tailing plant.

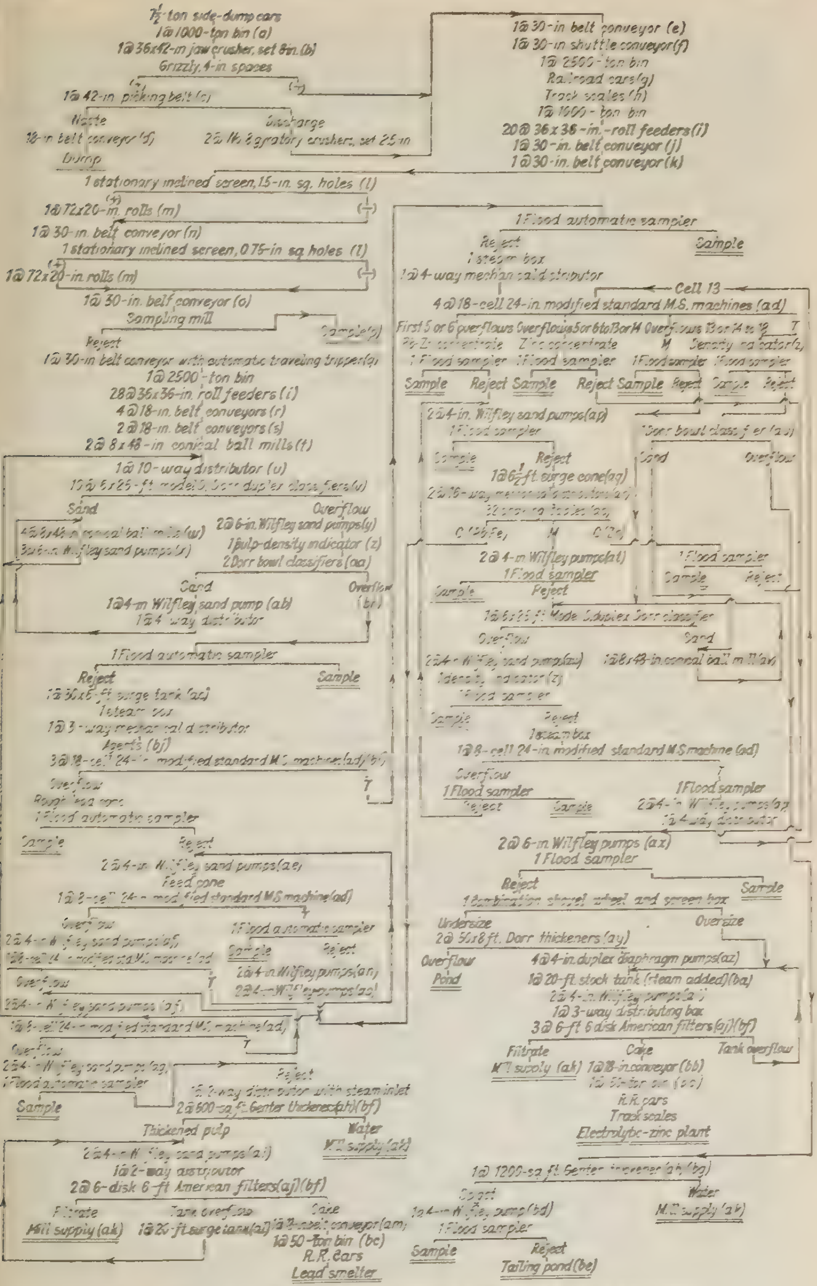


FIG. 109.—Consolidated Mining and Smelting Co. of Canada.

The feature of the plant, other than the outstanding one that it is the first large substantially all-flotation differential mill on this continent, is the complete elimination of bucket elevators by the use of Wilfley sand pumps for all fine wet pulp, and of belt conveyors for dry material and so arranging the machines that no coarse wet material need be elevated. This is distinctly modern practice, and with duplicate pumps, as in this mill, markedly decreases lost time due to breakdown of pulp-transporting equipment. The very complete automatic-sampling equipment is also admirable and should quickly pay for itself in better control of plant operation.

a, Air-operated gates. ***b,*** 155 r.p.m. ***c,*** See Table 78. ***d,*** See Table 78. ***e,*** See Table 78. ***f,*** See Table 78. ***g,*** Standard gage. 3½ miles. ***h,*** 150-ton. ***i,*** 0.57 r.p.m.

Table 78. Conveyors in Kimberly mill, Consolidated Mining and Smelting Co. of Canada. (*After Young*)

Reference letter	Width, inches	Length, feet	Slope, inches per foot	Speed, feet per minute	Plies
c	42	22.3	+3⅞	40	7
d	18	60	+3¼	100	6
e	30	123.5	+3¾	176	8
f	30	61.6	0	190	6
j	30	67	0	155	6
k	30	114	+3³⁄₁₆	182	6
n	30	45	+3³⁄₁₆	175	6
o	30	192	+3⁵⁄₁₆	190	6
q	30	133	0	217	6
r	18	38	0	204	6
s	18	38.5	+3⅜	164	6
am	18	185	0, +3	185	5
bb	18	205	0, +3	185	5

j, See Table 78. ***k,*** See Table 78. ***l,*** 2 ft. 3 in. (wide) × 5 ft. 3 in. 45° slope. ***m,*** 105 r.p.m. ***n,*** See Table 78. ***o,*** See Table 78. ***p,*** Final sample, 1 part in 10,000. ***q,*** See Table 78. ***r,*** See Table 78. ***s,*** See Table 78. ***t,*** 20.7 r.p.m. 40,000 lb. 3- and 4-in. forged-steel balls. See Sec. 4, Table 11. ***u,*** 97 r.p.m. ***v,*** 17 r.p.m. Slope, 2.5 in. per ft. ***w,*** 18.2 r.p.m. 40,000 lb. 1¾- and 2¼-in. chilled white-iron balls. See Sec. 4, Table 11. ***x,*** Lift, 37½ ft. 860 r.p.m. ***y,*** Lift, 40 ft. 860 r.p.m. ***z,*** Manometer type. ***aa,*** 3-ft. rake @ 16 strokes per min.; 10-ft. bowl, agitator 1.25 r.p.m. Slope, 2 in. per ft. ***ab,*** Lift, 29½ ft. 1150 r.p.m. ***ac,*** Cross-armed stirrer, 5 r.p.m. ***ad,*** 255.5 r.p.m. ***ae,*** Lift, 45 ft. 1150 r.p.m. ***af,*** Lift, 35 ft. 860 r.p.m. ***ag,*** Lift, 37 ft. 860 r.p.m. ***ah,*** "Diamond-Stiles" type. ***ai,*** Lift, 37½ ft. 1150 r.p.m. ***aj,*** 300 sq. ft. filtering area per machine. 0.2 to 0.4 r.p.m. ***ak,*** To sump feeding 1 @ 8-in. 2-stage centrifugal pump. 1160 r.p.m. Lift, 160 ft. ***al,*** Cross-armed stirrer, 7.5 r.p.m. ***am,*** See Table 78. ***an,*** Lift, 42 ft. 1150 r.p.m. ***ao,*** Lift, 59 ft. 1150 r.p.m. ***ap,*** Lift, 50 ft. 1150 r.p.m. ***aq,*** 9 ft. 10 in. deep. ***ar,*** 14 r.p.m. ***as,*** 24 Plat-O and 8 Wilfley. 295 r.p.m. ***at,*** Lift, 55½ ft. 1150 r.p.m. ***au,*** 3-ft. rake at 15 strokes per min.; 10-ft. bowl agitator, 1.25 r.p.m.; slope, 2 in. per ft. ***av,*** 18.2 r.p.m., 40,000 lb. 1¾- and 2¼-in. chilled white-iron balls. ***aw,*** Lift, 34 ft. 1150 r.p.m. ***ax,*** Lift, 44 ft. 1150 r.p.m. ***ay,*** 1 rev. in 2 min. ***az,*** 36.5 strokes per min. ***ba,*** Cross-armed stirrer, 7.5 r.p.m. ***bb,*** See Table 78. ***bc,*** Also provision for 2500-lb. stockpile with re-loading plant. ***bd,*** Lift, 20 ft. 900 r.p.m. ***be,*** About 40-yr. storage. ***bf,*** The two 600-sq. ft. Genter thickeners and the submerged sections of the 5 American filters are connected through a 5 × 11-ft. vacuum receiver and condensing trap with the main vacuum tank. The drying sections of the filter leaves are connected through a similar receiver with the same vacuum tank. Vacuum is provided by 2 @ 31 × 12-in. Type ER-1, Ingersoll-Rand vacuum pumps, 220 r.p.m., 2295 cu. ft. per min. displacement each. Filtrate discharged through barometric leg. Compressed air is supplied at 5 lb. per sq. in. by 2 @ 14 × 24-in. Connersville blowers, 330 r.p.m.; 1800 cu. ft. displacement. ***bg,*** 1 @ 31 × 12-in. ER-1 dry vacuum pump, 220 r.p.m., 2295 cu. ft. per min. displacement. Vacuum receiver discharged by barometric leg. ***bh,*** 95 per cent. −200-mesh. ***bi,*** Pulp strongly alkaline (7 lb. sodium carbonate per ton), 50 per cent. solids, 70° to 75° F. Oil mixture, 3 parts water-gas tar and 2 parts coal tar creosote, 0.5 lb. per ton with 0.1 lb. sodium or potassium cyanide. ***bj,*** Copper sulphate, 1 lb., (0.33 lb. per ton of original flotation feed) and water-gas tar, added at the head of the cells, and 0.05 to 0.1 lb. of a mixture of 9 parts water-gas tar and 1 part coal-tar creosote is added at the 13th cell.

Northern Ore Co. *(116 J 401.)*

Location: Edwards, N. Y.

Ore: Ferriferous sphalerite (black jack) and pyrite with a little galena, pyrrhotite and chalcopyrite in a gangue chiefly diopside, calcite, talc and serpentine. The sphalerite contains about 62 per cent. Zn and 5 per cent. Fe. Analysis of ore: 17 per cent. Zn, 9 per cent. Fe, 17 per cent. S, 0.12 per cent. Pb, 16 per cent. SiO_2, 11 per cent. CaO, 13 per cent. MgO, balance undetermined.

Products: Gravity concentrate, 25 to 30 per cent. Zn, raised to 45 per cent. in the magnetic plant. Jig tailing, 3 per cent. Zn; table and vanner tailing, 1 to 3 per cent.; magnetic tailing, 6 per cent. Zn.

Summary. CRUSHING: Jaw crusher from 8- in. to 2-in.; horizontal disk to about ½-in.; rolls in closed circuit with a trommel to 4-mesh. CONCENTRATION: Material coarser than 8-mesh jigged to make tailing and zinc-iron concentrate. Minus-8-mesh material classified and concentrated on shaking tables and vanners. Middling re-ground in rolls in closed circuit with 12- to 18-mesh Bunker Hill screens and returned to the primary classifier-table circuit. Tailing to waste. Jig and table concentrates are re-ground to −16-mesh by rolls in closed circuit with Bunker Hill screens and treated in a Weatherby high-intensity magnetic separator (Sec. 13, Art. 15) to remove blende.

Table 79. Composition of Bunker Hill and Sullivan ore. *(After Handy and Rickard)*

Chemical composition	Per cent.
Pb	10.3
Fe	18.0
Zn	2.4
Mn	1.5
S	3.6
CaO	1.2
SiO_2	40.0
O	6.2
CO_2	16.1
Ag	3.6 oz.
Mineralogical composition	
Galena (argentiferous)	11.89
Sphalerite	3.57
Pyrite	1.57
Siderite	35.80
Rhodochrosite	3.14
Calcite	3.30
Quartz	40.00

Bunker Hill and Sullivan Min. and Conc. Co. Fig. 101. *(Q; 120 P 485, 525.)*

Location: Kellogg, Idaho.

Ore: Lead-silver. See Table 79.

Capacity: 1200 tons per 24 hr.

Assays, June (1919): Feed, 10.05 per cent. Pb and 3.9 oz. Ag; concentrate, 68 per cent. Pb, 23.8 oz. Ag; tailing, 1.2 per cent. Pb. For analyses of products of individual machines see Tables 80 and 81.

Recovery: 90 per cent.

Ratio of concentration: 7.5 : 1.

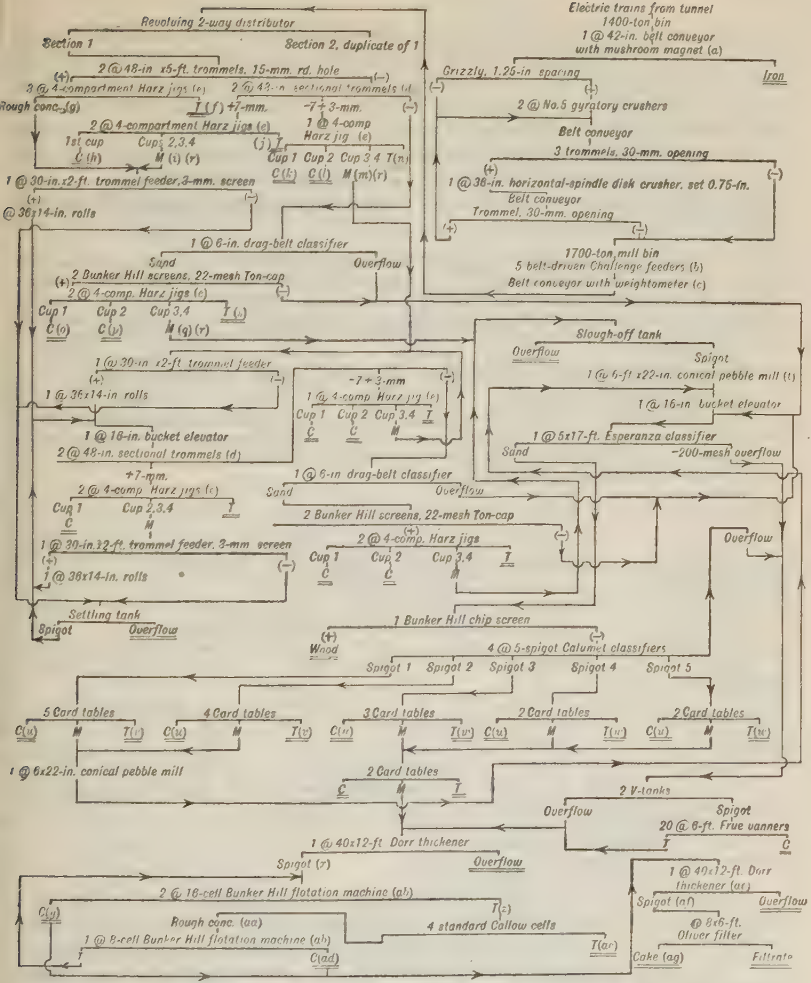
Table 80. Weights and assays of products of concentrators, West No. 2 mill, Bunker Hill and Sullivan M. & C. Co. *(After Rickard)*

Machine	Feed			Concentrate			Middling			Tailing		
	Tons	Per cent. Pb	Tons Pb	Tons	Per cent. Pb	Tons Pb	Tons	Per cent. Pb	Tons Pb	Tons	Per cent. Pb	Tons Pb
Jig, coarse	210	9.0	18.90	....	....	....	100	18.3	18.3	110	0.50	0.55
Jig, 7-15-mm., primary	100	11.5	11.50	8.0	69	5.5	45	13.0	5.8	45	1.00	0.45
Jig, 7-15-mm., secondary	100	8.0	8.00	5.0	73	3.6	40	10.0	4.0	55	1.20	0.70
Jig, 3-7-mm., primary	110	7.5	8.25	5.0	75	3.7	50	8.0	4.0	45	1.50	0.70
Jig, 3-7-mm., secondary	80	14.0	11.20	6.0	65	4.0	65	11.0	7.0	7	1.50	0.10
Jig, sand, primary	75	15.0	11.25	5.0	72	3.6	40	10.0	4.0	30	2.00	0.60
Jig, sand, secondary	80	12.0	9.60	4.5	73	3.3	40	14.0	5.6	25	2 25	0.56
Coarse-sand tables	185	15 0	27.63	20.0	74	14.8	60	14.0	10.3	75	2.00	1.50
Fine-sand tables	55	13.0	7.15	5.5	75	4.1	20	12.5	2.5	30	1.50	0.45
Middling tables	20	12.5	2.50	1.0	50	0.5	10	17.5	1.75	10	2.50	0.25
Vanners	100	11.0	11.00	8.0	67	5.4	90	6.5	6.0	....	....	....
Flotation	100	8.0	8.00	14.0	50	6.9	2	17.0	0.34	85	1.00	0.85
Total	....	....	....	82.0	67.5	55.4	....	....	....	517	1.20	6.21

Table 81. Screen analyses of mill products, Bunker Hill and Sullivan M. & C. Co. (*After Rickard*)

Screen	Mill feed	Coarse roll		Intermediate roll		Fine roll		Drag-belt classifier overflow	
		Feed	Product	Feed	Product	Feed	Product	Primary	Secondary
15-mm.	25.5	63.2	5.5	4.4	....	....	....	....	....
7-mm.	26.6	33.4	51.1	83.9	20.1	10.9	0.9	....	....
3-mm.	20.0	3.3	29.3	11.5	53.7	84.9	40.1	....	....
20-mesh	11.3	....	....	....	14.9	....	30.9	....	....
28-mesh	3.0	....	....	....	2.6	....	5.7	1.6	1.7
35-mesh	1.7	....	....	....	1.2	....	3.1	1.7	1.7
48-mesh	1.6	....	....	....	1.2	....	2.7	3.0	3.1
65-mesh	1.2	....	....	....	....	....	....	3.5	3.6
100-mesh	1.3	....	....	....	....	....	....	4.2	6.2
150-mesh	0.4	....	....	....	....	....	....	3.8	4.8
200-mesh	1.4	....	....	....	....	....	....	10.4	11.5
Through last screen	6.0	0.1	13.9	0.2	6.3	4.2	16.6	71.8	67.4

Screen	Calumet classifier products						Primary pebble mill		Esperanza classifier	
	Spigot 1	Spigot 2	Spigot 3	Spigot 4	Spigot 5	Overflow	Feed	Product	Sand	Overflow
15-mm.	....	....	....	....	....	....	....	....	....	....
7-mm.	....	....	....	....	....	....	....	....	....	....
3-mm.	....	....	....	....	....	....	....	....	....	....
20-mesh	1.7	0.1	....	....	....	....	....	....	....	....
28-mesh	10.1	0.9	....	....	....	....	94.5	0.3	7.6	....
35-mesh	9.8	1.7	....	....	....	....	1.4	1.3	8.4	....
48-mesh	13.9	3.8	0.4	....	....	....	0.5	5.0	12.8	....
65-mesh	14.5	10.0	2.3	....	....	....	0.6	8.2	13.1	....
100-mesh	20.4	29.5	14.5	1.5	0.4	....	0.5	17.9	18.5	....
150-mesh	6.4	14.4	11.8	2.9	0.4	0.6	0.6	7.9	10.4	....
200-mesh	14.9	27.8	46.6	14.6	5.0	1.3	0.8	25.2	16.7	5.0
Through last screen	8.3	11.8	24.4	81.3	94.2	98.1	1.1	34.2	12.5	95.0



a, 17 to 55 ft. per min., Reeves double-cone variable-speed drive. *b*, ¼-in. wearing plate on disk lasts one year. Feed rate remarkably uniform. Driven from tail pulley of following conveyor. *c*, Inclined +22°-30′. Blake-Dennison weigher. Weigher error less than 0.5 per cent. *d*, Sections 30 in. long. Screens on first 3 sections 3 × 6-mm. slots; on last section 7 × 12 7-mm. slots, long diameter of slot in plane at right angles to axis. *e*, Discharge cups at center of discharge ends of compartments. Hutch products routed, in general, with corresponding cup discharges. See also Sec. 9, Art. 3. *f*, 0.4 to 0.5 per cent. Pb. *g*, 18 to 25 per cent. Pb. *h*, 65 per cent. Pb. *i*, 10 to 13 per cent. Pb. *j*, 0.7 to 1 per cent. Pb. *k*, 75 per cent. Pb. *l*, 65 per cent. Pb. *m*, 7 to 10 per cent. Pb. *n*, 1.0 to 1.2 per cent. Pb. *o*, 77 per cent. Pb. *p*, 70 per cent. Pb. *q*, 8 to 10 per cent. Pb. *r*, Normally routed as shown. Occasionally sent to smelter as flux on account of high iron content. *s*, 1.3 to 1.5 per cent. *t*, Grinding to 20 mesh, open circuit. *u*, 72 to 74 per cent. Pb. *v*, 1.75 to 2 per cent. Pb. *w*, 1.5 per cent. Pb. *x*, 50 per cent. solids. 95 tons solid per 24 hr. *y*, 60 to 65 per cent. Pb. *z*, 3.5 to 4 per cent. Pb. *aa*, 30 per cent. Pb. *ab*, See Sec. 12, Art. 13 (Parker machine). *ac*, 1 per cent. Pb. *ad*, 45 per cent. Pb. *ae*, Automatic intermittent discharge regulated by oscillating gate. *af*, 35 per cent. moisture. *ag*, 11 per cent. moisture.

Fig. 101.—Bunker Hill and Sullivan Mining Co.

Summary. CRUSHING: Gyratory, 10- to 1.25-in.; disk crusher, 1.5- to 0.75-in.; rolls, 1- to 0.5-in.; rolls, 0.6- to 0.25-in.; rolls, 0.3- to 0.15-in.; 2-stage reduction in open-circuit pebble mills to 200-mesh with screening, or classification and concentration intervening between all crushing steps following the disk crusher. CONCENTRATION: Jigging at 4 sizes on primary feed and 3 sizes on re-crushed middling with removal of concentrate and tailing; tabling of −14-mesh sands after classification, making both concentrate and tailing; heavy mineral taken out of slime by vanners; vanner tailing and finest slime floated, combination routing.

This flow-sheet is typically a graded-crushing and graded-concentration scheme with flotation added to clean up the slime, but taking a minor part in the flow-sheet as a whole. Further development is to be expected along the line of elimination of primary hydraulic classification, tabling of unclassified feed, cleaning the rough concentrate on tables, re-grinding all tailing, except, perhaps, that from the coarsest jigs, to flotation size and floating.

Silver King Coalition Mines Co. Fig. 102. (*116 J 369.*)

Location: Park City, Utah.

Ore: Both sulphide and carbonate treated. SULPHIDE: galena with tetrahedrite, pyrite and a little blende in quartz and dolomite. CARBONATE: anglesite and cerussite in the same gangue. About 1000 tons per month of carbonate ore.

Capacity: 300 tons per 24 hr.

Assays:

	Au, oz.	Ag, oz.	Pb, per cent.	Cu, per cent.	Zn, per cent.	Fe, per cent.	SiO_2, per cent.
Sulphide ore:							
Feed	0.02	5.9	6.8	0.10	3.4	8.5	50.4
Concentrate	0.05	21.4	30.9	0.20	2.8	16.4	15.6
Iron middling	0.04	10.4	6.3	0.25	5.6	30.9	14.4
Tailing	0.005	2.0	1.3	Tr.	1.5	2.4	66.2
Carbonate ore:							
Feed	0.02	5.0	5.1	0.10	4.2	2.6	56.2
Concentrate	......	25.5	25.1	0.60	6.0	4.5	32.8
Tailing	0.01	2.0	1.6	......	3.0	1.5	68.0

Recovery: Carbonate ore, 75 per cent. Sulphide ore, without flotation, 85 per cent; with flotation, 90 per cent. (est.).

Ratio of concentration: Sulphide ore, 3 : 1.

Power: 34 hp.-hr. per ton milled.

Costs: About $0.90 per ton milled (1923).

Summary. Gravity concentration by jigs and tables followed by flotation. CRUSHING: Gyratory crusher from 8- to 1.5-in.; rolls from 1.5- to 0.5-in.; ball mills from 0.5-in. to 16-mesh. CONCENTRATION: Jigging of closely sized feed on Harz jigs; tabling of closely classified fine material on shaking tables; sulphide slimes floated in pneumatic cells, rougher-cleaner routing.

Notes to Fig. 102.

a, Surface ribbed by means of 1 × 2-in. angles bolted on with 1-in. leg projecting. Wear practically *nil*. Feeders staggered in 2 rows on 10-ft. centers along bottom of bin. Maximum size of feed particle about 8 in. ***b,*** 18 × 8 × 8½-in. buckets, 20-in. centers, 201 ft. per min. ***c,*** 5-ply, 258 ft. per min. ***d,*** Capacity, 1200 tons sulphide ore and 100 tons carbonate ore. Discharged through narrow lengthwise slots in the bottom. ***e,*** Two for sulphide ores and one for carbonates. 5-ply belt. 15 ft. per min. maximum speed. Ratchet drive. ***f,*** 5-ply. 236 ft. per min. +20° slope. Ore rolled back when speed was 250 ft. per min. ***g,*** 1 @ 6-in. section removed to cut sample. ***h,*** About 5 per cent. of feed by weight. ***i,*** 67 r.p.m. ***j,*** 12 × 7-in. buckets spaced 16 in. 5-ply belt. 354 ft. per min. ***k,*** 19½ × 30-in. compartments, 5-mesh screen, 165 @ 1-in. strokes per min. ***l,*** 19½ × 30-in. sieves, 4-mesh screen, 215 @ ⅝-in. strokes per min. ***m,*** 19½ × 30-in. sieves, 4-mesh screen, 240 @ ½-in. strokes per min. ***n,*** 19½ × 30-in. sieves, 5-mesh screen, 270 @ ⅜-in. strokes per min. ***o,*** 19½ × 30-in. sieves, 5-mesh screen, 280 @ 5/16-in.

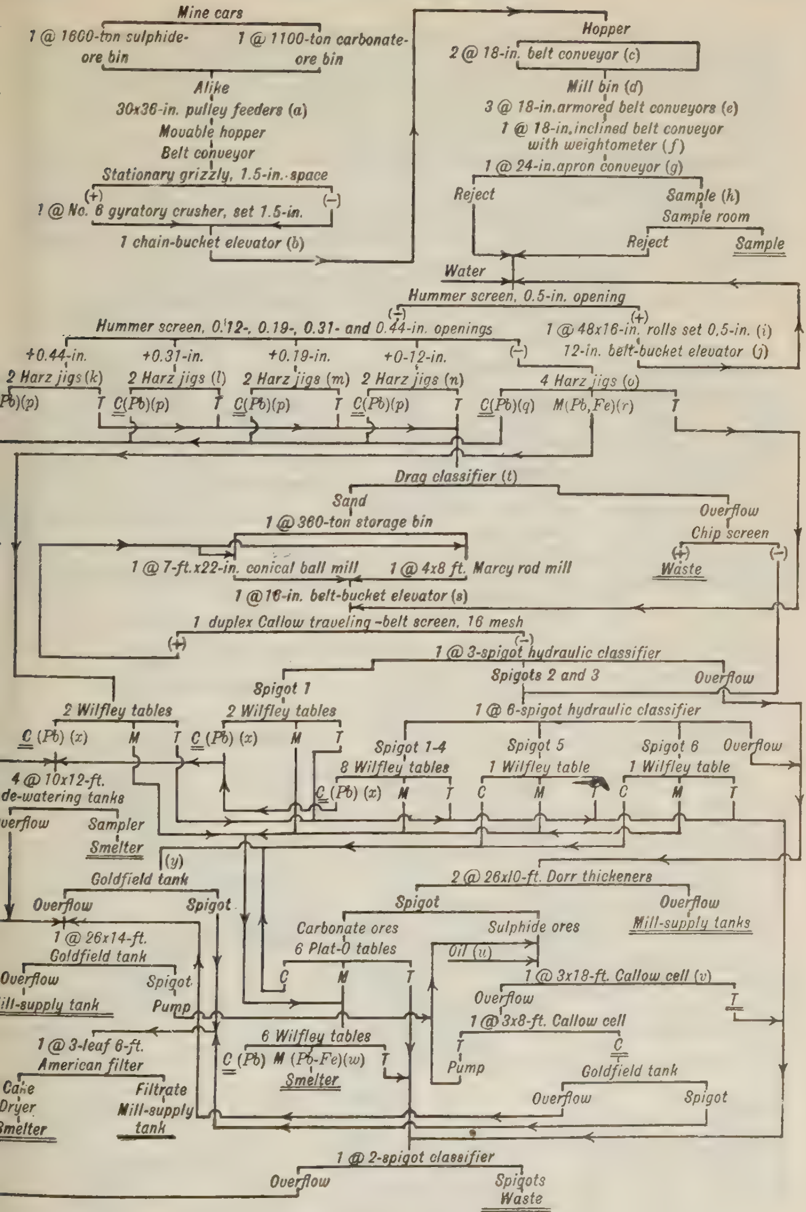


FIG. 102.—Silver King Coalition Mines Co.

strokes per min. *p*, Average assay, first-compartment discharge, 42.4 oz. Ag, 59.6 per cent. Pb; second-compartment discharge, 34 oz. Ag, 32 per cent. Pb. *q*, First-compartment dis-

charge, 31 oz. Ag, 53 per cent. Pb. *r*, Second-compartment discharge, 11 oz. Ag, 15 per cent. Pb, 26 per cent. Fe. *s*, 5-ply belt, 16 × 7-in. buckets spaced 16 in. 375 ft. per min. *t*, 16-in. belt. 2 × 2 × ¼-in. angles spaced 12 in. Slope, 40°. 3.15 ft. per min. *u*, 0.1 lb. pine oil and 2 lb. Barrett No. 4 (coal-tar creosote) per ton of ore. *v*, Feed, 20 per cent. solid. *w*, 10 oz. Ag, 6 per cent. Pb, 30 per cent. Fe. *x*, Average assay, 50 to 60 per cent. Pb and 27 to 35 oz. Ag. *y*, Average assay, 21 per cent. Pb, 20 oz. Ag.

Federal Mining and Smelting Co. Morning mill. Fig. 103. (*Q; 120 J 422.*)

Location: Wallace, Idaho.

Ore: Complex. Mineralogical analysis of average ore, per cent.: Argentiferous galena, 10.2; sphalerite, 7.2; siderite, 35.4; calcite, 2.5; rhodochrosite, 3.9; quartzite minerals, 40.8. Pyrite, pyrrhotite, stibnite, barite and other minerals are found in varying amounts.

Capacity: 1500 tons per 24 hr. About 300 tons removed in the sorting plant.

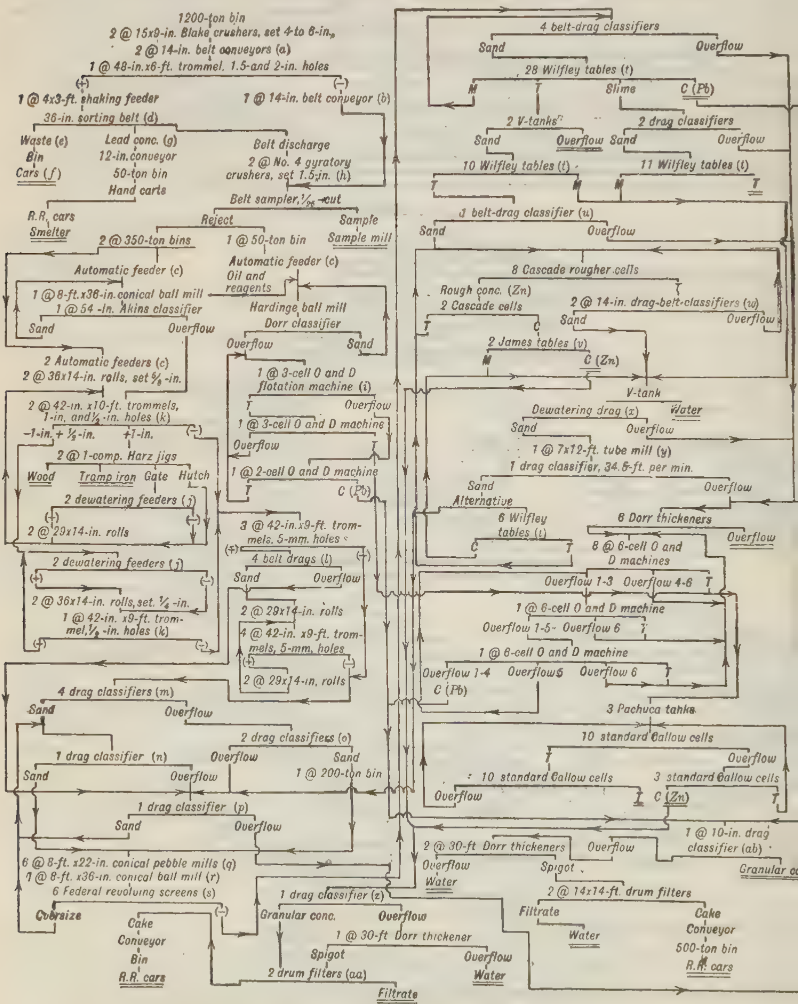


Fig. 103.—Federal Mining and Smelting Co., Morning mill.

a, 208 ft. per min. *b*, 142 ft. per min. Rise, 2½ in. per ft. *c*, Cam and spring, 102 strokes per min. *d*, 88 ft. long. 8-ply. ⅛-in. cover. 40 ft. per min. 10 pickers. 0.34 ton removed per man-hr. See Sec. 7, Art. 3. *e*, 100 tons picked. *f*, Electric haulage to dump or mine. *g*, 200 tons picked. *h*, See Sec. 3, Table 15. *i*, Horizontal-drum, centrifugal-type bubble-column machine. 80 r.p.m. *j*, 245 @ 1-in. strokes per min. *k*, 15 r.p.m. *l*, Slope, 1 in 6¾. Belt speed, 94 ft. per min. *m*, Slope, 1 in 6. Belt speed, 56 ft. per min. *n*, Slope, 1 in 6. Belt speed, 70 ft. per min. *o*, Slope, 3½ in 12. 25 ft. per min. *p*, 54 ft. per min. *q*, 23 r.p.m. See Sec. 4, Table 77. *r*, 19 r.p.m. See Sec. 4, Table 11. *s*, Drum type. 16-mesh. 15 r.p.m. *t*, 240 r.p.m. *u*, 16-in. belt. 75 ft. per min. *v*, 300 r.p.m. *w*, 24 ft. per min. *x*, Slope, 1 in 2. 5 ft. per min. *y*, 22 r.p.m. See Sec. 4, Table 65. *z*, Slope, 7½ in 12. 60 ft. per min. *aa*, 1 @ 6 × 8-ft.; 1 @ 10 × 11-ft. *ab*, 43 ft. per min.

Distances: Mine to mill, 2.5-mile tunnel; mill to lead smelter, 260 miles; to zinc smelter, 1500 miles. Water transported one mile by flume. Hydro-electric power from Montana Power Co., 24 miles at 100,000 volts, then 5 miles at 16,500 volts.

General: Sloping mill site.

Summary. Hand sorting, tabling and differential flotation. CRUSHING: Careful graded crushing in jaw and gyratory crushers, rolls, and cylinder mills in closed circuit with screens to 16-mesh. CONCENTRATION: Hand sorting at −6 +2-in. Tabling of −16-mesh sand to make lead concentrate. Middling, re-ground in tube mills, joins primary slimes and is floated in centrifugal and pneumatic bubble-column machines to save lead and zinc differentially.

Central Mine. Fig. 104. (*28 IMM 5.*)

Location: Broken Hill, Australia.

Ore: Galena, low-grade sphalerite and silver in gangue composed principally of quartz, rhodonite, rhodochrosite and garnetiferous sandstone. Minerals are freed at 0.012-in. (0.32-mm.).

Capacity: 600–700 tons per 24 hr.

Assays:

	Ag, oz.	Pb, per cent.	Zn, per cent.	Weight, per cent.
Feed	11.8	14.0	15.6	100.0
Lead concentrate (*a*)	40.8	64.6	9.0	17.4
Lead concentrate (*b*)	39.3	58.1	13.8	1.3
Zinc concentrate	12 8	5.1	47.9	27.2
Tailing	1.3	1.1	1.5	54.1

a From jig and cascade flotation. *b* From sub-aeration machine and Wilfley table.

Recovery, per cent.: Ag, 64.4; Pb, 85.6; Zn, 83.3.

Summary. Jigging for coarse galena (−⅛-in.), differential flotation for bulk of fine galena, collective flotation for zinc and balance of fine galena with separation of collective-flotation concentrate on Wilfley tables.

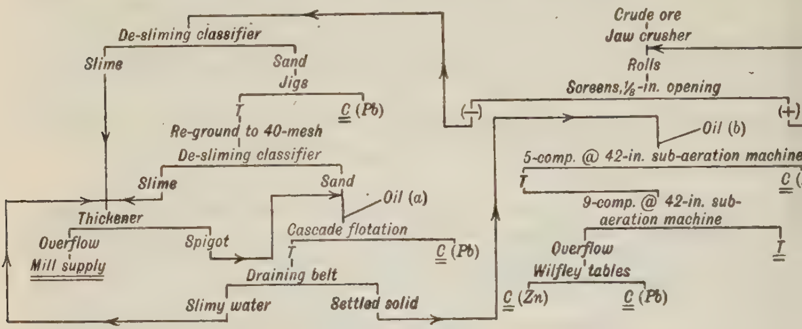
This flow-sheet is a development from another in which, substantially, shaking tables and vanners occupied the place now taken by cascade flotation. Representative results in the older operation are given in Table 82. The selective flow-sheet shows marked

Table 82. Performance of Central mine gravity-and-flotation plant. (*After Harvey*)

Assays	Ag, ounces	Pb, per cent.	Zn, per cent.	Weight, per cent.
Feed	11.6	14.5	16.4	100.0
Lead concentrate (*a*)	32.9	67.6	6.2	15.5
Lead concentrate (*b*)	43.3	60.6	13.8	1.3
Zinc concentrate	16.1	8.3	45.8	30.8
Tailing	2.3	1.8	2.9	52.4
Recovery, per cent.	49.2	77.6	85.8	....

a From primary tables and vanners. *b* From tables treating flotation concentrate.

increase in silver and lead recovery with somewhat lower zinc recovery. The grade of the present lead concentrate is lower, but the zinc concentrate is better. Economic results should be distinctly in favor of the present plant, as the increased recovery of lead and silver, the bettered grade of zinc concentrate, and probable decrease in labor and maintenance should much more than counterbalance the lower grade of lead concentrate and ower zinc recovery.



a, Aver. 0.125 oz. per ton of a mixture of coal tar and eucalyptus oil, varied as necessary.
b, Pine oil and wood-tar oil (and probably sulphuric acid) added as needed.

FIG. 104.—Central mine, Broken Hill.

Roseberry concentrator. Fig. 105. (*114 J 677; 101 J 463.*)

Location: Roseberry, Slocan District, B. C.

Ore: Argentiferous galena and sphalerite with considerable pyrite and siderite in quartz and slate. Silver occurs native and as tetrahedrite, proustite and argentite, very finely disseminated in the galena and blende.

Capacity: 150 tons per 24 hr.

Assays: Surprise ore (1915). Gravity mill only.

	Ag, oz.	Pb, per cent.	Zn, per cent.
Feed	27.7	8.6	19.2
Concentrate, Pb	118.0	62.7	9.7
Concentrate, Zn (*a*)	23.8	3.4	39.0
Tailing (*a*)	5.6	0.3	9.1

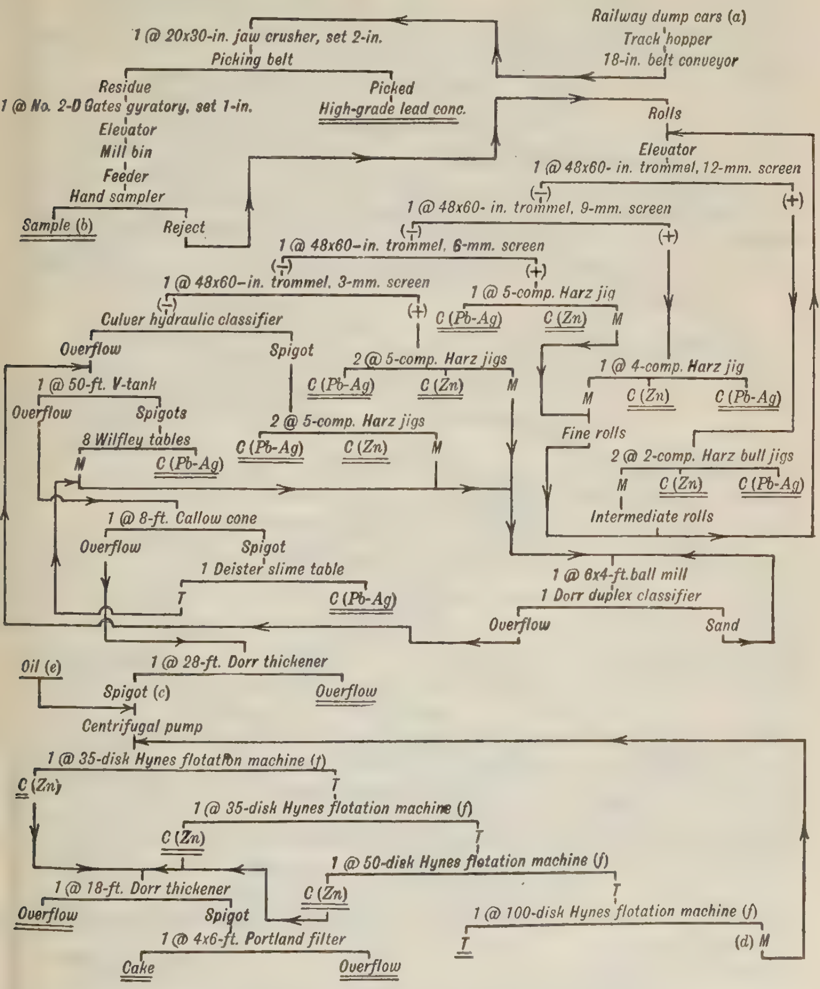
a For present flotation concentrate and tailing see Table 83.

Table 83. Flotation results at Roseberry mill. (*After McFadden*)

Ore	Feed			Concentrate			Tailing		
	Ag, oz.	Pb, per cent.	Zn, per cent.	Ag, oz.	Pb, per cent.	Zn, per cent.	Ag, oz.	Pb, per cent.	Zn, per cent.
Bosun	57.7	0.9	12.6	95	2.9	44	1.4	0	0.6
Surprise	22.3	1.2	11.7	94.5	5.0	42.6	3.7	0	3.0
Ivanhoe	12.0	1.1	9.8	45	3.3	45	0.6	0	0.8
Canadian	12.0	1.0	9.6	53	3.9	44.9	0.6	0	0.6

Summary. Gravity concentration by jigs and tables for lead and bulk of silver followed by flotation for zinc and balance of silver. CRUSHING: Jaw crusher, gyratory and 3-stage roll crushing for jigs and tables with re-grinding

of −6-mm. tailing to flotation size in ball mills. CONCENTRATION: 5-stage jigging after sizing and classification with tabling of de-slimed sands. Collective flotation of primary slime and re-ground middling.



a, Gathering from aerial tramways from 4 widely separated mines. *b*, Full stream caught for 30-second period every 15 min. Alternate-interval samples weighed for tonnage. *c*, 30 to 35 per cent. solid. *d*, 10 to 12 per cent. Zn. *e*, 3 or 4 parts water-gas tar to 1 of hardwood creosote. Total oil from 1 to 2 lb. per ton. Some added to machines, if desirable. About 0.75 lb. per ton of copper sulphate. *f*, 30-in. disks of 12-gage plate, 0.5-in. perforations. Rotor speed, 90 r.p.m.

FIG. 105.—Roseberry concentrator.

United States Smelting, Refining and Mining Co., Midvale plant. Fig. 106. (*Q*; 73 *A* 342.)

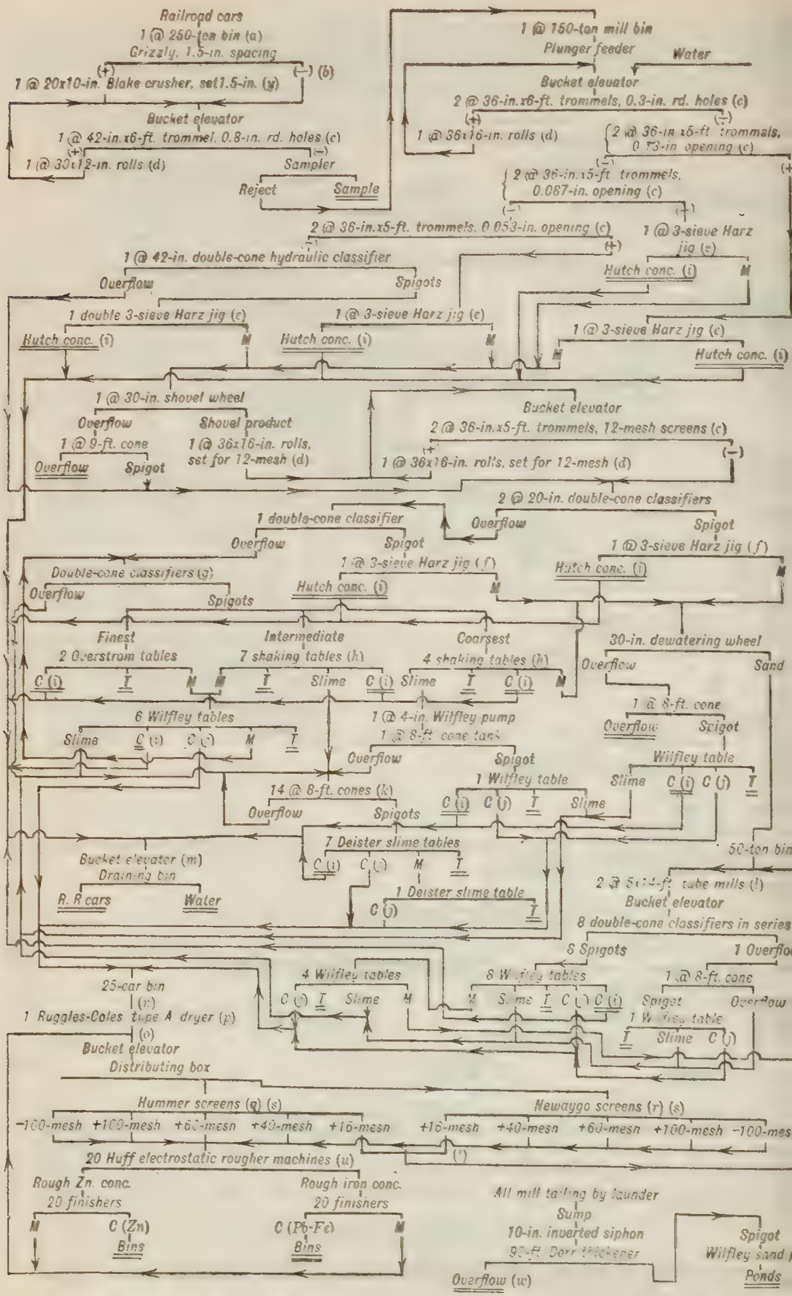


FIG. 106.—United States Smelting, Refining and Mining Co., Midvale plant.

a, Discharged through rack-and-pinion gate, hand operation. *b,* About 25 per cent. *c,* See Sec. 5, Table 26. *d,* See Sec. 3, Table 22. *e,* 36 × 24-in. sieves. Sieve openings larger than largest particle of feed; all concentrate from hutch. See Sec. 9, Art. 3. Bedding replenished by hand as necessary. Continuous spigot discharge from fine jig; intermittent, through manually operated molasses gates, from others. *f,* 24 × 36-in. sieves. See Sec. 9, Art. 3. *g,* A series of these ranging from 30- to 42-in. diameter. *h,* Wilfley and Deister Plat-O. See Sec. 10, Table 1. *i,* Lead-iron. *j,* Zinc-iron. *k,* In series. *l,* About 130 tons per 24 hr. One mill only used except when considerable quartzite is present, when 2 are required. Operated to just liberate mineral and produce as little slime as possible. *m,* Some of the lead-iron conc. cannot reach this elevator by gravity and is manually handled into cars and thence to the R.R. cars. *n,* 12.5 per cent. moisture. *o,* Bone-dry, 280° F. *p,* Capacity, 100 tons per 24 hr., 75-lb. slack coal per ton dried. Forced draft by 45-in. fan to 30-in. × 48-ft. stack. See Sec. 18, Art. 4. *q,* 75 per cent. of feed. Two @ 2-surface screens in series. *r,* 25 per cent. of feed. Two 2-surface screens in series. *s,* Screens: 16-mesh steel, 0.0445-in. sq. opening; 40-mesh brass, 0.014-in. sq. opening; 60-mesh brass, 0.007-in.; 100-mesh brass, 0.0045-in. *t,* About 2 per cent. of screen feed. *u,* 14 Type F consisting of 1 rougher and 2 finishers in each unit. 18 Type D (remodeled), 3 of which are required to make a 1-rougher-2-finisher unit. One 7½-kw. generator direct-connected to 5-hp. motor furnishes electrical energy. Voltage stepped up from 120 at generator to 18,000 to 22,000 at machines. *w,* 55 per cent. of feed tonnage. *y,* See Sec. 3, Table 5.

Location: Midvale, Utah.

Ore: Argentiferous galena, auriferous pyrite, chalcopyrite, and sphalerite in quartz, quartzite, limestone and porphyry. Sulphides range from coarse aggregation to very fine dissemination.

Capacity: 465 tons per 24 hr.

Assays:

	Pb, per cent.	Fe, per cent.	Zn, per cent.	Insol., per cent.	Ag, oz.	Au, oz.
Lead-iron conc. from jigs......	25.0	24.0	10.0	3.5	12.0	0.15
Zinc conc., electrostatic.......	1.8	6.2	49.5	8.2	1.4	0.03
Iron conc., electrostatic.......	4.9	35.0	8.0	2.0	4.5	0.12
Tailing......................	1.7	3.5	5.25	......	1.1	0.02

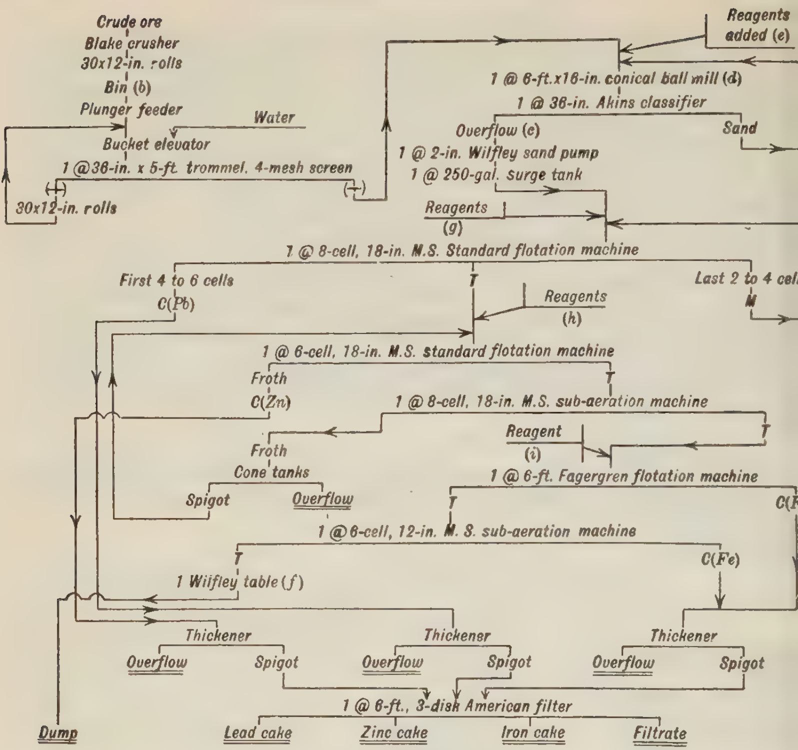
Recovery, per cent.: Pb, 91; Zn, 40; Ag, 87; Au, 87.

General: Distance mine to mill, 18 miles.

Summary. Graded crushing, jigging of sized products and tabling of classified products. Electrostatic separation of zinc-iron table middling. CRUSHING: Jaw crusher, 9- to 1.5-in.; rolls, 1.5- to 0.75-in.; rolls, 0.75- to 0.25-in.; 2 rolls in series, 0.25- in. to 12-mesh; tube mill to 28-mesh, open circuit. Screens precede all crushers. CONCENTRATION: Jigging starts at 0.25-in. with 5 steps down to 12-mesh. Jigs make lead-iron concentrate and a middling only. Tabling starts at about 16-mesh (1-mm.) and continues to the finest sizes. Electrostatic machines treat −16-mesh material in 4 sizes.

This is a relatively complex ore, but the individual mineral grains are sufficiently coarse to permit severing to an economic extent by crushing to about 0.5 mm. and separation of lead from zinc by gravity concentration. Recoveries are satisfactory for this type of ore, except the zinc recovery.

Differential flotation, according to the flow-sheet, Fig. 107, is expected to replace the old flow-sheet. This is an experimental plant running in competition with the regular gravity-concentration and electrostatic plant. Capacity, 50 tons per 24 hr. Recoveries and grades have been satisfactory except with ores containing considerable amounts of oxidized minerals.



b, 0.5 in. *c,* 20 per cent. solid. All −65-mesh, 65 per cent. −200-mesh. *d,* 7500 lb. @ 2½-, 2- and 1½-in. cast-iron balls; consumption, 1.7 lb. per ton crushed. Titanite lining, life 8 mo. *e,* Barrett No. 4 coal-tar creosote or similar oil, steam-distilled pine oil sodium sulphite. On some ores sodium sulphide and sodium silicate are also added here. *f,* Pilot. *g,* Potassium xanthate. Soda ash with some ores. *h,* Pine oil, Barrett No. 4 or similar oil, hardwood creosote, potassium xanthate, copper sulphate. *i,* Sodium sulphide. A small amount of pine oil and Barrett No. 4 with some ores.

A separate leaf of the filter works on each concentrate.

FIG. 107.—Differential flotation plant at U. S. S. R. & M. Co., Midvale plant.

Sunnyside Mining and Milling Co. Fig. 108. *(Q; 105 J 193; 109 J 838; 121 J 757.)*

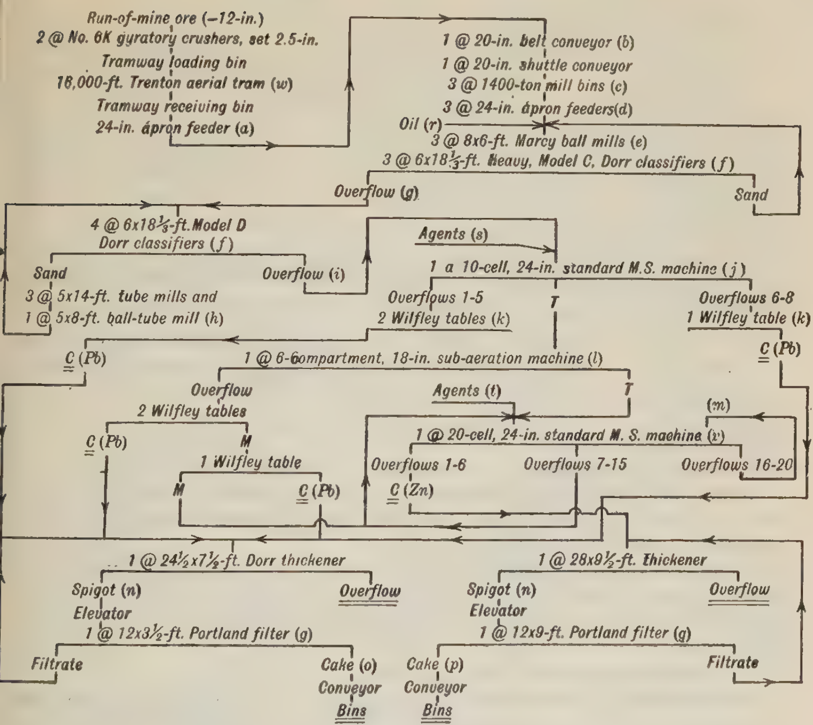
Location: Eureka, Colo.

Ore: Galena, sphalerite, chalcopyrite, pyrite, silver and gold in quartz-calcite-rhodonite gangue. Sulphides finely disseminated.

Capacity: 750 tons per 24 hr.

Assays:

	Au, oz.	Ag, oz.	Pb, per cent.	Cu, per cent.	Zn, per cent.	Fe, per cent.	Insol., per cent.
Feed	0.06–0.07	3.5– 4.5	4.8	0.4–0.5	5.5–7.5	3.0–4.0	.....
Concentrate, Pb	0.50–0.75	25.0–35.0	46.0–48.0	2.8–3.5	13.0	7 0	5.0–6.
Concentrate, Zn	0.08–0.14	7.0–11.0	5.5–6.0	1.2–1.5	50.0–54.0	4.0–4.8	2.0–5.
Middling (see Summary)	0.34	16.0	15.0	3.2	26.5	18.0	.....
Tailing	0.01–0.02	1.0	0.6	0.08–0.13	0.8–1.1	.....	.....



a, 33 ft. per min. *b*, 213 ft. per min. 7½-hp. motor. *c*, Steel and concrete, 30 ft. (diam.) × 36 ft. Flat bottom. *d*, 6 ft. per min. *e*, See Sec. 4, Table 5. *f*, See Sec. 6, Table 40. *g*, 55 to 60 per cent. solids, 22 per cent + 48-mesh, 30 to 35 per cent. − 200-mesh. *h*, See Sec. 4, Table 65. *i*, 20 per cent. solids, 7 per cent. +100-mesh, 68 per cent. − 200-mesh. *j*, 232 r.p.m. Maximum tonnage, 525 at 20 per cent. solids. Pulp temperature, 55° to 60° F. High-level type. Assays of froth from different cells follow:

	No. 1	No. 5	No. 6	No. 10
Au, oz. per ton...	0.60	....	0.96	0.84
Ag, oz. per ton...	23.0	26.2	....	22.5
Pb, per cent......	53.5	....	....	25.8
Cu, per cent.....	4.2 in every cell			
Zn, per cent......	9.3	....	....	23.8
Fe, per cent......	6.4	....	....	10.8
Insol., per cent...	5.0	....	....	9.0

k, Pilot tables, which also serve to break down froth. *l*, 425 r.p.m. *m*, Enters cell No. 8. *n*, 25 to 40 per cent. moisture. *o*, 6 to 8 per cent. moisture, 60 tons per 24 hr. *p*, 7 to 9 per cent. moisture, 65 tons per 24 hr. *q*, Operated about 6 hr. per day. See Sec. 17, Table 3. *r*, 0.15 lb. coal tar, 0.05 lb. hardwood creosote, 0.02 lb. crude naphthalene; sodium sulphite, 0.25 lb. per ton. *s*, Sodium silicate, 0.75 lb. per ton; soda ash (58 per cent.), 1 lb.; potassium xanthate, 0.05 lb; pine oil, minute amounts as necessary. *t*, Coal tar, 0.08 lb. per ton; hardwood creosote, 0.04 lb.; pine oil, 0.05 lb.; copper sulphate, 1.5 lb.; potassium xanthate, 0.18 lb. *v*, 250 r.p.m. *w*, Trenton, 10-cu.-ft. buckets, 45 tons per hr.

FIG. 108.—Sunnyside Mining and Milling Co.

Recovery:

	Au, per cent.	Ag, per cent.	Pb, per cent.	Cu, per cent.	Zn, per cent.
Lead concentrate	55	50	71	42	13
Zinc concentrate	10	16	11	21	66
Middling	11	11	8	15	9

Labor: 10 tons per man-shift, total.

Water: 4.9 tons per ton milled.

Power: 43 hp.-hr. per ton milled.

Distances: Mine to mill, 3 miles; mill to zinc smelter, 700 miles; mill to lead smelter, 54 miles; water transported, 1 mile by gravity pipe line; power transmitted, 54 miles at 50,000 and 17,000 volts.

Costs, cents per ton (1925):

	Labor	Reagents	Other Supplies	Power	Total
Lead flotation	3	9	1	2	15
Zinc flotation	3	17	....	4	24

Summary. Differential flotation. CRUSHING: Gyratory from 12- to 3-in.; ball mill from 3-in. to 20-mesh; tube mill from 20-mesh to 65-mesh. CONCENTRATION: differential flotation.

In 1925–26 about 15 tons per day of a mixed lead-zinc middling product was made and sold, additional to the regular lead and zinc concentrate. This was made possible by a peculiar local smelting need and enabled the production of higher-grade zinc and lead concentrates than otherwise.

Zinc carbonate ores are relatively rare.

In the Highland, Wis., district (*99 J 906*) small deposits of mixed carbonate and sulphide ore are worked by leasers. The ore is hand picked and the high-grade material stacked on the surface until sufficient has accumulated or until weather conditions are suitable and is then crushed in a set of slow-speed rolls, roughly concentrated in a log washer and cleaned on a power hand jig. The mixed concentrate carries about 40 per cent. Zn. Recovery is, naturally, low, but the deposits are too small to warrant more elaborate equipment. At MONTEPONI, Sardinia (*83 J 1094*) a 500-ton plant treating calamine in dolomite with some zinc, lead and iron sulphides consists of a 4-in. grizzly and a series of shaking screens of 1.25-, 0.8-, 0.55-, 0.4-, 0.28-, 0.2-, 0.12-, and 0.08-in. apertures to prepare feed for a picking belt and a series of jigs. Concentrate and tailing are rejected at all sizes and middling is re-ground and treated by magnetic separation to remove iron.

New Jersey Zinc Co. Franklin mill. Fig. 109. (*Q.*)

Location: Franklin, N. J.

Ore: Willemite, franklinite and zincite together with zinciferous manganese silicates in calcite.

Capacity: 100 tons per hr. (2-shift work).

Assays, per cent. Zn; Feed, 17; concentrate; willemite, 48; franklinite, 17; tailing, 2.25.

Recovery: 93 per cent.

Labor: Tons per man-shift, operating, 8.6; repairs, 179.

Power: 20 hp.-hr. per ton.

Water: 8 tons per ton of ore milled, approximately 99 per cent. recovered.

General: Level mill site. Concentrate shipped 90 miles. Tailing filled back into the mine.

Summary. High-intensity magnetic concentration to collect franklinite; willemite and zincite recovered on jigs and shaking tables. CRUSHING: Careful graded dry crushing in gyratory and rolls to $-\frac{1}{10}$-in. CONCENTRATION: Magnetic concentration follows extremely close dry sizing, coarse-sized non-magnetic material is jigged, fine is carefully classified and tabled.

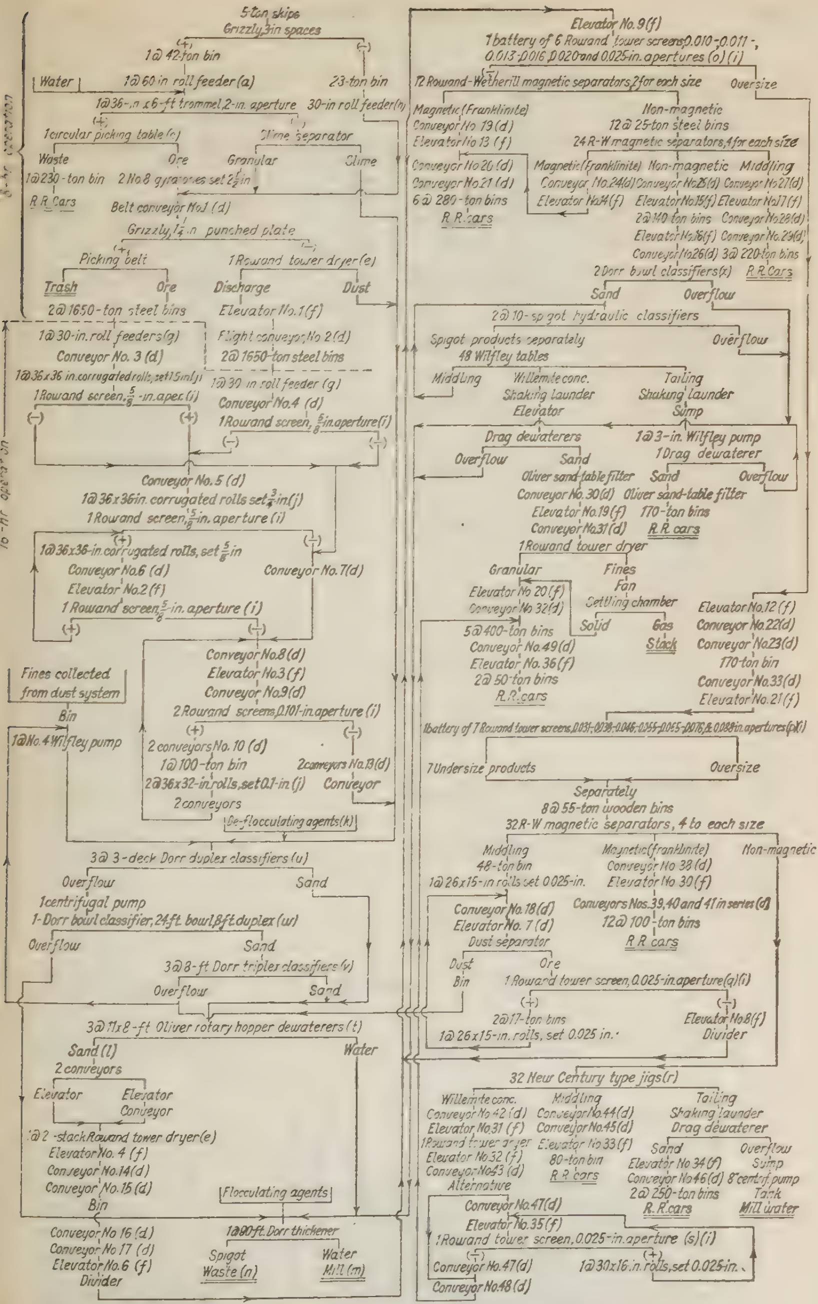


FIG. 109.—New Jersey Zinc Co., Franklin mill. (See p. 188 for notes.)

Table 84. Conveyors at N. J. Zinc Co., Franklin mill

Conveyor number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Type	Belt	Flight	Belt	Belt	Belt	Belt	Belt	Belt	Belt	Belt	Belt	Belt	Belt	Belt	Flight	Belt
Length c. c. pulleys, ft.	301	27	59	60	57	12	57	51	42	39	29	22	49	98	59	28
Width, in.	30	41	24	18	24	18	13	24	30	18	24	24	18	18	50	18
Rise, ft., in.	81–6	4–1	......	......	16 6	1 0	......	8–1	8–2	......	......	7 2	......	12 0	......	......
Slope in degrees	15	9	......	......	16	5	......	9	11	......	......	18	......	7	......	......
Dia. head pulley or sprocket	48	39	30	23	25	18	30	24	31	24	18	23	24	18	39	18
Dia. tail pulley or sprocket.	30	39	30	30	23	18	24	24	24	24	23	18	24	18	39	18
Spacing trough. idlers, ft., in.	4–0	......	4–0	4 0	4–0	4–0	4 0	4–0	4–0	4–0	4–0	4–0	4 0	4–0	......	4–0
Spacing return idlers, ft., in	8 0	......	8 0	8 0	8–0	8–0	8 0	8–0	8–0	8 0	8–0	8–0	8–0	8–0	......	8–0
Tonnage carried per hr.	200	100	50	50	60	25	90	100	200	50	100	100	50	100	100	90
Method of driving	Belt	Gear	Chain	Chain	Chain	Belt	Chain	Chain	Belt	Belt	Bevel	Belt	Belt	Chain	Gear	Bevel
Size mat. carried max., in.	3	1¼	3	1¼	5/8	5/8	5/8	5/8	5/8	5/8	1/10	1/10	1/10	1/10	1/10	1/10
Percentage moisture carried	3	......	2	......	2	2	2	2	......	......	......	......	......	......	......	......
Speed, ft. per min.	372	45	95	308	296	250	353	459	495	408	368	367	405	495	48	150
Belt, No. of plies	8	......	5	5	5	5	5	5	5	5	5	5	5	5	......	4
Character of plies	32 oz.	......	32 oz.	32 oz.	32 oz.	32 oz.	32 oz.	32 oz.	32 oz.	32 oz.	32 oz.	32 oz.	32 oz.	32 oz.	......	30 oz.
Thickness of cover	1/8	......	1/8	1/8	3/16	1/8	1/8	3/16	3/16	1/8	3/16	1/8	1/8	3/16	......	1/8
Life in days	1900	......	500	400	300	200	600	225	200	325	100	150	609	200	......	300
Screws, flights, pans: Mat.	......	St.	......	......	......	......	......	......	......	......	......	......	......	......	St.	......
Life in days	......	1500	......	......	......	......	......	......	......	......	......	......	......	......	1500	......
Life of idlers, days	......	......	......	......	......	......	......	......	......	......	......	......	......	......	......	......
Life of tracks, days	......	1500	......	......	......	......	......	......	......	......	......	......	......	......	1500	......
Life of trough, days	......	No wear	......	......	......	......	......	......	......	......	......	......	......	......	No wear	......
Life of sprockets, days	......	1500	......	......	......	......	......	......	......	......	......	......	......	......	2400	......

Conveyor number	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
Type	Belt	Belt	Belt	Belt	Flight	Belt	Belt	Belt	Belt	Belt	Belt	Belt	Belt	Flight	Flight	Flight
Length c. e. pulleys, ft.	69	24	106	100	92	114	112	114	105	35	105	105	32	15	35	119
Width, in.	18	14	14	14	23	14	14	14	14	11	14	14	14	12	17	17
Rise, ft., in.	5–9	......	......	......	......	......	......	......	......	8–5	......	......	......	7–0	......	......
Slope in degrees	5	......	......	......	......	......	......	......	......	13	......	......	......	25	......	......
Dia. head pulley or sprocket	18	18	24	24	39	24	24	24	24	24	24	24	24	30	39	39
Dia. tail pulley or sprocket.	18	18	18	18	39	18	18	18	18	18	18	18	18	24	39	39
Spacing trough. idlers, ft., in.	4 0	4–0	4–0	4–0	......	4 0	4 0	4–0	4–0	4–0	4 0	4 0	4–0	......	......	......
Spacing return idlers, ft., in.	8–0	8–0	8–0	8–0	......	8–0	8 0	8 0	8 0	8–0	8–0	8–0	8–0	......	......	......
Tonnage carried per hr.	90	10	7	14	14	60	60	4	7	7	7	14	14	3	6	6
Method of driving	Belt	Belt	Belt	Belt	Gear	Chain	Belt	Belt	Belt	Belt	Belt	Belt	Belt	Chain	Chain	Chain

Size mat. carried max., in.	1/10	1/40	1/40	1/40	1/40	1/10	1/10	1/10	1/40	1/40	1/40	1/10	1/40	1/40	1/40	1/40
Percentage moisture carried	......	......	......	......	......	......	......	......	......	......	......	......	......	8	8	......
Speed, ft. per min.	155	94	101	148	50	251	251	94	94	150	94	176	226	50	20	30
Belt, No. of plies	5	4	4	4	......	4	4	3	3	4	5	5	5	4	......	......
Character of plies	32 oz.	32 oz.	32 oz.	32 oz.	......	32 oz.	32 oz.	30 oz.	30 oz.	32 oz.	32 oz.	32 oz.	32 oz.	32 oz.	......	......
Thickness of cover	1/8	1/8	1/8	1/8	......	1/8	1/8	1/8	1/8	1/8	1/8	1/8	1/8	1/8	......	......
Life in days	200	500	2000	600	......	1000	600	2000	2000	1500	1200	600	900	1500	......	......
Screws, flights, pans: Mat.	......	......	......	......	St.	......	......	......	......	......	......	......	......	St.	St.	St.
Life in days	......	......	......	......	1500	......	......	......	......	......	......	......	......	1500	1500	3000
Life of idlers, days	......	......	......	......	......	......	......	......	......	......	......	......	......	......	......	......
Life of tracks, days	......	......	......	......	1500	......	......	......	......	......	......	......	......	......	1500	3000
Life of trough, days	......	......	......	......	No wear	......	......	......	......	......	......	......	......	No wear	No wear	No wear
Life of sprockets, days	......	......	......	......	1500	......	......	......	......	......	......	......	......	......	1500	3000

Conveyor number	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49
Type	Belt	Belt	Belt	Belt	Belt	Belt	Belt	Flight	Flight	Flight	Flight	Flight	Flight	Belt	Flight	Flight	Belt
Length c. c. pulleys, ft.	5	28	125	17	114	122	76	94	59	142	102	133	109	106	14	74	91
Width, in.	16	18	14	14	14	14	14	22	22	18	28	10	10	16	23	23	18
Rise, ft., in.	......	......	......	4-7	13-6	......	20-4	2-9	......	*a*	11-8	*b*	......	35-2	......	......	......
Slope in degrees	......	......	......	15	7	......	15	2	......	*a*	7	*b*		18	......	......	......
Dia. head pulley or sprocket	18	18	20	18	24	18	14	28	28	20	39	16	16	48	39	39	18
Dia. tail pulley or sprocket	18	18	14	18	18	18	18	28	28	20	39	16	16	28	39	39	18
Spacing trough. idlers, ft., in.	4-0	4-0	4-0	4-0	4-0	4-0	4-0	......	......	......	......	......	......	4-0	......	......	4-0
Spacing return idlers, ft., in.	8-0	8-0	8-0	8-0	8-0	8-0	8-0	......	......	......	......	......	......	8-0	......	......	4-8
Tonnage carried per hr.	60	60	5	5	10	10	20	20	20	6	12	2	3	14	17	17	17
Method of driving	Belt	Belt	Belt	Belt	Belt	Belt	Belt	Gear	Chain	Chain	Chain	Chain	Belt	Belt	Chain	Gear	Chain
Size mat. carried max., in.	1/10	1/10	1/10	1/10	1/10	1/10	1/10	1/10	1/10	1/10	1/10	1/10	1/10	1/10	1/10	1/10	1/10
Percentage moisture carried	......	......	......	......	......	......	......	......	......	10	......	30	30	......	......	......	......
Speed, ft. per min.	245	250	274	179	233	264	220	51	51	24	54	30	58	192	50	50	250
Belt, No. of plies	5	5	3	3	3	3	4	......	......	......	......	......	......	4	......	......	5
Character of plies	32 oz.	32 oz.	30 oz.	30 oz.	30 oz.	30 oz.	32 oz.	......	......	......	......	......	......	32 oz.	......	......	32 oz.
Thickness of cover	1/8	1/8	1/8	1/8	1/8	1/8	1/8	......	......	......	......	......	......	1/8	......	......	1/8
Life in days	300	300	300	1200	600	2000	2000	......	......	......	......	......	......	250	......	......	1500
Screws, flights, pans: Mat.	......	......	......	......	......	......	......	St.	St.	St.	St.	St.	St.	......	St.	St.	......
Life in days	......	......	......	......	......	......	......	3000	3000	1000	1000	1000	1000	......	1000	1000	......
Life of idlers, days	......	......	......	......	......	......	......	......	......	......	......	......	......	......	......	......	......
Life of tracks, days	......	......	......	......	......	......	......	3000	3000	1500	1500	1500	1500	......	1500	1500	......
Life of trough, days	......	......	......	......	......	......	......	No wear	No wear	No wear	No wear	No wear	No wear	......	No wear	No wear	......
Life of sprockets, days	......	......	......	......	......	......	......	3000	3000	1500	1500	1500	1500	......	1500	1500	......

a Horizontal until within 28′-0″ of end when it rises 6′-3″. *b* Horizontal until within 9′-0″ of end when it rises 6′-0″.

Table 85. Elevators, N. J. Zinc Co., Franklin mill.

Elevator number	1	2	3	4	5	6	7	8	9	10	11	12
Type	Chain	Chain	Chain	Chain	Chain	Belt	Belt	Belt	Belt	Belt	Belt	Belt
Length c.c. pulleys, ft.	86	40	95	95	49	71	82	54	63	63	63	27
Vertical lift, ft. (a)	86	40	95	95	49	71	82	54	63	63	63	27
Size of bucket, length × width × depth, in.	19×15×23	16×15×10	19×15×23	19×15×23	16×10×17	16×6×6	10×6×6	10×6×6	14×6×6	14×6×6	14×6×6	12×6×6
Spacing of buckets, in.	24	18	24	24	18	18	18	18	18	18	18	18
Dia. head pulley or sprocket	46	29	39	39	29	30	30	30	28	28	28	28
Dia. tail pulley or sprocket.	46	29	39	39	29	28	28	28	28	28	28	26
Material used in housing (b)	St.	St.	St.	St.	St.	St.	St.	St.	St.	St.	St.	St.
Solid tons per hour	100	25	100	200	100	90	10	10	50	45	40	30
Method of driving	Chain	Chain	Gear	Gear	Chain	Chain	Belt	Belt	Belt	Belt	Belt	Belt
Speed in feet per minute	95	63	58	112	153	250	235	337	396	396	396	272
Max. size mat. carried, in.	1¼	¾	⅝	⅝	1/10	1/10	1/10	1/40	1/10	1/10	1/10	1/10
Percentage moisture	......	2	2	......	......	......	......	......	......	......	......	......
Belt: Width, in.	......	......	......	......	......	18	12	12	16	16	16	14
Material	......	......	......	......	......	Rub.	Rub.	Rub.	Rub.	Rub.	Rub.	Rub.
Number of plies	......	......	......	......	......	10	10	6	8	8	8	6
Thickness of cover, in.	......	......	......	......	......	1/16	1/16	1/16	⅛	⅛	⅛	1/16
Life in days	......	......	......	......	......	1000	1000	1500	1000	1000	1000	2000
Chain, material	St.	St.	St.	St.	St.	......	......	......	......	......	......	......
Life in days	1000	1000	1000	1000	1000	......	......	......	......	......	......	......
Buckets, material	St.	St.	St.	St.	St.	St.	St.	St.	St.	St.	St.	St.
Life in days	1000	1000	1000	1000	1000	1000	1000	1500	600	600	600	1000

Elevator number	13	14	15	16	17	18	19	20	21	22	23	24
...e	Belt	Belt	Belt	Belt	Belt	Belt	Belt	Chain	Belt	Belt	Belt	Belt
...th c.c. pulleys, ft.	38	57	53	47	54	43	67	67	30	58	46	46
...cal lift, ft. (a)	38	57	53	47	54	43	67	67	30	58	46	46
... of bucket, length × ...th × depth, in.	8×5×5	12×6×6	8×5×5	8×5×5	8×5×5	16×8×8	10×6×6	10×6×9	14×6×6	12×6×6	12×6×6	12×6×6
...ng of buckets, in.	18	18	18	18	18	18	18	9	18	20	24	24
...ead pulley or sprocket	28	30	28	28	28	36	32	35	30	30	30	30
...il pulley or sprocket.	26	28	26	26	26	24	28	35	24	18	18	18
...al used in housing (b)	St.	St.	St.	St.	St.	St.	St.	St.	Wood	Wood	Wood	Wood
...ons per hour	4	4	7	7	7	3	3	6	60	60	62	54

Method of driving	Belt	Belt	Belt	Belt	Belt	Belt	Belt	Bevel	Belt	Belt	Belt	Belt
Speed in feet per minute	250	264	250	250	240	300	335	260	300	314	314	314
Max. size mat. carried, in.	1/40	1/40	1/40	1/40	1/40	1/40	1/40	1/40	1/10	1/10	1/10	1/10
Percentage moisture	......	......	......	......	......	300	100	......	......	......	......	......
Belt: Width, in.	10	14	10	10	10	18	12	......	16	16	14	14
Material	Rub.	Rub.	Rub.	Rub.	Rub.	Rub.	Rub.	......	Gandy	Gandy	Rub.	Rub.
Number of plies	7	8	8	8	8	10	10	......	10	10	8	8
Thickness of cover, in.	1/16	1/8	1/8	1/8	1/8	1/8	1/8	......	......	......	1/8	1/8
Life in days	2000	1200	1000	1000	1000	300	60	......	200	700	900	900
Chain, material	......	......	......	......	......	......	......	St.	......	......	......	......
Life in days	......	......	......	......	......	......	......	1000	......	......	......	......
Buckets, material	St.	St.	St.	St.	St.	St.	St.	St.	St.	St.	St.	St.
Life in days	1000	600	600	600	600	300	300	1000	200	700	300	300

Elevator number	25	26	27	28	29	30	31	32	33	34	35	36
Type	Belt	Belt	Belt	Belt	Belt	Belt	Belt	Chain	Belt	Belt	Belt	Belt
Length c.c. pulleys, ft	46	40	46	46	43	43	52	50	52	28	60	70
Vertical lift, ft. (a)	46	40	46	46	43	43	52	50	52	28	60	70
Size of bucket, length × width × depth, in.	10×6×6	10×6×6	10×6×6	10×6×6	10×6×6	10×6×6	12×6×6	10×17×10	8×5×5	12×6×6	8×5×5	16×6×6
Spacing of buckets in.	24	24	24	24	18	18	19	18	18	20	18	18
Dia. head pulley or sprocket	30	30	30	30	28	30	30	29	28	30	30	35
Dia. tail pulley or sprocket	18	18	20	24	24	24	18	21	26	18	24	30
Material used in housing (b)	Wood	Wood	Wood	Wood	Wood	Wood	Wood	Steel	Wood	Wood	Steel	Steel
Solid tons per hour	47	38	30	20	5	10	6	6	3	14	25	17
Method of driving	Belt	Belt	Belt	Belt	Belt	Belt	Belt	Bevel	Belt	Belt	Chain	......
Speed in feet per minute	314	314	314	314	291	365	165	91	200	212	270	275
Max. size mat. carried, in.	1/10	1/10	1/10	1/10	1/10	1/10	1/10	1/10	1/10	1/10	1/10	1/40
Percentage moisture	......	......	......	......	......	......	20	......	20	40	......	......
Belt: Width, in.	12	12	12	12	12	12	14	......	10	14	10	18
Material	Gandy	Gandy	Gandy	Rub.	Rub.	Gandy	Rub.	......	Rub.	Rub.	Rub.	Rub.
Number of plies	10	10	10	8	8	10	8	......	8	8	8	10
Thickness of cover, in.	......	......	......	1/8	1/8	......	1/8	......	1/8	1/8	1/8	1/8
Life in days	600	600	600	900	900	3000	900	......	900	900	900	900
Chain, material	......	......	......	......	......	......	......	St.	......	......	......	......
Life in days	......	......	......	......	......	......	......	1000	......	......	......	......
Buckets, material	St.	St.	St.	St.	St.	St.	St.	St.	St.	St.	St.	St.
Life in days	300	300	300	300	600	600	600	600	600	600	600	600

a All vertical. *b* 3-in. bucket clearance in all.

Notes to Fig. 109.

a, ¼ r.p.m. *c,* 30 ft. per min., 6 pickers, 1½ tons waste removed per man per hr. = about 4 per cent. of total feed; 3- to 18-in. lumps. Good natural light. *d,* See Table 84. *e,* See Sec. 18, Art. 3. Feed, 3 per cent. moisture. *f,* See Table 85. *g,* 1.5 r.p.m. *h,* ⅔ r.p.m. *i,* See Sec. 5, Table 23. *j,* See Sec. 3, Table 22. *k,* See U. S. patents 1,446,375 to 1,446,378, incl., and 1,448,514, 5. *l,* With canvas cloth, 10 to 12 per cent. moisture; with woven-wire screen, 4 to 5 per cent. moisture. *m,* Wash water on jigs or dilution water at head of de-colloiding section. *n,* 800 to 1200 tons per mo. 6 to 8 per cent. Zn. *o,* Each screen has 135 sq. ft. of screening area. Elevators Nos. 10 and 11 (Table 84) follow screens Nos. 2 and 4, respectively. Feed passes over successively coarser screens. *p,* All except the 0.088- and 0.055-in. screens have 135 sq. ft. screening surface, these have 41 and 68 sq. ft., respectively. Elevators Nos. 23 to 28 (Table 85) follow 0.031- to 0.076-in. screens inclusive, respectively. Feed passes over successively coarser screens. *q,* 135 sq. ft. of screen area. *r,* See Sec. 9, Table 22. *s,* 58 sq. ft. of screen area. *t,* 11 × 8-ft. Oliver filters with baskets 15 in. deep for holding granular material. 1 rev. in 27 min. *u,* 8 ft. wide, 24 strokes per min., 2½ in. per ft. slope. *v,* 12 strokes per min., 2 in. per ft. slope. *w,* Rakes, 12 strokes per min.; bowl mechanism, 2 r.p.m.; slope, 2 in. per ft. *x,* 4-ft. bowls on 2-ft. simplex tanks. Stirrer speed, 8 r.p.m.; rake speed, 16 @ 12-in. strokes per min.

American Smelters Securities Co. Fig. 110. (*112 J 1050.*)

Location: Santa Barbara, Chihuahua, Mex.

Ore: Lead carbonate with small amounts of gold and silver; silicious gangue with some lime.

Capacity: 500 tons per 24 hr.

Summary. Table concentration after careful hydraulic classification. Compare with Shattuck-Arizona.

Shattuck-Arizona Copper Co. Fig. 111. (*Q; 110 J 759.*)

Location: Bisbee, Ariz.

Ore: Cerussite, cerargyrite, gold and a small amount of lead sulphate (anglesite) in a silicious gangue with some specular hematite and limonite. Sulphur about 0.1 per cent.

Capacity: 400 tons per 24 hr. 500 tons has been handled with but little loss in efficiency.

Assays:

	Au, oz.	Ag, oz.	Pb, per cent.	Fe, per cent.	Insol., per cent.
Feed	0.059	7.22	6.27	13.73	65.99
Concentrate	0.227	28.53	32.92	20.6	22.3
Tailing	0.022	2.52	0.43	....	....

Recovery, per cent.: Au, 69.4; Ag, 71.3; Pb, 94.4.

Ratio of concentration: 5.56 : 1.

Labor: Tons per man-day (24 hr.), operating, 25; repairs, 36.4; total, including outside surface men, 9.1.

Power: 24 to 32 hp.-hr. per ton milled.

Water: 2 to 3 tons fresh water per ton milled.

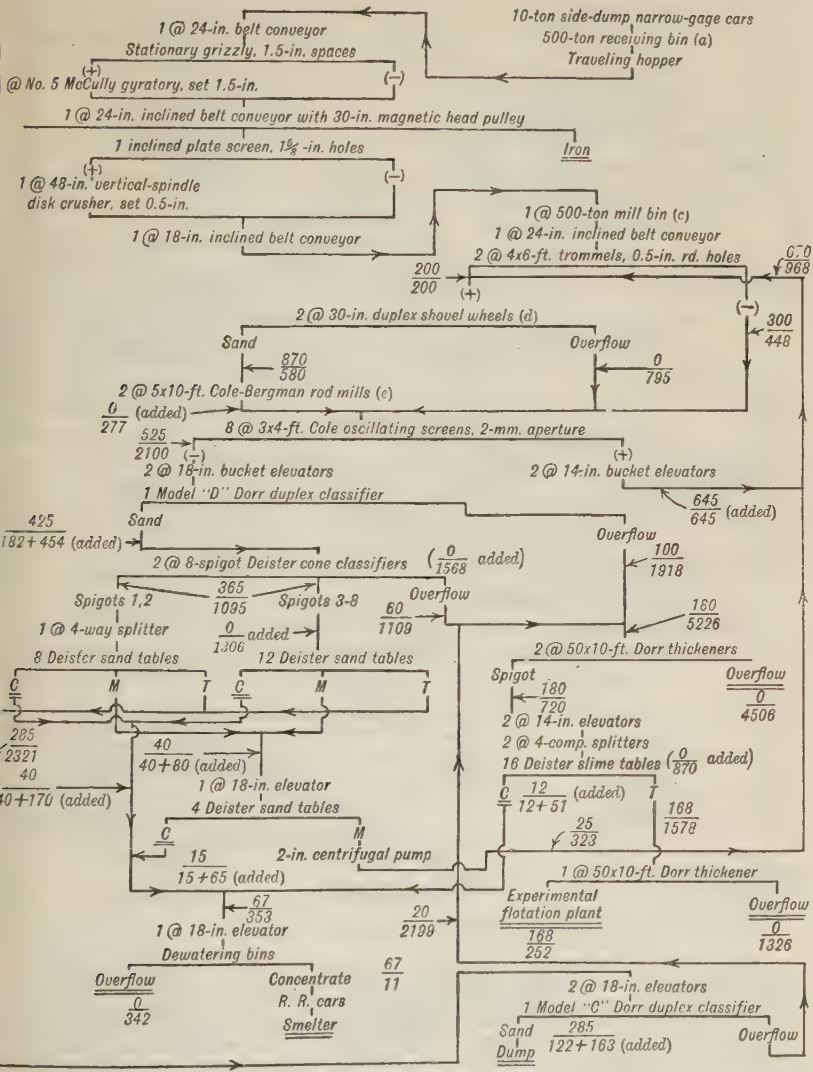
Running time: 76 per cent. of possible. Stoppages due to lack of ore, water and power and to repairs.

Distances: Mine to mill, 3½ miles; mill to smelter, 250 miles; power from a power company; water from adjacent mine and from city (Naco) supply.

General: Sloping mill site with crushers at track level and inclined conveyor to top of mill bin. (See photograph *110 J 760.*)

Summary. Crushing: From run-of-mine to 35-mesh as follows: Gyratory from 8- to 1.5-in.; rolls from 2- to 0.5-in.; ball mills and ball-tube mills respectively grinding −⅝-in. + 4-mesh and −4-mesh to 35-mesh. Concentration: One-stage tabling at −4-mesh, unclassified, and 2-stage tabling at −35-mesh, de-slimed; sulphidizing flotation of slimes by concentrate-middling routing.

This is simple, efficient and comparatively cheap treatment of a difficult ore. It depends for its success on quick and effective surface sulphidizing of the finely ground oxidized



a, Flat bottom, concrete. 8 bottom outlets. *c*, Redwood tank, 30 ft. (diam.) × 24 ft. with hopper and rack-and-pinion gate at center of bottom. *d*, See Sec. 16, Art. 2. *e*, 19.8 r.p.m., 100-hp. motor, Lenix drive.

Fig. 110.—American Smelters Securities Co.

lead and silver minerals. Few oxidized lead-silver ores yield so readily to sulphidizing treatment and, failing this, gravity concentration of the slimes is expensive and inefficient. Another apparently successful method of treatment is practiced at Chief Consolidated Mining Co. (Fig. 113).

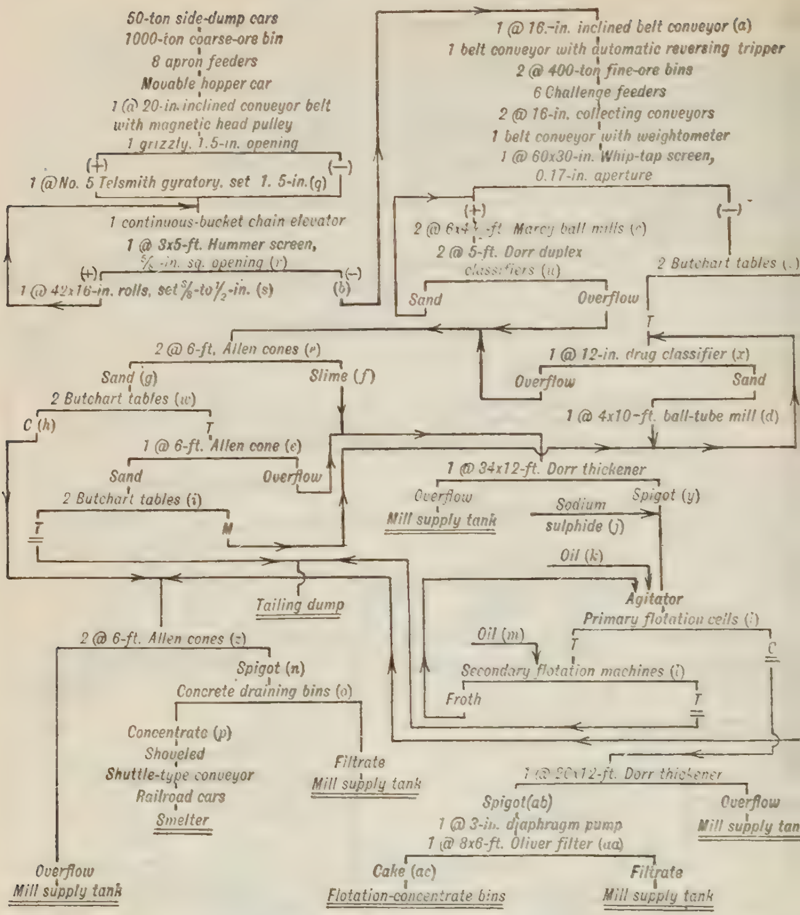


FIG. 111.—Shattuck-Arizona Copper Co.

a, $+21\frac{1}{3}°$. *b*, 20 per cent. +3-mesh, 75 per cent. −48-mesh (cumulative), 14 per cent. −200-mesh. *c*, See Sec. 4, Table 5. *d*, See Sec. 4, Table 4. *e*, See Sec. 6, Table 34 *f*, Approx. 90 per cent. −200-mesh. *g*, Approx. 10 per cent. +48-mesh, 16 per cent −200-mesh. *h*, Approx. 30 per cent. −200-mesh. *i*, To save last of gold and silver values *j*, Weak solution of ordinary commercial, 60-per cent. Na_2S, fused, to film the lead carbonate with sulphide; 3 to 4 lb. per ton. *k*, Mixture of 60 per cent. Cleveland Cliffs hardwood creosote and any good coal-tar creosotes such as Barrett Nos. 4, 6, or 600, Lewis oil blast-furnace oil, etc. Total primary and secondary (*m*) oil, 0.6 to 1 lb. per ton. Occasionally a small amount of steam-distilled pine oil (G. N. S. No 5) or eucalyptus (amygdalena) oil. Disk feeder. *l*, 3 @ 10-ft. K. and K machines and 1 @ 10-ft. Rork machine Temp. of mill water, Feb. 18, 1920, was 102° F. *m*, Oil tar, 57 per cent; Pensacola Tar and Turpentine Co, No. 350, 43 per cent. Added as needed *n*, 21 per cent moisture *o*, Sand-filter bottom. *p*, 10 per cent. moisture. *q*, See Sec. 3, Table 16. *r*, See Sec. 5 Table 38. *s*, See Sec. 3, Table 22. *u*, See Sec. 6, Table 40 *v*, See Sec. 10, Art. 4 Feed about 15 per cent. −200-mesh. Average recovery, Au, 45 per cent; Ag, 35 per cent. Pb, 70 per cent. *w*, See Sec. 10, Art. 4.

x, See Sec. 6, Table 52. *y*, 30 per cent. solids. *z*, See Sec. 6, Table 34. *aa*, See Sec. 17, Table 3. *ab*, 40 per cent. moisture. *ac*, 16 per cent. moisture.

Assays:

	Au, oz.	Ag, oz.	Pb, per cent.	Fe, per cent.	Insol., per cent.
Feed	0.04	5.8	2.9	8.0	82.0
Concentrate	0.22	21.0	15.6	20.4	47.8
Tailing	0.02	3.1	0.6	....	....

Recovery, per cent.: Au, 55; Ag, 55; Pb, 83.

Royal Asturiana Mining Co. Fig. 112. (*115 J 395.*)

Location: Reocin, Spain.

Ore: Sphalerite, 15 to 30 per cent.; calamine, 1 to 4 per cent.; galena, 1 to 7 per cent.; iron as oxide and sulphide, 3 to 7 per cent.; gangue practically all dolomite.

Capacity: Crushing plant, 800 tons per 24 hr.; concentrator, 200 tons per 24 hr.

Labor: 11 tons per man-shift, operating.

Power: 24 hp.-hr. per ton.

Water: 7 tons per ton milled. A large part is returned.

Summary. Tables, flotation, tabling of flotation concentrate. CRUSHING: Jaw crusher from 13- to 3-in.; jaw crusher from 3- to 1-in.; ball mill in closed circuit with screen from 1.5-in. to 0.08-in.; ball mill in closed circuit with Dorr classifier from 2-mm. to 0.2-mm. CONCENTRATION: Tables treating de-slimed −2-mm. feed and making lead and zinc concentrate and a tailing for re-grinding and flotation; collective flotation (concentrate-middling routing) with separation of lead and zinc in concentrate by shaking tables.

This flow-sheet was adopted in 1918–19 after careful tests of all-flotation treatment. The combined treatment yielded higher lead recovery, and, when zinc oxides were present, more zinc.

Chief Consolidated Mining Co. Fig. 113. (73 A 200.)

Location: Eureka, Utah.

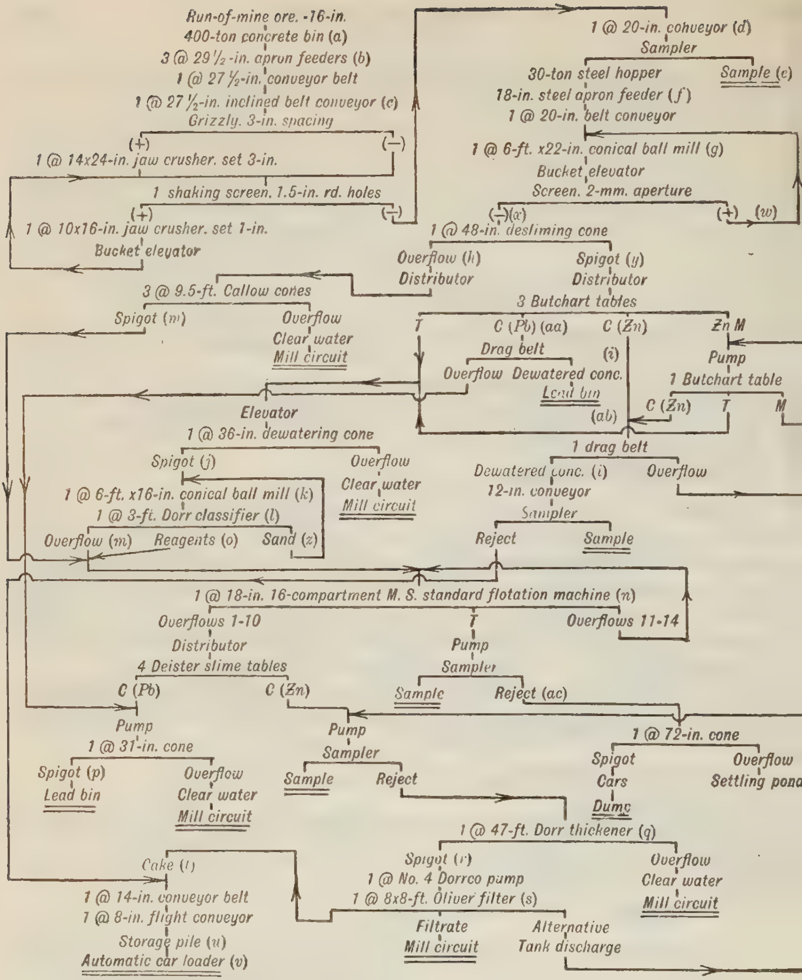
Ore: Silicious replacements in limestone, carrying gold and sulphide and oxide minerals of lead, silver, iron and zinc.

Capacity: Volatilization plant, 250 tons per 24 hr.; concentrator, 300 tons per 24 hr.; crushing plant, 65 tons per hr.

Assays:

	Au, oz.	Ag, oz.	Pb, per cent.	Zn, per cent.	Fe, per cent.	Insol., per cent.	S, per cent.	CaO, per cent.
EXPERIMENTAL-MILL RESULTS								
Ore	0.055	25.2	6.0	4.6	7.0	63.8	4.8	2.6
Concentrate	0.23	172.6	39.8	7.9	11.3	11.6	15.2	1.0
Tailing (furnace feed)	0.044	15.9	3.9	....	....	....	....	..
Fume (furnace conc.)	0.18	60.5	14.0	4.0	5.3	46.4	1.8	4.0
Calcine (furnace tailing)	0.005	1.6	0.12	0.0	5.0	89.2	0.1	1.4
VOLATILIZATION-PLANT RESULTS								
Kiln feed	0.035	11.4	4.7	....	....	....	....	
Fume	0.30	64.0	27.8	....	...	..	.	.
Calcine	Tr.	1.2	0.2	...	....	.	....	

Summary. CRUSHING: Gyratory from 12- to 2.5-in. and rolls from 2.5- to 1-in. in open circuit. Two-stage ball milling to 20- and 65-mesh respectively, each mill in closed circuit with its own classifier. CONCENTRATION: Wilfley tables on 20-mesh unclassified feed making concentrate and middling, latter

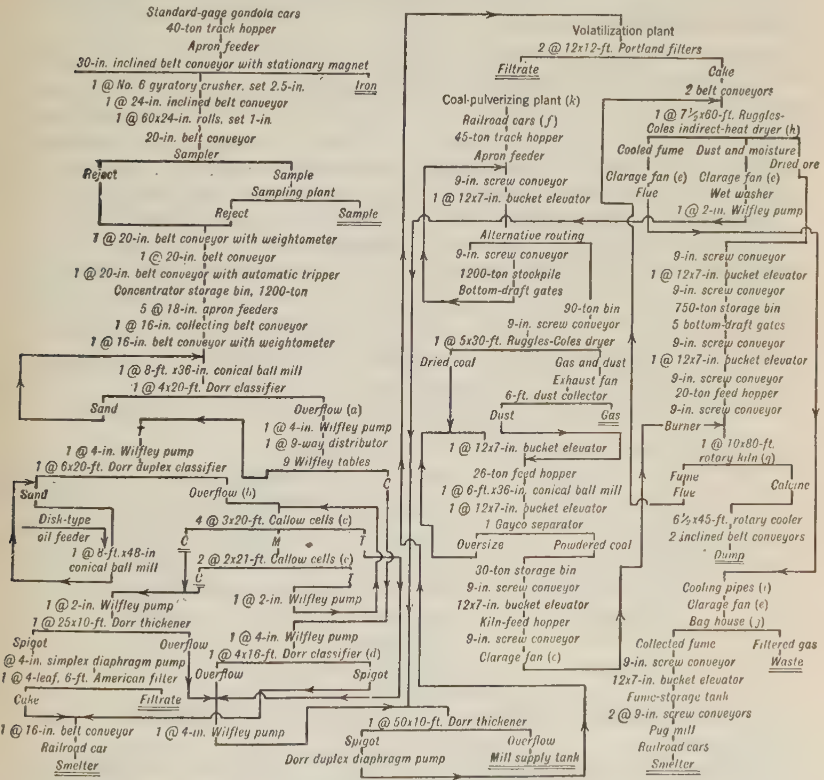


a, Three 24 × 24-in. gates. *b*, 20 ft. per min. Capacity, 25 tons per hr. *c*, +20°. *d*, 108 ft. long, +21°. *e*, 1 in 2000. *f*, 4 ft. per min. *g*, 5 tons balls, 27 r.p.m., draws 45 hp. 40 per cent. solids in mill. *h*, 65 tons per 24 hr., 0.1 per cent. +100-mesh, 83 per cent. −200-mesh. Feed all −1.5-in., 12 per cent. −0.08-in. (2-mm.) Capacity falls off badly, if ore contains less than 15 per cent. Zn. *i*, About 15 per cent. water. *j*, 50 per cent. solids. *k*, 9000 lb. @ 2-in. steel balls, 38 hp. *l*, 28 strokes per min. *m*, 25 per cent. solids, 2 per cent. +72-mesh. *n*, 14 frothing cells, 2 mixing cells. *o*, 3.3 lb. coal-tar creosote; aver., 0.13 lb. steam-distilled pine oil (G. N. S. No. 5) added as required; 0.66 lb. copper sulphate. *p*, 25 per cent. moisture. *q*, 6 min. per rev. *r*, 65 per cent. solids. *s*, Operates 10 hr. to 24 hr. for balance of plant. *t*, 10 to 12 per cent. moisture. 51 to 54 per cent. Zn, 2 to 3 per cent. Pb, 7 per cent. dolomite. (Assays on dry weight.) *u*, 100 ft. long, 40 ft. wide. *v*, Barber-Greene. *w*, 30 tons per 24 hr., contains 20 per cent. of −2-mm. sand. *x*, 10 per cent. +1-mm., 22 per cent. −200-mesh. *y*, 135 tons per 24 hr.; 68 per cent. moisture; 19 per cent. +1-mm.; 2 per cent. −200-mesh; 25 per cent. Zn. *z*, 46 per cent. −72-mesh. *aa*, Recovery about 70 per cent. of the lead in the feed. *ab*, 42 to 51 per cent. Zn (depending on amount of pyrite in feed), 2 to 4 per cent. Pb, 9 per cent. dolomite. Recovery about 65 per cent. of the zinc in the feed. *ac*, 1 to 4 per cent. Zn according to oxidized zinc in feed.

FIG. 112.—Royal Asturiana Mining Co.

re-ground and floated in pneumatic cells with combination routing. Flotation tailing dried and volatilized.

This plant is a pioneer in volatilization of lead-silver-gold ores without addition of volatilizing agents. The process is commendable in principle and the results cited are excellent. Mechanically the plant would appear to be very weak in the large number of bucket elevators and screw conveyors and it is a relatively safe prediction that lost time will be high until these are largely replaced by belt conveyors.



a, 20-mesh. *b,* 65-mesh. *c,* Two No. 7 Sturtevant blowers furnish air at 4 lb. pressure. *d,* Vacuum-filter bottom near sand-discharge end. *e,* Model 90. *f,* Raw bituminous coal, − 1-in. *g,* Variable-speed d.-c. motor with storage-battery equipment sufficient to continue operation for a short time in case of power interruption and prevent sticking of hot charge. A water-cooled plow is provided to plow out accretions as necessary. Plowing requires about 20 min. Coal about 30 per cent. of furnace feed. *h,* Hot fume does not come in contact with ore. *i,* 20,000 sq. ft. *j,* 42,000 sq. ft. *k,* Capacity sufficient to supply 75 tons powdered coal per 8 hr.

FIG. 113.—Chief Consolidated Mining Co.

23. Manganese, Mn

Properties. Metal; reddish-white, lustrous, hard, slightly magnetic. (See also Table 1.) AT. WGT., 54.9. Only slightly attacked by air when pure. Dissolves readily in acids forming corresponding bi-valent salts. Forms a great diversity of compounds. Ion is bi-, quadri-, sexa- and septa-valent. The lower oxidation products form bases, the higher, acids. Alloys with other elements.

Uses. Manganese is not used commercially in the free state. Its greatest use is in the steel industry where it is introduced in the form of the iron alloys, ferro-manganese or spiegeleisen. Probably the next most important use of manganese is as the dioxide (MnO_2) in the making of dry batteries. This calls for a very high-grade product containing about 80 per cent. MnO_2 and less than 1 per cent. iron. Other uses of manganese compounds are: oxidizing agent in the manufacture of chlorine, bromine, and disinfectants; dryer in paints and varnishes, coloring agent in calico printing and dyeing, in making glass, pottery, brick and paints. Manganese bronze is an alloy with copper which may or may not contain some iron; silver bronze, an alloy with aluminum, zinc, copper and silver; manganese-titanium, an alloy of these metals with iron for use in the manufacture of special steels.

Ores. The economic minerals are pyrolusite, psilomelane, braunite and wad. The specific gravity of the manganese minerals as mined is 3.5 to 5 and averages between 3.5 and 4. These minerals occur as nodules, lumps, pockets, stringers or lenticular masses irregularly scattered through residual clays and weathered rocks. Domestic deposits are small. The common associates, beside the clay, are limonite, barite, ocher and bauxite. Manganiferous iron, manganiferous silver, and manganiferous zinc ores also occur.

Production. Manganese is used in three forms, viz.: high-grade ore (+35 per cent. Mn) for making ferro-manganese and for chemical uses; manganiferous iron ores, containing 10 to 35 per cent. Mn; and iron ores containing 5 to 10 per cent. Mn. Only the first class is concentrated. Production of this class of ore in the United States is confined to the few states named in Table 86. The United States produces only from 1.5 to 3.5 per cent. of the world production. The principal foreign producers are Brazil, British India, Gold Coast, Russia and Georgia, Cuba, China and Spain.

Table 86. Production of high-grade (+35 per cent. Mn) manganese ores in United States (long tons) (a)

State	1921	1922	1923	1924
Arkansas	728	2,264	3,768	8,400
Colorado	490	....	2,278	b
Georgia	....	....	1,502	1,893
Montana	11,129	9,751	21,916	29,445
Virginia	717	800	987	1,565
Washington				5,000
Others (c)	467	589	1,049	10,012
Total	13,531	13,404	31,500	56,315

a USGS. *b* Included under "Others." *c* Ala., Ariz., Cal., Nev., N. M., Tenn., Utah.

Selling. High-grade domestic ore (concentrate) for ferro-manganese is sold on the basis of the content in long-ton units (22.4 lb. per ton) of metallic manganese with a fixed lower limit as to manganese content, frequently between 45 and 50 per cent., and maximum limits of 8 per cent. silica and 0.2 per cent. phosphorus, with premiums for higher manganese and lower silica and phosphorus. Lime in such concentrate is not undesirable, but too much iron is. The ore should be in lump form and hard enough not to crumble in the blast furnace. Chemical ore should contain from 70 to 90 per cent. MnO_2 (it is the available oxygen that is important), and should be low in lime, iron, copper, nickel, cobalt and arsenic, but may contain more phosphorus than the limit for metallurgical ore. Prices in 1924 ranged from \$0.38 to \$0.46 per long-ton unit for metallurgical ore; in the spring of 1926 it was quoted \$0.42 @ \$0.44. At the same time chemical ore, 82 to 87 per cent. MnO_2 was quoted \$70 to \$80 per ton at New York (*Eng. and Min. Jour.-Pr.*). Demand is increasing and the United States is a large importer.

Treatment. The method of concentrating depends upon the character of the ore deposit. When the ore is nodular, in easily disintegrated clay, and the nodules are high-grade, simple washing by log washer or wash trommel is all that is ordinarily necessary.

At EUREKA CC., Batesville, Ark., the ore is washed in a 30-ft., 2-log washer at 20 r.p.m. The log product is sent to a wash trommel (16-mesh), then sized at ½-in. into coarse and fine shipping grades. The washed product contains about 42.5 Mn.

When silicious and other foreign matter occurs in the nodules, a more complicated plant involving crushing is necessary. The CRIMORA mill, Fig. 114, is typical. When the ore occurs in rock, which must be mined with the manganese minerals, the flow-sheet is commonly graded crushing followed by jigging and tabling. The PHILIPSBURG MINING Co. plant is an ingenious departure.

WASH ORES, to be readily amenable to treatment, should carry better than 3 per cent. of manganese mineral. Leaner material can be handled, if it can be graded up by picking, but in general the wash ores average about 10 per cent. manganese content. Some manganese deposits run as high as 25 to 35 per cent. Mn. The character of the clay is important; if tenacious, capacity and grade of product will be much reduced and tailing loss high.

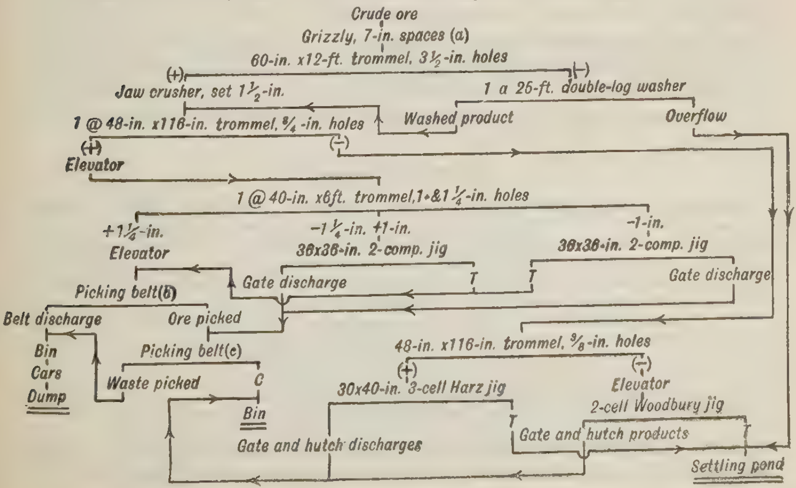
Philipsburg Mining Co. *(Bul. 725 C, USGS 152; Bul. 734 USGS; Bul. 173, USBM 51; 116 J 181.)*

Location: Philipsburg, Mont.
Capacity: 75 to 100 tons per day.
Ore: Nodules of soft pyrolusite and psilomelane in a gangue of quartz, kaolin, calcite and iron oxides.
Assays: Feed, 30 to 40 per cent. MnO_2; concentrate, 72 to 80 per cent. MnO_2; tailing, 10 per cent. MnO_2, upward.

Summary. Graded crushing in jaw crusher and two roll-trommel circuits to 5-mesh. Mechanical classifier to reject −60-mesh material and de-water the sand, which is dried and sized to −5+10-mesh and −10+100-mesh; −100-mesh to dump storage. Wetherill magnetic separators working the two screened grades separately take out a high-iron product on the first pole, which is rejected; make concentrate on the remaining five poles, and reject tailing.

Crimora mill, Virginia. Fig. 114. *(Bul. 173 USBM 48.)*

Ore: High-grade manganese nodules in residual clay and sand; lean.



a, All material sledged through. *b,* 24 in. wide, 22 ft. long. *c,* 30 in. wide, 22 ft. long.

FIG. 114.—Crimora mill.

Summary. Rough concentration in log washers. Log product crushed to about 2-in. and sized into five grades. Coarsest size hand picked; finer sizes jigged separately.

24. Mercury, Hg

Properties. Metal; silver-white, lustrous, liquid under atmospheric pressures at ordinary temperatures. (See also Table 1.) AT. WGT., 200.6. Not acted upon by dry air at ordinary temperatures. Does not decompose water, but that it is slightly soluble in water at ordinary temperatures without decomposition of the water can be reasoned and has been proven experimentally. Oxidizes when heated in the air and the oxide can be broken down by further heating. Insoluble in hydrochloric and in cold sulphuric acids; dissolves in hot concentrated sulphuric acid and is easily soluble in nitric acid. Ion is of basic character only and may be either mono- or bi-valent. Mercury alloys freely with other metals forming AMALGAMS.

Uses. The drug and chemical trades are the principal consumers. Oxides and fulminates are used in making paints and explosives respectively. Smaller amounts are used in electrical apparatus, gold (amalgamating) mills, scientific instruments (thermometers, barometers, and the like). If the mercury boiler proves practical from a power standpoint, consumption will increase markedly.

Ores. Cinnabar is the only economic mineral. It occurs as veins, disseminations or masses of irregular form, not confined to any special type of rock, although igneous rocks are often found in the vicinity. The common gangue minerals are silica and calcite; pyrite or marcasite is usually, and bitumen often, present.

Production. World production averages about 90,000 flasks (75 lb. each) per year. Of this Italy produces slightly more than 50 per cent., Spain nearly 40 per cent. and the United States between 8 and 10 per cent. The foreign deposits are much higher grade than the domestic, hence competition is overwhelming. The principal domestic mines are in California and Texas.

Price fluctuates widely and rapidly. Average yearly prices per 75-lb. flask at New York in recent years were (*Eng. and Min. Jour.-Pr.*): 1919, $92.15; 1920, $81.12; 1921, $45.46; 1922, $58.95; 1923, $66.70; 1924, $69.76.

Treatment. Ores carrying as low as 0.25 per cent. Hg can be treated by roasting more economically than by concentration followed by roasting. The ore is crushed to a size that will permit oxidation of the cinnabar and vaporization of the metal, ordinarily to 1- or 2-in., dried, and then roasted. At some mines hand sorting precedes crushing. Concentration of ore carrying 0.10 to 0.20 per cent. Hg by gravity methods and flotation has been successful experimentally (*120 P 117; Bul. 222 USBM*) but the domestic deposits are too small to permit large-scale treatment and small-scale treatment is not economical.

25. Molybdenum, Mo

Properties. Metal; silver-white, ductile, malleable, difficultly fusible. Ductility increases above red heat. (See also Table 1.) AT. WGT., 96.0. Unchanged in air at ordinary temperatures, but oxidizes on heating. Not attacked by dilute acids, but is attacked by concentrated sulphuric and nitric acids. Ion enters into combination both as a base and as an acid, is tri-, quadri- and sexa-valent, and the compounds in which the valence is highest are the most stable. Alloys with other metals.

Uses. The principal use is in alloy steels containing from 0.15 to 0.5 per cent. Mo in automobile manufacture. Steels containing up to 3 per cent. Mo, usually containing also one or more of the following: chromium, nickel, cobalt, manganese, tungsten or vanadium, are used for permanent magnets, high-speed tools, stainless steels, etc. Molybdenum is also used as a substitute for platinum in jewelry, dental work and gas-engine ignition. Molybdenum chemicals are used in dyeing, ceramics, and fire-proofing textiles.

Ores. The economic minerals are molybdenite, wulfenite, and molybdite. They occur as irregular masses or disseminations in crystalline rocks, fre-

quently associated with bismuth and tungsten minerals, pyrite, pyrrhotite and magnetite. Ores containing from 0.5 per cent. molybdenum upward may be of commercial grade.

Production. World production since 1918 has fluctuated from a high point of 875 tons in that year through a low of 5.5 tons in 1921 to 230 tons in 1924 (*33 MI 500*). The United States produced more than 60 per cent. of the total in 1924, the balance coming from Norway, Canada and Australia. The United States is the principal consumer. High-grade deposits are small and erratic. The principal domestic deposits are in Arizona, Nevada, California, Colorado, New Mexico, Montana and Washington.

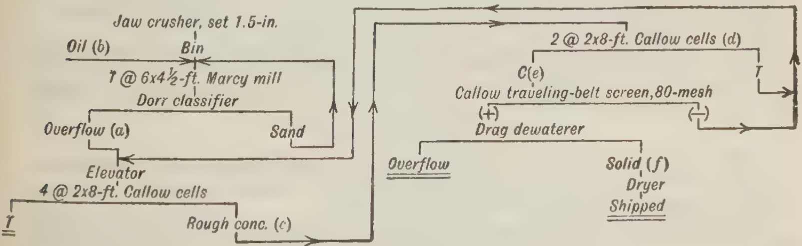
Selling. Concentrate is sold either on the short-ton unit or the percentage basis, reckoned on Mo, MoS_2 or MoO_3. Molybdenite concentrate should contain more than 65 per cent. MoS_2, but concentrate containing 65 per cent. MoS_2 and 25 per cent. pyrite can be successfully converted into ferro-molybdenum (*Spurr and Wormser*). The usual demand is a minimum of 85 per cent. MoS_2 and frequently for 90 to 95 per cent. Arsenic, antimony, barium, bismuth (0.5 per cent.), calcium, copper (0.2 to 0.5 per cent.), phosphorus and tin (0.5 per cent.) are objectionable. PRICES per pound of MoS_2 in 85 per cent. concentrate substantially free from arsenic, bismuth, copper and tin ranged from about $0.40 in 1919 to $0.85 in 1924. Potential production is in excess of present or prospective demand.

Treatment. Wulfenite ores concentrate readily by gravity methods. Molybdenite does not respond to gravity concentration on account of its flaky character and resistance to wetting. Present successful methods of recovery employ flotation. In the past hand sorting, electrostatic separation, air jigging, skin flotation and Elmore vacuum flotation have been used, but none of the plants were highly successful.

Dominion Molybdenite Co. Fig. 115. (*109 J 840.*)

Location: Quyon, Que., Canada.
Ore: Molybdenite and pyrite in quartz diorite. Dissemination, coarse to very fine flakes.
Capacity: 150 tons per 24 hr.
Assays, per cent. MoS_2: Feed, 0.7; concentrate, 90; tailing, trace.
Recovery: 98 per cent.
Ratio of concentration: 125 : 1.

Summary. Flotation (rougher-cleaner routing) with sizing of clean concentrate and rejection of fines. CRUSHING: Jaw crusher and 1-stage ball milling to 40-mesh.



a, −40-mesh. *b*, 0.5 lb. pine oil (G. N. S. No. 5) and 1 lb. kerosene per ton. *c*, 10 to 15 per cent. MoS_2 and about the same in FeS_2. *d*, Sides raised to prevent overflow. Bubble column carried 14 to 18 in. deep. Overflow through 1-in. slot across end. *e*, 60 to 70 per cent. MoS_2. *f*, 85 to 95 per cent. MoS, 2 to 3 per cent. FeS_2.

FIG. 115.—Dominion Molybdenite Co.

It is interesting to compare the relatively coarse flotation feed at this plant with the fine feed (120-mesh) at AMERICAN METAL Co., treating an ore containing granular molybdenite. At the latter plant a somewhat similar flotation operation and 3 or 4 successive cleanings make 65 to 70 per cent. concentrate (MoS_2) and only 82 per cent. sulphide recovery (*61 A 171*). Coghill and Bonardi (*109 J 1210*) working on PRIMOS CHEMICAL Co. ore found that fine grinding flake molybdenum greatly increased tailing loss, and in light of their work the Primos flow-sheet was changed so that 35- or 40-mesh feed was sent to the rougher-flotation machines and the sandy part of the tailing from these was re-ground and re-treated. By this change and a change in oils the tailing was lowered from 0.2 or 0.25 per cent. Mo to 0.12 per cent.

Boykin and Hereford wulfenite mill. Fig. 116. (*Bul. 111, USBM.*)

Location: Mammoth, Ariz.
Ore: Cyanide-plant tailing.
Capacity: 200 tons per day.
Assays: Feed, 1 to 2 per cent. wulfenite; concentrate, 22.6 per cent. MoO_3, 61.8 per cent. PbO.
Ratio of concentration: 100 : 1.

Summary. Gravity concentration by classification and tabling of −4-mesh feed.

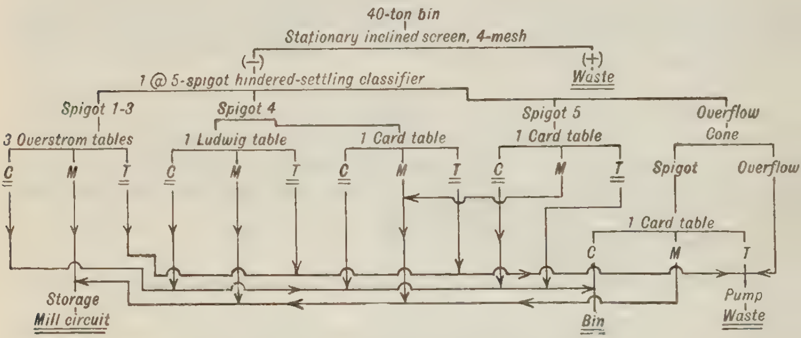


FIG. 116.—Boykin and Hereford wulfenite mill.

26. Monazite

Properties. Monazite is a phosphate of the cerium metals, (Ce, La, Di) PO_4, but is commercially valuable for the content of thoria (ThO_2) which is frequently present in an amount between 5 and 10 per cent. COLOR is red-brown to yellow-brown, greenish or grayish; LUSTER, resinous; HARDNESS, 5 to 5.5; SP. GR., about 5.

Uses. The important use is in the manufacture of thorium nitrate for gas mantles, which are 99 per cent. thoria. Thorium metal is alloyed with tungsten for electric-light filaments. Cerium compounds have a limited use in making a sparking alloy, flaming-arc electrodes, ceramics, dyeing, tanning, etc. Mesothorium, of which minute amounts are obtainable from monazite, is used as a substitute for radium.

Ores are placer deposits in which monazite sand occurs with other heavy minerals such as zircon, rutile, garnet, ilmenite, magnetite, tourmaline and also, of course, with quartz, feldspars and mica.

Production. The principal producing countries are India and Brazil. World production is from 1500 to 3000 tons annually. The principal domestic deposits are in North and South Carolina.

Selling. Concentrate should contain 90 to 95 per cent. monazite and 6 to 9 per cent. ThO_2. PRICE of the Indian mineral was from \$12 to \$15 per unit of thoria in 1924. In the spring of 1926 the quotation (*Eng. and Min. Jour.*-

Pr.) was \$120 per ton for sand containing 6 per cent. ThO_2. Demand is small and potential production in excess.

Treatment consists in rough concentration to reject the lighter sands, followed by careful separation of the heavier sands by gravity concentration, or electrostatic or electromagnetic methods. Sluice concentrate contains 20 to 60 per cent. monazite. When this is dried and closely sized (20-, 50-, 80- and 100-mesh) and treated carefully and by a low-intensity, then a Wetherill high-intensity separator, the products are: low-intensity magnet, magnetite; high intensity; first pole, ilmenite, hematite; second pole, garnet, platinum, epidote, apatite, olivine, tourmaline; third pole, monazite with small amounts of zircon, rutile, epidote, etc.; fourth pole, fine grains of monazite; non-magnetic, gold, zircon, rutile, quartz, feldspar, etc. The gold, zircon and rutile can be separated from the tailing by shaking tables (*TP 110, USBM*; see also *Ladoo*).

27. Nickel, Ni

Properties. Metal; white, lustrous, hard, tenacious, magnetic. (See also Table 1.) AT. WGT., 58.7. Not acted upon by air or water. Attacked with difficulty by hydrochloric and sulphuric acids but readily by nitric acid, forming the corresponding salts in which nickel is a bi-valent base. Tri-valent nickel compounds are unstable and little known. Nickel also forms complex ions, especially with nitrogen compounds as $K_2Ni(CN)_4 \cdot H_2O$. It alloys freely with other metals.

Uses. The principal use is as nickel steel, which is particularly strong, tough and easily machined. Its principal use in the past was for armor plate and projectiles. At present the automotive and steam-engineering industries take large amounts. Nickel is largely used for plating other metals where corrosion is to be resisted. A new important use is in PERMALLOY, used in the manufacture of ocean cable for high-speed transmission.

Ores. The economic minerals are niccolite, millerite, nickeliferous pyrrhotite, pentlandite, garnierite. The two most important deposits of the world are at Sudbury, Ontario, Canada, and on the island of New Caledonia. At Sudbury the ore consists of enormous masses of nickeliferous pyrrhotite segregated at the bottom of a quartz-diorite intrusion, and as scattered, irregular masses in the diorite. The ore contains 1 to 6 per cent. nickel and 1 to 2 per cent. copper. The ore in New Caledonia is garnierite. No nickel mines are worked in the United States, although some few hundred tons of the metal are produced annually as a by-product in the smelting of other ores.

Production. The peak production was 52,000 tons in 1918. In 1922 it was 10,000 tons and in 1924, 38,000 tons (*33 MI 506*).

Price in the spring of 1926 was \$0.35 per lb. for ingot metal (about 98.5 per cent. Ni) to \$0.39 for electrolytic (99.75 per cent. Ni). There is no competition and potential production is greatly in excess of demand.

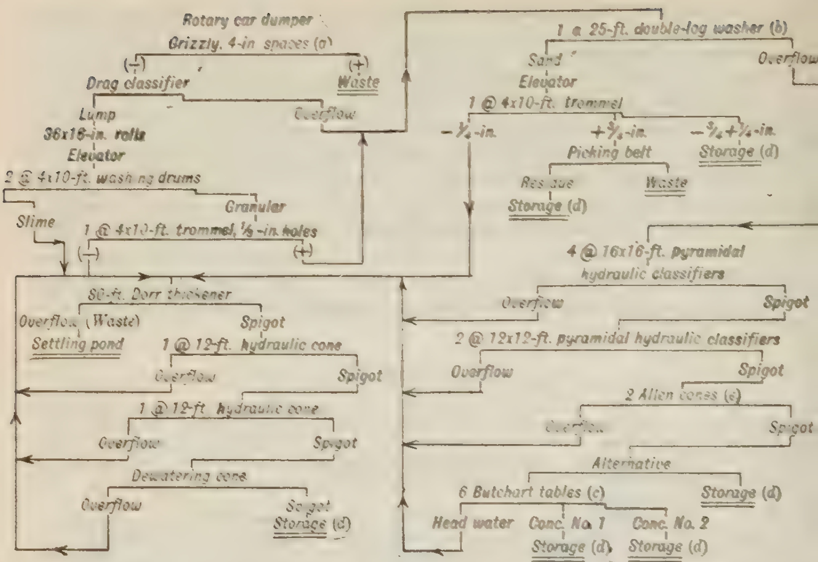
Treatment. Both the Canadian and New Caledonian ores are smelted directly.

28. Phosphate

Properties. Phosphate rock is an amorphous material, principally BONE PHOSPHATE, of the approximate composition $Ca_3P_2O_8$. HARDNESS ranges from 2 to 5. SPECIFIC GRAVITY is from 2.5 to 2.8.

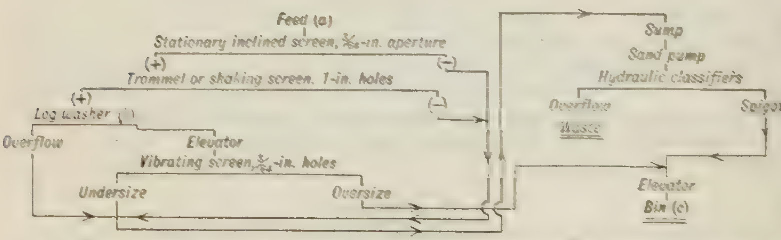
Uses. The principal use is as the raw material for acid-phosphate, super-phosphate and soluble phosphate to form a base for mixed fertilizers. Small quantities are used as a source of phosphorus and in making ferro-phosphorus.

Ores. The principal occurrences are of three types, *viz.:* (1) pebble deposits, in which rounded particles of the phosphate, usually less than 1 in. in size, occur mixed with fine quartz sand and clay in irregular beds under a



a. Organic waste picked. Other material sledged or washed through. *b.* Usually 20 to 30 ft. long, 1 to $1\frac{3}{4}$ in. per ft. slope. *c.* Such separation as is effected depends on the fact that the quartz is smaller than the phosphate and therefore rides higher on the table. *d.* Dried, usually in rotary driers, fed at the cold end, coal- or oil-fired. Usually 4 or 5 by 30 to 50 ft. Capacity, about 100 tons sand or 120 tons rock per 10 hr. from 7 or 8 to 1 or 3 per cent. moisture. Coal consumption, 5 to 6 tons per 10 hr.; power, 5 hp., one man to two driers ([illegible]). *e.* Dorr bowl thickeners have replaced these cones in some washers and saved more fine phosphate.

FIG. 117.—Charleston Mining and Manufacturing Co., phosphate washer (115 J 225).



a. Delivered by centrifugal pump in a pulp containing about 10 per cent. solids. Contains 35 to 50 per cent. −3/64-in., assaying upwards of 68 per cent. B. P. L. and the balance about 35 per cent. B. P. L. *b.* Usually 2-log, 20 to 30 ft. long, 1 to $1\frac{1}{2}$ in. per ft. slope. *c.* See note (*d*), Fig. 117.

Usual capacity, 1500 to 2000 tons per 10 hr. Recovery is usually not over [illegible] per cent. of the phosphate content of the feed.

FIG. 118.—Typical Florida pebble-phosphate washer.

few feet of soil; (2) hard-rock deposits in which the phosphate rock is either in place or in the form of boulders and nodules, in either case mixed with and overlain by residual clays, sand and soil; (3) bedded deposits, *i.e.*, beds of phosphate rock interstratified with sedimentary rocks.

Production. World production was 4,100,000 metric tons in 1919, 7,700,000 in 1924 and in the intervening years ranged between these two extremes. The United States and Tunis together produced about 75 per cent. of this total, each producing about the same amount. Algeria, Morocco, Nauru Island and Ocean Island were the other big producers. The Pacific island deposits are exceptionally high grade (80 to 85 per cent. bone phosphate, B. P. L.). The domestic production comes principally from Florida and Tennessee. The bedded deposits of the western states, Utah, Idaho, Wyoming, and Montana, are enormous (estimated at 6,000,000,000 tons) and readily mined, but cannot compete with the eastern product on account of freight.

Selling. The acid-phosphate market requires a phosphate content (B. P. L. = tri-calcium phosphate) between 70 and 80 per cent. with a maximum of 4.0 to 6.5 per cent. combined $Al_2O_3 + Fe_2O_3$. A usual procedure is to credit 2 per cent. B. P. L. for each per cent. of combined iron oxide plus alumina below an agreed maximum and penalize similarly for excess. Moisture content is usually below 3 per cent. PRICES for domestic consumption, f.o.b. mines, quoted in the spring of 1926 were: Florida, pebble; 76 @ 77 per cent. B. P. L., $5.75 per long ton; 75 per cent., $5; 74 @ 75 per cent., $4.75; 70 per cent. $3.25. Tennessee, rock; lump ($-\frac{3}{4}$-in. screen), base 75 per cent. B. P. L., $6; ground, 95 per cent. −200-mesh, base 65 per cent. B. P. L., $7.25; 70 per cent. base, $8.25 (*J, Mar. 6, 1926*).

Treatment consists in washing in log washers and screen washers. Flowsheets vary somewhat according to whether the ore is pebble phosphate or rock phosphate. See Figs. 117 and 118.

29. Platinum, Pt

Properties. Metal; gray-white, heavy, tough, malleable, ductile, capable of welding at red heat. (See also Table 1.) AT. WGT., 195.2. Not acted upon by air or water, nor appreciably by pure acids. Dissolves rather slowly in AQUA REGIA. Also attacked by fused caustic potash or soda and by phosphorus at red heat. Forms easily-fusible alloys with readily reducible metals. Finely divided platinum occludes enormous quantities of oxygen and hydrogen and exhibits marked catalytic properties, especially in the acceleration of gas reactions. Platinum acts both as a base- and as an acid-forming element, the basic ion having a valence of either two or four.

Uses. Catalytic agent in contact-process sulphuric-acid plants; in jewelry and dental supplies; in scientific apparatus; in incandescent electric-light bulbs. It is being replaced in dental supplies by other alloys and in incandescent lights by alloys possessing the same coefficient of expansion with heat as glass.

Ores. The economic mineral is metallic platinum, usually alloyed with one or more of the elements iron, iridium, rhodium, palladium, osmium and ruthenium. It has usually been recovered from placer deposits near areas of basic igneous rocks; peridotites, pyroxenites, dunites and serpentine. Gravels that carry platinum are usually rich in chromite and olivine. Recently what appears to be a large primary deposit has been found in dunite in South Africa.

Production. World production in 1912 was about 280,000 troy ounces of which 250,000 came from Russia. In 1921, with Russia practically eliminated (5500 oz.) world production dropped to a minimum of 48,000 oz. In 1924 it was 104,500 oz., of which Colombia produced 51,300 oz.; Russia, 40,000;

Canada, 9200; United States, 3500, and Australia, 500. Of the domestic production only 315 oz. was crude mine platinum; of this, 285 oz. was a by-product of California gold dredges.

Prices. The restricted supply has caused an increase in price to an average of $119.09 per troy oz. in 1925. The price in 1908 was $16.32; in 1913, $44.88, and in 1919, $114.61. For a discussion of the South African discovery and the possible effect on prices see *33 MI 583*.

Treatment. Platinum is recovered from gravels in sluices, gold tables and the like (or by pan, batea, rocker, etc., in the case of rich placers) in the same way as placer gold is recovered (see p. 120). It does not amalgamate readily. The platinum that is recovered from gold dredges is caught with black sand and amalgam in the sluices, the amalgam is separated, the residue ground fine and then concentrated in a long tom or rocker until the bulk is small. The final clean-up is made dry by careful and repeated blowing and treatment with a hand magnet.

In refining, the platinum metals are separated. These sell at prices ranging from about $50 per oz. for ruthenium (1924) to as high as $400 per oz. for iridium (1925).

30. Tin, Sn

Properties. Metal; white, lustrous, crystalline, malleable, very ductile at ordinary temperatures; when exposed for a considerable time to low temperatures, the ordinary form changes into a grayish powder of sp. gr. 5.8, known as gray tin. (See also Table 1.) AT. WGT., 118.7. Not acted upon by air or water at ordinary temperatures. When heated in air above the melting point it oxidizes to stannic oxide. At red heat tin decomposes steam with evolution of hydrogen. Tin is attacked by the common mineral acids, both dilute and concentrated, also by strongly basic hydroxides. The ion is both bi- and quadrivalent and acts to form both bases and acids. Tin alloys freely with other metals.

Uses. On account of its resistance to corrosion by air, water and weak acids, tin surfaces are widely used where such corrosive action is to be resisted. Hence tin-plate, which is sheet iron covered with a thin layer of tin, is used for roofs, kitchen utensils and containers for canned goods. Cooking utensils made of copper are tinned to prevent the formation of poisonous copper salts. Important alloys of tin are: soft solder, an alloy of tin and lead which has a lower melting point than either the tin or lead; bronze, bell metal and speculum metal, principally copper and tin; Britannia metal, consisting of tin and antimony, with sometimes copper and zinc; bearing metal and pewter. Lead-tin alloys are used for foil, collapsible tubes and the like. Tin salts are used to weight silks.

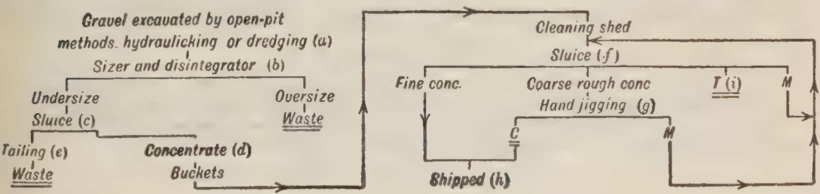
Ores. The principal economic mineral is cassiterite. Stannite is relatively rare. The largest production comes from cassiterite gravels, but primary ores in which cassiterite and sulphides occur in pegmatitic veins are also important.

Production. World production amounts to from 120,000 to 130,000 long tons per year. Of this the placer deposits of the Federated Malay states and of the Dutch East Indies (Banka and Billiton) produce about 50 per cent. and vein deposits in Bolivia 20 to 25 per cent. Other important producers are Australia, China, Nigeria and Siam and, of less importance, Great Britain, Congo, Czecho-Slovakia, India, Unfederated Malay States, and Union of South Africa.

Selling. Tin concentrate should contain 50 to 70 per cent. Sn. Placer-tin concentrate is usually relatively pure. Bolivian concentrate contains impurities such as lead, arsenic, bismuth, antimony, copper, iron and sulphur that are difficult to eliminate and hence highly objectionable. Average yearly PRICES of metallic tin (99 per cent. Sn) at New York were: 44.25 cents per lb. in 1913; 63.33 in 1919; 48.27 in 1920; 28.58 in 1921; 31.83 in 1922; 41.80 in 1923; 49.67 in 1924; and 56.79 in 1925 (*121 J 145*).

Treatment. Cassiterite placers are mined and treated by the usual placer methods. The Bolivian vein deposits are of several different varieties and degrees of complexity. The tin mineral is invariably cassiterite. In the simple oxidized ores the accompanying minerals are principally quartz, feldspars and iron oxides, and separation is simple. The sulphide ores, frequently occurring in the same mines below the oxidized ores, contain, in addition to cassiterite and the gangue minerals, part or all of the following: pyrite, chalcopyrite, bornite, arsenopyrite, wolframite, bismuth minerals, silver minerals, galena, and sphalerite. Many of these ores are of high grade and the cassiterite occurs in relatively coarse particles, but the complexity of the ores makes concentration difficult and the inaccessibility of the district makes changes in methods slow to be effected. In Cornwall the ores are low grade, more or less complex and the cassiterite is very finely dispersed. Many of the mills are ancient, as mills go, owners and operators are highly conservative, and most of the flow-sheets, judged from the viewpoint of American operators, are antiquated.

Placer tin (Fig. 119). The ores (gravels) are typically low-grade, representing tailing from previous placer operations on higher-grade gravels or low-grade ground that was passed over in the earlier work. In the typical flow-sheet coarse material is removed on a screen, a relatively large amount of low-grade concentrate is made in sluices and this concentrate is cleaned up by hand on small film-sizing tables and hand jigs.



a, See *Peele*, Sec. 10, Placer Mining Methods. *b*, On dredges a revolving stone screen with 0.5- to 0.75-in. round holes is ordinarily used here; in hydraulic mining a sluice grizzly and in open-pit mining a disintegrator somewhat resembling a log washer or one consisting of a horizontal pan in which the material is harrowed by stakes depending from revolving radial arms. *c*, 4 to 6 ft. wide, 60 to 300 ft. long; grade, 1 in 24 to 1 in 40, depending upon percentage of moisture in feed pulp. No riffles, but 3 × 2-in. stops are placed across the bottom at 6- to 10-ft. intervals, and as concentrate builds up behind these the height is increased to 9 or 12 in. *d*, Collected at intervals of several hours to a week (depending upon rate of accumulation) by shutting off feed, turning in a small stream of clear water, and shoveling over carefully. (See Sec. 8, Art. 11.) The crude concentrate finally collected contains from 10 to 50 per cent. cassiterite, the balance being black sand, ilmenite, pyrite, etc. *e*, Probably contains 10 to 15 per cent. of the tin in the feed. *f*, 10 to 12 ft. long, 9 to 10 in. deep, converging from 3 or 5 ft. at the head end to about 15 in. at the discharge end. Water supplied full width at the head end over a weir board 6 to 7 in. high. Operated first with a fairly strong stream of water and hoeing of the material to remove the bulk of the sand tailing. The residue is then washed with a weak stream and the coarse material collected at the lower end. The remainder is finally washed again with a heavier stream and at the same time brushed up-slope, yielding fine concentrate and middling. *g*, Tin-plate sieve, 12 in. diam., 3 in. deep, 60- to 80-mesh apertures. *h*, Assays 70 to 76.5 per cent. Sn. *i*, About 0.1 per cent. Sn.

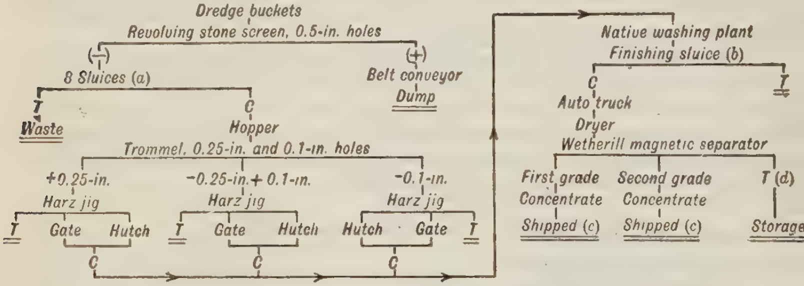
Fig. 119.—Typical Malay placer-tin plant. (*15 MM 327*; *34 IMM 357*).

Summary. No crushing. Concentration by 2-stage sluicing and hand jigging of coarse finishing-sluice concentrate.

On two of the Malay dredges of the YUKON GOLD CO., screen undersize is sent to a specially designed 4-compartment hydraulic spitzkasten, 6 × 18 ft. with a strong horizontal stream of hydraulic water flowing under the main pulp stream. Overflow goes to waste and the spigot products are fed into four 4-compartment Harz jigs (36 × 48-in. sieves) bedded with 0.5-in. lumps of tin or iron ore and making hutch concentrate. Products of the first two hutches of the three coarse jigs are sent to a native cleaning shed and sluiced as described above; products from third and fourth compartments go to a high-speed jig that treats the fourth-spigot product of the classifier and makes finished concentrate and tailing. Recovery of 97 to 98 per cent. of the tin in the feed is claimed, but at the expenditure of 30 to 40 hp. additional, as compared to the ordinary sluice, and with the requirement of specially skilled labor and closer supervision.

Portuguese American Tin Co. Fig. 120. *(104 J 1109.)*

Location: Guia, Portugal.
Ore: Placer tin; cassiterite in gravel.
Capacity: 40 to 50 cu. yd. per hr.
Power: 25 hp. for jigs and screens; 85 hp. for pumps.



a, 13 in. wide, 6 in. deep, 80 to 100 ft. long. Transverse iron riffles, 1⅝ in. deep and 2½ in. apart. Six sluices constantly in use while the other two are being cleaned up. *b,* Narrow, deep launder with slight slope and strong water current. Material hoed over and up-stream. *c,* Averages 68 per cent. Sn. *d,* Principally ilmenite.

FIG. 120.—Portuguese American Tin Co.

Simple low-grade tin ores

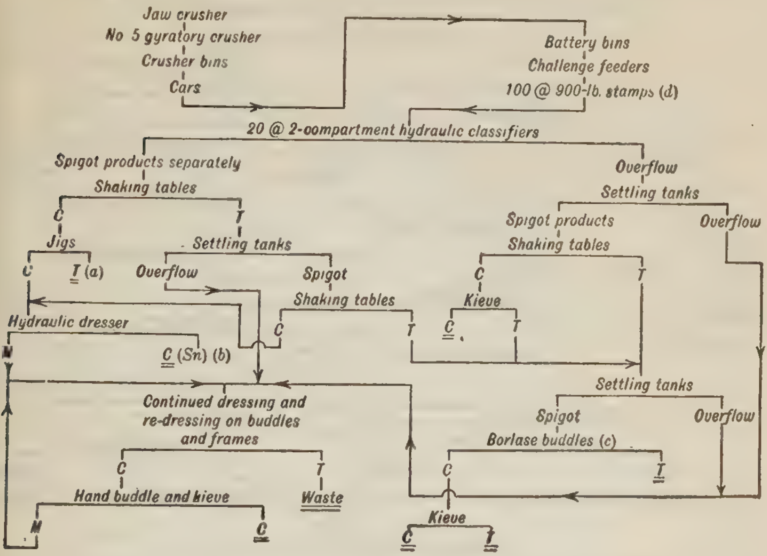
The method of treatment is typified by the flow-sheet of the ANCHOR mine and the primary machines in the EAST POOL mill. It consists in preliminary fine grinding (−0.5-mm.) followed by repeated passage of the tin-bearing pulp over slime-gravity concentrators with re-treatment of the rough concentrate to raise the grade, middling being returned to the circuit. It is a surprising element of these flow-sheets that no re-grinding of tailing is practiced, notwithstanding the usually very fine dissemination of the cassiterite, with the result that substantially no tin not freed in the primary grinding is saved.

Anchor mine. Fig. 121. *(109 P 65.)*

Location: Lottah, Tasmania.
Ore: Low grade. Cassiterite in granite. More than half the cassiterite will pass 200-mesh.
Capacity: About 300 tons per 24 hr.
Assays, per cent. Sn: Feed, 0.10 to 0.16; concentrate, 68 to 73; tailing, 0.025 to 0.03.
Recovery: 70 to 75 per cent.
Ratio of concentration: 550 : 1.

Summary. Gravity concentration by repeated roughing of roughly classified products with cleaning and re-cleaning of concentrate and re-treatment of middling without re-grinding.

This is an exceptionally low-grade ore and the repeated re-treatment could only be justified economically with a valuable metal. As it is, the lower limit of tin assay in the feed (0.10 per cent.) is just about the lower economic limit.



a, Topaz, heavy sand, iron, etc. *b,* 73 per cent. Sn. All other concentrate worked up to 68 to 70 per cent. *c,* Building type. *d,* 0.054-in. Ton-cap.

Fig. 121.—Anchor mine.

Complex tin ores

There are four types, *viz.:* (1) Bolivian sulphide ores, low in lead and silver and containing no other minerals of economic value (see Avicaya mill and Compañia Estañifera de Llallagua); (2) the same, but high in silver and usually containing economic amounts of gold, lead and copper (Socavon de Oruro); (3) Cornish ores, carrying wolfram and arsenic in economic quantities in addition to tin (East Pool); and (4) ores containing tin, lead and wolfram in economic quantities (Butler mine). The summaries of the following flow-sheets set forth the typical features.

Avicaya mill. Fig. 122. *(114 P 774; 115 P 343.)*

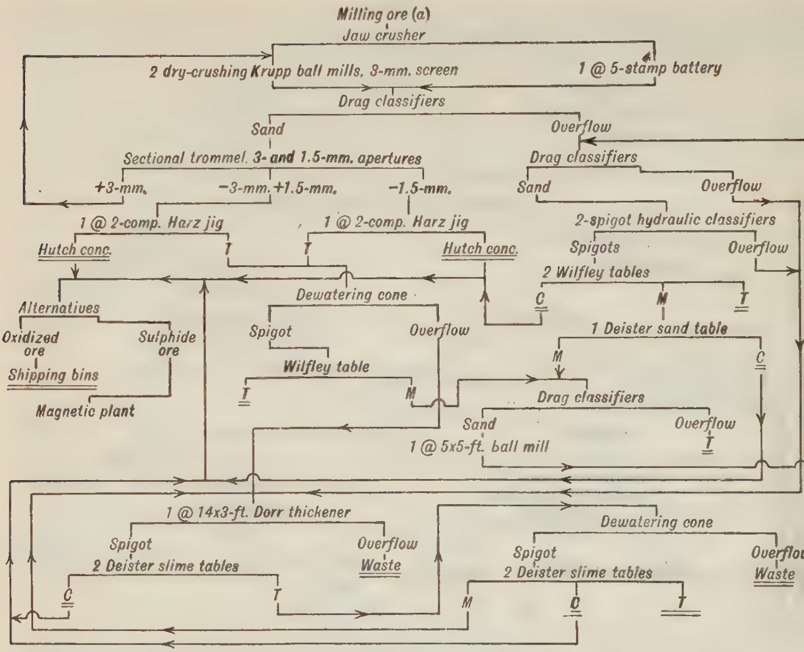
Location: Pazna district, Bolivia.

Ore: Oxidized: cassiterite in a silicious gangue with some iron oxides. Sulphide ore: cassiterite and pyrite in quartz with minor amounts of chalcopyrite and other sulphides and some feldspar and tourmaline.

Capacity: 50 tons per day.

Assays, per cent. Sn: Feed, 5; concentrate (oxidized ores), 60 to 65; sulphides, 50 to 55; tailing, −1.

Summary. Step gravity concentration beginning at 3-mm. with repeated treatment of fine sand making at each step a concentrate for magnetic treatment and a middling for re-grinding or re-concentration, according to fineness. Magnetic treatment is unnecessary with oxidized ores.



a, Comprises upwards of 50 per cent. of the mine product, the balance, both waste and lump cassiterite, having been cobbed out on sorting floors.

FIG. 122.—Avicaya mill.

Compañia Estañifera de Llallagua, magnetic plant. Fig. 123. *(100 J 513.)*

Location: Llallagua, Bolivia.

Ore: Cassiterite-arsenical pyrite concentrate from gravity-concentration mill. (For type of gravity-concentration mill, see above.)

Capacity: 10 tons per 24 hr.

Assays:

	Sn, per cent.	Fe, per cent.	S, per cent.
Feed	31.80	24.56	25.00
Concentrate (Table 87)	68.05	2.90	2.09
Tailing	3.0	....	....

Recovery: 93 to 95 per cent.

Ratio of concentration: about 2 : 1.

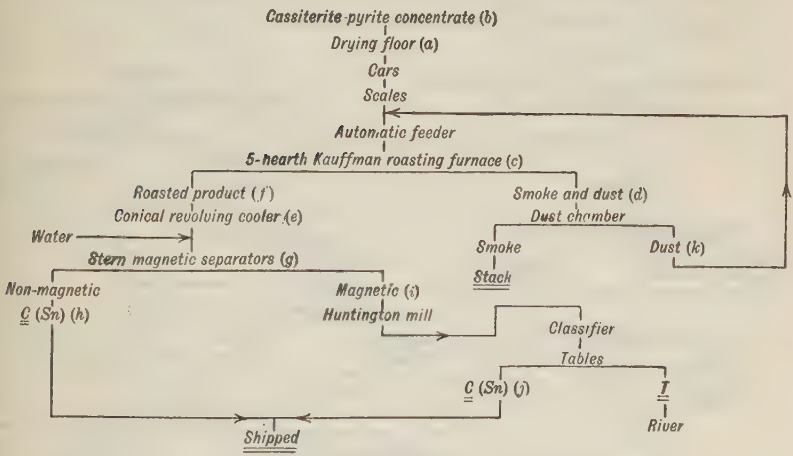
Cost (3 mo., 1914) per ton milled was $1.94 to $2.65.

Summary. Roasting to render pyrite magnetic, followed by magnetic separation of iron- from tin-bearing minerals.

At some plants the iron is "flash-roasted" only.

Table 87. Analysis of Llallagua tin concentrate. (*After Copeland and Hollister*)

Material	Per cent.
Tin oxide (68.32 per cent. Sn)	86.670
Fe	4.746
Ni	0.023
Cu	0.060
Bi	0.360
Pb	0.070
Zn	0.225
Ag	0.006
TiO_2	0.400
WO_3	0.560
CaO	0.130
SiO_2	2.240
S	3.586
O and loss	0.924
	100.00



a, Concrete floor on which material is hand raked until it contains about 5 per cent. moisture. *b,* Mostly smaller than 1-mm. *c,* Similar to McDougall furnace. Roasted until sulphur content is lowered from 25 or 27 per cent. to 10 or 12 per cent. Burning sulphur furnishes necessary heat. Properly roasted material appears black with metallic luster. Reddish, over-roasted material causes poor work on the magnetic separators and too much iron in concentrate. *d,* Less than 5 per cent. of furnace feed. *e,* Similar to short conical trommel, but with un-perforated jacket. Serves also to equalize feed rate to separators. *f,* Assay: 33.3 per cent. Sn, 12.76 per cent. S, 29.3 per cent. Fe. *g,* Draws about 6 amp. at 110 volts. 14 r.p.m. About 18 gal. per min. required to feed and remove concentrate from one separator. *h,* Assay, 68.05 per cent. Sn. Separator recovery = 93 to 95 per cent., equivalent to +90 per cent. of tin entering furnace. *i,* Assay: 3.8 per cent. Sn, 21.22 per cent. S, 50.07 per cent. Fe. *j,* Table recovery about 70 per cent., equivalent to 3.5 to 5 per cent. of tin entering furnace. *k,* Rarely over 5 per cent. Sn.

FIG. 123.—Compañia Estañifera de Llallagua, magnetic plant.

Socavon de Oruro. Fig. 124. (*6 MM 301.*)

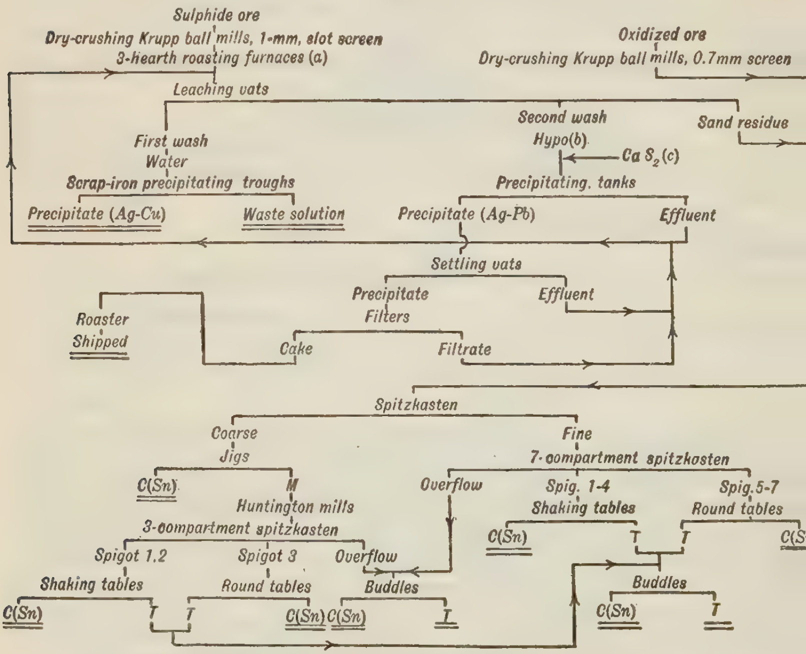
Location: Machacamarca, Bolivia.

Ore: Complex sulphide and oxide ores containing tin, silver, gold, lead, copper, iron, antimony and arsenic in a silicious gangue.

Assays:

	Ag, oz.	Au, oz.	Sn, per cent.	Pb, per cent.	Cu, per cent.
Feed	39	....	3–10	....	....
Ag-Cu precipitate	250	....	....	....	50
Ag-Pb precipitate	35	4	....	23	2
Concentrate (Sn):					
Jig and sand table	....	....	+60	....	....
Slime table	....	....	30–60	....	....
Buddle	....	....	−30	....	....

Summary. Two-stage leaching to recover silver, gold, copper and lead followed by gravity concentration of leaching-plant tailing. Crushing: The notable feature is dry-crushing in screen-discharge ball mills. This is typical of what is probably the best practice in relatively fine granulation for roasting. The Huntington mills would be replaced in more modern practice by a revolving cylinder mill (rod, ball or tube).



a, Sulphur reduced from 30 to 3 per cent. About 4 per cent. sodium chloride added on last hearth. *b*, 0.5 per cent. sodium hyposulphite. 10 to 12 days. *c*, Made on the ground by boiling together sulphur and lime, both obtained locally.

Fig. 124.—Socavon de Oruro.

East Pool mill. Fig. 125. *(6 MM 115.)*

Location: Cornwall, Wales.
Ore: Complex mixture containing cassiterite, wolframite, arsenopyrite and chalcopyrite in silicious gangue. Cassiterite finely disseminated.
Assays: Feed, 0.5 to 1 per cent. Sn, about the same amount of As_2O_3 and half as much WO_3.
Recovery: 60 to 70 per cent. Sn.

Summary. Gravity concentration to reject rocky tailing; roasting to recover arsenic and render iron magnetic; leaching to remove soluble iron, and two-stage magnetic separation to separate tin and wolfram.

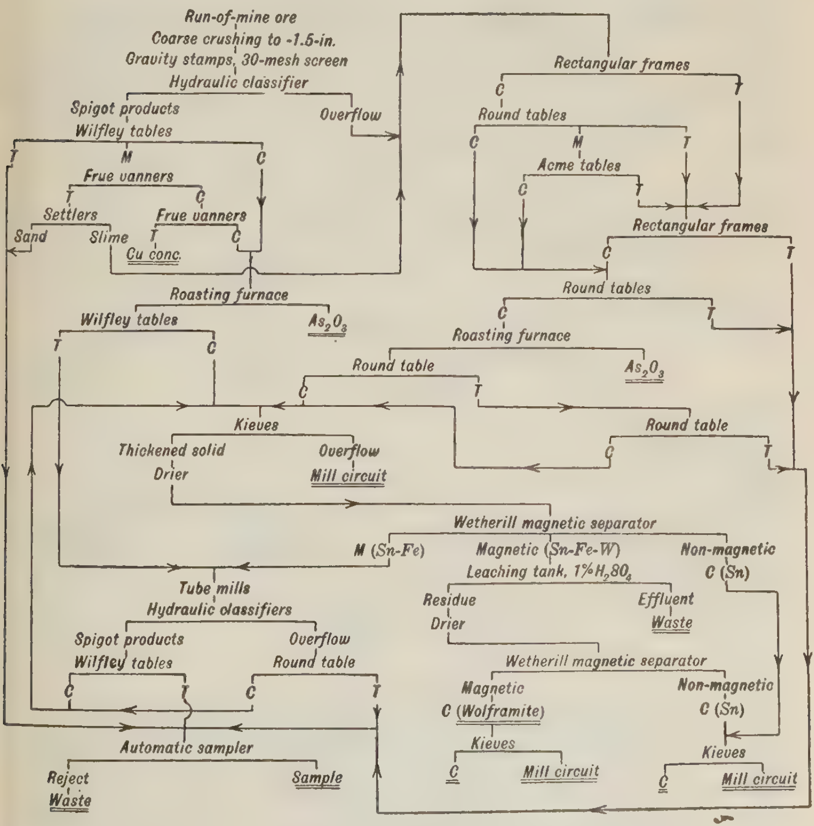
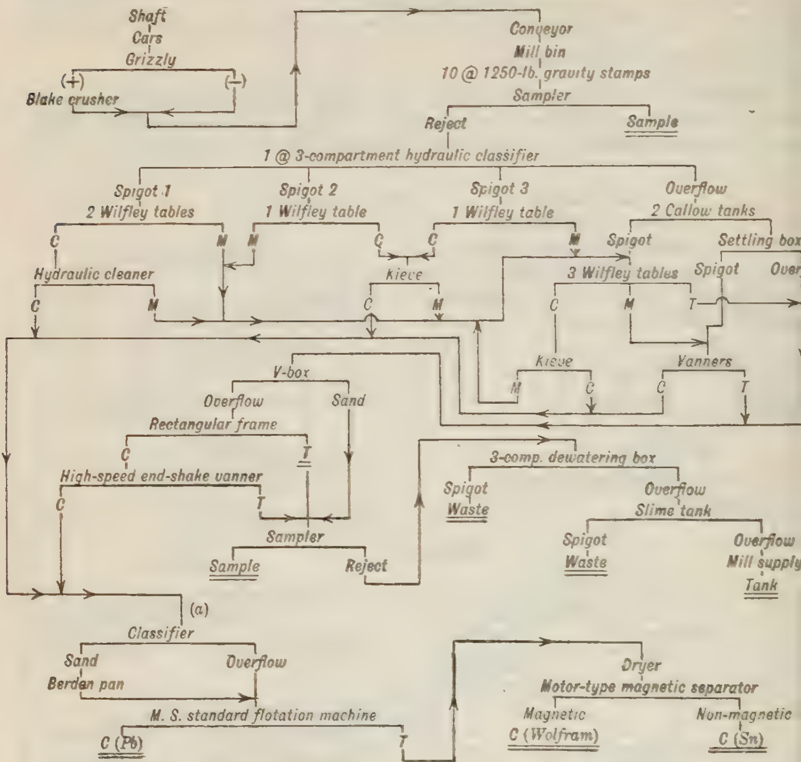


Fig. 125.—East Pool mill.

Butler Mine. Fig. 126. *(106 J 530.)*

Location: Torrington, N. S. W.
Ore: Complex tin-wolfram-lead.
Recovery: Sn, 80 per cent.; WO_3, 68 per cent.; Pb, 66 per cent. in gravity mill and 98 per cent. by flotation = 64.5 per cent.

Summary. Collective gravity concentration on tables, vanners and frames with flotation of gravity concentrate to separate lead, and magnetic separation of flotation tailing to separate tin and wolfram.



a, Average, 53 per cent. Sn, 12 per cent. WO_3, 5 per cent. Pb.

Fig. 126.—Butler mine.

31. Titanium, Ti

Properties. Metal: steel gray, lustrous, non-crystalline structure: hard, brittle, non-ductile. (See also Table 1.) At. wgt., 48.1. Oxidizes to TiO_2 when heated in air. Powdered metal reacts slightly with boiling water with evolution of hydrogen. Dissolves in warm hydrochloric and cold dilute sulphuric and nitric acids. In the important compounds the ion is quadri-valent, both base- and acid-forming. Ion is also tri-valent.

Uses. The principal use has been as a de-oxidizer and cleanser in steel making. Titanium-treated steels are said to be stronger and tougher than untreated steels. They are used for axles, tires, and other forgings; gears and pinions, plates, castings and rails. The tonnage of the latter has decreased from over 250,000 in 1910 to 2?,000 in 1916, due to the increasing use of open-hearth over Bessemer steel for rail manufacture and the uncertainty as to the benefit of titanium in open-hearth smelting. At present the demand for ferro-titanium is small in comparison to the supply. Other titanium alloys are those with aluminum and copper. Titanium-aluminum bronze is said to have physical properties equal to phosphor and manganese bronzes and to be lighter. Titanium salts are used in textile and leather industries for dyeing, mordanting and bleaching. There is also a limited

use of titanium compounds in the ceramic, dental and metal-ceramic industries and in the making of arc-lamp electrodes. The use of titanium oxide as a paint pigment is small but increasing.

Ores. The economic minerals are rutile and ilmenite. Ilmenite occurs rather widely associated with magnetite in deposits which, if the smelting problem were solved, would be of commercial importance. These deposits are not of the enormous size popularly believed, but two of them, one in New York and one in Wyoming, are reported by Singewald (*Bul. 64, USBM*) to be of such extent as to be of importance as iron producers. Rutile occurs in igneous, sedimentary and metamorphic rocks as microscopic grains included in the silicate rock-forming minerals; as scattered, separate grains, and as masses. The latter occurrence is the only one of economic importance. Only four deposits of commercial size are known; in the United States, in Virginia; in Quebec, Canada; in Norway; and in southern Australia.

Production of rutile concentrate, 95 per cent. TiO_2, at the Virginia mine ranges from less than 100 to less than 600 tons per annum and ilmenite concentrate (52 per cent. TiO_2) is produced in the same plant in slightly greater quantity. Rutile and ilmenite are also obtained as by-products in the treatment of monazite sand.

Price of high-grade rutile concentrate (94 to 96 per cent. TiO_2) varies from $0.10 to $0.25 per lb. ($0.12 to $0.15 for granular Virginia and $0.17 to $0.30 for −100-mesh in March, 1926). Ilmenite (52 per cent. TiO_2) was quoted (*J. Mar., 1926*) at $0.015 per lb. f.o.b. Virginia points and $60 per short ton, Florida mines.

Treatment at the Virginia mines consists in wet-gravity concentration to collect a mixed rutile-ilmenite concentrate and high-intensity magnetic separation to take out rutile concentrate and leave ilmenite concentrate.

32. Tungsten, W

Properties. Metallic gray, hard, dense, malleable and remarkably tenacious and ductile. It can be drawn into finer wire than any other metal. The fine wire has exceptionally high tensile strength. (See also Table 11, Art. 136.) Not affected by air at ordinary temperatures but burns to the trioxide if heated sufficiently. Not attacked by water or the common acids. Dissolves slowly in alkali hydroxides and in fused alkali nitrates, nitrites and peroxides. It is hexa-valent. The metal alloys readily with nickel, cobalt, molybdenum, chromium, iron, manganese, vanadium, uranium, aluminum and titanium.

Uses. More than 95 per cent. of all tungster mined goes into the manufacture of ferro-alloys and tungsten steels. Tungsten is an essential element of high-speed tool steels which are indispensable in present-day manufacturing. Other uses for the metal are for magnet steel, alloys with aluminum, copper, zinc, nickel, cobalt and other metals, incandescent lamp filaments, electrical apparatus, X-ray tubes, coating for sound reproduction, and as a catalyzer in production of ammonia from atmospheric nitrogen. Tungsten salts are used for fire-proofing cloth, as mordants in dyeing, and for coloring glass and porcelain.

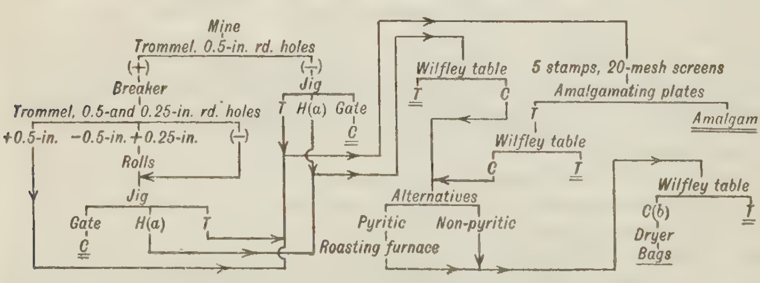
Ores. The economic minerals are scheelite, hübnerite, ferberite and wolframite. Deposits are usually placers or fissure veins, less commonly in pegmatites and contact-metamorphic zones. The usual gangue minerals are quartz, fluorite, cassiterite, tourmaline, mica, etc., occasionally sulphides. The country rock is commonly granite, less frequently quartzite, limestone and metamorphic rocks.

Production. World production of tungsten concentrate (60 per cent. WO_3) in 1914 was 8000 metric tons. The peak production was 32,000 tons in 1918. Production in the years 1921 to 1924 was 5800, 9700, 8000 and 550 tons respectively. Of this 55 to 65 per cent. comes from China and 12 to 15 per cent. from Burma. The United States is a small producer under normal circumstances, although in 1918 United States production was about 4000 tons (*33 MI 721*).

Selling. Tungsten concentrate for making ferro-tungsten should carry upwards of 65 per cent. WO_3 and not more than 0.5 per cent. each of Sn, Cu, P, As and S. Lower grades may be used in chemical manufacture (*Spurr and Wormser*). Sales are usually based on the 20-lb. unit. Prices of concentrate on this basis, in the United States, have ranged from a high of $34 per unit for 60-per cent. concentrate for the year 1916 to a low of $3.20 in 1921. The quotation in March, 1926 (*J*), for wolframite concentrate at New York was $11.50 to $12.50 per unit. Foreign prices are much lower, the American price being artificially maintained by the tariff.

Treatment. There are two general types of mills, designed to treat the two entirely different general classes of ore in which tungsten occurs. The American and some Australian ores consist essentially of the tungsten mineral, usually scheelite or ferberite, in a silicious gangue, while most of the foreign ores that are being worked are complex and carry wolframite with cassiterite and iron and lead sulphides. The simple ores are usually of higher grade than the complex and the tungsten mineral more coarsely disseminated. The Glenorchy flow-sheet is typical of the older practice in treating the simple ores. Variations comprise hand sorting ahead of the jigs, rough classification prior to tabling, and magnetic separation of roasted concentrate. The Tungsten Mines Co. mill is typical of modern practice on low-grade simple ores. Mills treating higher-grade ores usually include jigging and may include hand sorting. This modern practice recognizes the friability of the tungsten minerals and the whole endeavor is to crush so as to produce as little slime as possible and to begin removal of concentrate as soon as the mineral is free. In treating higher-grade ore, tailing from slime tables and vanners is usually run over rag frames and the frame concentrate cleaned or shipped to electric-furnace plants or mixed with the higher-grade coarse concentrate. Recoveries on simple ores range from 65 to 85 per cent., depending on the grade of feed and elaborateness of plant. The S. and M., the Burma Queensland, and the East Pool flow-sheets are illustrative of methods of separating wolframite from complex ores. In all cases the practice is to make a collective gravity concentrate containing the wolfram and to utilize the permeability of wolframite to effect its separation from the non-magnetic portion of the roasted concentrate. When, as is the case at East Pool, the collective concentrate contains iron, magnetic separation of wolframite from the iron is difficult or impossible. At East Pool the difficulty is overcome by leaching out the iron oxide with sulphuric acid.

Glenorchy Scheelite Mine Co. Fig. 127. (*113 P 136.*)



a. 20-mesh. *b.* 70 per cent. WO_3.

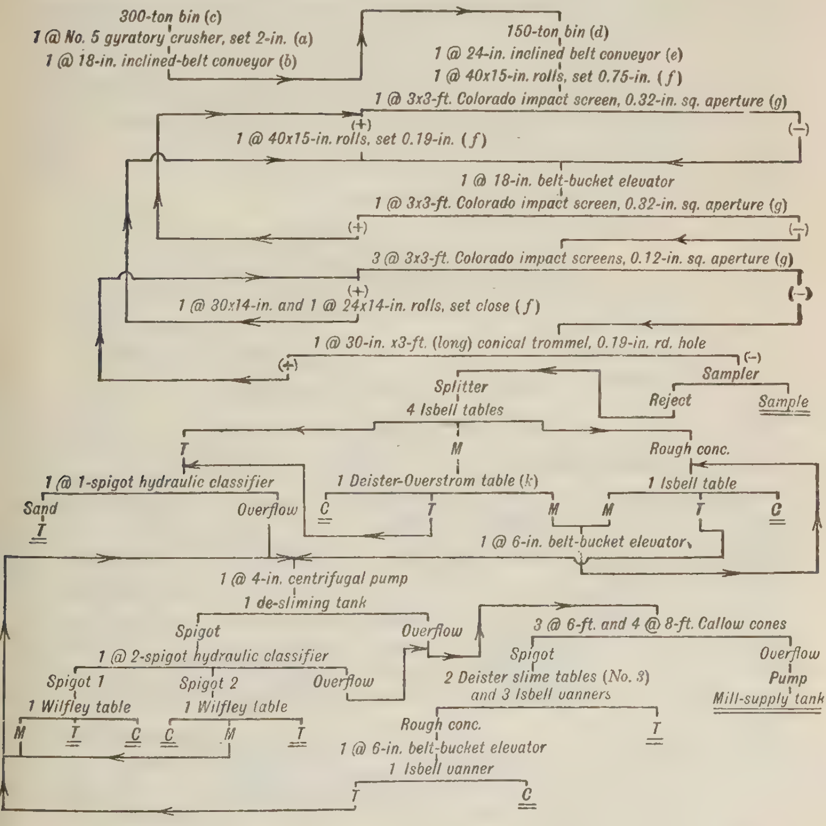
Fig. 127.—Glenorchy Scheelite Mine Co.

Location: Glenorchy, New Zealand.
Ore: Scheelite with pyrite, arsenopyrite and gold in quartz veins in hard, crystalline schist.
Assay: concentrate, 70 per cent. WO_3.
Recovery: 7.1 per cent. of feed as WO_3.
Cost (1907 to 1914 aver.): $1.10 per ton milled.

Summary. Gravity concentration by jigs and tables, beginning at 0.5-in. Harder part of feed and jig middling re-ground to 20-mesh and concentrated (rougher-cleaner routing) on tables. Roasting is practiced to convert pyrite to porous iron oxide and thus lower its effective specific weight and render it separable from the scheelite by gravity (table) concentration.

At another New Zealand mill pyritic scheelite concentrate is given a flash roast only and the now permeable pyrite particles are separated magnetically.

Tungsten Mines Co. Fig. 128. (*Q.*)



a, See Sec. 3, Table 15. *b,* 23 ft. long; slope, +4 in. per ft.; 60 ft. per min; 20-in. head and tail pulleys; 5-ply belt. *c,* Flat-bottom. Live capacity, 100 tons. *d,* Flat-bottom. Live capacity, 70 tons, center-bottom draw. *e,* 19 ft. long, slope, +3.25 in. per ft.; 12 ft. per min.; 20-in. head and tail pulleys; 5-ply belt. *f,* See Sec. 3, Table 22. *g,* See Sec. 5, Table 34. *k,* See Sec. 10, Art. 19.

Fig. 128.—Tungsten Mines Co.

Location: Bishop, Cal.

Ore: Scheelite in a gangue principally (90 per cent.) garnetite with small amounts of epidote, tremolite, calcite, etc.

Capacity: Aver., 240 tons per 24 hr.; max., 350 tons.

Assays, per cent. WO_3: Feed, 0.4; concentrate, 62; tailing, 0.09.

Recovery: 65 to 71 per cent.

Ratio of concentration: 200 : 1.

Labor: 16 tons per man-shift, operating; 60 tons per man-shift on repairs.

Water: 6 to 6.5 tons per ton milled.

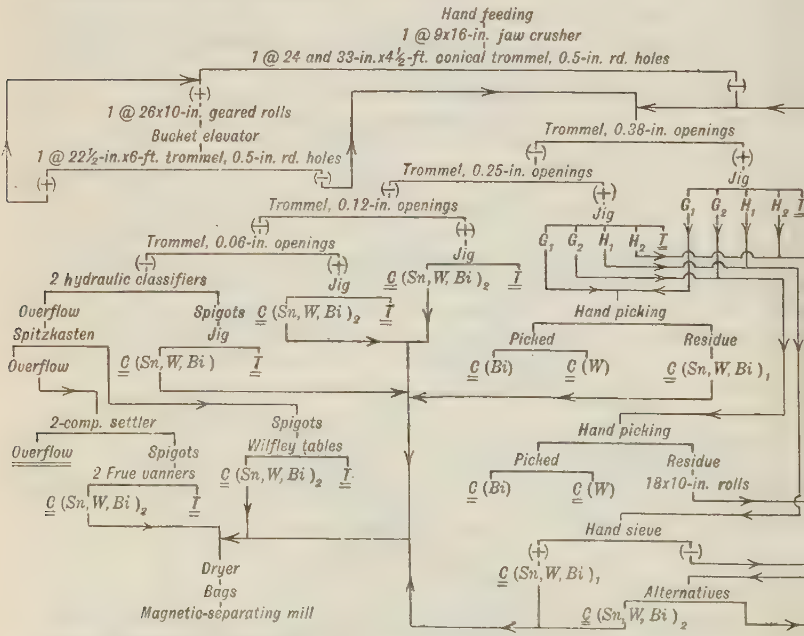
Running time, 70 per cent. of possible: High delay due to repairs caused by abrasive character of ore.

Distances: Mine to mill, 2500 ft., mule tramming; mill to market, 13 miles to railroad, thence to eastern U. S.; water transported 2 miles by open ditch and gravity pipe line; power transmitted 4 miles at 33,000 volts.

General: Slightly sloping mill site.

Summary. Gravity concentration on tables and vanners after careful graded crushing to 0.12-in. CRUSHING: Gyratory from 11- to 2-in.; rolls from 2- to 0.75-in., open circuit; rolls from 0.75- to 0.32-in., closed circuit; rolls from 0.32- to 0.19-in., closed circuit. CONCENTRATION: Roughing unclassified pulp on shaking tables and cleaning rough concentrate and primary middling separately on shaking tables. Primary-table tailing de-slimed, sand rejected, slime joined with primary-cleaner tailing, classified, the sands treated on Wilfley tables and the slimes on Deister slimers and vanners. Slime concentrate cleaned on a vanner.

S. and M. mine. Fig. 129. (*120 P 229.*)



NOTE: Subscripts indicate first- and second-grade concentrate. The principal difference is in the pyrite content, the "firsts" containing but little while the "seconds" contain considerable. Neither contains much gangue.

FIG. 129.—S. and M. mine.

Location: Monia, Tasmania.

Ore: Cassiterite, wolframite, scheelite, bismuthinite, bismutite and pyrite in quartz gangue with some fluorite, mica and topaz. The quartz is very friable and the economic minerals occur in coarse grains and patches so that 0.5-in. crushing frees most of the values.

Capacity: 100 tons per 24 hr.

Assays of concentrate: "Firsts" (see note on flow-sheet)—Sn, 35 per cent.; WO_3, 35 per cent.; Bi, 2.5 per cent. "Seconds"—Sn, 12 per cent; WO_3, 10 per cent.; Bi, 3.5 per cent.

General: Mill 18 miles from railroad. Concentrate shipped to Launceston.

Summary. Gravity concentration by jigs, tables and vanners after careful sizing and classification. Hand sorting and magnetic concentration to separate the collective concentrate thus obtained.

Burma Queensland Corporation. Fig. 130. (*123 P 821.*)

Location: Wolfram, N. Q., Australia.

Ore: Molybdenite, wolframite, metallic bismuth, bismuthinite, bismutite and a little scheelite, in quartz and granite. The economic minerals occur in coarse grains and patches.

Capacity: 110 to 120 tons per 24 hr.

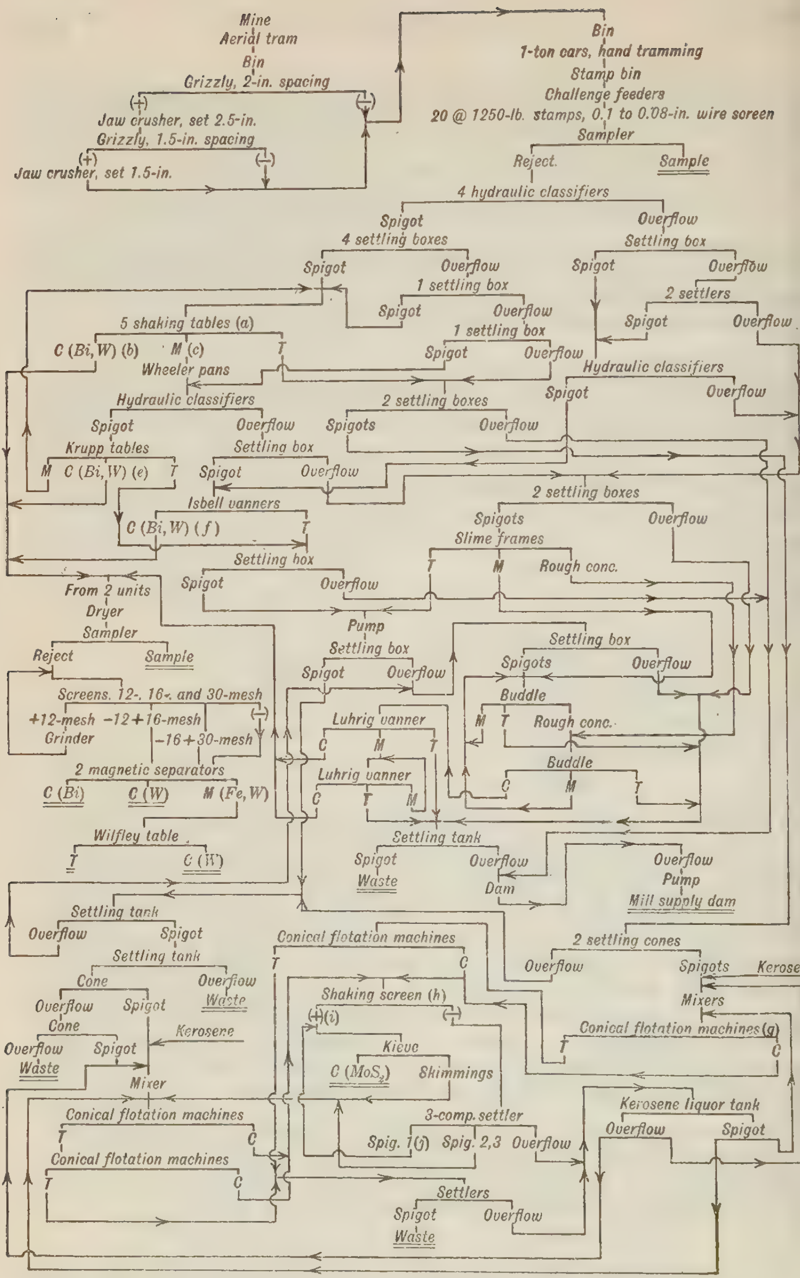
Assays: See Table 88.

Table 88. Assays of intermediate and final products, Burma Queensland Corp.

Reference letter	Product	Assay, per cent.		
		WO_3	Bi	MoS_2
b	Primary-table concentrate	30–45	5–7	....
c	Primary-table middling	8–10	....	5
	Primary-table tailing	0.1–0.13	....	....
e	Secondary-table concentrate	25–40	3–5	....
f	Primary-vanner concentrate	15–25	3–4	....
i	Coarse flotation concentrate	....	....	85–94
j	Fine flotation concentrate	....	....	80–85

Summary. Gravity concentration to make a collective tungsten-bismuth concentrate followed by magnetic concentration to separate tungsten from bismuth. Gravity tailing floated, yielding molybdenum concentrate and final tailing. Crushing: Two-stage reduction in jaw crushers followed by gravity stamps to reduce pulp to 8-mesh. Pans used to re-grind primary-table middling. Plus-12-mesh gravity concentrate ground through 12-mesh for magnetic separation. No other grinding. Concentration starts on 8-mesh pulp, which is classified to an excessive degree in preparation for shaking-table, vanner, frame and buddle treatment. Flotation feed (gravity-concentration tailing) is −8-mesh size. Magnetic-plant feed (gravity concentrate) is −12-mesh and is treated in 3 sizes. An iron-wolfram reject from the magnetic machines is tabled to recover wolframite.

The outstanding feature of the plant is the excessive number of settling boxes, entailing, as they do, great loss of head-room and, undoubtedly, much lost time in operation due to clogging of the spigot discharges. Modern practice would substitute mechanical classifiers or diaphragm cones for the sand-slime separation and Dorr thickeners for de-watering, thus greatly decreasing the number of such steps and simplifying the flow-sheet.



a, Wilfley and Buss types. *b, c, d, e, f, i, j*, See Table 88. *g*, Cones 4.75 ft. diam., 3.25 ft. deep. *h*, Box, 3 × 2-ft. × 6 in. deep with 80-mesh cloth bottom. Dewatered oversize removed periodically.

Fig. 130.—Burma Queensland Corporation.

33. Selling ores and mill products

Some ores as mined contain sufficiently large quantities of valuable mineral per ton to make concentration unnecessary. Certain ores, notably those in which gold and silver are the only commercially valuable constituents, likewise are not subject to concentration. These ores and the concentrate produced in milling plants must be treated by some chemical process in order to obtain metals in a form useful in manufacturing and the arts. The gold and silver ores mentioned may be treated by the mining company, or, especially in case of small mines, may be sold to a custom milling plant. High-grade base-metal ores and mill concentrates are treated by smelting. Smelting plants usually buy the products that they treat, although occasionally contracts are made in which the smelting plant returns the metal recovered to the miner, making a charge for smelting.

Custom mills are built and operated in the same fashion as company mills. They differ from company mills only in that the mill equipment must be fitted to treat ores of somewhat diverse character, while the ordinary company mill is built to handle a special ore.

Methods of payment for gold and silver ores are simple. The ores are ordinarily shipped but short distances and are, consequently, all subject to substantially the same freight rate. In such case the custom mill makes a flat rate to cover freight and treatment and pays for the precious-metal content on the basis of agreed assays, less a flat rate covering freight and treatment, and of course, a certain profit on the transaction. Table 89 gives a

Table 89. Milling rates on Cripple Creek ores, including freight charge. (Full assay value in gold paid for at $20 per ounce)

Value of ore per ton	Charge per ton for treatment	Value of ore per ton	Charge per ton for treatment
Up to $10	$4.00	$25 to $30	$6.50
$10 to 15	5.00	30 to 40	7.00
15 to 20	6.00	Over 40	8.00
20 to 25	6.25		

typical schedule of custom-mill charges in the Cripple Creek district of Colorado in 1914. The charge increases with increase in value of ore. This is done in order to stimulate mining of low-grade ores and thus maintain a reasonably continuous supply of ore to the mill. High-grade ore is made to pay a greater profit than low-grade ore in order that a reasonable average profit may be maintained.

Smelters have a more complicated problem in the purchase of base-metal ores with or without a precious-metal content, consequently their schedules of charges and methods of payment are more complicated than is the case with custom mills. The smelter buys ores on the basis of the agreed assay, paying for valuable metals contained therein at prices current in principal metal-market centers, either at date of purchase or at some agreed date thereafter meant to be the probable date of sale, less a charge covering the cost of treatment and profit thereon. The treatment charge must include the cost of delivering ore to the smelter, sampling, smelting, freight on crude metal to the refinery, refining, selling, and a carrying charge on metal from the time of purchase to the time of disposal. Various methods of assessing these charges and the profit on operations are followed. In the case of some metals all is included in a treatment charge; in other cases a part only, namely, smelt-

ing, is included in the base treatment charge, the balance being taken care of in the price at which metal is paid for after certain deductions from the market price. Various methods are illustrated by examples on subsequent pages. All methods have as the fundamental basis of charge the cost of the various items above enumerated.

Freight is easily determined by reference to published tariffs available to the shipper. Rates per ton are usually higher, the higher the grade of ore. The quantity shipped, when less than carload lots, affects the rate. The charge is based on the gross weight at the shipping point while the smelter pays for dry ore, hence the less moisture in the ore as shipped the less the charge for transportation of water against dry ore. Moisture may run as high as 30 per cent. on some products, notably flotation concentrate, and as low as 3 or 4 per cent. on hard non-porous ores. (See also Sec. 23.)

Sampling. For discussion of methods see Sec. 21. Moisture samples should be taken as soon as possible after the ore is weighed in, especially if in open cars. Wetting after weighing in and before sampling results in a charge against the lot as weighed of too much moisture and hence operates against the shipper. Exposure to a hot dry atmosphere for a few days between weighing and sampling works similarly against the smelter. Sampling for chemical analysis should yield four final pulp samples; one for the smelter laboratory, one for the shipper, one for umpire, and one as an emergency reserve for either of the three. SPLITTING LIMITS between mine and smelter assays are given in Sec. 21, Art. 1. When umpiring is resorted to, the umpire's results are usually taken when they lie between the disputed results. The disputed result nearest the umpire's is taken when the umpire's result lies outside the disputed results. Analysis should determine not only the valuable metals but also constituents such as iron, lime, silica, magnesia, alumina and sulphur, upon which the common bonuses and penalties are based, and also any other substances that are specified in special instances. Analyses to determine these constituents should be just as carefully done as those for the valuable metals, as the penalty or bonus may make an important difference in the value of the ore. Silica should be determined by fusion rather than as "insoluble," as the latter is almost invariably high, as much as 15 to 20 per cent. with ores of high insoluble content and frequently 4 to 6 per cent. (*TP 83, USBM*). It is sometimes insisted also that iron and lime be determined by fusion. Ores that must be roasted and therefore crushed fine are usually sampled by machine, but those that are to be smelted in a blast furnace directly, such as oxide ores, are commonly sampled manually. The COST of sampling varies with the kind of ore and the method; it should rarely exceed \$1.00 per ton and averages near \$0.50 per ton with wages for common labor at \$4.00 per day.

Smelting is a chemical process in which reactions between various constituents present are brought about in a molten mass under the influence of heat. The method of smelting differs with the principal base-metal constituent of ore. When this is LEAD, the process consists essentially in mixing together a charge of such composition that when melted the worthless minerals will combine together to form a complex silicate, principally of iron and calcium, called SLAG, while metallic lead is freed and settles to the bottom of the molten mass, carrying with it in solution any precious metals present. If COPPER is also present it, together with some iron, combines with sulphur, forming an artificial copper-iron sulphide, called MATTE, that also settles out and comes to rest in the furnace between the lead and the slag. The various products are drawn from the furnace separately, slag is run to waste, lead is cast as crude lead, which must be refined in order to make it marketable and to recover the precious-metal contents, and the matte is further treated as described below to recover the copper and any precious metals present. In COPPER SMELTING a charge is similarly compounded to slag off the gangue and form matte that settles out, carrying with it precious metals. Matte is treated in a converter, a large pot in which the molten substance is subjected to air blown through in order to carry off sulphur in the form of

sulphur dioxide and leave crude copper containing varying amounts of iron, precious metals, etc. This crude or "blister" copper is cast into ingots and sent to the refinery where it is purified electrolytically. In ZINC SMELTING the ore, after being roasted to drive off sulphur, is placed with carbonaceous material in a retort and heated. Zinc is thus volatilized and passes into suitable collecting chambers, where it is condensed.

Penalties and bonuses. It is necessary in lead and copper smelting that the slag be sufficiently fluid to allow the metal or matte to settle out freely and to permit easy withdrawal of slag from the furnace. In order to obtain the desired fluidity the silicate formed must be of a particular composition. The basic smelter charge is founded upon the cost of smelting a charge of such composition and includes fuel, labor and all overhead costs. If the ore to be treated is not of the desired composition, it is necessary to make up any deficiency. In districts where the majority of the ores sent to the smelter are silicious, iron or lime, or both, must be added. In order to cover the cost of addition of such substances, a penalty is imposed for silica present in the ore bought. If, on the other hand, the prevailing ores of the district are calcitic or ferruginous, then silica must be added and iron and lime are penalized to make up the cost of such addition. When silica is penalized iron and lime are usually granted a corresponding bonus; silica is given the bonus when iron and lime are penalized. Penalties and bonuses are quoted at so much per unit. A UNIT is one per cent. or 20 lb. per ton (22.4 lb. per long ton). SILICA PENALTY ranges from 10¢ to 15¢ per unit for all excess over a stated amount, which should be that required for a self-fluxing charge; IRON BONUS ranges from 5¢ to 10¢ per unit in excess. A bonus is not ordinarily paid for LIME, but should be, especially when silica is penalized. One part of iron or lime fluxes one part of silica in copper smelting; in lead smelting, two parts iron and lime are required for one part silica. Hence in copper smelting the bonus for iron and lime should be equal to the silica penalty and *vice versa*. In lead smelting there is justification in a silica penalty higher than the iron or lime bonus. When silicious ores are wanted at a smelter a FLAT RATE may be made for their treatment in place of allowing a bonus for silica. In this case the charge usually increases with the grade of ore, as is the case at custom mills, and for the same reason. Table 90 shows such a

Table 90. Flat schedule for silicious ores in Colorado district (1914)

Value of ore per ton	Charge per ton for treatment	Value of ore per ton	Charge per ton for treatment
$14 and less	$5.00	$35 to $40	$8.00
14 to $20	6.00	40 to 45	8.50
20 to 25	6.50	45 to 50	9.00
25 to 30	7.00	50 and more	10.00
30 to 35	7.50		

schedule. In lead smelting, the formation of any considerable amount of matte is undesirable and consequently it is necessary to roast sulphide ores prior to their introduction into the smelting furnace. For this reason sulphur, being a source of expense to the smelter must be paid for by the ore seller and is usually charged in the form of a penalty for sulphur. The usual contract penalizes SULPHUR in excess of 2 or 3 per cent., ordinarily at a rate of 15 to 25¢ per unit. Sulphur is not penalized in copper smelting, since here it is desirable. ZINC, in lead and copper ores, causes slag to be thick and viscous, thus increasing metal losses therein and lowering furnace capacity.

It also causes fumes that foul the furnace walls, and, by volatilization, causes losses of other metals. For this reason, zinc over a certain amount is frequently penalized. Various forms of contract covering zinc in lead and copper ores are as follows: Penalty, 30¢ to 50¢ per unit for all excess above 10 per cent. (*TP 83, USBM*). Eight per cent. free, excess penalized at 30¢ per unit. Twelve per cent. free, excess penalized at 30¢ per unit (*85 J 992* [*1908*]). The limit may run as low as 5 per cent. and the penalty as low as 15¢ per unit. ARSENIC is decidedly undesirable in lead smelting and for that reason all over 1 per cent. is commonly penalized; ANTIMONY, BISMUTH and TIN are also objectionable; the penalty for these four combined may run as high as $1 per unit for the excess over 0.5 per cent. An arsenic penalty is less frequently imposed in copper smelting. Iron is highly undesirable in zinc smelting; more than 10 per cent. makes the ore substantially unfit for treatment. The PENALTY FOR IRON is normally imposed by setting a certain price for zinc ore of a given zinc content and penalizing it so much per unit for each unit below standard. In such cases a corresponding bonus is ordinarily given for each unit of zinc above standard.

Basic smelting costs. Walker (*Peele*) states that basic smelting costs as of 1926 for large plants are $3 to $5 per ton of charge in lead smelting and $2 to $3 in copper smelting.

The following are base charges for LEAD SMELTING taken from various contracts (*85 J 992* [*1908*]); (*a*) $7.50 per ton for all ores; (*b*) $7.50 per ton for ores carrying less than 25 per cent. lead, $7 per ton from 25 to 30 per cent. lead, $6.50 per ton for ores carrying more than 30 per cent.; (*c*) $8 per ton for ores carrying less than 30 per cent. of lead and $6 per ton for ores carrying 30 per cent. or more; (*d*) as in Table 91. These figures should be increased 30 to 50 per cent. for 1926. Usually the smelting cost per ton decreases with increase in grade, because there is less gangue to be slagged off in the higher-grade ores, hence less fuel need be used per pound of metal produced; furnace capacity is higher so that labor cost and overhead per pound of metal produced are lower.

Table 91. Schedule of treatment charges on lead ores

Lead content, per cent.	Treatment charge, dollars per ton	
	Contract A	Contract B
5 to 10	7.25	9.00
10 to 15	6.00	8.00
15 to 20	5.00	7.00
20 to 25	4.00	6.00
25 to 30	3.00	5.00
30 to 35	2.25	4.00
35 to 40	1.35	3.00
40 to 45	0.50	2.00
Over 45	0.00	1.00
45 to 50	0.00	....
50 to 55	Bonus 0.75	....
Over 55	Bonus 1.25	....

A common basic charge for COPPER ORES is $4 to $4.50 per ton.

Freight on crude metal, refining, and selling are usually taken care of in smelter contracts by deductions from the price per pound paid for metal. The costs (1926) of these various operations, according to Walker (*Peele*) are:

for refining lead, \$8 to \$12 per ton of bullion; for converting copper matte, about \$10 per ton of blister copper produced; for refining copper, \$15 to \$23 per ton of bullion. The usual deduction in copper contracts is 3¢ per lb. from the New York quotation for electrolytic copper. Similarly from 1 to 1½¢ per lb. is deducted from the lead price.

Losses include metal in slags, either as shot or combined, and losses by volatilization. Walker gives the following metal losses in smelting and refining; lead, 4 to 15 per cent., depending upon the grade of ore and its refractoriness; the lower limit is for a charge carrying about 40 per cent. lead; the higher, for one carrying 10 to 12 per cent. lead. Copper loss sometimes reaches 30 per cent. in lead smelting. In smelting copper ore the loss varies with the percentage of copper, but is always much less than the loss of lead in lead smelting. Silver loss is normally not over 2 to 5 per cent. Gold loss is not generally considered.

Fulton (*T. P. 83, USBM*) gives the following actual losses based on studies of smelter performances. In LEAD SMELTING: lead, 5 to 20 per cent. of contents, the low figure on a high-lead charge, 30 to 35 per cent. or upwards, and containing but a small amount of roasted material; the higher, on a charge containing less than 10 per cent. lead or a large amount of roasted material. Normal loss with a charge containing 10 to 15 per cent. lead is 8 to 11 per cent. Copper, 1½ to 4 lb. per ton of charge. Silver, 1.5 per cent. loss to 1.75 per cent. gain. The gain is, of course, apparent only, and follows from the practice of charging the smelter with silver only in those ores containing more than a certain minimum amount. Gold, 0.3 per cent. loss to 4.5 per cent. gain. The gain here also is apparent only and arises from the same reason as given for apparent gain in silver. In COPPER SMELTING: copper, 3 to 11 lb. per ton of ore smelted or 5 to 15 per cent. on the copper present, the higher figure on the lower grade ore. Silver, 0.5 to 10 per cent. of contents. Gold, 1.5 per cent. loss to 4 per cent. gain. Barbour (*92 J 314*) states that the silver loss in good lead smelting should not exceed 3 per cent. He says the usual zinc loss in ZINC SMELTING is about 12.5 per cent.

Lead losses are sometimes taken care of, wholly or in part, by basing payment for lead on a fire assay. This practice is based on the assumption that fire assaying is smelting on a small scale and recovers the same amount of lead that will be recovered in the furnace. The fire assay for lead runs about 0.5 to 1 per cent. below the wet assay unless such metals as zinc, copper, antimony, bismuth or arsenic are present, in which case fire assay man run high. To cover this contingency some contracts call for wet analysis of the button obtained by fire assay and base lead payment on the assay thus obtained. Common practice at present is to make wet analysis and deduct therefrom 1 to 1.5 per cent., calling this DRY ASSAY or FIRE ASSAY and use it as basis for settlement. On the basis of a 10-per cent. loss of lead in smelting a 1.5-per cent. deduction from the wet assay more than covers the loss on ores carrying less than 15 per cent. lead but does not cover with higher-grade ores. However, the loss on higher-grade ores is a smaller percentage of the total lead present and it is probable, therefore, that a deduction of 1.5 per cent. from the wet assay more than covers loss in all cases of gold lead-smelting practice. Lead losses are further taken care of by paying for less than the whole amount of lead determined by either method of assay as, for instance, by paying for 90 per cent. of the lead determined by dry assay. Further, the deduction from the market price is oftentimes made greater than the cost of freight on crude metal, refining, and selling, in order to further insure the smelter against losses.

Copper losses are similarly taken care of, although the use of the dry assay is less common than with lead. "Dry copper" means from 0.75 to 1.5 per cent. less than the wet assay.

Gold and silver losses are provided for by deducting up to 5 per cent. of the assay value for gold and 5 to 10 per cent. for silver, or by paying less than the full price for all of the metal present, or both.

Value of ore is stated in some monetary unit per unit of weight. In the United States the respective units are dollars and short tons; in most British possessions shillings and long tons; in certain Latin countries, the metric ton may be used with the local monetary unit. Locally almost any system of stating value may be met. Value is determined by multiplying together units of weight of valuable material per ton by price per unit of weight at some accepted market center, usually New York, and deducting from the product thus obtained the cost of producing refined metal from a ton of ore and the price of the valuable material lost in the operation. MARKET PRICES of metals vary widely. Ranges and averages are given in this section. The unit of weight for precious metals in the United States is the

troy ounce; in other countries pennyweight, grain, or gram may be used. English-speaking countries use pound or ton as the unit for base metals; countries using metric system employ metric units.

Payment for metals

Gold is usually paid for at from \$19 to \$20 per oz. for all gold, if in excess of some certain minimum, usually, from 0.02 to 0.05 oz. per ton. Oftentimes the contract calls for payment for only the excess of gold over the minimum. The price may vary according to the amount present, thus \$19 per oz. may be paid for all of the gold or the excess over the minimum amount, if the total amount is less than 0.25 oz.; \$19.50 if the total amount is between 0.25 and, say, 1 oz.; and \$20 per oz., if the total present is more than 1 oz.

Silver. The usual form of contract is to pay for all silver at 95 per cent. of New York quotation at some date ranging from the time of purchase to, say, 3 or 4 months thereafter. The assumption is, in case of deferred payment, that the smelter does not then have to gamble on the price of metal at the time that it is sold and that consequently the price paid to the shipper, or the amount of metal paid for, or the treatment charge, can be so adjusted as to repay the shipper for taking the risk of price fluctuation. Another common form of contract is to pay for 90 to 95 per cent. of silver present at 90 to 95 per cent. of the New York quotation at the date agreed upon. Of course, in this form of contract, the smelter is more liberally insured against losses and more liberally paid for recovery of silver than in the first form of contract. If such a contract is made, a corresponding advantage should accrue to the shipper in some other part of the contract.

Lead. Three methods are followed in paying for lead.

METHOD No. 1. Payment is made on the basis of the wet assay less 1 to 1.5 per cent. at certain prices per unit arrived at by the following method. First, a certain base price per unit, varying with the metal content and with a QUOTATION is set.

Quotation is 90 per cent. of New York price when this is \$4 per 100 lb. of lead or less If the New York price is between \$4 and \$4.50, the quotation is 90 per cent. of \$4 plus one-half of the excess of price over \$4. If New York price exceeds \$4.50, the quotation is 90 per cent. of \$4 plus half of the excess between \$4 and \$4.50 plus all of the excess over \$4.50. Table 92 presents a set of prices based on a quotation of \$4 per 100 lb. of lead. For each change of 5¢ in quotation a corresponding change of 1¢ is made in the price per unit.

Table 92. Unit prices for lead on "Quotation" of \$4 per 100 lbs.

Lead content of ore, dry basis, per cent.	Price per unit (*a*)	Lead content of ore, dry basis, per cent.	Price per unit (*a*)
5 to 10	\$0.35	25 to 30	\$0.49
10 to 15	0.40	30 to 35	0.51
15 to 20	0.45	Over 35	0.51
20 to 25	0.47		

a Unit equals 1 per cent., or 20 lbs. per ton.

Example: Ore contains 40 per cent. lead by wet assay and New York price is \$4.40 per 100 lb. Quotation is $0.90 \times (\$4)$ plus $0.5 \times (\$4.40 - \$4) = \$3.80$. This is 20¢ below the quotation on which Table 92 is based, hence 4¢ must be deducted from the price per unit in the table, corresponding to a dry assay of 38.5 per cent. The price paid per unit is, therefore, 47¢. The lead paid for is 770 lb. per ton. The price paid per ton is $(770/20) \times 0.47 = \$18.09$. On a New York price of \$5, the quotation would be $0.90 \times (\$4) + 0.5 \times (\$4.50 - \$4) + 0.50 = \$4.35$; the price per unit would be \$0.58, and price per ton \$22.33.

METHOD No. 2. Payment is usually for 90 per cent. of the fire assay at from 1¢ to 1.5¢ per lb. off New York price, usually the higher figure if the ore carries 5 to 25 per cent. lead and 1.25¢ if the ore carries over 25 per cent. lead. Less than 5 per cent. of lead is not paid for. In some cases as much as 1.75¢ is deducted from New York price.

METHOD No. 3. Payment for 90 per cent. of fire assay at from 90 to 95 per cent. of New York price. Less than 5 per cent. of lead is not paid for.

BONUS FOR LEAD may be allowed on ores carrying less than 5 per cent. This bonus is usually at the same rate as that for iron, but may run up to 15¢ per unit where iron pays only 10¢ per unit.

In some cases a fixed price is set for lead equivalent to a deduction of from 1.5¢ to 2¢ per lb. from the probable average New York price over the term of the contract. This, of course, puts the burden of fluctuation in lead prices on the smelter and the deduction from the probable market price is, therefore, greater than would be the case, if the risk of fluctuation were carried by the seller. Such a method of payment is rarely introduced into large contracts, but is used with small shippers who desire to know definitely at time of sale what the receipts from the shipment will be.

Lead schedules. The following schedules are typical:

Schedule No. 1. Allow: Gold, if 0.1 oz. or over, $19 per oz. for full assay; silver, if 1.5 oz. or over, 95 per cent. at New York quotation on date of assay; lead, if 5 per cent., 50¢ per unit on quotation of $4 per 100 lb with variation of 1¢ per unit for every 5¢ change in quotation. (This is method No. 1 above); iron, pay for all at 4¢ per unit; lime, pay for all CaO at 6¢ per unit, if over 5 per cent, except that lime combined with fluorine is not paid for. Deduct: all insoluble at 10¢ per unit; sulphur, 1 per cent. free, excess at 50¢ per unit; arsenic, antimony and bismuth, if a total of 3 per cent. or over is contained, all at 50¢ per unit. Base charge, if 5 per cent. of lead or under, $5 per ton of 2000 lb.; if over 5 per cent. lead, deduct 10¢ per unit for each unit of lead in excess. Sampling, free on lots over two tons; $3 per lot, if under 2 tons. Add $1.50 per ton for concentrates or ores requiring briquetting. Moisture, 1 per cent. minimum in all cases (*Mines and Methods*, Nov., 1909).

Schedule No. 2. Utah, in the neighborhood of SALT LAKE. Pay for all gold at $19 per oz. and all silver at 95 per cent. of New York price at date of assay. Pay for 90 per cent. of lead by fire assay, at New York price less 1.25¢. Pay for iron at 10¢ per unit and impose a penalty on silica of 12¢ per unit. Treatment charge, $2.50 per ton for 30-per cent. lead ore with a credit of 5¢ per unit for each unit of lead in excess and a penalty of 8¢ per unit for each unit deficit (*92 J 314*).

Schedule No. 3. LEAD-SILVER ORE Pay for 85 per cent. of lead on fire assay at 85 per cent. of New York price, if New York price is $4 or less; if more than $4, pay 85 per cent. of $4 plus one-third of difference between $4 and New York price. Pay for 95 per cent. of silver at 95 per cent. of New York price. Treatment charge, including freight, for ore containing 45 per cent. lead, $10, with 10¢ per unit penalty for lower lead content and corresponding bonus for excess.

Schedule No. 4. TRAIL, B. C. Base, $8.50 to $9.50 per ton for 70 per cent. lead ore plus 10¢ for each unit under 70 to a maximum of $10.50 to $11.50. The lower rate is the contract rate and the higher, the open rate. Lead, if over 5 per cent., is paid for on the basis of 90 per cent. of fire assay at the London price less 1¢. 95 per cent. of gold is paid for at $20 per oz. All silver is paid for at 95 per cent. of New York quotation (*92 J 314* [1911])

Schedule No. 5. Pay for 90 per cent. of lead at St. Louis price less a treatment charge of $6 to $8 per ton of ore (*92 J 314* [*1911*]).

Schedule No. 6. NEUTRAL SCHEDULE, used chiefly in COLORADO. Gold, $19.50 per oz., if 0.05 oz. or over per ton. Silver, 95 per cent. of contents at New York quotation, date of assay. Lead (dry), as in No. 7, prices based on "quotation" of $4. Copper: for dry copper (1.5 per cent. off wet), at 6¢ off per lb. on Western Union quotation for casting copper. Zinc, 10 per cent. allowed; 50¢ penalty per unit for zinc in excess of 10 per cent. Silica penalty, 10¢ per unit; iron bonus, 10¢ per unit.

Schedule No. 7. NEUTRAL SCHEDULE used in UTAH. Credits: Gold, $19 per oz. if the product contains 0.03 oz. or over per ton. Silver, 95 per cent. of contents, at New York quotation, date of assay. Copper, as per wet assay, less 1 per cent. or unit, at the price for cathode copper quoted in the *Engineering and Mining Journal-Press*, issue of the week previous to date of receipt, less 5¢ per lb. Lead, 90 per cent. of contents (dry assay) to be paid for at the quotation in the *Engineering and Mining Journal-Press*, issue of week previous to date of arrival, less 1.25¢ per lb. Iron, paid for at 10¢ per unit. Charges:

Insoluble, 12¢ penalty per unit. Zinc, 10 per cent. allowed free; excess at 30¢ per unit penalty. Speiss (arsenic), 10 per cent. allowed free; excess at 20¢ per unit. Sulphur, 25¢ penalty per unit; maximum penalty imposed, $2.50 per ton. Treatment charge, $2 per ton, based on an ore carrying 30 per cent. lead; for each unit of lead above 30, a credit of 5¢ to be allowed; for each unit of lead below 30, a charge of 8¢ to be made.

Schedule No. 8. FLAT SCHEDULE. Payment made on metals as in Table 93.

Table 93. Flat schedule for lead in lead ores and lead-copper ores

Lead content, per cent.	Value per unit	Smelting charge, per ton	Lead content, per cent.	Value per unit	Smelting charge, per ton (a)
5 to 10	$0.40	$10.00	35 to 40	$0.52	$2.00
10 to 15	0.43	8.50	40 to 45	0.52	1.00
15 to 20	0.45	7.00	45 to 50	0.53	1.00
20 to 25	0.47	5.50	50 to 55	0.54	1.00
25 to 30	0.49	4.50	Over 55	0.55	1.00
30 to 35	0.51	3.50			

a For concentrate containing more than 30 per cent. lead, apply flat schedule, as above, or neutral schedule, Table 95, whichever will give a larger return to the shipper, except that a gold content of 0.05 ounce to 2 ounces is at $19 and a content of more than 2 ounces at $19.50.

Schedule No. 9. For LEAD CONCENTRATE. Gold, $19 per oz. when gold content is 0.05 to 2 oz. per ton; when content is more than 2 oz., $19.50 per ton. Silver and copper, as in neutral schedule, Table 94. Lead, as in Table 95, prices based on "quotation" of $4. Silica, limit, 10 per cent.; 10¢ penalty for each unit in excess of 10 per cent. Zinc, limit, 10 per cent.; 50¢ penalty for each unit in excess of 10 per cent.

Table 94. Neutral schedule for lead in lead ores and lead-copper ores

Lead content, per cent.	Value per unit	Smelting charge per ton (a)	Lead content, per cent.	Value per unit	Smelting charge, per ton (a)
5 to 10	$0.40	$6.00	35 to 40	$0.52	$1.50
10 to 15	0.43	5.00	40 to 45	0.52	1.00
15 to 20	0.45	4.00	45 to 50	0.53	1.00
20 to 25	0.47	3.00	50 to 55	0.54	1.00
25 to 30	0.49	3.00	Over 55	0.55	1.00
30 to 35	0.51	2.00			

a Includes freight charge from a certain district in Colorado, a comparatively short distance from the smelter.

Table 95. Schedule for lead in lead concentrate

Lead content, per cent.	Value per unit	Smelting charge, per ton	Lead content, per cent.	Value per unit	Smelting charge, per ton
5 to 10	$0.40	$3.75	20 to 25	$0.47	$2.25
10 to 15 *a*	0.43	3.00	25 to 30	0.49	2.25
15 to 20	0.45	2.50			

a More than 10 per cent.

Copper schedules. Payment is made on the basis of the wet assay less from 1 to 1.5 per cent. at prices representing certain deductions from New York prices for refined copper. New York quotations are for three grades: electrolytic copper, as ingots or wire bars; cathode copper, which is electrolytic

copper not re-melted into ingot form; and casting copper, which is not electrolytically refined. Cathode copper sells at about 0.1¢ less and casting copper at about 0.2¢ less per lb. than ingot copper. The usual smelter deductions

Table 96. Smelter deductions from market prices for copper

Kind of ore	Copper content of ore (a)	Deductions per pound
Lead ore containing copper...	1.5 to 5	6 cents off casting-copper quotation
	5 to 10	5 cents off casting-copper quotation
	More than 10	4 cents off casting-copper quotation
Copper ores..................	1.5 to 5	5 cents off electrolytic-copper quotation
	5 to 10	4½ cents off electrolytic-copper quotation
	10 to 20	4 cents off electrolytic-copper quotation
	20 to 30	3½ cents of electrolytic-copper quotation
	More than 30	3 cents off electrolytic-copper quotation

a Dry copper content represents content as determined by wet analysis less 1.5 per cent.

from the price paid for copper in lead and copper ores are shown in Table 96 (*TP 83, USBM*). The following are typical schedules.

Schedule No. 1. UTAH contract. Pay for all copper shown by wet assay less 1 per cent. at New York quotation for electrolytic wire bars less 2.5¢. 1 per cent. off selling price for selling. Base charge, $2.30 per ton of ore. Silica penalty, 12¢ per unit. Iron bonus, 10¢ per unit. Freight charge, on sliding scale (this is for camps in the region surrounding the smelter) as follows: Net value of ore after all deductions under $10, $1.75 per ton; $10 to $20, $2 per ton; $20 to $30, $2.50 per ton (*92 J 314* [*1911*]).

Schedule No. 2. Schedule on copper ores containing gold and silver: NEUTRAL SCHEDULE. Credits: Gold, $19 per oz. if product contains 0.03 oz. or over per ton. Silver, 95 per cent. of contents at New York quotation, date of assay. Copper, as per wet assay, less 1 per cent. or unit, at price for cathode copper quoted in *Engineering and Mining Journal-Press*, issue of week previous to date of receipt, less 2.5¢ per lb. Iron, paid for at 10c per unit. Charges: Insoluble, 10¢ penalty per unit. Zinc, 10 per cent. allowed free; excess at 25¢ per unit. Speiss (arsenic), 10 per cent. allowed free; excess at 25¢ per unit. Treatment charge, $3 per ton.

Table 97 summarizes copper-smelting contracts at various North American smelters.

Zinc Schedules. (*92 J 314* [*1911*]) Middle West zinc ores are usually bought outright by smelters and ore buyers on a base price for 60-per cent. sulphide ore or 30-per cent. carbonate ore with a penalty of $1 for each per cent. below the base and a bonus of the same amount for each per cent. above. In some cases a penalty is imposed for iron and in some also for lead. The iron penalty may run as high as $1 for each unit in excess of one. Lime also is sometimes penalized. Wisconsin carbonate zinc-ores, which are sold on a base of 30 per cent. zinc, carry a premium of 65¢ per unit in excess. Base prices are quoted in mining journals weekly.

Sliding scale for zinc has been tried, but was not favored by the miners. It was based on a price per ton of blende containing 60 per cent. zinc, less than 2 per cent. iron and 5 per cent. lead, of $37 to $41 per ton with spelter at 5¢ at St. Louis. A penalty of $1 per unit of zinc was imposed for each unit under 60 and a corresponding bonus allowed for each unit in excess of 60. The base price increased or decreased $8.50 per ton, respectively, with each corresponding difference of 1¢ in price of spelter (*92 J 314*).

European zinc-ore purchases are made by a formula $V = 0.95\ P(T - 8)/100 - R$, where V = the value per ton in any monetary unit, P = price of spelter at London in the same unit, T = units of zinc, and R = treatment charge in the monetary unit adopted.

Australian zinc ore. At BROKEN HILL zinc ores are penalized if they contain less than 5 per cent. of lead; nothing is paid for lead between 5 and 8 per cent.; a bonus is paid, if lead is in excess of 8 per cent.

Table 97. Summary of various

Plant	A	B	C	D
Gold	$19 per oz. if under 0.5 oz. per ton. $19.50 per oz. if over 0.5 oz. per ton.	All at $20 per oz. if over 0.5 oz. per ton. No payment for less than 0.5 oz.	95% at $20 per oz.	$19 per oz.
Silver	90% at New York quotation. No payment for less than one oz. per ton.	95% at New York quotation.	90% at New York quotation.	90% at New York quotation.
Copper	Wet assay, less 1.3 at Engineering and Mining Journal electrolytic quotation, less 4¢.	Full electrolytic assay less 15 lb. copper per ton of ore at Engineering and Mining Journal electrolytic quotation less 3¢.	Wet assay, less 1.3 at Engineering and Mining Journal quotation for electrolytic, less 3¢.	Dry assay or electrolytic, less 1.3 at New York quotation for electrolytic, less 3¢.
Insoluble	Insoluble, no charge.	....	....	....
Iron	No payment.	....	....	....
Lime	....	....	....	....
Arsenic, Antimony and bismuth	Arsenical and antimonial ores not received.	....	....	....
Treatment charge	$4 per dry ton.	$4 per dry ton.	$4 per ton if ore does not exceed $50 in value, and 25¢ per ton additional for every $5 or fraction thereof increase in ore value.	$2 per dry ton.
Briquetting	For concentrates and fine ores $1 per ton.	....	....	....

Note. A, B, C are California smelteries. D is a British Columbia smeltery.

copper-smelting contracts. (92 J 314)

E	F	G	H	I
If one ounce or more, $19 per oz.	Gold not paid for in quantities less than 0.5 oz. per ton	....	....	$19 per oz. if 0.02 oz. per ton or over, no payment for less.
95% of New York quotation, if one ounce or over per ton.				95% at New York quotations. No payment for less than one oz. per ton.
If 3% or more pay for 90% of full dry assay at quotation for electrolytic cathodes less 3¢.	Wet assay, less 1.3, at New York quotations less 2¢	Wet assay, less 1.3, at New York quotations, for electrolytic, less .... if ...% Cu. 5¢ if 1.4 to 5 4½¢ if 5 to 10 4¢ if 10 to 20 3½¢ if 20 to 30 3¢ if over 30	Wet assay, less 1.0 at New York quotations less 1%, less 2½¢.	Wet assay, less 0.85, at New York quotations for electrolytic cathode, less 2½¢.
Insoluble, all at 10¢ per unit penalty.			Insoluble, all at 12¢ per unit penalty.	Insoluble, all at 7¢ per unit penalty.
All paid for at 5¢ per unit.		....	All paid for at 10¢ per unit.	All paid for at 7¢ per unit.
If over 3% all at 6¢ per unit bonus.	....			....
3% free, excess at 50¢ per unit penalty.		If the silica excess is: Up to 25%, $7.50; 25 to 50%, $8; 50 to 75%, $8.50; above 75%, $9. Iron-oxide ore containing copper, $7. All these per dry ton.	....	....
On 3% copper $3 base, deduct 10¢ per each unit of copper contained in excess of 3%.	$12.50 per dry ton.		$2.40 per dry ton.	Freight and treatment, $4.25 per ton dry weight (freight $2.90 per wet ton).
$1.50 per ton when any lot contains 25% passing ¼-mesh.	....	....	....	....

E is a Southwestern plant. F is a Northwestern plant. G, H, I are Utah smelteries.

Silver ores. Table 98 (*92 J 364*) gives special schedules for treatment of silver ores from the Cobalt district. These ores are high-grade and contain large amounts of cobalt, nickel and arsenic. The schedules are special schedules for this particular type of ore. Most silver in this country is recovered by amalgamation and cyanidation or as a by-product in smelting base-metal ores and concentrates.

Table 98. Summary of schedules for Cobalt silver ores (*92 J 364*)

Plant	Balbach Smelting and Refining Company	Pennsylvania Smelting Company	Beer, Sondheimer and Co.
Silver	Pay for 93.5% at New York quotation.	95% at New York quotation less 1¢ per oz. for refining.	94% at New York quotation.
Arsenic	Penalty of 45¢ per unit in excess of 6.	....	....
Insoluble	Penalty of 6¢ per unit in excess of iron.	....	....
Treatment charge	$4 per ton on ores containing 1000 to 1500 oz. Ag. $20 when 1500 to 2000 oz. $19 when over 2000 oz.	$8 per ton.	$30 per ton.

Plant	Canadian Copper Company		Coniagas Reduction Company		Delora Mining and Reduction Company
Silver	Pay at New York quotation for		Pay at New York quotation for		Pay for 98% at New York quotation, less 1¢ per oz. for refining.
	Per cent. Ag	When over oz.	Per cent. Ag	When over oz.	
	75	100	75	20	
	84	200	84	200	
	86	300	86	300	
	87	400	89	500	
	89	500	91	750	
	90	600	93	1000	
	92	800	93.5	1500	
	93	1000	94.5	2000	
	93.25	1300	95	3000	
	93.5	1600			
	94.5	2000			
	94.75	3000			
Cobalt	Under 6%, no payment. 6 to 8%, pay $10 per ton of ore. 8 to 12%, $20. Above 12%, $30.		Under 6%, no payment. 6 to 8%, pay 8¢ per lb. 8 to 10%, 10¢. Above 10%, 12¢.		Under 6%, no payment. Over 6%, 10¢ per lb.
Treatment charge	....		$10 on ores containing less than 100 oz., unless the ore contains 12% or over of nickel and cobalt.		$20 per ton.

SECTION 3

COARSE AND INTERMEDIATE CRUSHING

CRUSHING PLANTS

1. Introduction

Ore invariably comes from the mine in lumps that are large in comparison to those that are amenable to concentration of any kind other than hand sorting. Crushing is, therefore, a part of all flow-sheets. Further, since the method of crushing, at least in the larger sizes, has little or no effect on the character or performance of the subsequent mill processes, crushing flow-sheets may be studied without regard to the mineral content of the ore.

The determining elements in every coarse-crushing flow-sheet are: (1) the nature of the material, especially as regards hardness and moisture content; (2) the dimensions of the largest lumps of run-of-mine ore; (3) the size of the largest particles that are to be sent to the fine-crushing plant; (4) the tonnage to be treated; (5) the daily running time; and (6) the location with respect to the mine and the balance of the mill. Table 1, arranged from a study of the effects of explosives on rocks (*Snelling, 28 Pro. Eng. Soc. W. Pa., No. 8*) is useful for estimating probable relative resistance to coarse crushing. The arrangement of machines will be affected by the type of plant construction, the kind of power and the method of distribution, and by considerations of accessibility and consequent economy of operation and maintenance. As a general principle, all crushers should have receiving openings of sufficient size to take the largest lump of feed without any aid from the operator, but under certain circumstances this rule may be departed from in the case of the initial crusher, which must have an operator continuously in attendance anyway to pick out powder, wood, rope ends, steel, etc., and this operator may be called upon to sledge an occasional oversize lump. The ratio of size of the largest lump entering the plant to that of the largest particle leaving determines in a general way the number of steps in the size reduction. Best practice rarely exceeds a reduction ratio of 4 : 1 in maximum size and ratios

of 3 : 1 and 2 : 1 are common. The number of hours per day during which ore is delivered from the mine, the storage and handling facilities, the daily and hourly tonnage, and the facilities for repairs, influence both size of machines and the number of reduction steps. Few underground mines hoist during all three shifts and few hoist continuously during the hoisting shifts. In many, if not most open pits, loading takes place only during daylight. Hence, unless ample storage is provided between the mine and coarse-crushing plant (and bin storage of this material is avoided where possible because of the tendency of coarse ore to clog in gates and chutes of any reasonable size) the capacity of the plant must be adapted to the rate of ore delivery from the mine. This calls for machines whose hourly capacity is from 1½ to 3 times that of the fine-crushing and concentrating equipment and for the provision of sufficient storage capacity between the coarse-crushing plant and mill proper to equalize the flow over 24 hr.

Table 1. Relative toughness of rocks and their resistance to fracture. (*After Snelling*)

Rock	Relative toughness (limestone = 1)
Limestone	1.0
Dolomite	1.0
Hornblende gneiss	1.0
Augite syenite	1.0
Biotite granite	1.0
Mica schist	1.0
Amphibolite	1.0
Andesite	1.1
Granite gneiss	1.2
Peridotite	1.2
Slate	1.2
Granite	1.5
Calcareous sandstone	1.5
Chert	1.5
Gabbro	1.6
Feldspathic sandstone	1.7
Altered basalt	1.7
Augite diorite	1.9
Biotite gneiss	1.9
Quartzite	1.9
Rhyolite	2.0
Hornblende granite	2.1
Diorite	2.1
Hornblende schist	2.1
Basalt	2.3
Altered diabase	2.4
Sandstone	2.6
Pyroxene quartzite	2.7
Fresh diabase	3.0

Coarse-crushing machinery is subject to enormous strain and considerable wear, and, notwithstanding its rugged character, must frequently be shut down for repairs. For this reason and in order that the shut-downs may not interfere either with mining or milling operations, the size and number of machines is usually such that part-time operation only is necessary to crush the daily tonnage, thus affording time for daily inspection and the making of minor repairs.

2. Flow-sheets

Examples of several types of coarse-crushing flow-sheets are given in Figs. 1 to 20, incl. These are all large-capacity plants, planned for one- or two-shift operation. (For examples of smaller plants see the flow-sheets in Sec. 2.) The flow-sheets fall naturally into two groups, depending upon the size of feed, which, in turn, depends upon the method of mining. When the feed is extremely coarse and bouldery, the product of steam-shovel mining, the plants contain two breakers of the jaw or gyratory type in series, followed by one or more intermediate crushers of the roll or disk type. With relatively fine feed (−18-in. or smaller), the product of underground mining, one of the breakers is frequently dispensed with, the rest of the plant being of the same general character as the plants for steam-shovel mines. Most of the plants close the circuit on the last intermediate crusher with a screen to insure that all of the material in the bins is less than the desired limiting size. The plants

at underground mines usually interpose a bin between the skip dump and the primary crusher. This permits steady feed to the primary crusher and hence to the balance of the plant, and smaller crushers can be used than if they were to be intermittently buried under an avalanche of several tons from the dumping skip. At steam-shovel plants bins are not ordinarily used ahead of the primary crusher because of the great difficulty of moving large lumps through the gates without excessive clogging. Hence the primary crusher at these plants must have a receiving opening large enough to take the largest lumps that the steam shovel can handle, and with these lumps the major part of the quarry-car load, and must be able to keep on crushing steadily when the receiving opening is buried. The primary requirements of such a crusher are, therefore, a large receiving opening and ability to utilize large overdrafts of power. It is rarely that the capacity required is greater than that of a jaw crusher of the necessary area of receiving opening and for this reason jaw crushers are usually placed in this position (see also Art. 9).

3. Steam-shovel mines

Plants characterized by direct feeding from cars and the use of two breakers in series.

Valhalla crushing plant, N. Y. City Water Supply. Fig. 1. (*36 MEW 451.*)

Capacity: 500 cu. yd. per hr. (est.)

Summary. No receiving bin. Jaw crusher, 60- to 9-in.; jaw crusher, 9- to 4-in.; rolls, 4.5- to 2-in. Rotary screen to close the circuit on the rolls. Bucket elevator.

The unusual feature of this plant is the use of a jaw crusher for the second crushing unit. A gyratory crusher of equal weight has greater capacity, smaller power consumption per ton crushed and a more favorable nip angle. The only apparent reason for use of the jaw-type machine is that it saved considerable head room and made it unnecessary to sink the elevator boot pulley any deeper into the ground. Since the plant was a temporary one, built to work only during the building of the Kensico dam, it may be that the higher running expense and greater initial cost of the jaw crusher were more than offset by the saving on the elevator. The lack of balance in reduction ratios in large and small jaw crushers is due to the inadequate capacity of the second crusher. To most designers the use of a bucket elevator for such coarse material is poor practice. It is true that its use gave a more compact and probably less expensive plant than would have been possible had belt conveyors been used for elevation (Fig. 2), but almost universal experience shows that the conveyors would have been cheaper and smoother in operation. The bin and pan conveyor ahead of the rolls are excellent design, insuring, as they do, against overfeeding and equalizing, to a certain extent, the fluctuations in tonnage rate incident upon intermittent feeding of carload lots to the primary crusher.

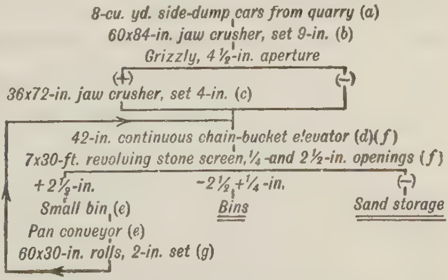


Fig. 1.—N. Y. City Water Supply, Valhalla crushing plant.

a, Dumped one at a time directly into the crusher jaws. *b,* Rope drive (sheave with 15 @ 1½-in. grooves, tension carriage and idler sheave) from a jack shaft belt-driven through a friction-clutch pulley from a 300-hp. @ 560-r.p.m. mill-type slip-ring induction motor with 250 per cent. full-load starting torque. Crusher, cast-steel; weight, 450,000 lb. Flywheels, 12 ft. diameter; weight, 15 tons each. Manganese-steel jaw and cheek plates in interchangeable sections. All bearings water cooled. Speed, 90 r.p.m. *c,* Cast steel. Weight, 210,000 lb. 200 r.p.m. Motor, 150-hp. @ 560-r.p.m., same type as above, belted

direct to crusher with 20-in. endless double-leather belt. *d*, Buckets, 42 × 18 × 19-in. of ¼-in. pressed steel with ¾ × 2-in. reinforcing bar strap, each carried on a shaft with steel-bushed rollers that travel on 30-lb. rail. Roller-shafts linked by double bars, ½ × 3 × 18-in., each side. Speed, 95 ft. per min. *e*, To equalize roll-feed rate. *f*, Elevator and trommel driven by a 50-hp. @ 850-r.p.m. squirrel-cage induction motor. *g*, Smooth shells, 50 r.p.m. Capacity, 300 cu. yd per hr. from 4-in to 2-in. Motor, 100-hp., 560-r.p.m., same type as crusher motors.

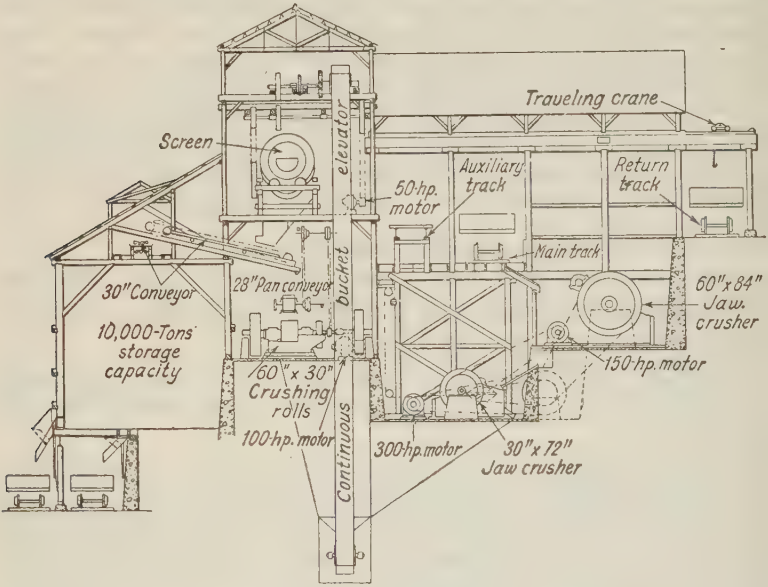


FIG. 2. —Elevation of Valhalla crushing plant.

New Haven Trap Rock Co., North Branford, Conn. Fig. 3. (*73 EN 582.*) Crushing trap rock from steam-shovel size to −2-in. at 350 to 400 tons per hr., 1-shift operation. Diagrammatic layout of plant is shown in Fig. 4.

Summary. No receiving bin. Jaw crusher, 48- to 10-in.; gyratory, 10- to 5-in.; gyratory, 5- to 1.75-in. Circuit closed by revolving screen and disk crusher. All plant transport and elevation by belt conveyors.

This flow-sheet shows the more usual arrangement for medium-large tonnages, *i.e.*, a large jaw crusher for the initial machine followed by a gyratory. The weight of these two crushers is less than that of jaw crushers for the same crushing capacity and the power consumption per ton crushed is also less. The arrangement that allows either of the first two crushers to be cut out without stopping the whole plant (see Fig. 4) is admirable. The great bulk of the crushing is done in the jaw and gyratory crushers and but little load goes to the disks, which serve essentially merely as accessories to the secondary gyratories, with the advantage, however, that they may be set down to increase, slightly, the tonnage of finer sizes, when there is such a demand. The four No. 6 gyratories are used in this plant to do substantially the same work done by one set of 60 × 30-in. rolls at the Valhalla plant. The gyratories weigh more, require more power and more transmission machinery, and probably, on the whole, cost more for repairs. They have the advantage that plant operation does not have to be stopped completely while repairs are being made, as is the case with the single-roll unit at Valhalla.

a, Dumped one at a time directly into jaws of primary crusher. Can be dumped into gyratory, if desired. *b*, 250-hp. motor drives jaw crusher, primary trommel and 30- and 36-in. conveyors; crusher draws 200 hp. Lenix drive, 10-ft. centers. *c*, Jaw crusher can discharge directly to 36-in. conveyor. *d*, 150-hp. motor. *e*, 150-hp. motor for the four. *f*, No. 2 trommels and 20- and 24-in. conveyors driven by 1 @ 75-hp. motor. *g*, Setting varied according to sizes of finished rock in demand. 1 @ 100-hp. motor. *h*, 20- and 30-in. conveyors and 6 screens driven by 150-hp. motor. *i*, 6-compartment, 3000 cu. yd. total capacity. Loads directly to cars through tilting chutes on the front or through bottom gates and an inclined conveyor to an auxiliary loading bin. (See Fig. 4.) The shipping yard has trackage for 40 @ 50-ton standard-gage empties ahead of the loading point and 40 loaded cars following. Cars move by gravity from empty yard to loaded yard. Tidewater shipping bins 6 miles distant by privately owned, single-track railroad. *j*, All screen jackets of manganese steel.

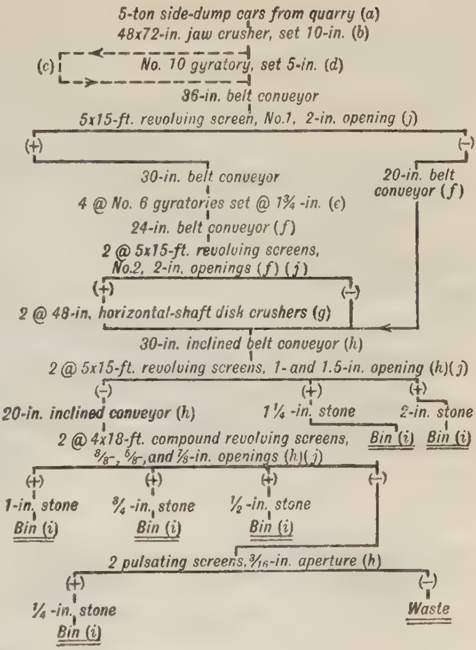


FIG. 3.—New Haven Trap Rock Co.

New Cornelia Copper Co., Ajo, Ariz. Fig. 5. (*108 J 465.*)

Crushing hard, bouldery ore from 4½ × 4½ × 10-ft. maximum to ¼-in. at 400 to 500 tons per hr., 2-shift crushing.

Summary. No receiving bin. Gyratory, 54- to 6-in.; gyratory, 9- to 3-in.; disk, 3- to 0.75-in.; disk, from 0.75- to 0.38-in. Circuit closed on final disk by means of screens. Coarse-ore transport by belt conveyors, −0.75-in. product raised in bucket elevator.

The primary gyratory was oversize for this plant when making the normal reduction to 8- or 9-in. and reduced a carload of material so rapidly that it flooded the secondary gyratories. The primary machine was, therefore, made to do more work by reducing the discharge opening to 6 in., thereby relieving the secondary machines, but, more important yet, distributing the feed to the secondary machines over a greater part of the total working time. Considerable trouble is caused by lumps too large to enter the primary machine. On the information available it would appear that a 60 × 84-in. jaw crusher followed by a flow-equalizing bin and feeder would give more satisfactory service at less cost than the big gyratory. A grizzly was subsequently put in ahead of the secondary gyratories and its use cut the repair costs on these machines materially.

The choice of disk crushers was dictated in part, at least, by the fact that the final product of these machines contained less fine material than that from rolls and the coarser product was more desirable for leaching. Comparing the product of this plant with that of the Utah Copper Co. rolls, the −20-mesh material in −⅜-in. screen product amounts to only 19 per cent. against 15 per cent. in the −1-in. product of the Utah plant.

In general disks cost more in supplies and time lost for repairs than do rolls, and they are frowned upon by many designers on this account. For coarse feeds and relatively small tonnages per unit, however, the weight and power consumption are so much smaller than for rolls that the disks are entitled to serious consideration.

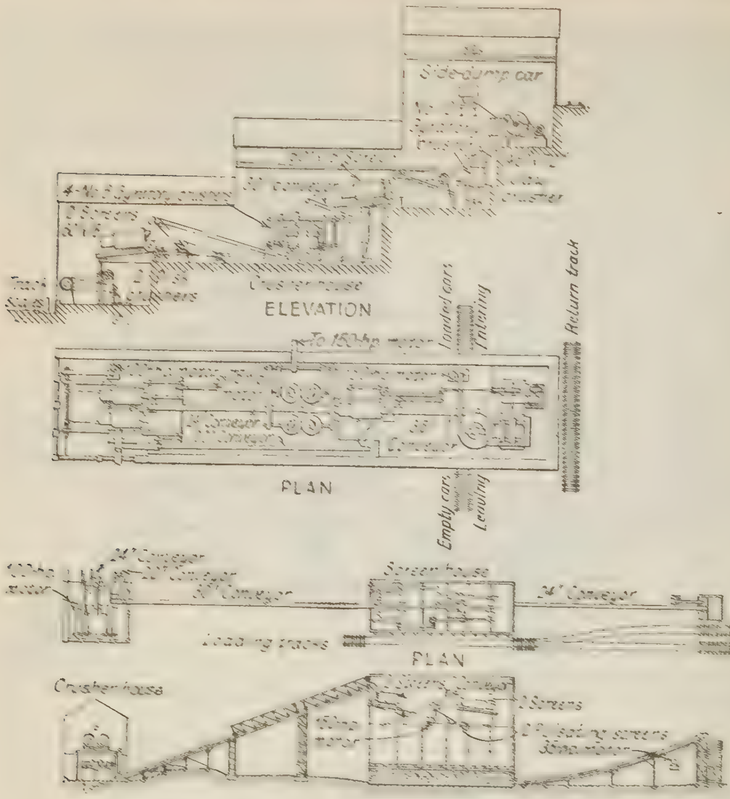


FIG. 4.—Plant of New Haven Trap-rock Co.

The use of bucket elevators here to handle −0.75-in. dry material is a relic of disappearing practice. While data are not available to support the statement, it is a safe prediction that the excess of lost time and repair charges due to the elevator over that due to belt conveyors in the same position, will, in the life of the plant, much more than counterbalance the excess in first cost that would have attended a conveyor installation.

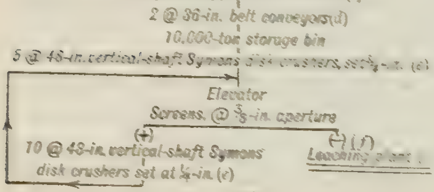


FIG. 5.—New Cornelia Copper Co.

a, Capacity, 35 to 37 tons each. Dumped one at a time directly into crusher hopper. *b*, Capacity of this crusher with head and concaves set for 9-in. discharge was too great (500 tons per hr.) for the gyratories following, hence lower ring of mantle and concaves was made thicker so that the discharge size was reduced to 6-in. and capacity became 400 to 450 tons per hr. *c*, Originally set for 4-in. discharge and "flooded" at 500 tons per hr. (125 tons each) of − 9-in. product of primary breaker. Capacity to −3-in. on −6-in. material from primary breaker is in excess of 112 tons per hr. *d*, Large

and powerful magnets are suspended over each belt and magnet head pulleys are used. The latter alone would not remove all tramp iron. *e*, Coarse crusher has 4-in. annular grinding surface on upper disk; lower, 6-in. Repairs low on coarse disk but high on fine disks. Capacities are 100 tons and 50 tons each per hr., respectively. One 75-hp. induction motor on each machine. *f*, Average screen test, per cent: +3-mesh, 26.3; +4, 16.6; +6, 12.8; +8, 9.1; +10, 7.0; +14, 5.0; +20, 4.2; −20-mesh, 19.0. *g*, Fine dust caused serious trouble. Wetting down in cars allayed slightly, but dust-collecting system had to be added. *h*, See Sec. 15, Fig. 4.

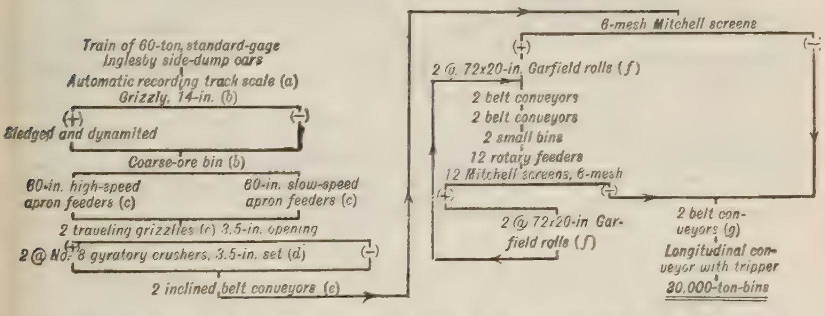
Nevada Consolidated Copper Co., McGill, Nev. Fig. 6. *(123 P 326.)* Steam-shovel mine, ore very soft with considerable clayey material. 13,000 tons per day crushed from steam-shovel size to 6-mesh.

Summary. Receiving bin. Sledging to 14-in.; gyratory, 14- to 3.5-in.; two sets of rolls in series from 3.5- to 0.12-in. Circuit closed on final rolls by means of vibrating screens. All transport and elevation by belt conveyors.

In this flow-sheet the usual arrangement following steam-shovel mining is departed from, a coarse-ore bin precedes the initial crusher and one breaker step only precedes the intermediate crushing. The reasons are: (1) the ore is very soft, breaks relatively small at the mine, and requires but little sledging and mud-capping on the grizzlies to reduce it to pass a 14-in. opening; (2) yard space for storage and switching ahead of the crusher was more expensive to build than a bin; (3) a small amount of hand work in preliminary breaking saved the cost of a large primary crusher.

The reduction made in two steps by the rolls is unusually large. Taking the maximum safe reduction in the first rolls (30° nip angle) would set them at 0.9 in., which leaves a reduction ratio of 7.5 : 1 for the fine rolls. Such work as this at the high capacities maintained would be possible only with very soft ore.

Cost in 1921 was $0.053 per ton.



a, Streeter-Amet. Each car weighed automatically at train rate of 3 miles per hr. Permanent stamped record automatically kept. *b*, Single track on top of a bin 288 ft. long, 29 ft. wide and 14 ft. deep. Horizontal grizzly each side of track, 70-lb. rail, base up. 2-in. compressed-air pipes ending on hopper bottom about 3 ft. from gates aid discharge greatly. *c*, 24 feeders, 8 ft. 9 in. long, set under hoppers spaced along longitudinal center line. All but two center feeders run toward transverse center line, two center feeders run toward ends. The two feeders (oppositely directed) in each half of the bin nearest transverse center line are high speed (about 5 ft. per min.) and discharge onto 60-in. / 118-ft. transverse traveling-belt grizzlies that feed the two gyratory crushers. All other feeders on each side feed onto a 60-in. pan conveyor, thence to the above-mentioned traveling grizzlies. Speed of these feeders is 2.5 ft. per min. *d*, 100-hp. motor, direct-connected to pinion shaft. *e*, 42 in. wide, 120 ft. long; slope, +19° 20′; speed, 250 ft. per min. *f*, Both drive pulleys same side, outboard bearing on fixed roll. 150-hp. motor to each set of rolls. *g*, 42 in. wide, 270 ft. long; slope, +16°; 275 ft. per min.

Fig. 6.—Nevada Consolidated Copper Co., coarse-crushing plant.

Motors. Total hp. @ 860. Every machine except feeders has an individual motor.

Conveyors. All are 42 in. wide; belts, 8-ply with 1/8-in. rubber cover; ball-bearing troughing idlers and Hyatt roller-bearing return idlers.

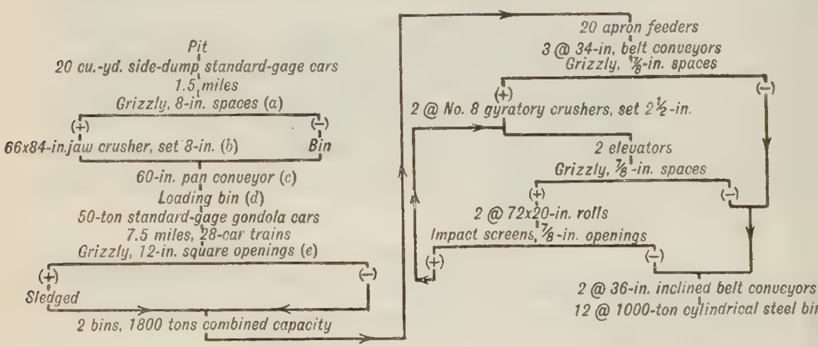
Chutes. All made of sheet steel lined with cast iron or boiler plate.

Chino Cons. Copper Co., Santa Rita, N. M. Fig. 7. (*116 J 1120; 117 J 13.*)
Crushing from steam-shovel size to ⅞-in. at the rate of 12,000 tons per 24 hr. For layout at mine see Fig. 8.

Summary. No receiving bin. Jaw crusher, 60- to 8-in.; gyratory, 8- to 2.5-in.; rolls, 2.5- to 0.88-in. Circuit closed on final rolls by means of impact screens. Bucket elevators used for −2.5-in. material.

This plant is remarkable only in the fact that the initial breaker and secondary breaker are separated by several miles of rail travel. The jaw crusher is the proper initial crusher for the relatively small tonnage of coarse material. Compare with the Utah plants treating a much larger tonnage of similarly sized though softer ore. The use of the bucket elevator for −2.5-in. material marks the age of the plant; without question conveyors would be used for this service in case of rebuilding.

Cost of initial crushing (1924) was $0.0125 per ton.



a, I-beams with manganese-steel caps. *b,* 48 × 60-in. was too light. Now in reserve. Weight, 475,000 lb. Throw at throat, 3¼ in. 80 r.p.m. *c,* Incline, +15°. *d,* 40 ft. long, 14½ ft. wide, 23 ft. high; 567 tons capacity. 6 chutes each side. 50-ton car loaded in 15 min. Louvres over gates. *e,* 60-lb. rail grid under track, over bins.

FIG. 7.—Chino Consolidated Copper Co.

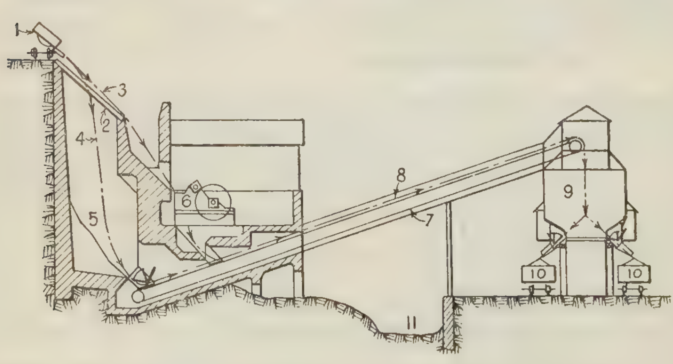


FIG. 8.—Lay-out of coarse-crushing and loading plant, Chino Copper Co.

1. Dump-car train from steam shovel. *2.* Grizzly with 8-in. openings. *3.* Oversize. *4.* Undersize. *5.* Fine-ore bins, 1500-ton capacity. *6.* 84 × 66-in. jaw crusher, 80 r.p.m. *7.* 2 @ 5-ft. steel pan conveyors, 60 ft. per min. *8.* −8-in. *9.* 1450-ton loading bins. *10.* 50-ton cars. *11.* Creek bed.

4. Underground mines

Plants characterized by the use of one breaker only, preceded by a coarse-ore bin.

Ohio Copper Co., Lark, Utah. Fig. 9. *(99 J 749.)*

Crushing from 14-in. to −⅜-in. at 125 tons per hr., one-shift crushing.

Summary. Receiving bin. Jaw crusher, 14- to 2-in.; rolls, 2- to 0.55-in.; rolls, 0.55- to 0.19-in. Circuit closed on final rolls by means of stationary screen. Conveyor and bucket-elevator transport.

This plant is typical of good reduction practice for relatively low tonnages, using a jaw crusher for primary breaking, a pan conveyor to feed the crusher, and removal of fines ahead of the crusher. A belt conveyor in place of the belt-bucket elevator would have cost more for construction, but would easily pay for itself in smoother and more continuous operation. Vibrating screens would undoubtedly replace the stationary screens in a present-day design.

a, Set on slope to elevate from bin to crusher. 15-hp. motor for conveyor and two feeders. ***b,*** Cast-steel T-shape bars; top tapers from 2½ in. at head end to 1½ in. at lower end, thus making openings 1 in. wide at the upper end and 2 in. wide at the lower end of the grizzly. This prevents clogging. ***c,*** Blake type. 150-hp. motor drives this and coarse roll. ***d,*** 70 r.p.m. (See note *c*.) ***e,*** 75-ft. lift, 410 ft. per min. 60-in. head pulley, 42-in. boot pulley. Large concrete pit with broad stairs. Lower part of housing counterweighted to swing away easily. 12-ply belt. 2 rows 9 × 9 × 18-in. buckets staggered 30 in. apart. 3 × 6-in. knocking timbers suspended over the head pulley to jar buckets and loosen sticky ore. 50-hp. motor. ***f,*** Set at 45°. Heavy wire cloth with ⅜ × 1-in. openings. Easily turned end-for-end or over. Efficient except with damp, sticky ore. ***g,*** 70 r.p.m. 75-hp. motor on each. ***h,*** Roll shells 6 in. thick; in two sections, one 10 in. and the other 14 in. wide. Shrunk on. Heat from dry crushing loosens shells unless carefully shrunk. ***i,*** 10-hp. motor.

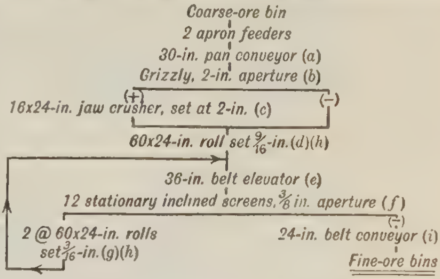


FIG. 9.—Ohio Copper Co., coarse-crushing plant.

United Verde Copper Co., Hopewell, Ariz. Fig. 10. *(104 J 117.)*

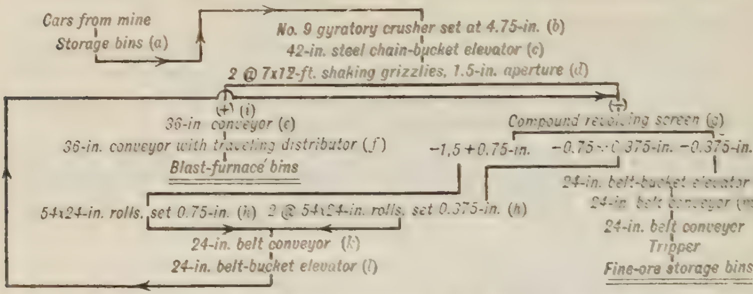
Crushing from 18-in. to −⅜-in. at 400 to 450 tons per hr.

Summary. Receiving bin. One initial gyratory crusher, 18- to 4.75-in.; +1.5-in. lumps not further crushed; rolls, 1.5- to 0.75-in.; rolls, 0.75- to 0.375-in. Circuit closed on final rolls with revolving screens. Conveyor and bucket-elevator transport.

This plant would probably have one or more gyratory crushers to take the oversize of the shaking grizzlies were it not for the fact that this coarse material is wanted for the blast furnaces. The flow-sheet would then become the two-breaker type, or, by making a larger reduction in the primary breaker and using either larger rolls, disk crushers or reduction gyratories for the second machines, the present type of flow-sheet could be preserved.

The secondary crushing is directly comparable with that at NEW CORNELIA (p. 233), eliminating the question of desirability of a granular product at the latter plant. At Hopewell three 54 × 24-in. rolls are used to crush 300 tons per hr. from −1.5-in. to −⅜-in., while at NEW CORNELIA fifteen 48-in. disks crush 425 (aver.) tons per hr. from −3-in. to −⅜-in. Ordinary smooth-faced rolls could not be used to break −3-in. feed to −0.75-in., as is done by the coarse-crushing disks at NEW CORNELIA, on account of nip-angle limitations. It would require 100-in. (diam.) rolls to make this reduction and 78-in. is the largest made (1925). A roll plant for the NEW CORNELIA tonnage, based on Hopewell practice, would require one pair of 78 × 20-in. rolls from −3-in. to −1.5-in., one pair of 60 × 24-in. from −1.5- to −0.75-in., and 3 pairs of 54 × 24-in. rolls to −⅜-in. This is 5 rolls against

15 disks, with a marked advantage for the rolls in first cost and operating expense. See also the comment on the Clarkdale plant of United Verde.



a. One 2-compartment bin, 3600-ton capacity discharged by air-operated slide gates and one 6800-ton auxiliary bin discharging to a 54-in. shuttle belt conveyor with 18 (diam.) × 58-in. 16-section 16-pole magnetic head pulley, thence to a 42-in. pan conveyor, lined with 4-in. maple blocks and running at 40 ft. per min. delivering to crusher chute. Motors: one 5-hp. on shuttle; one 50-hp. on pan conveyor. *b.* 150-hp. slip-ring induction motor. *c.* 68-ft. lift, 45 ft. per min. 75-hp. slip-ring motor. *d.* 11½° slope, 1⅝-in. eccentric, 220 strokes per min., the two driven by a 75-hp. slip-ring motor. Capacity of each grizzly, 400 tons per hr. *e.* 300 ft. per min., 25-hp. induction motor. *f.* 15-hp. motor. *g.* Inside screen, 5 × 18-ft., 0.75-in. holes; outside 80-in. × 15-ft., 0.375-in. holes. Both screening surfaces made of manganese-steel quadrantal segments. *h.* Anaconda type, 1040 ft. per min. peripheral speed, 150-hp. motor. *i.* Amounts to about 30 per cent. of total feed. *j.* 26-in. belt, 250 ft. per min. *k.* 15-hp. motor. *l.* 35-hp. induction motor. *m.* 300 ft. per min. Normal capacity, 200 tons per hr.; possible overload capacity, 150 per cent. *n,* This plant was later superseded by the Clarkdale plant.

Fig. 10.—United Verde Copper Co., Hopewell plant (*n*).

United Verde Copper Co., Clarkdale, Ariz. Fig. 11. (*112 J 833*.)

Crushing from about 18-in. to −3-in. and −¼-in. at 500 tons per hr. For arrangement of machines, see Fig. 12.

Summary. Receiving bin. Four initial jaw crushers, 24- to 3-in.; disk crushers, 3- to 0.75-in.; rolls, 0.75- to 0.25-in. Circuit closed on final rolls by means of screens. Conveyor for coarse and bucket elevator for −0.75-in. material.

This plant is of particular interest when compared with the Hopewell plant of the same company, which it replaces. Four initial crushers of the jaw type replace one gyratory. This increases both first cost and operating cost, when operations are at full capacity, but permits more economical low-capacity operation. The jaw-type crusher is better fitted to handle the clayey ore than the gyratory. Disk crushers replace rolls for intermediate work. This is an economy both in first cost and power cost for the relatively small unit capacities planned and is an example of the use of the disk in the position to which it is ideally suited, *i.e.*, taking relatively coarse feed at a relatively low rate (as compared with big rolls) with less metal and less installed power required per ton of daily capacity. Fine crushing (to 0.25-in.) is done in rolls, which is undoubtedly better practice from the crushing standpoint alone than to use disks. Conveyors have replaced the chain-bucket elevator for handling crusher product but bucket elevators are retained for −0.75-in. material. Storage capacity has been largely and cheaply increased by the use of the stock pile.

Motors. All over 50-hp. are 2200-volt, those under are 440-volt. All motors have push-button control. Remote and multiple push-buttons are used on crushers and long conveyors. All conveyors and feeders on a given ore stream can be stopped by a main button. An electrical interlock controls starting and prevents any piling up of material. Power consumption, 3 kw.-hr. per ton.

Labor. 35 to 40 men; cost about $0.05 per ton.

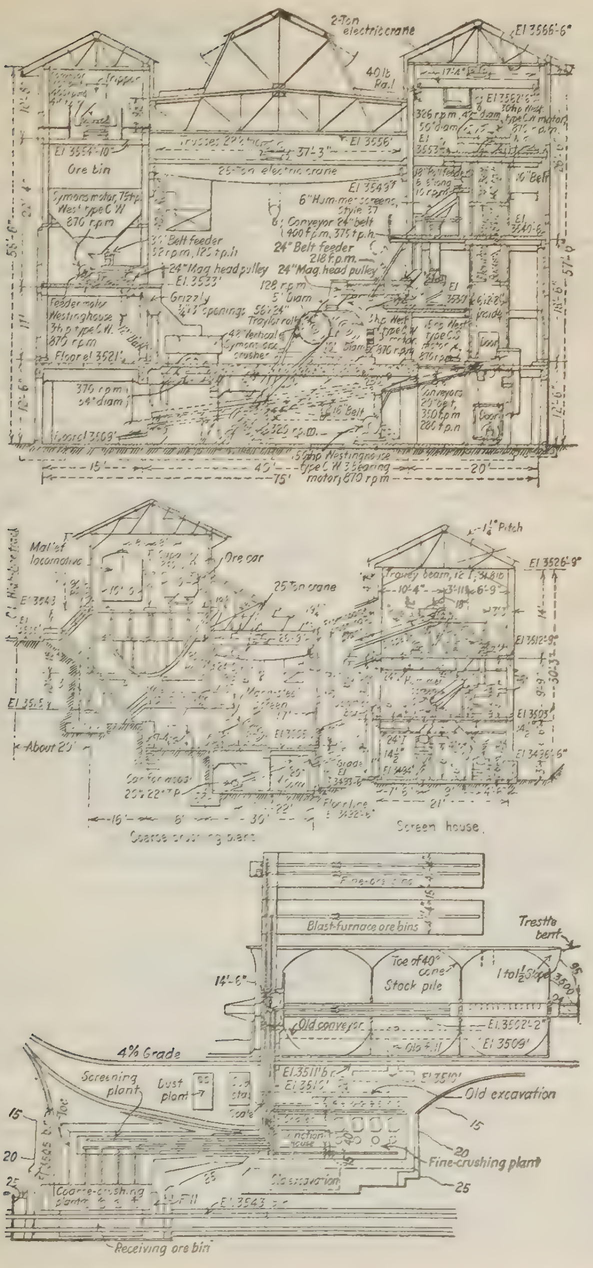
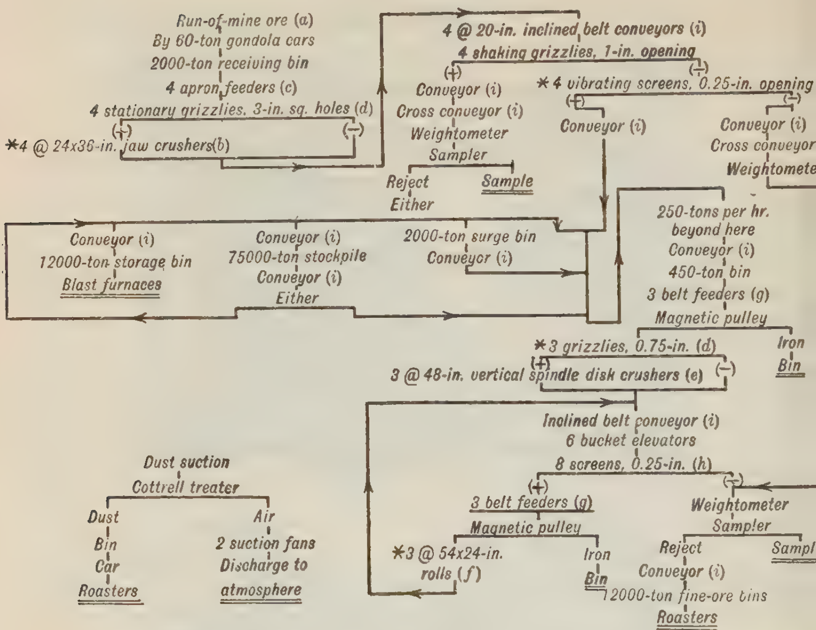


FIG. 12.—Clarkdale crushing plant, United Verde Copper Co.



* Connects with dust-collecting system. *a,* Includes hard, massive sulphide from underground workings and both silicious ore and soft clayey material from steam-shovel pits, the hard preponderating. *b,* Each driven by a 125-hp. motor through short-center belts with tighteners. *c,* Driven by individual variable-speed motors with speed reducers. *d,* Manganese steel. *e,* Foundations for a fourth in place; 75-hp. motors for each. *f,* Foundations for a fourth in place; 150-hp. motors for each. Open belts provided with tighteners. *g,* Driven by individual variable-speed motors with speed reducers. Magnetic head pulleys. *h,* 4 Mitchell and 4 Hummer. *i,* Sizes, 20-, 24- and 30-in.; speeds, 250 to 380 ft. per min, depending upon width and capacity. Wide belts are used for coarse material. Short belts are chain-driven from gear speed reducers, intermediate lengths are direct-geared, the long inclined conveyors have tandem drives. Inclines limited to 19°. Conveyors following crushing machines are fed over grizzlies in order to put fines on belt first.

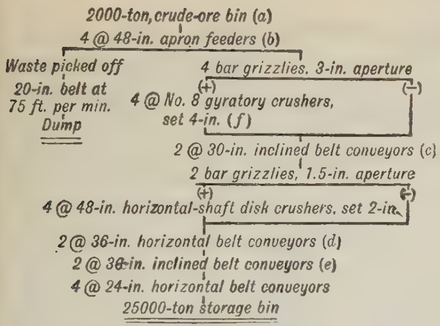
FIG. 11.—United Verde Copper Co., Clarkdale plant.

Inspiration Cons. Cop. Co., Miami, Ariz. Fig. 13. (*55 A 707.*)

Crushing medium-hard granitic porphyry and schist from −12-in. to −2-in. at 1000 tons per hr. 2-shift operation. Arrangement of machines is shown in Fig. 14.

Summary. Receiving bin. Four initial gyratory crushers from 12- to 4-in.; disk crushers from 4- to 2-in. Open circuit. All-conveyor transport.

This plant shows a smaller first cost for crushing machinery and a greater tonnage crushed per unit of installed power than any of the other plants discussed. (See Table 2.) These low-cost figures are due to the fine feed and coarse product and to the fact that the circuit is not closed. On the other hand, labor cost is relatively high due to the fact that four primary crushers require four attendants to pick the feed. The plant has the great advantage that it is built in four substantially independent units so that capacity may be reduced without lowering the efficiency, and 24-hr. operation is possible with only slight loss of capacity for repairs. Floor space is 1.3 sq. ft. per ton of daily capacity. Actual power consumption is about 0.33 kw.-hr. per ton crushed.



a, Steel, double-hopper bottom. *b*, 25 ft. 6 in. long, 7 ft. per min. 20-in. belt conveyors at 7 ft. per min. placed underneath catch drip and spill and deliver to grizzly-undersize chute. *c*, Magnetic head pulleys. *d*, Short cross conveyors. *e*, 300 ft. long, 350 ft. per min. *f*, Each unit of the 4 shown is driven by one 200-hp. motor, placed in dust-proof rooms.

FIG. 13.—Inspiration Cons. Copper Co., coarse-crushing plant.

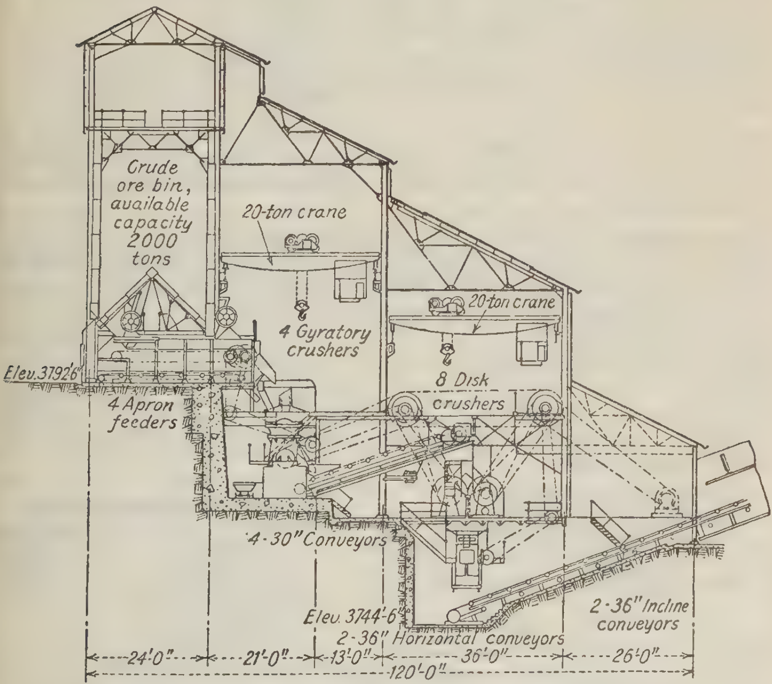


FIG. 14.—Section of coarse-crushing plant at Inspiration.

Alaska Gastineau Mining Co., Juneau, Alaska. Fig. 15. *(63 A 492.)*

Ore: Quartz lenses in slate, schist and metagabbro; 450 tons per hr.

Summary. Receiving bins. Two initial jaw crushers, 30- to 5-in.; gyratory crushers, 10- to 2-in.; rolls, 2.5- to 1-in.; rolls, 1- to 0.1-in. Circuit closed on intermediate and final rolls by means of impact screens. Automatic skip hoists for elevation.

Table 2. Comparison of coarse-crushing plants

Plant	Feed, maximum, inches	Product, maximum, inches	Tons per hour	Approximate weight of crushing machinery	Approximate horse-power installed for crushers only	Tons crushing machinery per ton of hourly capacity	Tons crushed per horse-power-hour (installed)
Valhalla	48–54	2½	675	860,000	550	0.6	1.2
New Haven	36–42	2	375	690,000	600	0.9	0.6
Utah Copper, Arthur	48–54	1	1650	1,360,000	1300	0.4	1.3
Utah Copper, Magna	48–54	−0.75	1000	1,980,000	2750	1.0	0.4
New Cornelia	48–54	⅜	425	1,540,000	1825	1.8	0.2
Ohio Copper	14	⅜	125	340,000	300	1.4	0.4
United Verde, Hopewell	18	⅜*a*	425	325,000	300	0.4	1.4
United Verde, Hopewell (*b*)	18	⅜	425	545,000	575	0.6	0.7
Inspiration	12	2	1000	480,000	640	0.2	1.6
United Verde, Clarkdale	18*c*	¼	500	1,080,000*d*	1950*d*	1.1	0.3
Nevada Consolidated	14	6-mesh	500	650,000	800	0.6	0.6
Alaska Gastineau	30*c*	10-mesh	450	1,650,000	2200	1.8	0.2
Chino Cons. Cop. Co.	54–60	⅞	500	885,000	800	0.8	0.6

a But note that about 130 tons is rejected at − 4.75 + 1.5-in. *b* Estimate based on roll crushing all feed to pass ⅜-in. *c* Estimated. *d* Estimated on basis of 500 tons per hour down to ¼-in. Present capacity is 500 tons per hour through the initial crushers and 250 tons beyond this.

This, although an underground mine, can handle lumps into the mill-ore trains of a size approximating those from steam-shovel mines. The coarse-crushing flow-sheet is, therefore, of the steam-shovel-mine type with 2 breakers in series ahead of the intermediate crushers. The use of a bin for + 10-in. material ahead of the initial crushers is unusual and fraught with danger of clogging in the discharge gates. The breakers are installed on shelves on the side of a great hole cut in solid rock and material flows by gravity from the tipples to the bin storing the − 2½-in. size Reduction in final rolls is larger than is customary. The use of automatic hoists is unique. Several of the men intimately and responsibly connected with their operation vouch (*PC*) for their smooth and satisfactory operation, after the troubles incident to starting were overcome. They have the advantage over belt conveyors of making a very compact plant, particularly where large vertical lift is necessary; and with the mechanical difficulties incident to a new type of machine overcome, should be much superior to bucket elevators.

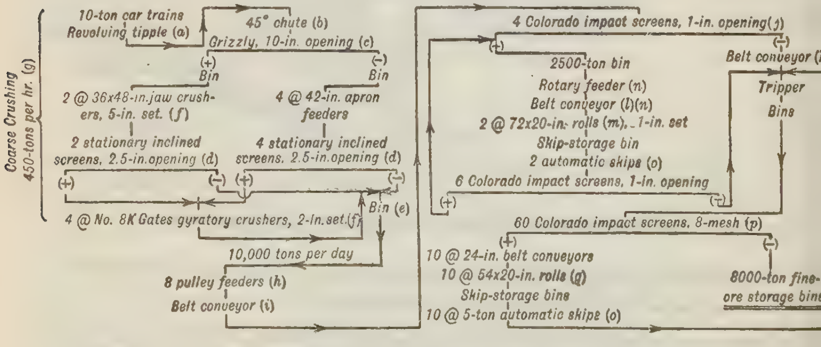


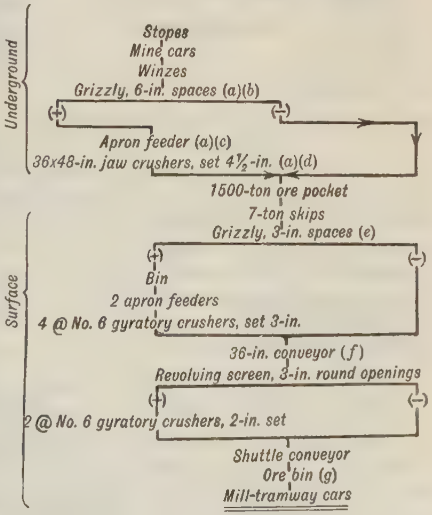
FIG. 15.—Coarse-crushing plant of Alaska Gastineau Mining Co.

a, Capacity, 4 cars. 2 tipples operated by 50-hp. motor. Friction clutch between motor countershaft and tipple mechanism, band brake on tipple. *b*, Lined with worn-out roll shells, 1 in. thick, straightened at red heat by steam hammer. Cheaper than boiler

plate and has longer life. *c,* 8-in. steel I-beams with manganese-steel caps. Several beams failed by bending after 6,000,000 tons, but caps were still good. *d,* 3 ft. (wide) × 14 ft.; 45° slope; crimped-wire cloth. *e,* Underground, 8000 tons cap. *f,* One jaw and two gyratory crushers with grizzlies and screens constitute a separate unit. Crushers are clutch-driven from a countershaft which, in turn, is driven by a 200-hp. motor. *g,* Crew: 1 crusher man, 1 feeder man, 1 oiler each shift; 1 repair man and helper on day shift; rigger crew as necessary for changing jaw plates, mantles and concaves. *h,* 36 × 36-in. Different feeders operated at different times to overcome size segregation in bin. *i,* 42-in., 1216 ft. long, 8-ply, slight down slope. *j,* All steel, extra heavy. *k,* 42-in.; 8-ply belt. *l,* 36-in. *m,* 80 r.p.m. 300-hp. motor for rolls, conveyor and feeder. *n,* Driven from roll shaft. Capacity: oversize from daily mill feed of 12,000 tons per 24 hr. *o,* Each 5-ton capacity. 75- to 135-hp. Westinghouse hoist motor operates one hoist drum for 2 skips in balance. Air cylinder operates skip-loading gate through 3-way valve on a shaft actuated by the descending skip. Loading time (11 sec.) determined by an oil dash pot that throws the motor switch. Lift, 100 ft. Maintenance cost $0.009 per ton milled. *p,* 0.032-in. aperture (some are 9-mesh, 0.028-in. aperture). Steel-wire cloth. 600 vibrations per min. Manganese-steel cams and tappets. *q,* Set to grind to 10-mesh. Roll feed sprayed to lay dust. One 300-hp. motor direct-connected to a line shaft for each two sets of rolls. Capacity, 1100 tons per 24 hr. from 1-in. to 10-mesh.

5. Underground crushing plants

Underground crushing plants are used to lessen the difficulty of loading large lumps into the hoisting skips, and thereby decrease shaft delays. They are to be considered only when the character of the ore and the method of mining produce large lumps at the stope chutes and skip-hoisting is employed. Under such circumstances placing the primary breaker underground may result in sufficient time and labor saving in skip-loading to counterbalance the disadvantages, of which the principal ones are the high cost of installation; separation of the initial crusher from those following, with consequent increase in labor cost and superintendence; and dust production. Where ore haulage takes place on several levels, the output is usually sent to one crusher by long steeply inclined chutes through which ore travels by gravity. At HOMESTAKE (see below) the working levels extend 1500 to 1800 ft. vertically, necessitating 3 crusher levels, to save excessively long ore passes and excessive loss of elevation of the ore. The disadvantages of this arrangement are the multiplication of initial crushers and their wide separation, with the attendant multiplication of labor and difficulty of supervision. At the Lebanon plant



a, One each at 800-ft., 1400-ft. and 2000-ft. level. (See Fig. 17.) *b,* Made of 6-in. steel shafting on cast-iron supports. *c,* 5 ft. wide × 5 ft. 3 in. long. *d,* Frame, semi-steel, cast in halves which are held together by keys and heavy steel bands. Motor, 125-hp., 720 r.p.m., 2200-volt, slip-ring type. Lenix drive. Crusher capacity, about 200 tons per hr. *e,* Hard cast-iron wedge bars, 6 in. deep, 1½ in. wide at top and 1 in. at bottom. *f,* Magnetic head pulley. *g,* 100 ft. long, 20 ft. wide, 60 ft. high.

FIG. 16.—Homestake Mining Co.

of Bethlehem Steel Co. the primary crusher is placed at the bottom of the open pit and crushed product hauled away through a tunnel. Fig. 18 shows the flow-sheet, with underground crusher in an open pit at the Cornwall mine (*88 J 725* [*1909*]). At the Creighton mine, Ontario (*99 J 192*) a 30 × 42-in. jaw crusher was installed on the lowest level, arranged to be fed from this and the upper levels, as shown in Fig. 19. Another arrangement, used at Crown Mines, Ltd., on the Rand (*96 J 118*), is shown in Fig. 20.

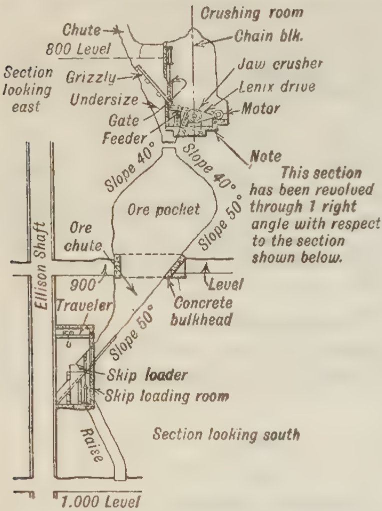


Fig. 17.—Underground crushing station at Homestake.

Homestake Mining Co., Lead, S. D. Fig. 16. (*122 P 539.*)

Ore: Hard quartz and slate.

Summary. Receiving bin (ore pass). Jaw crusher from 30- to 4.5-in.; gyratory crushers from 6- to 3-in.; gyratory crushers from 3- to 2-in. Open circuit. Conveyor and skip transport.

6. Stage crushing

It is an axiom of modern crushing practice to perform the reduction in size from mill feed to final tailing in stages, irrespective of the demand for stage crushing imposed by metallurgical requirements. Experience has taught that this makes for economy, both in power consumption and maintenance. Argall recommended a maximum reduction ratio of 4 to 1 for breakers, *e.g.*, 8-in. to 2-in. maximum size, and condemned the use of stamps crushing from 2-in. to 0.02-in. (100 : 1) as very inefficient. He pointed out that certain machines are best adapted to work within a given range of size reduction, *e.g.*, breakers down to 2-in. and rolls for the next step but not for coarser material. Barring occasional exceptions, such as the greater efficiency of Chilean mills on ¾-in. feed than on ¼-in. feed or finer and of tube mills on ¼-in. feed as compared to 10- or 20-mesh, the smaller the reduction ratio down to, say, 2 : 1, the greater the efficiency of the machines. In coarse crushing plants this fact has been long recognized; reduction ratios in a given

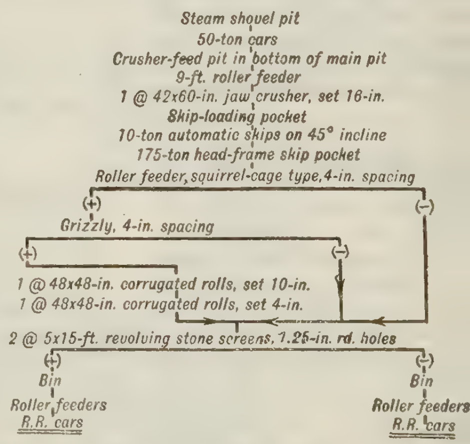


Fig. 18.—Coarse-crushing plant, Cornwall mine.

machine rarely exceed 4 : 1, except occasionally in case of the primary breaker, and are generally less than 3 : 1.

Closing the circuit on a coarse crusher results in making it work through too great a range of initial sizes and imposes an additional task that could

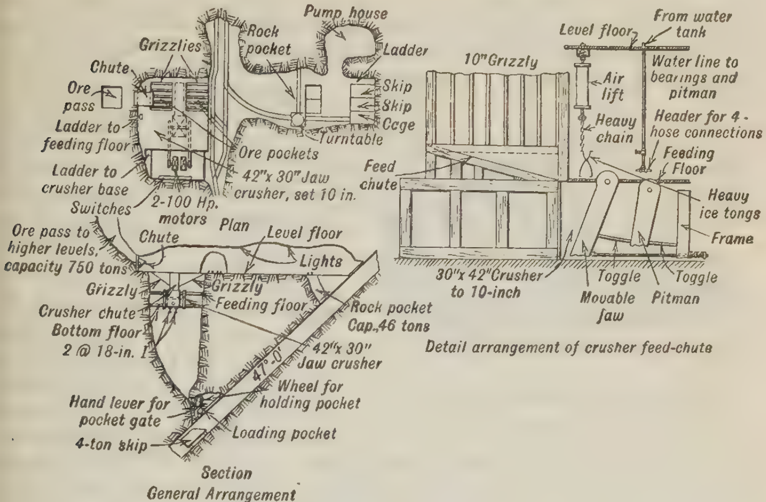


Fig. 19.—Underground crushing station at Creighton mine.

be much better performed by a crusher adapted particularly to breaking the material returned, hence coarse-crushing circuits are closed only when necessary and then only with respect to the final crusher. This requires more crushers, but the crushers work more efficiently, screens and transporting equipment are reduced in number and amount, and overall economy is increased.

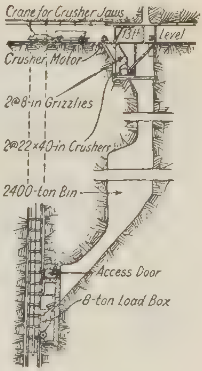


Fig. 20.—Crushing station on 13th level, Crown Mines, Ltd.

Sending natural feed to a crusher results in giving it material that it is not adapted to crush and increases wear and power consumption, therefore undersize is usually screened out ahead of crushers except where this would cause too great complexity of design.

When rolls are used in fine-intermediate crushing, say, to produce 10-mesh roughing-table feed prior to flotation, an apparent exception to the rules against large reduction ratio and closed circuit is found in the practice of feeding material as coarse as 1-in., setting the roll faces close and building up a large return circuit with a screen. But this is done in order to establish a condition of choke-crushing, otherwise impossible to attain with any degree of economy, and, as a matter of fact, such rolls are to be considered as grinders, in which larger reduction ratios are desirable than in coarse and intermediate crushing. Stamps were for many years looked upon as fine

grinders, and experience seemed to show greater economy in crushing throug say, 16-to 20-mesh screen, if the feed was 1- to 2-in. maximum size than if it w

Table 8. Classification of common crushing machines

Machine	Maximum feed, inches	Minimum product (a)
Jaw crusher	66b	2-in.(c)
Gyratory crusher	52d	2-in.(c)
Reduction gyratory	8	$\frac{1}{4}$-in.(c)
Disk	$4\frac{1}{2}$	$\frac{3}{8}$-in.(c)
Rolls (smooth-face)	3–4	10-mesh
Stamps	$1\frac{1}{2}$–$2\frac{1}{2}$	$\frac{1}{2}$-in. to 10-mesh
Rod mills	1	10- to 35-mesh
Ball mills	$\frac{3}{4}$–3	65- to 200-mesh
Pebble mills	$\frac{1}{2}$	65- to 300-mesh

a Meaning the size of the largest particle in the product when the particular type of machine is set to crush as small as is economically desirable. Where a range of sizes is indicated the machine is suited to the production of any maximum size within the range. *b* In a 66 × 84-in. machine. An 84 × 120-in. machine is advertised by one maker but none has been built. *c* This size does not, of course, correspond to the feed size in the preceding column, but to a smaller machine. Finer crushing in this machine may be justified on the ground of simplicity or cheapness, in small plants. *d* This is the minimum dimension of the largest particle that a No. 27 machine (54-in. gape) will take. Machines of 72-in. gape are advertised by one maker but none has been built.

$\frac{1}{2}$- to $\frac{3}{4}$-in. or finer. B with the introduction of tu mills it was quickly an clearly demonstrated th the two machines in seri were more economical in pr ducing fine material th stamps alone (and, until th work of Davis, *et al.* (21 988; see also Sec. 4, Art. than either alone) so th stamps were relegated to th position of fine-intermedia crushers, breaking from 2- 3-in. maximum to $\frac{1}{2}$- or in. and the balance of t crushing to 0.01-in. and fin is done in tube mills. Dav (see Sec. 4, Art. 8) has de nitely proved experimental the advantage of stage crus ing in ball milling. A simil conclusion has been reach at every plant where caref comparison of 1-stage *vs.* mu tiple-stage reduction from or 3-in. maximum to flotation feed (0.2-mm.) or finer has been made. Tab 8 gives the range of sizes through which the different common crushi machines are used.

CRUSHING MACHINES

Classification of crushers. Coarse crushers are those that take ru of-mine ore and discharge material much coarser than ordinary concentrati size. Fine crushers or grinders discharge products crushed to concentra ing size, usually 2-mm. or under. Intermediate crushers are those install between the coarse and fine crushers in a mill. This latter designation based rather upon the place of the machine in the mill flow-sheet than up its characteristics as a crusher.

Coarse crushers are built large enough to take any rock that can handled by a steam shovel and the term is used also to describe crushing m chines, the feed to which is not, in general, less than 3-in. They are of fo general types, (a) jaw crushers, (b) gyratory crushers, (c) disk crushers, a (d) rolls.

7. Jaw crushers

Jaw crushers consist essentially of two crushing surfaces set nearly v tically, one fixed, the other movable and caused to alternately approa and recede from the fixed surface. The best known types are the Bla with movable jaw pivoted at top, and the Dodge with movable jaw pivo at the bottom. Mill crushers are almost without exception of the Blake ty

Blake breaker (Fig. 21) consists of a main frame (*a*) carrying a fixed jaw (*b*) and a movable jaw (*c*), the latter pivoted at the top on the swing-jaw shaft (*d*). The movable jaw is caused to oscillate by the action of toggles (*e*) and pitman (*f*) actuated by the eccentric (*g*) through the medium of pulleys (*h*) mounted on the drive shaft. One pulley only is used on small crushers. The movable jaw is held up against the toggles by tension rod (*j*) and spring (*k*). The rear toggle is seated against adjusting block (*l*). The horizontal position of the rear toggle seat is changed by raising or lowering the block (*m*) by means of the bolt shown, thus determining the distance between the fixed and movable jaws. The throw of the swing jaw may be varied slightly by raising or lowering block (*l*). Heavy fly-wheels (*n*) are mounted on the drive shaft for the purpose of lessening the intermittent character of the load on the prime mover. A jaw crusher breaks rock only during that half of each revolution in which the movable jaw is approaching the fixed jaw; during the other half revolution the only work done is that in overcoming friction. Without fly-wheels a driving belt flaps badly and there is a tendency for the prime mover to run away on the unloaded half of the revolution while it is subject to an enormous power draft on the loaded half. Fly-wheels average the load by storing energy during the unloaded half of the revolution and returning it during the loaded half. Table 4 presents essential data concerning Blake crushers, taken from manufacturers' catalogs. For most sizes the figures given have been checked against operating data and found to be conservative.

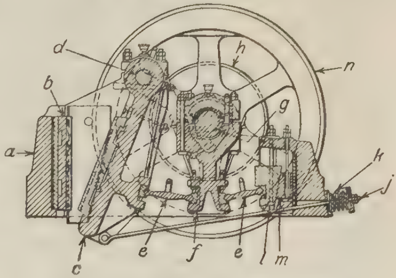


Fig. 21.—Blake jaw crusher.

Main frame must withstand constant vibration and heavy shock loads which produce tensile stresses around the mouth and near the base and compression under the main bearings. There is considerable racking strain due to uneven loading of the jaws. In small crushers (up to 15 × 30-in.) the frame is usually made in one piece, of cast iron (tensile strength, 20,000 to 30,000), cast semi-steel (28,000 to 35,000), or cast steel (60,000 to 75,000). In large crushers a cast-steel frame, thoroughly annealed, is made in four pieces, two ends and two sides. These pieces are fitted together with carefully machined tongue-and-groove joints and secured by heavy bolts and tie-rods of high-carbon steel (100,000 to 150,000 tensile strength) put in hot. Castings are heavily ribbed to give strength, while allowing considerable saving in weight. The principal tensile stresses are longitudinal and the longitudinal tie-rods are made sufficiently heavy to withstand these stresses without aid from the sides of the frame. Bearings for the pitman and swing-jaw shafts are best cast integral with the frame to aid in shaft alignment. The sides should be strengthened at these bearings by ribs, and in addition, by cross tie-rods or by collars set in grooves or screwed onto the shafts. In the largest crushers the side pieces are sometimes made in two parts, split horizontally and fastened together with bolts of alloy steel. Cast-iron and semi-steel one-piece frames are sometimes reinforced with steel, shrunk on around the jaw openings and base. The Calumet and Hecla-type crusher has a one-piece ribbed frame reinforced by 3 @ 5-in. longitudinal bolts each side, 3 @ 4-in. bolts across the front end and 2 @ 4-in. bolts across the rear end. By this expedient the weight of a 24 × 48-in. crusher is kept down to 80,000 lb.

Swing jaw is subjected almost entirely to bending loads. It is made of the same material as the frame, and, in large crushers, is similarly ribbed. It is shrunk or otherwise rigidly fastened to the swing-jaw shaft in order to bring the movement into bearings that are readily accessible for lubrication. The face of the jaw is machined to give an even bearing for the jaw plates. Some manufacturers make removable toe plates for the swing jaw so

Table. 4. Blake-crusher data from manufacturers' catalogs

Size of receiving opening, inches	Approximate capacity in tons per hour to sizes stated, inches									
	Size	Tons	Size	Tons	Size	Tons	Size	Tons	Size	Tons
7×10	0.75	1.5–2	1.0	2.5	1.5	4	2	5–6	....	....
9×15	1	5–6	1.5	5.5–11	2	8–10	2.5	10–12.5	....	....
10×20	1.5	10–15	2	15–17	2.5	17.5–22	3	20	....	....
12×24	1.5	20	2	20–30	2.5	25	3	30	4	35
15×24	1.5	15	2	17–24	2.5	25	3	30–33	4	37.5–45
15×30	2	20–27	3	35–40	4	45	5	50	....	....
18×24	2	24	2.5	30	3	35	7	70	....	....
18×30	2	25–35	3	37–45	4	45	5	50	7	80
18×36	2	40–45	2.5	30–50	3	39–55	3.5	50–60	....	....
24×30	1.5	35	2	40	2.5	45	3	50	7	90
24×36	2	25–50	2.5	38–55	3	41–70	4	60–90	5	75–105
30×36	2.5	48	5	90	6	105–120	7	125	....	....
30×42	3	60–72.5	4	90–115	5	110–145	6	120–175	8	235
30×48	4	100	5	120–125	6	150–190	7	225	....	....
30×72	4	150	5	180	6	220	.....	....	....	....
36×42	4	76	5	108	6	144–175	8	235	10	290
36×48	4	100–130	5	130–165	6	150–200	8	260	10	325
42×48	5	118–140	6	150	8	260	10	320	....	....
42×60	5	175–185	6	225	7	245–260	9	320	16	350
48×60	5	175–180	6	175–235	8	230–450	9	290	10	320–713
48×72	6	210–246	7	315	8	280–360	10	350–450	....	....
60×84	6	380	7	285–375	9	360–500	10	450	11	495–625
66×86	8	330–510	9	420	10	415–778	12	495–1110	13	600
84×120	10	1340	12	1970	14	2840	.....	....	....	....

Size of receiving opening, inches	R.p.m.		Hp.		Weight, pounds		Tons per horse-power-hour (c)
	Range	Average	Range	Average	Range	Average	
7×10	250–300	275	7–8	7.5	6,000– 8,400	7,250	0.33
9×15	250–300	275	10–15	12.5	7,500– 16,900	15,000*a*	0.66
10×20	250–300	275	14–20	16	8,800– 22,300	19,500*a*	0.81
12×24	250–300	275	20–25	22.5	22,600– 45,000	24,000*b*	1.11
15×24	250–300	275	25–32	29	15,000– 33,000	31,500*a*	0.86
15×30	250	250	35–55	43	17,000– 40,000	39,000*a*	0.70
18×24	230–300	260	30–40	35	32,700– 54,100	41,200	1.00
18×30	240–300	266	40–55	47	41,500– 55,500	48,300	0.87
18×36	225–300	262	56–65	60	59,000– 63,000	61,000	0.78
24×30	250–300	275	50–65	58	57,000– 67,000	61,000	1.15
24×36	180–300	231	60–80	72	56,000–100,000	74,500	1.04
30×36	200–300	250	60–80	72	58,000– 85,000	74,500	1.25
30×42	175–300	225	90–115	104	62,500–130,000	116,500*a*	1.23
30×48	175–300	225	100–225	112	92,000–180,000	121,500	1.09
30×72	175–195	185	150	150	120,000–162,000	138,000	1.10
36×42	175–300	187	90–115	105	93,000–131,000	113,000	1.43
36×48	175–300	229	100–150	128	95,000–215,000	156,000	1.37
42×48	150–250	181	110–150	134	155,000–218,000	173,000	1.53
42×60	125–250	183	140–165	152	180,000–227,000	199,000	1.66
48×60	125–200	154	90–200	190*a*	205,000–245,000	221,000	1.79
48×72	125–150	133	150–215	186	240,000–257,000	246,000	1.72
60×84	80–100	91	100–300	246	415,000–500,000	456,000	1.83
66×86	80–90	85	275–300	288	460,000–680,000	550,000	2.43
84×120	75	....	500	....	875,000	....	5.68

a Excludes one exceptionally low figure. *b* Excludes highest figure. *c* Reduction ratio, 6 : 1.

that they can be replaced when worn, this being the only part of the swing jaw that is subjected to excessive wear.

Pitman stresses are almost wholly tensile. In all but the smallest crushers the pitman is made of cast steel, thoroughly annealed. Every attempt is made to make it of the least weight consonant with the required strength. This end is ordinarily attained by the use of a heavy ribbed casting of box section, but one manufacturer makes a cast-steel cap and a toggle support separately and joins them together with wrought-steel tension rods. Several manufacturers of large crushers support the pitman on a nest of springs resting on a heavy cross-head on the frame. This reduces friction and consequent heating of the bearings, thereby saving power and lubricants and lessening or eliminating the necessity for pitman cooling. At CALUMET AND HECLA (*100 J 11*) substitution of a spring-supported pitman for the old style reduced the power consumption on a given crusher from 29 to 16 hp. Bolts holding down the pitman cap should have fine threads to aid adjustment.

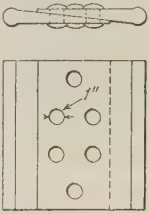


FIG. 22.—Split toggle for jaw crusher.

Toggles are subjected to compression only. The ends of the toggles, rolling or sliding in the toggle seats, are difficult, if not impossible to properly lubricate, and since substantially the full crushing force is concentrated here they must be specially hardened to resist wear. Toggles are made of cast iron in small, cheap crushers. Small crushers of better grade have cast semi-steel or steel toggles with chilled ends. Large crushers are fitted with cast-steel toggles with chilled ends or with toggles made of alloy steels. In many crushers the rear toggle is made the BREAKING POINT to relieve strain when steel gets into the jaws. In some cases this is accomplished by splitting the toggle along a diagonal plane as in Fig. 22 and riveting together with just enough metal to withstand all normal strains, but insufficient to stand an excessive load. At TALISMAN MINE, N. Z., six 1-in. rivets in a 16 × 10-in. crusher sheared with a hammer head. At LYELL COMSTOCK, Mt. Lyell, twelve 1-in. rivets were used in a 12 × 20-in. crusher. Some manufacturers make the rear toggle itself of such light section that it fails when excessive load is applied.

Life of toggles varies from 90 days to several years, the majority less than a year. One mill reports the use of toggles with replaceable ends, and change of ends necessary after 4 to 6 months.

Toggle seats are subjected to the same wear as toggle ends, but are less easily replaced and hence greater care is taken to insure long life. In small crushers they are made of hard, high-carbon steel; in large crushers of manganese or chrome steel. From its properties, a chrome steel containing 1 to 2 per cent. chromium and about 1 per cent. carbon, hardened, should be the best possible material for this service. A rolling toggle, such as illustrated in Fig. 23*a*, with chrome-steel toggle-block and liners should be superior to any form of sliding toggle such as illustrated in Fig. 23*b*.

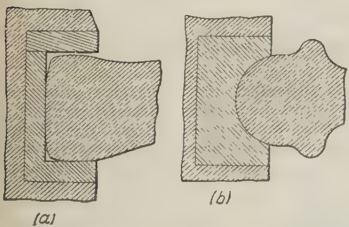


FIG. 23.—Toggles and toggle seats.

Life reported in the mills ranges from 90 days to several years. Manganese-steel blocks on large crushers under fairly heavy load lasted from 1 to 2 years.

Shafts are subjected to enormous bending stresses and should be of large diameter to prevent deflection and consequent heating of bearings, as well as to guard against breakage. In all high-grade crushers they are made of best quality high-carbon steel forgings, heat-treated and tempered, then turned and polished.

Bearings are heavily loaded even when the crusher is running light. They should, therefore, be of large diameter and as long as possible. In one-piece frames the bearings should be cast integral with the frame to aid in proper alignment of shafts. When frames are cast sectional, proper alignment is obtained by the use of ball-and-socket bearings. The pitman-eccentric bearing is water-cooled in all good crushers except those in which the pitman is spring-supported. The pitman-shaft end bearings are often likewise water-cooled in large crushers (30-in. opening and upward). The best quality of hard babbitt is used.

Fly-wheels are subjected to heavy strains by reason of the rapidly and greatly varying load on the crusher and must, therefore, be built especially strong. They are usually made of cast iron or semi-steel. When cast in one piece special precautions must be taken to insure against cooling strains. Some manufacturers cast rim, hub and arms separately

and thus eliminate shrinkage strain One manufacturer keys the fly-wheel to the shaft with compression keys that allow slip with overloading, but this is probably an unnecessary precaution so far as fly-wheel breakage is concerned and insufficient provision as a breaking point to save the crusher bearings in case of entrance of tramp iron. Rims on all large crushers are cored for pockets to allow barring over and are crowned when the fly-wheels are to be used as drive pulleys.

At Calumet and Hecla the end of the shaft is marked with an arrow to show the position of the eccentric and thus make it apparent which way to turn in order to ease off when the crusher is clogged.

Heat treatment of crusher steel is highly important. Harder (*115 J 314*) tested parts of jaw and gyratory crushers and rolls that had failed in service and found in all cases that the tensile and impact strengths, toughness and hardness were markedly increased by simple heat treatment.

Drive of small crushers is by a single pulley carried on an extension of the eccentric shaft or bolted to a fly-wheel. The usual practice is to use two pulleys on crushers with greater than 24-in. width of receiving opening, if drive is from a counter-shaft, but when direct belt drive from a motor is employed, one pulley only is used. At Creighton mines Ont., a 30 × 42-in. crusher is driven by two 100-hp. motors. The principal objection to single-pulley drive is the unbalanced side pull on the bearings. This is not serious, if the

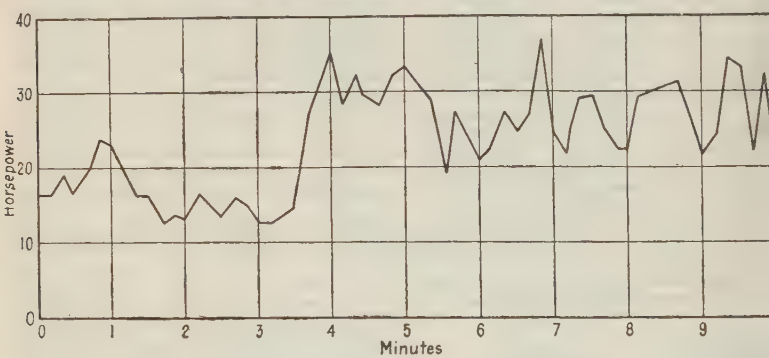


Fig. 24.—Power draft of 18 × 36-in. Blake jaw crusher.

shaft and bearings are properly designed. In some large crushers the fly-wheels are replaced by extra-heavy flanged drive pulleys or rope sheaves, in some cases counter-weighted to balance the pitman. Lenix system is frequently used for driving crushers of all sizes. When two drive pulleys are used, each should be sufficient for independent drive. Pulley dimensions are generally based on transmission of 1 hp. per inch of drive belt traveling at 1000 ft. per min. Installed horsepower is generally well in excess of that actually used, in order to take care of the starting load. One mill reports a 150-hp. motor installed on a 24 × 36-in. crusher drawing 186 hp. at starting and 92 hp. running under regular load. Tests at Witherbee Sherman and Co. mills (Fig. 24) indicate fluctuations from 19 to 3 hp. within a minute in the power draft of an 18 × 36-in. crusher running under full load (185 r.p.m.; 8- to 10-in. max. size of feed; −4-in. product; 102 tons per hr.). Crushers must, therefore, be over-powered, if serious speed reduction and clogging are not to result from peaks in the power draft.

Lubrication of jaw crushers is difficult on account of the great pressures on the bearings and the large amount of dust and grit present. All well-built crushers use forced feed of either grease or oil. Because of the way in which exuding grease forms a collar that excludes dust and grit, most manufacturers use grease fed by means of compression cups with automatic or manual feed. Some crushers are fitted with compression feed to the under side of the pitman-eccentric and pitman-shaft bearings and large grease or oil reservoirs on top for gravity feed. Toggle seats are most difficult to lubricate and that design of toggle and seat is best that minimizes the need for lubrication.

Consumption of lubricant as reported by various mills is shown in Table 10. The enormous differences are apparently due to differences in crusher duty. High consumption of lubricant corresponds to heavily loaded crushers and *vice versa*.

Jaw plates. The principal wear on the crusher comes on these and on the cheek plates. They are, therefore, made replaceable. Further, wear on the jaw plates is uneven and i

order to lessen the amount of metal discarded in the form of worn plates, they are made reversible In the case of large crushers with sectional jaw plates, they are also made interchangeable Thus in a small crusher, when the jaw plates become worn at the throat, where wear is greatest, they are turned end for end and their life is practically doubled. When plates are sectionalized horizontally, two wears are added for each such sectionalizing. When the form of surface corrugation permits, further increase in life of plates is gained by also sectionalizing vertically, when four wears at the throat can be had from each section. In addition to longer life, sectionalizing makes for ease in handling, which is a great advantage in large crushers. Materials used for jaw plates are chilled iron, white iron, high-carbon cast steel, forged steel, manganese and chrome steels. The great majority of plants use manganese-steel plates. This material is particularly fitted for such service on account of the fact that it is tough and that surface abrasion produces rapid and marked surface hardening. One special form of jaw plate is made of forged and rolled chrome-steel bars cast-welded into a back of open-hearth steel and subsequently tempered. This gives a hard chrome-steel crushing surface, while the untempered back is tough and resists cracking. Chilled steel wears unevenly because it is initially uneven in hardness. CALUMET AND HECLA uses chilled cast-iron plates for crushing soft amygdaloid and manganese steel for the hard conglomerate. The main frame and swing jaw are carefully surfaced to give full, even bearing to the jaw plates, or the plates may be backed by zinc or hard babbitt. Plates in small crushers are usually wedged in; those in large crushers are bolted in. One maker places a buffer plate between the wearing plates and frame to take up any wear on frame due to the abrasive action of grit, in case the liner plates loosen and vibrate.

Life of jaw plates varies according to material used and service required. (See Table 10.) Three to six months is an average life for manganese-steel plates in ordinary service. One plant reports 40 days' life for manganese-steel plates in an 18 × 30-in. crusher handling 1500 tons per 24 hr. of hard granitic ore. Another reports two to four years for plates of the same material in a 36 × 42-in. crusher handling 4000 to 5000 tons per 24 hr. of hard slate with some quartzite. The consumption of manganese steel, including waste on rejection, is from 0.01 to 0.06 lb. per ton of rock crushed. The consumption of chilled iron in small crushers ranges from 0.02 to 0.2 lb. per ton and averages about 0.1 lb. per ton. Johnson (*101 J 907*) gives the following comparative figures on chrome and manganese steel in the same service: chrome steel, weight of plates, 921 lb.; cost f.o.b. mill, $96.93; life, 70,206 tons; manganese steel, weight, 740 lb.; cost, $72.62; life, 86,478 tons. It must be remembered that manganese steel, as well as all other metals, varies markedly in grade and wearing qualities. Waterhouse (*38 Aa 107*) cites a set of manganese-steel jaw plates of Sheffield manufacture that served to crush 6100 tons in a small crusher against a life of 4500 tons for local Australian plates of substantially the same manganese content. Del Mar (*40 MEW 687*) recommends 13 per cent. Mn as the best alloy.

The time required to change jaw and cheek plates is from one to three hours for crushers up to 24 × 36-in. One mill using a 36 × 42-in. crusher reports eight hours to make a change of plates.

The crushing surface of jaw plates is made in a variety of forms. For fine crushing and brittle rock, plane surfaces are best; for all-around coarse work, a surface corrugated vertically with 90° ridges as shown in Fig. 25 is best, but in crushing soft, tough rock the ridges are likely to pulverize locally without effecting a break. A waved surface has the advantage of concentrating the breaking load without the disadvantage of local pulverization. It discharges more readily than the corrugated, thus increasing capacity. It is especially suited to tough, slabby rock, like slate. Waved plates are used in several Lake Superior copper mills in preference to corrugated, since they allow mass copper that has caught to be freed by working sideways, which cannot be done with corrugated plates.

FIG. 25.—Corrugated jaw plate.

Cheek plates take wear on the sides of the crushing opening. The materials used are ordinarily the same as for jaw plates. The surfaces are plane. The plates are wedged in in small crushers and bolted through in large. The life of manganese-steel cheek plates is the same or slightly less than that of jaw plates of the same material and metal consumption is usually less than 0.01 lb. per ton crushed. Chilled-iron plates have about one-quarter to one-third the life of manganese-steel.

Adjustments of Blake crushers are, (1) width of discharge opening, (2) THROW, *i.e.*, the distance traveled in each direction by the jaw at each revolution of the drive shaft, and (3) speed.

Width of discharge opening is adjusted by changing the length of the

toggles, changing from worn to new jaw plates, or by a wedge or shim adjustment of the rear toggle seat. Ordinarily sufficient wedge or shim adjustment is provided to compensate for jaw-plate wear and this adjustment is made from time to time as plates are worn down. Change in length of toggles is usually made only when the duty of the crusher changes and a wholly different size of product is desired.

Throw, in Blake crushers, is measured at the throat. It is adjustable in some crushers by a device for raising or lowering the main toggle block and thus changing the angularity of the toggles, but ordinarily this adjustment involves a change in the eccentric and requires a new pitman or, at the least, a new eccentric. Throw ranges from about $\frac{3}{8}$ in. as the minimum in small crushers up to 1 in. minimum in large crushers. The maximum throw is about twice the minimum figures. The principal factor determining the length of throw is the character of rock to be crushed. If the rock is hard and brittle, so that the jaws do not pulverize locally and deformation prior to fracture is not great, the minimum throw may properly be employed. If the rock is of such character that the reverse situation prevails, and there is local pulverization and a tendency for the rock to crack and be deformed under load but not to fall apart, then the maximum throw should be used. Firm quartzitic and acid rocks generally require only the minimum throw. Tough, basic rocks, highly crystalline rocks, slabby rock, and decomposed rocks in general require the greater throws.

Speed can be varied only by change in the speed of the prime mover or by a change in pulley ratios in the power-transmission chain. Change in speed affects capacity and power consumption, but has no marked effect on the size of product. Lake Superior rock-house practice varies the speed according to the hardness of rock crushed (*100 J 55*). At QUINCY, breaking soft amygdaloid, the speed is 140 r.p.m.; at CALUMET AND HECLA, with a feed of hard conglomerate, the crusher is run at 175 r.p.m. and at COPPER RANGE, crushing dense amygdaloid, the speed is 185 r.p.m. Excessive speed causes heating. At MOOSE MOUNTAIN (*99 J 974*) a 24 × 36-in. crusher overheated when run at 250 r.p.m. but gave no trouble when run at 180. There was, however, an accompanying decrease in capacity.

Reduction ratio is the ratio between size of feed and size of product in a crushing operation. The sizes considered are usually maximum sizes, although in some instances consideration of average sizes is more informative. NIP ANGLE in jaw crushers is the angle formed by the jaw faces. Table 5 gives data concerning the nip angle and maximum reduction ratios in Blake crushers estimated from manufacturers' data. The average reduction ratio for crushers of all sizes, with minimum setting, is 8.7; for maximum setting, 5.4. Larger reduction ratios are allowable in the smaller crushers by reason of the fact that the nip angle corresponding to these large ratios is smaller in these crushers. Maximum nip angle corresponding to the above ratios is less than 24°. In the mills the average reduction ratio for 20 crushers ranging from 6 × 20-in. to 66 × 84-in. size was 7.46 (see Table 10); the maximum is 18.6 and the minimum 2.6. The corresponding nip angles range from 18° to 23°, the maximum angle corresponding to the largest crusher. It is to be noted that the above reduction ratios are those of which the crushers are capable according to their size and setting and not, necessarily, those under which they are called on to perform. In general the maximum particle of feed is somewhat less than the gape of the receiving opening and the average size of the feed is much less. The average reduction ratio is, therefore, less in

every case than the figure given. The nip angle increases as the jaw moves forward, hence, if the angle is too large with the jaws open the unfavorable condition is aggravated as the jaw closes.

Table 5. Reduction ratio and nip angle in Blake crushers

Size of crusher, inches	Minimum setting			Intermediate setting			Maximum setting		
	Inches	Reduction ratio	Nip angle, degrees	Inches	Reduction ratio	Nip angle, degrees	Inches	Reduction ratio	Nip angle, degrees
7×10	0.75	9.3	13.5	1	7	12.1	1.5	4.7	11.1
10×20	0.75	13.3	21.1	1.25	8	20.0	1.5	6.7	19.5
15×24	2	7.5	21.5	3	5	20.0	3.5	4.3	19.2
18×30	2.5	7.2	20.2	3	6	19.6	3.5	5.1	19.0
20×24	2.5	8	23.1	3	6.7	22.5	3.5	5.7	22.1
24×36	3	8	22.0	4	6	21.0	5	4.8	20.1
28×36	3	9.3	23.4	4	7	22.5	5	5.6	21.7
36×42	4	9	23.1	5	7.2	22.5	6	6	21.8
42×48	5	8.4	23.8	6	7	23.2	8	5.2	22.0
48×60	6	8	23.0	8	6	22.0	10	4.8	21.0
56×72	7	8	23.8	8	7	23.4	10	5.6	22.5
66×84	8	8.2	23.0	10	6.6	22.2	12	5.5	21.5
84×120	10	8.4	23.2	12	7	22.6	14	6	22.1
Average..	......	8.7	....	......	6.6	....	......	5.4	

Capacity of Blake crushers depends primarily upon the character of ore, size of feed, and discharge setting. It is materially affected, also, by the throw, speed, angularity of jaws, and character of surface of the jaw plates. Capacity figures averaged from manufacturers' catalogs are given in Table 4. These figures are based on rock such as limestone, which is easy to crush. On the other hand, they are generally given in terms of a feed none of which will pass through the crusher without breaking, while the ordinary feed to a crusher contains a considerable percentage of material that requires no breaking but passes directly through. Quartz, quartzitic ores, and firm brittle ores generally can be crushed at a rate equal to catalog figures. Tough ores, such as basic silicates, traps and diorites, do not crush so readily. Hersam (*68 A 463*) shows that if the capacity of a given crusher on quartz is taken as 100, the capacity on a granite such as he tested would be 89.5 and on a trap, 83. Materials of low specific gravity such as coal, fibrous materials such as asbestos, and clayey materials, can be crushed only at rates markedly below catalog figures. The effect of variation in density is indicated by Table 6. The column headed "Relative volumes per hour" shows that there is a distinct

Table 6. Effect of density of feed on capacity of jaw crushers. (*After Hersam*)

Material	Sp. gr.	Relative tons per hour	Relative volumes per hour
Coke....................	1.11	100	100
Coal....................	1.91	170	100
Granite..................	2.66	381	159
Stibnite in quartz...........	3.03	436	159
Chalcocite in quartz.........	4.40	641	162
Galena in quartz............	6.15	950	172

trend toward the passage of a larger volume of dense material through the crusher in a given length of time. Sticking of clayey materials may be considerably lessened by allowing a trickle of water to run into the crusher with the feed. This tends to lubricate the crushing faces sufficiently to allow the compressed material to slide forward as the movable jaw recedes and thus to work through the crusher. In crushing slate, which cleaves easily and tends to discharge from the crusher in slabs much larger in one or two dimensions than would be expected from the crusher setting, there may be considerable reduction in capacity, due to the measures that must be taken to prevent such discharge.

At one plant crushing slate discharge of slabs was prevented by attaching prongs at 6-in. intervals to the bottom of the swing jaw, these prongs projecting across the throat and under the fixed jaw plate. The feed to the crusher ranged from pieces that would just enter the 15 × 24-in. receiving opening, to pieces that could pass through without any crushing. The crusher was set for a minimum opening of 1⁵⁄₁₆ in. and had a throw of 1 in. The capacity without prongs was 25 tons per hr. and the product contained about 2 per cent. of slabs that stayed on a 4-in. screen. The capacity with prongs was 20 tons per hr., but there was no slabby material in the product. The average size of the product was not greatly altered.

There is a marked progressive increase in capacity with decrease in reduction ratio. With a crusher having a minimum throat opening of 0.24 in. and a throw of 0.21 in., crushing a uniform granite of various sizes, Hersam reports the results summarized in Table 7.

Table 7. Effect of reduction ratio on capacity of jaw crushers. (*After Hersam*)

Size of feed, inches	Reduction ratio	Relative tons per hour
3 to 4	8.9	100
2 to 3	6.7	170
1 to 2	4.4	182
0.5 to 1	2.2	232
0.125 to 0.5	1.1	419

Capacity increases with increase in speed up to a certain limit although the increase is not a proportionate one. See Table 8.

Increase in nip angle decreases capacity somewhat, as shown by Hersam's tests summarized in Table 9. Note, however, that the effect is small and that when the angles are near those common in practice (see Table 5), but little effect is to be noticed. In crushing granite Hersam found that greater tonnage could be handled with smooth than with medium or rough jaw faces and that the tonnage with medium faces was greater than with rough.

Table 8. Effect of speed on capacity of jaw crushers. (*After Hersam*)

Revolutions per minute	Relative tons per hour
160	100
255	144
304	171
348	174
534	179
629	246

Table 9. Effect of nip angle on capacity of jaw crushers. (*After Hersam*)

Angle of nip, degrees	Relative tons per hour
30	100
27	102
20	116
14	114

Increase in throw produces marked increase in capacity, provided the same minimum opening is maintained.

Capacity formulas. Hersam develops theoretically the formula

$$T = 54nwdsk\frac{(a^2 - b^2)10^{-6}}{(c - b)}$$

where T = tons per hr.; n = r.p.m.; a = distance between jaws at bottom, when open; b = distance between jaws at bottom, when closed; d = vertical depth of jaws; c = distance between jaws at the top when closed (width of receiving opening); w = length of discharge opening; s = sp. gr. of rock; k is a factor varying with changing operating conditions but averaging about 0.75. All dimensions are to be taken in inches. This formula does not check well with practice or with catalog figures.

An empirical formula that is relatively accurate for all except the smallest and the largest crushers is

$$T = \frac{0.6A}{R} = 0.6LS,$$

where T = tons per hr.; A = area of receiving opening in sq. in.; R = reduction ratio; L = length of receiving opening and S = width of discharge opening (set), both in inches. For small crushers the answer will be high; for large crushers, low.

Performances as reported from the mills are presented in Table 10.

At La Rose No. 2 mill (E. & M. J. 1912) one 9 × 15-in. followed by two 6 × 12-in. crushers break very hard conglomerate ore to 1-in. at the rate of 100 tons per 24 hr. Repair cost, 2 ¢ per ton. At Chuquicamata Mining Co. [illegible] a 10 × 16-in. machine breaks 135 tons per day to 2-in. at a cost of [illegible] per ton, of which [illegible] is labor; steel forms the bulk of the supply cost. At Tonopah (E. & M. J. [illegible]) a 15 × 20-in. machine crushes 15 tons per hr. to 1.5-in. [illegible] (E. & M. J. [illegible]) [illegible] 11 × 20-in. machines to break run-of-mine rock to 1.5-in. at the rate of [illegible] tons per 24 hr. At Zinc Corporation, one 12 × 30-in. crusher running at 275 r.p.m. breaks from 15-in. to 4-in. at the rate of 25 tons per hr.; one 12 × 20-in. machine set at 2 in., [illegible] the 4-in. material [illegible] a product with the following screen analysis: 31.1 per cent on 1-in., 21.7 on [illegible], 7.8 on [illegible], 5.5 on [illegible] and 10.5 per cent through [illegible] (E. & M. J. [illegible]). At Chino Copper Co. one 18 × 24-in. machine crushes 1000 tons of hard ore per 8 hr. to 2-in. (E. & M. J. [illegible]). At Mt. Morgan, one 16 × 24-in. breaks fair quartzose feed to 1.5- to 2-in. at the rate of 10 to 15 tons per hr. (E. & M. J. [illegible]). At Inspiration Copper Mining Co., an 18 × 36-in. crusher breaks 60 to 100 tons per hr. through 3-in. [illegible] with a 150-hp. motor [illegible]. At [illegible] mill, [illegible], B. C., a 24 × 36-in. machine crushes 150 tons per day to 2-in. (E. & M. J. [illegible]). At [illegible] Iron and Steel Co., Mt. Hope, N. J., a 24 × 36-in. crusher breaks 25 tons per hr. to 3-in. [illegible]. At St. Joseph L. and Z. Co., National plant, one 36 × 48-in. machine set at 4-in. breaks 4000 to 5000 tons per 24 hr. of [illegible] feed, [illegible] r.p.m., 60 to 100 hp. [illegible]. At [illegible] Corporation a 32 × 72-in. crusher breaks 160 tons per hr. to 10-in. (E. & M. J. [illegible]). At Johns-Manville a 60 × 84-in. machine handles [illegible] tons per 8 hr. to 10-in. [illegible] is held to 18° on account of the slippery character of the feed. Weight of crusher, [illegible] lb. (E. & M. J. [illegible]). At Chile Copper Co. two 60 × 84-in. crushers with rope drive are used for 10,000 tons per day; the capacity of each is 1200 tons per hr. to 10-in. [illegible]

Power consumption per ton of ore crushed is considerably greater in small than in large crushers. Up to 36-in. width of receiving opening (gape) the tons per hp.-hr. consumed ranges from 0.7 in the smaller sizes to 1.6 in the 36-in. machines. From 36-in. to 60-in. gape, the tons per hp.-hr. increases from about 1.3 to 1.8. For 60-in. gape, 2.5 tons per hp.-hr. is a fairly conservative figure. These figures are all based on a reduction ratio of 6 : 1 and on a continuously busy crusher. If a smaller reduction is practiced, tons per hp.-hr. will be increased, roughly in inverse ratio.

At Mt. Morgan a 24 × 16-in. machine set to discharge at 1.5- to 2-in. required 20 to [illegible] hp. to crush 10 to 15 tons per hr. If the crusher ran idle for any considerable portion

Table 10. Performance

Plant	Hedley G. M. Co.	Pittsburgh Dolores	Federal M. & S. (Morning)
Gape of receiving opening, in.	6	8	9
Length of receiving opening, in.	20	12	15
Set, in.	1.5	1.5	3.5–4
Throw at throat, in.	1	....	1
Speed, r.p.m.	220	....	280
Tons feed per hour	25	4.2	83.3
Maximum size of feed, in.	3	....	6
Apparent reduction ratio (*a*)	4	5.3	2.4
Actual reduction ratio (*b*)	2	....	1.6
Power installed, hp.	25	10	25
Power consumed, hp.	....	....	20
Tons per horsepower-hour, installed	1	0.42	3.3
Tons per horsepower-hour, consumed	....	....	4.2
Running time per 24 hours	8	8	18
Lost time, average per cent	12*c*	10*c*	1
Lubricants, pounds per shift	15	0.5	2
Life of wearing parts:			
Toggles, days	*d*	....	150
Toggle seats, days	*d*	....	200
Jaw plates, days	120	180	120
Material	*M*	Steel	*M*
Cheek plates, days	120	360	100
Material	*M*	Steel	*M*
Time required to change cheek and jaw plates, hr.	1	....	1

Plant	Timber Butte	Tonopah-Extension	Butte and Superior
Gape of receiving opening, in.	12	14	18
Length of receiving opening, in.	24	24	30
Set, in.	1.5	0.75	1.5
Throw at throat, in.	....	....	0.88
Speed, r.p.m.	....	....	....
Tons feed per hour	62.5	25	62.5
Maximum size of feed, in.	12	....	....
Apparent reduction ratio (*a*)	8	18.7	12
Actual reduction ratio (*b*)	8	....	....
Power installed, hp.	....	35	50
Power consumed, hp.	....	....	45
Tons per horsepower-hour, installed	....	0.71	1.25
Tons per horsepower-hour, consumed	....	....	1.40
Running time per 24 hours	....	16	16
Lost time, average per cent	....	....	....
Lubricants, pounds per shift	....	....	2.5
Life of wearing parts:			
Toggles, days	....	180	90*f*
Toggle seats, days	120	90	365*M*
Jaw plates, days	120–180	90	40
Material	Steel	*Ch*	*M*
Cheek plates, days	60	....	....
Material	*f*	....	*W*
Time required to change cheek and jaw plates, hr.	6	2	2

a Gape ÷ set. *b* Maximum size of feed ÷ set. *c* For causes not chargeable to crusher. *d* Several years. *e* Stationary; 171, swing. *f* Cast iron. *g* Minimum.

of Blake crushers

U. S. S. R. & M. (Midvale)	Tonopah-Extension	Cananea Consolidated	U. S. S. R. & M. (Midvale)	Hedley G. M. Co.	Tonopah-Extension	Consolidated Arizona Smelting Co.
9	10	10	10	10	12	12
15	16	20	20	20	20	24
1.5	0.75	1.5	1.5	2.5	3	3
0.5	....	1.5	0.5	1.25	....	0.75
252	....	250	287	220	....	250
7.9	16.7	3.3	10.5	25	16.7	25
....	....	10	....	6	....	....
6	13.3	6.7	6.7	4	4	4
....	....	6.7	....	2.4	....	....
20	20	25	20	25	35	....
10	....	20	15	....	....	....
0.4	0.84	0.13	0.52	1	0.48	....
0.8	....	0.16	0.70	....	....	....
24	16	8	24	8	16	....
3	....	10*c*	3	12*c*	....	0.77
1.25	....	1.5	1.25	15	....	....
200	180	90	180	*d*	180	....
245	90	120	225	*d*	90	....
97	90	180	82	106	90	112*e*
M	*Ch*]	Steel	*M*	*M*	*Ch*	*M*
8	....	60	71	76	....	....
CCI	....	*f*	*CCI*	*M*	....	....
....	2	1	1.25	1	2	....

Witherbee, Sherman	Engels C. M. Co.	Hedley G. M. Co.	Witherbee, Sherman	Calumet and Hecla	Alaska Gastineau	Chino C. C. Co.
18	24	24	24	24	36	66
30	36	36	36	48	42	84
3	4.5*g*	3.5	4	3	8.12	8
0.75	1	1.5	1	1	1.12	1
125–195	260	150	233	170–180	206	80
75	42	100	100	140	290	500
....	....	....	24	....	....	....
6	4.8	6.8	6	8	4.4	8.2
....	....	....	6	....	....	....
35	150	75	100	50	47	300
22.5	92	40	....	@ 50	27	....
2.14	0.28	1.33	1	2.8	6.2	1.67
3.33	0.46	2.5	....	@ 2.8	10.7	....
9	16	8	18	24	24	18
....	1	12	....	....	....	....
8	2	32	4	....	5	112
600	120–180*h*	*d*	300	180*f*	1440	....
1200	120–180	*d*	365	720*M*	1440	....
150	720	100	300	300	720	....
M	*M*	*M*	*M*	*M*	*M*	....
300	1080+	70	300	300	....	....
f	*M*	*M*	*M*	*M*	*M*	....
4	24–36	2	3	2	8	....

h Removable ends. *M* = manganese steel, *CCI* = chilled cast iron. *Ch* = chrome steel. *W* = white iron.

of the total running time, tons per hp.-hr. dropped materially. A 24 by 36-in. crusher set at 4.5-in. minimum with 1-in. throw and capable of crushing 80 to 90 tons per hr. was run at one plant 10 hr. per day and crushed 42 tons per hr. Average power consumption was 92 hp., giving but 0.45 ton per hp.-hr. Heller (*99 J 399*) states that the average jaw crusher consumes about 70 per cent. of its total power draft while idling, 50 per cent. is probably nearer the correct figure.

Fig. 24 represents the results of a test showing the fluctuation in power drawn by an 18 × 30-in. Blake jaw crusher. For the second and third minutes practically no feed was coming to the crusher and the average power drawn was about 14 hp. From the fourth minute to the end of the test the crusher was breaking from 8- to 10-in. maximum lumps to pass a 4-in. screen at the rate of 100 tons per hr. with an average power consumption of about 28 hp. Fifty per cent. of the average full-load power is, therefore, consumed in running the crusher light and must be charged against tons crushed.

Fall through the machine proper is about twice the gape. To this must be added the additional drop necessary to get feed to the machine and product away. If chutes are used, they may not safely be set at less than 45°.

Calumet and Hecla (*100 J 11*) uses a special crusher with 3-ft. 6-in. fall for 24-in. gape and 4-in. set. This has been found a great aid in freeing the crusher when mass copper clogs the jaws.

Lost time. The average percentage of lost time in the mills due to causes chargeable to the jaw crushers themselves, such as repairs and renewals, clogging and its attendant difficulties, is less than one per cent. Renewal of jaw and cheek plates and re-babbitting of bearings are the principal causes of lost time. When the crusher is planned to run one or two shifts per day only, as is done in a majority of mills, renewals and repairs are handled in the off shifts. In such cases substantially no delay is chargeable to the jaw crushers in most plants.

Attendance. The usual practice is a man to each machine. Rarely one man attends two machines. The principal duty of the attendant is to regulate the feed rate and to pick powder, steel, and waste. When the crusher is fed by dumping carloads or skip-loads of rock directly into the jaws no picking is possible, but the attendant must see that material does not bridge, or break up such jams as occur.

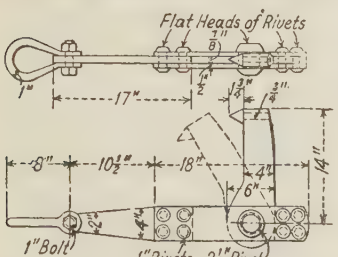


Fig. 26.—Hook for jaw-crusher feeding.

Crane service. See Sec. 23, Art. 12, for general discussion. Less than 50 per cent. of mills reporting had such service.

Feeding jaw crushers. Jaw crushers should be fed regularly and up to capacity, if possible. It is well not to bury them, since bridging at the mouth occurs easily and this necessitates digging out in order to get the flow of rock started again. Fig. 26 shows a power-actuated hook for removing lumps from the crusher jaws. Some such provision is an economy. Ample provision should be made to prevent pieces of rock from dropping into the actuating mechanism, or serious accidents may occur.

Breaking point is provided in most coarse crushers to take care of excessive loads caused by introduction of material, such as metal, that cannot be handled. In Blake breakers the usual breaking point is one of the toggles.

In crushers not specially provided, the usual effect of a stoppage under full working load with power on is either to crush the babbitt of the pitman or to throw the belt or both. If the belt does not throw off and there is not a satisfactory overload circuit breaker on the driving motor, a burned-out motor will result. Recently the provision of an overload circuit breaker on the motor, which, in case of an overdraft of power, due to clogging, cuts off the power and stops the motor, has been frequent.

Size of product. With ordinary rocks about 20 per cent. of the product will stay on a square-mesh screen whose aperture is equal to the average set of the crusher (mean between minimum and maximum throat openings); about half of the product will pass a square-mesh screen whose aperture is half the size of the crusher set. Experience at COPPER RANGE (*95 J 947*) is that primary jaw-crusher product (– 4-in.) is more even-sized if fines are not screened out ahead of the crusher, since slabs do not pass through so readily under this circumstance.

Cost of jaw crushers (1925) was 9.5 to 10 cents per pound for small machines and 8 to 9 cents for large machines. Operating cost is best estimated from the data given on power consumption, attendance and wear. These items make up about 90 per cent. of the total cost. For rough estimates, 8 to 10 cents per ton for small crushers to 2 to 3 cents for large are outside figures.

Dodge breaker (Fig. 27) differs essentially from the Blake in that the movable jaw is pivoted at the bottom. In the original Dodge type shown in the figure the movable jaw was mounted on the short arm of a lever whose fulcrum was the swing-jaw shaft (*K*) and the long arm (*B*) was actuated by an eccentric (*L*) on the drive shaft. Adjustment of the size of discharge was effected by shims (*S*). In later designs toggles and pitman have been used to actuate the swing jaw, as in the Blake type and the only difference between the two types becomes the essential and important one of the place at which the swing jaw is pivoted. Sizes and essential operating data of Dodge breakers are given in Table 11.

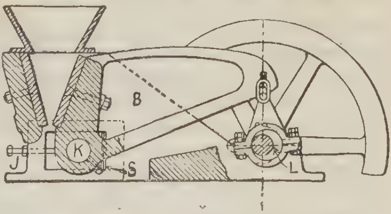


FIG. 27.—Dodge jaw crusher.

Table 11. Operating data for Dodge crushers

Size of receiving opening, in.	Weight, pounds	Capacity, tons per hour				Revolutions per minute	Horse-power
		Size, inch	Tons	Size, inches	Tons		
4×6	1,100	0.75	0.5	....	....	300	3
6×9	3,200	0.75	1.5–2.5	1.5	3–5	250–300	4–6
7×11	5,500	....	....	1.5	6–8	250–300	7
8×12	5,900	0.75	3–5	1.5	7–10	250–300	10
11×15	14,000	0.75	6–8	1.5	10–20	250–300	15

The Dodge breaker, due to the manner of pivoting the swing jaw, is forced to do its greatest work at a point on the working end of the lever farthest from the fulcrum. This makes it most uneconomical in the use of power. The capacity is low as compared to the Blake by reason of the rela-

tively small movement at the throat. It has the further disadvantage that there is difficulty in nipping the material being crushed and as a result lumps tend to fly out of the jaws.

The Dodge-type breaker is made in smaller sizes than the Blake and is, therefore, suitable for small sample plants where power consumption is so small as to be unimportant, and where uniformity in size of product, caused by the small throw at the throat, is of distinct advantage.

8. Gyratory crushers

The gyratory crusher consists essentially of a fixed crushing surface in the form of a frustum of an inverted cone around the axis of which gyrates a movable crushing surface, which has the shape of a conical frustum in erect position. The material to be broken is fed into the downwardly-converging annular space between these two crushing surfaces, it is crushed when the surfaces approach, and the crushed material falls through when they recede. The machine is built in three types, known respectively as the suspended-spindle type, the supported-spindle type and the fixed-spindle type. The first is the best known and most used; the second is fast disappearing; the third is coming into favor and in the future will dispute the field with the first type.

The sizes of suspended- and supported-spindle gyratory crushers are indicated by numbers which are usually one-half or slightly less than one-half the gape of the receiving opening, expressed in inches. Whether the spindle is suspended or supported is indicated by a letter suffix to the number, *K* usually indicating suspended spindle and *D* supported. In the fixed-spindle type the number is equal to the gape in inches. Length of receiving opening is stated as the circumferential distance, measured along the outer edge of the receiving opening, between adjacent faces of the spider arms, multiplied by the number of spider arms. This length is approximately eight times the gape in the lever-type crushers in sizes below No. 9 and seven times the gape in the larger sizes. Thus a No. 4-D crusher is of the supported-spindle type with a receiving opening roughly 8 × 64 in. and a No. 21-K is suspended-spindle type with 42 × 294-in. receiving opening.

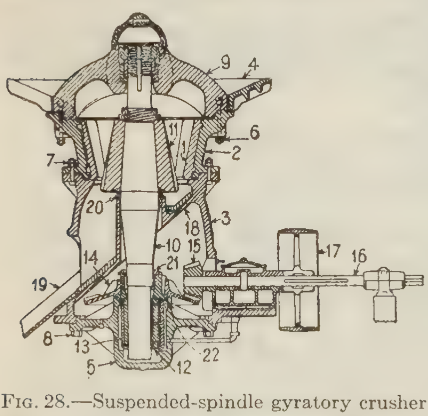


Fig. 28.—Suspended-spindle gyratory crusher.

Suspended-spindle gyratory is shown in Fig. 28. The main frame carrying the concaves (*1*) is made up of the upper shell (*2*), the lower shell (*3*), the hopper (*4*), and the bottom plate (*5*). These four parts are rigidly bolted together with heavy bolts (*6*), (*7*) and (*8*). The upper shell carries the concaves (*1*) and the two-armed spider (*9*). The spindle (*10*), carrying the breaking head (*11*), is suspended at its upper end from the spider. The lower end of the spindle passes through the eccentric sleeve (*12*), which runs in a vertical bearing (*13*) set in the bottom plate. Rotation of the eccentric sleeve is accom-

plished by means of bevel gear (*14*), bevel pinion (*15*), shaft (*16*), and pulley (*17*), driven by belt from the source of power. The axis of the spindle thus suspended and driven describes the surface of an acute cone whose apex is within the spider and the amplitude of whose base is determined by the eccentricity of the sleeve (*12*). At the same time, the spindle, being free to rotate around its own axis, rotates slowly; in the same direction as the travel of the spindle when the crusher is empty, in the reverse direction when the crusher is working. Rock to be crushed is fed into hopper (*4*), and slides down into the converging space between the concaves and the breaking head. The pieces fall as far as their size allows, then seat against the crushing surfaces, where they are broken by the movement of the mantle toward the concaves. As the mantle recedes the fragments fall to a new seat and are again crushed as the mantle approaches the concaves at the point where they are resting. When material is finally broken so that it will pass the lowest annular space between the breaking head and concaves, it falls into the annular chute (*18*), in the lower shell, and discharges through chute (*19*). Dust and grit are kept away from the gears and eccentric bearing by the dust cap (*20*), resting on the diaphragm (*18*), which is cast in the lower shell, and also by the dust collar (*21*) placed directly above the eccentric bearing. Access to the gear chamber is made possible by removable doors in the bottom shell. Access to the eccentric bearing is gained by lowering the bottom plate.

Shell is built up of heavy iron castings. The joint between the upper and lower portions is a tapered fit and is heavily flanged. The bolt holes in these flanges are drilled in such a manner that the upper shell may be turned on the lower shell, if desired. The lower shell is heavily flanged at the bottom to form a base for supporting the crusher and for bolting to foundations. All parts of the shell that are subject to wear are lined with replaceable parts. The bottom shell should be so cast that the space between the bottom of the breaking head and concaves and the highest point of the diaphragm or annular chute is as large as possible, in order to prevent clogging by slabs of rock. Discharge chute on the opposite side from the drive, as shown in Fig. 28, is known as STANDARD ARRANGEMENT. RIGHT-ANGLE DISCHARGE, with the center line of the discharge chute in a plane at right angles to the axis of the driving shaft, is furnished if desired. With such discharge, a RIGHT-HAND CRUSHER is one in which the driving pulley is on the right when the discharge chute is faced; a LEFT-HAND CRUSHER is the reverse of this.

Bottom plate is made of cast iron, heavily ribbed, in order to furnish a strong and rigid bearing for the gear-driven eccentric sleeve. It is so bolted to the flange of the lower shell as to allow it to be dropped in the foundations by means of long threaded bolts depending from the lower half of the shell. A track is provided in the foundation on which the bottom plate may be slid out to one side after dropping, to facilitate work on the gear and eccentric.

Hopper is made of cast iron heavily ribbed. In the larger crushers it is sectionalized so that only the inner ring need be removed in order to allow removal of the spider. In most crushers the hopper is so arranged that it is not necessary to remove it in order to remove the concaves; in others the inner section of the hopper projects over the concaves, the argument for such construction being that it lessens bridging of rock above the crushing zone. In large crushers the hopper is lined, usually with cast-steel sectional plates.

Spider is made of cast iron in small crushers and of cast steel in the largest crushers. Since it carries the fulcrum of the crushing lever and is two-armed only, it must be very heavy and be strongly bolted to the top shell in order to prevent overturning or breakage under lateral stresses. It should be so shaped that it forms an arch over the receiving opening of such height that the largest piece of rock that can enter the receiving opening can pass under it freely, if bridging and clogging of the crusher are to be prevented. In some crushers the base of the spider is a continuous ring forming the inner circle of the hopper, but such construction results in more breakage than when the spider is not an integral part of the hopper (*113 P 336*). Sometimes the spider arms are broadened out at the ends to give a large base for secure bolting to the upper shell. Spider arms should be protected by wearing shields unless the crusher is so fed that only occasional particles of rock come into contact with them.

Spindle or main shaft is the lever by means of which the crushing force transmitted through the eccentric bearing is applied to the rock. It must, therefore, be of great strength

and capable of withstanding a continuous succession of shock loads. It is made in most crushers of hammered, open-hearth steel, specially heat-treated, turned and polished. For extremely heavy work it may be made of special alloy steel. It is usually made to taper toward both ends, thus giving maximum cross-section and strength at the point of greatest stress. One maker hollow-bores the spindle in order to remove any defects or cracks due to the original forging, that might work outward. Threads are cut at the upper end to accommodate the adjusting nut and also at a place near the upper end to accommodate the nuts that lock the mantle. Key seats are ordinarily cut under the breaking head and adjusting nut.

Suspension bearing is of different detail in different makes of crushers, although of the same principle in all. The underlying idea is to bring the suspending surface as near to the point of no-movement as possible. In the suspension bearing shown in Fig. 29 a steel sleeve (*a*) held in place vertically by the adjusting nut (*b*) rides on the wearing ring (*c*) within a wearing sleeve or bushing (*d*), both carried by the spider. Adjusting nut (*b*) can be keyed into any desired position on the shaft.

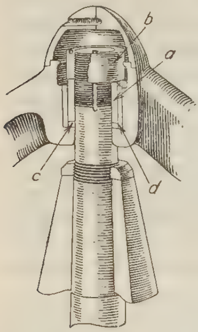


FIG. 29. — Suspension bearing for gyratory crusher.

Eccentric sleeve transmits the crushing force to the lower end of the spindle. It therefore works under high pressures and should have as large a bearing surface as possible. In all crushers the sleeve is carried in a well in the bottom plate. In most makes the eccentric is babbitted inside and out with special hard babbitt; in one make, bronze bushings are substituted for babbitt. The inner surface of the bottom plate surrounding the eccentric sleeve is bushed. The gear is keyed or riveted to the eccentric sleeve. The eccentric sleeve rides on a wearing ring of brass, bronze, or steel. In most crushers the eccentric sleeve has cylindrical faces, but in one make the inner face is spherical and engages a spherical ball on the spindle. The argument in favor of this construction is that it is self-aligning and affords a greater area of contact than the cylindrical eccentric.

Gears are made of semi-steel or high-carbon steel, cast. Provision is made, by varying the thickness of the wearing rings or by adjustment of the countershaft bearing, for taking up a small amount of wear in gear and pinion. Fitting the main gear down over the head of the eccentric sleeve and keying it thereto would seem to be better construction than that in which the gear is riveted onto the eccentric.

One manufacturer makes a gearless crusher, in which the usual gear is replaced by a horizontal pulley (*112 J 593*). This arrangement allows the spindle to be driven at a higher speed than with gears, which is an advantage in secondary-crushing service. On the other hand, the drive is bad mechanically. The crusher has not had sufficient trial to determine its utility.

A high-speed gyratory (*109 J 194*), actuated by a mechanism similar to that employed in the Mitchell screen (Sec. 5, Art. 7) has been exhibited in model size and in that size had high capacity and remarkably low power consumption. It has not, however, progressed beyond the experimental stage.

Countershaft bearing is made extra long and is cast as an integral part of the bottom shell or may be made adjustable in order to take up wear on gear and pinion. An outboard bearing of the usual ball-and-socket type is furnished.

Drive pulley in older crushers was loose on the shaft and transmitted power through pins to an auxiliary hub keyed to the shaft. The purpose of this arrangement was to furnish a breaking point. Modern crusher construction, however, eliminates this feature and the driving pulley is made extra heavy, with clamped hub keyed onto the drive shaft. In the largest crushers rope drive is sometimes used. Small crushers are belted to transmit one horsepower per inch of belt traveling at 1000 to 2000 ft. per min., reckoned on installed horsepower; the corresponding figures for large crushers are 400 to 500 ft. per min.

Data as to the different sizes of suspended-spindle gyratories available, compiled from the publications of the principal manufacturers, are given in Table 12.

Supported-spindle gyratory has the spindle supported on a wearing button (*1*) (Fig. 30) in the bottom plate, the spider serving merely to prevent horizontal movement of the upper end. The amplitude of the movement of the spindle on its support is greater in this than in the suspended-spindle type with consequent greater wear and power consumption. As a result,

this type is disappearing. Table 13 shows the sizes available. Weights and rated capacities are less than for corresponding sizes in the suspended-spindle type.

A supported-spindle machine with eccentric at the upper end of the spindle appears in patent literature, but has never advanced beyond the paper stage.

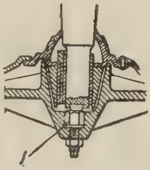


Fig. 30.—Support of spindle in supported-spindle gyratory.

Fixed-spindle gyratory (Fig. 31) is at present made by one manufacturer only (Smith Engineering Works) and goes under the name of Telsmith breaker. It differs from the types already described in that spindle (*a*) is rigidly fixed top and bottom and the movement of the crushing head (*b*) is effected by an eccentric sleeve (*c*) running between the spindle and the crushing head itself. Discharge of rock is vertical; a discharge chute is no essential part of the crusher proper. Lubrication is effected by a pump (*d*) operating in an oil well (*f*), oil being thus forced by pipes to all sliding surfaces. The frame and spider are made of cast steel and as a result of this fact and the vertical shortening made possible by applying the moving force directly under the crushing head rather than at the end of a spindle acting as a lever (as in the other types of gyratory), the weight for a given size-reduction and capacity are less than in the other types.

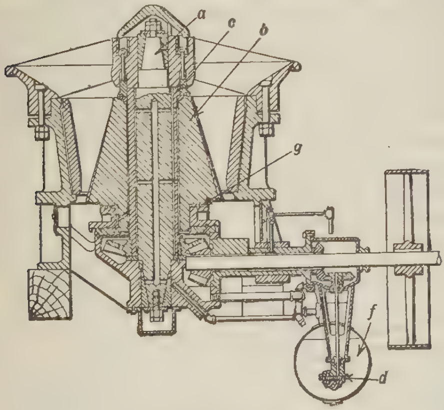


Fig. 31.—Fixed-spindle gyratory crusher.

The fixed-spindle gyratory is becoming a recognized competitor of the suspended-spindle machine. It has been installed as a primary or secondary coarse crusher in many recent plants and has given satisfaction. Its relatively small height lends itself to rugged construction and the short spindle cuts down the clear height that must be left above the machine for convenience in repair work. The fact that the length of stroke is the same on both large and small pieces is an advantage when soft, tough material is being crushed but is unnecessary and may be disadvantageous with hard and brittle materials.

The sizes available and performances to be expected are given in Table 14, compiled from manufacturer's data.

Concaves for gyratory crushers are made almost without exception of manganese steel, very occasionally of chilled iron for light service. In larger crushers each stave of the concaves is sectionalized, allowing discard of one part while retaining the rest. At some plants the staves are sectionalized in two parts, making the upper two-thirds of chilled iron and the lower third of manganese steel. Concaves are supported either at the bottom by a sectional removable iron rim resting on a shelf on the lower shell or by means of lugs or ribs that are cast on the back near the top and fit into a groove in the top shell. After being set in place the concaves are backed by hard zinc or hard babbitt poured in between them and the shell. Frequently the lower edge of the concaves is beveled in order to increase the amount of metal behind the point at which maximum crushing is done. Life

Table 12. Data on suspended-spindle gyratory

Number	Size of receiving opening, inches	Average weight, pounds	Capacity,					
			⅜	¾	1	1¼	1½	1¾
0	2½ × 28	1,000	0.6	......	......	......	......	......
1F	3½ × 45	7,000	......	3–4	......	......	......	......
2F	4 × 56	10,000	......	5–6	......	......	......	......
3F	4½ × 64	15,500	......	......	8–10	......	......	......
4F	5 × 77	22,000	......	......	11–14	......	......	......
1	5 × 50	7,000	......	......	5	6	7	8
5F	6 × 93	35,000	......	......	......	18–22	......	......
2	6 × 50	10,250	......	......	6.5	8	9	10
3	7 × 56	17,000	......	......	......	11	12.5	......
3	8 × 60	15,500	......	......	......	10	12	......
4	8 × 60	22,000	......	......	......	......	15	20
4	8 × 68	23,000	......	......	......	......	20	......
......	8 × 74	16,500	......	......	*b* 16 27	....	24 41	....
6F	8 × 100	48,000	......	......	......	......	30–37	......
4	9 × 70	22,500	......	......	......	......	22	......
7½F	9 × 129	71,500	......	......	......	......	......	37–46
5	10 × 80	37,000	......	......	......	......	......	30
5	11 × 82	36,000	......	......	......	......	......	35
8F	11 × 158	100,000	......	......	......	......	......	62–65
6	12 × 88	48,000	......	......	......	......	......	......
......	12 × 92	29,000	......	......	......	......	*b* 28 49	....
6	12½ × 90	49,000	......	......	......	......	......	......
7½	14 × 104	68,000	......	......	......	......	......	......
......	14 × 110	36,500	......	......	......	......	......	......
7½	15 × 110	72,000	......	......	......	......	......	......
......	16 × 126	53,500	......	......	......	......	......	......
......	18 × 136	103,000	......	......	......	......	......	......
8	19 × 138	106,000	......	......	......	......	......	......
......	20 × 160	83,000	......	......	......	......	......	......
9	21 × 152	160,000	......	......	......	......	......	......
......	22 × 154	159,000	......	......	......	......	......	......
......	24 × 168	175,000	......	......	......	......	......	......
......	25 × 200	210,000	......	......	......	......	......	......
......	26 × 200	143,000	......	......	......	......	......	......
......	27 × 184	180,000	......	......	......	......	......	......
......	36 × 260–272	265,000 405,000	......	......	......	......	......	......
......	42 × 272	425,000	......	......	......	......	......	......
......	42 × 306	325,000	......	......	......	......	......	......
......	48 × 320–332	470,000	......	......	......	......	......	......
......	60 × 380–420	750,000	......	......	......	......	......	......
......	72 × 484	1,000,000	......	......	......	......	......	......

a First size under which a capacity figure is given is minimum setting. *b* Upper line,

crushers from manufacturers' catalogs

tons per hour to .. in.(a)

2	2¼	2½	2¾	3	3½	4	4½	5	5½
....	....	....	....	....	....	....	....	....	....
....	....	....	....	....	....	....	....	....	....
....	....	....	....	....	....	....	....	....	....
....	....	....	....	....	....	....	....	....	....
....	....	....	....	....	....	....	....	....	....
9	....	....	....	....	....	....	....	....	....
....	....	....	....	....	....	....	....	....	....
11	12.5	....	....	....	....	....	....	....	....
17	....	21	25	....	....	....	....	....	....
16	....	20	....	....	....	....	....	....	....
25	....	30	....	40	....	....	....	....	....
30	....	40	....	....	48	....	....	....	....
32	36	}	....	....	....	....	....	....	....
....	....								
....	....	....	....	....	....	....	....	....	....
32	....	42	....	....	....	....	....	....	....
....	....	....	....	....	....	....	....	....	....
40	....	50	....	60	70	75	....	....	....
43	....	60	....	....	....	....	....	....	....
....	....	....	....	....	....	....	....	....	....
50	....	70	....	80	90	....	120	....	....
40	....	51	....	63	75	}	....	....	....
65	....	....	....	....	....				
50	65	80	....	....	....	....	....	....	....
....	....	80	....	90	100	120	....	....	....
*b*52	....	64	....	77	89	102	} 115	....	....
83	....	105	....	....	....	....			
....	....	75–80	....	125	139	153	166	180	....
..... {	*b* 93	100	....	117	134	151	} 168	185	....
	132	....	....	175	....	....			
....	....	....	110	....	130	150	....	....	250
....	....	....	....	125	200	....	....	....	....
....	....	....	...... {	*b*157	191	225	} 259	295	330
				187	....	250			
....	....	....	....	160	205	250	....	300	....
....	....	....	....	....	....	200	260	320	380
....	....	....	....	....	210	250	200	330	370
....	....	....	....	....	....	....	....	300	400
....	....				...... {	*b*289	336	383	} 450
						328	....	430	
....	....	....	....	....	....	260	310	355	400
....	....	....	....	....	....	....	525–595	600	....
....	....	....	....	....	....	....	....	....	700
....	....	....	....	....	....	....	....	930	....
....	....	....	....	....	....	....	....	....	650–1158
....	....	....	....	....	....	....	....	....	....
....	....	....	....	....	....	....	....	....	....

run of mine rock; lower, oversize return with maximum size not over ½ dimension of gape.
F = equipped with short head and concaves for fine crushing.

Table 12. Data on suspended-spindle gyratory

Number	Size of receiving opening, inches	Average weight, pounds	Capacity, tons per hour				
			6	6½	7	8	9
0	2½ × 28	1,000	....	....	....	....	....
1F	3½ × 45	7,000	....	....	....	....	....
2F	4 × 56	10,000	....	....	....	....	....
3F	4½ × 64	15,500	....	....	....	....	....
4F	5 × 77	22,000	....	....	....	....	....
1	5 × 50	7,000	....	....	....	....	....
5F	6 × 93	35,000	....	....	....	....	....
2	6 × 50	10,250	....	....	....	....	....
3	7 × 56	17,000	....	....	....	....	....
3	8 × 60	15,500	....	....	....	....	....
4	8 × 60	22,000	....	....	....	....	....
4	8 × 68	23,000	....	....	....	....	....
....	8 × 74	16,500	....	....	....	....	....
6F	8 × 100	48,000	....	....	....	....	....
4	9 × 70	22,500	....	....	....	....	....
7½F	9 × 129	71,500	....	....	....	....	....
5	10 × 80	37,000	....	....	....	....	....
5	11 × 82	36,000	....	....	....	....	....
8F	11 × 158	100,000	....	....	....	....	....
6	12 × 88	48,000	....	....	....	....	....
....	12 × 92	29,000	....	....	....	....	....
6	12½ × 90	49,000	....	....	....	....	....
7½	14 × 104	68,000	....	....	....	....	....
....	14 × 110	36,500	....	....	....	....	....
7½	15 × 110	72,000	....	....	....	....	....
....	16 × 126	53,500	....	....	....	....	....
....	18 × 136	103,000	....	....	....	....	....
8	19 × 138	106,000	....	....	....	....	....
....	20 × 160	83,000	....	....	....	....	....
9	21 × 152	160,000	350	....	....	....	....
....	22 × 154	159,000	440	500	....	....	....
....	24 × 168	175,000	410	450	....	....	....
....	25 × 200	210,000	500	600	700	....	....
....	26 × 200	143,000	477	525	....	....	....
....	27 × 184	180,000	450	500	550	....	....
....	36 × 260–272	{ 265,000 405,000	} 770	....	940	1000–1100	....
....	42 × 272	425,000	790	870	960	1080	1300
....	42 × 306	325,000	1090	....	1250	1425	1600
....	48 × 320–332	470,000	715–1260	780–1360	850–1460	975–1675	1100–1890
....	60 × 380–420	750,000	....	750	810–1678	985–1900	1060–2150
....	72 × 484	1,000,000	....	....	....	....	2572

[a] First size under which a capacity figure is given is minimum setting.

crushers from manufacturers' catalogs. *Continued*

to....in.(*a*)			Average revolutions per minute	Horse-power	Fall through crusher, average, feet-inches	Reduction ratio, maximum	Fall ÷ gape
10	11	12					
....	....	....	700	2–4	1–7	6.7	7.6
....	....	....	700	4–5	4–9	4.7	....
....	....	....	600	6–9	5–4	5.3	....
....	....	....	550	10–13	5–10	4.5	....
....	....	....	525	12–17	6–9	5	....
....	....	....	600	4–6	4–9	5	11.5
....	....	....	500	17–23	8–0	4.8	....
....	....	....	500	6–10	5–4	6	10.7
....	....	....	475	10–15	5–10	5.6	10
....	....	....	525	10–15	5–11	6.4	8.9
....	....	....	400	14–21	6–5	5.3	9.6
....	....	....	450	12–20	6–9	5.3	10.2
....	....	....	500	12–25	6–4	8	9.5
			600	15–40			
....	....	....	525	23–35	8–9	5.3	....
....	....	....	475	12–20	6–6	6	8.7
....	....	....	500	45–60	10–2	5.1	....
....	....	....	400	20–30	7–10	5.7	9.4
....	....	....	450	20–25	7–11	5.7	8.6
....	....	....	475	65–85	12–0	5.7	....
....	....	....	375	25–45	8–9	6	9.5
....	....	....	365	25–45	7–9	8	7.8
			400	30–60			
....	....	....	425	25–40	8–8	6.2	8.3
....	....	....	350	50–75	9–6	5.6	8.1
....	....	....	345	40–75	8–7	7	7.4
			375	50–90			
....	....	....	375	45–70	10–1	6	8.1
....	....	....	340	60–100	10–1	7.1	7.6
			360	75–125			
....	....	....	350	65–100	11–9	6.5	7.8
....	....	....	375	65–100	11–9	6.3	7.4
....	....	....	330	90–150	12–4	6.7	7.4
			345	100–180			
....	....	....	325	100–150	13–6	7	7.7
....	....	....	350	100–140	13–3	5.5	7.2
....	....	....	350	115–160	14–5	6.9	7.2
....	....	....	350	125–175	14–4	5	6.9
....	....	....	320	125–200	14–10	6.5	6.8
			340	150–250			
....	....	....	350	130–180	....	6.8	....
....	....	....	300	150–250	18–1+	8.0	6.0
....	....	....	300	225–280	....	7.6	....
....	....	....	275	175–275	19–5	8.4	5.5
....	....	....	250–300	150–350	21–9	8.7	5.4
1200–2400	....	....	200–240	250–400	29–3	9.2	5.9
2860	3150	3432	175	300–500	34–7	8	6.9

F = equipped with short head and concaves for fine crushing.

of concaves varies according to the material used. Figures from practice are given in Table 15.

Table 13. Supported-spindle type gyratory crushers

Number	Size of receiving opening, inches	Weight, pounds	Capacity, tons per hour with 2.5-in. setting	Minimum setting, inches	Revolutions per minute of driving pulley	Horse-power	Fall through crusher, feet-inches
F	2×12	650	....	⅜	700	1–1.5	....
0	4×30	3,850	2	¾	500	3–4	3–11
1	5×36	5,900	4	1	475	5–8	4–2
2	6×42	8,400	6	1⅛	450	7–12	4–7
3	7×45	14,480	10	1⅜	425	10–16	5–8
4	8×54	21,700	15	1½	400	14–21	7–0
5	10×60	30,700	25	2	375	22–30	7–9
6	11×72	43,000	30	2¼	350	28–45	9–1
7½	14×90	48,000	75	2½	350	50–75	10–5

Breaking head is, like the concaves, made either of chilled iron or manganese steel. Chilled-iron breaking heads are solid and keyed directly to the spindle. When manganese steel is used, a soft iron or semi-steel core is keyed to the spindle and a mantle of manganese steel is slipped on over this. The mantle is held down on the core by means of some self-tightening arrangement. Two devices are commonly used for this purpose. GUN-LOCK MANTLE is illustrated in Fig. 32. The core itself is keyed to the spindle by means of feathers in keyways (*a*). On the outside of the core two grooves (*b*) are cut at 180°, and the bottom of the core is finished to a curve whose highest point is at the bottom of grooves (*b*). The mantle is ground to fit closely on the head and is cast with lugs (*c*) that slip down the grooves (*b*) and pull the mantle tightly onto the core as the mantle turns with respect to the spindle. Another SELF-TIGHTENING MANTLE is shown in Fig. 33. In one form the nut is made of three pieces: piece (*1*), pinned to the top of the mantle and carrying on its inner surface a left-hand thread; piece (*2*) having a corresponding left-hand thread on the outside to engage piece (*1*) and a right-hand thread on the inside to engage the thread on the spindle; and piece (*3*) with a right-hand thread inside. When the mantle loosens it works round on the core and imparts its motion to nut (*1*). If the direction of motion is such as to cause the left-hand thread to unscrew, the mantle is thereby pressed down onto the core. If, on the other hand, the motion is such as to cause the left-hand thread to tighten, then nut (*2*) is caused to move down on the spindle, again pressing the mantle down on the core. With this type of mantle zinc is used to make a tight joint between the mantle and the core. Some manufacturers recommend the use of zinc or babbitt keys to prevent the solid heads or cores from turning on the spindle; others use an ordinary steel feather key. One maker uses self-tightening nuts to force the core down on a tapered spindle and eliminates keys. The use of keys would, however, seem to be the better practice, notwithstanding the difficulty in drawing keys when a head is to be changed. A DISADVANTAGE of the use of self-tightening lock-nuts is that large pieces of ore jammed in the mouth of the breaker may loosen the control nut. This happened frequently on a primary gyratory at MOOSE MOUNTAIN (*99 J 973*).

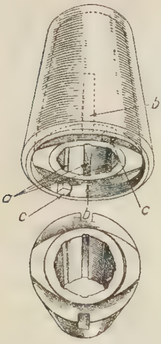


FIG. 32.—Gun-lock mantle for gyratory crusher.

In large crushers the mantles are frequently made in two parts. The greatest wear comes at the bottom and the amount of metal that must be wasted can be decreased by making the bottom part replaceable while retaining the old upper part. Mantles are made corrugated or smooth. The corrugated head is best for coarse breaking; a smooth head is best if a large proportion of fines is desired or if the feed to the crusher is already relatively fine. Life of mantles is given in Table 15.

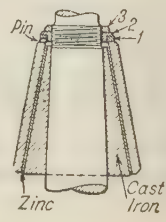


FIG. 33. — Self-tightening lock-nut for gyratory mantle

Lubrication. In all crushers the eccentric bearing runs submerged in oil. Capillarity is depended upon in some crushers to draw oil from the bath

into the bearing surfaces while in others a forced feed is used. The main countershaft bearing is generally ring- or chain-oiled with overflow from the reservoir into an oil sump in the bottom plate. It is probable that the best lubricating system is one in which an efficient forced feed of heavy oil is employed. Extrusion of this oil around the bearings aids to keep grit and dust out.

Adjustments of gyratory crushers are (1) width of discharge opening, (2) throw, and (3) speed. Width of discharge opening is changed in the suspended-spindle crusher by raising or lowering the spindle by means of the adjusting nut (*b*), Fig. 29. A ring bolt

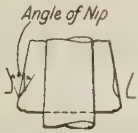


Fig. 34.—Change in nip angle with wear of breaking head and concaves.

is furnished to engage the top of the spindle, which is then hoisted by appropriate means, allowing the adjusting nut to be readily moved as desired. The change is made in the supported-spindle type by raising the supporting nut (*1*) (Fig. 30). In the fixed-spindle type the adjustment is made by changing the thickness of shims (*g*), Fig. 31.

Range of adjustment in width of opening is limited because of the fact that the nip angle is markedly increased as a result of wear of the breaking head. This is shown in Fig. 34. This increase in nip angle causes a great decrease in ability of the crusher to discharge, with corresponding decrease in capacity. If more than a small adjustment in width of discharge is required, it must be obtained by using thicker and shorter concaves and breaking head.

Throw is less than in jaw crushers. It is adjustable only by changing the eccentric sleeves. It should be greater in large crushers than in small and should be greater for relatively soft and tough rocks than for hard and brittle rocks.

Table 14. Fixed-spindle gyratory, catalog data

Number	Size of receiving opening, inches	Weight, pounds	Capacity, tons per hour to . . in.									Revolutions per minute	Horsepower (*a*)	Fall through crusher, feet-inches
			1	1¼	1½	1¾	2	2½	3	3½	4			
6	6¾ × 70	9,650	12–13	13–14	14–16	. . . .	. . . .	. . . .	. . . .	. . . .	. . . .	475	15	5–9
8	8 × 82	12,850	. . . .	22–23	23–24	24–26	. . . .	. . . .	. . . .	. . . .	. . . .	450	20	6–2
10	10 × 102	18,400	. . . .	. . . .	28–31	31–34	34–37	37–40	. . . .	. . . .	. . . .	425	25	6–8
13	13 × 118	28,000	. . . .	. . . .	. . . .	. . . .	60–65	65–70	70–75	. . . .	. . . .	340	40	9–0
16	16 × 148	47,650	. . . .	. . . .	. . . .	. . . .	. . . .	100–110	110–120	120–130	. . . .	275	60	10–3
20	20 × 176	68,000	. . . .	. . . .	. . . .	. . . .	. . . .	. . . .	175–190	190–210	210–225	260	80	11–3
25	25 × 212	110,000	. . . .	. . . .	. . . .	. . . .	. . . .	. . . .	. . . .	275–300	300–325	260	100	13–6

a Crushing limestone.

Table 15. Performance of lever-type gyratory crushers

Plant	Tonopah Belmont	St. Joseph Lead, Rivermines	Federal M. & S., Morning	Witherbee-Sherman No. 5	Witherbee-Sherman No. 4	Tungsten Mines Co.
Gape, in.	5	7	8	10	10	10
Set, in.	1	1.25	1.5	1.5	1.5	2
Throw, in.	0.5	0.75	0.75	1.25	1.25	0.75
Maximum size of feed, in.	2	4	4	4×8–12	4×8–12	R.O.M.
Apparent reduction ratio	5	5.6	5.3	6.7	6.7	....
Actual reduction ratio	2	3.2	2.7	2	2	....
Capacity, tons per hour	50	25	21	67	50	16.7
Power installed, hp.	....	15	20	37.5	....	40
Power consumed, hp.	....	12	15	11 max. 6 min.	11 max. 6 min.	30
Tons per hp.-hr., installed	....	1.67	1	1.8	....	0.42
Tons per hp.-hr., consumed	....	2.1	1.4	7.9	5.9	0.56
Speed, driving pulley, r.p.m.	410	610	460	350	365	316
Speed, spindle, r.p.m.	165	220	185	140	146	126
Oil consumed, pounds per shift	....	....	2	8	8	2
Life of wearing parts, days						
Eccentric	730	70	180	150	150	90–180
Gears	730	600	365	300	300	180 Cast iron 450–540 Steel
Mantle	720	300	180	300	250	520
Material	*Mn*	*Mn*	*Mn*	*Mn*	*Mn*	*Mn*
Concaves	720	180	270	150	175	90-120, lower; 300, upper, and still in service
Material	*Mn*	*Mn*	*Mn*	*Mn*	*Mn*	*Mn*
Chute liners	730	180	150	365	365	270
Material	*Mn*	Cast iron	Cast iron	*Mn*	*Mn*	Cast iron
Changing concaves, hr.	....	....	....	....	....	18, 5 men
Hours run per 24 hr.	8	4–6	18	9	18	24
Per cent. lost time	....	1	1	....	....	1

Mn Manganese steel.

Table 15. Performance of lever-type gyratory crushers—*Continued*

Plant	St. Joseph Lead, Bonne Terre	St. Joseph Lead, Bonne Terre	Melones	Engels Copper Mining Co.	Phelps-Dodge, Moctezuma	Chino Copper Co.	Bunker Hill and Sullivan
Gape, in.	10	10	10	10	10	12	12
Set, in.	2	2	1.75	1.75	1.5	3	2.5
Throw, in.	...	...	...	0.75	1	1	...
Maximum size of feed, in.	10	7	...	4	4	12	12
Apparent reduction ratio	5	7	5.7	5.7	5.7	4	4.8
Actual reduction ratio	3.3	3.5	...	2.3	2.7	4	4.8
Capacity, tons per hour	40	33	12.5–21	40	33	290	62.5
Power installed, hp.	40	40	...	...	37.5	40	...
Power consumed, hp.	37.5	37.5	40	30	25	25	15
Tons per hp.-hr., installed	1.00	0.82	...	...	0.88	7.25	...
Tons per hp.-hr., consumed	1.07	0.88	0.31–0.52	1.33	1.32	11.6	4.2
Speed, driving pulley, r.p.m.	375	375	...	340	575	445	466
Speed, spindle, r.p.m.	...	...	...	142	200	130	171
Oil consumed, pounds per shift	0.5	1.0	1	2	6	1.33	3.2
Life of wearing parts, days							
Eccentric	45	72	720	90	40	365	5 yr.
Gears	...	...	1600	...	200	7 yr.+	5 yr.
Mantle	...	...	540	360–720	180	365	288
Material	...	...	*Mn*	*Mn*	*Mn*	*Mn*	Cast iron
Concaves	130	...	540	Lower, 270	180*a*	Top, 21 mo.; bottom, 210	365
Material	Cast iron	*Mn*	*Mn*	*Mn*	*Mn*	*Mn*	*Mn*
Chute liners	50	60	360	200	150–180	120–150	960
Material	Cast iron	Cast iron	Sheet steel	White iron	Cast iron	Grizzly bars	Cast iron
Changing concaves, hr.	...	...	...	...	...	...	8
Hours run per 24 hr.	24	24	16	16	16	22	6.5
Per cent. lost time	−0.1	−0.1	Very small	0.4	1	8*b*	Practically none

Mn Manganese steel. *a* Increased at least 50 days by use of oversize heads. *b* Clogged chutes.

Table 15. Performance of lever-type gyratory crushers—*Continued*

Plant	Belmont-Surf Inlet	Nevada Packard	Mexican cyanide plant	Mexican cyanide plant	Tonopah Belmont	Phelps-Dodge, Morenci
Gape, in.	12	12	13	13	14	14
Set, in.	2	1.5	5	2	2	2.5
Throw, in.	....	0.75	1	1	2	0.75
Maximum size of feed, in.	....	−12	13	6	9	14
Apparent reduction ratio	6	8	2.6	6.5	7	5.6
Actual reduction ratio	....	8	2.6	3	4.5	5.6
Capacity, tons per hour	....	12.5	100	100	100	250
Power installed, hp.	50	....	35	35	....	....
Power consumed, hp.	15–50	....	35	35	....	....
Tons per hp.-hr., installed	....	....	2.85	2.85	....	....
Tons per hp.-hr., consumed	....	....	2.85	2.85	....	....
Speed, driving pulley, r.p.m.	....	....	350	350	350	365
Speed, spindle, r.p.m.	....	....	....	....	140	150
Oil consumed, pounds per shift	....	....	....	....	2	....
Life of wearing parts, days						
Eccentric	....	250	100–200	100–200	....	....
Gears	....	380	180	180	....	....
Mantle	250	500	360–540	180–240	540	630 of 8 hr.
Material	*Mn*	*Mn*	*Mn*	*Mn*	*Mn*	*Mn*
Concaves	200	375	360	360	360	Upper, 730 of 8 hr.; lower, 240 of 8 hr.
Material	*Mn*	*Mn*	*Mn*	*Mn*	*Mn*	*Mn*
Chute liners	....	....	....	....	730	240 of 8 hr.
Material	....	....	....	....	Cast iron	Cast iron
Changing concaves, hr.	6	19	7, with spare ring	7, with spare ring	....	....
Hours run per 24 hr.	8	9	24	24	8	8
Per cent. lost time	....	....	....	....	1	Practically none

Mn Manganese steel

Table 15. Performance of lever-type gyratory crushers—*Continued*

Plant	McIntyre Porcupine	Witherbee-Sherman No. 3	Alaska-Gastineau	Phelps-Dodge, Moctezuma	Witherbee, Sherman No. 3	Engels Copper Mining Co.	American Zinc Lead and Smelting Co.
Gape, in.	14	16	18	18	18	20	24
Set, in.	4	1.5	2.25	4	3	4	4
Throw, in.	....	2.25	0.56	2	2	1.37	1.25
Maximum size of feed, in.	R.O.M.	....	9	R.O.M.	....	12	24
Apparent reduction ratio	3.5	10.6	8	4.5	6	5	6
Actual reduction ratio	....	....	4	....	....	3	6
Capacity, tons per hour	80	75	100	167	150	100	208
Power installed, hp.	50	....	76.5	75	35	....	75
Power consumed, hp.	47	11 max. 6 min.	44	65	50 max. 12 min.	20	32
Tons per hp.-hr., installed	1.6	....	1.3	2.22	4.3	....	2.78
Tons per hp.-hr., consumed	1.7	8.8	2.27	2.57	....	5	6.5
Speed, driving pulley, r.p.m.	350	340	391	340	207	290	385
Speed, spindle, r.p.m.	....	136	154	120	83	98	146
Oil consumed, pounds per shift	....	8	27	16	8	2	4
Life of wearing parts, days							
Eccentric	250	150	52	180	300	360	80–100
Gears	350	450	375	360	600	....	360
Mantle	250	300	375	300	300	3–4 yr.	720
Material	*Mn*	*Mn*	*Mn*	*Mn*	*Mn*	*Mn*	*Mn*
Concaves	300	250	300	300*a*	480	Upper, indefinitely; lower, 2 to 5 yr.	400
Material	Upper, chilled iron; lower, *Mn*	*Mn*	*Mn*	*Mn*	*Mn*	*Mn*	*Mn*
Chute liners	200	400	679	150	600	90	360
Material	Steel plate	....	*Mn*	Hard cast iron	....	White-iron	Cast iron
Changing concaves, hr.	16	....	12	....	....	....	....
Hours run per 24 hr.	8	18	24	16	18	16	16(av. 10)
Per cent. lost time	1	....	....	1	0	0.25	0

Mn Manganese steel. *a* Increased at least 50 days by use of oversize heads.

Speed may be varied throughout wide limits. Increased speed is not accompanied by the marked increase in vibration and shock that occurs with jaw crushers. The lowest speed compatible with the capacity required is most economical within certain operating limits. A tendency to clog by reason of the sticky character of ores may often be overcome by increase in speed, thus increasing the sharpness with which the head recedes from the concaves.

Speed, throw, capacity, reduction ratio and power consumption are closely related. If the reduction ratio is increased when the crusher is working near maximum capacity, the speed of the crusher must be increased to keep up the capacity. This will be accompanied by a considerable increase in power consumption. If the machine is sufficiently over-motored so that the change brings no perceptible strain on the driving equipment, it is well to watch for heating of the eccentric, as any marked increase in the direction above noted over the figures recommended by the manufacturers is likely to result in burned-out bearings. If increase in reduction ratio is to be effected without mechanical troubles, it should be accompanied normally by decrease in speed and a decrease in the amount of material in the crushing zone. This latter decrease is accomplished by shortening the head and concaves and by decreasing capacity.

Table 16. Performance of fixed-spindle gyratory as a primary crusher

Plant	Shattuck Arizona	Liberty Bell	Burro Mountain	Federal Lead Mills Nos. 3, 4	United Eastern
Gape, in.	10	12	12	14	18
Set, in.	1.5*a*	2	3.5	3	1.75
Throw, in.	0.75	0.75	1	$1\frac{1}{16}$	0.5
Size of largest particle of feed, in.	−10	−12	−12	−14	10
Apparent reduction ratio	6.7	6	3.4	4.7	5.7
Actual reduction ratio	4	6	....	3.5	5
Capacity, tons per hour	25*b*	100	250	125	150–250
Power installed, hp.	25	40	40	50	50
Power consumed, hp.	25−	....	20	41	13.4
Tons per hp.-hr. installed	1	2.5	6.3	2.5	3–5
Tons per hp.-hr. consumed	....	....	12.5	3.0	14.9 av.
Speed, driving pulley, r.p.m.	....	335	350	250	320
Speed, spindle, r.p.m.	....	132	70	100	140
Oil consumption, pounds per shift	....	8	....	5	8
Life of wearing parts, days:					
Eccentric	180	2000+	....	90–120	....
Gears	900*a*	1100	....	1800	....
Mantle	90*d*	360–540	....	1100	....
Material	*MS*	*MS*	....	*CCI*	*MS*
Concaves	90*e*	360	....	1500	....
Material	*MS*	*MS*	....	*MS*	*k*
Chute liners	180	180	....	....	....
Material	*f*	*h*	....	....	....
Changing concaves, hr.	10–15	12–18	....	....	6
Hours per day operated	15	8–12	8	24	5–8
Per cent. lost time	Very little	....	....	2*i*	....
	g			*j*	

a When new. Wears to 2 or 2½ in. *b* Very hard rock. *c* Still in good condition. *d* With one set of concaves; used another 90 days with a second set of concaves. *e* Set up or renewed, according to wear. *f* Old ball-mill liners. *g* Superintendent reports that the machine requires almost no attendance. It has stalled on steel several times without breakage. *h* ¼-in. sheet iron. *i* Babbitting eccentric and changing mantles and concaves. *j* Have given complete satisfaction. *k* Upper ring chilled cast iron, lower manganese steel. *MS* Manganese steel. *CCI* Chilled cast iron.

Data on mill performances are given in Tables 15 and 16 and in the following notes.

At Roseberry mill, Slocan, B. C., a No. 2-D Gates (6 × 50-in.) crushes 150 tons per 24 hr. from 2-in. to 1-in. (*114 J 677*). At the Tonopah Belmont mill a No. 4-B short-head machine (5 × 77-in.) is used to break from 2-in. to 1-in. at the rate of 500 tons per 24 hr. The concaves for this machine weigh 666 lb., cost $88 (1915), and wear for two years, crushing 75,000 tons. They are set out at the end of 6 months and turned end-for-end at 12 months. The mantle weighs 536 lb. and has the same life (*52 A 96*). A No. 5 crusher (10 × 80-in.) at the Trojan mill, So. Dak. (*114 J 763*), set to 1.75-in., crushes 400 tons per 24 hr. of the oversize on a 1.5-in. grizzly. At the Santa Barbara (Mex.) mill of the American Smelters Securities Co. a No. 5 McCully crusher (suspended-spindle) set at 1.5-in. breaks the run-of-mine oversize on a 1-in. grizzly at the rate of 500 tons per 24 hr. (*112 J 1050*). At St. Joseph Lead Co. mill, Bonne Terre, Mo., three No. 5 gyratories are used to crush 2100 tons run-of-mine ore per 24 hr. The ore, principally dolomite, sizes to 11.1 per cent. on a 10-in. screen, 12.6 per cent. on 8-in., 7.6 per cent. on 6-in., 32.5 per cent. on 4-in. and 36.2 per cent. through 4-in. It is split on a 7.5-in. grizzly and the oversize is sent to one No. 5-K (suspended-spindle type) gyratory while the undersize is split between two No. 5-D (supported-spindle type) machines. A sizing test of the combined discharge is 12.9 per cent. on 3-in. screen, 11.4 per cent. on 2.5-in., 17.6 per cent. on 2-in., 15.7 per cent. on 1.5-in., 14.9 per cent. on 0.35-in., 9.9 per cent. on 0.079-in., 3.5 per cent. on 0.0041-in. and 1.6 per cent. through 0.0041-in. (*57 A 423*). (See also Table 12.) At the Shattuck-Arizona mill a No. 5 Telsmith (fixed-spindle) crusher set at 1 5-in. breaks 400 tons per 24 hr. of +1.5-in. run-of-mine ore. This crusher has handled 500 tons per day (*110 J 759*). At Barnes-King Development Co. mill (*60 A 98*) a No. 5 Telsmith breaks 15.3 tons per hr. to 0.4 per cent. on 3-in. screen with a power consumption of 1.1 kw.-hr. per ton and at a cost (1917) of $0.046 per ton. At the head house of the Ellison shaft of the Homestake mines four No. 6 gyratories set at 3-in. are used to crush about 25 tons per hr. each of rock passing a jaw crusher set at 4.5-in. and remaining on a 3-in. grizzly (*114 J 760*). At New Modderfontein (So. Africa) four No. 6 gyratories set to 2-in. take 1200 tons per 24 hr. run-of-mine ore. Each crusher has a 60-hp. motor (*120 P 789*). At the Silver King Coalition mill (Utah) the primary crusher is a No. 6 gyratory set to 1.5-in. and handles 300 tons per day of +1.5-in. run-of-mine ore, 8-in. maximum size (*116 J 370*). At United Eastern a No. 6 Telsmith set to 2-in. takes 35 tons per hr. run-of-mine ore, 10-in. maximum size, and draws 10 kw. average load. By placing a 1.75-in. grizzly ahead of the crusher, repairs were materially decreased (*63 A 551*). United Comstock uses one No. 7½ gyratory set at 3-in. for the oversize, on a 3-in. grizzly, from 2000 tons per 24 hr. run-of-mine ore (*114 J 846*). At Hollinger three No. 7½ gyratories set at 3.5-in. handle an average of 70 tons per hr. each (90 tons per hr. maximum), when running at 144 r.p.m. of spindle and drawing 50 hp. Two work on the oversize of a 7.5-in. grizzly screening run-of-mine ore through a 14-in. grizzly, the other crushes the undersize of the 7.5-in. grizzly. This routing of ore is said to increase the gross capacity of the three crushers, especially when the ore is wet. Concaves are sectionalized transversely: the upper, of white iron, last 12 months; the lower, of manganese steel, 8 months. They are re-set every 4 months and require 1700 lb. of zinc per setting. Head centers are of cast steel, mantles of manganese steel. The latter are drawn on the shaft at 125 tons pressure after warming to 120° F. Life is 7 mo. Poured-zinc keys are used to keep the mantle and head center (core) from turning on the spindle. The product of the primary crushers is sent over a 2-in. grizzly and the oversize sent to three No. 5 crushers, two set at 2.5-in. and one at 1.5-in. The total feed to the three machines is about 35 tons per hr. Speed of spindle, 157 r.p.m. Each crusher consumes about 25 hp. The upper ring of concaves is white iron and the lower manganese steel. Life of the upper ring is 12 months and of the lower, 3 months. The concaves are re-set every four or five weeks; 900 lb. of zinc is required for a setting. (*1922 Bul. CMI 340.*) At Tonopah Belmont manganese-steel concaves for a No. 7½ crusher weigh 1120 lb. and crush 150,000 tons; a mantle weighs 1994 lb. and will crush 225,000 tons (*52 A 110*). At Alaska Gastineau four No. 8-K machines set at 2.5-in. are used to crush −5-in. jaw-crusher product at the rate of 110 tons per hr. each. Mantles are of gun-lock type and both mantles and concaves are manganese steel. Lower concaves last 8 months, crushing 16 hr. per day. Changing concaves and re-zincing requires 8 hr. The crushers are driven by clutch pulley from a line shaft (*63 A 493*) (see also Table 12). Chile Copper Co., Chuquicamata, Chile, originally put in two No. 10 crushers for primary service to handle 275 tons per hr. each from 16-in. maximum to 3-in. discharge size. They proved too small and are now preceded by two 84 × 60-in. jaw crushers (*101 J 23*). Alaska Treadwell used two No. 12-K Gates crushers followed by four No. 6-K set at 2-in. to crush about 4500 tons per day of run-of-mine ore. Sizing analysis of the product follows: 6 per cent. on 3-in. screen; 28.2 per cent. on 2-in ; 29.25 per cent. on 1-in.; 10.11 on 0.5-in.; 11.65 per cent. on 0.185-in.; 7.80 per cent. on 0.0328-in.; 2.23 per cent. of 0.0164-in.; 0.80 per cent. on 0.0116 in.; 0.65 per cent.

on 0.0082 in.; 0.65 per cent. on 0.0058 in.; 0.80 per cent. on 0.0041-in.; 0.53 per cent. on 0.0029-in. and 1.33 per cent. through 0.0029-in. (*114 P 410*). The primary crusher at New Cornelia is a No. 24-K Gates gyratory. It will take boulders 3.5 × 4.5 × 10-ft. and reduce them 9-in. at the rate of 500 tons per hr. and the discharge rate at this setting has run as high as 800 tons per hr. when the feed was free of boulders that caused clogging in the bowl. Capacity was reduced to 400 to 450 tons per hr. to 6-in. size by setting out the concaves and using a thicker mantle. Average manganese-steel consumption in this and four No. 8's following is 0.0032 lb. per ton in mantles and 0.081 lb. per ton in concaves. Average power consumption for the five machines is 0.21 kw.-hr. per ton. Four No. 8-K crushers set at slightly less than 3-in. take the oversize of a 3-in. grizzly screening the product of the primary crusher set at 6-in. The feed to each machine is between 100 and 125 tons per hr. and this is a little less than full load. These same crushers set at 4-in. and taking 9-in. feed were unable to handle the feed at the rate of 200 tons per hr. each (*63 A 509*). At Utah Copper Co. a No. 27 double-discharge machine is used to break from 54-in. maximum size to 4.5 in. (*118 P 469*).

Reduction ratio, according to manufacturers' ratings, ranges from a minimum of 5, excluding fine-crushing gyratories, to a maximum of about 9 with a general average of about 6. A greater ratio is recommended for large crushers than for small. See Table 12. In practice the apparent ratio, reckoned as the ratio of the gape to the minimum opening at the throat averages 5.78 for twenty-five crushers (see Table 15). The actual ratio, reckoned as the ratio of maximum size of feed to set of crusher averages 3.04 for nineteen suspended-spindle crushers and 4.6 for four fixed-spindle machines. The maximum apparent reduction ratio recorded from the mills is 10.6; the maximum actual ratio is 8. The minimum apparent ratio is 2.6; the minimum actual ratio 2.

Angle of nip in gyratory crushers ranges from 21° to 23° with an average very near 22°.

Capacity depends primarily upon character of ore, size of feed and discharge setting. Throw, speed, angularity of jaws, and character of crushing surfaces have a material effect. Capacities for different sized crushers according to makers' catalogs, are given in Tables 12, 13 and 14. (See Art. 7 for a discussion of the applicability of these figures to various ores.) The discussion of the effect of various factors upon the capacity of jaw crushers, as given in Art. 7, holds for gyratory crushers. The Hersam formula is supposed to be applicable to gyratory crushers as well as jaw crushers. There is no simple empirical formula that answers to results in gyratory crushing as does that given for jaw crushing. Capacities given in makers' tables are frequently far exceeded in mills although usually the excess capacity results from the feed of material much smaller than the maximum that the crusher will take, with consequent freedom from bridging and clogging. The lower figures in the lines bracketed with the parenthesis in Table 12 refer to this character of feed and are conservative for any rock other than the most tough. In such service the actual reduction ratio will rarely exceed 3.

Power consumption. Tons crushed per horsepower-hour consumed in the mills ranges from 0.31 to 11.6 in suspended-spindle machines (Table 15) and as high as 14.9 on a soft ore in a fixed-spindle crusher (Table 13). The average for 20 lever-type machines is 3.56. The high figures represent machines in secondary service when worked well up to the limit of capacity. According to field practice no greater number of tons per consumed horsepower is to be expected from large crushers than from small ones.

Figs. 35 to 36 represent the results of a power test on a gyratory crusher in actual operation. The data reported on Fig. 35 appear in columns 4 and 5 of Table 15. These figures show that the power installations called for by the manufacturers, in Table 12, are higher than need be counted on for running; they are needed, however, for starting.

Fall through machine proper is about ten times the gape for machines up to 12-in. gape. For larger machines the figure decreases with increase in

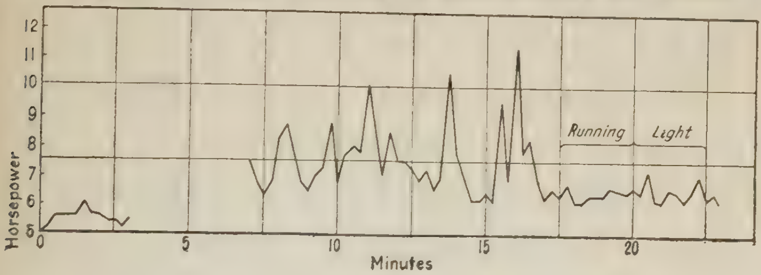
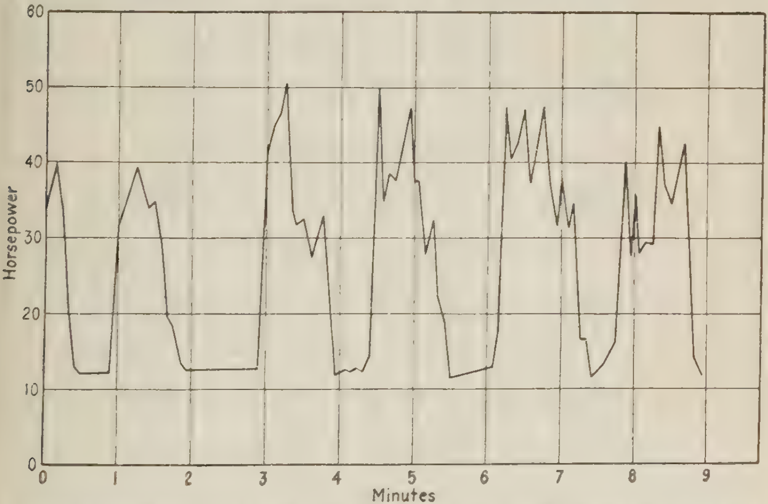


FIG. 35.—Load curve of No. 6-K Gates gyratory crusher.

gape from 8 to 6. Closer figures for fall are given in Tables 12, 13, and 14. These figures will come within a few inches of the exact ones for crushers made by any of the leading manufacturers.



Speed of pinion shaft, 207 r.p.m.; maximum size of feed, 24-in.; largest particle in product, 4-in.; rate of feed during test, 99.6 tons per hr.; average hp., 26.2 (decrease chart readings by 2.7 hp. for power draft of a conveyor driven by the same motor).

FIG. 36.—Load curve of No. 8 Gates gyratory crusher.

Lost time. Average percentage of lost time in the mills due to causes chargeable to the crushers themselves, such as repairs and renewals, clogging and its attendant difficulties, is less than 1 per cent. Renewal of mantles, concaves and chute liners and re-babbitting of the eccentric bearing are the principal causes stated. One plant reports 8 per cent. lost time due to clogged chutes. In this plant a 12-in. crusher was discharging a product, a considerable proportion of which was more than 4 in. in size; an extremely large tonnage was being put through and clogging was to be expected unless a large drop was allowed from the concaves to the highest point of the diaphragm. The majority of the crushers reported are planned to work but one or two shifts per 24 hr., thus leaving at least one shift free for all minor repairs. With good planning and with a sufficient supply of repair parts on hand, such operation will practically eliminate lost time.

Table 17. Comparison of jaw and gyratory crushers

Crushing from — to — in.	6-1		9-1.5		12-2		18-3		24-4	
Type of crusher	J	G	J	G	J	G	J	G	J	G
Size of receiving opening, in.	7×10	8×74	10×20	12×92	15×24	14×110	20×24	20×160	28×36	26×200
Capacity, tons per hour	4	16	10	28	17	52	34	157	66	310
Weight of machine, tons	3.5	7.75	10	14	14.9	19	18.9	46.5	36	76.5
Installed horsepower	7	12	15	25	25	40	35	90	70	135
Price, cents per pound (1925)	16	18	15	17	14.2	16	13.6	11	11	10.6
Fall through machine, ft.	2.33	6.25	2.5	7.75	2.75	8.6	3.5	12.33	4.8	15
Hourly capacity ÷ tons weight	1.14	2.06	1.00	2.00	1.14	2.74	1.80	3.38	1.83	4.05
Hourly capacity ÷ installed horsepower	0.57	1.33	0.67	1.12	0.68	1.30	0.97	1.74	0.94	2.30
Price, dollars per ton hourly capacity	280	174	300	170	249	170	151	65	120	52.4
Relative capacity ÷ ton weight, G/J	1.8		2.0		2.4		1.9		2.2	
Relative capacity ÷ horsepower, G/J	2.3		1.7		1.9		1.8		2.4	
Relative price per ton of hourly capacity, G/J	0.62		0.57		0.68		0.43		0.44	
Relative price, G/J	2.5		1.6		1.4		2.0		2.0	
Relative capacity per dollar of price, G/J	1.6		1.8		1.5		2.3		2.3	
Price per ton of hourly jaw capacity	....	696	....	476	....	520	....	300	....	246
Approximate actual power full load, horsepower	3.5	7.2	7.5	15	12.5	24	17.5	54	35	81.0
Approximate actual power idling, horsepower	1.2	2.3	3.8	4.5	6.2	7.2	8.8	16	18	24
Tons per horsepower-hour, based on J capacity	1.1	1.1	1.3	1.2	1.4	1.3	1.9	1.4	1.9	1.8

J = Blake-type jaw crusher. G = suspended-spindle gyratory.

Table 17. Comparison of jaw and gyratory crushers—*Continued*

Crushing from — to — in.	30–5		36–6		48–8		60–10		72–12	
Type of crusher	J	G	J	G	J	G	J	G	J	G
Size of receiving opening, in.	36×42	36×272	42×48	42×306	56×72	60×420	66×86	72×484	84×120	72×484
Capacity, tons per hour	108	680	150	1015	475	1920	778	3000	1970	3432
Weight of machine, tons	58	150	77.5	180	155	390	230	625	437.5	625
Installed horsepower	105	150	115	200	200	320	300	400	500	500
Price, cents per pound (1925)	13	10	12.5	9.8	10.9	8	9	8	8.5	8
Fall through machine, ft	6.25	18.1	7.0	19.5	9.25	29.25	11	36	14.5	36
Hourly capacity ÷ tons weight	1.86	4.40	1.93	5.64	3.06	4.92	3.38	4.80	4.50	5.50
Hourly capacity ÷ installed horsepower	1.03	4.40	1.30	5.08	2.38	6.00	2.59	7.50	3.94	6.86
Price, dollars per ton hourly capacity	140	45.5	129	34.8	71.2	32.4	53.2	33.3	37.8	29.1
Relative capacity ÷ ton weight, G/J	2.4		2.9		1.6		1.4		1.2	
Relative capacity ÷ horsepower, G/J	4.3		3.9		2.5		2.9		1.7	
Relative price per ton of hourly capacity, G/J	0.33		0.27		0.46		0.63		0.77	
Relative price, G/J	2.0		1.8		1.8		2.4		1.3	
Relative capacity per dollar of price, G/J	3.1		3.7		2.2		1.6		1.3	
Price per ton of hourly jaw capacity	......	278	......	235	......	131	......	128	......	51
Approximate actual power full load, horsepower	52.5	90	57.5	120	100	1920	150	240	250	300
Approximate actual power idling, horsepower	26	27	29	36	50	58	75	72	125	90
Tons per horsepower-hour, based on J capacity	2.1	2.8	2.6	3.1	4.8	5.2	5.2	6.8	7.9	9.4

J = Flake-type jaw crusher. G = suspended-spindle gyratory.

Attendance. Usual practice is one man per machine to two or three machines per man. Where the gyratory crusher is the primary crusher there should be an attendant for each machine to remove waste. Where the gyratory is a secondary crusher little or no actual attendance, apart from oiling and watching for trouble, is necessary.

Crane service. See Sec. 23, Art. 12, for general discussion. Substantially 50 per cent. of the mills reporting had crane service to the gyratory crushers and 30 per cent. more were served with a crawl and chain block. There can be no doubt that some such service should be provided.

Feeding. The crusher should be fed regularly and as nearly as possible up to capacity. If the feed contains particles near the largest that the crusher will receive, it is wise not to bury the crusher, as bridging may easily occur and necessitate digging aside a lot of heavy material. However, bridging is much less likely to occur in gyratory crushers than in jaw crushers and many more of the former are fed so as to be buried. The majority of plants report the crushers fed by chutes or over stationary grizzlies. Other methods reported are by belt or pan conveyors, drum feeders and shaking grizzlies. Pan conveyors are probbly the most satisfactory method for feeding initial crushers. The chute-fed crushers mostly occupy a secondary position in the mills, taking feed from a primary crusher.

Breaking point is not generally provided in modern gyratory breakers. The big breakers will pass almost any piece of steel that gets to them without stalling or breakage. Small crushers will generally slip a belt before breaking and if the motor is properly safe-guarded against overload, this is a fairly satisfactory method of procedure. In older crushers a breaking pin was provided in the driving pulley. Overload circuit breakers are probably the best method of providing against damage with machines directly connected to a motor.

Size of product. Between 10 and 20 per cent. of the product of a gyratory will remain on a square-mesh screen of the same aperture as the set, the higher percentage with slabby material; between 50 and 55 per cent. of the product will remain on a square-mesh screen whose aperture is half the set.

Cost. Price of gyratory crushers in 1925 ranged from 7.5¢ per lb. for No. 21 size, and 8¢ for No. 7½ to 18¢ for No. 2. Upwards of 90 per cent. of the cost of crushing is made up of power, labor and wearing parts. If crushers are run to capacity the crushing cost should range below 6¢ to 8¢ per ton for the small crushers and 1.5¢ to 2.5¢ for large.

9. Comparison of jaw and gyratory crushers

Table 17 compares the crushers of the two types built by one manufacturer. It shows that, when worked to capacity, the price of a gyratory varies from 27 to 77 per cent. of that of a jaw crusher capable of the same reduction in particle size; also, that under the same circumstances, the gyratory will crush from 1.7 to 4.3 times as much rock as the jaw crusher per installed horsepower. But, when the quantity of rock to be crushed per hour is within the capacity of one jaw crusher of the proper gape, then the price of a gyratory for the same work ranges from 1.3 to 2.5 times that of a jaw crusher. The work at WITHERBEE-SHERMAN presented in Figs. 24, 35 and 36 indicates that the power consumption of a jaw crusher idling is roughly 50 per cent. of that at full load and the full-load consumption about 50 per cent. of the installed horsepower; corresponding percentages for the gyratory are 30 and 60 per cent. Applying these figures to Table 17 it appears that for machines reducing to 4-in. or less, power consumption per ton crushed is less in the jaw crusher. In coarser crushing, products from 5- to 12-in. sizes, power consumption per ton crushed is less for gyratory than for jaw crushing. Repairs, cost of installation, and loss of head are all greater for the gyratory than for the jaw crusher at all sizes. It may, therefore, be concluded that for crushing quantities that can be handled by one jaw crusher the jaw crusher is the more economical installation on all counts; that for coarser crushing the gyratory will consume less power per ton crushed, but will cost more to install, to keep repaired, and will require greater expenditure for elevation of material. It has the advantage of allowing expansion in capacity, it can be set higher than a jaw crusher on account of absence of vibration, and it can be fed from all sides. On the other hand, if the rock is clayey or fibrous or does not break freely, it is more likely to clog. The shape of the crushing zone of the gyratory prevents the discharge of large slabs unbroken, such as can pass through a jaw crusher, and hence the gyratory product will be more uniform than that of a jaw crusher.

Analysis of good practice shows that the following empirical relation will give a rough indication of of the proper crusher to use, based on tonnage and size considerations only, *viz.:* If the hourly tonnage to be crushed divided by the square of the gape expressed in inches yields a quotient less than 0.115, use a jaw crusher; otherwise a gyratory.

INTERMEDIATE CRUSHING

Intermediate crushers are those used to take the product of the breakers and reduce it to sizes best suited for feeding to fine crushers and grinding

machines. The group includes stamps, rolls, disk crushers and reduction gyratories. Under certain rather unusual circumstances any of these may be used as a final crusher, *e.g.*, steam stamps in Lake Superior copper mills, gravity stamps in amalgamation plants, rolls in dry magnetic plants, disk crushers preparing oxidized copper ore for leaching, etc. Rolls are most widely used. Disk crushers are direct competitors of rolls; they are relatively recent members of the group and for certain types of work are superior to the older form (see p. 312). Gravity stamps are the oldest type and were for many years pre-eminent as a combined intermediate and final crusher in gold mills. They still hold an important position because of the large number yet operating in mills built ten to thirty years ago, but with the exception above noted they have now been relegated entirely to the field of intermediate crushers where they come into direct and generally unsuccessful competition with rolls, and disk crushers. The reduction gyratory is the latest addition to the group and is so new that its importance cannot yet be accurately adjudged.

10. Reduction gyratory. Cone crusher

Reduction gyratory is made both in the suspended-spindle and the pillar-shaft (fixed-spindle) types. The latter type is similar to the pillar-shaft breaker except that the flare of the breaking head is greater in the reduction gyratory and the concaves flare downward instead of converging, as in the cone crusher (Fig. 37), in order to maintain the required nip angle. This construction has the advantage of giving a large area of discharge opening for a given set of the crusher and consequently a larger capacity than otherwise. The usual sizes are 4-, 6- and 10-in. gape. This is a new machine and its capacity is not established. Capacities given by one manufacturer range from 24 tons per hr. to ¾-in. to 48 tons to 1½-in. for a 6-in. machine and from 80 tons per hr. to 1½-in. to 135 tons per hr. to 2½-in. for a 10-in. machine.

Symons cone crusher (Fig. 37) is of the flaring-bowl gyratory type of intermediate crusher. The crushing head (*a*) is supported on a large hemispherical bearing (*b*) and is gyrated by a long, ball-supported eccentric (*c*), driven by the usual bevel gearing. The stationary crushing surface (*g*) is attached to the main frame (*h*) by means of bolts (*j*) that act against a nest of springs (*i*) which compress and allow the head to lift when an uncrushable body enters. A distributing plate (*d*) is mounted on the upper end of the spindle under the feed opening (*e*). Set may be adjusted while running by loosening lock nuts and revolving the head (*f*) by means of nuts (*k*). Sizes, capacities and power consumption, as stated by the manufacturer, are shown in Table 17*a*.

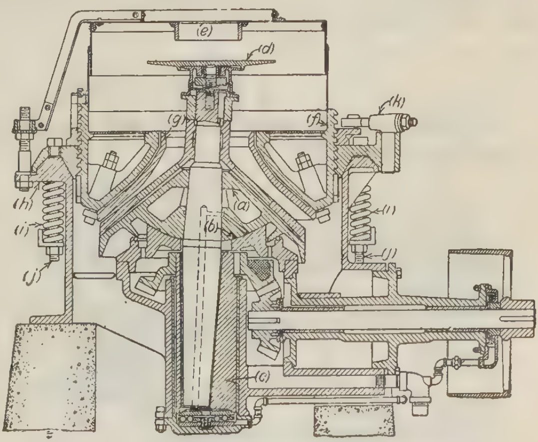


Fig. 37.—Cone crusher.

Performance. This crusher has not yet (Jan., 1927) been in the field long enough to supply many data on performance, but those given below show it a likely competitor of fine-crushing gyratories, disk crushers and rolls. At Utah Apex a No. 4 machine crushes −4+1-in. feed to −1-in. at the rate of 60 tons per hr. It is driven at 525 r.p.m. (driving pulley) by a 75-hp. motor and is estimated to draw 50 hp. No weaknesses have developed in 3 months' running. At St. Louis Smelting and Refining Co. a 7-ft. machine set ¼ to

Table 17*a*. Symons cone crusher (*Symons Bros. Co.*)

Rated size (max. diam. of cone)	Gape, in.	Capacity, tons per hr.										Hp.	Wgt., lb.
		Set, in.											
		⅛	3/16	¼	⅜	½	⅝	¾	1	1¼	1½		
2 -ft.	1⅝	7	10	14	20	25	....	....	....	....	....	25–	
2 -ft.	3	....	....	....	....	25	30	35	45	50	....	30	10,500
3 -ft.	2¾	14	20	25	35	40	....	....	....	....	....	50–	
3 -ft.	4½	....	....	....	....	40	55	70	80	100	....	60	21,000
4 -ft.	4½	....	30	40	60	80	....	....	....	....	....	75–	
4 -ft.	6¾	....	....	....	....	80	100	120	150	180	....	100	35,000
5½-ft.	6	....	50	65	100	130	....	....	....	....	....	125–	
5½-ft.	9	....	....	....	....	130	160	200	275	300	....	150	70,000
7 -ft.	6¾	....	....	110	160	225	280	....	....	....	....	150–	
7 -ft.	......	....	....	....	....	225	....	330	450	560	600	200	110,000

5/16 in. takes the product from a jaw crusher set 4 to 5 in., containing slabs up to 14 in. square, at the rate of 150 tons per hr. with a consumption of 130 hp. Capacity falls to 140 tons per hr. with damp ore. Screen test of product is: On 0.742-in., 3.5 per cent.; 0.525, 14.7; 0.371, 21.8; 0.263, 13.2; 0.185, 10.7; 0.131, 8.0; 0.0082, 24.8; −0.0082-in., 3.3 per cent. The +⅜-in. material is all flaky with the minimum dimension less than the crusher set. With a feed passing 3-in. ring capacity reaches 180 tons per hr., the +⅜-in. material in the product decreases and the power draft is 100 hp. The machine has passed large pieces of steel without choking or breakage. A 4-ft. machine at the same plant set ¼ to 5/16 in. crushes −3-in. gyratory product containing 28% − ⅜-in. material at the rate of 100 tons per hr. and draws 65 hp. The product sizes: +0.742-in., 2.2 per cent.; 0.525, 9.4; 0.371, 19.6; 0.263, 16.4; 0.185, 12.5; 0.131, 7.8; 0.0082, 27.1; −0.0082, 5.0 per cent. C. G. Dresser (*PC*) states that it is contemplated at this plant to replace four No. 6 gyratories and two 54 × 24-in. rolls by two 7-ft. cones.

11. Disk crushers

The disk crusher is made in two forms, known respectively as the horizontal and vertical types.

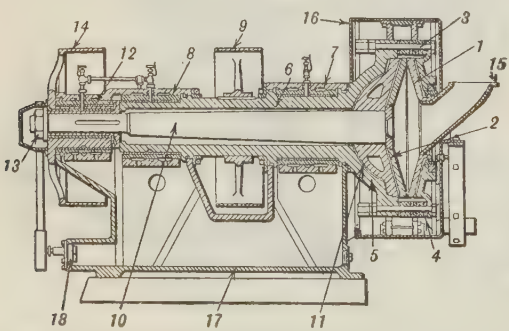


Fig. 38.—Disk crusher, horizontal-spindle type.

Horizontal disk crusher (Fig. 38) consists essentially of two opposed concave conical crushing surfaces (*1*) and (*2*), of large apex angle, rotating in the same direction at substantially the same speed around horizontal axes. The mechanism is so arranged that any two given points at corresponding positions on the surfaces of the opposing cones alternately approach and recede from each other. This action is accomplished as follows: Disk (*1*) is attached by means of bolts (*3*)

and spacers (*4*) to the bell-shaped end (*5*) of the hollow horizontal shaft (*6*). This shaft is carried in bearings (*7, 8*) in the main frame of the machine and is positively driven by means of pulley (*9*). Disk (*2*) is carried on the end of the solid shaft (*10*). This shaft and the disk are supported by the hemispherical bearing (*11*), eccentric (*12*) and thrust nut (*13*). The solid shaft is not positively rotated. The center of eccentric sleeve (*12*) is not coincident with the center of revolution of shaft (*10*). Pulley (*14*) rotates in a direction opposite to that of pulley (*9*) and at a speed somewhat more than twice greater. With no rock in the machine the inner disk rotates only to the extent that motion is induced by bearing friction, but as soon as rock is introduced the movement of the outer disk is transmitted through the rock to the inner disk and the speed of rotation of the inner disk comes up substantially to that of the outer.

With the end of shaft (*10*) furthest from the disk set eccentrically to the center of shaft (*4*), each point on the surface of disk (*2*) would approach and recede from the corresponding point on the surface of disk (*1*) once in each revolution of the disks, if there were no independent motion of the eccentric. By driving the eccentric in the opposite direction to that of the outer disk and at the same rate of speed, each point on the surface of either disk would approach and recede from corresponding points on the other disk twice per revolution. Further increase in relative motion between the two pulleys causes further increase in the number of approaches and recessions of corresponding points on the disks per unit of time or per revolution of the main shaft. Since the eccentric pulley is normally driven at something over twice the rate of the main pulley, there are normally something over three crushing actions at each point on the surface of the disk for each revolution of the main pulley. Feed is introduced by pipe (*15*) through the center of the outer disk and drops to a bearing between the disks. Centrifugal force, friction, and the pinch of the disks then act on the particles with the result that some material at least, is carried up and, after crushing, thrown out at the periphery well above the lower segment. This increases the area of discharge from the crushing zone and therefore increases capacity. It is to be noted also that the cross-sectional area of the crushing zone increases toward the region of finer crushing, so that crowding is lessened and production of fines minimized. Discharged material is caught by the housing (*16*) and discharged through a chute at the bottom. The lower part of main frame (*17*) is made oil- and dust-tight and acts as a reservoir in which oil, circulated by means of pump (*18*) is collected after seeping through the various bearing passages. The crusher is made in four sizes, named by the diameter of the disks as 18-, 24-, 36- and 48-in.

Table 18. Symons disk crusher, catalog data

Diameter of disks, inches	Maximum size of feed, inches	Capacity, tons per hour, to . . in.								Revolutions per minute, main pulley	Revolutions per minute, eccentric pulley	Horsepower	Weight, pounds
		Size(*b*)	Tons	Size	Tons	Size	Tons	Size	Tons				
18	1.5	3/8	5–8	1/2	8–10	3/4	10–12	1	12–15	200	450	12–18	5,600
24	2.5	1/2	12–15	3/4	18–20	1	20–25	1½(*a*)	25–30	200	400	18–25	9,300
36	3.5	3/4	25–30	1	30–45	1½	45–60	2(*a*)	50–65	133	300	30–40	23,500
48	6.5	1	45–60	1½	60–80	2(*a*)	80–100	2½(*a*)	100–120	100	250	50–65	39,000

a Special disk. *b* Minimum setting.

Table 19. Performances of horizontal disk crushers

Plant	Timber Butte	Bunker Hill & Sullivan	Federal Lead No. 3	Phelps-Dodge Co.		Consolidated Arizona Smelting Co.	American Zinc, Lead & Smelting Co., Mascot	Braden	Replogle Steel Co.	Chile Copper Co. (*h*)	Moose Mountain (*j*)
				Burro Mountain	Morenci						
Size, in.	36	36	48	48	48	48	48	48	48	48	48
Disk setting, in.	0.5–0.75	1.25	1	1	0.75	0.62	1 5/16	1	2	1.25	7/16
Tons per hour	62.5	37.5*a*	50	60	50	50	108	20	116*g*	100	50*k*
Horsepower, installed	....	....	50	50	50	....	100	75	....	....	*l*
Horsepower, consumed	....	....	....	40	35	....	30	....	....	29–47.9	....
Tons per horsepower-hour	....	....	1.0*e*	1.5*f*	1.4*f*	....	3.6*f*	0.27*e*	....	3.4–2.1	....
Speed of disk, r.p.m.	133	131	90	100	100	100	102	100	100	....	....
Speed of eccentric, r.p.m.	305	300	275	200	200	260	254	....	250	....	....
Size of feed, in.	−3+0.75	+1.25	4*c*	....	3.5×6	61%+2	*d*	*d*	*d*	*i*	−3+1 3/8
Size of product, in.	....	−1.25	1*c*	....	0.75×2.5	....	*d*	*d*	*d*	*i*	....
Oil consumption, pounds per shift	....	14	8	....	....	....	16	....	....	....	....
Attendance, machines per man	....	*b*	4	3	3	....	2	....	....	....	....
Life of parts, days (24 hr.):											
Disks	....	180	720	....	160	200	720	....	....	....	....
Material	*MS*	*MS*	*MS*	....	*MS*	*MS*	*MS*	....	....	....	....
Eccentrics	....	....	300	....	....	....	120	....	....	....	....
Time to change disks, hr.	8	8	8	....	12	....	8–12	....	....	....	....

MS Manganese steel. *a* Original feed. Crusher is in closed circuit with a 30-mm. screen. *b* Part time of one man. *c* Maximum. *d* See Table 19*a*. *e* Based on installed horsepower. *f* Based on horsepower consumed. *g* Average; maximum, 165. *h* *99 J 692.* *i* Feed : 20 per cent., 4- to 6-in.; 50 per cent., 2- to 4-in.; 25 per cent., 1- to 1.5-in. Product all through 1.5-in., 22 per cent. −0.5-in. *j* *99 J 975.* *k* Plus about 12.5 tons oversize return from circuit. *l* 1 @ 50- and 1 @ 30-hp. motor.

Performance. Catalog capacities to various sizes with makers' estimates of power consumption are given in Table 18. Performances of 36-in. and 48-in. machines at various plants are given in Tables 19 and 19*a*. Comparison of Tables 18 and 19 shows that capacities as stated by the manufacturers are conservative.

Table 19*a*. Sizing tests of feed and products of horizontal disk crushers in Table 19

On screen, aperture	Weights, per cent.					
	American Zinc, Lead & Smelting Co., Mascot		Braden Copper Co.		Replogle Steel Co.	
	Feed	Discharge	Feed	Discharge	Feed	Discharge
2 -in.	....	....	17	....	Material through 1⅛ × 2½-in. slot and product from jaw crusher set 3-in.	16.80
1.5 -in.	45.1	23.0	24	....		....
1.25-in.	6.4	9.3	....	....		36.14
1.0 -in.	8.1	10.1	26	9		....
0.75-in.	....	....	13	18		26.48
0.5 -in.	15.6	22.6	11	29		....
9.42-mm.	1.6	2.2	4	12		....
6.68-mm.	4.5	6.2	2	2		10.64
4.70-mm.	....	....	1	7		....
3.33-mm.	6.3	8.8	1	5		2.58
2.36-mm.	....	....	1	4		....
1.65-mm.	....	5.2	....	3		0.59
1.17-mm.	....	....	....	2		....
0.83-mm.	....	2.6	....	2		1.55
0.59-mm.	....	....	....	1		....
0.42-mm.	....	2.1	....	1		2.33
0.30-mm.	....	....	....	1		....
0.21-mm.	....	1.4	....	0		2.42
0.10-mm.	....	1.8	....	....		....
0.07-mm.	....	0.6	....	....		....
0.04-mm.	....	0.6	....	....		....
Through last screen	12.4	3.5	....	4		0.47
Totals.........	100.0	100.0	100	100		100.00

In the earlier disk machines much trouble was experienced by breakage of disks by steel and other unbreakable material in the feed. All disk machines in practice should be guarded against tramp iron by magnets. The use of manganese and chrome steel for disks has done away with much of the breakage trouble and any iron that gets past powerful guard magnets is readily taken care of. Non-magnetic manganese steel from steam-shovel dipper teeth cannot, of course, be caught by magnets.

At New Cornelia this is guarded against by checking up on dipper teeth as each train load of ore leaves the shovel and redoubling the watch at the crushing house when a tooth is missing. Guard magnets cannot be used with magnetite ores, which is a serious impediment to the use of disk crushers on such material. At Replogle Steel Co. fine-grinding disks are guarded by a screen in closed circuit with rolls. Steel large enough to hurt the disks is diverted to the rolls and kept in the roll circuit until broken down to a size that the disks can handle. At Moose Mountain (*99 J 975*) a man watches the feed. Steel that gets past him has never broken an important part, but stalls the machine and blows the motor fuses.

Size of product. From 10 to 30 per cent. of the product of the horizontal-spindle machine remains on a square-mesh aperture equal to the disk setting, and from 55 to 65 per cent. is coarser than half the disk setting.

Vertical disk machine is shown in Fig. 39. The upper disk (*1*) is carried by means of bolts (*2*) and spacers (*3*) on the table (*4*), which is carried on rollers and driven by gear (*5*) and pinion (*6*) from the main shaft (*7*). The lower disk (*8*) is carried on plate (*9*), the lower surface of which is a spherical segment and rests on the spherical wedge (*10*). Both (*9*) and (*10*) are carried along by rotation of table (*4*). The stem (*11*) of wedge (*10*) is moved in a small horizontal circle by eccentric (*12*) through the gears (*13*) and (*14*) from shaft (*7*). The direction of rotation of the eccentric is opposite that of the upper disk, hence the relative motion of the disks is the same in this machine as in the horizontal-spindle type. The upper disk is held to its work by springs (*15*), designed to compress when an unbreakable substance enters the machine and thus guard against disk breakage. This provision, together with a magnet ahead of the machine, prevents most of the troubles chargeable against the early disk machines. Feed is introduced through spout (*16*) onto the lower disk and thrown out by centrifugal force until it wedges between the disks. After breaking, it again travels outward and is subjected to further crushing operations until it finally can escape into the outer chamber and discharge through spouts. The eccentric speed is so high (350 r.p.m.) and the resulting number of crushing movements so great that but few particles escape being subjected to a crushing pinch near the periphery of the disks and hence the discharge size is substantially that of the minimum setting of the disks. Gears and bearings are enclosed in dust-proof housings as in the case of the horizontal machine and are lubricated by a forced feed from pump (*17*). The vertical-spindle machine is made with 36-in. and

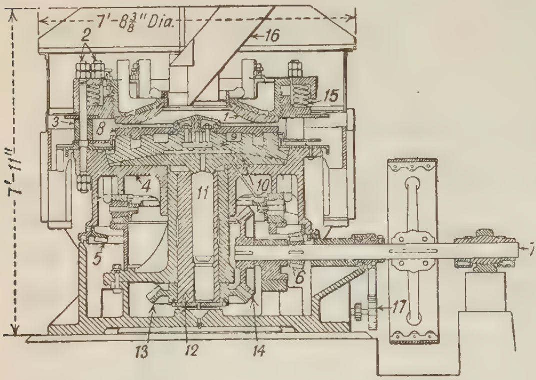


Fig. 39.—Disk crusher, vertical-spindle type.

Table 20. Screen tests of feed and products of vertical disk crushers

Screen aperture, inches	Weight, per cent. New Cornelia Copper Co. Product	Replogle Steel Co. Feed	Replogle Steel Co. Product
1.25	....	4.65	....
0.75	....	32.40	....
0.263	26.7	53.90	....
0.185	17.3	....	....
0.131	12.7	7.02	17.43
0.093	9.0	....	....
0.065	7.0	0.25	9.84
0.046	4.9	....	....
0.0328	4.1	0.41	23.15
0.0164	....	....	24.65
0.0082	....	....	19.96
0.0058	....	....	3.99
Through last screen	18.3	1.37	0.98

48-in. disks. By use of different disks it may be fitted for either coarse or fine crushing.

Performance. CHILE COPPER Co uses twenty-four 48-in. vertical-spindle disks to crush 13,000 tons per 24 hr. from 1.5-in. to 12 per cent. on ⅜-in. (*123 P 849*). NEW CORNELIA uses one 48-in. vertical-spindle machine set at ¾-in. and two 48-in. machines set at ¼-in., taking the oversize on a ⅜-in. screen from the product of the first, to crush 90 to 100 tons per hr. from 3.5-in. to 3-mesh. A sizing test of the product is given in Table 20. Repairs are light on the coarse-crushing disk but heavy on the fine-crushing machines. Average power consumption for five such 3-machine sets is 0.7 kw.-hr. per ton; average steel consumption, 0.059 lb. per ton crushed; babbitt, 0.0097 lb. per ton. Cost in May, 1920, including power, repairs, and labor and including also screening and conveying was $0.09 per ton (*63 A 507*; *123 P 849*). At REPLOGLE STEEL Co. 48-in. disk crushers average 25.2 tons per hr. (maximum, 36 tons per hr.). Screen tests of feed and product are given in Table 20. Average power draft, 15 kw.; minimum, 7.5 kw.; maximum, 25 kw. At Santa Barbara mill of AMERICAN SMELTERS SECURITIES Co. a 48-in. machine set at 0.5-in. crushes the oversize on a 0.75-in. screen from 500 tons per day of −1.5-in. material.

12. Rolls

Rolls are of two general types, known as rigid rolls and spring rolls. RIGID ROLLS are the older type but are rarely used at the present time. They differ from the type to be described in that the bearings for both rolls are rigidly fixed on the frame and the rolls must stall, or something bend or break when an uncrushable particle enters.

Spring rolls are illustrated in Fig. 40. They consist essentially of two cylinders (*1*) mounted on horizontal shafts which are driven in opposite directions so that corresponding points on the cylinder faces above the horizontal plane through the shaft centers are moving toward each other. The main frame (*3*) carries the fixed bearings (*4*) and movable bearings (*5*). These bearings carry shafts (*2*) and (*6*). Near the centers of the shafts are the cores or hearts (*7*), fixed, and (*8*), removable, for holding the shells (*9*). The fixed roll is driven by pulley (*10*) and the movable roll by a smaller pulley (*11*). The movable roll is held up to the fixed roll by means of two heavy tension rods (*12*), carrying nuts that bear at one end against the movable-roll bearings, and at the other end against a nest of springs (*13*), which are seated against the main frame. The minimum distance between roll faces is fixed by the shims (*14*). The rolls are encased in a housing consisting of fixed sides (*15*), and covers (*16*), which are easily removed to permit inspection of the roll faces. Beneath the rolls a hopper (*17*) is provided to guide the discharge to a chute. A feed hopper (*18*) is bolted to the top of the housing. This hopper is fitted with replaceable distributing plates that spread the stream of feed

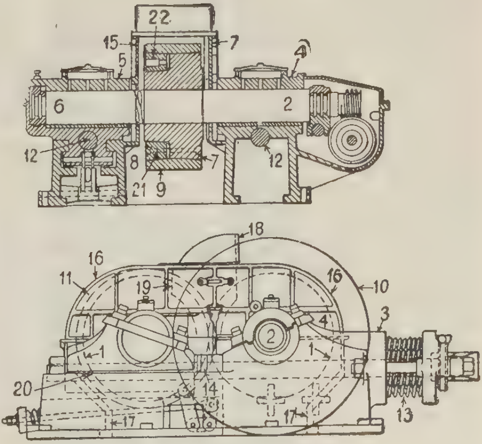


FIG. 40.—Spring rolls.

out over the full width of the roll faces. Cheek plates (*19*) are provided to protect the housing from wear of the feed stream and to prevent material from passing between the sides of the rolls and the housing without crushing. Cheek plates, roll shells, and hopper liners are made replaceable. Replaceable steel strips (*20*) are provided to protect the frame from wear due to movement of the movable-roll bearing. Table 21 is generalized from catalogs of the

Table 21. Summary of catalog data concerning crushing rolls

Size, diameter ×face, inches	Number of makers *a*	Weight, pounds		Speed recommended, revolutions per minute		Horsepower installation recommended		Approximate overall dimensions, inches		
		Minimum *b*	Maximum *b*	Minimum *c*	Maximum *c*	Minimum *d*	Maximum *d*	Length	Width	Height to top of feed hopper
12×12	1	3,700	......	......	230	6	......	......	......	......
18×10	1	6,500	......	250	300	8	......	65	71	29
20×12	1	6,000	......	150	225	7	......	......	......	......
24×8	1	10,500	......	100	160	7.5	......	......	......	......
24×10	2	10,600	11,000	90	160	8	11	......	......	......
24×12	3	10,900	11,600	90	230	8	11	82	78	36
24×14	3	11,200	12,000	90	160	8.5	11	......	......	......
26×15	1	16,500	......	75	125	......	......	......	......	......
27×14	1	10,800	......	125	200	10	......	......	......	......
30×10	2	16,000	16,500	66	130	10	15	......	......	......
30×12	2	16,900	17,000	66	130	10	15	......	......	......
30×14	5	11,300	19,200	66	190	10.5	15	100	94	46
30×16	2	18,600	19,600	75	180	10.5	15	100	98	46
36×12	3	21,200	23,100	50	100	12	18	......	......	......
36×14	3	23,000	31,000	51	150	12.5	20	118	106	54
36×15	1	33,000	......	40	100	......	......	......	......	......
36×16	5	18,500	31,500	50	175	13	25	118	108	54
40×15	1	38,000	......	50	100	......	......	......	......	......
40×16	1	23,000	......	80	100	25	......	......	......	......
40×20	1	42,000	......	50	100	......	......	......	......	......
40×30	1	50,000	......	50	100	......	......	......	......	......
40×36	1	52,000	......	50	100	......	......	......	......	......
42×12	1	35,200	......	41	......	20	......	......	......	......
42×14	2	36,100	37,000	41	100	20	30	131	107	59
42×16	4	35,000	62,000	41	100	20	50	147	123	72
42×18	1	63,500	......	95	120	55	......	147	126	72
48×12	1	49,000	......	70	90	28	......	......	......	......
48×14	2	50,500	57,600	33	90	23	31	......	......	......
48×16	3	51,800	75,000	33	100	23	31	......	......	......
48×18	2	60,000	75,000	33	105	23	60	176	135	74
48×20	3	50,000	80,000	33	105	23	65	176	136	74
54×16	2	51,800	88,000	50	90	35	......	......	......	......
54×18	1	101,000	......	28	......	25	......	......	......	......
54×20	5	55,000	103,000	28	95	25	70	189	141	79
54×24	5	60,000	136,000	28	95	25	75	189	145	79
60×20	1	85,000	......	50	60	60	......	......	......	......
60×24	2	90,000	150,000	50	85	70	90	208	173	91
60×30	1	165,000	......	65	85	100	......	208	179	91
72×20	4	118,000	220,000	40	120	90	100	......	......	......
72×24	4	133,000	230,000	40	120	100	125	233	194	111
72×30	2	205,000	235,000	50	75	125	......	233	200	111

a The catalogs of five principal makers are summarized. This column gives the number of makers who manufacture a given size. *b* The lightest and heaviest of the size listed. *c* Not usually by the same maker. Low speeds ordinarily correspond to the light-weight rolls. *d* The lower figure corresponds to light rolls at low speed and *vice versa*.

principal makers and gives data as to sizes, weights, speed and power consumption of rolls available without special design. Note that there is a wide diversity in weight of a given size of roll as offered by different manufacturers. The lighter rolls should be chosen only for small capacities or very easy crushing or where the first cost is of paramount importance. Heavy rolls for heavy service will repay the additional first cost in a short time in lower repair costs and continuity of operation.

Frame is cast by all makers in a single piece, usually with the lower half of the fixed bearing, the spring seat and the discharge hopper cast integral therewith. The sides of all but the smallest rolls are of box section, thus forming heavy box girders along both sides of the rolls under the bearings. The tops of the sides are machined at one end to form slides for the movable bearings and are fitted with removable steel wearing plates that protect them from the wear of the movable bearing. Cast-iron and semi-steel are the materials used for frames. The bottoms of the frames of the better rolls are planed in order to give a proper bearing on the foundation.

Bearings are made extra long and heavy in order to distribute properly the heavy forces brought upon them. Normally they are split along planes at an angle of 20° to 30° to the horizontal as shown in Fig. 40. This is done in order to prevent shearing stresses on the bearing-cap bolts by bringing these bolts into lines substantially parallel to the direction of the resultant of the forces exerted in the bearings by the roll shafts. Movable bearings are made with a long base in order to aid in keeping the movable roll in close alignment with the fixed roll. Bearings are self-aligning, allowing the rolls to swivel in order to permit a hard object to go through on one side without bending the roll shafts. Babbitt or phosphor bronze is used for lining bearings, some users preferring one and some the other. In the smaller rolls babbitted bearings are plain babbitted, in large rolls babbitted bushings are used in the lower half. With this provision the bearings can be rebabbitted by lifting the shafts but a small distance and slipping the bushings in under them. Some manufacturers provide a holding-down device for the movable bearing that prevents chattering and lessens wear on slides and bearings. Bearing caps are fitted with large reservoirs for grease or oiled waste and generous oil passages should be cut in the babbitt to allow ready circulation to all parts of the bearing.

Tension rods are made of best forged steel of ample cross-section. One end is fitted with a heavy hexagonal nut with provision for pinning in position. The other end is threaded with special thread for transmitting power and fitted with a special nut or a gear wheel that works against the spring cage. Tension rods are placed as near the shafts as possible in order to bring them as near as possible in the line of the forces resisting crushing.

Springs are of the helical type carried in cages and set to the proper tension at the factory. The tension varies from 4000 to 30,000 lb. per lineal inch of roll face. With these high pressures the rolls are practically rigid and while there is enough give when an unbreakable article passes through to save bent shafts and broken frames in most cases, there is no doubt that there are more cracked frames with high spring pressures than with low. The springs do not give until a pressure well in excess of ordinary resistance to crushing is exceeded. They take no part, therefore, in ordinary crushing. Springs are held in the cages by two or four heavy bolts to each cage. These bolts need not be disturbed when spacing the shells nor at any other time except in the event of replacing a broken cage or spring or when it is desired to change the spring pressure. Increase in spring pressure should be made with caution, as it is usually set by the maker as high as is safe.

Shafts are made of forged steel turned and polished and keyed for pulleys. In some makes of rolls shafts are interchangeable and reversible. In other makes the ends are grooved for thrust collars and fitted for lateral adjusting devices in such a way as to prohibit the interchangeable feature. Normally, the diameter of the shaft is greater in the core than elsewhere, in order to put the most metal at the place of greatest strain.

Pulleys are made of wood, cast iron, cast semi-steel, and with steel disk and cast-iron or cast-steel rim. The cast pulleys are made with split hub key-seated and with solid rim. In all cases the pulleys are balanced as perfectly as is possible. Wood pulleys or steel-disk pulleys are recommended for heavy vibrating duty such as is encountered in coarse crushing. Most makers furnish a large pulley on the fixed-roll shaft of sufficient size to transmit all of the power necessary for crushing and place on the movable roll a smaller pulley whose principal function is to keep this roll moving when there is no material passing through. If the rolls are not of the reversible type, with equal pulleys on both shafts, they must be ordered right-hand or left-hand, according to the way in which it is desired to drive them. In general, the designation right- or left-hand is made on the basis of the position of the large pulley when standing at the end of the rolls carrying the spring cages. Standing thus and facing the rolls, if the large pulley is on the left the rolls are left-hand, and *vice*

Table 22. Performance

Plant	Tungsten Mines Co.	N. J. Zinc Co., Ogdensburg	N. J. Zinc Co., Franklin
Size, diameter × face, in.	24×14	24×14	26×15
Set, in.	*Cl*	0.3*W*	0.025
Speed, r.p.m.	60	140	98
Peripheral speed, ft. per min.	382	880	668
Tons new feed per 24 hr.	....	48	120
Tons total feed per 24 hr.	200*a*	192	120
Actual capacity, per cent. theoretical (*Y*)	....	16.4	151
Horsepower installed	....	11	100
Horsepower consumed	....	9	75
Tons per hp.-hr. consumed (*H*)	....	0.22	0.067
Moisture in feed, per cent.	*w*	Dry	Dry
Size of feed (*P*)	....	*31*	*23*
Size of product (*P*)	....	*31*	*23*
Angle of nip (*V*)	....	....	7° 12′
Reduction ratio (*e*)	....	....	1.6
Attendance, machines per man	1.5+	6	4
Lost time, per cent.	....	5	5
Principal causes of lost time	*b*	Choking	Repairs
Oil consumption, pounds per shift	1	1*g*	0.5*g*
Method of feeding	....	Chute	Chute
Method of preventing corrugation	....	*MLS, T*	*MLS, T*
Time to change shells, hr.	....	8	8
Life of parts:			
Shells, days	....	900	600
Thickness, new, in.	....	3.12	2.25
Thickness, discarded, in.	....	0.5	1.5
Material	....	Stl.	Stl.
Method of setting	....	....	....
Cheek plates, days	....	1500	1500
Material	....	....	Stl.
Shafts, days	....	5000	5000
Bearings, days	....	3000	1200
Springs, days	....	3000	3000

Plant	Federal M. & S. Morning	St. Joseph Lead Co., Bonne Terre	St. Joseph Lead Co., Bonne Terre
Size, diameter × face, in.	30×14	30×14	30×14
Set, in.	*Cl*	*Cl*	*Cl*
Speed, r.p.m.	108	110	100
Peripheral speed, ft. per min.	859	875	795
Tons new feed per 24 hr.	45	....	....
Tons total feed per 24 hr.	75	200	60
Actual capacity, per cent. theoretical (*Y*)	12.4	....	10.7
Horsepower installed	....	21	20
Horsepower consumed	....	20	16
Tons per hp.-hr. consumed (*H*)	....	0.42	0.16
Moisture in feed, per cent.	40	Dry	15
Size of feed (*P*)	*3*	*5*	*6*
Size of product (*P*)	*3*	....	*6*
Angle of nip (*V*)	13° 24′	35° 30′	13° 48′
Reduction ratio (*e*)	3.0	....	2.8
Attendance, machines per man	4+	13	13
Lost time, per cent.	0.008	0.1	0
Principal causes of lost time	*s*	*m*	....
Oil consumption, pounds per shift	1*g*	....	....
Method of feeding	....	Challenge	Launder
Method of preventing corrugation	*E, k*	None	None
Time to change shells, hr.	3	6–8	8
Life of parts:			
Shells, days	180	*n*	300
Thickness, new, in.	4	....	....
Thickness, discarded, in.	0.75–1	....	....
Material	*Cr*	*n*	Stl.
Method of setting	*d*	*d*	....
Cheek plates, days	440	....	....
Material	*CI*	....	....
Shafts, days	2100	....	....
Bearings, days	2000	....	300
Springs, days	3600	....	....

For explanation of reference

of rolls

Cananea Consolidated Copper Co.	U. S. S. R. & M., Midvale	U. S. S. R. & M., Midvale	U. S. S. R. & M., Midvale	Tungsten Mines Co.	Federal M. & S., Morning	Federal M. & S., Morning
27×14	30×12	30×12	30×12	30×14	30×14	30×14
....	*f*	*j*	*f*	*Cl*	0.25	*Cl*
120	80	105	80	60	51	100
859	637	835	637	478	406	795
500	....	....	....	....	160	140
....	235	160	100	200	220	....
....	10	11.2	4.2	....	18.5	18.8
50	20	15	25	....	....	....
21	15	18	15	....	....	....
0.99	0.65	0.37	0.28	....	....	....
20	Dry	18	Dry	*w*	5	9
....	−1.5-in.	−0.62-in.	−1.5-in.	....	*1*	*2*
....	−0.62-in.	−0.25-in.	−0.62-in.	....	*1*	*2*
....	21° 24′	13° 36′	21° 24′	....	23° 36′	13° 24′
....	2.6	2.5	2.6	....	1.3	2.0
4	....	....	....	1.5+	6+	4+
....	4	7.5	2	....	0.008	0.008
s	*h*	*h*	*h*	*b*	*s*	*s*
5*g*	18*g*	18*g*	18*g*	1	1*g*	1*g*
....	Choke	*l*	Choke	....	....	....
....	*E*	....	....	*k*	*E, k*	*E, k*
8	8	10	....	....	3	3
34	148	122	380	120–180	170	180
....	3.5	3.5	3.5	2.5	4	4
....	0.5	0.5	0.5	0.38	0.75–1	0.75–1
c	*c*	*c*	*c*	*HCS*	*Cr*	*Cr*
d	*d*	*d*	*d*	Shrunk	*d*	*d*
....	100	100	90	120–180	440	440
....	*CCI*	*CCI*	*CCI*	Old shells	*CI*	*CI*
....	No repl.	360	No repl.	No repl.	2100	2100
....	150	120	310	360	2000	2000
....	350	400	No repl.	No repl.	3600	3600

N. J. Zinc Co., Franklin	Copper Range	Braden Copper Co. Co. (*Z*)	Bunker Hill and Sullivan	Bunker Hill and Sullivan	Bunker Hill and Sullivan	Federal M. & S., Morning
30×16	36×10	36×12	36×14	36×14	36×14	36×14
0.025	0.12	*Cl*	0.75*W*	0.5*W*	0.25*W*	0.5
145	145	120	92	84	84	46
1140	1383	1130	880	803	803	440
192	....	458	100	45	40	500
192	125	....	180	105	106	....
134	9.0	....	2.3	2.2	4.5	19.5
50	....	60	....	....	....	....
25	15	47	....	....	....	....
0.32	0.35	0.41	....	....	....	....
Dry	67	....	22	7	1.6	2.5
24	−0.62-in.	*37*	−30-mm.	−15-mm.	−7-mm.	*4*
24	−0.25-in.	*37*	−15-mm.	−7-mm.	−3-mm.	*4*
5° 36′	13° 36′	....	....	....	....	16° 48′
1.3	....	....	....	....	....	1.8
2	....	....	6+	6+	6+	6+
5	0.28	....	0.1	0.1	0.1	0.008
Repairs	*s*	....	*s*	*s*	*s*	*s*
0.5*g*	....	....	....	....	....	1*g*
Chute	*o*	....	Trommel	Trommel	Trommel	....
AF	*MLS*	....	*E, MLS*	*E, MLS*	*E, MLS*	*E, MLS*
8	10	....	8	8	8	3
....	600	....	600	540	420	720
....	4	....	3.25	3.25	3.25	4
....	0.75	....	0.75	0.5	0.38	0.75–1
....	*Mn*	....	*Mn*	*Mn*	*r*	*Cr*
....	*p*	....	*q*	*q*	*q*	Shrunk
....	150	....	332	....	....	440
....	*CCI*	....	*CI*	....	....	*CI*
....	....	....	No repl.	....	....	2100
....	....	....	....	....	....	2000
....	....	....	No repl.	....	....	3600

letters, see page 296.

Table 22. Performance

Plant	St. Joseph Lead Co., Rivermines	U. S. S. R. & M., Midvale	U. S. S R. & M., Midvale
Size, diameter × face, in.	36×15	36×16	36×16
Set, in.	0.25	*j*	*t*
Speed, r.p.m.	90	90	90
Peripheral speed, ft. per min.	860	860	860
Tons new feed per 24 hr.	400	270	135
Tons total feed per 24 hr.	600	270	135
Actual capacity, per cent. theoretical (Y)	22.3	13.8	39.1
Horsepower installed	50	50	50
Horsepower consumed	40	22	22
Tons per hp.-hr. consumed (H)	0.42	0.53	0.41
Moisture in feed, per cent.	Dry	20	20
Size of feed (P)	1.25-in. max.	−0.62-in.	−0.25-in.
Size of product (P)	−9-mm.	−0.25-in.	−0.053-in.
Angle of nip (V)	19° 0′	12° 48′	9° 12′
Reduction ratio (e)	....	....	....
Attendance, machines per man	11	2	2
Lost time, per cent.	1	8	8
Principal causes of lost time	Choking	*h*	*h*
Oil consumption, pounds per shift	2	18*g*	18*g*
Method of feeding	Chute	*l*	*l*
Method of preventing corrugation	....	*E*	*E*
Time to change shells, hr.	6	10	10
Life of parts:			
Shells, days	900	87	87
Thickness, new, in.	5	3.5	3.5
Thickness, discarded, in.	1	0.75	0.75
Material	*Mn*	*c*	*c*
Method of setting	....	*d*	*d*
Cheek plates, days	270	80	80
Material	*Mn*	*CCI*	*CCI*
Shafts, days	No repl.	300	300
Bearings, days	270	85	85
Springs, days	No repl.	310	310

Plant	N. J. Zinc Co., Ogdensburg	N. J. Zinc Co., Ogdensburg	N. J. Zinc Co., Ogdensburg
Size, diameter × face, in.	36×16	36×16	36×16
Set, in.	0.09	0.12	0.19
Speed, r.p.m.	89	89	77
Peripheral speed, ft. per min.	840	840	725
Tons new feed per 24 hr.	144	192	216
Tons total feed per 24 hr.	576	768	960
Actual capacity, per cent. theoretical (Y)	151	151	138
Horsepower installed	25	25	28
Horsepower consumed	21	21	23
Tons per hp.-hr. consumed (H)	0.19	0.38	0.39
Moisture in feed, per cent.	Dry	Dry	Dry
Size of feed (P)	*28*	*29*	*30*
Size of product (P)	*28*	*29*	*30*
Angle of nip (V)	6° 12′	12° 0′	17° 12′
Reduction ratio (e)	4.0	....	....
Attendance, machines per man	6	6	6
Lost time, per cent.	5	5	5
Principal causes of lost time	Choking	Choking	Choking
Oil consumption, pounds per shift	2*g*	2*g*	2*g*
Method of feeding	Chute	Chute	Chute
Method of preventing corrugation	*MLS, T*	*MLS, T*	*MLS, T*
Time to change shells, hr.	8	8	8
Life of parts:			
Shells, days	203	456	260
Thickness, new, in.	3.5	3.5	3.5
Thickness, discarded, in.	0.5	0.5	0.5
Material	Stl.	Stl.	Stl.
Method of setting	....	....	....
Cheek plates, days	1500	1500	1500
Material	*Mn*	*Mn*	....
Shafts, days	5000	5000	5000
Bearings, days	450	900	900
Springs, days	3000	3000	3000

For explanation of reference

of rolls—*Continued*

Braden CopperCo.(*Z*)	McIntyre Porcupine	Cananea Consolidated Copper Co.	Calumet & Hecla	Federal Lead Co., Mill No. 4	N. J. Zinc Co., Ogdensburg	N. J. Zinc Co., Ogdensburg
36×16	36×16	36×16	36×16*v*	36×16	36×16	36×16
CI	0.75	....	0.062	*CI*	0.62	0.06
120	65	85	97	85	72	107
1130	621	812	927	812	680	1010
468	550	500	175	400	360	96
....	....	....	240	....	1440	384
....	8.8	....	32.3	147	67.1	125
60	25	50	40	50	24	29
49	15	....	30	40	20	24
....	1.53	....	0.24	0.42	0.75	0.16
....	Dry	20	42	15	Dry	Dry
9	*7*	−1.5-in.	*10*	−15-mm.	*26*	*27*
9	*7*	....	*10*	−2-mm.	*26*	*27*
....	27° 0′	....	19° 48′	15° 54′	20° 12′	7° 48′
....	1.3	....	8.0	....	1.4	2.2
....	....	4	8	4	6	6
....	....	5	2.96	3	5	5
....	....	*s*	*b*	*b*	Choking	Choking
....	....	5*g*	....	1*g*	2*g*	2*g*
....	Chute	*u*	Grizzly	Chute	Chute	Chute
....	*E*	*MLS*	*MLS*	*MLS*	*MLS, T*	*MLS, T*
....	36	8	7	8	8	8
....	300	60	310	360	260	304
....	....	....	3.5	3	3.5	3.5
....	....	....	1	1	0.5	0.5
....	*HCS*	*c*	*Mn*	Stl.	Stl.	Stl.
....	....	*d*	....	*p*	....	....
....	....	60	....	360	900	1500
....	....	*CI*	*CI*	*CI*	*Mn*	*Mn*
....	....	6 yr.	No repl.	No repl.	5000	5000
....	....	120	No repl.	360	600	600
....	....	6 yr.	No repl.	No repl.	3000	3000

Federal Lead Co., Mill No. 3	N. J. Zinc Co., Franklin	N. J. Zinc Co., Franklin	N. J. Zinc Co., Franklin	N. J. Zinc Co., Franklin	Nevada Packard	Witherbee Sherman, Mill No. 4
36×32	36×32	36×36*S*	36×36*S*	36×36*U*	37.5×16	40×15
CI	0.1	1.5	0.75	0.62	0.38	0.25
80	104	133	130	141	92	93
765	980	1250	1220	1330	914	990
600	1200	1200	1440	288	300	....
....	2400	1200	1440	600	....	47*t*
118	242	5.7	13.8	6.33	6.6	15.2
50	100	50	50	75	....	....
40	50	25	25	40	....	13.7*x*
0.63	1.0	2.0	2.4	0.3	....	1.44
15	Dry	Dry	Dry	Dry	Dry	Dry
−12-mm.	*22*	*19*	*20*	*21*	−2+0.38-in.	*11*
−2-mm.	*22*	*19*	*20*	*21*	−0.38-in.	*11*
14° 24′	12° 36′	....	....	14° 12′	23° 36′	12° 0′
....	1.2	1.6	2.8	1.9	....	1.5
....	2	1	1	2	....	6
3	5	5	5	5	2	....
b	Repairs	Repairs	Repairs	Repairs	Choking	....
1.5*g*	30*g*	10*g*	10*g*	10*g*	....	5*g*
Chute	Chute	Chute	Chute	Chute	....	Chute
MLS	*MLS, T*	*MLS, T*	*MLS, T*	*MLS, T*	*k*	*z*
8	8	8	8	8	3.4	10
240	600	150	150	200	400	230
4	3.5	3.37	3.37	3.37	5	3
1	1.5	1.62	1.62	1.62	0.75	0.5
Stl.	*Mn*	*Mn*	*Mn*	*Mn*	*Mn*	*Cr*
p	....	....	....	....	*p*	*p*
360	600	1000	1000	1200	300	150
CI	*Mn*	*Mn*	*Mn*	*Mn*	*CCI*	*CI*
No repl.	5000	5000	5000	5000	No repl.	No repl.*A*
360	200	600	600	600	No repl.	150*y*
No repl.	3000	....	....	3000	No repl.	No repl.

etters, see page 296

Table 22. Performance

Plant	Tungsten Mines Co.	Tungsten Mines Co.	Consolidated Arizona Smelting Co.
Size, diameter × face, in.	40×15	40×15	40×16
Set, in.	0.75	0.19	0.5
Speed, r.p.m.	54	54	*B*
Peripheral speed, ft. per min.	573	573	...
Tons new feed per 24 hr.	...	300	...
Tons total feed per 24 hr.	240	400	...
Actual capacity, per cent. theoretical (*Y*)	10.6	24.4	...
Horsepower installed	20	30	35
Horsepower consumed	20	26	...
Tons per hp.-hr. consumed (*H*)	0.50	0.36	...
Moisture in feed, per cent.	Dry	*w*	Dry
Size of feed (*P*)	...	...	...
Size of product (*P*)	...	...	*R*
Angle of nip (*V*)	...	...	...
Reduction ratio (*e*)	...	...	...
Attendance, machines per man	1.5+	1.5+	1+
Lost time, per cent.	2	2	0.95
Principal causes of lost time	*b*	*b*	Power (*s*)
Oil consumption, pounds per shift	1.5	1.5	...
Method of feeding	...	...	Trommel
Method of preventing corrugation	*E, k*	*E, k*	*E*
Time to change shells, hr.	18	18	14
Life of parts:			
Shells, days	210	90	190
Thickness, new, in.	3.5	3.5	4
Thickness, discarded, in.	0.25–1	0.25–0.75	0.25–0.5
Material	*HCS*	*HCS*	*Cr*
Method of setting	Shrunk	Shrunk	...
Cheek plates, days	210	90	...
Material	Old shells	Old shells	...
Shafts, days	No repl.	No repl.	...
Bearings, days	360–490	180	...
Springs, days	No repl.	No repl.	...

Plant	Replogle Steel Co.	St. Joseph Lead Co., Rivermines	Porphyry Copper
Size, diameter × face, in.	42×16	42×18	43.5×16
Set, in.	...	0.25	*CI*
Speed, r.p.m.	120	90	125
Peripheral speed, ft. per min.	1320	1000	1440
Tons new feed per 24 hr.	300	800	750
Tons total feed per 24 hr.	...	1100	1400
Actual capacity, per cent. theoretical (*Y*)	...	26.3	181
Horsepower installed	...	...	...
Horsepower consumed	23	65	55–65
Tons per hp.-hr. consumed (*H*)	0.54	0.51	0.52
Moisture in feed, per cent.	Dry	Dry	35
Size of feed (*P*)	*34*	−1.25-in. max.	*13*
Size of product (*P*)	*34*	−9-mm.	*13*
Angle of nip (*V*)	...	17° 36′	23° 6′
Reduction ratio (*e*)	...	...	16.8
Attendance, machines per man	...	11	8
Lost time, per cent.	...	1	1
Principal causes of lost time	...	Choking	*s*
Oil consumption, pounds per shift	...	2	6
Method of feeding	...	Chute	Launder
Method of preventing corrugation	...	*k*	*k, MLS*
Time to change shells, hr.	...	6	4
Life of parts:			
Shells, days	...	900	33
Thickness, new, in.	...	5	4.75
Thickness, discarded, in.	...	1	1
Material	...	*Mn*	*Cr*
Method of setting	...	...	Shrunk
Cheek plates, days	...	270	33
Material	...	*Mn*	*CI*
Shafts, days	...	No repl.	No repl.
Bearings, days	...	270	66
Springs, days	...	No repl.	No repl.

For explanation of reference

of rolls—*Continued*

Phelps-Dodge Morenci	Phelps-Dodge, Morenci	Witherbee Sherman, Mill No. 4	Shattuck Arizona	Moctezuma Copper Co.	Moctezuma Copper Co.
40×16	40×16	42×16	42×16	42×16	42×16
0.25	0.25	0.5	0.38–0.5	0.19	*CI*
80	80	75	120	80	80
850	850	835	1336	890	890
350	200	....	480–600	....	625
....	400	486	1440–1800	1000	1000
12.4	14.1	8.7	23	44.2	....
....	....	....	....	50	50
18	18	15.4*l*	40–50*G*	16.7	16.7
0.81	0.46	1.31	0.5	2.45	1.53
Dry	20	Dry	Dry	*w*	*w*
−0.75×2.5-in.	−0.5×1-in.	*12*	−2.5+0.62-in.	−1.5-in.	−7-mm.
−0.5×1-in.	−0.25-in.	*12*	....	....	....
17° 12′	11° 36′*X*	20° 36′	26° 6′	21° 0′	22° 12′
....	....	1.4	....	....	....
6	6	6	1+	3	3
2	2	....	@6	1	1
s	*s*	....	*C*	*F*	*F*
....	....	5*g*	....	1.7*g*	1.7*g*
Chute	*l*	Chute	Chute	Launder	Launder
MLS	*MLS*	*z*	*k, D*	*AF*	*AF*
8	8	5–10	32–40	3.5	3.5
210	135	200	60–75	70	70
3.5	3.5	4	5	5	5
0.5	0.5	0.75	0.75	0.5–0.75	0.5–0.75
HCS	*HCS*	*Cr*	*Cr*	*Mn*	*Mn*
....	....	*p*	Shrunk	*q*	*q*
420	420	150	....	70	70
CI	*CI*	*CI*	*CI*	*CI*	*CI*
....	....	No repl.	No repl.	No repl.	No repl.
210	135	150	60	70	70
....	....	No repl.	No repl.	No repl.	No repl.

Chino Consolidated Copper Co.	American Zinc, Lead & Smelting Co., Mascot	Braden Copper Co. (*Z*)	Federal Lead Co., Mill No. 4	Timber Butte	Granitic Zinc Ore	American Zinc, Lead & Smelting Co., Mascot
43.5×16	43.5×16	48×18	48×24	54×18	54×20	54×20
CI	*CI*	0.12	*CI*	*CI*	*CI*	*CI*
120	92	90	65	87	108	92
1385	1060	1130	828	1246	1547	1317
1250	350	605	1000	....	700	1400
2500	1000	....	....	711	2100	2800
225	37.2	....	40.2	....	271	66.9
300	....	100	50	110	100	150
75–100	....	65–90	50	75–90	70	95
0.60	....	....	0.83	0.36	0.42	0.61
15–20	28	....	15	*w*	45	Dry
14	....	*36*	0.75-in. max.	....	*16*	*15*
14	....	*36*	−12-mm.	....	*16*	*15*
13° 48′	....	....	15° 36′	....	....	23° 54′
2.8	....	....	....	....	3.4	6.0
2	....	....	4	2	4	2
1	....	....	3	2	....	5
s	....	....	*b*	Choking	*s*	*L*
10.9	2	....	2*g*	....	12	8
Chute	Screen	....	Chute	....	Hopper	Conveyor
MLS	*MLS*	....	*MLS*	*MLS*	*MLS*	*MLS*
3.5	10	....	8	16	8	10
30	240	....	360	30	45	120
5	4.5	....	4	5	5.5	5
0.75	1	....	1	2.25	1	1.5
J	*HCS*	....	Stl.	*Cr*	*HCS*	*HCS*
Shrunk	Shrunk	....	*p*	....	Shrunk	Shrunk
365	1110	....	360	30	90	1100
CI	*CI*	....	*CI*	*CI*	*CI*	*CI*
No repl.	No repl.	....	No repl.	No repl.	No repl.	No repl.
365	No repl.	....	360	No repl.	270	No repl.
No repl.	No repl.	....	No repl.	No repl.	No repl.	No repl.

letters, see page 296.

Table 22. Performance

Plant	Alaska Gastineau	Replogle Steel Co.	Replogle Steel Co.
Size, diameter × face, in.	54×20	54×20	54×20
Set, in.	0.19	@1	....
Speed, r.p.m.	104	86	104
Peripheral speed, ft. per min.	1490	1215	147
Tons new feed per 24 hr.	1000	1354	1440
Tons total feed per 24 hr.	5000	....	....
Actual capacity, per cent. theoretical (*Y*)	106	17.6	....
Horsepower installed	150	....	....
Horsepower consumed	148	....	50
Tons per hp.-hr. consumed (*H*)	0.38	....	1.2
Moisture in feed, per cent.	Dry	Dry	Dry
Size of feed (*P*)	*17*	*32*	*33*
Size of product (*P*)	*17*	*32*	*33*
Angle of nip (*V*)	....	15° 24′	....
Reduction ratio (*e*)	15.1	....	....
Attendance, machines per man	3.33	....	....
Lost time, per cent.	....	....	....
Principal causes of lost time	....	....	....
Oil consumption, pounds per shift	12	....	....
Method of feeding	....	....	....
Method of preventing corrugation	*k, MLS*	....	....
Time to change shells, hr.	2.5	....	....
Life of parts:			
Shells, days	60*M*	....	....
Thickness, new, in.	5	....	....
Thickness, discarded, in.	1.12	....	....
Material	*HCS*	....	....
Method of setting	Shrunk (*p*)	....	....
Cheek plates, days	60	....	....
Material	*CI*	....	....
Shafts, days	No repl.	....	....
Bearings, days	160	....	....
Springs, days	No repl.	....	....

a Overloaded. *b* Choking and changing shells. *c* Midvale steel. *d* Wooden wedges. *e* Ratio of average size of feed to average size of product. *f* To pass ⅝-in. ring. *g* Grease. *h* Changing shells, babbitting bearings, alignment, choking. *i* Average. Maximum = 22; minimum = 11.8. *j* To pass 7-mm. round hole. *k* Regular shifting of feed. *l* Shaking launder. *m* Changing shells; broken gears; tramp iron. *n* One shell cast iron, life 400 days; other steel, life 300 days. Steel shell has one transverse groove, 1.5 × 0.75 in. to aid nipping. *o* Horizontal slot with fingers. *p* Tapered cores. *q* Wood and steel wedges. *r* Mild steel. *s* Changing shells. *t* To pass 0.053-in. round hole. *u* Shovel wheel. *v* Rigid. *w* Wet; per cent. water unknown. *x* Average. 18.3 maximum; 7.0 minimum. *y* Bronze. *z* No corrugations. Center grooving eliminated by turning in lathe. *A* After seven years' service ends turned down and fitted with steel bushings. *AF* Automatic fleet. *B* Fixed, 70; movable, 84. *C* Hot bearings, breaking of foundation bolts, choking. *CI* Cast iron. *CCI* Chilled cast iron. *Cl* Close, faces in contact. *Cr* Chrome steel. *D* Rolls flange and corrugate. Flanges burned off with acetylene torch. *E* Emery bricks kept on all rolls. *F* Shells slipping when thin; clutch shoes wearing.

versa. Occasionally both pulleys are placed on the same side, the stationary shaft furnished with an outboard bearing. This arrangement is much superior from the standpoint of safety and convenience.

Roll centers, cores or hearts are made of cast iron. A typical form for large rolls is shown in Fig. 40. The fixed heart (*7*) has a long tapered hub. It is pressed onto the shaft under hydraulic pressure running up to 300 tons per square inch. The movable heart (*8*) is split and drawn onto the hub of the fixed heart by bolts (*21*). The outer surface of the hearts is tapered toward a least diameter along a circumference at the central transverse plane of the roll shell when in place. The roll shell is correspondingly tapered inside. Shells are put on by first removing the movable heart and slipping the shell over the fixed heart, then drawing the movable heart tightly in place by means of bolts (*21*). Frequently the shell is expanded by heating at the time the cores are drawn together so that when cool it shrinks down on the cores. A special bolt (*22*) is provided for backing away the movable heart when necessary. One maker puts the taper on the shaft and makes the outer surface of the hearts and the inner surface of the roll shells cylindrical. The advantages claimed

of rolls—*Continued*

Phelps-Dodge, Burro Mountain	N. J. Zinc Co., Ogdensburg	McIntyre Porcupine	Chino Consolidated Copper Co.	Alaska Gastineau	Alaska Gastineau	Braden Copper Co. (*Z*)
54×24	54×24	60×30	72×20	72×20	72×20	72×21
0.5	0.62	1	1	1.25	0.62	0.25
100	58	52	72	108	108	88
1430	820	827	1373	2060	2060	1660
....	600	2000	....	6000	6000	1382
3000	1200	....	5000	....	8000	....
21	30.9	9.6	21.8	13.9	37.2	....
90	....	150	300	150	150	200
75	....	59	50–60	80	80	141
1.67	....	1.42	3.78	3.12	3.12	0.40
....	Dry	Dry	Dry	Dry	Dry	Dry
....	*25*	*8*	−3-in.	−3.5+1.25-in.	*18*	*35*
....	*25*	*8*	....	....	*18*	*35*
....	16° 24′	26° 0′	17° 0′	17° 24′	14° 36′	....
....	1.8	3.4	....	....	1.7	....
2	....	1	3	2	2	....
....	....	....	0	....	....	....
....	....	....	2.5	12	12	....
....	Chute	....	Chute	....	....	....
MLS	*AF*	*E*	*MLS*	*k, MLS*	*k, MLS*	....
....	....	100	18–30	2.5	2.5	....
....	....	563	365	80*N*	80*N*	....
....	4.25	4	6	6	6	....
....	....	0.75	0.75	0.62–0.75	0.62–0.75	....
....	Stl.	*HCS*	*J*	*HCS*	*HCS*	....
....	....	....	Shrunk	Shrunk	Shrunk	....
....	....	100	365	80	80	....
....	....	*Mn*	*CI*	*CI*	*CI*	....
....	....	No repl.	No repl.	No repl.	No repl.	....
....	....	300	365	160	160	....
....	....	No repl.	No repl.	No repl.	No repl.	....

G Estimated. *H* Based on original tonnage where available, otherwise on total. *HCS* High-carbon steel. *J* Crucible steel. *L* Broken shells; jammed threads on tension-rod bolts; loose pulleys. *M* 0.1252 lb. per ton. *MLS* Manual lateral shifting. *Mn* Manganese steel. *N* 0.0543 lb. per ton. *P* Numbers in italics refer to columns in Table 21*a*. *R* See sizing test, conical ball-mill feed, Table 55*a*, No. 16. *S* Edison type, rigid, corrugated shells. *T* High spots burned off at intervals. *U* Edison type, spring, corrugated shells. *V* Minimum dimension of largest particles of feed taken as 0.6 times aperture of screen that passes feed. Where set of rolls is not given it is taken as 0.6 times aperture of screen that passes all of product. *W* These settings do not accord with size of product. *X* Feed size reckoned at 0.9 times short dimension of slot. *Y* Based on total tons passing. Where set of rolls is not given it is taken as 0.6 times aperture of screen passing all product. When set close the mean opening is taken as one-half of 0.6 times (= 0.3 times) the aperture of the screen passing all product. *Z 29 IMM 238*. + Indicates that attendant has other duties.

for this method of construction are strengthening of the shaft at the point of maximum strain and greater ease in rolling the shells. A serious disadvantage lies in the fact that the expensive shaft is subjected to a possibility of considerable wear with resulting necessity for replacement. In smaller rolls wood or wood and steel wedges are used between the shell and a solid cylindrical core.

Shells are made of high-carbon steel rolled similarly to locomotive tires, or of forged chrome steel rolled or bored to size and taper, or, less frequently, of manganese steel ground to proper shape. In old practice the shells were made of chilled iron, but on account of the impossibility of making a uniformly hard shell, this material pits rapidly and badly with resultant loss in crushing efficiency (see Sec. 4, Art. 27). For light service, rolled high-carbon steel shells are entirely satisfactory. An advantage claimed for them is that, due to the fact that the surfaces are not extremely hard, there is less failure to nip than is encountered with harder shells. Likewise they do not crack readily and therefore can be worn down very thin before replacing.

There is great difference in steels from various sources. The products of local or little known shops are particularly to be regarded with skepticism. Ferguson (*50 A 290*) gives

Table 22a. Sizing tests

(Figures under the headings F

Reference Numbers			1		2		3		4	
Plant			Federal M. & S.		Federal M. & S.		Federal M. & S.		Federal M. & S.	
Screen aperture										
Mesh	In.	Mm.	F	P	F	P	F	P	F	P
	3.0									
	2.5									
	2.0									
	1.75	44.40								
	1.5	38.10	0						0	
	1.25									
	1.05	26.67								
	1		3.2	0					18.0	1.6
		20								
	0.742	18.83								
	0.62									
	0.525	13.33								
	0.5									
		12	38.5	18.3					19.7	14.4
		10								
	0.371	9.42								
		9	49.6	49.1					14.5	15.7
		8								
3	0.263	6.68								
	0.25									
		6								
		5							15.8	17.1
4	0.185	4.70	7.5	21.8	60.5	14.4	14.6			
		4								
6	0.131	3.33	1.2	5.8	30.6	27.2	51.0	5.5		
		3							11.4	17.6
8	0.093	2.36								
		2								
10	0.065	1.65								
14	0.046	1.17								
16		1		3.9	8.9	34.7	34.4	63.5	12.0	18.5
20	0.0328	0.83								
28	0.0232	0.59								
30										
35	0.0164	0.42								
40				0.7		9.6		23.7	3.5	7.4
48	0.0116	0.29								
50										
60										
65	0.0082	0.21								
80										
100	0.0058	0.15		0.3		7.3			2.5	3.9
120										
150	0.0041	0.10								
200	0.0029	0.07								
280										
300										
Through last screen				0.1		6.8		7.3	2.6	3.8
Average diameter,(b) in.			0.405	0.310	0.156	0.078	0.107	0.035	0.374	0.202
Average diameter,(b) mm			10.29	7.88	3.96	1.98	2.72	0.89	9.50	5.13

a Principally slabs. *b* Av. $D = \frac{\Sigma DW}{\Sigma W}$, where D = screen aperture and W = per-

F = feed.

referred to in Table 22
and P are weight per cent.)

5		6		7		8		9		10		11	
St. Joseph Lead Co., Bonne Terre		St. Joseph Lead Co., Bonne Terre		McIntyre Porcupine		McIntyre Porcupine		Braden		Calumet & Hecla		Witherbee-Sherman, Mill No. 4	
F	–	F	P	F	P	F	P	F	P	F	P	F	P
12.9*a*						51.4*c*							
11.4*a*						15.0							
17.6						3.7	2.2						
15.7						6.5	7.2						
14.9					10.0	5.5	28.7						
								1.6					
					21.0	4.7	15.4					0	0
										0			
					16.0	3.6	14.7						
								14.6	3.5				
												68.6	34.8
12.5													
								28.7	15.9				
		1.7											
					19.6	3.4	30.8					30.8	15.2
								12.8	10.9				
		9.8								100			
								12.2	12.2				
		11.2	0.5									0.6	34.1
		25.9	3.6		13.9								3.0
9.9								19.0	33.9				
		31.9	17.6		3.0						55.1		1.5
		14.3	17.6										
													3.0
			10.4	See No. 8, P.	4.7						24.9		0.6
			10.0										
					3.9						9.6		
			12.2										
									17.4		0.6		
			2.7		3.0								
			6.2		0.5						0.3		
					1.3								
3.5			1.9										
1.6			17.3		3.1	6.2	1.0	11.1	6.2		9.5		7.8
		0.089	0.032	0.704	0.403	2.48	0.704	0.233	0.137			0.333	0.217
		2.25	0.80	17.88	10.24	64.50	17.88	5.92	3.48			8.44	5.51

centage weight on screen. *c* Includes 16 per cent. +4-in. and 20.9 per cent. + 3½-in.

P = products.

Table 22*a*. Sizing tests referred

Reference Numbers			12		13		14		15	
Plant / Screen aperture			Witherbee-Sherman, Mill No. 4		Porphyry Copper		Chino Consolidated Copper Co.		American Zinc, Lead & Smelting Co.	
Mesh	In.	Mm.	F	P	F	P	F	P	F	P
	3.0									
	2.5									
	2.0		0	0						
	1.75	44.40							22.5	
	1.5	38.10							12.8	
	1.25									
	1.05	26.67			7.7					
	1								12.2	
		20								
	0.742	18.83	78.9	42.9	21.5					
	0.62									
	0.525	13.33			16.8					
	0.5								21.2	0.2
		12								
		10								
	0.371	9.42			11.4					
		9	21.1	38.5			52.1			7.2
		8								
3	0.263	6.68								28.7
	0.25				8.0					
		6		4.5						
		5								
4	0.185	4.70			5.9					
		4								
6	0.131	3.33			4.5			52.7		28.3
		3					33.4			
8	0.093	2.36			3.6	2.2				
		2		3.2						
10	0.065	1.65			3.0	12.6		13.9		13.4
14	0.046	1.17			2.0	11.5	2.4	10.7		
16		1					2.3			
20	0.0328	0.83		3.2	1.6	10.0		5.6		6.6
28	0.0232	0.59			1.4	11.2	1.9	3.3		
30							1.6			
35	0.0164	0.42			1.2	9.8	1.4	2.3		4.4
40										
48	0.0116	0.29			0.3	5.9	0.9	1.6		
50										
60										
65	0.0082	0.21			1.7	5.1	0.8	1.5		3.2
80										
100	0.0058	0.15			1.5	5.2	0.8	1.4		
120										
150	0.0041	0.10			0.6	2.9	0.6	1.3		2.7
200	0.0029	0.07			0.8	2.7	0.3	0.8		1.0
280										
300										1.8
Through last screen				7.7	6.5	20.9	1.5	4.9	31.3	2.5
Average diameter,(*b*) in.			0.663	0.481	0.416	0.0248	0.241	0.086	0.80	0.152
Average diameter,(*b*) mm.			16.84	12.14	10.57	0.63	6.12	2.19	20.34	3.87

b Av. $D = \frac{\Sigma DW}{\Sigma W}$, where D = screen aperture and

to in Table 22—*Continued*

16 Granitic Zinc Ore		17 Alaska Gastineau		18 Alaska Gastineau		19 N. J. Zinc Co., Franklin		20 N. J. Zinc Co., Franklin		21 N. J. Zinc Co., Franklin		22 N. J. Zinc Co., Franklin	
F	P	F	P	F	P	F	P	F	P	F	P	F	P
.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
.....	.....	.....	.....	18.6	.....	.....	.....	.....	.....	.....	.....	.....	.....
.....	.....	.....	.....	.....	.....	65.9	25.9	65.9	4.3	9.5	.....	.....	.....
.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
.....	.....	.....	.....	.....	.....	9.7	16.7	19.0	16.3	21.5	4.4	.....	.....
.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
.....	.....	.....	.....	.....	.....	4.9	9.6	9.0	10.7	46.4	26.0	.....	.....
.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
.....	.....	.....	.....	.....	.....	3.5	10.0	2.8	12.4	19.5	26.2	2.9	.....
.....	.....	65.2	.....	75.8	85.6	.....	.....	.....	.....	.....	.....	.....	.....
.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
.....	.....	.....	.....	.....	.....	2.0	5.9	0.9	7.2	1.5	12.8	1.6	0.9
.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
.....	.....	.....	.....	2.8	5.0	1.6	3.6	0.3	6.5	0.3	7.9	9.3	5.1
.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
.....	.....	.....	.....	.....	.....	1.2	2.5	0.1	4.8	0.1	4.4	38.6	26.6
.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
.....	.....	14.8	.....	0.9	3.1	1.0	2.1	0.1	3.7	0.0	2.8	37.2	46.0
.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
80.1	2.2	.....	7.1	0.2	0.9	1.2	2.1	0.1	4.6	0.0	2.6	9.5	17.1
.....	.....	.....	.....	.....	.....	1.1	2.6	0.1	4.3	0.0	2.1	0.6	2.8
.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
7.3	25.6	.....	.....	.....	.....	1.0	2.6	0.1	4.2	0.0	1.8	0.1	0.9
1.5	12.6	.....	42.6	.....	.....	1.0	2.9	0.1	3.9	0.0	1.5	0.1	0.3
.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
.....	.....	.....	.....	.....	.....	1.0	2.9	0.1	3.7	0.0	1.4	0.1	0.1
.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
4.7	18.8	11.3	10.4	.....	.....	1.0	2.8	0.1	2.8	0.0	1.1	.....	0.1
.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
.....	.....	.....	.....	.....	.....	1.4	3.7	0.2	2.6	0.1	1.0	.....	0.1
1.9	6.1	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
0.6	4.5	.....	11.2	.....	.....	1.0	1.3	0.3	2.3	0.1	0.9	.....	.....
.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
1.1	4.9	.....	.....	.....	.....	0.6	1.4	0.2	2.4	0.2	0.9	.....	.....
0.5	5.6	4.3	7.0	.....	.....	0.5	0.8	0.3	1.4	0.3	1.0	.....	.....
0.1	2.8	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
2.2	16.9	4.4	21.7	1.7	5.4	0.4	0.6	0.3	1.9	0.5	1.2	.....	.....
.055	0.016	0.257	0.017	0.568	0.333	0.816	0.509	0.889	0.317	0.540	0.288	0.124	0.103
1.41	0.41	6.53	0.43	14.42	8.47	20.71	12.92	22.58	8.07	13.73	7.31	3.15	2.60

V = percentage weight on screen. *F* = feed. *P* = product.

Table 22a. Sizing tests referred

Reference Numbers			23		24		25		26	
Plant			N. J. Zinc Co., Franklin		N. J. Zinc Co., Franklin		N. J. Zinc Co., Ogdensburg		N. J. Zinc Co., Ogdensburg	
Screen aperture										
Mesh	In.	Mm.	F	P	F	P	F	P	F	P
	3.0									
	2.5									
	2.0									
	1.75	44.40								
	1.5	38.10								
	1.25									
	1.05	26.67					60.9	13.7	49.0	9.6
	1									
		20								
	0.742	18.83					25.6	19.8	29.8	37.9
	0.62									
	0.525	13.33					8.7	20.5	17.0	29.8
	0.5									
		12								
		10								
	0.371	9.42					2.1	12.4	2.0	11.4
		9								
		8								
3	0.263	6.68					0.2	8.3	0.8	3.3
	0.25									
		6								
		5								
4	0.185	4.70					0.2	6.0	0.2	1.8
		4								
6	0.131	3.33	0.4	0.1			0.1	2.1	0.1	0.9
		3								
8	0.093	2.36	5.6	0.2	0.5	0.4	0.1	2.4	0.1	0.8
		2								
10	0.065	1.65	19.1	3.8	8.8	7.7	0.1	2.6	0.0	0.7
14	0.046	1.17	17.4	10.7	22.0	16.0	0.1	1.7	0.0	0.5
16		1								
20	0.0328	0.83	19.7	19.4	36.8	23.8	0.1	1.7	0.0	0.4
28	0.0232	0.59	18.3	22.2	26.7	19.5	0.1	1.3	0.0	0.3
30										
35	0.0164	0.42	12.0	14.2	4.6	9.1	0.1	1.1	0.0	0.3
40										
48	0.0116	0.29	4.0	7.6	0.3	5.7	0.2	1.3	0.0	0.1
50										
60										
65	0.0082	0.21	1.5	5.7	0.1	4.8	0.3	1.3	0.1	0.2
80										
100	0.0058	0.15	0.7	4.4	0.1	3.5	0.3	1.2	0.1	0.2
120										
150	0.0041	0.10	0.3	3.1	0.0	2.6	0.2	0.9	0.1	0.2
200	0.0029	0.07	0.3	3.1		2.3	0.3	0.7	0.1	0.2
280										
300										
Through last screen			0.7	5.5	0.1	4.6	0.3	1.0	0.6	1.3
Average diameter, (b) in.			0.039	0.024	0.035	0.028	0.885	0.486	0.836	0.600
Average diameter, (b) mm			1.00	0.60	0.90	0.71	22.47	12.35	21.22	15.24

b Av. $D = \frac{\Sigma DW}{\Sigma W}$, where D = screen aperture and

to in Table 22—*Continued*

27 N. J. Zinc Co., Ogdensburg		28 N. J. Zinc Co., Ogdensburg		29 N. J. Zinc Co., Ogdensburg		30 N. J. Zinc Co., Ogdensburg		31 N. J. Zinc Co., Ogdensburg		32 Replogle Steel Co.		33 Replogle Steel Co.	
F	P	F	P	F	P	F	P	F	P	F	P	F	P
.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	35.1	.....	.....	.....
.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	53.1	10.0	.....	.....
.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	11.4	27.8	.....	.....
.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
.....	.....	.....	.....	.....	.....	8.0	0.7	.....	.....	.....	.....	.....	.....
.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
.....	.....	.....	.....	.....	.....	20.5	5.2	.....	.....	.....	.....	.....	.....
.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
.....	.....	.....	.....	3.7	0.6	37.6	25.3	.....	.....	.....	27.0	9.8	.....
.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
.....	.....	0.3	0.5	26.2	4.4	29.5	37.7	.....	.....	.....	.....	.....	.....
.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
0.6	0.5	16.4	3.4	56.8	15.5	3.7	18.7	.....	.....	.....	.....	.....	.....
.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
22.3	2.8	56.6	6.7	11.2	18.4	0.0	4.4	0.1	.....	.....	9.0	60.7	.....
.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
62.4	10.8	19.1	5.8	0.9	15.5	0.0	1.9	6.1	1.2	.....	.....	.....	.....
.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
11.7	18.9	2.8	9.5	0.1	15.1	0.1	1.4	35.4	15.4	.....	1.9	17.7	0.2
0.6	20.5	1.1	8.9	0.1	8.4	0.1	0.9	35.9	35.5	.....	.....	.....	.....
.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
0.3	15.4	0.8	11.4	0.0	5.5	0.0	0.8	19.1	30.5	.....	5.2	8.7	30.4
0.2	8.2	0.5	9.2	0.1	3.2	0.0	0.4	2.6	8.5	.....	.....	.....	.....
.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
0.2	5.5	0.5	8.7	0.0	3.0	0.0	0.4	0.4	3.7	.....	.....	.....	.....
.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	7.1	0.5	28.7
0.1	2.8	0.3	5.6	0.0	2.2	0.0	0.3	0.1	1.6	.....	.....	.....	.....
.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	4.8	0.1	18.3
0.1	2.4	0.2	4.5	0.1	1.6	0.0	0.2	0.0	1.1	.....	.....	.....	.....
.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	1.7	.....	6.1
0.2	2.3	0.2	5.1	0.1	1.6	0.1	0.3	0.1	0.8	.....	1.2	.....	2.4
.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
0.2	1.6	0.1	3.8	0.0	1.0	0.0	0.2	0.0	0.4	.....	.....	.....	.....
0.2	1.6	0.1	3.9	0.1	1.0	0.0	0.3	0.1	0.4	.....	.....	.....	.....
.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
0.9	6.7	1.0	13.0	0.6	3.0	0.4	0.9	0.1	0.9	0.4	4.3	2.5	13.9
.096	0.045	0.126	0.032	0.204	0.098	0.391	0.273	0.052	0.040	1.585	0.467	0.128	0.017
2.45	1.14	3.20	0.80	5.17	2.50	9.93	6.95	1.33	1.02	40.23	11.87	3.26	0.42

W = percentage weight on screen. *F* = feed. *P* = product.

Table 22*a*. Sizing tests referred to in Table 22—*Continued*

Reference Numbers / Plant			34 Replogle Steel Co.		35 Braden Copper Co.		36 Braden Copper Co.		37 Braden Copper Co.	
Screen aperture										
Mesh	In.	Mm.	F	P	F	P	F	P	F	P
	3.0									
	2.5									
	2.0				12.8					
	1.75	44.40								
	1.5	38.10								
	1.25									
	1.05	26.67								
	1				37.2	8.1	8.9			
		20			9.7	7.2	8.5	0.7		
	0.742	18.83								
	0.62									
	0.525	13.33								
	0.5									
		12								
		10			20.6	35.3	39.9	11.4	2.9	
	0.371	9.42								
		9								
		8			6.2	17.2	19.9	21.8	10.0	4.7
3	0.263	6.68								
	0.25									
		6			1.8	5.0	6.0	10.7	13.1	5.8
		5								
4	0.185	4.70								
		4			1.5	4.4	4.8	8.9	15.3	10.1
6	0.131	3.33								
		3								
8	0.093	2.36								
		2			3.8	9.6	5.0	21.7	47.9	51.7
10	0.065	1.65	33.4	26.1						
14	0.046	1.17								
16		1								
20	0.0328	0.83	45.0	25.9						
28	0.0232	0.59								
30										
35	0.0164	0.42								
40			11.8	12.9						
48	0.0116	0.29								
50										
60			5.5	22.3						22.0
65	0.0082	0.21								
80			1.1	2.5						
100	0.0058	0.15	0.5	1.3						
120										
150	0.0041	0.10								
200	0.0029	0.07								
280										
300										
Through last screen			2.7	9.0	6.4	13.2	7.0	24.8	10.8	5.7
Average diameter, (*b*) in.			0.039	0.030	0.878	0.357	0.401	0.176	0.136	0.087
Average diameter, (*b*) mm.			0.98	0.75	22.30	9.06	10.18	4.47	3.45	2.21

b Av. $D = \frac{\Sigma DW}{\Sigma W}$, where D = screen aperture and W = percentage weight on screen.
F = feed. ***P*** = product.

the following chemical analysis and physical properties of a good rolled-steel tire: C, 0.65 to 0.80 per cent.; Si, 0.15 to 0.30 per cent.; S, 0.04 per cent.; P, 0.05 per cent.; Mn, 0.6 to 0.85 per cent.; ultimate stress, 125,000 lb. per sq. in.; yield point, 72,000 lb. per sq. in.; elongation, 10 per cent.; reduction in area, 15 per cent. On three tests with these tires the wear was 1.06, 3.8 and 4.25 lb. per ton crushed. These figures are all much above normal. (See Table 22.) The six new tires weighed 4494 lb. total and 1640 lb. total when discarded. Manganese-steel shells have longer life on coarse feed than chrome-steel, but on fine feed there is much less difference and the advantage is with the cheaper chrome steel. Life of shells under varying conditions of service is given in Table 22.

In a mill where rolls are used for crushing different sizes of material, partly worn shells from fine rolls may be transferred to the coarse rolls, provided these are of the same size, since the loss of efficiency in coarse rolls, due to slight pitting, is much less than in fine rolls. Flanging and pitting of roll shells are prevented or materially slowed down by proper attention to lateral adjustment. Emery bricks held constantly against the face of the rolls diminish corrugation. At Van Roi mill (*101 J 465*) the use of such bricks tripled the life of fine-roll shells. In South-eastern Missouri flanged shells are taken off and ground or turned down in a lathe, according to whether they are manganese or rolled steel. A 54 × 20-in. shell can be turned down in 20 to 30 hr. dependent upon the extent of corrugation (*57 A 346*). For rolls with wide faces the shells are sometimes made in two rings. At Ohio Copper Co. (*99 J 748*) shells for 60 × 24-in. rolls are in two sections, one 10-in. width and the other 14-in. This construction induces grooving at the joint.

Housing is made of cast iron or sheet steel. In most cases the lower part is cast integral with the frame and provided with a flange for bolting on the upper part. Most makers provide the upper part with cast-iron ribbed sides and sheet-steel cover. Hinged inspection doors of sheet steel or of canvas ribbed with steel straps are provided over both rolls. The housing is made as nearly dust-proof as is practicable. The shaft openings are covered with special devices to prevent emission of dust and grit. The sides of the housing are made sufficiently strong and stiff to carry the weight of the feed hopper and in some cases also to carry a feeder.

Cheek plates are made of hard iron or, rarely, of special steel. They are bolted to the inside of the housing in the hopper-shaped opening formed by the sides of the housing and the upper surfaces of the rolls. They should be made capable of lateral adjustment by means of bolts projecting through the housing so that they can be properly crowded up against the edges of the rolls to prevent the passage of uncrushed material between the sides of the rolls and the housing. Life of cheek plates is given in Table 22.

Feed hopper is placed to one side of the opening between the rolls in order to deliver the stream as nearly as possible to the center of the opening. It should be furnished with distributing plates for spreading the stream of feed across the full width of the roll faces. Adjustable side plates are also a convenience. Liner plates for the hopper are made of hard iron or manganese steel. The life of liner plates is from 30 days to several years. Commonly they are changed with the cheek plates.

Adjustments possible in well-designed rolls are (*a*) the distance between roll faces, and (*b*) lateral adjustment of one or both roll shafts. Adjustment of the distance between faces, or roll setting, is accomplished by changing the distance between the faces of the nuts at the two ends of the tension rods and by change of the total thickness of the shims placed between the forward end of the movable bearing and the frame. In making this adjustment it is important that the shafts be kept parallel. In most rolls this is accomplished by pinning the nuts at one end of the tension rods and so arranging the nuts at the other end that they are both moved equally and dependently by the adjusting mechanism. Fig. 41 illustrates the adjusting mechanism on one make of rolls in which the adjusting nuts are turned by spur gears operated by intermediate gears from a common pinion. This mechanism draws

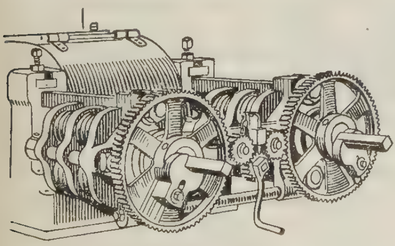


Fig. 41.—End adjustment for setting rolls.

the movable-roll bearing up against a block of shims whose total thickness is that necessary to keep the roll faces a predetermined distance apart. As roll shells wear down the shims are changed to allow the roll shafts to be drawn closer together and thus compensate for wear. This end-adjustment mechanism provides for backing the movable roll away from the fixed roll in order to free the rolls in case of clogging. It also allows the rolls to be backed away from or drawn up to the shims while running.

Side adjustment is necessary to prevent flanging and circumferential corrugation. In order to prevent flanging the range of side adjustment must be such that either edge of both rolls can be made to run for a part of the time, at least, on the face of the other roll. To prevent corrugation rolls should be shifted through a distance of about 0.6 times the diameter of the largest particles in the feed. Lateral adjustment is accomplished manually or automatically. At SILVER KING COALITION (*99 J 615*) the life of shells on fine-crushing rolls was increased from 90 days to 2 years by installing rolls with manual fleet and adjusting every 16 hr. The objection to manual adjustment is that it is likely to be forgotten or purposely neglected by the roll operator and that a short period of neglect may result in ruining the surface of a pair of shells. The only objection to automatic lateral shifting lies in the difficulty of making a simple, durable and certain shifting mechanism. This objection bids fair to be overcome.

Several operators report satisfactory performance of the Traylor mechanism shown in Fig. 42. In this device the worm (*C*) cut on one end of the fixed-roll shaft (*B*) operates the worm wheel (*D*) on the transverse shaft (*E*), set below the main shaft. The worm (*T*) on shaft (*E*) in turn drives the worm wheel (*G*) on shaft (*H*). A worm on the other end of shaft (*H*) drives the worm wheel (*K*) on shaft (*L*). Shaft (*L*) carries an eccentric sleeve (*N*), fitted to a slide in a machined recess in the thrust collar (*O*). This collar carries grooves into which fit corresponding collars cut on the end of shaft (*B*). Lateral motion of the eccentric sleeve (*N*) is thereby transmitted to the shaft (*B*) which is caused to move backward and forward along the line of its axis through a distance determined by the throw of the eccentric (*N*). This mechanism is enclosed in a dust-proof case cast integral with one side of the roll frame. The case carries bearings for the various shafts. Variation in amount of lateral movement, or FLEET, is accomplished by changing the eccentric shaft (*L*). The chain of gears is such that one complete cycle of roll-shaft movement is completed in about 30 min.

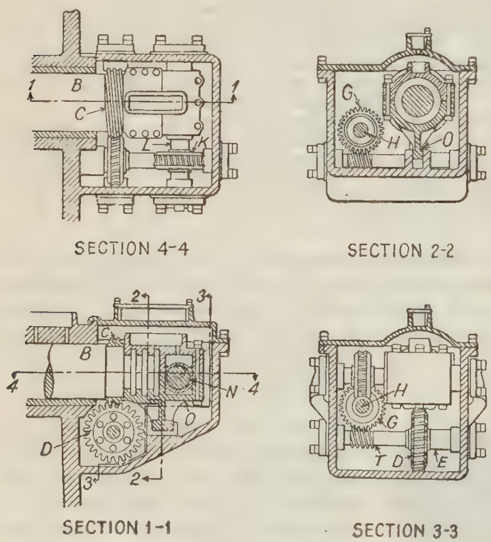


FIG. 42.—Traylor automatic lateral adjustment for rolls.

Sectionalizing. Most makers will furnish rolls up to 30-in. diameter sectionalized so that the heaviest piece does not exceed 300 or 350 lb.

Requirements for ideal rolls are that they should be rugged, simple in construction, compact and the working parts should be readily accessible; worn parts should be capable of easy change with as little dismantling of the apparatus as possible. Springs should exert a pressure sufficient to crush the hardest rock and yet should be sufficiently flexible to pass unbreakable substances without bending the shafts or breaking the castings. A substantially dust-proof housing and large dust- and grit-proof, well lubricated bearings should be provided. The mechanism for adjustment for distance between roll faces should be capable of rapid and easy operation, in order to facilitate clearing the rolls in case of clogging, it should not necessitate a change in spring pressure, and it should advance both sides simultaneously in order to maintain proper alignment of shafts. If possible there should be automatic lateral adjustment of one of the roll shafts. Fleeting devices, however, add considerably to the first cost and thorough investigation of their trustworthiness should be made before purchase.

Performances at a number of mills are shown in Table 22.

Angle of nip (n) is the angle formed by the tangents to the roll faces at the points of contact therewith of particles to be crushed. This angle is shown as angle ACB, Fig. 43, a. Particle P, which is to be crushed, is assumed to be spherical. If r is the radius and D the diameter of the rolls, d the diameter of particle and s the distance apart of roll faces along the line joining the centers of the rolls, the following relation holds:

$$\cos\frac{n}{2} = \frac{r + s/2}{r + d/2} = \frac{D + s}{D + d}.$$

Neglecting gravity, the particle is acted upon by forces applied at the points of contact in directions indicated by the lines F (Fig. 43, b). These forces can be resolved (considering one side only) into a normal force N and tangential force T. If the normal and tangential forces are resolved into their horizontal and vertical components respectively, it will be seen that the particle will be drawn down when the vertical component of T, acting downward, exceeds the vertical component of N, acting upward. The limiting condition is reached when the vertical components of T and N are equal but opposite in direction. Under this condition the particle will neither be nipped nor thrown out of the rolls but will ride in the hopper formed by the converging faces. With this condition the following equations may be written:—$N_v/N = \sin n/2$; $T_v/T = \cos n/2$. Dividing the first equation by the second, $\tan n/2 = TN_v/NT_v$. But under the assumption $T_v = N_v$. Therefore $\tan n/2 = T/N$. From the ordinary relations of mechanics T/N = tangent of the angle of friction. For stone on iron the coefficient of friction (= tangent of the angle of friction) is about 0.3. Substituting this value in the above equation, $\tan n/2 = 0.3$; $n/2 = 16° 42'$ and $n = 33° 24'$.

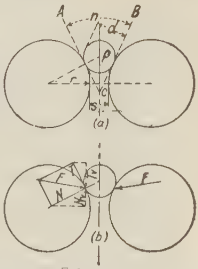


Fig. 43.—Nip angle of rolls.

In practice the nip angle rarely exceeds 25°. The average nip angle in 45 sets of rolls reported from the mills was 17° 26′. The range in angle was from 5° 36′ to 35° 30′. The angle averaged 23° 21′ for feeds coarser than 2 in. 19° 26′ for feeds between 1 and 2 in., 14° 38′ for feeds between 0.5 and 1 in., and 11° 25′ for feeds smaller than 0.5 in. The variation is due, how-

ever, to the use of large rolls with small feeds in order to get capacity and the angles used with coarse feeds may be taken as the safe average figure.

The angle of nip varies with the diameter of the rolls, the diameter of the particle and the set of the rolls. In order to nip large particles, rolls of large diameter must be used, or the reduction ratio must be small. When feed of a given size is not being nipped, the usual alternatives are to install larger rolls or to increase the distance between the roll faces and consequently increase the size of product. A third expedient reported by one plant was to cut a transverse groove in one of the roll shells. The effect was, of course, to increase the nip angle at this particular point.

At American Graphite Co. smooth shells would not nip. The difficulty was eliminated by drilling eight sets of four 1.5-in. holes at equal angular distances around the roll faces (*120 P 569*).

Diameter of rolls required for various sizes of feed is given in Table 23. The largest commercial roll is 78-in. diameter and rolls smaller than 24-in. are rarely used outside of the laboratory.

Table 23. Diameter of rolls required for various sizes of feed

Diameter of largest feed particle, inches	Minimum diameter of roll, in.(*a*)				
	Reduction ratio				
	6 : 1	5 : 1	4 : 1	3 : 1	2 : 1
6	....	....	....	....	121
5	....	....	....	....	100
4	....	....	....	....	80
3.5	....	....	....	95	70
3	....	....	....	81	60
2.5	....	....	77	68	50
2	68	65	61	54	40
1.75	60	57	54	48	35
1.5	52	50	46	41	31
1.25	43	41	38	34	25
1	35	33	31	27	20
0.75	26	24	23	20	15
0.5	17	16	16	14	10
0.38	13	12	11	10	8
0.25	9	8	8	7	5
0.12	4	4	4	3	2

a Allowing 25° nip angle.

Reduction ratio is a phrase commonly used in discussing crushing performances. It has no exact quantitative significance, but may convey useful qualitative information. As defined by *Truscott* in connection with rolls, it is the ratio of the size of the largest feed particle to the smallest distance between the roll faces, *i.e.*, the set. As used frequently in the field, it is the ratio of the smallest aperture passing all of the feed to that passing all of the product. Another basis of expression is the ratio of the average size of feed to the average size of product. (See Sec. 22, Art. 5.) In the relation of size reduction to angle of nip and strain on crushing machinery, the first expression is the most significant; in stating the duty of rolls and their performance in a flow-sheet, the second; in judging the relation between duty and power consumed, the third. When rolls are set close, the first method is, of course, inapplicable, without knowledge of the mean roll spacing while crushing, and

this is not, normally, available. The mean ratio of reduction in average size from Table 22 is 3.4; individual figures range from 1.2 to 16.8.

Speed of rolls is limited by the ability to nip and by the weight and ruggedness of the rolls. Hence the allowable speed is affected by the diameter of rolls, the kind of ore, method of feed, reduction ratio, and size of feed. Speed should be lower for hard, tough rock than for soft and brittle rock, less for dry feed than for wet feed, less for coarse feed than for fine, and less for a large reduction ratio than for a small, nip being the controlling factor in each case. The speeds reported range from 382 ft. per min. for rolls 24-in. diameter, to 2060 ft. per min. for rolls 72-in. diameter. Practice tends to keep below 900 ft. per min. with rolls up to 36-in. diameter, below 1000 ft. per min. for 42-in. rolls, and not to exceed 1500 ft. per min. with 56- and 72-in. machines, more or less independently of the other factors; higher speeds are dangerous to springs, shafts, frames and foundations.

At Miami Copper Co. 55-in. rolls taking −3.5-in. feed are run at 100 r.p.m. and the same size roll taking −2-in. feed is run at 115 r.p.m. At Engels (*123 P 183*), crushing to 1 in. in 54 × 24-in. rolls, capacity was increased and power decreased 40 per cent. by a decrease in speed from 110 to 54 r.p.m.

Cornish rolls, gear-driven at 50 to 100 ft. per min. peripheral speeds, are occasionally met. The allowable nip angle is much greater at these slow speeds and the product is likely to contain more fines than the product of high-speed rolls because of the tendency toward choke feeding and restriction of discharge.

Capacity of rolls. The theoretical capacity in tons per hr. is given by the equation

$$C = \frac{NDWSG}{18{,}300},$$

where N = the number of revolutions per minute; D = the diameter of the rolls, W = the width of face and S = the set, all in inches; and G = the specific gravity of the rock being crushed. The development of this equation is based on the assumption of a solid ribbon of crushed material whose length is 60 times the distance traveled by a point on the roll face in one minute, whose width is the width of the roll faces and whose thickness is equal to the set of the rolls. With open setting the actual capacity never reaches the weight of the "theoretical ribbon." The theoretical ribbon is more nearly approached the smaller the set. Table 22 indicates that for rolls set coarser than 1-in., a capacity of about 5 per cent. of the theoretical ribbon is to be expected. For sets between 0.25 in. and 1 in. the average performance is 15 to 20 per cent. of theoretical, while for sets less than 0.25-in. the average is 20 to 30 per cent. of theoretical with free feeding. With choke feeding in closed circuit with a screen, from 100 to 250 per cent. of theoretical ribbon is to be expected, the set of the rolls being assumed as 0.3 times the screen aperture. The rolls may be set in actual contact or with some small space between faces, but the mean cross-section of the ribbon is, of course, greater than the setting, since the rolls recede against the spring pressure at short, irregular intervals. The percentage of theoretical ribbon that can be crushed is about 50 per cent. greater in the case of soft, easily crushed rocks than with hard, tough rocks.

Power consumption, in terms of tons of new feed per horsepower-hour, depends upon the kind of rock being crushed, the size reduction, the size of

product and the amount of circulating load, if any. Average figures from Table 22 are 0.4 ton per hp.-hr. to less than 0.25-in. size; 0.6 ton per hp.-hr. to a size between 0.25- and 0.75-in.; 0.75 ton per hp.-hr. to a size between 0.75-in. and 1-in.; 1.6 tons per hp.-hr. for a product between 1- and 1.5-in. and 2.8 tons of a product between 1.5- and 2-in. These figures disregard the size reduction and the character of the rock, on the basis that in the number of examples given these average. Reference should be made directly to the table when the character of the rock to be crushed is known to be similar to that at one of the mills reporting, since variations from the average are large.

Feeding. Rolls must be fed at a constant rate and with the stream distributed over the full width of face in order to get maximum capacity and efficiency. If the feed stream is not distributed over the full face, circumferential grooving occurs under the feed point and the amount of crushing done at one pass rapidly decreases. If the feed is not constant in quantity but comes in surges or rushes, the rolls are liable to choke and stall or, if the driving equipment is sufficiently heavy to prevent this, they spring apart and pass a mass of material only partially crushed. This causes chattering and excessive wear on slides and babbitt; it produces a greater tonnage of circulating feed, if the rolls are run in closed circuit with a screen; and makes it impossible to reach maximum capacity. Types of feeders designed to insure constant feed rate are described in Sec. 20. It is good practice to drive the feeder from the roll shaft so that when the roll stops the feed will stop, but in such a case the residue in the feeder must be removed before the roll starts again. If the feed contains a large percentage of clayey material, rolls are liable to choke.

It is best to balance the speed of the rolls and height of drop of the feed in such a way as to bring the feed particles to the roll faces at a speed as near that of the roll faces as possible. In this way there is no differential movement of particles and roll faces with resulting polishing and difficulty in nipping.

Rolls may be FREE-FED or CHOKE-FED, the former phrase indicating freedom of movement between particles resting in the V of the rolls prior to nipping, the latter that the particles at this point are piled up to such a depth that no free movement exists. In free feeding each particle is broken substantially individually and crushing is practically uniform and continuous; in choke feeding masses of material are rolled through intermittently, the roll faces springing apart to permit their passage. In this compression of the mass of rock there is much abrasion between particles which results in a less granular product than that from free crushing. Except in the case of the largest rolls, choke feeding can be practiced only with material already crushed to ¼-in. or less.

Rolls are ordinarily run dry, especially at the present time when they have been displaced by ball and rod mills for re-grinding middling.

The fine rolls at MIAMI were originally designed for wet crushing, but in operation they worked better with dry feed, so the water was cut off the feed and added to the discharge (*115 P 565*). These rolls have now been removed.

Dust production with dry feeding is a distinct disadvantage; the dust is difficult to control and causes trouble with belts and bearings.

Size of product. The lower limit of size for efficient roll crushing is not clearly established. Where a product passing a 10-mesh screen is all that is desired, it is probably more economical to complete the crushing in rolls than

to install rod or ball mills. If a product finer than this is desired, ball or rod mills will crush more economically than rolls and the weight of practice is to carry roll crushing not finer than 0.75-in. In many cases ball mills are taking feed much coarser than this. Rolls can be made to crush to 20-mesh or finer, but to do so, must be set close and run "choked." Such practice is highly uneconomical, on account of wear and tear and power consumption.

Graded crushing is an operation of gradual reduction in size by means of a series of crushers, each set with a smaller discharge aperture than the preceding, with material fine enough to pass the following crusher removed between the crushing steps. The purpose is to minimize production of slimes. The size reduction in successive steps in graded crushing is usually small, i.e., of the order of 2 or 3. The alternative extreme is to break down with as big steps in reduction ratio as the size and strength of the crushing machines will permit, with no removal of fines ahead of successive crushers, except that the last crusher in series is in closed circuit with a limiting screen.

During the years before the introduction of flotation processes in base-metal milling, when minimum sliming was essential to maximum recovery, the tradition that graded crushing was necessary was established, apparently with very little experimental evidence. Recently an exhaustive investigation by the New Jersey Zinc Co., crushing a sphalerite ore with granitic gangue from 1-in. to 0.1-in. maximum size in rolls, showed that the amount of −0.025-in. material produced was the same, within a range of about 2 per cent. of the weight crushed, irrespective of the number of steps or the presence or absence of intermediate screening. Tests in the laboratory at Columbia University have shown that the sizing test of the product of a pair of rolls with a given set is substantially the same with a given rock, irrespective of the size of feed, provided only that the rolls are free crushing, that they will nip the particles, and that there is no undersize present in the feed. The significance of the last restriction lies in the fact that if the various feeds contain different amounts of undersize these will have different effects on the screen tests of the products, even though they pass through without any breaking. These facts would seem to establish definitely that there is no advantage from graded crushing and intermediate screening in free crushing in rolls.

Corrugated rolls differ from the plane rolls already described in that the shells are corrugated transversely. Corrugated shells are used in a few metal mills, usually where the feed is too large to be nipped by rolls with plane shells.

At Hollinger (*1922 Bul. CMI 343*) a set of 40 × 20-in. transversely-corrugated rolls set at 0.75-in., running at 110 r.p.m. and drawing 45 hp. takes the product from three gyratories, one set at 1.5-in. and the others at 2.5-in., at the rate of 125 tons per hr. Finger gears are used on these rolls to keep the corrugations in mesh. The driving motor has a double-throw switch to allow reversal in case of clogging. The shells are manganese steel, 4 in. thick. One set weighs 4730 lb. and crushes 200,000 tons, so that steel consumption is 0.024 lb. per ton crushed.

Character of roll product. Two cases arise, *viz.*: (*a*) The rolls are set spaced a definite distance between faces. (*b*) The faces are set close.

In the first case the feed ranges, in general, upwards of 75 per cent. coarser than the set of the rolls (average of nineteen cases at random was 83 per cent.); the product ranged in the cases investigated from 4 per cent. coarser than the set to 78 per cent. coarser, average 45 per cent., which gives an average reduction in the percentage of material coarser than the set of 45 per cent. In the same operations the percentage of material finer than half of the roll setting

averaged 30 and ranged from 2 to 66 when there was an average of less than 5 per cent of such material in the roll feed.

With close setting the average reduction in maximum particle size is close to one-half, this average applying as well where the maximum size in the feed is 20-mm. as when 1.5-mm. The average percentage of material in the product that is less than half of the maximum particle in size is about 60; the range is between 30 and 80. In the cases investigated this average represented an increase in such material over that present in the feed of about 2½ times.

Applying these generalizations to specific problems: (*a*) Given a feed containing 75 per cent. +1-in. material to be crushed in rolls set 1-in.; the average product would contain 45 per cent. +1-in. material and 30 per cent. −½-in. material. (*b*) Given a feed containing 5 per cent. of +20-mm. material to be crushed in rolls set close; the average product would contain about 5 per cent. +10-mm. size and about 60 per cent. −5-mm. material.

Cost of roll crushing. The elements of cost are power, labor, repairs and lubrication. See p. 309 for data on power consumption. One man can attend to from 3 to 12 sets of rolls; the average in 20 plants investigated, where the roll tender had no other duties, was 6. Repairs may be estimated at about twice the cost of roll shells. Consumption of lubricant ranges from about 2 to 30 lb. per 24 hr. On the basis of these quantities the cost of crushing to −¼-in. should not exceed $0.07 per ton in small rolls (36-in. or less diameter) nor $0.045 in large. Coarse crushing will cost considerably less on account of smaller power consumption and labor cost.

Applicability. Rolls are the most widely used intermediate crushers for handling feeds smaller than 1.5-in. and delivering products down to 0.1-in. In such service they have large capacity, low power consumption and relatively low repair costs. They are rugged, reliable, simple in construction and easy to repair.

Selection of rolls. 1. *Open-circuit crushing.* Determine the set necessary (p. 311). From this and the small dimension of the largest feed particles (This averages 0.6 × aperture of the hand screen that passes the particle, if of granular shape; 0.4 to 0.5 times, if slabby.), calculate the diameter required for an allowable nip angle (p. 307). Determine from Table 21 (p. 288) the widths of face available; choose a suitable speed (p. 309; Tables 21, 22); then substitute in the equation for theoretical capacity (p. 309) Allow for the difference between actual and theoretical capacity (p. 309; Table 22), and determine the number of machines needed. If one or less, the size chosen is indicated, but with a large reduction ratio and small tonnage, two smaller rolls in series may cost less both for maintenance and operation. If more than one machine is indicated the alternatives are: two or more smaller machines, in series (which will change diameter requirements) or in parallel, or fewer and larger machines, with more favorable nip angle.

2. *Closed-circuit crushing.* The set must be less than 0.7 times the aperture of the screen closing the circuit (pp. 311, 516). The percentage of oversize in the original feed to the circuit that will return after one pass through rolls and screen will vary, depending on the ore, roll setting, ratio of roll setting to screen aperture, and screen efficiency; it is rarely less than 40 per cent. and sometimes above 80 per cent. The circulating load may be estimated (based on a percentage return experimentally determined, or estimated) from $S = a/(1 - r)$, (a = tonnage of oversize in original feed, per unit of time, r = percentage return expressed as a decimal, and S = tonnage of circulating load). Total load to the first machine in the circuit is S + *tonnage of original feed.* Tonnage in closed roll circuits varies (Table 22) from 1.3 to 5 times the original feed; average, 2.5 times. Proceed as in (1).

13. Rolls *vs.* disk crushers

Smooth-faced rolls are limited in service at the coarse end to 3.5- to 4-in. feed, taking 25° as the allowable angle of nip and 72- to 78-in. as the largest diameter manufactured, and even these rolls are limited to a 2 : 1 reduction

ratio on such feed (see Table 23). The capacity of such rolls is enormous, but so also are the weight, cost and power consumption. A disk crusher, on the other hand, will take coarser feed, if necessary, will make a larger reduction on 3- to 4-in. feed, and, although of smaller capacity than rolls, is of much less weight, cost and power consumption. Hence in the coarse range of intermediate crushing the disk crusher is preferable to rolls, and particularly so in plants of relatively small capacity. In the intermediate range, taking feeds of 2- or 2.5-in. maximum size and making a 4 or 6 : 1 size reduction, both rolls and disk crushers are available. Here the advantage is probably with rolls if the rock is hard and the tonnages large and with disks under the reverse conditions, it being borne in mind that rolls are the more rugged machines and will probably cost less in repairs and delay due to shut-downs. For fine crushing, delivering at, say, ¼-in. or finer, the advantage is all with rolls.

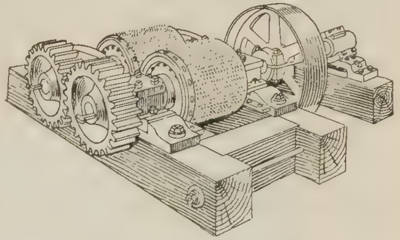


FIG. 44.—Toothed rolls.

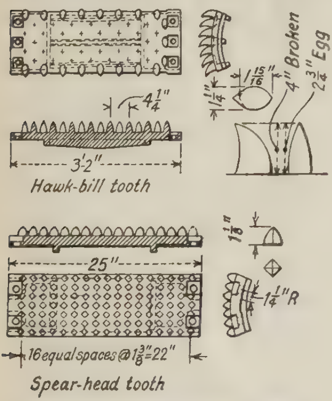
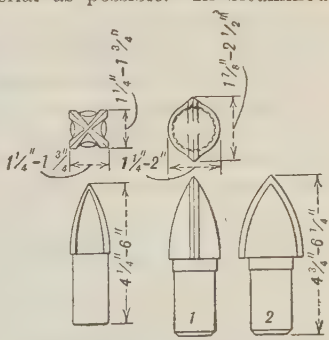


FIG. 45.—Toothed-roll shells (*66 A 422*).

14. Rolls for coal breaking

These are to be considered from an entirely different point of view than the rolls used in rock crushing. The duty required from anthracite rolls is breakage through a relatively coarse screen (rarely less than 1½-in. and usually 3⅜-in. aperture) with as little oversize and as little −¾-in. material as possible. In bituminous work 2½- to 3½-in. is usually as fine as rolls are called upon to crush. The breaking work is relatively easy and toothed rolls are ordinarily used. The reduction ratio is small and the question of nip angle is unimportant, hence the rolls required for a given capacity may be much lighter and smaller than rock-breaking rolls. Fig. 44 is an assembly of a typical set of coal-breaking rolls with toothed shells. The rolls are geared to insure that the teeth mesh properly. The shells are usually cylindrical segments, bolted to the core, and in most modern forms are fitted with replaceable teeth, usually made of manganese steel.



Johnson tooth. Spear-head tooth.

FIG. 46.—Details of roll teeth (*after Ashmead*).

The shape and spacing of the teeth are of the utmost importance in determining the performance of the rolls. Fig. 45

Table 25. Performance of toothed rolls in

Test Number	Breaker	Rolls		Speed		Size of feed
		Size Number	Shells	Revolutions per minute	Feet per minute	
1	Short Mountain	2	*a*	115	918	*B*
2	Williamstown	2	*b*	80	....	*B*
3	Williamstown	2	*b*	80	....	*B*
4	Williamstown	2	*b*	80	....	*B*
5	Williamstown	2	*b*	80	....	*B*
6	Wm. Penn	2	*Mn*	....	301	*S*
7	Wm. Penn	1	*Mn*	....	905	*L*
8	Cameron	2	*Mn*	....	256	*S*
9	Cameron	1	*Mn*	....	250	*L*
10	Cameron	3	*Mn*	....	246	*B*
11	Luke Fidler	2	*Mn*	....	233	*S*
12	Luke Fidler	1	*Mn*	....	1107	*L*
13	Luke Fidler	3	*Mn*	....	281	*B*
14	Scott	1	*Mn*	....	1008	*L*
15	Scott	2	*Mn*	....	364	*S*
16	Pennsylvania	1	*Mn*	....	240	*L*
17	Pennsylvania	2	*Mn*	....	345	*S*
18	Richards	1	*Mn*	....	288	*L*
19	Richards	3	*Mn*	....	289	*B*
20	Jeddo No. 4 (*c*)	1	Lloyd	135	....	*L*
21	Jeddo No. 4 (*d*)	2	Lloyd	135	....	*S, B*
22	Highland No. 5	1	Lloyd	135	....	*L*
23	Highland No. 5	2	Lloyd	135	....	*S,B*
24	Lansford No. 5	1	Lloyd	135	....	*L*
25	Lansford No. 5	1	Lloyd	135	....	*L*
26	Lansford No. 5	1	Lloyd(*e*)	135	....	*L*
27	Lansford No. 5	1	Lloyd(*e*)	135	....	*L*
28	Lansford No. 6	2	Lloyd	135	....	*S*
29	Lansford No. 6	2	Lloyd	135	....	*S*
30	Rahn No. 11	2	*Mn*	135	....	*S*
31	Rahn No. 11	2	*Mn*	135	....	*S*
32	Rahn No. 11	2	*Mn*	135	....	*S*
33	Rahn No. 11	2	*Mn* (*e*)	135	....	*B*
34	Tamaqua No. 14	2	Lloyd	135	....	*S*
35	Tamaqua No. 14	2	Lloyd(*e*)	135	....	*S*

a Solid cast; 896 teeth arranged diagonally and alternate. Teeth in fair condition. *b* Manganese steel; 840 alternate teeth. Good condition. *c* Average of 7 tests. *d* Average of 10 tests. *e* Johnson hollow-ground teeth. ***B*** = broken. B_1 = buckwheat No. 1.

shows two common forms of tooth used in anthracite breakers. Fig. 46 shows details of the usual spearhead tooth for different sizes of rolls and also an improved form of tooth that is claimed to produce a greater percentage of domestic-size coal than the usual forms.

Table 25 gives performances of rolls at a number of anthracite breakers. The tests at Lansford No. 5, which are directly comparable, show a distinct advantage for the Johnson tooth. The tests at Rahn No. 11 are not comparable, but comparison of tests Nos. *31* and *32* at Rahn No. 11 with test No. *35* at Tamaqua No. 14 would seem to indicate, if the coals are not greatly unlike in breaking characteristics, that in breaking from steamboat to egg size the Johnson tooth produces both less fines and less oversize than the usual forms. Tests Nos. *34* and *35* are directly comparable and show a marked advantage in crushing with the Johnson tooth in that it produces no more steam-size coal, in fact a little less, in breaking from steamboat to egg size than the ordinary tooth in breaking from steamboat to broken size. Comparison of tests *6*, *8*, *11*, *14*, *16*, and *18* indicates that in general high peripheral speeds cause higher production of steam sizes than low speeds, but test *18* is not concordant. The tests presented in Fig. 47, as well as the results in Table 25 taken as a

anthracite breakers. (*After Ashmead, 66 A 422*)

Size distribution of product, per cent.										Per cent. by weight	
S	B	E	St	N	P	B_1	R	Ba	B_4	Domestic sizes	Steam sizes
......	......	47.5	21.0	13.5	6.0	5.5	3.5	1.8	1.2	82.0	18.0
......	1.3	37.3	27.7	16.2	5.0	6.0	3.6	1.2	1.7	82.5	17.5
......	9.0	39.5	22.9	13.2	4.5	4.6	3.0	1.5	1.8	84.6	15.4
......	7.0	39.4	22.8	14.8	5.0	4.5	3.3	1.7	1.5	84.0	16.0
......	3.8	40.0	23.0	14.9	5.9	5.6	3.7	1.4	1.7	81.7	18.3
......	43.0	21.0	12.5	9.0	6.0	3.5	4.0	1.0	.....	85.5	14.5
51.0	21.0	8.5	6.0	4.8	3.0	3.2	3.0	0.5	.....	90.3	9.7
12.5	38.8	12.5	13.7	9.2	4.2	4.3	2.0	2.5	1.3	86.7	13.3
64.5	6.5	4.5	7.8	6.0	3.5	3.7	1.0	1.5	1.0	89.3	10.7
......	15.3	38.7	18.0	15.8	4.5	2.2	3.0	1.5	1.0	87.8	12.2
2.9	26.2	18.0	18.4	16.0	6.8	4.9	3.4	2.4	1.0	81.5	18.5
38.1	16.7	12.4	9.5	10.0	6.0	3.3	1.9	1.4	0.7	86.7	13.3
......	5.0	26.0	33.0	23.0	5.0	2.5	2.5	1.5	1.5	87.0	13.0
30.0	21.9	14.7	11.7	10.0	4.2	3.7	2.1	1.1	0.6	88.3	11.7
......	42.7	19.0	12.7	11.7	4.7	4.2	2.6	1.5	0.9	86.1	13.9
36.0	20.5	15.3	11.3	10.0	3.0	2.5	0.8	0.5	0.2	93.0	7.0
......	29.0	35.0	21.0	11.2	1.5	1.2	0.5	0.3	0.3	96.2	3.8
43.3	17.8	12.3	8.1	6.3	4.3	2.7	2.3	1.8	1.1	87.8	12.2
......	35.0	27.7	15.6	11.1	3.8	2.8	1.9	1.4	0.7	89.4	10.6
38.0	20.0	19.0	8.0	6.0	3.0	2.0	2.0	1.0	1.0	91.0	9.0
......	......	52.0	28.0	10.0	3.0	3.0	2.0	1.0	1.0	90.0	10.0
41.8	29.0	20.0	4.5	2.0	1.0	0.7	0.7	.....	0.3	97.3	2.7
......	......	63.4	19.2	9.6	2.8	2.0	1.7	1.0	0.3	92.2	7.8
41.4	19.8	11.1	8.2	8.3	3.6	.....	7.6	.....	.....	88.8	11.2
30.5	24.0	15.5	9.2	10.1	3.8	.....	6.9	.....	.....	89.3	10.7
45.1	23.6	7.6	7.9	8.6	2.6	.....	4.6	.....	.....	92.8	7.2
46.1	22.5	9.6	4.9	7.3	3.4	.....	6.1	.....	.....	90.5	9.5
9.6	38.7	20.9	10.6	9.3	3.8	.....	7.1	.....	.....	89.1	10.9
0.6	43.6	27.4	9.8	8.7	3.4	.....	6.5	.....	.....	90.1	9.9
21.0	35.4	16.6	8.6	7.2	4.0	.....	7.2	.....	.....	88.8	11.2
......	4.4	37.2	28.3	16.2	3.5	.....	10.4	.....	.....	86.1	13.9
......	5.9	51.8	20.5	11.0	3.8	.....	7.0	.....	.....	89.2	10.8
......	26.0	39.0	15.3	10.6	3.9	.....	5.2	.....	.....	90.9	9.1
......	36.4	32.8	11.5	8.9	3.5	.....	6.9	.....	.....	89.6	10.4
......	0.5	54.8	22.8	12.0	4.1	.....	5.8	.....	.....	90.1	9.9

B_4 = buckwheat No. 4. *Ba* = barley. *E* = egg. *L* = lump. *Mn* = manganese steel. *N* = nut. *P* = pea. *R* = rice. *S* = steamboat. *St* = stove.

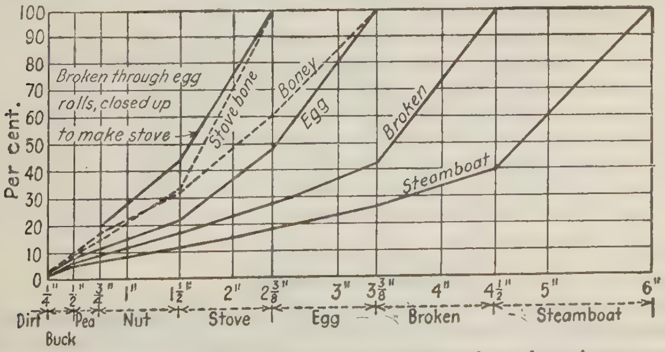


FIG. 47.—Percentages of different sizes in coal broken through various screens by toothed rolls (*after Ashmead*).

whole, show, as is to be expected, that the finer the domestic size crushed to, the greater the percentage of steam sizes.

Test on toothed rolls at CRANBERRY CREEK COAL Co. anthracite breaker (*19 CA 311*): No. 1 rolls, 33-in. diameter with 4-in. Johnson hollow-ground teeth staggered in both directions. Rolls were set 42¾ in. center to center. Speed, 33 r.p.m. No. 2 rolls, same diameter and speed, but teeth were 2¼ in. high and rolls were set 41¼ in. center to center. No. 3 rolls, same diameter and speed, height of teeth the same as in No. 2 rolls but setting was 40¾ in. Feed to the series of rolls was +6-in. lump coal. Flow-sheet was as shown in Fig. 48. Results were as shown in Table 26. Adjustment of the rolls raised the percentage of domestic sizes in the final product from 79.3 to the figure (82.6) given in the table. Lincoln (*11 Bul. UI No. 9*) notes that the product of toothed rolls crushing bituminous coal has substantially the same size distribution as that of coal as broken in mining. Comparative screen tests are given in Table 27.

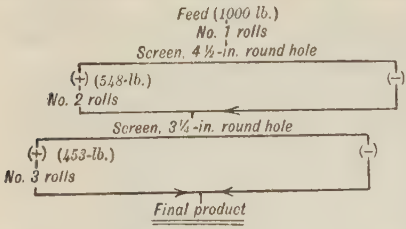


FIG. 48.—Flow-sheet for test on toothed rolls.

Table 26. Results of 3-stage crushing of anthracite in toothed rolls

Size		Product, weight, per cent.			
Name	Screen, inches (rd.-hole)	No. 1 rolls	No. 2 rolls	No. 3 rolls	Final
Steamboat	4½	54.8	1.8	....	....
Broken	3¼	18.5	47.0	22.3	10.1
Egg	2⁵⁄₁₆	7.9	27.4	40.6	41.3
Stove	1⅝	5.0	8.7	20.1	18.9
Nut	⅞	4.9	6.4	8.6	12.3
Pea	⁹⁄₁₆	2.9	2.7	2.6	5.6
Buckwheat	⁵⁄₁₆	2.2	2.6	2.0	4.5
Rice	³⁄₁₆	1.6	1.6	1.3	3.1
Barley	¹⁄₁₆	1.3	1.0	1.0	2.3
Culm	−¹⁄₁₆	0.7	0.4	1.0	1.3
Total domestic size	+⅞-in.	91.1	91.3	91.6	82.6

Table 27. Performance of toothed rolls crushing bituminous coal. (*After Lincoln*)

Screen size, inches		Weight, per cent.	
Through	On	Run-of-mine	Toothed-roll product
3¾	3	9.0	10.0
3	1¾	15.8	17.2
1¾	1¼	12.4	15.0
1¼	¾	15.6	9.7
¾	¼	27.2	29.7
¼	....	20.0	18.4

15. Single-roll crusher

This machine (Fig. 49) is used for relatively soft materials such as coal, stratified shale, limestone and the like, where a large size reduction is required in one passage. The elements of the machine are the toothed roll (*a*) and

the stationary breaking plate (*b*). The roll is driven by gear (*c*) and pinion (*d*, by means of a belt and drive pulley (*e*). The breaking plate, hinged on (*h*), is held in position by two heavy bolts (*f*), one on each side, with a nest of compression springs (*g*) under the upper nuts to permit the plate to move away when an unbreakable substance is fed. The roll shell is renewable and, in the form shown, has renewable teeth as well. Renewable wearing plates, usually corrugated, are also provided for the fixed breaking plate. Sizes vary from 24 × 48-in. (roll 24-in. diameter by 48-in. long) of 60 to 200 tons capacity per hour, according to the material and setting, to 60 × 84-in. with a rated capacity of 500 to 1500 tons per hr.

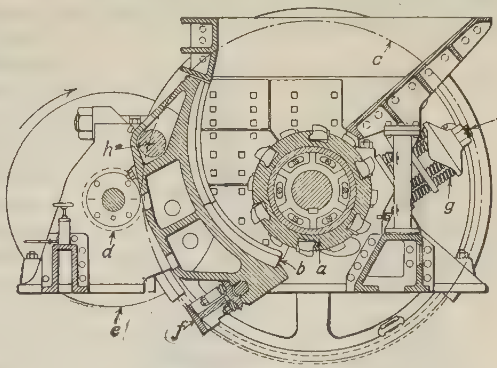


Fig. 49.—Single-roll crusher.

STAMPS

Stamps are a mechanical form of the ancient mortar and pestle. Two types have had long use, *viz.*: the steam stamp and the gravity stamp. A third type, actuated in various ways by crank or eccentric has appeared in many different forms, but has not been widely used. The use of steam stamps has been limited, with the exception of a few experimental installations, to the native-copper mills of the Lake Superior district. Gravity stamps have found their principal field as final grinders of precious-metal ores for amalgamation. They are unexcelled in preparing ores for amalgamation and as machines in which amalgamation can be performed simultaneously with crushing. Sporadic attempts to place them in base-metal mills in the days before flotation failed on account of their slime-producing proclivity. With the introduction of the cyanide process, particularly the all-slime process, finer grinding than could be economically performed in stamps with one pass of the material became necessary and since stamps are unfitted for closed-circuit grinding, and the dilution necessary is bad for cyanide work, tube mills were installed for final grinding and the stamps were relegated to the position of intermediate crushers. In this service they come into competition with more efficient machines such as rolls, disk crushers, and ball and rod mills, and are probably on the way to complete disappearance except in isolated special instances of hard ores and low mill capacities, where their flexibility and ability to make a large size reduction outweigh their disadvantages, or where discard from existing plants would involve greater capital charges than are warranted.

16. Steam stamp

Description. A steam stamp (Fig. 50) consists essentially of a die resting in a mortar (*a*) with perforate walls, and a pestle (*b*) connected at the upper end with a piston rod (*c*) actuated up and down by a piston in a steam cylinder (*d*). The mortar rests on a heavy metal anvil block which in turn rests on an enor-

mous block of concrete. The steam end is carried on a frame (*e*) which is entirely separate from the mortar. The stamp stem is kept in alignment by guides (*f*), with phosphor-bronze boxes, carried on transverse supports attached to the legs of frame (*e*). The steam end may be simple, cross-compound or steeple-compound. The CROSS-COMPOUND is an intermediate form in the development from the old simple form which was recklessly inefficient in steam consumption, to the modern efficient steeple-compound type. Valves, are driven by a cam-and-eccentric mechanism from shaft (*g*) which, in turn, is driven by an independent motor. The cylinder of the simple stamp has become standard at 20-in. diameter by 24-in. length. Simple stamps are operated condensing, taking steam at 115 to 120 lb. per sq. in. and making 105 to 110 strokes per minute. Many are of the LEAVITT TYPE (Fig. 51) with two pistons (*a*) and (*b*) of different diameters. Steam is admitted by means of a griddle slide valve through port (*c*) to the upper side of the larger piston and exhausted through a similar valve and port (*d*) to the condenser. The space (*e*) surrounding the lower cylinder and connecting with its interior through ports (*f*) is a receiver into which steam is admitted under constant pressure, to which pressure the under side of piston (*b*) is subject at all times.

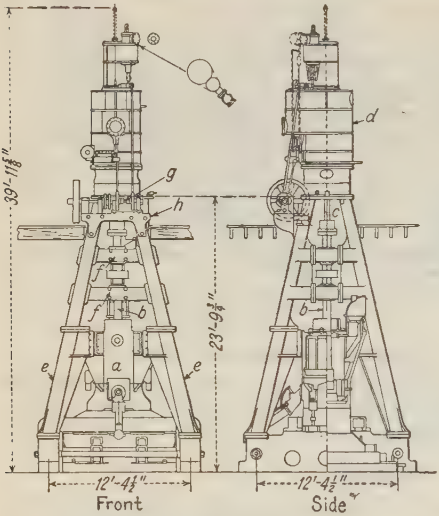


FIG. 50.—Steam stamp.

The lower part of the upper cylinder and upper part of the lower cylinder are at all times connected to the vacuum of the condenser. The constant pressure on the under side of (*b*) is sufficient to lift the stamp, but the pressure on top of (*a*) is greatly in excess of this and makes the force of the blow enormous. The economy of operation of the simple stamp is low because of the large clearance, amounting to as much as 20 per cent., made necessary by reason of the variable thickness of ore on the dies and the wear of shoes and dies. In the STEEPLE-COMPOUND, the high-pressure cylinder is 15½ × 24-in. and the low-pressure cylinder, mounted above in order to facilitate removal of pistons, is 32 × 24-in. A re-heater is installed on the steam line between the high- and low-pressure cylinders. The high-pressure cylinder has four valves, two inlet and two exhaust, each driven by a separate eccentric, the lower or lifting valves from a constant-speed drive shaft, the upper or driving valves from a separate shaft driven from the first at a non-uniform rate through a drag-link mechanism, which permits fast opening and closing. The low-pressure cylinder has two valves only, connecting with the upper side of the

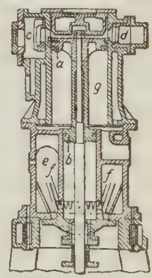


FIG. 51.—Cylinder of Leavitt steam stamp.

piston; the under side of the piston is connected constantly with the condenser, which eliminates considerable clearance. The usual steam pressures are 140 to 160 lb. per sq. in. in the high-pressure cylinder and 35 to 40 in the low.

Recently exhaust steam from the stamps has been used, mixed with boiler steam, if necessary, to run steam turbines. This is a considerable advance in steam economy.

A typical STEM has an oblong detachable shoe and rubber-packed dash-pot connection with the piston rod to prevent breakage of the rod; a swelling or "BONNET" at the top of the stem serves a secondary purpose by striking against a dash-pot buffer in the entablature (*h*) (Fig. 50), when the stamp is lifted too high, and thus preventing the piston from knocking out the upper cylinder head. Similarly it strikes a warning bell when the layer of feed on the die becomes too thin. SHOE is made of chilled cast iron, weighs 750 to 800 lb. new and 300 to 400 lb. when discarded. The life is 4 days to 2 weeks. The complete stamp stem, shoe, piston and piston rod, weighs 5500 to 7900 lb. The lifting velocity is 8 to 10 ft. per sec. and falling velocity 20 to 24 ft. per sec. The striking force of the shoe is of the order of 25 to 50 tons. The MORTAR may be circular or rectangular. The bottom is protected by a false die on which the crushing die rests, centered by a die ring which is wedged in place by sectional side liners; a key or king stave bolted through the mortar locks the whole lining in place. LINERS and DIE are made of chilled cast iron. The die weighs about 800 lb. new and about 500 lb. when discarded. The life is 6 months to 1 year. SCREENS surround 50 to 75 per cent. of the periphery of the mortar above the liners. The usual height of the bottom of the screen above the top of a new die is 9 to 10 in.; with a worn die the height increases to 13 to 16 in. Screens are punched plate with 5⁄16-in. to 5⁄8-in. round openings. One panel is usually covered with screen having 50 per cent. larger apertures than the balance in order to pass copper that is free but cannot escape through either the finer screen or the HYDRAULIC DISCHARGE. The latter, in its simplest form is an inclined pipe-like opening through the mortar wall, up which water is introduced into the mortar at such velocity that only the largest particles of metallic copper can settle against the current. Another form has a small plunger jig, arranged to feed from a slot in the side liner just below the mortar grate. Either type will remove copper between 5⁄8-in. and 4-in. size. Larger lumps must be manually removed; smaller pass the screens. The MORTAR JIG has a 1-in. screen and discharges fine copper from the hutch, coarse copper from the screen and middling over the tail board. The mortar rests on a solid cast-iron ANVIL BLOCK which in turn may rest either on hardwood spring timbers or directly on a concrete foundation slab. The latter is more recent practice; it increases capacity and lessens repair costs.

Operation. Steam stamps are fed by hand through sloping chutes from feed bins containing −4-in. jaw-crusher product. The operator controls the flow by means of a hook or hoe so as to keep the thinnest safe layer of material on the dies. He picks mass copper, wood and mine waste from the feed stream. Water is introduced with the feed and also through the hydraulic discharges. The total amount introduced is from 3 to 7 tons per ton of feed.

Performance. Table 28 gives typical data on simple and compound stamps working on conglomerate and amygdaloid respectively.

The capacity of a simple stamp crushing amygdaloid through 5⁄8-in. screen at QUINCY is 450 to 500 tons per 24 hr. and the capacity of a steeple-compound in the same service at the same plant is 700 to 800 tons. When crushing amygdaloid through 5⁄8-in. screen

about 70 per cent. of the stamp product is less than half the screen aperture in size and 40 per cent. is smaller than 0.1-in. Cost of crushing in steam stamps ranged from $0.15 to $0.30 per ton (1907).

Table 28. Performance of steam stamps

Mill	Calumet and Hecla	Baltic
Type of stamp	Leavitt, simple	Nordberg, compound
Weight of reciprocating part, lb.	5500	7860
Steam pressure, lb. per sq. in.	115	....
Tons crushed per 24 hr.	350	600
Kind of rock	Conglomerate	Amygdaloid
Size of feed	−3-in.	−3½-in.
Aperture in battery screens, round	3/16-in.	⅝-in.
Horsepower, indicated	210	265
Tons per horsepower-hour	0.07	0.09
Attendance, men per stamp	1.5	1.5
Lost time, per cent	2	4
Lost time, cause	*a*	*b*
Water added, tons per ton of ore	3	3.8
Die, material	***CCI***	***CCI***
Die, weight, new, lb.	900	798
Die, weight, discarded, lb.	300	415
Die, life, days	270	222
Die, time to replace, hr.	5	10
Shoe, material	***CWI***	***CCI***
Shoe, dimensions, in.	16×22×6.5	16×22×8
Shoe, weight, new, lb.	750	845
Shoe, weight, discarded, lb.	330	490
Shoe, life, days	4	13
Shoe, time to replace, hr.	0.75	1
Liners, material	***CCI***	***CCI***
Liners, life, days	270–360	344
Screens, material	*c*	*d*
Screens, area, sq. ft.	47.0	45.1
Screens, aperture, in. diam.	3/16	⅝
Screens, life, days	70	218

a Changing shoes and other wearing parts. *b* Hopper choked, changing shoes, inspection. *c* Punched No. 10 hardened-steel plate. *d* Punched open-hearth high-carbon tempered steel plate, 3/16 in. thick. ***CCI*** Chilled cast iron. ***CWI*** Chilled white iron.

Comparison of simple and steeple-compound stamps was made at Osceola mill (*84 J 349* [*1907*]). Results of a 24-day test are shown in Table 29.

Table 29. Comparison of simple and compound steam stamps

	Compound	Simple
Tons per 24 hr. actual	686.6	543.3
Tons rock per ton of coal	88.3	62.8
Lost time, per cent	2.8	1.8
Steam pressure, lb. per sq. in.	148	118

17. Gravity stamp

Description. A gravity-stamp battery is shown in Fig. 52. The essential parts are the frame (A), mortar block (B), mortar (C) containing die (D) on which ore is broken by a pestle composed of shoe (E), boss head (F), stem (G) and tappet (H). The pestle is lifted through the tappet by means of cams

(I) carried on cam shaft (J) driven by pulley (K) and belt (L) from pulley (M) on countershaft (N). Ore to be crushed passes from the bin (O) through the automatic feeder (P) to the mortar and is discharged, when fine enough, through screen (Q). Stamps are rated on the weight of the falling part. The usual weights in American mills are from 1250 to 1500 lb. In South Africa 1500- to 2000-lb. falling weights are more usual. Old California practice was to use 850- to 1050-lb. stamps and many of these are still found.

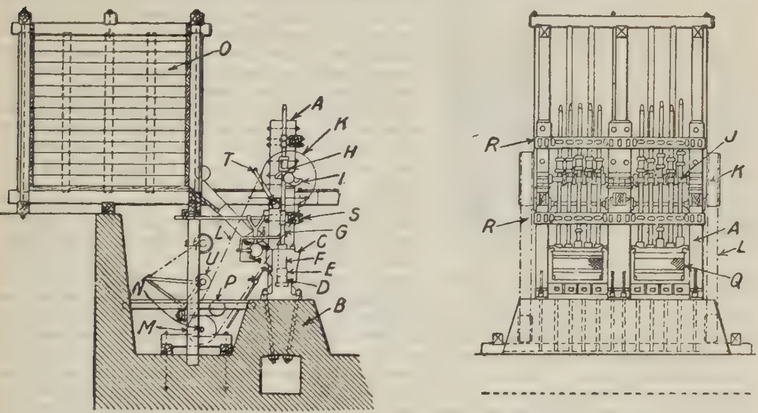


Fig. 52.—Typical 10-stamp battery.

The weight of a complete 5-stamp battery, excluding wood of frame, ranges from 20 times the weight of an individual stamp for batteries under 1000-lb. falling weight to about 15 times for heavy stamps. Of this total the weight of the falling parts and mortar is about 55 to 65 per cent. for stamps under 1000-lb. increasing to 75 to 80 per cent. for the heaviest stamps.

Frames are commonly made of timber, but cast iron, structural iron and reinforced concrete have also been used. Timber frames are made in three forms. The back-knee frame (Fig. 52) is tied to the ore bin directly behind by means of sills and girts as shown. The main posts (A) carry guide timbers (S) which in turn carry guides for directing and confining the up-and-down motion of the stamp. The main sills are tied together by cross sills and rest on mud sills. Finger bars (T) resting on a jack shaft are arranged to slide under the tappets and hold them stationary above the reach of the cams. The front-knee frame differs from the back-knee frame in that a horizontal member for supporting the drive shaft is framed in front of the main timbers at about the level of the cam shaft and is supported on the outer end by posts; the frame is further stiffened by means of diagonal members running from these posts to the top of the main posts. The front-knee frame allows horizontal drive and does away with the use of the tightener (U, Fig. 52) that is necessary when the drive countershaft is placed on the sills, but unless individual drive is used a clutch pulley is required on the driving shaft to permit an individual battery to be thrown out. The A-frame is stiffened entirely by an inclined post in front. This frame is the least rigid of the three and the least used. The drive countershaft is placed on the main sills either in front of or behind the posts and a belt tightener must be used. The front-knee frame gives the most floor space and easy access to the cams but is expensive and obstructs light.

Main posts are commonly 12 × 24-in. or 12 × 26-in.; sills and girts 10 × 12-in. or 12 × 12-in., and guide timbers 12 × 12-in. and 12 × 14-in.

When 10 stamps are carried on the same cam shaft, as is usual, common practice is to carry one center cam-shaft bearing on a single central post and to drive from one end. Cam shafts will vibrate less and last longer if a 4-post frame is used with the bull wheel placed between the two central posts. At Suan, Korea (*119 P 916*), shafts broke frequently, even with this arrangement. The difficulty was overcome by cutting the shafts at the center, placing the two driving pulleys side by side and driving both with the same belt. Pulley faces were turned to bring a common crown at the center.

Drive is usually from a countershaft, belt-driven from one large motor, but the latest practice is to use a back-geared motor belted directly to each 5- or 10-stamp battery. The greater initial cost of the unit installation is considered to be more than overbalanced by its greater flexibility and convenience and by elimination of the difficulty of maintaining alignment of the long countershaft. At AURORA, Nev. (*99 J 97*), the motor is sufficiently powerful to start the stamps from any position so that it is not necessary to hang up.

Mortar block should be as rigid as possible in order to obtain the maximum crushing effect. The old practice was to use wood; modern blocks are almost invariably concrete. They are more rigid, more easily built and have much longer life; stamps set on them have greater capacity than those set on wood. They should be footed on rock, if possible, but if rock cannot be reached at less than 16 to 18 ft., they may be footed safely on gravel or firm soil, if given a broad base. In any case the block should have a generous batter, at least 30° from the vertical; the ends are extended at the proper height for the main sills and where more than ten stamps are installed it is the usual practice to make the blocks continuous below the sills. Details of typical concrete mortar blocks are shown in Fig. 53. Bolts should never be cast into a concrete block on account of the difficulty of replacing them when broken. The form with bolts coming to the outside gives trouble with heavy stamps because of the difficulty of casting the projections (*97 J 664*). A good mixture for a concrete block is 4 of 2½-in. rock, 2 of sand and 1 of cement. The top 6 or 8 in. should be made somewhat richer than this to prevent cracking.

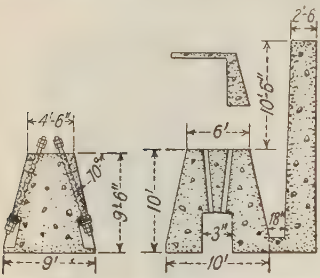


FIG. 53.—Concrete mortar block.

Anvil block is a heavy cast-iron sub-base sometimes used between the mortar and a wooden mortar block. With a wooden foundation it increases stamp capacity and lessens vibration but RAND experience is summed up by Bosqui (*52 A 24*) in the statement that its use caused excessive stem and cam-shaft breakage. Caldecott (*19 IMM 57* [*1909-10*]) gives a stamp duty of 6.84 tons with a heavy cast-iron anvil on a wooden block, against 6.78 tons with the mortar directly on concrete, with 26.75 and 25 per cent. of +0.01-in. material in the products, respectively.

Mortar is usually set on the mortar block on a pad of rubber or hair felt ¼ in. to ⅜ in. thick, and bolted down by 6 or 8 bolts of 1½- to 2¼-in. diameter. Since the use of concrete mortar blocks has become general many designers omit the pad. Mortars commonly hold five dies. A typical mortar is shown in Fig. 54. Variation in detail of mortars depends upon the purpose for which they are to be used and upon the kind of mortar block. The weight of the mortar varies with the weight of the stamps, ranging from 6 times the falling weight with light stamps to 9 or 10 times with heavy stamps. The narrow, straight-backed mortar shown in Fig. 54 is designed for rapid discharge and high capacity. It is cast roughly as a deep box with feed opening (*A*) at the back and discharge opening at the front. The bottom is planed to give an even bearing. The base is broad on mortars meant for concrete mortar blocks and narrow for wooden blocks. The screen (*C*) is held in the discharge opening by means of wedges (*D*) which press the screen frame tightly against planed surfaces on the mortar. Chuck blocks (*E*), held in place by wedges, are inserted for the purpose of varying the height of the screen. Where amalgamation is practiced it is usual to mount a copper amalgamating plate (*F*) on the chuck block in order to catch some gold in the battery. A wooden cover (*G*) to prevent splash rests on a shelf cast on the walls of the mortar. It is provided with holes for the stamp stems. Liner plates (*H*) in the feed chute, sides and ends protect the casting from wear and are easily replaceable. Amalgamating mortars are wider and are provided with a rear amalgamating plate as well as a chuck-block plate; provision is made by means of a removable cover for access to the back plate without lifting the stamps. Crushing is not so rapid as in the narrow mortar; such a mortar is used when amalgamation in the battery is more important than rapid crushing. DOUBLE-DISCHARGE MORTARS arranged with screens front and rear, with provision for conducting pulp discharged at the rear through the mortar base to join pulp discharged at the front, were devised to increase the crushing rate. Caldecott (*19 IMM 57*) showed substantially identical duty for 1350-lb. stamps working in single- and double-discharge mortars on RAND

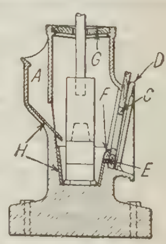


FIG. 54. — Narrow straight-backed mortar (Homestake type) with broad base for concrete mortar block.

ores, 5.8 tons through 0.024-in. screen in both cases. The double-discharge mortar had 3¾-in. discharge height against 2¾-in. in the single-discharge mortar, it used 10.7 tons of water per ton to 7.7 in the single-discharge and with a feed containing 3 per cent. more − ¾-in. material, the product contained 3 per cent. less +0.01-in. sand. A so-called OPEN-FRONT MORTAR is made, with removable front, making for ease in removing shoes, boss and stems.

The mortar is made of tough, gray cast iron. The feed opening usually extends the length of the three center stamps but in some mortars is made full length. Feed is supplied in a narrow stream at the center and is distributed by the swash of pulp. The choice of a mortar is influenced not only by capacity requirements but also by considerations of recovery of precious metals. The design of a mortar has a considerable influence on operation and there is little adjustment possible. It is therefore important that the proper choice be made in the first instance. With ores containing minute particles of rusty gold which require burnishing before amalgamation can be accomplished, fine grinding is necessary and a wide mortar with back and front plates and a fine screen should be used, if the battery is dependend upon for all of the crushing. With a free-milling ore containing coarse gold that is easily separated from the gangue, a narrow, single-discharge mortar is properly used. This mortar should also be used where the purpose is high capacity, and later machines are depended upon for completion of the grinding. The limit of narrowness is imposed by the requirement of a space between the dies and front and back walls greater than that of the largest particles of feed in order to prevent wedging therein with consequent strain on the stem. For maximum capacity with minimum stem breakage the back should be about 2½ in. from the back of the dies and the screen about 5½ in. from the front of the dies. The slope of the back is set at about 75° from horizontal and the screen at 75° to 80°.

King (*11 MM 378*) reports that at a plant in WESTERN AUSTRALIA when cyaniding replaced amalgamation, the old wide mortars were narrowed by steel-plate liners backed with hardwood, with resulting duties for 1050-lb. stamps on a quartz ore containing 50 to 60 per cent. granite of 4.8 tons through 30-mesh, 6.3 tons through 20-mesh and 8.9 tons through 10-mesh. These figures represented increases of 50 per cent. over those obtained with the wide amalgamating mortars.

Mortar liners were not used in the majority of mills when fine crushing was practiced, but the mortar was made sufficiently heavy to last 3 to 6 years. The average life of 5500-lb. mortars at HOMESTAKE (4.5-ton stamp duty) was 3 years. Recent practice in high-duty coarse-crushing batteries is to use both end and side liners. At NIPISSING (*48 A 13*) chilled cast-iron liners 1 in. thick lasted 30 days; manganese steel lasted 165 days. At SUAN (*119 P 916*) back liners are made in upper and lower sections, since the upper part wears much more quickly than the lower. At BELMONT-SHAWMUT (*121 P 659*) cast-iron front liners 1 in. thick last 4 months, back liners ¼ in. thick and end liners 1 in. thick last 6 months.

Some mortars are provided with lugs for holding a SPLASH BOARD in front of the screen. This is particularly necessary where the pulp is thin and a coarse screen is used. An amalgamating plate is sometimes attached to the splash board.

Sectional mortars are made for shipment into inaccessible regions. The mortar is divided into sections transversely and each joint is faced and grooved in order to make a water-tight and mercury-tight joint. The whole is tied together by means of tie bolts that fit into reamed holes. The top is made of steel plate riveted together or of hardwood back and front bolted against cast-iron ends. Riveted joints tend to loosen. Hardwood can be tightened up from time to time by taking up on the bolts. The sections are made so that no piece much exceeds 300 lb. weight. Stems are made somewhat lighter than standard to go with sectional mortars and a hollow cam shaft is likewise furnished.

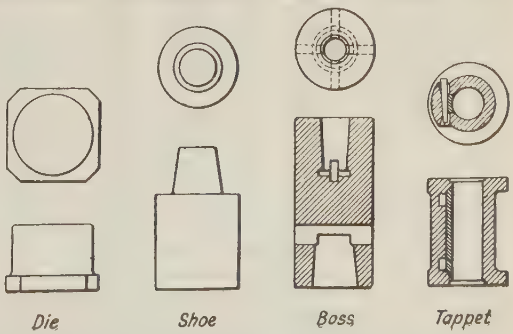


FIG. 55.—Stamp parts.

Dies (Fig. 55) are inserted in the bottom of the mortar. They are subject to hard wear and are therefore made of special wear-resisting material. Chilled cast iron, semi-steel, forged steel, chrome steel and manganese steel are the usual metals employed. Normally the shoe and die are made of the same material but some

operators have reported good results with cast-iron or semi-steel dies working against forged- or chrome-steel shoes. The die shown is the ordinary form with square base and clipped corner to permit the insertion of a bar to pry the die out of the mortar. Dimensions and weights of dies for different weights of stamps are given in Table 30.

Table 30. Sizes and weights of stamp-battery parts (*a*)

Weight, pounds	Stem		Tappet		Head		Shoe	
	Size	Weight	Size, in.	Weight	Size, in.	Weight	Size, in.	Weight
	in. ft.							
250	....	....	....	....	....	....	....	......
550	2⅝ × 11	200	8¾ × 11½	100	8 × 16	170	8 × 6	108
750	3 × 13	310	9¼ × 10	130	8½ × 13	160	8½ × 8	150
850	3⅛ × 14	365	9¼ × 12½	145	8½ × 15	190	8½ × 8	150
950	3¼ × 14	395	9¼ × 12½	145	9 × 18	280	9 × 8	167
1000	3¼ × 14	395	9¼ × 12½	145	9 × 18	280	9 × 8	167
1050	3½ × 14	458	9¼ × 12½	145	9 × 18	280	9 × 8	167
1250	3¾ × 14	525	9½ × 14	170	9 × 20	367	9 × 9	188
1400	3⅞ × 16	640	9½ × 16	205	9 × 20	367	9 × 9	188
1500	4 × 16	670	10 × 14	170	9¼ × 24	415	9¼ × 12	245
1550	4⅛ × 16	720	10 × 14	170	9¼ × 24	415	9¼ × 12	245
1600	4¼ × 16	770	10 × 14	170	9¼ × 24	415	9¼ × 12	245
1650	4⅜ × 16	820	10 × 14	170	9¼ × 24	415	9¼ × 12	245
1700	4⅜ × 16	820	10 × 14	170	9¼ × 28	465	9¼ × 12	245
1800	4⅜ × 16	820	10 × 16	233	9¼ × 30	502	9¼ × 12	245
2000	4½ × 16	870	10 × 20	275	9½ × 30	565	9½ × 12	290
2250	4¾ × 16	970	10 × 20	325	9½ × 34	665	9½ × 12	290

Weight, pounds	Die		Cam shaft				Pulley, in.	
			5-Stamp		10-Stamp			
	Size, in.	Weight	Size	Weight	Size	Weight	5-Stamp	10-Stamp
			in. ft. in.		in. ft. in.			
250	....	......	....	....	....	....	......	......
550	8½ × 6	100	3⅞ × 7–10	314	3⅞ × 13–8	550	60 × 11	72 × 13
750	8¾ × 6	104	4⅞ × 8– 0	510	4⅞ × 14–0	890	60 × 11	72 × 13
850	8¾ × 7½	109	5⅜ × 8– 0	620	5⅜ × 14–0	1080	60 × 11	72 × 15
950	9¼ × 8	157	5⅞ × 8– 0	740	5⅝ × 14–6	1340	60 × 11	72 × 17
1000	9¼ × 8	157	6 × 8– 3	800	6 × 14–6	1400	60 × 13	72 × 17
1050	9¼ × 8	157	6 × 8– 3	800	6 × 14–6	1400	60 × 13	72 × 17
1250	9¼ × 8	157	6½ × 8– 6	960	6½ × 15–0	1700	60 × 15	72 × 19
1400	9¼ × 8	157	6¾ × 8– 6	1040	6¾ × 15–0	1820	60 × 15	72 × 19
1500	9½ × 8	162	7 × 8– 9	1140	7 × 16–0	2100	72 × 13	84 × 21
1550	9½ × 8	162	7 × 8– 9	1140	7 × 16–0	2100	72 × 13	84 × 21
1600	9½ × 8	162	7¼ × 8– 9	1230	7¼ × 16–0	2250	72 × 15	84 × 23
1650	9½ × 8	162	7¼ × 8– 9	1230	7¼ × 16–0	2250	72 × 15	84 × 23
1700	9½ × 8	162	7½ × 8– 9	1320	7½ × 16–0	2400	72 × 15	84 × 23
1800	9½ × 8	162	7½ × 9– 0	1350	7½ × 16–6	2460	84 × 13	84 × 25
2000	9¾ × 8	170	7¾ × 9– 0	1530	7¾ × 17–0	2750	84 × 15	90 × 25
2250	9¾ × 8	170	8 × 9– 0	1620	8 × 17–0	2900	84 × 17	90 × 29

a Taken complete from Power & Mining Machinery Co., Bull. 41. Dimensions and weights of standard equipment of other makers differ slightly but not essentially. Compare Table 31, from practice.

It is good practice to replace dies before they are worn to full depth, for the reason that the surface becomes very irregular with wear and that crushing efficiency is thereby lost. By making dies ¼ in. larger than the shoes, contact of the shoe with the die over the whole face of the shoe is insured even after considerable stem and die wear. This

makes for more even wear of shoes and dies and higher crushing duty. In order to save in amount of metal thus wasted, a sectional die with replaceable face is sometimes used. At SUAN, Korea (*119 P 916*), half-worn shoes are used for the upper part of sectional dies; this practice has an incidental advantage that there is less variation in height of discharge between new and worn-out dies than is the case with solid dies. The average life for solid dies on ordinary ores is about 60 days (Table 31). Three-year average for hard cast-iron

Table 31. Wear of parts, gravity stamps

	Liberty Bell	Hedley G. M. Co.	Melones
Weight of stamp, lb.	900	1050	1050
Dies:			
Material	*CCI*	*FS*	*WI*
Weight, new, lb.	115	120	96
Weight, discarded, lb.	35	....	35
Life, days	60	60	90
Consumption, lb. per ton crushed	0.27	....	0.12
Shoes:			
Material	*Cr*	*FS*	*S*
Weight, new, lb.	155	240	235
Weight, discarded, lb.	27	....	30
Life, days	90	60	210
Consumption, lb. per ton crushed	0.28	....	0.20
Screens:			
Type	*WWS*	*WWT*	*WWS*
Material	*S*	*S*	*S*
Aperture	0.04 to 0.06 in.	⅛×½-in.	20-mesh
Life, days	40	14	3
Life, tons crushed	1000	385	83

	Tonopah Belmont	Tonopah Extension	Mexican gold mill
Weight of stamp, lb.	1250	1300	1500
Dies:			
Material	*FS*	*WI*	*FS*
Weight, new, lb.	159	150	175
Weight, discarded, lb.	55	40	50
Life, days	66	60	65
Consumption, lb. per ton crushed	0.27	0.24	0.12
Shoes:			
Material	*Cr*	*Cr*	*FS*
Weight, new, lb.	183	172	260
Weight, discarded, lb.	35	25	32
Life, days	72	90	65
Consumption, lb. per ton crushed	0.29	0.23	0.22
Screens:			
Type	*WWT*	*WWS*	*WWS*
Material	*I*	*S*	*S*
Aperture	3/16-in.	3-mesh	0.5-in.
Life, days	Var.	42	30
Life, tons crushed	....	1575	2400

CCI Chilled cast iron. *Cr* Chrome steel. *S* Steel. *FS* Forged steel. *WI* White iron. *I* Iron. *WWS* Woven wire, square aperture. *WWT* Woven wire, elongated opening.

dies at HOMESTAKE (900-lb. stamps) was 30 to 35 days (*22 IMM 74*). At NIPISSING (*48 A 13*) forged chrome-steel dies lasted 130 days with very hard tough ore; at SUAN (*119 P 916*) the same material lasts 100 to 120 days. At CHURCHILL WONDER (*52 A 128*) the consumption of forged chrome-steel dies on hard tough quartz was 0.107 lb. per ton crushed through ⅜-in. screen. At RAINBOW chrome-steel dies last 100 to 120 days with 1050-lb. stamps crushing through 4-mesh. Consumption of cast-iron dies at ALASKA

Treadwell (*102 J 63*) was 0.17 to 0.24 lb. per ton crushed. At Zaruma, Ecuador (*111 J 583*), the consumption of chrome-steel dies was 0.12 lb. per ton.

Dies are ordinarily set in the mortar without fastening, but at one plant dies were grouted into place with a mixture of 2 cement and 3 sand up to the level of the chuck block; notwithstanding that several days were required for the cement to set, increased duty, due to the time saved in battery clean-up and increased crushing rate counterbalanced the lost time; further advantages claimed are better inside amalgamation due to the fact that amalgam and quicksilver are kept up near the inside plates; smaller amount of battery sand to clean up; no necessity to remove dies at clean-up time; rigidity of dies. The concrete wears with the dies and is mostly gone when the latter are worn out, so that they are not hard to remove.

Shoe (Fig. 55) is the lowest part of the stamp and is subjected to great wear. Shoes are therefore made replaceable and of wear-resisting material such as cast iron, chilled iron, cast steel, forged steel, chromium or manganese steel. Common practice is to use forged or alloy steel. The question of economy depends upon the first cost and the extent to which the shoes can be worn down. With a local foundry and high freight cost, the choice between cast shoes and shoes of special steel may be a close one. At Waihi, N. Z. (*16 Aa 124*), local cast-steel dies showed 50 per cent. economy over imported forged steel but forged-steel shoes were used. The liability of cast shoes to breakage, and the cost of replacement must be borne in mind in making any decision in favor of the cast product. Dixon (*18 JCM 240*) cites a mill record showing 10.9 per cent. shank breakage with cast shoes against 3.5 per cent. with forged. Castings containing more than 1 per cent. carbon are most likely to break. At Passagem (*20 IMM 16*) the life of locally cast shoes and dies was about half that of imported forged steel.

The shank of the shoe fits loosely into a cored recess in the bottom of the boss head and is wedged in by means of hardwood wedges which swell when wet and make a firm joint. In dry crushing steel wedges are used. Weights and dimensions of shoes for various weights of stamps are given in Table 30.

Wear of shoes varies considerably with the character of ore and kind of material. An average life of from 60 to 90 days for forged- or chrome-steel shoes working on medium ores is a fair figure. The three-year average for chilled cast-iron shoes on 900-lb. stamps at Homestake was 60 to 90 days (*22 IMM 74*). Forged chrome-steel shoes at Nipissing lasted 105 days (*48 A 13*) and 100 to 120 days at Suan (*119 P 916*). Consumption of the same material at Churchill-Wonder (*52 A 128*) crushing hard tough quartz through ⅜-in. screen was 0.232 lb. per ton. At Rainbow (*99 J 1104*) the life of chrome-steel shoes was 80 to 90 days with 1050-lb. stamps and 4-mesh screen. Consumption of chrome-steel shoes at Alaska Treadwell (*102 J 46*) was 0.37 to 0.41 lb. per ton of ore crushed. At Zaruma (*111 J 583*) the consumption of chrome-steel shoes was 0.29 lb. per ton. Table 31 gives detailed data on wear at various mills. According to Wraight (*30 IMM 201*) the figures in Table 32 are average for wear of shoes and dies of different materials. He recommends specifications as given in Table 33. Del Mar (*40 MEW 687*) records the failure of

Table 32. Average wear of shoes and dies. (*After Wraight*)

Material	Wear in pounds per ton crushed	
	Shoes	Dies
Cast iron	1.5	0.4–1.5
Cast steel	0.5–0.75	0.3–0.7
Forged steel	0.3	0.21
Chrome steel	0.29	0.16

Table 33. Composition of steels for shoes and dies. (*After Wraight*)

Element	Per cent. by weight		
	Cast or forged steel	Manganese steel	Nickel-chrome steel
C	0.55–0.65	0.9–1.1	0.28–0.32
Si	Up to 0.25	Up to 0.5	Up to 0.3
S	0.06 max.	0.08 max.	0.04 max.
P	0.06 max.	0.10 max.	
Mn	0.35 max.	11–13	0.35–0.45
Cr	@ 0.5	May contain 0.5 max.	0.55–0.65
Ni	....	....	3.25–3.75

manganese-steel shoes by deep chipping. Foote (*122 P 739*) states that at North Star two sets of five shoes all failed by casting a flat disk 1 to 3 in. thick and the diameter of the shoe. Loring (*28 A 553*) gives a life of 296 days for manganese-steel shoes working against cast-iron dies (life 120 days) at Utica mills with 5-ton stamp duty. Alloy-steel shoes and dies with exceptionally slow wear may not be economical, if the wear results in uneven crushing surfaces, because of the resulting loss in crushing duty.

Boss head, sometimes called boss or head, is shown in Fig. 55. It forms a link between stem and shoe and also adds materially to the falling weight of the stamp. It is made of cast iron, cast steel, or chrome steel, cored out at the bottom to receive the shank of the shoe and bored to a taper at the top for receiving the tapered end of the stem. Slots are cored at right angles at the bottom of the tapered openings to receive drift keys for driving out the stem and shoe. Weights and dimensions of boss heads for various weights of stamps are given in Table 30. Cast-iron boss heads are sometimes reinforced with wrought-iron bands, shrunk on, to prevent cracking. This was particularly necessary in dry crushing practice when steel wedges were used for holding on the shoe. The stem is likely, also, to split cast-iron boss heads. Life is indefinitely long and is terminated by breakage. Figures reported by mills are 330 days; 400 to 600 days; 4 years; five boss heads for 40 stamps in 14 years. Homestake average for cast-iron bosses with 900-lb. stamps is 6 years (*22 IMM 74*). At Suan (*119 P 916*) chrome-steel bosses on 1050-lb. stamps last 2 to 4 years.

Stems are made of hammered iron or mild steel, turned and polished and tapered both ends so as to make them reversible in case of breakage. Wraight (*30 IMM 201*) recommends a steel analyzing: C, 0.15 to 0.25 per cent.; Si, up to 0.15 per cent.; S + P, 0.06 per cent. max.; Mn, up to 0.6 per cent. Dimensions for stamps of different weights are given in Table 30.

The amount of breakage depends upon the length, the location and condition of guides, and the weight of the tappet. Long stems with heavy tappets and worn guides break in a short time while under reverse conditions breakage is almost unknown. Steel and occasional large lumps in the feed cause eccentric strains that produce much breakage. The break usually occurs near the boss. Broken stems are turned end-for-end. Broken ends may be turned down and the stem again used, if not too short. Annealing before turning down will defer subsequent breakage. In starting, the center stamp should be started first to save pounding on empty dies with consequent stem breakage. Average life at Homestake over a 3-year period for wrought-iron stems 3⅛-in. diameter was 4 months. Mild-steel stems 3$\frac{7}{16}$-in. diameter at Suan, Korea (*119 P 916*) on 1050-lb. stamps last 4 years.

Tappets (Fig. 55) are made of cast iron, cast steel, or alloy steel. The purpose of the tappet is to convert the rotary motion of the cam-shaft into rectilinear motion of the stamp. Tappets are cast with a cored opening which is bored out to fit the stem. A recess in the opening receives a steel gib that is bored out with the tappet. The gib is pressed against the stem by keys driven through the keyways shown. Some makers counterbore the hole through the tappet slightly off the center of the original bore in order to give a three-line bearing of the tappet on the stem when the gib is tightened. Tappets are faced both ends and counterbored to a width slightly less than the face of the cams in order to prevent shouldering and consequent side thrust Both ends of the tappet are alike and it is therefore reversible. Two or three keys are used. Three-key tappets should be used on heavy stamps and are less likely to slip in any case, but are somewhat more trouble to set up. Dimensions and weights of tappets for various weights of stamps are given in Table 30. Life is indefinite. Figures reported are: 320 days; 650 days; about 4 years; 100 tappets in 14 years for 40 stamps, first 40 tappets of chrome steel broke in a short time. Chrome-steel tappets at Suan (*119 P 916*) last 2 to 4 years. At Homestake cast-iron tappets averaged upward of 600 days life (*22 IMM 74*).

Compensating weights are split rings weighing 20 to 40 lb. each, made to bolt around the stem either just above the boss or above the tappet. Without their use the falling weight with a worn shoe and a short stem may be as much as 10 per cent. less than the new weight and there will be an accompanying decrease in stamp duty. They are best placed at the boss; at the tappet they increase vibration and stem breakage.

Cams, mounted on a shaft, working against the tappets, lift and drop the stamps. Cams are made two-armed and are ordinarily designed to lift the stamps at constant speed; the cam surface is, therefore, the involute of a circle, somewhat flattened, however, at the two ends. Another form, designed for constant acceleration of the rising stamp was used at North Star. Cams are made of cast iron with chilled faces or of cast or special steels. The best material should be used in order to get strength without clumsiness and undue weight. At Homestake (*114 J 761*) chrome adamantine steel is used for cams with 10-in. drop and cast iron for 8- to 8.5-in. drop. Cams are designed for a short drop (6 to 8 in.) or for a long drop (14 to 16 in.) according to the particular practice for which they are desired. It is important that the cam be closely designed for the required drop length. The design should be such that the vertical tangent to the inscribing circle passes

through the point of contact of the cam with the tappet, or the radius of the inscribing circle should equal the distance between cam-shaft and stem centers. If the stroke length is greatly shortened without changing cams, the stamps are picked up at a point on the cam face that is moving with excessive velocity, with the result of much noise and undue strain on the cam, cam-shaft and tappet fastenings.

Cams were keyed to the shaft in the older mills but present practice is to use a special form of fastener so designed that the load on the cam tends to tighten it on the shaft, while it may be easily loosened by a blow in the reverse direction. Two forms of such fasteners are shown in Fig. 56. Fastener (*A*), known as the Blanton, consists of a curved wedge (*a*) with two pins (*b*) that fit into holes drilled in the shaft. The cam is bored eccentrically to slip over this wedge and tighten thereon when turned on the shaft in the direction opposite to the direction of rotation. This is the fastener most commonly used. The Davis fastener (Fig. 56, *B*), has a lozenge-shaped piece (*c*) that fits into a correspondingly shaped depression in the shaft. A channel is cut in the cam to allow it to slide over the lozenge. Eccentric channels are also cut on the inner face of the cam bore in such a way that when the cam turns on the shaft in a direction opposite to the direction of rotation thereof, it wedges tightly. These special fasteners require less metal to be cut out of the cam-shaft than must be taken for a keyway and therefore weaken the shaft less. Cracks are likely to start from the holes drilled for Blanton fasteners and from keyway corners. Fasteners that do not require sharp-edged cuts in the shaft are best. Cams are made right-hand and left-hand. A RIGHT-HAND CAM has the hub projecting toward the right when the cam stands before the observer in a position such that rotation would carry the upper ear away from the observer; a LEFT-HAND CAM has the hub projection toward the left when standing in like position. The shape of the cam face depends upon the height to which the cam is lifted and the necessary distance between the center of the stem and the center of the cam-shaft.

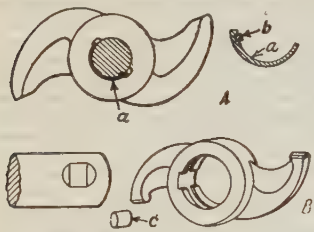


FIG. 56.—Cam fasteners.

Life of cams is indefinite. Figures reported are 500 days; 700 days; 6 months to 3 years; 4 years. Average for cast iron at HOMESTAKE with 900-lb. stamps was upward of 2 years (*22 IMM 74*). Breakage is normally due to CAMMING, *i.e.*, pick-up of the tappet by the cam before the stamp has completed its fall. This is caused by too long stroke or too many strokes per minute.

Height of drop may be varied within small limits, with a given cam, by changing the position of the tappet on the stem. The amount of variation is, however, small, if the tappet is to be picked up at the place on the face of the cam designed for this service. As dies and shoes wear, the height of drop is kept as nearly constant as possible by changing the position of the tappet on the stem to compensate for wear. A skilful operator can make this change while the stamp is in operation by loosening the tappet keys slightly and allowing the upward blow of the cam to slide the tappet on the shaft but this may result in cutting the stem or gib and it is best to hang up the stamp when a change in height of drop is to be made. In general, the height of drop of the stamp in front of the feeder, usually the center stamp, is made about 1 inch greater than that of the others on account of the greater depth of ore at that point. The height of drop of the end stamps is likewise normally made somewhat greater than that of the adjacent stamps in order to overcome the tendency for the ore to pile up in the ends of the mortar.

This arrangement is, however, sometimes varied, thus at SUAN, Korea (*119 P 916*), the center stamp has 9-in. drop; stamps 2 and 4, 8¾-in. and the end stamps, 8½-in.

Drop sequence. Cams are spaced at equal intervals on the cam shaft, 36° or 72° apart, depending upon whether the shaft carries 10 or 5 cams, in order to equalize the load. The sequence in which the stamps in an individual mortar drop has a marked effect on performance. If, numbering the stamps from the pulley end, the drop sequence is 1, 2, 3, 4, 5, ore will be piled up under No. 5

stamp. This will decrease the height of drop and, consequently, the crushing effect and thus will result in more heaping up and eventual stoppage of this stamp. Clogging will then progress along the battery until the latter is completely clogged. Reasoning from this fact, the following rules governing sequence have been set down: (1) No two adjacent stamps should fall in succession. (2) When one stamp is falling its neighbor should be rising. The common sequences, aimed to satisfy this rule, are the HOMESTAKE, **1, 3, 5, 2, 4**, which, stated backward is **1, 4, 2, 5, 3**; CALIFORNIA, **1, 4, 2, 3, 5** = **1, 5, 2, 4, 3**; and modifications of the latter such as **1, 5, 3, 4, 2**; and **1, 5, 3, 2, 4**. The sequence **1, 3, 5, 2, 4** comes nearest to satisfying the theoretical requirements, but many operators claim that the California sequence gives better distribution of pulp on the dies and a swash of pulp in the battery that is better fitted to cause material to pass through the screen.

Clark and Sharwood (*22 IMM 73*) report that at HOMESTAKE the sequence 1, 3, 5, 2, 4 gave slightly greater tonnage than 1, 5, 3, 2, 4. At the GOLD CROSS mill (*88 J 1034* [*1909*]) the drop 1, 4, 2, 3, 5 gave good results while change to the drop 1, 3, 5, 2, 4 caused immediate trouble. The probability is that both drops can be made to work equally well by proper adjustment of the height of drop of individual stamps.

When a 10-stamp battery is used the sequence 1, 3, 5, 2, 4 becomes **1, 7, 3, 9, 5, 2, 8, 4, 10, 6** and 1, 5, 2, 4, 3 becomes **1, 6, 5, 10, 2, 7, 4, 9, 3, 8**.

Cam-shaft is made of a diameter sufficient to withstand the bending stresses imposed by the weight of the cams and the lifting shocks, when supported over a span equal to the distance between the main posts of the frame. It is made of a metal that will withstand the tendency to crystallization brought about by repeated shocks. Mild steel or iron, hammered out and turned to the desired diameter, are the usual metals. Foote (*122 P 739*) says that in operating 1050-lb. stamps at 110 drops per min. 11 shafts were broken by 40 stamps in one year. Forged wrought iron was superior to chrome-nickel steel, vanadium steel, mild or machinery steel, or "Flyer iron." The cam-shaft varies from 5- to 8-in. diameter according to the weight of stamps. At SONS OF GWALIA mine, W. A. (*101 J 224*), a 5½-in. shaft designed for 1000-lb. stamps broke frequently when the falling weight was increased to 1225 lb. Shafts are made to serve batteries of 5 or 10 stamps, never more. The shaft is keyed at one end for a pulley and runs in open boxes. Cam-shafts are designated as right-hand and left-hand, but the terminology is not universal in this respect. A logical method of designation is that in which, when the cams on the shaft are looked at in such a way that the upper ears are moving away from the observer, if the pulley is at the right end of the shaft, the shaft is a RIGHT-HAND SHAFT; if at the left end, a LEFT-HAND SHAFT.

Life of cam-shafts is reported as from 6 months to 5 years. One mill reports 6 months as the average for good shafts and less for poor ones, but this is a poor record. The average for wrought-iron shafts 5.36-in. diameter with 900-lb. stamps at HOMESTAKE was 4 years. Damping shafts by means of wooden blocks pressed down on the shaft over the boxes with a force of 600 to 1000 lb. lengthens the life (*98 J 743*). Broken shafts can be repaired by thermit welding, but must be dismounted for the job (*110 J 1092*) and one or more extra shafts should be kept on hand to reduce delays.

Cam-shaft boxes are made of cast iron or steel bored out to fit the shaft and machined plane on back and bottom. Some makers babbitt the boxes but this is not common practice; broken babbitt may fall into the mortar and contaminate the amalgam. The boxes are half boxes, cast with rims around the base to form an oil reservoir and thus prevent lubricant from dropping into the battery and interfering with amalgamation; canvas covers are frequently used to keep out grit. At some African mills using heavy stamps, bearings were placed on both sides of all cams, the boxes being carried on heavy castings, but alignment was difficult and the castings broke so frequently that the construction was found uneconomical.

Pulley is subjected to heavy service and experience has shown that this service is best withstood by a built-up wooden pulley. A common form consists of a cast-iron flanged hub faced and keyed inside and out, and a movable flange bored to fit the outside of the hub and provided with a key-seat corresponding to that in the outside of the hub. Between these two flanges the pulley is built up of 2-in. plank laid up in linseed oil and white lead, and faced with ⅞-in. plank, all securely bolted together, the flanges bolted through, and the whole turned true and balanced as accurately as possible. Cast-iron pulleys will not stand the strain. Pulleys are finished 7- to 8-ft. diameter by 16- to 20-in. face. The flange is normally 36- to 42-in. diameter for such pulleys. Slipping of the pulley on the shaft gives

considerable trouble in some cases. At HOLLINGER a cast-steel flange was pressed onto the cam shaft with hydraulic jack to overcome trouble from slipping.

Belt tightener must be used when the driving countershaft is placed on the sills. Two forms of tightener are in common use. RACK-AND-PINION TIGHTENER has a tightening pulley carried on a sliding frame that may be racked backward and forward by means of a pinion and hand wheel. SUSPENDED TIGHTENER has the tightening pulley on one end of a lever. The other end of the lever carries a chain that may be wound around a drum fitted with a handwheel. (See Fig. 52.)

Guides are used to hold stems in place and in alignment. They are fastened to guide timbers (S) (Fig. 52). The commonest form is made of maple or other hard wood (Fig. 57). The two halves are bolted together with shims between and bored out to the size of the stems. The shims are planed down as the guides wear, in order to maintain a snug fit against the stems and prevent vibration. Oak guides at HOMESTAKE lasted about 18 months with 850-lb. stamps; hard pine, 4 months. Guide bolts loosen quickly in wooden guides under the vibration to which the battery frame is subjected and there is a considerable amount of friction. Another disadvantage of the ordinary wooden guide is that when a stamp stem is to be removed it is necessary to remove the entire front half of the guide, which necessitates taking out a considerable number of bolts. IRON GUIDES and SECTIONAL WOODEN GUIDES, made to permit removal from one stamp at a time, have been introduced to overcome the various disadvantages of the wooden guides outlined above. With sectional guides it is possible to remove a stem by the removal of two bolts only, and individual adjustment of guides to compensate for the different wear of individual stems is possible. Clark and Sharwood (*22 IMM 73*) report that at HOMESTAKE cast-iron guides reduced friction and renewal cost as compared with maple or oak. Several forms of iron guides have been put on the market; in a typical one solid iron cylinders are bolted individually to a back plate by means of two bolts. The back plate is cast with a bottom rail for insuring alignment of the guides horizontally and bolt holes are carefully drilled to insure vertical alignment. The IDEAL GUIDE has split boxes which are tapered downward and dropped into converging sockets in a cross casting. These may be removed without removing any bolt and are theoretically self-tightening. It is not unknown, however, for them to be lifted by the stem, thereby permitting excessive play and vibration. After iron guides have been polished there is considerably less friction between them and the stems than with wooden guides. The Ideal guide has the sectional advantage likewise. Sectional guides are also made as individual half-boxes for bolting individually onto guide timbers. There is, however, considerable difficulty in getting them into alignment and keeping them there. Another form of iron guide has bushings locked into frames by gib and key. This does away with dependence on bolts and nuts which tend to loosen by reason of the vibration of the frame, but is less easy to handle. Cast-iron guides without babbitt or bushing have the serious disadvantage that they wear the stems badly and that vibration of the worn stems causes excessive stem breakage. Babbitt or bronze bushings obviate this difficulty, but, if battery amalgamation is practiced, the amalgam is frequently badly contaminated by cuttings from the guide liners. The lower guides should be placed as near the boss head and the upper as near the tappets as possible to lessen vibration and stem breakage.

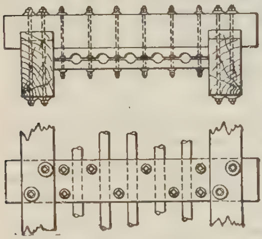


FIG. 57.—Wooden guides.

Finger bars are used to hang up stamps while the cam-shaft is in motion, in order that one or more stamps in the battery may be shut down while others are operating and also in order that the cam-shaft may be started up with no load but friction and the inertia of the shaft to overcome. They are made of hardwood with a leather-lined cast-iron shoe on one end bored out to fit a jack shaft carried between the posts, and at the other end a piece of iron to take the blow of the falling cam. A handle is provided for moving them. A cam stick, consisting of a long acute iron wedge tapering from 1 in. to ½ in. in 18 in. or thereabouts, and flexibly fastened to a handle, is used for getting the tappets on and off the cams. The procedure for lifting a tappet onto the finger bars is to lay the cam stick on top of a rising cam in such a way as to bring it between the surface of the cam and the under surface of the tappet and, as the tappet is raised the additional height, to push the corresponding finger bar forward against the stem so that when the stamp falls the tappet comes to rest on top of the finger bar and out of reach of the revolving cam. To drop the tappet the procedure is similar except that as the tappet is raised by reason of the presence of the cam stick between the cam face and the tappet, the finger bar is removed, allowing the stamp to fall. The handle should be offset in order to keep the fingers of the operators from being caught between the cam and the finger bar.

Screens for the mortar are made up by tacking woven wire or punched plate to heavy wooden frames. Panel pieces are used, if necessary, to prevent excessive bulging of the screen surface. Screens are subjected to considerable wear and the life depends upon the hardness of the ore, acidity of water, material of which the screen is made, type of screen, style of perforation, etc. Discharge through the screen is dependent upon the type of perforation, percentage of opening, and size of opening. The old practice was to use punched tinned plate with circular apertures. This plate is thin, quick-discharging and cheap. If heavy enough to withstand breakage, it is probably the best available, but with narrow, high-discharge mortars failure by breakage is excessive. For heavier service slotted punched plate is used. Present practice, where high capacity with coarse discharge is sought, is to use woven wire, which has a greater percentage of opening (see Sec. 5, Art 3). Brass, copper, bronze and steel are the materials commonly used. Tinned-iron screens are annealed before using by heating them to redness and cooling slowly. Steel wears best but fails quickly with acid water. Screens for fine-crushing stamps are ordinarily renewed before failure on account of the increase in size of aperture through wear, therefore for such crushing heavy plate and heavy wire are not particularly economical, especially in view of the fact that the use of large wire or thick plate reduces capacity by reducing the area of opening. Coarse-wire and heavy-plate screens also are subject to more clogging than lighter screens. Capacity requirements dictate screens of moderate weight even though such practice involves more frequent replacement. Percentage of opening is about the same for medium-weight wire-cloth screens, irrespective of aperture. It is greater for fine cloth than for fine punched plate, but punched plate is stronger. Round holes in plates strain the plate worse than slots and clog more by reason of the fact that they offer three bearing places to grains that attempt to pass. Slotted screens pass a more uneven product than round-hole screens but are preferable for flaky ores. Theoretically, diagonal or horizontal slots should give the freest discharge but swash in the battery modifies this. Punched plate has a burr that should be placed inside. Slotted screens are commonly designated by the needle sizes or the woven-wire meshes to which they correspond.

At Suan, Korea (*119 P 916*), screen with 0.04-in. aperture is used on the lower half of the screen frame and 0.035-in. aperture above.

Life of screens is extremely variable. It ranges from 2 or 3 days to perhaps 2 weeks for fine screens and from 2 weeks to 2 months for coarse screens.

At Homestake (*22 IMM 74*) cold-rolled re-annealed open-hearth steel with diagonal needle slots 0.035 in. wide averaged 10 to 16 days' life and failed by breaking due to wood in the mortar. High-carbon cast crucible-steel wire cloth at Nipissing, 0.25- and 0.4-in. aperture, showed practically no wear after 3 months' service (*48 A 13*).

Prospecting mills. Most manufacturers make one-stamp, two-stamp, three-stamp and five-stamp prospecting mills with stamps ranging in weight from 250 to 450 lb., sectionalized so that no piece exceeds 300 to 350 lb. weight. Such mills require from 2-hp. to 6-hp. engines with boiler rated at 25 to 33 per cent. in excess of the engine. The boiler will weigh so much more than any part of the stamp that much of the advantage of sectionalizing is lost.

Performance of gravity stamps at a number of mills is given in Table 34.

18. Operation of gravity stamps

Duty is the tons crushed per stamp per 24 hr. It varies, according to Table 34, from 1.8 for a 750-lb. stamp crushing through 0.022-in. screen to 21.1 for a 1550-lb. stamp crushing through 0.2- to 0.28-in. screen. Duty

Table 34. Performance

Mill	Passagem	Baker Mines
Nominal falling weight, lb.	750	850
Drops per minute	96	102
Height of drop, in.	8	7
Size of feed	....	−2-in.
Screen aperture	0.022-in.	0.025-in.
Height of discharge, in.	3.5	2
Moisture in discharge, per cent.	92	....
Drop sequence	....	1, 3, 5, 2, 4
Size of product (*d*)	*b*	....
Duty, tons per stamp per 24 hr.	2.9	3.4
Horsepower per stamp	1.8*f*	2
Tons per horsepower-hour	0.067	0.071
Lost time, per cent.	....	....
Reference	*20 IMM 16*	*13 CME 947*

Mill	Hedley G. M. Co.	Melones
Nominal falling weight, lb.	1050	1050
Drops per minute	108	107
Height of drop, in.	7½	7
Size of feed	....	−1.75-in.
Screen aperture	0.125×0.5-in.	0.03-in.
Height of discharge, in.	....	....
Moisture in discharge, per cent.	89	86
Drop sequence	1, 5, 2, 4, 3	1, 5, 3, 4, 2
Size of product (*d*)	....	....
Duty, tons per stamp per 24 hr.	5.5	5–6
Horsepower per stamp	3	3.4
Tons per horsepower-hour	0.077	0.062–0.074
Lost time, per cent.	7.4	5
Reference	*Q*	*Q*

Mill	Belmont-Shawmut	Bullfinch Proprietary
Nominal falling weight, lb.	50 @ 1050 20 @ 1250	1200
Drops per minute	112	106
Height of drop, in.	6	7
Size of feed	−2-in.	....
Screen aperture	0.065-in.	0.08 and 0.2-in.
Height of discharge, in.	2–3	....
Moisture in discharge, per cent.	82	....
Drop sequence	....	....
Size of product (*d*)	....	*6*
Duty, tons per stamp per 24 hr.	7	11.1
Horsepower per stamp	2.3*f*	3.3
Tons per horsepower-hour	0.127	0.140
Lost time, per cent.	....	....
Reference	*121 P 659*	*13 CME 331*

Mill	Tonopah Extension	Churchill Wonder
Nominal falling weight, lb.	1300	1400
Drops per minute	99–103	96
Height of drop, in.	7	6½
Size of feed	−3-in.	*7*
Screen aperture	0.26-in.	0.375
Height of discharge, in.	....	....
Moisture in discharge, per cent.	81	....
Drop sequence	1, 3, 5, 2, 4	....
Size of product (*d*)	....	*8*
Duty, tons per stamp per 24 hr.	7–8	13.3
Horsepower per stamp	2.7	2.8*f*
Tons per horsepower-hour	0.107–0.122	0.198
Lost time, per cent.	....	....
Reference	*Q*	*52 A 128*

a With 19-mesh Rek-tang (0.03-in. aperture) = 8.5 to 9.5. *b* 77 per cent. sand. *c* 6.5 to 7 tons on hard ore and 9 tons on soft. *d* Italic number refers to corresponding column in Table 34*a*. *e* Less than theoretical. *f* Estimated from equation HP = $\frac{1.25WNH}{33,000}$

of gravity stamps

Homestake	Homestake	Homestake	Waihi	Liberty Bell
900	900	900	900–1250	922
89	89	89	102–105	98–110
10	10	10	7–8	7
....	....	....	....	− 3 to 4-in.
0.022×0.5-in.	0.022×0.5-in.	0.28-in.	0.07-in.	0.038-0.06-in.
8	10	1	2–5	....
91	92.5	....	88–89	85
....	....	....	....	1, 5, 2, 4, 3
2	*3*	*4*	*5*	....
4.5	4.2	8	5.6 av.	4–6
2*e*	2.5 *f*	2.5 *f*	....	2
0.094	0.070	0.133	....	0.08–0.12
....	....	....	....	1
109 J 833	*22 IMM 73*	*22 IMM 73*	*16 Aa 124*	*Q*

Suan	Oriental Consolidated	Nugget	Rainbow	Pittsburg-Silver Peak
1050	1050	1050	1050	1050
104	102	100	106	105*i*
8½–9	7½	6½	6¾	6½–7
....	− 1.75-in.	− 1.5-in.	....	....
0.04-in.	0.038-in.	0.25-in.	4-mesh No. 16 wire	0.0164-in.
2½	4	....	2–3	....
82–85	....	75	....	85
1, 4, 2, 5, 3	....	....	....	....
....	....	....	....	....
c	3.3–4.9	9–9.8	6–8	....
3*f*	2.5*f*	2.2*f*	2.4*f*	4
0.094–0.125	0.055–0.082	0.17–0.185	0.104–0.139	....
....	....	....	....	....
119 P 916	*118 P 422*	*124 P 52*	*99 J 1104*	*29 M & M 569*

Sons of Gwalia	Alaska Treadwell	Tonopah Belmont	Edna May	Plymouth
1225	1240	1250	1250	1250
....	100	104	108–110	100
....	9	6	6½	7–7½
....	*9*	− 2-in.	− 2-in.	....
0.31	20-mesh	0.187	8-mesh No. 18 ga. wire	0.087-in.
....	6	....	....	5.5–6.5
82	83	85	....	....
....	1, 3, 5, 4, 2	1, 5, 3, 2, 4	1, 3, 5, 2, 4	1, 3, 5, 2, 4
....	*10*	....	....	....
10	4.7	9	10–11	12.5–16*a*
....	3.5*f*	2.6	2.8*f*	2.8
....	0.056	0.144	0.149–0.164	0.186–0.238*g*
....	....	11	....	....
101 J 224	*114 P 410*	*Q*	*34 Aa 20*	*13 CME 618*

Mexican cyanide mill	Nipissing	Santa Gertrudis	Rochester	Homestake
1500	1500	1550	1550	1550
102	96	102	100	100
8	8	7½	6	8
− 2.5-in.	....	....	− 1.5-in.	....
0.5-in.	0.261 and 0.408-in.	0.2–0.28-in.	4-mesh	0.5
....	1	....	....	....
92–94	88	91	....	....
1, 5, 2, 4, 3	....	....	....	....
....	*1*	....	....	....
16	6.7	21.1	12	15
3.8	3.1	3.5	2.9*f*	3.9*f*
0.176	0.090	0.251	0.172	0.160
4	....	....	....	....
Q	*48 A 43*	*55 A 397*	*103 J 133*	*114 J 761*

g With 0.03-in. aperture = 0.126–0.141. *i* Reduction to 96 drops lowered repair and power costs without affecting tonnage.

Table 34*a*. Screen analyses of gravity-stamp products

Reference number (from Table 30)	1	2	3	4	5	6	7*b*	8	9	10
Screen aperture, mesh	Weight, per cent.									
3-in.									6.0	
2-in.									28.2	
1-in.									29.2	
½-in.									10.1	
⅜-in.							37.8			
4									11.6	
8	25.1									
10						5.6				
20	29.8			37	16.7*a*	16.2	38.6	46.9	7.8	6.2
35									2.2	25.0
40					12.5	14.8	6.6	14.5		
48									0.8	12.5
50		8								
60	17.9				9.8	10.2	4.4	8.7		
65									0.6	12.5
80					8.5	3.4				
100	5.9	17	24	28	4.0		3.1	7.5	0.6	8.4
120					3.3					
150	1.9					7.2	2.1	5.0	0.8	12.5
200	1.6	22	17	11	6.7	3.0	1.2	1.5	0.5	6.2
Through last screen	16.7	53	59	24	38.6	39.6	6.4	15.9	1.3	16.6

a = I.M.M. screens. *b* = Feed.

depends upon the character of the ore, size of feed and product, weight of stamps, number of drops per minute, height of drop, drop sequence, shape of mortar, character of mortar foundations and condition of shoes and dies.

Capacities of stamps of various weights treating different kinds of ore are given in Table 34. Capacities of stamps of different weights on the same ore are given in Table 35.

Table 35. Effect of weight of stamp on capacity. (*After Caldecott, 19 IMM 57*)

Test number	Weight of stamp, pounds	Screen aperture, inch	Height of discharge, inches	Tons water per ton of ore	Tons crushed per stamp per 24 hr.	Per cent. of + .01-in. material in product
1	1196	0.021	3	5.80	5.88	22.63
2	1279	0.021	3	5.80	6.58	22.23
3	1531	0.021	3	5.70	6.74	20.86
4	1216	0.016	11	5.40	4.26	5.16
5	1288	0.016	11	5.30	4.29	4.91
6	1293	0.016	11	5.43	4.55	9.49
7	1337	0.016	11	6.27	4.96	6.66
8	1562	0.016	11	5.05	5.17	....
9	1605	0.016	11	6.30	6.02	11.66

Drop = 8 in. in all cases.

Size of feed. The effect of size of feed on stamp duty is a moot question. Beaver (*81 J 748* [*1906*]) gives the results shown in Table 36 for tests on a 1050-lb. stamp, showing maximum duty with feed between 1- and 1.5-in. But while this is the general trend of experience, the location of the maximum

point is not definitely fixed. It will, of course, vary with the weight of the stamp. Caldecott (*19 IMM 57*) thinks 1.75-in. the maximum economical size for unaltered Rand banket and says that fine feed distributes the blow and is, therefore, uneconomical. Way, commenting on Caldecott's data, suggests the possibility of increasing efficiency by feeding fine roll-crushed material and using light stamps and a large number of short drops. This practice would, however, entail such a complicated plant, consisting of breakers, rolls, stamps and tube mills, that any increased crushing efficiency would be far outweighed by the disadvantages of complexity. Several plants using coarse battery screens and finishing in tube mills report an increase in tons finished per horsepower-hour following the installation of a by-pass screen ahead of the battery with covering of the same aperture as the battery screen. Clark (*109 J 834*) says that a 900-lb. stamp can handle 3-in. feed and that no gain follows from decreasing the feed size below 2-in.

Table 36. Effect of size of feed on capacity of stamps. (*After Beaver*)

Size of feed; all pass. . . .-in. aperture	Duty, tons per 24 hr.
2.5	4.04
2	4.10
1.5	4.60
1	4.74
0.5	4.35
0.25	4.10

Size of product. Fine crushing in the battery reduces capacity by reason of the fact that material must be kept in the mortar a longer time and subjected to a greater number of blows of the stamp before discharge, therefore all of the factors that tend to produce a finer product tend to reduce capacity. The size of product is determined by the aperture of the screen, the quantity of water, the height of discharge, and the character of the screening surface.

Table 37. Effect of battery screen on stamp duty. (*After Caldecott, 19 IMM 57*)

Battery screen, aperture, inch	Duty per nominal stamp(*a*)	Tons of −90-mesh product per 24 hr. per nominal stamp
0.404	15.68	3.85
0.272	13.33	3.69
0.197	12.34	3.84
0.135	12.74	3.83
0.121	11.00	3.83
0.053	7.24	3.15
0.028	6.49	3.36

a A nominal stamp or NOMINAL CRUSHING UNIT (*N.C.U.*) is defined by Caldecott as a stamp with a running weight of 1250 lb. and an assumed new weight of 1350 lb. running at 100 drops of 8-in. set height (7½ in. actual) per minute. Stamps of other weight represent *N.C.U.* proportional to their weight, *e.g.*, a 1400-lb. stamp = 1.12 *N.C.U.*

Screens. Increase in screen aperture increases stamp duty, but the increase is substantially all in the coarse sizes. (See Table 37.) Table 38

Table 38. Effect of screen aperture on stamp performance. (*After Schmitt*)

Screen aperture, inch	Tons per 1400-lb. stamp per 24 hr.	Sizing test of product			Tons −90-mesh per stamp per 24 hr.
		+60-mesh	+90-mesh	−90-mesh	
0.27	15	62	10	28	4.2
0.20	14	59	11	30	4.2
0.15	13	56	12	32	4.15
0.09	10	47	14	39	3.90
0.07	9	43	14	43	3.87
0.028	7	31	16	53	3.72
0.02	6	22	18	60	3.58

shows that increase in aperture from 0.02- to 0.27-in. increased duty from 6 to 15 tons but the corresponding change in tons of −90-mesh product per day was only from 3.58 to 4.2.

At Homestake a similar change from 0.022- to 0.25-in. screen increased duty from 4.5 to 8 tons but decreased −200-mesh product from 2.4 to 1.9 tons per stamp per 24 hr. At the Giant mine, Rhodesia (*87 J 543* [*1909*]), the amount of −120-mesh product increased from 20 to 31 per cent. with decrease in size of battery screen from ½- to ⅜-in. Hardinge (*89 J 221* [*1910*]) states that the duty of 1000-lb. stamps crushing hard quartz was increased from 2.5 to 10 tons on increasing the aperture of the battery screen from 20-mesh to 0.25-in. The sizing test of the product with the larger screen was 13 per cent. on 10-mesh, 11 per cent. on 20-, 17 per cent. on 40-, 10 per cent. on 60-, 17 per cent. on 100- and 32 per cent. through 100-mesh. At Nipissing (*48 A 14*) change from 20 stamps with 0.077-in. and 20 with 0.178-in. screens to 20 with 0.261-in. and 20 with 0.408-in. screens increased the duty from 5.7 to 6.7 tons.

Screening surfaces that have a large percentage of opening produce a coarser discharge than those having a small percentage of opening because with restricted egress, material must be crushed smaller to pass through readily. Apertures convergent outward tend to clog and thus not only reduce the size of the aperture but also the percentage of opening and both of these factors produce finer product. The area of the screen surface does not seem to be of great importance, judged from the fact that multiple-discharge mortars do not increase capacity sufficiently to pay for the increased cost of screen wear. Most of the screening is done in a narrow strip at the lower edge of the screen and any given part of this area is worked only about one-fifth of the total time. The layer of feed has time to drain away between successive presentations of pulp to be screened so that each new presentation is made to a bare screening surface and the area provided in the usual single-discharge 5-stamp battery is sufficient. Munroe (*9 A 84* [*1880*]) points out that if this were not so, *i. e.*, if crushing capacity were in excess of screen capacity, increase in falling weight, which is not accompanied by any sensible increase in falling velocity, would not increase capacity, because it would in no way affect screening capacity. This is not rigorously true, because increase in the amount crushed per blow increases the proportion of fines in the pulp splashed against the screens, which, of course, makes the task of the screens easier.

Height of discharge is the vertical distance from the top of the die to the top of the lower rail of the screen frame. The height of discharge increases as the dies wear. In order to keep it constant, as should be done, chuck blocks of different heights are provided which vary the height of the screen above the bottom of the screen opening. For closer regulation slats 1 to 1½ in. thick are used between the bottom of the screen and the top of the chuck block. The height of discharge may also be varied by use of a false bottom under the dies but this practice is not favored because of the effect on the character of impact.

High discharge results in low capacity by reason of the tendency of the coarser material to settle and not be lifted to the screen surface. Small discharge height should be used for large capacity and high discharge when a fine product is desired at the expense of high capacity. The tests shown in Table 39 consistently bear out the conclusion that, with a given screen, increase in height of discharge results in decreased tonnage and increased fineness of product. The effect is least with the finest screen. With high discharge, the size of screen aperture, within the limits of these tests, has no appreciable effect on either tonnage or size of product, but with low

discharge the tonnage is materially decreased and fineness materially increased by change to a finer screen.

Table 39. Effect of height of discharge on stamp crushing. (*After Meinke, 100 J 763*)

Test number	Screen aperture, inch	Height of discharge, inches	Tons crushed per stamp per 24 hr.	Screen analysis					
				+40	+50	+60	+80	+100	−100
1	0.024	8	3.05	8.3	7.9	4.9	5.7	24.0	49.2
2	0.024	8	3.00	3.8	7.5	6.0	5.4	26.3	51.0
3	0.020	8	2.90	5.5	6.8	4.5	5.5	26.0	51.7
4	0.024	5	3.48	8.3	14.5	6.0	4.9	23.4	42.9
5	0.024	5	3.34	8.7	10.9	6.7	10.7	24.6	38.4
6	0.024	5	2.97	2.3	6.7	6.9	6.3	26.9	50.9

Water. If the quantity of water fed to the battery is small the discharge is fine because the velocity of water-flow through the battery is not sufficient to carry the heavier particles in suspension and there is a lack of water to wash particles through the screen. The high viscosity of thick pulp reduces the rate of flow through the screen apertures, but, conversely, increases the carrying power for coarse material. If water is fed on top of the stamps, as is the most usual practice, part of the stream flows down over the screen and washes fine material away from the apertures. This effect is more marked the finer the screen. Individual jets introduced through the back of the mortar at the level of each die are probably the most effective in the case of coarse screens, but with fine screens such jets, unless operated at low velocity, wash coarse material against the screens and cause blinding.

Pitt (*8 JCM 373*) recommends for fine and medium crushing that the water be so introduced as to flow down the back of the mortar and thence forward across the dies. West (*116 P 7*), stating the duty of 950-lb. stamps on the Mother Lode, Cal., as 7.5 tons through 20-mesh screen, when making 110 @ 7½-in. drops per min., notes a water-solid ratio of 2.4 to 1 and states that increase in water would have increased the capacity. He also states that coarse crushing requires more water than fine. Recent practice is to use coarse screens and a small amount of water. Carpenter (*116 P 288*) records a duty for 1400-lb. stamps of 20 tons per 24 hr. through 1-in. screen with 2.6 tons of solution per ton of ore and expresses the opinion that doubling the solution would not increase duty 10 per cent.

Weight of stamps varies from about 900 to 2000 lb., this weight being that of the falling part including shoe, boss head, stem and tappet. The distribution of weight between the various parts is given in Table 30. The trend in the United States is toward stamps weighing from 1250 to 1500 lb. South African practice goes toward yet heavier stamps up to 2000 lb. Increase in weight causes increase in capacity (see Tables 34 and 35) but has little effect on size of product, all other things being constant. Tests 1 to 3, Table 35, indicate considerably more effect on tonnage crushed of the 80-lb. difference between 1 and 2 than for the 250-lb. difference between 2 and 3. On the other hand, as between tests 4, 6 and 8 the rate of increase in capacity with increase in falling weight is substantially uniform. Within the range covered this is probably the proper conclusion. Increase in weight is accompanied by increased breakage of cam shafts and stems, greater screen wear and metal consumption.

Height of drop. The amount crushed per drop increases with increase in height of drop. Meinke reports that 900-lb. stamps making 100 @ 6-in. drops per min. with 8-in. discharge height crushed 2.66 tons per stamp and with

7-in. drops, 3.05 tons per stamp per 24 hr. This is substantially direct increase. Munroe has shown, however, that more ore is crushed per blow of a given foot-pound value with a short drop and heavy stamp than under the reverse conditions. Louis (*28 A 553 [1898]*) has shown that the actual falling velocity is less than theoretical by a sensible amount (see Fig. 58), and has taken a series of stamp-indicator cards which show that the actual ultimate velocities range from 85 per cent. of theoretical with 82 @ 6½-in. drops per minute to only 63 per cent. with 93 @ 8-in. drops. Since the force of the blow struck varies as the square of the ultimate velocity, these figures correspond to losses of 30 to 60 per cent. due to friction and pulp resistance, the loss being greater with the higher drop. Further, the short drop permits more drops per minute, hence the line of development has been toward heavy highspeed stamps. High drop causes greater splash in the battery and, therefore, more rapid discharge.

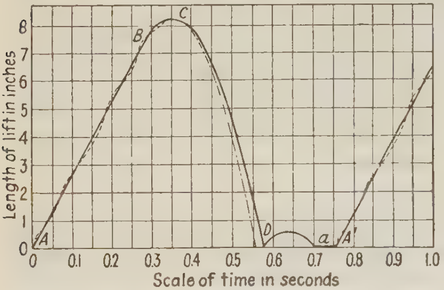


Fig. 58.—Stamp-mill diagram.

Eight-in. lift, 80 drops per min. Dotted line, actual rising path; dash-dot line, theoretical graph of fall.

Speed. The amount crushed per unit of time increases directly with the number of drops per minute. The amount discharged shows a similar increase in wet-crushing batteries, but in dry-crushing batteries the increase in amount discharged is out of all proportion to the increase in speed. Munroe reports an increase in yield of 122 per cent. for an increase from 60 to 90 drops per minute and an increase of 55 per cent. for a speed change from 90 to 102 drops.

The number of drops per minute and height per drop are interdependent, as may be seen by reference to Fig. 58, in which the portion of the curve from (A) to (B) represents the time when the cam is in contact with the tappet, that from (B) to (C) the time required for the upward momentum to be lost, that from (C) to (D) the falling time, that from (D) to (a) the time for rebound, and that from (a) to (A') rest time. Clearly, the time for a cycle with a given drop cannot be shortened beyond the point where the stamp is picked up on the rebound without danger of camming. If, however, the height of drop is lessened, the time for completion of a cycle is correspondingly decreased and the number of cycles or drops per minute can be increased. The force of the blow struck by the falling stamp can then be kept up by increasing the weight of the stamp. As above stated, the trend of modern practice in which stamps are heavier, has been to make drops shorter and speeds higher than formerly. The limit is reached when the force of the falling stamp is insufficient to break the largest particle of ore in the feed. The average of present-day practice is 100 @ 6- to 8-in. drops per min.

Condition of shoes and dies has a great effect on the capacity of the battery. These parts tend to wear unevenly; when wear has passed a certain point the capacity is greatly decreased.

Power for stamps varies with the weight of stamp, height of drop, and number of drops per minute. The theoretical power required may be cal-

culated by the formula $HP = WHN/(12 \times 33{,}000)$, where W is the weight per stamp in pounds, H is the height of drop in inches and N is the number of drops per stamp per minute. The total theoretical power for a battery is this figure multiplied by the number of stamps. The actual power consumption exceeds the theoretical by from 16 to 70 per cent. An allowance of 25 to 30 per cent. excess is safe for purposes of estimate. Tons crushed per horsepower-hour averages, according to Table 34, 0.074 with battery screens finer than 0.05-in. aperture, 0.138 for apertures from 0.05- to 0.25-in. and 0.164 for apertures coarser than 0.25-in. Truscott is quoted (*111 J 200*) to the effect that South African performance averages 0.05 ton per hp.-hr. from −2-in. through 30- to 40-mesh screen, 0.1 ton through 12- or 16-mesh, and 0.2 ton through 3- or 4-mesh. These figures are of the same general order as the preceding.

Moisture in product. Present-day practice is universally wet crushing. Table 34 shows a range in the percentage of moisture in pulp discharged through the screen from 75 to 94 per cent. The average of Rand practice is about 85 per cent. with a range of 50 to 90 per cent. The quantity of water used per ton is less with coarse screens than with fine and less the smaller the height of discharge.

When lime-bearing solution is being fed, all water lines should be of generous cross-section, nozzles should be at least 1-in. diameter, and all lines should be in duplicate with provision for easy replacement.

Feeding is universally done by means of automatic feeders in order to secure a supply of ore to the stamps properly regulated in proportion to the crushing done. Irregular feed not only decreases capacity, but increases breakage of stems and cam shafts. The Challenge feeder (Sec. 20, Art. 4) is most commonly used. The Tullock feeder is occasionally used. The Challenge feeder is the most satisfactory, especially on wet and sticky ores. It is operated by means of a small tappet on one, usually the center, stamp stem. The length of stroke of the feeder and consequently the quantity fed are determined by the distance that the stamp falls. Therefore, when material piles up in the battery the stroke is shortened and the quantity of feed automatically lessened and *vice versa*. The feeder is set up either suspended or supported, but in either case is arranged to slide out of the way when access to the space behind the stamps is desired.

Lost time in stamp mills is due to breakage of parts, principally stems, shoes, cams and cam shafts; dropping of boss heads or shoes; slipping of tappets, pulleys, or cam-shaft collars; renewal of shoes, dies and screens; regulation of height of discharge and height of drop; and dressing or cleaning plates inside and outside the battery. The latter cause is not directly chargeable to the stamp as a crushing device. Losses due to the other causes range from 1 to 11 per cent. (see Table 34).

Cost of crushing in gravity stamps ranges from \$0.15 to \$0.50 per ton.

At Homestake with 1000 @ 900-lb. stamps the cost in 1913 was \$0.36 per ton, making a product 81 per cent. − 100-mesh. Comparative costs at Simmer and Jack Proprietary with heavy stamps and tube mills was substantially the same but the ore was much harder (*22 IMM 185*). The cost of stamping alone at Simmer and Jack East in 1907 with 0.016-in. screen aperture, making a product containing 11 per cent. +0.01-in. material at the rate of 5 tons per 1350-lb. stamp per 24 hr. was \$0.45 per ton. Placing 0.057- and 0.035-in. screens on the stamps, thus raising the duty to 8.33 tons and re-grinding in tube mills reduced the percentage of +0.01-in. material to 1.6 without changing the cost of grinding (*19 IMM 57*). Cost at Alaska-Treadwell in 1915 ranged from \$0.17 to \$0.21 per ton in the 5 mills. At Churchill Wonder (1913–1914) (*52 A 128*) cost was \$0.081 for labor; \$0.073 for power; \$0.031 for supplies; total, \$0.185 per ton. At nipissing (*31 Ont. Dep. Mines 259*) forty

1500-lb. stamps crushing from 1½-in. to 4-mesh cost $0.10 for labor, $0.07 for supplies, $0.11 for power, $0.03 for shop; total, $0.31 per ton (aver. 1913–1922).

19. Nissen stamp

Description. The Nissen stamp unit consists of a single stamp falling in an individual mortar which is cylindrical in horizontal section. Two stamps mounted in a frame similar to that used with the 5-stamp battery constitute a unit and require about the same floor space as one 5-stamp battery. The screen extends more than half-way around the mortar and has an area of 3.75 sq. ft. which is about 3½ times as much per stamp as in an ordinary single-discharge 5-stamp battery. The mortar is made of semi-steel with manganese-steel liners. The total weight of a steel-frame battery ranges from about ten times the falling weight for a one-stamp unit to five times for a four-stamp unit.

Performance. Nissen (*12 JCM 357*) claims the performances given in Table 40.

Table 40. Performance of Nissen stamps. (*After Nissen*)

Falling weight, pounds	Mine	Ore	Screen aperture	Tons crushed per 24 hr.
1100	....	....	35-mesh	9
1500	....	....	30-mesh	10.7
1500	Leeuwpoont...............	Hard, tough	17-mesh	13
1700	....	....	30-mesh	11
1750	Linnet....................	Soft	26-mesh	20
1980	....	....	35-mesh	10.7
2000	Shamva...................	Medium	⅜-in.	35
2000	Falcon....................	Medium	9/64-in.	19.5
2000	Linnet....................	Soft	28-mesh	20
2000	Bustick...................	Soft	3/16-in.	25
2000	Machavie.................	Hard qtz.	17-mesh	15
2000	Fairview..................	Medium	3-mesh	33
2000	Modderfontein B...........	Hard qtz.	3-mesh	29
2000	New Modderfontein.........	Hard qtz.	3-mesh	30

An exhaustive competitive run of Nissen *vs.* ordinary gravity stamps was made at the City Deep mill on the Rand in 1911. The results are given in Table 41. A less exhaustive test at the Northern Customs Concentrator Co., Cobalt, Ont. (*31 CMJ 87*), showed a duty of 5.5 tons per 24 hr. through 30-mesh screens for a 1650-lb. Nissen stamp against 2.25 tons per 1250-lb. stamp in a 5-stamp battery; the power consumed by two of the Nissen stamps was 40 to 50 per cent. less than that for the five 1250-lb. stamps; the feed was a hard conglomerate crushed through 1.5-in. The +40-mesh in the Nissen-stamp product was 8.6 per cent. and the −200-mesh, 43.4 per cent. against 5.2 per cent. and 48.8 per cent., respectively for the 5-stamp battery. A similar comparative test at Tincroft, Cornwall (*11 MM 378*) showed 11.4 tons per stamp day for a 2000-lb. Nissen *vs.* 7.2 tons for a 1250-lb. standard stamp; the respective tons per hp.-hr. were 0.09 and 0.08; the products were substantially the same, *viz.*: 13 to 16 per cent. +20-mesh and 26 to 27 per cent. −200-mesh. The 5-stamp mortar was particularly narrow with straight back and steep front. At Modderfontein B (*114 J 836*) California stamps crushed through 6-mesh Ton-cap with a consumption of 6.5 hp.-hr. per ton; Nissen, through 9-mesh Ton-cap, 4.4 hp.-hr. per ton.

Advantages claimed by Nissen over the ordinary 5-stamp battery are as follows: (1) It is difficult to distribute feed evenly to the dies in the 5-stamp mortar while in the Nissen the feed is led directly onto the single die and each rise of the stamp causes the pulp in the mortar to flow toward the center of the die. (2) The area of screen per stamp is greater in the Nissen and the pulp is presented to the screen along a normal to the surface instead of at an angle, as is the case with most of the pulp in the 5-stamp mortar. Both

differences make for more rapid screening and hence greater capacity in the Nissen. (3) Cam-shaft breakage is less in the Nissen because of the smaller loading (two stamps to be lifted between bearings *vs.* five in the ordinary battery). The diameter of cam-shafts in the ordinary battery cannot be further increased owing to the resulting increased velocity of pick-up. (4) The limit of weight (and, therefore, capacity) of a 5-stamp battery has been reached on account of the incre se in size of mortar and in length of cam-shaft necessarily attendant upon further increase in diameter of falling parts or, if the increase is gained by increase in length of head and weight of tappets, stem breakage is increased, due to the great length below the lower guide and to vibration caused by the heavy tappets. A larger mortar means greatly increased difficulty in anchoring the mortar and also greater strains in the mortar itself. In the Nissen stamp the weight is concentrated near the shoe by use of a shouldered boss, consequently vibration is lessened. (5) Shoes and dies wear more evenly in the Nissen stamp, due to even return of the uncrushed material to the die and great rotation of the stamp, hence the faces are flatter and the capacity correspondingly greater. (6) Power and steel consumption are less on account of the higher discharge rate gained by favorable screen placing.

Table 41. Summary of competitive test of Nissen and standard gravity stamps at City Deep mill, Johannesburg. (*12 JCM 111*)

Test number	1		2		3		4		5	
Kind of stamp	*N*	*S*	*N*	*S*	*N*	*S*	*N*	*S*	*N*	*S*
Falling weight, lb.	1932	1863	1927	1859	2245	1855	1993	1775	1991	1773
Drops per minute	103	100	103	100	103	100	103	100	103	100
Height of drop, in.	8½	8½	8½	8½	8¾	8¾	8½	8½	8½	8½
Height of discharge, in.	2⅝	2¼	2	2⅜	2	2½	2¼	2¼	2¼	2¼
Screen, kind	*T*	*T*	*T*	*T*	*W*	*W*	*W*	*W*	*W*	*W*
Screen, aperture, width, in.	0.205	0.205	0.205	0.205	0.375	0.375	0.277	0.277	0.375	0.375
Screen, aperture, length, in.	0.536	0.536	0.536	0.536	0.375	0.375	0.277	0.277	0.375	0.375
Per cent. solids in discharge	15.6	27.1	19.2	29.4	24.4	31.8	23.3	23.3	24.8	23.5
Size of feed (*b*)	1	2	3	4	5	6	7	8	9	10
Size of product (*b*)	1	2	3	4	5	6	7	8	9	10
Tons crushed per stamp per 24 hr.	24.5	18.3	27.7	19.9	36.7	22.7	29.8	21.0	37.7	24.3
Horsepower per stamp (*a*)	4.1	4.2	4.1	4.2	5.0	4.2	4.2	4.6	4.2	4.6
Tons crushed per horse-power-hour	0.25	0.18	0.28	0.20	0.30	0.22	0.30	0.19	0.37	0.22
Wear of shoes per ton crushed, lb.	0.13	0.14	0.14	0.24	0.20	0.19	0.15	.....	0.11	0.08

N = Nissen. *S* = Standard 5-stamp battery. *T* = Tyler Ton-cap. *W* = Square-mesh No. 14 (0.08-in. wire).

a Excluding belt, shaft and motor losses. *b* Numbers refer to screen analyses on Table 41*a*.

Table 41*a*. Screen analyses in connection with Table 41

Test number	1	2	3	4	5	6	7	8	9	10
Feed:										
+2-in. mesh	36.9	12.1	14.96	12.4	26.9	18.2	39.0	14.1	23.7	27.1
1½-in. mesh	18.6	14.4	14.50	11.35	18.75	16.0	19.7	18.0	20.9	17.1
½-in. mesh	15.1	20.2	20.40	19.35	19.3	18.15	15.0	21.3	18.8	19.1
¼-in. mesh	11.7	12.7	16.95	17.70	13.05	16.25	9.3	16.0	12.7	12.6
−¼-in. mesh	27.5	37.9	33.20	39.20	22.0	31.4	17.0	30.6	23.9	24.1
Discharge:										
+60-mesh	65.2	63.4	64.9	63.4	67.24	69.33	66.6	66.5	70.7	70.7
+90-mesh	8.3	9.1	8.46	8.70	8.62	9.09	9.9	10.2	10.1	10.3
−90-mesh	9.7	12.0	8.46	9.10	10.84	9.82	8.8	10.9	10.2	9.2
−200-mesh	16.7	10.7	18.10	19.5	13.3	11.76	14.7	12.4	9.0	9.8

20. Pneumatic stamps

Description. Pneumatic stamps, also called CRANK STAMPS and HIGH-SPEED STAMPS, have had limited use. The Holman (Fig. 59) is the best known. It consists essentially of a cylinder (*a*) attached by means of trunnions (*b*) and connecting rods to the driving mechanism. A crushing member consisting of shoe (*c*) and boss-head (*d*) is mounted on the lower end of a piston rod (*e*) that runs through the cylinder and carries piston (*f*). Air ports are provided in the cylinder walls, one set for use with new shoes and dies, the other when shoes and dies are half-worn. The piston itself acts as a valve, closing the lower ports on the up-stroke and the upper ports on the downstroke and thus providing air cushions that protect the cylinder heads. The stamp is run at 120 to 140 @ 12-in. strokes per min. and requires from 30 to 35 hp. per unit of two stamps.

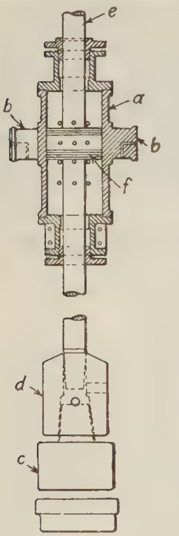


Fig. 59.—Pneumatic stamp.

Performance. At EAST POOL, Cornwall (*86 J 213*) one 2-stamp unit crushed 21 tons per 24-hr. of −3-in. feed through a 25-mesh screen. The capacity of a 950-lb. gravity stamp on the same feed is 1.4 to 1.5 tons per 24 hr. At MOUNTAIN QUEEN, West Australia (*95 J 108*) a 2-stamp unit crushed 135 to 160 tons per day of fairly hard ore through a 10-mesh screen. Metal consumption was 0.45 lb. per ton; water, 1200 gal. per ton. At BABILONIA GOLD MINES, Nicaragua (*113 P 911*), a 2-stamp unit making 145 to 150 drops per min. crushed −2.5-in. feed through 6- and 9-mesh wire screens at the rate of 25 tons per stamp per 24 hr. Water consumption was 9 tons per ton of ore. Mortar liners lasted 4 months, stems 3 months, screens 3 days. Wear of shoes was 0.48 lb. per ton and of dies, 0.17 lb. per ton. The product contained 50 per cent. +20-mesh and 12 per cent. −200-mesh.

21. Stamps *vs.* other crushers

Stamps *vs.* stamps plus tube mills. Table 42 shows the reasons that caused the change from stamps to stamps plus tube mills for fine grinding

Table 42. Stamps *vs.* stamps plus tube mills at Simmer and Jack East

Crushing machines	Stamps	Stamps + tubes
Running weight of stamps, average pounds	1350	1500
Speed, drops per minute	96	96
Height of drop, in., average	8	8
Height of discharge, in., average	9	3.5
Screen aperture, in	0.016–0.017	0.035–0.057
Ratio water to ore by weight	8 : 1	6.46 : 1
Per cent. +0.01-in. material in product	10.9	1.6
Cost, dollars per ton	0.3515	0.3578

for cyanide work. By putting coarser screens on the stamps (0.057- and 0.035-in. instead of 0.016- and 0.017-in.) and sending the sands to tube mills for regrinding, the stamp duty was increased from 5 tons per 24 hr. to 8.33 tons and the percentage of material remaining on a 0.01-in. screen was reduced from 10.9 per cent. with stamps alone to 1.6 per cent. with stamps plus tube mills, with an increase in cost per ton ground of 0.63¢. Later

practice uses still coarser screens on the stamps and more tube mills, with resulting increased production of fine material per unit of cost. As early as 1909 the Mines Trials Committee on the Rand reported (*So. Af. Min. Jour., Oct. 9, 1909*) that crushing to 3-mesh in stamps with final reduction in tube mills would give a stamp duty of 15 tons per 24 hr. with a 1400-lb. stamp and represented the maximum economy with these machines.

Stamps vs. rolls. Reid (*17 CMI 56*), analyzing the factors determining the choice between stamps and rolls as intermediate crushers for Cobalt mills, says that stamps were used because of the anticipated short life and low capacity of the mills, while the ore was tough and required fine crushing to free the mineral. Rolls alone could not finish to the required fineness and several in series would have been needed, together with a final fine grinder, to accomplish the size reduction possible with one set of stamps. This would have meant a complicated flow-sheet and more slime would have been produced than with the stamps. Furthermore, on account of the small capacity of stamp units, heavy stamps can be used with resultant economy, while rolls of the size and weight necessary for economical operation would have great excess capacity.

Stamps vs. ball mills. The general and probably the correct conclusion on this question, when all-slime cyanidation is the metallurgical aim, is that stamps are superior for small tonnages of hard ore, but that for any tonnage that will justify the use of a 6-ft. ball mill or larger, ball mills followed by tube mills will be the most economical combination. The stamp has the advantage for small tonnages that it can finish to slime from as coarse as 3-in. feed at one operation, which a small ball mill cannot do and a large one cannot do economically; the stamp unit is of small capacity and hence the most efficient size can be chosen, even for small mills; and the small capacity per unit permits one unit to be cut out for repairs without serious diminution of mill capacity. On the other hand, for large tonnages, when the choice is between stamps plus tube mills and ball mills plus tube mills, the ball mill does more crushing per unit of power input than the stamp, has a much smaller percentage of lost time, takes less attendance, has fewer parts and is, therefore, less complicated; the foundations are simpler, it takes less mill room, the first cost is less, and it is less noisy. Steel consumption probably averages less in the stamp, but if the stamp is charged with broken stems, cams, camshafts and discarded tappets, the difference in its favor quickly disappears.

At a Canadian mill (*113 P 260*) eighty 1250-lb. stamps crushed 800 tons per day through ⅜-in. screen with a consumption of 7.2 hp.-hr. per ton milled. One 6-ft. ball mill crushed 498 tons per day to substantially the same size, consumed 5.3 hp.-hr. per ton, and required about the same floor space as 10 stamps but less mill height and mill construction.

SECTION 4

FINE GRINDING. CRUSHING EFFICIENCY

In present-day terminology, fine-grinding machines are those that deliver the finished product well under a millimeter in maximum size. The machines most commonly used for this work are cylinder mills, including ball mills, tube mills and rod mills; gravity stamps; and, in comparatively rare cases, Chilean mills, Huntington mills, and grinding pans. The great bulk of fine grinding in concentrating mills is done by the first three types. Stamps as fine grinders are now found only in occasional gold mills of small size. Huntington mills have practically disappeared. Chilean mills have a limited use in small, isolated precious-metal mills. Pans have never had wide use and have been supplanted by more modern machines in practically all places. Their only claim to consideration at the present day is small first cost.

CYLINDER MILLS

The term cylinder mills is herein used to describe machines that consist essentially of hollow containers of circular cross-section, mounted with the axis substantially horizontal, and partially filled with crushing bodies that are caused to tumble, under the influence of gravity, by revolution of the container. There are several different types. The usually accepted primary classification is based on the character of the crushing bodies and groups the machines as: (1) ball mills, in which the crushing bodies are usually metal spheres, although other shapes such as cones, cubes, octahedra, short cylinders and the like have been tried; (2) pebble mills, using flint pebbles or selected lumps of local hard rock or ore as crushing media; (3) rod mills, in which straight metal rods of cylindrical or polyhedral section and substantially as long as the cylinder form the tumbling mass. Ball mills are further classified as (*a*) cylindrical mills with trunnion feed and simple trunnion discharge, called CENTER-DISCHARGE OR OVERFLOW MILLS; (*b*) cylindrical mills with

trunnion feed and an accelerated trunnion discharge accomplished by some type of pulp elevator at the discharge end, called PERIPHERAL-DISCHARGE or GRATE MILLS; (3) conical mills; (4) cylindrical mills with trunnion feed and discharge through a peripheral screen, often called KRUPP MILLS. Pebble mills with cylindrical shells are called TUBE MILLS and those with cylindro-conical shells, CONICAL PEBBLE MILLS. A rod mill commonly has a cylindrical shell but the cylindro-conical shell has been tried successfully.

1. Center-discharge mills (Overflow mills)

These consist essentially of a cylindrical shell of heavy metal partially closed at both ends by metal heads carrying hollow trunnions. The cylinder is mounted horizontally, usually on trunnion bearings but occasionally on a tire and rollers at one or both ends. A gear mounted on the outside of the cylinder is driven by a pinion shaft, suitably mounted at one side and actuated by various means. Balls up to 7-in. diameter are charged until the mill is filled to within a few inches of the axis. Feed is introduced through one of the trunnions and the ground product flows out at the other end. Mills range from 3 to 8 ft. in diameter and from 2 to 8 ft. in length. Ordinary sizes are given in Table 1.

Table 1. Cylindrical ball mills. Size, weight and operating data, from makers' catalogs

Diameter, feet	Length, feet	Ball charge, pounds	Speed, revolutions per minute	Power, horsepower		Weight of liner, pounds	Weight of mill and liner, pounds
				Required	Installed		
3	2	1000	....	6–8	10	....	7500
3	3	875–1200	32–40	5–10	15	2000	8400
3	4	1400	32	8	10	....	....
3	5	1800–2600	31.5–40	10–15	20–25	....	12,000–13,000
4	3	2400–3000	27–29	14–20	20–25	....	14,000–23,000
4	4	3000	27–32	18–25	30–40	3600	23,000–25,000
4	5	3600–5000	27–29	22–28	40	....	27,000
4	6	4200–6000	27–29	26–33	40	....	29,000
4	7	7000	27	39	50	....	30,500
4	8	8000	27	44	60	....	32,500
4½	4½	4480	27	27	....	5300	....
5	3	4000	26	24	....	....	....
5	4	4000–6900	24–28	30 40	50–60	....	24,000–34,000
5	5	6400–8600	24–26	36–50	75	6500	36,000–37,000
5	6	7,600–10,400	24–26	42–55	75	....	39,000
5	7	12,100	24	67	100	....	41,000
5	8	13,800	24	75	100	....	43,000
6	3	6000	23	50	75	....	35,000
6	4	8000–9800	22–24	58 62	75–85	....	36,000–48,000
6	4½	7000	22	50–75	75	....	43,000
6	5	10,000–12,000	22–24	72–80	100	....	37,000–52,000
6	6	12,000–14,700	22–24	86–100	100 125	12,000	49,000–56,000
6	7	17,000	22	100	125	....	52,000
6	8	19,600	22	114	125	....	55,000
7	5	16,750–18,600	20–20.5	100–150	125–150	....	59,000–70,000
7	6	20,000–23,000	20–20.5	127–135	150	....	63,000–73,000
7	7	19,450	23	140	....	15,400	....
8	5	22,500–25,000	18–19	145–150	175	....	79,500–89,500
8	6	27,000–30,000	18–19	150–225	200–225	28,000	82,000–10,000

A 9 × 6-ft. mill is being tested at Inspiration; see page 359.

Shell of most ball mills is cast iron, cast semi-steel or cast steel, made extra thick, and flanged at the ends for attachment to the heads. The joint between head and shell is

made with tongue and groove in order to distribute shearing stresses and take some of the load off the bolts. Long shells are usually segmented with flanged joints similar to those at the ends. The Marcy mill is cast in two parts with heads integral and has a flanged joint at the center of the barrel. The shell is drilled for liner bolts. Some manufacturers cast the shell with one and some with two manholes. In short mills the manholes are frequently placed in the heads. Manholes are made large enough to allow for the introduction of liner plates. Power is saved by providing manholes on opposite sides of the mill.

Heads are cast, of iron, semi-steel or steel, and heavily ribbed. Some manufacturers cast the trunnions integral with the heads, others cast heads and trunnions separately, the latter with a heavy flange, and after facing both head and trunnion accurately, bolt the two together. The assembled mill should be swung in a lathe and the trunnions turned true with the mill axis, then polished. Some manufacturers provide trunnions with steel sleeves pressed on before turning down, to protect the main casting from wear. This is probably an unnecessary refinement.

Trunnion bearings are made very large and heavy. Some makers provide rigid bearings, other ball-and-socket type. It is probable that the latter give better service. The lower half of the bearings is babbitted, the upper half is cored out to form a large pocket for lubricant. Some manufacturers furnish a tire-and-roller bearing instead of a trunnion at the discharge end. This type of bearing takes less power than the trunnion bearing when new, but it is difficult to maintain even wear on tires and rollers and, as a consequence, after a short time such a mill is likely to get out of alignment, then vibration and power consumption increase markedly. Renewal of tires and rollers is expensive and slow as compared with the renewal of babbitt in ordinary trunnion boxes, particularly as compared with the renewal of removable babbitt linings, where such are used.

Lining. Shells are usually lined with replaceable metal plates bolted to the interior surface. The best design is one that allows easy replacement of individual sections. Liner materials are commonly hard iron or manganese or chrome steel. Hard iron is unsatisfactory because of breakage, when balls larger than 2-in. diameter are used. Cast manganese steel is widely used and has been particularly successful with heavy charges of large balls. Cheaper materials, having less strength and toughness than manganese steel are satisfactory with light loads and will probably be considerably cheaper. Manganese steel for linings must be very tough. It may be of satisfactory chemical composition, yet, through improper heat treatment, wear even more quickly than cast iron (*122 P 465*).

Plates are made in a considerable variety of sections. Typical sections are those that give a smooth interior surface and those that give a ribbed surface with ribs parallel to the axis of the mill (see Fig. 1). The liners (*A*) and (*B*) (Fig. 1) consist of individual plates bolted directly to the shell, while in (*C*) and (*D*), the plates are held in by wedge-shaped bars. The advantage of the latter construction is greater ease in drawing plates into place.

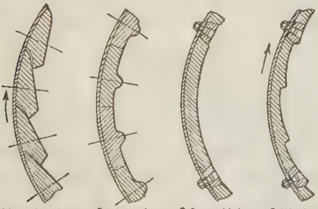


Fig. 1.—Liners for ball mills.

End liners of the same metal as the shell liners are used to protect the heads. Liners are also used for feed and discharge trunnions. These latter may be made with smooth inner surfaces or may have a helical groove cast inside, the feed-trunnion liner having the groove directed to facilitate entrance of feed, while the discharge-trunnion liner has the grooves directed to return coarse material into the mill and to aid in feeding balls into the mill against the flow of discharging pulp. A grooved liner for the feed trunnion has the disadvantage that it limits capacity when a feeder of

excess capacity is used in the attempt to force large quantities through the mill. THICKNESS OF LINERS ranges from 2 in. in small mills to as much as 5 in. in mills of large diameter.

A special lining for a 4 × 10-ft. ball-tube mill at LIBERTY BELL (see Table 4) was made by cementing in alternate courses of 1¾ × 2 × 5 × 48-in. hard-chilled cast-iron bars and 4½ × 5 × 8½-in. quartzite blocks, set with the 4½-in. dimension radial; no bolts were used. For one lining, 200 quartzite blocks, 42 ribs, 9 sacks of cement and 9 sacks of sand were required. Consumption was 0.65 lb. quartzite and 1.25 lb. iron per ton of new feed.

Fig. 2 (*122 P 465*, *112 J 778*) shows a lining made of steel rail, designed to pick up a ball layer in the same way that the El Oro lining picks up pebbles (see p. 427). Lining for a 6 × 5-ft. grate mill using 4-in. balls and crushing −1-in. material is composed of 50-lb. re-laying rails with ⅞-in. bolts. A complete liner cost about $250 (1921), which was one-tenth the cost of a manganese-steel liner. It lasted 5 months on open-circuit work grinding to −⅛-in. The cost of balls lodged in the liner must also be charged. A similar lining in a 5 × 6-ft. center-discharge re-grinding mill (−⅛-in. to 95 per cent. −100-mesh) lasts about 15 months using 2-in. balls. A white-iron plate lining in the same mill lasts about 5 months and costs twice as much.

FIG. 2.—Rail liner for ball mill.

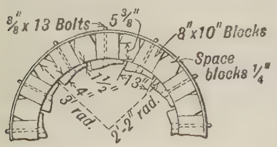


FIG. 3.—Method of reducing diameter of a cylinder mill (*after Daman*).

A ball mill at CATEMU, Chile (*123 P 884*), was lined with steel rails set in cement. The lining gave no trouble and wore well.

Liners are sometimes blocked out from the shell by wooden filling in order to reduce the internal diameter of the mill. Fig. 3 (*103 J 111*) shows the method employed at NEVADA PACKARD in changing a 6 × 5-ft. pebble mill into a ball mill. Twelve hours were required to empty the pebble mill, change liners and re-fill with balls.

Balls are made of forged high-carbon or chrome steel or of cast iron or manganese steel. Wraight (*30 IMM 208*) gives the analyses in Table 2 for three different samples. Lawler (*112 J 5*) states that most balls are made of assorted scrap of unknown composition with carbon content varying from 0.2 to 1 per cent when the proper limits are 0.6 to 1 per cent., that the chrome in chrome-steel balls varies from *nil* to 0.05 per cent. and that balls from most small forges will vary greatly from shipment to shipment. The usual sizes are from 2- to 6-in. diameter. Fairchild (*116 J 197*) shows that if the balls are all of one size, the weight per cubic foot with close packing is 363 lb., corresponding to 26 per cent. voids. Davis (*61 A 276*) thinks 35 per cent. voids more nearly correct for calculating ball-mill loads. Tillson (*116 J 357*) says that the void in the charge of a cement mill has been found to be 44 per cent.

Table 2. Analyses of grinding balls.

	Cast iron	Cast iron	Manganoid
C.	0.84%	1.17%	1.21%
Si.	0.42%	0.40%	0.80%
S.	0.031%	0.034%	0.031%
P.	0.031%	0.067%	0.098%
Cr.	0.06%	0.03%	0.06%
Mn.	0.56%	0.89%	10.97%
Brinell.	3.5 mm.	3.3 mm.	4.1 mm.
Scleroscope.	45	55	27

Ball load. The most efficient ball load is probably all that a mill will hold which is normally about 55 per cent. of internal volume. The weight of a full charge may be roughly estimated from equation $W = 80\ D^2L$, in which W = weight in lb., D = nominal diameter and L = length, both in feet. In many mills, however, the ball load is carried as much as 8 to 10 in. below

the axis of the mill. It is usual to have a charge of mixed sizes, the maximum size depending upon the maximum size of feed. Table 3 shows common practice as to maximum size and proportion of various sizes of balls with different sizes of feed. The use of as small balls as will crush the largest rock is often recommended, but Young (*58 A 126*) found the best efficiency for 5-in. balls at all feed sizes. Balls added to compensate for wear are generally of the largest size only. The weight added is usually a fixed amount daily, determined by average consumption and tonnage treated. A better method of regulation is to determine the ammeter reading corresponding to maximum grinding efficiency and then to load balls as needed to maintain this reading. When maintaining the ball load near the mill axis, a drop in the ammeter reading following the addition of balls usually corresponds to an overload. For ball wear see p. 384. For other shapes than spheres see p. 400.

Table 3. Proportions of balls of various sizes in new ball loads. (*From Marcy catalog*)

Diameter of balls, inches	Percentage of total load	
	Coarse feed, 1-in. to 3-in. maximum	Fine feed, ⅛-in. to ½-in. maximum
2½	....	10
3	10	20
3½	20	40
4	30	30
5	40	....

Feeders are of three general types, known respectively as one-way scoop, three-way scoop, and drum. Occasionally a two-way scoop is used. ONE-WAY SCOOP (Fig. 4) consists of a single spiral with open end and central side opening for delivery into the feed trunnion. Extreme radii of spirals are 15, 24, 27, 30, 36, 42, and 48 in., depending on size of mill, size and tonnage of feed, and amount of elevation desired. The cross-section of the spiral passage is rectangular, and should be sufficiently large to pass freely the largest ball that is used in the mill. A steel-plate scoop was used at MIAMI COPPER CO. A THREE-WAY SCOOP contains three spirals of but one-third turn each and is designed to give greater capacity than a single scoop (but see Hines below). It is not made of as great radius as the larger single scoops. Both types revolve with the mill in a rectangular feed box from which they pick up a certain portion of material at each revolution. Spiral feeders serve the useful purpose of elevating feed as well as introducing it into the mill and therefore make possible a lower feed-delivery point. This is of distinct advantage when mills are run in closed circuit with mechanical classifiers. If feed is brought into the feed box on the up-coming side in the plane of revolution, the capacity of the feeder is considerably greater than when feed enters at right angles to this plane.

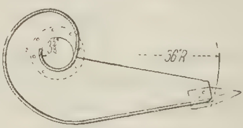


FIG. 4.—One-way scoop feeder.

DRUM FEEDER consists of a cylindro-conical receptable, open at both ends, with inside helix to transfer material from the feed side into the mill trunnion. It is superior to the scoop types for coarse feeds because it is more positive and less subject to breakage, but it requires to be fed at a higher elevation. The combination drum-and-scoop feeder (Fig. 5) is used when the original feed to the mill is coarse and can be delivered at the height of the mill axis and when sand oversize is delivered from a mechanical classifier. With this feeder, the usual box is made for receipt of classifier sand and from this sand is picked up by the scoop while coarse original feed is discharged directly into the drum.

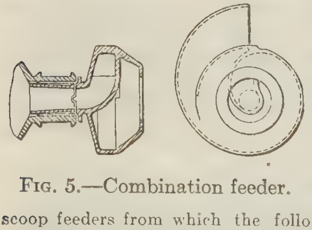


FIG. 5.—Combination feeder.

Hines (*59 A 249*) reports a series of tests on scoop feeders from which the following conclusions were drawn: (1) The capacity of a spiral feeder is proportional to the length of the spiral. (2) The capacity of a single-scoop feeder is twice to four times as great as that of a double-scoop, according to the form of the latter. (3) The capacity of a 3-way scoop is about half that of the single

scoop. (4) With coarse feeds the capacity of the scoop is limited by the trunnion and trunnion capacity is less with a spiral liner than with a smooth liner flaring into the mill. For largest tonnages of coarse material the trunnion must be short and of large diameter.

At ASTURIANA DE MINAS (*115 J 399*) the capacity of a ball mill taking −1.5-in. feed was limited by the feeder to 175 tons per 24 hr. The smallest section of the spiral was 5½ × 6-in. and the opening into the mill trunnion was 7-in. diameter. Capacity was increased to 200 tons by making the thimble between the scoop and the trunnion cylindrical instead of conical and then increasing the diameter of the central discharge opening from the feeder to 9 in. and placing a helix of 10-in. pitch through the feed trunnion.

Feeders are made either of cast iron or cast steel or built up of steel plate. It is well to provide for bolting either side to the trunnion, thus making the feeder reversible, a cover plate being provided for the outside. This is, of course, impossible with the drum and combination feeders. Replaceable lips of hard cast iron or manganese steel are usually provided for scoop feeders and in some cases liners for the spiral are also used. The great advantage in the use of the combination feeder lies in the fact that coarse material is kept out of the feed box. This prevents jamming of large particles between the moving scoop and the sides and bottom of the box with consequent racking of the box and strain on feeder and bolts.

Fig. 6 shows a special form of elbow feeder for ball mills. A screw feeder is sometimes used for soft, dry feed; at the T. P. KELLEY AND CO. graphite plant (*114 J 325*) scoop feeders failed while this type was successful.

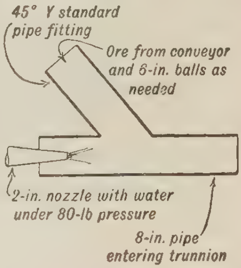


FIG. 6.—Sketch of ball-mill feeder at Hollinger.

Feed box is required for scoop feeders. It consists of a rectangular box built around the scoop, fitted closely around the thimble between the feed trunnion and scoop proper, with clearance all around the scoop greater than the largest lump of ore or the largest ball (or pebble) fed. Feed is introduced through suitable openings. The top should be protected with a grating.

Discharge trunnion is normally of slightly greater diameter than the feed trunnion in order to induce flow of feed through the mill. The end of the trunnion is fitted with a bell-shaped lip with, in some cases, drip rings to prevent sand from working back into the bearing. As previously noted, the trunnion liner may have a reverse helix cast on the inner surface.

Drive. The smallest mills may be driven directly by belt to a pulley mounted on the shell. This is cheap but most unsatisfactory, as the belt is likely to slip badly, especially at starting. All other mills have a gear mounted on the mill shell driven by a pinion on a countershaft at about the level of the mill axis. The cheapest drive, from the standpoint of first cost, is one with a machine-molded cast-iron spur gear on the mill, cast-steel cut pinion, and pulley-driven countershaft. A cast-steel cut gear is mechanically better but more expensive. Gears are reversible so that two wears can be taken on the teeth. A long-center horizontal belt is best from mechanical considerations, but takes up a lot of floor space. This difficulty may be obviated by using some form of short-belt drive with tightening pulley, such as the Lenix. With this drive the motor may be placed in any position with respect to the driven pulley, but on account of splash is best not placed below the mill axis. On account of the heavy starting load, clutches must be used with belt drives unless the belt and motor are greatly oversize. Any standard friction clutch is satisfactory, but it should be comfortab y oversize (50 to 100 per cent.) to withstand excessive starting strains and be well protected against grit. One form has the pulley mounted on a quill with a friction clutch. This makes for easier alignment than the ordinary friction-clutch pulley. Rarely a bevel gear is used in place of a spur gear on the mill but end thrust on the pinion shaft causes difficulty. Silent-chain drive of the mill countershaft has the same advantage as the short b lt and no slip at starting, but is more expensive. No clutch is required for the drive itse' , but one must be used in lieu of an oversize motor. Gear sets in various combinations are frequently used. The cheapest has the usual spur gear on the mill and a large gear on the mill countershaft, driven, through a flexible coupling, by a pinion on the motor. A more compact but somewhat more expensive arrangement uses a standard gear-speed reducer on the motor, attached through a flexible coupling to the mill countershaft. The gears in the speed reducer are enclosed, run in a bath of oil, and are more efficient and longer-lived than the open gear set first described. Both permit the use of high-speed motors. The most expensive but most efficient drive is a herringbone gear on the mill with a slow-speed motor direct-connected to the pinion shaft through a flexible coupling, ordinarily of the pin-and-bushing type. Speed reduction can be as great as 20 to 1, so that the motor speed required is from 300 to 600 r.p.m., according to

Table 4. Performance of center-discharge cylindrical ball-mills

Plant	Liberty Bell	Shattuck Arizona	Nipissing	Le Roi No. 2(*o*)	Le Roi No. 2(*o*)	Mexican cyanide plant	St. Joseph Lead, Bonne Terre	Cons. Arizona Smelting Co.
Size, diameter × length, ft.	4×10	4×10	4×20	5×4	5×4	5×10*n*	6×4	6×6
Speed, r.p.m.	28	31.8		26	26	28	22	23½
Tons new feed per 24 hr.	25	120	105	45-50	*s*	150	450	288
Tons total feed per 24 hr.	50	360–420*u*	240			350–425	450	288
Method of closing circuit	Cone	Drag	*aa*	Open	Open	*DC*	Open	Open
Installed horsepower	40	40				65	100	125
Actual horsepower	35	−40		35	20	98	68	74
Horsepower per ton of balls	4.4	5.7		11.7	10	8.9	11.3	12.3
Tons new feed crushed per horsepower-hour	0.03	0.125		0.057		0.064	0.276	0.162
Moisture in mill, per cent	35	28–45		43	50	35-40	50	
Size of feed (*a*)	*1*	*9*	40%-200-mesh	−1.5-in.	*q*	*6*	*2*	*3*
Size of product (*a*)	*1*	*9t*	98.7%-200-mesh	*7*	*8*	*6m*	*2*	*3*
Attendance, machines per man	10					6	*e*	2
Lost time, per cent	3	Small				4		
Principal causes of lost time	Re-lining	*v*				*l*		Power
Lubricant, kind/pounds per shift	O 1							
Feeder, type	*SS*	Comb.		*SS*		Drum	Drum	*SS*
Feeder, material	Steel	*CI*, *w*				*CI*	*CI*	Stl. plate
Feeder, life, days	100	*w*				800		150
Liners, type	*b*	Block		*WB*		Lifter-bar	Ship-lap	
Liners, material	*b*	*WI*		Semi-steel		*WI*	*Mn*	*g*
Liners, life, days	180	*x*				150		*g*
Liners, consumption, pounds per ton	*b*	0.63		1.25	1.25*r*	0.8		*g*
Time for re-lining, hr.	72					16	16	16
Number of men for re-lining	3					8	3	6
Balls, material	*Cr*	*CI* and *Cr*	*CI*	Semi-steel	*CI*	Steel	Steel	Steel
Balls, new charge, total weight, lb.	16,000	14,000		6000	4000	22,000	12,000	12,000
Balls, size, in. @ weight, lb.	2 @ 8000	3 @ 2100		5	3	4	5 @ 1000	5 @ 12,000
Balls, size, in. @ weight, lb.	1½ @ 4800	2 @ 9800				3½	4 @ 2500	
Balls, size, in. @ weight, lb.	7/8 @ 3200	*y* @ 2100					2½ @ 5000	
Balls, size, in. @ weight, lb.							2 @ 3500	
Balls, size added to compensate wear, in.		*z*	1				4 and 5	5
Balls, method of determining addition	*d*	10.4 lb. per hr.				*d*	*f*	*d*
Balls, consumption, pounds per ton	9-10*c*	2.3		4.5*p*	1.5*r*	1.8	0.17	1.5

Table 4. Performance of center-discharge cylindrical ball-mills—*Continued*

Plant	Granitic zinc ore	Granitic zinc ore	St. Joseph Lead Co., Rivermines
Size, diameter × length, ft.	7×10	8×6	8×6
Speed, r.p.m.	17	20	22
Tons new feed per 24 hr.	600	300	900
Tons total feed per 24 hr.	600	900	1200
Method of closing circuit	Open	....	Cones
Installed horsepower	150	200	225
Actual horsepower	135	246	210
Horsepower per ton of balls	11.2	18.9	16.8
Tons new feed crushed per horsepower-hour	0.185	0.051	0.178
Moisture in mill, per cent	36.8	30.1	....
Size of feed (*a*)	*4*	*5*	60%+2-mm.
Size of product (*a*)	*4*	*5*	86%−2-mm.
Attendance, machines per man	10	4	....
Lost time, per cent	....	....	1
Principal causes of lost time	Re-lining	Re-lining	Re-lining
Lubricant, kind/pounds per shift	O/1.33 G/0.75	O/1.33 G/0.75	O/1
Feeder, type	3-way	3-way	3-way
Feeder, material	***CI, h***	***CI, h***	***CI***
Feeder, life, days	360	850	***i***
Liners, type	Komata	Ring	Ship-lap
Liners, material	***WI***	***WI***	***Mn***
Liners, life, days	....	....	***j***
Liners, consumption, pounds per ton	....	....	....
Time for re-lining, hr.	16	32	....
Number of men for re-lining	4	6	....
Balls, material	Mang.	*Cr*	*Cr*
Balls, new charge, total weight, lb.	24,000	26,000	25,000
Balls, size, in. @ weight, lb.	3 @ 12,000	5	5
Balls, size, in. @ weight, lb.	2½ @ 6000	4	....
Balls, size, in. @ weight, lb.	2 @ 6000	....	....
Balls, size, in. @ weight, lb.	....	....	....
Balls, size added to compensate wear, in.	....	....	5
Balls, method of determining addition	....	....	***k***
Balls, consumption, pounds per ton	2.05	1.03	....

a Italic numbers refer to column numbers in Table 4*a*. ***b*** See page 347. ***c*** Very high due to extremely fine grinding and abrasive character of the pyritic concentrate ground. Small balls gave finer product but greatly increased ball consumption. ***d*** Fixed amount added daily. ***e*** One man attends 3 ball mills, rolls and conveyors and does miscellaneous floor work. ***f*** 40 lb. @ 5-in. and 24 lb. @ 4-in. added daily. ***g*** White iron, 35 days, 1.34 lb. per ton; semi-steel, 59 days, 0.81 lb. per ton; chrome steel, 81 days, 0.48 lb. per ton; manganese steel, 134 days, 0.50 lb. per ton. ***h*** White-iron tips. ***i*** In good condition after 3 years' service. ***j*** In good condition after 2½ years' service. ***k*** Added weekly to bring load up to desired level. ***l*** Inspection and re-lining. ***m*** Classifier overflow contains 0.1 per cent. on 10-mesh and 44.6 per cent. − 200-mesh. ***n*** Originally 5 × 16-ft. pebble mills. Converted by use of a false head of wood, 8 in. thick with standard 10-in. pipe delivering discharge to mill trunnion. ***o***, *114 J 1119.* Ore augite porphyrite; tougher and harder than any other American ore. ***p*** Lay estimates that ball consumption could be decreased by at least 50 per cent. by using chrome steel. The cast balls cost $0.06 per pound (1922). At the time they were bought chrome-steel balls cost three times as much. ***q*** Tailing from tables treating hydraulic-classifier spigot products from 7, Table 4*a*. ***r*** Per ton of feed to primary mill. ***s*** 45 to 50 tons less sand-table concentrate and slime from primary mill. See preceding column. ***t*** Drag overflow contains 3.2 per cent. on 65-mesh. See Sec. 6, Table 50. ***u*** Estimated. ***v*** Replacing scoop lips. ***w*** With manganese-steel lip. Life of lip, 8 to 12 days. ***x*** Liners and bolts weigh 16,000 lb. new. Set of liners furnished with mill came out 37.7 per cent. scrap. End liners wore out before shell liners and were patched with steel plates. Life of shell liners, 36 days. ***y*** 1½- to ½-in. scrap from Marcy mills (chrome and forged steel) or 1½-in. chrome-steel balls. ***z*** Same as original charge. ***aa*** Two 15-ft. Dorr bowl classifiers. ***CI*** Cast iron. ***Cr*** Chrome steel. ***DC*** Dorr classifier. **G,** Grease. **Mang.,** Manganoid. ***Mn*** Manganese steel. **O,** Oil. ***SS*** Spiral scoop, one-way. ***WB*** Wedge bar. ***WI*** White iron.

Table 4: Sizing tests referred to in Table 4

(Figures under the headings F and P are weight per cent.)

Reference number			1		2		3		4	
Plant			Liberty Bell		St. Joseph Lead, Bonne Terre		Consolidated Arizona Smelting Co.			
Screen aperture										
Mesh	In	Mm	F	P	F	P	F	P	F	P
	0.52	13.33								
	0.37	9.42					1.5			
3	0.26	6.68			4.9					
4	0.18	4.70			18.5		63.5	1.3		
6	0.13	3.33			10.4					
8	0.093	2.36			23.2	1.4	12.8	2.8		
10	0.065	1.65			21.5	3.7			7.9	0.4
14	0.046	1.17	0		15.5	6.1	5.8	7.6	9.4	0.1
20	0.033	0.83			4.8	8.1				
28	0.023	0.59			0.9	12.0			39.4	8.9
35	0.016	0.42	9.0			8.2	4.4	14.4	15.1	12.2
40										
48	0.012	0.30				10.6				
60										
65	0.008	0.21	15.2			9.1	4.4	27.2	25.8	28.7
80										
100	0.006	0.15	20.1			13.2		9.4	2.8	12.2
120										
150	0.004	0.10				10.5		4.5	0.6	7.0
200	0.003	0.07	32.5	0		1.9		7.4		
280									0.5	12.2
300			18.0	4.8						
Through last screen			5.2	95.2	0.3	12.2	7.6	25.4	0.6	18.4

Reference number			5		6		7	8	9	
Plant							Le Roi No. 2	Le Roi No. 2	Shattuck Arizona	
Screen aperture										
Mesh	In.	Mm.	F	P	F	P	P	P	F	P
	0.52	13.33	35.5							
	0.37	9.42	24.2		3.3					
3	0.26	6.68	16.5		5.6					
4	0.18	4.70	9.0		3.4	0.8			3.5	2.1
6	0.13	3.33	5.6		4.1	0.6				2.3
8	0.093	2.36	4.1		3.5	0.9				2.1
10	0.065	1.65	2.8	0.7	3.0	1.5	0			2.5
14	0.046	1.17	1.0	3.8	11.9	7.2				2.4
20	0.033	0.83	0.4	4.8	11.0	8.9	5.3			3.7
28	0.023	0.59	0.2	5.1	11.8	10.8				4.4
35	0.016	0.42	0.1	8.9	18.4	20.9				6.7
40							20.0			
48	0.012	0.30		8.1	10.8	13.6				12.4
60							4.2	4.1		
65	0.008	0.21		9.5	6.6	10.4			42.3	14.5
80							10.7	3.6		
100	0.006	0.15		9.7	2.6	6.4				16.3
120							11.5	6.6		
150	0.004	0.10		8.5	0.8	3.3				7.4
200	0.003	0.07		4.1	0.6	3.1			37.3	7.2
280										
300										
Through last screen			0.4	36.7	2.8	12.0	48.3	85.7	16.9	16.0

F = feed. *P* = product

the diameter of the mill. This drive showed 10 to 15 per cent. saving in power over ordinary belt-driven countershaft and spur gears on conical mills at CALUMET AND HECLA (*109 P 759*). Wormser (*114 J 763*) says that the opinion is commonly expressed at mills that he has visited that herringbone gears easily make up for the added cost in increased smoothness of operation. Dirt must be carefully excluded. A difficulty with direct connection is the end surge of the mill consequent upon the clearance that must be left between trunnion shoulders and mill bearings to compensate for expansion with temperature rise.

The pinion shaft is made of wrought iron or mild steel, of large diameter, set in long bearings with special provision made for aligning the countershaft with the mill axis. With herringbone gears some manufacturers attach the pinion-shaft bearings rigidly to the sole plate of the trunnio bearings, and attain alignment of mill and pinion shafts by moving the trunnion bearings on he sole plates.

Motors must be of a type that will stand heavy continuous duty at constant speed under constant load and must have large starting torque, ranging fr m 150 per cent. of full-load torque for long tube mills to 200 per cent. for short bal mills and rod mills. Wound-rotor motors are substantially necessary for large mills. Squirrel-cage motors designed for especially high starting torque may be sed for smaller mills. When clutches are used load conditions are ideal for synchronous motors. (See also Sec. 23, Art. 7.)

Individual drive is usual and, except in the case of a number of small mills, undoubtedly best from all points of view except economy in first cost. Group drive is more complicated and subject to mechanical difficulties, causes greater loss in plant capacity when out of repair, is more dangerous and more obstructive of headroom and floor space.

Performance of center-discharge ball mills is given in Tables 4 and 4*a*. Capacity ranges from 45 to 50 tons per 24 hr. in open circuit, reducing very hard ore from -1.5-in. to pass 10-mesh, to 900 tons per 24 hr. crushing soft ore from -9-mm. to -3-mm. Power consumption varies from 4.4 hp. per ton of balls in a 4×10-ft. mill to 18.9 hp. per ton in an 8×6-ft. mill; average for mills of typical shape (p. 345) is 13.2 hp. Tons crushed per horsepower-hour ranges from 0.162 to 0.276 in open-circuit crushing making 4- to 8-mesh product and is from 0.05 to 0.06 when making 10- to 20-mesh product.

2. Peripheral-discharge mills (Grate mills)

Cylindrical ball mill with peripheral discharge has a perforated diaphragm mounted near the discharge end and a pulp elevator located between the diaphragm and the discharge head. The original purpose of the grate was to hold back oversize material until ground to a size predetermined by the grate apertures, but this was quickly found to be impracticable and the function of the grate as now used is to confine the ball charge and to form a part of a quick-discharge device. In the type shown in Fig. 7 the discharge diaphragm (*a*) carrying gratings (*b*) is held against the discharge-end head by means of bolts (*f*). The diaphragm is protected by plates (*g*) and grating bars (*b*). The plates (*g*) are held by bolts (*e*) through the head and the grate bars are held in place by wedge blocks (*c*) which are in turn drawn up by bolts (*d*). The diaphragm carries a central discharge pipe (*h*), projecting through the center of the trunnion, which acts as an emergency overflow. If no such provision is made, an overloaded mill will discharge back through the feed trunnion and may require as much as two hours to work back to normal. Ribs (*k*) are cast

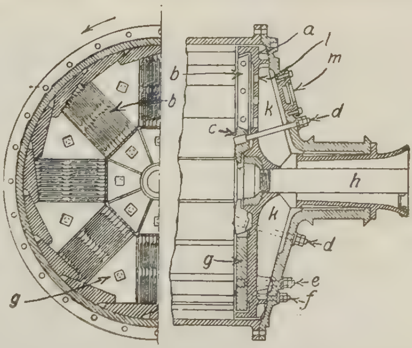


FIG. 7.—Grate discharge for Allis-Chalmers ball mill.

Table 5. Performance of peripheral-

Plant	Elko Prince	*D–E*	Catemu *V*
Size, diameter × length, ft	4×3½	4×4	5×4
Speed, r.p.m.	32	32.5	25
Tons new feed per 24 hr	50–60	50	*W*
Tons total feed per 24 hr	....	....	250
Method of closing circuit	Open	Open	*DC*
Installed horsepower	....	20	....
Actual horsepower	25*e*	....	50*e*
Horsepower per ton of ball charge	20.0	....	17.9
Tons new feed crushed per horsepower-hour	0.10	0.104	....
Moisture in mill, per cent	40	50	*X*
Size of feed (*a*)	−2-in.	*10*	−1.5-in.
Size of product (*a*)	*1*	*10*	*W*
Attendance, machines per man	....	....	....
Lost time, per cent	5–6	....	....
Principal causes of lost time	Re-lining	....	....
Lubricant, kind/pounds per shift	*Gr*	....	....
Feeder, type	Drum	Comb.	Comb.
Feeder, material	*CI*	....	....
Feeder, life, days	200*c*	....	....
Liners, type	Ship-lap	Step	Ship-lap
Liners, material	*Mn*	*Cr*	*Mn*
Liners, life, days	110–200	....	....
Liners, consumption, pound per ton	0.511	....	0.65
Time for re-lining, hr	8	....	....
Number of men for re-lining	3	....	....
Grates, type	....	....	....
Grates, material	*Cr*	....	....
Grates, opening, in	3/16	....	....
Grates, life, days	350	....	....
Grates, consumption, pounds per ton	....	....	....
Balls, material	*Cr*	*Cr*	*Cr*
Balls, new charge, total pounds	2000	2500	5600
Balls, size, inches @ weight, lb	5	3	2–4
Balls, size, inches @ weight, lb	4	4	....
Balls, size, inches @ weight, lb	....	5	....
Balls, size, inches @ weight, lb	....	....	....
Balls, added to compensate wear, in	5	....	....
Balls, method of determining addition	*d*	....	....
Balls, consumption, pounds per ton	1.25	....	1.43

a Italic numerals refer to column numbers in Table 5*a*. ***aa*** *109 J 845*. Ore very soft, slimes easily. ***ab*** Classifier overflow all passes 65-mesh (See Column 63, Table 40*a*, Sec. 6); 82 per cent. −200-mesh (*59 A 554*). ***ac*** With manganese-steel lip. ***ad*** Fixed amount added daily as calculated from tonnage and wear per ton. ***ae*** Classifier overflow, 0.2 per cent. on 48-mesh, 50 per cent. −200-mesh. ***b*** Classifier overflow, 2.8 per cent. on 20-mesh, 35 per cent. −200-mesh. ***c*** No liners. ***d*** Fixed weight added each day. ***e*** Estimated. ***f*** No regular attendant. ***g*** This is a 6 × 6-ft. tube mill lagged down with wood to 5 ft. in order to take balls. See Fig. 3. ***h*** Replaceable lip. Life of lip, 300 days. ***j*** Life of lip, 8 to 10 days. ***k*** Scoop lip of manganese steel. ***l*** Includes scrap. ***m*** Variations in amount of fines in mill feed cause range from 72 to 264 tons per 24 hr. new feed. ***n*** Interior diameter of shell 7 ft. Liners set in 6 in. and backed by concrete. ***o*** Changing grates and inspection. ***p*** Mills stopped every Monday morning to inspect ball load. Surface is measured and charted and subsequent ball addition made accordingly. ***q*** About 21 hp. consumed by resistance coils used to lower speed. ***r*** Classifier overflow all −35-mesh, 35 to 40 per cent. −200-mesh. ***t*** High-carbon forged steel, 3.2 lb. per ton; chrome steel, 1.9 lb. per ton; based on new feed. ***u*** Balls added as required to keep ammeter reading constant. ***v*** Loose bolts and re-lining. ***w*** Shell, 0.27 lb. per ton; end, 0.06 lb. per ton; total 0.33 lb. ***x*** Overload on return circuit. ***y*** 450 tons when taking coarse product from classifier. See note *c*, Table 5*a*. ***z*** Solid cast-iron scoops with no liners failed when half worn, in from 70 to 90 days. Sectional cast-iron scoops with hard-iron liners were in first-class condition after running

discharge ball mills (grate mills)

G–E	Nevada Packard	*F–E*	*H–E*	United Eastern	*I–E*	United Eastern
5×5	5×6*g*	5×6	5×6	5×6	6×4	6×4½
28	26	28	28	28	23.6	26
285	100	90	90	90	450	280
...	...	...	...	209	...	747
Open	Open	DC	*DC*	*DC*	*J*	*DC*
...	...	...	...	75	...	100
45	38	60	64	63	62.5	90
10.0	...	12.0	16.0	15.7	10.4	20.0
0.264	0.110	0.104	0.059	0.060	0.300	0.130
75	40	31	30	28–33	60	28–30
12	...	*11*	*13*	*23*	*14*	−2.5-in.
12	...	*11*	*13*	*23ab*	*14*	*2b*
...	*f*	...	...	5	...	5
...	6	...	...	...	...	...
...	Re-lining	...	...	...	...	Power
...	...	...	...	...	...	...
...	*SS*	Comb.	Comb.	*SS*	Drum	Comb.
...	Stl. *h*	...	...	*CI,ac*	...	*CI,k*
...	70*h*	...	...	...	...	360
...	Wave	Step	...	Wave	...	Ship-lap
...	*WI*	*Cr*	*Cr*	*Cr*	*Mn*	*Mn*
...	210	...	...	330	...	190
...	...	0.55	0.12	0.173	0.05	0.140
...	20	...	...	13–15	...	20–22
...	3	...	...	3	...	3
...	Slot	...	...	Radial	...	
...	*WI*	...	...		...	*Cr*
...	...	...	...	...	3/16	...
...	400	...	...	330	...	190
...	...	...	...	...	...	...
...	*WI*	*FS*	*FS*	*Cr*	*FS*	*Cr*
9000	...	10,000	8000	8000	12,000	9000
...	3	1½	1½	2	5	5
...	...	1¼	2	...	4	...
...	...	...	...	...	3	...
...	...	...	...	...	...	...
...	3	...	...	2	...	5
...	*d*	...	...	*ad*	...	*d*
...	1.25	3.8	2.1	3.2	0.13	0.91

50 days. *A* Consumption of 4-in. and 3-in. manganoid balls with 5-in. chrome-steel balls, in original charge, was excessive. *B* Classifier overflow, 0.7 on 48-mesh, 75 per cent. −200-mesh. *C* Classifier overflow, 1 per cent. on 28-mesh, 45 per cent. −200-mesh. *CI* Cast iron. *Cr* Chrome steel. *CS* Cast steel. *D* Hard quartz. *DC* Dorr classifier. *E* Allis-Chalmers catalog. *F* Quartz and calcite. *FS* Forged steel. *G* Silicious quartz-porphyry, very hard. *Gr* Grease *H* Calcite and quartz with some andesite. *I* Dolomite. *J* Circuit closed with elevator and screen. *K* Gold quartz ore. *L* Chalcopyrite in alaskite porphyry. *M* Marcy-mill catalog. *Mang.,* Manganoid. *Mn* Manganese steel. *N* Hardinge Co. Compare 6-ft. × 22-in. conical ball mill at the same plant (Table 11). Ore exceptionally hard and tough (*104 J 71*). O, Oil. *O′* Marcy, (*16 CME 344*). *P Bul. CMI, Mar., 1922.* *Q* End liners made 3¼ in. thick at place of greatest wear. Weight of a complete set of liners, 20,000 lb. *R* ⅜ × 1 15/16-in. slots. Slotted area 24 in. diameter. *S* Except for oversize of a conical screen with ⅜ × ⅝-in. openings attached to discharge end. Weight of returned material, 7 to 8 tons per 24 hr. *SS* Spiral scoop. *T* See Fig. 6. *U, 57 A 436.* *V, 123 P 888.* *W* 50 tons per day to 2 to 5 per cent. +100-mesh with soft ores and 2 per cent. + 48-mesh with hard ores. With −2-in. feed, the capacity on hard ores dropped to 42 tons. Capacity was a maximum with mill discharging at the periphery. *WI* White iron. *X* With 35 to 45 per cent. moisture the mill discharged at periphery (see note *W*). With 20 to 25 per cent. it could be made to discharge through central pipe. *Y, 99 J 693.*

Table 5. Performance of peripheral-discharge

Plant	Shattuck Arizona	Belmont Surf Inlet	*F–E*
Size, diameter × length, ft	6×4½	6×5	6×5
Speed, r.p.m.	23½	24	24
Tons new feed per 24 hr	200*m*	200	280
Tons total feed per 24 hr	600	300	....
Method of closing circuit	*DC*	*DC*	*DC*
Installed horsepower	80	100	....
Actual horsepower	75	90	90
Horsepower per ton of ball charge	13.6	11.2	11.6
Tons new feed crushed per horsepower-hour	0.111	0.092	0.130
Moisture in mill, per cent	30	50	38
Size of feed (*a*)	−0.5-in.	....	*15*
Size of product (*a*)	*3C*	....	*15*
Attendance, machines per man	3	2	....
Lost time, per cent	Small	6	....
Principal causes of lost time	Re-lining	Re-lining	....
Lubricant, kind/pounds per shift	....	....	....
Feeder, type	*SS*	Comb.	Comb.
Feeder, material	*CI,k*	*WI*	....
Feeder, life, days	270*j*	600	....
Liners, type	Block	Step	Ship-lap
Liners, material	*Mn*	*Cr*	*Cr*
Liners, life, days	154	120–150	....
Liners, consumption, pound per ton	0.60*l*	0.38	0.42
Time for re-lining, hr	30	....	....
Number of men for re-lining	....	....	....
Grates, type	Marcy		....
Grates, material	*Cr*	*Cr*	....
Grates, opening, in	¼	....	....
Grates, life, days	110	100	....
Grates, consumption, pounds per ton	....	....	....
Balls, material	*FS*	*Cr*	*FS*
Balls, new charge, total pounds	11,000	16,000	17,000
Balls, size, inches @ weight, lb	4	5	5
Balls, size, inches @ weight, lb	....	....	4
Balls, size, inches @ weight, lb	....	....	3
Balls, size, inches @ weight, lb	....	....	....
Balls, size added to compensate wear, in	4	5	....
Balls, method of determining addition	....	....	....
Balls, consumption, pounds per ton	2.65	2.4	1.2

on the back of the diaphragm and serve as lifters to elevate pulp discharged through the diaphragm to such a height that it can flow out through the trunnion. The level of pulp at the discharge end of the mill is determined by the distance of openings (*l*) from the periphery of the mill. If all openings are free, the pulp level at the discharge end of the mill will be relatively low and the rate of flow of pulp through the mill will be a maximum for a given feed rate and moisture content. If it is desired to slow down the rate of flow, with consequent finer grinding at one passage, plugs are introduced into the outer ring of openings (*l*), working through hand-holes (*m*), or the pulp density in the mill is increased. In another type of grate mill variation in rate of discharge is got by adjusting the width of

ball mills (grate mills)—*Continued*

K-E	Consolidated Arizona Smelting Co. (*N*)	Swansea Lease (*aa*)	Chino Consolidated Copper Co.	Engels C. M. Co.	*L–E*	Hollinger (*P*)
6×6	6×6	6×6	6×10*n*	6×12	7×6	7×6
24	23.5	26½	20	24	20	25
188	257	250	600	230	275	600
....	....	....	....	750	....	....
DC	Open	*DC*	Open	*DC*	Open	Open (*S*)
....	....	90	100	150	....	....
94	67.4	....	130	140	125	145
10.7	11.2	....	7.2	13.3	11.3	12.1–13.2
0.063	....	....	0.192	0.068	0.092	0.173
26.3	....	....	30	33	45	20
16	*20*	−2.5-in.	*4*	*24*	*17*	70% − 1-in.
16	*20*	90% −100-mesh	*4*	*24ae*	*17*	15% −200-mesh
....	....	....	4	3	....	....
....	....	....	1	1.2	....	....
....	....	....	*o*	Re-lining	....	....
....	....	....	*Gr*/0.7 O/0.6	*Gr*/0.8	....	....
Comb.	....	....	3-way	*SS*	Comb.	*T*
....	....	....	*CI,k*	*CI*	....	....
....	....	....	....	350	....	....
Ship-lap	Wave	....	Step	Rib	....	Step *Q*
Mn	*CI*	....	*WI*	*CI*	*WI*	*Mn*
....	....	....	365	300	....	240
0.24	....	....	....	0.29	....	....
....	....	....	8	12	....	....
....	....	....	6	5	....	....
....	....	....	Slot	....	....	Slot (*R*)
....	....	....	*WI*	*CI*	....	*CS*
....	....	....	½	....	....	....
....	....	....	30–60	90	....	180
FS	*FS*	*Cr*	*Mn*	Duraloid	*FS*	*Cr*
17,500	12,000	12,000-14,000	36,000	21,000	22,000	22,000-24,000
5	5	2	2 @ 24,480	2	5	....
4	....	....	1½ @ 4320	....	4	....
3	....	....	1¼ @ 3600	....	....	....
....	....	....	1 @ 3600	....	....	....
....	....	....	....	2	....	6
....	....	....	*p*	*ad*	....	....
1.35	1.34	0.33–0.5	1.114	2.3	2.0	0.67

radial lifting bars attached to the discharge-end head (Chalmers and Williams, adjustable quick-discharge ball mill). In yet other mills the rate of discharge is varied by varying the space between grate bars (Marcy).

The discharge grating is made of cast iron or cast steel, liner plates are normally made of manganese steel and grate bars of high-carbon or chrome steel. Grate bars may be rectangular in cross-section, or slightly wedge-shaped. Wedge-shaped bars are set with the greatest width toward the head end of the mill. There is less danger of clogging with wedge-shaped bars; on the other hand, the width of opening between bars increases as bars wear. Tangential setting of bars is favored by some manufacturers as giving freer discharge than radial setting. With this setting the pulp level may be varied

Table 5. Performance of peripheral-discharge

Plant	Utah Copper (*M*)	Inspiration (*M*)	Engels C. M. Co.
Size, diameter × length, ft	8×5	8×5	8×6
Speed, r.p.m.	....	....	20.7
Tons new feed per 24 hr	800–850	340	444
Tons total feed per 24 hr	....	....	....
Method of closing circuit	Open	*DC*	*DC*
Installed horsepower	....	....	225
Actual horsepower	130*Y*	....	272*q*
Horsepower per ton of ball charge	....	....	21.8
Tons new feed crushed per horsepower-hour	0.266	....	0.068
Moisture in mill, per cent	....	....	24
Size of feed (*a*)	*18*	*19*	*5*
Size of product (*a*)	*18*	*19*	*5B*
Attendance, machines per man	....	....	4
Lost time, per cent	....	....	3.5
Principal causes of lost time	....	....	Re-lining
Lubricant, kind/pounds per shift	....	....	*Gr*/1.2
Feeder, type	....	....	Comb.
Feeder, material	....	....	*CI*
Feeder, life, days	....	....	360
Liners, type	....	....	Block
Liners, material	....	....	*Mn*
Liners, life, days	....	....	273
Liners, consumption, pound per ton	0.12*Y*	....	0.221
Time for re-lining, hr.	....	....	15
Number of men for re-lining	....	....	7
Grates, type	....	....	Bar
Grates, material	....	....	*Cr*
Grates, opening, in.	....	....	....
Grates, life, days	....	....	210
Grates, consumption, pounds per ton	....	....	....
Balls, material	....	....	*Cr*
Balls, new charge, total pounds	....	....	25,000
Balls, size, inches @ weight, lb.	....	....	5
Balls, size, inches @ weight, lb.	....	....	....
Balls, size, inches @ weight, lb.	....	....	....
Balls, size, inches @ weight, lb.	....	....	....
Balls, size added to compensate wear, in.	....	....	5
Balls, method of determining addition	....	....	....
Balls, consumption, pounds per ton	0.62*Y*	....	1.4

by replacing wedge-shaped bars near the periphery by rectangular bars, thus slowing down pulp egress. Ordinary spacing of bars is from 3/32 in. to 1/4 in., or, in rare cases, more; depth is made 3 to 3½ in.

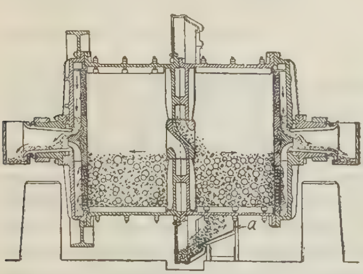


Fig. 8.—Fairchild double ball-mill.

Fairchild mill (Fig. 8) is a recent form of grate ball mill which aims, by central feed and double discharge to increase the tons ground per horsepower-hour by eliminating over-grinding. The feeder is of the double-spiral drum type and the grate and discharge show no variation in principle from standard forms.

Performances of grate mills at a number of plants are given in Tables 5 and 5*a*.

ball mills (grate mills)—*Continued*

Mexican cyanide mill	U. S. S. R. & M. Co., Midvale	U. S. S. R. & M. Co., Loreto	U. S. S. R. & M. Co., Guerrero	Sunnyside M. & M. Co.	Inspiration (*O'*)	St. Joseph Lead Co. (*U*)
8×6	8×6	8×6	8×6	8×6	8×6	8×6
23	23	22½	24	23	....	....
450	480	400	....	450*y*	475	750
....	1000	....	....	620	....	....
Open	*DC*	Open	Open	*DC*	*DC*	Open
225	225	225	225	225	....	....
225	190	225	225	210	257	200
14.5	13.6	14.5	14.5	14.0–17.5	....	14.3
0.083	0.105	0.074	....	0.089	0.047	0.160
25–35	20	25–35	25–35	30–35	....	65
6	−2.5-in.	*7*	*8*	*9*	*21*	*22*
6	*r*	*7*	*8*	*9*	*21*	*22*
0.67	3	0.067	0.5	3	....	....
8	....	8.5	....	....	....	....
Re-lining	....	*v*	....	*x*	....	....
0/0.16	0/1	0/0.16	0/0.16	0/0.5	....	....
Comb.	Comb.	*SS*	*SS*	Comb.	....	....
CI	....	*CI*	*CI*	*CI*	....	....
60–90	....	60	....	*z*	....	....
Cascade	....	Cascade	Cascade	Step	....	....
Mn	*Mn*	*Mn*	*Mn*	*Mn*	....	....
200	240	180	....	150	....	....
0.5	0.5	0.33*w*	....	0.65	....	....
44	....	44	....	....	....	....
14	....	14	....	....	....	....
Marcy		Marcy	Marcy	....	....	....
Cr	*Cr*	*Cr*	*Cr*	*Cr*	....	....
....	....	....	....	....	....	....
200	....	....	....	180	....	....
....	....	....	....	....	....	....
FS	*t*	*FS*	*FS*	*Cr,A*	....	....
31,000	28,000	31,000	31,000	24,000–30,000	....	28,000
5 @ 15,000	5	5 @ 15,000	5 @ 15,000	5	....	5
4 @ 10,000	....	4 @ 10,000	4 @ 10,000	....	....	4
3 @ 6000	....	3 @ 6000	3 @ 6000	....	....	3
....	....	....	....	....	....	....
....	5	....	....	....	....	5
....	*u*	....	....	*u*	....	....
2	*t*	3.5	....	2.3	....	....

At Afterthought Copper Co. (*119 P 154*) a 6 × 4½-ft. Marcy mill grinds 150 tons per 24 hr. from 6 per cent. on 10-mesh with 20 per cent. − 200-mesh to 5 per cent. on 60-mesh with 67 per cent. − 200-mesh. Moisture, 25.4 per cent. Sand return is 803 tons. At Trojan plant (*114 J 763*) a 6 × 6-ft. mill with 5-in. balls grinds 400 tons per 24 hr. from − 2-in. to 3 or 4 per cent. on 10-mesh.

Capacity, according to the figures from practice given in Table 5, ranges from 50 tons per 24 hr. in a 4 × 4-ft. mill grinding in open circuit from −2- or −3-in. to 8- or 10-mesh to 800 to 850 tons in an 8 × 6-ft. mill grinding in open circuit from 1-in. to 10-mesh. At Inspiration a 9 × 6-ft. mill grinds 535 tons per 24 hr. from 2½- or 3-in. to 48-mesh. Power consumption is from 7.2 hp. per ton of ball load in a 7 × 10-ft. mill to 20 hp. in a 6 × 4½-ft. mill and averages 12.9 hp. per ton. Tons per horsepower-hour ranges from 0.063 to 0.300, crushing to table or tube-mill feed, the average of 14 mills is 0.147; crushing to flotation size the range is from 0.047 to 0.111 and the average of 6 mills, 0.080.

Manufacturers of cylindrical ball mills. Abbé Engineering Co., Allis-Chalmers Mfg. Co., Chalmers and Williams, Denver Eng. Works Co., Mine and Smelter Supply Co., Power and Mining Machinery Co., Traylor Engineering Manufacturing Co.

Table 5*a*. Sizing tests referred to in Table 5.
(Figures under headings F and P are weight per cent.)

Reference number / Plant			1 Elko Prince	2 United Eastern	3 Shattuck Arizona	4 Chino Cons. Copper Co.		5 Engels C. M. Co.		6 Mexican cyanide plant		7 U. S. S. R. & M. Co., Loreto		8 U. S. S. R & M. Co., Guerrero	
Screen aperture: Mesh	Inch	Mm.	P	P	P	F	P	F	P	F	P	F	P	F	P
	3														
	2.5														
	2									12.1				27.9	
	1.5														
	1.25														
	1.05	26.67						41.2		40.5		33.0		24.0	
	0.74	18.83						5.8							
	0.52	13.33						9.4		29.6		16.0		17.6	
	0.37	9.42						7.4							
		7													
3	0.26	6.68						3.9							
		5			3.1										
4	0.18	4.70		0.8	2.8			4.5	4.1			16.5			
6	0.13	3.33		1.2	3.3			3.8	2.8	10.2	3.5		2.7	15.1	6.9
		3													
8	0.093	2.36		5.5	2.4			2.8	2.4			12.0			
		2													
10	0.065	1.65	19.2	9.0	3.9	5.4	0.1	2.8	3.2						
12	0.052									2.9	8.5	5.3		3.3	9.5
14	0.046	1.17		11.0	4.0	12.4	0.5	2.4	3.3						
20	0.033	0.83	13.1	10.0	7.3	18.7	1.3	2.0	3.3	1.3	8.3	5.8	16.5	2.0	9.5
28	0.023	0.59			9.2	22.2	3.3	2.0	3.5						
30															
35	0.016	0.42	15.3		12.1	19.4	4.9	1.7	4.8						
40	0.014									0.6	16.9	5.6	16.9	2.2	18.2
48	0.012	0.30		25.5	11.4	8.7	7.9	1.6	5.4						
60															
65	0.008	0.21	13.7	5.0	9.8	5.1	9.9	1.4	5.8						
80												1.2		2.6	22.9
100	0.006	0.15	7.7	5.0	7.6	2.9	12.4	1.8	7.6	0.5	21.7				5.5
120													22.5		
150	0.004	0.10	8.5	4.0	3.5	1.5	11.3	1.1	5.7	0.5	7.0		7.3		4.0
200	0.003	0.07	5.2	4.0	4.2	0.6	5.1	1.8	10.0	0.4	2.8		3.3		
240						0.3	1.4								
Through last screen			17.3	19.0	15.4	2.6	41.9	2.5	38.2	0.8	31.3	4.6	30.8	5.3	23.5

F = feed. P = product.

Plant			9			10		11				12		13		14		15	
Screen aperture			Sunnyside M. and M. Co.																
Mesh	Inch	Mm.	F(a)	P(b)	P(c)	F	P	F(d)	S(e)	P	O	F	P	F	P(f)	F	P	F	P(g)
	3		2.0																
	2.5					9.6													
	2		7.5																
	1.5					18.2												100	
	1.25																		
	1.05	26.67	9.0																
	0.74	18.83				20.8													
	0.52	13.33				17.8						0							
	0.37	9.42																	
		7														7.4			
3	0.26	6.68				10.6													
		5														20.4			
4	0.18	4.70																	
6	0.13	3.33																	
		3														32.9	6.4		
8	0.093	2.36																	
		2																	
10	0.065	1.65				9.3	0.5	5.4	11.0							15.9	16.0		11.0
12	0.052																		
14	0.046	1.17												0					
20	0.033	0.83				4.4	10.6					77.5	13.5	4		9.7	14.7		
28	0.023	0.59						10.5	24.4	1.2				6					20.6
30						1.4	11.2												
35	0.016	0.42												7	1		29.7		
40	0.014					1.2	12.8					14.0	27.0			10.5			
48	0.012	0.30			22.2			14.4	27.0	13.4				7	3				21.8
60						1.7	15.3					3.5	10.0			1.8	12.9		
65	0.008	0.21		0.3	12.0									11	7				
80				0.7		0.8	5.6												
100	0.006	0.15		5.0	23.2	2.0	10.2	28.2	27.1	41.3	11.6			20	14	0.6	9.1		22.8
120																			
150	0.004	0.10		7.3				6.2	4.2	13.3	13.6			29	25	0.3	3.5		6.2
200	0.003	0.07		11.2	10.0	0.9	13.3	3.4	2.2	3.0	11.6	3.5	32.5	16	18	0.5	2.7		3.1
240																			
Through last screen			71.5	75.6	32.6	2.3	20.5	31.6	3.8	27.8	63.0	1.0	17.0		32		5.0		14.5

a Size varies considerably with type of ore and condition of coarse crushers. The analysis given is a 30-day composite. *b* Classifier overflow with 240 tons per 24 hr. of new feed and 20 to 22 per cent. solids in overflow. *c* Classifier overflow with 450 tons per 24 hr. new feed and 60 per cent. solids in overflow. *d* Feed to classifier. *e* Feed to mill. *f* Classifier overflow, 2 per cent. on 100-mesh, 59 per cent. −200-mesh. *g* Classifier overflow, 6.3 per cent. on 100-mesh, 70.3 per cent. −200-mesh. O = Classifier overflow. S = Classifier sand.

Table 5a. Sizing tests referred to in Table 5—*Continued*

Reference number			16		17		18		19		20		21		22		23		24	
Plant							Utah Copper Co.		Inspiration		Cons. Ariz. Sm. Co.		Inspiration		St. Joseph Lead Co.		United Eastern		Engels C. M. Co.	
Screen aperture																				
Mesh	Inch	Mm.	F	P(*h*)	F	P	F	P	F	P	F	P	F	P	F	P	F	P	F	P
	3		2.0																	
	2.5																			
	2		15.2		20															
	1.5				15				4.7				17.7							
	1.25														27.7					
	1.05	26.67	36.5		15		1.7		16.4		9.3		16.5							
	0.74	18.83	7.5																	
	0.52	13.33	7.9																	
	0.37	9.42							35.2		19.7		24.7		41.3					
		7																		
3	0.26	6.68	12.2				39.3		7.0				7.3							
		5																		
4	0.18	4.70									49.3	0.8							8.4	2.2
6	0.13	3.33							7.4				7.7						5.2	0.7
		3													14.1	20.1				
8	0.093	2.36					24.2	2.5	3.4		7.6	3.4	2.2						5.1	0.7
		2													7.8	9.1				
10	0.065	1.65		11.0	50	7.7													5.9	0.7
12	0.052																			
14	0.046	1.17					11.2	13.0	3.6		3.7	7.6	4.8						5.4	0.7
20	0.033	0.83		18.5		14.2									4.4	21.9	4.0		5.3	1.0
28	0.023	0.59					5.8	16.7	3.4				3.6			8.4	5.4	0.4	6.0	1.5
30				18.0																
35	0.016	0.42					1.6	7.7			2.2	13.4			2.0	5.7	7.0	0.5	7.8	3.5
40	0.014			10.0		21.8														
48	0.012	0.30					1.5	6.9	2.6	0.6			2.9	2.9		5.2	7.6	2.6	7.5	6.2
60				13.3		13.5														
65	0.008	0.21					1.4	6.4		6.0	2.6	26.3		7.9	0.9	4.0	10.0	7.5	8.2	9.9
80				4.0																
100	0.006	0.15		2.2		10.5	2.0	6.0	2.4	14.4		10.2	2.3	12.6		3.9	20.0	13.0	10.3	17.5
120																				
150	0.004	0.10		7.0			1.7	6.1				1.1		12.6	0.8	4.0	26.0	25.0	6.9	15.1
200	0.003	0.07		6.0		7.0	1.4	3.2	8.0	19.1		13.4	1.5	5.8		3.0	12.0	17.0	7.6	14.3
240																				
Through last screen			18.7	10.0	5	25.3	8.2	31.5	5.9	59.9	5.6	23.8	8.8	58.2	1.0	14.7	8.0	34.0	10.3	26.0

h Classifier overflow, 0.7 per cent. on 30-mesh, 47.3 per cent. — 200-mesh.

3. Grate mills *vs.* center-discharge mills

Grate mills consume more power, with a given ball load than simple trunnion-discharge mills, and steel consumption and repair costs per mill are also higher, but on most ores the capacity is sufficiently greater to more than counterbalance the higher operating cost. As a result the majority of cylindrical ball mills at the present time are grate mills. Tellam (*106 J 583*) states, on the other hand, that at Smuggler Union Mining Co. the capacity of a grate mill from 4- to 60-mesh was 3.03 tons per 24 hr. per hp. consumed against 4.67 tons in a center-discharge mill. Consumption of cast-iron balls in the grate mill was 2.5 lb. per ton and in the center-discharge mill 2 lb. As a result the grate was removed and the mill operated as a center-discharge mill. A similar conclusion was reached from a different point of attack at St. Joseph Lead Co. (*66 A 108*). Here the end sought was maximum production of −10-mesh material with minimum production of −150-mesh slime. Experiments showed that the desired condition corresponded to the maximum tonnage that could be forced through the mill, the circuit being closed on a 2-mm. screen. With a 6 × 4-ft. grate mill the feed rate was limited to 440 tons of −9+2-mm. feed, producing 340 tons −10-mesh material per day, by the fact that with more than 25 per cent. of +10-mesh material in the discharge the mill choked. The same mill without grate took 600 tons per day total solid and produced 390 tons of −10-mesh material. Del Mar (*106 J 14*) presents the data in Table 6 to show the effect of grate and lifters

Table 6. Comparison of products of a 4 × 4-ft. ball mill with and without quick discharge. (*After Del Mar*)

Screen, Tyler mesh	Weight, per cent.					
	A	*B*	*C*	*D*	*E*	*F*
4	9.5	....	....	....	....	....
6	3.1	....	....	....	....	....
8	5.5	....	....	....	....	....
10	6.0	....	....	....	....	....
14	4.7	....	....	....	....	....
20	4.7	....	....	....	....	....
28	4.7	....	1.2	....	....	....
35	7.2	6.5	0.9	....	....	....
48	8.3	28.5	1.6	0.5	....	1
65	6.2	23.5	1.3	4.0	....	7
100	11.7	17.5	13.8	31.5	6.9	25
150	14.1	13.5	45.0	34.0	24.8	25
200	4.6	3.5	17.4	12.0	8.3	17
−200	9.7	7.0	18.5	18.0	60.0	25

A Quick discharge with lifters set at maximum. *B* Grate and lifters removed. *C* Grate in, one lifter operating, other 3 out. *D* Grate in, all four lifters out. *E* Classifier overflow, mill operating with quick discharge. *F* Classifier overflow, mill with grate and lifters removed.

on the character of the mill product. He states that the maximum capacity of the mill with quick-discharge mechanism was 56 tons per day, while it ran up to 88 tons as a center-discharge mill. He attributes the difference to the large circulating load in the grate mill (see Test A) and to under-work near the discharge end due to low pulp level. His conclusions are not wholly convincing, due to lack of data concerning other operating conditions.

Differential grinding. It is frequently claimed that in closed-circuit grinding the sulphide mineral, being more friable than the gangue, is slimed more than the gangue and, further, that the center-discharge mill is a worse offender in this regard than the grate mill. Neither of these contentions is borne out by the available data. In Table 7 it is seen that in the tube-mill feed, considering the -200-mesh material, 14.7 per cent. of the total solid contains 25.7 per cent. of the total sulphide, while in the classifier overflow 81.3 per cent. of the total solid carries only 85.1 per cent. of the total sulphide, a distinct impoverishment of the slime, relative to the coarser grades, in the process of grinding and classifying. Comparing the results in Tables 8 and 9, and considering -200-mesh material, the first shows 0.87 per cent. of the total lead in 0.2 per cent. of the total solid in the mill feed, while in the product 23.8 per cent. of the total solid contains only 44.97 per cent. of the total lead. Considering the oversize and undersize on 100-mesh, the former assays 6.03 per cent. Pb and the latter 11.8 per cent. in the feed; the former 4.52 per cent. and the latter 13.6 per cent. in the product. Here there is undoubtedly some concentration, indicating differential grinding. Taking the ratios of the assays as a measure of the concentration, the ratio of assays of fine to coarse in the feed is 1.96, and in the product, 3.01. The ratio of these ratios is, then, an index of the differential grinding in this center-discharge mill, and is 1.53. Similar analysis of Table 9 shows that the -150-mesh material in the feed to the grate mill comprises 1 per cent. of the total weight and contains 3.1 per

Table 7. Effect of tube milling and classification on distribution of sulphides, McIntyre-Porcupine mill (*20 CMI 100*)

Screen mesh	Conical ball mill discharge			Tube-mill feed		
	Per cent. weight	Per cent. FeS_2	Per cent. total FeS_2	Per cent. weight	Per cent. FeS_2	Per cent. total FeS_2
+20	10.0	7.0	6.3	7.0	16.0	6.3
40	23.2	8.5	17.8	12.8	14.0	10.0
60	13.0	11.5	13.5	14.3	11.0	8.9
80	4.3	12.0	4.6	9.6	11.0	6.0
100	9.0	14.0	11.4	21.4	15.0	12.4
200	6.7	13.0	7.9	20.2	27.0	30.7
−200	33.8	12.5	38.5	14.7	24.0	25.7
Total	....	11.30	....	....	17.73	....

Screen mesh	Tube-mill discharge			Final pulp, classifier overflow		
	Per cent. weight	Per cent. FeS_2	Per cent. total FeS_2	Per cent. weight	Per cent. FeS_2	Per cent. total FeS_2
+20	....	....	....	....	....	....
40	2.4	13.0	2.0	....	....	....
60	12.1	14.0	11.2	....	....	....
80	7.2	13.0	6.1	....	....	....
100	18.2	15.0	18.0	7.4	8.0	5.5
200	20.4	19.0	25.6	11.3	9.0	9.4
−200	39.7	14.0	37.1	81.3	11.2	85.1
Total	....	15.1	....	....	10.7	....

cent. of the total lead while in the product it comprises 17.7 per cent. of the total and contains 37.7 per cent. of the total lead. The oversize of the 65-mesh screen in the feed assays 0.98 per cent. Pb and the undersize 2.33 per cent.; in the product, oversize assays 0.61 per cent. and the undersize 2.12 per cent.

Table 8. Concentration of sulphide in different sizes in the product of a center-discharge ball mill

Screen, mm.	Feed				Product			
	Weight, per cent.	Assay, per cent. Pb	Tons Pb per 100 tons feed	Per cent. of total lead	Weight, per cent.	Assay, per cent. Pb	Tons Pb per 100 tons feed	Per cent. of total lead
6.68	2.6	5.7	0.147	2.41	....	....	....	....
4.70	11.8	5.1	0.601	9.87	0.7	0.56	0.004	0.05
3.33	20.4	5.0	1.019	16.72	2.1	1.09	0.023	0.30
2.36	22.0	2.9	0.638	10.47	2.9	0.88	0.025	0.33
1.65	14.1	3.5	0.493	8.10	4.5	1.30	0.058	0.76
1.17	11.9	6.8	0.809	13.27	6.6	1.44	0.095	1.24
0.83	4.8	12.6	0.405	6.65	6.5	2.70	0.175	2.28
0.59	3.5	18.6	0.651	10.68	8.5	3.20	0.272	3.54
0.42	2.6	18.0	0.468	7.68	8.4	5.30	0.445	5.79
0.30	2.2	18.0	0.396	6.50	8.5	6.00	0.510	6.64
0.21	1.5	14.0	0.210	3.45	6.7	7.70	0.516	6.72
0.15	1.6	8.6	0.138	2.27	9.0	8.70	0.783	10.19
0.10	0.7	7.8	0.055	0.90	7.3	10.60	0.773	10.06
0.07	0.1	10.4	0.010	0.16	4.5	12.20	0.548	7.13
−0.07	0.2	26.4	0.053	0.87	23.8	14.50	3.455	44.97

Table 9. Concentration of sulphide in different sizes in product of a grate ball mill

Screen aperture	Feed			Product		
	Weight, per cent.	Relative assay, per cent. Pb	Per cent. of total Pb	Weight, per cent.	Relative assay, per cent. Pb	Per cent. of total Pb
1.25-in..........	27.7	0.98	27.1	....	....	....
9-mm............	41.3	1.08	44.5	....	....	....
3-mm............	14.1	1.17	16.5	20.1	0.26	5.2
2-mm............	7.8	0.44	3.4	9.1	0.40	3.6
20-mesh.........	4.4	0.34	1.5	21.9	0.46	10.0
28..............	....	....	....	8.4	0.81	6.8
35..............	2.0	0.75	1.5	5.7	0.97	5.5
48..............	....	....	....	5.2	1.48	7.7
65..............	0.9	1.33	1.2	4.0	1.70	6.8
100.............	....	....	....	3.9	1.97	7.7
150.............	0.8	1.50	1.2	4.0	2.25	9.0
200.............	....	....	....	3.0	2.03	6.1
Through last screen.........	1.0	3.10	3.1	14.7	2.15	31.6

Differential grinding is indicated here likewise. Applying the same measure as before, the ratio of assays in the feed is 2.38, in the product 3.48 and the mill index 1.46. Hence there is no substantial difference in the relative action of the two mills toward gangue and friable sulphide.

4. Conical ball mill

One form is shown in Fig. 9. The principal structural difference between this and the center-discharge cylindrical mill is in the shape of the shell. Standard sizes are given in Table 10. This mill was developed from the conception that as the size of the material in the mill diminishes toward the discharge end, the crushing forces should likewise be diminished, and that the cylindro-conical shell would effect such diminution, both by causing a graduated size classification of the grinding media with the largest in the section of greatest diameter, and by graduated variation in the length and character of drop of the balls from parabolic fall of maximum length at the section of greatest diameter to cascade (roll) of minimum length near the discharge trunnion. Size segregation of the crushing bodies is not sharp, but there is distinct concentration of the larger sizes at the longest diameter and *vice versa*, as is shown by Fig. 10 and also by the fact (*124 P 209*) that in changing from a 2-in. to a 4-in. ball load, charging 8 tons of 4-in. balls forced out an equal tonnage of the smallest sizes. Another and, possibly, more important effect of the discharge-end cone is the aid that it affords to egress of ground material. Mills are usually lined in such a way that the inner surface of the cone is ribbed parallel to the mill axis. While the ribs are rising below the pulp level they act as lifters for the pulp and when in a position a short distance above the mill axis they form the lower sides of troughs directed toward the discharge trunnion, down which the lifted pulp flows. This quick-discharge effect is somewhat preferential and makes for rapid discharge of finished material from the mill, thus increasing capacity.

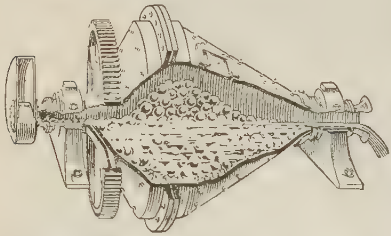


Fig. 9.—Hardinge conical ball mill.

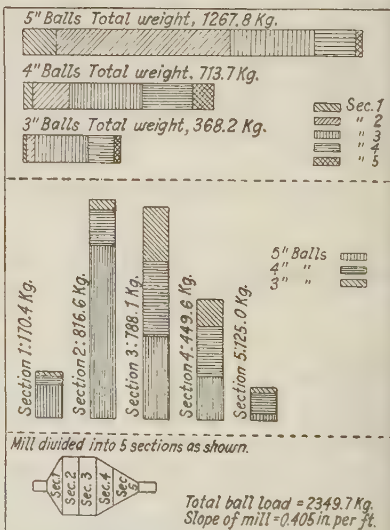


Fig. 10.—Arrangement of balls by sizes in a conical ball mill.

Construction. The shell is usually made of cast iron or cast steel, in two parts, with a tongue-and-groove flanged joint in the cylindrical section. Trunnions are cast integral with these sections and after the flanges have been faced the two parts are bolted together and the trunnions turned down in a lathe. A lighter form of mill is made of riveted boiler plate, with cast trunnions bolted on and then turned down Gears and bearings are similar to those on cylindrical mills (see p. 346). An incidental structural advantage of the conical shape is the fact that it allows a driving pulley to be placed on the pinion shaft between trunnions near the discharge end. This makes for compactness. When, however, the

[illegible]

Manufacturer. Hardinge Co.

Table 10. Conical ball mills (Hardinge Co. catalog)

Size of mill	Floor space	Approximate weights, pounds			Horse-power required	Size of motor, horsepower
		Mill	Lining	Ball charge		
2' × 8"	[illegible]	750	375	[illegible]	1	2
3' × 8"	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	7½
4½' × 16"	7' × 19'	6,500	4,300	[illegible]	[illegible]	25
6' × 22"	9' × 19'	10,200	7,800	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	14,000	14,000	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

Performance is shown in Tables 11 and 11a.

[illegible] circuit, to tube-mill feed and consumes 110 hp.

Capacity, according to Table 11, ranges from 30 tons per 24 hr. in a [illegible] mill grinding [illegible] feed in open circuit to [illegible]-mesh to 1015 tons in an [illegible] mill grinding [illegible] feed to [illegible] per cent. on [illegible]-mesh. Power consumption varies from [illegible] hp. per ton of ball load to [illegible] hp., the [illegible] figure corresponding to an undersized of balls. Average for mills of all sizes with normal loads is [illegible] hp. per ton of balls and the range for such mills is not over 2 hp. either side of the average. Tons crushed to table and tube-mill feed sizes, per hp.-hr. consumed, ranges from [illegible] to 0.417, average [illegible]; and from [illegible] to [illegible], average [illegible], in grinding to flotation size.

5. Conical ball mill vs. cylindrical ball mill

At [illegible] Copper Co. a conical mill running side-by-side with a grate mill averaged 250 tons per 24 hr. against 210 tons for the grate mill. Power consumption for the conical mill was 160 hp. and for the grate mill [illegible] hp.; [illegible] tons per horse-power-hour were [illegible] and [illegible] respectively. The conical mill lost much less time and was less sensitive to feed changes. The feed to the grate mill had to be watched continuously and occasionally shut off for 30 minutes or more to allow it to work out of a choked-up condition [illegible] wood chips. There was slightly more fine material in the open-circuit discharge of the grate mill, but the classifier overflows were substantially the

Table 11. Performance of conical ball mills

Plant	St. Joseph Lead Co., Bonne Terre	Vipond Porcupine	Catemu (*at*)	Pittsburg Dolores	McIntyre Porcupine	U. S. S. R. & M., Midvale	Miami Copper Co.
Size, outside diam. in ft. × length of cylinder in in.	3×8	4½×13	4½×16	5×16	6×16	6×16	6×16
Speed, r.p.m.	37	33	34	28	29	27½	28
Tons new feed per 24 hr.	30	48	20	60	150 *ao*	135	351
Tons total feed per 24 hr.							
Method of closing circuit	Open	Open	*DC*	Open	Open	Open	Open
Installed horsepower				50	50		
Actual horsepower	7.5	15 17	18		40	41	35
Horsepower per ton of ball charge	15.0	8.0	10.3		10.8	10.0	8.8
Tons new feed crushed per horsepower-hour	0.166	0.125	0.046		0.156	0.137	0.417
Moisture in mill, per cent	50	50	30–35	40	35	35–42	44
Size of feed (*a*)	–05-in.	*37*	–1.5-in.	–1.25	*1 ao*	*2*	*25*
Size of product (*a*)	–6-mesh	*37*	12%+80-mesh(*au*)	–12-mesh	*1 ao*	*2*	*25*
Attendance, machines per man	3+			1	7		
Lost time, per cent					2		
Principal causes of lost time					Re-lining		
Lubricant, kind/pounds per shift							
Feeder, type	Drum			*SS*	*SS*		
Feeder, material	*CI*			Stl.	*CI*		
Feeder, life, days					160		
Liners, type				Ribbed	*WB*		
Liners, material	*CI*			*CI*	*Mang.*		
Liners, life, days				60	120		
Liners, consumption, pounds per ton					0.3		
Time for re-lining, hr.	4			8	16		
Number of men for re-lining	2			2	4		
Balls, material	*CI*		*Cr*	*CI*	*FS*	*FS*	
Balls, new charge, total weight, lb.	1000	4000	3500	6000	7500	8100	8000
Balls, size, inches @ weight, lb.	2 @ 300		4	5 @ 4000	5	2½ @ 5000	
Balls, size, inches @ weight, lb.	1½ @ 700		2	2 @ 2000		2 @ 3100	
Balls, size, inches @ weight, lb.							
Balls, size, inches @ weight, lb.							
Balls, size added to compensate wear, in.	2				5	2½	
Balls, method of determining addition					*c*	*c*	
Balls, consumption, pounds per ton	0.4			7*b*	0.8	0.8 0.9	

Plant	Britannia M. & S. Co.	Buckhorn Mines Co.	Tul Mi Chung (*ap*)	St. Joseph Lead Co., Bonne Terre	Bunker Hill & Sullivan	Cons. Arizona Sm. Co.	Miami Copper Co.	Asturiana de Minas (*as*)
Size, outside diam. in ft. × length of cylinder in in.	6×16	6×16	6×16	6×22	6×22	6×22	6×22	6×22
Speed, r.p.m.	28	28	27.5	25	26	24	26	27
Tons new feed per 24 hr.	251	160	155	275	160	120 *ac*	106	200
Tons total feed per 24 hr.								30
Method of closing circuit	Open	Open	*DC*	Open	Open	Open	Drag	2-mm.scr.
Installed horsepower			60	50		75		
Actual horsepower	38–40	33.2	39	47	45	40	45	45
Horsepower per ton of ball charge	9.5	8.3	7.8	7.8	7.5	11.4	7.5	9.0
Tons new feed crushed per horsepower-hour	0.268	0.201	0.166	0.239	0.148	0.125	0.098	0.185
Moisture in mill, per cent.	40	80	33	45	70		46	
Size of feed(*a*)	*38*	*39*	*36*	*35*	*3*	*4*	*21*	*41*
Size of product(*a*)	*38*	*39*	*36* *aq*	*35*	*3*	*4*	*21*	*41*
Attendance, machines per man				3+	*d*	2		
Lost time, per cent.								
Principal causes of lost time				Re-lining	Re-lining	Power		
Lubricant, kind/pounds per shift								
Feeder, type				Drum	*SS*	*SS*		
Feeder, material			*CCr*	*CI*	*CI*	*f*		
Feeder, life, days					360	150		
Liners, type					*WB*	*WB*		
Liners, material				*Mn*	*Mn*	*Mn*		
Liners, life, days					*e*	201		
Liners, consumption, pounds per ton			0.208 *ar*					
Time for re-lining, hr.				12	16	12		
Number of men for re-lining				2	2	6		
Balls, material	*CI*		*Cr*	*FS*	*Mang.*	*FS*		*FS*
Balls, new charge, total weight, lb	8200	8000	10,000	12,000	12,000	7000	12,000	10,000
Balls, size, inches @ weight, lb.				2½ @ 6000	3 @ 7200	5	4	
Balls, size, inches @ weight, lb.				2 @ 4000	2½ @ 2400			
Balls, size, inches @ weight, lb.				1½ @ 2000	1 @ 2400			
Balls, size, inches @ weight, lb.								
Balls, size added to compensate wear, in.				2½		5		
Balls, method of determining addition				*c*		*c*		
Balls, consumption, pounds per ton	0.72	0.45	0.47	0.33	1.6	0.9–1.2	2	

For explanation of reference letters, see page 380.

Table 11. Performance of conical ball mills—*Continued*

Plant	Inspiration	Anaconda (*y*)	Anaconda (*y*)	Miami Copper Co. (*al*)	Mesabi Iron Co.	Mesabi Iron Co.	Mesabi Iron Co.	Mesabi Iron Co.
Size, outside diam. in ft. × length of cylinder in in.	6×48	7½×72*z*	7½×72*z*	8×22	8×22	8×22	8×22	8×22
Speed, r.p.m.	24	20	15	21	23.8	23.8	23.8	23.8
Tons new feed per 24 hr.	134	300	425	396–443	367	302	259	177
Tons total feed per 24 hr.	...	...	...	...	...	...	...	...
Method of closing circuit	Drag	*DC*	*DC*	Open	*DC*	*DC*	*DC*	*DC*
Installed horsepower	...	...	...	...	...	...	...	...
Actual horsepower	78	198	136	75	145	145	146	145
Horsepower per ton of ball charge	...	...	8.5	12.5	10.4	10.4	10.4	10.4
Tons new feed crushed per horsepower-hour	0.072	0.063	0.135	0.233	0.105	0.087	0.074	0.051
Moisture in mill, per cent	30	56	50–60	45	...	...	...	30
Size of feed(*a*)	*20*	*13*	*14*	*24*	*26*	*27*	*28*	*29*
Size of product(*a*)	*20*	*13*	*14*	*24*	*26*	*27*	*28*	*29*
Attendance, machines per man	...	...	...	...	...	...	...	...
Lost time, per cent	...	...	...	...	...	...	...	...
Principal causes of lost time	...	...	...	...	...	...	...	...
Lubricant, kind/pounds per shift	...	...	...	...	...	...	...	...
Feeder, type	...	...	...	...	...	...	...	...
Feeder, material	...	...	...	...	...	...	...	...
Feeder, life, days	...	...	...	...	...	...	...	...
Liners, type	...	...	*ab*	Ribbed	...	...	...	...
Liners, material	*Mn*	...	...	*CI*	...	...	...	...
Liners, life, days	...	...	...	...	...	...	...	...
Liners, consumption, pounds per ton	...	...	...	...	...	...	...	...
Time for re-lining, hr.	...	...	...	...	...	...	...	...
Number of men for re-lining	...	...	...	...	...	...	...	...
Balls, material	*Mang.*	*CI*	*Mang.*	*Mang.*	...	...	...	...
Balls, new charge, total weight, lb.	...	...	32,000	12,000	28,000	28,000	28,000	28,000
Balls, size, inches @ weight, lb.	...	...	...	3	3	2⅜	2½ to 5	2⅜
Balls, size, inches @ weight, lb.	...	...	...	2	2	2	...	2
Balls, size, inches @ weight, lb.	...	...	...	...	...	...	...	...
Balls, size, inches @ weight, lb.	...	...	...	...	...	...	...	...
Balls, size added to compensate wear, in.	...	...	...	...	...	...	...	...
Balls, method of determining addition	...	...	...	...	...	...	...	...
Balls, consumption, pounds per ton	2.6	3.0	2.75*aa*	1.25	...	...	...	...

Plant	Mesabi Iron Co.	Cananea Cons. Copper Co.	Federal Lead, Mill No. 4	Utah Leasing Co. (*u*)	Dome	Nevada Cons. Copper Co.	Nevada Cons. Copper Co.	Copper Range
Size, outside diam. in ft. × length of cylinder in in.	8×22	8×28	8×30	8×30	8×30	8×30	8×30	8×30
Speed, r p m	23.8	25	27	17*x*	24	24½	28	27
Tons new feed per 24 hr	157	160*ae*	855	270	496	408	451	150
Tons total feed per 24 hr								
Method of closing circuit	*DC*	Drag	Open	*DC*		*DC*	*DC*	Open
Installed horsepower		125	125	125				50
Actual horsepower	145	110*i*	116	100*v*	113.2	119.8	150	65
Horsepower per ton of ball charge	10.4	13.7	7.2	6.2		9.6	12.0	14.4
Tons new feed crushed per horsepower-hour	0.045	0.061	0.307	0.112	0.182	0.204	0.125	0.096
Moisture in mill, per cent		70	35	25.5	46	30.3	28	55
Size of feed (*a*)	*30*	*5*	*6*	*11*	*18*	*31*	*32*	−0.25
Size of product (*a*)	*30*	*5*	*6*	*11w*	*18*	*31aj*	32*ak*	*ax*
Attendance, machines per man		5	*k*					36
Lost time, per cent		2	2−					
Principal causes of lost time		Re-lining	*l*					Re-lining
Lubricant, kind/pounds per shift		*O/2*	*O/2* *G/1*					
Feeder, type		*SS*	*SS*					*SS*
Feeder, material		*g*	Stl.					*CI*
Feeder, life, days		180						600
Liners, type			*WB*					
Liners, material		*h*	*Mn*		*Cr*			*Mn*
Liners, life, days		325						154
Liners, consumption, pounds per ton		0.175						0.25
Time for re-lining, hr		3 days						27
Number of men for re-lining		7						4
Balls, material		*CI*	*Cr*	*Mang.*	Stl.	*CI*	*CI*	*Mang.*
Balls, new charge, total weight, lb	28,000	10,000	32,000	32,000		25,000	25,000	9000
Balls, size, inches @ weight, lb	2	3	4 @ 14,400	2½		3 @ 10,000	4 @ 10,000	3
Balls, size, inches @ weight, lb	1		3 @ 14,400			2½ @ 8500	3 @ 8500	
Balls, size, inches @ weight, lb			2 @ 3200			1½ @ 6500	2½ @ 6500	
Balls, size, inches @ weight, lb								
Balls, size added to compensate wear, in			*m*					
Balls, method of determining addition		*c*	*c*					*c*
Balls, consumption, pounds per ton		4.8–5.2*j*	0.6		0.4	1.34	1.55	0.75

For explanation of reference letters, see page 380.

Table 11. Performance of conical ball mills—*Continued*

Plant	Old Dominion	Engels C. M. Co.	Federal M. & S., Morning	McIntyre Porcupine	Cons. Arizona Sm. Co.	Cons. Arizona Sm. Co.	Homestake	Miami Copper Co.
Size, outside diam. in ft. × length of cylinder in in.	8×36	8×36	8×36	8×36	8×36	8×36	8×36	8×36*ah*
Speed, r.p.m.		20.6	27	22	26	16	24	20.6
Tons new feed per 24 hr.	250	236	198	650	150	141.4	378	967*ag*
Tons total feed per 24 hr.			670					
Method of closing circuit	*DC*	Open	Drag	Open	*DC*	*DC*	*DC*	Open
Installed horsepower	150	150	150	150				
Actual horsepower		140	132	148	97.5	100.5	170.3	136.4
Horsepower per ton of ball charge		11.2	12.0		8.5	7.2	12.6	8.0
Tons new feed crushed per horsepower-hour		0.070	0.062	0.183	0.064	0.059	0.092	0.295
Moisture in mill, per cent	32	35	30	35	50	65	37	54.5
Size of feed(*a*)	*n*	*7*	*8*	*9*	*15*	*16*	*19*	*22*
Size of product(*a*)	*n*	*7*	*8*	*9*	*15*	*16*	*19*	*22ag*
Attendance, machines per man		4	6	*k*				
Lost time, per cent		6	1					
Principal causes of lost time		Re-lining	Re-lining	Re-lining				
Lubricant, kind/pounds per shift		*G*/0.8	*O*/2					
Feeder, type	*SS*	Comb.	*SS*	Schmidt				
Feeder, material	*CI*	*CI*	*CI*	*CS*				
Feeder, life, days		360	180					
Liners, type	*WB*	*WB*		*WB*	*WB*	*WB*		
Liners, material		*CCI*	*CI*	*Mn*				*Mn*
Liners, life, days		110	90		150			
Liners, consumption, pounds per ton		0.6	1.0*q*					
Time for re-lining, hr.		*o*	8	12				
Number of men for re-lining		*o*	4	5				
Balls, material	*Mang.*	*Cr*	*CI*					
Balls, new charge, total weight, lb.	32,000	25,000	22,000		23,000	28,000	27,100	31,000
Balls, size, inches @ weight, lb.	2	5	2½ @ 8800	5				4
Balls, size, inches @ weight, lb.			2 @ 8800					
Balls, size, inches @ weight, lb.			1 @ 4400					
Balls, size, inches @ weight, lb.								
Balls, size added to compensate wear, in.		5		5				
Balls, method of determining addition		*c*	*r*	*c*				
Balls, consumption, pounds per ton		1.5	3.0			2.41	1.6	0.47

Plant	Miami Copper Co.	Ohio Copper Co. (*al*)	Ohio Copper Co. (*al*)	Braden (*av*)	Mexican Gold Mill	Arizona Copper Co.	Cons. M. & S., Canada	Cons. M. & S., Canada
Size, outside diam. in ft. × length of cylinder in in.	8×36*ah*	8×36	8×36	8×36	8×48	8×48	8×48	8×48
Speed, r.p.m.	20.6	23	23	....	21.5	22.5	20.7	18.2
Tons new feed per 24 hr.	274*ag*	533	....	389	500–600	575	1015	500
Tons total feed per 24 hr.	....	....	313	....	....	....	....	....
Method of closing circuit	***DC***	Open	***DC***	Open	Open	Open	Open	***DC***
Installed horsepower	....	....	....	....	200	175	....	....
Actual horsepower	146	130	....	73.5	180	....	145*af*	165
Horsepower per ton of ball charge	8.6	10.8	....	19.1	9.6	10.3	7.3	8.2
Tons new feed crushed per horsepower-hour	0.078	0.171	....	0.220	0.127 (aver.)	0.137	0.292	0.126
Moisture in mill, per cent	66.1	40	21	34	35–40	40	....	....
Size of feed (*a*)	*23*	*33*	*34*	*42*	*10*	*12*	− 1-in.	*17*
Size of product (*a*)	*23ag*	*33*	*34am*	*42*	*10*	*12*	*17*	***an***
Attendance, machines per man	....	....	....	....	1	....	....	....
Lost time, per cent	....	....	....	....	4	....	....	....
Principal causes of lost time	....	....	....	....	***s***	....	....	....
Lubricant, kind/pounds per shift	....	....	....	....	***G***/1; ***O***/1	....	....	....
Feeder, type	....	....	....	....	Schmidt	....	....	....
Feeder, material	....	....	....	....	***CI***	....	....	....
Feeder, life, days	....	....	....	....	1000	....	....	....
Liners, type	....	....	....	Britannia (*aw*)	***WB***	....	....	....
Liners, material	***CI***	*Mn*	....		***t***	***CI***	***CI***	....
Liners, life, days	....	....	....	....	***t***	....	....	....
Liners, consumption, pounds per ton	....	....	....	....	***t***	....	0.095	....
Time for re-lining, hr.	....	....	....	....	24	....	....	....
Number of men for re-lining	....	....	....	....	6	....	....	....
Balls, material	***Mang.***	....	....	***CI***	***FS***	***CI***	***FS***	***CCI***
Balls, new charge, total weight, lb.	34,000	24,000	....	7700	37,400	34,000	39,500	40,000
Balls, size, inches @ weight, lb.	2	....	....	3½	5	3½	4	2¼
Balls, size, inches @ weight, lb.	....	....	....	....	....	....	3	1¾
Balls, size, inches @ weight, lb.	....	....	....	....	....	....	....	....
Balls, size, inches @ weight, lb.	....	....	....	....	....	....	....	....
Balls, size added to compensate wear, in.	....	....	....	....	....	....	....	....
Balls, method of determining addition	....	....	....	....	***c***	....	....	....
Balls, consumption, pounds per ton	....	6	....	....	2.1	0.5	0.273	....

For explanation of reference letters, see page 380.

Table 11*a*. Sizing tests referred to in Table 11.
(Figures under the headings F and P are weight per cent.)

Reference Numbers			1		2		3		4		5		6		7	
Plant			McIntyre Porcupine		U. S. S. R. & M., Midvale		Bunker Hill & Sullivan		Consolidated Arizona Smelting Co.		Cananea Con-Consolidated Copper Co.		Federal Lead, Mill No. 4		Engels C. M. Co.	
Screen aperture																
Mesh	In.	Mm.	F	P	F	P	F	P	F	P	F	P	F	P	F	P
	1.25															
	1.05	26.67	10.0												55.6	
	0.74	18.83	21.0												7.5	
	0.52	13.33	16.0												8.1	
		10	19.6										3.2			
	0.37	9.42							1.5						5.7	
		8											11.2			
3	0.26	6.68													1.6	
		6											16.3	2.0		
4	0.18	4.70			20.6				63.5	0.9	3.4				3.6	
		4											22.3	4.4		
6	0.13	3.33			21.0						9.0				2.6	
8	0.093	2.36	13.9		27.0		14.2		12.8	2.6	11.8				1.9	0.4
		2											26.4	14.7		
10	0.065	1.65	3.0	1.4			24.3				9.4				1.8	0.9
		1.5											3.3	4.1		
14	0.046	1.17					13.5		5.8	8.2	7.6	1.0			1.4	1.9
20	0.033	0.83	4.7	8.6	19.5	8.0	5.8	2.2			7.6		10.3	13.7	1.1	3.3
28	0.023	0.59					4.8	4.9			7.6	6.6			1.0	4.4
30					3.1	18.7										
35	0.016	0.42					3.6	6.3	4.4	16.3	12.4	10.2			1.1	7.0
40			3.9	21.0	2.3	11.2							4.9	18.4		
48	0.012	0.30					5.3	9.7			12.0	13.8			1.0	8.3
60			3.0	9.5	2.9	15.9							1.0	8.3		
65	0.008	0.21					8.6	11.9	4.4	27.0	9.4	14.0			0.8	8.4
80			0.5	10.6		6.2								3.8		
100	0.006	0.15	1.3	4.0		9.7	10.3	15.7		8.9	6.0	13.6		4.4	0.7	9.0
115																
150	0.004	0.10				7.5	5.5	11.9		5.0	1.4	6.0		5.7	0.6	7.2
170																
200	0.003	0.07		6.8		6.7	2.9	9.7		7.1	1.2	7.0		2.6	1.0	10.4
Through last screen			3.1	37.9	3.6	16.1	1.2	27.7	7.6	24.0	1.2	27.8	1.0	18.1	2.7	38.8

Reference Numbers			8		9		10		11		12		13		14	
Plant			Federal M. & S., Morning		McIntyre Porcupine				Utah Leasing Co.		Arizona Copper Co.		Anaconda		Anaconda	
Screen aperture																
Mesh	In.	Mm.	F	P	F	P	F	P	F	P	F	P	F	P	F	P
...	1.25	...														
...	1.05	26.67			5.9		25.4									
...	0.74	18.83			13.4											
...	0.52	13.33			25.3											
...	...	10														
...	0.37	9.42					37.3									
...	...	8														
3	0.26	6.68	17.6				9.4									
...	...	6														
4	0.18	4.70	27.0				5.6	0.1			0.2					
...	...	4														
6	0.13	3.33	9.0				3.8	0.2			9.4					
8	0.093	2.36	4.3				2.2	0.3	5		19.3		2.2		2.2	
...	...	2														
10	0.065	1.65	3.9				1.1	4.0			19.6	2.7	13.2		13.2	
...	...	1.5														
14	0.046	1.17	1.8			12.0	3.4	7.5	13	1	13.2	3.5	21.4		21.4	
20	0.033	0.83	3.2	1.0			2.2	6.7			10.7	7.6	16.0		16.0	
28	0.023	0.59	7.5	1.3		19.0	1.0	6.7	18	4	7.6	10.6	13.6	0.8	13.6	0.5
30	...	...														
35	0.016	0.42	7.9	2.2		8.8	1.4	8.9	12	7	7.3	16.5	9.3	2.9	9.3	2.8
40	...	...														
48	0.012	0.30	10.8	8.0			1.2	7.1	13	14	4.1	14.2	5.7	6.9	5.7	8.8
60	...	...														
65	0.008	0.21	3.8	10.7		16.3	1.2	7.6	11	15	2.8	13.2	4.2	10.6	4.2	15.1
80	...	...					0.5	4.4								
100	0.006	0.15	2.6	15.8		8.0	0.5	3.6	8	16	1.7	13.0	3.7	15.6	3.7	18.1
115	...	...					0.3	2.6								
150	0.004	0.10	0.3	15.2		3.5	0.3	2.2	5	18	1.2	7.6	5.4	10.4	5.4	11.2
170	...	...					0.4	3.0								
200	0.003	0.07	0.0	10.1		2.4	0.3	3.2			0.3	1.3	1.2	14.2	1.2	14.6
Through last screen			0.3	35.7	55.4	30.0	2.8	32.0	15	25	2.4	9.7	3.6	38.6	3.6	28.9

F = feed. *P* = product.

Table 11*a*. Sizing tests referred to in Table 11—*Continued*

Reference Numbers			15		16		17		18		19		20		21	
Plant			Consolidated Arizona Smelting Co.		Consolidated Arizona Smelting Co.		Consolidated Mg. & S. Co., Canada		Dome		Homestake		Inspiration		Miami	
Screen aperture																
Mesh	In.	Mm.	F	P	F	P	F	P	F	P	F	P	F	P	F	P
	1.25															
	1.05	26.67			27.9				46.2		48.0					
	0.74	18.83													3.1	
	0.52	13.33							24.6		16.0		1.2		16.6	
		10														
	0.37	9.42			31.0								4.2		12.1	
		8														
3	0.26	6.68	6.3										13.3		11.0	
		6														
4	0.18	4.70			21.3						10.0		14.4		7.1	
		4														
6	0.13	3.33	46.2										9.6		10.6	
8	0.093	2.36			7.1				12.7				9.2			
		2														
10	0.065	1.65	17.5							14.5	11.0		7.7		7.3	
		1.5														
14	0.046	1.17			3.6								5.4		4.2	
20	0.033	0.83	7.5						6.7	14.2			4.0		3.4	
28	0.023	0.59						3.0		6.2			3.6		3.3	
30																
35	0.016	0.42			2.5				4.8	5.8			3.6		2.4	
40																
48	0.012	0.30	6.3	2.5		4.0				3.8	7.0	15.1	2.4	2.6	2.5	
60																
65	0.008	0.21		10.0	2.6	6.5			0.5	5.7	1.0	9.5	2.3	7.8	1.9	7.6
80																
100	0.006	0.15	6.2	12.5		12.9			0.9	7.4	1.0	14.0	2.0	10.2	2.1	17.7
115																
150	0.004	0.10		10.0		7.9				3.7	1.0	7.8	1.8	11.4	1.9	16.1
170																
200	0.003	0.07		15.0		16.3		55.0	0.4	2.5	1.0	12.9	0.8	4.0	0.9	6.6
Through last screen			10.0	50 0	4.0	52.4		42.0	3.2	36.2	4.0	40.7	14.5	64.0	9.6	52.0

Reference Numbers			22		23		24		25		26		27		28	
Plant			Miami		Miami		Miami		Miami		Mesabi Iron Co.		Mesabi Iron Co.		Mesabi Iron Co.	
Screen aperture																
Mesh	In.	Mm.	F	P	F	P	F	P	F	P	F	P	F	P	F	P
	1.25															
	1.05	26.67	0.4													
	0.74	18.83	2.7						6.7							
	0.52	13.33	16.6						19.7							
		10														
	0.37	9.42	12.1													
		8														
3	0.26	6.68	11.0	0.6	2.2		3.9		28.5	5.2						
		6														
4	0.18	4.70	7.1	1.6	2.4		10.5				29.2		47.8		22.7	
		4														
6	0.13	3.33	10.6	3.1	4.9		20.0									
8	0.093	2.36		5.1	8.1		17.0				26.2		19.1		32.0	
		2														
10	0.065	1.65	7.3	6.5	10.9		13.3		22.3	16.3						
		1.5														
14	0.046	1.17	4.2	7.6	13.1		10.1	0.8			14.1		10.2		14.6	
20	0.033	0.83	3.4	6.6	14.3		7.0	3.1	7.5	20.3						
28	0.023	0.59	3.3	8.6	14.6		5.6	8.1	3.0	11.9	8.6		6.0		6.6	
30																
35	0.016	0.42	2.4	5.2	9.2		2.1	10.7	1.1	4.9						
40																
48	0.012	0.30	2.5	7.3	6.9	0.1	1.4	13.2			4.7	4.3	3.3		6.6	
60																
65	0.008	0.21	1.9	4.7	3.9	3.9	0.6	9.0	1.6	6.8						
80																
100	0.006	0.15	2.1	5.1	2.9	13.0	1.0	11.7	1.6	6.2	3.3	15.8	2.6	8.7	4.1	11.5
115																
150	0.004	0.10	1.9	5.0	1.6	17.5	0.9	9.0	0.9	3.1						
170																
200	0.003	0.07	0.9	2.2	0.7	5.7	0.5	3.8	1.0	3.5	3.3	18.8	2.5	18.7	3.9	20.8
Through last screen			9.6	30.8	4.3	59.8	6.1	30.6	6.1	21.8	10.6	61.1	8.5	72.6	9.5	67.7

F = feed. P = product.

Table 11*a*. Sizing tests referred to in Table 11—*Continued*

Reference Numbers			29		30		31		32		33		34		35	
Plant			Mesabi Iron Co.		Mesabi Iron Co.		Nevada Consolidated Copper Co.		Nevada Consolidated Copper Co.		Ohio Copper Co.		Ohio Copper Co.		St. Joseph Lead Co.	
Screen aperture																
Mesh	In.	Mm.	F	P	F	P	F	P	F	P	F	P	F	P	F	P
	1.25						3.1		3.5							
	1.05	26.67														
	0.74	18.83									0					
	0.52	13.33					17.8		17.3							
		10														
	0.37	9.42														
		8														
3	0.26	6.68													2.6	
		6														
4	0.18	4.70	29.2				23.4	3.4	23.6	2.1					11.8	0.7
		4														
6	0.13	3.33													20.4	2.1
8	0.093	2.36	27.1				8.8	2.0	8.7	2.3					22.0	2.9
		2														
10	0.065	1.65					5.2	2.0	5.4	3.5					14.1	4.5
		1.5														
14	0.046	1.17	15.4		0.3		3.6	2.4	3.5	3.7					11.9	6.6
20	0.033	0.83					3.8	4.4	3.7	7.0	90.0	35.4	61.8	12.4	4.8	6.5
28	0.023	0.59	7.9		0.3		3.5	7.0	3.4	11.1	0.5	3.7	7.8	5.6	3.5	8.5
30																
35	0.016	0.42					3.4	14.0	3.4	14.6	0.6	6.4	6.2	10.2	2.6	8.4
40																
48	0.012	0.30	4.5		5.4		2.7	11.0	2.6	9.6	0.7	2.1	6.4	9.9	2.2	8.5
60																
65	0.008	0.21					2.8	10.4	2.8	8.5	1.1	6.4	6.8	15.5	1.5	6.7
80																
100	0.006	0.15	3.7	1.4	39.0	2.4	3.7	9.6	3.6	8.2	1.6	3.9	4.6	15.0	1.6	9.0
115																
150	0.004	0.10					2.4	5.4	2.4	4.3	0.7	1.6	2.4	3.4	0.7	7.3
170																
200	0.003	0.07	2.8	11.8	38.0	13.3	1.6	3.0	1.6	2.4	0.9	1.6	1.0	4.0	0.1	4.5
Through last screen			9.4	86.8	17.0	84.3	14.2	25.4	14.5	22.7	4.0	38.8	2.9	24.1	0.2	23.8

Reference Numbers			36		37		38		39		40		41		42	
Plant			Tul Mi Chung		Vipond Porcupine		Britannia M. & S. Co.		Buckhorn Mines Co.		McIntyre Porcupine		Asturiana de Minas		Braden	
Screen aperture																
Mesh	In.	Mm.	F	P	F	P	F	P	F	P	F	P	F	P	F	P
	1.25				0											
	1.05	26.67	18.2		5.5						10.0		30.0			
	0.74	18.83			28.0						21.0					
	0.52	13.33			30.0				35.2		16.0					
		10									19.6					
	0.37	9.42							12.2				28.0			
		8														
3	0.26	6.68			19.7				13.2			3.3				
		6														
4	0.18	4.70	45.6													
		4													5	
6	0.13	3.33	4.2						13.8							
8	0.093	2.36	3.2								13.9	9.1	30.0			
		2														
10	0.065	1.65	2.6	0.7	10.9	2.1	60.4	0.4			3.0	6.8			33	
		1.5														
14	0.046	1.17	2.0	1.7					12.2	3.0				10.0		
20	0.033	0.83	1.9	3.1	2.4	8.0	28.8	5.5			4.7	19.7			37	30
28	0.023	0.59	1.7	4.8												
30							5.8	10.9								
35	0.016	0.42	1.1	8.6												
40						22.7	1.1	6.0		12.0	3.9	19.2		14.0	14	24
48	0.012	0.30	1.4	8.6												
60						10.5	1.9	18.0			3.0	6.5			4	10
65	0.008	0.21	0.7	9.7												
80						11.8				31.0	0.5	2.6			1	5
100	0.006	0.15	1.6	12.0		3.9	1.2	23.3		10.0	1.3	5.2		34.0	1	2
115																
150	0.004	0.10	1.4	10.2						16.0				12.0	1	4
170																
200	0.003	0.07	0.9	7.6								4.2		6.0	1	3
Through last screen			3.5	33.0	3.5	40.2	1.1	34.2	13.4	28.0	3.1	23.4	12.0	22.0	3	22

F = feed. P = product.

NOTES TO TABLE 11

a Italic numbers refer to column numbers in Table 11*a*. *b* Balls chipped and broke badly. Forged-steel balls substituted, consumption 1.1 lb. per ton. *c* Constant quantity added daily. *d* One man to six 36 × 14-in. rolls, 3 ball mills and 2 drag classifiers. *e* See Fig. 14. *f* Mild-steel plate. *g* Cast-iron and steel plate. *h* Midvale steel. With cast iron, life 58 days; consumption, 1.23 lb per ton. *i* At 17 r.p.m. consumes 92 hp. and has the same capacity. *j* Consumption of chrome-steel balls in mill making 17 r.p.m. is 2.1 lb. per ton. *k* Included with other work. *l* Flexible coupling. Tightening liner bolts. *m* 60 per cent. @ 4-in. and 40 per cent. @ 3-in. *n* Feed is de-slimed sand-table tailing, 23 per cent. +48-mesh; mill discharge, 8 per cent. +48-mesh; classifier overflow, all −48-mesh. *o* Individual liners replaced as worn out. *q* 0.55 lb. consumed, 0.45 lb. waste. *r* Sufficient added each shift to keep ammeter at constant reading. *s* Inspection and re-lining. *t* Cast iron, 120 days, 0.4 lb. per ton; Mn, Cr, life, 300 days. *u Salt Lake Min. Rev., Nov. 15, 1918*. *v* 250 hp. required momentarily at starting. *w* Classifier overflow is 0.5 per cent. +35-mesh, 44 per cent. −150-mesh. *x* Change from 26½ r.p.m. to 17 r.p.m. reduced power consumption from 150 to 100 hp., reduced ball consumption $0.05 per ton of feed, and made corresponding reduction in liner consumption, without affecting grinding. *y* Compare 5 × 10-ft. rod mill at same plant, Table 50. *z* Dimensions inside lining. *aa* Would probably be lower with lower moisture content. *ab* Manganese-steel, wave-type, life 85 days; 23,275 tons ground from −8-mesh to 80 per cent. −65-mesh. Cost of liner, $0.125 per pound f.o.b. factory. Liner flowed and was hard to remove. Forbes-type cast-iron liners, life 88 days, 24,200 tons as above. Cost, $0.0349 per pound f.o.b. concentrator. Re-lining, 4 men, 3 hr. *ac* Has run up to 170. *ad* Compare 6 × 6-ft. grate mill at same plant, Table 5. *ae* In 1920 handled over 200 tons per 24 hr. of crude ore from 7.8 per cent. +1-in. to 3.4 per cent. +48-mesh, but crude is not so hard to grind as this middling. *af* Dropped from 165 hp. by improvement in gear-redu tion set. *ag* Increasing feed rate to 1100 tons on primary mills lowered the amount of −200-mesh in primary-mill discharge to 21.4 per cent. and in the secondary-mill discharge increased the +48-mesh to 3.5 per cent. and decreased the −200-mesh to 47.9 per cent. *ah* In Section 5 one primary and two secondary 8-ft. × 30-in. mills handled 1027 tons from 1.3 per cent. +18.85-mm. to 0.2 per cent. on 48-mesh and 59.4 per cent. −200-mesh with a total power consumption of 360 hp. Ore varies considerably from day to day and this performance is for softer ore than the performance of the 8-ft. × 36-in. mills in Section 6, here give . *ai* Compare 8-ft. × 22-in. pebble mill at Miami, Table 77. *aj* Classifier overflow is 1.8 per cent. +35-mesh, 47.8 per cent. −200-mesh. *ak* Classifier overflow is 0.5 per cent. +20-mesh, 45.7 per cent. −200-mesh. *al* Two mills in series, the first discharging to a Dorr classifier that is in closed circuit with the second. *am* Classifier overflow is 0.6 per cent. +20-mesh, 41.3 per cent. −200-mesh. *an* 0.5 per cent. +100-mesh, and 85 per cent. −200-mesh. *ao* In *22 CMI 98* the product shown in screen test No. 39, Table 11*a*, is given as corresponding to 200 tons per 24 hr. of this same feed. *ap 119 P 808*. *aq* Classifier overflow contains 5.5 per cent. +65-mesh and 73 per cent. −200-mesh. Sands to conical pebble mill. See Table 77. *ar* Shell and feed-end liners, 30,000 tons; discharge-end liner 50,000 to 60,000 tons; total consumption, 0.208 lb. per ton of ore. *as 115 J 398*. *at 123 P 886*. *au* Contains 60 per cent. −200-mesh. *av 101 J 316*. *aw* Weight, 15,200 lb. *ax* 6 per cent. +40-mesh, 35 per cent. −200-mesh. ***CI*** Cast iron. ***CCI*** Chilled cast iron. ***CCr*** Cast chrome steel. ***Cr*** Chrome steel. ***CS*** Cast steel. ***DC***, Dorr classifier. ***FS*** High-carbon forged steel. ***G*** Grease. ***Mang.*** Manganoid. ***Mn*** Manganese steel. ***O*** Oil. ***SS*** Spiral scoop, one-way. ***WB*** Wedge-bar. Cast plates and forged chrome-steel wedge bars.

same. See also comparison based on Tables 4, 5, and 11, given in Table 12. On

Table 12. Comparison of ball mills

Mill	Horsepower per ton of balls			Tons product per horsepower-hour					
				Table and tube-mill feed size			Flotation size		
	High	Low	Average	High	Low	Average	High	Low	Average
Conical	19.1	6.2	9.3	0.417	0.062	0.195	0.166	0.045	0.087
Grate	20.0	7.2	12.9	0.300	0.063	0.147	0.111	0.047	0.080
Center-discharge	18.9	4.4	13.2	0.276	0.05	0.139	......	......	......

the basis of comparison used, *viz.*: tons product of a given size per horsepower-hour, the conical mill is clearly superior, particularly in the coarser range. At NEVADA CONSOLIDATED four cylindrical mills and eight conical mills were used in re-grinding −2-mm. classified roll product. Comparative performances are given in Table 13. The product of the conical mill was slightly finer while power and metal consumption were distinctly less.

Table 13. Comparative performances of conical and cylindrical ball mills at Nevada Consolidated

	Cylindrical	Conical
Size of mill	7×6-ft.	8-ft.×30-in.
Feed rate, dry tons per 24 hr., average	359	451
Moisture, per cent	27.9	26.9
Horsepower consumed, full-load average	128.75	115
Tons per horsepower-hour	0.116	0.163
Metal consumption, pounds per ton (*a*)	1.583	1.072

Sizing tests, screen aperture, mesh	Weight, per cent.			
	Feed	Product	Feed	Product
10	3.7	....	3.7	0.2
14	13.8	1.0	13.8	0.8
20	22.4	2.0	22.4	2.0
28	20.2	5.0	20.2	3.4
35	16.2	6.6	16.2	5.7
48	10.6	9.0	10.6	7.7
65	6.5	9.2	6.5	10.0
100	3.6	13.6	3.6	14.4
150	1.2	11.4	1.2	10.8
200	0.6	5.6	0.6	7.6
−200	1.2	36.8	1.2	37.4

a Cast-iron liner plates, chrome-steel lifting bars, cast-iron balls.

6. Mechanics of the ball mill

Davis (*61 A 250*) gives an exhaustive mathematical analysis of the action in a ball mill. A summary of his conclusions follows:

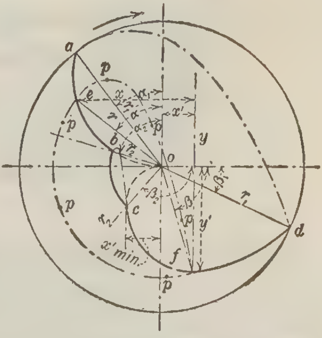


FIG. 11.—Chart of theoretical paths of balls in a ball mill (*after Davis*).

Notation: (See Fig. 11) r = radius in ft., drawn to any particle p at the instant that its path changes from circular, under the impulse of the mill shell, to parabolic under the influence of its acquired momentum and gravity, or *vice versa*. r_1 = radius of mill drawn to the point where the outer layer of balls changes from circular to parabolic motion or *vice versa*. r_2 = radius drawn to the point of motion change of the inner layer of that part of the charge which, at any given instant, is following a circular path. R = radius of gyration of charge near line aO. α = angle in degrees between vertical and the radius r drawn to the point of change from circular to parabolic motion. α_1 = angle between vertical and the corresponding radius r_1. α_2 = angle between vertical and the corresponding radius r_2. α_R = angle between vertical and R. β = angle

between horizontal and the radius r drawn to the point of change from parabolic to circular motion. β_1 = angle between horizontal and corresponding radius r_1. β_2 = angle between horizontal and corresp nding radius r_2. n = speed of mill in rev. per sec. N = speed of mill in r.p.m. N_1 = critical speed of mill in r.p.m. V_b = velocity of particle relative to striking point. w = weight of any given portion of charge, *e.g.*, one ball, in lb. W = weight of entire charge in lb. P = fraction of mill volume occupied by charge (voids included). g = acceleration due to gravity = 32.2 ft. per sec. per sec. k = a constant for any given mill speed $= 4\pi^2n^2/g = 1.226n^2$. K = a constant $= r_2/r_1$. H = height of charge in mill at rest, in ft. E = Kinetic energy in ft.-lb. T = time for a ball to complete one cycle, in sec. T_r = time for one rev. of mill, in sec. C_n = cycles of ball travel per rev. r_c = radius of circular arc aO along which change from circular to parabolic motion occurs. x, y, See Fig. 11.

Equations:

$$\cos\alpha = kr = 1.226rn^2, \quad (1) \qquad r_c = \frac{0.408}{n^2}, \quad (2)$$

$$\cos\alpha_R = 0.867r_1n^2\sqrt{1+K^2}, \quad (3) \qquad N_1 = \frac{54.19}{\sqrt{r_1}}, \quad (4)$$

$$y = x\tan\alpha - \frac{0.613n^2x^2}{\cos^4\alpha}, \quad (5) \qquad \beta = 3\alpha - 90^\circ, \quad (6)$$

$$V_b^2 = 16rg\cos\alpha\sin^4\alpha. \quad (7)$$

For the best theoretical efficiency

$$V_{b\,\text{max.}} = \frac{7.88}{n}, \quad (8) \qquad V_{b\,\text{min.}} = \sqrt{16Kr_1g\cos\alpha_2\sin^4\alpha_2}, \quad (9)$$

$$\alpha_R = 54^\circ\,44', \quad (10) \qquad K = \sqrt{\frac{0.443}{r_1^2n^4} - 1}, \quad (11)$$

$$N = \frac{48.95}{\sqrt{r_1}\sqrt[4]{1+K^2}}, \quad (12) \qquad \cos\alpha = \frac{0.8165r}{r_1\sqrt{1+K^2}}, \quad (13)$$

$$\cos\alpha_1 = \frac{0.8165}{\sqrt{1+K^2}}, \quad (14) \qquad \cos\alpha_2 = K\cos\alpha_1, \quad (15)$$

$$C_n = \frac{T_r}{T} = 1.444 \text{ (considering all balls to have the same cycle as if their radius of revolution were the radius of gyration)}. \quad (16)$$

$$K = -0.024 + 0.39\sqrt{7 - 10P} \text{ (very nearly)}, \quad (17)$$

$$\text{HP.} = Wr_1^{\frac{3}{2}}\left[0.004467\frac{1-K^3}{(1+K^2)^{\frac{1}{8}}} - 0.0037\frac{1-K^5}{(1+K^2)^{\frac{3}{8}}} + 0.00088\frac{1-K^7}{(1+K^2)^{\frac{5}{8}}}\right] \quad (18)$$

$$r_2 = Kr_1. \quad (19)$$

Examples of application of Davis' equations. An 8 × 6-ft cylindrical mill charged with 28,000 lb. of steel balls. Assuming 35 per cent. of voids in the ball charge, the weight per cu. ft. of charge is 325 lb. and the volume of the charge, 28,000/325 = 86 cu. ft. The interior volume of a 8 × 6-ft. mill is 301 cu. ft. (according to Davis. This figure is based on the assumption of 8 × 6-ft. inside. The actual internal volume of an 8 × 6-ft. mill is probably 20 per cent. less than this on account of the space occupied by the liners), hence the charge occupies 86/301 = 28.6 per cent. of volume = P. Then from equation (17), K = 0.770; from equation (12), the best speed = N = 21.8 r.p.m.; and from equation (18), HP = 153. (This is low for an 8 × 6-ft. mill.) If 8000 to 10,000 lb. is added for

the weight of the pulp and W is made the total load in mill = 36,000 to 38,000 lb., HP becomes 197 to 208, which is about correct (see Table 4). From equation (14), $\alpha_1 = 49°$ $30'$ and from equation (15), $\alpha_2 = 60° 20'$. From equation (19), $r_2 = 3.05$ ft. By equation (6), $\beta_1 = 58° 30'$ and $\beta_2 = 91°$. Values of r for values of α between α_1 and α_2 may be obtained from equation (1) and for values of β between β_1 and β_2 from (6) and (1). By plotting these values, curves $a - b$ and $c - d$ in Fig. 11 may be obtained. Then the concentric circular arcs represent the circular paths of the balls and the parabolas the free-moving paths. The value of r for the start and finish of any particle on its parabolic path is the same. The equation of the parabolic path of any given particle, taking its point of starting on the parabolic path as origin of co-ordinates, is given by equation (5). Thus the complete theoretical path of the ball charge can be plotted as in Fig. 12.

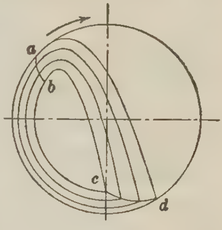
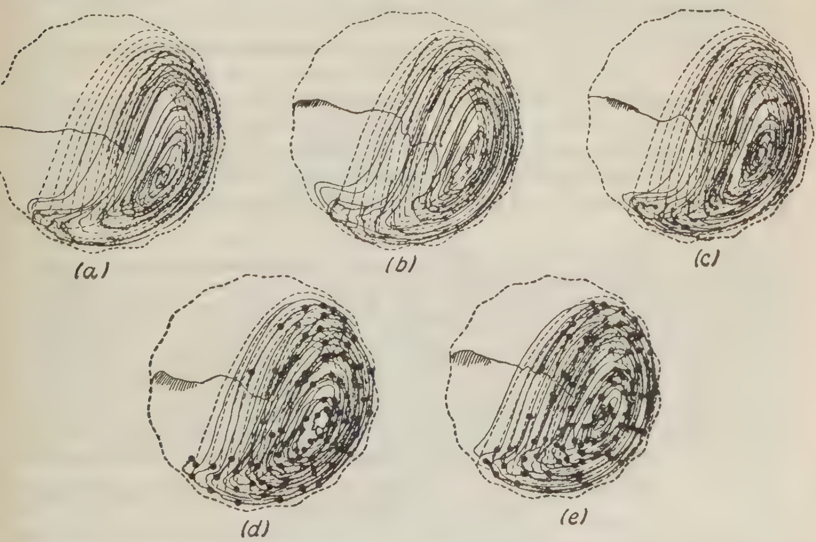


Fig. 12.—Chart of ball paths in 8 x 6-ft. mill at 21.8 r.p.m. with 28,000-lb. ball (*after Davis*).

Haultain and Dyer (*69 A 198*) question Davis' conclusions as to ball paths on the basis of a remarkable set of moving pictures portraying the performance in a glass-ended cylinder 2 ft. diam. by 2½ in. long, but Davis' discussion of their results confirms his original conclusions. Fig. 13 shows five different conditions of operation graphically summarized by Davis from the films. The dash lines in the figures are the theoretical paths. The lines carrying round black dots represent actual paths of individual balls as traced from the films. In Figs. 13*a* and 13*d*



a. Small balls, no rock. *b*. Small balls, some quartz. *c*. Small balls, full charge of quartz. *d*. Large balls, no rock. *e*. Large balls, full charge of quartz.

Fig. 13.—Actual ball paths in tube mills.

departure of the actual paths from the theoretical is greatest. In these operations balls and water only were present, and the discrepancy is due to slip between ball load and lining, amounting to 8 to 10 per cent., resulting in failure to carry balls to their full theoretical height. This slip eliminated

practically all free fall in Fig. 13*a*, and materially decreased the amount of free fall in Fig. 13*d*. But in Figs 13*c* and 13*e* sand (corresponding to ore) was present, slip was practically zero, and the actual paths approximated the theoretical very closely.

7. Wear of balls and liners

General. Wear depends primarily upon the materials of which balls and liners are made. With given ball and liner material the wear is, of course, greater with hard than with soft ore; it is usually greater with fine feed than with coarse, although this is undoubtedly because fine feed connotes fine product while coarse feed does not necessarily do so. (If material is to be finished to the same size, wear is greater for coarse than for fine feed.) Wear increases with increase in mill speed and with increase in moisture content in the mill, and it is greater for open-circuit than for closed-circuit crushing (per ton of original mill feed), when the endeavor is to finish to the same size in both cases.

Ball wear. The most resistant material in common use for balls is forged chrome steel. Wear of chrome-steel balls with ordinary ores is from 1 to 2 lb. per ton grinding from 1.5- or 2-in. to 20- to 48-mesh, closed-circuit, and from 0.5 to 1.5 lb. per ton grinding from the same feed size to 6- to 10-mesh, open circuit. Corresponding figures for forged high-carbon steel balls are 1.5 to 3 lb., closed-circuit, and 1 to 2 lb., open-circuit.

At Engels C. M. Co. the consumption of forged- and chrome-steel balls under similar conditions was 3.2 and 1.9 lb. per ton, respectively.

Consumption of cast-iron balls is usually from 1.5 to 2 times that of forged steel, if the balls are 2-in. or smaller; large balls are excessively consumed due to breakage. Manganoid and Duraloid (cast manganese-steel) balls wear about the same as forged high-carbon steel and are cheaper. They break readily, however, and are not likely to be economical when large sizes are necessary.

At Utah Leasing Co. the consumption of 2½-in. Duraloid balls in an 8-ft. × 30-in. conical mill was 2.19 lb. per ton (cost $0.1018 per ton). At Miami Copper Co. the consumption of 2-in. chrome-steel balls in 8-ft. × 36-in. conical re-grinding mills was 0.8015 lb. per ton for the year 1916. With manganoid balls of the same size in the same service, consumption was 1.1928 lb. per ton. Prices were $0.05 and $0.025 per lb., respectively. At United Eastern (*63 A 556*) 1½-in. chrome-steel balls in fine-grinding service (20-mesh to 85 per cent. −200-mesh) were consumed at the rate of 3.18 lb. per ton while cast-iron balls the same size wore at the rate of 4.35 lb. per ton. At Inspiration (1916) the wear of chrome-steel balls making 8-mesh product, open circuit, in Marcy mills was 0.3 to 0.7 lb. per ton; making 48-mesh product in closed circuit the wear was 1.7 lb. per ton (1.82 lb. in 1918). Table 14 gives comparative data on consumption of chrome-steel, forged-steel and

Table 14. Comparison of manganoid and chrome-steel balls in 8 × 6-ft. Marcy mills at Inspiration; 1920; 44-day run

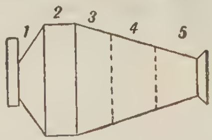
Material	Manganoid	Chrome steel
Diameter of balls, in.	4½	4
Consumption, pounds per ton	3.164	1.66
Cost of balls, cents per pound	4.54	5.15
Cost of balls, cents per ton ground	14.36	8.55

Comparative consumption of chrome-steel and forged high-carbon steel balls over a one-year period was: Chrome-steel, 1.75 lb. per ton; forged-steel, 1.82 lb.

manganoid balls at the same plant. At Anaconda consumption of cast-iron balls in a 7½-ft. × 72-in. (inside lining) mill was 5.5 lb. per ton at 23 r.p.m. and 2.75 lb. at 15 r.p.m. At Nevada Consolidated consumption of 3-in. machine-cast balls (Nov., 1923) was 1.74 lb. per ton; 3-in. sand-cast, 1.83 lb.; 2-in. drop-forged steel, 1.0 lb.

Size of balls has a small effect on ball wear. In 5 × 6-ft. ball-pebble mills at United Eastern (*63 A 556*), grinding from 20-mesh to 85 per cent. − 200-mesh, the consumption of 2-in. chrome-steel balls was 3.00 lb. per ton and of 1½-in. was 3.18 lb. per ton. At St. Joseph Lead Co. (*66 A 107*) consumption of cast-iron balls in a 6-ft. × 22-in. conical mill crushing to 10-mesh was 1.218 lb. per ton and of steel balls of substantially the same size (1.5- to 2.5-in.), 0.332 lb. In the same mill the wear of 5- to 2.5-in. steel balls was only 0.12 lb. per ton.

Liner wear varies with liner material, position of liner, kind of ore, tonnage, size of feed and product, whether open- or closed-circuit grinding, ball load, speed, and pulp consistency. Life of liners ranges from 4 months to 3 years but the average for all kinds of liners is close to 6 months. Consumption of manganese-steel liners, which are most commonly used, averages (from Tables 4, 5 and 11) 0.35 lb. per ton when grinding to sizes between 6- and 20-mesh, mostly in open circuit, and 0.50 lb. per ton in grinding in closed circuit to flotation size. Chrome-steel liners are more resistant, if the averages based on the meager data in Table 5 are dependable, the consumption being 0.18 lb. per ton in coarse grinding and 0.39 lb. per ton in fine grinding. Cast-iron liners are least resistant, the wear in coarse grinding averaging 0.69 lb. per ton. Comparative consumption of different metals on the same ore is given in note *g* Table 4, and note *t*, Table 11. Wear on liners in different parts of the mill is shown by Fig. 14; note *ar*, Table 11; note *w*, Table 5; and by Table 15. A considerable part of the liner consumption is scrap, hence the heavier the new liner, the smaller the proportion of waste rejected. At United Eastern (Table 15) rejection averaged 43 to 45 per cent. of the new weight of all liners. Progress is in the direction of heavier liners. It is commonly stated and probably true that the consumption on hard ore is greater than on soft, but comparative data on this score, uncomplicated by variability in other factors, are not available in Tables 4, 5 and 11. It is, of course, true that the abrasive character of the hard ore is the greater and that, on account of slower crushing rate, more motion of the liner with respect to the load is necessary per ton of hard ore ground, hence on both grounds more wear is to be expected. Life of a liner will decrease with increase in mill tonnage rate but consumption per ton of ore will generally decrease on account of the increased cushioning effect of the greater tonnage. Wear is usually greater with coarse than with fine feed, as is indicated by the greater wear of liners near the inlet end of a center-discharge cylindrical mill. This effect may be obscured in a peripheral-discharge mill running with thin feed and low pulp level by excessive wear near the grate occasioned by lack of pulp to cushion the ball charge. Wear is distinctly greater in fine closed-circuit grinding than in coarse open-circuit work as shown by the comparative figures



Number	Life	Tons ground	Number of pieces
1	10 mo	45 000	16
2	8 mo.	36 000	16
3	9 mo.	40 000	13
4	1–3 yr.	....	12
5	Indefinitely long	......	8

Fig. 14.—Liner wear in conical ball mill at Bunker Hill and Sullivan Mining Co.

on wear of manganese-steel liners previously cited (0.50 and 0.35 lb. per ton of feed, respectively). Liner wear is greater with a heavy ball load than with a light load and, all other things being equal, greater with large balls than with small. Probably the best evidence of this fact, complicated, it is true, by the fact that the size of feed is different in the different places, is the comparative wear in the different sections of the conical mill, shown in Fig. 14. Wear increases with increase of speed and with increase in moisture content of the pulp.

At Wright-Hargreaves (115 *J* 881) the wear in an 8-ft. × 30-in. conical mill was excessive at 22 r.p.m., when the feed rate was 200 tons per day, open-circuit, but fell to normal when the speed was reduced to 19½ r.p.m. At Nevada Consolidated (see Table 11) the wear of cast-iron balls (which is proportional to liner wear) was 1.55 lb. per ton in an 8-ft. mill at 28 r.p.m. and 1.34 lb. per ton at 24½ r.p.m.

Table 15. Liner wear in ball mills at United Eastern. (*After North*)

Liner	Material	Consumption, pounds per ton (including scrap)	Per cent scrap	Cost per ton milled (1917, 8)
		No. 64½ Marcy		
Feed-end	Manganese steel	0.072	32.2	$0.0110
Shell (step type)	Manganese steel	0.163	45.5	0.0268
Discharge grate	Chrome steel	0.055	48.4	0.0137
Bolts, clamp bars, center liners, etc.		0.022	63.8	0.0068
Total		0.312	43	$0.0583
		5 × 6-ft. ball-peb mills		
Feed-end	Chrome steel	0.040	52	$0.0072
Throat	Chrome steel	0.007	40	0.0012
Shell	Chrome steel	0.115	28	0.0218
Discharge grates	Tool steel	0.005	40	0.0014
Discharge wedges	Chrome steel	0.017	41	0.0031
Total		0.184	45	$0.0347

Liner wear must be watched carefully in order to insure that the shell is not reached and perforated. With bolted liners the plates usually break when too thin, which allows the bolt to loosen and causes leakage, thus giving warning. At many plants mills are stopped periodically for inspection. At some plants, *e.g.*, United Eastern (65 A 557), after a wear record has been established, the mill is stopped and opened only when records of past performances show that a particular part of the liner should be reaching the danger point.

Cost of maintenance. The cost of balls and liners is 80 to 90 per cent. of the total maintenance cost of ball mills. The following figures for a 4 × 3½-ft. Marcy mill at Elko Prince (1916–1918 incl.) are typical of the distribution. Total maintenance, $0.2460; liner, $0.1042 (= 42.5 per cent.); balls, $0.1011 (= 41.0 per cent.); miscellaneous supplies, $0.0253 (= 10.3 per cent.); labor, $0.0154 (= 6.2 per cent.).

8. Operation of ball mills

Capacity. The meaning of the word capacity is not closely defined in the literature of cylinder mills. The ability of a mill without grates to pass material from inlet to outlet is limited only by the ability of the feeding device to force the material into the mill. The mill will reduce the average size of the material passing through it, more or less, depending upon the rate of passage and also upon a number of other factors such as nature of material ground, size of feed, size of product required, percentage of solids in feed, type of liner, size of balls, weight of charge, speed of mill, and whether or not in closed circuit with a classifier. With grate mills the grate openings also affect capacity. Probably the best method of stating capacity is in terms of tons of material of the desired maximum size and smaller actually produced by the mill per unit of time. Tables 16 and 17, generalized from operating

Table 16. Capacity of cylindrical ball mills

Size, diameter × length, feet	From	To mesh	Tons per 24 hr	Size, diameter × length, feet	From	To mesh	Tons per 24 hr.
3× 3	2-in.	14	15–30	*	0.5	28	200
....	2	48	10–20	....	2.5	48	75–150
3× 5	2	14	22–40	....	2.5	65	50–100
....	2	48	15–30	*6× 5	2	8	280
4× 3	2	14	30–60	....	2	14	200–350
....	2	28	25–50	....	2	28	140–250
....	2	48	15–40	....	2	48	75–150
....	2	65	10–30	....	2	65	60–120
*4× 3½	2	8	50–60	*6× 6	1	4	257
*4× 4	3	10	50	*	3	8	188
....	2	14	50–75	*	2	8	288*c*
....	2	28	25–50	....	2	14	300–400
....	2	48	20–40	....	2	28	200–300
....	2	65	15–30	....	2	48	100–200
4× 6	2	14	70–100	....	2	65	80–160
*4×10	14-mesh	200	25*a*	*	2.5	100	250
5× 4	2-in.	14	100–150	*6×10	10-mesh	28	600*d*
....	2	28	70–100	7× 5	2-in.	14	350–600
....	2	48	40–60	....	2	28	250–400
....	2	65	25–50	....	2	48	150–225
*	1.5	100	42–50	....	2	65	100–200
5×5	2	14	125–175	*7× 6	2	10	275–600
*	0.5	20	285	*7×10	8-mesh	28	600
....	2	28	80–125	*8× 5	1-in.	8	800–850
....	2	48	50–75	*	2	48	340
....	2	65	30–60	*8× 6	2.5-in.	6	400–450*c*
*5× 6	20-mesh	35	90	*	1.5	6	750
*	3-in.	100	90	*	0.4	8	900*b*
*5×10	0.5	10	150	*	0.75	10	300
*6× 4	0.25	10	450*b*	....	2	14	500–800
....	2	14	175–275	....	2	28	400–600
....	2	28	100–150	*	2.5	35	480
....	2	48	65–130	*	2	48	440–480
....	2	65	40–80	*	3	65	450
*6× 4½	2.5	20	280				

* Marks data from actual operation. Other figures generalized from manufacturers' catalogs. *a* Hard, pyritic concentrate. *b* Soft calcareous lead ore. *c* Hard, tough ore *d* Soft porphyry ore.

Table 17. Capacity of conical ball mills

Size, diameter cylinder, feet × length cylinder, inches	From	To mesh	Tons per 24 hr.	Size, diameter cylinder, feet × length cylinder, inches	From	To mesh	Tons per 24 hr.
*3×8	0.5-in.	6	30*a*	*8×28	0.25	14	160
....	1.5	14	10	*8×30	0.4-in.	3	855*a*
....	1.5	28	9	*	2	8	496
....	1.5	48	8	*	8-mesh	14	270
....	0.5	65	8	*8×30	1.5-in.	20	450
*4½×13	1.5	10	48	*	1.5	35	408
4½×16	1.5	8	48	*8×36	1	4	1000
....	1.5	28	30	....	1.5	8	430
....	1.5	48	24	*	0.75	10	533
*	1.5	65	20	*	4-mesh	10	389
....	0.5	65	24	*	1.25-in.	12	650
*5×16	1.25	12	60	*	2	14	236
*6×16	0.75	3	351	*	0.4	20	198
*	1.5	10	175–200	*	3-mesh	20	396–443
*	0.75	14	160	*	2	35	378
*	0.3	20	135	*	10	20	313
*	1.5	65	155	*	1.5-in.	48	141–336
*6×22	0.5	6	275*a*	*	3-mesh	48	150
*	0.5	8	120*b*	*	3	65	274
....	1.5	8	144	....	0.5-in.	65	384
....	1.5	14	130	8×48	1.5	8	576
*	6-mesh	20	160	*	1.5	10	500–600
....	1.5-in.	28	120	*	1.0	28	1015
....	1.5	48	96	....	1.5	48	456
*	0.75	48	106	*	0.5	65	500
....	0.5	65	75	*	28-mesh	100	500
*6×48	0.5	48	134	9×48	1.5-in.	8	790
7½×72	8-mesh	28	300–425	....	1.5	48	620
8×22	1.5-in.	8	336	....	0.5	65	700
....	1.5	48	225	10×48	1.5	8	1080
*	0.25	48	367	....	1.5	48	840
....	0.5	65	240	....	0.5	65	900
*	0.25	65	150–300				

* Data from actual operation. Other figures generalized from manufacturers' catalogs.
a Soft ore. *b* Hard, tough ore.

Table 18. Effect of feed rate on capacity of an 8-ft. × 22-in. conical ball mill, open-circuit crushing. (*After Davis*)

Feed rate, tons per 24 hr.	Tons −200-mesh material actually produced	Tons −200-mesh material produced per horsepower-hour
88	53.8	0.019
178	90.5	0.032
264	116.3	0.041
360	144.8	0.051
432	184.0	0.065

Iron ore, 35 per cent. magnetite, balance quartzite and iron silicates. All through 0.25-in. screen. Moisture @ 50 per cent. 28,000 lb. of 5-, 4-, 3- and 2½-in. balls; 19.7 r.p.m.

data and manufacturers' catalogs, give figures that are reliable for estimating purposes for ordinary ores. For hard and tough ores the capacity may be reduced 50 per cent.; with soft, decomposed feed the figures can be increased somewhat. Decrease in size of feed will cause an increase in capacity, greater the smaller the diameter of the mill.

Rate of feed to mill. In open-circuit grinding the amount of material of any given size produced by a single mill increases with the rate of feed. (See Table 18 and Fig. 15.) Production passes through a maximum which is reached more quickly the finer the desired size. (See Tables 19 and 20.) On the other hand, the amount of oversize discharged likewise increases with increase in the feed rate and, since the object of the grinding operation is to reduce all of the material to the limiting size, high-capacity open-circuit work is justifiable only when, taken in conjunction with the finishing grinding, a lower cost per ton of finished product is attained than is otherwise possible. If the feed rate in open-circuit grinding is cut down to the point where the discharge is all of a desired fine size, capacity, particularly in connection with power consumption (tons per horsepower-hour) is extravagantly low, consumption of grinding media is high, and wear of lining at the discharge end of the mill is excessive. Much of the material must be over-ground to insure

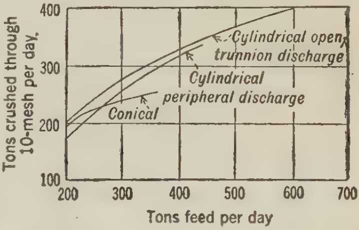


Fig. 15.—Effect of feed rate on capacity of ball mills (*after Delano and Rabling*).

Table 19. Effect of feed rate on capacity of 4½-ft. × 16-in. conical ball mill, open-circuit crushing

Feed rate, tons per 24 hr.(*a*)	−200-mesh		−65-mesh		−48-mesh	
	Per cent. in product	Tons produced per 24 hr.	Per cent. in product	Tons produced per 24 hr.	Per cent. in product	Tons produced per 24 hr.
18	21.6	3.68	48.9	7.88	59.8	9.53
36	15.2	4.96	39.7	12.45	50.2	15.60
72	8.4	5.04	26.0	15.05	32.9	18.70
108	5.5	4.32	19.9	16.00	26.5	21.20
144	4.6	4.60	17.3	17.65	23.2	23.40

a Quartzite; 0.7 per cent. on 1.5-in. screen, moisture, 39 per cent.

that all is sufficiently ground. Hence in order to take advantage of the ability of a cylinder mill to handle large tonnages and produce therefrom a large amount of finished product per unit of power, the mills are run in closed circuit with a sizing device that returns oversize to the mill feed. The effect of closed-circuit crushing is clearly shown, by comparison of Table 18 with Table 21, to be to increase the amount of −200-mesh material produced per horsepower-hour at all tonnages throughout the common range of the two experiments.

At Tough Oakes (*101 J 691*) a conical ball mill run open-circuit to produce tube-mill feed was fitted with a double-cone screen in the discharge trunnion, the converging part

of this screen being a continuation of the surface of the discharge-end cone of the mill, so that the level at which coarse material could discharge was raised. The ball load could be and was increased from 8500 to 11,500 lb. Oversize was returned to the head of the mill by belt conveyor. Capacity was increased from 82.7 to 102.3 tons per 24 hr., power decreased from 40 to 35 hp., tons per hp.-hr. increased from 0.086 to 0.119, and ball consumption decreased from 2.45 to 1.80 lb. per ton.

Table 20. Relation between feed rate and ground product in a 6 × 4-ft. ball mill. (*After Hines*)

Feed, tons per 24 hr.	Product					
	−200-mesh		−100-mesh		−65-mesh	
	Per cent. weight	Tons per 24 hr.	Per cent. weight	Tons per 24 hr.	Per cent. weight	Tons per 24 hr.
120	40	48	52	62.4	56	67.2
135	37	50	50	67.5	54	72.9
250	28	70	40	100	45	112.3
300	21	63	28	84	32	96
400	14	56	23	92	27	108

Feed, tons per 24 hr.	Product					
	−48-mesh		−20-mesh		−10-mesh	
	Per cent. weight	Tons per 24 hr.	Per cent. weight	Tons per 24 hr.	Per cent. weight	Tons per 24 hr.
120	60	72	78	93.6	87	104.3
135	57	77	72	97.1	84	113.3
250	50	125	64	160	76	190
300	37	111	53	159	67	201
400	32	128	49	196	63	252

Table 21. Effect of feed rate on capacity of an 8-ft. × 22-in. conical ball mill, closed-circuit crushing. (*After Davis*)

Feed, rate, tons per 24 hr. (*a*)	Tons −200-mesh produced per 24 hr.	Tons −200-mesh produced per horsepower-hour
111	96.1	0.028
120	104.1	0.030
132	111.0	0.032
156	128.1	0.037
177	135.2	0.039
264	163.8	0.047
302	193.7	0.056
368	218.5	0.063

a Iron ore, 35 per cent. magnetite, balance quartzite and silicates; all through 0.25-in. screen. 40 per cent. moisture, 23.8 r.p.m., 28,000 lb. @ 3- and 2-in. balls.

It is not to be understood, however, that closed-circuit grinding to a given size is always more economical than open-circuit. In the first place, tests in Tables 18 and 21 do not give a true comparison, if the ordinary mean-

ing of the term closed-circuit is taken, because the material was not all finished to the size taken as the basis for comparison in either the closed-circuit or open-circuit tests. Had it been, it is apparent that the tons produced per horsepower-hour would have been considerably reduced, particularly in the tests at the higher tonnages, in which the percentage of final oversize was large. At these tonnages, therefore, the production of −200-mesh material per unit of power would have been greater in the open-circuit work. This discrepancy becomes larger the coarser the feed and leads to stage reduction.

Truscott recommends feeding coarse material to short ball mills at such a rate (in a 50 per cent. pulp) that the available volume (which he takes as the mill volume below overflow level minus the ball volume) is re-filled every 1 to 2 min. This figure accords reasonably with good practice when the ball load is all that an open-trunnion mill will stand, but applied generally it would call for a higher feed rate to an undercharged mill than to one with full charge, which, of course, should not be the case.

Stage-reduction. Whenever ball-mill feed is coarser than 0.75-in. maximum size, or, with even finer feed than this, when the tonnage to be crushed is sufficient to require more than one mill, crushing will usually be cheaper if the reduction is done in two or three stages. In such work usually only the final mill is run in closed circuit and in the other mills advantage is taken of the ability to handle large tonnages and produce, in connection, it is true, with a large percentage of over-size, a large tonnage of finished product per unit of power expended.

Tables 22 and 23 present the results of a test comparing one- and two-stage crushing of −0.25-in. iron ore to 100-mesh (*Davis, 61 A 253*), both mills being in closed circuit. In

Table 22. Performance of 8-ft. × 22-in. conical ball mill crushing from 0.25-i to 100-mesh in one step. (*After Davis*)

Closed circuit with Dorr classifier. Original feed, 177 tons per 24 hr., return, 792 tons; total 969 tons. 28,000 lb. @ 2⅜-in. and 2-in. balls. 23.8 r.p.m. 30 per cent. moisture. 145 hp.

Sizing test of feed and product

Screen, mesh	Feed, per cent. weight	Classifier overflow, per cent. weight
4	29.2	....
8	27.1	....
14	15.4	....
28	7.9	....
48	4.5	....
100	3.7	1.4
200	2.8	11.8
300	1.6	15.0
−300	7.8	71.8

the single-stage operation the final product contained 1.4 per cent. oversize on the limiting screen against 2.4 per cent. in the two-stage treatment. Considering both products satisfactory, single-stage crushing produced finished material at the rate of 177 tons per 24 hr. or 0.049 ton per hp.-hr. In the 2-stage operation −100-mesh material was produced in the first stage at the rate of 294 tons per 24 hr. or 0.085 ton per hp.-hr. In the second stage finished material was produced at the rate of 157 tons per 24 hr. or 0.045 ton per hp.-hr., which is nearly as good as was done in the single-stage operation, yet Davis states that this second mill was distinctly underloaded and estimates that it could have produced 230 tons of finished material per 24 hr. which would have been 0.066 ton per hp.-hr. As it was, the 2-stage operation as a whole produced 368 tons of finished material per 24 hr. or 0.053 ton per hp.-hr.

Table 23. Performance of 8-ft. × 22-in. conical ball mill crushing from 0.25-in. to 100-mesh in two steps. (*After Davis*)

FIRST STAGE: Closed circuit with Dorr classifier. Original feed, 368 tons per 24 hr.; return, 768 tons; total, 1136 tons. 28,000 lb. @ 3- and 2-in. balls; 23.8 r.p.m.; 145 hp.

SECOND STAGE: Closed circuit with Dorr classifier. Original feed, 157 tons per 24 hr.; return, 120 tons; total, 277 tons. 28,000 lb. @ 2- and 1-in. balls; 23.8 r.p.m.; 145 hp.

Sizing tests of feed and product

Screen, mesh	Weight, per cent.			
	First stage		Second stage	
	Feed	Classifier overflow	Feed	Classifier overflow
4	29.2	....	....	....
8	26.2	....	....	....
14	14.1	....	0.3	....
28	8.6	....	0.3	....
48	4.7	4.3	5.4	....
100	3.3	15.8	39.0	2.4
200	3.3	18.8	38.0	13.3
300	1.7	11.4	8.2	17.7
−300	8.9	49.8	8.8	66.6

The performance of 8-ft. × 36-in. conical ball mills at MIAMI (Table 11) is a good example of the economy of 2-stage grinding with one primary mill in open circuit followed by two secondary closed-circuit mills in parallel. The capacity of the three mills as arranged was 967 tons per 24 hr. from 0.4 per cent. on 1-in. to 0.1 per cent. on 48-mesh, or 0.079 ton per hp.-hr. Their capacity in parallel on 3-mesh feed was 822 tons per 24 hr. to the same size, or 0.068 ton per hp.-hr. The capacity of the three mills treating primary feed (test 22, Table 11) in closed circuit would have been less than this because the primary feed was coarser. The primary mill in open circuit produced 304 tons per 24 hr. of −48-mesh material, or 0.093 ton per hp.-hr. against 237 tons each (0.068 ton per hp.-hr.) in the secondary mills in closed circuit.

North (*63 A 554*), commenting on the UNITED EASTERN flow-sheet, states that 2-stage ball milling is about 15 per cent. cheaper than stamps plus tube mills in the district and that single-stage ball milling shows about the same supply and labor costs as 2-stage, but about 20 per cent. greater power consumption.

Circulating tonnage in closed-circuit crushing ranges from 50 to upwards of 500 per cent. of the original feed tonnage. The average return is between 150 and 200 per cent. of the original. A greater percentage would ordinarily overload the classifier.

At the MESABI IRON CO., the experimental flow-sheet was arranged to permit the use of one to four Dorr classifiers with one 8-ft. × 22-in. conical ball mill. Results of a test run are shown in Table 24; they show distinct increase in capacity and fineness with increase in number of classifiers. Against this advantage, however, must be balanced a decided inconvenience in handling the circulating load and a much larger floor space devoted to grinding; as a result multiplication of classifiers has not yet been adopted in grinding practice.

Nature of material ground has a marked effect on ball-mill grinding in the case of certain materials, comparing, *e.g.*, coal, cement copper, gypsum, cocoanut shells, ores, brass ashes, etc., but considering ores alone, the physical character has much less effect on the amount of crushing done than operating factors such as size of feed, moisture content, ball load, etc. Thus in Table 4, comparing the 6 × 4-ft. mill at ST. JOSEPH LEAD CO. with the 6 × 6-ft. mill at CONSOLIDATED ARIZONA SMELTING CO., dolomite gangue appears to grind from 3- to 8-mesh much more readily, based on tons per horsepower-hour,

than quartzitic rock from 2- to 4-mesh, and this is in line with the reasonable expectation that limestone is more easily ground than quartz. On the other hand, substantially the same amount of −200-mesh material was produced by both mills, at the rate of 0.034 ton per hp.-hr. from dolomite and 0.033 ton from quartz. In Table 5, comparing the 6 × 4½-ft. mills at United Eastern and Shattuck Arizona, the quartzitic ore at United Eastern is crushed from 2.5-in. to 4-mesh at a higher rate per horsepower-hour than Shattuck Arizona limestone is crushed from 0.5-in. to 3-mesh and −200-mesh material is produced from the quartz ore at the rate of 0.024 ton per hp.-hr. against

Table 24. Tonnage test with 8-ft. × 22-in. conical ball mill and two to four 8-ft. Dorr classifiers

	2-classifier	3-classifier	4-classifier
Tons per hour to ball mill......	17.7	20.6	21.4
Tons −150-mesh per hour to ball mill......................	15.38	18.02	19.2
Total circulating load in tons per hour......................	175.4	222	260
Solids in ball-mill product, per cent......................	67.75	70.30	73.90
Solids in classifier overflow, per cent......................	13.90	10.65	9.52

Sizing tests

Size, mesh	Feed, per cent. weight	Sands, per cent. weight	Overflow, per cent. weight	Sands, per cent. weight	Overflow, per cent. weight	Sands, per cent. weight	Overflow, per cent. weight
8	1.4	....	....	0.4	....	1.0	....
14	3.7	1.1	....	1.6	....	3.4	....
28	7.3	5.0	....	7.3	....	7.5	....
48	11.2	13.5	....	14.9	0.1	11.8	....
100	27.2	34.7	3.4	34.0	4.3	27.6	3.1
115	10.5	11.1	4.2	10.0	3.5	10.4	3.0
150	8.5	9.0	5.5	7.9	4.6	8.8	3.6
200	13.1	11.5	12.8	10.5	11.8	12.9	9.8
325	7.3	5.0	17.6	5.4	15.9	7.4	16.3
−325	9.8	9.0	56.6	8.0	59.7	9.1	64.1

a rate of 0.020 ton from the limestone. The same conclusion is indicated by Table 25, compiled from Lennox' work (*61 A 237*). The column "Comparative grinding resistance" (as given by Lennox) indicates but little difference between a number of ores in the middle range, and when the "Grinding order," which is based on another method of analysis of the same data, is compared with this column, it is seen that this more direct method of weighting the results causes marked changes in Lennox' estimate of the ease of grinding the different materials. Thus while, *e.g.*, by both methods of estimating, ores 34 and 13 fall together and likewise ores 18 and 20, yet Lennox' method makes the first two slightly more resistant than the second two while the other separates the two pairs a considerable distance in the scale and indicates that 18 and 20 are much more resistant than 34 and 13. Both methods agree in placing ores like 28 and 11 at one end of the scale and 22, 32, 30 and 48 at the other and experience accords with the conclusion thus pointed, *viz.*: that there are certain ores so very hard and tough that they are ground with

much more difficulty than the average and certain others that are easier to grind than the average. It would not be safe to take more than 50 to 75 per cent. of the capacity figures in Tables 16 and 17 for the first type of ore, and 150 to 200 per cent. can safely be taken for the second, but no distinct difference in ease of grinding should be expected between, say, ores 44 and 42 notwithstanding they appear widely separated in the table.

Table 25. Ball-grinding resistance of various ores. (*After Lennox*)

Sample number (*a*)	Comparative grinding resistance (*b*)	Grinding Order (*c*)
28	1.33	1
12	1.16	6
11	1.00	2
44	0.94	8
36	0.92	18
38	0.83	10
7	0.81	3
34	0.80	14
13	0.80	13
18	0.79	4
20	0.71	5
52	0.71	9
16	0.70	15
40	0.69	12
24	0.63	16
46	0.63	17
30	0.61	22
48	0.53	21
42	0.53	11
26	0.52	7
50	0.46	19
22	0.38	23
32	0.37	20

a 7, Butte and Superior; 11, Portland mill feed, phonolite, syenite, breccia and some granite; very hard and tough; 12, British Columbia, Tonopah-Belmont Development Co.; 13, Tonopah-Belmont; 16, Miami Copper Co.; 18, Goldfield Consolidated, cyanide feed; 20, Goldfield Consolidated, flotation feed; 22, Utah Copper Co.; 24, Homestake, unaltered ore; 26, Alaska Treadwell, schist; 28, Calumet and Hecla, jig tailing; 30, Nevada Consolidated, pit ore; 32, Ray Consolidated; 34, Humboldt mine, Smuggler Union; 36, New Cornelia Copper Co.; 38, [Liberty Bell; 40, Alaska Gold Mines Co.; 42, Calumet and Arizona, roaster ore; 44, Smuggler Union; 46, Arizona Copper Co.; 48, Phelps-Dodge, Morenci; 50, Copper Queen, mill feed; 52, Alaska Treadwell, quartz and schist. *b* Ratio of average number of mesh tons (Art. 27) produced by 5- and by 10-min. grinding. (*After Lennox.*) *c* Numerical order of increasing "grindability" of corresponding samples, as indicated by increasing amounts passing the 200-, 65- and 20-mesh and decreasing amounts remaining on a 10-mesh screen in the products obtained by 5- and 10-min. batch-crushing tests on a standard feed in a small ball mill.

Such differences as do exist are more marked in grinding to very fine sizes than in the coarser range. Thus Young's work (*58 A 126*), summarized in Table 26, shows little difference in the amount of −48-mesh material produced

Table 26. Effect of nature of feed on capacity of ball mill(*a*)

Material	Per cent of feed on 13.33-mm. screen	Per cent. −200-mesh		Per cent. −48-mesh	
		Feed	Product	Feed	Product
Quartzite.........	44.5	1.4	24.4	6.9	66.5
Trap.............	44.7	2.4	32.9	5.3	67.0

a Feed rate in both runs, 18 tons per 24 hr. Moisture, 36 per cent.

from trap and quartzite under similar conditions, but does show a distinct difference in the amount of −200-mesh product. Hanson (*120 P 17*) classes Inspiration ore (which is the same as Miami ore, sample No. 16, Table 25) as a soft ore and warns that the capacities there attained in an 8 × 6-ft. grate

mill (475 tons per 24 hr. from 4.7 per cent. +1.5-in. to 0.6 per cent. on 48-mesh) are not to be expected on harder ores.

Size of feed ranges in practice from 3- or 4-in. maximum size down to, say, 20-mesh (see Tables 4, 5 and 11). For any given ball mill and ball charge there is probably a feed size that will give, in connection with the preceding crushers, maximum tonnage of a given desired size per horsepower-hour, but what this size is is rarely, if ever, worked out exhaustively in a plant. Too frequently it is not examined into at all, but the ball mill is set the task of handling a given feed whose size is determined by other and often less important considerations than the resulting efficiency of the mill. The problem is dodged so frequently on account of the difficulty of obtaining convincing evidence.

The classic dissimilarity in practice lies between Miami Copper Co. and Inspiration. These are adjoining mines with similar ores that enter the coarse-crushing plant at about the same maximum size, *viz.*: 18-in., and are ground to substantially the same size (48-mesh) for flotation. At Inspiration, crusher product, 3- or 4-in. maximum size, goes directly to 8 × 6-ft. grate mills; at Miami 8-ft. × 36-in. conical mills are fed with −¾-in. roll product. Table 27, compiled from the booklet issued by Miami Copper Co., at the time

Table 27. Comparison of crushing and grinding at Miami and Inspiration, 1916

	Miami	Inspiration
Grinding mills used	8 @ 8-ft. × 30-in. conical ball mills 3 @ 8-ft. × 66-in. conical pebble mills 1 @ 6-ft. × 22-in. conical ball mill	40 @ 8 × 6-ft. grate mills
Daily tonnage	6000–7000	16,000–17,000
Product, per cent. on 48-mesh	12.5*c*	3.1
Power, kw.-hr. per ton	7.86	10.27
Power, cost per kw.-hr. cents	1.1	0.7
Power, cost per ton, actual, cents	8.63	7.18
Power, cost per ton, comparative, cents (*a*)	8.63	11.30
Labor to re-line, man-shifts	6	48
Labor, operation, mills per man	10	4
Supplies, balls	2.07 lb. cast iron @ 5¢	1.79 lb. chrome steel @ 6.5¢
Supplies, lining	0.1 lb. manganese steel @ 15.6¢ and 1.15 lb. cast iron @ 5¢	0.3 lb. manganese steel @ 11¢
Supplies, grates	None	0.47¢ per ton
Total cost per ton coarse crushing and grinding, actual	20.7*c*	27.80*b*

a Reckoning Inspiration on the same basis as Miami, viz., 1.1¢ per kw.-hr. *b* With power cost raised as above this would be higher. *c* For April, 1917, grinding was to 3 per cent. + 48-mesh.

of the Arizona meeting of the A.I.M.E., 1916, and from a report of Inspiration operations (*102 J 678*) and from miscellaneous notes, shows the essential results at the two mills. It indicates a distinct advantage either for the finer feed at Miami or for the conical mill or both. The extreme in roll reduction prior to ball milling is found at Canada Copper Corp., where roll crushing is carried to 10-mesh, followed by 2-stage ball milling to 100-mesh. At Engels (*123 P 189*) an 8 × 6-ft. Marcy mill and one 6 × 12-ft. tube mill in series handled −2-in. feed to flotation size at the rate of 444 tons per 24 hr. Total power consumed was 377 hp. so that the tonnage ground per hp.-hr. was 0.049. With −1-in. feed the ball mill followed by two tube mills in parallel handled 866 tons per 24 hr. with 519 hp. or 0.069 ton per hp.-hr. Not over 75 hp. would be required to break the +1-in. material in this feed to −1-in., which would make the tons per hp.-hr. from 2-in. 0.061 by the stage operation. At Consolidated Arizona Smelting Co. (*104 J 71*) a 6-ft. cylindrical mill and a 6-ft. conical mill installed to grind to 8-mesh in open circuit utterly failed to deliver

the expected tonnage with −1.5-in. feed. Many ⅜-in. to ½-in. pebbles were discharged. Reducing feed to −1-in. increased tonnage considerably. Further decrease to −0.75-in. doubled the capacity over that on −1-in. feed. Lay (*114 J 1123*), commenting on the performance of the 5 × 4-ft. center-discharge mills at Le Roi No. 2 (see Table 4), says that further reduction of the primary-mill feed (−1.5-in.) is clearly indicated.

The present-day trend is toward finer initial feed to primary ball mills and toward stage ball or tube milling, except in the cases where limited tonnage will not justify the more elaborate equipment.

Size of product is the most important factor in determining capacity of a ball mill. Tons per horsepower-hour, based on Tables 4, 5 and 11 average 0.200 for mills delivering 8-mesh and coarser products, 0.146 for mills delivering at 10- to 20-mesh, 0.090 when grinding to between 28- and 48-mesh, and 0.080 when grinding to 65-mesh and finer.

Moisture. Changes in moisture content of the pulp in a ball mill affect its grinding capacity in several different ways and the net result is not invariably susceptible of correct prediction. The moisture content affects the fluidity of the pulp and also its transporting power. Perfectly dry rock, when ground fine, is surprisingly fluid, flows readily through a cylinder mill, and has great transporting power. Its fluidity is evidenced by the fact that it readily discharges from a mill with horizontal axis; its high transporting power is illustrated by the large particles in the discharge (see Table 28). When the moisture content is between 8 and 15 per cent., or thereabouts, especially if the solid contains clayey matter, a stiff mud is formed that cannot be forced through the mill and effectually prevents operation. With upwards of 20 per cent. moisture, ordinary pulps are sufficiently fluid to pass readily through the mill, the fluidity increasing with increase in moisture content. For a given moisture content the apparent wetness is greater the coarser the solid particles. In such wet pulps varying moisture content has different and somewhat contradictory effects on the performance of a mill. In the first place the transporting power of the pulp increases with the decrease in moisture content (see Sec. 20, Art. 10), hence, all other things being equal, the thicker the pulp the larger the largest particles arriving at the discharge end of the mill. In the center-discharge types these coarse particles appear in the discharged products, but in a grate mill the thick, coarse pulp is held back by the grate. Hence from this one standpoint, a thick pulp will cause the production of large oversize

Table 28. Results of test of a 4½-ft. × 16-in. conical ball mill, dry-grinding

Material	Trap
Feed rate, tons per 24 hr.	18
Speed, r.p.m.	28
Ball load; 5-, 4-, 3-, 1¾-in. balls, lb.	4500
Power consumed, horsepower	22.1

Screen aperture, mm.	Weight, per cent.	
	Feed	Product
38.1	2.2	...
26.7	70.9	6.7
18.8	24.4	5.8
13.3	1.6	2.3
9.4	0.2	0.8
6.7	0.1	0.5
4.7	...	0.3
3.3	...	0.5
2.4	...	0.3
1.6	...	0.5
1.2	...	0.8
0.83	...	1.5
0.59	...	2.9
0.42	...	3.7
0.29	...	5.2
0.21	...	5.9
0.15	...	6.9
0.10	...	9.4
0.07	...	7.5
Through last screen	1.7	38.6

in a center-discharge mill, while in a grate mill it will result in reduction in the amount of coarse oversize. Conversely, increased dilution will produce a more granular discharge from a grate mill while it may result in increased fineness in the discharge from a center-discharge mill (see Table 29).

Table 29. Effect of moisture on ball-mill crushing (*58 A 126*)

Moisture, per cent.	Horse-power	−200-mesh		−65-mesh	
		Per cent. in product	Tons produced per horsepower-hour	Per cent. in product	Tons produced per horsepower-hour
0	19.2	15.4	0.0055	37.6	0.0127
18.8	17.7	14.6	0.0056	34.4	0.0124
25.0	17.7	20.4	0.0080	43.8	0.0164
38.5	17.7	21.6	0.0086	48.9	0.0185
51.8	19.3	26.2	0.0096	58.2	0.0206
68.2	20.1	26.1	0.0092	60.0	0.0204

At Asturiana de Minas (*115 J 396*) when a 4.9-ft. (diam.) conical mill with 2-ton ball load was fed with −2-in. feed at the rate of 50 tons per 24 hr., the product with 30 per cent. moisture in the mill contained 1 per cent. +2-mm., 83 per cent. −40-mesh, and 17 per cent. −200-mesh; while with 50 per cent. moisture the corresponding quantities were trace, 92 and 28 per cent. The experimenter found that increase in moisture to 80 per cent. caused further increase in fineness of discharge and that progressive thickening below 30 per cent. moisture caused production of a progressively coarser product until grinding ceased altogether. At Catemu (*123 P 886*) a 4½-ft. × 16-in. conical mill (see Table 11) was operated at from 20 to 60 per cent. moisture. The best results were obtained at about 35 per cent. with soft, clayey ore and 30 per cent. with hard, compact ore. When moisture dropped below 30 per cent. the mill discharged much coarse material.

From another standpoint thick pulp will tend to increase the tonnage of fine material produced in both types of mill. This follows from a consideration of the behavior of thick and of dilute pulps at the ball surfaces. With a thick pulp all of the balls in the mill are coated with a layer of solid all the time, whether they are above or below the general pulp level in the mill, and, therefore, every impact and every relative movement of balls in contact will result in crushing, provided only that the balls are heavy enough. With dilute pulp, on the other hand, the balls above the general pulp surface are substantially free of pulp and little grinding takes place in this part of the load; also the concentration of solid per unit of volume is less below the pulp surface, and hence the crushing done for any given relative ball movement tends to be less. On the other side of this picture, however, is the fact that the cushioning effect of dilute pulps is not so great as that of thick pulps and hence the loss in momentum of falling balls after striking the pulp surface is not so great in dilute pulps. Finally, the volume of pulp corresponding to a given tonnage of solid matter is greater for dilute than for thick pulps and with equal tonnages of solid passing through the mill the rate of passage is higher with the dilute pulp and less opportunities to be crushed are, therefore, afforded. The usual result of this combination of effects is to increase the tonnage of fine material produced by decreasing the moisture content and simultaneously increasing the tonnage of solid passed through the mill. The point of maximum grinding efficiency (maximum tonnage of desired product per mill per day) is found in most cases to lie between 25 and 35 per cent.

moisture for coarse feeds (0.5 in. maximum and upwards) and between 30 and 50 per cent. moisture for finer feeds. Table 30 (*66 A 105*) shows in detail results in accord with this conclusion. Table 29 (*58 A 126*) shows the less common but nevertheless not exceptional case. Table 29 shows also that the power consumption per mill is higher with high moisture content than with low. Thick pulp crowds the ball charge toward the mill axis and lessens the lever arm of the load.

Table 30. Effect of moisture content on performance of conical ball mill. (*After Delano and Rabling*)

Tons solid feed per 24 hr.	240	190
Moisture, per cent.	65.8	48.1
Horsepower consumed	51.6	51.6
Tons − 10-mesh product	216	181
Tons − 10-mesh product per horse-power-hour	0.174	0.146
Per cent. − 150-mesh in − 10-mesh product	23.8	33.4

Screen aperture, Tyler mesh	Weight, per cent.		
	Average feed	Products.	
3	3.1	...	...
4	11.9	1.0	...
6	17.6	1.7	0.3
8	26.0	3.8	1.0
10	24.7	7.8	3.5
14	12.4	8.8	5.2
20	4.3	9.0	7.0
28	...	9.7	9.3
35	...	7.8	8.3
48	...	6.7	7.9
65	...	6.0	7.1
100	...	8.3	8.0
150	...	8.0	10.6
200	...	3.9	5.3
− 200	...	17.5	26.5

In most cases water is added to the mill to bring pulp to the proper consistency. Commonly this water is necessary to make the classifier-sand return flow in the launder. Dowsett (*108 J 185*) recommends that, in conical mills, at least, as much of the new water as possible be added from the discharge end about 1½ to 2 ft. back in the discharge cone. This increases the fluidity of the pulp in this part of the mill and lessens discharge of oversize.

Wet *vs.* dry grinding. All wet-concentrating mills grind wet but many industrial plants grind dry. Capacity, power consumption, and ball (or pebble) and liner wear are greater in wet than in dry grinding. Wet grinding permits more efficient closed-circuit work for all fine sizes (less than 10-mesh) than dry and has the added advantage that it eliminates dust troubles. Hardinge (*114 J 935*) limits the capacity increase in wet grinding to 15 to 25 per cent. and states that pebble and liner wear are less than half as great in dry grinding, and capital cost less, but even so believes that wet grinding is cheaper on account of the greater efficiency of the wet closed circuit.

Liners with rough surfaces produce more tumbling action of balls

than smooth-surfaced liners and therefore are better for coarse and intermediate grinding; but for finest grinding, experience shows that smooth liners are best. Rough-surfaced liners can be run at lower speed, to produce a given amount of tumbling, and, although for a given speed they require more power than smooth liners, yet the power consumed for a given amount of tumbling is less.

Size of balls should be proportioned to the work to be done, *i.e.*, the size of particle that must be broken and the size of product desired. Large and hard particles are best broken by impact, hence coarse feeds and hard ores require larger balls than finer feeds and softer ore. But impact is a function both of the weight of the falling body and the distance through which it falls, so with mills of large diameter or with high mill speeds and consequent free parabolic fall, balls need not be as large as with the reverse conditions. The usual maximum size is 5-in., but 6- and even 7-in. balls have been used in a few places.

At ALASKA-JUNEAU 5-in. balls failed to crush the ore, but 7-in. balls succeeded (*120 P 17*). At CATEMU (*123 P 886*) a 4½-ft. × 16-in. conical mill was used to crush −1.5-in. feed (see Table 11). Increase in size of balls from 4-in. maximum to 5½-in. decreased the amount of coarse material discharged but did not increase capacity to 12 per cent. +80-mesh. With feed coarser than 0.75-in. common practice is to feed nothing smaller than 5-in. balls, and at some plants all balls smaller than 3-in. are removed from the mill periodically. At SHATTUCK ARIZONA (*110 J 760*) balls smaller than 2-in. are removed when possible. At TIMBER BUTTE all balls smaller than 3-in. are sorted out.

Balls smaller than 1-in. tend to discharge automatically from non-grate mills when the pulp is thick and the ball load is kept well up to the center of the mill. Removal of small balls is based on the reasoning that a large particle is just as likely to be struck by a small ball as by a large, and that if the small ball is unable to crush it the blow is wasted, hence there should be no ball present that is incapable of crushing the largest particle in the feed. With feeds finer than 0.75-in. the maximum size of ball may be less than in the above instances. Davis (*61 A 255*) tested for the proper size of ball for −0.25-in. feed. His tests (see Table 31) indicate that on such feed a mixture of 2⅜- and 2-in. balls is superior to a mixture of 5-, 4-, 3- and 2½-in. balls both in tons of −200-mesh material produced per 24 hr. and per hp.-hr. The test also indicates that heaping up of one or two sizes in the classifier sands, such as at 48- and 100-mesh in the test with large balls, signifies improper operation from one cause or another, while more regular size distribution of the sands signifies proper operation. This conclusion is confirmed by the tests presented in Table 36 from which Davis draws the further conclusions that if heaping occurs in the coarser sizes the balls are too small or the mill speed too low, or both, while if heaping occurs in the finer sizes the balls are too large or mill speed too high, or both. The effect of small change in ball size is clearly shown in comparing tests 8 and 9 of Table 36. The average size of ball in test 8 was 2¼-in.

Reasoning along the line that the amount of crushing done is in proportion to the number of crushing blows struck, then the smaller the balls the more the crushing, provided each ball is large enough to strike a crushing blow. From another viewpoint, if the amount of crushing done is proportional to the effective crushing surface, then the smaller the balls the greater the surface for a given weight of charge, provided again that the balls are heavy enough to provide effective crushing surface. This conclusion seems to be borne out in practice.

Table 31. Comparison of performance of large and small balls in 8-ft. × 22-in. conical mill. (*After Davis*)

	Large balls	Small balls
Feed rate, tons per 24 hr	259	302
Classifier	Drag	Dorr
Ball load, lb	28,000	28,000
Size of balls, in	5, 4, 3, 2½	$2\frac{3}{8}$, 2
Speed, r.p.m	23.8	23.8
Horsepower consumed	146	145
Moisture in feed, per cent	40	40
Circulating load, tons per 24 hr	960	648
Tons −200-mesh per 24 hr	151	194
Tons −200-mesh per horsepower-hour	0.043	0.057

Sizing test, screen, mesh	Weight, per cent.			Weight, per cent.		
	Original feed	Classifier sand	Classifier overflow	Original feed	Classifier sand	Classifier overflow
4	22.7	......	......	47.8	10.3	......
8	32.0	0.7	......	19.1	9.6	......
14	14.6	2.8	......	10.2	9.9	......
28	6.6	12.5	......	6.0	14.4	......
48	6.6	31.6	......	3.3	18.1	......
100	4.1	36.6	11.5	2.6	19.8	8.6
200	3.9	7.5	20.8	2.4	10.1	18.7
300	1.1	4.2	10.8	1.5	2.4	12.8
−300	8.4	4.1	56.8	7.0	5.5	59.8

Delano and Rabling (*66 A 106*), crushing −9+2-mm. feed through 10-mesh in a conical ball mill, found highest capacity and lowest slime production with the largest proportion of 5-in. balls in the mill. At Miami Copper Co. in 8-ft. × 22-in. conical mills with 14,000- to 15,000-lb. load, taking a feed containing 2 per cent. on 3-mesh, a ball charge consisting originally of 4000 lb. of 4-in. and 10,000 lb. of 2-in. balls, with 4-in. balls fed daily, discharged a product containing 24 per cent. more +48-mesh material than the discharge from a parallel mill with 2-in. ball charge, but otherwise operating identically.

Table 32. Sizing test of ball charge in 8-ft. × 22-in. conical mill at Miami. (*After Davis*)

Diameter, inches	Weight, per cent.
2.0 to 1.8	27.5
1.8 to 1.6	21.0
1.6 to 1.4	17.7
1.4 to 1.2	12.8
1.2 to 1.0	9.0
1.0 to 0.8	5.8
0.8 to 0.6	3.4
0.6 to 0.4	1.9
Below 0.4	0.9

Actual size of balls in an operating mill is, of course, quite different from the original charge and from the daily addition.

Table 32 shows the size distribution in the charge from an 8-ft. × 22-in. conical mill at Miami originally charged with 14,800 lb. of 2-in. balls and kept up by the addition of 400 lb. of 2-in. balls daily. Table 33 shows the charge in a 6 × 6-ft. dry-crushing cylindrical mill at Golden Cycle after 694 hr. operation with a charge of about 6500 lb. of balls, originally 5½-in., with daily addition of 5½-in. balls and periodic removal of all less than 3-in.

Shape of grinding medium. Most operators have noticed the flattened shapes assumed by the worn balls in a ball mill, and, reasoning therefrom that the greater efficiency of an established mill as compared to one newly started is due to the non-spherical shape of the old balls, have advocated the

use of such shapes as cones, single and base-to-base; cubes, disks, short cylinders, dished balls, etc. The intent in every case is to increase the area of contact between the crushing media and thereby increase, particularly, abrasion. Detailed experimental data are lacking.

Police (*116 J 553*) says that double pyramids proved best in cylindrical mills, that cubes were highly efficient for very hard and abrasive material, that single cones were better than spheres for conical mills, and that, in a rod mill, short rods were better than long. Feust (*116 J 553*) recommends disks. Hardinge (*116 J 641*) has experimented with the disk in sliming. Cubes have been tried at several plants (Chino, Nacozari, and others) but results have not been published. MacDonald (*118 J 446*) infers that results with cubes have been superior to those with rods in grinding to 48-mesh. Failing definite experimental evidence to the contrary, the moving pictures of Haultain and Dyer (*25 CMI 651*) would seem to point directly to the conclusion that balls or short cylinders, which in the rising mass of the load most readily assume rotary movement around their own axes are, therefore, most active within the load and the most efficient shapes for fine feeds. The conical and cubical shapes should be carried higher in the mill at a given speed and, therefore, cause more active cascading and more free parabolic fall and so might do more impact crushing than the sphere, which would be an advantage with coarser feeds. Scobey (*110 J 11*) tells of the trial of mill scrap, such as 6-in. lengths of 4-in. shafting, necks of stamp shoes, etc., as substitutes for balls in a 6-ft. conical mill running in parallel with gravity stamps. Capacity fell off badly. Helipebs, made of heavy steel wire coiled in the form of a helix 1¼ in. long and ¾ in. external diameter and afterward hardened, have been used in English cement plants as a substitute for balls and pebbles. Weight is 1.25 oz. each (*109 J 202*). They give large crushing surface for little weight, but in the absence of conclusive data showing efficient performance are not to be recommended.

Table 33. Sizing test of ball charge in 6 × 6-ft. cylindrical ball mill at Golden Cycle, dry-crushing. (*After Davis*)

Diameter, inches	Weight, per cent.
5.5 to 5.3	12.8
5.3 to 5.1	12.5
5.1 to 4.9	13.2
4.9 to 4.7	8.6
4.7 to 4.5	8.6
4.5 to 4.3	8.4
4.3 to 4.1	6.8
4.1 to 3.9	5.6
3.9 to 3.7	4.2
3.7 to 3.5	5.4
3.5 to 3.3	5.2
3.3 to 3.1	4.5
3.1 to 2.9	4.2

Weight of charge. The usual weight is such that the struck volume of the load is between 30 and 50 per cent. of the mill volume.

Davis (*61 A 284*) recommends a load between 25 and 50 per cent. of the mill volume, stating that with a load of more than 60 per cent., interference in the falling zone is excessive, while with a load of less than 20 per cent., slip is excessive. Hines (*59 A 249*) says that maximum crushing per unit of power is attained with ball loads of about 33 per cent. of mill volume. At Miami Copper Co. when the change was made from pebble to ball charges the power installed was insufficient to run the 8-ft. × 22-in. conical mills with full ball load (30,000 lb.) and but 15,000 lb. were charged. In tests against a 6½-ft. (inside) by 42-in. conical mill (made by lagging an 8-ft. × 22-in. mill), charged with 26,000-lb. balls, the smaller mill did slightly better work in open circuit but not such good work in closed circuit. Comparative efficiencies (Stadler method) were 100 and 92. Liner consumption was much less in the small mill but ball consumption was about the same in both. At Asturiana de Minas (*115 J 395*) comparative tests were run with charges of different weights (see Table 34). The product of the lighter charge contained 17 per cent. +40-mesh against 14 per cent. from the heavier charge, but fine material was produced at a much higher rate per unit of power with the lighter charge. Table 35, showing the effect of smaller percentage variations in weight of load, shows lower ball consumption with the smaller ball load, but while the trend of the grinding data is the same as at Asturiana, the details are not wholly consistent. *Truscott's* recommendation that the load for coarse feed should be less than that for fine is out of line with these results. Young's experiments (*58 A 126*) indicate that increase in ball load causes a decrease in the power consumed per ton of load (see Fig. 16) and a finer product, from which facts the conclusion is that the maximum load that a mill will carry is best. It is to be noted, however, that at Catemu (*123 P 888*) it was found impossible to introduce feed into a 5 × 4-ft. grate mill with ball-load up to the axis.

Table 34. Effect of weight of ball charge on product of a 4.9-ft. (diameter) conical ball mill. (*After Barcena y Diaz*)

Weight of ball charge, lb.		7000	4000
Power, consumed, hp.		32	24
Screen analyses; weight, per cent.			
Aperture	Feed	Product	Product
2-mm.	60	2	1
1-mm.	....	5	6
40-mesh	26	7	10
100-mesh	5	38	44
200-mesh	5	27	22
−200-mesh	4	21	17
Tons produced per horsepower-hour (*b*)			
−2-mm.		0.045	0.061
−40-mesh		0.056	0.072
−200-mesh		0.013	0.013

a Feed −2-in. *b* Average feed rate, 60 tons per 24 hr.

Speed depends primarily upon the size of feed and diameter of mill. Movement of balls in a properly operated mill is indicated by the moving pictures of Haultain and Dyer (*25 CMI 651*) to consist of two distinct varieties of motion: (1) rotation around their own axes parallel to the mill axis, (2) cascading or free-falling. Fig. 17 represents the action in a mill operating at proper speed for crushing coarse feed. In the lower part of the mill balls are in layers concentric with the mill shell. The layer in contact with the shell itself is moving at substantially the same rate as the shell; the rougher the mill lining the less the slip. Any ball in this layer, between lines *a–a* and *b–b*, is subject to two sets of forces, one applied at the point of contact with

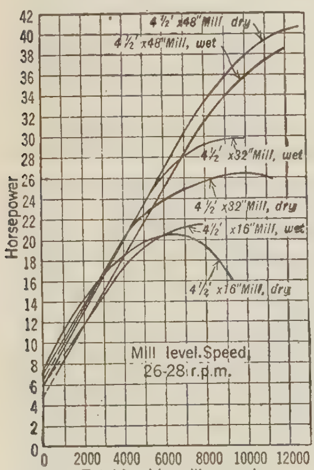


FIG. 16.—Variation in horsepower with weight of ball load. Conical mill, 4½-ft. × 16-in.

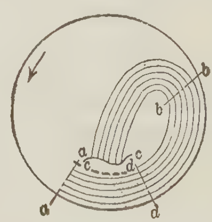


FIG. 17.—Diagram of ball action in a properly-operated coarse-crushing ball mill (*after Haultain and Dyer*).

...ball load in 8-ft. × 36-in. conical mill at Homestake

Weight of balls, pounds	Tons per 24 hr.	Feed		Product		Horsepower consumed	Tons −200-mesh produced per horsepower-hour	Balls consumed, pounds per ton
		+1-in.	−200-mesh	+48-mesh	−200-mesh			
22,054	314	56.0	5.0	10.0	39.0	156.2	0.028	1.18
23,813	318	58.0	5.0	8.0	45.0	160.7	0.033	1.45
25,141	307	48.0	4.0	8.0	42.0	166.2	0.029	1.63
25,645	330	57.1	2.7	12.1	44.3	167.5	0.034	1.61
27,112	295	56.6	4.7	6.0	43.0	170.3	0.028	1.60
27,112	378	48.0	4.0	15.1	40.7	170.3	0.034	1.60
28,608	340	61.7	7.2	7.8	46.6	172 e	0.032	1.67

e Estimated.

Table 36. Tests showing relation between size of balls, speed of mill and size distribution in classifier sands; 8-ft. × 22-in. conical ball mill. (*After Davis*)

Test number	1	2	3	4	5	6	7	8	9
Size of balls, in.	5, 4, 3, 2½			4, 3, 2½, 2			2½, 2, 1½		2⅝ aver.
Speed of mill, r.p.m.	16.6	19.7	23.8	19.8	21.1	23.8	19.8	23.8	23.8
Classifier sands	Weight, per cent.								
On 4-mesh	3.8	...	...	1.8	1.5	...	34.1	10.3	0.8
8	6.3	1.0	...	1.3	0.8	0.7	14.1	9.6	3.2
14	5.6	4.3	3.5	1.9	1.1	1.3	9.5	9.9	7.4
28	7.4	15.3	12.5	4.3	3.3	4.2	10.2	14.4	18.8
48	14.7	30.4	31.6	9.2	8.8	9.6	10.9	18.1	31.0
100	35.4	33.2	36.6	25.8	26.1	29.4	11.6	19.8	25.4
200	20.2	10.1	7.5	34.9	35.6	36.2	6.3	10.1	7.9
300	2.5	1.4	4.2	11.9	11.5	11.0	1.5	2.4	1.9
−300	4.2	4.2	4.1	8.9	11.3	7.5	1.8	5.5	3.4

Tests 7–9 inclusive were run at higher feed rates (300 tons per 24 hr.) than tests 1–6 inclusive. In test 7 the circulating load became so large that the test had to be discontinued.

the mill shell, in a direction tangent to the shell and (in Fig. 17) counterclockwise, the other applied on the opposite side of the ball and oppositely directed. This pair of forces acting on any one ball constitutes a couple and, since the ball is constrained by contact with the shell and with its neighbors, it rotates around an axis perpendicular to the plane of the couple, *i.e.*, parallel to the mill axis. The balls in the adjacent layer are similarly acted upon and similarly rotate and every ball in the zone between *a–a* and *b–b* has similar rotation under the action of similar couples. This motion of individual balls is shown in Fig. 18. The result is grinding by abrasion. In the zone from *b–b* to *a–a*, reckoning counterclockwise, in a mill properly operating on coarse feed, there is substantial free fall of balls out of contact with each other and no grinding or breaking action whatever. At the surface *c–c* there is crushing by impact between the falling balls and the balls below this surface, which are supported by the mill shell. In the zone *a–a*, *c–c*, *d–d*, there is most intense and turbulent motion, appearing, in the moving pictures, to consist of violent tumbling in the region above the heavy dotted line and rapid shear of the mass along the dotted line with the portion of the balls below the line moving rapidly with the mill shell and those above appearing, as a mass, to be stationary, although each individual in this mass is in rapid movement with respect to its neighbors. Haultain and Dyer describe the zone *a–a*, *c–c*, *d–d* as the "TOE"; it is apparently the most active region in the mill and the place where the most grinding is done.

FIG. 18.—Action of crushing bodies in the layers adjacent to the shell of a cylinder mill.

Hines (*59 A 249*) takes the view that substantially all the crushing is done by impact, that abrasion is unimportant, and that there is substantially no rotation of the balls, but the moving pictures are practically conclusive on this point, and the effectiveness of fine-grinding mills at speeds far below those necessary for parabolic fall points to abrasion as a dominant factor in fine crushing.

If a mill is operated at a sufficiently high speed, all of the balls will cling in concentric layers within the shell and there will, of course, be no crushing. The speed at which cling of the outer layer just occurs is given by Davis (*61 A 266*) as $N = 54.19/\sqrt{r}$, where N = r.p.m. and r = the radius inside the shell in feet. Usual speeds in the mills range from 60 to 80 per cent. of this figure.

Davis (*61 A 257*) shows that the speed of a mill is closely related to the size of balls and that proper correlation is indicated by the sizing test of the return sands from the mill classifier. His tests are summarized in Table 36. With any given size of ball, increase in speed results in decrease of coarse material in the classifier sands, *i.e.*, in increased crushing of the coarser part of the mill feed. This fact appears in each of the three series of tests summarized (1 to 3 incl.; 4 to 6 incl.; and 7 to 8), but it is most striking in the third series. According to Davis, if speed is too low, the ball size being right for the feed, there is heaping up of material in the coarser sizes of classifier sand; if too high, heaping up in the finer sizes.

Haultain and Dyer (*25 CMI 651*) show that the fine material in the pulp in the mill segregates against the shell at high mill speeds and at the inactive spot near the center of the load at low speeds. The speed of best dispersion is considerably below that of parabolic fall of balls. For fine grinding the best speed will be that at which dispersion is best.

Reduction in speed is one of the most important advances in ball-mill practice. Originally 8-ft. mills were run at 22 to 26 r.p.m. Reduction to 15 or 16 r.p.m. resulted in decreasing power consumption by 25 to 33 per cent., substantially halving ball wear, and markedly decreasing liner wear, without changing capacity to any appreciable extent.

Pennick (*120 P 459*) recommends that the speed be made dependent on the size of product desired and suggests 375 to 400 ft. per min. peripheral speed (in an 8-ft. mill) for 48-mesh product and 550 to 625 ft. per min. for 8-mesh product. Wiggin (*59 A 245*) says that reducing the speed of the 7½-ft. × 72-in. conical mill at Anaconda from 23 to 15 r.p.m. reduced power consumption from 215 to 140 hp. and ball consumption from 4.5 to 3.25 lb. per ton crushed, while feed rate and size of product were substantially unchanged. A similar result is reported at Asturiana de Minas (*115 J 395*). A 4.9-ft. conical mill drew 40 hp. at 34 r.p.m. and 32 hp. at 30 r.p.m. Decrease in speed produced no appreciable difference in product with the mill crushing 60 tons per 24 hr. of −2-in. feed. See also Table 11, Anaconda, 7½-ft. × 72-in. and Consol. Ariz. Smelting Co., 8-ft. × 36-in. mills. Dowsett (*108 J 185*) says that at Bluestone Mining and Smelting Co., where it was possible, by means of a water rheostat to control mill speed very closely, variations of as little as ½ to ¼ r.p.m. made marked difference in capacity. At Miami results in closed-circuit grinding in underloaded 8-ft. × 22-in. conical mills was slightly improved by decreasing the speed and increasing the ball load from 15,000 lb. to an amount that held the power at the same figure as with the higher speed and lighter load. At American Graphite Co. (*120 P 569*) lack of crushing in a 6-ft. × 22-in. conical mill, due to the lubricating character of the feed, was remedied by increasing the speed. At Nevada Cons. Cop. Co. an 8-ft. × 30-in. conical ball mill with 30,000-lb. ball charge drew 130-hp. at 28 r.p.m. and made a product containing 8 per cent. on 48-mesh; at 21½ r.p.m. it ground 451 tons of −6-mesh feed per 24 hr. to 19.8 per cent. +48-mesh with an expenditure of 115.1 hp.; the same size mill at 17 r.p.m. ground 494 tons per 24 hr. from 18.6 per cent. + 1-in. to 5.2 per cent. +48-mesh, with the consumption of 109 hp. and at a cost of $0.059 per ton.

Shape of mill. The diameter of a ball mill is principally determined by the size of the largest particles to be crushed, while the length is, to a considerable extent, dependent upon the size of product desired. Diameter of mill, size of balls and speed together determine the force of the crushing blows. With balls of the usual size (5-in. maximum) a mill run at a speed to permit some free parabolic fall should be at least 3 ft. diameter for particles up to 0.5-in. in size, 5 to 6 ft. diameter for particles of 1-in. size and 8 ft. for 2- to 4-in. particles. The amount of coarse oversize in the discharge will be materially lessened if a mill of more than the minimum diameters given is used for any given size of material. Increase in length increases the amount of fine material produced in a single pass, but since power consumption also increases with increased length the amount of such material produced per horsepower-hour does not necessarily increase.

Table 37 shows the results of a test on a 4½-ft. mill with three different lengths of cylindrical section. With the feed rate constant, the tonnage of −200-mesh material produced per hp.-hr. shows a maximum with 32-in. cylinder, while the amount of −65-mesh per hp.-hr. is a maximum at 16-in. Tables 84 and 86, showing the performance of 8-ft. × 22-in. and 8-ft. × 66-in. conical ball mills at Miami Copper Co., indicate that the shorter mill is the more efficient in producing both −48-mesh and −100-mesh material. It is to be noted, however, that the longer mill was in re-grinding service, with the harder feed, and that it was making the finer product. These tests and practical experience both show that the finer the product desired the longer the mill necessary and that an excessive increase in mill length reduces mill efficiency.

The usual ratio of diameter to length in modern cylindrical mills ranges from 1 to 1.6, the greater ratio corresponding to the coarser product. Conical mills have a ratio of diameter to length of cylindrical section varying usually between the limits of 2.5 and 4.3. In closed-circuit grinding a small deficiency in either length or diameter can be overcome by increase in the amount of circulating load. For this reason mills of smaller diameter

than otherwise can be used for coarse feed and mills of smaller length than otherwise to produce fine products. But this does not alter the fact that wrong proportions will not give the most economical operation.

Table 37. Effect of length of cylindrical section on performance of conical ball mill (58 A 126)

Length of cylindrical section, inches	Ball charge, pounds	Horsepower	Product			
			−200-mesh		−65-mesh	
			Per cent. weight	Tons produced per horsepower-hour	Per cent. weight	Tons produced per horsepower-hour
16	1412	13.7	13.1	0.0072	35.4	0.0194
32	5167	26.6	27.5	0.0078	61.2	0.0173
48	6654	33.5	29.2	0.0065	65.4	0.0147

Mill, $4\frac{1}{2}$-ft. nominal diameter. Moisture, @ 40 per cent.; feed rate, 18 tons per 24 hr.

Slope. In the early days of ball-milling the mills, particularly conical mills, were set with the axis slightly sloping toward the discharge end. The purpose served was to produce a more granular product than could be made with the axis horizontal, all other conditions being the same.

Table 38 presents the results of a test with three different slopes. It would appear to show the most granular product at the intermediate slope, but the moisture content of the pulp in this test was considerably lower than in the last test and since other tests in the series indicated that fineness of product increased with increase in pulp moisture (see Table 29), it is probable that higher moisture would have placed the second test intermediate in results between the two others.

Table 38. Effect of slope on performance of $4\frac{1}{2}$-ft. × 16-in. conical ball mill (a)

Slope, in. in mill length	0	1.25	2.62
Moisture, per cent.	36.5	31.7	38.5
Weight of ball load, lb. (5, 4 and 3-in.)	4264	3550	2819
Horsepower	20.1	20.1	17.7
Coarsest product	0.2 per cent. on 6.68-mm.	0.2 per cent. on 6.68-mm.	0.1 per cent. on 9.42-mm.
Per cent. −65-mesh in product	55.2	46.7	48.9
Per cent. −200-mesh in product	24.4	19.5	21.6
Average size of product, mm.	0.246	0.316	0.288

a Feed, 18 tons per 24 hr., 0.7 per cent. on 1.5-in.

Mills are rarely sloped in present-day practice, due to the fact that the same result can ordinarily be obtained by regulation of the classifier and the feed rate and that sloping causes undue thrust on trunnion bearings, difficulty in countershaft alignment, and excessive wear of gears.

Grate opening may be made to control capacity and the amount of oversize in the mill product. Fig. 19 (59 A 2.?) shows the result of change in this factor on tonnage and size of product when all other conditions of operation were kept constant. It is better, however, to use a grate with large

openings and depend on outside control of product size. This is especially true when the changed result desired is one that would call for a smaller grate opening. The grate is not intended to be a sizing device and should not be used for one.

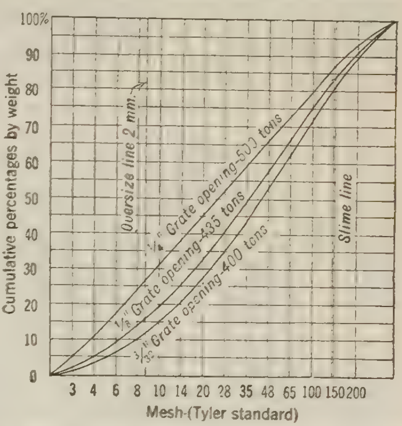


Fig. 19.—Effect of grate opening on ball-mill operation.

Power consumption. (See also Art. 6.) Figures from practice are given in Tables 4, 5 and 11. Power consumption increases with increase in ball load until the load reaches a point at or slightly above the axis, after which further loading may result in decrease of power (see Fig. 16).

At the Quincy mill an 8-ft. × 36-in. conical mill taking 350 tons of feed per 24 hr. in open circuit consumed 128 hp. at 28 r.p.m. with 20,000 lb. of balls (actually 1¼-in. drill-steel scrap in 4- to 6-in. lengths) or 12.8 hp. per ton. With 31,000 lb. of balls, power consumption was 12.4 hp. per ton, and with 32,000 lb. 12.1 hp. per ton. At 18 r.p.m. and 20,000 lb. of balls, consumption was 8 hp. per ton and at 20 r.p.m. and 32,000 lb. of balls, 7.6 hp. per ton.

Power consumption is higher with thin pulp than with thick because of segregation of balls near the periphery of the mill and consequent displacement of the center of gravity of the total load toward the periphery.

Power consumption increases with increase in speed within operating limits. (See "Speed" p. 402.)

Power with rough liners is more than with smooth, is greater per pound of ball load with large mills than with small, and is greater with balls of mixed sizes than with balls of one size only.

Power consumed per ton of ball load is somewhat different with different types of mills. The range with conical mills is from 6.2 hp. per ton of load to 15, average 10.2 hp. With simple trunnion mills the corresponding figures are 11.2, 18.9 and 14.1 hp. (excluding two mills whose length is twice the diameter, or more). With grate mills the figures are 7.2, 21.8 and 13.8 hp.

Much of the power consumed in ball milling is transformed into heat and becomes apparent in rise of temperature of the pulp in passing through the mill. (See Table 39.)

Table 39. Pulp temperatures in ball mills at Nevada Consolidated Copper Co.

Mill number	Temperature of feed, degrees C.	Temperature of discharge, degrees C.	Increase, degrees C.
1	25	35.5	10.5
2	25	40	15
3	26	39	13
4	26	39	13
5	26	37	11
6	27	49	22
7	22	43	21
8	21	40	19
9	18	40	22
Average	24	40.3	16.3

The amount of temperature rise is proportional to the percentage of solids in the pulp. Occasionally, in the Nevada Consolidated mills, when the percentage of solids rises to between 75 and 80, the temperature rises to 50° C.

Selection of a ball mill. Hines (*59 A 249*) suggests an ingenious method for choosing a ball mill for a given service. After making a reliable determination (or the best possible estimate) of the tons per horsepower-hour to be expected in grinding from the given feed size to the desired size of product, divide this figure into the hourly tonnage to get the power requirement, then choose a mill of such size that, with the proper volume of balls, the power rating will correspond.

Example. Desired a ball mill to grind from 2-in. to 8-mesh. at the rate of 5 tons per hr. From p. 396 the average "Tons per hp.-hr." grinding from 2- or 3-in. to 6- or 8-mesh is 0.200. The power required is therefore $5/0.200 = 25$ hp. This corresponds to a load of 3600 lb. of balls in a grate mill or 4900 lb. in a conical mill. The corresponding mill sizes (Tables 10 and 11) are a 5-ft. × 22-in. conical and 4 × 5-ft. or 5 × 4-ft. grate mill. With coarse feed the mills of larger diameter are the better. According to Tables 16 and 17, either of the 5-ft. mills would do the work, if the rock was not exceptionally hard and tough.

Another method of attack is to take the weight of charge in the mill at, say, 300 lb. per cu. ft. and ball charge as occupying, say, 40 per cent. of mill volume inside of grates and liners. Then the desired mill volume is $3600/30(0.40) = 30$ cu. ft. Since at least a 5-ft. mill is needed for 2-in. feed, the cross-sectional area inside the liners will be about 15 sq. ft. and the least required length 2 ft. The 5 × 4-ft. mill has somewhat in excess of the required volume, inside grates and liners. Care should be taken not to choose a mill too large for the duty required, since such a mill over-grinds, wastes power, and steel consumption is excessive. If, however, this condition must be met, power and steel consumption can be reduced by reducing speed and ball load.

Operating ball mills. Tonnage, size of feed, moisture content, and ball load must be kept as nearly constant as possible, if maximum efficiency of operation is to be attained. A moisture flask (see Sec. 22, Art. 23) and an ammeter on the driving motor are useful aids. The ammeter will indicate relatively small changes in tonnage or ball load and is sensitive also to somewhat larger changes in moisture content. If the ball load is below the mill axis, a drop in the ammeter reading means decrease in ball load, increase in feed tonnage or decrease in moisture. In starting a new mill, run empty for some time to make sure that all is right mechanically, then stop and charge in the grinding media with plenty of ore, preferably fine, to act as a cushion and prevent excessive hammering of the lining when the mill starts. Hammering will start leaks in the liner bolts and with cast balls or lining may cause breakage.

The following difficulties encountered in operation at ALASKA-JUNEAU (*122 P 633*) were all due to structural defects, which should be guarded against in designing. The feed trunnions were too long and the diameter too small for coarse feed. The feed scoop was too small. The discharge bell had too little flare so that grit worked back into the discharge-end bearing. The clutch pulley on the pinion shaft slipped and had to be replaced by a keyed pulley with belts and motor sufficiently large to start the load. The floor space allowed was too small for convenient repairing. The crane carriage did not extend over the feed end, hence scoop changes had to be made with hand tackle. Other frequent faults of construction that cause operating difficulties are: Lack of sufficient slope in feed and discharge launders (see Sec. 20, Art. 10) and failure to provide convenient methods for handling dumped loads, whether from mill or classifier, back into the circuit.

Arrangement of ball mill and mechanical classifier in closed circuit. The important points to be observed are launder slopes, protection of motors and gears against splash, adequate protection of operators, particularly from feed scoop and gears, and clearance around the mill for handling liners.

When open-trunnion mills are operating in closed circuit with a classifier, provision should be made to divert wood chips and worn balls from the classifier. This may be done by placing a flat stationary screen over the mill-discharge box (in which case the screen must be cleaned periodically by an attendant) or a revolving screen may be placed on the end of the discharge spout.

Cost of ball milling. At McIntyre-Porcupine the average for the 15 months ending June 30, 1917, was \$0.1469 per ton, made up: labor \$0.0228, supplies \$0.0763, power \$0.0414, shop \$0.0064. *(Official report.)* Poirier *(51 A 128)* says, in the light of experience at Vipond-Porcupine, that attendance and maintenance of a conical ball mill taking −2-in. feed and grinding to 8-mesh should not exceed \$0.10 per ton (1914). At Elko Prince the average cost over the three years ending 1918 on a 43½ Marcy mill (see Table 5) was \$0.246, as follows: repairs, \$0.0253; labor, \$0.0154; liners, \$0.1042; balls, \$0.1011. Costs at United Eastern *(63 A 553)* in 1917 and 1918 are given in Table 40.

Table 40. Costs of ball milling at United Eastern, 1917 and 1918. *(After North)*

Mill	Year	Grinding		Costs, dollars per ton					
		From	To	Operating labor	Repair labor	Supplies	Power	Miscellaneous	Total
Marcy, No. 64½	1917	2.5-in.	20-mesh	0.0429	0.0140	0.5097	0.1183	0.0001	0.2850
Marcy, No. 64½	1918	2.5-in.	20-mesh	0.0387	0.0165	0.1424	0.1465	0.0021	0.3462
5×6-ft. ball-peb.	1917	20-mesh	65-mesh	0.0553	0.0328	0.1862	0.1899	.....	0.4642
5×6-ft. ball-peb.	1918	20-mesh	65-mesh	0.0678	0.0171	0.2393	0.2055	....	0.5297

9. Ball mills *vs.* other intermediate and fine grinders

General. The principal competitors of the ball mill are rolls, disk crushers and gravity stamps in the range from 3- or 4-in. to 0.5-in., rod mills throughout the ball-mill range, and pebble mills in the range from 0.25-in. to slimes. Roller mills and grinding pans, while cheaper in first cost, are not real competitors, on account of low capacity and high attendance and maintenance charges.

Ball mills *vs.* rolls or disk crushers. This battle has not yet been fought out to the entire satisfaction of mill men generally. The performances at Miami, Inspiration, Engels, and Consolidated Arizona Smelting Co., cited on p. 395, surely favor the Miami practice and indicate that rolls (or disk crushers) are cheaper than ball mills in crushing from 2- or 4-in. to 0.5- or 0.75-in., and should be installed for this service, notwithstanding the greater complexity of the flow-sheet, when the capacity of the plant is sufficient to keep both machines busy.

Ball mills *vs.* gravity stamps. It is not to be concluded from the preponderance of stamps in gold mills that it is the world-wide judgment of mill men that the stamp is superior to the ball mill in gold milling. Most of the world's big gold mills date back beyond the modern development of the ball mill and were equipped with stamps before the ball mill was seriously considered as a competitor. Since stamps are expensive and long-lived machines, their rejection from an operating mill is to be considered only on a clear showing of enormous superiority by the competing machine and then would be justified only in large plants. For new plants the comparative advantages and disadvantages of the two machines are as follows: (1) The gravity stamp has a smaller unit capacity and the efficient, large-sized units can be used for small mill tonnages. A heavy (1750- to 2000-lb.) gravity stamp can crush −4-in. feed to pass a 0.5-in. or 0.38-in. screen at the rate of, say, 15 tons per stamp per 24 hr. and one stamp (Nissen) can be installed as an efficient unit. A ball mill 8 ft. diameter with a daily capacity of, say, 1000 tons per 24 hr. is required for the same service. Furthermore, the stamp can be made to grind down to a size suitable for cyaniding many ores (65-mesh) at the rate of 4 to 6 tons per stamp per 24 hr. or, say, 0.05 ton per hp.-hr. while the ball mill will not grind at this rate of power consumption, if crushing less than 250 tons per day. Hence for small tonnages of coarse, hard feed the stamp

has undoubted superiority. (2) When large capacities are to be handled from crusher product (2- to 4-in.) to cyanide size (say, 1 to 10 per cent. on 100-mesh), and both machines are used as intermediate crushers only, 2-stage crushing with a pebble mill the second stage in both cases is the basis of comparison.

Table 41. Screen tests of feed and products of ball and stamp mills in San Rafael competition

Screen aperture, Mesh	Screen aperture, Inches	Weight, per cent., Feed, both mills	Weight, per cent., Product, Stamp	Weight, per cent., Product, Ball mill
....	2.0	20.1	....	....
....	1.25	24.9	....	....
4	0.200	33.5	....	....
6	0.132	6.9	9.2	7.4
10	0.075	....	10.2	9.7
20	0.034	....	15.8	19.9
50	0.011	....	16.2	21.0
100	0.0056	....	15.2	12.5
150	0.0041	....	6.2	5.3
200	0.0029	....	2.1	2.6
Through last screen.....		14.7	24.9	22.5

Table 42. Comparative costs of ball-mill and gravity-stamp crushing at San Rafael, Pachuca

Item	Cost, dollars per metric ton, Stamps	Cost, dollars per metric ton, Ball mill
Labor...	0.355	0.065
Supplies.	0.132	0.330
Power...	0.153	0.110
Total..	0.640	0.505

At San Rafael mill, Pachuca, Hidalgo, Mex., a 20 @ 1250-lb. stamp battery fitted with 0.27-in. screen was run for three months against a 6 × 5-ft. grate mill. Screen tests of the common feed and both products are given in Table 41. Both machines were run at their normal rates. For the ball mill this was 250 short tons per day, open circuit. Relative costs are given in Table 42. Since the products were so nearly of the same size, the costs of pebble milling were the same in both cases At the Socorro Mining and Milling Co., Mogollon, N. M., stamps and conical ball mills came into competition in grinding a mixture of andesite, rhyolite, trachyte and quartz. Comparative data are given in Table 43.

Table 43. Comparative performances of gravity stamps and conical ball mills at Socorro Mining and Milling Co.

	Stamp-mill section	Ball-mill section
Equipment, primary........	30 @ 1000-lb.	2 @ 6-ft.×22-in.
Equipment, secondary.......	2 @ 5×16-ft. tube mills	1 @ 5×16-ft. tube mill
Tons feed per 24 hr..........	225	200
Power consumption, total....	186	115
Stamps.................	86	...
Ball mills................	...	74
Tube mills...............	82	41
Accessories...............	18	...
Size of feed...............	−2-in.	−2.5-in.
Product..................	−150-mesh	−150-mesh
Tons per horsepower-hour....	0.050	0.072

Wile (*99 J 693*) in a comparison based on the performances of an 8 × 5-ft. Marcy mill at Utah Copper Co. and gravity stamps at Treadwell, concludes that relative average costs of ball-milling and stamping, for breaking from − 1.5-in. to tube-mill feed are $0.08 and $0.13 per ton respectively. He takes repairs to be the same (4¢ per ton) but makes 2.5¢ per ton difference in favor of the ball mill on both power and labor items. These comparative costs are of the same order of difference as those at San Rafael (Table 42) but it is probable that the cost of repairs (including steel renewal) is higher with a ball mill than with stamps notwithstanding the greater complexity of the latter and the more frequent breakage of parts, and that the correctness of Wile's conclusions follows because the discrepancies in power and labor requirements are greater than he allowed for. Morse's summary (*113 P 938*) is rather generally supported by the facts in the case of large installations. He says that the ball mill is cheaper in first cost, cheaper to set up, requires less floor space and head room, consumes less power, is simpler to operate and maintain, and is more flexible in performance when run in closed circuit. Its only disadvantage is the higher steel consumption.

Ball mills *vs.* rod mills. See Art. 12.
Ball mills *vs.* pebble mills. See Art. 16.

10. Screen-discharge ball mills

Krupp ball mill, is shown in Fig. 20. The inner shell containing the grinding media is built up of hard-iron or alloy-steel wearing plates (*a*), containing coarse perforations as shown. These plates are bolted between heavy cast heads which are, in turn, fastened to a heavy through shaft. One end of the shaft carries a large gear, driven by means of the pinion and pulley shown. Surrounding the perforated plates (*a*), are circumferential sections of punched-plate screen (*b*) with relatively coarse aperture, and surrounding this a ring of fine screens (*c*). Both sets of screens are bolted to the heads and revolve with them. The revolving part is all contained in a sheet-iron housing (*d*), having a hopper bottom for discharge of screened material. In operation feed is introduced at one end of the cylinder at the center around the shaft. When sufficiently ground by the balls to pass the large apertures in the grinding plates (*a*), the material passes through to screen (*b*). That part of the ground material that will pass the relatively coarse meshes in screen (*b*) does so, and is subjected to screening by the fine screens (*c*). The material that passes through these fine screens discharges from the hopper finished. That material that will not pass through screen (*c*) is carried around in the space between (*b*) and (*c*) to the position marked (*e*), when it falls through opening (*f*) into chamber (*g*) and joins material that passed the heavy liner plates but would not pass screen (*b*). From chamber (*g*) both materials pass through an opening at (*h*) back into the crushing chamber and are there again subjected to crushing action by the balls. Table 44 gives data on commercial sizes of this

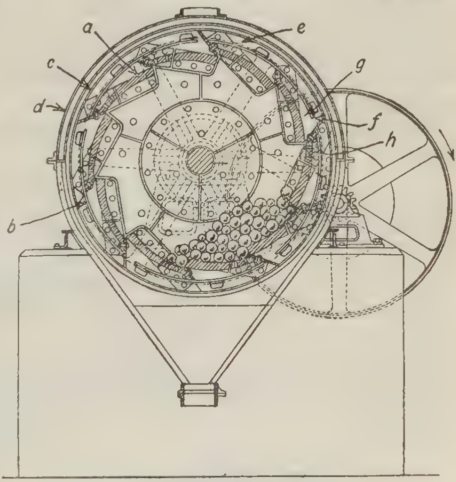


Fig. 20.—Krupp ball mill.

machine as furnished by one manufacturer. The balls used are the same as those used in other ball mills.

Table 44. Sizes and weights of Krupp ball mills. (*Catalog data*)

Size numbers	Weight without charge, pounds	Weight of balls, pounds	Capacity on Portland cement clinker to 20-mesh	Horsepower required
7	29,500	3000	12 to 16 bbl. per hour	30 to 40
8	41,100	4500	18 to 24 bbl. per hour	40 to 50

From 100 to 120 per cent. additional power is required momentarily in starting. When pulverizing to pass 20-mesh, from 30 to 40 per cent. will pass a 100-mesh sieve.

These mills are made for either dry or wet crushing. In the latter case the bottom of the housing is a spitzkasten in which water stands at such a level that the fine screen dips into it at the lower part of its revolution and is washed. The lifters that return oversize of the fine screen also serve as water lifters and thus introduce water into the grinding zone.

A special field for these mills has arisen in sampling plants handling materials containing a large percentage of metallics. With such material this mill is peculiarly suitable, in that it will discharge the metallics that are sufficiently fine to be susceptible to ordinary sampling methods, but retains those particles that are too coarse and collects them so that they are available for melting and sampling by bullion methods.

Wet mills have been very little used because of high metal and screen consumption.

Allen (*114 P 362*) tells of a 5-ft. mill wet-crushing − 1.5-in. feed through 20-mesh at the rate of 60 tons per 24 hr. and drawing 20 hp. The rock was exceptionally hard. As compared with dry crushing the greater capacity with no increase in power consumption more than compensated for the greater metal consumption.

The dry mill has been rather extensively used in industrial grinding, particularly in cement plants. Its use in metallurgical work has been restricted to plants that crush dry for roasting.

At Mt. Morgan (*102 J 755*), No. 5 mills were used to crush dry from − 2-in. through 50-mesh screens. The product contained 10 to 15 per cent. + 60-mesh. Charge, 2000 lb. of 6-in. balls; speed, 20 r.p.m.; 15 hp.; capacity, 22 to 24 tons per 24 hr. James (*111 J 199*) reviews West Australian practice and states that at South Kalgoorlie No. 5 mills crushed from − 2.5-in. through 30- to 40-mesh screens at the rate of 100 tons per 24 hr. with a ball consumption of 0.5 lb. per ton. Liners lasted 8 months. Screening ahead of the mills increased mill capacity markedly as it prevented overloading the mill screens. With 20-mesh screens, sizing analysis of product with this friable ore was as shown in Table 45 (*83 J 475*). At the Perseverance mill, Kalgoorlie (*16 MM 204*), No. 8 mills (about 9 ft. diam. by 4 ft. long) with 4400 lb. @ 4½-in. forged-steel balls ground 100 tons per 24 hr. from − 1.5-in. through 28-mesh, dry. Speed, 24 r.p.m.; power draft, 55 to 60 hp. Ball consumption, including − 2-in. balls rejected, was 0.5 lb. per ton milled and life of manganese-steel liners was 6 to 9 months.

Table 45. Sizing test of Krupp mill product, dry grinding

Screen, mesh	Weight, per cent.
40	24.1
60	9.4
80	6.2
100	6.6
150	3.2
200	4.2
− 200	46.3

Krupp ball mills *vs.* **rolls.** At a Bolivian silver-tin mine (*64 A 676*) the crushing problem was dry grinding from 2-in. maximum to approximately 10-mesh for roasting. The original installation was a No. 4 Krupp ball mill which, fitted with 1.2-mm. screen, ground to the

size shown in Table 46. This was displaced by two 36 × 12-in. rolls in series, the first in closed circuit with a 3/8-in. trommel, the second with a No. 8 Newaygo screen with 0.078-in. opening. The sizing test of the product is given in Table 46. Roll crushing was at the rate of 0.085 ton per hp.-hr. with a steel consumption of 0.38 lb. per ton of finished product. The cost of the roll product was less than one-third that of the ball-mill product.

Table 46. Comparative performances of Krupp ball mills and rolls at a Bolivian mill. (*After Söhnlein.*)

Screen, mesh	Weight, per cent.	
	Ball-mill	Rolls
10	3.0	1.5
20	17.0	16.0
40	24.0	19.5
60	8.5	23.0
80	7.0	8.0
100	5.0	5.5
150	3.5	4.5
−150	32.0	22.0

Ferraris mill (Fig. 21) is a grate mill with a screen (*a*) forming an extension of the mill shell at the discharge end. The screening compartment has a conical imperforate inner surface (*b*) and closed end (*c*) and is compartmented by means of radial partitions. Material flows through grate (*d*) into the screen compartment at the lower part of the mill, is elevated and screened therein and oversize passes back into the mill when the compartment reaches top position. The mill is made in 5- and 6-ft. diameters. The smaller mill is rated to crush 50 tons per day through 10-mesh or 25 tons through 30-mesh from −2-in. feed and the larger mill double these figures. The mill has been very little used and is principally interesting as foreshadowing the present-day grate mill.

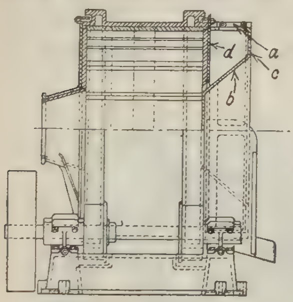


Fig. 21.—Ferraris ball mill.

Herman mill (Fig. 22) is built on the same basic principle as the Krupp mill, *i.e.*, trunnion feed and peripheral screen-controlled discharge, but is simpler in construction. The heads are joined by a heavy sectional grid with coarse openings tapering outward. Around this is fastened a heavy punched-plate screen on the outside of which is strapped the fine screen that determines discharge size. The heavy grid serves several ends. It is the structural link between the heads, it is a rough-surfaced liner, acting to a small extent like the ribbed liners of the El Oro type and protecting itself by the balls so held, and it holds back the largest of the feed particles from the punched plate.

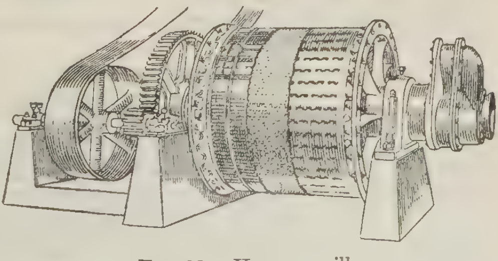


Fig. 22.—Herman mill.

Schmidt Kominuter is a peripheral screen-discharge ball mill of the usual type with screens set on a conical surface surrounding the perforated grinding cylinder. Oversize moves into a lifting compartment in the feed-end head and joins new feed. The mill may be run either wet or dry.

Performance. At Golden Cycle (*60 A 119*) the mills are fitted with diagonal-slotted screens having 3.6 × 12.7-mm. openings. Screen tests of products (dry-crushing) with new and with worn screens are given in Table 47. The effect of difference in screen size is shown principally in the coarser sizes. Wear of manganese-steel balls was 0.7 lb. per ton (*113 P 938*).

Table 47. Sizing test of product of Schmidt Kominuters at Golden Cycle (dry-crushing). (*After Blomfield and Trott*)

Screen aperture	Per cent. weight	
	Worn screens	New screens
4.12-mm.	6.1	1.1
3.18-mm.	5.1	2.4
10-mesh	24.0	16.6
20	22.2	26.2
30	8.0	10.2
40	5.2	7.6
60	6.4	9.8
80	1.2	1.2
100	2.5	3.0
150	2.2	3.9
200	4.2	4.4
−200-mesh	12.9	13.6

11. Rod mill

Description. The rod mill consists of a heavy steel cylinder, usually with a length between two and three times the diameter, filled with rods up to somewhat below the mill axis, and slowly revolved. The feed end of the cylinder is closed with a head provided with a central opening for introduction of feed. The discharge end is treated differently according to the type of mill. In trunnion mills it is closed by a head provided with a central opening through the trunnion for pulp discharge. In open-end mills, the discharge end is carried on rollers, and the discharge-end plate is a relatively narrow ring beveled inward at the center and loosely closed by a circular swinging door, separately supported. (See Fig. 23.) Space is left between plate and door for discharge of pulp. In this open-end mill the pulp level is considerably lower than in the trunnion-discharge mill. A quick-discharge grate mill, similar to the grate ball mill is also on the market.

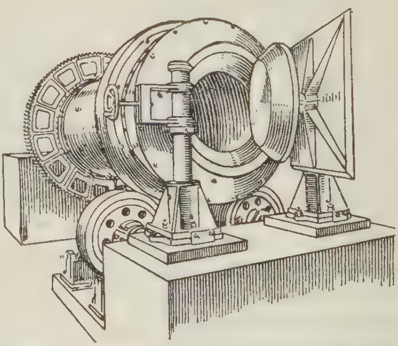


Fig. 23.—Marcy open-end rod mill, discharge end.

Shell is made of heavy steel plate with welded butt joints reinforced with heavy, double-riveted butt straps. Heavy cast flanges for attachment of heads are double-riveted to the ends of the shell, carefully faced for tongue-and-groove joint and drilled for bolts. Heads are usually cast semi-steel, machined and drilled to fit the cylinder flanges. No manholes are provided. In open-end mills, the discharge opening is large enough to permit introduction of liners and in trunnion mills the discharge head is removed for this purpose. Trunnions are cast integral with the heads and carefully turned to the mill axis. Tires and rollers are of hardened steel, and in one make the rollers are carried on ball bearings. Bearings and drive are similar to those used on ball mills. Liners are usually of the ship-lap or wave type (see Fig. 1) made of manganese steel, although cast iron and chrome steel are sometimes used. With smooth liners the load oscillates and wears out the liner in a few days (*112 J 1052*).

Rods are made of high-carbon steel, best 0.8 to 1.0 per cent. carbon and not less than 0.6 per cent., 1½- to 4-in. diameter, and from 1 to 2 in. shorter than the length of the mill inside the head liners. Mild-steel rods are unsuitable for the reason that they bend and kink after wearing down to a certain minimum diameter and snarl up the whole rod load

so that little or no crushing is done. The hardened-steel rods break up when they get to this size and are eventually discharged in small pieces without snarling. Rods used in primary mills, are 3- and 4-in. diameter, 1½- and 2-in. rods are used in secondary mills.

At Iron Cap (111 J 897) 3-in. rods are charged in the primary mill and removed at 2-in. These worn rods are then charged into the secondary mills, from which they are removed at ⅛-in. to prevent kinking.

Manufacturers' data are given in Tables 48 and 49.

Table 48. Trunnion-type rod mill. (*Allis-Chalmers catalog*)

Size of mill, diameter × length feet	Weight of average rod charge pounds	Approximate capacity in tons per 24 hr.; material, medium-hard quartz		Horse-power to run	Horse-power of motor recom-mended	Revolu-tions per minute of mill	Approximate weights complete	
		1-in. to 20-mesh	½-in. to 60-mesh				Pulley and spur-gear drive, without rod charge	Wuest gear drive without motor or rod charge
3× 6	4,900	95	45	15	20	34	22,500	20,500
3× 8	6,500	125	60	20	25	34	24,500	23,000
4× 8	12,000	240	110	36	40	26	39,000	38,500
4×10	15,000	300	140	45	50	26	43,000	42,500
5×10	26,500	500	225	80	100	20	57,500	55,500
6×12	47,500	800	350	135	150	18	81,500	79,500

Table 49. Open-end rod mill. (*Mine and Smelter Supply Co. catalog*)

Size, diameter ×length, feet	Usual charge, tons rod	Horsepower to run	Size of motor recommended, horsepower	Revolutions per minute mill	Approximate shipping weight, pounds
1× 2*a*	300 lb.	1–1½	1½	35	1,100
2× 4	¾	5–7	7½	33	7,000
3× 6	2–2½	15–18	20	30	21,000
3× 8	3–4	24–28	30	30	25,000
4× 8	6–7	45–50	50	25	40,000
4×10	8–9	55–60	60	25	44,000
5×10	12–13	95–100	100	20	80,000
5×12	14–15	115–120	125	20	90,000
6×12	18–19	140–150	150	17	110,000
7×15	24–25	185–200	200	15	165,000

a Laboratory size.

Performance at various plants is given in Table 50.

Cole-Bergman mill at New Cornelia (111 J 901) 5 × 10-ft. inside, with 32,000 lb. of 2-in. and smaller rods drew 90 hp. (= 5.6 hp. per ton of rods) at 19 r.p.m. and crushed 250 tons per 24 hr. from − 1-in. to − 48-mesh (= 0.116 ton per hp.-hr.). Central discharge was found best for wet work. This mill carries 85 per cent. of the load on roller bearings at the discharge end, although it is a trunnion-type mill. The roller bearings are ball-bearing and it is claimed save 16 per cent. of power over other types of bearings. At D. W. Jones Co., Denver, Colo. (111 J 901) a 4 × 10-ft. mill with grate discharge crushed dry from − 0.75-in. to 22 per cent. +40-mesh at the rate of 125 tons per 24 hr. (0.13 ton per hp.-hr.). Rod charge was 8000 lb. and the mill drew 40 hp. (= 10 hp. per ton of balls) at 26 r.p.m.

Capacity depends upon the same factors that influence ball-mill capacity (see p. 387). The range, according to the practice cited, is from 98 tons per

Table 50. Performance

Plant	Phelps-Dodge, Morenci (*am*)	Phelps-Dodge, Morenci (*am*)	Phelps-Dodge, Morenci
Size, diameter×length, ft	3×7	3×7	3×8
Speed, r.p.m.	30	30	29
Tons new feed per 24 hr	236	440	98*b*
Tons total feed per 24 hr	....	....	....
Method of closing circuit	Open	Open	Open
Installed horsepower	....	....	....
Actual horsepower	18.6	22.5	25
Horsepower per ton of charge	5.8	6.3	7.1
Tons of new feed crushed per horsepower-hour	0.528	0.815	0.163
Moisture in mill, per cent	63	36	35
Size of feed(*a*)	*12*	*13*	*1*
Size of product(*a*)	*12*	*13*	*1*
Attendance, machines per man	....	....	3
Lost time, per cent	6	5.8	2.5
Principal causes of lost time	....	....	*c*
Lubricant, kind/pounds per shift	....	....	....
Feeder, type	....	....	Scoop
Feeder, material	....	....	....
Feeder, life, days	....	....	270
Liners, type	Wave	Wave	....
Liners, material	*CI*	*CI*	*CI*
Liners, life, days	82	72	135*d*
Liners, consumption, pounds per ton	0.23	0.14	....
Time for re-lining, hr	....	....	....
Number of men for re-lining	....	....	....
Rods, material	....	....	....
Rods, new charge, total weight, lb	6400	7100	7000
Rods, diameter, in	2 to ½	2 to ½	1¼
Rods, consumption, pounds per ton	0.64	0.39	....
Rods, addition	....	....	*g*

Plant	*ad, ab*	Anaconda(*o*)	Iron Cap
Size, diameter×length, ft	4×10	5×10	5×10*p*
Speed, r.p.m.	....	15	....
Tons new feed per 24 hr	315	375	254
Tons total feed per 24 hr	....	....	....
Method of closing circuit	Open	*DC*	*p*
Installed horsepower	....	150	....
Actual horsepower	45	90	80 each
Horsepower per ton of charge	....	4.3	....
Tons of new feed crushed per horsepower-hour	0.291	0.172	0.132
Moisture in mill, per cent	45	....	....
Size of feed(*a*)	*11*	*5*	*p*
Size of product(*a*)	*11*	*5*	*p*
Attendance, machines per man	....	....	....
Lost time, per cent	....	....	....
Principal causes of lost time	....	....	....
Lubricant, kind/pounds per shift	....	....	....
Feeder, type	Drum	....	....
Feeder, material	....	....	....
Feeder, life, days	....	....	....
Liners, type	....	....	....
Liners, material	*Cr*	....	....
Liners, life, days	....	....	....
Liners, consumption, pounds per ton	0.059 *ac*	....	*r*
Time for re-lining, hr	....	....	....
Number of men for re-lining	....	....	....
Rods, material	*HCS*	....	....
Rods, new charge, total weight, lb	....	42,000	....
Rods, diameter, in	3½, 2¾	3, 2	*q*
Rods, consumption, pounds per ton	0.272	....	....
Rods, addition	....	....	....

For explanation of reference

of rod mills

Phelps-Dodge, Morenci	Old Dominion	Phelps-Dodge, Burro Mtn.	Federal Lead, Mill No. 4	Desloge Consolidated Lead Co (y)	*ad, ab*
4×8	4×8	4×8	4×9*n*	4×10	4×10
28–30	25	28½	25	...	26
445*h*	210	*j*	400–450	450	300
...	...	*j*	...	...	...
Open	Open	*j*	Open	Open	Open
...	50	50	75	...	...
40	39	48	66	40	45–50
5.3	4.9	6.0	9.4	6.1	...
0.464	0.224	*j*	0.252–0.284	0.469	0.250–0.278
35	26	28	35	45	40
2h	*6*	*j*	*4*	*8*	*10*
2h	*6*	*j*	*4*	*8*	*10*
3	*l*	4	*l*	...	...
2.5	Low	...	−1	...	...
c	Re-lining	*k*	...	...	...
...	...	...	*O*, 2; *G*, 1	...	...
Scoop	3-way	...	3-way	...	Drum
...	...	...	*CCI*	...	...
270	...	...	...	...	...
...	...	...	Step	...	...
CI	*Mn*	*CI*	*CCI*	...	*CI*
135*d*	120–150	...	...	...	...
0.153*e*	...	0.17	...	...	0.14*ac*
...	...	...	...	...	...
...	...	...	...	...	...
...	...	...	...	...	*HCS*
15,000	16,000	16,000	14,000	13,000	...
1¼	...	1¼	*m*	4, 3, 1½	2
1.461*f*	1.72	0.75–1.0	0.12	...	0.20
g	...	...	2-in.	...	...

aa, ab	*s*	Phelps-Dodge Copper Queen	Moctezuma Copper Co. (*y*)	Moctezuma Copper Co. (*ae*)	Mexican Corp. (*an*)
5×10	6×12	6×12	6×12	6×12	6×12
20	17	17.5	17.5*af*	17.5	...
216	607	720*v*	668*z*	300*ag*	325
...	...	...	...	...	...
DC	...	*v*	Open	*DC*	*DBC*
100	...	...	150	...	150
68	154	137 each	153	...	140
6.8	8.1	...	8.5	...	...
0.132	0.164	0.219	0.181	...	0.097
26	...	...	...	...	24
9	−1.5-in.	1.5%+1.5-in.	*7*	3.7%+10-in.	*14*
9	*t*	3%+48-in.	*7*	*ah*	*14ao*
...	...	...	...	...	...
...	...	...	...	...	...
...	...	...	...	...	...
Comb.	...	...	*al*	Scoop	...
...	...	...	...	...	...
...	...	...	...	...	...
Ship-lap	...	...	Wave	Wave	Wave
Cr	*CCI*	*Mn, CI*	*CI*	*CI*	*CI*
...	...	...	...	...	*ap*
0.14*ac*	*u*	*u*	*ai*	*ai*	1.65*ap*
...	...	...	...	...	...
...	...	...	...	...	...
HCS	*HCS*	*HCS*	*HCS, ak*	*HCS, ak*	*HCS*
20,000	38,000	...	36,000	36,000	47,000
3	...	...	...	3	1½ to 2½
1.2	2.95	*x*	*aj*	*aj*	2.34
...	...	...	...	...	...

letters, see page 420.

Table 50*a*. Sizing tests referred to in Table 50
(Figures under the headings F and P are weight per cent.)

Reference numbers			1		2		3		4		5		6		7	
Plant			Phelps-Dodge, Morenci		Phelps-Dodge, Morenci		Phelps-Dodge, Morenci		Federal Lead, Mill No. 4		Anaconda		Old Dominion		Moctezuma Copper Co.	
Screen aperture																
Mesh	In.	Mm.	*F*	*P*	*F*	*P*	*F*	*P*	*F*	*P*	*F*	*P*	*F*	*P*	*F*	*P*
......	1.5	......													3.3	
......	1.05	26.67													13.5	
......	0.74	18.83														
......	0.52	13.33														
......	0.37	9.42													31.0	
3	0.26	6.68											3.2		2.3	
......	......	6							6.2							
4	0.18	4.70			4.8		4.9						10.8		5.9	
......	......	4							16.0							
6	0.13	3.33	0.1		18.7		16.7						18.0		8.4	0.8
8	0.093	2.36	0.2		16.5		12.6				2.2		15.4		4.9	1.4
......	......	2							32.9	13.5						
10	0.065	1.65	2.5		13.5	0.1	10.4				13.2		11.6		4.6	4.5
......	......	1.5							6.3	6.6						
14	0.046	1.17	8.1		10.1	0.7	7.9	0.3			21.4		8.9	0.1	3.6	5.9
20	0.033	0.83	12.0		8.8	3.8	7.9	1.6	21.0	24.5	16.0		7.6	0.8	3.4	8.4
28	0.023	0.59	13.5	0.1	7.1	10.1	7.3	5.0			13.6	0.9	7.3	4.4	2.5	9.2
30	......	......														
35	0.016	0.42	14.0	1.4	5.3	14.0	6.6	9.7			9.3	2.2	4.8	9.1	2.3	9.7
40	......	......							12.2	24.5						
48	0.012	0.30	13.7	6.0	3.9	13.4	5.6	12.6			5.7	6.3	3.7	13.7	2.4	9.4
60	......	......							2.7	8.0						
65	0.008	0.21	15.0	19.0	3.2	11.8	4.6	13.9			4.2	10.5	2.5	13.2	1.8	7.6
80	......	......								3.7						
100	0.006	0.15	10.2	19.6	2.0	8.7	2.9	11.3		3.5	3.7	15.0	2.0	12.6	1.4	6.8
150	0.004	0.10	6.2	18.0	1.6	7 8	2.5	10.9		4.1	5.4	12.0	1.5	10.1	1.2	4.7
200	0.003	0.07	1.2	4.1	0.5	2.7	0.8	3.3		1.8	1.2	16.5	0.6	5.5	1.1	4.2
Through last screen			3.4	31.8	4.0	26.9	9.4	31.4	2.8	9.8	3.6	36.6	2.2	30 5	6.4	27.4

Reference numbers			8		9				10		11		12		13		14	
Plant			Desloge Consolidated Lead Co.										Phelps-Dodge, Morenci		Phelps-Dodge, Morenci		Mexican Corporation	
Screen aperture																		
Mesh	In.	Mm.	*F*	*P*	*F*	*D*	*S*	*O*	*F*	*P*	*F*	*P*	*F*	*P*	*F*	*P*	*F*	*P*
	1.5																	
	1.05	26.67																
	0.74	18.83															30.5	
	0.52	13.33							0								19.2	
	0.37	9.42																
3	0.26	6.68	4.8								5.0						31.8	
		6																
4	0.18	4.70	16.3								11.1		0.1		0.1			
		4																
6	0.13	3.33	11.3	0.3							12.4	1.0	6.5		9.4		6.4	
8	0.093	2.36	12.4	2.6							14.4	4.8	17.1		21.6	0.8		
		2							100									
10	0.065	1.65			39					0.8	7.7	5.4	13.4	0.3	15.9	4.9	1.9	
		1.5																
14	0.046	1.17	36.3	27.6						2.7	28.5	24.0	10.4	2.1	13.6	12.0		
20	0.033	0.83	11.2	18.3						5.8	10.1	15.8	8.5	7.1	11.1	15.6	1.8	
28	0.023	0.59	3.9	12.2						10.8	4.5	10.7	8.4	14.8	10.4	16.8		
30					33	3	5											
35	0.016	0.42	1.6	8.8						11.3	2.4	8.2	6.6	14.4	7.0	12.5		
40																	2.3	2.0
48	0.012	0.30	0.6	5.3	9	16	30	2		10.6	1.2	5.1	5.3	11.7	3.9	8.5		
60																	0.9	6.5
65	0.008	0.21	0.3	4.3						8.2	0.7	4.2	3.9	8.7	2.0	5.4		
80					6	26	32	10									0.4	9.0
100	0.006	0.15	0.2	3.8	3	9	9	7		10.0	0.6	5.6	3.8	8.0	1.1	4.7	0.7	15.0
150	0.004	0.10	0.1	1.9	3	12	10	14		11.4	0.2	2.0	2.5	5.2	0.7	2.8	0.4	4.5
200	0.003	0.07	0.2	4.1	3	9	6	14		4.1	0.4	4.5	1.3	2.7	0.4	1.7	0.5	1.5
Through last screen			0.7	10.8	4	25	8	53		24.3	0.8	8.7	12.2	25.1	2.9	14.2	3.2	61.5

D Mill discharge. *F* Feed. *P* Product. *S* Classifier sand. *O* Classifier overflow.

NOTES TO TABLE 50

a Italic numbers refer to column numbers in Table 50*a*. ***b*** Usual rate is between 125 and 175; product becomes gradually coarser as feed rate increases. ***c*** Changing rods and liners. ***d*** Hard cast-iron liners locally cast outlast manganese steel and, therefore, cost less. ***e*** Includes 0.129 lb. consumed and 0.024 lb. discarded. ***f*** Includes 1.238 lb. consumed and 0.223 lb. discarded. ***g*** Mill stopped and charge overhauled every half month; small and bent rods removed. Constant quantity added each day to keep up load. ***h*** See column 3, Table 50*a*, for sizing tests with 205 tons per 24 hr., open circuit. ***i*** One man attends table, rolls and one rod mill. ***j*** 500 tons per 24 hr. from 0.5-in. to 14-mesh (= 0.433 tons per horsepower-hour.); 280 tons per 24 hr. from 14-mesh to 48-mesh (= 0.243 ton per horsepower-hour). ***k*** Changing rods. ***l*** Included in other work. ***m*** 3080 lb. @ 2-in., 10,920 lb. @ 1¼-in. ***n*** Trunnion type; roller type unsatisfactory on account of mechanical troubles. ***o*** Compare 7½-ft. × 72-in. conical ball mill at the same plant, Table 11. ***p*** Two of these mills in series, the first taking − ¾-in. feed in closed circuit to table size, the second re-grinding table tailing (about 200 tons) to 0.6 per cent. +48-mesh and 38.5 per cent. −200-mesh. ***q***. Primary mill, 1.46 lb. per ton; secondary, 1.65 lb. ***r*** Primary and secondary the same. Feed-head liner semi-steel, weight 900 lb., cost at plant $159; life 81 days, 0.043 lb. per ton; discharge-head liner, semi-steel, weight 2000 lb., cost $424, 113 days, 0.097 lb. per ton; shell liner, cast steel, weight 15,000 lb., cost $2028, life 113 days, 0.52 lb. per ton. Cost of rods and liners, $0.30 per ton out of a total cost of $0.425 per ton. ***s*** Hard quartz ore. *Mine and Smelter Supply Co.* ***t*** 5 per cent. +10-mesh, 30 per cent. −200-mesh. ***u*** End liners, 0.06 lb. per ton; shell, 0.59 lb. per ton. ***v*** Two mills in series, the second in closed circuit with a classifier. ***w*** Primary, 0.48 lb. per ton (= $0.028); secondary, 0.33 lb. per ton (= $0.0175). ***x*** Primary, 1.06 lb. per ton (= $0.0374); secondary, 1.3 lb. per ton (= $0.0455). ***y*** *111 J 899.* ***z*** With a feed containing 0.2 per cent. +2-in., 1.1 per cent. +1.5-in., 4.3 per cent. +1-in., 3.9 per cent. +0.5-in. and 24 per cent. +4-mesh, total on 4-mesh = 33.6 per cent., the average daily tonnage for a month to the same fineness was 1027, ***aa*** Gold-bearing quartz and schist of medium hardness. ***ab*** *Allis-Chalmers, Bul. 1821-A.* ***ac*** Shell liner only. ***ad*** Soft limestone and galena. ***ae*** *118 J 446.* ***af*** Speeds of 15.5 and 16.5 r.p.m. were unsatisfactory. ***ag*** Estimated, on the basis that 60 per cent. of the discharge of the primary mills (which is 50 per cent. +48-mesh, see column 7, Table 50*a*) comes back to the secondary mills as sand-table tailing. ***ah*** 12 per cent. +48-mesh, 52 per cent. −200-mesh. ***ai*** Total for both primary and secondary mills = 1.6 lb. per ton of original primary-mill feed, including 30 to 35 per cent. scrap loss. ***aj*** Total for both primary and secondary mills = 2.32 lb. per ton of original primary-mill feed, including 10 per cent. scrap loss. ***ak*** 0.9 to 1.1 per cent. carbon. ***al*** Gravity chute. ***am*** *55 A 678.* ***an*** *Private communication.* ***ao*** This is selective grinding of soft country rock. The hard quartz containing the values is mostly found in the +100-mesh sizes and the flow-sheet was changed to rod mills in open circuit followed by conical ball mills in closed circuit with Dorr bowl classifiers to take care of this material. ***ap*** Discharge-end liner, 40 days; shell and feed-end liners, 81 days. Original weight of complete set, 28,000 lb., scrap 54 per cent. ***CCI*** Chilled cast iron. ***CI*** Cast iron. ***Cr*** Chrome steel. ***DBC*** Dorr bowl classifier. ***DC*** Dorr classifier. ***G*** Grease. ***HCS*** High-carbon steel. ***Mn*** Manganese steel. ***O*** Oil.

24 hr. in a 3 × 8-ft. mill to 350 tons in a 6 × 12-ft. mill grinding from 6- or 8-mesh to 48-mesh and from 440 tons in a 3 × 7-ft. mill to 720 in a 6 × 12-ft. mill grinding from about 1-in. to 8- or 10-mesh.

Power consumption ranges from 4.9 hp. per ton of rod charge in a 4-ft. mill to 8.5 hp. in a 6-ft. mill, both types having the usual bearings. Power for the Cole-Bergman mill with discharge end supported on ball-bearing tire rollers is less. Thus tons per horsepower-hour average 0.18 when crushing from 6- or 8-mesh to rather coarse flotation feed (2 to 9.5 per cent. +48-mesh) and 0.35 when crushing from 1-in. to 6- or 10-mesh for roughing-table work. The reason for the smaller power draft of the rod mill is that more rods than balls can be loaded into a given space and hence, with a given crushing load, the center of gravity is nearer the axis of revolution.

Size of feed. The rod mill should not be fed with material coarser than 1-in., and with very hard ore the feed should be finer than this. Larger lumps cause the rods to be spread so far apart at the feed end that they fail to do efficient crushing for a considerable part of their length. When fine feed (8- or 10-mesh) is introduced with the idea of sliming, the mill comes into competition with ball-tube mills and pebble mills, and is outclassed. In grinding from 1-in. to 48-mesh the work is most efficiently done in two stages.

Thus two 6 × 12-ft. mills at Copper Queen (Table 50) crushed from 1.5-in. to 48-mesh at the rate of 0.219 ton per hp.-hr., while the 5 × 10-ft. Cole-Bergman mill at New Cornelia crushed from 1-in. to 48-mesh at the rate of only 0.116 ton per hp.-hr.

Size of product. The product of a rod mill is remarkable for its granular character and the large percentage near the limiting screen size. This is particularly true in open-circuit work or when the circuit is closed by a screen. It is noticeable, also, that the product of open-circuit work contains very little "accidental" oversize, *i.e.*, substantially unground feed particles, even when the capacity is raised considerably above normal. Fig. 24 (*57 A 360*) shows that the result of an increase in feed rate is a relatively uniform increase in size of product, with minimum increase in the coarsest sizes.

Rod-mill product is remarkably similar in size distribution to roll product as may be seen from Fig. 25. (*57 A 361*). This similarity is, of course, to be

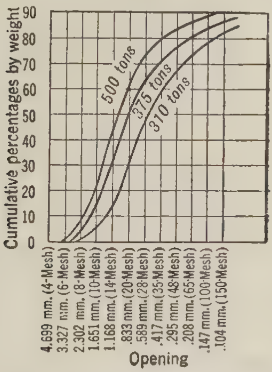


FIG. 24.—Effect of change in feed rate on product of rod mill (*after Watt*).

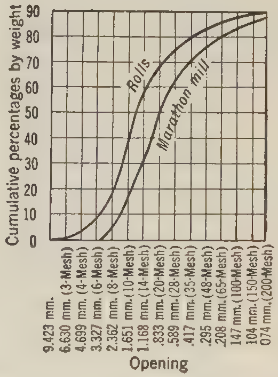


FIG. 25.—Comparison of roll and rod-mill products (*after Watt*).

expected, in the light of Haultain and Dyer's work (see p. 402), since the rods in a slow-moving mill act almost entirely like a large number of close-set rolls. Coghill and Anderson (*117 J 1006*) warn, that, under certain circumstances the rod mill may, even while making a product that is not, as a whole, unduly slimed, nevertheless slime the mineral constituents excessively, and that the usual statement that the rod mill is a granulator and not a slimer is to be accepted with caution and discrimination. Table 51, giving the results of a thorough sizing-assay test on a rod mill crushing from 0.5- or 0.75-in. feed to 20-mesh, is cited by the authors in support of this contention. Comparing the percentage of −200-mesh in the product with that in products ground to the same limiting size in ball mills (Tables 4, 5 and 11) it is apparent that the rod-mill product contains more slime and that the criticism is justified. But sliming has not been differential, as the authors infer; actually relatively more gangue than sulphide has been slimed in the case of silver, gangue and sulphide have been substantially equally slimed in the case of zinc, and only in the case of the lead has sulphide been slimed more than gangue; even here the differential is small. On the other hand, there has been impoverishment of the sizes from 20-mesh to 200-mesh inclusive, which are the sizes in which table concentration is most effective in making the differential separation required on this ore.

Table 51. Sizing-assay test of rod-mill grinding. (*After Coghill and Anderson*)

Material	Screen, mesh	Solids, per cent. weight	Assay, per cent. Zn	Per cent. of total Zn	Assay, per cent. Pb	Per cent. of total Pb	Assay, oz. Ag per ton	Per cent. of total Ag
Feed	3	36.9	1.84	21.4	1.00	20.4	1.66	24.1
	4	13.2	3.04	12.6	1.88	13.6	3.00	15.5
	6	6.3	3.60	7.2	1.85	6.4	2.74	6.8
	8	6.2	3.33	6.5	1.86	6.3	2.24	5.5
	10	5.4	3.37	5.7	2.47	7.3	2.80	5.9
	14	3.4	3.79	4.1	2.82	5.3	2.87	3.9
	20	3.8	4.02	4.8	3.10	6.4	2.50	3.7
	35	4.8	5.75	8.7	3.50	9.3	3.35	6.3
	48	2.3	6.63	4.7	3.00	3.7	3.20	2.8
	65	1.9	5.75	3.4	2.55	2.6	3.00	2.2
	100	2.2	5.40	3.8	2.10	2.6	2.60	2.3
	150	2.4	5.50	4.2	1.80	2.4	2.70	2.6
	200	1.0	5.75	1.7	1.43	0.7	2.52	1.0
	260	1.2	6.20	2.3	2.50	1.6	3.80	1.7
	325	0.7	4.40	1.0	2.00	0.8	4.10	1.2
	−325	8.4	3.00	7.9	2.25	10.4	4.40	14.5
Total......	....	100.0	3.17	100.0	1.82	100.0	2.54	100.0
Product	20	0.9	1.49	0.5	0.44	0.2	1.52	0.6
	35	8.9	1.85	5.7	0.52	2.9	1.26	4.7
	48	10.6	2.47	9.1	0.78	5.1	1.44	6.4
	65	11.1	2.73	10.5	0.98	6.9	1.60	7.6
	100	12.6	3.24	14.2	1.55	12.3	1.94	10.4
	150	11.2	3.40	13.3	1.91	13.6	2.22	10.6
	200	3.9	3.29	4.4	1.32	3.2	1.74	2.9
	260	5.6	3.60	7.0	2.34	8.3	2.50	6.0
	325	3.2	3.55	4.0	2.03	4.1	2.46	3.4
	−325	31.9	2.83	31.3	2.14	43.3	3.50	47.5
Total......	....	100.0	2.88	100.0	1.58	100.0	2.35	100.0

12. Rod mill *vs.* ball mill

These machines are in competition throughout their range of crushing ability; both can, if forced to do so, take 3- to 4-in. feed and both can, in closed circuit with a mechanical classifier, produce −100-mesh product. But, while the ball mill may, under certain circumstances, justify its use on such coarse feed and may, at the other end of the scale, successfully compete with pebble mills the rod mill has, by common consent, failed to handle feeds coarser than 1-in., or at the coarsest 1.5-in., or to grind finer than 48-mesh. This is to be expected from consideration of the crushing bodies. There is little free fall of rods and hence insufficient impact to break large particles, nor is there, except near the bottom of the load, sufficient force (weight) to crush these particles by roll action. The amount of fine grinding done is a function of the extent of surface of the crushing media. This is much less in the rod than in the ball mill, for a given weight of charge, and therefore the amount of fine grinding is less. The weight of a single rod, or of a superincumbent layer transmitted through a single rod, is more distributed than is the case with balls; in consequence the crushing force is less and there is less tendency to crush large particles or to grind small ones very fine. Furthermore, while the ball mill may, with strong claims to economy, be used for one-stage reduction through a considerable range, as at INSPIRATION, the rod mill works much more efficiently when the reduction per step is small. But in the intermediate size

range, taking, say, −1-in. feed and delivering anywhere from 8- to 48-mesh product, averages based on Tables 4, 5, 11 and 50 indicate that the rod mill grinds about twice as many tons per horsepower-hour as the ball mill; that liner wear in the rod mill is about 60 per cent. that in the ball mill and rod wear about 75 per cent. of ball wear (based on high-carbon steel in both cases). Rods normally cost less than balls of the same material. Capacity per square foot of mill floor space required for the installation favors the ball mill slightly, but not enough to be an important factor in the comparison. Handling rods to and into the mill is somewhat less convenient than handling balls and the rod mill should be opened for inspection and the charge picked over the more frequently. The product of the rod-mill is not so sensitive to changes in feed rate as that of the ball mill, if accidental oversize is undesirable, as in preparing table feed, but the ball mill is the less sensitive to changing conditions when preparing flotation feed. Consideration involving the performance of the two types of mill on a particular ore, causes the conclusion to fall one way or the other, according to the ore and to the type of ball mill.

At Consolidated Mg. & Sm. Co. of Canada, Kimberley mill, a 5 × 10-ft. rod mill crushed 625 tons daily from − 1-in. to 14 per cent. + 28-mesh (and 28 per cent. − 200-mesh) with 72 hp. and a rod consumption of 0.46 lb. per ton. Compared with the 8-ft. × 48-in. conical ball mills at the same plant (see Table 11) the number of tons ground per horsepower-hour open-circuit (0.35) was greater than in the conical mill (0.29). Ball consumption was 0.273 lb. per ton. The conical-mill product was finer, which lessened the load on the re-grinding (second-stage) mills.

The following comparison between conical ball mills, Marcy ball mills and Cole-Bergman rod mills is based upon crushing hard quartz and diorite with rod mills, crushing the same ore with Marcy mills, crushing soft porphyry ore in Marcy mills and in conical ball mills. The hard ore was crushed from − 1-in. to the size shown in Table 52 in two 5 × 10-ft. rod mills in series, the first crushing to approximately 10-mesh in open circuit, the second finishing in closed circuit. The 10-mesh product was roughed on tables between the two stages of grinding. The same ore, coarse-crushed to the same size (− 1-in.) was ground in 8 × 6-ft. Marcy ball mills to the size shown in Table 52. Comparative costs, based on crushing 55,000 tons in the rod mills and 29,000 tons in the ball mills are shown in Table 53. The

Table 52. Comparative screen analyses of classifier overflows with rod mills and Marcy ball mills grinding hard quartz-diorite ore

Screen, Tyler mesh	Weight, per cent.	
	Rod mills	Marcy mills
48	1.0	1.6
65	8.2	6.4
100	14.4	12.0
150	14.1	14.0
200	8.5	6.4
−200	53.8	59.6

Table 53. Comparative costs of grinding hard quartz-diorite in rod mills and Marcy ball mills

Item	Rod mills			Marcy ball mills		
	Quantity per ton	Unit cost	Cost per ton	Quantity per ton	Unit cost	Cost per ton
Power	9.03 kw.-hr.	$0.011	$0.0993	13.79 kw.-hr.	$0.011	$0.1517
Rods (*a*)	2.92 lb.	0.0448	0.1308	....	....	....
Balls (*b*)	....	....	....	2.89 lb.	0.0529	0.1529
Liners (*c*)	1.09 lb.	0.1324	0.1443	0.76 lb.	0.1592	0.1210
Totals	....	....	0.3744	....	....	0.4256

a 1920-21 prices. *b* 1920 prices. *c* Cast steel for rod mills, manganese steel for ball mills.

advantage is distinctly with the rod mill. In Table 54 actual costs with the Marcy mill treating the hard and soft ores are compared with actual costs with the conical mill crushing

Table 54. Comparative costs of operating conical ball mills, grate mills and rod mills

Kind of ore	Soft porphyry		Hard quartz and diorite		
Kind of mill	Conical	Grate	Rod	Grate	Conical (*c*)
Power consumption, kw.-hr. per ton:					
Coarse crushing	0.813	0.480	0.710	0.710	....
Grinding	7.592	9.170	9.500	13.790	....
Total	8.405	9.650	10.210	14.500	12.640
Ball consumption, pounds per ton	2.21*a*	1.76*b*	....	2.89	3.63
Rod consumption, pounds per ton	....	....	2.92	....	....
Liner, kind	*CSS, Mn*	*Mn*	*CS*	*Mn*	*CSS, Mn*
Consumption, pounds per ton	0.156, 0.089	0.420	1.09	0.760	0.268, 0.153
Costs, coarse crushing and conveying, except power	$0.04187	$0.02850	$0.04750	$0.04750	$0.07187
Crushing and grinding, power (*d*)	0.11162	0.12815	0.13559	0.19256	0.16786
Balls	0.10385*a*	0.08395 *b*	....	0.13785	0.17052
Rods	....	....	0.13080*e*	....	....
Liners (*f*)	0.01886	0.05811	0.12542	0.10515	0.03234
Total	0.27620	0.29871	0.43931	0.48306	0.44259

a Cast semi-steel, 4-in. @ $0.04165 per pound, 2-in. @ $0.04855 per pound. *b* Forged steel, 5-in. @ $0.0477 per pound. *c* Estimated on basis that the relative capacities of conical mills on soft and on hard ore are as 1000 : 582. This was the ratio of Marcy mills on the soft and hard ores respectively. *d* At $0.01328 per kw.-hr. *e* At $0.04480 per pound. *f* *Mn* @ $0.13835 per pound; *CS* @ $0.115 per pound; *CSS* at $0.04165 per pound. *CSS* Cast semi-steel. *CS* Cast steel. *Mn* Manganese steel.

the soft ore and the rod mill, crushing the hard ore. With these, and particularly with the rod mill, is compared an estimate of the cost of crushing the hard ore in the conical mill. This table indicates that the cost of crushing and grinding the soft ore with the conical mill is $0.02 per ton less than with the cylindrical ball mill; that rod-mill grinding of the hard ore is distinctly cheaper than Marcy ball milling, and that the difference between rod mills and conical mills on the hard ore is too small to be conclusive on estimate.

At Mill No. 4 of Federal Lead Co. an 8-ft. × 30-in. conical ball mill and a 4 × 9-ft. rod mill are working on somewhat different feeds (see Table 55). Under these circumstances the

Table 55. Comparison of conical ball mill and rod mill at Federal Lead Co.

Mill	8-ft. × 30-in. conical	4-ft. × 9-ft. rod	Ratio: conical ÷ rod
Size of feed	3% + 10-mm.	6% + 6-mm.	....
Tons crushed per 24 hr.	855	460	1.86
Tons per horsepower-hour, total feed	0.302	0.291	1.04
Tons per horsepower-hour of − 10-mesh produced	0.178	0.119	1.49
Tons per horsepower-hour of − 20-mesh produced	0.166	0.109	1.52
Tons per horsepower-hour of − 60-mesh produced	0.103	0.058	1.77

tonnage of feed passed through the mills per horsepower-hour is substantially the same, but the tonnage ground to any given mesh per horsepower-hour is distinctly in favor of the ball mill and increasingly so as finer sizes are investigated. Ball consumption per ton is half of rod consumption.

Forrester-Rexman mill (Fig. 26) is a cylinder 4 to 5 ft. diameter by 4 to 5 ft. long, trunnion-fed and discharged through fine screens at the ends with peripheral lifters in the mill shell for oversize return. It is run at about 28 to 36 r.p.m. according to diameter. The grinding media are rods cradled in the quadrants of the mill, supported on spider arms which are suitably protected by renewable liners. The mill is provided at each end with a drum feeder and a spiral in the trunnion, which delivers feed to the center of the mill

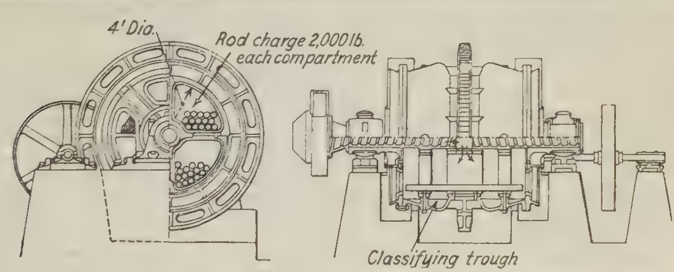


FIG. 26.—Forrester-Rexman rod mill.

whence it is delivered successively, due to the rotation of the mill, to one after another of the rod groups and the screen until it is finally crushed fine enough to pass the screen and leave the mill. With the rod load divided as illustrated the amount of power required to drive a given weight is claimed to be only about 25 per cent. that in the ordinary rod mill. On the other hand, it is very probable that the crushing effect per rod is less. The mill has not had sufficient use in metallurgical plants to furnish reliable data as to performance.

Allen (*113 J 987*) gives the following data of a test run on a 4 × 4.5-ft. mill. Weight of mill, 19,400 lb. Rod load, 8562 lb. @ 2½-, 2¾- and 3-in., 36 r.p.m. Feed: granite, −0.5 + 0.12-in. Product all through 0.06-in. (10-mesh). Feed rate, 182 tons per 24 hr. Power consumed, 19.5 to 21 hp. This is at the rate of 4.6 to 4.9 hp. per ton of rods and 0.38 ton per hp.-hr. crushed to 10-mesh. This performance is about the same as that of the ordinary type of rod mill (Table 50).

PEBBLE MILLS

Pebble mills are of the same general nature as ball mills but differ from them in that rock fragments instead of metal bodies are used for the crushing medium. Since pebbles are lighter than balls, the crushing effect of the tumbling mass in a pebble mill is less than in a ball mill, feed must be finer, and capacity and power consumption for a given size of mill are less.

There are two types of pebble mill, *viz.*: the cylindrical mill or tube mill and the conical pebble mill (Hardinge).

13. Tube mills

General. Tube mills were the forerunners of center-discharge ball mills. They were adapted from cement practice into cyanide practice, and from cyaniding were taken over into concentration shortly before the adoption of flotation. They are used for finer grinding than ball mills. This end is accomplished by increasing the length of the mill. At the same time, being used on finer feeds than the ball mill, it is unnecessary to have as great diameter or as heavy grinding media. The standard tube mill, so-called because of its substantially uniform adoption in South African cyanide practice, is 5-ft.

6-in. diameter by 22-ft. length inside the shell. The development of the tube mill in concentration plants has been toward a shorter mill of somewhat greater diameter. This is a logical development of the more perfect methods of classification that have been developed. Originally it was necessary to finish the grinding in one passage through the mill. When this is done, great length is necessary, notwithstanding that the great bulk of the grinding is done in the first few feet of the mill as shown in Fig. 27.

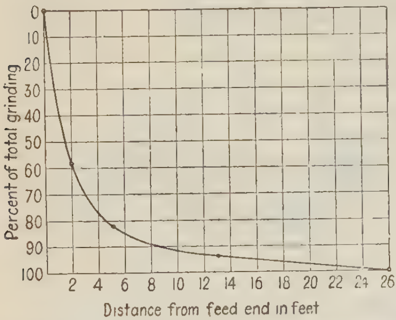


FIG. 27.—Amount of grinding at different points in a 6 × 22-ft. tube mill, dry-grinding talc.

Shell of tube mills is made of ⅝- to ¾-in. steel plate with double-riveted butt-strap joints. Rarely the joints are welded. Cast-iron flanges for attachment of heads are double-riveted to the ends of the shell after which the latter is swung in a lathe and the bases of the flanges are accurately machined at right angles to the shell axis. Bolt holes in the flanges are drilled by jig. Manholes, 13 to 15 × 18 to 20 in., are cut in the shell on opposite sides near the ends and fitted with cast seats for the manhole cover.

Heads are made of cast iron, about 1½ in. thick at the edges and thicker at the center, heavily ribbed, usually with the trunnion cast integral. After attachment to the shell the best practice is to swing the shell in a lathe and turn the trunnions true to the axis of the shell. Where this is not done it is difficult to bring the axis of the trunnions coincident with the axis of the mill and as a result the mill does not revolve truly and additional strain is brought onto all parts. The feed trunnion should be 9 to 12 in. inside diameter with 3- to 3½-in. walls and the length should be sufficient to give a bearing pressure of 100 to 150 lb. per sq. in. of projected area. The discharge trunnion of overflow mills is usually 4 to 10 in. greater diameter than the feed trunnion to afford gradient for pulp flow.

Linings for tube mills (see also ball-mill linings, p. 346) are of many varieties including iron or steel plates, silex, ironite, El Oro, Komata, Osborn, Forbes, Britannia, and many other types less well known.

Plate linings are made of hard white cast iron, cast steel, manganese or chrome steel with smooth or ribbed surfaces. Thickness is usually from ⅞ to 1¾ in. Plates give the largest possible internal diameter in the mill with consequent greater capacity and efficiency throughout the life of the liner. Smooth white-iron plates, 1¼ in. thick, 8 in. wide and 48 in. long in a 5 × 22-ft. mill taking 2½-mesh feed at the BUTTERS VIRGINIA CITY mill lasted 9 months, which was as good as the El Oro would do and the plate lining was the cheaper. (*89 J 905.*) At RAINBOW mill (*99 J 1104*) the 1½-in. cast-iron feed-end liner lasted 6 months. The discharge-end liner lasted longer; 1½-in. side liners lasted 13 months. Re-lining required 14 hours.

Ribbed plates allow less slip of the pebble load than smooth plates and are more suitable for coarse feed. They are sometimes made reversible, so that when one side of the ribs wears and the lifting effect is lessened the other side can be turned up.

Jones (*52 A 95*) says that in tube mills at TONOPAH BELMONT manganese-steel liners cost $0.064 per ton and lasted 16⅓ months while hard white iron lasted about 2 years and cost only $0.0173 per ton. Addition of new ribs to the manganese liners increased the life 10 months and decreased the cost to $0.0457 per ton.

Silex lining is composed of blocks or bricks of hard flint, normally 8 in. long, 4 in. wide, and from 4 to 7 in. thick. Uniformity in thickness is the important requirement, considerable variation in length and width are allowable. They are cemented into the mill with Portland cement. Joints are staggered to prevent circumferential grooving. Re-lining a 5½ × 22-ft. mill on the Rand has been done in 18 hr., but the usual time is 24 hr. from stopping to re-starting, including 4 hr. steaming. (*RMP.*) The cost of installing silex lining (4 × 4 × 8-in. blocks) in a 5 × 18-ft. mill at TONOPAH BELMONT in 1913 (*52 A 112*) was: Silex, 12,300 lb., $258; cement, 33 sacks, $36; labor, $102; total, $396. Silex lining

can be worn down to 1½ or 2 in. before it is necessary to replace it. Average life on the Rand was from 60 to 150 days, depending on the thickness and the conditions of operation. At the TREASURY MINE (*So. Af. Ass'n Eng'r's, Apr., 1905*) silex lining lasted 2½ times as long as smooth cast-iron and cost half as much. With thick blocks, which have the longer life, the diameter of the mill is materially decreased when the blocks are new as compared with the diameter when the blocks are old and there is consequently a marked change in peripheral speed of the mill during the life of the liner. With a new liner, horsepower, pebble capacity, grinding capacity and grinding efficiency are less than with a worn or thinner liner. It is necessary, therefore, to balance the cost of more frequent renewals attendant upon the use of thinner blocks against the decreased efficiency in the early part of the life of thick blocks. To compensate for greater wear at the feed end one plant (*7 JCM 368*) lined the first 7 ft. with 8-in. block, then 7½-in. for 1 ft., 7-in. for 1 ft., and 6-in. for the balance of the length. IRONITE is similar to silex, except that the material used consists of blocks of hard trap rock. In some localities where hard, close-grained rocks are obtainable locally, these are used in place of silex. Silex was more generally used than plate liners. The principal DISADVANTAGES of the stone-block type of liner are the great reduction in mill volume and the variation in volume, power consumption, capacity and efficiency between new and old liners. These are sufficient to bar such liners in most modern plants.

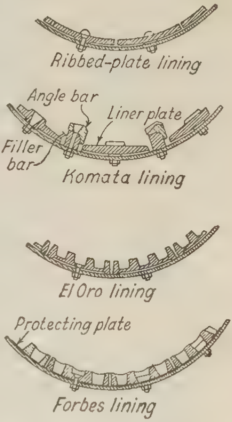


FIG. 28.—Liners for tube mills.

El Oro, Forbes and Komata liners are most used in American mills. El Oro liner is shown in Fig. 28. It consists of grooved plates bolted to the shell of the mill, the width of the grooves being about the medium dimension of the larger pebbles used. In operation these larger pebbles wedge into the grooves and the liner surface soon becomes an irregular one of pebbles that protect the metal ribs from wear. As pebbles wear down and are broken they fall out and are replaced by other pebbles. El Oro liner plates are made of cast iron, cast steel, or special steels.

Osborn liner (Fig. 29) is a modification of the El Oro in which hard-steel ribs are wedged into place by mild-steel wedges, both being set in cement mortar. There are no bolts. Pebbles fill the spaces between the ribs and take most of the wear. This liner has displaced all others in South Africa. At SIMMER AND JACK (*RMP*) a mill with Osborn liner had a duty of 139.5 tons containing 57.2 per cent. − 90-mesh while an adjoining silex-lined mill ground 133.5 tons per day to 54.6 per cent. − 90-mesh. Ribs wear most rapidly at the fed end and after initial wear at this end are reversed. Sectional bars (three lengths to a course) permit more economical replacement for head-end wear. Life is about 300 days. One man can re-line in 12 hr. (*97 J 465*).

Erect plates 4 in. deep, tapering from 1¾ in. to ¾ in. in width; wedge bars 2½ to 3 in. wide and ½ to ¾ in. thick. (*52 A 42*).

FIG. 29.—Osborn liner.

Forbes lining (Fig. 28.) is the same in principle and similar in appearance to the El Oro lining. It differs from the El Oro in that the plates are cast in the form of a curved grid and are bolted against a back plate that protects the shell at the bottom of the openings in the grid. Life at NEVADA PACKARD in a 6 × 10-ft. mill making 75 per cent. − 200-mesh was 345 days compared to 220 days for a step liner of approximately the same weight.

Globe lining (*98 J 393*) is of the general El Oro type but with circumferential grooves tapering toward the base and also tapering longitudinally in a direction opposite to the direction of rotation. The original pebble load should be a good mixture of all sizes in order to insure quick filling of the liner grooves. Life at HOLLINGER is reported as 3 months greater than that of El Oro lining. Weight of metal for a 5 × 20-ft. mill is 15,500 lb. Re-lining requires 6 men 12 hr.

Komata lining is substantially a ribbed-plate lining with replaceable ribs. See Fig. 28. It was first introduced in New Zealand and is now used almost exclusively there both for pebble charges and for ball charges up to 3-in. size, both wet and dry grinding (*24 CMI 192*). It is made up of plate liners alternating with rib liners about 18 to 20 in. apart. The rib bars are made up of two parts, a cast-iron base and a special steel wearing top that is reversible and readily replaceable. In one type (wedge-bar) the rib bars are wedge-shaped and hold in the plates so that the number of bolts is reduced. Plates are ⅜ in. thick at the edges

and 1 in. at the center. Maximum wear comes on the angle bars and the use of separate wearing parts effects a considerable economy in the amount of metal that must be discarded. Raised bosses are provided around the bolt holes to prevent cupping, and joints in the bars are staggered to prevent circumferential grooving. With cast-iron plates a low boss is put on the back to protect the plate against breakage when the bolts are tightened Brown, the inventor, recommends (*104 P 206*) lower speed and lower pebble load than with smooth liners of the El Oro type on account of the greater lifting effect of the ribs. He cites (*104 J 177*) a competitive test between Komata and El Oro liners in 4½ × 20-ft. mills. First cost was about the same. Metal wear was much less with the Komata and power consumption slightly greater. Sizing tests are given in Table 56. The Komata appears

Table 56. Comparative grinding with Komata and El Oro liners

Screen, mesh	Weight, per cent.			
	El Oro		Komata	
	Feed	Product	Feed	Product
28	14.6	0.6	18.5	0.1
35	13.7	2.3	16.8	0.7
65	38.7	11.4	35.3	11.6
80	7.3	5.5	8.0	8.4
150	18.1	37.5	15.5	35.5
280	5.3	15.1	3.7	18.5
−280	2.3	27.6	2.2	25.2
	Tons produced per 100 tons of feed			
−28-mesh	14		18.4	
−65-mesh	52.7		57.2	
−150-mesh	35.7		37.8	
−280-mesh	25.3		23.0	

to have been slightly better in producing material down to 150-mesh but the El Oro produced slightly more −280-mesh material. At TONOPAH BELMONT (*52 A 112*) Komata manganese-steel lining cost (1913) $1785 installed, including $145 for labor. After 16⅓ months new ribs were put in at a cost of $273 and lasted 10 months, when the whole lining was removed. Cost per ton ground for this lining was $0.0457. A locally-cast hard white-iron ribbed liner costing $712 in place lasted about 2 yrs. and cost $0.0173 per ton. Silex (4 × 4 × 8-in. blocks) cost $396 in place and lasted 8 months, making the cost $0.0583 per ton ground.

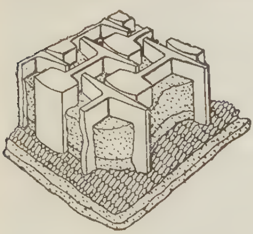


FIG. 30.—Britannia tube mill lining.

Britannia lining (Fig. 30) consists of 3- to 4-in. pieces of old railroad rails set on end in cement. Line the mill in sections as follows (*99 J 239*): First spread a layer 1 in. thick of cement mortar (1 cement to 2 of sharp, clean sand), press down sections (not to exceed 1 × 2 ft.) of 4- to 8-mesh wire screen, then a second 1-in. layer of mortar into which the rail ends are forced as far as possible, then more mortar to within 1 in. of the top of the rails. Allow to set, then turn the mill to line another section. When starting up, run for a few hours with lump ore but no balls or pebbles to allow crevices to fill. Cement wears sufficiently more rapidly than steel so that the ends of the rails project and make the lining surface irregular.

Barry lining consists of silex blocks cemented into a sectional iron honeycomb. Comparative performances of the Barry and Komata liners at the TALISMAN mine, N.Z., in two 4 × 12-ft. mills running side-by-side are shown in Table 57. Neither mill did good work but the mill with Komata lining produced 6.8 tons of −60-mesh material per 100 tons of feed in open circuit against 4.8 tons in the other mill. At 100-mesh the corresponding produc-

Table 57. Comparative performances of 4 × 12-ft. tube mills with Barry and with Komata lining

Mills run at 26-27 r.p.m., with about 50 per cent. moisture, in closed circuit with cone classifier.

Screen, I.M.M. mesh	Weight, per cent.			
	Barry lining		Komata lining	
	Feed	Discharge	Feed	Discharge
20	2.0	0.4	2.6	0.4
40	12.3	7.9	13.4	9.0
60	24.6	25.3	25.7	25.9
80	26.6	31.0	28.6	32.7
100	9.6	9.9	9.1	9.1
120	3.6	4.0	3.1	3.9
150	2.8	3.2	2.1	2.5
−150	18.2	17.5	14.7	16.2
Tons −60-mesh produced per 100 tons feed	4.8		6.8	
Tons −100-mesh produced per 100 tons feed	0.1		2.7	

tions were 2.7 and 0.1 tons per 100 tons respectively (*16 Aa 366*). The Barry has the distinct disadvantage, in common with all cemented liners, that a large part can fall out, with subsequent wear on the shell, and no warning given to operators. Leakage at bolt holes usually warns of breaks or wear-throughs in the case of plate or plate-and-rib liners.

Plymouth lining (*101 J 263*), shown in Fig. 31, is made of pieces of semi-steel battery liner, roughly 4 in. wide and 12 to 36 in. long, and pebbles, cemented in as shown.

Fig. 31.—Plymouth lining.

Rubber liners have been tried at the Nipissing mill. A 4 × 20-ft. ball-tube mill carrying 20 tons of 1¼-in. cast-iron balls and lined with ⅝-in. vulcanized rubber sheet with two layers of fabric was run in regular re-grinding service from Jan. to Aug., 1923, at which time, after study of the data, Parsons (*116 J 489*) reported as follows: Feed, −60-mesh, 40 to 80 per cent. −200-mesh sand; product, substantially all −200-mesh. Moisture, 44 per cent. Speed, 32 r.p.m. Change from ½-in. smooth-iron liners to ⅝-in. rubber increased slime-making capacity 15 per cent. and made it possible to shut down one of the primary 6 × 20-ft. tube mills and reduce the speed of the rubber-lined mill from 32 to 28½ r.p.m. The 4 × 20-ft. mill, rubber-lined, produced 11 per cent. more −200-mesh material from −4-mesh stamp-battery product than was produced on equal tonnage by a 6 × 20-ft. iron-lined pebble-charged mill running in parallel with it. Cost of ⅝-in. rubber liner for a 4 × 20-ft. mill is somewhat less than that of a 2-in. chilled cast-iron liner and the weight of the rubber is about 5 per cent. that of the iron. The relative life of the two was not determined but it was indicated that rubber would probably outlast iron on −4-mesh feed. Re-lining with rubber is easier and quicker than with iron. Ball consumption is less with the rubber liner.

Another rubber liner was tried in a ball mill, but was vulcanized too hard and wear was excessive.

Rubber lining has also been tried in Rand tube mills (*118 J 490*) with distinctly encouraging results. Wear conditions are harder than at Nipissing on account of the coarse, hard, sharp-edged lumps charged for grinding media and it was found that a soft pure rubber was better than a vulcanized composition. Cost of liner 1½ in thick, which thickness is contemplated from a structural standpoint, is estimated at about $4000 against $800 for 4-in. iron liner in a standard (5½ × 22-ft.) mill, but the life of the rubber is estimated at 2 years against 6 months for the iron. At Sandusky Cement Co. (*B. W. Rogers, 1925 meeting, Portland Cement Ass'n*) smooth and wave-type rubber liners were tested against smooth-iron liners in 5 × 22-ft. tube mills charged with balls, grinding cement slurry in a

pulp containing 60 to 65 per cent. solids. The feed was all − 4-mesh. Results are given in Table 58.

Table 58. Rubber vs. iron liners in tube mills grinding cement

Type of lining	Iron			Smooth rubber	Wave rubber		
Ball charge, per cent. mill volume	45–50	46	45–50	45–50	30	40	46
Speed, r.p.m.	27	27	24.4	27	27	27	27
Dry tons per hour	8.15	8.5	7.4	9.8	9.4	10.3	11.8
Product, per cent. − 200-mesh	93.2	93.2	93.5	94.1	93.4	92.6	92.5
Kw.-hr. per ton	14.9	14.9	16.2	13.5	9.8	11.6	11.1

For further data on wear of various types of liners see Tables 65 and 77.

Head liners are made of the same material as shell liners. Wear of end liners is frequently much greater than that of the shell liners. See notes (*ag*), (*ay*) and (*bv*), Table 65, and data on HOLLINGER performance, p. 437. Allen (*106 J 867*) warns against unsuitable liners and shows (Fig. 32) a wrongly- and a rightly-designed liner. The first wore through at the junction of the trunnion and head liner, grit and sand worked in behind and scoured until the mill collapsed by failure of the trunnion. This possibility was prevented in the re-designed (second) form.

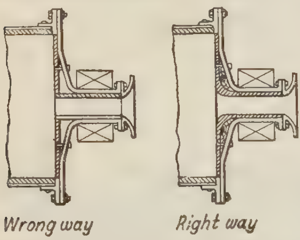


FIG. 32.—Tube-mill head liners.

Porcelain and wood blocks have been used in special instances for liners.

Choice of liner. The type of liner used depends to a considerable extent upon the character of the feed. When the feed is relatively coarse and relatively coarse product is desired, the rough-surface liners, such as the El Oro, Komata, or ribbed-plate are used; with finer feeds, or where an extra fine product is desired, silex or metal-plate liners are used. A smooth-iron liner will give the finest product and lowest power consumption for a given mill speed; a rough liner will give higher capacity unless the finished size is very fine. Comparison involves consideration of the ability to produce the material desired, life, effect on capacity, time to re-line and power consumption, as well as initial cost.

Grinding charge is composed ordinarily of flint pebbles in sizes varying from 2- in. to 6-in. The best flint pebbles come from the seashore in Denmark. Inferior imported flint pebbles come from France. French pebbles are usually light brown to gray on fracture compared to dark brown to black of the Danish pebble. Newfoundland pebbles are graywacke. They are hard but break readily. Pebbles range from 1- to 7-in. diameter and are graded by number as in Table 59. The characteristics of a desirable pebble are: (*a*) shape as nearly spherical as possible; (*b*) toughness; *c*) hardness; (*d*) high specific gravity; (*e*) chemical composition that is harmless in the mill product; (*f*) low cost. In using imported pebbles the first cost is usually only a small part of the total cost, hence it is economy to buy the best. In 1915 the best grade cost $11 per ton ready

Table 59. Trade numbers and sizes of imported pebbles

Number	Size, inches
0	1 to 1½
1	1¼ to 2
2	1¾ to 2½
3	2⅜ to 3¼
4	3⅛ to 3¾
5	3½ to 4
6	3¾ to 6
7	4¾ to 7

for export, \$15 per ton *ex* steamer New York, about \$35 at Tonopah, Nevada, and \$62.50 at Round Mountain, Nevada, 72 miles from Tonapah by wagon road (*110 P 139*). No domestic source of pebbles yields a product as good as the imported variety.

At the West End mill (*110 P 139*) artificially-rounded (tumbled) locally-mined flint was consumed in a mill with smooth-iron lining in the proportion of 1.05 to 1 of Danish pebbles in a mill with Komata lining and the cost per ton ground was only ⅐ of the cost of imported pebbles. Local pebbles wore flat and occasionally stuck in the grate. At Hedley Gold Mining Co. (*114 J 1057*) Danish pebbles at \$33 per ton delivered cost, on the average, \$2600 per month. Substitution of local pebbles at \$4 per ton cut the cost to \$600 per month.

In localities where freight charges on imported pebbles are excessive, lumps of local hard rock or hard ore have been used for grinding media. The advantage of this procedure is the apparent small cost of the material. The disadvantages are, however, several. Grinding is likely to fall off because the angular shape of the particles when first introduced is not good for grinding and because the pieces tend to wear flat and flat shapes are not so good as rounded.

But Graham (*7 JCM 317*) reports a competitive test in a Rand plant between mills loaded with banket and with Danish pebbles in which, with split feeds, the product of the former contained 9.7 per cent. +60-mesh and 64 per cent. −90-mesh against 11.3 per cent. +60-mesh and 61.4 per cent. −90-mesh for the latter. Bevington (*7 JCM 368*), discussing Graham's paper, agrees with his conclusions as to relative grinding efficiency and also with a conclusion that the banket is less destructive of silex lining. Allen (*105 J 1034*), tested Danish flint against roughly-rounded local felsite. His results are given in Table 60. The

Table 60. Comparative results of tube milling with imported flint and local felsite pebbles. (*After Allen*)

Screen, mesh	Weight, per cent.			
	Felsite		Flint	
	Feed	Product	Feed	Product
10	15.0	....	5.8	....
20	12.8	0.4	4.3	0.4
30	6.2	1.4	3.0	0.8
60	20.8	15.0	16.6	9.4
90	31.9	36.4	45.8	41.4
120	4.7	7.2	9.2	11.2
150	3.0	7.2	4.3	5.4
−150	6.4	32.4	11.0	31.4
Tons feed per 24 hr....................	176		175	
Consumption, pounds per ton...........	11		2.8	
Cost, dollars per ton..................	0.027		0.089	

felsite was much cheaper per ton of pebble and, therefore, per ton ground. He also tested the felsite against quartzitic mine rock and found (Table 61) that the felsite ground more material and that less was consumed. Breakage is likely to be excessive, resulting in formation of small angular and flat pieces which do little or no grinding and themselves are liable to further breakage.

When hand picking is already practiced the pebble material may be selected at little or no cost, but frequently a special operator must be put on for this purpose. His capacity will be 15 to 20 tons per day when picking from a belt,

if the supply of material is plentiful, much less under any other mode of selection. Handling from the sorting place into the mill need be no more expensive than the handling of imported pebbles, per unit weight, and may be even less if special provision is made, but the consumption of local materials may run up to well over 100 lb. per ton of ore ground, so that the handling cost per ton of daily capacity will usually be much higher than with pebbles. Large consumption causes corresponding decrease in capacity of the mill and the feed to the balance of the plant is materially diluted by the introduction of barren material. Recovery may also fall off. Quartano (*91 J 1017*) says that mine rock does not wedge into ribbed linings as well as flint pebbles and that liner wear is correspondingly higher. He suggests the daily addition of some flints to furnish liners and says that they will be selected by the mill, and that apparent economy may be gained by this practice in certain cases.

Table 61. Comparative performances of quartz and felsite pebbles. (*After Allen*)

Screen, mesh	Weight, per cent.			
	Mine rock (quartz)		Felsite	
	Feed	Product	Feed	Product
30	19.5	0.5	28.5	0.5
60	15.5	6.0	19.5	10.5
90	37.5	35.0	32.0	37.0
150	13.0	16.5	10.5	12.0
−150	14.5	42.0	9.5	40.0
Tons of feed per 24 hr.	101		133	
Comparative pebble consumption per ton ground	2		1	

Pebble charge is ordinarily carried at slightly above the mill axis, averaging perhaps 55 per cent. of the total internal capacity. The weight of pebbles for this charge is given roughly by the equation $W = 40\ D^2L$, where W = weight of charge in lb., D = nominal diameter of mill in ft. and L = length of mill in ft. For a charge of any depth, the weight W per ft. of length, in a mill charged up to or below the axis, is given by the equation

$$W = 4Vw/3\sqrt{(0.626V)^2 + V(2R - V)},$$

where V = depth of charge in ft., R = radius of mill in ft., and w = weight of charging material in lb. per cu. ft., struck volume. This is 100 to 105 lb. per cu. ft. for pebbles and 300 to 325 lb. per cu. ft. for iron or steel balls. If the top of the charge is above the axis, take V as the depth of the uncharged portion and deduct W from the weight for a cylinder of radius = R, and length = 1, fully charged.

Burt and Caetani (*37 A 3*) concluded that efficiency increased with increasing pebble load. On the other hand at Belmont Shawmut mill (*121 P 660*) it was found that although there was no change in grinding, there was an increase in power consumption corresponding to decrease in pebble volume and that consumption of Danish flint pebbles in a 5 × 18-ft. mill was 0.3 lb. per ton in grinding −10-mesh feed to 4 per cent. on 48-mesh, 56 per cent. −200-mesh, when the pebble load was carried 10 to 12 in. below center line, while with the pebble load at the center, consumption was 1.3 lb. per ton. Decrease in consumption was gradual with decrease in charge.

Testing tube-mill pebbles. A large part of pebble consumption is due to chipping, which occurs principally at the feed end of the mill. This has a double disadvantage because it reduces the size of pebbles at the point where maximum size is desired and also makes it necessary for the chips to pass the full length of the mill before they can be discharged, and so consumes considerable power in grinding. Allen (*105 J 1033; 124 P 405*) contends that toughness is the important property of suitable pebble material and that a tough soft rock may be superior to a hard but brittle flint. He cites the test given in Table 61 in support of this contention. " Mine rock " was a hard but friable quartz. The felsite was comparatively soft, but tough. Both were hand picked and roughly cobbed.

Allen suggests testing pebbles for hardness, toughness, and abrasion by the method practiced for testing road material. The hardness test consists in measuring the dimunition in weight of a standard cylinder of rock (about 25 mm. diam. by 100 mm. long) when abraded against a revolving steel disk, using quartz sand as the abrasive, the pressure against the disk and the number of revolutions thereof being constant for all tests. An arbitrary coefficient of hardness is obtained by dividing the loss in weight, expressed in grams, by 3, and subtracting the result from 20. The range is from 19.7 for the hardest varieties of quartzite to 0 for very soft limestones and sandstones.

Toughness is tested by measuring resistance to impact. A cylinder of rock with plane ends is placed in an impact machine and subjected to the blow of a 2-kg. weight falling from increasing heights varying by 1 cm. The height in cm. of the last blow preceding the breaking blow is taken as the toughness coefficient. The experimental range with different rocks is from 2 or 3 to 60.

Resistance to abrasion is tested by tumbling 50 pieces of uniform size, averaging about 100 gm. each, in a cast-iron cylinder 20 cm. inside diameter revolved 10,000 times at 30 r.p.m. The material is removed from the mill after tumbling and sized on a 16-mesh screen and the oversize weighed. The percentage loss in weight is called " Per cent. of wear," and 40/" Per cent. of wear " = " FRENCH COEFFICIENT OF WEAR." French coefficient varies for different specimens from as low as 1 to as high as 40, the higher figure indicating the least abrasion, *i.e.*, the greatest resistance to abrasion.

Table 62. Wearing characteristics of rocks (toughness 18 or over). (*After Allen*)

Rock	Samples tested	Abrasion, " French coefficient "			Hardness, average
		Low	High	Average	
Quartzite	14	7.8	24.5	17.5	18.7
Rhyolite	1	...	...	23.0	18.7
Diorite	3	10.8	23.8	17.4	18.8
Basalt	7	13.8	24.1	20.2	18.5
Andesite	2	13.4	14.2	13.8	18.0
Trachyte	1	...	...	12.7	19.1
Diabase	13	9.7	36.4	19.4	18.6
Chert	3	5.6	14.9	8.8	19.3
Limestone	1	...	...	4.6	16.2
Schist	5	8.5	31.7	19.6	18.4
Granite	1	...	...	25.3	18.9
Dolomite	1	...	...	7.4	18.2

Table 62 gives abrasion and hardness coefficients on a variety of rocks having a toughness of 18 or over. After study of a large number of tests Allen concludes that for pebble-mill work toughness should be more than 18, that the French coefficient should not fall below 16, and that, these coefficients being equal for two rocks, the one with the greater hardness coefficient will make the better pebble. For detailed procedure in testing see *Bull. 347, U. S. Dept. Agriculture.*

Hotchkiss (*104 J 288*) reports competitive abrasion tests on imported flints and Baraboo (Wis.) quartzite pebbles in which the quartzite showed considerably greater resistance. There are no corresponding tests of behavior in actual tube milling.

Pebble consumption varies according to the kind of pebble, hardness of the ore, size of feed and product, type of liner, pulp dilution and total feed rate. When mine rock is used, the consumption ranges from 15 to 200 lb. per ton of feed. With flint pebbles the average, according to Table 65, is about 2 lb. per ton and the range from 0.5 to 8.0. Hard ore, naturally, causes greater consumption than soft. Coarse feed consumes more than fine.

At West End mill (*110 P 139*) feed was battery product through 0.27- and 0.19-in. screens, hard but not tough quartz. Mill product was 80 per cent. – 200-mesh. Pebble consumption averaged 7.1 lb. per ton; range was from 4.8 lb. with the finer feed and Danish pebbles to 8.7 lb. with coarse feed and French pebbles. Consumption at three other mills in the district taking finer feed and finishing to the same size was 4.2, 4.4, and 4.7 lb. per ton.

Fine grinding consumes more pebbles than coarse, as is quite consistently indicated by Table 65. Consumption in closed-circuit grinding is greater than in open-circuit work, as a general thing, but it is probable that this is due to the fact that the product of closed-circuit work is generally the finer. Ribbed liners cause higher pebble consumption than smooth on account of the greater tumbling that they produce. High moisture content causes high pebble consumption. Consumption per ton of initial feed is lower the greater the circulating load. Table 63 (*98 J 471*) illustrates this fact.

Table 63. Effect of total mill tonnage on pebble consumption. (*After Mishler*)

Tons initial feed per 24 hr.	26.1	40.3	42.0	50.0
Tons total feed per 24 hr.	35	80	85	200
Pebble consumption, pounds per ton	127.9*a*	15.8	15.3	8.2

a Includes 110 lb. of local hard rock.

At Miami (*47 A 54*) it was found that increase in length of cylinder of a 6-ft. conical pebble mill decreased pebble consumption by nearly 50 per cent. It was also found that pebble consumption was dependent on size of feed, amounting to 1.85 lb. per ton on – 2.5-mm. feed against 3.60 lb. with – 6.5-mm. feed.

Small iron balls (BALL-PEBS) have replaced pebbles in tube-mill practice in some plants with marked increase in capacity. The change usually requires change of liners and change in drive to transmit the additional power, or the mill must be run with an undercharge of balls. Increase in capacity is roughly in proportion to increase in power consumption. See Tables 84, 86 and 87. Substitution of balls for pebbles must be carefully made, however, because of the considerable increase in strain on the mill, the shell and bearings of which were designed for a pebble load.

Feeders are of three general varieties known respectively as scoop, elbow and squirt. The scoop feeder is of the spiral type, ordinarily one-way, similar in every way to ball-mill feeders except not ordinarily so heavily made nor of such large opening.

At Nevada Packard the scoop feeder has removable side plates which permits inspection of the spirals.

The elbow feeder consists of a flanged elbow leading into the trunnion and packed to prevent leakage. The squirt feeder is used to introduce relatively fine liquid feed from a classifier spigot or similar source and consists of a nozzle pipe passing through a stuffing box into the trunnion.

Discharge from tube mills may be of simple overflow (center-discharge) type, in which case the discharge-trunnion opening is frequently of larger diameter than the feed trunnion; or a quick discharge with sand elevator is used.

A slotted grate is frequently placed near the discharge end to prevent discharge of pebbles. The grate is usually placed in the mill proper close to the trunnion and is bolted to the shell. This makes it impossible to observe the inside of the mill and the grate wedges badly and is hard to remove.

Allen (Fig. 33) shows a form of trunnion grate that screws against the trunnion liner; it does not blind readily on account of the large peripheral discharge space (a) and is easily removed.

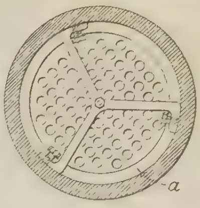


Fig. 33.—Trunnion grate for tube mill *(103 J 1072)*.

Scoop discharge is a quick-discharge mechanism for tube mills, consisting of a spiral placed between a grate and the discharge-end head of a tube mill. It picks up material discharged through the grate and delivers it through the discharge trunnion.

Graham *(16 JCM 59)* presents the data in Table 64 summarizing tests on mills with and without scoop discharges at City Deep Mill, Rand. The table shows that the tons of −90-mesh produced per horsepower-hour is substantially the same both with and without scoops and irrespective of the size of scoop, that capacity is greater with than without the scoop and greater with a scoop of large (58-in.) diameter than with a small (34-in.) one, but that pebble (and presumably liner) wear are greater with scoops than without and greatest with the scoop of greatest diameter. The conclusion from the tests is that if pebble and liner wear cost more per ton than the capital expense of additional mills, use the center-discharge mill, otherwise, the scoop mill.

An incidental advantage of the scoop discharge, which may become important in certain instances, is the fact that it permits the use of smaller discharge trunnions and thus saves head room. Mueller *(98 J 564)* commenting on his experience with quick-discharge mills grinding through 35-mesh at Cobalt, in open-circuit, condemns both diaphragm and quick discharge, the former on the ground of uselessness and the latter on the grounds that it discharges too much oversize and causes excessive pebble consumption. He admits the possible usefulness of the quick discharge in closed-circuit work with large circulating load. The quick discharge has also been used when it was wished to use the tube mill as a granulator for table feed *(99 J 694)*.

At Gold Hunter a 5 × 14-ft. mill with radial adjustable shutters ground 130 tons of jig middling per day in closed circuit with a 20-mesh Callow screen. The product contained 30 per cent. −200-mesh; power consumption was 39 hp.; pebble consumption, 0.9 lb. per ton. Without the quick discharge the capacity was 112 tons to 40 per cent. −200-mesh; power consumption, 33 hp.; and pebble consumption 1.2 lb. per ton.

A bell-shaped discharge lip, similar to that used in ball mills, and drip rings to prevent grit from working back into the discharge bearing are used in some cases. With open-end discharge it is common practice to fit the discharge bell with a trommel with ⅛- to ¼-in. screen which serves to separate wood chips and pebbles yet passes pulp readily.

Pebbles are fed by throwing them through the discharge trunnion of center-discharge mills. In other cases the discharge-trunnion liner has a reverse helix cast on the inner surface which serves to turn back pebbles and also to carry in pebbles fed through the discharge trunnion. In grate mills pebbles must be fed through the feed trunnion. This is readily done with scoop feeders but not so readily with the elbow type. When local rock is used for a grinding medium the tonnage necessarily fed is large and special provision is frequently made for charging.

Support. Tube mills are almost invariably supported at the head end by a trunnion bearing. The discharge end is supported by trunnion bearing or, less frequently, by tire

Table 64. Effect of scoop discharge on tube-mill performance.(a) (*After Graham*)

Diameter of scoop, in.	0*d*		58		58		58		58		58		58	
Total feed, tons per 24 hr.(*b*)	300		320		335		346		400		270		290	
Moisture, per cent.	32.0		37.5		31.0		30.0		31.0		33.5		32.0	
Pebble load, pounds	21,200		21,200		21,200		21,200		21,200		16,400		16,400	
Pebble load, height in mill	*e*		*e*		*e*		*e*		*e*		*g*		*g*	
Power consumed, horsepower	115		148		148		148		148		130		136	
Screen analysis, mesh(*f*)	Weight, per cent.													
	F	*P*	*F*	*P*	*F*	*P*	*F*	*P*	*F*	*P*	*F*	*P*	*F*	*P*
60	72.1	23.3	83.2	19.6	72.6	20.0	80.6	21.8	80.6	26.6	88.0	14.6	77.4	13.8
90	17.6	25.6	9.4	22.4	18.0	24.0	12.2	25.0	12.2	24.8	7.6	23.4	14.0	25.0
−90	10.3	51.1	7.4	58.0	9.4	56.0	7.2	53.2	7.2	48.6	4.4	62.0	8.6	61.2
Tons −90-mesh produced per 24 hr.	122.4		161.9		156.1		159.1		165.6		155		152.5	
Tons −90-mesh produced per horsepower-hour	0.044		0.045		0.044		0.045		0 047		0.050		0.047	
Pebble consumption, tons per 24 hr(*c*)	7		30		30		30		30		30		30	
Diameter of scoop, in.	24		24		24		24		24		24		24	
Total feed, tons per 24 hr.(*b*)	295		320		350		285		320		360		400	
Moisture, per cent.	32.0		32.0		32.0		32–33		32		32		32 33	
Pebble load, pounds	21,200		21,200		21,200		16,400		16,400		16,400		16,400	
Pebble load, height in mill	*e*		*e*		*e*		*g*		*g*		*g*		*g*	
Power consumed, horsepower	125		125		126		102		110		108		109	
Screen analysis, mesh(*f*)	Weight, per cent.													
	F	*P*	*F*	*P*	*F*	*P*	*F*	*P*	*F*	*P*	*F*	*P*	*F*	*P*
60	72.0	14.4	73.0	16.6	78.8	24.8	70.2	18.2	75.0	26.2	79.6	29.4	79.6	33.8
90	18.0	27.4	16.8	27.4	14.4	26 8	20.0	27.6	15.8	26.6	14.2	29.4	14.2	27.4
−90	10.0	58.2	10.2	56.0	6.8	48.4	9.8	56.2	9.2	47.2	6.2	41.2	6.2	38.8
Tons −90-mesh produced per 24 hr.	142.2		148.9		145.6		132.2		121.6		126.0		128.8	
Tons −90-mesh produced per horsepower-hour	0.047		0.050		0.048		0.054		0.046		0.049		0.049	
Pebble consumption, tons per 24 hr(*c*)	10 12		10 12		10 12		10 12		10–12		10–12		10–12	

a Standard $5\frac{1}{2} \times 22$-ft. mill. Diameter inside liners = 5 ft. Speed, 32 r.p.m. *b* Includes sand return. *c* Lump ore. *d* No scoop. Center-discharge. *e* At center. *f* All feed: −8- to 10-mesh battery screen. *g* 8-in. below center. *F* = feed. *P* = product.

and rollers. Dowling (*RMP*) says that trunnion mills on the Rand have required less maintenance than tire-and-roller mills.

Drive problem is substantially the same as for ball mills (p. 349).

Bulkley (*54 A 137*), writing of SOUTH AFRICAN practice, says that belt drive to pinion shafts from 500- to 600-r.p.m. slip-ring motors with high starting torque (150 to 200 per cent. of full load) is usual and that with such a rig there is no necessity for a clutch or belt-shifting devices. Motors (250-r.p.m.) direct-connected to the pinion shaft through a flexible coupling failed through breaking down of the motor winding on account of vibration transmitted through the coupling. At NIPISSING (*48 A 16*) the belt from 125-hp. motor with 24-in. (diam.) pulley to the 96-in. pulley on the mill countershaft is 18-in. 8-ply endless rubber. The pulleys range from 19-ft. 4-in. to 24-ft. 4-in. centers horizontally on different installations in this plant. The belts were highly satisfactory and did not have to be cut or spliced in the first 12 months.

Setting up. For detailed instructions see Labbe, 101 *J* 777.
Manufacturers. Same as cylindrical ball mills (see p. 359).
Performance of tube mills is shown in Tables 65 and 65*a*.

At BELMONT SURF-INLET a 5 × 16-ft. mill in closed circuit grinds 80 tons per 24 hr. to 70 per cent. − 200-mesh for 60 hp. with a consumption of 0.6 lb. white-iron liner and 5- to 6-lb. Danish pebbles per ton. At HOLLINGER (*Bul. CMI, Mar. 1923*) two 6 × 16-ft. mills charged with 11 tons of pebbles each (5 in. above axis), running at 24 r.p.m. crush 300 tons each of new feed per 24 hr. to 70 per cent. − 200-mesh. The total tonnage is three times the original. Power consumed = 115 hp. each. Pebble consumption = 1.33 lb. of 4-in. flints per ton of new feed. Ribbed white-iron shell liners last 22 months; end liners, 16 months; cast-steel grates (3/8 × 3/4-in. slots) last 6 months. At the same plant 5 × 20-ft. tubes following the stamps crush 160 tons new feed per day to 70 per cent. − 200-mesh. Total tonnage is about 500 for each mill. Charge, 9 to 9.5 tons of flint pebbles to 5 in. above the mill axis; 28½ r.p.m.; 75 hp. for one mill and one Dorr classifier. Pebble consumption, 5 lb. @ 4-in. per ton of new feed. At NEW MODDERFONTEIN (*109 J 255*) 6 × 20½-ft. mills at 28 r.p.m. with Osborn liners and scoop discharges, in closed circuit with cones, grind 240 tons per day from − 3-mesh battery product to 60-mesh. Each mill has a 150-hp. motor.

14. Operation of tube mills

Capacity depends upon the physical character of the ore, the size of mill, size of feed and product, weight of charge and size of pebbles, speed, liner, and whether the mill is in closed or open circuit. The effects of the several variables are much the same as in the ball mill. (See pp. 387–409.) Table 66, which is generalized from Table 65 and from manufacturers' catalogs gives figures that are reasonably safe for estimating purposes on average ores.

James (*21 IMM 3*) gives a generalized figure for capacity of Rand mills of 10 tons per day per ton of pebble load or 1.5 tons per horsepower-day (0.062 ton per horsepower-hour) from − 3/8-in. to − 90-mesh. Ball's generalized figure (*21 IMM 3*) is 150 tons per 24 hr. to 60-mesh from a 5½ × 22-ft. mill drawing 75 to 90 hp., or 0.069 to 0.083 ton per hp.-hr. At EL ORO the capacity is about 1.2 tons per hp.-day (0.05 ton per hp.-hr.) from sand to 150- or 200-mesh. Gieser (*97 J 463*) gives 0.087 ton per hp.-hr. to 90 per cent. − 100-mesh on this ore and about the same when grinding from 7 per cent. + 60-mesh to all-through 200 mesh. Average tons per horsepower-hr. from Table 65 are: 0.179 grinding to table sizes (10- to 20-mesh); 0.073 ton per hp.-hr. to flotation size (48- to 65-mesh); and 0.040 ton to 100-mesh. Brown (*105 J 246*) says that a tube mill can safely be fed 0.75 ton per cu. ft. of mill volume per 24 hr., including classifier return. This figure, developed from cyanide milling, is low for flotation practice and low for the most advanced cyanide practice at the present time, but accords well with practice in older mills.

Character of ore has, in general, more to do with the grinding done in tube mills than in ball mills for the reason that tube-mill feed and product are normally both finer than ball-mill and the bulk of the work of the tube lies in

Table 65. Performance of cylindrical tube mills

Plant	Granitic zinc ore	U.S.S.R. & M., Loreto	U.S.S.R. & M., Guerrero	Mexican gold mill	Waihi(*at*)	U.S.S.R. & M., Midvale
Size, diameter × length, ft	4½×20	*q*	*r*	*K*	4¾×18	5×8*m*
Speed, r.p.m.	20	32	32	32	25–26	26
Tons of new feed per 24 hr.	170	50 average	50 average	75 average	110–120	60
Tons of total feed per 24 hr.	....	175–200	175–200	175 average	280–340	....
Method of closing circuit	Open	*MC*	*MC*	*L*	*aw*	*DC*
Installed horsepower	50	69 average	64 average	70 average	....	75
Actual horsepower	46	59 average	58 average	65 average	55	60–65
Horsepower per ton of pebbles	4.8	6.7	6.6	8.1	10	6.7–7.2
Tons of new feed crushed per horsepower-hour	0.154	0.035	0.036	0.024	0.083–0.091	0.042–0.038
Moisture in mill, per cent	32	....	....	35–40	50	40
Size of feed(*a*)	2	4, 5*s*	6, 7*t*	*M*	26	*n*
Size of product(*a*)	2	4, 5*s*	6, 7*t*	*M*	26	*n*
Attendance, machines per man	10	1.3	1.4	0.8	....	4
Lost time, per cent	....	2.0	6.6	2	....	....
Principal causes of lost time	Re-lining	Re-lining	Re-lining	Re-lining	....	....
Lubricant, kind/pounds per shift	*Cr* 2.3	*O*/0.12; *Gr*/1.3	*O* 0.12; *Gr*/1.3	....	....	....
Feeder, type	*SS*	*SS*	*SS*	*SS*	....	Comb.
Feeder, material	*CI*, *i*	*CI*	*CI*	*CI*	....	*CI*
Feeder, life, days	....	720	720	720	....	....
Liner, type	Komata	Cascade	*u*	Cascade	Ribbed	*o*
Liner, material	*WI*	*WI*	*WI*	*WI*	*CI*	*CI*
Liner, life, days	....	*v*	*w*	200	540	....
Liner, consumption, pounds per ton	....	....	....	2.0	....	....
Time for re-lining, hr.	24	16	16	16	16	....
Number of men for re-lining	4	20	20	20	2	....
Grate, type	Screen	*x*	*x*	Slot	....	None
Grate, material	*WI*	....	*CI*	*CI*	*CI*	....
Grate, life, days	....	....	180	....	....	....
Grate, consumption, pounds per ton	....	....	....	....	....	....
Grinding media, kind	*DF*	*MR*	*MR*	*MR*	....	*p*
Grinding charge, new, total weight, lb.	19,000	17,700	17,700	16,000 average	11,000 average	18,000
Grinding charge, size, in. @ weight, lb.	4	6	*y*	−8 +4-in.	....	2 to 5
Grinding charge, size, in. @ weight, lb.	3	....	....	....	....	....
Grinding charge, size, in. @ weight, lb.	....	....	....	....	....	....
Grinding media, size added to compensate wear, in.	....	6	*y*	−8+4-in.	....	....
Grinding media, method of addition	....	*l*	*l*	*l*	....	*l*
Grinding media, consumption, pounds per ton	3.5	200	200	150	26	45

For explanation of reference letters, see page 447.

Table 65. Performance of cylindrical tube-mills—*Continued*

Plant	Elko Prince	Sunnyside M. & M. Co.	McKinley Darragh(*aq*)	Lucky Tiger(*bg*)	Tonopah Extension	McIntyre Porcupine
Size, diameter × length, ft.	5×14	5×14	5×14	5×14	5×16*G*	5×16
Speed, r.p.m.	22	29	28	27	27–29	30
Tons of new feed per 24 hr.	50–60	90	35*be*	*bh*	70–80	110*as*
Tons of total feed per 24 hr.	97	....	....	....	95	350
Method of closing circuit	*DC*	*DC*	....	....	*DC*	*DC*
Installed horsepower	....	75	....	60	*H*	100
Actual horsepower	35	65–70	35–37.5	45.5	*H*	65
Horsepower per ton of pebbles	....	5.9–6.4	....	6.5	....	13.0
Tons of new feed crushed per horsepower-hour	....	0.054–0.058	0.042	....	....	0.083
Moisture in mill, per cent	35	35–40	....	*bh*	31–34	28
Size of feed(*a*)	*8*	*9C*	*23*	*bh*	*I*	*10as*
Size of product(*a*)	*8z*	*9C*	*23be*	*bh*	*I*	*10as*
Attendance, machines per man	....	4	....	....	5	4
Lost time, per cent	−1*B*	Small	....	....	....	1
Principal causes of lost time	Re-lining	*D*	....	....	Re-lining	Re-lining
Lubricant, kind/pounds per shift	....	*O*/0.25	....	....	....	....
Feeder, type	*SS*	Comb.	....	*SS*	*SS*	*SS*
Feeder, material	*CI*	*CI*	....	....	*CI*	*CI*
Feeder, life, days	1080	....	....	....	180	540
Liner, type	Komata	*o*	Globe	El Oro	Smooth	El Oro
Liner, material	*A*	*E*	....	*Mn*	*WI*	*CI*
Liner, life, days	1022	*E*	....	....	240	420
Liner, consumption, pounds per ton	0.283*B*	*E*	....	....	....	0.26
Time for re-lining, hr.	10*B*	....	....	....	8	20
Number of men for re-lining	3	....	....	....	8	8
Grate, type	None	None	½-in. slot	....	....	None
Grate, material	....	....	....	....	*WI*	....
Grate, life, days	....	....	....	....	....	....
Grate, consumption, pounds per ton	....	....	....	....	....	....
Grinding media, kind	*DF*	*MR*, *F*	*FF*	*DF*	*J*	*DF*
Grinding charge, new, total weight, lb.	13,500	11,000	....	14,000	....	10,000
Grinding charge, size, in. @ weight, lb.	4	....	1	....	....	4
Grinding charge, size, in. @ weight, lb.	3	....	....	....	....	....
Grinding charge, size, in. @ weight, lb.	....	....	....	....	....	....
Grinding media, size added to compensate wear, in.	3 and 4	....	....	....	....	4
Grinding media, method of addition	*l*	*l*	....	....	*l*	*l*
Grinding media, consumption, pounds per ton	3.6*B*	20	6.5	....	15*J*	4

For explanation of reference letters, see page 447.

Table 65. Performance of cylindrical tube mills—*Continued*

Plant	Witherbee-Sherman, No. 6	Mt. Lyell (*ax*)	Barnes King(*bq*)	Pittsburg Dolores	Tonopah Belmont	Calumet & Hecla(*ar*)
Size, diameter × length, ft	5×16	5×16	5×16	5×18	5×18	5×18
Speed, r.p.m.	22			28	28	
Tons of new feed per 24 hr.	50	50	45	60	70	54.5
Tons of total feed per 24 hr.				100	130	
Method of closing circuit		Open	*DC*	*DC*	*DC*	Open
Installed horsepower	75	75		50	50	
Actual horsepower					57.8	40
Horsepower per ton of pebbles					5.5	6.7
Tons of new feed crushed per horsepower-hour			0.07		0.050	0.57
Moisture in mill, per cent	Dry	26		40	35–38	
Size of feed(*a*)	−8-mesh	*27*	*35*	−0.25-in.	*11*	*24*
Size of product(*a*)	−80-mesh	*27*	*35br*	O_1	*11R*	*24*
Attendance, machines per man	1			*P*	8	
Lost time, per cent					11	
Principal causes of lost time					*Q*	
Lubricant, kind/pounds per shift					*Q*.2	
Feeder, type				*SS*	*SS*	
Feeder, material				*CI*	*CI*	
Feeder, life, days						
Liner, type		Ribbed		Ribbed	Ribbed	
Liner, material		*CI*		*CI*	*WI*	
Liner, life, days		500*ay*		420–540	583	
Liner, consumption, pounds per ton					0.49	
Time for re-lining, hr.				24	24	
Number of men for re-lining				3	7	
Grate, type				None	None	
Grate, material						
Grate, life, days						
Grate, consumption, pounds per ton						
Grinding media, kind	*DF*	*az*		*MR*	*DF*	
Grinding charge, new, total weight, lb.	*N*				21,000	12,000
Grinding charge, size, in. @ weight, lb.		2½ to 4		4	4	
Grinding charge, size, in. @ weight, lb.						
Grinding charge, size, in. @ weight, lb.						
Grinding media, size added to compensate wear, in.				4	4	
Grinding media, method of addition	*l*			*l*	*l*	
Grinding media, consumption, pounds per ton	5	11		16–24	4 2	4

For explanation of reference letters, see page 447.

Table 65. Performance of cylindrical tube mills—*Continued*

Plant	Mexican gold mill	McKinley Dorragh(*aq*)	Mexican gold mill	Hedley G. M. Co.	Liberty Bell	Rand (average)
Size, diameter × length, ft.	5×20	5×20	5×22	5×22	5×22	5.5×22
Speed, r.p.m.	28	24½	30	28	24	31-32
Tons of new feed per 24 hr.	125	50	125	40–50	100	130
Tons of total feed per 24 hr.	200-275	....	200-275	130	....	400
Method of closing circuit	*DC*	Open	*DC*	*DC*	Open	Cones
Installed horsepower	65	....	100	75	50	125–175
Actual horsepower	88	31.1	90	....	....	75-140
Horsepower per ton of pebbles	8.8	....	9.0	....	....	9.4-11.2
Tons of new feed crushed per horsepower-hour	0.059	0.067	0.058	....	....	0.039-0.072
Moisture in mill, per cent	35–40	80	35–40	30–40	38	38-40
Size of feed(*a*)	*12*	*31*	*12*	*13*	*14*	– ¾-in.
Size of product(*a*)	*12S*	*31*	*12S*	*13V*	*14*	– 90-mesh
Attendance, machines per man	1.5	....	1.5	*W*	10	....
Lost time, per cent	5	....	5	7	4	....
Principal causes of lost time	*T*	....	*T*	*X*	*Z*	....
Lubricant, kind/pounds per shift	....	....	....	....	*Gr*0.08	....
Feeder, type	Schmidt	....	Schmidt	*SS*	*SS*	....
Feeder, material	*CI*	....	*CI*	*Y*	*CI*	....
Feeder, life, days	1000	....	1000	720	*aa*	....
Liner, type	El Oro	....	El Oro	El Oro	*ab*	Osborn
Liner, material	*WI*	....	*WI*	*CCI*	*ab*	....
Liner, life, days	300	....	300	480	200	360
Liner, consumption, pounds per ton	0.7	....	0.7	....	*ab*	....
Time for re-lining, hr.	16	....	16	48	*ab*	....
Number of men for re-lining	8	....	8	3 to 5	*ab*	....
Grate, type	None	None	None	Slot	None	Slot
Grate, material	....	....	....	*Mn*	....	....
Grate, life, days	....	....	....	180	....	....
Grate, consumption, pounds per ton	....	....	....	....	....	....
Grinding media, kind	*MR*	....	*MR*	*DF*	*DF*	*MR*
Grinding charge, new, total weight, lb.	20,000	*bf*	20,000	....	18,000	16,000–25,000
Grinding charge, size, in. @ weight, lb.	*U*	....	*U*	3–4	4	....
Grinding charge, size, in. @ weight, lb.	....	....	....	....	....	....
Grinding charge, size, in. @ weight, lb.	....	....	....	....	....	....
Grinding media, size added to compensate wear, in.	*U*	....	*U*	3–4	4	....
Grinding media, method of addition	*l*	....	*l*	*l*	*l*	....
Grinding media, consumption, pounds per ton	70	7.6	70	7-8	1.7	....

For explanation of reference letters, see page 447.

Table 65. Comparison of cylindrical tube mills—*Continued*

Plant	Nevada Packard	Arizona Copper Co.(*ap*)	Granitic zinc ore	Zaruma(*bs*)	Nipissing	Nipissing(*bl*)
Size, diameter × length, ft.	2 @ *ba*	6×9	6×12	6×14	6×20	6×20
Speed, r.p.m.			20	28 30	26*bj*	25
Tons of new feed per 24 hr.	90	145	200	90	68	122
Tons of total feed per 24 hr.					74	
Method of closing circuit	*ba*	Open	Open	*DC*	*DC*	Open
Installed horsepower	100		100			125
Actual horsepower		50	62			114
Horsepower per ton of pebbles			5.2			
Tons of new feed crushed per horsepower-hour		0.121	0.134			0.045
Moisture in mill, per cent			34	40	48	40
Size of feed(*a*)	*bd*	*22*	*16*	*36*	*32*	*33*
Size of product(*a*)	*bd*	*22*	*16*	*36bt*	*32bi*	*33bo*
Attendance, machines per man			10			
Lost time, per cent						
Principal causes of lost time			Re-lining			
Lubricant, kind/pounds per shift						
Feeder, type	*SS*, *bb*		*SS*	*SS*		*SS*. *bm*
Feeder, material	*CI*		*CI*			*CI*
Feeder, life, days	*bb*					
Liner, type			Komata	Tonopah		Silex, 4-in.
Liner, material			*WI*	*WI*		
Liner, life, days				*bv*		
Liner, consumption, pounds per ton		*ao*				
Time for re-lining, hr.			16			
Number of men for re-lining			4			
Grate, type						0.5-in. holes
Grate, material			*WI*			*CCI*
Grate, life, days						
Grate, consumption, pounds per ton						
Grinding media, kind	*bc*		*DF*	*bu*		*bn*
Grinding charge, new, total weight, lb.			24,000			*bn*
Grinding charge, size, in. @ weight, lb.			3–4		3–5	
Grinding charge, size, in. @ weight, lb.						
Grinding charge, size, in. @ weight, lb.						
Grinding media, size added to compensate wear, in.			3–4			
Grinding media, method of addition			*l*			
Grinding media, consumption, pounds per ton		0.862*an*	1.6	*bu*		*bn*

For explanation of reference letters, see page 447.

Table 65. Performance of cylindrical tube mills—*Continued*

Plant	Nipissing (*bl*)	Granitic zinc ore	Chino Cons. Copper Co.	Alaska Gastineau	Amer. Z. L. & S., Mascot	Federal M. & S., Morning
Size, diameter × length, ft.	6×20	7×10	7×10	7×10	7×10*ah*	7×12
Speed, r.p.m.	25	20	23	20	22	24
Tons of new feed per 24 hr.	122	300	300	600	350	260
Tons of total feed per 24 hr.	.....	.....	.....	.....	550	.....
Method of closing circuit	*DC*	Open	Open	Open	*ai*	Open
Installed horsepower	125	150	100	75	100	150
Actual horsepower	114	91	75	69	95	90
Horsepower per ton of pebbles	.....	7.6	9.4	9.2	8.6	10
Tons of new feed crushed per horsepower-hour	0.045	0.137	0.167	0.362	0.153	0.120
Moisture in mill, per cent	40	40	30	33	45	49
Size of feed(*a*)	*34*	*17*	*18*	*19*	*20*	*21*
Size of product(*a*)	*34bp*	*17*	*18*	*19*	*20aj*	*21*
Attendance, machines per man	.....	10	4	20	*ak*	1
Lost time, per cent	.....	.....	1	.....	.....	1.5
Principal causes of lost time	.....	Re-lining	*ad*	.....	.....	Re-lining
Lubricant, kind/pounds per shift	.....	*Gr.*/2.3	*O*/0.6; *Gr.*/0.7	*O*/0.7	*O*/1	*O*/2
Feeder, type	*SS*	*SS*	3-way	3-way	3-way	*SS*
Feeder, material	*CI*	*CI*	*CI*	*CI*	*CI*	*CI*
Feeder, life, days	.....	.....	.....	1250	720	365
Liner, type	Silex, 4-in.	Komata	Komata	*ag*	Forbes	Forbes
Liner, material	.....	*WI*	*WI*	*ag*	*CCI*	*CI*
Liner, life, days	.....	.....	*ae*	*ag*	720	300
Liner, consumption, pounds per ton	.....	.....	.....	.....	0.1	0.16
Time for re-lining, hr.	.....	16	8	14	16	16
Number of men for re-lining	.....	4	6	4	8	3
Grate, type	0.5-in. holes	.....	½-in. slot	.....	Slot	None
Grate, material	*CCI*	*WI*	*WI*	*CI*	*CI*	.....
Grate, life, days	.....	.....	365	110	90	.....
Grate, consumption, pounds per ton	.....	.....	.....	.....	.....	.....
Grinding media, kind	*DF*	*DF*	*DF, af*	Flint cubes	*DF*	*DF*
Grinding charge, new, total weight, lb.	*bn*	24,000	16,000	15,000	22,000	18,000
Grinding charge, size, in. @ weight, lb.	.....	3–4	2½	3	2 to 5	4 @ 8000
Grinding charge, size, in. @ weight, lb.	.....	.....	.....	.....	.....	3 @ 6000
Grinding charge, size, in. @ weight, lb.	.....	.....	.....	.....	.....	2 @ 4000
Grinding media, size added to compensate wear, in.	.....	3 and 4	No. 2	3	2–5	.....
Grinding media, method of addition	.....	*l*	*l*	*l*	*al*	*am*
Grinding media, consumption, pounds per ton	4.3	1.7	1.234	1.3	0.5	1.8

For explanation of reference letters, see page 447.

Table 65[a]. Sizing tests

(Figures under the headings F

Reference numbers / Plant			2 Granitic zinc ore		4 U. S. S. R. & M., Loreto		5 U. S. S. R. & M., Loreto		6 U. S. S. R. & M., Guerrero	
Screen aperture										
Mesh	In.	Mm.	*F*	*P*	*F*	*P*	*F*	*P*	*F*	*P*
4	0.18	4.70			10 2					
6	0.13	3.33							22.7	
8	0.093	2.36			21.4					
10	0.065	1.65								
12					25.0				27.0	
14	0.046	1.17	0.2							
20	0.033	0.83	0.4		26.6		1.0		14.5	3.3
28	0.023	0.59	1.5							
35	0.016	0.42	6.2	0.3						
40					14.8		18.0		17.0	10.5
48	0.012	0.30	11.2	1.4						
60										
65	0.008	0.21	17.0	4.0						
80			11.6	5.8		13.2	12.3	8.8	13.1	37.5
90										
100	0.006	0.15	14.6	11.0		30.4	52.0	45.6	2.2	10.4
120										
150	0.004	0.10	20.5	21.8		10.4	7.2	15.2	1.0	6.8
160										
200	0.003	0.07	9.4	17.0		4.2	1.6	4.0		
240										
280			0.7	1.6						
Through last screen			6.5	37.1	2.0	41.8	7.9	26 4	2.5	31.5

Reference numbers / Plant			16 Granitic zinc ore		17 Granitic zinc ore		18 Chino Cons. Copper Co.		19 Alaska Gastineau	
Screen aperture										
Mesh	In.	Mm.	*F*	*P*	*F*	*P*	*F*	*P*	*F*	*P*
4	0.18	4.70								
6	0.13	3.33								
8	0.093	2.36								
10	0.065	1.65	12.1	2.3	7.9	0.4	5.1	0.1	12.2	0.9
12										
14	0.046	1.17	21.6	5.9	9.4	0.1	12.0	0.8		
20	0.033	0.83	25.4	14.2			18.8	0.8	44.5	12.8
28	0.023	0.59	18.3	17.2	39.4	8.9	22.1	2.2	17.7	18.6
35	0.016	0.42	13.9	19.7	15.1	12.2	15.2	5.1	7.8	9.0
40										
48	0.012	0.30	4.9	11.6			7.0	7.0	7.4	11.8
60										
65	0.008	0.21	1.7	7.3	23.8	28.7	12.9	11.3	5.4	11.0
80										
90										
100	0.006	0.15	0.7	5.3	2.8	12.2	2.0	13.9	2.6	9.6
120										
150	0.004	0.10	0.4	4.2	0.6	7.0	1.7	11.8	0.6	2.8
160										
200	0.003	0.07	0.2	2.4			0.5	5.6	0.6	6.2
240										
280					0.5	12.2				
Through last screen			0.8	9.8	0.6	18.4	2.7	41.9	1.2	17.4

[a] I.M.M. screens.

referred to in Table 65

and P are weight per cent.)

7 U. S. S. R. & M., Guerrero		8 Elko Prince		9 Sunny-side		10 McIntyre Porcupine		11 Tonopah Belmont		12 Mexican gold mill		13 Hedley G. M. Co.		14 Liberty Bell	
F	*P*	*F*	*P*	*F*	*P*	*F*	*P*	*F*	*P*	*F*	*P*	*F*	*P*	*F*	*P*
						4.0									
		8.5						5.4		0.1					
								5.2		0.4					
		8.7				9.5		5.3		0.9	0.3				
								5.9		1.6	0.2				
		11.4	2.5					4.9	1.0	6.3	1.7			18.0	0.3
6.2	2.5					25.9									
								6.5	3.1	10.8	9.7			25.7	2.7
						5.6	2.2								
		15.8	9.9					8.1	6.5	25.2	19.2			22.9	6.2
50.5	34.3					26.2	15.4					30			
23.0	26.1	14.8	13.0	58.0	7.0	11.1	15.0	20.0	20.0	32.1	27.9	3		13.5	21.7
10.0	13.0	22.9	22.3					24.4	28.6	12.5	14.7	19	3.5		
		9.7	19.5	34.0	25.0	9.0	5.0	7.8	12.7	4.8	9.9	10	2.0	8.5	28.7
10.3	21.1	8.2	32.8	8.0	68.0	9.0	52.1	6.5	28.1	5.2	21.3	38	94.5	11.4	40.4

20 American Z. L. & S. Co.		21 Federal M. & S., Morning		22 Arizona Copper Co.		23 McKinley Darragh		24 Calu-met & Hecla		25 McIntyre Porcupine		26 Waihi		27 Mt. Lyell		28 Nevada Packard	
F	*P*	*F*	*P*	*F*	*P*	*F*	*P*	*F*	*P*	*F*	*P*	*F*	*P*	*F*	*P*	*F*	*P*
								Fine-jig and table tailing		7.3						33.9	0.2
0.3						32.8						(a)		(a)			
				0.9		22.0				5.2							
10.0	0.6			26.0	0.1	17.4				6.5						21.7	0.9
				17.7	0.5	11.1											
18.9	9.8	1.0		13.9	1.5					11.6		16.7		62.2		9.8	2.3
				13.1	6.6												
38.9	30.3			8.9	10.0												
		6.8							3.7	16.4	5.6	12.5		12.6	2.1		
				8.0	16.1		9.6									5.6	11.4
									5.2	12.9	12.0	9.8	0.2	5.5	6.2		
19.3	26.1			4.5	11.9		13.2									2.9	10.4
		38.2	14.2							4.4	7.1	8.5	1.8				
														10.2	15.4		
		20.6	15.5	2.9	10.9		16.9		19.7	18.0	22.1	4.0	2.6			2.5	6.4
												3.3	7.6	1.2	8.7		
7.1	12.6			1.5	8.5		13.9		17.1					6.4	3.1	2.7	6.9
		19.2	23.3														
3.0	6.7			0.6	4.5				9.4	11.2	31.1	6.7	8.6			0.9	2.6
		12.0	15.2														
2.5	13.9	2.2	31.8	2.0	29.4	16.5	46.5		44.9	6.5	22.1	38.6	79.4	1.9	64.5	19.6	57.8

F = Feed. *P* = Product.

Table 65*a*. Sizing tests referred to in Table 65—*Continued*

Reference numbers / Plant			29 Nevada Packard		30 McKinley Darragh		31 McKinley Darragh		32 Nipissing	
Screen aperture										
Mesh	In.	Mm.	*F*	*P*	*F*	*P*	*F*	*P*	*F*	*P*
4	0.18	4.70	0.2	......	1.7	......	......	......	......	......
6	0.13	3.33	......	......	40.3	......	......	......	2.5	......
8	0.093	2.36	......	......	17.2	......	......	......	6.3	......
10	0.065	1.65	2.5	......	16.0	......	12.4	......	8.6	......
12	....	....	......	......	......	......	......	......	......	......
14	0.046	1.17	......	......	10.7	0.2	12.6	......	6.3	......
20	0.033	0.83	3.9	......	7.2	0.2	11.8	......	6.1	......
28	0.023	0.59	......	......	......	0.6	15.6	......	5.7	......
35	0.016	0.42	......	......	......	1.8	12.5	......	6.0	......
40	....	....	......	......	......	......	......	......	......	......
48	0.012	0.30	17.8	1.6	......	3.0	12.4	......	4.3	0.2
60	....	....	......	......	......	......	......	......	......	......
65	0.008	0.21	22.8	11.8	......	6.2	9.1	......	4.2	0.6
80	....	....	......	......	......	......	......	......	......	......
90	....	....	......	......	......	......	......	......	......	......
100	0.006	0.15	20.2	17.8	......	12.9	......	32.7	9.2	4.1
120	....	....	......	......	......	......	......	......	......	......
150	0.004	0.10	12.9	16.2	......	14.9	......	15.6	6.7	7.5
160	....	....	......	......	......	......	......	......	......	......
200	0.003	0.07	3.5	5.8	......	10.7	......	11.8	22.3	20.1
240	....	....	......	......	......	......	......	......	......	......
280	....	....	......	......	......	......	......	......	......	......
Through last screen..........			15.9	46.2	6.9	49.5	13.6	39.7	11.8	67.5

Reference numbers / Plant			33 Nipissing		34 Nipissing		35 Barnes King		36 Zaruma	
Screen aperture										
Mesh	In.	Mm.	*F*	*P*	*F*	*P*	*F*	*P*	*F*	*P*
4	0.18	4.70	......	......	......	......	......	......	0.2	......
6	0.13	3.33	......	......	......	......	......	......	1.5	......
8	0.093	2.36	......	......	......	......	......	......	3.8	......
10	0.065	1.65	......	......	......	......	......	......	4.3	......
12	....	....	......	......	......	......	......	......	......	......
14	0.046	1.17	......	......	......	......	......	......	3.6	......
20	0.033	0.83	57.0	5.8	2.1	0.4	0	......	5.5	......
28	0.023	0.59	......	......	......	......	......	......	5.0	......
35	0.016	0.42	......	......	......	......	......	......	6.6	0.4
40	....	....	14.7	5.6	1.9	0.9	......	......	......	......
48	0.012	0.30	......	......	......	......	26.6	......	12.7	0.6
60	....	....	5.2	6.0	2.4	1.0	......	......	......	......
65	0.008	0.21	......	......	......	......	17.7	1.1	11.5	2.0
80	....	....	4.3	6.2	5.1	7.8	......	......	......	......
90	....	....	......	......	......	......	......	......	......	......
100	0.006	0.15	4.7	9.2	12.1	2.7	23.2	4.2	18.9	10.2
120	....	....	3.5	7.6	11.1	6.5	......	......	......	......
150	0.004	0.10	0.3	0.4	1.4	0.4	13.6	9.9	14.4	20.0
160	....	....	......	......	......	......	......	......	......	......
200	0.003	0.07	3.4	6.9	18.4	12.9	8.7	12.4	3.6	6.4
240	....	....	......	......	......	......	......	......	......	......
280	....	....	......	......	......	......	......	......	......	......
Through last screen..........			6.9	51.5	44.3	66.5	10.2	72.4	8.4	60.4

F Feed. *P* Product.

NOTES TO TABLE 65

a Italic numbers refer to column numbers in Table 65*a*. *i* White-iron tips. *l* Fixed amount added daily as calculated from tonnage and wear per ton. *m* One 5 × 8-ft. and three 5 × 14-ft. *n* Feed is classifier sand, all through 35-mesh, 90 per cent. +100-mesh; product of classifier, 75 per cent. −200-mesh. *o* El Oro, Komata and wave in different mills. *p* Mine rock, largely rhodonite. *q* Seven 5 × 16-ft.,one 5 × 20-ft., two 4½ × 15-ft., one 4 × 20-ft., eight 4½ × 20-ft. *r* Ten 4½ × 20-ft., one 5 × 20-ft., three 4½ × 15-ft. *s* Primary mills *4*, secondary *5*. Product in both columns is classifier overflow. *t* Primary mills *6*, secondary *7*. Product in both columns is classifier overflow. *u* Cascade and El Oro. *v* Primary mills, 6 months; secondary mills, 12 months; see also note (*s*). *w* Primary, cascade type, 5 months; secondary, El Oro type, 16 months. See also notes (*t*) and (*u*). *x* Primary mills have no grates. Secondary mills have baffles to hold back grinding media. *y* 6-in. in primary mills, 3-in. in secondary. *z* Classifier overflow is 1.5 per cent. +100-mesh and 78 per cent. −200-mesh. *A* Hard-iron and manganese-steel bars. *B* Average of 3-yr. operation: labor on repairs, $0.0036; supplies, $0.0044; liners, 0.0233; pebbles, $0.0771; total, $0.1084. *C* Screen test of feed (*F*) is a composite of original plus classifier return; product (*P*) is classifier overflow. *CI* Cast iron. *CCI* Chilled cast iron. *Cr* Chrome steel. *D* Leaky liner bolts. *DC* Dorr classifier. *DF* Danish flint. *E* El Oro ribs of hard cast iron, partially replaced after 133 days; consumption, 0.60 lb. per ton of mill feed. *F* Largely rhodonite and quartz. *FF* French flints. *G* Three 5 × 16-ft.and two 5 × 18-ft. Tonnage of new feed to 18-ft. mills = 80 per 24 hr.; total, 100 tons per 24 hr. *Gr* Grease. *H* 50-hp. motors on 16-ft. mills; actual power 66, 75 and 76 hp. respectively on the 3 mills. One 50-hp. motor on one 18-ft. mill draws 79 hp.; one 100-hp. motor on the other 18-ft. mill draws 84 hp. *I* Classifier feed all passes 3-mesh, 20 per cent. −200-mesh; classifier sand to tube mills; classifier overflow, 70 per cent. −200-mesh. *J* Local quartz and chalcedony pebbles. High consumption due to poor quality. *K* One 5 × 20-ft., seven 5 × 16-ft., eighteen 4½ × 20-ft., one 4 × 20-ft., arranged as the second and third steps in a 3-stage series of which stamps and Marcy ball mills in parallel are the first stage. *L* Dorr and Esperanza classifiers. *M* Feed is ball-mill and gravity-stamp discharge, all through 2-mesh. Product of secondary mill classifiers is 75 per cent. −200-mesh. *MC* Various types of mechanical classifiers, *e.g.* Dorr, Akins, Ovoca, Esperanza. *Mn* Manganese steel. *MR* Mine rock. *N* 40 per cent. of mill fall. *O* Oil. O_1 Classifier overflow, 80 per cent. −200-mesh. *P* Rolls, ball mill, tube mill, screens, and Dorr classifier tended by one man per shift. *Q* Power repairs, overcrowding of cyanide department. *R* Classifier overflow is 1 per cent. +100-mesh, 72.3 per cent. −200-mesh. *S* Classifier overflow is 0.5 per cent. + 48-mesh, 68.2 per cent. −200-mesh. *SS* Spiral scoop, one-way. *T* Inspection and re-lining. *U* 3-in. to 7-in.; weight, 7 to 16 lb. a piece. *V* Classifier overflow. *W* Tended by battery man. *WI* White iron. *X* Power shortage; re-lining. *Y* Cast-iron sole with plate sides. *Z* Repairing tires and rollers. *aa* Indefinitely long. *ab* See page 347. Same except that silex blocks replace quartzite. Consumption of silex, 0.16 lb. per ton; iron 0.097 lb. per ton. Material for 5 × 22-ft. mill: 900 @ 4½ × 5 × 8½-in. silex blocks, weighing about 13 lb each; 19 cu. ft. each Portland cement and fine sand; 10 rows cast-iron ribs, 47½ in. long 4½ in. deep, 2 in. at base tapered to 1¾ in., weight 110 lb. each. Two men line the mill in three shifts. One shift for lower half, one shift live steam, one shift cooling with draft; then one shift for ¼ of lining followed by the same drying and cooling; then one shift for the final ¼, followed by 12 hours' steaming after which the pebble load is put in and the mill started. The addition of soda ash to the mortar hastens setting and permits the mill to be put into use in 8 or 10 hr. The use of the iron ribs has greatly lengthened life of lining. Present life 2½ to 3 years (*102 J 27*). *ad* Inspection and changing grates. *ae* 365 and still good. Hard white-iron lifter bars last 9 months. *af* Adamant blocks (quartzite cubes) from Iowa and pebbles from California are a fair substitute for Danish pebbles. The consumption is almost twice as great but the cost per ton ground is about the same. *ag* Shell is lined with cast-iron plates and filler bars and manganese-steel lifting bars. ⅝ × 7-in. ship-lap false liners used between the shell and the shell liners. The outer circle of the end liners is Forbes type with flint cubes concreted in, all backed by ¼-in. boiler plate. The inner circle end liners are cast iron. Trunnion liners are manganese steel. Life of shell liners, 450 days; Forbes-type end liners, 430 days; feed-end inner circle, 210 days; discharge-end inner circle, 600 days. Discarded high-carbon-steel roll shells, approximately 1¼ in. thick, when cut and shaped to replace cast-iron shell liners, have double the life of the latter. *ah* Two 7 × 10-ft. and one 7 × 12-ft. Data refer to both. *ai* Anaconda classifier and bucket elevator. *aj* Classifier overflow is 1.3 per cent. +20-mesh, 18 per cent. − 200-mesh. *ak* Three mills and two jigs operated by one man. *al* Once a week through man-hole. *am* Loaded to keep ammeter constant. *an* Cost per ton, $0.0116. *ao* Cost per ton, $0.0105. *ap* Compare with 8-ft. × 36-in. conical pebble mill at same plant, Table 77. *aq 98 J 564*. Compare 8-ft. × 18-in. conical pebble mill, Table 77. *ar* Compare 8-ft. × 20-in. conical pebble mill at same plant, Table 77. Against the slightly better grinding shown by the cylindrical mill, the conical mill requires 40 per cent. less floor space, has lower pebble and liner cost, and is less sensitive to inequality in feed rate. *as* The same screen tests are given at *20 CMI 98* as corresponding to 150 tons new feed per 24 hr. In the same article screen tests (*25*), Table 65*a*, are given as corresponding to 200 tons new feed per 24 hr. *at 16 Aa 126* Ore notoriously hard (*97 J 464*). *au* Ribs 4 in. wide, spaced 15 in. and when new stand 3¼ in. above plate, which is 1½ in. thick. *av* Fills mill to 2 in. above axis, which has been determined to be the best height. *aw* Pyramidal diaphragm boxes. See Sec. 6, Art. 5. *ax 123 P 88*. *ay* Wear on ends was more severe than on shell. *az* Tasmanian quartzite.

ba One mill 6 × 5-ft. and one 6 × 10-ft. in series. The first mill is in open circuit and discharges to a Dorr duplex classifier; the second is fed with the classifier sand and runs in closed circuit with the classifier. The first has since been changed to a ball mill lagged down to 5 ft. diameter. See Table 5. ***bb*** Removable side plates. White-iron digging lip, life 6 months. ***bc*** One-half silex and one-half rhyolite. ***bd*** Column 28, Table 65*a* gives feed and discharge of first mill and column 29 of second. Classifier overflow contains 0.4 per cent. +60-mesh and 83.3 per cent. −200-mesh. ***be*** Screen tests of a run at 30 tons per 24 hr. with diaphragm removed are given in Column 30, Table 65*a*. ***bf*** 8 in. below discharge level. ***bg*** *98 J 469*. ***bh*** See Table 73. ***bi*** Classifier overflow is 0.2 per cent. +150-mesh, 96.4 per cent. −200-mesh. ***bj*** Later reduced to 16 r.p.m. without change in capacity or grinding but with reduction of 33 per cent. in power. ***bl*** *48 A 16*. ***bm*** 6 × 6-in. inside. ***bn*** Flint and hard run-of-mine. Charge carried 2 to 3 in. above axis. Consumption 1.8 lb. of flint plus 2.0 lb. of hard ore per ton of feed. ***bo*** The mill in parallel with this discharged, at the time of sampling, a product containing considerably less coarse sand but 9 per cent. less −200-mesh. ***bp*** Classifier overflow, 1 per cent. +200-mesh. ***bq*** *60 A 103*. ***br*** Classifier overflow. ***bs*** *111 J 58*. ***bt*** Classifier overflow, 80 per cent. −200-mesh. ***bu*** Lumps of hard quartz, hard silicious rhyolite and scrap iron all failed to grind satisfactorily. Run-of-mine ore (soft calcite) wore too rapidly. River pebbles most satisfactory available material, but consumption was 110 lb. per ton and they cost $0.13 per ton ground. ***bv*** Shell about 8 mo. Pebbles will not wedge so all wear is taken on the iron. End-liners 4 months.

Table 66. Capacity of tube mills

Size, diameter × length, feet	From mesh	To mesh	Tons per 24 hr.	Size, diameter × length, feet	From mesh	To mesh	Tons per 24 hr.
3⅓×12	8	20	62	5×18	8	40	170
....	8	40	52	....	8	60	122
....	8	60	31	....	8	100	64–90
....	8	100	23	*	3	100	60
3⅓×14	8	20	74	*	10	100	70
....	8	100	27	5×20	8	60	136
3⅓×16	8	40	60	*	8	65	50
....	8	100	31	*	20	65	125
4×8	8	20	66	....	8	100	70–100
....	8	100	25	*5×22	14	35	100
4×12	8	20	94	*	20	65	125
....	8	40	66	....	8	100	75–110
....	8	60	47	*	60	150	40–50
....	8	100	29	*5½×22	−¾-in.	90	130
4×16	8	40	86	*6×9	8-mesh	20	145
....	8	60	61	6×10	8	20	200
....	8	100	38–45	....	8	40	139
4×20	8	60	75	....	8	100	63
....	8	100	40–55	*6×12	6	10	200
*	35	200	105	....	8	20	238
*4½×20	14	35	170	....	8	40	166
*4¾×18	8	60	110–120	....	8	60	119
5×7	8	20	90	....	8	100	71
5×8	8	100	42	6×14	8	20	276
5×12	8	20	160	....	8	40	193
....	8	40	110	....	8	60	138
....	8	60	80	....	8	100	80
....	8	100	50	*	4	100	90
5×14	8	20	188	*6×20	4	20	122
*	4	28	35	....	8	60	195
....	8	40	130	....	8	100	107–144
*	8	60	90	*	6	150	68
....	8	100	54	*	20	200	122
*	10	100	50–60	*7×10	6	10	600
*5×16	10	35	50	*	8	10	300
....	8	40	150	*	8	14	300
*	3	60	70–80	*	6	20	350
*	8	65	110	....	8	100	85
*	28	65	45	*7×12	20	60	260
....	8	100	59–80	....	8	100	94

* Marks data from actual operations. Other figures generalized from makers' catalogs.

grinding the ultimate crystals of the rock-forming minerals, while much of the work of the ball mill lies in breaking the rock along the parting surfaces between the crystals. Resistance to such parting is much the same in many rocks, hence the resistance of such rocks to ball-mill grinding is similar. But the ultimate mineral crystals differ widely in hardness and toughness and there is consequent variable resistance to grinding in the tube mill. Accurate data on this score, which will form a basis for comparison between plants, are substantially impossible to collect and present, but the possible differences in grinding resistance must be borne in mind in estimating capacity, and final estimates in important cases should always await the test of actual trial.

Size of mill affects directly rather the size of feed that can be handled, size of product, and mill efficiency, than capacity, except that it is true that the larger mill has the larger capacity, from a given size of feed to a given product size, roughly in proportion to the comparative volumes. The diameter of the mill is one of the factors affecting the force of impact of the pebbles and also the superincumbent weight on the revolving pebbles in the grinding mass, hence it affects the work done by these forces. The length of the mill and feed rate determine the duration of the crushing action on any given volume of feed and for a given size of feed and product, length and feed rate vary directly. Table 67 arranged from Chalmers and Williams' catalog represents one school of thought as to the relation between diameter of mill and feed size. Brown (*104 P 206*) states that as a result of careful tests he has concluded that it is a waste of power to use larger than 4-ft. working diameter for −6-mesh feed; that small feed could be best crushed in smaller mills except for inconvenience in repairs; and that 5 ft. inside liners is big enough for −⅛-in. material. Gieser (*97 J 463*) summarizes the literature of American and foreign practice in cyanide plants and concludes that notwithstanding published results to the effect that the smaller-sized mills are the more efficient, especially on finer feeds, the majority of American mills are 5 × 18-ft. and the African mills 5½ × 22-ft., irrespective of the feed size. In the use of tube mills for preparing flotation feed, 6- and 7-ft. mills have been used for 10-mesh feed, on account of the shorter lengths used (10 to 12 ft.) and the high capacities demanded.

Table 67. Relation between tube-mill diameter and size of largest feed particles as recommended by Chalmers and Williams

Size of largest feed particles, mm.	Diameter of mill recommended, inches	
	For 100-mesh product	For 20- to 60-mesh product
1	40	....
1.5	....	40
2	48	....
3	....	48
4	60	....
5	....	60
6	72	....
8	....	72
10	....	84

It is the common experience of all who have worked with tube mills that the bulk of the crushing is done in the head end of the mill and that, in a long mill, very little crushing is done in the last few feet. Fig. 27 shows this in an operating mill and Fig. 34 (*L. C. Pan, Columbia Univ., 1924*) shows the same thing in batch grinding. Table 68 shows that in a Rand tube mill, wet-grinding, crushing of coarse particles proceeds at a rapidly falling rate along the mill and that substantially 80 per cent. is completed in the first half. Production of −90-mesh is likewise performed at the greatest rate near the head end and 74 per cent. of the total sliming is done in the first half, but on the other

hand 20 per cent. of the total is done in the last 4 ft. The reason for the apparent inefficiency of the 12th to 17th foot inclusive is not apparent from the data. When, however, a mill is run open circuit, great length must be provided in order to crush the last of the oversize grains, and the finer the final product the greater the length that must be provided. Even for closed-circuit grinding, in which the classifier or other guard machine sends back the oversize, manufacturers recommend longer mills for finer products. Thus Chalmers and Williams recommend 6.5- to 14-ft. lengths for 20-mesh product; 9- to 18-ft. for 40-mesh; 12- to 20-ft. for 60-mesh; and 16- to 22-ft. for 100-mesh for closed-circuit work and 2 ft. above the maximum lengths recommended above for open-circuit work to the same sizes. Recent experience has shown that, if sufficient classifier capacity is provided to handle a large return tonnage, the shortest mills above listed are long enough for the respective sizes, and since power consumption is proportional to pebble load and hence to length, these are the most efficient.

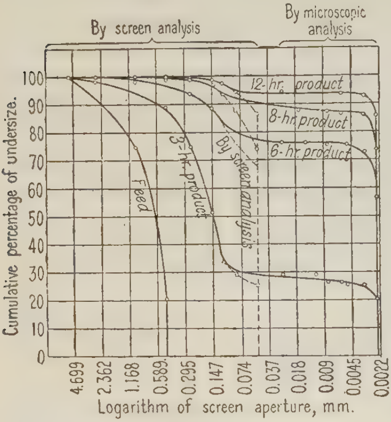


FIG. 34.—Effect of grinding time (length of mill) on product of a tube mill.

Table 68. Rate of grinding in 5½×22-ft. tube mill. (*After Dowling, RMP*)

Location of sample	Sizing test, per cent. weight			Grinding, per cent. of total reduction, effected	
	+60	+90	−90	+60	−90
Feed	73	20	7	0	0
4 ft. from feed end	46	29	25	45.8	36.0
11 ft. from feed end	26	30	44	79.7	74.0
18 ft. from feed end	19	34	47	91.5	80.0
Discharge	14	29	57	100	100

Robbins and Hanson (*109 J 194*) report a test at HOLLINGER in which 6 × 15-ft. tire-and-trunnion mills with 42-in. discharge opening were run on ⅛-in. disk-crusher product in competition with 5 × 20-ft. mills. All mills choked at normal tonnages. When feed was prepared in a ball mill, pebble and power consumption and general maintenance were all higher per ton ground in the 6-ft. mills. The writers attribute this reversal of recent South African experience to the fact that Hollinger ore is soft and friable while the Rand ore is hard and tough and to the fact that an all-slime product was sought while South African practice is to make a 60-mesh product.

Size of feed. Within limits, the amount of fine material produced per mill per unit of time increases with the size of feed. This general trend is shown in Table 69. Table 73 apparently emphasizes this fact but the fine feed there was a sand-table middling containing the hardest fraction of the original feed.

Nevertheless, by no means the whole difference is due to the mineralogical character.

Table 69. Performance of $5\frac{1}{2} \times 22$-ft. tube mills on Rand ore, using 6- to 9-in. cubes of banket for grinding medium

Aperture of battery screen, linear mesh	Tons feed per 24 hr.	−90-mesh in final product, per cent.	Tons −90-mesh per day in final product
2.24	145	79.3	115
2.83	150	85.1	128
3	125	84.6	106
3	138	85.2	118
3	129	84.2	109
4	113	79.1	89
5	146	80.8	118
5.1	126	77.6	98
6.85	107	64.0	69
7.94	132	76.4	101
8	144	85.7	123
8	146	81.4	119
8.06	143	81.0	116
8.12	146	77.3	113
8.5	137	78.4	107
8.5	130	79.2	103
10	145	83.0	120
12	126	75.5	95
12.6	130	81.5	106
12.7	131	75.7	99
15.8	119	76.4	91

In 1910 the duty of Rand tube mills (*71 A 983*) treating stamp product through 15- to 25-(linear) mesh screens was not more than 80 tons of −90-mesh product per 24 hr. In 1919 a duty of 125 to 135 tons of −90-mesh was good current practice, the feed being stamp product through 0.75-in. screens. Total load, including circulating, with this duty was about 400 tons. This later practice involved the discharge and return to the stamp-mill bins of rejected rounded pebbles passing 1-in. circular holes and amounting to 0.5 to 2 per cent. of the total mill feed. Gieser (*97 J 466*) cites a South African mill that was divided for experimental purposes into two sections, one with 60 @ 1500-lb. stamps and 4 tube mills crushing about 16 tons per stamp per day through 0.3-in. and finishing to 65 or 75 per cent. −90-mesh in the 4 tubes; the other with 160 @ 1500-lb. stamps and 4 tubes, crushing 6 tons pe r stamp per day through 30-mesh and finishing in tubes. There was very little difference in the size of the final product in the two sections. The latest practice is the New State Areas plant (*113 J 1038*) in which ore is crushed by jaw and disk crushers to −1-in. and then sent to $6\frac{1}{2} \times 20$-ft. tube mills, driven by 250-hp. motors. Feed rate is 120 to 130 tons per day. Davis, Willey and Ewing experimented with yet coarser feeds and found that by increasing the circulating load, increasing the size and number of apertures in the discharge screen and making a reject-pebble discharge of about 25 tons per mill per day, the duty was increased to 130 to 140 tons of −90-mesh product with $-1\frac{1}{2}$-in. feed. By further raising the feed size to $-1\frac{3}{4}$-in. and rejecting 15 to 20 per cent. of the total feed as coarse pebbles the duty rose to 145 to 150 tons of −90-mesh product and the product contained a high percentage of −200-mesh material. With yet coarser feed (12.6 per cent. on 2-in.), 1.75-in. slots in the discharge screen and screen oversize broken in a set of 36 × 16-in. rolls and returned to the mill, 135 tons of ore and 12.9 tons of pebble rock were fed to a 5 × 22-ft. mill daily and produced 144.32 tons −90-mesh, leaving 2.3 per cent. on 90-mesh. The product contained 62.5 per cent. −200-mesh which was 15 per cent. less than in subsequent products when the mill circuit was closed by a Dorr classifier instead of diaphragm cones. With slightly finer material (2.1 per cent. on 2-in.) the mill feed was 144.2 tons plus 14.2 tons of pebble rock; the product contained 0.2 per cent. on 60-mesh, 4.2 on 90-mesh and 60.4 per cent.−200-mesh, and the duty was 151.48 tons −90-mesh per 24 hr. Increased duty with coarser feeds is ascribed by the above workers to be due to the ease with which the corners are broken and rubbed off irregular pieces, thus readily producing fine material, and to the free rejection of small rounded lumps and their re-cracking to irregularly shaped pieces before re-introduction to the mill. This process,

started in regular operation at the SPRINGS MINES in June, 1923, produced during the first month an average of 147.4 tons −90-mesh material per 5½ × 22-ft. mill in a product containing 2.1 per cent. +90-mesh and 71.4 per cent. −200-mesh. The circuit was closed by a Dorr classifier. It is to be noted that the economic efficiency of this method is probably limited to operations in which ore is used as a grinding medium. Using flint pebbles the pebble consumption would be too high.

Burt and Caetani (*37 A 3*) found that efficiency increased with coarseness of sand fed to the mill, up to 25-mesh. In the coarser range, at the old GOLDFIELD CONSOLIDATED mill in which everything was ground to pass 200-mesh, the most efficient work was done by stage grinding, from 1.5-in. to 4-mesh in 1050-lb. stamps, thence to 16-mesh in Chilean mills, and finishing in 5 × 22-ft. tube mills. (*102 P 616*.)

Size of product. Capacity and grinding efficiency both decrease with decrease in the size of product. Davis, *et al.*, found in working through a relatively narrow size range of product that the percentage increase in total tonnage ground was about twice the percentage decrease in amount of −200-mesh material in the product. Table 65 shows 0.040 ton per hp.-hr. grinding to 100-mesh and finer, 0.073 ton to 48- to 65-mesh and 0.179 ton to 10- to 20-mesh.

Weight of charge. The normal charge is close to 50 per cent. of mill volume. A somewhat greater charge is usually carried in the quick-discharge mills. Smaller charges are carried when the daily tonnage is less than the full mill capacity or when motor capacity is deficient.

Table 70 (*21 IMM 3*) gives the results of three tests with different charges in a 2.83 × 3.5-ft. laboratory mill operated open-circuit. These indicate equal grinding efficiencies

Table 70. Effect of weight of pebble load on performance of tube mill. (*After Ball*)

Load in terms of mill volume	⅜		½		⅝	
Load, weight, lb.	900		1200		1500	
Horsepower consumed	4.9		5.1		6.9	
Horsepower consumed per ton of pebbles	10.9		8.5		9.2	
Tons of new feed per horsepower-hour	0.107		0.103		0.076	
Screen analyses, I.M.M. mesh	Weight, per cent.					
	F	P	F	P	F	P
20	12.0	1.0	11.0	0.8	12.0	1.0
30	21.0	6.0	19.5	5.0	20.0	3.0
50	21.5	17.0	21.0	15.0	22.0	14.0
80	18.5	23.5	19.0	23.0	19.0	22.5
120	9.5	13.5	10.5	14.8	10.0	15.0
200	8.5	13.0	9.2	14.0	8.0	14.0
−200	9.0	26.0	9.8	27.0	9.0	30.5
Tons −80-mesh produced per horsepower-hour	0.027		0.027		0.017	
Tons −200-mesh produced per horsepower-hour	0.018		0.018		0.016	

All tests at 12.6 tons per 24 hr. feed rate, 41 r.p.m. and 38 per cent. moisture.

with ⅜- and ½-loads with falling off at ⅝-load. The ⅝-load produces, however, the greatest tonnage of −200-mesh material. The decrease in grinding efficiency is due to an increase in power consumption which is entirely disproportionate and does not accord with Young's data (*58 A 126*) for the ball mill (p. 402) and is probably wrong, in which case Ball's maximum efficiency would correspond to greatest load. Fox (*28 M&M 537*) working with a 5 × 23-ft. mill, found that power consumption increased with weight of pebble load up to 23,000 lb. or 68.5 per cent. of mill volume and that the best load, on the basis of

tons of desired size produced per horsepower-hour was 18,000 lb. or 53 per cent. of the mill volume. Further increase in pebble load did not increase grinding. Graham (*16 JCM 57*) (see Table 64) shows that a light load of pebbles grinds slightly more per horsepower-hour than a heavy load, but that the heavier load grinds slightly finer at a given rate and size of feed.

An incidental advantage of the high charge is the fact that it insures automatic discharge of small pebbles that, as indicated by their discharge, are more or less floating in the pulp and doing no useful work.

Size of pebbles depends upon the size of the largest particles in the feed. The reasoning as to proper size of balls in ball mills (p. 339) applies here, but it is even more important to reject undersize pebbles than balls on account of their smaller specific weight and their consequent greater loss in grinding effect in a dense pulp with decrease in size. The consensus is that the charge should contain a mixture of sizes, the largest depending on the size of feed.

Chalmers and Williams recommend 2- to 3-in. maximum pebble size for 1/16-in. feed 3- to 4-in. for 1/8-in. feed, 4- to 5-in. for 3/16-in. feed, 5- to 6-in. for 1/4-in. feed and 6-in. for 3/8-in. feed, but the largest sizes are difficult to obtain. Brown (*104 P 206*) says that 2- to 3-in. pebbles are large enough for −6-mesh feed while for coarse feed (−1/3-in.) 5-in. pebbles should be used.

When pieces of mine rock or inferior pebbles are used they should be as large as possible (up to 7- or 8-in. lumps); the chances of obtaining pebbles of crushing size when they break will be greater, they will have a longer crushing life with consequent smaller dilution of the ore, and the consumption (and consequent cost) per ton will be less.

Speed depends primarily on the diameter of the mill. The best speed for a given diameter, according to White (*5 JCM 290*), is given by the equation $N = 215/\sqrt{d}$ where N = r.p.m. and d = internal diameter of the lining in inches. If d is taken as the nominal diameter of the mill in inches the formula agrees fairly well with American practice until recently. Present practice in the best-run mills is much below these figures, especially for fine grinding.

At Nipissing, grinding to 96 to 99 per cent. −200-mesh, the speed of 5 × 20-ft. mills with smooth liners was decreased from 26 to 21 and then to 16 r.p.m. with a total saving as between 26 and 16 r.p.m. of 33 per cent. in power consumption when handling identical tonnages to identical sizes. Speed should be slightly greater with relatively coarse feed and products than with fine since in coarse grinding maximum cascading of pebbles is desired. According to Brown (*105 J 246*) speed should be greater for thick pulp than for thin. Table 71 (*21 IMM 3*) indicates a distinct maximum efficiency in a 2.83 × 3.5-ft. laboratory mill at between 33 and 41 r.p.m., apparently nearer the lower figure, but substantially all of the difference is due to the power minimum at 37 r.p.m. and the accuracy of this measurement may be questioned.

Leupold (*Pro. So. Af. Ass'n Eng'rs, Apr., 1905*) states that at too low or too high speeds liner and pebble wear were excessive and the pebbles became flattened disks unsuitable for crushing.

Liner. (See also p. 426.) For fine feeds and very fine grinding smooth liners give highest capacity.

Careful tests at Nipissing where the endeavor was to make a final product with not over 2 per cent. +200-mesh, showed that highest capacity on −4-mesh feed was obtained with smooth iron lining and 60 per cent. moisture.

For somewhat coarser product a liner of the El Oro type gives greater capacity and the maintenance cost is less. With coarser feed or for relatively high tonnage a lining with lifter bars is essential for the best work. Of these the Komata is probably the most used and the most economical.

Open *vs.* closed-circuit grinding. The principles involved are the same as those discussed under ball milling (p. 391). The kind of device used to close the circuit is of great importance in determining capacity and size of product of a given mill.

Table 71. Effect of speed on performance of tube mill. (*After Ball*)

Speed, r.p.m.	33		37		41		46	
Horsepower consumed	6.8		5.1		6.3		7.5	
Tons of total feed per horsepower-hour	0.038		0.051		0.041		0.034	
Screen analyses, I.M.M. mesh	Weight, per cent.							
	F	P	F	P	F	P	F	P
20	13.5	0.5	13.0	0.5	11.0	0.5	14.0	0.5
30	19.5	2.0	20.0	2.0	19.5	2.0	20.5	2.5
50	21.0	9.0	21.5	8.5	21.0	9.0	21.5	9.5
80	18.5	18.0	18.5	19.5	18.5	20.0	18.0	20.5
120	6.5	15.0	8.5	14.5	7.5	15.5	6.5	14.5
200	11.5	17.0	14.0	16.5	8.5	17.0	10.0	16.5
−200	9.5	38.5	14.5	38.5	14.0	36.0	9.5	36.0
Tons −80-mesh produced per horsepower-hour	0.019		0.024		0.018		0.019	
Tons −200-mesh produced per horsepower-hour	0.013		0.014		0.010		0.011	

Feed rate, all tests, 7.2 tons per 24 hr.; 1200 lb. pebbles; 38 per cent. moisture.

Davis, *et al.*, found that while the capacity of a 5½ × 22-ft. mill grinding to 1 or 2 per cent. on 90-mesh was the same whether the circuit was closed by diaphragm cones or a Dorr classifier, the classifier overflow, representing the final product of the grinding, contained about 15 per cent. more −200-mesh when the Dorr classifier was used. Ball's work (*21 IMM 3*) on open-circuit grinding in a laboratory mill (2.83 × 3.5-ft.) is given in Table 72. From these data it would appear that grinding efficiency, measured in tons of desired

Table 72. Effect of feed rate on performance of a tube mill operating open-circuit. (*After Ball*)

Feed rate, tons per 24 hr.	7.2		9.6		12.6		14.4		18.6		23.0	
Horsepower consumed	6.3		5.0		5.1		5.6		4.8		6.6	
Tons of new feed per horsepower-hour	0.048		0.080		0.103		0.107		0.162		0.145	
Screen analysis I.M.M. mesh	Weight, per cent.											
	F	P	F	P	F	P	F	P	F	P	F	P
20	11.0	0.5	12.0	0.5	11.0	0.8	12.0	1.0	11.0	1.0	13.5	2.0
30	19.5	2.0	20.0	3.0	19.5	5.0	21.0	6.0	19.5	7.5	19.5	9.0
50	21.0	9.0	21.5	12.0	21.0	15.0	22.0	16.5	21.5	18.5	21.0	19.0
80	18.5	20.0	18.5	22.0	19.0	23.0	19.0	23.0	18.0	24.0	17.5	21.5
120	7.5	15.5	9.5	16.0	10.5	14.8	9.5	14.0	10.5	14.5	7.0	10.5
200	8.5	17.0	9.0	16.5	9.2	14.0	8.0	14.0	9.5	13.5	7.5	14.0
−200	14.0	36.0	9.5	30.0	9.8	27.0	8.5	25.5	10.0	21.0	14.0	24.0
Tons −80-mesh produced per horsepower-hour	0.018		0.028		0.027		0.029		0.031		0.029	
Tons −200-mesh produced per horsepower-hour	0.010		0.016		0.018		0.018		0.018		0.015	

All tests at 41 r.p.m., 38 per cent. moisture and 1200-lb. pebble load.

size produced per horsepower-hour, increased to a broad maximum somewhere between 14.4 and 23 tons per 24 hr. for that mill. The appearance of a maximum at 18.6 tons is clearly due to a corresponding minimum in the power curve. Fox (*28 M&M 537*) found that (within limits) power decreased with increase in feed rate and Mishler (*98 J 469*) that change in feed rate from 22 to 85 tons per 24 hr. did not affect power consumption. In the light of these data Ball's power measurements are not sufficiently convincing to justify the acceptance of this particular minimum. It is probable that there is considerable latitude in the allowable feed rate to a tube mill operated in open circuit, through which the tonnage of a desired fine size produced per unit of power remains substantially constant. This conclusion from Ball's data is confirmed by Table 73 which shows a maximum somewhere between 80 and 693 tons of fine feed per 24 hr. The product is progressively coarser as the feed rate increases and the choice of rate depends, therefore, on the character of product wanted. Burt and Caetani (*37 A 3*) found that efficiency increased proportionately to feed rate in open-circuit grinding and that in closed-circuit work it was higher with a large than with a small circulating load. Table 73, representing the results of work at the LUCKY TIGER mill (*98 J 469*) shows a continuing increase in tons ground in closed circuit per unit of power, with an increasingly coarse product, as the total tonnage of a 5 × 14-ft. mill was increased from 22 to 85 per 24 hr. on coarse feed and from 67 to 693 tons on fine feed. At the highest tonnage, although very little grinding was apparently done, judged from screen analysis of feed and product, yet greater tonnages of −100- and −200-mesh material were produced than with the apparently better grinding at the 67-ton rate. When, following these experiments, the original feed to the tube mills was increased from 26 to 50 tons per day per mill, the final classifier overflow remained unchanged at 80 per cent. −200-mesh and the power consumption per ton of initial feed ground was halved, becoming 22 hp.-hr. per ton. Costs of grinding, classifying, concentrating and circulating decreased from $1.88 with light feed (26 tons per 24 hr.) to $0.97 at 41 tons and $0.68 at 50 tons.

Tonnage of circulating load increases rapidly as tonnage of original feed increases, if the product size is held constant.

In Table 73, with 37 tons of original fine feed per 24 hr., the total load was 216 per cent. of the original feed; with 50 tons original feed, 478 per cent; and with 75 tons, 925 per cent. According to Mishler, when the total load at LUCKY TIGER exceeded 400 per cent. of the original feed the cost of elevating and concentrating the circulating material exceeded the saving due to greater mill capacity. When sands are not treated and the classifier acts as an elevator, the most economical total load may easily exceed this figure. Easton (*13 CME 89*) found that the capacity to a given size was greatly increased by closed circuit and furthermore, what has been frequently observed by other operators but is rarely mentioned in the literature, that the capacity of the circuit is frequently determined by the classifier rather than the mill. Using a 6 × 20-ft. quick-discharge mill grinding from −6-mesh to about 8 per cent. + 100-mesh and 70 per cent. −200-mesh for flotation, when the feed rate was 144 tons per 24 hr. there was practically no sand return while an increase in feed rate resulted in sand return, so that 144 tons may be taken as the maximum open-circuit capacity of the mill. With the same classifier, the overflow when the mill was fed 240 tons per 24 hr. contained 15 per cent. + 100-mesh and 56 per cent. −200-mesh, but with a larger classifier the overflow at 240 tons was the same size as at 144 tons. Easton found also that the power draft decreased with increase in tonnage passing through the mill. Thus at 144 tons per day original feed, 40 per cent. moisture and, as above noted, but little sand return, the power draft was 75 kw.; at 180 tons and 38 per cent. moisture, the draft was 70 kw.; at 240 tons and 38 per cent. moisture, 65 kw. and at 350 tons and 40 per cent.

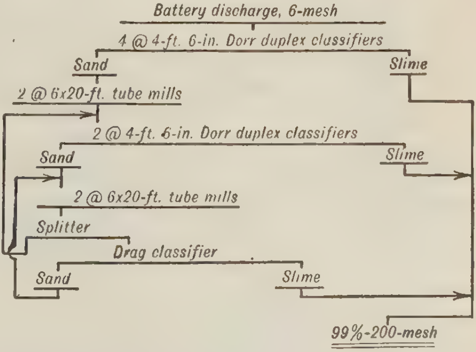


FIG. 35.—Flow-sheet of 2-stage tube milling and classification at Nipissing. (Bowl classifiers later substituted.)

Table 73. Effect of feed rate on performance of 5 × 14-ft. tube mill. (*After Mishler*)

	Coarse feed								Fine feed							
Feed rate, tons per 24 hr., original + circulating	22		47		70		85		67		80		239		693	
Tons original feed per 24 hr.	22		30		37		37		37		37		50		75	
Moisture, per cent	47.6		48.5		45.6		47.3		31.0		30.6		31.4		35.5	
Screen analyses	Weight, per cent.															
Mesh	*F*	*P*	*F*	*P*	*F*	*P*	*F*	*P*	*F*	*P*	*F*	*P*	*F*	*P*	*F*	*P*
20	11.2	0.3	21.6	0.2	18.4	......	22.4	......	0.2	......	0.3	......	0.5	......	1.0	0.2
40	28.0	0.6	39.6	0.6	45.2	0.4	39.4	0.6	2.5	......	3.1	0.4	3.3	1.4	6.8	4.6
60	25.6	1.4	23.0	0.8	19.6	4.0	20.6	5.2	4.4	1.1	7.2	1.8	6.4	4.6	11.5	10.8
80	13.4	0.4	3.8	2.2	5.0	4.2	4.2	6.2	8.6	2.8	12.5	4.2	12.8	9.0	12.8	13.0
100	6.0	1.2	2.4	2.9	2.8	5.4	2.6	6.8	25.8	14.8	28.4	19.0	30.7	27.5	28.0	28.5
150	4.6	2.4	2.2	5.4	1.8	8.4	1.2	9.6	30.2	25.8	27.2	29.3	25.0	27.0	24.9	24.6
200	3.6	2.6	1.0	4.9	1.2	6.2	0.8	4.8	17.9	25.5	12.8	15.1	12.6	14.4	9.6	10.3
− 200	7.6	91.1	6.4	83.0	6.0	71.4	8.8	66.8	10.5	30.0	8.5	30.2	8.7	16.0	5.5	8.1
Tons produced per horse-power-hour (*a*)																
− 100-mesh	0.016		0.035		0.048		0.055		0.014		0.019		0.025		0.018	
− 200-mesh	0.016		0.032		0.041		0.043		0.012		0.016		0.016		0.015	

a Power constant throughout at 47 hp. in coarse-feed tests and at 44.8 hp. in fine-feed tests. *F* = Feed. *P* = Product.

moisture, 55 kw. Thus with increased grinding and decreased power consumption accompanying increase in feed rate, there is marked increase in efficiency and decrease in cost. At NIPISSING (*48 A 16*) six 4-ft. 6-in. duplex Dorr classifiers and one drag classifier were used with four 6 × 20-ft. tube mills in grinding from 6-mesh to 1 per cent. + 200-mesh. The flow-sheet is shown in Fig. 35.

Moisture. The percentage of moisture in the mill feed should be from 30 to 50. This is slightly greater than is best for coarse ball-mill feed, owing to the fact that with the finer material a given percentage of moisture gives thicker pulp and consequently more water must be used to maintain the proper pulp consistency.

Table 74 (*21 IMM 3*) shows an apparent maximum grinding efficiency with fine feed at 38 per cent. moisture, but the change in the range from 30 to 58 per cent. moisture is not

Table 74. Effect of moisture on performance of tube mills in open circuit. (*After Ball*)

Moisture, per cent	30		38		50		58	
Horsepower consumed	5.7		5.1		6.2		6.0	
Tons of new feed per horsepower-hour	0.092		0.103		0.086		0.089	
Screen analyses, I.M.M. mesh	Weight, per cent.							
	F	P	F	P	F	P	F	P
20	12.5	1.0	11.0	0.8	12.5	0.5	11.5	1.0
30	19.0	5.0	19.5	5.0	20.0	4.5	21.5	5.5
50	20.5	14.5	21.0	15.0	21.0	15.0	21.5	15.5
80	18.5	22.5	19.0	23.0	18.5	22.5	18.5	23.5
120	10.5	14.5	10.5	14.8	9.5	15.5	9.5	14.5
200	9.5	14.0	9.2	14.0	9.0	15.0	8.5	14.5
−200	9.5	28.5	9.8	27.0	9.5	27.0	9.0	25.5
Tons −80-mesh produced per horsepower-hour	0.025		0.027		0.025		0.024	
Tons −200-mesh produced per horsepower-hour	0.018		0.018		0.015		0.014	

All runs at 12.6 tons feed per 24 hr.; 1200-lb. charge; 41 r.p.m.; mill 2.83 × 3.5 ft.

great. Smart (*10 JCM 443, 11 JCM 15*) found that the best moisture content varied with the total tons of solid passing through the mill and that 27 per cent. was best with 200 tons per 24 hr. while 39 per cent. was best with 400 tons per 24 hr. Fox (*28 M&M 537*), working with a 5 × 23-ft. silex-lined mill on − 12-mesh feed found that with 39.6 per cent. moisture and grinding to 50 per cent. through 150-mesh the mill took 55 hp. while with 54.6 per cent. moisture, grinding to substantially the same product, 62.3 hp. was consumed. Sherrod (*Mex. IMM, Dec., 1909*) found a maximum capacity between 40 and 45 per cent. moisture with a very fine (7 per cent. +60-mesh) feed.

Ball's figures apparently show that the percentage of fines in the mill discharge decreases with increase in moisture content. This is probably the usual experience but in the copper mills of LAKE SUPERIOR high moisture is used to get fine product and similar experience elsewhere is far from unusual. When the tube mill is of the overflow type, increase in moisture should yield a finer product by analogy with the Dorr classifier, while in a quick-discharge mill high moisture content should cause coarse discharge.

Power consumption depends upon weight of pebble load, diameter of mill, speed, type of lining, and consistency of pulp. For estimates, an average figure of 6 to 7 hp. per ton of pebble load for a 5-ft. center-discharge mill with silex lining, loaded to the axis and grinding a pulp containing 35 to 40

per cent. water, is safe, if an additional allowance of 50 per cent. is made to cover starting overload. For a smaller load in the same mill the power per ton is greater on account of the increased moment arm. Scoop-discharge mills require 25 to 30 per cent. more power. Smooth metal liner will require 5 to 10 per cent. less power, El Oro-type about 5 per cent. more and ribbed liners from 10 to 20 per cent. more. James (*21 IMM 3*) gives 7 hp. per ton of pebbles for Rand mills ($5\frac{1}{2} \times 22$-ft.) with silex lining. The power requirement under like conditions for mills of other diameters can be obtained from the equation $Hp. = 1.5\ DT$ where D = nominal diameter in ft. and T = tons of pebble load. The power requirement for mills of given diameter and loading varies roughly in proportion to the speed, within the limits of speeds ordinarily employed. A thin pulp requires more power than a thick because it allows the relatively dense pebbles to crowd further away from the axis of revolution. Merton (*38 MEW 1129*) recommends as best that speed at which $Hp./r.p.m.$ is a maximum.

Operation. Feed rate, size of feed, and moisture content should be kept constant, the pebble load must be kept up to weight and worn and cracked pebbles rejected from the circuit, if the mill is to operate efficiently. Close classifier regulation is essential as it affects both the allowable feed rate and the product size. The mill should be opened for inspection at sufficiently frequent intervals to insure that the liner does not become dangerously thin, and, in the case of ribbed liners, that the ribs have not worn down so much that their lifting effect is lost. Discharge grates should be kept open. An ammeter is a great aid to steady operation; any decided variation from the normal reading indicating abnormal work of the mill. A high reading ordinarily indicates an increase in moisture content, or, in the case of a surcharged mill, a deficiency of pebbles. With an undercharged mill a high reading indicates more than the normal pebble load or an increase in feed rate. Lubrication must, of course, be regularly attended to. Lack of lubricant will show first on the ammeter reading. Grease is used on trunnion bearings and gears and oil on the pinion shaft. Consumption of grease averages about 1 lb. per shift, oil less than ¼ pint.

Cost of tube milling should average between \$0.08 and \$0.12 per ton in grinding to table size; \$0.15 to \$0.25 to flotation size and \$0.25 to \$0.40 to 150- or 200-mesh. Individual costs may depart considerably from these figures due to local conditions.

At WAIHI, N. Z. (*16 Aa 126*) (see Table 65) the average cost to 60-mesh including labor, supplies, power and overhead in 1912 was \$0.1766 per ton. RAND cost in 1914 grinding to a few per cent. on 60-mesh was about \$0.12 per ton. (*97 J 467.*) At CHURCHILL MILLING Co. (*52 A 129*) the cost (1914) of grinding hard, tough quartz ore, closed-circuit, in a 5 × 22-ft. tube mill from 20-mesh to 95 per cent. − 100-mesh was: labor, \$0.029; supplies, \$0.119; power, \$0.107; total, \$0.255. At BARNES KING DEVELOPMENT Co. (*60 A 104*) the cost per ton ground from − 20-mesh to 1 per cent. on 65-mesh (1917) was: labor, \$0.0500; supplies, \$0.1223; power, \$0.0702; total, \$0.2425. At GOLDFIELD CONSOLIDATED (*89 J 1230* [1910]) the cost per ton ground from − ¼-in. to 200-mesh was \$0.202. At NIPISSING (*31 Ont. Dep. Mines 259*) the average cost for the years 1913 to 1922 incl. for two-stage reduction from − 4-mesh to − 200-mesh, including classification, was: labor, \$0.13; supplies, \$0.22; power, \$0.32; shops, \$0.05; total, \$0.73 per ton. At COBALT REDUCTION Co. (*ibid.*), grinding from − 3-mesh to − 40-mesh (1922) the cost per ton was: operating labor, \$0.023; repair labor, \$0.012; balls, \$0.144; liners, \$0.092; power, \$0.122; total, \$0.393.

Tube mill *vs.* grinding pan. Table 75 shows the result of a competitive test at Homestake (*22 IMM 93*). The tonnage of fine material produced by the pan per hp.-hr. is more than that produced by the tube mill when fed at the regular rate but the tube mill is so superior in low consumption of grinding medium, in capacity per unit, and in water consumption (permitting mate-

rial reduction in dewatering equipment) that its advantages far outweigh its larger power consumption per unit weight crushed.

Table 75. Comparative performances of tube mill and grinding pans at Homestake. (*After Clark & Sharwood*)

	5-ft. pan		5 × 14-ft. tube mill			
			Regular feed		Heavy feed	
Tons of new feed per 24 hr.	19.3		73		110	
Horsepower consumed	8.4		37.5		37.7	
Tons consumed per horsepower-hour	0.096		0.081		0.122	
Moisture, per cent	80–90		38		38.4	
Consumption, pounds per ton	4.23 metal		1.66 pebble		1.30 pebble	
Screen analyses, mesh	Weight, per cent.					
	F	P	F	P	F	P
50	47	6	39	5	18	7
80	34	14	38	12	49	15
100	9	14	12	13	17	14
200	6	26	7	28	11	26
−200	4	40	4	42	5	38
Tons −100-mesh produced per horsepower-hour	0.053		0.048		0.058	
Tons −200-mesh produced per horsepower-hour	0.034		0.031		0.040	

15. Conical pebble mill

Description. The conical pebble mill is similar in construction to the conical ball mill except that it is lighter and usually differently lined. The shell is made of steel plate with butt joints and double-riveted butt straps. Heavy cast-iron flanges are riveted at the end and carefully faced for attachment of the cast-iron trunnions. The usual sizes with weights, power required and capacity, according to the manufacturer's catalog, are given in Table 76.

Table 76. Specifications for conical pebble mills. (*Hardinge Co. catalog*)

Size of mill	Floor space	Approximate weights, pounds			Horsepower to run	Size of motor, horsepower
		Mill	Lining	Charge		
3′ × 8″	5′ × 7′	3,800	1,500	300	3	5
4½′ × 16″	7′ × 10′	6,600	3,500	2,500	8	10
5′ × 22″	8′ × 11′	10,200	4,500	3,500	12	15
6′ × 22″	9′ × 11′	10,000	7,500	4,500	18	25
6′ × 48″	9′ × 13′	12,000	9,000	5,500	27	35
7′ × 30″	10′ × 13′	13,000	10,000	9,000	35	50
7′ × 48″	10′ × 14′	14,000	12,000	10,000	40	50
8′ × 36″	11′ × 15′	16,000	16,000	12,000	55	60
8′ × 48″	11′ × 16′	17,000	20,000	14,000	65	75

Manufacturer. Hardinge Co.
Performance is shown in Tables 77 and 77*a*.

Table 77. Performance of conical pebble mills

Plant	Bunker Hill & Sullivan	Vipond Porcupine	Calumet & Hecla(*w*)	McKinley Darragh(*v*)	Braden(*ak*)	Federal M. & S., Morning
Size, outside diameter in ft. × length of cylinder in in.	6×22	6×72	8×18	8×18*an*	8×18	8×22
Speed, r.p.m.	32	27	26	26	27	29
Tons of new feed per 24 hr.	75	40	45*ap*	56.2	40–45	75
Tons of total feed per 24 hr.						
Method of closing circuit	Open	*ai*	Open(*ar*)			Open
Installed horsepower			40	50		50
Actual horsepower	20	30	26	37-40	34–37	45
Horsepower per ton of pebble load		6.7	5.8		11.3–12.3	6.4
Tons new feed crushed per horsepower-hour	0.156	0.055	0.072	0.058–0.063	0.049–0.055	0.069
Moisture in mill, per cent		50	75		40	50
Size of feed(*a*)	*1*	*20*	*9*	*8*	*21*	*2*
Size of product(*a*)	*1*	*20*	*9*	*8*	*21*	*2*
Attendance, machines per man	*b*		32			6
Lost time, per cent			3.5			1.5
Principal causes of lost time			Re-lining			Re-lining
Lubricant, kind/pounds per shift						***O*/2**
Feeder, type	***SS***		***SS***			***SS***
Feeder, material	***CI***		***CI***			***CI***
Feeder, life, days			1080			365
Liners, type			Smooth			***WB***
Liners, material	***Mn*(*c*)**		Silex	*ao*		***CI***
Liners, life, days			165			300
Liners, consumption, pounds per ton						0.378
Time for re-lining, hr.	12		32*aq*			14
Number of men for re-lining	2		2			3
Grinding medium, material	Jasper		***DF***			***DF***
Grinding charge, new, total weight, lb.		9000	9000		6000	14,000
Grinding charge, size, inches @ weight, lb	4		2½-5			4 @ 7000
Grinding charge, size, inches @ weight, lb						3 @ 4200
Grinding charge, size, inches @ weight, lb						2 @ 2800
Grinding medium, size added to compensate wear, in.						
Grinding medium, method of determining addition			*d*			*e*
Grinding medium, consumption, pounds per ton			3.5–4			3.1

Plant	Miami Copper Co.(*x*)	Miami Copper Co.	Braden Copper Co.	Federal M. & S., Morning	Vieille Montagne Zinc Co.	Granitic zinc ore
Size, outside diameter in ft. × length of cylinder in in.	8×22	8×22	8×22	8×22	8×22	8×30
Speed, r.p.m.	27	27	27	32	27	16
Tons of new feed per 24 hr.	180	101	171	95	120	150
Tons of total feed per 24 hr.						
Method of closing circuit	Open	Open	Open	Open	Open	Open
Installed horsepower						50
Actual horsepower	36	36	53	53	35	47.5
Horsepower per ton of pebble load		7.2	9.6	10.6	11.7	
Tons new feed crushed per horsepower-hour	0.208	0.117	0.134	0.075	0.143	0.131
Moisture in mill, per cent.	62.5	63	38 average	55.1	40	32
Size of feed(*a*)	*10*	*11*	*13*	*16*	*18*	*3*
Size of product(*a*)	*10*	*11*	*13*	*16*	*18ah*	*3*
Attendance, machines per man						10
Lost time, per cent.						
Principal causes of lost time						Re-lining
Lubricant, kind/pounds per shift						*O*/0.7
Feeder, type						*SS*
Feeder, material						
Feeder, life, days						
Liners, type	*aj*		*ad*	Silex		
Liners, material						
Liners, life, days			*ad*	390		
Liners, consumption, pounds per ton						
Time for re-lining, hr.						
Number of men for re-lining						
Grinding medium, material	*DF*		*ae*		*DF*	
Grinding charge, new, total weight, lb.		10,000	11,000*af*	10,000	6000	
Grinding charge, size, inches @ weight, lb.	4					
Grinding charge, size, inches @ weight, lb.						
Grinding charge, size, inches @ weight, lb.						
Grinding medium, size added to compensate wear, in.						
Grinding medium, method of determining addition						
Grinding medium, consumption, pounds per ton			10 average	2		

For explanation of reference letters, see page 466.

Table 77. Performance of conical pebble mills—*Continued*

Plant	Utah Leasing Co. (*l*)	Utah Leasing Co. (*l*)	Braden Copper Co. (*as*)	Old Dominion	Consolidated Arizona Smelting Co.	Arizona Copper Co. (*u*)
Size, outside diameter in ft. × length of cylinder in in.	8×30	8×30*r*	8×30	8×36	8×36	8×36
Speed, r.p.m.	28½	28½	....	30	20	....
Tons of new feed per 24 hr.	250	200	86.4	120	400	217
Tons of total feed per 24 hr.	....	....	....	....	....	....
Method of closing circuit	*DC*	*DC*	*DC*	Open	Open	Open
Installed horsepower	75	75	....	75	75 100	....
Actual horsepower	....	....	52	60	50 60	59.4
Horsepower per ton of pebble load	....	....	9.0	6.7	10 12	....
Tons new feed crushed per horsepower-hour	0.139	0.111	0.069	0.083	0.278–0.333	0.152
Moisture in mill, per cent	42.6	41	32	30	35	55.6
Size of feed (*a*)	*5*	*6*	*22*	*g*	*4*	*7*
Size of product (*a*)	*5m*	*6q*	*22*	*g*	*4*	*7*
Attendance, machines per man	3	....	....	5	2	....
Lost time, per cent	Small	....	....	4	....	....
Principal causes of lost time	Re-lining	....	....	Re-lining	....	....
Lubricant, kind/pounds per shift	*O*/0.7	....	....	....	....	....
Feeder, type	2-way	....	....	3-way	*SS*	....
Feeder, material	*CI*	....	....	*CI*	Stl. plate	....
Feeder, life, days	720	....	....	720	200	....
Liners, type	*n*	....	Silex	*h*	*i*	....
Liners, material	*CI*	....	....	*h*	*i*	....
Liners, life, days	116	....	....	....	*i*	....
Liners, consumption, pounds per ton	....	....	....	....	*i*	*s*
Time for re-lining, hr.	6	....	....	....	*i*	....
Number of men for re-lining	4	....	....	....	*i*	....
Grinding medium, material	*o*	*DF*	*DF*	*DF*	*j*	....
Grinding charge, new, total weight, lb.	8000	18,000	11,500	18,000	10,000	....
Grinding charge, size, inches @ weight, lb.	4–5	....	....	3–6	4–5	....
Grinding charge, size, inches @ weight, lb.	....	....	....	....	....	....
Grinding charge, size, inches @ weight, lb.	....	....	....	....	....	....
Grinding medium, size added to compensate wear, in.	....	....	....	5 or 6	....	....
Grinding medium, method of determining addition	*d*	....	....	*d*	*k*	....
Grinding medium, consumption, pounds per ton	10.4*p*	4.1	....	3.8	40	1.15*t*

Plant	Consolidated Arizona Smelting Co. (*y*)	Braden Copper Co.	Bridgeport Wood Finishing Co.	Tul Mi Chung (*al*)	Braden Copper Co.	Calumet & Hecla
Size, outside diameter in ft. × length of cylinder in in.	8×36	8×36	8×36	8×36	8×48	8×72
Speed, r.p.m.	27*ac*	26	27	26	26	26
Tons of new feed per 24 hr.	120	235	48	120	186	110
Tons of total feed per 24 hr.	....	....	....	....	....	....
Method of closing circuit	*z*	Open	*DC*	*DC*	Open	Open
Installed horsepower	....	....	....	....	....	100
Actual horsepower	74.6*ac*	65.2	50	55	85.9	83
Horsepower per ton of pebble load	....	11.8	12.5	....	15.6	10.4
Tons new feed crushed per horsepower-hour	0.067	0.150	0.040	0.091	0.090	0.055
Moisture in mill, per cent	35	38 average	35	33	38 average	75
Size of feed (*a*)	*12ac*	*14*	*17ag*	*19*	*15*	*9*
Size of product (*a*)	*12ac*	*14*	*17*	*19am*	*15*	*9*
Attendance, machines per man	....	....	....	....	....	12
Lost time, per cent	....	....	....	....	....	3.5
Principal causes of lost time	....	....	....	....	....	Re-lining
Lubricant, kind/pounds per shift	....	....	....	....	....	....
Feeder, type	....	....	....	....	....	*SS*
Feeder, material	....	....	....	....	....	*CI*
Feeder, life, days	....	....	....	....	....	1080
Liners, type	*aa*	*ad*	....	....	*ad*	Rail
Liners, material	....	....	....	....	....	Steel
Liners, life, days	....	*ad*	....	....	*ad*	200–250
Liners, consumption, pounds per ton	....	....	....	....	....	....
Time for re-lining, hr	....	....	....	....	....	32
Number of men for re-lining	....	....	....	....	....	2
Grinding medium, material	*ab*	*ae*	*DF*	....	*ae*	*DF*
Grinding charge, new, total weight, lb.	....	11,000*af*	8000	....	11,000*af*	16,000
Grinding charge, size, inches @ weight, lb	....	....	....	....	....	2½–5
Grinding charge, size, inches @ weight, lb	....	....	....	....	....	....
Grinding charge, size, inches @ weight, lb	....	....	....	....	....	....
Grinding medium, size added to compensate wear, in.	....	....	....	....	....	....
Grinding medium, method of determining addition	....	....	....	....	....	*d*
Grinding medium, consumption, pounds per ton	68.8	10 average	....	2.3	10 average	4

For explanation of reference letters, see page 466.

Table 77*a*. Sizing tests

(Figures under the headings F

Reference numbers			1		2		3		4	
Plant			Bunker Hill & Sullivan		Federal M. & S., Morning		Granitic zinc ore		Cons. Ariz. Sm. Co.	
Screen aperture										
Mesh	In.	Mm.	*F*	*P*	*F*	*P*	*F*	*P*	*F*	*P*
3	0.26	6.68								
4	0.18	4.70								
6	0.13	3.33	1.6		9.1					
8	0.093	2.36	11.7							
10	0.065	1.65	18.0				1.0			
14	0.046	1.17	9.5				2.9		1.3	
16					62.6					
20	0.033	0.83	5.0							
28	0.023	0.59	4.1	0.4			10.6			
30										
35	0.016	0.42	2.2	1.7			12.2	1.8	8.0	3.3
40					21.5					
48	0.012	0.30	3.1	5.6			14.6	5.2		
60										
65	0.008	0.21	3.8	9.2			16.3	9.3	46.4	40.4
80					5.5	7.0	10.5	8.6		
100	0.006	0.15	10.3	15.4	0.5	8.6	7.4	9.1	18.4	20.8
140						15.4				
150	0.004	0.10	3.9	7.7			13.1	19.9	6.4	7.6
200	0.003	0.07	13.3	24.2	0.6	17.0	5.8	9.8	6.1	8.3
240						8.6				
280							2.7	9.8		
300										
Through last screen			13.5	35.8	0.2	43.4	3.0	26.6	11.5	19.6

Reference numbers			12		13		14		15	
Plant			Cons. Ariz. Sm. Co.		Braden Copper Co.		Braden Copper Co.		Braden Copper Co.	
Screen aperture										
Mesh	In.	Mm.	*F*	*P*	*F*	*P*	*F*	*P*	*F*	*P*
3	0.26	6.68					0			
4	0.18	4.70	14.9							
6	0.13	3.33								
8	0.093	2.36	22.6							
10	0.065	1.65			5		46	11	8	
14	0.046	1.17	10.3							
16										
20	0.033	0.83			24	3	33	22	28	
28	0.023	0.59								
30										
35	0.016	0.42	11.7	3.6						
40					31	14	10	19	37	18
48	0.012	0.30								
60					14	17	2	9	11	20
65	0.008	0.21	15.9	14.3						
80					5	10	1	4	3	6
100	0.006	0.15	9.9	14.7	5	8	1	3	3	7
140										
150	0.004	0.10			3	6	1	5	2	5
200	0.003	0.07		26.0	2	5	1	2	1	4
240										
280										
300										
Through last screen			14.7	40.7	11	37	5	25	7	40

F - Feed

referred to in Table 77

and P are weight per cent.)

5 Utah Leasing Co.		6 Utah Leasing Co.		7 Arizona Copper Co.		8 McKinley Darragh		9 Calumet & Hecla		10 Miami Copper Co.		11 Miami Copper Co.	
F	*P*	*F*	*P*	*F*	*P*	*F*	*P*	*F*	*P*	*F*	*P*	*F*	*P*
												10.7	
0.5						5.2				0.4			
						36.0		0.2					
3.5		5		0.5		18.4		2.7					
				26.0	0.1	14.6		5.1		45.0		28.8	
26.0	1.0	13	2	17.8	0.6	8.9							
				14.5	2.1	7.5		25.7		32.7	2.2	37.0	3.4
25.0	4.0	18	2	13.2	8.3		3.5			8.4	6.8	11.4	9.2
9.0	9.0	12	4	8.0	10.9		5.8			1.7	7.0	1.8	7.3
								36.4	2.0				
12.0	26.0	13	10	6.9	14.5		13.7						
								19.4	4.7				
7.0	13.0	11	13	3.7	10.7		12.8			1.8	15.5	1.5	13.9
			7					7.0					
6.0	15.0	8	14	2.7	9.9		14.5	1.4	19.7	1.5	16.6	1.0	15.0
1.0	13.0	5	18	1.9	7.9		10.9	1.4	21.5	0.8	8.5	0.5	7.3
2.0	8.0			0.8	4.2		6.2	0.7	8.3	0.4	5.3	0.7	8.0
8.0	11.0	15	30	4.0	30.8	9.4	32.6	0.2	43.8	7.3	38.1	6.6	35.9

16 Federal M. & S. Co.		17 Bridgeport Wood Fin. Co.		18 Vieille Montagne		19 Tul Mi Chung		20 Vipond Porcupine		21 Braden Copper Co.		22 Braden Copper Co.	
F	*P*	*F*	*P*	*F*	*P*	*F*	*P*	*F*	*P*	*F*	*P*	*F*	*P*
		48.0		25.0		0.1							
				30.0		0.3				0.2			
		19.0		34.0		0.7				2.7			
41.2				7.5		0.9		16.6		5.1			
		10.1				2.7							
45.8					5.0	4.7		27.4		25.7		24.3	
		6.0				9.5							
8.0	0.5												
						13.5	2.3						
3.0	6.0				18.0			27.0		36.4	0.2	45.5	12.0
		3.3				11.5	5.7						
1.3	14.5				17.9			8.9		19.4	2.1	15.3	11.2
						13.5	13.4						
0.3	11.5				15.4			7.0	0.2	7.0	6.6	5.3	8.8
	12.0	2.6			4.4	14.8	25.5	5.0	2.0	1.4	6.0	3.1	9.2
	4.5					10.8	18.2	5.0		1.4	16.9		
0.3	11.0	2.5			25.8	6.8	9.9	2.0	29.1	0.7	26.7		10.4
		3.5	3.2										
0.1	40.0	5.0	96.8	3.0	13.5	10.2	25.0	1.1	68.7	0.2	40.8	6.0	48.4

P = Product.

NOTES TO TABLE 77

a Italic numbers refer to column numbers in Table 77*a*. *b* One man for 6 rolls, 3 pebble mills and 2 drag classifiers. *c* Worn liners from ball mills. *d* Added daily at a rate, determined by consumption, to keep load at a constant average. *e* Added as needed to keep ammeter reading constant. *g* Feed, 87 per cent. +48-mesh; product, 22 per cent. +48-mesh. *h* Ends, silex; cylinder, El Oro, cast iron. *i* One mill has manganoid lining with lifting bars of the Komata type, life 245 days, consumption, 0.722 lb. per ton; re-lining takes 6 men 99 hours: the other, manganese-steel lining without lifting bars, life, 256 days, consumption, 0.46 lb. per ton; re-lining takes 6 men 55 hr. *j* Local hard ore. Costs about $2 per ton extra to pick out but contributes to concentrate and would have to be milled anyway. Saving in pebble cost is $0.15 per ton (*104 J 71*). *k* Hourly according to operator's judgment of grinding. *l Questionnaire and Salt Lake Min. Rev., Nov. 15, 1918.* *m* Classifier overflow contains 1 per cent. on 28-mesh and 34 per cent. −200-mesh. *n* Ribbed in cylinder, smooth in cones. *o* Local quartzite. *p* Average cost (1918), $0.0141 per ton. *q* Classifier overflow is 2 per cent. on 35-mesh and 35 per cent. −150-mesh. *r* Compare with preceding column and also with conical ball mill at the same plant, Table 11. *s* Cost per ton, $0.0062. *t* Cost per ton, $0.0155. *u* Compare with 6 × 9-ft. tube mill at the same plant, Table 65. *v 98 J 564.* Compare tube mill at the same plant, Table 65. *w* Compare 5 × 18-ft. tube mill at the same plant, Table 65. *x* Compare 8-ft. × 22-in. ball mill at Miami, Table 11. *y 13 CME 897.* *z* In closed circuit with trommel and cone. Screen analysis of discharge is of actual discharge and not of final product of closed circuit. *aa* Pebbles set in cement. *ab* Crude ore. *ac* No. 2 mill in the same plant, running at 23 r.p.m. took 65 hp. and crushed a slightly coarser feed slightly finer at the same feed rate. *ad* Silex blocks, 8 months; pebbles set on end in cement, 4 months; local cut blocks, 2 months; Komata and Britannia also used but life not available. *ae* Local pebbles. *af* Average of 22-in., 30-in., 36-in. and 48-in. mills. *ag* Heated and quenched in water before crushing. *ah* Feed end elevated 4 in. to obtain granular product. *ai* Colbath classifier. *aj* Combination of ribbed cast iron and pebbles set in cement. *ak 20 IMM 44.* *al 119 P 808.* *am* Classifier overflow contains 0.6 per cent. +65-mesh and 56 per cent. −200-mesh. *an* Feed end elevated 6½ in. *ao* El Oro and ribbed plate, alternating. *ap* 8-ft. × 72-in. pebble mills at Tamarack plant take 100-hp. motors and have about three-times the capacity of these mills (*117 J 282*). See also 8-ft. × 72-in mill at Calumet and Hecla, this table. *aq* 64 mills set in two rows are all served by a 15-ton crane that can pick up a full mill, carry it to a re-lining floor, dump the load into a spare mill standing there, set the old mill on a standard and set the spare into place on the operating floor, all in one hour. Two extra mills are sufficient for the entire plant (*117 J 282*). *ar* Open circuit necessary because native copper builds up and slimes excessively in closed circuit (*117 J 282*). *as 101 J 316.* *CI* Cast iron. *DC* Dorr classifier. *DF* Danish flints. *Mn* Manganese steel. *O* Oil. *SS* Spiral scoop, one-way. *WB* Wedge bar.

At Inspiration a 10-ft. × 28-in. mill drawing 90 hp. ground 190 tons per day of feed containing 43.7 per cent. on 8-mesh and 26.7 per cent. −48-mesh to 2.3 per cent. on 48-mesh.

Capacity is dependent upon the same factors as in tube mills. (See p. 437 for discussion.) Table 78 gives capacity figures from manufacturer's catalog and from actual practice. These figures are reasonably safe for estimate with average ores. Most of the figures from practice are for open-circuit work and could be exceeded somewhat in closed circuit.

Shape of mill. Increasing the length of the cylindrical section increases capacity, power consumption and fineness of grinding.

At Miami (*115 P 568*) a 6-ft. × 38-in. mill re-ground as much as two 6-ft. × 22-in. mills. At the same plant an 8-ft. × 66-in. mill re-ground 294 tons per 24 hr. from 10- to 28-mesh for 74 hp. while an 8-ft. × 22-in. mill ground 178 tons from 3- to 10-mesh for 47 hp. (see Tables 85 and 86). The longer mill had the harder material to grind. At Calumet and Hecla an 8-ft. × 72-in. mill ground 2½ to 3 times as much as an 8-ft. × 18-in. mill but drew three times as much power. It is probable that about 48 in. is the maximum economical length of cylindrical section for 8-ft. mills and smaller, that fineness should be attained by closing the circuit and capacity by installation of another mill, probably best arranged for stage reduction.

Power consumption ranges from 5 to 7 hp. per ton of pebble load for 6-ft. mills and from 6 to 12 hp. for 8-ft. mills, the higher figures in both cases corresponding to a long cylindrical section and an under-charge of pebbles. Tons crushed per horsepower-hour averages about 0.15 to table size (10- to

20-mesh), 0.10 to a coarse flotation size (28- to 35-mesh) and 0.050 to 100-mesh.

Table 78. Capacity of conical pebble mills

Size, diameter, feet × length of cylinder, inches	From mesh	To mesh	Tons per 24 hr.	Size, diameter, feet × length of cylinder, inches	From mesh	To mesh	Tons per 24 hr.
3×8	0.5-in.	28	8	*	4	35	95
....	0.5-in.	48	6	*	6	65	75
....	8	200	3	*8×30	4	28	250
4½×16	0.5-in.	28	24	*	8	35	150–200
....	0.5-in.	48	16	*8×36	4	10	235
....	8	200	8	*	8	20	217
6×22	0.5-in.	28	32	....	0.5-in.	28	168
*	6	28	75	*	3	35	120
....	0.5-in.	48	40	....	0.5-in.	48	120
....	8	200	24	*	8	65	120
6×48	0.5-in.	28	72	....	8	200	60
....	0.5-in.	48	60	*	0.5-in.	300	48
....	8	200	32	8×48	0.5-in.	28	200
*6×72	6	100	40	*	10	35	186
*8×18	4	28	56	....	0.5-in.	48	144
*	6	35	40–45	....	8	200	72
*8×22	3	20	100–120	*8×72	8	40	110
*	8	20	170–180	*10×28	4	48	190

* From Table 77.

Field of conical pebble mill in ore milling is fairly restricted at present. When first put on the market as a competitor of tube mills and roller mills in grinding for concentration the conical pebble mill was superior and supplanted them. As a slime producer, however, it has never been able to compete successfully with the tube mill except on very soft ores. As a grinder for table or flotation concentration it has been generally superseded by ball or rod mills on account of their greater capacity. Only under certain special circumstances, *e.g.*, in re-grinding very hard ores such as Lake Superior copper conglomerate, where the consumption of balls and metal liners is excessive, or where granular nature of the product is more important than cost of grinding, is the conical pebble mill now the best machine.

16. Conical pebble mill *vs.* other grinders

Conical pebble mill *vs.* Chilean mill. Results of an exhaustive test at Miami Copper Co. are presented by Franke (*47 A 50*). The product desired was sand below 60-mesh for gravity concentration. The conical mill yielded 47 per cent. of such material against 36.2 per cent. from the Chilean and yet did not produce quite so much − 200-mesh slime, which was very difficult to treat. The conical mill was smoother running, required less attendance, used less water and consequently required less de-sliming equipment. Results of the competition are shown in Table 79. A similar situation developed at Moctezuma Copper Co. before the introduction of flotation. The feed to the mills assayed 1.8 per cent. copper and the best size range for gravity concentration was from 20- to 200-mesh. An 8-ft. pebble mill ground 71.5 tons per 24 hr. to 1.8 per cent. on 20-mesh. A 6-ft. Chilean mill ground 83.5 tons to 33 per cent. on 20-mesh. The pebble mill product contained 54.8 tons of the desired product containing 71.8 per cent. of the total copper against 30.3 tons containing but 34.3 per cent. of the total copper from the Chilean. Detailed results are given in Table 80. Results of competitive work in producing flotation feed at Braden (*29 IMM 204*) are

Table 79. Conical pebble mill vs. Chilean mill at Miami Copper Co. (*After Franke*)

	Conical		Chilean	
Size	8-ft. × 22-in.		6-ft., 3-roller (*c*)	
Delays, per cent. possible running time	1.29*a*		2.11*b*	
Tons crushed per horsepower-hour	0.104		0.093	
Maintenance cost, dollars per ton crushed	0.0636		0.0709	
Depreciation, dollars per ton crushed	0.007*d*		0.042*e*	
Screen, mesh	Weight, per cent.			
	Feed	Product	Feed	Product
4	12.9	....	13.9	....
10	47.3	....	47.5	....
20	26.8	0.2	22.9	2.3
30	5.0	3.2	5.2	11.8
40	0.8	4.9	0.9	6.7
60	0.8	13.8	1.0	11.4
80	0.4	10.4	0.5	6.7
100	0.3	8.6	0.4	5.4
150	0.3	8.0	0.5	6.3
200	0.5	10.0	0.7	7.2
−200 sand (*f*)	0.8	10.0	1.2	10.6
−200 slime	4.1	30.9	5.3	31.6

a Relining, 0.71 per cent.; repairs, 0.58 per cent. *b* Changing screens, 0.57 per cent.; repairs, 1.54 per cent. *c* High speed. Screens, 0.037-in. aperture, 240 tons per 24 hr. at 75 hp. Could be increased to 250 to 300 tons but power increased to 88-90 hp. *d* Estimated on basis of 10-yr. life. *e* Actual. *f* Quick settling.

Table 80. Conical pebble mill vs. Chilean mill at Moctezuma Copper Co.

Screen, mesh	Feed	Product: Conical, Per cent. weight	Product: Conical, Gm. copper per 100 gm. total product	Product: Chilean, Per cent. weight	Product: Chilean, Gm. copper per 100 gm. total product
4	38.5	....	....	....	....
8	51.3	....	....	4.1	0.0152
12	7.6	0.1	....	16.2	0.0842
20	1.9	1.7	0.0072	13.1	0.0812
30	0.4	12.5	0.0563	7.6	0.0540
40	....	13.0	0.1014	3.9	0.0308
60	....	18.2	0.2548	5.3	0.0461
80	....	11.1	0.2220	4.4	0.0502
120	....	14.4	0.3384	9.7	0.1901
200	....	7.5	0.1950	5.4	0.1593
Through last screen	0.3	21.5	0.4515	30.3	0.8333

shown in Table 81. The Huntington mill was clearly outclassed by both the Chilean and the conical mill and the performance of the conical mill was better than that of the Chilean,

even if an allowance is made to the Chilean for its coarser feed. As a matter of fact, the conical mill would have done substantially the same work on the Chilean-mill feed as it did on the feed provided it.

Table 81. Comparative performance of conical pebble mills and roller mills at Braden. (*After Broadbridge*)

Mill	Huntington	Chilean	8-ft. × 30-in. conical
Tons of feed per 24 hr.	63.6	81.9	90
Moisture, per cent.	90.2	90	47.6
Costs per ton ground (1913-5:)			
Labor, operating and repair	$0.0919	$0.0402	$0.0133
Supplies	0.1570	0.0574	0.0638
Power (*a*)	0.0346	0.0424	0.0328
Total	0.2835	0.1400	0.1099

Screen	Weight, per cent.					
	F	P	F	P	F	P
5-mm.	....	....	12.0	....	....	....
4-mm.	13.7	....	10.8	....	....	....
10-mesh	37.3	0.7	49.9	....	3.0	....
20-mesh	35.0	4.6	22.5	12.2	34.9	1.2
40-mesh	9.5	31.2	3.9	24.3	42.7	18.4
60-mesh	....	11.5	....	8.9	13.1	18.3
80-mesh	....	6.1	....	5.6	....	10.4
Through last screen	4.5	45.9	0.9	49.0	6.3	51.7

a At $36–$39 per hp.-yr. *F* = Feed. *P* = Product.

Conical pebble mills *vs.* tube mills. Comparing the two mills on the basis of tons crushed per horsepower-hour from the data in Tables 65 and 77, the tube mill grinds more (0.179 against 0.150 ton) open-circuit to table sizes; substantially the same, allowing for differences in the particular sizing tests, to flotation size; and somewhat less (0.040 against 0.050 ton) to cyanide sizes. But the available data on the conical mill in sliming are too few to support the apparent conclusion of superiority in cyanide grinding, and this fact in itself is a strong argument against that conclusion, indicating, as it does, the inability of the conical pebble mill to make a place for itself in this service against the tube mill in the fifteen years that it has contended.

At Anaconda (*59 A 245*) a 10-ft. × 48-in. pebble mill was tested against an 8 × 12-ft. tube mill grinding −2-mm. sand tailing to 48-mesh in closed circuit with Dorr classifiers. The conical mill crushed 163 tons per 24 hr. with a power consumption of 15.9 hp.-hr. per ton and pebble consumption (Danish flints) of 7.2 lb. per ton. The tube mill crushed 177 tons per 24 hr., consumed 17.7 hp.-hr. per ton and 14.0 lb. of Danish and French pebbles. Wiggin says that they considered the mills on a par as to efficiency and cost.

Pebble mills *vs.* ball and rod mills. The use of balls as grinding media instead of pebbles permits a greater weight of charge in the same mill volume, a greater number of crushing units of the same size, each with greater striking force, and a greater area of crushing surface. As a result, the capacity of a given mill is greatly increased by such substitution and the product when grinding to table or flotation size is usually finer. On the other hand, power consumption is increased rather more than in proportion to the increased weight of charge, liner wear is increased, the cost of grinding medium per pound is usually greater, while consumption per ton ground is the same or

greater. Table 82, summarized from Tables 4, 5, 11, 50, 65 and 77, indicates the rod mill superior in point of power consumption to both ball and pebble mills in producing table and coarse flotation feed (with the limitation as to size of mill feed discussed in Art. 12). Pebble mills, on the other hand, are superior or equal to ball mills in power economy in the same range, subject, however, to a similar limitation in feed size. Table 83 shows equal power con-

Table 82. Comparative power consumption of cylinder mills

Mill	Horsepower per ton of grinding medium	Tons ground per horsepower-hour		
		To table size, 10-20-mesh	To flotation size, 35-65-mesh	To slime, −100-mesh
Cylindrical ball mill, center discharge	14.1 aver.	0.089	....	....
Cylindrical ball mill, peripheral discharge	13.8 aver.	0.143	0.083	....
Conical ball mill	10.2 aver.	0.163	0.087	....
Rod mill	4.9–8.5	0.35*a*	0.18*a*	....
Tube mill	6–8	0.179	0.073	0.040
Conical pebble mill	5–12	0.150	0.100*a*	0.050

a These products are coarser than the others in the same column.

Table 83. Comparative performances of balls and pebbles in conical mills in open circuit at Timber Butte

	Ball mill	Pebble mill
Size, diam. ft. × length cyl., in.	6 × 56*a*	8 × 30
Speed, r.p.m.	24½	24½
Tons of new feed per 24 hr.	228	119
Horsepower consumed	105	55
Tons per horsepower-hour	0.090	0.090
Moisture, per cent.	54	65
Grinding charge, material	Manganoid	Basalt
Weight and size	21,000*b*	10,000
Consumption, pounds per ton	2.75	2.89
Liner, material	Cast iron	Hard iron
Liner, life	*c*	4½ mo.

Screen aperture, Tyler mesh	Weight, per cent.			
	Feed	Product	Feed	Product
10	5.2	....	1.2	....
14	14.8	0.8	3.2	....
20	16.2	1.0	4.4	0.6
28	16.2	1.8	7.2	0.6
35	13.6	2.4	10.0	1.2
48	9.8	4.2	13.8	2.8
65	5.8	5.6	15.6	6.0
100	3.8	8.0	13.0	10.8
150	2.0	9.0	13.2	16.8
200	0.6	8.6	4.6	9.4
−200	12.0	58.6	13.6	51.8

a 8-ft. × 30-in. pebble mill lagged down. *b* 43 per cent. @ 1½-in., 31 per cent. @ 1¼-in., 26 per cent. @ 1-in. *c* Cylindrical portion, 3½ mo.; conical portion, 7½ mo.

sumption as between balls and pebbles per ton ground but finer product from coarser feed and greater capacity for the ball mill. Ball and ball-mill liner wear in this case cost at least 4 times as much as pebble and pebble-mill liner consumption.

At COPPER RANGE an 8-ft. × 30-in. mill carrying 10,000 lb. of pebbles ground 65 tons per 24 hr. Change to 9000 lb. of balls raised the capacity to 150 tons without substantial change in screen test of product. At MIAMI COPPER CO. change in load of an 8-ft. × 22-in. conical mill from 8000 lb. of pebbles to 15,000 lb. of balls, open-circuit grinding, increased capacity from 178 to 300 tons per 24 hr. and produced a slightly finer product at a slightly lower rate per horsepower-hour. Pebble consumption was 1.14 lb. per ton and consumption of 2-in. manganoid balls 1.21 lb. per ton (see Table 84.) Closing the circuit on the ball

Table 84. Comparison of 8-ft. × 22-in. conical ball and pebble mills at Miami Copper Co.

Mill	Pebble		Ball		Ball	
Circuit	Open		Open		Closed	
Tons of original feed per 24 hr.	178		300		298	
Moisture, per cent	48.7		46.9		34.5	
Speed, r.p.m.	27.5		22.0		22.0	
Charge, lb.	8000		15,000		15,000	
Power consumed, horsepower	46.8		87		85.7	
Tons ground per horsepower-hour	0.159		0.144		0.145	
Tons produced per horsepower-hour:						
−20-mesh	0.112		0.107		0.116	
−48-mesh	0.081		0.086		0.101	
−100-mesh	0.055		0.061		0.072	
Screen, Tyler mesh	Weight, per cent.					
	F	P	F	P	F	P
3	1.7	....	1.7	....	2.3	....
4	4.3	....	6.4	....	4.8	....
6	15.8	....	18.9	....	15.4	....
8	18.4	....	19.0	....	18.7	....
10	15.6	0.5	14.8	0.4	18.3	....
14	12.0	1.7	10.5	1.4	12.0	....
20	9.3	4.3	8.2	3.0	8.7	0.1
28	6.6	9.5	5.8	6.6	5.6	0.8
35	2.6	9.7	2.3	7.6	2.1	5.1
48	1.9	11.5	1.7	10.4	1.7	13.9
65	1.1	8.3	1.0	8.1	1.0	10.7
100	1.3	10.1	1.1	11.2	1.0	11.8
150	1.1	8.0	1.0	9.4	0.9	10.6
200	0.7	4.1	0.6	5.2	0.5	5.1
−200	7.6	32.3	7.0	36.7	7.0	41.9

mill with a Dorr classifier gave a yet finer product at substantially the same power consumption. Comparative costs are given in Table 85. Power for ball milling cost 8.2 per cent. more than for pebble-mill grinding; grinding medium, 251 per cent. more and lining 152 per cent. more; labor cost was about the same for both. At the same plant, changing the grinding load in an 8-ft. × 66-in. conical mill from pebbles to balls increased average capacity from 250 to 330 tons per 24 hr. but doubled the per-ton power consumption and more than doubled the cost per ton for re-grinding (see Tables 86 and 87). On the other hand, the product re-ground in the ball mill contained 97.6 per cent. −65-mesh material while that from the pebble mill contained only 48.2 per cent. and was unsuitable for flotation feed. Further, the capacity of a mill section with a ball re-grinding mill was 822 tons per 24 hr. against 714 tons with a pebble mill. Increased recovery and tonnage on the ball-mill

Table 85. Comparative costs of 8-ft. × 22-in. ball- and pebble-mill operation
MIAMI COPPER Co., Oct., 1915 to Mar., 1916, incl.[a]

Mill	Ball	Pebble
Total tonnage ground	371,754	167,532
Tons per mill per 24 hr.	283.6	174.5
Delays, per cent. total time	3.0	1.4
Tons of crude ore per man-shift operating	1061	933
Tons of crude ore per man-shift, repairs	619	736
Tons of crude ore per man-shift	391	396
Horsepower per mill	81.9	46.7
Horsepower-hour per ton ground	0.144	0.156
Ball or pebble consumption, pounds per ton ground	1.510	1.428[b]
Cast-iron lining consumption, pounds per ton ground	0.454	0.015
Total cost per ton ground	$0.1591	$0.1036

a 8-ft. × 22-in. conical mills. Ball mills underloaded on account of insufficient motor equipment. *b* Includes pebbles used in lining.

Table 86. Comparison of 8-ft. × 66-in. conical ball and pebble mills at Miami Copper Co.

Mill	Pebble	Ball	Ball
Circuit	Open	Open	Closed
Tons of feed per 24 hr.	294	324	333
Moisture, per cent.	41.1	38.9	27.9
Speed, r.p.m.	28	20.5	20.5
Charge, lb.	15,000	29,000	29,000
Power consumed, horsepower	73.7	160	162
Tons ground per horsepower-hour	0.166	0.084	0.086
Tons produced per horsepower-hour:			
−48-mesh	0.065	0.061	0.063
−100-mesh	0.046	0.058	0.061

Screen, Tyler mesh	Weight, per cent.					
	F	P	F	P	F	P
10	1.0	....	1.5	....	1.9	....
14	2.3	0.1	3.5	....	5.1	....
20	6.0	0.7	7.9	0.2	8.9	....
28	18.3	4.5	23.6	1.1	19.6	....
35	24.7	11.6	23.9	2.4	20.2	....
48	23.6	20.0	21.5	6.0	18.8	0.5
65	11.6	15.0	9.8	6.9	7.8	1.9
100	8.4	16.1	5.7	11.5	8.9	17.3
150	2.8	9.1	1.5	11.9	2.7	16.7
200	0.4	4.1	0.2	7.9	1.4	8.2
−200	0.9	18.9	0.9	52.1	4.7	55.4

section more than compensated for increased re-grinding costs. CALUMET AND HECLA is the only large plant in this country that has continued to use pebble mills in re-grinding for concentration. Their stand is clearly justified by Benedict's statement (*117 J 282*) that on the hard conglomerate ore pebble wear is 4 to 5 lb. per ton and ball wear almost as great; pebbles cost $0.0075 per lb. and balls $0.035 per lb.; and the difference in cost on this score alone is $0.10 per ton in favor of pebbles. At WRIGHT-HARGREAVES (*115 J 884*) there was an actual saving in power attendant upon a change from pebbles to balls. Two 5 × 16-ft. cylindrical pebble mills with El Oro liners ground ball-mill product to 60 per cent. −200-mesh at the rate of 125 to 150 tons per 24 hr. and consumed 70 to 75 hp. (= 0.045 to 0.050 tons −200-mesh per hp.-hr.) Change to ball loads (20,000 lb. each mill) increased the daily

capacity to 200 to 260 tons containing 80 to 85 per cent. −200-mesh and power consumption at 28 r.p.m. was 90 to 100 hp. (= 0.078 to 0.087 ton −200-mesh per hp.-hr.)

Table 87. Comparative costs of 8-ft. × 66-in. ball- and pebble-mill operation at Miami Copper Co., Oct., 1915 to Mar., 1916, incl.

Mill	Ball	Pebble
Total tonnage crude ore	147,709	639,770
Tons of actual feed per mill per 24 hr	330	250
Delays, per cent. possible time	2.0	2.8
Tons of crude ore per man-shift, operating	1644	1424
Tons of crude ore per man-shift, repairs	4804	4162
Tons of crude ore per man-shift	1225	1061
Horsepower per mill	164.5	73.3
Horsepower per ton ground	0.084	0.142
Ball or pebble consumption, pounds per ton ground	2.526	1.229
Cast-iron lining consumption, pounds per ton ground	0.320	0.240
Total cost per ton ground	0.2308	0.1037

17. Chilean mill

The Chilean mill developed from the ancient arrastre (Art. 20). The first step was a single stone roller running on a stone pavement. The modern counterpart of this step is the slow-speed mill. In the years 1900 to 1910 the high-speed form, representing the highest development, had considerable vogue in gravity-concentrating mills, but ball and pebble mills have now almost completely superseded it in concentrating practice. Slow-speed mills have been put forth by a few engineers as a one-step machine from rock breakers to amalgamating table in competition with stamps alone or stamps plus tube mills, but they have been adopted only in a few small plants or in backward countries where substantially fool-proof operation is essential. Repairs are frequent and slow to make and the parts are heavy and difficult of access. Lennox *(63 A 521)* states the now generally accepted conclusion, *viz.*: that the Chilean mill is not a fine grinder and hence not a true competitor of ball and pebble mills.

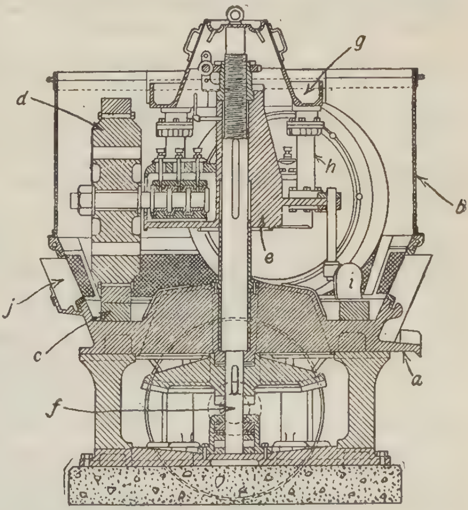


FIG. 36.—High-speed Chilean mill.

High-speed mill. A typical high-speed mill is shown in Fig. 36. The essential parts are a circular cast-iron pan (*a*) with screen and sheet-iron wall (*b*), a die ring (*c*) and three steel-shod circular rollers (*d*). The method of mounting and driving the rollers differs in different makes. In the figure, the rollers are rigidly mounted on axles carrying thrust bearings trunnion-

mounted in an adjustable yoke (*e*) driven by feathers from spindle (*f*). The vertical position of the yoke on the spindle is adjustable by means of nuts on the threaded upper portion. This permits lowering the axles as the tires and die wear and also, by inclining the axles downward toward the spindle, utilizes, in crushing, a certain proportion of the centrifugal force developed. The feed stream, introduced into the annular box (*g*) is led by pipes (*h*) to a point just ahead of the advancing rollers. Scrapers (*i*), following the rollers, churn up the material on the dies and help to keep the fines in suspension. Travel of the rollers causes a circular swash of pulp against the screen through which ground material discharges into the annular launder (*j*) and is led away. Each roller is free to lift independently when an uncrushable lump is encountered. In another form the thrust bearings are rigid in the yoke and the latter is carried on a spherical bearing on the spindle. The whole table tilts when one roller passes over a large particle. Both types of mill are made, infrequently, with overdrive and also with fixed axles and the journals in the roller hubs. Over-drive is clumsy and less rigid than under-drive. Hub journals are difficult to lubricate and protect against grit.

Sizes. High-speed mills are made in sizes from 2- to 8-ft. diameter, weighing 6000 to 125,000 lb. respectively. The commoner sizes are the 5- and 6-ft. weighing respectively about 30,000 and 50,000 lb.

Low-speed mill. The modern form of low-speed mill has two to four rollers running on a die ring 6- to 10-ft. diameter. The rollers are made very heavy, or, in one form (LANE MILL) are on axles whose bearings are attached rigidly to the yoke while the latter carries a large tank that may be loaded with several tons of ore or pig iron in order to add to the crushing load. The SIZE OF PRODUCT is determined by the height of the overflow lip and moisture content, no screen being used.

Mantey offset. In some slow-speed mills the center line of the roller shaft is offset slightly behind the center line of the spindle. This accentuates slipping of the roller face on the die surface and increases abrasive action.

Drive. Mills should be fitted with an oversize clutch on account of the great inertia of the heavy revolving parts and the consequent heavy power draft at starting.

Performance of high-speed mills in American concentrating practice is given in Table 88.

At the GAIKA PLANT (*20 IMM 161*), Rhodesia, a 5-ft. Akron (high-speed) mill with 30-mesh screen was run at 43 r.p.m. The feed was all through 1.5-in., trommel and battery screen, 10.4 per cent. on 1.25-in.; product, 1.5 to 3.5 per cent. on 40-mesh, 38 to 49 per cent. − 150-mesh according to whether the dies were new or worn (new dies did finer crushing but at lower capacity). Power consumed, 25 hp.; consumption of chrome-steel tires and die, 0.646 lb. per ton, aver. life 53 days. At GREAT GOLD BELT mine, Amboy, Calif., (*92 J 305*) a Lane 4-roller slow-speed mill with 7-ft. (O.D.) die-ring, running at 10 r.p.m. with 4-in. discharge height, crushed − 1-in. feed (hard, close-grained quartz-hematite) at the rate of 20 tons per 24 hr. to substantially 40-mesh, with 86 per cent. through 100-mesh The weight tank contained 4½ tons of rock. The size of product was determined entirely by height of discharge and moisture content. Inside amalgamation was practiced. This mill was installed to answer the demand for a low-cost unit of small capacity and low water and power consumption. At the SANTA ELENA plant, Guanajuato, Mex. (*90 J 967*) a slow-speed mill with Mantey offset crushed 27 tons per 24 hr. of − 2-in. average quartz ore through 40-mesh wire screen. The product contained 1 per cent. on 60-mesh and 67 per cent. − 200-mesh. Speed was 15 r.p.m.; moisture 89 to 91 per cent.; horsepower, 15 (0.075 ton per hp.-hr.). Ton-cap screen of the same aperture gave notably higher capacity than square-mesh. Dies and tires lasted 18 mo. COST was $0.235 per ton, of which power was 62 per cent. and tires and dies 28 per cent. At HACIENDA DE LA UNION (*86 J 989*) a slow-speed 2-roller mill with overflow discharge 23 in. high ground quartz ore from − 1¼-in. to 0.4 per cent. + 80-mesh and 80 per cent. − 200-mesh at the rate of 15 tons per 24 hr.

The wheels were 8-ft. diam., 16-in. face, weighed about 8 tons and were mounted with Mantey offset. The die ring was about 4-ft. diameter. Speed was 10 r.p.m.; power consumption, 10 hp. Cost was about $0.35 per ton. Bayldon (*20 IMM 125*) describes 7- to 10½-ft. 2-runner slow-speed mills used in Russia for crushing and amalgamating relatively soft gold ores. Taking – 3- or 4-in. breaker product, these mills crushed from 16 to 26 tons per 24 hr. through 0.5- to 1-mm. screens with a consumption of 7 to 12 hp. Speed was 11 to 14 r.p.m. The product contained less than 1 per cent. on 30-mesh and 63 to 88 per cent. – 200-mesh. Tons per horsepower-hour ranged from 0.065 to 0.095. Steel consumption averaged 1.2 lb. per ton. In an improved type of mill (8-ft. 6-in. mean diameter of die ring) with 2 runners weighing 5 tons each and making 14 to 16 r.p.m., 3-in. crusher product was ground to 30-mesh (70 to 80 per cent. – 200-mesh) at a rate of 40 tons per mill per 24 hr. or about 0.1 ton per hp.-hr. Weight of mill was 23 tons. At La Union (*93 J 393*) a Pachuca-type mill ground from 1½-in. to 80 per cent. – 200-mesh at one pass at the rate of 0.071 ton per hp.-hr.

The low-speed mill is a competitor of the modern high-speed mill and of stamps plus tube mills. Compared with the former (see Table 88), the tons per horsepower-hour to 30-mesh is low, but the feed is much coarser than any sent to the high-speed mills. Bayldon states that the capacity of one of his mills was increased 63 per cent. by a speed increase of 14 per cent. while the power increase was less than 62 per cent. Maintenance charges are, however, greatly increased by increase in speed and the fool-proof character is largely lost. Stamps plus tube mills crush through the same range at the rate of about 0.05 ton per horsepower-hour. On this basis alone the Chilean mill is superior, but except in small plants, the number of units required is so great and lost time so large that labor and maintenance charges outweigh the apparent power saving.

Speed of mill affects size of product, capacity and power consumption. High speed makes coarser products and consumes more power, if the feed rate is the same. If the product is held to the same screen test the mill with higher speed has the greater capacity. Thus at the Portland plant (*63 A 523*) a 6-ft. mill at 37 r.p.m. ground 282 tons coarse feed per 24 hr. through 6-mesh screen (see Table 89). The same mill at 29 r.p.m., grinding to an almost identical screen analysis, handled 186 tons per 24 hr. only and consumed 93 hp.

Size of feed. Capacity through a given screen is considerably greater with feed of 0.75- to 1.5-in. maximum size than with, say, 3-mesh feed, (*33 CMJ 832*) the apparent explanation being that the coarse particles break up the swirl and cause sand to be deposited on the tire for grinding. At Portland comparative tests on 2-in. and 10-mesh feeds with 18-mesh screen on the mill (Table 90) showed more tons produced per horsepower-hour at every screen size with coarse than with fine feed, the discrepancy growing less the finer the screen taken. It was found impossible to raise the power draft with fine feed to more than 80 per cent. of that with coarse feed.

Size of product is determined by the screen aperture, height of screen above die ring, speed, moisture content, and whether the mill is operated in closed- or open-circuit. Finer screens produce, of course, finer products (see Table 89), but both tons per machine and tons per horsepower-hour are less with fine screens. On the other hand, the amounts of the finer sizes produced per horsepower-hour may increase with decrease in screen size, *e.g.*, the tons of – 20-mesh product is greater in Table 89 with 18-mesh screen than with 6-mesh screen. The tonnage of the finest sizes (65- to 200-mesh) per horsepower-hour is not greatly affected by change in mill screen. For the finest grinding, slow-speed mills without screens but with high discharge and relatively high moisture content are used. Running in closed-circuit with a classifier makes finer product with a given screen than open-circuit grinding (see Table 91).

Open *vs.* closed-circuit. Table 91 shows the results of comparative tests at the Portland mill. Closed-circuit and open-circuit work produce substantially equal tonnages of – 28-mesh material per horsepower-hour but closed-circuit work produces more – 200-mesh. Further tests with 6-mesh screens on the mill showed that with an original feed of 251 tons and sand return of 217 tons per 24 hr. (46.3 per cent.) the number of tons per horsepower-hour of coarse feed (see Table 90) ground was 0.089, tons – 65-mesh product per horsepower-hour was 0.058 and tons – 200-mesh, 0.035. With 198 tons original feed and 310 tons return (61.0 per cent.) the corresponding tonnages were 0.078, 0.056 and 0.035. This shows economy in the smaller return.

Table 88. Performance

Mill	Porphyry copper	Phelps-Dodge, Morenci	Phelps-Dodge, Moctezuma
Size, nominal diameter of die ring, ft.	6	5*f*	6*f*
Speed, r.p.m.	32	40	35
Tons of new feed per 24 hr.	200	150	225
Tons of total feed per 24 hr.	300	...	300
Moisture, per cent	60	72	65–75
Power installed, hp.	...	...	...
Power consumed, hp.	30–35	35	45
Tons crushed per horsepower-hour	0.238–0.278	0.170	0.208
Attendance, machines per man	8	...	4
Lost time, per cent	1.5	...	...
Principal causes of lost time	*a*	*a, c, d*	*c*
Oil consumption, pounds per shift	2	...	2
Screens, type	***TC***	Plate	***RT***
Screens, material	Steel	$\frac{1}{16}$-in. steel	Steel
Screens, aperture	0.0325-in.	0.059-in.	0.080-in.
Screens, life, days	6.5	1	15
Dies, material	*b*	***CS***	***Mn***
Dies, weight new, lb.	2990	1722	3000
Dies, life, days	159	65	170–180
Dies, consumption, pounds per ton	0.048	*e*	0.075
Roller tires, material	*b*	***RS***	***Mn***
Roller tires, weight of each new, lb.	1550	842	1530
Roller tires, life, days	167	65	140
Roller tires, consumption, pounds per ton	0.075	*e*	0.133

Screens			Sizing					
Mesh	In.	Mm.	F	P	F	P	F	P
...	3	...	...	...	...	...		
...	2	...	...	...	...	...		
...	1.5	...	...	...	...	...		
...	1.05	26.67	...	...	...	...		
...	0.74	18.85	...	...	...	...		
...	0.52	13.33	...	...	...	...		
...	0.37	9.42	...	...	...	...		
3	0.26	6.68	...	...	...	...		
4	0.18	4.70	...	...	1.3	...		
6	0.13	3.32	...	...	11.2	...		
8	0.093	2.36	...	...	12.6	0.1		
10	0.065	1.65	3.1	...	11.7	0.5		
14	0.046	1.17	6.5	...	9.5	3.0		
20	0.033	0.83	12.1	2.9	8.5	4.2	−7-mm.	−2-mm.
28	0.023	0.59	25.2	13.9	6.7	5.2		
35	0.016	0.42	13.0	11.3	5.1	6.1		
40	...	...	...	...	...	...		
48	0.012	0.30	5.3	6.9	3.9	6.7		
60	...	...	...	...	...	...		
65	0.008	0.21	3.5	4.6	3.4	7.2		
80	...	...	...	...	...	...		
100	0.006	0.15	3.8	6.4	2.7	6.5		
120	...	...	...	...	...	...		
150	0.004	0.10	1.8	3.4	3.0	7.8		
200	0.003	0.07	1.4	3.6	1.1	2.5		
Through last screen			24.3	46.9	19.4	50.0		

a Changing screens. *b* Latrobe brand steel. *c* Changing tires and dies. *d* Oiling. *e* Tires plus dies: consumed, 0.3549; scrap, 0.0662; total, 0.4211. *f* Replaced by rod mills. *g* Crucible steel. *h* Mechanical troubles. *i* Hardened steel. *j* *52 A 129.* *k* 0.50 lb. per ton. *l* *60 A 98.* *m* 0.41 lb. per ton. *n* *63 A 511.* *o* Slot 0.130 × 0.438 in.

of Chilean mills

Chino Consolidated Copper Co.	Calumet & Hecla	Churchill Milling Co. (j)	Barnes-King Development Co. (l)	Portland (n)	Le Roi No. 2 (p)
6	6	6	10	6	6
32	30	30	8–10	37	
225	40	133	65	275–300	50
300					
60–70	75		84	75	75
				100	
50–75	35			113	25
0.125–0.187	0.048			0.101–0.111	
7½	9				
2	@ 10			3.5	
a, c, d	*h*			*a, c, d*	
6¼					
TC	Plate	*TC* No. 693			Plate
g	*i*				Steel
0.0325-in.	20-mesh			6-mesh (*o*)	0.05-in.
4.1	35			2–3	*q*
g	*HCS*	*Cr*			*Cr*
2875	2300				
135	160				
0.09	0.4	*k*	*m*	0.284	*r*
g	*HCS*	*Cr*			*Cr*
1500	1300				
148	160				
0.14	0.6	*k*	*m*	0.406	*r*

tests

F	P	F	P	F	P	F	P	F	P	F	P
						0.4					
						2.9		12.2			
						7.5		14.0			
						17.7		15.8		0	
								12.8			
								12.0			
				0							
						43.3					
								17.2			
								5.5			
7.2	0.1							3.0	2.2		0
12.4	1.3							0.9	5.6		
14.4	7.1	−3/16-in.	−20-mesh	46.9	9.7	22.9	0	0.9	11.7		1.8
17.2	11.5							0.9	8.5		
14.0	11.1							0.7	7.5		
				14.5	16.2						5.9
5.6	7.7					2.5	14.0	0.7	7.4		
				8.7	14.7						3.0
3.6	7.7						11.4	0.5	8.3		
											6.0
3.0	7.6			7.5	12.8		13.8	0.6	8.3		
											12.0
2.4	6.7			5.0	9.4		10.2	0.4	6.1		
1.2	3.7			1.5	6.0		7.5	0.4	6.2		
19.0	35.5			15.9	31.2	2.8	43.1	1.6	28.2	100	71.3

p 114 *J* 1119. *q* Cost \$0.011 per ton. *r* 1.64 lb. per ton total; cost, \$0.274 per ton. *Cr* Chrome steel. *CS* Cast steel. *HCS* High-carbon steel. *Mn* Manganese steel. *RS* Rolled steel. *RT* Rek-tang. *TC* Ton-cap. F = Feed. P = Product.

Moisture. Increase in moisture increases capacity but causes discharge of a coarser product. Comparative tests at Portland mill are shown in Table 92. Tons −65-mesh and −200-mesh, product per horsepower-hour are substantially the same with the two moistures tested. The higher moisture with corresponding higher tonnage is indicated as the better practice, if the coarser product is satisfactory metallurgically or if the following re-grinding machines reduce sands as cheaply as the Chilean mill. Lennox' conclusion from mill operation was that 75 per cent. moisture is best; usual practice, according to Table 88, lies between 60 and 75 per cent.

Table 89. Effect of screen aperture on performance of Chilean mill. (*After Lennox*)

Screen aperture, mesh	6	18	30
Tons of feed per horsepower-hour	282	256	175
Speed, r.p.m.	37	37	37
Moisture, per cent.	75	75	75
Power consumption, hp.	112.8	114.2	100.0

Screen mesh (*a*)	Weight, per cent.		
	Product (*a*)	Product (*a*)	Product (*a*)
10	2.2	....	....
14	5.6	0.8	....
20	11.7	5.1	1.1
28	8.5	9.1	5.1
35	7.5	9.8	9.0
48	7.4	10.2	11.8
65	8.3	10.0	10.0
100	8.3	10.2	10.9
150	6.1	6.3	8.1
200	6.2	7.5	8.5
−200	28.2	31.0	35.5
Tons of feed per horsepower-hour	0.108	0.093	0.073
Tons −20-mesh produced per horsepower-hour	0.078	0.083	0.068
Tons −65-mesh produced per horsepower-hour	0.048	0.048	0.044
Tons −200-mesh produced per horsepower-hour	0.028	0.028	0.025

a For feed analysis, see Coarse feed, Table 90.

Power consumption with a given feed increases with the feed rate, all other things remaining constant. Lennox found that with a feed rate of 256 tons per 24 hr. and power draft of 114.2 hp. the tons −20-mesh product per horsepower-hour was 0.083 and the −200-mesh, 0.028 (Table 90). Reducing the power draft to 84.1 hp., by reducing the feed rate to 171 tons reduced the production of −20-mesh material to 0.075 ton per hp.-hr. while −200-mesh, remained the same. With softer ores, tons per horsepower-hour through 6-mesh screens averages about 0.2. At Miami (*115 P 568*) two 6-ft. Chilean mills handled 480 tons per 24 hr. with 150 hp. By increasing the feed rate to 550 to 600 tons, power increased to 175 hp. or over and the product was markedly coarser.

Wear of tires and dies when crushing relatively soft copper ores from 6- or 10-mesh to 14- to 20-mesh in high-speed mills averages about 0.25 lb. per ton. On hard conglomerate ore the consumption is 1 lb. Ferguson (*50 A 298*) gives a consumption of 1.06 lb. per ton on very hard silicious ore in Montana and 0.24 to 0.30 lb. on softer ores in Colorado and Mexico. These figures represent the usual extremes.

Cost. At Churchill Milling Co. (see Table 88) cost (1914) was: labor, \$0.026; supplies, \$0.053; power, \$0.063; total, \$0.142 per ton. At Barnes-King Development Co. (see Table 88) cost (1917) was: labor, \$0.1253; supplies, \$0.0689; power, at \$0.07 per kw.hr., \$0.0617; total, \$0.2559. At Portland (*63 A 531*) cost per ton (1918) was; labor, \$0.0231; supplies, \$0.0975; power, \$0.0764; total, \$0.1970.

Chilean mills *vs.* Huntington mills *vs.* rolls. At Nevada Consolidated mill (*123 P 325*) comparison was possible between 6-ft. Chilean mills, 6-ft.

Huntington mills and 35 × 16-in. rolls set close, all re-grinding −3⁄16-in. table tailing. Comparative performances are shown in Table 93. The results indicate the Chilean mill the best and the Huntington poorest in production of −30-mesh material (table feed) per horsepower-hour although the Huntington-mill product was the more granular (least −200-mesh material produced per ton of feed or per ton of −30-mesh produced). Repairs were highest on the Huntington mills. Only one roll was driven, which, in part, explains the high slime production.

Slow-speed mills *vs.* stamps, stamps and tubes, etc. McLaren (*101 J 15*) presents a strong argument for the slow-speed mill. In comparing it to stamps he cites two western U. S. mills working on medium-hard quartz ores.

Table 90. Effect of size of feed on performance of Chilean mill. (*After Lennox*)

	Coarse feed		Fine feed	
Tons of feed per 24 hr.	256		306	
Power consumption, hp.	114.2		78	
Speed, r.p.m.	37		37	
Screen aperture, mesh	18		18	
Screen, aperture	Weight, per cent.			
	Feed	Product	Feed	Product
2-in.	12.2	....	....	....
1.5	14.0	....	....	....
1.0	15.8	....	....	....
0.75	12.8	....	....	....
0.5	12.0	....	....	....
4-mesh	17.2	....	....	....
6	5.5	....	....	....
10	3.0	....	16.3	....
14	0.9	0.8	8.5	1.4
20	0.9	5.1	10.3	7.1
28	0.9	9.1	14.6	14.4
35	0.7	9.8	12.2	12.6
48	0.7	10.2	13.2	13.3
65	0.5	10.0	8.9	11.0
100	0.6	10.2	6.6	10.0
150	0.4	6.3	2.7	5.4
200	0.4	7.5	2.2	5.2
−200	1.6	31.0	4.5	19.6
Tons ground per horsepower-hour	0.093		0.163	
Tons −20-mesh produced per horsepower-hour	0.082		0.043	
Tons −65-mesh produced per horsepower-hour	0.048		0.039	
Tons −200-mesh produced per horsepower-hour	0.027		0.025	

The first mill used ten 1440-lb. stamps and one 5 × 16-ft. tube mill. Feed was 80 tons −2-in. material per 24 hr.; battery screens were No. 12 Ton-cap, and battery product contained 25 per cent. −200-mesh, which was removed ahead of the tube mill. Stamps drew 34 hp. and the tube mill 40 hp., so that the tons per horsepower-hour was 0.098 for the stamps and 0.045 for stamps plus tubes. In the other plant three 10-ft. Lane mills drawing about 12 hp. each crushed 48 to 50 tons each per 24 hr. from −2-in. to 40 per cent. −200-mesh with discharge height of 6½ in. Tons per horsepower-hour was 0.167 to 0.174, or about 70 per cent. more than that for the stamp battery, and the product contained 60 per cent. more −200-mesh, which would take 20 per cent. of the load off the tube mill in the first plant. McLaren also compares a 10-ft. Lane mill at 8 r.p.m. crushing 40 tons per

Table 91. Open- vs. closed-circuit grinding in Chilean mills. (*After Lennox*)

	Open circuit	Closed circuit
Tons per 24 hr.	225	184
Tons return sand per 24 hr.	....	190
Speed, r.p.m.	37	37
Screen aperture, mesh	18	18
Power, consumed, hp.	105.7	107

Screen aperture (*a*)	Weight, per cent.		
	Mill discharge	Discharge	Classifier overflow
20	8.0	8.0	....
28	8.9	11.2	0.8
35	8.8	12.1	4.2
48	9.3	11.9	7.4
65	9.0	10.2	10.6
100	8.9	8.8	11.8
150	7.1	6.9	8.9
200	7.0	5.9	9.3
−200	33.0	25.4	47.0
Tons original feed per horsepower-hour	0.089	0.072	
Tons −28-mesh produced per horsepower-hour	0.069	0.068	
Tons −200-mesh produced per horsepower-hour	0.028	0.033	

a For feed analysis, see Coarse feed, Table 90.

Table 92. Effect of moisture on performance of Chilean mill. (*After Lennox*)

Moisture in feed, per cent	62.7	78.5
Tons of feed per 24 hr	215	241
Speed, r.p.m.	37	37
Screen aperture, mesh	18	18
Power consumption, hp.	110.4	110.5

Screen mesh (*a*)	Weight, per cent.	
	Discharge (*a*)	Discharge (*a*)
35	23.8	27.2
48	9.0	9.9
65	8.1	8.2
100	9.4	9.4
150	6.4	6.2
200	7.2	6.9
−200	36.1	32.2
Tons of feed per horsepower-hour	0.081	0.091
Tons −65-mesh produced per horsepower-hour	0.048	0.050
Tons −200-mesh produced per horsepower-hour	0.029	0.029

a For feed analysis, see Coarse feed, Table 90.

Table 93. Comparative performances of Chilean and Huntington mills and rolls

Machine	Huntington	Roll	Chilean
Tons per machine per 24 hr.	185	260	140
Part ground through 30-mesh, per cent	27.3	26.9	55.8
Power consumed, hp.	26	31.5	32
Tons −30-mesh produced per horsepower-hour	0.081	0.093	0.103
Tons −200-mesh produced per machine per day	13.7	26.2	23.3
Tons −200-mesh produced per horsepower-hour	0.022	0.035	0.030

24 hr. of −1.5-in. feed to 53 per cent. −200-mesh and drawing 12 hp. (0.139 ton per hp.-hr.) with the first two steps in the 3-stage reduction at GOLDFIELD CONSOLIDATED where 1050-lb. stamps crushing −1.5-in. feed through 4-mesh screen were followed by high-speed Chilean mills with 16-mesh screens and these by 5 × 22-ft. tube mills. Stamp duty was 8.5 tons per 24 hr. for 2.4 hp. (0.148 ton per hp.-hr.) and the product contained 20 per cent. −200-mesh; Chilean mills handled 75 tons per 24 hr. for 35 hp. (0.089 ton per hp.-hr.) to 30 per cent. −200-mesh. Tube mills ground 95 tons per 24 hr. to 95 per cent. −200-mesh and drew 60 hp. (0.066 ton per hp.-hr.) Taking the first two stages as a unit, 0.056 ton were ground per horsepower-hour to 44 per cent. −200-mesh, which is substantially half that of the slow-speed mill. At BONITA MILL, Barron, Wash. (*85 J 1053*) a 5-stamp battery with 25-mesh screens ran side-by-side with a 5-stamp battery with 4-mesh (No. 16-wire) screen and a 10-ft. Lane mill in series. Discharge height in the mill was 6 in.; speed, 6 r.p.m.; power consumed, 12 hp. Stamps alone crushed 16 tons per 24 hr. and the product contained 10 per cent. on 30-mesh and 14 per cent. −100-mesh. Stamps plus Chilean mill crushed 40 tons per 24 hr.; product contained 4 per cent. on 30-mesh and 36 per cent. −100-mesh. Extraction on plates and tables was much better on the finer pulp.

Chilean mills *vs.* conical pebble mills. See Art. 16.

Chilean mills *vs.* Huntington mills. See Art. 16.

18. Huntington mill

Construction. Huntington mill (Fig. 37) consists of a cylindrical cast-iron tub (*a*), 3½ to 6 ft. diameter, with screened openings (*b*) in the walls to permit controlled egress of ground pulp; a die ring (*c*) wedged in place against the wall near the bottom; and four rollers (*d*) suspended like governor arms from the yoke (*e*) which is keyed onto the upper end of the driving spindle (*f*). As (*f*) is revolved through bevel gearing from driving pulley (*g*), the rollers (MULLERS) are pressed against the die ring by centrifugal force and roll around, crushing material between them and the dies. Feed is introduced with water at hopper (*h*), the swirl set up by the revolving mullers throws it against the dies to be crushed and against the screens for discharge. The principal wearing parts are the roller tires, ring dies and screens. Tires and dies are usually made of chrome steel. Screens are woven wire or punched plate.

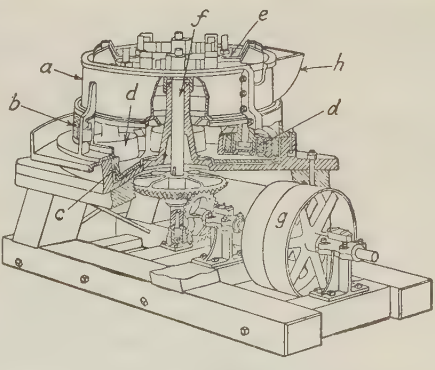


FIG. 37.—Huntington mill.

At ANACONDA, crushing de-slimed 10-mesh middling to 20-mesh, a 6-ft. mill with punched-slot screen crushed 74 tons per 24 hr. against 58 tons with woven wire. Average life of the punched plate was 12.4 days and of the wire screen, 13.3 days.

Weight ranges from 7000 lb. for 3½-ft. mill to 50,000 lb. for the heavy 6-ft. mill.

Operation. Mills are usually run at a peripheral SPEED of 1000 ft. per min., based on the nominal die-ring diameter but Semple (*42 A 602*) reports that increase in speed of a 5-ft. mill from 70 r.p.m. to 90 increased capacity from 50 to 75 tons per 24 hr. Low MOISTURE CONTENT makes for fine grinding and low steel consumption but reduces capacity greatly; the usual practice is 75 to 85 per cent. moisture, which supplies sufficient water to wash material through the screens readily. The die ring and mullers both wear unevenly and this causes excessive pounding and rapid destruction of the mill. Metal shoes made to replace a muller are used to grind down the die ring and muller shells should be removed and turned down when the faces get flat. Average STEEL CONSUMPTION is from 0.75 to 1.5 lb. per ton of feed. POWER CONSUMPTION varies from 5 hp. for 3½-ft. mills to 25 for a heavy 6-ft. mill with heavy feed. FEED should be fine (0.25-in. or under) unless the ore is very soft. Coarse feed causes excessive bumping because the crushing force is insufficient to break large lumps and the muller is forced to pass over them and then flies back against the die. CAPACITY to 20-mesh ranges from about 8 tons per 24 hr. for a 3½-ft. mill with coarse (−0.75-in.) feed to as high as 185 tons in a heavy 6-ft. machine re-grinding soft table middling from 10- to 20-mesh (see Table 94). The product is granular and well suited to table concentration. This is one of the principal ADVANTAGES of the machine. Another is the fact that it is especially adapted to sectionalizing for mule-back transport and can therefore be used in inaccessible regions. The DISADVANTAGES are the close attendance required and high repair and maintenance charges. The Chilean mill was an almost universally successful competitor before the advent of ball and tube mills, which latter have driven both mills out of the field. COST at TONOPAH MINING CO. (*91 J 1213*) grinding through 0.02-in. rectangular mesh was $0.52 per ton, of which $0.21 was for repairs.

Table 94. Feed and product of 6-ft. Huntington mill at Anaconda

Screen, mesh	Weight, per cent.	
	Feed	Product
6	0.3	....
8	0.4	....
10	1.2	....
20	52.1	7.1
40	35.2	33.5
60	5.9	13.8
80	2.8	11.5
100	1.2	7.7
150	0.3	3.7
200	0.4	6.2
−200	0.3	16.6

Punched-plate screen, 14-mesh. Feed rate, 74 tons per 24 hr.

19. Grinding pan

Construction. The grinding pan (Fig. 38) consists essentially of a cast-iron tub, usually about 5 ft. diam. and 2½ to 3 ft. deep, carrying a broad annular die ring on the bottom, on which heavy shoes are dragged by means of a yoke; this in turn, is driven by a spindle from bevel gears and a belt-driven counter-shaft below the pan bottom. Shoes and dies are ordinarily of gray cast iron which wears down with a rough scored surface. White iron and alloy steels are unsuitable because the wearing faces become smooth and polished with accompanying reduction in capacity.

At GREAT FINGALL, Australia (*11 CME 291*) shoes and dies corrugated circumferentially had higher capacity than plane shoes and dies, metal consumption was lower (2.9 to 3.0 lb. per ton against 3.6 to 3.8) and power consumption was less on account of the lower speed.

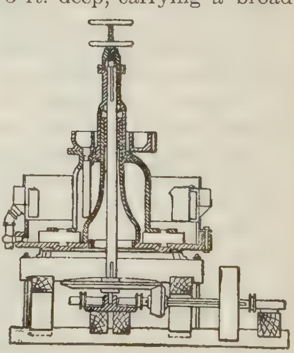


FIG. 38.—Grinding pan.

An adjusting screw with locking wheel is provided for adjustment of the height of shoes. Mullers, which carry the shoes, should be attached to the

yoke arms by a flexible fitting in the nature of a universal joint; if a rigid joint like that in an amalgamating or clean-up pan is used, the shoes will often chatter, capacity be reduced and breakage increased. Die ring and shoe circle are sometimes continuous, but ordinarily short spaces are left both between the shoes and the die segments. These form channels into which pulp flows and from which the crushing faces are fed. New shoes weigh 75 to 200 lb. and the crushing force is limited to that exerted by this weight when dragged over the die. Compensating weights are sometimes used to keep the crushing force up to normal as the shoes wear.

Pans are of two general types, *viz.:* ordinary and positive. In the ordinary pan the feed is introduced at any point, although preferably this is near the center, and discharge is by peripheral overflow or its equivalent. Fineness of product is controlled by height of discharge. Baffles are placed on the inner wall of the pan to return pulp to the center. Capacity is low and power consumption relatively high on account of the large body of pulp that is kept in motion. The POSITIVE PAN is fed through a central cylinder discharging near the bottom of the pan at the inner edge of the die ring. Pulp flows outward through the shoe and die channels and is then forced upward along the walls of the pan by centrifugal force induced by the swirling action. Discharge is low. Size of product is controlled by outside classification (or screening) or by a screen (COBBE-MIDDLETON), or a classifier (FORWOOD-DOWN) attached to the pan walls.

Performance. At PERSEVERANCE mine, Kalgoorlie (*16 MM 300*) 8-ft. Wheeler pans (ordinary type) 3½ ft. deep, with a small spitzkasten on the outside of the pan wall fed near the top by a row of ¾-in. holes and having a slot 3 × 2-in. at the apex to return sand, were used. The speed of different pans varied from 25 to 30 r.p.m. A higher speed could be used with slime-free feed. Capacity was 40 to 45 tons per pan per 24 hr.; power draft, 7 hp. each; tons per horsepower-hour, 0.24 to 0.27. Feed contained 50 per cent. +150-mesh and product 30 per cent. Amalgamation was practiced in the pans. Shoes and dies were gray cast iron, 2½ in. thick; life of shoes, 3 mo.; dies, 6 to 9 mo., each being worn to about 1-in. thickness; consumption, 0.35 lb. per ton ground. At HOMESTAKE (*22 IMM 93*) modified Wheeler pans, 5-ft. diameter, with peripheral overflow adjustable as to height were used to re-grind and amalgamate. Feed was received in a cup forming part of the muller carriage and delivered by pipes directly in the path of the shoes. Total revolving weight was 2200 lb.; power draft, 8.4 hp. at 58 r.p.m. Capacity was about 20 tons per 24 hr. or 0.099 ton per hp.-hr. Screen tests of feed and product are given in Table 95. At BECK MINING CO., Wyo. (*108 P 417*) a 5-ft. ordinary pan at 68 r.p.m. in closed circuit with a Dorr classifier ground 26 tons per 24 hr. from 10-mesh to 80-mesh. (Table 95.) Pulp in the pan contained only 30 per cent. moisture when operating best. At KYLOE, N. S. W. (*22 IMM 16*) 8-ft. Forwood-Down pans at 30 r.p.m. with classifying discharge were used to grind jig tailing to flotation feed. (Table 95.) The product contained too much +40-mesh material. Raising the overflow increased the percentage of slime produced but did not greatly affect the amount of +40-mesh. Finally the classifier discharge was removed and the machines were put in closed circuit with King screens, which solved oversize troubles. At BROKEN HILL (*100 J 152*) 8-ft. Forwood-Down pans at 32 r.p.m. ground 2 to 2½ tons per hr. from −3-mm. to 10 per cent. +40-mesh; 5-ft. pans of the same type at 50 r.p.m. ground 1½ tons per hr. through the same range with a consumption of 8 hp. (0.187 ton per hp.-hr.). Cast-iron shoes and dies lasted 6 to 8 weeks, equivalent to 2.5 to 3 lb. wear per ton ground. Stage crushing was found preferable to single-step and tube mills much cheaper than pans. At the ZINC CORPORATION (*118 P 90*) −7-mesh +20-mesh jig tailing was ground to flotation size (40-mesh) in 5-ft. pans at the rate of 1.25 tons per hr., return sand bringing the total up to 3 tons. These pans were originally installed as non-sliming fine grinders for gravity concentration and were not as good as tube mills for flotation feed. At PRESTEA BLOCK A, West Africa (*106 J 54; 22 JCM 149*) 5-ft. Cobbe-Middleton pans, having a weight-and-lever device for holding the shoes against the dies, were used for grinding and amalgamating pyritic concentrate. The duty on −10-mesh feed with 14-mesh screen was 8.2 tons per 24 hr. of −60-mesh product or 5.7 tons −90-mesh. This was at the rate of 0.048 ton −90-mesh per hp.-hr. and is comparable to 0.052 ton −90-mesh per hp.-hr. in Rand tube mills on similar ore. (See Table 95.) Wear of shoes and dies including waste, which was large, was 0.9 lb. per ton ground. The pan had maximum capacity with new shoes and dies.

Table 95. Screen tests of feed and product of grinding pans.

Mill	Homestake		Kyloe		Prestea		Bolivian tin, ordinary pan, launder feed, 32 r.p.m.		Bolivian tin, modified pan, pipe feed, 40 r.p.m.		Bolivian tin, as preceding, but 50 r.p.m.		Bolivian tin, positive pan			Beck Mining Co.		
Screens, mesh	Weight, per cent.																	
	F	*P*	*F*	*P*	*F*	*P*	*F*	*P*	*F*	*P*	*F*	*P*	*F*	*D*	*CO*	*F*	*D*	*CO*
16			10								As preceding							
20			19.7	0.3														
30					25.8	2.6						3.0	20.0	1.0		21.3	13.3	
40			10.5	11.0			2.0		6.2	1.5								
50	38.4	4.0										9.5	21.5	12.5				
60			18.2	30.9	22.3	34.4	8.5	4.0	16.8	6.5		22.0	24.0	22.5	1.2	45.6	13.9	1.1
80	42.0	12.5					24.4	8.2	27.0	10.8								
90					12.2	16.5						20.5	9.5	9.5	5.6	17.0	8.8	13.0
100	9.9	13.5					20.3	16.0	19.7	23.0								
120					5.0	6.2												
130			17.0	23.4								16.5	13.0	11.0	4.5	4.0	5.9	10.0
150							26.0	23.4	18.3	20.2			7.5	11.5	10.5			
200	6.8	28.0										28.5	4.5	32.0	78.2	12.1	58.1	75.9
—last screen	2.9	42.0	24.8	34.4	34.8	40.3	18.8	48.4	12.0	38.0								

F = feed. *P* = product. *D* = pan discharge. *CO* = classifier overflow.

Positive *vs.* ordinary pan. Söhnlein (*96 J 581*) reports comparative performances of positive and ordinary types. At a Bolivian tin mine, 5-ft. pans of the ordinary type run at 32 r.p.m. ground 2.5 tons per 24 hr. as shown in Table 95. At 60 r.p.m. practically no grinding occurred in a pulp containing 20 per cent. solid. Variation in discharge height from 2 to 3 ft. made no appreciable difference in character of product (at 32 r.p.m.) and the lower height was adopted because of lower power consumption. By changing the feed from a launder discharging near the center of the pan to an annular cup with pipes leading down to the inside of the shoe circle, the speed could be increased to 40 r.p.m. and capacity was raised to 7 tons per 24 hr. (See Table 95.) Feed and product were both coarser than in the unmodified pan but the tonnage of fines produced by the modified pan is much greater. At 50 r.p.m. (see Table 95) the discharge was too coarse. Further change to a positive pan with central feed cylinder, 20-in. overflow height and outside classification (Dorr) with return of oversize increased capacity to 38 tons per 24 hr. at 60 r.p.m. (See Table 95.) Consumption of cast-iron shoes and dies was 0.67 lb. per ton in the positive pan. Cost was \$0.09 for metal, \$0.19 for power (at \$0.0375 per hp. hr.), labor and lubrication, \$0.02; total, \$0.30.

Pan *vs.* tube mill. See Art. 14.

20. Arrastre

The arrastre is a primitive machine much used in early Mexican gold-milling for grinding from −1-in. size to slime. Practice in building varied, but the usual form was a circular pit, walled with rock or logs and chinked with clay, the bottom lined with flat stones similarly chinked, over which flat-

bottomed cubical stones weighing 100 to 1000 lb. were dragged by means of a horse-drawn sweep. Ground pulp overflowed through pipes at about 6 in. above the floor. Amalgamation was commonly practiced in the machine. CAPACITY varies from 200 or 300 lb. per day of soft ore in machines 3 or 4 ft. in diameter to 2 to 5 tons in 8- to 12-ft. mills.

Storms (*91 J 1053*) gives the following details of construction of the double arrastre shown in Fig. 39. Floor (*a*) was a double layer of 2-in. plank. Walls (*b*) were built of circular segments of 2-in. plank nailed together. The bottom of the basin was filled with a layer of firmly-rammed clay, then 6 to 8 in. of sand in which the floor blocks of hard, tough stone were laid. The edges of the blocks were left rough so that the spaces between them would afford settling places for amalgam. The floor sloped about one in 8 toward the sluice gate, which was 6 in. wide, extended 4 in. below the top of the rock floor and was closed by a sliding door. The discharge height was varied by 1-in. strips laid in cleats. Drags weighed 800 to 1000 lb. Massive diabase, diorite or granite served well for both floors and drags. The drags were fastened to chains by ⅝-in. eye-bolts wedged into holes 5 in. deep, and were set to sweep pulp into the path of the following stone. The front end of the drag stone should be lifted about 1 in. in order to nip large particles. The water wheel was of the hurdy-gurdy type; pipe line 7-in. diameter with 2-in. nozzle; water available, 18 cu. ft. per min. under 100 ft. head. Pinions were 3-ft. diameter with 13 hardwood pins 2-in. diameter, 4 in. high and 4 in. center-to-center. The horizontal pin wheels of 2-in. plank spiked and bound with strap iron were 6-ft. diameter with 51 pins similarly shaped and spaced. The main shaft revolved on 3-in. iron gudgeons. The bearing for the center post was made of 4-in. square steel 7 to 8 in. long with a spherical depression 3-in. diameter by 1½ in. deep, smoothly finished, tempered at bronze color and set in the upper end of the lower post, which was bound with an iron collar to prevent splitting. The gudgeon in the foot of the revolving post was forged to fit.

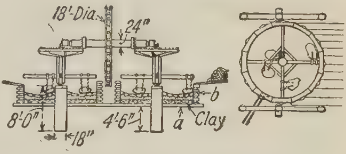


FIG. 39.—Arrastre.

Operation. When amalgamating in this machine batch operation was practiced. Ore was charged a little at a time with enough water to make a thin mud (40 to 60 per cent.) and ground for 5 to 7 hr., or until fine enough to add mercury, after which addition grinding was continued 2 or 3 hr. or until panning showed all free gold amalgamated. The speed of drags during amalgation should not exceed 360 ft. per min.; higher speed could be used during crushing, but repairs were less when the desired capacity was attained with heavy drags and low speed. During the latter part of the amalgamation period the drags were slowed down to 150 ft. per min. and pulp diluted to allow coarse grains to settle and be ground and amalgam to settle in cracks. When panning showed the pulp free of amalgam, the basin was raked and sluiced out gently and the floor cleaned up in the usual manner (see Sec. 8, Art. 11).

21. Swing-hammer pulverizer

The swing-hammer pulverizer (Fig. 40) is probably the most widely used of the crushers that depend upon impact to break 2- or 4-in. lumps to ½- or ¼-in. size. The essential parts are a plurality of disks (*a*) mounted on a rapidly-revolving shaft, each disk carrying a plurality of hinged beater arms (*b*). This mechanism is enclosed in a cylindrical chamber (*c*) lined with heavy breaking plates (*d*) and a grid or cage (*e*). Feed entering hopper (*f*) is struck by the beater arms and thrown against the breaking plates and against the cage until broken fine enough to fall through the cage openings. The cage and toothed breaker

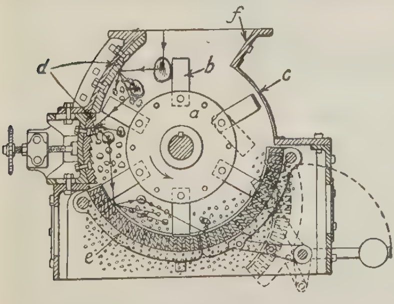


FIG. 40.—Swing-hammer pulverizer.

plates are adjustable as to distance from the central shaft, which permits take-up to compensate for wear and adjustment for size of discharge. Hammers, liners and grids are made of high-carbon tool or manganese steel. The machine is made in many SIZES and is used for crushing many different materials, mostly in the industrial field. A few of the sizes with data from actual operations are given in Table 96. Some hammer machines take 5- to 6-in. feed but the usual practice is to feed them with 1- to 3-in. material.

Table 96. Swing-hammer pulverizers. (*From Jeffrey Mfg. Co. catalog*)

Size, diameter × length, inches	Weight, pounds	Speed, revolutions per minute	Approximate horsepower	Grid spacing, inches	Capacity		
					Material	Tons per hour	Size product
18 × 9	....	....	6	1	Dolomite	0.33	50% − 20-m.
24 × 18	2,700	1200–1500	20	1	Dolomite	5.5	50% − 20-m.
30 × 24	4,700	1000–1200	35	0.06	Sea coal	3	75% − 40-m.
36 × 18	5,800	1000	45	0.25	Limestone	8	60% − 20-m.
36 × 24	6,400	1000	60	0.06	Limestone	9	80% − 24-m.
36 × 30	7,600	1000	80	0.06	Limestone	10	50% − 60-m.
36 × 30	7,600	1000	80	0.125	Limestone	18	90% − 20-m.
36 × 30	7,600	1000	80	1	Limestone	25	− 0.25-in.
36 × 30	7,600	1000	80	1	Culm	50	− 0.25-in.
36 × 30	7,450	1000	80	1	Bit. coal	50	− 0.25-in.
42 × 36	8,700	800–1000	100	1	Bit. coal	75	− 0.25-in.
42 × 48	17,000	800–1000	110	1	Bit. coal	100	− 0.25-in.
48 × 48	19,000	700–800	125	1	Bit. coal	125	− 0.25-in.
48 × 60	24,000	700–800	200	1	Bit. coal	180	− 0.25-in.

In the hammer-bar machine U-shaped bars take the place of the hammers. They are generally used for somewhat finer feed (0.5-in.) than the swing-hammer machines.

According to Williamson (*37 TAIEE 1550*) hammer mills (all types) crush 0.14 to 0.16 ton per hp.-hr. from 2½-in. maximum to 20-mesh.

CAPACITY is markedly affected by moisture. At one plant a machine that crushed 75 tons per hr. of bituminous coal through a ¾-in. screen when the coal contained 4 or 5 per cent. moisture would crush only 50 tons of coal containing 9 to 12 per cent. moisture. In asbestos work the discharge grid holds back coarse fiber and causes it to be over-crushed. Tramp iron causes serious breakdowns. A machine handling 15 to 20 tons per hr. of 3-in. rock draws 100 hp. (*123 P 934*). Lincoln (*Bul. 11 UI No. 9*) says that a No. 00 Williams pulverizer at 2200 r.p.m. reduced Illinois bituminous coal to ⅜-in. at the rate of 1.25 tons per hr.

Ring pulverizer is a variation of the swing hammer machine in which alternate hammers are replaced by heavy rings of such diameter that the diameter of the circle described by their outer faces is the same as that described by the beater tips.

22. Kent cracker

This machine (Fig. 41) consists of a pair of toothed rolls fitted above with a set of reciprocating steel picks in a cross-head crank-driven from one of the roll shafts. These picks both crack large particles and force them down to be nipped by the rolls. The machine is suitable only for cracking soft material that comes in lumps too large to be nipped by ordinary toothed rolls. It is used for breaking chalk at BENJAMIN MOORE Co., Carteret, N. J. (*116 J 415*).

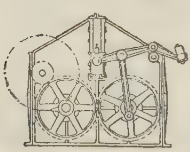


FIG. 41.—Kent cracker.

23. Jumbo mill

The Jumbo mill (Fig. 42) consists of a shaft carrying hinged beater arms and revolving at high speed (400 to 800 r.p.m.) in a closed cylinder. Feed is introduced at one end, material is carried forward by deflection of the arms and the product is discharged at the other end. In another mill of the same general type, the beater arms are more closely spaced and arms carried on the cylinder walls project between them. The cylinder is split in half longitudinally, the lower half hinged to swing down and allow easy access for repairs.

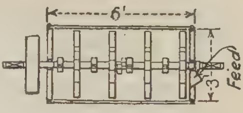


FIG. 42.—Jumbo mill.

24. Cyclone mill

This machine (Fig. 43) has two heavy (1000-lb.) 2- to 4-armed propeller-shaped beaters (*a*) revolving at high speed (2000 to 2200 r.p.m.) in a closed chamber. The direction of rotation of the two beaters may be the same or opposite; the latter gives the greater disintegrating effect. Feed enters on both sides at (*c*), directly over the beaters, and is discharged by suction from the top of the chamber.

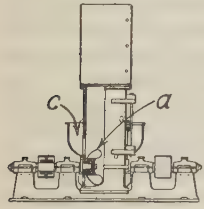


FIG. 43.—Cyclone mill.

The machine has been used in asbestos work, but on account of the great wear of propellers (lives as low as two days are recorded) has been generally replaced by other machines. FEED is – 1-in. size. CAPACITY ranges from 2.5 tons per hr. on hard asbestos rock to twice this figure on soft rock. In crushing soft rock from 2-in. to 0.25-in. the capacity may go up to 15 tons per hr. POWER DRAFT is 65 hp. Overfeeding causes material to accumulate on the bottom of the machine and be over-crushed. Tramp iron does great damage.

Torrey cyclone has a vertical ball-bearing shaft carrying two sets of vanes, one above the other, all suspended in a box-like chamber lined with manganese steel plates. The shaft is driven at 1000 to 2000 r.p.m. Material about 3-in. maximum size is fed onto a distributing plate at the top, falls onto the top impeller, is hurled against the liner plates, and broken. It then falls to the lower impeller, where it is further broken and then discharged. Tramp iron does not do much damage under ordinary circumstances. Hubbard (*123 P 934*) states that 17 hp. will run a machine handling 12 to 15 tons per hr. of asbestos rock.

25. Dry grinders

In addition to ball and tube mills, there are a number of mills of the ring-roller type used in this service, particularly in grinding coal and cement clinker. (See *37 TAIEE 1540*.) The FULLER MILL has heavy steel balls which are pushed around at high speed on the inner periphery of a circular die ring. In the coarse grinder, finished product is fanned out through a screen surrounding the upper part of the grinding chamber; in the fine grinder air classification is induced above the grinding chamber by connection with an external suction fan. The capacity with four or five 12- to 14-in. balls is 6 to 8 tons per hr. of coal or similar material from ½-in. to 95 per cent. – 100-mesh, drawing 35 to 60 hp. at speeds of 90 to 130 r.p.m. In the KENT MILL a vertical ring die revolves around 3 rollers that are held against it by powerful spring pressure. The crushing range is from – 1-in. to 20-mesh. Power consumption is about 1 hp.-hr. per barrel on cement clinker, 3 to 5 hp.-hr. per ton on cement rock. The BONNOT MILL is substantially a vertical-ring Huntington, 36-in. die ring, 75 hp. at 180 r.p.m., 4 to 5 tons per hr. from – ⅝-in. to 97 per cent. – 100-mesh. The GRIFFIN MILL resembles the Huntington in the placing of die and screens but has a single grinding roller with pendulum-like suspension. The RAYMOND MILL is similar to the Huntington but with an air-classifying attachment for raising finished material out of the grinding zone. CAPACITY, according to Williamson is about one ton per hr. per roll from ¼- or ½-in. feed to 95 per cent. – 100-mesh. Power consumption of a 2-roll mill is 45 to 50 hp.; 3-roll, 55 to 60 hp ; 4-roll, 75 to 80 hp.; 5-roll, 85 to 90 hp. None of these mills is used in concentrating practice.

26. Operation of crushing machinery

For maximum economy of power consumption machines must be kept up to full capacity without overloading. In closed-circuit work the return circuit should be carefully watched. Increase in tonnage of this circuit above normal will probably cause reduction in capacity. The usual causes of excessive circulating loads, depending on the flow-sheet, are: wear of crushing faces or a slacking off in their spacing, blinding of screens, reduction of rising current in hydraulic classifiers and, in the case of cylinder mills, change in moisture content of mill or classifier pulp. Steady feed is necessary for the most economical operation of all crushers; it is absolutely essential for cylinder mills; on the other hand the difficulty of attainment may excuse the lack in the case of coarse crushers. Liners and replaceable crushing surfaces should be replaced before they are worn through. Neglect causes an increasing diminution in crushing efficiency and, aggravated, may result in destruction of the machine structure itself, *e.g.*, the head, shell, or trunnion of ball or tube mills; the main frame of jaw or gyratory crushers or rolls, etc. Lubrication is of immense importance, both from the point of view of power consumption and that of bearing wear. Dry-crushing machinery working, necessarily, in a dusty atmosphere, should be provided with grit-proof bearings where possible and lubricated by a pressure system that forces lubricant out through any crevices through which grit might enter. Heavily-loaded bearings, such as those of cylinder mills, require a lubricant that cannot be squeezed out and the discharge bearing particularly must be protected against splash and the working back of pulp from the discharge bell. The usual expedients are a discharge bell of goodly length and wide flare, which reduces splash and increases the distance from splash to bearing, provision of one or more drip rings between the edge of the discharge bell and the bearing, and some kind of cloth fabric rubbing against the outside of the bell between the lip and the bearing to wipe off pulp. This wiper, however, rapidly becomes dirty and useless.

27. Crushing efficiency

Considerable work has been done in the attempt to find a means for determining the efficiency of crushing. True mechanical efficiency is not capable of determination because it is impossible, in the present state of knowledge, to measure the useful work done. Determination of relative efficiency is also highly elusive. One of the four methods described below is usually employed.

Cost comparison. This method consists in actual competitive tests of the machines that are to be compared, under operating conditions, keeping close records of tonnages crushed to the desired size, cost of operation, and, where advisable, of metallurgical results, from which records, by suitable comparison, the merits of the competing machines can be determined. Costs should include, in addition to power, operating labor, supplies, maintenance and supervision, charges for interest, depreciation and amortization. Such comparison as this is expensive and is justified only in the largest plants. Even in such plants the method is difficult and uncertain, since the test installation is rarely run on as large a scale as the finished plant and the test runs are rarely of sufficient duration to allow the most economical operation.

Tons crushed per horsepower-hour. This is the simplest of the purely physical methods of comparing crushing machines. It requires determination of the tonnage, power consumption, and screen analysis of the material produced. If the feed to competing machines is of the same character in all respects, and if the size of the product is unimportant except for the one requirement that it shall all pass a given limiting screen, and if maintenance and capital charges can be neglected, this method is satisfactory. But if the amounts of material in the different sizes below the limiting size are important, the method does not necessarily give a correct measure of the comparative performances of the crushers. It may be modified by calculating the average size of product from the screen analysis and stating performance in tons of a given average size produced per horsepower-hour, but in this case it is usually necessary to stipulate in addition that all material has passed the limiting screen, and the final figures that are reached are incapable of accurate comparison because of the fact that average size is not a definitely significant quantity (see Sec. 22, Art. 5) and that the average sizes of the products as estimated will probably be different. Thus if two competing crushers break a given feed through, say, a 1-mm. screen, and the first produces 5 tons per hp.-hr. of material having an average size of 0.5 mm., while the other produces 4.5 tons per hp.-hr. of material of 0.3-mm. average size, no conclusion as to relative efficiency of the crushers by this method is possible. If concentration showed that both products were equally amenable to treatment, a conclusion could be reached on the basis of costs. If the crushers that are being compared are not fitted to take the same size of feed or to make a product that will pass the same limiting screen, then, clearly, no such comparison as that described in this paragraph is possible.

Relative mechanical efficiency implies a rating for crushing machines similar to that employed for rating prime movers. Such rating involves the ratio of useful work done to energy expended. Unfortunately, as has been stated above, it is impossible to determine the amount of work usefully expended in crushing. Work expended in overcoming friction and inertia of the machine parts, in rubbing between particles and between particles and crushing surfaces, in producing deformation without breaking, and in making noise and generating electricity in the crusher is not useful, as the word is used in discussion of efficiency. An indication of useful work can, however, be reached by measuring the reduction in size of particles in the crushing operation and attributing thereto, in arbitrary units, certain energy values. When this is done, a statement of relative efficiency can be made as follows: $E = WT/P$. Where E is the relative mechanical efficiency, W is the number of units of size reduction per unit weight of material crushed, T is the number of units of weight of material crushed per unit of time, and P is the number of units of power used.

Opinion is divided as to the method to be employed in assessing the equivalence of useful work and size reduction. Two methods have been proposed. These are named respectively Kick and Rittinger from their proponents.

Kick's law states: "The energy required for producing analogous changes of configuration of geometrically similar bodies of equal technological state varies as the volumes or weights of these bodies." Stadler (*19 IMM 471*) has worked out a method of applying this law to rock crushing and has developed ORDINAL NUMBERS, corresponding to different sizes of crushed material, that, accepting the law, are proportional to the amount of useful work done

to produce the given size from a certain unit size. Stadler's reasoning is summarized in the following quotations from his article.

"The area of fracture over which the cohesion of the molecules has to be destroyed, multiplied by a coefficient determining the resistance which the molecules oppose to their separation by the exercise of any stress (crushing, tensile, shearing, etc.) represents the *force* required to cause the fracture.

"In order to perform mechanical work, this force has to run through a *distance*, represented by the deformation which the body can stand before reaching the breaking point. It is in this connection immaterial that this distance of deformation within the limits of elasticity and plasticity is, for not perfectly homogeneous bodies, subject to variations, which for highly elastic and unplastic bodies as quartz, glass, etc., are too insignificant to be considered, and in addition they are, by the nature of our crushing appliances averaged to such an extent, that these averages are as good as exactly defined figures. Dealing with relative values only, we have not even to care about the exact extent of this deformation, and all that we need is, to be satisfied that this factor is a constant function of the diameter of the particles to be crushed.

"The *mechanical work done* is represented by the product of the *force by the distance*, but as in a regular scale of reduction by volume the diameters of the particles decrease at the same ratio as the area of fracture increases, the product, or the mechanical work for reducing the volume (or weight) of the unit from one grade to the next following, *is a constant for each grade*, called the crushing or ENERGY UNIT (EU).

"The volumes of the particles decrease from grade to grade in the same ratio as the number of the particles, constituting in their total the volume of the unit[y] increases, and the product of the volumes into the number of the particles of that grade is, therefore, constant for each grade. As in conformity to the above law, the amount of energy absorbed is proportional to the volume of the body to be crushed, it follows again also that the total energy required for reducing the weight of the unit *is constant for each grade*.

"The ORDINAL NUMBERS of any arithmetical progression given to these grades represent consequently the relative values of the energy which has to be spent upon producing this respective grade from the initial unit, or the *mechanical value* of the grade.

"For obtaining the *mechanical value of mixed sands*, we need only to multiply the percentages of the gradings by the number of the energy units of the respective grade and add the products.

"The *useful work done per unit* by any crushing machine is determined by the difference between the mechanical values of the samples taken at the inlet and the discharge of the machine, and for obtaining the *total work done* this difference has to be multiplied by the tonnage dealt with."

Stadler's formulas for determining ordinal numbers and various other factors regarding crushed material are presented in Table 97 (*48 A 153*). If the screens used in making the sizing test vary in aperture from screen to screen by a constant multiplier, the ordinal numbers will be whole numbers in an arithmetical progression. Numerical values of the quantities in Table 97 for Tyler standard screen-scale sieves are given in Table 98.

The following is an example of the application of Stadler's method to a crushing problem.

Example. A 4.5-ft. × 16-in. conical ball mill fed with quartzite having the screen test shown in column 2, Table 99, breaks to the size shown in column 3 at the rate of 36 tons per 24 hours with a consumption of 18.5 hp. Total energy units of feed = 487.82 (column 5) and of product = 1376.66 (column 6). The relative work done per unit weight of feed equals 1376.66 − 487.82 = 888.84.

$$E = \frac{888.84 \times 36}{100 \times 18.5} = 17.30.$$

Rittinger's law states that the work required to produce material of a given size from a larger size is proportional to the new surface produced. Richards (*TB*) develops the application as follows: The number of planes of fracture necessary to break a cube whose edge is one unit in length into

Table 97. Formulas for crushed-rock constants

	P	S	V	F	N
P		$\frac{U^3}{S^3}$	$\frac{U^3}{V}$	$\frac{(F + 3U^2)^3}{27U^6}$	R^N
S	$\frac{U}{P^{1/3}}$		$V^{1/3}$	$\frac{3U^3}{F + 3U^2}$	$\frac{U}{R^{\frac{N}{3}}}$
V	$\frac{U^3}{P}$	S^3		$\frac{27U^9}{(F + 3U^2)^3}$	$\frac{U^3}{R^N}$
F	$3U^2(P^{1/3} - 1)$	$3U^2\left(\frac{U}{S} - 1\right)$	$3U^2\left(\frac{U}{V^{1/3}} - 1\right)$		$3U^2\left(R^{\frac{N}{3}} - 1\right)$
N	$\frac{\log P}{\log R}$	$\frac{3(\log U - \log S)}{\log R}$	$\frac{3\log U - \log V}{\log R}$	$\frac{3[\log(F + 3U^2) - \log 3 - 2\log U]}{\log R}$	

U = length of edge of assumed unit cube. S = length of edge of any given cube (= screen aperture). P = number of cubes of edge S in unit cube. V = volume of cube of edge S. F = area of fracture or new surface produced in breaking unit cube into P cubes of side S. R = ratio of increase in volume of particles from one screen size to the next in the sieve scale used. N = ordinal number of screen size.

Table 98. Crushed-rock constants for Tyler standard screen-scale sieves

Apertures, S		Volumes, V		Area of fracture, F		Number of pieces, P	Ordinal numbers
In.	Mm.	Cu. in.	Cu. mm.	Sq. in.	Sq. mm.		
1.050	26.67	1.158	19,000	0	0	0	0
0.742	18.85	0.409	6,700	1.37	884	2.84	1
0.525	13.33	0.145	2,370	3.30	2,130	8.00	2
0.371	9.423	0.0511	837	6.03	3,890	22.7	3
0.263	6.680	0.0182	298	9.89	6,380	63.6	4
0.185	4.699	0.00633	104	15.5	9,970	183	5
0.131	3.327	0.00225	36.8	23.2	15,000	515	6
0.093	2.362	0.00080	13.2	34.0	22,000	1,440	7
0.065	1.651	0.00027	4.50	50.1	32,300	4,220	8
0.046	1.168	0.000097	1.59	72.2	46,600	11,900	9
0.0328	0.833	0.000035	0.578	103	66,200	32,800	10
0.0232	0.589	0.000012	0.204	146	94,400	92,700	11
0.0164	0.417	0.0000044	0.0725	208	134,000	262,000	12
0.0116	0.295	0.0000016	0.0257	296	191,000	742,000	13
0.0082	0.208	0.00000055	0.0090	420	271,000	2,110,000	14
0.0058	0.147	0.00000020	0.0032	595	384,000	5,930,000	15
0.0041	0.104	0.000000069	0.0011	844	544,000	16,800,000	16
0.0029	0.074	0.000000024	0.0004	1190	770,000	47,500,000	17
0.0014*a*	0.037	0.0000000027	0.00005	2480	1,598,000	380,000,000	19

a Assumed as average size of material through last screen.

Table 99. Application of Stadler's ordinal numbers to a crushing test

1	2	3	4	5*a*	6*b*
Screen aperture, mm.	Percentage on screen		Ordinal number	Energy units	
	Feed	Product		Feed	Product
37.70	0.66	....	−1	−0.66	....
26.67	5.51	....	0	0.00	....
18.85	16.36	....	1	16.36	....
13.30	22.00	....	2	44.00	....
9.423	11.18	0.02	3	33.54	0.06
6.680	7.46	0.02	4	29.84	0.08
4.699	5.49	....	5	27.45	0.00
3.327	3.62	0.29	6	21.72	1.74
2.362	2.97	0.83	7	20.79	5.81
1.651	3.30	2.45	8	26.40	19.60
1.168	2.94	4.34	9	26.46	39.06
0.833	3.18	6.92	10	31.80	69.20
0.589	3.55	12.37	11	39.05	136.07
0.417	2.37	10.15	12	28.44	121.80
0.295	2.48	12.40	13	32.24	161.20
0.208	1.85	10.55	14	25.90	147.70
0.147	1.72	10.88	15	25.80	163.20
0.104	1.25	8.48	16	20.05	135.68
0.074	0.70	5.12	17	11.90	87.04
Through 0.074	1.41	15.18	19	26.79	288.42
Totals	100.00	100.00	....	487.82	1376.66

a Column (5) = Column (2) times column (4). *b* Column (6) = column (3) times column (4).

n^3 smaller cubes with edges $1/n$ units in length is $3(n-1)$. See Fig. 44. The area of each such fracture plane is one. If the work done to produce fracture along one such plane is B, then the total work done in the subdivision indicated is $3B(n-1)$. Generalizing, if D is the edge of the original cube and d that of the resulting cubes, n becomes D/d; the area of each fracture plane is D^2, and the formula for work done becomes

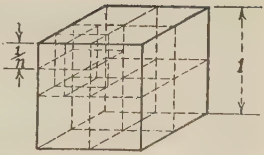


FIG. 44.—Theoretical breakage of a cube.

$$W = 3BD^2\left(\frac{D}{d} - 1\right).$$

The number of cubes D inches on an edge contained in a 1-inch cube is $1/D^3$ and the work per cubic inch for breaking cubes of any size D to cubes of any smaller size d is

$$W = \frac{1}{D^3}\cdot 3BD^2\left(\frac{D}{d} - 1\right) = 3B\left(\frac{1}{d} - \frac{1}{D}\right).$$

When, as is the case in rock crushing, the particles are irregular in shape, a multiplier K must be used, representing the ratio between the surface of a mass of ore consisting of particles that will pass a given rectangular opening and the same mass existing in such theoretical cubes as will just pass the same opening. The formula then becomes

$$W = 3KB\left(\frac{1}{d} - \frac{1}{D}\right).$$

Richards states that K varies from ½ to ¼. He also gives the following method for determining B in any particular case.

Example. Let S = sp. gr. of rock. Then $55320/S$ is the number of cubic inches of rock of sp. gr. S in a ton.

$$W = \frac{55320}{S}\cdot 3\,KB\left(\frac{1}{d} - \frac{1}{D}\right) \text{ ft.-lb. per ton}$$

$$= \frac{0.08382\ KB}{S}\left(\frac{1}{d} - \frac{1}{D}\right) \text{ hp.-hr. per ton.}$$

If now, by experiment, the number of hp.-hr. per ton expended in crushing the ore in question from an average diameter D to an average diameter d is determined and substituted for W in the last equation, and proper values for K and S are likewise introduced, the value of B for the given ore may be obtained.

It is to be noted that the figure substituted by Richards for W is not the useful work done, but includes also the work done in overcoming friction. B, therefore, as thus determined, applies only to the particular experiment.

Del Mar (*94 J 1129*) gives the following statement of Rittinger's law: "The work required to crush rock is very nearly proportional to the reciprocals of the diameters crushed to." He applies this law to a crushing problem as follows:

Example 1. Comparison of efficiency of two mills. Mill No. 1 crushes 2.5 tons per hp.-hr. and mill No. 2, 3.0 tons per hp.-hr. Sizing tests of feed and product and energy calculations are given in Table 100. The surface produced per unit weight of material by mill No. 1 is 21,111 minus 16,012 or 5099 units; similarly by mill No. 2, 3737 units. The units of reduction produced per hp.-hr. by mill No. 1 equals 5099 times 2.5 or 12,747.5; by mill No. 2, 11,211. Mill No. 1 is, therefore, more efficient by 13.7 per cent., based on the performance of mill No. 2.

Table 100. Calculation of energy by Del Mar method

Screen			Reciprocal of mean diameter (Energy factor)	Mill No. 1				Mill No. 2			
				Feed		Product		Feed		Product	
Mesh	Aperture, inch	Mean diameter of material on, inch		Per cent. weight	Relative surface	Per cent. weight	Relative surface	Per cent. weight	Relative surface	Per cent. weight	Relative surface
20	0.0335	0.0376	26.5	3.1	82	....	....	15.1	400	3.5	92
40	0.0147	0.0171	41.4	15.5	641	3.2	132	35.6	1,473	21.2	877
60	0.0091	0.01005	83.8	15.6	1,304	4.5	376	10.4	919	14.1	1,178
80	0.00675	0.00792	138	14.6	2,015	10.5	1,449	11.1	1,532	17.9	2,470
100	0.0055	0.00612	163	9.6	1,564	15.8	2,575	6.4	1,043	11.2	1,825
120	0.0043	0.0049	204	2.4	489	2.4	489	0.7	143	1.6	326
Through 120	....	0.00395	253	39.2	9,917	63.6	16,090	20.7	5,237	30.5	7,716
Total units of work..........			....	....	16,012	....	21,111	....	10,747	....	14,484

Table 101. Comparison of efficiencies through various screens by Del Mar method

Screen		Reciprocal of aperture	Mill No. 1				Mill No. 2				Comparison of unit efficiencies	
			Feed		Product		Feed		Product			
Mesh	Aperture, inch		Cumulative per cent. through screen	Relative surface	Cumulative per cent. through screen	Relative surface	Cumulative per cent. through screen	Relative surface	Cumulative per cent. through screen	Relative surface	Mill No. 1	Mill No. 2
20	0.0335	29.8	96.9	2887	100	2,980	84.9	2530	96.5	2875	228	1035
40	0.0147	68	81.4	5535	96.8	6,582	49.3	3352	75.3	5120	2,617	5304
60	0.0091	110	65.8	7238	92.3	10,153	38.9	4279	61.2	6732	7,287	7359
80	0.00675	148	51.2	7577	81.8	12,106	27.8	4114	43.3	6408	11,574	6882
100	0.0055	182	41.6	7571	66.0	12,012	21.4	3894	32.1	5842	11,102	5844
120	0.0043	232.5	39.2	9114	63.6	14,787	20.7	4812	30.5	7091	14,182	6837
(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(i)	(j)	(k)	(l)	(m)

Example 2. Comparison of efficiencies through various screens. If it is desired to know which of the mills compared in the preceding example is most efficient in crushing through a given screen, with no weight given to the character of material passing the screen in question, the method of calculation is shown in Table 101. Column (*l*) in the table is obtained by multiplying the differences between the corresponding numbers in columns (*e*) and (*g*) by 2.5, the tonnage crushed per hp.-hr. Similarly column (*m*) is obtained by multiplying the differences between columns (*i*) and (*k*) by 3.0. The table indicates that in crushing through 20-mesh, 40-mesh and 60-mesh, mill No. 2 is the more efficient; crushing through finer meshes, mill No. 1 is better.

Gates' crushing-surface diagram. In presenting a graphical method of comparing crushing efficiencies according to Rittinger's law, Gates (*95 J 1039*) says:

" . . . the energy absorbed by a lot of given sized particles is proportional to the product of their surface by their weight, which can be shown graphically by an area. And if we have a series of these areas, placed side by side, and representing the summation of the energies of the different sizes produced by a crushing operation, the total area is proportional to the energy expended on the rock. And when the cumulative weights of the different sizes are plotted consecutively, as in Fig. 45, the area between the sizing-analysis curve and the zero lines is proportional to the work expended on the rock in breaking it down from the infinite mass. And then as further crushing takes place on all or part of this rock and the new sizing-analysis is plotted, the area between the two curves is proportional to the further work done. This is the crushing-surface diagram."

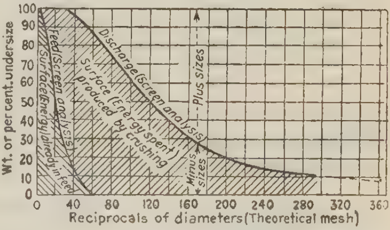


Fig. 45.—Crushing-surface diagram (*after Gates*).

In another article (*97 J 795*) Gates introduces a method for applying the crushing-surface diagram to a determination of crushing efficiencies. He defines a MESH-GRAM as a surface unit

" produced by a theoretical crushing operation in which one gram of particles of the same diameter is all reduced to a diameter whose reciprocal is one greater than before reduction."

Thus one gram of particles 0.1-in. diameter (reciprocal, 10), if crushed to particles 0.05-in diameter (reciprocal, 20), would result in the production of 10 mesh-grams of new surface. If the quantity so crushed were 1 ton, the new surface produced would be 10 mesh-tons. If, now, sizing analyses of the feed and product of a crushing operation are plotted with cumulative percentages as ordinates and reciprocals of diameters as abscissas (see Fig. 45) and the area between the feed and product curves is measured, this area, in terms of the scales used, will represent a certain number of MESH-PER CENT. of new surface produced, equal to 100 times the change in average size expressed in THEORETICAL MESH (= reciprocal of diameter). This figure multiplied by the tons crushed per horsepower-day gives MESH-TONS PER HORSEPOWER-DAY.

Example. Fig. 46 is plotted from data given in Table 102. The area between curves H_p and H_f and the ordinate 333 is 95.7 unit squares on the diagram and that between the same ordinate and curves C_p and C_f is 78.3 unit squares. Each unit square represents 200 mesh-per cent. Hence the new surface exposed by the Chilean-mill crushing is 15,660 mesh-per cent. and that by conical mill crushing, 19,140 mesh-per cent. The Chilean mill crushed 2.25 tons per hp.-day and the conical mill, 2.50. Multiplying 15,660 by 2.25 and 19,140 by 2.50, dividing in each case, also, by 100 to eliminate percentage, gives 392 mesh-tons per hp.-day as the performance of the Chilean mill, compared with 478 mesh-tons per hp.-day for the conical mill.

Table 102. Comparative screen analyses of conical pebble mills and Chilean mills at Miami Copper Co. *(47 A 50)*

Mesh	Reciprocal of aperture	Cumulative per cent. through			
		Conical pebble mill		Chilean mills	
		Feed	Product	Feed	Product
4	4.9	87.1	100.0	86.1	100.0
10	13.3	39.8	100.0	38.6	100.0
20	29.4	13.0	99.8	15.7	97.7
30	50.5	8.0	96.6	10.5	85.9
40	66.7	7.2	91.7	9.6	79.2
60	115	6.4	76.9	8.6	67.8
80	147	6.0	66.5	8.1	61.1
100	182	5.7	57.9	7.7	55.7
150	272.5	5.4	49.9	6.2	49.4
200	333	4.9	40.9	6.5	42.2

Kick *vs.* **Rittinger.** Controversy has raged for several years as to the correctness of the volume or surface method in estimating the useful work done in crushing. Details of the arguments may be had by reading the papers previously cited in this article and the citations in this paragraph. Convincing experimental data are lacking. The advocates of Rittinger's law in no place show that they consider the factor of distance in the equation "*work = the force applied multiplied by the distance through which it acts*," and the printed argument apparently neglects this essential element. Advocates of Kick's law must admit that the relation "*work done is proportional to the volume ratio of the particles in the feed and product*" can only be true if the rock breaks at or near the elastic limit, and that the further from the elastic limit fracture occurs, the weaker becomes the theoretical foundation for the methods based on Kick's law. But in any case, up to the elastic limit, Kick's law is in accord with accepted physical conceptions, and its weakness beyond the elastic limit in no way strengthens the position of the Rittinger advocates. The methods based on Rittinger's law credit the crusher with a much greater amount of useful work in producing very fine material than do those based on Kick, and, therefore, show higher efficiencies for fine crushing. Bell *(57 A 133)* presents the experimental data and calculations summarized in Table 103 to show that a gyratory crusher breaking from 3.5-in. to 1.0-in. and rolls breaking from 0.07-in. to 0.04-in. are working with substantially equal efficiency, according to the Rittinger method of calculation, while according to the Kick method the efficiencies are 710 and 52 respectively. He argues from this that because the Rittinger method indi-

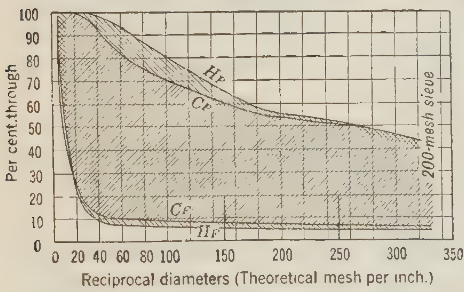


FIG. 46.—Application of Gates method to the data in Table 102.

cates that the amount of crushing done by a measured horsepower is the same whether the material being crushed is fine or coarse while the Kick method indicates more crushing per measured horsepower in breaking coarse material, the Rittinger method is correct and the Kick wrong. But the inefficiency of rolls crushing from 0.07-in. to 0.04-in. is notorious and the comparative efficiencies indicated by the Kick figures are much the more reasonable. Haultain, who has probably done more experimental work on the subject than anyone else, summarizes his conclusions (*69 A 183*) as follows: The energy usefully employed, neglecting friction and inertia losses in the machinery, is absorbed principally as (1) energy of new surface produced, (2) heat attendant upon deformation without breaking, (3) heat resulting from friction between particles and between particles and crushing surfaces. Neither method measures this absorbed energy accurately; the mean of the two is probably more nearly correct than either for roll crushing; no simple formula is applicable to all rocks and all methods of crushing. See also A. O.

Table 103. Comparison of work of crushing calculated according to Stadler, Kick and Rittinger methods by Bell

Crusher	Diameter of feed, inches	Crusher opening, inches	Work done per A.E.H.P.(*a*) per 24 hr.	
			Measured in Stadler E.U. (Kick)	Measured in Rittinger S.U.
Gyratory	3.5	1.0	710	947
Dodge, jaw	1.20	0.5	520	1030
Rolls	0.50	0.24	286	1128
Rolls	0.29	0.17	138	1000
Rolls	0.18	0.04	68	1022
Rolls	0.11	0.04	89	1187
Rolls	0.07	0.04	52	823

a Apparent effective horsepower.

Gates (*52 A 875*). A. M. Gaudin (*73 A 253*) recommends using the ratio of the energy of the new surface produced to the energy input as a measure of efficiency, thus charging the machine with items (2) and (3) above. The difficulty with this method lies in evaluating the surface energy of the broken rock. Based on figures for surface energy of solid quartz (*Fourth rep. on colloid chemistry, Brit. Assoc. Adv. Sci., p. 281*), Gaudin's figure for the efficiency of dry ball-milling from −0.5 +0.065-in. to 98 per cent. −200-mesh is 0.6 per cent. Bureau of Mines investigators, using a falling ball as the crushing force and rate of solubility as a measure of new surface produced have found that the relation between the force applied and the new surface produced is linear.

Choice of methods of comparison. When the operations to be compared are closely similar, both as to size of feed and size of product, it does not make any difference in the conclusion reached whether the Kick or Rittinger method is used. When, however, there is considerable difference in the character of the material entering and leaving the competing machines, the comparative efficiency of the finer crusher will be relatively higher by the Rittinger method. This will lead to the conclusion that economy will be gained in a crushing installation by putting more and more of the burden of crushing on the fine-crushing machines, a conclusion that is open to most serious question.

SECTION 5

SIZING

1. Introduction

Definitions. SIZING is the process of division of a mixture of grains of different sizes into groups or GRADES whose characteristic is that the particles therein are more or less nearly of the same size, that all have passed an aperture of certain dimensions and failed to pass through some smaller aperture. The screen through which the particles have passed is called the LIMITING SCREEN; that which has retained them is sometimes called the RETAINING SCREEN. CLOSE SIZING is practiced when the limiting and retaining screens are of nearly the same aperture. A mass of particles is said to be a NATURAL PRODUCT when it has all passed a given limiting screen but has been subjected to no further treatment. Material that stays on a given screen is the OVERSIZE or PLUS (+) of that screen; that passing is the UNDERSIZE or MINUS (−). A SIEVE SCALE is the list of apertures of successively smaller screens in a step-sizing operation. The SIEVE RATIO is the ratio of the aperture of a given screen in a given sieve scale to the aperture of the next finer screen. PERCENTAGE OF OPENING is the ratio of the combined area of the openings in a given area of screening surface to the total area of the surface. CLASSIFICATION is a process of approximate sizing in which a natural product is caused to settle through a fluid, usually water, but sometimes air, at rest or in motion. The settled products of a classifier are called SANDS or SPIGOT PRODUCTS according to whether the classifier is of the mechanical or hydraulic type, the fine overflowing material is called OVERFLOW or SLIME. Sizing with classifiers is discussed in Section 6.

Purposes of sizing: (*a*) to grade broken stone such as road metal, concrete aggregate, rock powders, etc.; (*b*) to remove fine material from crusher feeds and thus save power and prevent over-grinding; (*c*) to perform a step in a concentration process, *e.g.*, to size before jigging (see Sec. 9, Art. 1).

Principles. The fundamental elements in screening are the passage of undersize particles through the screen apertures and the rejection of oversize particles. In order to effect this choice, the particles must be brought to the openings and must be presented thereto at such a velocity and in such direction that passage of the undersize will not be hindered or prevented by rebound from the edges or walls of the opening. If every particle of undersize could be brought to an opening individually, at substantially zero velocity, in a direction perpendicular to the plane of the opening and with the center of its least projected cross-section in the line of the center of the aperture, and if

the screening surface had zero thickness, there would be immediate passage of every such particle. But tonnage demands forbid individual and low-velocity presentation, while mechanical considerations prevent perpendicular presentation and the use of screening surfaces of gossamer thickness. Practically, particles are crowded and continually interfering at the apertures, they are presented at considerably in excess of zero velocity, in a direction nearly parallel to the plane of the screen surface, with their maximum rather than their minimum projected surfaces parallel to the plane of the screen, and the screen opening has a depth frequently greater than the greatest dimension of the particle of undersize. As a result many undersize particles are prevented for a considerable time from access to an opening, others come to the opening from such a direction or with such orientation or at such velocity that they fail to enter, others entering are delayed in passage or stopped by friction against the walls of the opening.

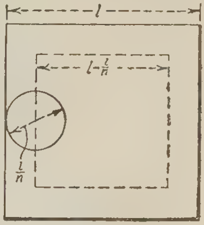


Fig. 1.

Chance rules the approach of a particle to hole or imperforate surface and the probability that the lowermost point of the approaching particle will strike hole rather than screen fabric is proportional to the percentage of opening in the fabric. Chance coupled with the relative dimensions of particle and opening rules in determining the passage of a particle presented to an opening. The problem may be stated in mathematical form as follows: If, in Fig. 1, the full-line square whose side is l represents a square screen aperture and the circle a spherical particle whose diameter is l/n (n being any number greater than l), then the probability of passage without touching the side of the aperture is proportional to the ratio of the area of the smaller square to the area of the larger, $= P = \left(\frac{n-1}{n}\right)^2$ and the probability that a grain will touch the side on presentation is $(1 - P)$. On the assumption that if the center of the particle falls within the inner square in Fig. 1, the particle will surely pass, while if it falls without this square, it will fail to pass, the values of P for different values of l/n are measures of the ease of screening particles of l/n diameter through l-sized square apertures and $1/P$ gives the number of such apertures side-by-side that should be provided. The first part of the assumption is true if l^2 is the area of the opening projected on a plane at right angles to the path of the particle; the second part must be modified in favor of passage, because although a particle presented with its center in the space between the squares will rebound in a direction and through a horizontal distance dependent upon the

Table 1. Probabilities in screening

Size of grain, l/n-in.	Probable chance per 1000 for unrestricted passage through l-in. square	Probable number of apertures in path of particle required to effect passage
0.001	998	1
0.01	980	2
0.1	810	2
0.2	640	2
0.3	490	2
0.4	360	3
0.5	250	4
0.6	140	7
0.7	82	12
0.8	40	25
0.9	9.8	100
0.95	2.0	500
0.99	0.1	10,000
0.999	0.001	1,000,000

Table 2. Time required to batch-screen material on 4.699-mm. gyrating screen. (*After Warner*)

Total time	Total undersize, gm.			Undersize percentages							Increment per second		
				On 3.327-mm.			Through 3.327-mm.			Of total feed	Grams		
	On 3.327-mm.	Through 3.327-mm.	Total	Of total feed	Of total undersize	Of total material +3.327-mm.	Of total feed	Of total undersize	Of total material through 3.327-mm.		+3.327-mm.	−3.327-mm.	Total
2 sec.	21.3	239.0	260.6	0.98	1.14	2.86	10.87	12.56	20.83	11.85	10.8	119.5	130.3
4 sec.	42.6	464.0	506.6	1.94	2.24	5.64	21.10	14.39	40.44	23.04	10.5	112.5	123.0
8 sec.	86.5	735.0	821.5	3.93	4.54	11.45	33.41	38.63	64.05	37.34	10.9	67.8	78.7
16 sec.	202.0	1048.0	1250.0	9.19	10.62	26.75	47.65	55.11	91.41	56.84	14.4	39.1	53.5
32 sec.	487.1	1144.1	1631.2	22.14	25.61	64.52	52.05	60.14	99.75	74.19	17.8	6.0	23.8
1 min.	606.2	1147.1	1753.3	27.56	31.87	80.30	52.19	60.34	100.00	79.75	8.5	0.2	8.7
2 min.	659.4	1147.1	1806.5	29.99	34.65	87.33	52.19	60.34	100.00	82.18	0.89	0	0.89
4 min.	682.4	1147.1	1829.5	31.04	35.87	90.35	52.19	60.34	100.00	83.23	0.19	0	0.19
8 min.	701.5	1147.1	1848.6	31.90	36.86	92.90	52.19	60.34	100.00	84.09	0.08	0	0.08
16 min.	714.4	1147.1	1861.5	32.50	37.55	94.65	52.19	60.34	100.00	84.69	0.03	0	0.03
32 min.	724.9	1147.1	1872.0	32.97	38.09	96.05	52.19	60.34	100.00	85.16	0.01	0	0.01
64 min.	738.4	1147.1	1885.5	33.58	38.83	97.80	52.19	60.34	100.00	85.77	0.007	0	0.007
128 min.	747.6	1147.1	1894.7	34.00	39.29	99.00	52.19	60.34	100.00	86.19	0.002	0	0.002
256 min.	755.0	1147.1	1902.1	34.33	39.68	100.00	52.19	60.34	100.00	86.52	0.001	0	0.001

Total sample 2197.7 gm.

shape of the edge of the aperture, the angle of approach, the velocity of approach and the shape of the particle, nevertheless some of the rebounding particles will fall with their centers within the inner square and pass. On the other hand, reduction in size of effective aperture by reason of screen inclination, acuteness of angle of approach of particle, interference by another particle of undersize simultaneously presented, and covering of part of the opening by oversize particles, all decrease the opportunities for passage. None of these modifying factors is capable of mathematical statement, hence the equation serves merely as a means of indicating the probable effect on rate of screening of the ratio of particle size to aperture. To this extent Table 1, derived from the formula, is illuminating. It indicates that as the particle size approaches that of the aperture the difficulty of passage becomes extremely great, even excluding all factors other than pure chance that militate against passage. Warner (*70 A 631*) gives the data summarized in Table 2 as the results of a test in which a sample of broken rock was screened on a standard 4.699-mm. testing sieve in a Ro-Tap shaker (Sec. 22, Art. 2) and the undersize products passed during the successive screening intervals were separately

collected, weighed and subjected to analysis on a 3.327-mm. screen. This work is qualitative confirmation of the theory, modified, however, by the practical elements of crowding, particle velocity, etc., as previously enumerated. The column headed "Increment per sec., grams +3.327-mm." holds the key to the operation from the point of view of screening rate. In the first three intervals the passage of even the smallest particles was interfered with by the mass of smaller material going through. By the time the fifth interval was reached most of the −3.327-mm. material had passed and by the end of this interval the bulk of the finer part of the +3.327-mm. material had also gone through. From that time there was a marked decrease in the rate of screening corresponding remarkably in order to the last four items in Table 1 and indicating that the material passing the screen in this part of the operation was of a size about 0.9-times the diameter of the screen opening and upwards.

The important fact as regards capacity in close sizing is the proportion of undersize or oversize that is of nearly the same size as the aperture. Undersize much smaller than the screen openings passes through very rapidly, so rapidly in fact that, if it is given free access to the holes, it goes through almost like water, and no normal variation in the amount of such material has any effect on screen capacity. Likewise particles much coarser than the openings, when sliding over a screening surface, have such large interstitial spaces with comparison to undersize particles that the latter pass down freely to the screen surface. But when the proportion of particles of a size near that of the screen aperture is large, these particles, which have interstitial spaces that are relatively small, hold back the fine material from the screen; the particles only slightly larger than the screen apertures wedge in and cause blinding, and those only slightly smaller than the aperture pass through with difficulty, therefore slowly. The result, as shown by Warner's work and the above theoretical analysis is marked reduction in screen capacity.

The velocity of approach of particles to the screen together with the direction of approach have a marked effect on screening rate. If a spherical particle is traveling at a given rate along a screening surface whose apertures are several times the diameter of the particle, as in Fig. 2(*a*), its path upon leaving the support of the screen fabric will be somewhat as illustrated and the particle will pass through. With a smaller aperture the same particle will strike the far edge of the opening. If the particle is perfectly spherical and elastic and the aperture has square edges and vertical sides, such particle, as pictured in Fig. 2(*b*), will also pass; but with irregular particles and with apertures having rounded edges and converging sides there is every reason to expect that a particle following the path illustrated would fail to pass through. If the initial approach of the particle is from above and at an angle to the screen surface as illustrated at (*1*) in Fig. 2(*c*), the direction of approach and high velocity both aid in passing the particle, if it strikes an opening, but if it fails to do so, as at (*2*), then high velocity, by causing the particle to bound, will tend to defer its opportunity to pass and thus decrease screening rate. Since the percentage of opening in screens is usually less than 50, the evil effect of bounding may be expected to overbalance the good.

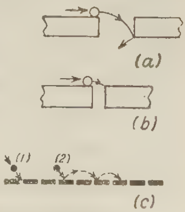


Fig. 2.

When, as in all practical screening operations, a mass of particles many grains deep is presented to a screen surface, coarse and fine grains are likely to be mixed indiscriminately, with the result that many oversize particles are

in contact with the screen and many undersize particles are supported in the mass above and away from the screen surface. If the particles are free to move among themselves, *i.e.*, if the material is not sticky, any subsequent movement of the mass will cause rearrangement of the particles. If the movement is sufficient to enliven the mass thoroughly without lifting it bodily from the screen, stratification occurs, with the finer particles at the bottom and the coarsest on top. Such stratification is essential to screening and is as much the purpose of the motion imparted to screening surfaces as are transport of oversize and prevention of blinding. Excessive movement, while it increases the ability of the screen to move material across its surface, defeats stratification and thereby decreases screen efficiency.

2. Screening efficiency.

Methods of reckoning screen efficiency are not uniformly established. The usual method is to express efficiency as the ratio of the weight of undersize obtained to the weight of undersize in the feed. This latter figure is determined by screening a feed sample on a hand screen of the same aperture as that of the screen under investigation, or by making a sizing test of the feed on the usual testing sieves, interpolating for the aperture of the operating screen and multiplying the percentage of undersize thus obtained by the tonnage treated. Weighing feed and undersize can be obviated by making screen analyses of representative samples of feed and oversize. The recovery formula (Sec. 22, Art. 15) then applies and efficiency $= E = 100\left(\frac{100(e - v)}{e(100 - v)}\right)$ where e and v are percentages of undersize in feed and oversize respectively. On this basis of calculation, Wiard (*14 CME 191*) states that the average efficiency in commercial practice is about 60 per cent., that 75 per cent. is extremely good work and 90 per cent. about as high as can be reached by hand screening. Warner criticizes the use of this formula on the basis of the experimental work shown in Table 2. He argues, correctly, that it does not take into consideration sufficiently the fact that 1-mm. material is readily passed through a 10-mm. screen while 9-mm. material passes the same screen with great difficulty, and suggests that the screen analysis of feed and of oversize concern itself only with the DIFFICULT GRAINS, which he defines for this purpose as those held on the next finer Tyler standard sieve-scale testing sieve whose aperture is less than 83 per cent. of the aperture of the sieve under investigation. He then applies the recovery formula in the form $E = \frac{U(F - O)}{F(U - O)}$, where F, U, and O are percentages of difficult grains in feed, undersize and oversize respectively.

In many places the practical efficiency of a screen is sufficiently measured by the percentage of undersize in the oversize. This is particularly the case when the screen is preparing feed for jigs, or returning oversize for re-grinding, or preparing for sale a product against which there are rigid specifications as to percentage of undersize.

Efficiency is markedly decreased by high feed rate, especially if accompanied by high percentages of oversize and difficult grains, because on a heavily loaded screen, fine material has difficulty in getting down to the screen surface and blinding is caused by wedging of particles into the screen openings on account of the weight of the bed.

Screen cloth should be tightly stretched, because material piles up and forms thick layers in depressions. Dry ore screens better if warm (100–150° F.).

3. Screening surfaces

There are three varieties of screening surfaces: parallel rods, punched plates, and woven-wire or silk cloth. The minimum clear space between the

Table 3. Wire-gage numbers and corresponding wire diameters in inches

Number	Steel-wire gage or Washburn and Moen	Birmingham or Stubs	American or Brown & Sharpe	United States or U. S.	Old English	Imperial or English standard
0000	.393	.454	.460	.40625	.454	.400
000	.362	.425	.40964	.375	.425	.372
00	.331	.380	.36480	.34375	.380	.348
0	.307	.340	.32495	.3125	.340	.324
1	.283	.300	.28930	.28125	.300	.300
2	.263	.284	.25763	.26562	.284	.276
3	.244	.259	.22942	.25	.259	.252
4	.225	.238	.20431	.234375	.238	.232
5	.207	.220	.18194	.21875	.220	.212
6	.192	.203	.16202	.203125	.203	.192
7	.177	.180	.14428	.1875	.180	.176
8	.162	.165	.12849	.171875	.165	.160
9	.148	.148	.11443	.15625	.148	.144
10	.135	.134	.10189	.140625	.134	.128
11	.120	.120	.09074	.125	.120	.116
12	.105	.109	.08081	.109375	.109	.104
13	.092	.095	.07196	.09375	.095	.092
14	.080	.083	.06408	.078125	.083	.080
15	.072	.072	.05707	.070312	.072	.072
16	.063	.065	.05082	.0625	.065	.064
17	.054	.058	.04525	.05625	.058	.056
18	.047	.049	.04030	.05	.049	.048
19	.041	.042	.03589	.04375	.040	.040
20	.035	.035	.03196	.0375	.035	.036
21	.032	.032	.02846	.034375	.0315	.032
22	.028	.028	.025347	.03125	.0295	.028
23	.025	.025	.022571	.028125	.027	.024
24	.023	.022	.0201	.025	.025	.022
25	.020	.020	.0179	.021875	.023	.020
26	.018	.018	.01594	.01875	.0205	.018
27	.017	.016	.014195	.0171875	.01875	.0164
28	.016	.014	.012641	.015625	.0165	.0148
29	.015	.013	.011257	.0140625	.0155	.0136
30	.014	.012	.010025	.0125	.01375	.0124
31	.0132	.010	.008928	.0109375	.01225	.0116
32	.0128	.009	.00795	.010156	.01125	.0108
33	.0118	.008	.00708	.009375	.01025	.0100
34	.0104	.007	.0063	.008593	.0095	.0092
35	.0095	.005	.00561	.007812	.009	.0084
36	.0090	.004	.005	.007031	.0075	.0076
37	.0085	....	.00445	.006640	.0065	.0068
38	.0080	....	.003965	.00625	.00575	.0060
39	.0075	....	.003531	....	.005	.0052
40	.0070	....	.003144	....	.0045	.0048
41	.0066	....	....	....	....	....
42	.0062	....	....	....	....	....
43	.0060	....	....	....	....	....
44	.0058	....	....	....	....	....
45	.0055	....	....	....	....	....
46	.0052	....	....	....	....	....
47	.0050	....	....	....	....	....
48	.0048	....	....	....	....	....
49	.0046	....	....	....	....	....
50	.0044	....	....	....	....	....

Table 4. Mesh and aperture of steel-wire screens

Mesh designation	Range in opening		Range in diameter of wire used		Number of screens in range	Range of wire numbers employed (a)	Range in percentage of opening	Approximate weight, pounds per square foot
	In.	Mm.	In.	Mm.				
4-in. *H*	4.0	101.6	1.0 –0.375	25.4 –9.53	8	1-in.–000	64.0–83.4	17.5 –2.9
3¾-in. *H*	3.75	95.2	1.0 –0.3125	25.4 –7.95	9	1-in.–0	62.3–85.0	20.4 –2.2
3½-in. *H*	3.5	88.8	1.0 –0.3125	25.4 –7.95	9	1-in.–0	60.5–84.1	21.8 –2.4
3¼-in. *H*	3.25	82.5	1.0 –0.3125	25.4 –7.95	9	1-in.–0	58.5–83.2	24.5 –2.5
3-in. *H*	3.0	76.2	1.0 –0.25	25.4 –6.35	10	1-in.–3	56.1–85.0	25.0 –1.8
2¾-in. *H*	2.75	69.8	1.0 –0.25	25.4 –6.35	10	1-in.–3	53.8–84.0	27.2 –1.9
2½-in. *H*	2.5	63.5	1.0 –0.225	25.4 –5.72	11	1-in.–4	51.1–84.1	29.7 –1.7
2¼-in. *H*	2.25	57.2	1.0 –0.207	25.4 –5.26	12	1-in.–5	47.9–83.9	32.7 –1.6
2-in. *H*	2.0	50.8	1.0 –0.192	25.4 –4.88	13	1-in.–6	44.5–83.2	36.7 –1.5
1¾-in. *H*	1.75	44.4	1.0 –0.192	25.4 –4.88	13	1-in.–6	40.6–81.7	41.8 –1.8
1½-in. *H*	1.5	38.1	1.0 –0.177	25.4 –4.50	14	1-in.–7	36.0–71.6	48.6 –1.7
1¼-in. *H*	1.25	31.8	0.75 –0.177	19.03 –4.50	13	¾-in.–7	39.0–76.8	33.0 –2.1
1-in. *H*	1.0	25.4	0.75 –0.162	19.03 –4.12	14	¾-in.–8	32.7–74.2	41.1 –2.1
1-in.	0.693–0.928	17.60–24.60	0.307 –0.072	7.81 –1.83	16	0–15	48.1–86.1	10 –0.5
⅞-in. *H*	0.875	22.2	0.625 –0.162	15.88 –4.12	12	⅝-in.–8	34.0–71.2	31.7 –2.4
¾-in. *H*	0.75	19.03	0.625 –0.148	15.88 –3.76	13	⅝-in.–9	29.8–69.7	40.6 –2.4
¾-in.	0.467–0.687	11.85–17.45	0.283 –0.063	7.19 –1.60	16	1–16	38.8–83.9	12 –0.5
⅝-in. *H*	0.625	15.88	0.5625–0.135	14.30 –3.43	13	9/16-in.–10	27.8–67.6	37.2 –2.3
⅝-in.	0.362–0.578	9.18–14.68	0.263 –0.047	6.68 –1.19	17	2–18	33.5–85.4	14 –0.3
½-in. *H*	0.5	12.69	0.4375–0.105	11.11 –2.67	13	7/16-in.–12	28.4–68.2	27.6 –1.7
7/16-in. *H*	0.4375	11.10	0.307 –0.105	7.81 –2.67	13	0–12	34.5–65.0	15.8 –2.0
2	0.275–0.459	6.98–11.67	0.225 –0.041	5.72 –1.04	16	4–19	30.2–84.2	13 –0.3
⅜-in. *H*	0.375	9.52	0.307 –0.105	7.81 –2.67	13	0–12	30.2–61.0	18.5 –2.2
5/16-in. *H*	0.3125	7.93	0.225 –0.105	5.72 –2.67	9	4–12	33.8–56.0	11.7 –2.8
¼-in. *H*	0.25	6.35	0.225 –0.092	5.72 –2.34	10	4–13	27.6–53.4	14.7 –2.5
2½	0.208–0.365	5.28–9.27	0.192 –0.035	4.88 –0.89	15	6–20	27.0–83.2	13 –0.3
3/16-in. *H*	0.1875	4.76	0.192 –0.092	4.88 –2.54	8	6–13	24.4–45.0	14.4 –3.3

3	0.171 –0.301	4.34 –7.65	0.162 –0.032	4.12 –0.81	14	8–21	26.4–81.7	9 –0.3
3½	0.138 –0.254	3.51 –6.45	0.148 –0.032	3.76 –0.81	13	9–21	23.3–78.8	12 –0.3
4	0.115 –0.222	2.92 –5.64	0.135 –0.028	3.43 –0.71	13	10–22	21.1–78.8	12 –0.3
4½	0.102 –0.197	2.59 –5.00	0.120 –0.025	3.05 –0.64	13	11–23	21.1–78.8	10 –0.25
5	0.095 –0.177	2.41 –4.50	0.105 –0.023	2.67 –0.56	13	12–24	22.5–78.3	8.5 –0.24
6	0.075 –0.147	1.90 –3.74	0.092 –0.020	2.34 –0.51	13	13–25	20.2–77.3	8.5 –0.22
7	0.063 –0.125	1.60 –3.18	0.080 –0.018	2.03 –0.46	13	14–26	19.4–76.3	7.5 –0.21
8	0.053 –0.108	1.35 –2.74	0.072 –0.017	1.83 –0.43	13	15–27	18.0–74.6	7.1 –0.22
9	0.048 –0.095	1.22 –2.41	0.063 –0.016	1.60 –0.41	13	16–28	18.8–73.2	6.2 –0.22
10	0.046 –0.085	1.17 –2.16	0.054 –0.015	1.37 –0.38	13	17–29	21.1–72.1	4.7 –0.21
12	0.036 –0.069	0.915–1.75	0.047 –0.014	1.19 –0.36	13	18–30	19.0–69.2	4.6 –0.21
14	0.030 –0.0606	0.712–1.54	0.041 –0.0104	1.04 –0.26	16	19–31	18.2–73.0	4.3 –0.15
16	0.0215–0.0530	0.546–1.35	0.041 –0.0095	1.04 –0.24	17	19–35	11.8–72.0	6.2 0.13
18	0.0206–0.0466	0.523–1.18	0.035 –0.0090	0.89 –0.228	17	20–36	13.7–70.2	4.5 –0.13
20	0.0220–0.0410	0.558–1.04	0.028 –0.0090	0.71 –0.228	15	22–36	19.4–67.2	2.7 –0.15
22	0.0175–0.0365	0.445–0.926	0.028 –0.0090	0.71 –0.228	15	22–36	14.8–64.2	3.5 –0.17
24	0.0187–0.0327	0.475–0.830	0.023 –0.0090	0.56 –0.228	13	24–36	20.0–61.4	2.1 –0.19
26	0.0205–0.0295	0.521–0.749	0.018 –0.0090	0.46 –0.228	11	26–36	28.3–58.7	1.1 –0.20
28	0.0187–0.0267	0.475–0.678	0.017 –0.0090	0.43 –0.228	10	27–36	27.5–56.0	1.1 –0.22
30	0.0163–0.0243	0.414–0.617	0.017 –0.0090	0.43 –0.228	10	27–36	24.0–53.2	1.3 –0.24
32	0.0153–0.0223	0.388–0.567	0.016 –0.0090	0.41 –0.228	9	28–36	23.9–50.8	1.3 –0.26
35	0.0126–0.0196	0.320–0.497	0.016 –0.0090	0.41 –0.228	9	28–36	19.4–46.8	1.6 –0.28
38	0.0123–0.0168	0.312–0.427	0.014 –0.0085	0.36 –0.216	6	30–35	21.9–40.8	1.2 –0.31
40	0.0118–0.0165	0.300–0.419	0.0132–0.0085	0.34 –0.216	7	31–37	22.3–43.5	1.1 –0.31
45	0.0094–0.0137	0.238–0.348	0.0128–0.0085	0.32 –0.216	6	32–37	18.0–38.2	1.2 –0.38
50	0.0096–0.0120	0.244–0.305	0.0104–0.0080	0.26 –0.203	5	34–38	23.0–36.0	0.85–0.36
55	0.0087–0.0102	0.221–0.259	0.0095–0.0080	0.24 –0.203	4	35–38	22.8–31.3	0.77–0.42
60	0.0072–0.0092	0.183–0.234	0.0095–0.0075	0.24 –0.190	5	35–39	18.5–30.2	0.93–0.49
64	0.0071–0.0081	0.180–0.206	0.0085–0.0075	0.216–0.190	3	37–39	20.8–27.0	0.74–0.57
70	0.0058–0.0068	0.147–0.172	0.0085–0.0075	0.216–0.190	3	37–39	16.4–22.6	0.93–0.68
74	0.0060–0.0065	0.152–0.165	0.0075–0.0070	0.190–0.178	2	39–40	19.8–23.2	0.79–0.59
80	0.0055–0.0059	0.140–0.150	0.0070–0.0066	0.178–0.167	2	40–41	19.4–22.3	0.68–0.49
90	0.0049	0.124	0.0062	0.157	1	42	19.6	0.59

a Washburn and Moen gage, see Table 3. *H* Heavy.

edges of the opening in a screening surface is called the APERTURE or SCREEN SIZE. Aperture is expressed in several ways. The best way, undoubtedly, is to give the dimension in inches or millimeters; the commonest, particularly with woven screens, is to express it as so many MESH, meaning the number of openings in the screen per linear inch. This latter method is definite only when coupled with a statement of the size of wire, or when referred to one of the testing-sieve scales (Sec. 22, Art. 2). When size of wire is given it should be expressed in ordinary units of measure, *i.e.*, inches or millimeters; if gage numbers are used the gage must be named, on account of the differences in dimensions corresponding to the same gage number in different gage scales. See Table 3. Given two of the quantities a = aperture, d = diameter of wire, and m = mesh, the percentage of opening, P, and the other quantity can be determined from the following relations:

$$P = a^2m^2 = \frac{a^2}{(a + d)^2} = (1 - md)^2;\ m = \frac{1}{a + d};\ a = \frac{1}{m} - d;\ d = \frac{1}{m} - a.$$

The mesh of punched-plate screens does not mean the number of openings per linear inch of punched plate but rather that the aperture has a dimension near that of some woven-wire screen of the given mesh. The equivalent woven-wire screen is usually a medium-weight screen. If the largest and smallest apertures corresponding to a given mesh in Table 4 are averaged, the result will be close to the punched-plate aperture called by that mesh. Certain districts, notably South Africa, express aperture as SQUARE MESH, *i.e.*, the number of apertures per square inch, which is even less informing than the usual American terminology. NEEDLE MESH is a number used to designate an aperture and corresponds to that of a needle of the same or nearly the same diameter as the dimension of the opening. See Table 5. This is

Table 5. Needle mesh

Needle number	Largest diameter of head, inch	Equivalent screen
1	0.0735	10-mesh.
2	0.0428	16-mesh; No. 3 slot.
3	0.0395	16- to 18-mesh; No. 3 to No. 4 slot.
4	0.0355	20- to 24-mesh; No. 4 to No. 5 slot (nearer the last).
5	0.0335	20- to 24-mesh; No. 4 to No. 5 slot (nearer the last).
6	0.0300	24-mesh; No. 5 to No. 6 slot.
7	0.0265	24- to 26-mesh; No. 6 to No. 7 slot.
8	0.0230	26-mesh; No. 8 slot.
9	0.0203	29-mesh; No. 9 slot.

the acme of inconvenience, although actually more precise than mesh designation. Table 4 gives dimensions of square-mesh steel-wire cloth corresponding to mesh designations and showing the possible range in aperture for a given mesh cloth of standard manufacture. Table 6 gives dimensions corresponding to wire-gauge numbers in column 7 of Table 4. To determine the aperture of intermediate screens within the ranges given in Table 4 proceed as follows: From Table 6 pick wire of a number included within the range of column 7 of Table 4. Substitute the corresponding diameter in the equation $a = 1/m - d$ (see above).

Table 6. Diameter, weight per foot, and feet per pound, steel wire. Washburn and Moen gauge. (*W. S. Tyler Co.*)

Number of wire	Inch	Millimeters	Pounds per foot	Feet per pound
0000	.3938	10.00	.4136	2.418
000	.3625	9.2075	.3505	2.853
00	.3310	8.407	.2922	3.422
0	.3065	7.785	.2506	3.991
1	.2830	7.188	.2136	4.681
2	.2625	6.668	.1838	5.441
3	.2437	6.190	.1584	6.313
4	.2253	5.723	.1354	7.386
5	.2070	5.258	.1143	8.750
6	.1920	4.877	.09832	10.17
7	.1770	4.496	.08356	11.97
8	.1620	4.115	.07000	14.29
9	.1483	3.767	.05866	17.05
10	.1350	3.429	.04861	20.57
11	.1205	3.061	.03873	25.82
12	.1055	2.680	.02969	33.69
13	.0915	2.324	.02233	44.78
14	.0800	2.032	.01707	58.58
15	.0720	1.829	.01383	72.32
16	.0625	1.588	.01042	95.98
17	.0540	1.372	.007778	128.60
18	.0475	1.207	.006018	166.20
19	.0410	1.041	.004484	223.00
20	.0348	.8839	.003230	309.60
21	.0317	.8052	.002680	373.10
22	.0286	.7264	.002182	458.4
23	.0258	.6553	.001775	563.3
24	.0230	.5842	.001411	708.7
25	.0204	.5182	.001110	900.9
26	.0181	.4597	.0008738	1,144
27	.0173	.4394	.0007983	1,253
28	.0162	.4115	.0007000	1,429
29	.0150	.3810	.0006001	1,666
30	.0140	.3556	.0005228	1,913
31	.0132	.3353	.0004647	2,152
32	.0128	.3251	.0004370	2,288
33	.0118	.2997	.0003714	2,693
34	.0104	.2642	.0002885	3,466
35	.0095	.2413	.0002407	4,154
36	.0090	.2286	.0002160	4,629
37	.0085	.2159	.0001927	5,189
38	.0080	.2032	.0001707	5,858
39	.0075	.1905	.0001500	6,665
40	.0070	.1778	.0001307	7,652
41	.0066	.1676	.0001162	8,607
42	.0062	.1575	.0001025	9,753
43	.0060	.1524	.00009602	10,415
44	.0058	.1473	.00008972	11,145
45	.0055	.1397	.00008068	12,394
46	.0052	.1321	.00007212	13,866
47	.0050	.1270	.00006668	14,997
48	.0048	.1219	.00006145	16,273
49	.0046	.1168	.00005644	17,718
50	.0044	.1118	.00005164	19,366

Materials. Square-mesh wire screens of substantially the same meshes and apertures as listed in Table 4 are obtainable in brass, copper, bronze, nickel and Monel metal, although all of the heavier screens are special and very rarely made. The screening surface must be strong enough to carry

the load and it should resist, so far as possible, abrasion and corrosion. At the same time it should be cheap. When corrosion is not a factor screens are normally made of high-carbon steel and in certain cases of special alloy steels. Steel is stronger and resists abrasion better than any other material available. When corrosion must be resisted, iron, copper, bronze, Monel metal and other alloys are used. Such materials cost much more than steel initially, and are not so highly resistant to abrasion. Their use is justified only when the final cost, by reason of their greater resistance to corrosion, is less than that of steel screens.

Punched plate was formerly used more widely than at present. It consists of metal plate symmetrically punched with apertures that will pass particles of the desired size and hold back all larger. Methods of arranging holes and various shapes of holes are shown in Fig. 3. Substantially any

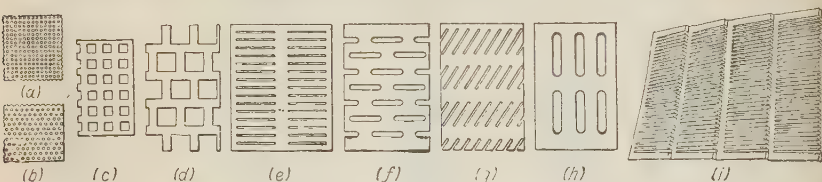


FIG. 3.—Types of punched-plate screen.

desired length is obtainable (limited only by the maximum lengths of rolled plate). The usual widths are 3 to 4 ft. but 6-ft. widths can be had. Maximum thickness is limited to about ½ in. by difficulty in punching. Apertures in punched plate are circular in most coarse screens, the circular form being used because it is cheaper to punch than the square and yet has 90 per cent. as much clear opening for a given strength of screen. Due, however, to the fact that the effective area of circular openings is so small in fine screens (see Table 7), slotted openings are most commonly used in these screens. The product of a slotted screen is not so irregular as the shape of the slot would indicate for the reason that most ores break in more or less granular fashion and there are relatively few needle-like or platy particles to take advantage of the slotted hole.

Very fine screens are so punched that the punch does not go completely through and remove a definite particle of metal but merely breaks or tears apart the plate, leaving a decided burr on the lower side. Holes in clear-punched plates diverge toward the die side. All punched-plate screens should be fed onto the side having the smaller opening in order to give a flaring passage that will tend to discharge freely all material that enters. Slotted screens are less easily clogged than square- or round-punched. Round holes retain smaller particles than square or slotted holes of the same clear aperture.

Roesler (*99 J 493*) showed that a square aperture in wire cloth would pass particles of the same size as a circular aperture whose diameter was 1.23 times the edge of the square opening.

The arrangement of round holes at the corners of equilateral triangles gives a greater percentage of opening than the arrangement with holes at the corners of squares. For diagonal spacing $P = 0.905d^2/(s + d)^2$; for square spacing $P = 0.785d^2/(s + d)^2$, where d = diameter of hole and s = least

Table 7. Typical punching for round-hole plate screens

Diameter of hole		Space between holes		Maximum thickness of plate		Percentage of opening		Weight of unpunched plate, pounds per square foot	Weight of punched plate, pounds per square foot	
In.	Mm.	In.	Mm.	Birmingham gage	In.	Diagonal spacing	Rectangular spacing		Diagonal spacing	Rectangular spacing
1⁄16	1.59	1⁄16	1.59	18	0.049	22.7	19.6	1.99	1.54	1.59
1⁄12	2.12	7⁄96	1.85	16	0.065	25.6	22.4	2.64	1.97	2.05
3⁄32	2.38	3⁄32	2.38	16	0.065	22.7	19.6	2.64	2.04	2.12
1⁄8	3.18	1⁄8	3.18	14	0.083	22.7	19.6	3.37	2.60	2.71
5⁄32	3.97	3⁄32	2.38	12	0.109	35.5	30.6	4.43	2.86	3.07
3⁄16	4.76	1⁄8	3.18	10	0.134	32.6	28.3	5.45	3.67	3.91
7⁄32	5.56	5⁄32	3.97	10	0.134	30.9	26.7	5.45	3.76	3.99
1⁄4	6.35	1⁄8	3.18	8	0.165	40.3	34.9	6.71	4.01	4.37
5⁄16	7.94	3⁄16	4.76	....	0.187	35.4	30.6	7.62	4.92	5.29
3⁄8	9.52	3⁄16	4.76	....	0.187	40.3	34.9	7.62	4.55	4.96
7⁄16	11.11	5⁄16	7.94	....	0.25	30.8	26.7	10.16	7.03	7.45
1⁄2	12.70	1⁄4	6.35	....	0.25	40.3	34.9	10.16	6.06	6.61
9⁄16	14.29	3⁄16	4.76	....	0.25	51.0	44.1	10.16	4.98	5.68
5⁄8	15.88	1⁄4	6.35	....	0.25	46.2	40.0	10.16	5.46	6.10
3⁄4	19.05	1⁄4	6.35	....	0.25	51.0	44.1	10.16	4.98	5.68
3⁄4	19.05	3⁄8	9.52	....	0.25	40.3	34.9	10.16	6.06	6.61
7⁄8	22.22	3⁄8	9.52	....	0.25	44.3	38.5	10.16	5.66	6.25
1	25.40	1⁄2	12.70	....	0.25	40.3	34.9	10.16	6.06	6.61
1 3⁄8	34.90	5⁄8	15.88	....	0.312	42.9	37.1	12.70	7.25	7.99
1 1⁄2	38.10	5⁄8	15.88	....	0.312	45.2	39.2	12.70	6.96	7.72
1 3⁄4	44.45	3⁄4	19.05	....	0.312	44.3	38.5	12.70	7.08	7.81
2	50.80	5⁄8	15.88	....	0.5	52.7	45.5	20.33	9.62	11.09
2 1⁄4	57.15	3⁄4	19.05	....	0.5	51.0	44.1	20.33	9.98	11.37
2 3⁄8	60.32	3⁄4	19.05	....	0.5	52.7	45.5	20.33	9.62	11.09
2 1⁄2	63.50	3⁄4	19.05	....	0.5	53.6	46.3	20.33	9.44	10.91
2 5⁄8	66.68	3⁄4	19.05	....	0.5	54.8	47.1	20.33	9.20	10.77
2 3⁄4	69.85	3⁄4	19.05	....	0.5	56.0	48.7	20.33	8.95	10.44
2 7⁄8	73.02	3⁄4	19.05	....	0.5	57.0	49.5	20.33	8.75	10.28
3	76.20	3⁄4	19.05	....	0.5	58.0	50.2	20.33	8.54	10.13

width of clear metal between holes. With holes ½-diameter apart, diagonal spacing gives 40 per cent. discharge area against 35 per cent. for square spacing; with holes one diameter apart the corresponding percentages are 22.5 and 20.

Table 7 gives the essential data concerning the usual sizes of round-hole punched-plate screen. It is usually more economical to use thinner plate than that given in the table except on coarse sizes, since such plate offers less resistance to the passage of undersize grains. Table 8 gives sizes and weights of needle-slot screens. Table 9 gives the usual sizes of holes and spacing in oblong-hole punched plate.

Table 8. Needle-slot screens (*Harrington and King Perforating Co.*)

Manufacturer's numbers	Mesh	Width of slot		Percentage of opening(a)			Usual plate		
		In.	Mm.	Diagonal punching	Straight punching	Hit-and-miss endway (b)	Gage	Thickness, inch	Weight, pounds per sq. ft.
1	12	0.058	1.47	45	28	....	16	0.0625	2.55
2	14	0.049	1.24	38	29	....	16	0.0625	2.55
3	16	0.042	1.07	33	25	....	18	0.05	2.04
4	18	0.035	0.89	35	29	....	18	0.05	2.04
5	20	0.032	0.81	32	26	....	18	0.05	2.04
6	28	0.028	0.71	28	23	....	20	0.0375	1.53
7	30	0.025	0.64	33	31	31	20	0.0375	1.53
....	......	0.0225	0.57	....	....	28	20	0.0375	1.53
8	35	0.022	0.56	29	26	....	20	0.0375	1.53
9	40	0.020	0.51	27	24	25	22	0.0312	1.28
10	50	0.018	0.46	24	22	23	23	0.028	1.15
....	60	0.016	0.41	....	....	20	23	0.028	1.15
11	65	0.015	0.38	20	18	....	24	0.025	1.02
....	70	0.0145	0.37	....	....	23	24	0.025	1.02
....	75	0.013	0.33	....	....	20	24	0.025	1.02
12	80	0.012	0.30	16	14	19	26	0.0187	0.76
....	90	0.011	0.28	....	....	17	26	0.0187	0.76
....	100	0.010	0.25	....	....	16	26	0.0187	0.76

a This is for usual punching. Other punching may be had on order. *b* Slots 1$\frac{1}{32}$ in. long.

Percentage of opening for hit-and-miss endways punching is

$$P = \frac{1}{4}\left[\frac{\pi d^2 + 4d(l - d)}{(s + d)(l + 0.866s - 0.134d)}\right],$$

where d = width of opening, s = spacing at sides and ends of slots and l = length of slots. For straight punching and hit-and-miss sideways (Fig. 3(*e*) and 3(*f*) respectively) the formula is

$$P = \frac{1}{4}\left[\frac{\pi d^2 + 4d(l - d)}{(s + d)(s + l)}\right].$$

Advantages of punched-plate screens are stiffness, strength, freedom from blinding, and long life. Long life is gained, however, by making the plates thick and allowing the screens to wear until the metal between holes breaks. There is thus a large variation in size of opening between new and old screens and close sizing is sacrificed. Blinding increases as the holes become hopper-shaped through wear. Punched plate is heavy, hard to handle,

Table 9. Oblong-punched plate screens

Width of opening		Length of opening		Spacing		Percentage of opening	
In.	Mm.	In.	Mm.	In.	Mm.	Hit-and-miss endways	Straight, Hit-and-miss sideways
1/8	3.18	1/2	12.7	1/8	3.18	40.2	37.8
1/8	3.18	1	25.4	3/16	4.76	34.0	33.1
1/8	3.18	1 1/4	31.8	1/4	6.35	28.1	27.2
5/32	3.97	5/8	15.9	5/32	3.97	39.9	38.9
5/32	3.97	3/4	19.0	5/32	3.97	41.4	39.5
5/32	3.97	1 1/4	31.8	7/32	5.55	35.6	34.4
3/16	4.76	7/16	11.1	1/8	3.18	45.6	42.1
3/16	4.76	3/4	19.0	3/16	4.76	39.8	37.6
3/16	4.76	1	25.4	3/16	4.76	42.0	40.2
3/16	4.76	1 1/2	38.2	3/16	4.76	44.4	43.1
1/4	6.35	1/2	12.7	1/4	6.35	32.4	29.4
1/4	6.35	1	25.4	1/4	6.35	39.8	37.6
1/4	6.35	1 1/4	31.8	1/4	6.35	41.7	39.8
5/16	7.93	5/8	15.9	3/16	4.76	47.0	43.0
3/8	9.52	1 1/4	31.8	5/16	7.93	44.4	40.8
3/8	9.52	2	50.8	5/16	7.93	47.1	45.3
7/16	11.1	7/8	22.2	5/16	7.93	41.3	38.4
1/2	12.7	1	25.4	1/4	6.35	51.7	47.5
1/2	12.7	1 1/2	38.2	1/4	6.35	56.2	53.0
9/16	14.3	2	50.8	1/4	6.35	58.2	57.8
5/8	15.9	1 1/4	31.8	5/16	7.93	51.8	47.8
3/4	19.0	1 1/2	38.2	3/8	9.52	51.6	47.5
3/4	19.0	2	50.8	3/8	9.52	55.0	51.5
7/8	22.2	1 3/4	44.5	5/8	15.9	40.7	38.3
1	25.4	2	50.8	3/8	9.52	62.1	57.2
1 1/4	31.8	2 1/2	63.5	1/2	12.7	57.7	53.1
1 3/8	35.0	2 3/4	70.0	1/2	12.7	60.0	55.4
1 1/2	38.2	2 1/2	63.5	5/8	15.9	54.0	49.2
2	50.8	3 1/2	88.9	3/4	19.0	57.5	52.5
2 1/2	63.5	3 1/2	88.9	7/8	22.2	55.9	50.1
3	76.2	6	152.4	1	25.4	62.3	57.5

Table 10. Standard sizes of punched-plate screen used in coal dressing. (*After Holbrook and Fraser*)

Diameter of round holes, inches	Distance apart of centers, inches	Size of oblong holes, inches	Usual thickness of plate
3/8	5/8	3/8×3/4 to 3/8×1 1/4	No. 10 gage.
1/2	3/4	1/2×1 to 1/2×1 1/4 to 1/2×2	No. 8 gage to 3/16 inch.
5/8	7/8	5/8×1 1/4 to 5/8×2	3/16 inch.
3/4	1 1/8	3/4×1 1/2 to 2 1/2	3/16 to 1/4 inch.
7/8	1 1/4	7/8×1 3/4	3/16 to 1/4 inch.
1	1 1/2	1×2 to 2 1/4	3/16 to 1/4 inch.
1 1/8	1 1/2	1 1/8×2 1/2	3/16 to 1/4 inch.
1 1/4	1 7/8	1 1/4×2 1/2	3/16 to 1/4 inch.
1 1/2	2 1/8	1 1/2×2 1/2	1/4 inch.
1 3/4	2 1/2	. . . .	1/4 inch.
2	2 5/8	2×3 1/2	1/4 inch.
2 1/4	3	. . . .	1/4 inch.
2 1/2	3 1/4	2 1/2×3 1/2	1/4 inch.
3	3 3/4	3×6	1/4 inch.
4	6	4×6	1/4 inch.
6	9	. . . .	3/8 to 1/2 inch.

and if it is to be used on trommels (Art. 5) or other curved screening surfaces, must usually be rolled to the proper curve at the factory, making a bulky shipment.

Punched plate is widely used for coarse screening, down to, say 0.5-in. aperture, and on certain machines such as coal shakers, the stamp battery, Chilean and Huntington mills, it is used for fine screening. The usual sizes and weights for coal-dressing are given in Table 10; plate for ore and rock screening is usually heavier on account of greater wear. Coarse screens for trommels are frequently made of cast manganese steel, usually with round holes.

Such covering for a 48-in. × 14-ft. trommel at TONOPAH BELMONT (*52 A 110*) weighed 3776 lb., cost $572.30 (1915), lasted two years, and screened 300,000 tons.

Comparison of slotted and round-hole punched plate. Handy (*94 J 1123*) reports comparative performances on trommels at the BUNKER HILL AND SULLIVAN mill. Table 11

Table 11. Comparative sizing tests of oversize on slotted and round-hole punched plate. (*After Handy*)

Sizing screen, mm.	Oversize from 10-mm. round-hole screen, weight, per cent.	Oversize from 7-mm. slotted hole, weight, per cent.
10	84.2	87.4
7	11.2	11.8
5	4.1	0.8
−5	0.5	....

shows screen tests of oversize from the two types of covering on trommels taking the same feed. This shows that the 7-mm. slot lets through more undersize than a 10-mm. round hole, thereby working at higher efficiency both in production of −7-mm. material and in producing clean oversize. Table 12 shows that a round-hole plate of a given opening retains

Table 12. Comparative performances of slotted and round-hole plate on trommels. (*After Handy*)

Sizing screen, mm.	Oversize of 7-mm. screens, weight, per cent.		Oversize of 3-mm. screens, weight, per cent.	
	Slotted	Round-hole	Slotted	Round-hole
7	89.6	64.6	....	....
3	....	....	97.7	89.2
Through last screen	10.4	35.1	2.3	10.8

3 to 4 times as much undersize as a slot-punched plate of the same opening. In all of these tests the slots were punched at right angles to the direction of the trommel axis and were of such length that the clear horizontal projection with the plate on a 20° slope was a circle. The slotted screens blinded much less than the round-hole.

Lip screen is shown in Fig. 3(*e*). It is a recently devised form of punched plate with slotted openings diverging in the direction of flow, the plate being bent downward near the lower end of each row of slots to form a drop in the general screen surface. The result is that particles that tend to wedge in the slots are pushed down to the bend and there readily freed. This type of screen surface is increasing in popularity for coal screening. It is thought to cause less breakage and abrasion than any other type.

Woven-wire screen is made with square and rectangular meshes. (See Fig. 4.) It is obtainable in any length and in widths up to 4 ft. Both warp

and filler wires are crimped to prevent distortion of the openings in service. Woven-wire cloth with slot-shaped apertures is manufactured and widely used at the present time under such names as TON-CAP and REKTANG. Fine screening on trommels, shaking and vibrating screens, etc., is nearly all done through woven-wire screening surfaces and woven wire is also used in place of punched plate

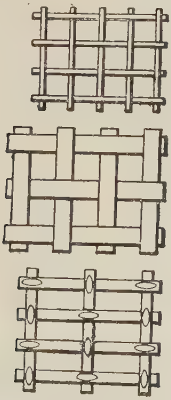


FIG. 4.—Double-crimped woven-wire screen cloth.

for much coarse screening. Table 4 gives the usual sizes and weights of square-mesh woven-wire screen and Table 13 gives sizes of one make of slotted mesh. There is very little slot-mesh woven-wire screen used coarser than 0.75-in. and very little finer than 48-mesh. The heavier screens in each size are used for rock and ore screening, the intermediate and lighter weights for coal screening, and the lightest for organic or very light mineral products. A covering of soft rubber tubing for the wires of coarse screens has been found at one or two plants to increase the life materially.

Table 13. Partial list of Ton-cap screen (*W. S. Tyler Co.*)

Width of opening, inch (*a*)	Mesh designation (*b*)	Extra heavy		Heavy		Medium		Medium light		Light	
		Manufacturer's numbers	Percentage of opening	Manufacturer's numbers	Percentage of opening	Manufacturer's numbers	Percentage of opening	Manufacturer's numbers	Percentage of opening	Manufacturer's numbers	Percentage of opening
0.750	....	1204	58	464	61	481	65	1098	69	1235	71
0.520	....	486	55	876	47	466	59	1242	63	1019	66
0.375	2	877	49	875	52	1036	55	963	58	652	62
0.270	3	995	44	468	46	461	49	1017	56	1026	58
0.190	4	904	40	390	42	924	47	407	55	965	58
0.125	6	23	38	514	45	624	51	622	53	736	57
0.090	8	368	37	35	43	755	45	341	52	906	53
0.063	10	898	35	691	41	47	44	302	52	1037	56
0.045	14	57	33	729	43	908	44	582	48	577	53
0.0325	20	79	30	910	35	89	39	95	42	796	48
0.023	28	615	27	295	32	677	37	858	42	434	47
0.016	35	912	27	438	31	542	34	162	37	775	43
0.012	48	158	24	159	26	778	30	182	33	215	43

a Length is two to four times width for openings greater than ⅛-in. and four to seven times for openings less than ⅛-in. *b* Nearest Tyler testing-sieve mesh; not actual openings per inch.

Percentage of opening. Table 14 shows comparative percentages of opening in different types of screening surfaces throughout the usual aperture

Table 14. Comparison of percentages of opening in different screening surfaces

Approximate aperture, inches	Woven-wire screens				Punched-plate screens				
	Square-mesh		Slot-mesh		Round-hole (b)	Slot-punch (a)	Needle-slot		
	Heavy	Light	Heavy (c)	Light (d)			Diagonal punch	Straight punch	Hit-and-miss endways
3	56	85	....	....	58	62	....	....	....
2	44	83	....	....	53	58	....	....	....
1	33	74	....	....	40	62	....	....	....
0.75	30	70	58	71	40	55	....	....	....
0.50	28	68	55	66	40	56	....	....	....
0.25	28	53	44	58	40	40	....	....	....
0.20	27	79	40	58	33	46	....	....	....
0.15	24	77	39	57	36	41	....	....	....
0.10	21	74	37	53	23	40	....	....	....
0.075	20	69	36	56	....	....	....	....	....
0.050	19	71	33	53	....	....	38	29	....
0.025	17	53	27	47	....	....	33	31	31
0.020	14	47	27	45	....	....	27	24	25
0.015	24	40	27	43	....	....	20	18	....
0.010	20	31	24	43	....	....	....	....	16

a Hit-and-miss endways. *b* Diagonal spacing. *c* Not so heavy as the heavy square-mesh. *d* Not so light as the light square-mesh.

ranges. In the 3-in. size there is little to choose between the several types of equal weight. From 2-in. to 0.5-in. inclusive there is considerable difference in favor of plate, as compared to square-mesh wire, the slot-punching giving the greatest opening, but in the 0.75-in. and 0.5-in. sizes the heavy slot-mesh has as much open space as the slot-punched plate. From 0.25-in. to 0.10-in. inclusive, slot-punched plate and heavy slot-mesh wire screen have substantially equal and the greatest amount of opening, with round-punched plate intermediate and heavy square-mesh wire the least. In the finer sizes, for screens of equal weight, slot-mesh woven wire has the greatest opening, needle-slot plate is intermediate, and square-mesh wire screen has the least, except in the two finest sizes. Throughout the range of sizes slotted screens have a greater percentage of opening than square- or round-hole.

Table 15 (*40 MEW 595*) shows the effect on screening capacity of difference in percentage of opening in screens of the same aperture. This test was made with different screens on a 5-stamp battery, the drop of stamps and water flow being maintained constant in the three cases.

Table 15. Effect of percentage of opening on screen capacity and product

Mesh	24	30	40
Aperture	0.0176	0.0173	0.0178
Percentage on 100-mesh, 0.005-in. aperture	34.7	36.3	40.2
Tons crushed per 24 hr., by 5 stamps	10.8	14.6	16.4

Cloth screens of silk are light, elastic, flat and have a relatively large percentage of opening but are expensive and fragile. They are used for very fine dry screening. Table 16 gives the usual sizes and numbers.

Table 16. Apertures in Dufour's bolting cloth (*19 IMM 500*)

Number	Mesh	Aperture		Per cent. opening
		In.	Mm.	
0000	18	0.0478	1.21	74.0
000	23	0.0342	0.87	61.9
00	29	0.0282	0.72	66.4
0	38	0.0204	0.52	60.1
1	48	0.0159	0.40	58.2
2	54	0.0131	0.33	50.0
3	58	0.0124	0.315	51.7
4	62	0.0116	0.295	51.7
5	66	0.0107	0.272	49.9
6	74	0.0096	0.244	50.5
7	82	0.0082	0.208	45.2
8	86	0.0072	0.183	38.3
9	97	0.0068	0.173	43.5
10	109	0.0058	0.147	40.0
11	116	0.0052	0.132	36.4
12	125	0.0049	0.124	36.0
13	129	0.0045	0.114	33.7
14	139	0.0039	0.099	29.4
15	150	0.0036	0.092	29.2
16	157	0.0035	0.089	30.2
17	163	0.0031	0.079	25.5
18	166	0.0028	0.071	21.6
19	169	0.0029	0.074	24.0
20	173	0.0030	0.076	26.9
21	178	0.0027	0.069	23.1
25	200	0.0026	0.066	27.0

At AMERICAN GRAPHITE CO. (*120 P 569*) it was found that for cleaning graphite-flotation concentrate on 100-mesh silk lasted much longer than wire.

Size of the product of a screen is by no means equal to the nominal dimension of the opening through which the product has passed. The maximum possible cubical particle that can be passed through a circular opening of diameter d has an edge $0.71d$. Larger platy or rounded particles can pass. Roessler has shown that on several different kinds of materials the average maximum size of the particles passing a round hole is only 81 per cent. of those passing the same size of square hole. When a screen is tilted the effective aperture is reduced substantially to that of the horizontal projection of the inner edges of the actual aperture. The amount of reduction in effective aperture depends upon the amount of tilt and the thickness of the screen plate or cloth.

Arthur Crowfoot (*PC*) states that at No. 6 Concentrator, MORENCI BRANCH, PHELPS-DODGE CORPORATION, a 0.280-in. aperture is used to produce a 0.185-in. product on Hummer screens set at 38° from the horizontal. Holbrook and Fraser (*Bul. 234 USBM*) state that ¼-in. screen at 45° made a ⅛-in. product. and that at 35° slope round-hole screens pass a product whose maximum-sized particles are about two-thirds the diameter of the hole. Referring to concentrating practice, they say that in one mill a 30-mesh wire cloth at 45° gave 50-mesh maximum undersize and that a 1-in. round-hole plate at 45° produced – ½-in. undersize. Under such circumstances, however, the undersize will contain TRAMP OVERSIZE, *i.e.*, occasional particles considerably larger than the average maximum, that work through because of some irregularity in the direction or velocity of presentation.

An apparent exception to the rule is presented by rectangular-mesh woven-wire screen which, if set with the long dimension of the aperture at right angles to the direction of slope, presents substantially the full aperture to material flowing down slope. This is shown in Fig. 5. Bland (*107 J 1114*) states that while 20-mesh cloth on some vibrating screens delivers −40-mesh product, the screening is incomplete, difficult grains are not passed, and while the oversize from hand screening will be shown to contain 40-mesh grains the undersize will contain 20-mesh grains. He states further that an inclination of 35° will decrease the effective dimension of the opening less than 10 per cent. rather than upwards of 18 per cent. as would be indicated by the geometrical solution. Table 17 is an empirical table, showing the sizes of maximum particles that are to be expected in a product screened through apertures of various diameters. In the smaller sizes particularly it is necessary to increase the screen apertures as much as 100 per cent., if the ore is damp and sticky, in order to pass the same maximum-size particles.

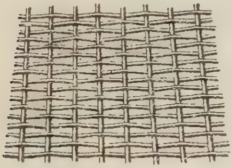


FIG. 5.—Rectangular-mesh wire cloth viewed from an acute angle.

Table 17. Relation between screen aperture and size of largest particle in product of various types of screens

Size of particle, inches (*a*)	Size of aperture					
	Round			Square		
	Flat surface	Steeply-sloping surface *b*	Revolving screen	Flat surface	Steeply-sloping surface	Revolving screen
0.25	0.35	0.50	0.5	0.28	0.38	0.40
0.375	0.55	0.75	0.75	0.45	0.57	0.60
0.50	0.75	1.0	0.88	0.62	0.75	0.75
0.75	1.0	1.50	1.25	0.81	1.15	1.15
1	1.5	2.0	1.88	1.15	1.50	1.50
1.25	1.75	2.50	2.25	1.40	2.0	1.75
1.5	2.0	2.75	2.5	1.62	2.25	2.0
1.75	2.5	3.25	3.0	2.0	2.75	2.5
2	2.75	3.75	3.5	2.25	3.0	2.75
2.5	3.5	4.75	4.0	2.88	3.75	3.25
3	4.25	5.50	5.0	3.5	4.5	4.0
3.5	5.0	6.50	6.0	4.0	5.25	4.75
4	5.75	7.50	7.25	4.75	6.0	6.0

a Intermediate dimension of roughly three-dimensional particles. *b* 40° to 45°.

TYPES OF SCREENS

Screens are classified on the basis of their method of support as FIXED and MOVING. Fixed screens are placed at any angle from 0° to 45° from the horizontal. When set at the smaller angles they are meant to retain the oversize for treatment such as sledging or hand-picking, and material must be worked over and through them manually. Fixed screens are used for both coarse and fine screening, although usually in the former service. Moving screens are of many forms, but most of them may be placed in one of the four classes: shaking, vibrating, revolving, traveling-belt. Fixed screens are almost invariably run dry; moving screens are run either wet or dry. Neither type will operate satisfactorily on damp material.

4. Grizzlies and fixed screens

Fixed screens with the screening surface made of parallel rods are known as grizzlies. Their economical use is limited to coarse screening (aperture 2-in. and upwards). A typical grizzly as sold by the supply houses is shown in Fig. 6. The bars are usually held together by bolts at right angles to their length and are spaced the desired distance apart by means of thimbles or sleeves on the bolts. The bolts should be spaced about 2 ft. with heavy, coarse ore, otherwise the bars will spread unless they are very stiff. The disadvantage of this type of screen is clogging due to the retarding effect of the cross pieces. This disadvantage may be overcome by making the bars of greater depth and holding them in comb-like cross plates with one through-bolt near the head end to keep them in place.

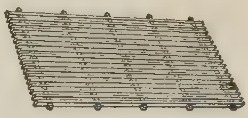


Fig. 6.—Fixed grizzly.

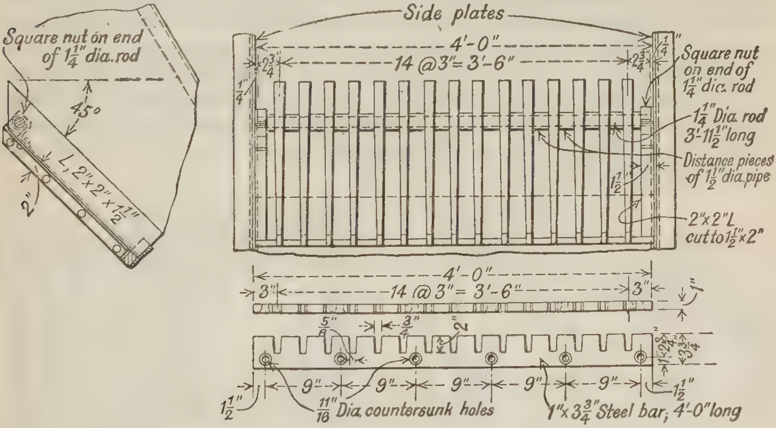


Fig. 7.—Tapered grizzly bars with depressed supports. (Miami Copper Co.)

Fig. 7 shows such an arrangement adopted by the Miami Copper Co. Another form is shown in Fig. 8. Soft steel bars 1 × 5-in. are clipped to heavy 4 × 4-in. cross supporting angles with heavy angle clips. This method of construction brings the cross bars well below the path of most particles and eliminates interference with the flow of particles. The particular grizzly shown was fed with maximum-size lumps weighing from 100 to 150 lb. and falling approximately 2 ft., yet the bars are said to have been capable of wearing to 2 in. in height before it was necessary to replace them. (*98 J 1045.*) Bars that taper along the run as shown in Fig. 7 tend to prevent clogging. Fig. 9 shows such a bar used by Ohio Copper Co. (*99 J 749.*) At Hol-

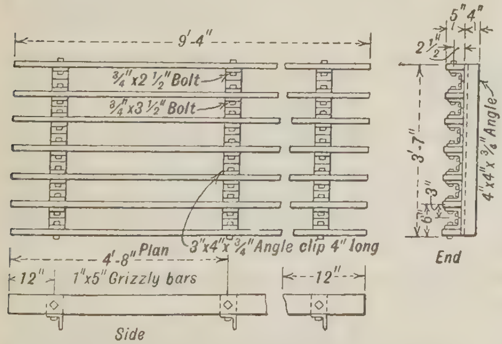


Fig. 8.—Grizzly with depressed cross bars.

LINGER MINES (*Bul. 119 CMI 341*) the bars are flared from 3.5-in. to 7.5-in. opening in a length of 8.5 ft.

Grizzly bars are made of wood, with or without metal protection, and of cast iron and steel. Various shapes are shown in Fig. 10. Other things being equal, the form of bar that gives minimum contact with the sliding material is best. The aperture of the standard taper-bar grizzly (Fig. 6) is favorable to screening, because the flare tends to pass everything that gets through the upper surface. The aperture changes as the bars wear, but as close sizing is not the aim in grizzly screening this fact is not so important as that when worn bars fail a considerable percentage of the weight of the new bar must be discarded. The diamond-headed form (Fig. 10, *f*) has the advantage of long wear with but little change in aperture or strength. A bar of this type with replaceable angle-shaped liners is probably the most economical form. At stamp mills, discarded stamp stems are commonly used. Around many mines old rails are made to serve. Normally these rails are merely sawed to length and set into the grizzly frames.

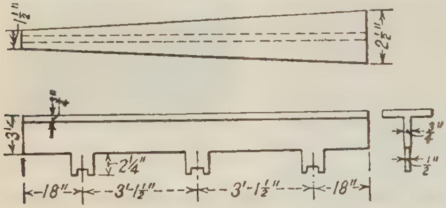


FIG. 9.—Tapered grizzly bar, OHIO COPPER CO.

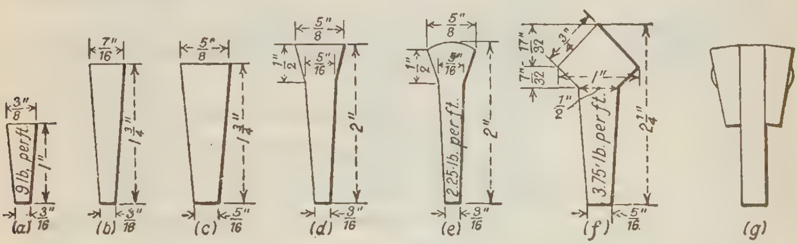


FIG. 10.—Grizzly bars.

At the HECLA MINE, Burke, Idaho (*106 J 24*), the flanges were cut from 85-lb. rails by means of an oxy-acetylene flame, leaving three wings 6 in. long and the full width of the rail flange, one at each end and one at the middle. These were bored and used for bolting the bars in place.

The use of such substitutes is, at best, a makeshift. They need more frequent renewal than the regular bars because they are made of milder steel, and the shape of the aperture is not ordinarily so favorable for screening.

Where the largest lumps in the feed are very much larger than the final size it is desired to pass through the grizzly, the arrangement employed in the QUINCY rock house (*100 J 103*) is useful. Skips dumped onto plates and the rock passed thence first over a short grizzly with 3-in. openings, then over a grizzly (in the same plane) with 6-in. round bars spaced 20-in. centers for taking out the largest lumps of feed. These passed to a drop hammer. The oversize of the 3-in. grizzly passed to the crushers and undersize to the stamp-rock bins. It was probably necessary to help some large lumps across the coarse grizzly, which was long (12 ft.) and set at 16° slope.

When heavy loads are to be dumped directly onto a grizzly, strength is an important factor in design.

At the Arthur mill of UTAH COPPER CO. (*118 P 469*) 50 to 80-ton cars are dumped by a revolving car-dumper directly onto a grizzly set at 35°. The bars are made of 12-in. @ 28.5-lb. I-beams, 32 ft. long, capped with special manganese-steel castings, set with 5-in. clear opening at the upper end and 6-in. at the lower. At ALASKA GASTINEAU (*63 A 493*)

10-ton cars were dumped four-at-a-time onto plates and thence passed over a 10-in. grizzly made of 8-in. I-beams with manganese-steel shoes. After 6,000,000 tons had passed over this grizzly some of the bars had to be replaced on account of bending but the shoes were still serviceable.

Slope of grizzly depends upon its purpose. A flat grizzly is used on top of ore bins fed by skips or cars in order to catch and hold lumps that are too large to pass the bin gates or enter the crusher until they can be sledged to proper size, as determined by the spacing of the grizzly bars. Grizzlies in the feed chute to primary crushers are set at such a slope that material will just slide over them (±30°) or will just not slide (±25°) so that the movement over them is readily controlled by the crusher tender, allowing him to remove waste of all kinds from the mill feed. To insure free movement of material, the slope should be from 35° to 45° or as high as 50° for sticky ores. A lower slope is used when material is delivered with an initial velocity in the direction of the slope.

Holbrook and Fraser (*Bul. 234 USBM*) have collected the following data on sliding angles that bears on the question of slope. Egg-size anthracite ($2\frac{5}{16}$ to $3\frac{1}{4}$-in.) will just slide uniformly on glass at $2\frac{1}{4}$ in. per ft., manganese bronze at $2\frac{5}{8}$ in., mild steel at 3 in. and cast iron at $3\frac{1}{2}$ in. per ft. (16° 15′). Slopes for anthracite are less than for bituminous coal and ores require more slope than either. Dry quartzite ran well on steel plate at 35°. Steel-lined chutes for Lake Superior copper ores are commonly 26° to 30°. Experimental results are given in Table 18. It must be remembered that blinding causes the screen surface to become rough and that steeper slopes than those given in the table mu t be used to overcome this difficulty. Table 19 shows common practice in coal dressing. Moist ores require 10 or 15° greater slope than dry. Concavely-curved bars have been used in coal-screening grizzlies, where it is desirable to discharge oversize with as low velocity as possible to prevent breakage.

Table 18. Sliding angles of various substances on bright steel. (*After Holbrook and Fraser*)

Material	Starting angle (static friction)	Angle on which material continued to run (kinetic friction)	Slopes used in practice
Pennsylvania anthracite:			
Egg size	15° 40′	14° 00′	14° to 16° chutes.
Chestnut size	16° 40′	15° 10′	18° to 20° chutes.
Southern Illinois coal:			
6-inch lump	21° 50′	20° 40′	22° 30′ chutes.
Run-of-mine	....	....	30° to 35° chutes.
Slack	25° 40′	22° 30′	30° to 45° chutes.
Oklahoma screenings	24° 00′	22° 30′	30° to 45° chutes.
Chestnut size (McAlester seam)	21° 00′	19° 00′	....
East Kentucky egg, 4×6 inches	21° 50′	20° 20′	26° 28′ bars.
Bituminous shale	21° 10′	20° 00′	....
Limestone-gangue ores	19° 40′	17° 30′	30° to 40° for steel 45° for wood.
Sandstone-gangue ores	22° 30′	19° 40′	30° to 40° for steel 45° for wood.
Hematite (Lake iron ore)	22° 40′	20° 40′	30° to 40° for steel 45° for wood.
Missouri galena	19° 20′	17° 20′	30° to 40° for steel 45° for wood.

Table 19. Slopes commonly used for grizzlies and steel-lined chutes in coal dressing. (*After Holbrook and Fraser*)

Size of coal	Slope, degrees
Run-of-mine	25–35
Standard lump	25–30*a*
Nut	26–37
Slack	30–45

a Chutes for lump coal range from 18 to 33° slope.

Size of grizzly depends upon the size of feed, percentage of undersize, slope and aperture. Width is governed by the same rule that governs width of chutes, *viz.:* the minimum width should be at least three times as great as the size of the largest particles in the feed, if the feed is fairly uniform in size, or twice as wide, if large lumps are rare. The width is usually governed, however, by other factors, such as chute width, width of car or skip supplying the feed, or width of crusher receiving opening. Length is determined by the amount of screening to be done, *i.e.*, if the aperture is small and the percentage of undersize large, more length must be provided than when reverse conditions obtain. Length of steep grizzlies should be greater than that of flat ones. No quantitative rules can be set down. Length is frequently made twice the width, but this relation has no logical foundation. If large particles slide along carrying fine material with them, they can be turned over by a drop in the grizzly surface or by dependent swinging bars, chains, ropes or the like placed about half-way along the length. These will, however, slow down the material and hence make it necessary to set the grizzly on a steeper slope than otherwise. Table 20 gives examples from practice. *Truscott* gives 12 to 15 ft. as usual lengths.

Average capacity may be taken as about 125 tons per sq. ft. of surface per 24 hr. with 1-in. opening and proportionately greater tonnages with greater apertures.

Table 20. Performances of fixed screens

	Braden Copper Co.	Braden Copper Co.	Braden Copper Co.
Type	Griz.	Griz.	Griz.
Dimensions of screening surface	30 in. × 18 ft.	30 in. × 12 ft.	16 × 6 ft.
Screen surface, type	...	...	...
Aperture, in.	1	1	½
Material	*Mn*	*Mn*	...
Life, days	...	...	...
Slope, inches per foot	...	...	...
Tons of feed per 24 hr.	4777	4777	680
Undersize in oversize, per cent	58	58	9
Tons per square foot per 24 hr.	106	159	85
Tons per square foot per 24 hr. per mm.	4.2	6.3	9.0

	Alaska Gastineau	Alaska Gastineau	Granitic zinc ore
Type	Griz.	Griz.	Inclined
Dimensions of screening surface	21 × 52 ft.	2⅔ × 13 ft.	3 × 6 ft.
Screen surface, type	*a*	*b*	Wire
Aperture, in.	9¾	4½ × 4¼	¾ × 2
Material	*a*	...	½-in. round iron
Life, days	3 yr.	106	21
Slope, inches per foot	9½	10	6
Tons of feed per 24 hr.	10,000	2500	...
Undersize in oversize, per cent	...	...	...
Tons per square foot per 24 hr.	114	139	...
Tons per square foot per 24 hr. per mm.	0.46	1.2	...

a A 10-in. I-beam capped with a 6-in. channel which was, in turn, capped by a manganese-steel shoe. *b* ¾ × 2¼-in. mild-steel bars with ⅞-in. space rods and manganese-steel spacers. *Mn* Manganese steel.

Holbrook and Fraser (*Bull. 234, USBM*) criticize this method of estimating the size necessary and give a rule of 500 tons run-of-mine coal per foot of width per 8 hr. without any statement of the aperture. This treats the grizzly simply as a chute.

Self-cleaning grizzly. Simcox and Humes (*117 J 307*) describe the grizzly illustrated in Fig. 11. The revolving arms prevent material from sticking between the bars. At

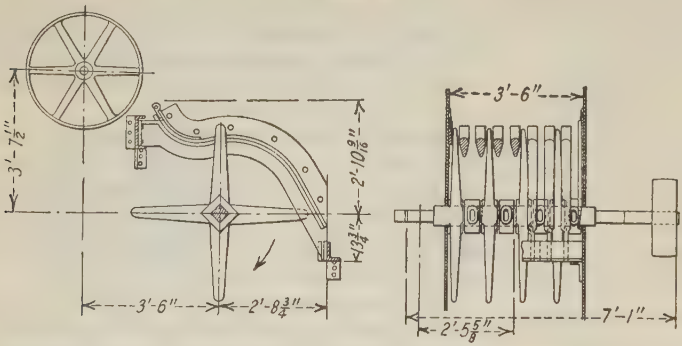


FIG. 11.—Self-cleaning grizzly.

COPPER QUEEN smelter this machine handled 196 tons per hr. of talcy ore containing 10 to 12 per cent. moisture which could not be handled by other types of grizzlies, and previously had been dried and re-handled at considerable expense.

ADVANTAGES of fixed grizzlies are simplicity and ruggedness. DISADVANTAGES are inefficiency, loss of head room, blinding, and, in the case of coal screening, breakage of oversize.

MOVING GRIZZLIES

Moving grizzlies of various types are used for the purpose of bettering screening, lessening breakage of material and saving head room. The first two purposes have determined their use in coal breakers, the second, in many ore-treatment plants.

Moving-bar grizzly (Fig. 12) is usual in coal breakers. Bars are mounted at one end on an eccentric, adjacent bars 180° apart, and are so driven that they move forward at the high position. Speed is about 50 r.p.m. The forward movement, together with a forward inclination of about 10° moves lump material gently along the grizzly and at the same time turns it over well and allows fines to drop through.

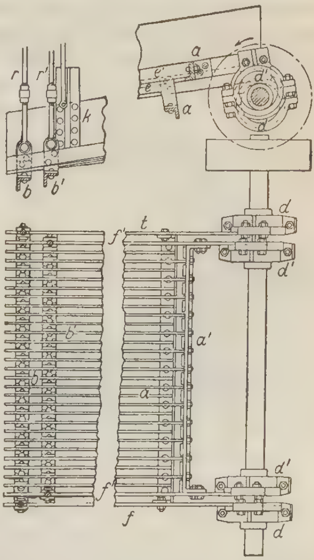


FIG. 12.—Briart moving-bar grizzly.

A chain grizzly was used between the tracks over the crude-ore bins at the ROWE MINE, Riverton, Minn. (*101 J 599*). It was built of old steam-shovel chains stretched endless over sprockets, the run of the chains being horizontal. Longitudinal rods in the plane of the upper chains were placed between the chains so that the

distance of a chain from a rod determined the aperture of the grizzly. Alternate chains were run at different speeds in order to break up lumps held together by clay and to prevent blinding. Oversize traveled on the chains to a chute feeding a primary crusher. The speed of the chains was 16 and 18 ft. per min. respectively, and it was possible to dump a 50-ton car directly onto the grizzly. Eleven supporting rollers were used between the head and tail rollers in a length of about 20 ft.

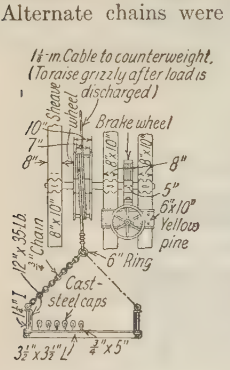


Fig. 13.—Movable sorting grizzly, Port Henry Iron Ore Co.

Movable sorting grizzly used at Port Henry Iron Ore Co. is shown in Fig. 13 (*88 J 884*). Its position is controlled by the band wheel. The counterweight is heavy enough to raise the empty grizzly to a horizontal position. The size is 6 ft. wide and 11 ft. long with 4-in. clear space between bars. The slope for discharging is 35°.

Traveling grizzly is shown in Fig. 14. It consists of short lengths of rail running across between two sprocket chains, the latter operating over head and tail sprockets, gear-driven. This grizzly is placed directly under a feed chute and is normally of adjustable speed to regulate feed to a crushing machine. Undersize passes through the upper run and is diverted both sides by chutes. Oversize is discharged over the head sprocket. Any material that tends to wedge between the bars is readily freed at the head sprocket because of spreading of the bars in passing around. Another form of this same grizzly is shown in Fig. 15(*a*). The drop of the bars on the lower run clears the surface and also allows the undersize chute to be placed below the return run. Fig. 15(*b*) shows the same type of grizzly used as a roller feeder. Speeds range, usually, between 10 and 20 ft. per min.

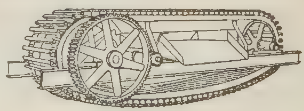


Fig. 14.—Traveling-bar grizzly.

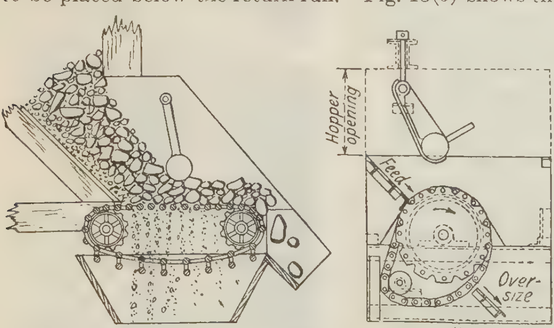


Fig. 15.—Drop-bar traveling grizzly.

Disk or roller type of grizzly consists of a series of disks properly spaced and mounted rigidly on a shaft; it replaces a roller feeder (see Sec. 19, Art. 4) at the discharge gate of a bin. The shaft is driven by a sprocket chain or by ratchet and pawl, oversize rides on the disks and passes to a crushing machine, while undersize drops through between the disks into a chute. The peripheral speed is dependent upon the rate of feed, but should not, in general, exceed 100 ft. per min.

Burchring grizzly (Fig. 16) is a special form of the roller type. It con-

sists of two disk-shaped heads (A) mounted on shaft (E) and tied together by rods (G) on which are mounted close-fitting rings (C), held in place by spacers (D), and alternate loose rings (B) spaced centrally between the rings (C) by the lugs (I). The principle lies in the fact that the loose rings always hang down in such a way that the space between adjacent rings at the bottom is greater than at the top and anything that passes between rings at the top is, therefore, readily discharged at the bottom.

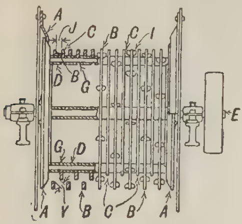


Fig. 16.—Burch ring grizzly.

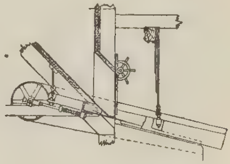


Fig. 17.—Shaking grizzly.

Shaking grizzly is shown in Fig. 17. It consists of grizzly bars carried in a suspended frame actuated by an eccentric. The surface of the bars is ordinarily inclined about 10° in the direction of flow, although this is not entirely necessary, if the method of suspension is that of the Ferraris screen. Speed should be about 80–100 strokes per min. Capacity is about 125 tons per sq. ft. per 24 hr. per in. of aperture.

At Empire Steel and Iron Co. (*99 J 560*) this type of grizzly with 2-in. spaces clogged badly and the bars were replaced by ¾-in. manganese steel plate with 2-in. round holes. This overcame the difficulty.

FIXED SCREENS

Fixed screens, as distinguished from fixed grizzlies, have an inclined screening surface consisting of punched plate or woven wire rather than of parallel bars. They size more closely than a grizzly on account of the fact that they limit the particle in two directions.

Holbrook and Frazer (*Bul. 234 USBM*) report that the undersize of a 1-in. grizzly yielded 24 per cent. on a 1-in. round-hole screen and that some of the material would not pass a 2-in. round hole.

Punched-plate screens may ordinarily be set on the same or a slightly steeper slope than a grizzly except that, when used for fine feeds, the slope must be greater on this account. Tables 21 and 22 show the effect of size of coal on sliding angles. Woven-wire cloth must be set on a considerably steeper slope (5° to 15°) than either bars or punched plate on account of the rougher surface. Capacity of fixed screens is 2 to 4 tons per sq. ft. per 24 hr. per mm. of aperture.

At Ohio Copper Co. (*99 J 749*) heavy wire screens, 2 ft. wide by 8 ft. long with 0.375 × 1-in. apertures, set on a 45° slope, handled 250 tons per 24 hr. or 1.1 tons per sq. ft. per 24 hr. per mm. At the old coarse-crushing plant of the Arthur mill of Utah Copper Co. (*117 P 715*) the 72 × 20-in. roll circuit was closed with stationary square-mesh woven-wire screens set at 40° slope, the aperture varying from 1 to 1.5 in. as the moisture varied from 5 to 13 per cent. In the new plant these screens have been replaced by Mitchell vibrating screens. At Alaska Gastineau (*63 A 493*) — 10-in. material was fed at the rate of 1000 tons per 8 hr. to a stationary woven-wire screen with 2.5-in. aperture, 3 ft. wide by 14 ft. long, set at 45° slope. This is 1.1 tons per sq. ft. per 24 hr. per mm.

Table 21. Slopes necessary for different sizes of dry anthracite. (*After Sterling, 42 A 264*)

Size (round hole, diameter in inches)	Glass chute: Minimum slope, in inches per foot, for sliding to begin	Glass chute: Minimum slope, in inches per foot, for sliding to continue	Steel-plate chute: Minimum slope, in inches per foot, for sliding to continue
Broken coal, 4½ to 3¼	2⅝	2¼	2.75
Egg coal, 3¼ to 2$\frac{5}{16}$	2⅝	2¼	2.94
Stove coal, 2$\frac{5}{16}$ to 1⅝	3	2½	3.12
Chestnut coal, 1⅝ to $\frac{15}{16}$	3	2½	3.50
Pea coal, $\frac{15}{16}$ to ⅝	3¼	2½	4.50
Buckwheat, No. 1, ⅝ to $\frac{7}{16}$	3⅝	3⅛	5.25
Buckwheat, No. 2, $\frac{7}{16}$ to ¼	3¾	3¼	6.00
Buckwheat, No. 3, ¼ to $\frac{3}{32}$	4⅜	3⅜	7.50
Buckwheat, No. 4, $\frac{3}{32}$ to 0	4⅞	4⅛	8.00

Table 22. Slopes necessary for different sizes of Illinois bituminous coal to slide on bright steel. (*After Holbrook and Fraser*)

Size	Minimum angle to start	Minimum angle to continue
6-inch lump on cleat	21° 50′	20° 30′
6-inch lump on bedding	21° 00′	19° 40′
6-inch lump on mother-of-coal	16° 25′	13° 44′
3-inch × 6-inch egg	21° 00′	19° 10′
3-inch × 2-inch nut (No. 1)	20° 30′	19° 00′
2-inch × 1¼-inch nut (No. 2)	20° 40′	19° 20′
1¼-inch × ¾-inch nut (No. 3)	21° 00′	19° 40′
¾-inch × ¼-inch nut (No. 4)	22° 00′	20° 00′
¼-inch to 0 nut (No. 5)	26° 34′	22° 52′

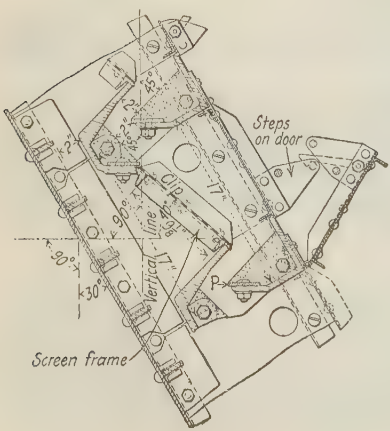


FIG. 18.—Rowand screen.

Screen with adjustable aperture is described by Watts (*40 CMJ 886*). It consists of a round-hole punched-plate fixed screen, the holes being punched with rectangular spacing, and a grizzly in contact with the under side of the plate, the grizzly bar being equal in width to the least width of metal between holes in the plate and the distance center to center of bars and holes being equal. When the centers of the bars register with the centers of the metal between holes in the plates the screen apertures are full open. By sliding the grizzly sideways the holes may be closed to any desired extent. This arrangement is useful on coarse screens when the moisture content of the feed is variable, but the same effect can probably be more easily obtained by changing screens.

Rowand screen (Fig. 18) is a modification of the old Edison screen (see *Richards*). Both screens

have been used to a considerable extent by the New Jersey Zinc Co. and subsidiaries for relatively fine screening of dry ores. The object of the apparatus is to feed ore in a thin regular stream over the whole screen surface and to break the flow of the ore a sufficient number of times to keep the velocity of the particles on the screen cloth low and to have them, therefore, slide rather than roll and bound. The essential elements in such screening are to have the feed substantially bone dry and to keep the aperture clear. Dryness both increases efficiency and markedly reduces wear on screening surfaces. Table 23 gives performances of Rowand screens. The

Table 23. Performance of Rowand screens, N. J. Zinc Co.

Square feet of screen surface	Aperture, inches	Slope, degrees	Tons per hour, new feed	Tons per hour, total	Tons per square foot per 24 hr.	Tons per square foot per 24 hr. at 1 mm.
24	1.0000	45	25	50	50	2
15	.875	45	28	112	179	8
120	.0945	40	40	40	8.0	3.33
80	.125	40	4	16	4.8	1.51
60	.1875	40	6	24	9.6	2.02
48	.3125	40	8	32	16	2.02
108	.013	40	0.96	0.96	0.22	0.67
108	.016	40	2.76	2.76	0.61	1.5
108	.02	40	3.28	3.28	0.73	1.44
108	.028	40	3.28	3.28	0.73	1.03
96	.055	40	4	4	1.0	0.72
84	.065	40	4	4	1.15	0.69
96	.075	40	6.8	6.8	1.7	0.89
84	.082	40	6.8	6.8	1.94	0.93
20	.625	40	20	20	24	1.51
20	.625	40	48	48	57.6	3.63
20	.625	40	12	12	14.40	0.91
360	.101	40	100	100	6.67	2.6
41	.088	37	9.8	9.8	5.7	2.6
67.5	.055	38.5	8.3	8.3	2.95	2.1
135	.025	40.5	7.7	8.6	1.53	2.4
135	.02	41	6.8	7.5	1.33	2.6
135	.016	41.5	5.7	6.3	1.12	2.8
135	.013	42	4.1	4.6	0.82	2.5
135	.011	42.5	1.4	1.5	0.27	0.97
135	.010	43	4	4.5	0.8	3.2
135	.025	43	11–13	11–13	2.1	3.3

average CAPACITY of the Rowand screen is 2 tons per sq. ft. per 24 hr. per mm. of aperture. The corresponding average of five Edison screens is 1.15 tons.

The ADVANTAGES of Rowand screens are lack of power consumption and low repair charges, due to absence of moving parts; the DISADVANTAGES are low capacity per square foot, excessive blinding, great loss of head room, and close dependence between efficiency and dryer performance. When the ore contains even so small an amount of moisture as to fail to allay dust, screen efficiency falls off badly and the circulating load builds up to enormous proportions. At NEW JERSEY ZINC Co. an alternating-current electromagnet mounted over a screen plate in a Rowand screen is locked electrically with a mechanism for regulating feed and with a timing arrangement, so that the feed is shut off for a few seconds every few minutes and the screen is vibrated vigorously. The result of the vibration is to clean the screen thoroughly. As a result capacity and efficiency have been greatly increased and a labor cost of upwards of $0.15 per ton screened for cleaning screens has been eliminated.

Drag screen is essentially a flight conveyor (Sec. 20, Art. 4) in a trough with perforated bottom. It may be set at any slope from a downward incline

less than the sliding angle of the material to a rise of the same inclination. The usual range in size is from 4 to 10 ft. wide and from 10 to 25 ft. long.

The usual CAPACITY of drag screens with ⅛- to $\frac{5}{16}$-in. round-hole plate is 1 ton per sq. ft. per hr.; the general limits are between 2 and 15 tons per sq. ft. per 24 hr. per mm. aperture. The usual speed is 60 to 100 ft. per min. but speeds from 40 to 240 ft. per min. are reported. Wear is not great with coal but is excessive with hard ore. The screen is used principally as a dewatering elevator or conveyor in coal washing, where screening is of secondary importance.

Lincoln (*11 Bul. UI, No. 9*) describes such a screen 2 ft. wide by 24 ft. long with 16 ft. of 1¾-in. slotted plate and 8 ft. of ⅜-in. square-hole plate which handled 40 tons per hr. of −3¾-in. raw coal. Another screen 3 ft. wide × 30 ft. long with 1½-in. grizzly bottom, sloped +26° for part of the run, then horizontal, handled 600 tons per 8 hr. at 80 ft. per min.

5. Revolving screens and trommels

Revolving screens have been more widely used than any other type of movable screen, but their popularity, particularly for screening finer than 1½- or 2-in., is on the wane at the present time. The ordinary cylindrical TROMMEL as used in ore-treatment plants is shown in Fig. 19. It consists essentially of a through-shaft carrying two or more 4- to 6-armed spiders, on the outer end of which are carried circular bands on which screen cloth is stretched. The main shaft is carried in bearings at the two ends. Several methods of drive are employed. The commonest is the so-called right-angle drive illustrated in the figure, consisting of a bevel gear mounted on the end of the trommel shaft and driven by a bevel pinion on the countershaft, which is, in turn, pulley-driven. A less common form of drive is one in which a pulley is carried directly on the end of the trommel shaft. Since the shaft of a cylindrical trommel is always on a slope, there is difficulty in keeping a belt on this pulley, especially as the trommel is a slow-speed machine. This difficulty is overcome, in some cases, by connecting the trommel shaft through a universal joint with a horizontal shaft in the same vertical plane and driving this directly by means of a pulley. Sprocket and chain may be substituted for belt and pulley in the last two drives and sprocket and chain are frequently used with the right-angle drive in dry screening, where the excessive dust would cause belt trouble. Common diameters are 24, 30, 36, 42, 48 and 60 in.; lengths, 4 to 12 ft.

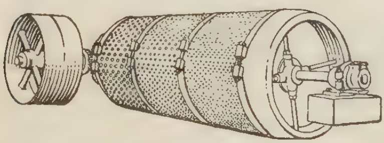


FIG. 19.—Cylindrical trommel.

Split spider hubs are superior to solid. The shaft for a 24 × 72-in. trommel should be $2\frac{11}{16}$-in., increasing to about $4\frac{7}{16}$-in. for the 48 × 72-in. size. For longer screens the shaft diameter must be increased considerably. The driving shaft should be fitted with a tight-and-loose pulley. The minimum slope of the undersize chute for wet work should be at least 2¾ in. per ft. for 2-mm. screen. The chute should slope at least 45° for dry screening. Wash-water boxes are better than spray pipes unless the water is clean and free from salts. Water should be applied on the upcoming side. Water consumption varies from 1 to 3.5 gal. per min. per ft. of length, the higher figures generally corresponding to fine screening. Water consumption in terms of tonnage of solid fed to the screen varies from 18 gal. per ton in coarse screening to 420 per ton in 1-mm. screening.

Wiard states that power consumption is about ¼ hp. per 2-ft. length for 48-in. trommels. Holbrook and Fraser give the formula $Hp. = tons\ per\ hr./10$; also $Hp. = DL/8$ where D and L are diameter and length in feet, respectively, for coal-screening trommel,

and $Hp. = DL/4$ for stone screens working on rock or ore. For light shaft-type trommels for fine screening in concentrating mills they give $Hp. = DL/12$.

Trommels for wet screening may be set up in banks of two to four in line, with intermediate spur-gear drive, but individual drive is better.

Compound trommels with two or more concentric screening surfaces on the same shaft, the coarsest inside, are used where several sizes are desired in the same screening operation and head room is at a premium.

The DISADVANTAGES are that it is necessary to remove the outer screens in order to make repairs on the inner, it is difficult to watch the inner screens for wear and blinding, and the area of the expensive fine-screen cloth is unnecessarily great.

Revolving stone screens, as distinguished from trommels, have no central shaft but are mounted on tires and rollers at both ends, or one end is so mounted and the other end carries a heavy head and gudgeon (Fig. 20, *b*). This type of screen is used for coarse screening only and is less frequently used in ore-treatment plants than in stone-crushing plants. The screen frame consists of heavy cast heads with four or more longitudinal members connecting

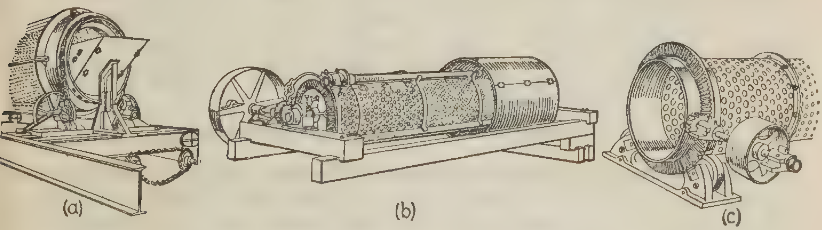


FIG. 20.—Revolving stone screens.

the heads and forming with them a strong truss. The screening surface is attached to the through members. The best arrangement is one in which the screens are in sections, each section bolted in between two adjacent through members as shown in Fig. 20, *b*. Commonly stone screens are SECTIONAL (Fig. 20, *b*), *i.e.*, plate or cloth sections of successively coarser aperture from feed to discharge end are used. Not infrequently, also, they are compound at one end. Right-angle drive is usual, but some forms are chain-driven by means of a sprocket bolted to the feed- or discharge-end casting (Fig. 20, *a*). The frame is sometimes built to carry a grid of heavy rods on the inside surface, parallel to the axis, with an aperture larger than the coarsest screen. This rides the largest lumps through the screen out of contact with the screening surface and thus protects the latter. Table 24 gives sizes and weights throughout the range of manufacture.

The principal DISADVANTAGE of this type of screen is the fact that all of the coarse material must pass over the finest plate, which is thereby subjected to excessive wear. This difficulty may be obviated, although at the expense of head room, by passing the original feed first to a heavy screen of intermediate size and passing each product of this screen to a separate sectional screen having suitable cloth.

Conical trommels are made with and without a through shaft. The latter type, called the Gilbert screen, has had considerable use, particularly in gravel-washing plants.

The ADVANTAGE of the conical trommel is that the axis is set horizontally and the inclination of the screening surface necessary to cause travel of material is obtained by the conical

shape. It is also economical of head room. The open-end type has ample space for feeding through the discharge end, but it is necessary that the feed be carried in suspension in water.

Table 24. Revolving stone screens. (*Traylor Engineering Co.*)

Size	Capacities, tons per hour, 3-inch perforation		Horse-power	Revolutions per minute, screen	Weight, lb.
	Scalper(*a*)	Sizer(*b*)			
24″ × 6′	20	9	3	26	3,000
24″ × 12′	45	20	4	26	3,700
32″ × 8′	40	18	4	22	5,150
32″ × 14′	80	35	5	22	6,300
40″ × 10′	65	30	8	18	7,950
40″ × 20′	135	65	12	18	10,700
48″ × 12′	100	45	10	16	13,000
48″ × 20′	180	85	14	16	15,650
60″ × 12′	120	55	12	14	21,000
60″ × 24′	250	115	18	14	28,000
72″ × 14′	180	85	16	12	30,000
72″ × 24′	300	150	20	12	40,000
84″ × 16′	250	125	20	10	40,000
84″ × 30′	500	250	25	10	52,000

a As guard screen on a crusher. *b* For clean oversize.

Prismatic trommels, usually hexagonal, have been used to a considerable extent in fine screening. The sides may have their elements parallel, in which case the axis is set on a slope, or the sides may be pyramidal and the axis set horizontal. The screening surface consists of a number of plane sections. This makes for ease in mounting and changing screen cloth, as the cloth is mounted on separate wooden frames that bolt onto the main screen frame. Cloth may be shipped flat, is lighter to handle than that for a cylindrical trommel, being in smaller pieces; and the amount of screening surface that need be replaced for a break in the screen is only one-sixth of the total.

Comparison of round and hexagonal trommels for fine screening was made at the mill of the Desloge Consolidated Lead Co., South-eastern Missouri (*94 J 833*). The material to be screened was undersize of a 10-mm. trommel and it was desired to remove from it material that would pass a 1-mm. opening. Comparison was between a round trommel of the conical type, 8 ft. long with diameters 3 and 4 ft., slope ¾ in. to the foot, speed 20 r.p.m.; and a hexagonal trommel 8 ft. long, pyramidal shape, largest diameter 4 ft., smallest diameter, 3 ft., speed, 20 r.p.m. Pulp in falling from one face of the hexagon to the other dropped 14 in. Screening surface used on both trommels was punched plate, 22-gage steel, 1-mm. round holes on 2.5-mm. centers. Outside spray was used in both cases. Feed rate, both trommels, was 150 tons per 24 hr. Duration of the test was several weeks. Results were as follows: Material finer than 1-mm. in oversize; conical trommel, 7.02 per cent; hexagonal trommel, 4.77 per cent. Life of screening surface; conical trommel, 7.1 days, 1065 tons; hexagonal trommel 12.85 days, 1927 tons. The experimenter calls attention to the fact that the difference in life of screening surfaces is due to the fact that the interior rings of the conical trommel cause pulp to back up behind them and that the greater weight of pulp sliding around at these points causes the screens to wear through there and fail. The higher efficiency of the hexagonal trommel was due to greater turning over of pulp and consequent more frequent presentation of all grains to the screening surface.

Variables in construction and operation of revolving screens are speed, slope of screen axis, aperture, percentage of oversize in feed, percentage of moisture in feed, quantity of feed per unit of time, diameter, length and character of screening surface.

Speed affects capacity and efficiency. Increase in speed up to the point where material is carried completely around by centrifugal force causes increase in capacity. Efficiency, however, passes through a maximum with increased speed.

Results of a test on a 36-in. trommel with varying speed (*98 J 305*) are given in Table 25 and Fig. 21, showing, for this size, maximum efficiency at 16 r.p.m. Since efficiency and not capacity is the end sought in most screening operations, this speed or one slightly greater, is the one that should be and is commonly used in practice for this size of trommel.

Diameter, 36 in. Length, 12 ft. Slope, 1¼ in. per foot. Perforations, ½-in. round holes, punched on lines intersecting at 60°. Percentage of opening, 40.3. Rate of feed, 22 to 24 tons per hour. Speed, 11 to 22 r.p.m. (Data plotted in Fig. 21.)

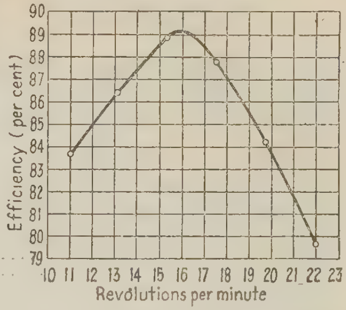


Fig. 21.—Relation of trommel speed and efficiency. (Numerical data in Table 25.)

Table 26 shows speeds of operating trommels in various mills. Average speeds are as follows: 2-ft., 17 r.p.m. (2 only); 2.5-ft., 16.4 r.p.m. (5 only); 3-ft., 16.75 r.p.m. (4 only); 3.5-ft., 15.5 r.p.m. (2 only); 4 ft., 15 r.p.m. (10 only); 4.5-ft., 15 r.p.m. (1 only). Table 27 shows the theoretical speed producing centrifugal cling and the usual speeds for ore and for coal screening, according to Holbrook and Fraser (*Bul. 234, USBM*).

Table 25. Relation between speed and efficiency of trommel. (*a*) (*After Roesler*)

Test number	1	2	3	4	5	6
Speed, r.p.m.	22.0	19.8	17.6	15.4	13.2	11.0
Diam. of drive pulley, in.	10	9	8	7	6	5
Oversize						
Per cent. on ½-in.	64.3	67.8	74.5	75.3	73.4	66.9
Per cent. through ½-in.	35.7	32.2	25.5	24.7	26.6	33.1
Feed						
Per cent. on ½-in.	26.9	25.1	26.2	25.3	27.4	24.9
Per cent. through ½-in.	73.1	74.9	73.8	74.7	72.6	75.1
Efficiency, per cent.	79.6	84.2	87.8	88.9	86.4	83.7

Slope of a trommel affects the speed of passage of material therethrough, and for a given tonnage determines the thickness of the bed on the screen. This factor in turn influences efficiency, the thinner the bed the greater the opportunity for material to have access to an aperture. Up to the point where the rate of travel of particles over the screening surface is too great for efficiency, increase in slope increases efficiency and capacity. The old practice was to use slopes of 0.5 to 0.75 in. per ft. Present practice, as indicated by Table 26, averages about 1.5 in. per ft. in ore treatment. The maximum is about 3 in. per ft. According to Holbrook and Fraser, 5° is standard slope for punched plate-trommels in coal treatment and slightly more for woven-wire.

Diameter of a trommel determines the thickness of the bed; the greater the diameter the thinner the bed. It is not advisable to use trommels less than 36 in. diameter on account of the difficulty of changing screens and making other repairs. The majority of trommels range from 36 to 48 in. diameter.

Table 26. Performances of

Name of mill	Pittsburg-Dolores Mining Co.	St. Joseph Lead Co., Rivermines plant	Nevada Packard
Dimensions, diameter × length, in.	24 × 48	24 × 144	30 × 111
Slope, inches per foot	....	1.5	1.75
Revolutions per minute	15	19	20
Screen surface, type	*W*	*P*	*W*
Aperture, in.	0.25	1.25	0.375
Material	*S*	*CI*	No. 10*S*
Life, tons of feed	14,400	180,000	144,000
Water, gallons per hour	....	Dry	....
Horsepower installed	3	5	....
Horsepower consumed	....	1.5	....
Tons of new feed per 24 hr.	60	1000	800
Tons of total feed per 24 hr.	80	1000	....
Tons per square foot per 24 hr.	3.2	13.2	11.0
Equivalent tons per square foot per 24 hr., 1-mm. aperture	0.50	0.41	1.15
Per cent. undersize in oversize	....	....	....
Efficiency, per cent.(*b*)	....	80	....

Name of mill	Moctezuma Copper Co.		Pittsburg-Dolores Mining Co.
Dimensions, diameter × length, in.	36 × 68	36 × 68	36 × 72
Slope, inches per foot	1.5	1.5	....
Revolutions per minute	36	36	15
Screen surface, type	*P*	*P*	*W*
Aperture, in.	0.67	0.275	0.5
Material	5/16-in.*S*	3/16-in.*S*	*S*
Life, tons of feed	148,000	32,000	10,800
Water, gallons per hour	1200	1200	....
Horsepower installed	....	....	5
Horsepower consumed	2	1.5	....
Tons of new feed per 24 hr.	600	300	60
Tons of total feed per 24 hr.	1800	800	60
Tons per square foot per 24 hr.	31	15	1.06
Equivalent tons per square foot per 24 hr., 1-mm. aperture	1.8	2.1	0.085
Per cent. undersize in oversize	12	12	....
Efficiency, per cent.(*b*)	....	....	....

Name of mill	Bunker Hill & Sullivan Mining Co.	Cananea Consolidated Copper Co.	American Zinc, Lead & Smelting Co.
Dimensions, diameter × length, in.	48 × 60	48 × 96	48 × 96
Slope, inches per foot	1	0.75	1.5
Revolutions per minute	17	16	15
Screen surface, type	*P*	*P*	*P*
Aperture, in.	0.5 × 1	0.375	0.5
Material	*S*	*S*	No. 10*S*
Life, tons of feed	36,000	37,800	65,500
Water, gallons per hour	....	3120	Dry
Horsepower installed	....	....	....
Horsepower consumed	2	1.5	....
Tons of new feed per 24 hr.	300	630	360
Tons of total feed per 24 hr.	300	....	720
Tons per square foot per 24 hr.	4.8	6.3	7.2
Equivalent tons per square foot per 24 hr., 1-mm. aperture	0.38	0.66	0.57
Per cent. undersize in oversize	....	5	32
Efficiency, per cent.(*b*)	....	....	....

b Recovery formula. *d* Diagonal slot. *CI* Cast iron.

trommels in milling plants

Federal M. & S. Co. Morning Mill				U. S. Sm. Ref. & Min. Co. Midvale Plant		
32×134	32×134	32×134	32×134	36×60	36×60	36×60
2	2	2	2	1.25	1.25	1.25
17	12	12	21	15	15	15
P	*P*	*P*	*P*	*W*	*W*	*W*
1.0	0.5	0.2	0.039	0.13	0.087	0.053
S	*S*	*S*	*S*	*S*	*S*	*S*
56,000	30,000	20,000	300	26,000	8700	5500
600	800	1000	1300	780	800	950
....	....	....	....	....	....	....
0.5	0.5	0.5	0.5	1	1	1
500	500	200	50	500	300	275
800	750	400	75	....	....	....
8.6	8.0	4.3	0.8	5.3	3.2	2.9
0.34	0.67	0.86	0.8	1.6	1.45	2.16
....	....	34	27	21	23	26
75	....	54	69	....	....	....

U. S. S. R. & M. Co., Midvale	Braden Copper Co.		Copper Range Consolidated Copper Co.	St. Joseph Lead Co., Bonne Terre	U. S. S. R. & M. Co., Midvale	Cananea Consolidated Copper Co.
36×72	36×72	36×72	36×78	36×96	42×72	48×60
1.25	1.8	1.2	1	1	1.25	0.75
15.67	19	18	18	36	15	16
P	*P*	*P*	*P*	*P*	*P*	*P*
0.28	0.158	0.1	0.25	0.35	0.59	0.625
S	*S*	*S*	*S*	*S*	*S*	*S*
57,500	....	....	27,500	20,500	45,000	48,000
740	....	....	12,500	....	Dry	6480
....	....	....	....	....	....	....
1.25	....	....	....	....	1.25	1.5
500	321	440	300	275	500	500
....	....	....	363	....	....	800
4.5	5.7	7.8	5.9	3.6	7.6	12.8
0.62	1.4	3.1	0.93	0.4	0.5	0.8
13	35	80	35	15	15	5
....	46	8	....	....	....	....

Federal Lead Co. Mill No. 3				Federal Lead Co., Mill No. 4	Braden Copper Co.
48×96	48×108	48×108	48×108	48×108	48×12
1.5	2	2	1.75	2	2.4
13	18	18	20	20	13
P	*P*	*P*	*P*	*P*	*P*
0.125	0.47	0.59	0.079	0.75	0.16*d*
No. 20*S*	*S*	*S*	*S*	*S*	*S*
4750	18,900	18,900	4500	52,500	....
Wet	....	....	....	....	....
....	....	....	....	....	10
....	3	3	3	3	....
100	450	450	375	1250	1400
250	....	....	....	....	....
2.5	4.0	4.0	3.3	11.0	11.1
0.79	0.33	0.27	1.6	0.58	2.8
40	@ 15	@ 15	@ 15	@ 15	22
....	....	....	....	....	72

P Punched plate. *S* Steel plate. *W* Woven wire.

Table 26. Performances of trommels in milling plants—*Continued*

Name of mill	Bunker Hill & Sullivan Mining Co.		Witherbee, Sherman Co. No. 4
Dimensions, diameter × length, in.	48 × 120 sectional		48 × 120
Slope, inches per foot	1		1.375
Revolutions per minute	17		16
Screen surface, type	*W*		*P*
Aperture, in.	0.25 × 0.5	0.125 × 0.25	0.75 × 2
Material	*S*		*S*
Life, tons of feed	18,000	6000	150,000
Water, gallons per hour	1700		Dry
Horsepower installed	...		...
Horsepower consumed	3		2.5
Tons of new feed per 24 hr.	200		2000
Tons of total feed per 24 hr.	200		2000
Tons per square foot per 24 hr.	3.2	3.2	15.9
Equivalent tons per square foot per 24 hr., 1-mm. aperture	0.5	1.0	0.84
Per cent. undersize in oversize	Less than 10		...
Efficiency, per cent. (*b*)	...		...

Name of mill	Witherbee, Sherman Co. No. 3	St. Joseph Lead Co., Rivermines	Phelps-Dodge Co., Morenci
Dimensions, diameter × length, in.	48 × 144	48 × 192	54 × 120
Slope, inches per foot	1.375	1.5	1.5
Revolutions per minute	14	14	15
Screen surface, type	*P*	*P*	*P*
Aperture, in.	1.25 × 2	0.354	0.75
Material	*S*	*S*	0.5-in. *S*
Life, tons of feed	90,000	96,000	150,000
Water, gallons per hour	Dry	Dry	Dry
Horsepower installed	...	...	...
Horsepower consumed	2.4	2	@ 4
Tons of new feed per 24 hr.	1300	500	2500
Tons of total feed per 24 hr.	1500	800	...
Tons per square foot per 24 hr.	10	4	17.7
Equivalent tons per square foot per 24 hr., 1-mm. aperture	0.32	0.44	0.93
Per cent. undersize in oversize	...	...	...
Efficiency, per cent. (*b*)	...	65	...

Name of mill	Empire Iron & Steel Co. (*e*)	Susquehanna mine (*f*)	Alaska Treadwell (*g*)
Dimensions, diameter × length, in.	32 × 72	60 × 288	60 × 168
Slope, inches per foot	1.25	1.75	...
Revolutions per minute	...	...	13
Screen surface, type	5/16-in. *P*	*P*	...
Aperture, in.	2	2	2
Material	*S*	*S*	...
Life, tons of feed	...	...	...
Water, gallons per hour	Dry	Dry	Dry
Horsepower installed	...	...	...
Horsepower consumed	...	...	...
Tons of new feed per 24 hr.	3000	5660	2250
Tons of total feed per 24 hr.	...	...	...
Tons per square foot per 24 hr.	60	15	10.2
Equivalent tons per square foot per 24 hr., 1-mm. aperture	2.4	0.30	0.20
Per cent. undersize in oversize	...	...	...
Efficiency, per cent. (*b*)	...	...	...

b Recovery formula.[1] ***e*** *99 J 560.* ***f*** *102 J 787.* ***g*** *114 P 410.* ***CI*** Cast iron. ***P*** Punched plate. *S* Steel plate. ***W*** Woven wire.

Table 27. Speed of trommels. (*After Holbrook and Fraser*)

Diameter of screens, inches	Revolutions per minute at which no screening takes place	Good practice			
		Ore		Coal	
		Revolutions per minute	Peripheral speeds, feet per minute	Revolutions per minute	Peripheral speeds, feet per minute
30	63	16 to 35	123 to 275	27 to 30	212 to 235
36	56	20 to 30	189 to 283	23 to 26	217 to 245
48	48	12 to 22	151 to 277	19 to 20	239 to 252
60	43	11 to 13½	173 to 212	15 to 16	236 to 252
72	40	11 to 13	207 to 245	12 to 13	226 to 245
84	35	10 to 11½	220 to 253	8 to 11	176 to 242

Length affects efficiency of screening; increased length resulting in more complete removal of fines. On the other hand, most of the screening is done in the first 2 feet. Present practice inclines to steep slopes and short lengths. *Truscott* gives 5 to 15 ft. for one separation, the longer for finer materials, but few trommels exceed 10 ft.

Capacity increases with increase in diameter, speed, slope and size of aperture, with decrease in percentage of difficult oversize in the feed, and is greater in wet than in dry screening. On the basis of Table 26 capacity is 0.6 ton per sq. ft. per 24 hr. per mm. of aperture for dry screening and 1.0 ton per sq. ft. per 24 hr. per mm. for wet screening. *Wiard* gives the rule for capacity of 48-in. trommels in open circuit, $C = 20\,d$ for round-hole punched plate and $C = 25\,d$ for square openings, where C = tons per 24 hr., d is the aperture in mm., and the feed contains 50 per cent. oversize. With a greater percentage of oversize, capacity will be greater to an extent substantially equal to the tonnage of excess above 50 per cent. oversize. Deduct 15 per cent. from the above capacities for each 6-in. decrease in trommel diameter. *Truscott's* figure is 0.5 ton per 24 hr. per sq. ft. per mm. Holbrook and Fraser give the rule for capacity on coal that 3 to 4 sq. ft. of screen surface is required per ton per hour per inch of aperture, with an increase of 50 per cent. in the area required if the coal is damp. Lincoln (*11 Bul. UI, No. 9*) gives data concerning the performance of a considerable number of revolving screens working in Illinois bituminous-coal washeries. Average capacity (19 screens) in sizing raw coal, 1-in. to 3-in. maximum size, over round-hole plate with ¾-in. to 2¾-in. openings was 0.36 ton per sq. ft. per 24 hr. per mm. of aperture and the range was from 0.05 to 1.20 tons. When re-sizing washed coal that had been sized prior to washing, the feed ranging from 3-in. down to ¾-in. maximum size and the screen apertures from 2½- to ¼-in. round-hole, the average capacity (41 screens) was again 0.36 ton per sq. ft. per 24 hr. per mm. and the range from 0.04 to 2.00 tons. In sizing washed coal, the feeds ranging from 3½- to ⅜-in. maximum, and the screen apertures from 2- to ¼-in. round-hole, the average capacity (29 screens) was 0.66 ton per sq. ft. per 24 hr. per mm. and the range from 0.12 to 2.80 tons. In draining or rinsing washed coal on 1⁄16- and 3⁄32-in. screens, capacities were 4.30 to 5.30 tons per sq. ft. per 24 hr. per mm.

Efficiency. Averages from Table 26 show that the oversize from coarse (plus-0.5-in.) dry trommels contains about 15 per cent. undersize, while that from fine (minus-0.25-in.) wet trommels contains about 30 per cent.

Efficiency reckoned as percentage recovery of undersize may run as high as 80 per cent. in ½-in. dry trommels. In wet trommels 45 to 70 per cent. is the usual range, but the figure may drop to 10 per cent. or lower when there is much material in the feed near the size of the screen opening, and the trommel is overloaded.

Table 28. Life of punched plate on trommel screening hard quartz ore. (*After Forbes*)

Diameter of holes, inches	Thickness of plate	Tons screened
⅝	5/16 in.	21,000
⅜	¼ in.	21,000
3/16	10 gage	18,860
1.5 mm.	16 gage	5,850

Life of covering on trommels is given in Table 26. Forbes (*43 A 487*) gives the data in Table 28.

Advantages of trommels are simplicity in construction and operation, freedom from vibration, small loss of head room and general ruggedness. Disadvantages are blinding, difficulty of repair, low capacity per square foot of screen surface and low efficiency.

Bunker Hill screen, developed and used at Bunker Hill and Sullivan Mining Co., is a variety of revolving screen in which a conical screening surface is made to revolve around an axis inclined 45°. (See Fig. 22.) It is for fine screening only and has not been used extensively. The screening surface is carried on a light framework attached to a trunnion, which revolves in an inclined bearing, revolution being accomplished by means of a bevel gear, bevel pinion and pulley. Feed is introduced through a box set to deliver pulp tangentially to the screen surface near the top of the down-coming side. Undersize collects in the conical housing and passes out through the chute shown, while oversize passes to the apex and out through a hollow trunnion. Water sprays are provided as shown, the interior one adjustable. Average speed is 20 r.p.m. Capacity, with different sizes of screen cloth is, according to the manufacturer, as given in Table 29.

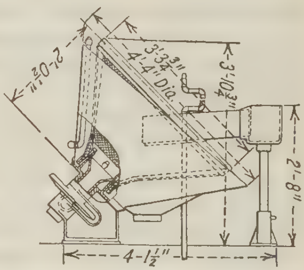


Fig. 22.—Bunker Hill screen.

Table 29. Average capacity of Bunker Hill screen. (*Hendric and Bolthoff*)

Mesh	Working load, tons per 24 hr.
6	150
16	100
22	55
30	40
Finer than 30-mesh	Up to 30

At Bunker Hill and Sullivan (*3 MM 45*) a screen with 22-mesh cloth handled 50 to 80 tons per 24 hr. of −2-mm. feed, or 4.5 to 7.2 tons per sq. ft. per 24 hr. per mm. The volume of feed should be kept down to 150 gal. per min. at 6-mesh, 100 at 22-mesh and 80 at finer meshes. Life of 22-mesh steel-wire screen at Bunker Hill and Sullivan was 13 days. Oversize contained 10 per cent. undersize.

6. Shaking screens

Description. Shaking screens consist essentially of a shallow rectangular box two to four times, or more, as long as wide, open at one end, fitted with a screen bottom and shaken by means of a suitable mechanism which alone, or in conjunction with the slope of the screen surface, moves oversize to the discharge end. The method of support of the frame and the means of shaking vary considerably. In the oldest and simplest form the frame is supported on four vertical rods or chains at the corners and actuated by an ordinary

eccentric. The motion under such circumstances is substantially straight-line harmonic motion and the screening surface must be given a slope of 10 to 15° in order to move material, or the forward stroke must be ended against a bumping block. The use of bumping blocks produces excessive rack and vibration on the screen frame and supports. By inclining the supporting rods backward from the vertical at an angle of about 15° (Ferraris support), the motion of the screen surface is upward at the end of the forward stroke and sharply downward at the beginning of the backward stroke, with the result that the screen drops away from under the load and the material moves ahead with a series of jumps. This motion is accented by shortening the suspending rods. The same effect, even more sharply accented, can be obtained by mounting the screen frame on rollers and shaping the track to give a sharp upward movement at the end of the forward stroke. Occasionally, with track support, the track is also turned up at the end of the backward stroke, in order to give some backward travel of the material and thus increase the length of path and multiply opportunites for undersize to pass. Table head motions, which impart a differential, quick-return movement, are suitable only for relatively short strokes (1.5 in. or less) and relatively small screens. With mechanisms producing differential movement the screen surface need be inclined only slightly or not at all in the direction of flow.

Speed, slope and length of stroke should be adjusted to produce rapid stratification of the feed, quick forward movement of the load and minimum blinding. If speed or length of stroke is too great, material is thrown bodily away from the screen surface and stratification is effected slowly, if at all. On the other hand, if there is too little throw, the screen blinds badly. Slope is adjusted so that with a proper speed and length of stroke there is sufficient throw of material away from the screen surface to prevent blinding. Speeds in practice range from about 60 or 70 @ 9-in. strokes per min. to about 400 @ ¾-in. strokes. The average for coal shakers is about 100 @ 6-in. strokes per min. A common rule for design of shaking coal screens is to make the product of stroke length in inches by strokes per minute equal 600 and to slope the screens about 14° for coarse coal and 18 to 20° for fine. Holbrook and Fraser state that these slopes should be reversed on account of the greater impulses, normal to the screen surface, required to clear coarse perforations. Assuming that the load moves forward with the screen but has no backward movement the general rule above gives the coal a forward velocity of 50 ft. per min. and some designers thus state the rule (*2 CA 352*).

If there is any considerable amount of lost motion or back-lash in the eccentric or suspending mechanism the action of material on the screen may be entirely different from that with the same mechanism with no back-lash. Suspending rods made of ash boards, 1- by 10- or 12-in. in section, attached rigidly at both ends are very durable and have a tendency to reduce back-lash. Table 30 gives data on shaking screens in various concentrating mills.

Capacity ranges from 2 to 8 tons per sq. ft. per 24 hr. per mm. of aperture. In de-sanding pebble phosphate 15 to 20 tons per sq. ft. per 24 hr. per mm. is sent over $\frac{3}{64}$-in. screens in a 3-in. bed with 400 to 500 gal. per min. of wash water, but the screening is not efficient. Wiard (*Liddell*) gives a formula that reduces to $T = AR/6P$ as a maximum theoretical tonnage for shaking screens where T = capacity in tons per hr. per ft. of width; A = average between screen aperture and preceding limiting-screen aperture, expressed in feet; R = rate of advance of material over the screen in inches per min. = shakes per min. × amplitude of shake; and P = ratio of weight of oversize to total

Table 30. Performances of shaking screens in metal-concentrating plants

Mill	Old Dominion	Phelps-Dodge Co., Morenci Branch		St. Joseph Lead Co.	Zinc Corporation (*b*)	Timber Butte (*c*)
Type	Home-made	Cole	Cole (*a*)	Ferraris	....	Pulsating
Dimensions, width × length, in.	24 × 48	28 × 48	28 × 48	29 × 89	24 × 93	24 × 96
Slope, inches per foot	4	1.25	1.25	Level	....	....
Strokes per minute	240	385–400	385–400	260	240	....
Length of stroke, in.	1.25	0.75	0.75	1.25	1.25	....
Screen surface	*P*	*P*	*W*	*P*	*P*	*P*
Aperture, in.	0.375 and 0.5	0.25 Rd	0.25	0.079	0.119	0.75
Material	*S*	⅛-in. *S*	No. 072 brass	18 ga. *S*	*S*	*S*
Life, tons of feed	12,000 and 18,000	15,750	5250	10,500	....	....
Horsepower consumed	....	1	1	0.75–1.5	....	....
Tons of new feed per 24 hr.	200	450	150	300	312	350–450
Tons of total feed per 24 hr.	....	....	250	....	....	....
Tons of total feed per square foot per 24 hr.	25	48	26.8	18.7	20.1	14.6–18.7
Equivalent total tons per square foot per 24 hr. per mm. of aperture	2.0 and 2.6	7.6	4.2	8.35	6.7	0.8–1.0

a Wet. See *51 A 417*. *b* *118 P 89*. *c* *52 A 915*. *P* = punched plate. *W* = Woven wire. *S* = Steel.

feed. This formula gives results lower than those shown in Table 30. In screening bituminous coal, capacity is usually considered to be a function of width only. Holbrook and Fraser (*Bul. 234, USBM*) give Table 31 as representative of common practice. The 1.5- to 2-in. slack screen is the minimum aperture. These writers also give a table used by certain designers of coal tipples which shows an allowance of 1 sq. ft. per ton per 24 hr. per mm. of aperture for screens handling 125 tons per hr., but for four times this capacity the area allowed is less than twice as much. Another table covering practice at 43 Central Interior mines (bituminous) shows 2 tons per sq. ft. per 24 hr. per mm. for slack screens at small mines (less than 1000 tons per 8 hr.) to 5 tons per sq. ft. per 24 hr. per mm. at mines handling upward of 4000 tons per 8 hr. The writers ascribe the higher tonnages handled per sq. ft. on the large screens at the large mines to uniform, well-distributed feed and the use of a coarse relief screen above the slack screen. Lincoln (*11 Bul. UI, No. 9*) gives data on a number of shaking-screen installations in Illinois coal washeries. The average capacity in tons per sq. ft. per 24 hr. per mm. of aperture for 23 screens with round-hole punched plate, ranging from ¼-in. to 2¼-in. openings, was 1.1, with an extreme range from 0.04 to 3.1 tons. For screens in dewatering service, with $\frac{1}{16}$- and ⅛-in. round-hole punched plate, capacity was from 8.4 to 10.1 tons per sq. ft. per 24 hr. per mm. Roberts and Schaeffer (*PC*) recommend 30 to 50 tons per hr. per ft. of width for picking and 50 to 75 tons for slack screens. Anthracite shakers are run at higher speed and with shorter stroke than bituminous shakers. Table 32 made up from Ashmead's description of the Susquehanna Collieries Co. (*66 A 422*) shows a much larger feed rate

per sq. ft. per 24 hr. mm. for the fine screening than for the coarse. This is partly due to the fact that specifications for fine sizes of anthracite are much less rigid than for the coarser sizes and partly due also to the fact that the coarser screens are under-loaded while the finer are overloaded. Areas of shaking screens in Dunmore breaker of Pennsylvania Coal Co. (*22 CA 785*) for 4000 tons per 8 hr. are: Lump size, 90 sq. ft.; grate size, 225 sq. ft.; egg, 609 sq. ft.; stove, 360 sq. ft.; chestnut, 648 sq. ft.; pea, 180 sq. ft.; No. 1 buckwheat, 180 sq. ft.; rice coal, 216 sq. ft.; barley, 216 sq. ft. Table 33 (*22 CA 5*) shows the areas of the shakers in another anthracite breaker, with the tonnages of feed and oversize handled.

Table 31. Width of screen for different tonnages of run-of-mine bituminous coal

Capacity per 8-hr. day, tons	Round-hole plate, feet	Lip screen, feet
1000 or less	5	4 to 5
1000 to 2000	7	5 to 6
2000 to 3000	8	6 to 7
3000 to 4000	9	7 to 8
Over 4000	10	8 to 10

Table 32. Tons of anthracite per square foot of shaker-screen surface per 24 hr. per mm. of aperture. (*After Ashmead*)

Screens for separating	Screen, aperture in.	Tons per square foot per 24 hr. per mm.
Egg	$2\frac{5}{16} \times 3\frac{1}{4}$	0.52
Stove	$1\frac{11}{16} \times 2\frac{5}{16}$	0.64
Chestnut	$1\frac{5}{16} \times 1\frac{11}{16}$	0.83
Pea	$\frac{5}{8} \times 1\frac{5}{16}$	1.20
Buckwheat	$\frac{3}{8} \times \frac{5}{8}$	1.68
Rice	$\frac{1}{4} \times \frac{7}{16}$	2.15
Barley	$\frac{3}{32} \times \frac{1}{4}$	5.00

Power consumption for screens handling coal averages from 0.04 to 0.05 hp. per sq. ft. of screen surface when the screen is run at average speed and with average load. Installed horsepower is 1.5 to 2 times the average consumption. Screens handling ore are ordinarily run at a much shorter stroke so that the screen travel in feet per minute is only about half of that of coal shakers, but the load per square foot is much heavier. From the meager data in Table 30 it would appear that power consumption is between 0.08 and 0.11 hp. per sq. ft. of screen surface.

Table 33. Areas of shaking screens at Seneca breaker of Lehigh Valley Coal Co.

Size	Dimensions	Tons per 8 hr.	
		Feed	Oversize
Broken	5×20-ft. 8 in.	1584	216
Egg	5×20-ft. 8 in.	1513	270
Egg	5×22 ft. 10 in.	834	140
Stove	5×22 ft. 10 in.	695	189
Nut	5×22 ft. 10 in.	506	259
Pea	5×22 ft. 10 in.	248	58
Buckwheat	5×22 ft. 10 in.	189	69
Rice	5×31 ft. 11 in.	120	48
Barley	5×31 ft. 11 in.	72	50

Screen banks making several different products are arranged either with the different screens in the same plane with all feed going over the finest screen, or with superimposed boxes driven by the same mechanism, with the coarsest screen in the uppermost box. In either case speed and amplitude must be a

compromise and since long slow stroke is necessary to move the coarse material, fine screening suffers.

Applicability. Shaking screens are used to the greatest extent in coal dressing, where they have substantially displaced all other types, and in treating certain non-metallic ores such as those of phosphate and asbestos. They are used both wet and dry, but consume excessive amounts of spray water when used wet. Many attempts have been made to set the screen so that it moves up and down through the surface of a body of water, but this involves mechanical removal of undersize, which has not proved successful. Shaking screens are not suitable for treating clayey ores on account of the readiness with which clay balls form on them. Their suitability for coal washing is based upon the fact that they load and discharge with but little vertical drop and do not subject the coal to much tumbling action during screening, hence there is but little breakage. Another advantage is that by extending a screen over three or four loading tracks, slack may be loaded into a car on the track nearest the feed end and progressively coarser sizes into cars on succeeding tracks. Blank plates cover the space between tracks. Thus mounted, the screen is both a sizing device and a loading conveyor and is frequently so used in bituminous-coal tipples.

The principal DISADVANTAGE of shaking screens is the high repair cost due to rack and vibration. This applies not only to the screen itself but to the supporting structure. In order to minimize the effect on the structure two screens are frequently mounted on opposite sides of the same drive shaft with eccentrics at 180°. Even this is insufficient, however, to balance the load because of the difference to be expected in load of material on the two screens at any given instant, hence many designers make the shaking-screen support independent of the building frame, in order to localize vibration and not rack the entire building.

Anti-gravity screen is a shaking screen with a deck about 3 ft. wide by 7 to 8 ft. long, fed at the mechanism end and sloping upward toward the discharge end at an angle of about 12° with the horizontal. Under the screen deck is a blank deck, which slopes upward toward the discharge end at an angle of about 7°. The decks are about 12 in. apart at the discharge end. Forward travel is attained by supporting the shaking frame on sloping cast-iron sliders, so that the deck falls away from under the materials on the backward stroke. The screen is run at 350 @ 1-in. strokes per min. Capacity at this speed at St. Louis, Rocky Mtn. and Pacific Co. plant at Raton, N. M. (*23 CA 791*) was 30 tons per hr. of − 1-in. feed over a battery of 5 screens in series, with ¾-in. and ½-in. square-mesh wire and ⅜-in., 3/16-in. and 1/16-in. Ton-cap screens respectively. At 300 strokes per min. the capacity was too low.

7. Vibrating screens

General. Vibrating screens developed from shaking screens in an attempt to overcome the blinding that the latter suffer when screening fine dry material. They differ from shaking screens in that the motion imparted to the screening surface is rapid, vigorous movement of small amplitude in a direction substantially at right-angles to the screening surface. Ordinarily the rate of vibration is very much greater than that of even the most rapid shakers. The early forms produced vibration by hammering the screen surface, either directly or through small anvil blocks fastened thereto, but in such mechanisms vibration was of low speed and decidedly localized. The next step in development was vibration of the screen frame as a whole. Several forms of pulsing, waving and vibrating screens were built on this principle. The Colorado impact screen was the most successful of these early types. The final development was attachment of the vibrating element of a high-speed

vibrator directly to the screen surface, usually near the center. Of this type, the Mitchell, Hum-mer and Leahy are the best known.

Table 34. Performances of Colorado impact screens in metal-concentrating plants

Plant	Braden Copper Co.	Tungsten Mines Co.,		
Size, width × length, in.	36 × 36	36 × 36	36 × 36	36 × 36
Slope, inches per foot	....	8	8	8.5
Speed, knocks per minute	600	504	504	504
Stroke, in.	....	0.375	0.375	0.375
Screen aperture, in.	0.5 × 0.5	0.323	0.323	0.120
Screen material	*W*	*W*	*W*	*W*
Screen life, tons treated	....	1680	6216	1665
Horsepower installed	....	....	....	....
Horsepower consumed	....	....	....	....
Tons of new feed per 24 hr.	282	240	240	80
Tons of total feed per 24 hr.	282	240	888	185
Water, gallons per hour	....	....	....	....
Equivalent tons per square foot per 24 hr., 1-mm. aperture	2.5	3.2	12.1	6.8
Per cent. undersize in oversize	23	....	....	....
Efficiency, per cent. (*b*)	66.4	....	82	65

Plant	Alaska Gastineau			St. Joseph Lead Co., River-mines
Size, width × length, in.	32.5 × 40	33.5 × 54	33.5 × 54	36 × 48
Slope, inches per foot	8.5	8.4	8.4	8
Speed, knocks per minute	498	660	660	450
Stroke, in.	0.125	0.125	0.125	0.25
Screen aperture, in.	0.083–0.108	1.25	1.25	0.079
Screen material	.028″–.035″ *W*	0.375″ *W*	0.375″ *W*	*W*
Screen life, tons treated	4165	75,000	124,000	3600
Horsepower installed	1.5	1.1	1.1	1.25
Horsepower consumed	0.75	1	1	0.5
Tons of new feed per 24 hr.	167	2500	1667	250
Tons of total feed per 24 hr.	833	2500	2750	450
Water, gallons per hour	....	....	....	900
Equivalent tons per square foot per 24 hr., 1-mm. aperture	4.4–3.4	6.3	6.9	18.8
Per cent. undersize in oversize	....	....	....	....
Efficiency, per cent. (*b*)	42	75	75	70

Plant	Porphyry copper mill	Braden Copper Co.	Chino Cons. Copper Co.	Granitic zinc ore	
Size, width × length, in.	36 × 48	36 × 48	36 × 48	36 × 48	36 × 48
Slope, inches per foot	8	......	8	6	6
Speed, knocks per minute	576	600	750	78*y*	60*y*
Stroke, in.	0.5	......	0.25	0.5	......
Screen aperture, in.	0.120	0.5	0.087	9-mesh	0.4 and 0.021
Screen material	0.080″ *W*	......	*W*	*W*	*W*
Screen life, tons treated	2700	......	5000–6000	660	240–180
Horsepower installed	....	......	1	0.75	0.75
Horsepower consumed	....	......	1	0.75	0.75
Tons of new feed per 24 hr.	200–250	360	300–400	166	60
Tons of total feed per 24 hr.	600	360	600–900	498	....
Water, gallons per hour	5000	......	4500	....	....
Equivalent tons per square foot per 24 hr., 1-mm. aperture	16.4	2.4	28	....	4.9–9.4
Per cent. undersize in oversize	....	......	....	....	....
Efficiency, per cent. (*b*)	....	......	44–50	....	....

b Recovery of undersize. *W* Steel-wire cloth. *y* Revolutions per minute of pulley.

Colorado impact screen is shown in Fig. 23. It consists essentially of the wooden frame (A) which carries the screen cloth and an undersize hopper (E), mounted on two elliptical wagon springs (B) which are fastened to the supporting frame. A shaft carrying multiple-armed cams (D) and pulley-driven is also mounted on the supporting framework. Revolution of the cam depresses the screen frame, which, on release, springs forward against bumping blocks on the framework. The usual speed is about 600 knocks per min. The screen surface is set at a slope of about 40° from the horizontal. The screen is made in two sizes, *viz.*: 3 ft. wide by 3 ft. long and 3 ft. wide by 4 ft. long. Screens may be superimposed, as shown in Fig. 23. Table 34 gives data concerning installations in several mills. It indicates an average capacity per square foot of screen surface per 24 hr. at 1-mm. aperture of about 5.5 tons for dry screening and about 20 tons for wet screening. An efficiency test on impact screens at Ray Cons. Copper Co. is shown in Table 35.

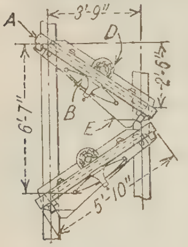


Fig. 23.—Colorado impact screen.

Table 35. Efficiency of Colorado impact screens at Ray Consolidated Copper Co.

Aperture, inch	Character of feed	Tons per 24 hr.	Undersize in feed, per cent.	Undersize in oversize, per cent.	Efficiency, per cent. (c,d)	Efficiency, average per cent.
0.086	a	73	30.8	15	60.8	68.3
0.086	b	465	55.8	49.3	22.9	29.1

a Original mill feed, open circuit. b Original mill feed, plus oversize from rolls c One test. d Recovery of undersize.

Mitchell screen (Fig. 24) consists essentially of the screen (a), stretched to drum-head tightness by means of tension rods (b), suspended by side plates (c) from the ends of the vibrator (d), which, in turn, is supported at the center only in cradle (e) from the main frame. Detail of the vibrator is shown in Fig. 25. It consists of a motor (A), in a dust-proof frame. Two ball cages (D) carried on the ends of the motor shaft revolve inside the cylindrical ball races (E). One set of balls only is placed in each ball cage and these sets are spaced at 180° on the shaft, as shown. When the motor shaft is revolved, the unbalanced load of the balls against the races combined with the gyroscopic action of the rotor causes each element of the motor housing to generate the surface of an acute double cone with common apex in the vertical plane through the center of the suspending cradle. This motion is transmitted through the side plates (c) (Fig. 24) to the screening surface. The motor speed is about 3400 to 3600 r.p.m., producing the same number of vibrations. Intensity of vibration is varied by changing

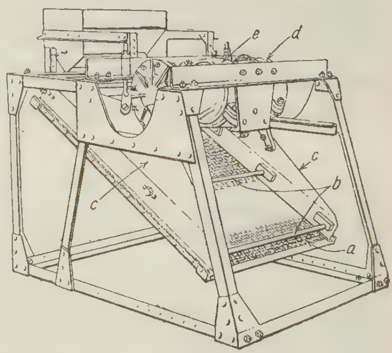


Fig. 24.—Mitchell screen.

Table 36. Tests of Mitchell screen handling coke and bituminous coal. (*After Holbrook and Fraser*)

Screen aperture	Slope, degrees	Feed rate, tons per hour	Undersize		Oversize		Per cent. true undersize in oversize	Tons per square foot per 24 hr. per mm.	Efficiency, per cent.	Tons through screen per hour	Tons through per square foot per 24 hr. per mm.
			Weight, per cent.	Moisture, per cent.	Weight, per cent.	Moisture, per cent.					
Tests on coke through 1.5-in. square-mesh screen											
0.25	28 to 19	28.9	16.2	18.1	83.8	6.9	0.8	4.6	96.1	4.7	0.7
	33 to 24	31.7	14.0	19.2	86.0	7.1	2.0	5.0	89.3	4.4	0.7
	36 to 27	35.6	15.4	18.8	84.6	7.4	2.5	5.6	88.3	5.5	0.9
	39 to 30	42.2	17.4	12.6	82.6	5.1	0.4	6.6	98.2	7.3	1.2
	39 to 30	48.7	17.8	11.9	82.2	5.1	0.8	7.7	96.6	8.7	1.4
	39 to 30	54.1	14.5	11.1	85.5	4.9	4.3	8.5	80.7	7.8	1.2
0.50	28 to 19	43.1	30.3	17.1	69.7	6.0	0.5	3.4	98.8	13.2	1.0
	29 to 20	47.0	30.3	18.1	69.7	6.4	0.7	3.7	98.3	14.4	1.1
	34 to 25	54.4	30.5	17.2	69.5	5.8	0.3	4.3	99.3	16.6	1.3
	34 to 25	59.1	30.1	12.2	69.9	5.8	0.8	4.6	98.2	17.8	1.4
	39 to 30	63.1	29.6	10.6	70.4	5.4	1.4	5.0	96.9	18.7	1.5
	39 to 30	65.2	28.2	10.1	71.8	4.9	2.6	5.1	93.9	18.4	1.4
0.75	28 to 19	34.7	67.2	12.4	32.8	5.6	0.6	1.8	99.7	23.3	1.2
	34 to 25	41.8	66.4	12.1	33.6	5.3	1.0	2.2	99.4	27.8	1.5
	34 to 25	49.2	66.6	11.8	33.4	5.3	1.2	2.6	99.4	32.2	1.7
	34 to 25	55.7	65.4	10.1	34.6	5.2	2.3	2.9	99.1	36.4	1.9
	39 to 30	62.8	64.6	9.8	35.4	5.4	3.2	3.3	98.2	40.6	2.1
	39 to 30	54.1	65.8	9.8	34.2	5.4	1.8	2.8	99.1	35.6	1.9
Tests on bituminous through 0.75-in. screen											
0.25	31 to 22	6.1	24.8	11.5	75.2	8.9	36.5	1.0	63.2	1.5	0.2
	31 to 22	26.3	59.0	6.1	41.0	4.2	2.2	4.1	98.7	15.5	2.4
	34 to 25	35.2	58.7	6.1	41.3	4.2	2.8	5.5	96.7	20.6	3.2
	37 to 28	46.4	58.4	5.2	40.6	3.8	3.0	7.3	97.3	27.2	4.3
0.50	33 to 24	32.3	74.8	5.8	25.2	3.4	1.8	2.5	99.6	24.2	1.9
	35 to 26	36.8	73.6	5.4	26.4	3.4	2.2	2.9	99.2	27.1	2.1
	37 to 28	41.6	73.9	5.4	26.1	3.1	2.4	3.3	99.2	30.8	2.4
	39 to 30	47.1	73.2	5.1	26.8	2.9	3.4	3.7	98.8	34.4	2.7
0.75*a*	33 to 24	42.2	63.1	4.5	36.9	2.8	1.7	2.2	98.9	26.6	1.4
a	35 to 26	48.6	61.6	4.1	38.4	2.8	2.6	2.6	98.3	30.0	1.6
b	34 to 25	36.2	91.2	2.9	8.8	2.0	3.1	1.9	98.8	33.0	1.7
b	39 to 30	41.3	90.2	2.9	9.8	2.0	4.2	2.2	99.7	37.2	2.0

a Feed – 1.5-in. grizzly. *b* Feed mixed nut and slack.

the size or number of balls in the ball cage. Power consumption is from 3/8 to 1/2 hp. for a 4 × 6-ft. screen. Slope of the screening surface is about 35 to 40° and may be varied while the screen is in operation. The path of the cross wires of the screen under the influence of the vibrator is circular, the direction of rotation being determined by the direction of rotation of the rotor. When the direction of the vibration of the wires is such that the movement at the top of the path is up-slope so that the wires are driven into the mass of downcoming material, maximum efficiency is obtained. Capacity on ores is about 11 tons per sq. ft. per 24 hr. per mm. of aperture.

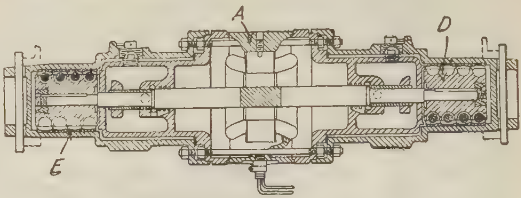


Fig. 25.—Vibrator of Mitchell screen.

Table 36 gives some tests on coke and bituminous coal. The tests on 0.25-in. screen show that the capacity on both coke and coal is about 8 tons per 24 hr. per sq. ft. per mm. The screen is clearly underloaded with the coarser cloth. The tests show also that, as the percentage of undersize in the feed increases, the tonnage that can be passed per millimeter of opening likewise increases. The tests with 0.25-in. coke indicate that a slope between 30° and 39° is better than a smaller slope. The effect of excessive moisture on screening is emphasized by the first test with bituminous coal. The experimenters noted that 50 to 60 per cent. of the holes in the screen were blinded by fine coal during this run. A given percentage of moisture is not so harmful with coke as with bituminous coal, due, no doubt, to the fact that much of the water is in the pores of the coke, and therefore has no effect on the behavior of the coke surface while much of the moisture in the coal is at the particle surfaces.

Hum-mer screen is a high-speed vibrating screen in which vibration is attained by means of a solenoid-buzzer mechanism with the moving part attached to the screen cloth. A diagrammatic sketch of the vibrator is shown in Fig. 26; (*c*) is the electro magnet, (*h*) the armature, to which is attached the post (*i*), which in turn, is attached through the foot (*j*) to the screen cloth. The amplitude of vibration may be varied by depressing the armature by means of the hand wheel (*a*). The upward motion of the screen terminates suddenly when the armature hits the striking block (*e*). The vibrating mechanism requires 15-cycle alternating current, hence ordinarily a special generator. A 1-hp. motor will drive a generator to furnish current to two vibrators. The usual size of a unit screening surface for ores and the like is 3 ft. wide by 5 ft. long, for coal 4 ft. wide by 5 or 7 ft. long. The 7-ft. length is sometimes built with two vibrators, an extra-powerful one to vibrate the upper 2½ ft. Screen tension is varied by means of tension rods. Both tension and amplitude adjustments may be made while the screen is in operation. Performance is materially affected by these changes. Vibration should be such that the material is lively, yet hugs the screen surface and travels rapidly.

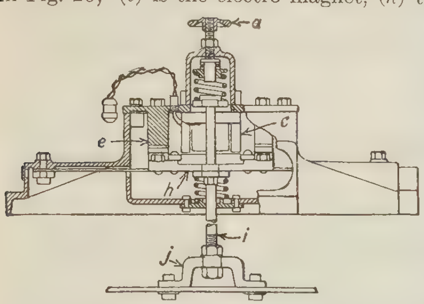


Fig. 26.—Vibrator for Hum-mer screen.

Table 38. Performances of Hum-mer screens

Mill	Shattuck-Arizona Copper Co.	Phelps-Dodge, Morenci (*c*)	U. S. S. R. & M. Co.	U. S. S. R. & M. Co.
Size, width × length, in.	36 × 60	36 × 60	72 × 60	72 × 60
Slope, degrees	35	38	31	31
Speed, vibrations per minute	±1800	...	...	...
Screen, aperture, in.	0.5 × 1	0.28*x*	0.0445 and 0.0140	0.0087 and 0.0057
Material	*w*	...	...	...
Life, tons treated	3120	...	*k*	*k*
Horsepower installed	1	...	*h*	*h*
Horsepower consumed	...	...	*i*	*i*
Tons of new feed per 24 hr.	720–840	1000	108*g*	*g*
Tons of total feed per 24 hr.	2080	...	144*g*	*g*
Water, gallons per hour	Dry	Dry	Dry	Dry
Equivalent tons per square foot per 24 hr., at 1-mm. aperture	11.3	11.5	4.2*n*	...
Per cent. undersize in oversize	...	...	*j*	*j*
Efficiency, per cent. (*b*)	High	93	...	...

Mill	Phillipburg Mining Co.	Talache Mines Co.	Braden Copper Co.	Braden Copper Co.
Size, width × length, in.	36 × 60	48 × 60	48 × 60*s*	48 × 60*s*
Slope, degrees	30	33	...	...
Speed, vibrations per minute	...	...	...	...
Screen, aperture, in.	0.150 0.084 0.0315	0.75	1.0*p*	1.25*p*
Material	*m*	...	¼-in. rod	...
Life, tons treated	*l*	4800–6700	...	...
Horsepower installed	...	...	...	...
Horsepower consumed	...	...	...	...
Tons of new feed per 24 hr.	408	1440	2040	4080
Tons of total feed per 24 hr.	No return	No return	3600	7200
Water, gallons per hour	Dry	Dry	Dry	Dry
Equivalent tons per square foot per 24 hr., at 1-mm. aperture	...	3.8 (*o*)	7.1	11.3
Per cent. undersize in oversize	...	...	...	...
Efficiency, per cent. (*b*)	...	...	...	...

Mill	Braden Copper Co.	Braden Copper Co.	Braden Copper Co.	Calumet & Arizona	Calumet & Arizona
Size, width × length, in.	48 × 60*s*	48 × 60*s*	48 × 60*s*	48 × 60*s*	48 × 60*t*
Slope, degrees	...	...	...	...	...
Speed, vibrations per minute	...	...	...	...	...
Screen, aperture, in.	0.338	1.25	⅝	⅜	⅜
Material	...	5/16-in. rod	0.207 wire	...	...
Life, tons treated	...	...	...	...	...
Horsepower installed	...	...	...	...	...
Horsepower consumed	...	...	...	...	...
Tons of new feed per 24 hr.	1015	...	...	...	...
Tons of total feed per 24 hr.	3000	5830	5830	2700	1680
Water, gallons per hour	Dry	Dry	Dry	Dry	Dry
Equivalent tons per square foot per 24 hr., at 1-mm. aperture	17.5	9.2	18.4	14.2*u*	8.9*u*
Per cent. undersize in oversize	...	*q*	*q*	...	...
Efficiency, per cent. (*b*)	...	95*r*	...	90	90

b Recovery of undersize. *c* Feed all through ¾-in. *g* 108 tons of new feed per hour to 0.0445-in. (16-mesh) scalper screen, undersizes in order to successively finer screens. *h* 2-hp. motor runs generator for vibrators. *i* About 0.5 hp. for screw-conveyor feeder. *j* See Table 38*b*. *k* 16-mesh, 135 da.; 40-mesh, 104 da.; 60-mesh, 107 da.; 100-mesh, 88 da. *l* 10-mesh screen has lasted 300 days. Screens usually break before wearing out. *m* Mixture of pyrolusite and psilomelane. *n* Upper screen. All screens underloaded. *o* Screen has large excess capacity. *p* Feed all −2-in. *q* See Table 38*a*. *r* At 0.75-in. *s* Heavy vibrator. *t* Light vibrator. *u* About 39½ tons per hour passes the screen in both of the Calumet and Arizona screens, but the second is in closed circuit, so that there is a larger percentage of "difficult grains," and it has a light, instead of a heavy vibrator. *w* Steel-wire cloth. *x* To get 0.185-in. product.

Bland (*107 J 1114*) states that in some tests with a screen having 24-mesh cloth (0.0287-in. opening) at 32° slope, the oversize contained 34 per cent. − 30-mesh (0.0230-in.) material when the cloth was vibrated heavily and 6 per cent. with light vibration. When the screen was inclined 20°, the corresponding figures were 26 per cent. and 3 per cent., yet sand remained on the screen longer at the lower slope with heavy vibration than at the higher slope with light vibration. He presents Table 37 as giving best conditions of slope, vibration, screen length and capacity for screening dry and damp quartz sand.

Table 37. Adjustments of Hum-mer screen for quartz sand. (*After Bland*)

Mesh	Dry					Damp				
	8	20	40	100	200	8	20	40	100	200
Vibration amplitude (per cent. of normal maximum)	20	5	10	30	40	30	15	40	80	100
Slope, deg.	26	26	29	32	36	26	26	26	29	32
Length of screen cloth, in.	14	20	24	30	36	20	24	38	38	50
Capacity, pounds undersize per square foot of screen area per hour	500	350	250	120	60	400	300	200	80	20

Performances of Hum-mer screens are presented in Table 38.

Table 38*a*. Sizing tests of feed and products of Hum-mer screens at Braden Copper Co. Reference letter (Table 39), *q*.

Screen	Weight, cumulative per cent.					
	F	O	U	F	O	U
2-in.	2	7	0	2	3	....
1½-in.	15	38	0	15	19	....
1-in.	38	79	....	38	48	....
¾-in.	57	96	....	57	74	....
½-in.	69	....	....	69	92	....
⅜-in.	74	....	31	74	98	23
3-mesh	80	....	39	80	98	32
4-mesh	83	....	45	83	....	40
6-mesh	85	....	52	85	....	50
8-mesh	87	....	57	87	....	57
10-mesh	88	97	62	88	99	63
65-mesh	93	99	81	93	99	88
Through last screen	7	1	19	7	1	12

Table 38*b*. Sizing tests on products of Hum-mer screen bank at U. S. S. R. & M. Co. plant. (*From C. A. Lemke*)

Screen, mesh	Feed to 40-mesh (0.0140-in.) screen	Oversize 40-mesh screen	Oversize 60-mesh screen	Oversize 100-mesh screen	Undersize 100-mesh
28	0.9	2.3	....	....	....
35	0.8	13.9	....	....	....
48	4.8	51.6	3.6	....	....
65	20.6	21.6	30.5	1.6	....
100	43.0	7.4	51.1	23.6	....
150	11.8	2.3	7.5	47.1	12.5
200	12.4	1.0	6.9	21.3	60.0
−200	5.5	0	0.4	6.5	27.5

Capacity on ores is 10 to 15 tons per sq. ft. per 24 hr. per. mm. of aperture. The capacity is much higher, up to 20 tons per sq. ft. per 24 hr. per mm., in gravel screening, where lower efficiency is acceptable.

Crowfoot (*69 A 176*) states that in the design for the new Phelps-Dodge mill at Morenci, Ariz., 1 sq. ft. of 6-mesh screen surface was allowed for each 1.5 tons of 6-mesh undersize per hour. Table 39 gives results of a test on a mixture of nut and slack bituminous coal,

Table 39. Capacity and efficiency tests on an electric vibrating screen treating bituminous coal. (*After Holbrook and Fraser*)

Screen aperture, inch	Feed rate, tons per hour	Moisture in feed, per cent.	Tons per square foot per 24 hr. per mm.	On ⅝-in.	On ½-in.	On ⅜-in.	On 3/16-in.	On ⅛-in.	On 1/16-in.	On 1/32-in.	On 1/60-in.	Through last screen
⅝	40	....	4.25	81.2	....	....	....	....	....	....	....	18.8
½	20	8.0	2.5	....	41.7	45.8	....	....	....	....	....	12.5
¼	15	6.8	3.8	....	....	11.7	84.8	....	....	....	....	3.5
⅛	7.2	4.8	3.8	....	....	....	14.1	....	81.0	....	....	4.9
1/16	4.4	4.4	4.4	....	....	....	....	0.8	....	87.6	....	11.6
1/32	1.8	....	3.6	....	....	....	....	....	1.5	....	87.6	10.9
1/60	0.9	....	3.4	....	....	....	....	....	....	11.8	73.7	14.4

preparing for subsequent concentration. Holbrook and Fraser report 18 to 20 tons per sq. ft. per 24 hr. per mm. screening raw bituminous coal over 3/16-in. cloth, and 10 tons per sq. ft. per 24 hr. per mm. screening anthracite culm on 8-mesh rectangular-weave cloth.

A W. S. Tyler Co. representative says (*25 CA 762*) that an 8 × 5-ft. Hum-mer screen with 3/16-in. apertures will handle 100 to 120 tons of bituminous coal per hour and that two 4 × 5-ft. screens in tandem screened 180 tons per hr. making −¾-in. undersize, using 1⅛-in. square-mesh cloth, with heavy vibration on the first unit and light on the second. The oversize contained 9 per cent. undersize (¾-in. round hole) of which 7 per cent. was +½-in. The undersize contained 4 to 4½ per cent. on ¾-in. round hole.

When screening coal with 12 per cent. moisture, the oversize on ¼-in. contained about 8 per cent. undersize; on ⅛-in., 20 per cent.; on 1/16-in. over 50 per cent. (Arms, *70A 772*.)

At West Canadian Collieries, Ltd., (*MCJ*, Jan., 1926) the capacities of Hum-mer screens used for preparing coal for dry cleaning were as in Table 40.

Table 40. Hum-mer screens on bituminous coal at West Canadian Collieries, Ltd. (*After Vissac*)

Screen, aperture, inch	Number of screens	Width of screens, feet	Tons per hour		
			Feed	Oversize	Undersize
¾	1	4	148	38	110
7/16	1	4	110	30	80
¼	2	8	80	24	56
⅛	1	8	56	18	38

Delameter (*22 CA 752*) states that a 6-ft. screen with 30 sq. ft. of screening area handled 80 to 110 tons of bituminous coal per hour on a screen with the equivalent of −3/16-in. clear square opening, with a consumption of 1 hp. The maximum permissible percentage of moisture for good screening is 10 per cent.

Leahy screen is one of the high-tension small-amplitude type, with a mechanical vibrator. The elements are a rectangular frame 3 or 4 ft. wide and 5 or 6 ft. long, with the lower cross bar adjustable endways by means of two tension bolts; a piece of screen cloth mounted on two cross bars and stretched tightly by means of the tension bolts above mentioned and two others at the upper end as shown; and a vibrating mechanism mounted cen-

trally on a bridge between the side bars. Details of the vibrating mechanism are shown in Fig. 27. It consists essentially of a multi-armed cam (*a*) of short lift, mounted on a pulley-driven shaft (*b*), which operates a tappet (*c*) and this actuates a tappet- or connecting-rod (*d*), the lower end of which is attached to a strap bolted to the screen frame. Sharpness of vibration is controlled by the coil spring (*e*), the tension of which is adjusted by nut (*f*). The usual maximum amplitude of vibration is 1/16 in. along a transverse line directly beneath the vibrator. The usual speed is 200 r.p.m., giving 1600 vibrations per min. The construction of the mechanism is such that the upstroke ends suddenly and the down stroke starts at maximum velocity, which tends to keep the meshes clear and imparts maximum liveliness to the load. The slope should be between 28 and 35°. Power requirement is between ¼ and ¾ hp. The weight of the 3 × 6-ft. size complete is 850 lb.

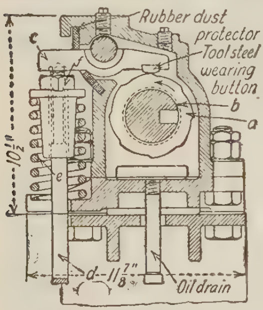


Fig. 27.—Vibrator for Leahy screen.

A 3 × 5-ft. screen at the Richard mill, Wharton, N. J., with 0.194-in. square aperture at 35° slope and 200 r.p.m. handled 28 tons of magnetite ore per hour, through ¾-in. round-hole plate, with 3 to 5 per cent moisture, at an efficiency of 92 per cent. With less than 1 per cent. moisture the efficiency rose to 98.5 per cent. At the Iron Mountain mill, Iron Mountain, Mo., a similar screen handled 19 tons per hr. of wet hematite ore with an efficiency of 99.6 per cent. and was underloaded. H. M. Roche (*PC*) believes that the screen could have handled 38 to 40 tons per hr. with 98 per cent. efficiency. These performances indicate a capacity of 9 to 12 tons per sq. ft. per 24 hr. per mm.

Lead-belt screen uses the principle of an unbalanced disk to cause vibration. The vibrator (Fig. 28) is mounted on two hickory rods (*19*) carried on the screen frame. Shaft (*5*), driven by pulley (*7*) carries, keyed to it, the heavy disk (*4*) which is unbalanced by coring out holes (*9*) and filling them with wooden plugs. The vibration caused by rapid rotation (1200 to 1250 r.p.m.) of the disk is transmitted through ball bearings (*6*) to the hammer (*1*) of which (*2*) is an integral part. A bronze bushing (*12*) and hardened-steel roller (*14*) are mounted on (*2*) and are surrounded loosely by the oblong ring (*15*) which is shrunk within the upper portion of transmitter (*16*), the lower end of which is attached to a cross-bar on the screen cloth. The roller (*14*) strikes the inside of ring (*15*) only at the top and bottom of its circular path, thereby transforming the circular motion of the hammer into up-and-down motion of the transmitter. The spring mechanism (*22*), (*23*), (*24*) and (*25*) is used on large screens to take some of the load off the transmitter.

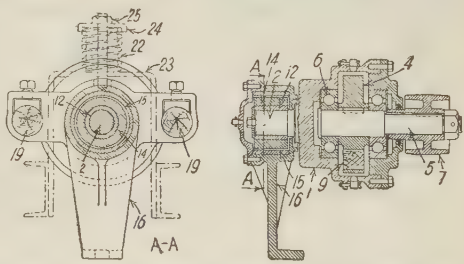


Fig. 28.—Vibrating mechanism of Lead-Belt screen.

At St. Joseph Lead Co. a 42 × 45-in. machine with 1.7-mm. wire cloth screens 25 tons per hr. wet, using spray water. The feed contains 45 per cent. −2-mm. material and the oversize 13.5 per cent. −2-mm. At the same plant a screen 33 in. wide × 7 ft. long with

9-mm. screen handles 33 tons per hr. of dry feed at 70 per cent. efficiency. At a St. Louis plant of the Laclede Gas Co. a screen with ⅜-in. cloth handles 25 tons of coke per hour.

Advantages of vibrating screens are high capacity, all of the screen surface is continually in use, there is less blinding than in other types, and greater ease of adjustment. The disadvantages are the danger of screen breakage if the cloth is too loose; flotation of fines at the surface of the bed by upward air currents in screens of large amplitude; loss of headroom in distributing feed and collecting products.

8. Traveling-belt screen

Callow traveling-belt screen (Fig. 29) is typical of this class. It consists essentially of an endless screen cloth (*a*) passing over rollers (*b*) so mounted that the upper surface is substantially horizontal. Rubber edgings are buttoned to the screen cloth to keep material on the screen. Feed is distributed over the width of the belt by means of a distributor, pulp is washed by sprays from box (*e*), undersize is carried away in chute (*f*) and oversize carried over the tail roller is washed off by spray from box (*g*) and discharged through chute (*h*). The standard size has two belts, each 2 ft. wide and 4 ft. center to center of rollers. The small size is single-belt, 2 ft. wide and about 2.5 ft. center to center of rollers. The usual speed is between 50 and 100 ft. per min., the lower speed for 8- to 20-mesh screening, the higher for 100- to 120-mesh.

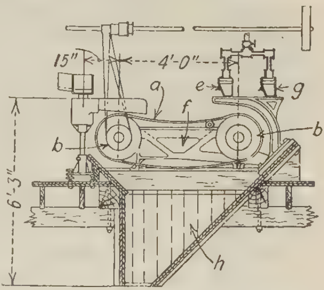


Fig. 29.—Callow traveling-belt screen.

Capacity, according to the manufacturer, is given by Table 41. *Truscott* rates the capacity at 4 tons per sq. ft. of total screen surface per 24 hr. per mm. aperture. This is substantially the average rate of the tests of Cox, Gibbons and Porter (*14 CMI 526*) as shown in Table 42.

Table 41. Capacity of Callow screen. (*Maker's catalog*)

Screen, mesh	Tons of feed per 24 hr. Standard duplex	Tons of feed per 24 hr. Small size, single-belt
20	250	62.5
30	200	50.0
40	150	37.5
60	125	31.2
100	75	18.7

Water required is 12 to 20 gal. per min. per duplex screen for 40- to 80-mesh screening. The maker's rating is somewhat lower at coarse sizes and higher at fine.

At Miami Copper Co. standard duplex screens equipped with 0.029-in. (0.74-mm.) rectangular-mesh cloth averaged 333 tons per screen per 24 hr. and for a month at a time handled as high as 435 tons per 24 hr. The pulp contained about 67 per cent. solids and no spray water was added. This is at the rate of 20.3 tons per 24 hr. per sq. ft. per mm. About 20 per cent. of the feed passed through the screen. Efficiency was about 60 per cent. The total cost in 1915 was $0.00598 per ton, made up as follows: operating labor, $0.00265; maintenance labor, $0.00078; screens, $0.00165; edgings, $0.00026; miscellaneous, $0.00064. At United Eastern Mining Co. two standard duplex screens, fitted with one 20-mesh and one 30-mesh belt on each screen, in closed circuit with Marcy mills took 120 tons per day each of original feed plus about 200 tons per day of circulating load or a total of 320 tons per day per screen, which is 22.3 tons per 24 hr. per sq. ft. per mm. The two screens were replaced by one Dorr simplex classifier and the capacity of the ball mills was increased to 260 tons. Comparative screen analyses are shown in Table 43. The efficiency of the

screen was 48 per cent. The finished product of the screen is somewhat smaller than that of the classifier, but against that fact is a set-off of $0.0175 per ton of original feed for screen and elevator operation, and 20 tons increase per day in ball-mill feed.

The tests of Cox, Gibbons and Porter (Table 42) seem to indicate that the efficiency of the screen is greater at 60- and 100-mesh than at 20-, 30- and 40-mesh; but the tests on the coarse screens were all run with relatively high percentages of solid and tests 7 to 10 inclusive indicate that dense pulps screen much less readily than thin pulps. It is probable that the difference in consistency explains the apparently discrepant drop in capacity per square foot per millimeter on the coarser screens. Tests 16 and 17 indicate that 65 ft. per min. is too low belt speed for 20-mesh screening.

Table 42. Performance of Callow screen. (*After Cox, Gibbons and Porter*)

Test number	Screen, aperture, mm.	Feed rate, tons per foot of width per hour	Feed rate, tons per square foot (*b*) per 24 hr. per mm.	Belt speed, feet per minute	Consistency of feed, per cent. solids	Undersize in feed, per cent.	Undersize in oversize, per cent.	Efficiency, per cent. (*a*)
1	0.21	0.53	4.7	90	21.7	55.5	40.0	46.7
2	0.21	0.48	4.3	86	20.9	36.4	14.1	71.2
3	0.21	0.46	4.1	86	15.6	40.2	16.9	69.9
4	0.21	0.43	3.9	86	12.2	39.8	13.6	76.2
5	0.21	0.94	8.4	86	22.2	21.4	8.8	64.5
6	0.21	0.35	3.1	86	19.6	42.4	18.2	69.7
7	0.13	0.32	4.6	100	16.9	35.3	13.6	61.5
8	0.13	0.54	7.8	100	23.8	13.1	7.0	51.7
9	0.13	0.28	4.0	100	14.5	5.2	1.7	67.7
10	0.13	0.27	3.9	100	14.5	13.9	4.4	71.4
11	0.21	0.53	4.7	90	20.9	20.2	10.3	54.5
12	0.32	0.80	4.7	80	26.3	34.4	23.5	41.6
13	0.32	0.66	3.9	80	25.6	28.8	14.0	59.6
14	0.42	0.96	4.3	80–110	29.4	42.1	27.7	47.4
15	0.42	0.70	3.1	80	23.2	39.8	23.2	54.3
16	0.63	0.96	2.8	65	29.4	73.4	61.1	43.2
17	0.63	0.96	2.8	95	27.0	76.9	62.5	50.0
18	0.63	0.69	2.1	80	24.4	74.5	55.6	57.4

a By recovery formula (Sec. 22, Art. 15). *b* Of total screen surface.

Table 43. Comparative screen analyses of Callow screens and Dorr classifier at United Eastern

Screen, mesh	Weight, per cent.				
	Feed	Dorr classifier		Callow screen	
		Sand	Overflow	Oversize	Undersize
14	27.0	55.0	....	....	....
20	11.0	13.0	1.9	53.8	1.9
28	9.0	9.0	9.6	16.7	5.1
35	7.0	5.0	10.3	9.8	8.8
48	7.0	4.0	9.8	5.7	8.5
65	6.5	2.0	8.4	3.7	8.7
100	5.0	2.0	7.2	2.3	8.0
150	5.0	1.5	6.8	1.5	9.5
200	2.5	1.5	7.5	1.1	8.4
−200	20.0	7.0	38.5	5.4	41.1

Breaking of screen cloth due to bending around the pulleys is the usual cause of failure, rather than wear.

9. Miscellaneous screens

A large number of screens of various descriptions has been advertised but few of them have found more than limited use in the mills. A few of the better known of these are as follows:

Pratt ore sizer. The screening surface is a six-sided pyramid with base up. Suspended within the pyramid is a vertical shaft carrying a box from which pulp is discharged through pipes and distributed over the screening surfaces. A diaphragm placed centrally collects oversize from the upper part of the screening surface and feeds it back into a box on the shaft from which it is again distributed over the lower part of the screen. Undersize is collected in a housing and spouted away from the machine, while oversize falls through another spout.

Drum screen consists of eight plane screening surfaces forming the sides of an octagonal prism carried by spiders mounted on a horizontal shaft which revolves slowly. Material is fed through a distributing chute onto the top screen, undersize passes through the screen and is collected in a trough and oversize is washed off into a hopper.

King screen, which has been used to some extent in Broken Hill mills (*27 MM 331*) is of the drum type with screen cloth supported between peripheral rods, thus forming the screening surface into outwardly concave cylindrical arcs or troughs. The usual number of troughs is 12 for a screen 3 ft. diameter by 3 ft. long. A screen covered with 30-mesh cloth, making 8 r.p.m., taking about 150 tons of feed per 24 hr. had an efficiency of 59 per cent.

Gyratory screens have been manufactured both for coarse and fine screening. Coxe screen consists of several superimposed sieves slanted toward one corner, all carried in a common gyrating frame driven at 130 to 180 r.p.m. It has found some use in coal sizing but the mechanism is relatively complicated and expensive and maintenance cost is high.

Screenless sizing may be effected by taking advantage of the fact that when a mass of particles of mixed sizes is shaken so as to loosen the mass, the smaller sizes work to the bottom. McKesson-Rice sizer (Fig. 30) is built on this principle. It consists of a series of corrugated decks carried on a framework that is shaken parallel to the corrugations by an eccentric mechanism, the decks being sloped at 30 to 40° from the horizontal in a direction at right angles to the corrugations. The standard size of deck is about 6 × 8-ft. Corrugations are of different sizes on the different steps or decks, the coarsest corrugations being used on the first deck. Rate of shaking is 250 to 300 strokes per min.; amplitude, ¼ to ¾ in. Under the influence of the shaking motion the particles stratify with the coarsest on top and these surface particles roll down the slope to the edge of the machine and are caught in the proper compartment. The finer material is carried forward by the shake and dropped onto the next deck having finer corrugations, when again the coarser material comes to the top and rolls down over the balance and is caught in its proper compartment. This process is repeated as many times as desired and a corresponding number of grades are obtained. Screenless sizers have not been employed in ore-treatment plants.

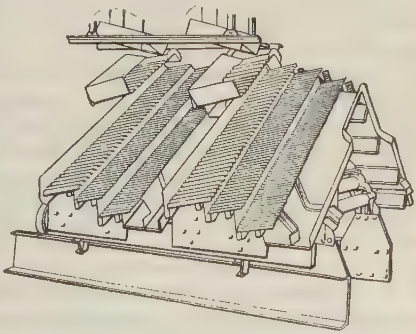


Fig. 30.—McKesson-Rice screenless sizer.

SECTION 6

CLASSIFICATION

1. Introduction

Classification is an operation in which a mass of grains of mixed sizes and different specific gravities is allowed to settle in a fluid which may be either in motion or substantially at rest. SORTING is another name for the same operation. The fluid ordinarily employed is water, but sometimes air is used. The velocity and direction of the fluid currents are closely controlled. The following general statements apply to the movements of the solid particles.

1. The relative falling velocities of particles of the same specific gravity and the same shape are dependent upon the sizes of the particles, the larger (and heavier) falling the more rapidly.

2. If particles are of the same size and shape but of different specific gravities, the most dense (heaviest) particles settle most rapidly.

3. If particles are of the same weight in a given fluid, but of different shapes, their falling velocities will probably differ; particles most nearly spherical will fall most rapidly, those most tabular most slowly.

4. Resistance to fall in a given fluid medium is dependent upon the velocity of the falling particle. Resistance varies as the ½-power of the velocity when the latter is very small, as the square when the velocity is large, and as some intermediate power or powers in the transition range.

5. Velocity of fall in a given fluid medium, all other things being equal, varies as the squares of the diameters of the particles when these are very small, as the ½-power of the diameters when the particles are relatively large, and as an intermediate power or powers in the transition range.

6. Resistance to fall increases with the density of the medium.

7. Resistance to fall increases with the viscosity of the medium. This increase is relatively greater the smaller the particle.

Formulas for falling bodies. When a body falls in a vacuum under the influence of gravity alone, its velocity v at any distance h from the starting point is given by the equation $v = \sqrt{2gh}$, g being the acceleration due to gravity. Inspection of the equation shows that the velocity at any point is dependent upon the distance from the starting point only. If the body falls in a fluid medium, its velocity at any given distance from the starting point is always less than had the fall been in a vacuum, due to the resistance offered by the medium. The nature of this resistance differs according to the velocity of the body. When velocity is low, no considerable disturbance is set up in the body of the fluid by the passage of the particle; the film or layer of fluid

in contact with the particle moves with it while the body of the fluid a short distance away is at rest. Substantially all of the resistance to movement is due to the viscosity of the fluid. Such resistance may be called VISCOUS RESISTANCE. When the velocity of the body is high the principal resistance to its motion is that offered by the fluid to bodily displacement from the path of the particle. The kinetic energy imparted to the fluid displaced is dissipated in eddying and turbulence. The effect of the viscosity of the medium is relatively small. This resistance is variously called EDDYING RESISTANCE and TURBULENT RESISTANCE. In all cases acceleration decreases rapidly and the body quickly attains a uniform terminal velocity.

Law of viscous resistance. Stokes, in 1850 (*Camb. Phil. Trans.*, IX) deduced theoretically a formula for the terminal velocity of a small solid sphere falling freely under the influence of gravity in a viscous fluid. At the beginning of such fall the pull of gravity on the sphere exceeds the resistance of the fluid and the particle is accelerated. With increasing velocity the frictional or viscous resistance of the fluid increases until a point is reached where the resistance is just equal to the gravity pull. Thereafter the body falls at uniform velocity. According to Stokes, at this stage resistance equals $6\pi\mu aV$, where μ = viscosity of the fluid, a = radius of the sphere, and V = the constant terminal velocity. The gravitational pull on the sphere is $\frac{4}{3}\pi a^3 g(\sigma - \rho)$ where g = acceleration constant, σ = specific gravity of the sphere and ρ = specific gravity of the fluid. Then $6\pi\mu aV = \frac{4}{3}\pi a^3 g(\sigma - \rho)$, and

$$V = \frac{2}{9}a^2 g\frac{(\sigma - \rho)}{\mu}. \qquad (1)$$

Allen (*50 Phil. Mag. 323, 519*) has confirmed Stokes' formula experimentally for the rise of air bubbles in water and in aniline and for the fall of solid spheres in the same liquids. When water is the medium, μ and ρ may be taken as constants and equation (1) becomes

$$V = KD^2(\sigma - 1), \qquad (2)$$

where K is a constant whose magnitude depends on the units employed.

Law of turbulent resistance. Newton (*Mathematical Principles of Natural Philosophy, Book II*) states that when a body falls in a non-viscous medium the resistance to fall is proportional to the square of the velocity of the body. Allen (*loc. cit.*) has proved experimentally that for the fall of steel spheres in water the resistance R after constant velocity has been attained is given by the equation

$$R = K\rho a^2 V^2, \qquad (3)$$

and that the value of the constant K for these particular conditions is 5.50×10^{-4}, when c.g.s. units are employed. By equating this value of the resistance to the gravitational pull on the falling body a formula for V may be derived. Thus

$$K\rho a^2 V^2 = \frac{4}{3}\pi a^3 g(\sigma - \rho),$$

and

$$V = \sqrt{\frac{4\pi a g(\sigma - \rho)}{3K\rho}} = C\sqrt{D\frac{(\sigma - \rho)}{\rho}}, \qquad (4)$$

where D = diameter of sphere and $C = \sqrt{\frac{2\pi g}{3K}}$.

Allen's equation for velocity of fall of spheres in the size range coarser than that covered by Stokes' law (Law of viscous resistance) and finer than that covered by Newton's law (Law of turbulent resistance) is

$$V = K\left(\frac{(\sigma - \rho)g}{\rho}\right)^{2/3}\frac{a - ha'}{\nu^{1/3}}, \quad (5)$$

where K and h are constants, a' is the critical radius marking the maximum size of sphere obeying Stokes' law, and ν is the kinematic coefficient of friction $= \mu/\rho$. Allen's experiments show that K = nearly 0.5, h = nearly 0.4 and $a' = 0.0085$ cm. Allen found this formula applicable to air bubbles varying from 0.13 to 1.10 mm. diameter when rising in water and in aniline; to paraffin spheres between 0.69 and 3.16 mm. diameter rising in aniline and to amber spheres from 1.14 to 3.46 mm. diameter falling in water. For none of the materials do the experimental data indicate that the upper size limit of particles obeying the linear equation is reached. The experiments with steel balls do, however, indicate that at velocities of about 750 mm. per second a change in the resistance to fall occurs and Allen infers that this may mark the point of change from the linear equation to the equation involving turbulent resistance alone. The data tell nothing as to the lower-size limit with paraffin and amber. With air bubbles smaller than 0.13 mm. marked divergence from the linear equation occurs, but this may be due to solution of the air.

Free fall of mineral fragments in water has been carefully investigated by Richards (*38 A 210*). His results on rates of settlement of quartz and galena are summarized in Fig. 1. The figures plotted are the averages of the falling rates of 100 grains for each size. Maxima and minima vary 15 to 50 per cent. from these averages. The curves show that for sizes larger than about 1.6-mm. the free-falling velocities of the particles in water vary as $D^{1/2}$. (See lines (a) and (d), Fig. 1.) By writing equations for lines (a) and (d) and equating the value of V to the value of V from equation (4), the values of C in the latter equations may be found and these equations equations become

$$V = 92\sqrt{D(\sigma - 1)} \text{ for quartz}, \quad (6)$$

and

$$V = 104\sqrt{D(\sigma - 1)} \text{ for galena}, \quad (7)$$

when D is in mm. and V in mm. per sec. For quartz between 0.16- and 0.015-mm. diameter and galena between 0.12- and 0.007-mm. diameter the free-falling velocities vary as D^2 (see lines (c) and (f), Fig. 1) and the equations of fall can be written

$$V = 450D^2(\sigma - 1) \text{ for quartz}, \quad (8)$$

and

$$V = 640D^2(\sigma - 1) \text{ for galena}, \quad (9)$$

with units the same as in the preceding equations. In the intermediate size ranges for the two minerals Allen's linear equation does not hold. The behavior of quartz particles is closely represented by the equation

$$V = 95D^{0.83}, \quad (10)$$

and galena

$$V = 263D^{0.60}. \quad (11)$$

The specific gravity of the solid is, of course, a factor in the falling velocity in this size range, as in the other ranges, but disappears into the constant and the exponent of D, due to the empirical method of developing the equation.

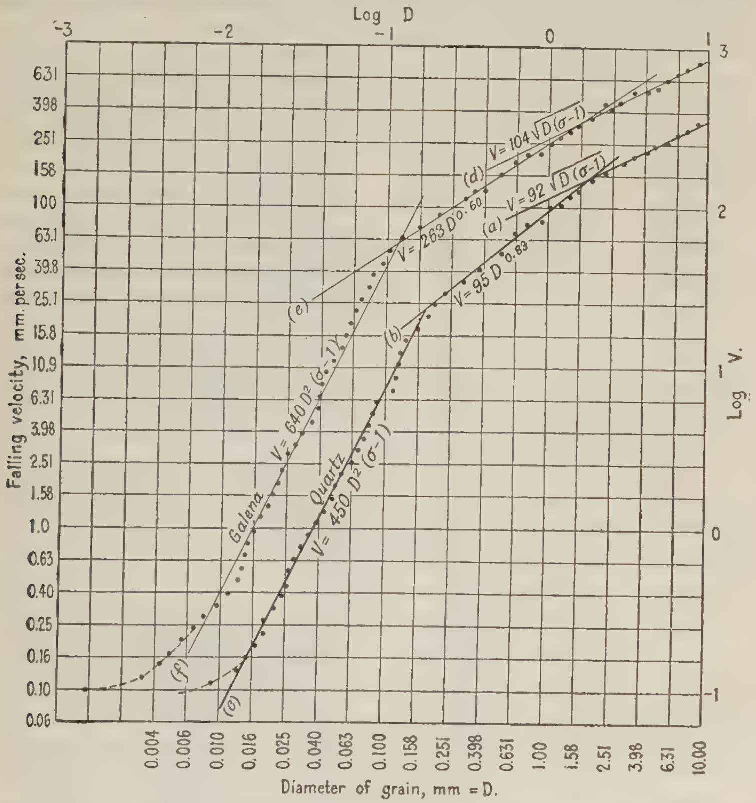


FIG. 1.—Free fall of mineral fragments in water.

Table 1 gives falling velocities in water, calculated from the appropriate formulas, for quartz particles of sizes corresponding to Tyler testing sieves.

For other minerals than quartz and galena the approximate falling velocities of particles can be obtained by use of the multipliers given in Table 2, which is calculated from experimental data given by Richards (*41 A 396*). To determine, *e.g.*, the falling velocity of a 1-mm. grain of cassiterite, divide by 0.34 to get the diameter of a quartz grain of corresponding settling velocity. Substitute the quotient for D in the equation heading the column and solve for V, which will be the figure desired. If the equivalent diameter of quartz is not within the range to which the equation heading the first-tried multiplier column is applicable, use the multiplier in the other column. The answer will be an approximation only, but well within experimental limits.

Free-settling ratio. If a mixture of mineral grains of different sizes and different specific gravities is subjected to a series of rising water currents of

diminishing velocities, the coarser grains will sink in the currents of higher velocity and the smaller in the slower currents. There will be a final overflow of the finest material. In each of the products except the first and the last there will be a distinct grouping of the minerals according to size, the coarser grains being the lighter mineral and the finer the heavy mineral. The ratio of the average size of light mineral in any of these products to the average size of the heavy mineral in the same product is the free-settling ratio. Table 3 (*41 A 396*) gives free-settling ratios of quartz and galena as determined experimentally by Richards and the theoretical average ratios determined from equations (6) and (7), (8) and (9), and (10) and (11). Estimates of probable free-settling ratios of any two minerals at any given velocity can be determined by use of Table 2.

Table 1. Theoretical free-settling velocities of quartz particles

Size, mm.		Velocity, mm. per second	
Retaining screen	Mean(a)		
....	11.38	398	$V = 92\sqrt{D(\sigma - 1)}$
9.423	....	362	
....	8.052	336	
6.680	....	305	
....	5.690	282	
4.699	....	256	
....	4.013	237	
3.327	....	216	
....	2.844	199	
2.362	....	182	
....	2.006	167	
1.651	....	152	
....	1.410	126	$V = 95D^{0.83}$
1.168	....	108	
....	1.000	95	
0.833	....	82	
....	0.711	72	
0.589	....	61	
....	0.503	54	
0.417	....	46	
....	0.356	40.2	
0.295	....	34.5	
....	0.252	30.3	
0.208	....	25.8	
....	0.192	24.2	
0.175	....	22.4	
....	0.161	19.2	$V = 450D^2(\sigma - 1)$
0.147	....	16.0	
....	0.136	13.7	
0.124	....	11.4	
....	0.114	9.65	
0.104	....	8.02	
....	0.096	6.83	
0.088	....	5.74	
....	0.081	4.86	
0.074	....	4.06	

a Reckoned as the arithmetical mean between the retaining screen and the screen preceding.

Example. To determine the probable free-settling ratio of quartz and chalcocite in a product that falls in a rising current of 100 mm. per sec. velocity and rises in a current of 150 mm. per sec. The largest quartz particle to settle will be, theoretically, 1.66-mm. diameter (from equation 6); the smallest, 0.94-mm. (from equation 10). The average diameter of quartz may be taken as the arithmetical mean of these extremes, or 1.30-mm. The largest chalcocite particle will be $0.44 \times 1.66 = 0.73$-mm. diameter and the smallest, $0.61 \times 0.94 = 0.57$-mm. The mean chalcocite diameter is 0.65-mm. and the free-settling ratio $1.30/0.65 = 2$.

Hindered-settling. If any constriction is placed in the tube in which a current of fluid is rising and solid particles are falling, the velocity of the fluid across the constricted portion is greater than that above and as a result certain of the solid particles fall to the constriction but cannot fall further. They remain, therefore, in a mass in the tube, above the constriction. If the particles are not too large and the ratio of area of constricted portion to tube cross-section not too small, the mass of particles above the constriction will teeter, as the solids in a quicksand. Under such circumstances particles settling are hindered by collision with particles in teeter and the mixture of solid and fluid acts as though it were a fluid of greater density than the fluid employed. The settling rate of solid particles in the tube is,

consequently, decreased. The decrease for any given particle is proportionately greater, the less the specific gravity, because, all other things being equal, V is proportional to $(\sigma - \rho)/\rho$.

Table 2. Multipliers for use in determining free-settling velocities of mineral grains. (*After Richards*)

Mineral	Sp. gr.	Multipliers for quartz diameters in equation	
		$V = 95D^{0.83}$	$V = 92\sqrt{D(\sigma - 1)}$
Anthracite	1.47	4.0	....
Epidote	3.38	0.86	0.65
Blende	4.05	0.83	0.61
Pyrrhotite	4.51	0.68	0.47
Chalcocite	5.33	0.61	0.44
Arsenopyrite	5.63	0.57	0.37
Cassiterite	6.26	0.51	0.34
Antimony	6.71	0.46	0.34
Wolframite	6.94	0.47	0.32
Copper	8.48	0.43	0.29

Table 4 gives hindered-settling velocities for several minerals as determined by Richards (*41 A 396*). In this table, under the caption "Hindered-settling" are given figures determined by dividing the volume of water, in cu. mm. per sec., passing through the sorting tube by the area of the tube in sq. mm. at the time when the solids were in FULL TEETER, *i.e.*, all grains in motion under the action of the rising current but none actually being carried upward away from the mass. The velocity figure is not, of course, the actual interstitial velocity of the water.

Table 3. Free-settling ratios of quartz and galena. (*After Richards*)

Diameter of particles		Particles fall in currents of . . mm. per sec.	Particles rise in currents of . . mm. per sec.	Free-settling ratio	Theoretical free-settling ratio
Quartz, mm.	Galena, mm.				
0.0301	0.0194	0.00	1.26	1.54	
0.0335	0.0198	1.26	2.51	1.68	
0.0568	0.0292	2.51	5.05	1.82	2.36
0.0772	0.0412	5.05	7.42	1.96	
0.0982	0.0488	7.42	10.01	2.09	
0.1423	0.0613	10.01	14.68	2.23	
0.1875	0.0721	14.68	19.80	2.35	
0.2254	0.1032	19.80	30.12	2.48	
0.3416	0.1305	30.12	40.37	2.61	
0.3880	0.1404	40.37	50.08	2.72	
0.5241	0.1708	50.08	60.09	2.82	
0.5892	0.1997	60.09	70.34	2.92	
0.6590	0.2381	70.34	80.28	3.03	3.74
0.8604	0.2750	80.28	90.21	3.12	
1.0234	0.3428	90.21	99.54	3.21	
1.4224	0.3504	99.54	110.09	3.29	
1.3216	0.3648	110.09	120.03	3.36	
1.1424	0.3776	120.03	130.43	3.42	
1.4256	0.4208	130.43	140.37	3.49	
1.6032	0.4560	140.37	150.31	3.54	
1.6848	0.4592	150.31	160.09	3.59	
1.7488	0.4624	160.09	169.95	3.63	4.0
1.8032	0.5248	169.95	180.57	3.66	
1.9746	0.5776	180.57	180.5	3.70	

The equations developed for free-settling are not applicable to hindered-settling by simple substitution of a value for ρ in the quantity $(\sigma - \rho)/\rho$.

Table 4. Hindered-settling velocities of mineral fragments. (*After Richards*)

Size of grain, mm.	Velocities, mm. per second						
	Galena, sp. gr. 7.5		Blende, sp. gr. 4.0		Quartz, sp. gr. 2.64		Anthracite sp. gr. 1.45 to 1.70
	Hindered settling	Free settling	Hindered settling	Free settling	Hindered settling	Free settling	Hindered settling
9.34	....	....	....	....	....	....	104
6.62	212	698	....	....	116	295	71
4.97	184	590	126	....	113	249	53
3.40	142	505	91	....	69	207	41
2.35	119	402	71	....	50	166	33
1.70	90	350	59	263	41	134	....
1.21	77	273	43	227	32	106	....
0.83	52	213	35	181	21	80	....
0.56	36	175	25	143	13	60	....
0.40	25	125	17	109	....	42	....
0.30	18	115	11	86	6.5	34	....
0.20	10	77	6.7	59	3.3	23	....
0.070	....	28	....	....	....	....	....

Such substitution in equation 4, *e.g.*, of values of V and D for galena from the first five horizontal lines of Table 4, gives a value of 4.8 for ρ, which would allow for but 41.5 per cent. voids. Corresponding figures for quartz are $\rho = 2.23$ and percentage of voids $= 25$. Since the densest packing of equal-sized spheres leaves 26 per cent. voids, these figures would require that sized mineral fragments of irregular shape in full teeter should be equally closely packed. This is not reasonable.

Table 5. Free- and hindered-settling ratios of various minerals with respect to quartz (*After Richards*)

	Free-settling ratios for 228.6 mm. per sec., fastest grains	Hindered-settling ratios
Copper	3.75	8.598
Galena	3.75	5.842
Wolframite	3.26	5.155
Antimony	3.00	4.897
Cassiterite	3.12	4.698
Arsenopyrite	2.94	3.737
Chalcocite	2.17	3.115
Pyrrhotite	2.08	2.808
Blende	1.56	2.127
Epidote	1.46	2.037
Anthracite	....	5.611*a*

a Anthracite to quartz.

Hindered-settling ratios are consistently larger than free-settling. Table 5 (*41 A 396*) compares the free- and hindered-settling ratios of various minerals with respect to quartz. In the same paper Richards records a test in which quartz-galena hindered-settling ratios as high as 8.0 were obtained in a small, carefully-run experimental classifier.

Types of classifiers

Classification is used in mills for several different purposes. The type employed depends upon the character of service. Hydraulic classifiers are used to divide a ground pulp into a number of GRADES or SORTS for subsequent treatment on gravity concentrating machines. These classifiers are so named because of the fact that fresh water, often called HYDRAULIC WATER,

is added to them in order to supply the rising current of water against which settlement of solid particles is effected. There are two general types, known respectively as free-settling and hindered-settling, and many varieties of each class. Sand-slime separators are used to determine the maximum size of particle discharged from a fine-grinding circuit, or to separate a ground product into sand and slime for different kinds of subsequent concentrating or metallurgical treatment. There are two principal types, known respectively as mechanical classifiers and cone classifiers. The first type is commonly preferred in the U. S.; the second was developed and has had wide use in South Africa. An automatic-discharge cone has recently met with considerable favor in this country.

2. Free-settling hydraulic classifiers

General. Classifiers of this type are characterized by the fact that the sorting column is of the same cross-sectional area throughout its length. A large number of types has been built and used. These may be grouped into two classes, *viz.:* launder type and tank type. LAUNDER CLASSIFIER is essentially a launder with sorting columns attached to the bottom at convenient intervals. If the upper end of the sorting column comes directly to the bottom of the launder, the machine is called by *Richards* a SHALLOW-POCKET CLASSIFIER; if the launder is deepened above the sorting pocket, *Richards* calls the machine a DEEP-POCKET CLASSIFIER. TANK CLASSIFIER consists of a relatively deep V-shaped trough or tank with the sorting columns attached to the bottom thereof.

Shallow-pocket free-settling classifiers include many of the early forms of hydraulic machine, such as the Lake Superior hog-trough, Calumet, Tamarack, Yeatman, Ferraris, Evans, etc. (*1 OD 390 et seq.*)

Evans classifier (Fig. 2) is typical. It consists of a launder (A) with inclined bottom to which are attached a plurality of pressure boxes (D) opening by means of adjustable rectangular transverse openings (B), (C), into the bottom of the launder. Water is introduced into the boxes (D) through pipes (F), a part flows out through spigot openings (G) and the balance rises through the openings (B, C) into the launder. Water input is regulated to give maximum rising velocity at the first box and minimum at the last. Feed enters as marked, with sufficient water (about 75 per cent. by weight) for ready transport, and flows along to the first sorting column where its horizontal rush is lessened by baffle (E). Some segregation of coarse and heavy solid to the bottom of the horizontal stream occurs in the flow along the launder. The largest and heaviest particles drop through the slots (B, C) and pass out through the spigot opening (G) while the solid unable to settle passes on to the second column. The coarsest and heaviest of the remaining particles are here taken out and the process continues until the lightest solids, unable to settle in the relatively slow current of the last sorting column, leave the classifier. Performance is shown in Table 6.

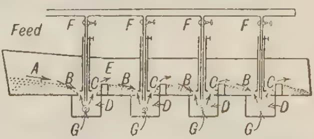


FIG. 2.—Evans classifier.

Calumet classifier (Fig. 3) is used at BUNKER HILL AND SULLIVAN MINING Co. to prepare the sand discharge of an Esperanza drag classifier for table treatment. It consists of an inclined trough (a) with a series of settling

pockets (*b*), through one side of which hydraulic water is led by pipes (*c*) while the spigot product discharges through the other side. A baffle (*d*) above the spigot discharge prevents short circuiting of material to the spigot.

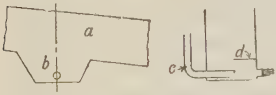


FIG. 3.—Calumet classifier.

The machine at the BUNKER HILL AND SULLIVAN mill treats 50 tons solid per day; water consumption, 600 gal. per ton. Screen tests of products are given in Table 7.

Table 6. Performance of Evans classifier. (*After Richards*)

Screen aperture, mm.	Weights, cumulative per cent.				
	Spigot 1	Spigot 2	Spigot 3	Spigot 4	Overflow
+2.69	0.2	....	....	....	....
1.89	6.6	2.0	0.4	....	....
1.49	18.5	6.6	1.8	0.3	....
0.945	44.8	23.0	9.0	2.4	....
0.667	65.8	41.8	22.0	8.2	....
0.493	82.9	63.0	44.4	23.0	....
0.371	86.8	67.6	49.4	29.0	0.2
0.270	92.5	79.4	65.2	47.0	0.5
0.158	98.0	94.0	89.0	80.0	5.2
0.119	98.8	97.4	94.2	90.9	10.1
0.073	99.3	99.2	98.8	98.3	25.9
0.069					28.2
0.047					41.1
0.034					50.5
0.025	0.7	0.8	1.2	1.7	57.1
0.019					65.7
0.012					74.4
−0.012					25.6

Table 7. Performance of Calumet classifier at Bunker Hill and Sullivan Mining Co.

	Feed	Spigot number					Overflow
		1	2	3	4	5	
Spigot, diameter, in. . .	....	⅝	½	⅜	5/16	¼	....
Moisture, per cent. . . .	42	56	67	73	86	94	99
Screen aperture, mm.	**Weight, per cent.**						
+0.833	....	1.2	....	....	....	....	....
0.589	7.6	18.0	2.8	....	....	....	....
0.417	8.4	16.3	6.9	....	....	....	....
0.295	12.8	17.6	13.8	3.7	....	....	....
0.208	13.0	17.1	18.2	8.4	....	....	....
0.147	18.6	15.5	28.7	30.3	6.2	....	....
0.104	10.4	7.8	16.6	33.0	21.2	....	....
0.074	16.7	4.5	9.3	17.8	47.8	32.4	5.3
0.074	12.5	2.0	3.7	6.8	24.8	67.6	94.7

Settling ratios obtained in shallow-pocket free-settling classifiers are far below the theoretical. Much slime appears in the earlier spigot products

because of the presentation of slime directly to the sorting column and much oversize goes to later spigots because the shallow pockets fail to hold it long enough to insure an opportunity to settle.

A free-settling launder-type classifier that can be made up readily around any plant is shown in Fig. 4 (*111 J 911*). Both the 6-in. and 3-in. pipes are welded to the cover plate and the hydraulic-water pipe is welded to the 6-in. pipe.

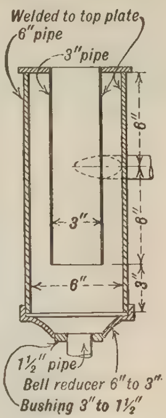


Fig. 4.—Simple free-settling launder-type classifier column.

Deep-pocket free-settling classifier is typified by the Richards vortex classifier (Fig. 5). It differs from the shallow-pocket apparatus by providing pockets above the sorting columns of sufficient size and depth to effect a rough segregation of the feed to the column, sending along immediately most of the material that could not settle in the sorting column proper and holding back material that might settle in the column long enough to give it an opportunity to do so. The vortex fitting used at the bottom of the sorting columns in this classifier is shown in Fig. 6. Water entering the tangential inlet pipe takes on a swirling motion around a vertical axis and retains this motion in rising through the sorting column, with the result that eddies around horizontal or inclined axes are eliminated. This elimination goes to prevent the presence of fine sand and slime in the spigot products. The ideal condition in the sorting column is a uniform current vertically upward,

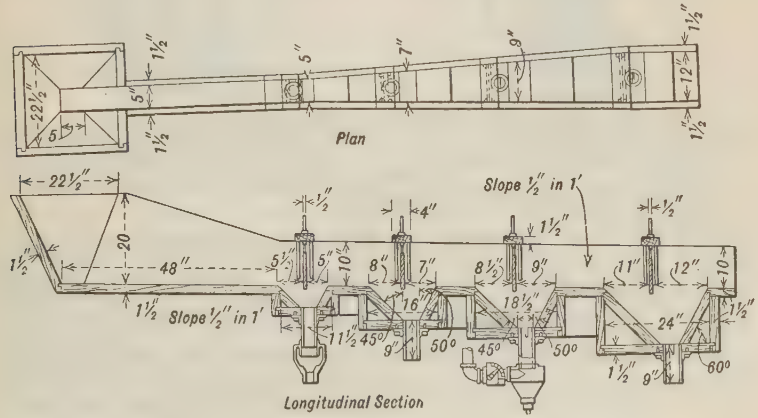


Fig. 5.—Richards launder-type vortex classifier.

but this cannot be attained. A home-made substitute for the vortex fitting is shown in Fig. 4. It is intended to bolt to the bottom of a roughing pocket. Performance of a 3-spigot vortex classifier is shown in Table 8.

Free-settling tank classifier is shown diagrammatically in Fig. 7. Sorting columns are of the same types as used on launder classifiers but the tank affords better opportunity for keeping slime away from the sorting columns. The use of free-settling hydraulic classifiers is past in well-designed mills

Table 8. Performance of Richards vortex classifier. (*After Richards*)

Screen aperture, mm.	Weights, cumulative per cent.				
	Feed	Spigot 1	Spigot 2	Spigot 3	Overflow
+0.907	0.4	0.0	1.5	0.0	0.0
0.566	21.0	52.0	19.4	0.4	0.1
0.427	35.8	76.6	39.3	1.9	0.4
0.351	48.2	90.9	62.9	5.6	0.8
0.277	58.1	96.1	81.1	12.5	1.6
0.206	67.1	98.7	90.7	24.6	2.0
0.137	77.1	99.7	98.9	54.1	5.7
0.130	80.7	....	99.7	62.3	6.3
0.107	84.8	0.3	0.3	37.7	6.6
−0.107	15.2	....	....	....	93.4

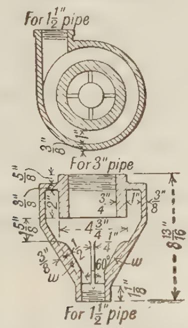


Fig. 6.—Richards vortex fitting.

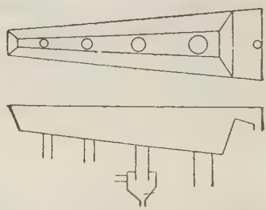


Fig. 7.—Sketch of free-settling tank classifier.

owing to their inefficiency and the superior results attained by hindered-settling machines. Mill operation of a free-settling classifier never yields settling ratios that approach the theoretical.

Fig. 8.—Methods of obtaining constriction in hindered-settling sorting columns.

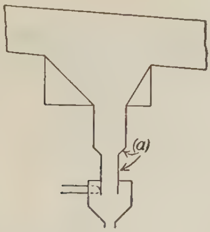
3. Hindered-settling hydraulic classifiers

General. Hindered-settling classifiers differ from free-settling in one characteristic only, *viz.:* that the sorting column is constricted at the lower end. This constriction may be obtained in any way. Several forms of hindered-settling sorting columns are shown in Fig. 8. The effect of the constriction has been considered in Art. 1. The same general types are found in hindered-settling as in free-settling mill machines, viz.: launder and tank.

Launder-type hindered-settling hydraulic classifiers

The simplest of these is a machine obtained by placing a reducer and a short pipe nipple of smaller size on the lower end of the sorting column of a Richards vortex classifier (Fig. 5), and placing the vortex fitting on the lower

end of the added nipple. Fig. 9 is a sketch of one spigot of such a classifier. Such a change in an existing classifier will result in a reduction in capacity, but by replacing the sorting column of the existing classifier by a teeter column of larger diameter, dropping the existing column to a position below the added member, capacity will be maintained. For discussion of the ratio of diameters of sorting column and teeter column, see Art. 4.



a. Reducer and nipple added to change classifier from free-settling to hindered-settling.

Fig. 9.—Sketch of one spigot of Richards hindered-settling vortex classifier, launder type.

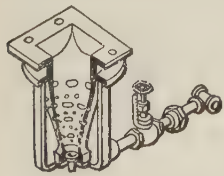


Fig. 11.—Richards hindered-settling sorting column.

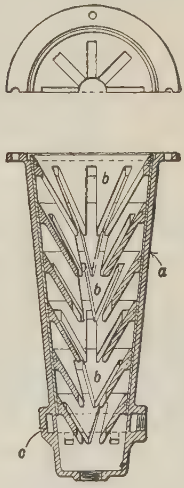


Fig. 10.—Deister cone-baffle classifier.

Deister cone-baffle classifier column (Fig. 10) consists of a downwardly-converging cylindrical casting (*a*) in which are suspended a number of slotted cone-shaped castings (*b*) with slots staggered vertically as shown. Water enters radially through a plurality of small holes from the pressure chamber (*c*). The column may be bolted directly to the bottom of a launder, but is better placed on the lower side of a roughing pocket.

This device had its greatest use in the gravity-concentration flow-sheet at Miami, prior to the adoption of flotation. A 12-spigot launder treated −10-mesh (1.651-mm.) drag-classifier sand. Table 9 gives sizing tests of the spigot product of a single-column machine under different operating conditions.

Richards hindered-settling launder-classifier column is shown in Fig. 11. The converging conical sorting chamber is perforated with vertically-alternating rows of radial and tangential water inlets which serve to maintain free movement and maximum fluidity in the mass of teetering grains.

Fig. 12 shows graphically the performance of a 10-spigot machine at Miami Copper Co.

At Clausthal (*100 J 426*) the 4-spigot launder-type hindered-settling classifier shown in Fig. 13 is used to treat −1.4-mm. undersize of trommels. Overflow is sent to a V-box, which is run without hydraulic water to prepare shaking-table feed.

Table 9. Performance of a 1-spigot Deister cone-baffle classifier at Miami Copper Co. (*108 P 1057*)

	Test No. 1	Test No. 2	Test No. 3
Tons of solid feed per 24 hr.	33.6	37.9	23.9
Per cent. of solids in spigot product	36.0	29.6	27.5
Hydraulic water, gallons per minute	17.5	22.4	....
Screen aperture, mesh	**Weights, cumulative per cent.**		
+10	0.1	0.1	0.0
20	4.8	6.2	12.7
30	42.3	44.8	57.4
40	66.6	64.6	76.6
60	88.6	84.0	91.1
80	93.6	89.7	94.8
100	96.5	93.8	97.0
−100	3.5	6.2	3.0

Richards pulsator classifier, launder type is shown in Fig. 14. Hindered-settling conditions are effected by the taper of the sorting column (*b*) which may be of the type in Fig. 11. Undue packing and sluggishness in teeter

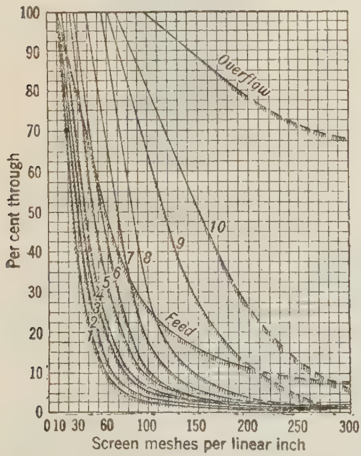


FIG. 12.—Performance of 10-spigot Richards hindered-settling launder classifier at Miami Copper Co. (*after Allis-Chalmers Co.*)

(Length of shading lines on curves represents copper content.)

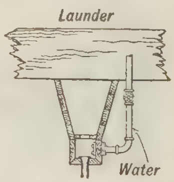


FIG. 13.—Hindered-settling launder classifier at Clausthal.

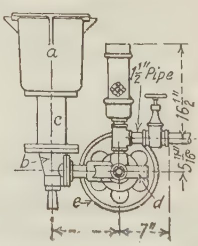


FIG. 14.—Richards pulsator classifier, launder type.

chamber (*c*) are prevented by the rotating valve (*d*) on the water inlet. This valve is opened and shut 200 to 400 times per min. by pulley (*e*), belt-driven, the higher speeds being used for the finest pulps. Water hammer is eliminated by the air chamber above the valve and the air compressed herein

when the valve is closed gives additional velocity to the water in the column as the valve opens. The assembled unit, carried by the trapezoidal roughing pocket (*a*), is bolted to the bottom of a launder. The machine is made in five sizes, based on the inner diameter of tube (*c*), *viz.*: 2½-, 3-, 4-, 5- and 6-in. Fall from the top of the roughing pocket to the lowest point of the spigot is 28 in. in the 2½-in. machine and 38 in. in the 6-in. machine. Capacity and water consumption may be estimated roughly from Figs. 15 and 16 re

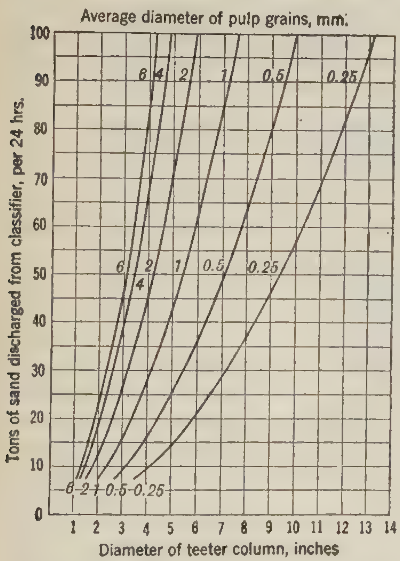


Fig. 15.—Capacity of Richards launder-type pulsator classifier (*after Denver Eng. Wks.*)

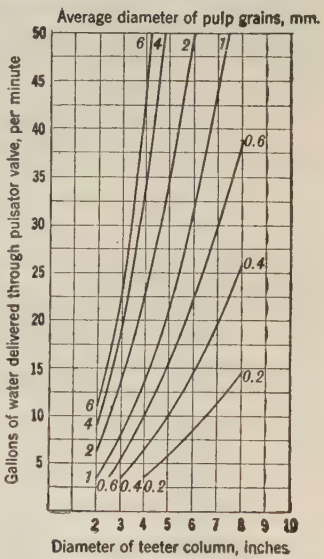


Fig. 16.—Water consumption of Richards launder-type pulsator classifier (*after Denver Eng. Wks.*)

spectively. Design should follow Art. 4. Performance of a 5-spigot machine is shown in Table 10.

Table 10. Performance of 5-spigot launder pulsator classifier at Akron Mines, White Pine, Colo. (*D. E. W. Co.*)

Screen aperture, mm.	Weight, per cent.						
	Feed	Spigot 1	Spigot 2	Spigot 3	Spigot 4	Spigot 5	Overflow
+0.833	2.2	6.0	0.2	0.0	0.0	0.0	0.0
0.417	14.2	26.8	9.2	0.7	0.0	0.0	0.0
0.208	33.6	49.2	41.6	12.7	1.0	1.5	0.0
0.147	21.6	14.2	32.6	32.0	7.0	2.9	0.7
0.104	5.6	2.4	6.8	10.1	7.0	2.9	0.7
0.074	15.2	1.4	9.2	38.7	47.0	16.2	7.2
−0.074	7.6	0	0.4	6.0	38.0	76.5	91.4

TANK-TYPE HINDERED-SETTLING HYDRAULIC CLASSIFIERS

These classifiers have been more generally used in recent gravity-concentration installations than the launder types. The best-known are the Richards, Richards-Janney, Richards inverted-pulsator, and Anaconda.

Richards hindered-settling tank classifier consists of the usual flaring V-tank, with inclined bottom, to which are attached a number of Richards hindered-settling sorting columns (Fig. 11).

Performance. At Moctezuma Copper Co. a 6-spigot machine treating Chilean mill discharge through 0.08-in. screen at the rate of 400 to 450 tons solid per 24 hr. in a pulp containing 75 per cent. moisture made spigot products as shown in Table 11. Moisture in the feed was about 75 per cent. At Liberty Bell a 3-spigot machine with 2-in., 2½-in. and 5-in. columns treating 40 tons per 24 hr. made the products shown in Table 12. Water consumption was about 1000 gal. per ton of feed.

Table 11. Performance of Richards hindered-settling tank classifier at Moctezuma Copper Co.

Legend	Spigot number					
	1	2	3	4	5	6
Diameter of spigot opening, in.	1	¾	⅝	⅝	⅝	½
Moisture, per cent.	62	80	82	84	86	92
Screen aperture, mm.	Weights on screen, cumulative per cent.					
2.362	4.2	2.5	0.3	....	....	....
1.651	23.3	18.2	4.1	....	....	....
1.168	42.7	38.0	13.5	....	....	....
0.833	60.5	60.1	30.0	2.7	....	....
0.589	73.3	76.7	50.1	9.4	....	....
0.417	83.0	88.7	71.8	26.8	7.0	0.3
0.295	89.4	94.8	87.3	52.8	30.1	3.3
0.208	93.3	97.6	94.8	77.2	70.5	17.0
0.147	96.3	99.0	98.4	92.6	92.5	52.4
0.104	98.1	99.5	99.5	97.9	98.1	74.6
0.074	98.8	99.8	99.8	98.3	98.8	82.0
−0.074	1.2	0.2	0.2	1.7	1.2	18.0

Table 12. Performance of Richards hindered-settling tank classifier at Liberty Bell Gold Mining Co.

	Feed	Spigot number			Overflow
		1	2	3	
Moisture, per cent.	74	71	75	81	....
Screen aperture, mm.	Weights on screens, cumulative per cent.				
0.417	7.7	18.9			....
0.295	12.5	42.5			....
0.208	33.1	64.1			0.7
0.147	45.1	75.0			6.0
0.074	59.1	83.1			22.5
−0.074	40.9	16.9			77.5

Richards-Janney classifier (Fig. 17) is of the tank type with hindered-settling sorting columns and various important features that give large capacity, steady operation, low water consumption and considerable flexibility. It consists of a number of pyramidal settling tanks, increasing in size from feed to discharge end, so joined that the tops of the dividing partitions formed by the junction of adjacent walls are well below the overflow level. To the bottom of each pyramidal frustum is attached the classifying mechanism proper, consisting of a cylindrical teeter chamber (*a*), converging at the bottom to the cylindrical glass-walled sorting column (*b*) below which is a tangential hydraulic-water inlet (*c*). These constitute the elements. The additional features are the rotating stirring arms (*d*) carried on a hollow spindle (*e*) depending from the worm gear (*f*) which is driven by a worm from shaft (*g*). Cams (*h*) on the upper side of gear (*f*) operate a lifting arm on the tappet rod (*i*) which carries on its lower end a rubber ball-valve that seats on a bushing (*j*). The valve is thereby periodically opened and discharges into the retarding chamber (*k*) which in turn discharges through a pipe-and-plug spigot (*l*). The rate of discharge is controlled by the size of the spigot opening and by the air cock (*n*). Water inlet to each column is regulated by a dial valve (*m*). The stirring arms make about 1.5 r.p.m. They prevent the formation of sand banks in the teeter column and also tend to prevent the formation of eddies with downward components of motion. The glass sorting column, by permitting observation of the actual sorting operation, makes regulation of the classifier easier than otherwise. The intermittent discharge effected by the ball valve makes it possible to discharge a thick spigot product through a large opening and thus decrease water consumption while eliminating clogged spigots. The retarding chamber prevents serious downward rushes of pulp, which would entirely disarrange classification, when the ball-valve opens.

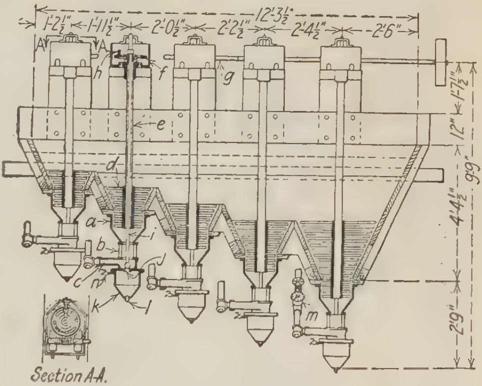


FIG. 17.—Richards-Janney classifier.

Performance is shown in Table 13. Both of the CHINO machines are apparently overloaded, with the result that there is much slime in the spigot products and much fine sand overflowing. The machine on zinc ore is doing the best work from the point of view of size differentiation in products. The moisture content of the products is reasonably consistent and characteristic and, compared with the spigot discharges from other classifiers, shows the advantage of intermittent retarded discharge.

Richards inverted pulsator classifier (Fig. 18) consists of a tank (*a*) with screen bottom, divided by partitions extending nearly to the screen into seven compartments of unequal area, six of the compartments being fitted with overflow spouts (*b*). The tank is set on top of a compartmented water-distributing tank or hutch (*c*), fed by a manifold from a pulsating cock. Each of the overflow compartments in the upper tank constitutes a settling

Table 13. Performance of Richards-Janney classifiers

Plant	Porphyry copper ore											
Character of feed	Primary, screen undersize											
Lost time, per cent	@ 1											
Hydraulic water, gallons per ton of feed	126											
Product	Feed		Spigot 1		Spigot 2		Spigot 3		Spigot 4		Overflow	
Tons per 24 hr	475											
Moisture, per cent	75		76		56		52		75		87	
Operation: Diameter of valve seats, in.			1¼		1⅛		1		1⅜			
Screen aperture, mesh	Per cent. weight	Per cent. Cu	Per cent. weight	Per cent. Cu	Per cent. weight	Per cent. Cu	Per cent. weight	Per cent. Cu	Per cent. weight	Per cent. Cu	Per cent. weight	Per cent. Cu
10			1.8	0.93	0.5	0.87						
14												
20	16.4	0.89	39.8	1.04	24.0	0.83	3.4	0.74				
28												
30	18.2	0.88	31.3	1.10	32.1	0.85	15.7	0.64	0.2	0.77		
35												
40	9.5	0.93	13.4	1.40	17.6	0.91	21.7	0.63	0.8	0.48		
48												
50	12.6	0.96	16.8	1.94	15.9	1.19	24.2	0.65	9.4	0.42		
60	3.8	0.98	1.3	2.44	2.8	1.62	8.9	0.76	7.9	0.40	0.1	1.86
65												
80	6.7	1.08	2.1	2.89	3.1	2.26	13.9	0.97	24.5	0.40	0.4	0.80
100	4.3	1.09	0.7	3.94	1.4	3.64	4.5	0.65	18.9	0.56	1.3	0.50
120	1.8	1.21	0.4	4.37	0.4	4.24	1.7	2.05	10.0	0.69	0.8	0.41
150	3.8	1.45	0.0	4.25	0.5	4.95	0.4	2.87	14.5	1.12	5.7	0.50
200	1.1	1.49	0.4	4.77	0.1	4.08	2.2	3.57	3.3	1.63	2.3	0.49
−200	21.8	1.72	2.0	2.65	1.6	2.68	3.4	3.93	10.5	4.03	89.4	1.34
Totals	100.0	1.14	100.0	1.30	100.0	1.08	100.0	0.88	100.0	0.99	100.0	1.22

Table 13. Performance of Richards-Janney classifiers—*Continued*

Plant										
Character of feed Lost time, per cent Hydraulic water, gallons per ton of feed	Porphyry copper ore									
Product	Chilean-mill discharge, feed to mill first 2 spigots of primary classifier @ 1 84									
	Feed		Spigot 1		Spigot 2		Spigot 3		Overflow	
Tons per 24 hr	475		...		...		...		...	
Moisture, per cent	68		45		60		63		88	
Operation: Diameter of valve seats, in.	...		1⅛		1		1		...	
Screen aperture, mesh	Per cent. weight	Per cent. Cu	Per cent. weight	Per cent. Cu	Per cent. weight	Per cent. Cu	Per cent. weight	Per cent. Cu	Per cent. weight	Per cent. Cu
20	2.5	0.44	5.6	0.47	1.7	0.44	1.4	0.45	...	...
28	...	...	...	...	...	...	...	...	...	...
30	15.0	0.53	42.0	0.83	26.5	0.53	8.6	0.48	...	...
35	...	...	...	...	...	...	...	...	...	...
40	12.5	0.70	23.3	1.43	25.2	0.63	10.1	0.49	...	...
48	...	...	...	...	...	...	...	...	...	...
50	10.7	0.83	14.3	2.17	19.4	0.95	13.2	0.50	...	...
60	3.8	0.98	4.0	3.08	6.8	1.30	7.4	0.52	0.4	0.89
65	...	...	...	...	...	...	...	...	...	...
80	6.9	1.11	4.4	4.17	8.0	1.84	16.0	0.61	0.3	0.55
100	5.2	1.40	2.0	5.82	4.2	2.59	13.9	0.89	1.4	0.42
120	2.5	1.43	0.5	7.15	1.2	3.05	5.1	1.05	0.9	0.41
150	1.0	1.34	1.1	6.45	2.7	3.28	3.0	1.42	5.7	0.48
200	6.9	2.11	0.2	5.41	0.5	3.29	9.2	2.16	2.4	1.54
−200	33.0	1.37	2.6	3.27	3.8	3.34	12.1	3.03	88.9	1.20
Totals	100.0	1.10	100.0	1.64	100.0	1.10	100.0	1.07	100.0	1.12

Table 13. Performance of Richards-Janney classifiers—*Continued*

Plant	Chino Copper Co.												
Character of feed	Primary, screen undersize						Secondary						
Power consumed, hp.	0.5						0.5						
Lost time, per cent.	1						1						
Hydraulic water, gallons per ton of feed	300–360						200–240						
Product	Feed	Spigot 1	Spigot 2	Spigot 3	Spigot 4	Overflow	Feed	Spigot 1	Spigot 2	Spigot 3	Spigot 4	Spigot 5	Overflow
Tons per 24 hr.	750	...	...	...	...	...	360	...	...	...	...	...	...
Moisture, per cent.	70–75	63	60	74	70	75–85	70–75	73	70	75	75	70	85–90
Operation:													
Diameter of valve seats, in.	...	1½	1¼	1¼	1¼	...	...	1	⅞	⅞	¾	⅝	...
Screen aperture, mesh	Per cent. weight												
10	1.1	11.2	3.0	1.0	...	...	...	...	...	...	...	...	...
14	4.0	20.2	12.0	3.0	1.0	...	1.0	1.7	...	...	...	...	...
20	5.6	19.5	19.2	7.0	5.2	...	5.2	12.2	3.3	0.5	0.4	0.8	3.2
28	7.6	16.2	21.7	14.5	12.3	2.4	12.3	32.0	12.5	1.9	2.3	2.0	2.0
30	...	...	...	...	...	...	...	...	...	...	...	...	...
35	10.0	8.8	14.5	25.7	14.0	4.5	14.0	25.2	23.7	6.5	12.0	9.2	1.5
40	...	...	...	...	...	...	...	...	...	...	...	...	...
48	6.5	3.2	5.2	17.2	7.2	5.0	7.2	7.5	11.5	11.0	15.5	8.8	2.0
50	...	...	...	...	...	...	...	...	...	...	...	...	...
60	...	...	...	...	...	...	...	...	...	...	...	...	...
65	6.3	1.7	3.3	10.5	5.8	6.0	5.8	3.7	7.0	12.5	12.5	7.0	3.0
80	...	...	...	...	...	...	...	...	...	...	...	...	...
100	6.3	1.5	2.5	5.8	6.2	8.3	6.2	2.5	5.3	14.8	11.0	7.8	4.7
120	...	...	...	...	...	...	...	...	...	...	...	...	...
150	5.7	1.5	2.0	3.0	5.7	9.7	5.7	1.5	4.2	12.0	7.7	9.7	7.3
200	3.7	0.8	1.0	1.0	3.2	3.3	3.2	0.7	1.8	3.5	3.2	6.0	3.7
−200	43.2	15.4	15.6	11.3	39.4	60.8	39.4	13.0	30.7	37.3	35.4	48.7	72.6
Totals	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0

Table 13. Performance of Richards-Janney classifiers—*Continued*

Plant	Granitic zinc ore							Alaska Gastineau					
Character of feed	Primary, screen undersize							Primary, screen undersize					
Power consumed, hp.	......							0.24					
Hydraulic water, gallons per ton of feed	220							270					
Product	Feed	Spigot 1	Spigot 2	Spigot 3	Spigot 4	Spigot 5	Overflow	Feed	Spigot 1	Spigot 2	Spigot 3	Spigot 4	Overflow
Tons per 24 hr.	300	75	65	60	50	40	10	300	......	......	......	......	......
Moisture, per cent	78	61	62	80	60	45	87	68	31	71	72.5	54	89.7
Operation:													
Diameter of valve seats, in.	......	1	¾	1	⅝	½	......	......	1	¾	⅝	½	......
Number of cams	......	4	3	3	2	2	......	......	......	......	......	......	......
Discharges per minute	......	6	4½	4½	2	2	......	......	......	......	......	......	......
Time open per discharge, seconds	......	8	9	7	10	10	......	......	......	......	......	......	......
Screen aperture, mesh	Per cent. weight												
10	2.6	6.0	3.3	......	......	......	......	1.6	2.3	1.0	......	......	......
14	4.0	9.4	4.5	0.4									
20	6.0	14.6	9.2	1.2	......	......	......	27.8	46.2	22.8	1.6	......	......
28	8.9	16.7	11.4	4.4	......	......	......	21.4	33.6	32.4	3.0	0.2	......
30	......	......	......	......	......	......	......	......	......	......	......	......	......
35	11.7	19.3	24.2	13.0	2.4	0.5	......						
40	......	......	......	......	......	......	......	......	......	......	......	......	......
48	12.7	14.9	20.4	21.5	9.5	1.3	......	7.8	7.8	17.0	10.8	0.8	......
50													
60	......	......	......	......	......	......	......	......	......	......	......	......	......
65	11.0	8.7	14.0	21.8	22.8	5.8	......	4.6	3.6	9.2	13.8	6.9	......
80	1.2	1.0	4.8	11.8	1.9	7.1	......	......	......	......	......	......	......
100	7.2	4.3	3.4	6.8	20.8	14.3	1.8	5.7	2.4	6.0	20.1	21.7	......
120	......	......	......	......	......	......	......	......	......	......	......	......	......
150	8.1	2.3	2.3	7.6	17.5	26.4	0.7	4.8	1.2	2.6	15.5	33.6	......
200	3.4	0.9	0.9	3.6	8.9	16.4	3.1	2.2	0.4	1.8	4.0	8.0	......
−200	23.2	1.9	1.6	7.9	16.2	28.2	94.4	24.1	2.5	7.2	31.2	28.8	......
Totals	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	......

column with a constriction at the bottom furnished by the screen. The velocity of the water rising in the individual compartments depends upon their cross-sectional area and upon the height of the overflow lip. Pulp introduced into the feed compartment passes to the bottom of the first compartment where the finest material is lifted out. The balance is moved along the screen to the next compartment under the horizontal impulse of new feed arriving. Each compartment removes its quota until the last is reached and here an extra-strong current induced by lowering the spout an inch or more below the others raises all but the coarsest heavy mineral. This may be discharged intermittently or continuously through spigot (*d*). Hutch spigots are provided to draw off any fine material that passes through the screen. Usual tank widths are 3- and 4-in.

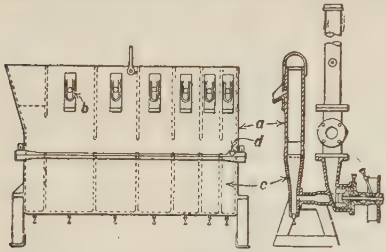


Fig. 18.—Richards inverted pulsator classifier.

Performance at Anaconda is shown in Table 14. Speed and water consumption in this test are higher than usual and tonnage is low. A 4-in. machine can probably treat 125 to 175 tons per 24 hr. of −2-mm. copper ore or 150 to 200 tons of lead ore at the same size. With finer feeds the capacity will be less.

This classifier can be made to do as nearly perfect work as any classifier that has ever been devised, but it is extremely sensitive to feed changes, requires rather close attention, and consumes a large amount of water, ranging from 1000 to 1500 gal. per ton of ore treated. Any dirt in the feed water, such as wood or plant fragments, lodges on the under side of the screen and by blinding it cuts down current velocities. The upper tank (*a*) must then be lifted off and the screen cleaned. A run of coarse material may cause building up at the bottom of the last compartment so that this compartment is cut off and the entire classification disarranged.

Richards pulsator classifier, tank type (Fig. 19) is used by the New Jersey Zinc Co. at the Franklin and Ogdensburg mills. Pulsating water rises through grids (*a*) in the bottom of the usual flaring V-tank and spigot products are drawn from the bottom of the hopper-shaped pressure boxes (*b*).

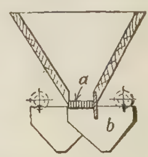


Fig. 19.—Richards pulsator classifier, tank-type, at New Jersey Zinc Co.

At Franklin each machine treats about 100 tons per 24 hr. and consumes about 450 gal. of water per ton of solids. Valves are run at 120 rev. (240 pulsations) per min. and each machine consumes 3 hp. Spigot openings vary from ½-in. diameter at the feed end to ⅛-in. at the overflow end. Choking causes about 5 per cent. lost time. Screen tests of feed and products are given in Table 15. At Ogdensburg the total load is 120 tons per 24 hr.; water consumption, 400 gal. per ton of feed and valve speed 130 r.p.m. Sizing tests of feed and products are given in Table 15.

Fahrenwald classifier (sizer) (Fig. 20) is of the hindered-settling tank type, but the spigot product is drawn from the bottom of the teeter chamber without passing through the constriction or being elevated to an overflow and the rate of discharge is automatically controlled by the pulp density in individual teeter chambers. (*A*) is a tank in which the bulk of the slime is roughed out. Below (*A*) is the trapezoidal tank (*B*), with vertical side walls, divided into compartments (*b*) by means of the partitions (*b′*). The perfor-

Table 14. Sizing-assay test of feed and products of Richards inverted pulsator classifier at Anaconda Copper Co.

Screen aperture, mm.	Feed		Overflow 1		Overflow 2		Overflow 3		Overflow 4		Overflow 5		Overflow 6	
	Per cent. weight	Per cent. Cu	Per cent. weight	Per cent. Cu	Per cent. weight	Per cent. Cu	Per cent. weight	Per cent. Cu	Per cent. weight	Per cent. Cu	Per cent. weight	Per cent. Cu	Per cent. weight	Per cent. Cu
5.56	0.8	2.12	......	......	......	......	......	......	......	......	......	......	1.6	1.38
3.99	10.0	1.51	......	......	......	......	......	......	......	......	5.9	1.22	14.0	1.74
2.79	17.0	1.62	......	......	......	......	......	......	......	......	11.4	1.29	20.1	1.81
2.03	13.3	1.10	......	......	......	......	......	......	......	......	12.5	1.34	17.2	1.99
1.40	10.3	1.61	......	......	......	......	......	......	......	......	13.3	1.38	14.2	2.20
1.02	5.3	1.69	......	......	......	......	......	......	......	......	9.4	1.48	7.9	2.43
0.686	6.0	1.73	......	......	......	......	......	......	......	......	14.9	1.48	9.0	2.78
0.508	3.2	1.81	......	......	......	......	......	......	......	......	9.4	1.66	4.1	3.14
0.351	1.6	2.08	2.1	1.25	1.8	0.87	1.8	1.72	11.4	0.82	5.0	1.89	1.7	4.39
0.242	4.7	2.25	3.5	1.17	4.1	0.82	8.6	0.67	19.5	0.98	8.5	2.44	3.7	6.53
0.154	4.5	2.71	6.1	1.74	7.1	1.17	18.1	0.96	17.1	1.62	3.3	4.30	2.1	9.42
0.129	2.0	3.03	3.5	2.17	3.5	1.88	7.3	1.65	5.7	2.41	0.8	5.18	0.6	9.35
−0.129	21.3	2.69	84.8	2.04	83.5	1.97	64.2	2.39	46.3	2.58	5.6	3.60	3.8	6.39
Total..................	100.0	1.90	100.0	1.98	100.0	1.84	100.0	1.97	100.0	1.95	100.0	1.70	100.0	2.70
Tons per 24 hr...........	83.35		4.05		3.13		2.88		2.35		48.72		22.22	
Moisture, per cent........	77.3		95.3		96.3		96.0		96.1		82.8		89.8	
Gallons fresh water added per ton...............	2331													

Pulsations per min. = 472.

ated constriction plate (*c*) separates the teeter chamber from the individual hydraulic-pressure chamber (*d*) below. Hydraulic water is supplied to the pressure chambers through individual pipes from a header. Observation of the individual teeter columns is afforded by means of glass windows. Spigot discharge is effected as follows: As classified sands accumulate in the teeter chamber the pressure rises in (*d*) and water is forced up through pipe (4), which connects with (*d*), into chamber (5) and pipe (10) until it stands at some level above (9). The pressure in chamber (5) due to this standing column of water raises the soft-rubber diaphragm (9) and with it pipe (10) and rod (11), which

Table 15. Performance of Richards pulsator classifier, tank type, at New Jersey Zinc Co.

Mill	Franklin											
Product	Feed	Spigot number										Overflow
		1	2	3	4	5	6	7	8	9	10	
Screen, mm.	Weight, per cent.											
1.651	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	....
1.168	.....	.....	0.4	0.1	0.2	0.1	0.1	.....	.....	.....	.....	....
0.833	0.6	1.8	1.8	1.0	1.0	0.5	0.8	0.4	0.4	0.2	.....	.....
0.589	7.1	15.5	15.4	11.1	10.0	6.8	8.2	5.4	6.2	3.4	.....	0.1
0.417	23.4	44.2	44.1	39.0	33.0	30.2	33.0	24.9	28.6	15.2	0.1	0.2
0.295	26.2	25.6	25.6	28.7	29.5	31.7	32.2	32.2	32.6	21.9	0.9	0.4
0.208	20.9	9.8	9.7	14.3	18.2	21.1	18.4	24.2	21.8	27.3	8.6	0.6
0.147	9.8	2.4	2.3	4.7	6.5	7.8	6.0	10.2	8.2	20.3	40.7	10.4
0.104	3.4	0.4	0.3	0.7	1.0	1.2	0.9	1.7	1.5	6.2	20.3	22 2
0.074	5.1	0.2	0.2	0.3	0.4	0.5	0.3	0.9	0.5	5.0	26.4	51.6
−0.074	3.5	0.1	0.2	0.1	0.2	0.1	0.1	0.1	0.2	0.5	3.0	14.3

Mill	Ogdensburg									
Product	Feed	Spigot number								Overflow
		1	2	3	4	5	6	7	8	
Screen, mm.	Weight, per cent.									
1.651	.....	0.4	0.1	......	......	......	....	....	......	......
1.168	0.3	0.9	0.1	......	......	......	....	....	......	......
0.833	2.0	16.0	0.9	0.1	......	......	....	....	......	......
0.589	4.0	27.2	16.2	1.8	0.3	0.1	....	....	......	......
0.417	15.8	30.9	24.4	28.6	6.2	1.0	0.1	0.1	......	......
0.295	18.1	16.0	34.7	29.8	31.8	14.6	2.6	0.5	0.2	0.4
0.208	22.9	7.2	18.1	28.6	43.2	50.9	31.1	11.3	4.4	3.0
0.147	20.6	0.5	4.3	9.0	15.6	26.9	48.5	47.6	39.4	41.5
0.104	9.5	0.3	0.4	0.9	2.1	5.0	14.4	27.8	37.3	21.5
0.074	3.9	0.2	0.4	0.7	0.3	0.8	3.0	8.6	12.4	17.1
−0.074	2.9	0.4	0.4	0.5	0.5	0.7	0.3	4.1	6.3	16.5

latter is pinned through hole (12) in such a way that rise of (10) engages the pin. Rod (11) carries ball valve (2) at its lower end and this ball, when lifted, permits classified sand to pass from above the constriction plate through pipe (14) out of the classifier. Drain plugs are provided at the bottoms of chambers (*d*) to discharge any sands that pass the constriction plate. The height of chamber (5) must be adjusted to correspond with the density of the pulp in the individual teeter chamber. This is done by loosening the set-screw (15), which clamps stem (7). For feeds passing 14-mesh the heights of the chambers above pulp level in (*A*) range from 10 to 16 in. in the first compartment to 3 to 5 in. in the last.

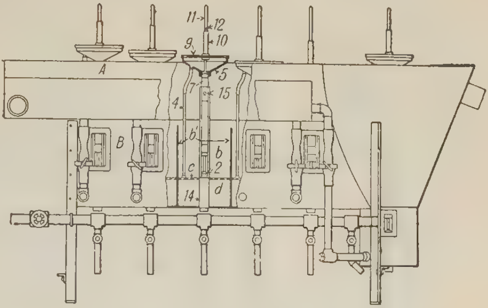
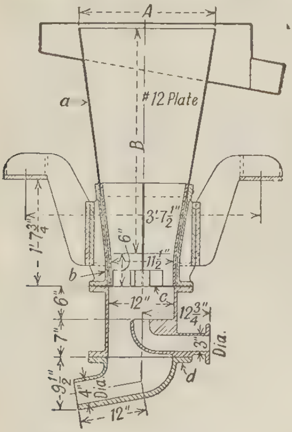


Fig. 20.—Fahrenwald classifier.

Performance. The inventor states (*U. S. Bur. of Mines, Reports of Investigations, Ser. No. 2618, June, 1924*) that a 10-spigot machine at Bunker Hill and Sullivan, West mill, treated 225 tons per 24 hr. with a water consumption of 320 to 480 gal. per ton. Sizing-assay test of the feed and products is shown in Table 16.

Anaconda classifier (Fig. 21) consists of a conical roughing chamber (*a*) atop a hindered-settling column (*b*) formed by placing a perforated plate (*c*)



Sizes			
A		B	
Ft.	In.	Ft.	In.
7	0	4	3
5	0	3	6
3	4	3	11
2	4	3	6
2	1	3	6
1	11	2	6

Fig. 21.—Anaconda classifier.

in the path of the rising water. At Anaconda the lower casting (*d*) and the length of the cylindrical part of the upper casting were made the same for all sizes of classifier but the shape of the cone was varied according to the table of dimensions given in the figure. Small, steep cones were used for coarse separation, larger and more obtuse cones for de-sliming.

Performance at Anaconda under various feed conditions is given by Ammon (*46 A 277*) and summarized in Table 17. The volume of feed pulp is the important factor in de-sliming

Table 16. **Performance of Fahrenwald classifier at Bunker Hill and Sullivan, West mill.** (*After Fahrenwald*)

	Feed		Spigot number					
			1		2		3	
Tons per 24 hr.......	223.48		5.7		14.9		22.8	
Per cent. of total feed	100		2.5		6.8		10.1	
Per cent. of total Pb.	100		37.00		10.02		10.40	
Screen aperture, mm.	Per cent. weight	Per cent. Pb	Per cent. weight	Per cent. Pb	Per cent. weight	Per cent. Pb	Per cent. weight	Per cent. Pb
+1.168	....	....	17.9	10.3	3.9	0.4	3.2	....
0.833	2.4	12.7	11.3	33.7	12.9	0.6	8.3	....
0.589	7.0	13.9	29.8	62.5	33.5	0.8	29.8	....
0.417	8.2	13.9	24.1	80.0	25.7	14.2	31.2	....
0.295	12.3	15.0	13.4	81.6	13.2	32.0	16.2	....
0.208	13.1	12.4	2.3	81.6	6.4	67.5	6.6	....
0.147	20.5	13.3	0.5	78.8	3.4	80.0	3.7	....
0.104	15.0	15.8	0.2	75.6	0.7	78.6	0.8	....
0.074	10.0	19.3	....	....	....	....	0.2	....
−0.074	11.5	28.2	0.5	71.5	0.3	50.1	....	....
Totals	100.0	16.3	100.0	57.5	100.0	15.8	100.0	16.4

	Spigot number							
	4		5		6		7	
Tons per 24 hr.......	32.6		18.8		22.7		37.8	
Per cent. of total feed	13.6		8.5		10.0		16.7	
Per cent. of total Pb.	9.28		5.77		4.96		4.45	
Screen aperture, mm.	Per cent. weight	Per cent. Pb	Per cent. weight	Per cent. Pb	Per cent. weight	Per cent. Pb	Per cent. weight	Per cent. Pb
+1.168	....	....	....	....	....	....	....	....
0.833	3.5	0.7	1.2	0.6	....	....	....	....
0.589	13.7	1.1	7.5	0.6	3.8	0.2	0.4	0.4
0.417	29.0	3.6	17.8	0.9	9.6	0.4	3.0	0.3
0.295	29.3	7.4	34.4	2.5	27.3	0.6	10.6	0.3
0.208	12.7	29.9	0.8	6.3	30.0	1.9	21.7	0.5
0.147	7.4	57.8	30.5	22.6	1.7	8.9	28.3	0.8
0.104	2.9	77.2	4.8	50.7	7.2	32.2	17.8	8.8
0.074	1.0	80.2	2.1	64.2	3.5	52.5	11.4	18.3
−0.074	0.5	58.8	0.9	56.9	1.7	63.6	6.8	41.9
Totals	100.0	14.77	100.0	9.0	100.0	7.69	100.0	6.87

	Spigot number						Overflow	
	8		9		10			
Tons per 24 hr.......	16.6		21.2		32.4		10.4	
Per cent. of total feed	7.6		5.6		14.2		4.5	
Per cent. of total Pb.	4.50		3.52		3.40		6.70	
Screen aperture, mm.	Per cent. weight	Per cent. Pb	Per cent. weight	Per cent. Pb	Per cent. weight	Per cent. Pb	Per cent. weight	Per cent. Pb
+1.168	....	....	....	....	....	....	....	....
0.833	....	....	....	....	....	....	....	....
0.589	....	....	....	....	....	....	....	....
0.417	4.3	0.1	....	....	....	....	....	....
0.295	7.2	0.2	3.0	0.1	....	....	....	....
0.208	21.0	0.4	8.5	0.2	9.1	0.3	....	....
0.147	34.9	1.0	28.5	0.4	22.5	0.3	....	....
0.104	15.7	8.5	25.6	2.4	30.0	1.0	1.0	2.3
0.074	10.6	20.9	22.0	5.9	23.4	6.0	51.7	4.4
−0.074	6.3	47.9	12.4	27.4	15.0	30.1	47.3	17.7
Totals	100.0	6.95	100.0	5.45	100.0	5.40	100.0	10.67

Table 17. Sizing-sorting test on Anaconda classifier fitted with feed cone, Anaconda C. M. Co. (*after Ammon*)

Test number	Screen aperture, mm.	Feed		Spigot product					Overflow			
					Tons per 24 hr.					Tons per 24 hr.		
		Per cent. weight	Tons per 24 hr.	Per cent. weight	Free mineral	Middling	Tailing	Total	Per cent. weight	Free mineral	Tailing	Total
1	+2.38	2.2	5.72	3.6	...	5.72	...	5.72	...	...	...	...
Maximum feed rate. Current velocity in constriction, 100 mm. per second. Settling ratio = 2.37(*a*)	1.41	20.1	51.20	32.2	3.33	6.76	41.11	51.20	...	...	...	...
	0.84	24.9	63.39	39.4	4.13	...	58.49	62.62	0.8	...	0.77	0.77
	0.50	13.6	34.61	17.6	3.72	...	24.27	27.99	6.9	0.39	6.23	6.62
	0.35	3.9	10.05	2.4	1.21	...	2.61	3.82	6.5	0.49	5.74	6.23
	0.17	9.4	23.94	3.9	2.76	...	3.44	6.20	18.5	1.56	16.18	17.74
	0.07	6.5	16.68	0.6	0.48	...	0.47	0.95	16.4	2.82	12.91	15.73
	−0.07	19.3	49.29	0.3	0.48	...	...	0.48	50.9	18.55	30.26	48.81
Total	...	100.0	254.88	100.0	16.11	12.48	130.39	158.98	100.0	23.81	72.09	95.90
2	+2.38	2.0	1.80	2.6	...	1.80	...	1.80	...	...	...	...
Minimum feed rate. Current velocity in constriction, 80 mm. per second. Settling ratio = 1.66(*a*)	1.41	39.2	35.57	51.5	1.03	34.54	...	35.57	...	...	...	...
	0.84	22.1	20.07	28.8	0.90	0.90	18.10	19.90	0.8	...	0.17	0.17
	0.50	9.9	8.97	11.2	0.91	...	6.83	7.74	5.7	0.04	1.19	1.23
	0.35	3.0	2.75	2.2	0.54	...	0.98	1.52	5.7	0.09	1.14	1.23
	0.17	7.6	6.85	2.8	0.78	...	1.15	1.93	22.8	0.86	4.06	4.92
	0.07	4.4	4.01	0.9	0.27	...	0.35	0.62	15.7	1.04	2.35	3.39
	−0.07	11.8	10.65	...	...	...	...	...	49.3	4.61	6.04	10.65
Total	...	100.0	90.67	100.0	4.43	37.24	27.41	69.08	100.00	6.64	14.95	21.59
3	+2.38	0.8	1.65	1.3	...	1.65	...	1.65	...	...	...	...
Average feed rate. Current velocity in constriction, 102 mm. per second. Settling ratio = 1.66(*a*)	1.41	24.8	48.21	38.0	1.69	46.52	...	48.21	...	...	...	...
	0.84	25.0	48.64	37.7	2.63	...	45.20	47.83	1.2	0.15	0.66	0.81
	0.50	13.5	26.21	16.4	3.68	...	17.13	20.81	8.0	0.23	5.17	5.40
	0.35	4.9	9.48	3.0	1.90	...	1.91	3.81	8.4	0.86	4.81	5.67
	0.17	10.0	19.42	2.9	2.82	...	0.86	3.68	23.3	3.94	11.80	15.74
	0.07	5.8	11.23	0.7	0.57	...	0.32	0.89	15.3	0.78	9.56	10.34
	−0.07	15.2	29.59	...	...	...	...	...	43.8	11.10	18.49	29.59
Total	...	100.0	194.43	100.0	13.29	48.17	65.42	126.88	100.0	17.06	50.49	67.55

a Area of constriction, 33.46 sq. in.

Table 18. Performances of Anaconda classifier

Mill	Anaconda Copper Mining Co.					
Character of feed	Primary feed			Roll product		
Size, diameter of cone, ft.	7			7		
Type of constriction plate	*a*			*a*		
Ratio of area of teeter to sorting columns	3.9			3.9		
Diameter of spigot, in.	1¼			¾		
Hydraulic water, gallons per ton of feed	905			409		
Rising velocity in constriction, mm. per second	226			84		
Product	*F*	*S*	*O*	*F*	*S*	*O*
Tons per 24 hr.	200	160	40	141	136	5
Moisture, per cent.	71.3	71.7	95.6	66.9	55.8	98.4
Assay, per cent. Cu	2.83	2.90	2.54	....	....	....
Screen analysis	Weight, per cent.					
+4.00 mm.	....	1.9	....	....	....	....
2.38	....	22.0	....	....	....	....
2.00	....	....	....	....	....	....
1.68	....	16.9	....	....	....	....
1.41	....	6.9	....	8.4	8.4	....
1.00	....	....	....	....	....	....
0.84	....	18.9	....	38.1	41.1	....
0.71	....	....	....	....	....	....
0.50	....	12.6	....	29.0	29.1	....
0.35	....	4.5	....	5.4	6.1	....
0.17	....	9.7	....	9.5	9.0	....
0.07	....	5.6	7.9*b*	4.5	4.3	0.2
−0.07	....	1.0	92.1	5.1	2.0	99.8
Mill	Anaconda Copper Mining Co.					
Character of feed	Huntington mill discharge			Huntington mill discharge		
Size, diameter of cone, ft.	7			7		
Type of constriction plate	*a*			*a*		
Ratio of area of teeter to sorting columns	3.9			3.9		
Diameter of spigot, in.	1⅛			1⅛		
Hydraulic water, gallons per ton of feed	590			1000		
Rising velocity in constriction, mm. per second	64			41		
Product	*F*	*S*	*O*	*F*	*S*	*O*
Tons per 24 hr.	205	154	51	111	89	22
Moisture, per cent.	91.4	74.2	97.7	88.4	85	97.3
Assay, per cent. Cu	1.10	1.37	1.01	....	....	....
Screen analysis	Weight, per cent.					
+4.00 mm.	....	....	....	....	....	....
2.38	....	....	....	....	....	....
2.00	....	....	....	....	....	....
1.68	....	....	....	....	....	....
1.41	3.4	5.4	....	5.9	5.1	....
1.00	9.0	11.7	....	12.1	12.0	....
0.84	13.5	16.0	....	15.9	18.5	....
0.71	10.9	13.9	....	9.8	13.0	....
0.50	11.7	13.2	....	9.7	13.5	....
0.35	9.9	14.7	....	6.2	7.9	....
0.17	12.4	15.5	....	10.9	14.7	....
0.07	8.7	8.6	9.4	6.7	10.9	1.2
−0.07	20.5	1.0	90.6	22.8	4.4	98.8

F = Feed. *S* = Spigot. *O* = Overflow.

a 1 pipe 6 in. in diameter, 2 in. long in plate. *b* Normal work is 2.3 per cent. +200-mesh. Too much hydraulic water used in this run.

Table 18. Performances of Anaconda classifier—*Continued*

Mill	Anaconda Copper Mining Co.					
Character of feed	De-slimed roll product			2.5-mm. screen undersize		
Size, diameter of cone, ft.	2			2*d*		
Type of constriction plate	*c*			*c*		
Ratio of area of teeter to sorting columns	2.8			2.8		
Diameter of spigot, in.	.....			1¼		
Hydraulic water, gallons per ton of feed	1165			1100		
Rising velocity in constriction, mm. per second	327			316		
Product	*F*	*S*	*O*	*F*	*S*	*O*
Tons per 24 hr.	270	102	168	250	157	93
Moisture, per cent.	75.8	83.3	90.8	78.2	78.5	94.1
Assay, per cent. Cu	...	...	...	2.69	2.56	2.88
Screen analysis	Weight, per cent.					
+4.00 mm.	...	...	...	...	...	...
2.38	...	2.6	...	...	...	...
2.00	...	...	...	7.4	12.3	...
1.68	...	...	...	12.0	18.0	...
1.41	...	34.3	3.4	11.1	13.8	...
1.00	...	27.7	8.4	13.4	21.0	...
0.84	...	21.2	17.4	11.7	14.8	6.7
0.71	...	...	...	7.1	9.2	6.1
0.50	...	12.0	33.5	7.6	6.1	11.0
0.35	...	2.2	10.0	7.5	2.9	16.0
0.17	...	...	18.9	10.5	1.9	27.8
0.07	...	...	6.1	8.9	...	26.1
−0.07	...	...	2.3	2.8	...	6.3

Mill	Anaconda Copper Mining Co.					
Character of feed	2.5-mm.-screen undersize			Primary 4-mm. de-slimed		
Size, diameter of cone, ft.	2*d*			2*d*		
Type of constriction plate	*c*			*c*		
Ratio of area of teeter to sorting columns	2.8			2.8		
Diameter of spigot, in.	1¼			1¼		
Hydraulic water, gallons per ton of feed	1050			445		
Rising velocity in constriction, mm. per second	373			136		
Product	*F*	*S*	*O*	*F*	*S*	*O*
Tons per 24 hr.	297	142	155	322	210	112
Moisture, per cent.	74.3	80.7	91.1	77.1	72.1	90.9
Assay, per cent. Cu	3.89	4.58	3.15	3.30	3.42	3.24
Screen analysis	Weight, per cent.					
+4.00 mm.	...	...	...	1.9	2.7	...
2.38	...	...	...	22.0	28.0	...
2.00	7.2	13.9	...	...	...	...
1.68	11.8	23.6	...	...	...	...
1.41	7.2	16.5	...	23.8	30.2	...
1.00	12.9	20.3	...	...	...	...
0.84	9.6	13.6	7.6	18.9	22.8	7.3
0.71	8.8	6.9	5.6	...	...	...
0.50	8.3	3.8	8.7	12.6	7.8	16.4
0.35	10.6	1.4	16.6	4.5	4.2	8.9
0.17	12.0	...	29.9	9.7	3.7	33.0
0.07	9.0	...	26.7	5.6	0.6	25.8
−0.07	2.6	...	4.9	1.0	...	8.6

F = Feed. *S* = Spigot. *O* = Overflow.
c 12 @ 2×2-in. nipples in plate = 40 sq. in. *d* Fitted with central feed cone.

Table 18. Performances of Anaconda classifier—*Continued*

Mill	American Zinc, Lead & Smelting Co.					
Character of feed	Tube-mill discharge, spigot returned to mill			Overflow of preceding		
Size, diameter of cone, ft.	3.33			5		
Type of constriction plate	*g*			*g*		
Ratio of area of teeter to sorting columns	4.7			4.7		
Diameter of spigot, in.	1¼			1¼		
Hydraulic water, gallons per ton of feed	264			600		
Rising velocity in constriction, mm. per second	......			......		
Product	*F*	*S*	*O*	*F*	*S*	*O*
Tons per 24 hr.	1091	548	543	484	233	251
Moisture, per cent.	61.7	52.4	78.0	78.0	72.6	89.1
Screen aperture, mm.	Weight, per cent.					
3.327	......	......	......	......	......	......
1.651	1.4	3.5	......	......	......	......
0.833	9.5	26.2	1.3	1.3	8.7	......
0.417	23.0	42.9	10.3	10.3	32.9	1.3
0.208	24.7	20.0	27.0	27.0	37.4	11.2
0.104	20.3	5.5	29.0	29.0	15.9	29.7
0.074	9.2	1.4	14.5	14.5	3.5	21.8
−0.074	11.9	0.5	17.9	17.9	1.6	36.0

F = Feed. *S* = Spigot. *O* = Overflow. *g* 16 @ 1¼-in. diameter × 2-in. nipples.

classifiers. Experiment at ANACONDA showed that this should not exceed 250,000 gal. per 24 hr. with a 7-ft. cone, if it was desired that the overflow should all pass 200-mesh. From 200 to 250 tons solid could be handled in this volume of pulp. The rising velocity necessary in the sorting column is dependent upon the size of the sands; it must be sufficient to maintain teeter. Table 17 shows the results of 3 out of 11 tests on a 28-in. machine fitted with a feed cone. The tests show that the performance of the machine is not greatly affected by large changes in feed tonnage. Had the amount of hydraulic water not been reduced in test No. 2 coincident with the increase in +1.41-mm. material, it is probable that the rise in proportion of spigot product and fall in proportion of total free mineral therein would not have occurred. The work of 40-in. classifiers at the AMERICAN ZINC, LEAD AND SMELTING Co. mill when working in closed circuit with tube mills, compared favorably with that of a Dorr classifier in the same service, the amount of water in the overflow was not excessive and the tube-mill feed could be thickened by the addition of drier material with the spigot product.* See Table 18.

Delano classifier (Fig. 22) is used in de-sliming service. Constriction of the sorting column is effected by the conical wooden plug at the apex of the cone. Capacity of a machine 5 ft. 6 in. diameter treating Missouri lead ores is 350 to 400 tons solid per 24 hr.; water consumption 120 gal. per min. (*57 A 430*). Spigot opening, 0.75 to 1 in.; moisture in spigot product, 50 per cent. Sizing tests of feed and products when thus working are given in Table 19.

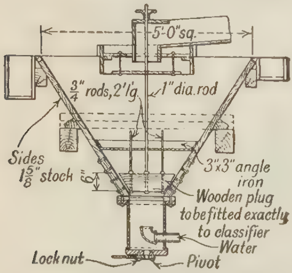


FIG. 22.—Delano classifier.

Table 19. Performance of Delano classifier, St. Joseph Lead Co.

Screen aperture, mm.	Weight, per cent.		
	Feed	Spigot	Overflow
+0.147	64.7	86.8	0.7
0.104	7.4	8.2	5.2
0.074	4.1	2.3	9.3
−0.074	23.8	2.7	84.8

Dalton classifier (Fig. 23) is used at the Morning mill of the Federal Mining and Smelting Co. The machine shown treats 60 tons solid per 24 hr. with a water consumption of 920 gal. per ton. Performance is shown in Table 20.

Table 20. Performance of Dalton classifier at Federal Mining & Smelting Co.

	Feed	Spigot number				Overflow
		1	2	3	4	
Diameter of spigot opening, in..	....	⅞	⅝	⅝	½	....
Moisture, per cent...........	30–60	60	67	72	76	93
Screen aperture, mesh	Weight, per cent.					
+ 40	7.1	11.1	5.1	0.0	0.0	....
100	43.5	57.2	54.0	12.7	4.4	....
160	19.2	12.0	14.6	18.1	10.6	....
−160	30.2	19.7	26.3	69.2	85.0	100

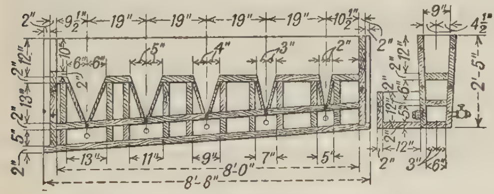


Fig. 23.—Dalton classifier.

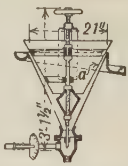


Fig. 24.—Double-cone classifier.

Double-cone classifier (Fig. 24) utilizes an adjustable inner cone (a) to increase and equalize the velocity of the rising current above the sorting column and thus enable overflow of sand without undue water consumption.

At the Midvale plant of the U. S. Smelting, Refining and Mining Co., a 20-in. machine handles 275 tons per 24 hr. of −1.3-mm. feed, discharging sand through a ¾-in. spigot with 70 per cent. moisture; a 30-in. machine treats 240 tons of −0.9-mm. feed; spigot, 1⅟₁₆-in.; moisture in sand discharge, 75 per cent.; a 40-in. machine is fed 200 tons of −0.5-mm. material and the sand discharge through a 9⁄16-in. spigot carries 83 per cent. water.

Pipe classifiers are pipe columns, usually vertical, fed through a branch fitting (Y or tee) let into the column. They make but two products, *viz.*: an overflow and one spigot product. A hindered-settling form was used at Bunker Hill and Sullivan (*2 MM 443*) for treating jig-hutch product. Sizing-assay test of feed is given in Table 21. Feed of this character assaying 55 per cent. Pb was divided by the classifier into a spigot product assaying 77 per cent. Pb and over-

Table 21. Sizing-assay test of feed to pipe classifier at Bunker Hill & Sullivan Mg. Co.

Screen aperture, mesh	Weight, per cent.	Pb, per cent.
+20	3.2	82.1
40	16.8	67.7
60	14.2	50.3
80	19.2	42.7
100	12.6	39.6
150	13.4	42.7
200	7.6	49.4
−200	13.0	58.8

flow containing 49 per cent. Feed from the second hutch of the jig, containing 25 per cent. Pb made a spigot product with 60 per cent. Pb and an overflow carrying 12 per cent.

4. Design of hydraulic classifiers

There are several cases to consider, *viz.:* (1) free-settling shallow-pocket launder type; (2) deep-pocket types; (3) tank types; (4) hindered-settling columns for any of the above. The first case involves design of the free-settling sorting column and the launder and consideration of the diameter of the spigot aperture; the second, that of a roughing pocket above the sorting column, whether the latter be free- or hindered-settling; the third, the design of the tank for removing slimes; the fourth, the relative areas of sorting column and teeter chamber.

Design of free-settling sorting column. The data required are: (1) tonnage of solid matter to be passed; (2) average size of all of the light-mineral grains that are to settle; (3) average diameter of the smallest light-mineral grains that are to settle. From Table 1 or from the proper formula (see Art. 1) determine the velocity of rising current necessary to raise all light-mineral grains smaller than (3) above and also the free-settling velocity of the average light-mineral grain. The difference between these two figures will give the average settling rate of the light-mineral particles. The volume of the tonnage to be passed per unit of time, reckoning all solid as having the specific gravity of the light-mineral, divided by the resultant falling velocity of the average grain, gives the average solid cross-section of the falling stream of solids. The area of the sorting column should be made about 50 per cent. more than this to allow for voids in the mass of settling grains. This allowance, plus the additional allowance that is inherent in the calculation because of the less volume and higher falling velocities of the heavy-mineral grains is sufficient.

Launder slope and size must be sufficient to cause the material to travel between sorting columns with the water available. General data on launder slopes are given in Sec. 20. Normally a slope between ½ and 1 in. per ft. will be sufficient.

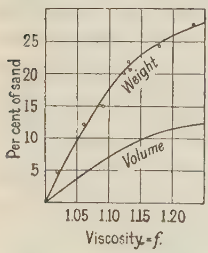


FIG. 25.—Relation between pulp consistency and viscosity (*after Richards and Dudley.*)

Spigot diameter, effective head on spigot, size of particles to be discharged and moisture content of spigot product are interrelated.

Flow of sandy pulps through pipe-and-plug spigots is different from that of water by reason of friction of the solid particles against the walls and inner friction of the mass. Richards and Dudley (*51 A 398*) call the sum of these effects the viscosity of the pulp and give the curves shown in Fig. 25 for determination of this factor, f. These curves are the result of experimental work with a silicious sand (sp. gr. = 2.72) ranging in size from about 0.1- to 1.4-mm. The experimenters used this value in the following formula derived from the fundamental hydraulic-flow equation:

$$Q = Ac\sqrt{2gh}/f, \qquad (12)$$

where Q = volume discharged, A = area of spigot opening, c is a coefficient of discharge depending on the shape of the orifice, ranging from 0.85 to 0.95 for ordinary pipe-and-plug spigots, g is the acceleration due to gravity and h

the head on the orifice. They give the following example of the use of the formula:

Example: Required the size of opening to discharge 40 tons of sand (0.1- to 1.4-mm. sp. gr. = 2.81) per 24 hr. as a pulp containing 25 per cent. solids through a pipe-and-plug spigot with conical receiving end under a head of 3 ft. The percentage of sand by volume is the governing factor in this case and is 10.6. From Fig. 25, f for this pulp is 1.17. Using c.g.s. units and substituting known quantities in equation (12)

$$A = \frac{1.17 \times 1430}{0.88\sqrt{2 \times 980 \times 914}} = 1.42 \text{ sq. cm.}$$

and the diameter of the spigot = 1.35 cm. = 0.53 in. For coarser sands, higher values of f must be used and *vice versa*. With sand of the size and sp. gr. experimented with the maximum volume percentage that could be continuously discharged without clogging was 13.

The spigot aperture should be as large as is consistent with the head and the volume of material to be passed. The limiting lower diameter is theoretically 3- to 5-times the diameter of the largest particle passing (see Sec. 20) but this factor rarely enters on account of the fact that tonnage requirements demand a larger opening.

Design of roughing pocket is the same for both free-and hindered-settling classifiers. The fundamental principle is that the rising velocity across the area $x\,y$ (Fig. 26) should be the same as that in the free-settling column below. The volume moving past this section is that of the incoming pulp plus the added rising water less that of all previously settled solid. It is wise to keep the section $x-y$ as nearly square as possible in order to reduce eddying. The slope of the sides of the pocket should be between 60 and 75° from the horizontal, the steeper the better, in order to prevent sands from banking. If banking occurs the work of the column will be disarranged every time the bank caves and this will be a frequent occurrence unless the feed is extraordinarily regular. Steepness of slope must be balanced against resulting loss of head room.

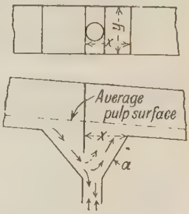


FIG. 26.—Roughing pocket.

Design of classifier tank is the same for both free- and hindered-settling machines. The tank is fundamentally a deep pocket for roughing out slimes and is, therefore, calculated to have a surface area such that the velocity of the rising current across the horizontal section at the level of the overflow lip will lift the coarsest particle that it is desired to overflow. The volume of rising current is the feed-pulp volume plus the total volume of hydraulic water rising in all sorting columns less the volumes of solid passing out of the spigots. The slope of the sides and discharge end should be at least 60° from the horizontal and better 70 to 75° to prevent banking. The feed end may slope as little as 45°, dependence being placed on the plunging effect of the incoming pulp to prevent banking here. The bottom slope should be 1½ to 3 in. per ft.; the greater slope gives greater capacity but tends to throw oversize into the later spigots. The sides of the tank necessarily flare toward the overflow end. Their slope is kept constant throughout the length. A safety spigot without rising current should be provided beyond the last hydraulic sorting column in order to discharge sand that is carried past this column but cannot be overflowed. This spigot may discharge continuously or intermittently according to the performance of the classifier. The product will be predominantly fine sand, but will contain considerable slime.

Design of hindered-settling column. The area of the constriction is determined in the same way as the area of a free-settling sorting column. The constriction may be a hole in a plate or at the bottom of a conical tube, if no rising current is to be employed, but with rising current a plurality of small holes is better than a single large one and experiments at ANACONDA (*46 A 266*) indicated that tubes a couple of inches in length tapped into a plate were better than a perforated plate or screen alone. The ratio of the area of the teeter chamber to the area of the constriction is important. Experiments at ANACONDA (*loc. cit.*) showed that for each size of grain there is a certain maximum density that can be maintained in the teeter chamber and yet permit discharge of solid through the constriction. If the ratio of the area of the teeter chamber to that of the constriction is too great, no sand can fall through the constriction when full teeter exists, and if the current is reduced to allow sand to fall, banking occurs in the teeter chamber and capacity is reduced; if the ratio is too small, teeter cannot be maintained but free settling will take place. The ANACONDA experiments indicated that the PERMISSIBLE DENSITY, meaning that co-existing at full teeter with free discharge through the constriction, varies inversely as the square root of the surface of a given weight of grains. The ratios of area of teeter chamber to area of constriction necessary to give the permissible density for different grain sizes may be read from Fig. 27.

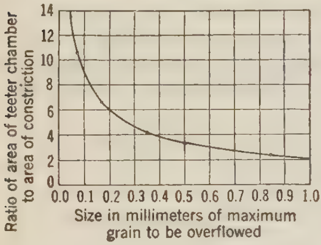


FIG. 27.—Ratio of areas of teeter chamber and constriction in a hindered-settling column.

Design of a hindered-settling tank classifier involves all of the elements of design of other types.

Example: To design such a machine with 4 spigots, to treat 200 tons per 24 hr. of −2.5-mm. feed in a pulp containing 25 per cent. solids, overflow −0.07-mm. material, and divide the sands into 4 products of such nature that the size range of the light-mineral particles will be the same in each spigot. Suppose that screen analysis of the material shows the following size-weight relations; −0.07-mm., 8 tons per 24 hr.; 0.07 to 0.68-mm., 72 tons; 0.68 to 1.29-mm., 56 tons; 1.29 to 1.90-mm., 32 tons; 1.90 to 2.5-mm., 32 tons.

To determine the constriction area for the first sorting column: the volume of solid to be passed per second, taking the sp. gr. of the light mineral at 2.6, = 4040 × 32 = 129,000 cu. mm. The mean diameter of the solids that are to settle = (1.90 + 2.50)/2 = 2.2 mm. The settling velocity of the mean particle = 178 mm. per sec. (From Table 1.) The settling velocity of the smallest particle = rising current to be supplied = 163 mm. per sec. The net falling velocity of the solid = 178 − 163 = 15 mm. per sec. The cross-section of the falling solid, considered as a solid bar, = 129,000/15 = 8600 sq. mm. The area of the constriction should be 1.5-times this area = 1.5 × 8600 = 12,900 sq. mm. In similar fashion the areas of the other constricted sections are found to be: column 2, 6680 sq. mm.; column 3, 14,100 sq. mm.; column 4, 11,030 sq. mm. If single pipes were to be used for the sorting column, 5-, 4-, 6- and 5-in. pipes respectively could be chosen. Most designers would furnish 5-in. pipes for the first two columns and 6-in. for the last two on account of greater simplicity in construction. The areas of teeter columns may be obtained by multiplying the constriction areas by the following factors taken from Fig. 27: No. 1, 1.5 (extrapolated); No. 2, 1.9; No. 3, 2.8; No. 4, 10.5; making the areas, on the basis of 5- and 6-in. sorting columns as above, 30, 38, 81 and 304 sq. in. respectively. The corresponding nearest standard pipe diameters are 6 in. 7 in., 10 in., and 20 in. The length of sorting column necessary increases with the size of material that is to be overflowed. Bardwell (*46 A 266*) states that 4 in. is sufficient for a de-slimer column and 8 in. for most cases. The area of the tank at the overflow level is computed to give a rising current of 4.1 mm. per sec. with the feed water plus the rising hydraulic water overflowing. The feed water is 600 tons per 24 hr. The volume of hydraulic water rising cannot be calculated accurately but may

be estimated with sufficient precision by assuming that 50 per cent. of the area of the constriction is unoccupied by solid. Under this assumption the available area in 5-in. standard pipe is 6450 sq. mm. and the volume rising, at 163 mm. per sec., is 1,050,000 cu. mm. per sec. The volumes from the other columns are 755,000, 642,000 and 38,200. The total overflow is 8,805,200 cu. mm. per sec. The required area = 8,805,200/4.1 = 2,140,000 sq. mm. = 23.2 sq. ft. A tank averaging about 2 to 2.5 ft. wide by 9 to 12 ft. in length is indicated. The depth of the tank and the size of spigot opening are interdependent. The first spigot must discharge 32 tons of solid per 24 hr. with, say, 96 tons of water. The determination of proper depth (head on discharge) and diameter of spigot is made by trial and error. Assume 4 ft. head. Then, from equation (12) (p. 580) the spigot diameter must be 1⅜ in. If a smaller spigot opening is desired the tank must be made larger and *vice versa*. The slope of the tank bottom should be at least 1½ in. per ft. and this slope, with the depth above the first spigot fixed, fixes the depth above the other spigots. From this depth the diameters of the other spigots may be computed.

5. De-sliming classifiers (non-mechanical)

Whole-current classifier is shown in Fig. 28. Pulp enters behind the perforated baffle (A) and is distributed in a series of streams, as indicated, across the transverse section of the classifier. In order that a particle may settle out of the stream it is necessary that it settle to the bottom before it is carried through the perforated baffle (B) at the discharge end. On the discharge side of baffle (B) the velocity of the rising current is $v = Q/xb$ and as x is less than l, all particles that failed to settle before reaching (B) will be readily lifted. If Q = the total volume of entering pulp; l, b and d = length, breadth and depth of the classifier respectively and H and V the horizontal and vertical resultant velocities, respectively, of a solid particle whose diameter is D and density δ, then d/V must be less than l/H for a particle to settle.

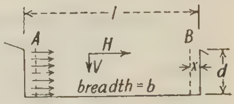


FIG. 28.—Whole-current classifier.

$$H = Q/bd, \qquad \qquad (13)$$

$$V = v_1 - v_2$$

where v_1 = the falling velocity of the particle in still water $= C\sqrt{D(\delta - 1)}$, and $v_2 = Q/bl$.

$$V = C\sqrt{D(\delta - 1)} - Q/bl. \qquad \qquad (14)$$

The limiting condition is when

$$\frac{d}{V} = \frac{l}{H} \qquad \qquad (15)$$

or

$$\frac{d}{C\sqrt{D(\delta - 1)} - Q/bl} = \frac{lbd}{Q} \qquad \qquad (16)$$

from which

$$D = \frac{4}{(\delta - 1)}\left(\frac{Q}{Clb}\right)^2 \qquad \qquad (17)$$

and

$$lb = \frac{2Q}{C\sqrt{D(\delta - 1)}} = \frac{2Q}{v_1}. \qquad \qquad (18)$$

It will be noted from equation (17) that the size of particle that can be separated in a given whole-current classifier is independent of the depth and of the relative proportions between length and breadth.

For particles below the "critical size" (Art. 1)

$$v_1 = KD^2(\delta - 1) \qquad \text{By equation (2).}$$

Then

$$V = KD^2(\delta - 1) - \frac{Q}{lb} \quad . \ . \ . \ . \ . \ . \ . \ . \ . \quad (19)$$

$$D = \sqrt{\frac{2Q}{blK(\delta - 1)}} \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \quad (20)$$

and

$$bl = \frac{2Q}{KD^2(\delta - 1)} = \frac{2Q}{v_1}. \quad . \ . \ . \ . \ . \ . \ . \ . \quad (21)$$

Whole-current classifiers are rarely used for the reason that they require more floor space for a given duty than do surface-current machines, as will appear below.

Sloughing-off boxes or V-boxes are the commonest form of whole-current classifier at the present time. They consist of a V-shaped box, with sides sloping 50 to 60° from the horizontal, 10 to 50 ft. long and 4 to 8 ft. deep. They are usually without partitions, but are discharged by means of spigots or goose-necks at several points along the length. Their principal use is as dewaterers and de-slimers where clear overflow or clean sand is not essential. Table 22 (*1 OD 463*) gives sizing tests of the combined spigot product and overflow of a 4-spigot tank 4 ft. deep by 17 ft. long.

Table 22. Performance of a 4-spigot V-box

Screen aperture, mm.	Weight, per cent.	
	Combined spigot products	Overflow
+0.37	1.2	....
0.27	2.1	....
0.16	12.9	....
0.12	7.9	....
0.07	13.7	....
−0.07	62.2	100.0

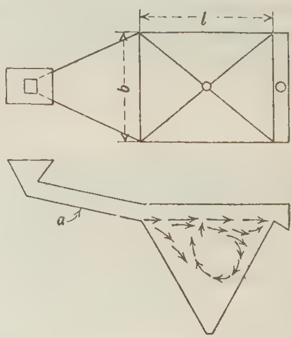


FIG. 29.—Surface-current classifier.

Surface-current classifier is shown diagrammatically in Fig. 29. Pulp enters over the feed sole (*a*) at the same level as the overflow. There is a distinct surface current toward the overflow lip, as indicated, but there are also many eddies of the general character indicated. Solid particles that fall out of the surface stream must settle against the rising current, which, while it varies from point to point over any horizontal section and may even be negative (*i.e.*, downward) at some points, nevertheless has an average upward velocity

$$v = Q/lb. \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \quad (22)$$

If v_1 is the settling velocity of a given particle, v_1 must be greater than v in order for the particle to appear in the spigot product.

For particles larger than the critical size (Art. 1) $v_1 = C\sqrt{D(\delta - 1)}$ (Art. 1, Eq. 4). In the limiting case $v = v_1$, hence

$$C\sqrt{D(\delta - 1)} = Q/bl \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

from which

$$D = \frac{1}{\delta - 1}\left(\frac{Q}{Cbl}\right)^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

and

$$bl = \frac{Q}{C\sqrt{D(\delta - 1)}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

For particles below the critical size:

$$D = \sqrt{\frac{Q}{blK(\delta - 1)}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)$$

and

$$bl = \frac{Q}{KD^2(\delta - 1)}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

Comparing equations (24) and (26) with equations (17) and (20) it will be seen that finer particles can be settled in a surface-current classifier of given horizontal cross-section than in a whole-current machine, and conversely, by comparing equations (25) and (27) with (18) and (21), to settle particles of a given size the surface-current classifier requires but half the surface area of the whole-current apparatus.

Rittinger spitzkasten is the typical form of surface-current classifier without hydraulic water. It consists of a series of pointed boxes, similar to Fig. 29, increasing in depth and area toward the discharge end. *Rittinger* recommended that the first box be made about one inch wide for each cubic foot of pulp fed per minute and that the width of each succeeding box be twice that of the preceding. For 20 cu. ft. of pulp per min. he recommended lengths of 6, 9, 12 and 15 ft. respectively for 4 boxes. The sides should slope at least 50° and better 60° from the horizontal. Slopes of the connecting launders should be: about ¼ in. per ft. for the feed launder, ⅛ in. per ft. from the first to the second box, $\frac{1}{16}$ in. per ft. from the second to the third, and $\frac{1}{32}$ in. per ft. from the third to the fourth. Richards' experiments (*27 A 249*) indicate that the feed sole should slope about 5° from the horizontal and should enter on the same level as the overflow. The overflow should be level. Eddying will invariably cause much slime to enter the spigot products. Discharge of coarse material is made through pipe-and-plug spigots at the apex of the box; fine material is discharged through goose-necks. In some cases successive boxes are joined so that the upper edge of the division wall is below the overflow level as in Fig. 17 and no connecting launders are needed.

Table 23. Performance of 2-box spitzkasten without hydraulic water.(*a*) (*After Richards*)

Screen aperture, mm.	Weight, per cent.		
	Spigot 1	Spigot 2	Overflow
+0.94	0.3	....	....
0.67	0.5	....	....
0.49	1.5	....	....
0.37	1.0	....	....
0.27	4.8	....	....
0.16	22.0	0.4	....
0.12	13.2	1.3	0.4
0.07	22.5	10.9	1.6
−0.07	34.2	87.4	98.0

a Baffle plate in second box to direct current downward.

Performance of a 2-box machine treating lead ore, the overflow from a hydraulic classifier, is given in Table 23 (*1 OD 462*).

Spitzkasten with hydraulic water are used in some Joplin zinc mills to prepare feed for table concentration. Table 24 (*57 A 453*) gives results of the operation of a 9-box classifier of this type.

Table 24. Performance of 9-compartment hydraulic spitzkasten on Joplin zinc ore

	Feed	Spigot numbers									Overflow
		1	2	3	4	5	6	7	8	9	
Size of box at top, in. square	......	16	21	22	28	30	35	41	48	45	......
Depth of box, in.	......	24	26	30	36	46	46	54	54	54	......
Spigot diameter, in.	......	1	⅞	⅞	⅞	¾	½	⅜	⅜	⅜	......
Tons per hour	....	0.73	0.80	0.67	0.37	0.47	0.24	0.14	0.22	0.08	......
Screen aperture, mm.	Weight, per cent.										
+0.833	0.4	0.8	0.7	...	...		...	....	....	....	......
0.417	15.0	25.8	21.5	16.1	9.3	3.2	0.7	0.2	0.1	....	3.8
0.208	32.1	38.5	43.0	40.3	35.4	25.6	11.2	5.4	1.3	1.0	0.1
0.147	19.0	15.8	15.7	18.2	21.5	27.0	26.2	16.4	7.3	4.8	4.5
0.104	18.7	10.8	11.3	14.3	18.2	24.6	34.2	33.8	24.8	27.0	4.4
0.074	6.5	3.6	3.6	5.0	7.6	10.0	13.3	16.6	21.9	20.3	3.0
−0.074	8.3	4.7	4.2	6.1	8.0	9.6	14.4	27.6	44.6	46.9	84.2

Callow tank (Fig. 30) has the form of an inverted cone with 60° apex angle. It is used either for de-sliming or dewatering. The principle is the same as that of all rising-current free-settling classifiers. Success has been due to simplicity and excellence of design. The overflow rim is a cylindrical steel band, cleated at intervals to the cone wall near the top, the joint made tight by first packing with oakum then pouring in a thin cement grout. An endless rubber belt stretched tightly around the metal band is readily leveled, after the tank is full, by tapping with a wooden mallet. Spigot discharge is effected through a bushing or plug cock at the apex or by a goose-neck siphon. Feed is introduced at the center through a pipe 5 to 12 in. diameter projecting 6 to 10 in. below the overflow level. The rating is based on the diameter of the overflow band. Weight varies from 165 lb. for the 2-ft. size to 650 lb. for the 8-ft.; rated capacity of the 2-ft. cone in de-sliming service is 24 tons per 24 hr.; 4-ft., 100 tons; 8-ft., 400 tons: in slime dewatering, 4-ft., 6 to 8 tons; 6-ft., 14 to 18 tons; 8-ft., 25 to 30 tons.

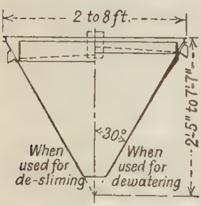


Fig. 30.—Callow tank.

Table 25. Screen tests of feed and products of 5-ft. Callow cones at Tonopah-Belmont Development Co.

Aperture, mm.	Weight, per cent.		
	Feed	Spigot	Overflow
0.147	1.0	2.0	....
0.104	12.5	15.0	....
0.074	14.2	11.0	....
−0.074	72.3	72.0	100.0

Performance. At Tonapah Belmont Development Co. 5-ft. Callow cones handle 70 tons solid per 24 hr. in a pulp containing 85 per cent. moisture. The spigot product contains 79 per cent. moisture and the overflow 87 per cent. Screen tests of feed and products are given in Table 25. At United Eastern (*63 A 554*) Dorr-classifier overflow

was sent tô 8-ft. Callow cones. The feed to each cone ranged from 16 tons solid and 1100 tons cyanide solution to 213 tons solid and 970 tons solution (87.3 and 82 per cent. solids respectively) per 24 hr. Spigot discharge was through a 1.5-in. goose-neck fitting 30 in. below the cone overflow. The spigot product at the maximum solid-feed rate given was 73 tons solid and 83 tons solution (54 per cent. moisture); overflow, 140 tons solid, 884 tons solution (86.3 per cent. moisture). Sizing tests of products are given in Table 26. Performance in dewatering service is given in Sec. 16, Art. 5.

Table 26. Sizing test of products of 8-ft. Callow cone, United Eastern

Aperture, mm.	Weight, per cent.	
	Spigot	Overflow
0.208	1.5	....
0.147	33.5	....
0.104	32.0	2.5
0.074	21.0	11.5
−0.074	12.0	86.0

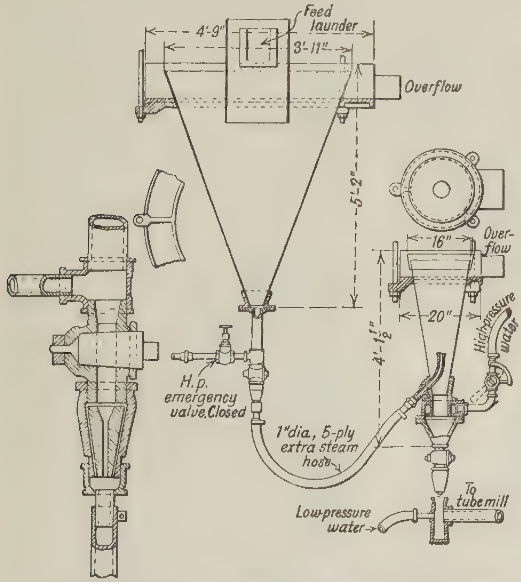


FIG. 31.—Series classification in cones at Homestake.

Homestake method of series classification in cones (*22 IMM 90*) is shown in Fig. 31. Feed enters the larger cone, which is run with a small discharge bushing so as to overflow about 88 per cent. of the total solids. The small cone re-works the spigot product of the first. Sizing-assay tests of products are given in Table 27.

Hindered-discharge cones

The difficulty in maintaining a continuous discharge of thick sandy pulps from settling tanks has led to several expedients. The first and most obvious

Table 27. Performance of series cones at Homestake

Aperture, mesh	Overflow from large cone		Small cone			
			Overflow		Sand	
	Weight, per cent.	Gold, dollars per ton	Weight, per cent.	Gold, dollars per ton	Weight, per cent.	Gold, dollars per ton
Total	8.8	....	4.5	....	7.5	....
On 50	....	....	....	....	47.0	2.00
80	5.5	1.09	12.5	0.53	34.0	2.97
100	3.5	1.45	11.0	0.88	10.0	4.66
200	22.0	1.36	28.0	1.36	6.0	5.40
−200	69.0	1.36	48.5	1.22	3.0	4.43

was intermittent manual discharge of the spigot product, allowing time between discharges for the coarser material to build up to a considerable depth and squeeze out water and suspended slime. This method was succeeded by a mechanically-operated intermittent slush gate. The Caldecott diaphragm in a steep-sided cone was the first successful device for continuous discharge of thickened sand. It was followed by the Allen cone, and Boylan and Wood classifiers, all of which are conical or pyramidal tanks with intermittent discharge automatically controlled.

Caldecott cone (Fig. 32) consists of a sheet-iron conical tank, with apex angle not more than 60° for coarse feed and about 40° for fine, having a disk-diaphragm near the apex, so supported as to leave an annular space between its edge and the cone wall. The cone is center-fed through an inlet pipe 6 to 8 in. diameter, carrying a disk-shaped baffle a few inches below the lower end. The area of exit between pipe and baffle should exceed the area of the pipe. The feed pipe should project 12 to 18 in. below the overflow level. The lower end of the cone wears most rapidly and is usually a hard-iron casting with removable bushings. It is fitted with an easily replaced sliding gate by means of which the flow can be regulated. The velocity of the rising current is predetermined within a certain range by means of the cylindrical sheet-iron ring surrounding the feed pipe. This fixes the maximum cross-section of the rising current. In operation the cone is allowed to fill with sand to a height about 2 ft. below the overflow lip at the center and extending nearly or quite to the overflow level at the periphery. The usual dimensions of the cone are 6 ft. diameter by 9 ft. deep and 8 ft. diameter by 10 ft. deep. The size and placing of the diaphragm are matters of experiment. With sandy feed the diaphragm is placed 12 to 18 in. above the spigot opening and the width of the annular opening is 0.75 to 2 in., the larger figure for finer material. For finest sand and slime the annular opening must be larger, in order to prevent bridging of thickened material. It is made 3 to 6 in. across, and the diaphragm must, therefore, be carried somewhat higher in the cone. Fig. 32 shows a double diaphragm for treating fine material.

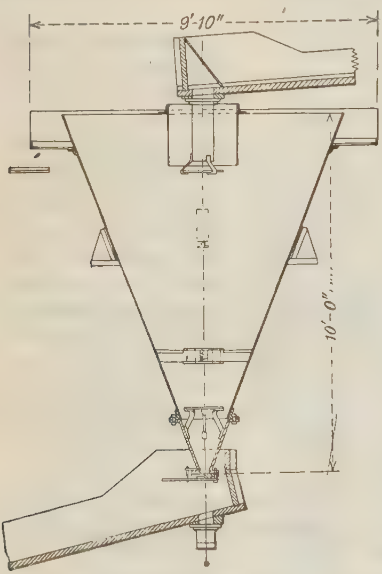


Fig. 32.—Caldecott cone with double diaphragm.

The purposes of the diaphragm are to slow down the flow of sand toward and through the spigot, and to prevent center-piping and bridging. Bridging cannot occur at the spigot because of the direction from which the solids approach, *i.e.*, obliquely convergent down the walls of the cone from the periphery of the diaphragm. Center-piping is prevented by the intervention of the diaphragm across the axis of the possible pipe. This eliminates rapid

settling and discharge of coarse material, which is the cause of piping in ordinary cones. Discharge through the spigot is slowed down by decrease in pressure effected by friction losses in the annular space surrounding the diaphragm, and the rate of approach to the spigot is limited at the same time by the same phenomenon.

The action in a diaphragm cone is substantially free settling with both horizontal and vertical currents. The construction of the feed pipe is such that the pulp stream starts flowing horizontally in all directions. Heavy, coarse sand is dropped near the center and finer sand near the periphery. With all conditions constant the surface of the settled solid stabilizes in such a position that the rising velocity in the annular section at the overflow level is just sufficient to lift the coarsest overflow particle, and settled solid is withdrawn through the spigot at a rate just equal to that of deposition.

Sizes and moisture content of the products relative to size and moisture content of the feed are affected by change in rate and character of feed and in size of spigot opening. If the size of particles and tonnage of solid matter in the feed remain constant, change in the amount of water sent to the classifier will not permanently affect either the size of products or the moisture content of the spigot discharge. Increase in quantity of feed water will cause instant increase in rising velocity of the overflow pulp with consequent scour of the sands, until the velocity over the increased cross-section is the same as it was before the change, when deposition and spigot discharge will again proceed at equal rates. Decrease in quantity of feed water will cause deposition and constriction of the overflow stream until deposition and spigot discharge again equalize. Decrease in size of feed particles without change in pulp volume will result in lessened deposition and, if discharge continues at the same rate, lowering of the sand surface and increase in cross-section of overflow stream. This will result in lessened velocity of the overflow stream and increased deposition until deposition and discharge rates equalize. At this time, since finer material is depositing, finer will also overflow. Similar reasoning shows that coarser feed will produce coarser overflow and coarser spigot product. The moisture content of the spigot discharge is dependent largely upon the percentage of voids in the settled solids, when packed as tightly as they will pack under the conditions of settlement. Coarse material has less voids than fine, hence less moisture; material containing particles of considerable range of sizes has less voids than that closely sized and hence less moisture. Concurrent change of moisture content and tonnage and/or size of feed particles may completely change the character of the discharge through a given spigot. Thus concurrent decrease in moisture content and solid tonnage will lessen overflow velocity and tend to cause deposition of finer material. At the same time the surface of the settled solids will lower by reason of excess of discharge over deposition, overflow velocity will be further reduced and yet finer material deposited. Assuming ultimate equalization of deposition and discharge, stabilization will occur with a large increase in the amount of fines depositing, and there will be corresponding increase in the fineness of both spigot and overflow products. The result, as concerns moisture content of the spigot product, cannot be predicted accurately; the greater size range will tend toward less moisture and the finer size toward more. It should be noted also, in this analysis, that if the reduction in moisture content is sufficiently great and there is enough fine solid present to produce a suspension that acts like a liquid of high specific gravity, the carrying power of the overflow may be sufficient to lift coarse material, and if the spigot is changed to prevent lowering

of the sand level, the size of the products and the moisture content of the spigot discharge may be maintained unchanged. When the water content of the feed decreases and solid tonnage increases, the respective effects are conflicting; the volume passing may increase or decrease, the carrying power of the current may increase by reason of an increase in specific gravity, and the sand level may either raise or lower, if the spigot size is maintained. The probable result is coarser overflow and an increased tonnage of coarser sand with slight decrease in moisture content. Concurrent decrease in moisture content and size of feed will usually result in marked decrease in size of spigot product and overflow, although, owing to increased apparent density of the overflow pulp, it may be possible to keep the overflow size up by decreasing the spigot opening. All other things remaining constant, increase in spigot opening will give finer products and *vice versa*. Since the spigot opening is ordinarily the only variable in control of the operator he thereby compensates for changes in quality and quantity of feed.

Performance. The Caldecott cone has had its greatest development in Rand gold mills, where it has been used to guard tube-mill discharge and return sands for re-grinding. In this service an 8-ft. cone will discharge 400 to 600 tons of +90-mesh (0.006-in.) quartzitic sand per 24 hr. in a pulp containing about 30 per cent. moisture when the feed contains about 40 to 50 per cent. of −90-mesh slime.

Results with an 8-ft. double-diaphragm cone at SIMMER AND JACK are given in Table 28. At a Mexican mill (*10 JCM 287*) a cone 4 ft. 10. in. deep by 4 ft. diam. fitted with a diaphragm 8 in. from the apex, leaving 1.5-in. annular space, was fed at 240 tons per 24 hr. Results are given in Table 29. At the ST. JOSEPH LEAD CO. Bonne Terre mill (*57 A 432*) a 6-ft. Caldecott cone handled 400 to 500 tons of −2-mm. sand tailing per 24 hr., made an overflow containing no material coarser than 0.1-mm., while the spigot product averaged 28 per cent. moisture. The sand level was held about 2 ft. below the overflow. A sizing test of the spigot product is given in Table 30. Mixed jig and table galena concentrate was discharged from a 6-ft. cone with 11 per cent. moisture, but intermittent discharge was necessary on account of the variable feed rate.

Table 28. Performance of 8-ft. double-diaphragm Caldecott cone at Simmer and Jack. (*11 JCM 323*)

Aperture, inch	Weight, per cent.		
	Sand	Overflow	Feed
Total	58	42	100.0
+0.01	11.8	....	6.8
0.006	31.4	....	18.2
0.003	43.1	2.0	25.8
−0.003	13.7	98.0	49.2

Table 29. Performance of 4-ft. Caldecott cone

Aperture, mesh	Weight, per cent.		
	Feed	Sand	Overflow
30	18.5	34.5	0.5
40	10.0	16.5	1.0
80	21.5	33.5	8.0
100	1.0	1.5	1.5
150	12.0	8.0	11.5
200	3.5	2.0	9.0
−200	33.5	3.5	68.5
Moisture, per cent.	90	34	93

Automatic diaphragm cone used at Mt. Lyell (*38 Aa 109*) is shown in Fig. 33. With no sand in the cone, the counterweight closes the spigot valve. When settled sand rises to the cylinder on the plug rod the drag of the sand pulls the cylinder down and opens the valve. Performance is shown in Table 31.

Table 30. Sizing test of spigot product of Caldecott cone dewatering tailing, St. Joseph Lead Co.

Aperture, mm.	Weight, per cent.
1.651	2.59
1.168	9.61
0.833	17.64
0.589	22.95
0.417	15.13
0.295	10.87
0.208	7.27
0.147	6.65
0.104	5.18
0.074	1.25
−0.074	0.86
	100.00

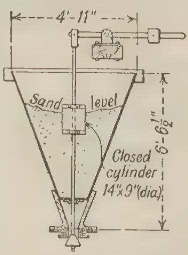


FIG. 33.—Automatic diaphragm cone at Mt. Lyell.

Table 31. Performance of automatic diaphragm cone at Mt. Lyell

Aperture, I.M.M. mesh	Weight, per cent.					
	Lyell Comstock ore			North Mt. Lyell ore		
	Feed	Sand	Overflow	Feed	Sand	Overflow
20	47.7	62.2	....	63.6	65.9	....
40	14.4	12.6	....	12.3	15.0	....
60	7.1	5.5	....	4.2	5.8	....
90	6.8	10.2	....	4.2	4.5	0.4
120	5.5	1.2	2.5	3.9	2.8	1.1
150	3.7	6.4	3.8	1.1	0.8	1.2
−150	14.8	1.9	93.7	10.7	5.2	97.3

Another automatic-discharge device is shown in Fig. 34 (*22 JCM 74*). The counterweight is set to discharge the proper tonnage at the desired percentage of moisture. If the pulp thins down, the velocity of discharge increases, with resultant increase in friction and pressure on the valve plug and the valve closes somewhat, thus retarding discharge and permitting solids to build up again. The device gave satisfaction at FERREIRA DEEP mill on tube-mill cones as compared with manual regulation of standard Caldecott cones. The normal moisture content of spigot product was 25 per cent. and a sizing test showed 57 per cent. +60-mesh, 33 per cent. +90-mesh and 10 per cent. −90-mesh.

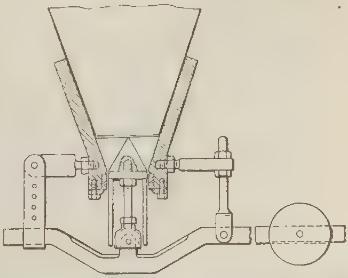


FIG. 34.—Automatic diaphragm cone.

Allen cone (Fig. 35) consists of a conical sheet-iron tank, center-fed, with peripheral overflow of slime and automatic regulation of sand discharge.

The spigot regulator consists of a spring-controlled link mechanism actuated by a float (F). Feed entering the center pipe (A) is distributed by the baffle (B), flows into the tank proper, the sand settles out and slime overflows the lip (L). When the upper surface of the settled sand reaches the position shown in the sketch, water backs up in chamber (C), float (F) is lifted, the valve rod is depressed and the ball valve (J) opened. Resistance to opening is varied by the weight (K). The valve seat is carried on a swinging plate, so arranged that when the spigot opening is swung to one side for inspection or replacement, the lower end of the cone is closed by a blank plate. New spigot liners can be dropped into place quickly, thus making it unnecessary to shut down. The cross-section of the overflow may be lessened by introducing a REDUCTION RING (R), which increases the rising velocity and consequently increases the size of particles overflowed. The effect is shown in Table 32. A diaphragm (D) is used to reduce the percentage of moisture in the sand discharge and to regulate the flow of solids. Standard sizes are given in Table 33.

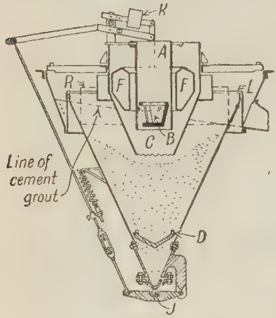


FIG. 35.—Allen cone.

Table 32. Effect of reduction ring on performance of Allen cone, Shannon Copper Co. (*Allen Cone Co.*)

Aperture, mm.	Weight, per cent.					
	Without reduction ring			With reduction ring		
	Feed	Sand	Overflow	Feed	Sand	Overflow
On 0.295	29.9	38.5	....	37.2	56.4	....
0.208	15.3	20.0	....	12.1	17.5	1.6
0.147	13.1	16.5	0.4	12.2	17.0	2.7
0.104	4.1	4.7	0.6	4.7	5.1	3.8
0.074	6.5	7.5	0.9	2.6	1.2	3.1
Through 0.074	31.1	12.8	98.1	31.2	2.8	88.8

Table 33. Standard sizes of Allen cones for concentrating mills. (*Allen Cone Co.*)

Manufacturers' numbers	Diameter at overflow lip	Fall, feed inlet to spigot	Weight, pounds
40–1	3′ 6″	5′ 2″	675
40–0	4′ 6″	6′ 2″	825
40–2	6′ 0″	7′ 9″	1050
40–3	8′ 0″	9′ 11″	1600

Performances are given in Table 34. Delano (*57 A 440*) says that a 6-ft. cone will treat galena concentrate ranging from 9-mm. down to 5 or 6 per cent. −200-mesh and carrying 83 per cent. water at the rate of 100 tons per 24 hr., make a spigot discharge containing 12 per cent. moisture and yield a clear-water overflow. On tailing ranging from 2-mm. to 0.1-mm. the same cone will treat 300 tons per 24 hr., yielding a spigot product containing 28 per cent. moisture and a clear overflow. He states that sand coarser than

3-mm. would be difficult to dewater unless it contained a large percentage of fines. Watt (*57 A 440*) says that a 4.5-ft. cone will handle 145 tons per 24 hr. of the undersize of a 10-mm. screen, yielding a spigot product containing 28 to 30 per cent. moisture and with less than 3 per cent. coarser than 200-mesh in the overflow solids.

Table 34. Performance of Allen cones.

Mine	Mine La Motte(*b*)	Mine La Motte(*b*)	Burro Mountain(*b*)
Character of service	De-sliming	De-sliming	De-sliming
Tons solid per 24 hr.	146	62	129
Per cent. solids in feed	12.5	7.7	18
Per cent. solids in sand	....	....	*c*
Per cent. solids in overflow	....	....	*c*
Size of feed	*a*	*a*	6% +65-m. 60% −200-m.
Size of sand product	*a*	*a*	....
Overflow, per cent. through 200-mesh	*a*	*a*	88
Diameter of tank, ft.-in.	4–6	4–6	4–6

Mine	Arizona Copper Co.(*b*)	Shannon Copper Co.(*b*)	St. Joseph Lead Co. Bonne Terre
Character of service	De-sliming	De-sliming	Dewater. conc.
Tons solid per 24 hr.	370	1442	120
Per cent. solids in feed	25	37	35
Per cent. solids in sand	....	....	88
Per cent. solids in overflow	....	....	0.02
Size of feed	All −6-m. 30% −200-m.	7% +10-m. 30% −200-m.	*a*
Size of sand product	....	....	*a*
Overflow, per cent. through 200-mesh	77	49	100
Diameter of tank, ft.-in.	4–6	4–6	6–0

Mine	Shattuck-Arizona Copper Co.	Shattuck-Arizona Copper Co.	Old Dominion(*d*)
Character of service	De-sliming	De-water conc.	*e*
Tons solid per 24 hr.	190	50	....
Per cent. solids in feed	35	....	±5
Per cent. solids in sand	70.5	80	72
Per cent. solids in overflow	14	0+	....
Size of feed	....	....	*a*
Size of sand product	*a*	*a*	*a*
Overflow, per cent. through 200-mesh	*a*	....	....
Diameter of tank, ft.-in.	6–0	6–0	4–6

a For screen tests, see Table 34*a*. *b* From Allen Cone Co., Bull. 25. *c* Blickensderfer (*104 J 76*) reports on similar feed 75 per cent. solids in sand and 4 per cent. solids in overflow. *d 102 J 508*. *e* Thickening and de-sliming table feed.

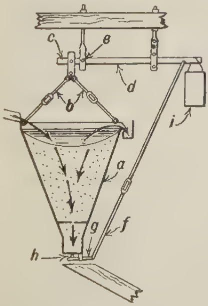


Fig. 36.—Boylan automatic cone.

Boylan automatic cone classifier (Fig. 36) consists of a conical or pyramidal tank (*a*) suspended by rods (*b*) from a knife edge on the short end (*c*) of lever (*d*). This lever is suspended at (*e*). The long arm connects by means of rod (*f*) to a short lever (*g*) whose fulcrum is carried on the lower part of the tank. The working arm of lever (*g*) carries a conical valve (*h*). Feed is introduced at the top at one side, sand settles out and slime overflows on the opposite side. When sufficient sand (about 60 to 75 per cent. of the volume of the tank) has collected, the weight of the tank causes rod (*f*) to be lifted, thus opening the valve (*h*) and permitting discharge of sand. The tripping point is determined by weights (*i*).

Table 34*a*. Sizing tests of feed and products of Allen cones

Aperture, mm.	Weight, per cent.										Old Dominion Copper Co.			
	Mine La Motte			Mine La Motte			St. Joseph Lead Co.	Shattuck-Arizona Copper Co.			Feed		Sand	
	Feed	Sand	Overflow	Feed	Sand	Overflow	De-watered conc.	Sand (*a*)	Overflow (*a*)	De-watered conc.	Weight Per cent.	Per cent. of total Cu	Weight Per cent.	Per cent. of total Cu
4.699	....	....	....	....	....	....	2.2	....	....	....	....	....	....	....
3.327	....	....	....	....	....	....	2.1	....	....	0.2	....	....	....	....
2.362	....	....	....	....	....	....	5.0	....	....	0.5	....	....	....	....
1.651	....	....	....	....	....	....	8.5	....	....	2.4	....	....	....	....
1.168	7.5	9.0	....	0.9	2.5	....	10.1	....	....	3.1	....	....	....	....
0.833	2.5	2.5	....	2.6	5.0	....	7.8	....	....	2.9	....	....	....	....
0.589	3.1	3.6	....	3.5	7.2	....	8.2	0.3	....	3.8	....	....	....	....
0.417	6.5	6.5	....	4.5	9.0	....	7.2	1.4	....	3.8	0.34	....	1.12	0.40
0.295	49.5	55.0	....	9.0	17.8	....	7.7	8.4	....	4.9	0.85	0.36	2.19	0.70
0.208	12.9	17.0	....	8.7	16.9	....	7.1	14.7	....	5.7	2.60	0.48	7.92	1.57
0.147	5.5	4.0	0.4	10.0	20.1	....	9.1	24.0	1.6	6.4	12.56	2.29	28.25	5.93
0.104	1.1	1.0	0.4	8.4	12.2	3.8	11.0	23.5	5.3	7.8	18.28	7.22	30.36	17.83
0.074	0.3	0.3	2.2	6.1	3.9	6.3	5.8	11.9	7.8	8.1	11.74	9.66	13.80	18.83
−0.074	11.1	1.1	97.0	46.3	5.4	89.9	8.2	15.8	85.3	50.4	53.63	79.99	16.36	49.51

a De-sliming cone

Sizing tests of feed and products of a Boylan cone treating rougher-jig tailing are given in Table 35 (*112 J 79*). The efficiency, reckoned on 20-mesh, is 54.5 per cent.

Table 35. Performance of Boylan cone

Screen, mesh	Weight, per cent.		
	Feed	Sand	Overflow
4	22.47	26.51	....
8	30.45	33.72	....
10	....	....	0.05
20	24.15	27.86	0.67
35	8.19	8.28	4.96
65	....	....	14.66
100	6.04	2.79	20.36
200	3.55	0.40	37.00
−200	5.15	0.44	22.30

Wood automatic classifier (Fig. 37) (*114 J 351*) consists of a pyramidal tank (1) hung by means of straps (4) and suspension rods (6) on knife-edge bearings (8) on the short arms of levers (5). The fulcrums of these levers at (9) are knife edges that support the entire mechanism. The long arms of the levers (5) are attached through a vertical connecting rod (10) to another lever (13) whose fulcrum (14) is mounted on the lower end of the hopper and whose working end carries a conical plug valve (15).

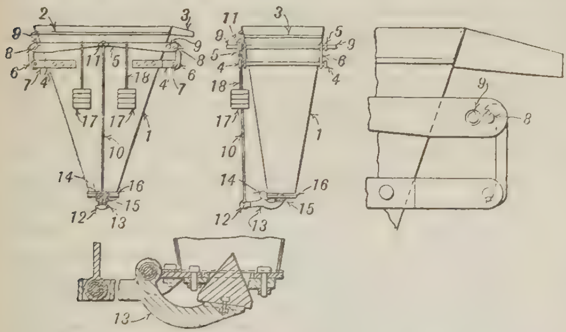


Fig. 37.—Wood automatic sand-slime separator.

When the cone lowers under the load of settled sand, (10) is lifted and the valve opened. Feed is introduced at (2) and slime overflows at (3). Rate of discharge is regulated by the position of the weights (17). Detail of the apex is shown in the large-scale figure. The inventor claims efficiencies of 70 to 90 per cent. with from 25 to 40 per cent. of moisture in the sand discharge.

A cleaner sand discharge and coarser slime overflow can be obtained in tank separators if gentle agitation is maintained in the settling zone.

At the Le Roi No. 2 mill, Rossland, B. C. (*114 J 1121*) a de-sliming cone was used to close the grinding circuit on a secondary ball mill; a short air-lift kept the upper part of the contents in gentle circulation. The degree of agitation was varied by vertical adjustment of the air-lift riser and by variation of air pressure.

6. Mechanical classifiers

Dorr classifier (Fig. 38) consists essentially of a rectangular tank (*a*) with sloping bottom, closed at the lower end by means of a vertical end wall (*b*), open at the upper end, and having suspended therein a mechanically-operated raking mechanism (*c*). The rakes are suspended by links (*d*) and (*e*) from arms of cranks (*f*) and (*g*) respectively. The pin of crank (*g*) is attached to the main frame of the apparatus, the pin of crank (*f*) is mounted on the arm of crank (*j*), whose position is regulated by means of the rope (*k*) and the drum (*l*), actuated by worm gear from crank (*m*). Power applied

to pulley (n) is transmitted by a pinion and gear to cam (p). The latter, working against roller (q) on the downwardly extending arm of crank (g) causes the arm of this crank to be raised and lowered and with it the upper end of the rakes. Simultaneously the arm of crank (f) is raised and lowered through connecting rod (r), similarly moving the lower end of the rakes. Coincident with the raising of the rakes, a crank attached to the shaft of cam (p) transmits motion through connecting rod (t) to the rakes in a plane parallel to the inclined bottom and directed down slope. Further revolution of the cam into the position shown in the figure causes the rakes to be dropped to the lowest position and in this position the rakes are dragged up slope.

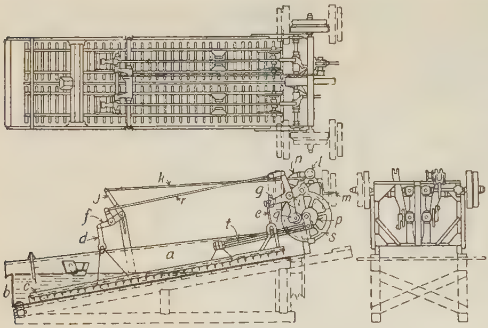


Fig. 38.—Dorr duplex classifier.

Classifiers are rated by giving the width of tank and the nominal length of the inclined bottom and are called simplex, duplex or quadruplex according to whether one, two or four sets of rakes are operated in the same tank.

Classifiers are made with tanks 2 ft. 3 in., 4 ft. 6 in. and 6 ft. wide, with one set of rakes in the narrowest tank, and two sets side-by-side in the wider tanks, driven from the same mechanism. The usual form is the two-rake or duplex type. The simplex type is used for small capacities. A special quadruplex form has been used at Calumet and Hecla for dewatering sand tailing, where handling the sand was the important feature and great raking capacity was necessary. The 4-ft. 6-in. duplex is designated "Model C"; the 6-ft., "Model D" machine, is of much heavier construction throughout and is to be recommended where large tonnages of sand must be handled.

The classifying action of the apparatus is the result of currents set up by the movement of the rakes and by the entrance and flow of liquid pulp through the machine. The rake movement and the plunge of the entering stream stir the pulp sufficiently to keep the smaller granular particles in suspension. The general flow of liquid through the settling basin to and over the overflow lip and the surge of pulp incident upon the movement of the rakes carry the suspended particles out of the machine. The more vigorous the stirring, the larger the particles prevented from settling; the greater the carrying power of the overflowing stream, the larger the particles overflowed. The factors affecting suspending and carrying power are: speed of rakes, slope of bottom, height of tail board, height of rakes, placing of baffle, volume of pulp, character of ore, pulp dilution, and size of solid particles. The character of the sand discharge is affected primarily by the character of the overflow, a fine overflow resulting in much fine material in the sand and *vice versa*. It is affected independently by the length of the bottom, slope of the bottom and quantity of wash or back water employed.

Slope of bottom together with height of tail board determine the area of the settling pond and therefore the average rising velocity of the pulp stream. These also determine the volume of the settling pond and consequently the vigor of the stirring effected by a given rake movement. Slope also affects performance by determining, to a considerable extent, the composition of the body of pulp remaining in the classifier at all times. With

steep slope the ratio of downward travel to upward travel of any given grain along the bottom for one cycle of the rakes is greater than with a flat slope because of the difficulty of holding sand grains, particularly the small ones, on the steep incline. Consequently the grains in the body of pulp in the settling pond are coarser with steep slope than with flat slope. This produces a coarser overflow, since grains that cannot discharge with the sands must discharge with the overflow, if the classifier is to continue to operate. Steep slope also, therefore, produces a coarser sand discharge than flat slope. Slopes range in mill practice from 1.5 in. per ft. for 200-mesh separation to 3 in. per ft. for 48-mesh separation. The Dorr Co. recommends even steeper slopes for 28-mesh separation. When a large tonnage of sand is to be separated from a comparatively small amount of slime, low slope may be employed to give a large settling pond, with high rake speed to move the settled sand and high dilution (small percentage of solids) in order to permit quick settlement of solids. Column 9, Table 40 shows this combination. Comparison of columns 3 and 8 shows how a larger tonnage may be handled without change in products by flattening the slope of classifier bottom.

Height of tail board should be such that with the bottom slope employed, a pond of sufficient size will be formed to allow settlement of all sand coarser than that which is to be overflowed and the pond should be of such depth that eddies and surging caused by the agitation will not throw coarse sand into the overflow. The standard depth in mill-size classifiers is from 21 to 24 inches. Increase in depth results in larger settling area and lessening of the surface agitation due to rake movement and pulp inflow, with the result that the overflow is finer (see Table 36) and more fine material is discharged with the sands. Table 38, however, shows that the effect is not great. (Columns 1 *vs.* 3 and 2 *vs.* 4.) If the same character of overflow can be taken as before the change, then greater tonnage can be handled with a higher tail board although usually at the expense of a higher water and slime content in the sand discharge. When the specific gravity of the solids is high, the tail board must be lower than with normal ores.

Table 36. Effect of height of tailboard on overflow product of Dorr classifier

Screen aperture, mesh	Weight, cumulative per cent.	
	Standard height of tailboard	Tailboard raised 4 in.
60	0.2	0.5
80	2.5	1.4
100	7.6	5.2
150	24.8	22.5
200	38.3	32.8
−200	61.7	67.2

Height of rakes. Raising the rakes decreases the effective depth of the settling pond, and increases the agitation near the surface. The result is to keep coarser sand in suspension and so produce a coarser overflow.

Length of classifier depends partly upon structural demands and partly upon the character of sand discharge desired. Where the classifier is to be operated in closed circuit with a grinding mill it is depended upon to elevate the sand return to a point that will allow this material to flow by gravity into the mill feed box. In such cases it may be necessary to lengthen the classifier beyond the point required for classification alone. Generally from 5 to 6 ft. of unsubmerged bottom or draining space is the minimum allowed where a sand product containing 20 to 25 per cent. moisture is desired. If less moisture is wanted in the sand a longer draining space or use of a dewatering lip is necessary.

Dewatering lip consists of a prolongation of the floor of the classifier at the sand-discharge end for about 6 in. on a slope of 30 to 45° with the horizontal. By making the discharging sands climb this additional slope, a considerable reduction in moisture can be attained.

Speed of rakes is determined by the number of strokes per minute and the length of stroke, and since the length of stroke in mill-size classifiers is usually fixed at 12 to 15 in., speed variation is effected by changing the number of strokes per min. Speed has several effects on performance.

(1) It affects the degree of agitation of the pulp in the settling pond and thereby determines the size of particles that can be kept in suspension. This in turn affects the size of overflow particles, since particles that cannot settle must overflow, if the classifier is to continue to operate. The specific gravity and viscosity of the fluid pulp are likewise increased by increased agitation. This results in increased suspending and carrying power of the overflow current.

(2) Speed affects the velocity of the overflowing current. The average rising velocity of the pulp current is determined by dividing the volume of overflowing pulp per unit of time by the surface area of the settling pond but the actual rising velocity effective to lift out suspended solid is much greater than the average, due to surge toward the overflow lip caused by the return rake movement. The velocity of this surge current is proportional to the speed of the rakes. The usual speeds vary from 12 strokes per min. for 200-mesh

separation to 30 strokes per min. for 28-mesh separation. The effect of speed change is greatest when the speed is already near the maximum allowable for a given separation. Thus Table 37 indicates an appreciable increase in size of products due to increase from

Table 37. Effect of speed change on Dorr-classifier performance

Screen aperture, mesh	Weights, cumulative per cent.			
	20 strokes per minute		12.5 strokes per minute	
	Sand	Overflow	Sand	Overflow
On 60	42.9	....	42.9	....
100	72.1	1.2	67.9	0.1
150	89.3	2.6	74.8	0.9
200	92.0	14.0	87.0	6.3
−200	8.0	86.0	13.0	93.7

12.5 to 20 strokes per min. when 100-mesh separation is being carried on while Table 38, columns 1 *vs.* 2 and 3 *vs.* 4 show inappreciable effects for changes in the same speed range at 20-mesh separation.

Shape of rakes. Rakes are commonly made of structual angles, but flat rakes made of flat strap clipped to the underside of the rake channel are used when it is desired to lessen agitation in the settling pond. Notched rakes, made by staggering 3-in. rectangular cut-outs taken from the vertical flanges of the angles are used to decrease the water and slime in the sand discharge by permitting freer drainage. Columns 5 and 6, Table 38, show the effect on size. One or two top rakes placed above the channels near the point that these emerge from the settling pond may be used to give additional agitation at this place and produce cleaner sand. Raking capacity can be increased by increasing the depth of the vertical blade.

Baffle. The position of the baffle at the overflow end determines the cross-sectional area of the overflowing stream and thereby the velocity, all other things being constant. The position of the baffle can in no way influence the size of particles suspended by the stirring of the rakes, but by moving the baffle close to the tail board it is possible to lift out particles that otherwise might not be discharged but would build up in the pond until the classifier stalled or the density increased sufficiently to carry them over.

Volume of pulp. In a given classifier, change in volume of pulp changes the velocity of the overflow current proportionately. If change in volume is not accompanied by change in specific gravity of the pulp in the settling pond, *e.g.*, if the tonnage of pulp only is changed, increase will produce coarser products and *vice versa*. If, however, increase in volume is gained by increasing the water fed without corresponding increase in solids, the carrying power of the pulp is lessened on account of the decrease in specific gravity and viscosity and the result is likely to be a net decrease in carrying power and consequently in average size of products. (See also paragraph on *Pulp dilution*.) When, however, coarse-mesh separation is being made, changes of as much as 10 per cent. in the volume of pulp passing, without material change in dilution, have no substantial effect on the size of the overflow particles. (See Table 38.)

Dilution of pulp is the most important operating variable. Together with speed of rakes, character of ore, and fineness of feed particles, it determines both the specific gravity and viscosity of the pulp in the settling pond and it is the only one of these factors in direct control of the operator. The specific gravity of the fluid in which settlement occurs enters as ρ in the quantity $(\sigma - \rho)/\rho$ in the equation for velocity of fall of bodies in resistant fluids (turbulent resistance). (Art. 1.) Hence increase in ρ causes decrease in settling velocity, which means that larger particles are lifted by a given current velocity. Viscosity of the fluid enters in the denominator of the settling equations, hence increase in viscosity likewise causes overflow of larger particles.

With normal ores the specific gravity and viscosity of pulps are not greatly affected by changes in the percentage of solids when the percentage is less than fifteen. Below this figure decrease in density with corresponding increase in current velocity may cause coarser overflow. Above this point the effect of changes in dilution is marked and satisfactory control of classifier products can be effected by control of dilution. Columns 5 and 15, Table 40, show this clearly. Comparison of columns 13 and 16 shows the same thing, although here increase in moisture content of overflow is accompanied by decrease in solid tonnage, which also tends to produce finer overflow. With ores containing large amounts of semi-collodial material, viscosity is considerably affected by even less than

Table 38. Tests on standard simplex Dorr classifier at Anaconda

Test number	1		2		3(*a*)			4(*a*)			5(*b*)			6		
Tons per 24 hr., feed	475		418		427			402			446			364		
Tons per 24 hr., overflow	359		296		320			287			225			230		
Tons per 24 hr., sand	116		122		107			115			221			134		
Speed, strokes per min.	17.5		12.5		17.5			12.5			17.5			19.3		
Percentage of solids in overflow	32.9		29.6		27.6			26.8			25.6			25.2		
Screen test: Cum. per cent.	F	O	F	O	F	O	S	F	O	S	F	O	S	F	O	S
On 0.83	1.7	0.4	0.8	0.2	2.0	0.9	2.4	1.1	0.4	1.4	1.5	0.5	2.7	1.3	.....	1.8
0.59	6.3	0.9	4.4	0.7	7.6	1.9	9.5	3.9	1.2	6.1	6.3	1.4	10.3	5.2	0.2	6.3
0.42	17.8	4.6	15.3	3.3	16.8	5.0	27.3	14.2	3.6	17.5	13.1	3.7	26.5	12.9	1.1	17.2
0.29	27.0	11.3	24.0	9.0	32.3	11.9	50.6	28.8	9.1	35.6	28.7	8.4	49.3	26.6	4.6	36.9
0.21	44.3	24.7	41.5	21.5	47.9	23.4	68.9	44.9	19.6	55.3	44.5	19.9	67.8	42.6	13.5	58.2
0.15	59.8	41.2	57.5	38.3	64.9	42.6	84.9	63.3	38.8	78.1	61.6	38.5	83.5	61.2	32.0	79.2
0.10	72.5	57.9	70.5	57.1	73.9	56.1	91.4	73.1	53.0	86.3	71.8	53.8	90.3	72.3	50.7	88.5
0.07	81.8	65.3	80.8	65.4	78.5	63.3	94.1	78.2	60.9	91.1	77.5	62.0	93.8	78.4	60.3	93.1
−0.07	18.2	34.7	19.2	34.5	21.5	36.7	5.9	21.8	39.1	8.9	22.5	38.0	6.2	21.6	39.7	6.9

a Height of tailboard increased 3 in. *b* Rakes notched with staggered cut-outs, 3 in. wide and full depth of vertical flange.
F, Feed. *O*, Overflow. *S*, Sand.

15 per cent. of solids and with such ores some variation in classifier products can be effected by dilution control even in dilute pulps.

Viscosity of the overflow stream has much greater effect on carrying power than either velocity or specific gravity. In the performances recorded in Table 39 the same tonnage of solid was treated in both runs. In the first run the volume of pulp passing through the machine was 2.25 times that in the second with correspondingly greater rising velocity. Notwithstanding the increased current velocity in the first run the product was markedly finer than in the second. The coarser product in the second run was due in small part only to the greater specific gravity of the pulps. For since V_D/V_T varies as $\rho_T(\sigma - \rho_D)/\rho_D(\sigma - \rho_T)$ it follows that on this score alone, the falling velocity of particles in the dilute pulp (V_D) is 1.18-times greater than the falling velocity of the same particles in a thick pulp (V_T) or that a rising velocity of dilute pulp 1.18-times as great as that of the thick pulp would overflow the same size particles. But since with a rising velocity of dilute pulp 2.25 times that of the thick pulp the particles lifted were smaller, it follows that the actuating cause was something other than the relative velocities or specific gravities of the pulps. Viscosity is the only remaining element in the settling equations to account for the difference in behavior. V_D/V_T varies as μ_T/μ_D. The viscosity of a pulp containing slime increases rapidly with increasing solid content beyond 15 per cent. although concordant experimental data are difficult to obtain. This fact explains why the much slower rising current of thick pulp in the test recorded in Table 39 lifted larger particles than the relatively rapid current of dilute pulp. Usual dilutions in mill practice are given in Table 40.

Table 39. Effect of pulp dilution on performance of Dorr classifier. (*Inspiration Consolidated Copper Co., from Dorr Co. catalog*)

Screen aperture		Weight, cumulative per cent.			
		11.1 per cent. solids in overflow		25 per cent. solids in overflow	
Mesh	Mm.	Sand	Overflow	Sand	Overflow
28	0.589	19.5	....	32.2	....
48	0.417	55.0	....	71.8	0.6
65	0.208	79.0	....	84.9	5.9
100	0.147	90.0	1.1	91.5	14.9
150	0.104	94.9	5.5	94.4	85.1
200	0.074	97.1	10.2	95.7	
−200	−0.074	2.9	89.8	4.3	

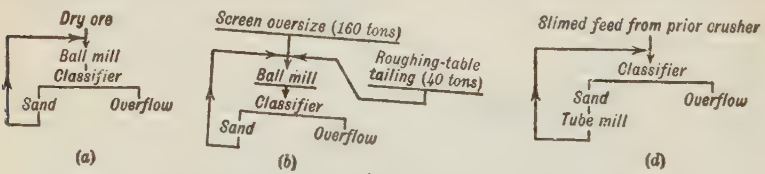


FIG. 38*a*.—Flow-sheets involving Dorr classifiers reported in Table 40.

Size of feed particle. The effect of solid particles on specific gravity and viscosity of a pulp is greater the smaller the particles. Hence the more fine material present in a pulp containing a given percentage of solids, the coarser the particles that can be overflowed. This is clearly shown by comparison of columns 10 and 11, Table 40, and the corresponding screen tests. If the feed is coarse and coarse overflow is desired, steep slope, high rake speed and low dilution are employed (columns 1 and 2, Table 40); for fine overflow, a flat slope, low rake speed and dilute pulp are usual (columns 21, 22). The low dilution in column 23 is exceptional and the effect in coarsening the overflow is apparent on comparison of screen tests 60 and 63, Table 40*a*.

Character of ore. Ores of high specific gravity must be treated in pulps of relatively high specific gravity and high viscosity in order to lift the desired particles over the tailboard. With such pulps clean separation is difficult or impossible and much material that should overflow must be sent with the sand discharge in order to prevent excessive amounts of oversize in the overflow. The performances of classifiers 11 and 12, Table 40, on heavy sulphide ores are typical of the trouble encountered.

Performances of Dorr classifiers under many varying conditions are given in Table 40.

Table 40. Performance of Dorr classifiers

Number	1	2	3	4	5	6
Mill	United Eastern	Shattuck Arizona	Miami Copper Co.	Miami Copper Co.	Miami Copper Co.	Miami Copper Co.
Size and type of classifier	*S*, 3×15–9	*CD*	*DD*	*DD*	*DD*	*DD*
Slope of bottom, inches per foot	3	3	3	3	3	3
Strokes per minute	27	27	25	25	25	25
Moisture, per cent., feed	48	....	32.1	47.1	31.4	49.1
Moisture, per cent., sand	20	30	23.4	22.7	23.6	24.5
Moisture, per cent., overflow	53	56	75.1	80.1	66.9	77.2
Tons overflow per 24 hr.	280	200	400	268	344	421
Tons sand per 24 hr.	435	346	862	688	935	679
Tons total feed per 24 hr.	715	546	126.2	956	1279	1100
Tons sand raked per foot of width per 24 hr.	193	77	143	115	155	113
Screen test, number. See Table 40*a*	1, 2, 3	4, 5, 6	7, 8, 9	10, 11, 12	13, 14, 15	16, 17, 18
Size at which separation is wanted, mm.	0.833	0.589	0.295	0.295	0.295	0.295
Character of overflow	97.2% − 20-m.	99.0% − 28-m.	92.2% − 48-m.	99.7% − 48-m.	95.3% − 48-m.	92.2% − 48-m.
Efficiency, at separation size(*e*)	69.6	41.3	59.7	66.0	43.7	71.5
Flow-sheet	*a*	*b*	*a*	*a*	*a*	*a*

Number	7	8	9	10	11	12
Mill	Moctezuma Copper Co.	Inspiration	Chino C. C. Co.	Engels C. M. Co.	Engels C. M. Co.	Consol. Ariz. Sm. Co.
Size and type of classifier	*DD*	*DD*, 6×27	*DD*	*DD*, 6×22–4	*DD*, 6×22–4	*DD*, 6×25
Slope of bottom, inches per foot	3	2⅝	1.5	2⅞	2⅞	3
Strokes per minute	27	25	30	23.4	23.4	22–23.5
Moisture, per cent., feed	25	25.4	55	63	63	....
Moisture, per cent., sand	20	21.9	30	17	17	20
Moisture, per cent., overflow	76	73.7	95	70	70	60
Tons overflow per 24 hr.	325	528	400	444	294	150
Tons sand per 24 hr.	1280	1456	2065	636	516	300
Tons total feed per 24 hr.	1605	1984	2465	1080	840	450
Tons sand raked per foot of width per 24 hr.	213	242	344	106	91	50
Screen test, number. See Table 40*a*	19, 20, 21	22, 23, 24	25, 26, 27	28, 29, 30	31, 32, 33	34, 35, 36
Size at which separation is wanted, mm.	0.295	0.295	0.295	0.295	0.295	0.295
Character of overflow	94.5% − 48-m.	93.4% − 48-m.	98.5% − 48-m.	99.3% − 48-m.	99.8% − 48-m.	96.4% − 48-m.
Efficiency at separation size(*e*)	22.4	63.4	53.5	60.6	38.0	2.5
Flow-sheet	*a*	*a*	*a*	*a*	*a*	*a*

For notes on reference letters see next page

Table 40. Performance of Dorr classifiers—*Continued*

Number	13	14	15	16	17	18
Mill	Sunnyside M. & M. Co.	Afterthought Copper Co.	Miami Copper Co.	Sunnyside M. & M. Co.	Inspiration	Burro Mountain
Size and type of classifier	*CD*, heavy	4–6×16–4	*DD*	*CD*, heavy	*DD*, 6×27	*D*, 5–6×18
Slope of bottom, inches per foot	3	3	3	3	2⅝	2
Strokes per minute	18	28	25	18	25	17
Moisture, per cent., feed	......	25.4	33.5	......	......	......
Moisture, per cent., sand	......	15.6	25.6	......	25.4	......
Moisture, per cent., overflow	40–45	65.0	70.8	78–80	78.3	80.5
Tons overflow per 24 hr.	450	150	340	210	495	150
Tons sand per 24 hr.	200	560	1249	384	2025	192
Tons total feed per 24 hr.	650	710	1589	624	2520	342
Tons sand raked per foot of width per 24 hr.	33	124	208	64	338	35
Screen test, number. See Table 40*a*	7, 38, 39	43, 44, 45	64, 65, 66	40, 41, 42	46, 47, 48	49, 50, 51
Size at which separation is wanted, mm.	0.295	0.295	0.208	0.208	0.208	0.208
Character of overflow	77.8% − 48-m.	97.5% − 48-m.	96.3% − 65-m.	99.7% − 65-m.	98% − 65-m.	98.3% − 65-m.
Efficiency at separation size(*e*)	......	35.5	48.2	60.2	58.2	63.0
Flow-sheet	*a*	*a*	*a*	*a*	*a*	*a*

Number	19	20	21	22	23	24
Mill	Miami Copper Co.	Timber Butte	Tonapah Belmont	United Eastern	Nipissing	Calumet & Hecla
Size and type of classifier	*CD*	*CD*	*CD*	*CD*	*CD*	Quadruplex
Slope of bottom, inches per foot	2	2.75	1.92	2.25	......	2
Strokes per minute	23	22.5	10.5	15	......	18
Moisture, per cent., feed	......	78.3	......	43	......	86
Moisture, per cent., sand	20.6	31.0	35	20	22.5	35
Moisture, per cent., overflow	73.8	83.9	85	53	90	96
Tons overflow per 24 hr.	333	134	70	90	68	......
Tons sand per 24 hr.	704	270	156	133	83	......
Tons total feed per 24 hr.	1037	404	226	223	151	750
Tons sand raked per foot of width per 24 hr.	117	60	34	30	18.4	......
Screen test, number. See Table 40*a*	52, 53, 54	55, 56, 57	58, 59, 60	61, 62, 63	67, 68, 69	70, 71, 72
Size at which separation is wanted, mm.	0.208	0.208	0.147	0.147	0.074	0.074
Character of overflow	97.6% − 65-m.	99.4% − 65-m.	99% − 100-m.	97.4% − 100-m.	92% − 200-m.	95% − 200-m.
Efficiency at separation size(*e*)	64.1	73.6	53.0	55.7	......	......
Flow-sheet	*a*	*c*	*d*	*d*	*d*	*f*

a, *b*. See Fig. 38a. *c*, See Sec. 2, Fig 99. ***CD***, Standard duplex, 4 ft. 6 in. wide by 16 to 18 ft. long. ***CS***, Standard simplex (1-rake), 2 ft. 3 in. wide by about 8 ft. long. *d*, See Fig. 38a. ***D***, Duplex. ***DD***, Heavy duplex (2-rake), 6 ft. wide by about 18 ft. long. *e*, See Sec. 22, Art. 20. *f*, See Sec. 2, Fig. 44. *S*, Simplex.

Table 40*a*. Screen tests of feed and products of Dorr classifiers
(For significance of column numbers, see Table 40)

Mill		United Eastern			Shattuck Ariz.			Miami Copper Co.			Miami Copper Co.		
		Weight, cumulative percentages											
Column No.		1	2	3	4	5	6	7	8	9	10	11	12
Mesh	Aperture, mm.	F	S	O	F	S	O	F	S	O	F	S	O
3	6.680	.....	.....	.....	.....	3.5	.....	1.1	.....	.....	2.1	0.5	.....
4	4.699	0.8	2.5	.....	5.7	7.6	.....	2.3	2.1	.....	4.3	2.4	.....
6	3.327	2.0	9.0	.....	.....	12.3	.....	3.5	3.6	.....	7.0	5.9	.....
8	2.362	7.5	19.2	.....	.....	17.0	.....	5.2	5.7	.....	12.8	11.3	.....
10	1.651	16.5	36.0	.....	.....	23.3	.....	7.2	8.3	.....	19.7	19.5	.....
14	1.168	27.5	54.8	.....	.....	29.5	.....	9.8	11.6	.....	27.5	30.5	.....
20	0.833	37.5	68.2	2.8	.....	39.5	.....	13.1	15.6	.....	34.4	42.9	.....
28	0.589	.....	.....	10.5	38.0	51.2	1.0	21.4	26.5	.....	44.4	57.8	.....
35	0.417		82.2	21.0	.....	63.7	4.6	32.0	43.0	.....	51.2	69.3	.....
48	0.295	63.0	86.3	31.5	.....	75.9	15.4	52.8	75.0	7.8	56.4	79.2	0.3
65	0.208	68.0	88.6	40.0	65.2	82.8	27.0	58.7	80.2	13.7	62.6	85.6	3.7
100	0.147	73.0	90.4	47.0	.....	87.7	40.2	67.3	87.9	27.5	67.8	90.4	8.2
150	0.104	77.0	91.6	55.0	.....	89.6	46.2	74.8	91.6	41.6	72.2	92.8	18.1
200	0.074	81.0	93.3	65.0	84.7	92.2	55.4	78.1	93.1	47.7	74.1	93.7	26.1
−200	0.074	19.0	6.7	35.0	15.3	7.8	44.6	21.9	6.9	52.3	25.9	6.3	73.9

Mill		Miami Copper Co.			Miami Copper Co.			Moctezuma			Inspiration		
		Weight, cumulative percentages											
Column No.		13	14	15	16	17	18	19	20	21	22	23	24
Mesh	Aperture, mm.	F	S	O	F	S	O	F	S	O	F	S	O
3	6.680	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
4	4.699	0.5	0.3	.....	0.2	.....	.....	.....	.....	.....	2.2	4.9	.....
6	3.327	1.2	0.8	.....	1.4	0.3	.....	.....	.....	.....	.....	.....	.....
8	2.362	2.4	1.7	.....	4.3	1.5	.....	.....	.....	.....	6.0	9.3	.....
10	1.651	4.0	3.2	.....	9.9	5.6	.....	10.1	14.5	.....	.....	.....	.....
14	1.168	6.1	5.9	.....	17.8	15.1	.....	15.8	21.7	.....	15.0	20.6	.....
20	0.833	8.7	10.4	.....	25.3	29.9	.....	25.0	33.3	.....	.....	.....	.....
28	0.589	15.7	20.5	.....	36.8	52.3	.....	35.9	46.7	.....	30.6	42.7	.....
35	0.417	26.7	36.9	.....	44.6	69.8	.....	50.3	63.5	.....	48.0	66.2	0.8
48	0.295	44.4	59.2	4.7	52.7	82.2	7.8	63.3	78.0	5.5	61.8	82.3	6.6
65	0.208	60.4	76.3	15.1	57.6	87.6	10.7	72.3	86.5	12.7	71.2	90.1	19.6
100	0.147	71.8	85.7	32.6	63.4	91.1	20.6	79.9	92.5	22.7	76.8	93.5	30.2
150	0.104	80.0	91.1	48.9	69.3	93.4	30.4	84.7	95.1	32.5	80.6	95.2	39.8
200	0.074	82.7	93.1	58.4	72.8	94.2	35.7	86.0	95.6	35.8	83.0	96.1	47.0
−200	0.074	17.3	6.9	41.6	27.2	5.8	64.3	14.0	4.4	64.2	17.0	3.9	53.0

Mill		Chino C. C. Co.			Engels C. M. Co.			Engels C. M. Co.			Cons. Ariz. Sm. Co.		
		Weight, cumulative percentages											
Column No.		25	26	27	28	29	30	31	32	33	34	35	36
Mesh	Aperture, mm.	F	S	O	F	S	O	F	S	O	F	S	O
3	6.680	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
4	4.699	.....	.....	.....	4.1	8.4	.....	.....	.....	.....	.....	.....	.....
6	3.327	.....	.....	.....	6.9	13.6	.....	.....	.....	.....	.....	.....	.....
8	2.362	.....	.....	.....	9.3	18.8	.....	.....	.....	.....	.....	.....	.....
10	1.651	3.0	5.2	.....	12.6	24.6	.....	.....	.....	.....	.....	.....	.....
14	1.168	13.4	17.4	.....	16.8	30.0	.....	.....	.....	.....	.....	.....	.....
20	0.833	29.6	36.2	.....	20.2	35.3	.....	0.2	0.3	.....	.....	.....	.....
28	0.589	44.9	58.2	.....	23.6	41.3	.....	1.3	1.1	.....	.....	.....	.....
35	0.417	63.7	75.5	.....	28.4	49.2	.....	4.6	3.2	.....	28.5	29.6	.....
48	0.295	70.2	83.5	1.5	33.8	56.7	0.7	10.1	15.4	0.2	45.2	45.8	3.6
65	0.208	80.5	92.6	4.1	39.6	64.9	3.0	20.3	31.8	3.1	58.3	59.5	10.2
100	0.147	84.9	95.0	18.6	47.1	75.2	7.8	38.9	53.4	16.4	74.5	75.7	22.5
150	0.104	87.7	96.6	34.7	52.8	82.1	12.6	50.4	67.2	30.4	81.2	82.9	29.1
200	0.074	88.7	97.2	43.6	62.8	89.7	21.7	71.9	76.2	50.6	91.0	92.7	44.8
−200	0.074	11.3	2.8	56.4	37.2	10.3	75.3	28.1	23.8	49.4	9.0	7.3	55.2

F = Feed. S = Sand. O = Overflow.

Table. 40*a* Screen tests of feed and products of Dorr classifiers—*Continued*
(For significance of column numbers, see Table 40)

Mill		Sunnyside M. & M. Co.			Sunnyside M. & M. Co.			Afterthought			Inspiration		
		Weight, cumulative percentages											
Column No.		37	38	39	40	41	42	43	44	45	46	47	48
Mesh	Aperture, mm.	F	S	O	F	S	O	F	S	O	F	S	O
3	6.680	All pass 3/8-in.	All pass 3/8-in.	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
4	4.699			.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
6	3.327			.....	3.9	6.4	.....	.....	.....	.....	4.2	7.0	.....
8	2.362			.....	.....	.....	.....	.....	.....	.....	6.0	9.2	.....
10	1.651			.....	6.4	10.5	.....	6.1	7.6	.....	.....	.....	.....
14	1.168			.....	.....	.....	.....	.....	.....	.....	11.8	16.4	.....
20	0.833			.....	.....	.....	.....	18.5	25.5	.....	.....	.....	.....
28	0.589			.....	11.3	18.3	.....	30.5	38.5	.....	29.2	35.4	.....
35	0.417			.....	.....	.....	.....	42.5	51.5	.....	.....	.....	.....
48	0.295			22.2	26.7	43.4	.....	51.7	62.5	2.5	57.8	73.0	0.4
65	0.208			34.2	36.9	59.8	0.3	60.5	73.0	6.0	67.0	83.0	1.6
100	0.147			57.4	49.9	77.3	6.0	72.2	86.9	18.2	74.2	89.8	8.0
150	0.104			.....	.....	.....	13.3	76.0	91.4	27.6	.....	.....	.....
200	0.074			67.4	66.3	92.5	24.5	80.6	94.0	32.6	82.0	95.0	25.4
−200	0.074			32.6	33.7	7.5	75.5	19.4	6.0	67.4	18.0	5.0	74.6

Mill		Burro Mtn.			Miami			Timber Butte			Tonopah Belmont		
		Weight, cumulative percentages											
Column No.		49	50	51	52	53	54	55	56	57	58	59	60
Mesh	Aperture, mm.	F	S	O	F	S	O	F	S	O	F	S	O
3	6.680	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
4	4.699	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
6	3.327	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
8	2.362	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
10	1.651	.....	.....	.....	1.0	1.4	.....	.....	.....	.....	3.1	5.4	.....
14	1.168	0.4	0.1	.....	3.2	4.5	.....	.....	.....	.....	.....	10.6	.....
20	0.833	0.5	0.7	.....	7.1	9.7	.....	.....	.....	.....	9.3	15.9	.....
28	0.589	3.0	4.5	.....	15.5	21.5	.....	1.8	4.2	.....	12.2	21.8	.....
35	0.417	9.0	14.0	.....	25.0	34.9	.....	4.2	10.2	.....	15.7	26.7	.....
48	0.295	22.7	36.3	.....	37.0	48.8	.....	7.4	20.4	.....	20.0	33.2	.....
65	0.208	33.7	58.8	1.7	43.4	68.9	2.4	13.8	39.8	0.6	26.5	41.3	.....
100	0.147	46.5	80.3	3.7	61.8	83.1	19.7	30.2	70.2	8.6	42.5	61.3	1.0
150	0.104	55.1	91.1	10.4	71.5	86.2	36.4	48.8	85.6	27.8	63.6	85.7	13.5
200	0.074	60.3	94.2	16.6	76.0	92.6	44.6	62.6	93.2	44.8	73.2	93.5	27.7
−200	0.074	39.7	5.9	83.4	24.0	7.4	55.4	37.4	6.8	55.2	26.8	6.5	72.3

Mill		United Eastern			Miami			Nipissing			Calumet & Hecla		
		Weight, cumulative percentages											
Column No.		61	62	63	64	65	66	67	68	69	70	71	72
Mesh	Aperture, mm.	F	S	O	F	S	O	F	S	O	F	S	O
3	6.680	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
4	4.699	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
6	3.327	.....	.....	.....	0.1	0.1	.....	.....	1.8	.....	.....	.....	.....
8	2.362	.....	.....	.....	0.3	0.3	.....	.....	6.6	.....	.....	.....	.....
10	1.651	.....	.....	.....	0.9	0.9	.....	.....	13.6	.....	.....	.....	.....
14	1.168	.....	.....	.....	2.3	2.2	.....	.....	19.4	.....	.....	.....	.....
20	0.833	1.1	4.0	.....	4.2	4.7	.....	.....	25.1	.....	.....	.....	.....
28	0.589	4.5	9.4	.....	10.0	11.8	.....	.....	30.6	.....	1.0	1.5	.....
35	0.417	9.0	16.4	.....	18.7	23.0	.....	.....	37.0	.....	.....	.....	.....
48	0.295	14.8	24.0	.....	34.5	46.0	.....	.....	41.4	.....	.....	.....	.....
65	0.208	22.7	34.0	.....	54.9	71.3	3.7	.....	46.8	.....	.....	.....	.....
100	0.147	33.3	54.0	2.6	72.4	86.3	18.4	.....	59.6	0.4	74.0	88.5	.....
150	0.104	51.4	80.0	21.7	80.6	93.0	35.8	.....	67.8	0.8	.....	.....	.....
200	0.074	65.6	92.0	41.5	84.4	94.9	44.7	.....	88.7	8.0	.....	.....	5.0
−200	0.074	34.4	8.0	58.5	15.6	5.1	55.3	.....	11.3	92.0	25.0	10.0	95.0

F = Feed. S = Sand. O = Overflow.

Capacity depends primarily upon the size at which separation is to be made and upon the character of the ore. Table 41, summarized from Table 40 and from other data not sufficiently complete to be included in Table 40, gives the capacities to be expected with normal ores, specific gravity 2.6 to 2.7, and not containing an undue proportion of clayey material. Heavy ores and ores containing considerable clayey material must be run at a lower rate, if overflow is closely limited as to maximum size; clean silicious ores can be treated at higher rates.

Table 41. Capacity of Dorr classifiers

Size at which separation is made		Slope of bottom, inches per foot	Strokes per minute	Percentage of solids in overflow	Tons per 24 hr. per foot of width	
Mesh	Mm.				Sands raked	Overflow
20	0.833	3 –3.75	27–30	45–50	200–250	90–100
28	0.589	3 –3.5	27–30	40–45	150–250	60– 90
48	0.295	2.6–3	20–25	25–30	150–225	50– 80
65	0.208	2 –3	17–25	20–25	100–200	40– 60
100	0.147	1.5–2.5	10–15	15–20	50–100	20– 40
200	0.074	1.5–2	10–12	5–10	30– 75	10– 20

Power consumption ranges from about ¼ hp. for a standard simplex machine under light load to not more than 1 hp. for a heavy duplex under large load. When individual drives are provided, it is wise to install not less than 1-hp. motor for the light machine and 2-hp. for the large machine in order to take care of rushes of sand and starting.

Repairs are substantially negligible. The only wearing part is the rake. The life reported for rakes varies from 90 days, in acid water, to several years in service where corrosion is not a factor. The tank bottom is protected by a layer of sand. At some mills, *e.g.*, UTAH COPPER, the tank is made of reinforced concrete. At the PHELPS DODGE, Morenci plant, the usual steel tank is cement lined. The mechanism is designed for rake speeds as high as 30 strokes per minute and no serious wear or breakage is experienced at this and lower speeds. SHATTUCK ARIZONA, operating at 27 strokes per min. and heavy load, charges delays to these facts.

Character of products. There is a distinct relation between consistency of overflow pulp and size of particles therein, as shown in Table 40. With normal ores these figures cannot be varied greatly. If thicker pulps are wanted at a given size, some dewatering device must follow the classifier. The moisture content of the sand may be lowered somewhat beyond the figures in Table 40 by special provisions. The dewatering lip described on p. 597 is highly efficient in some cases.

Söhnlein (*96 J 215*) cites an instance where the moisture content of the sand was reduced from 60 to 12 per cent. by such a device. A porous bottom near the sand-discharge end, connected to a vacuum pump, has been used in special instances. A water spray playing on the sand just above the pulp level removes appreciable amounts of slime, if the sand bed is not too thick, and the cleaner sand will drain to a lower moisture content. Wood chips are removed from the overflow by a screen placed substantially horizontally above the overflow-discharge box. At Hollinger (*98 J 213*) a push rod with a scraper was attached to the rake mechanism to clean such a screen.

Dorr multi-deck classifier is shown in Fig. 39. The original pulp feed is introduced at (*a*) and the first deck operates in the usual manner, discharging sands into the second tank by means of the upward prolongation

of the bottom. The overflow lip of the second tank is at (b) under the prolonged bottom of the first tank. Sand is similarly re-washed in the third tank. Sprays are provided for additional washing of sands, if desired. The principal use of this device is in hydrometallurgical processes, such, for instance, as in washing leached sands in cyaniding or in leaching ores containing highly soluble copper compounds. Each machine is a special job, hence there is no limitation to standard sizes. Hardwood and gun-metal rakes have been used for resisting the chemical action of solutions. The tank is usually made of wood.

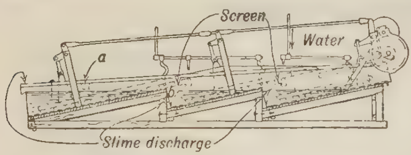


Fig. 39.—Triple-deck Dorr classifier.

Dorr bowl classifier (Fig. 40) is designed for separation at finer sizes than can be efficiently handled by the standard Dorr classifier, *viz.:* for 0.15-mm. and smaller. It consists of a shallow cylindrical pan or bowl (*a*), containing a thickener mechanism (*b*), (see Sec. 16, Art. 6), operated by belt from the classifier drive shaft, the whole mounted at the overflow end of the typical Dorr-classifier. A well, open at the bottom, extends from the floor of the bowl, at the center, into the classifier proper and terminates at a height that will clear the rakes. The sides of the classifier tank are high enough so that the level of liquid pulp in the tank can be carried above the level of the peripheral overflow lip of the bowl. Feed is introduced into a circular feed compartment at the center of the bowl. Liquid flows to the periphery and overflows, carrying the finest solid. Sands settle to the bottom and are raked to the central well by the thickener mechanism. Water, introduced through spray pipes (*d*), flows from the body of the classifier upward through the well and the solids settling from the bowl are subjected to a washing action that lifts slime back into the bowl for another chance to overflow. The sand that settles is raked up along the bottom of the classifier proper in the usual fashion. It is subjected to further working and washing by spray water which puts more slime into suspension to be carried back into the bowl.

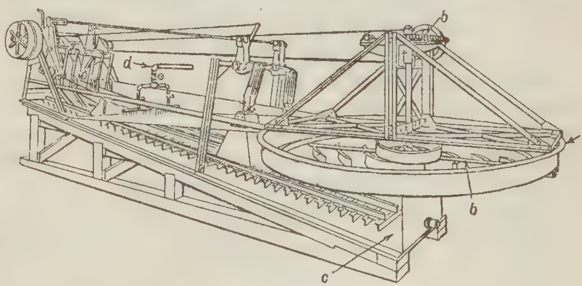


Fig. 40.—Dorr bowl classifier.

Size and speed. Bowls may be mounted on any size Dorr classifier. The diameter of the bowl depends upon the size of overflow product desired and the tonnage and dilution of the feed. Six- to 15-ft. diameter is the usual range. Rakes are operated at 12 to 25 strokes per minute, the same considerations governing as in the standard classifier. Too great speed, while tending to produce a very clean sand product, keeps a large amount of coarse material circulating between the bowl and the classifier proper and cuts down capacity proportionately. The revolving mechanism runs at 1.5 to 8 or more r.p.m. depending on the size of bowl and the amount of sand to be raked. Table 42 indicates that higher bowl speeds (in conjunction with higher rake speeds) increases capacity of the bowl and materially decreases the circulating load of finished material sent through the

grinding mill. The classifiers were 16-ft. bowls on 6 × 26⅔-ft. tanks, installed at the Springs mine, Witwatersrand; the ore makes very little slime. (*PP 1560-B A.*)

Table 42. Effect of speed on performance of bowl classifiers. (*After Bates*)

	Daily tonnage each classifier		Gradings, cumulative per cent.			
	Tons per 24 hr.	Percentage	+60	+90	+200	−200
Period *A*						
Reciprocating rakes 12 r.p.m.						
Bowl rakes 1.6 r.p.m.						
Enter bowl	800	100.00	0.26	10.68	38.10	61.9
Bowl overflow (slime)	454	56.70	....	0.70	8.60	91.4
Classifier underflow (sand)	346	43.30	0.50	19.40	76.80	23.2
Period *B*						
Reciprocating rakes 17 r.p.m.						
Bowl rakes 4 r.p.m.						
Enter bowl	679.2	100.00	0.26	11.40	37.12	62.88
Bowl overflow (slime)	461.7	68.00	....	0.60	12.40	87.60
Classifier underflow (sand)	217.5	32.00	0.80	34.30	89.60	10.40*a*

a Practically all sulphide.

Principle of operation is substantially the same as that of the standard classifier with the exception that the rising velocity in the bowl is not so much affected by the surge of pulp due to rake movement as is the overflow of the standard classifier. The speed of the bowl mechanism is not enough to cause centrifugal force to act effectively on the solid particles. The swirl of pulp aids in working sands in suspension toward the center by reason of the tendency of the solid particles to move toward the region of least velocity.

Performance. At Miami Copper Co. two machines having 13-ft. bowls mounted on 4½-ft. duplex classifiers treated flotation tailing ground to pass 48-mesh. The overflow was sent to waste and sand was concentrated on Plat-O sand tables. Sizing-assay tests of feed and products of one classifier are given in Table 43.

At Nipissing a bowl mounted on a simplex classifier treated 68 tons per 24 hr. of stamp-mill product, sending sand to a tube mill with tube-mill discharge returned to the classifier. The tube-mill circuit was 74 tons per 24 hr. The classifier overflow contained 8 per cent. solids and the sands 20 per cent. moisture. Screen tests of feed and products are given in Table 44. This classifier ran in parallel with standard 4½-ft. duplex classifiers. (See Table 40 for performance of the duplex machines.) The overflow of the primary classifier went to 4½-ft. duplex classifiers with 15-ft. bowls. The original feed to each was 120 tons per day; tube-mill return, 52.8 tons.

Treating a clay at the rate of 1.4 tons per sq. ft. of bowl area per 24 hr. in a 3-ft. bowl, with about 5 per cent. solids in the overflow, the latter contained 98.3 per cent. −330-mesh material. In this work the feed contained about 0.1 per cent. +100-mesh, sand discharge about 12 per cent. −330-mesh. Rake speed was 7 strokes per min.; revolving arms, 2.5 rev. per min.; slope, 2 in. per ft.

At the plant of the Benjamin Moore Co. (*116 J 418*) a 10-ft. bowl on a 2-ft. 3-in. × 18-ft. simplex classifier treats ground chalk rock. The rakes are run at 8 strokes per min., bowl rakes at 1.5 r.p.m., overflow contains 5.5 per cent. solids. Sizing tests of the products are given in Table 45. At St. Joseph Lead Co., Bonne Terre plant, a 6-ft. duplex classifier with 12-ft. bowl was used to dewater 2200 tons per 24 hr. of combined jig and table tailing. A sizing test of the dewatered product is given in Table 46. The rakes were run at about 30 strokes per min.; bottom slope, 2¼ in. per ft. The feed contained 65 per cent. water; the sand, 16 per cent.; the overflow, 98.5 per cent. Table 47, from the Dorr Co. catalog, shows results with several different bowl diameters treating a roasted ore. The capacities are probably considerably higher than on unroasted ore. At the Mesabi Iron Co. (see p. 148) a 9-ft. bowl in closed circuit with an 8-ft. × 22-in. conical ball mill overflows 8.6 tons per hr.; the sand circuit is 52.6 tons per hr.; sizing tests are shown in Table 48.

Table 43. Performance of bowl classifier at Miami Copper Co.

Product	Feed		
Tons solid per 24 hr	552		
Assay, per cent. of total Cu	0.235*a*		
Assay, per cent. oxide Cu	0.092		
Assay, per cent. sulphide Cu	0.143		
Moisture, per cent	. . . .		
Screen aperture, mm.	**Weight, cumulative per cent.**	**Assay, per cent. Cu**	**Per cent. total Cu**
0.295	0.9	. . . .	. . . .
0.208	6.6	0.38	10.4
0.147	19.7	0.36	20.2
0.104	28.4	0.25	9.3
0.074	42.3	0.22	13.1
−0.074	57.7	0.19	47.0

Product	Sand		
Tons solid per 24 hr	120		
Assay, per cent. of total Cu	0.375*a*		
Assay, per cent. oxide Cu	0.063		
Assay, per cent. sulphide Cu	0.312		
Moisture, per cent	30		
Screen aperture, mm.	**Weight, cumulative per cent.**	**Assay, per cent. Cu**	**Per cent. total Cu**
0.295	3.2	0.41	3.5
0.208	28.4	0.34	23.0
0.147	69.2	0.33	36.1
0.104	82.2	0.41	14.3
0.074	92.9	0.52	14.9
−0.074	7.1	0.43	8.2

Product	Overflow		
Tons solid per 24 hr	432		
Assay, per cent. of total Cu	0.165*a*		
Assay, per cent. oxide Cu	0.110		
Assay, per cent. sulphide Cu	0.055		
Moisture, per cent	72.3		
Screen aperture, mm.	**Weight, cumulative per cent.**	**Assay, per cent. Cu**	**Per cent. total Cu**
0.295	0.1	. . . .	. . . .
0.208	0.6	. . . .	. . . .
0.147	5.5	0.16	5.5
0.104	13.4	0.14	6.9
0.074	29.1	0.13	12.7
−0.074	70.9	0.17	74.9

a Lack of balance due to inaccuracy in sampling.

Table 44. Sizing tests of feed and products of Dorr bowl classifier at Nipissing

Classifier	Legend	Weights, cumulative per cent.												
	Screen aperture, mm.	3.327	2.362	1.651	1.168	0.833	0.589	0.417	0.295	0.208	0.147	0.104	0.074	−0.074
Simplex, primary	Original feed to classifier	1.7	5.9	12.5	18.2	24.2	30.8	38.7	44.7	50.8	57.8	60.0	65.0	35.0
	Tube-mill return	.....	.....	.....	.....	.....	.....	.....	0.2	0.8	4.9	12.4	32.5	67.5
	Classifier feed, total(*c*)	0.8	2.8	6.0	8.8	11.7	14.8	18.6	21.5	24.7	30.2	35.5	48.0	52.0
	Sand	2.5	8.8	7.4	23.8	29.9	35.6	41.6	45.9	50.1	59.2	65.9	88.2	11.8
	Overflow	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	0.2	3.4	96.4
Duplex, secondary, 15-ft. bowl	Original feed to classifier	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	10.6*a*	89.4
	Tube-mill return	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	14.6*a*	85.4
	Classifier feed, total(*c*)	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	11.8*a*	88.2
	Sand	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	60.3*a*	39.7
	Overflow	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	1.3*a*	98.7*b*

a, All +0.74-mm. material. *b*, Contains 82.3 per cent. slime. *c*, Calculated.

Table 45. Products of bowl classifier treating chalk

Screen aperture, mesh	Weights, cumulative per cent.	
	Sand	Overflow
+10	0.8	....
35	2.1	....
65	6.5	....
100	19.7	....
150	32.8	....
200	50.0	0.6
300	62.6	5.0
325	67.5	13.1
−325	32.5	86.9

Table 46. Screen test of sand dewatered in bowl classifier at the St. Joseph Lead Co.

Screen aperture, mm.	Weight, cumulative per cent.
6.680	3.7
4.699	17.4
3.327	29.9
2.362	43.9
1.651	52.2
1.168	61.2
0.833	70.7
0.589	79.5
0.417	85.5
0.295	90.5
0.208	94.0
0.147	97.0
0.104	99.0
0.074	99.3
−0.074	0.7

Table 47. Performance of Dorr bowl classifier treating roasted ore at Golden Cycle mill

Diameter of bowl, ft.	Tons solid per 24 hr.		Dilution overflow, per cent. solids	Tons back-water added per ton solid feed	Screen analyses — Weight, cumulative per cent. — Overflow				Sand							
	Feed	Overflow			+100	+150	+200	−200	+30	+40	+60	+80	+100	+150	+200	−200
4.83	400	150	14.1	0.31	2.0	5.5	20.5	79.5	4.0	15.5	55.3	60.8	74.0	86.8	95.5	4.5
4.83	350	140	13.2	0.34	1.5	4.0	17.0	83.0	3.5	13.0	52.0	56.0	70.0	83.5	94.2	5.8
6.0	500	200	14.9	.....	.....	1.0	15.0	85.0	8.5	27.5	59.5	67.5	81.5	94.0	96.0	4.0
6.0	400	150	13.2	0.67	.....	1.0	8.0	92.0	5.5	20.5	51.0	60.5	76.0	93.9	96.0	4.0
11.0	880	350	12.5	0.48	1.2	2.8	12.9	87.1	7.3	22.8	57.7	60.1	85.8	88.1	96.4	3.6
11.0	1000	400	12.5	0.42	1.4	5.3	13.0	87.0	6.7	20.5	60.0	61.4	77.4	88.6	96.1	3.9
11.0	1000	390	11.9	0.42	2.7	3.5	12.3	87.7	6.1	21.0	59.3	61.8	85.5	88.8	95.6	4.4
11.0	800	310	9.7	0.96	2.5	4.3	15.6	84.4	6.3	20.3	58.6	.....	84.1	86.4	96.4	3.6
11.0	700	245	12.5	0.60	0.1	0.5	5.5	94.5	5.5	18.5	50.0	55.5	69.0	82.5	94.5	5.5
11.0	800	290	11.1	0.93	0.6	1.7	7.7	92.3	6.3	20.3	54.6	57.2	75.3	86.2	96.1	3.9
11.0	580	205	11.0	0.72	.....	0.5	2.6	97.4	6.7	21.8	53.2	53.2	73.2	84.2	96.0	4.0
15.0	1300	520	9.9	.....	.....	0.5	4.3	95.7	14.8	31.3	58.5	62.5	73.1	84.7	94.6	5.4

Capacity and dilution. Capacity is dependent principally on the area of the bowl and the separating size.

Miami overflows 3.6 tons per sq. ft. per 24 hr. at 65-mesh separation; Golden Cycle, from 3 to 8 tons at 100-mesh and 2.2 to 7 tons at 150-mesh separation; at Nipissing about 1 ton per sq. ft. is overflowed with 200-mesh separation. The test on clay showing 1.5 tons per sq. ft. at 330-mesh separation was made on a feed containing relatively little coarse material so that the sand could be handled with low rake speed.

Table 48. Sizing tests of feed and products of bowl classifier at Mesabi Iron Company

Mesh	Feed, per cent. weight	Sand, per cent. weight	Overflow, per cent. weight
+14	60.01	11.41	....
+48	21.84	21.77	....
+100	4.85	29.41	4.66
+150	1.89	16.32	5.25
+200	0.44	9.82	7.37
+350	3.38	6.83	32.23
−350	7.59	4.44	50.49

Dilution for 65- to 100-mesh overflow should be 20 to 25 per cent. solids, for 200-mesh about 10 per cent. and for 300-mesh 5 to 6 per cent.

Use of chemical agents. (See also Sec. 16, Art. 4.) The settling rate of fine particles is markedly affected by certain chemical agents, some of which cause flocculation, others dispersion. When separation is to be made at 200-mesh or finer the effect of these reagents on the products is great. Flocculation increases the settling rate of the fine particles and coarse particles become entangled in the flocs, hence overflow will be finer and more fine material will be carried into the sand. Dispersion, on the other hand, increases the amount of slime that is carried over in the water going with the sand discharge, and may result in a coarser overflow than with flocculated pulp. In every case the result is a matter of experiment. It must be realized, however, that the chemicals necessarily present in cyanide treatment will affect settling and that preliminary grinding and settling-tests must, therefore, be made in solutions approximating those to be met in the operating mill.

Akins classifier (Fig. 41) consists essentially of a spiral ribbon revolving in a semi-cylindrical inclined tank closed at the lower end. The spiral is single and continuous from discharge end to the point at which the ribbon does not enter the settling pond; above this the spiral is double and interrupted. The principle of operation is the same as that of the Dorr classifier. For very fine separation, *i.e.*, at 150- or 200-mesh a larger settling pond is obtained in a submerged type of machine in which the tank is flared at the overflow end and the overflow lip raised so that the lower bearing must be carried in a stuffing box. For coarse overflow, increased agitation is affected by lessening the pitch of the standard spiral toward the overflow end and joining adjacent parts of the spiral ribbon by straps which serve to stir up the material settled in the pond and cause fine sand to overflow.

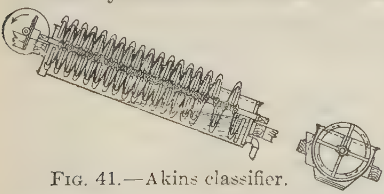


Fig. 41.—Akins classifier.

Performances, as given by the manufacturers, are shown in Table 49. Ten 54-in. machines were used at United Comstock to de-slime 2000 tons per 24 hr. of − ½-in. primary feed. (*114 J 850.*) At U. S. S. R. & M. Co., Midvale plant, a 30-in. machine was used to dewater 45 tons per 24 hr. of sands passing a 0.020-in. screen and containing no slime. Bottom slope was 2¼ in. per ft.; the spiral revolved 13 times per min. Feed contained 72 per cent. water; sand, 19 per cent.; the overflow was substantially clear water. Life of spiral, 518 days. At the Royal Asturiana Mining Co. plant (*115 J 396*) a 24-in. machine treating − 2-mm. feed (sp. gr. of solid 3.0 to 3.5) had to be run at 30 r.p.m. on a

Table 49. Performance of Akins classifier

Screen aperture, mesh	Weights, cumulative per cent.											
	48-mesh separation			65-mesh separation			80-mesh separation		100-mesh separation		150-mesh separation (a)	
	Feed	Sand	Overflow	Feed	Sand	Overflow	Sand	Overflow	Sand	Overflow	Sand	Overflow
20	24.2	58.5	.....	.....	.....	.....	.....	.....	.....	.....	4.5	.....
28	.....	.....	.....	28.0	60.2	...	.....	.....	.....	.....	19.0	.....
35	35.4	85.5	0.9	33.8	72.5	0.3	.....	.....	.....	.....	37.9	.....
48	40.3	92.1	3.5	39.2	81.8	1.8	.....	.....	.....	.....	56.0	.....
60	.....	.....	.....	.....	.....	.....	63.3	.....	57.0	.....	.....	.....
65	43.8	95.0	7.4	44.4	88.0	5.6	.....	.....	.....	.....	71.7	.....
80	.....	.....	.....	.....	.....	.....	82.6	0.4	69.8	.....	.....	.....
100	48.1	96.7	13.6	51.6	92.4	14.0	87.1	2.0	79.8	0.2	83.5	.....
150	52.7	98.5	20.8	57.8	94.6	22.9	.....	.....	.....	.....	92.2	1.2
200	55.8	98.8	25.7	61.4	95.6	29.4	97.7	28.4	94.2	11.5	96.2	6.0
−200	44.2	1.2	74.3	38.6	4.4	70.6	2.3	71.6	5.8	88.5	3.8	94.0

a Submerged type.

slope of 3 in. per ft. in order to obtain −48-mesh overflow when making 65 tons overflow per 24 hr. and under such conditions the sand contained 30 per cent. of undersize (−55-mesh). Sizing tests of feed and overflow are given in Table 50.

Effect of solutes on performance. Stowell (*117 J 362*) reports that the addition of 1 lb. sodium sulphide per ton of solid to the feed of an Akins classifier made it substantially impossible to overflow sand, with the result that, if the spiral was run at the speed for normal operation, sand built up in the settling basin until unclassified sand overflowed. Table 51 shows the effect on a 30-in. machine sloped 2 in. per ft. and making 12 r.p.m. when running in closed circuit with a 5 × 4-ft. ball mill.

Table 50. Performance of 24-in. Akins classifier at Royal Asturiana Mining Co.

Screen aperture	Weights, per cent.	
	Feed	Overflow
+2-mm.	0.6	....
1 mm.	3.4	....
55-mesh	27.5	3.0
100-mesh	27.0	14.5
−100-mesh	41.5	82.5

Table 51. Effect of solutes on performance of Akins classifier.

Screen aperture, mesh	Overflow, weight per cent.	
	Normal pulp, no solute	Sodium sulphide, @ 1 lb. per ton
+60	11.0	....
100	12.9	1.4
150	6.2	3.7
200	5.4	3.8
−200	64.5	91.1

Drag classifier (Fig. 42), sometimes called ESPERANZA CLASSIFIER, consists essentially of an endless band (*a*), which carries flights (*f*) and runs on wheels (*b*) and (*c*), all mounted as shown in a trough (*d*) with inclined bottom. The band (*a*) may consist of link chain or rubber belt. Feed enters the trough at the lower end, water carrying fine solids in suspension overflows the sides of the trough, sand settles to the bottom, is dragged up-slope by the flights, and discharged at the upper end. The principle of operation is the same as that

of the Dorr classifier and the adjustments to effect variation in character of products are similar.

Performances at several mills are given in Table 52.

In another type of drag classifier the sand settles on the drag belt and is carried by it above the pulp surface; slime overflows in the usual fashion. The disturbance is less than in the ordinary type and finer sand is, therefore, collected. At INSPIRATION two 18-in. belts are set side by side in one tank; the distance center to center of head pulleys is 39 ft.; the vertical distance from the overflow lip to the lower run of the belt is 5 ft. The machines are used to separate sand, for tabling, from flotation tailing (see Sec. 2, Fig. 61).

Dorr hydro-separator is a Dorr thickener of relatively small diameter fed at such a rate that slime instead of clear liquor overflows. It is used in the anthracite fields to separate −200-mesh slime from breaker slush, the spigot product being dewatered and used for mine fuel while the overflow is sent to a Dorr thickener and dewatered. Performance at one plant is shown in Sec. 16, Table 1.

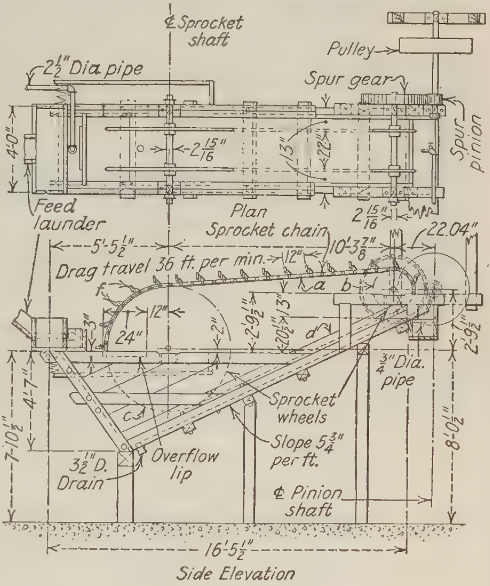


FIG. 42.—Drag classifier.

7. Comparison of classifiers

For separating sands into several grades hydraulic classifiers are far superior to all other types, principally on account of the fact that slimes are substantially excluded from the spigot products. Their great disadvantages are the large water consumption and the excessive dilution of the slime overflow. Hindered-settling classifiers yield spigot products better adapted to table concentration than those of free-settling classifiers, on account of the greater ratio of diameters of light- to heavy-mineral grains in the spigot products, but this ratio in most cases is far below the theoretical figures and below experimental results. The Richards-Janney is the most successful of the hindered-settling machines. Its principal advantages are steadiness of operation due to prevention of banking by the stirring mechanism, low water consumption by reason of the intermittent spigot discharge, and low moisture content of spigot product, which permits close control of the moisture content of the feed to the tables following. Launder-type machines have two great advantages over the tank type, *viz.:* (1) that the spacing of the sorting columns

Table 52. Performance of drag classifiers

Plant	Braden			Braden			Braden			Braden			Federal M. & S. Co.			Bunker Hill and Sullivan Mg. Co.		Shattuck-Arizona		Mexican cyanide plant		Federal Lead Co.		Federal Lead Co.		
Size	*a*			*a*			*c*			*d*			36-in.			4′–10″ × 22′-6″ tank(*l*)		*m*		4.5 × 20		See Fig. 42		See Fig. 42		
Slope of bottom, inches per foot	...			...			...			...			6			4		2		3¾		...		...		
Rake spacing, in.	18			18			18			18			12			...		12		...		...		...		
Belt speed, feet per minute	40			40			24–40			28			36			17		58		18		18		30		
Power installed, hp.	...			...			...			10			3			...		...		5		4		...		
Tons of solid feed per 24 hr.	833			1030			721			983			150			350		250		225–275		300		700		
Product	*F*	*S*	*O*	*F*	*S*	*O*	*F*	*S*	*O*	*F*	*S*	*O*	*F*	*S*	*O*	*S*	*O*	*S*	*O*	*S*	*O*	*S*	*O*	*F*	*S*	*O*
Moisture, per cent.	50	20	70	40	18	72	55	20	74	71	62	76	49	31	82	20	96.4	36	76	...	...			77	20	94
Sizing test, cumulative per cent.																										
On 6.680 mm.	2	3	...	10	12	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	All through 2-mm.	De-sliming service	All through 12-mm.	...	Dewatering service
4.699	4	6	...	15	18	...	1	1	...	...	1	...	...	...	...	...	...	...	...	...	...				...	
3.327	7	10	...	21	25	...	2	2	...	...	2	...	...	...	...	...	...	9.8	...	...	...				...	
2.362	11	16	...	28	33	1	3	4	...	...	4	...	...	...	...	...	...	14.8	...	...	...				...	
1.651	20	29	...	37	44	2	5	7	...	...	10	...	...	...	...	...	...	20.1	...	...	...				...	
1.168	28	41	...	45	54	4	8	12	...	...	18	...	...	...	...	...	...	24.4	...	...	...				...	
0.833	38	56	...	55	66	7	14	21	...	2	29	...	0.4*e*	0.4*e*	...	...	...	29.6	...	...	...				...	
0.589	48	71	...	66	79	11	25	37	1	4	46	...	...	...	...	7.6	...	35.6	...	...	...				...	
0.417	56	81	2	74	87	16	39	58	3	8	63	1	6.8*f*	9.6*i*	...	15.9	...	44.2	...	...	...				...	
0.295	63	88	8	79	92	25	53	76	9	16	76	4	...	...	...	28.7	...	54.9	...	...	...				...	
0.208	67	91	14	83	95	32	62	86	16	26	84	8	26.0*g*	41.6*g*	1.4*g*	41.8	...	67.9	3.2	...	...				...	
0.147	33*b*	9*b*	86*b*	7*b*	5*b*	68*b*	70	92	27	74*b*	16*b*	92*b*	34.6*h*	54.8*h*	1.8*h*	60.3	...	79.7	9.8	...	...				...	
0.104	...	...	...	...	...	...	74	94	35	...	...	...	44.8*i*	68.4*i*	5.2*i*	70.8	...	86.2	15.0	...	...				...	
0.074	...	...	...	...	...	...	76	95	40	...	...	...	54.8*j*	85.6*j*	12.6*j*	87.5	5.6	92.8	29.6	...	...				...	
−0.074	...	...	...	...	...	...	24	5	60	...	...	...	45.2*k*	14.4*k*	87.4*k*	12.5	94.4	7.2	70.4	...	75.0	...	...	...	...	...

a 42 × 82-in. settling area. 24 ft. center to center of sprockets; blades 3¼ × 32 × ¾-in. *b* −0.208-mm. *c* 42 × 90-in. settling area; 24 ft. center to center of sprockets; blades 3 × 36 × ½-in. *d* 48 × 150-in. settling area; 36 ft. center to center of sprockets; blades 4 × 42 × ½-in. *e* 20-mesh *f* 40-mesh. *g* 80-mesh. *h* 100-mesh. *i* 160-mesh. *j* 240-mesh. *k* −240-mesh. *l* Centrally-placed pan 26 × 60-in. for overflow making total length of overflow lip 172 in. Blades of wood, 4 × 1 × 56-in., steel-shod. *m* 30-in. overflow, 12-in. belt.

F = Feed *S* = Sand. *O* = Overflow.

may be adapted to the location of the concentrators following, thus eliminating a maze of launders radiating from a group of closely-spaced spigots; and (2) saving in head room. It is more difficult, however, to keep slime out of the spigot product of the launder classifier and, if slime is present in the feed, more coarse sand is carried into the later spigots. It is fairly well agreed that de-sliming the feed to any type of hydraulic classifier improves the performance of the machine and also separates a much less dilute slime.

For sand-slime separation and for dewatering sands, mechanical classifiers are most widely used, but diaphragm cones, particularly of the automatic type, are strong competitors. When the service required is to de-slime grinding-mill discharge and return sand to the mill, mechanical classifiers have unquestioned advantage because of the fact that they do their own elevating of the sand in the course of the separation, while cones require separate elevators. Cones require no power for their operation, but cause considerable loss of head room for the sand product, and the power required for re-elevation, when necessary, must be charged to the cone. This power will usually exceed that necessary to operate a mechanical classifier. Further, the gritty character of the material to be elevated causes excessive wear on all kinds of elevating apparatus.

Bates (*73 A 239*) gives a comparison of Caldecott cones, Dorr duplex classifiers and Dorr bowl classifiers at the GEDULD mill, Witwatersrand. The classifiers were working in closed circuit with tube mills, but the tests were not strictly parallel; reduced to terms of performances with a 5½ × 22-ft. tube mill, the results were as follows:

Classifier	Tons produced by tube mills per 24 hr.	
	−90-mesh	−200-mesh
Cones	140.0	109.5
Dorr classifier	160.3	127.1
Bowl classifier	163.4	137.9

On the basis of separating efficiencies alone, cones are generally superior to mechanical classifiers; they give a sand product containing less undersize and, in general less tramp oversize in the slime. They are also extremely efficient in dewatering. Comparative efficiencies are given in Table 53.

Comparison of mechanical classifiers. On the basis of the figures in Table 53, it would appear that the Akins yields the cleanest sands and that the overflow of the Dorr contains the least oversize. The chain-drag is the poorest on both counts. The efficiency of the Akins is best, so far as the tests given in the table go, but the number of performances included is small and these are presumably the best. Table 54, published by J. V. N. Dorr (*14 CME 296*) in answer to a generalization similar to that above made, seems to show that under strictly competitive conditions the superiority of the Akins sand is obtained at serious sacrifice of the character of overflow and that when the tonnage to the Akins is reduced to the point where overflows are of the same general character in both Akins and Dorr machines, the sand products are likewise similar and the capacity of the Akins much less than that of the Dorr.

Power consumption is so small for all types as to be an unimportant factor in the comparison; it is greatest in the chain-drags. Wear is greatest in the drags and least in the Dorr but is not excessive in any of the three. Bearings are furthest from the pulp in the Dorr, which is a point of considerable impor-

Table 53. Comparative efficiencies of sand-slime separators

Machine	Mill	Water: Per cent. in sand	Water: Per cent. of total in overflow	Separating size	Per cent. undersize in sand product	Per cent. oversize in overflow	Efficiency: At separating size, per cent.	Efficiency: At 200-mesh, per cent.
Caldecott, 2-diaphragm cone....	Simmer and Jack........	.....	.....	0.074-mm.	13.7	2.0	82.2	82.2
Caldecott, 2-diaphragm cone....	Springs.........	.....	.....	90-mesh	18.5*a*	0.4	66.6	72.6
Caldecott, 2-diaphragm cone....	Mexico.......	34	98.0	80-mesh	15.0	9.5	75.0	72.5
Automatic diaphragm......	Mt. Lyell.....	.....	.....	120-mesh	8.3	2.5	60.0	......
Automatic diaphragm......	Mt. Lyell.....	....	....	90-mesh	8.8	0.4	48.2	......
Allen............	Mo. lead......	12	97.3	Dewatering conc.			......	......
Allen............	Shannon......	.....	....	0.074-mm.	12.8	1.9	67.1	67.1
Allen............	Shannon......	.....	.....	0.208-mm.	26.1	1.6	65.0	88.7
Allen............	Mine La Motte	.....	.....	0.147-mm.	2.4	0.4	82.8	90.8
Allen............	Mine La Motte	.....	.....	0.104-mm.	9.3	3.8	87.2	85.0
Boylan..........	Joplin........	.....	.....	0.833-mm.	11.9	0.7	54.5	76.0
Dorr............	United Eastern	20	93.7	0.833-mm.	31.8	2.8	69.6	50.8
Dorr............	Shattuck-Arizona.....	30	.....	0.589-mm.	48.8	1.0	41.3	46.1
Dorr............	Miami........	23.4	39.3	0.295-mm.	25.0	7.8	59.9	61.6
Dorr............	Miami........	22.7	72.2	0.295-mm.	20.8	0.3	66.0	72.6
Dorr............	Miami........	23.6	38.3	0.295-mm.	40.8	4.7	43.7	50.0
Dorr............	Miami........	24.5	73.4	0.295-mm.	17.8	7.8	71.5	68.6
Dorr............	Moctezuma...	20	27.1	0.295-mm.	22.0	5.5	22.4	66.9
Dorr............	Inspiration....	21.9	19.6	0.295-mm.	17.7	6.6	63.4	67.6
Dorr............	Chino........	30	66.5	0.295-mm.	16.5	1.5	53.5	71.3
Dorr............	Engels........	17	73	0.295-mm.	43.3	0.7	60.5	70.2
Dorr............	Engels........	17	73	0.295-mm.	84.6	0.2	38.0	17.7
Dorr............	Cons. Arizona.	20	.....	0.295-mm.	44.2	3.6	2.4	20.0
Dorr............	Afterthought.	15.6	50.8	0.295-mm.	37.5	2.5	35.5	67.0
Dorr............	Inspiration....	25.4	.....	0.208-mm.	17	1.6	58.2	71.7
Dorr............	Burro Mtn....	.....	.....	0.208-mm.	41.2	1.7	63.0	79.7
Dorr............	Miami........	20.6	.....	0.208-mm.	31.1	2.4	64.1	59.6
Dorr............	Timber Butte..	31.0	90.4	0.208-mm.	60.2	0.6	73.6	48.1
Dorr............	Tonopah Belmont.....	35	.....	0.147-mm.	38.7	1.0	53.0	71.5
Dorr............	United Eastern	20	93.8	0.147-mm.	46.0	2.6	55.7	55.9
Dorr............	Miami........	25.6	33.3	0.208-mm.	28.7	3.7	48.2	63.0
Bowl...........	Miami........	30	.....	0.147-mm.	30.8	5.5	69.8	52.6
Bowl............	Nipissing.....	20	.....	0.074-mm.	11.8	3.4	84.6	84.6
Bowl............	Nipissing......	.....	.....	0.074-mm.	39.7	1.3	83.0	83.0
Bowl............	St. Joseph Lead	16	90	Dewatering jig and table conc.				
Bowl...........	Springs.......	.....	.....	90-mesh	10.5*a*	0.5	71.6	71.3
Drag............	Braden	20	84	0.417-mm.	19	2	69.4	......
Drag............	Braden.......	18	73.4	1.651-mm.	56	2	25.0	......
Drag............	Braden.......	20	87.2	0.589-mm.	63	1	42.7	68.3
Drag............	Braden.......	62	68.8	0.417-mm.	37	1	84.5	......
Drag............	Federal M. & S.	31	59.1	80-mesh	58.4	1.4	49.6	......
Akins...........	U. S. S. R. & M.	19	90.9	Dewatering, −0.02-in. sand				......
Akins...........	Royal Asturiana....	.....	.....	55-mesh	30	3	75.9	......
Akins...........	Manufacturer..	.....	.....	0.295-mm.	7.9	3.5	89.4	71.8
Akins...........	Manufacturer..	.....	.....	0.208-mm.	12.0	5.6	83.1	69.8

a −200-mesh.

tance. The drag type has the distinct advantage for small or temporary plants that it can be built locally and will probably cost less installed than either of the others. Local construction also permits considerable latitude in area of settling pond and length of draining space. In de-sliming service, the drag type is usually reported as less efficient than either of the other types.

The bowl-type is superior to all of the others for separation at 100-mesh or finer.

Table 54. Comparison of Dorr and Akins classifiers. (*After Dorr*)

Test number...	1						2			
Classifier......	Dorr			Akins			Dorr		Akins	
Tons per 24 hr..	145			145			145		75	
Cumulative per cent. on screens	*F*	*S*	*O*	*F*	*S*	*O*	*S*	*O*	*S*	*O*
40-mesh	12.8	34.7	......	9.4	36.4	......	33.1	......	26.7	......
60-mesh	39.6	63.8	......	29.7	78.5	......	62.8	......	60.8	......
100-mesh	48.9	78.0	0.2	41.3	88.4	14.3	75.3	0.9	76.1	2.9
200-mesh	62.3	91.5	8.9	52.9	95.8	31.8	91.5	6.7	92.9	14.1
−200-mesh	47.7	8.5	91.1	47.1	4.2	68.2	8.5	93.3	7.1	85.9

F, Feed. *S*, Sand. *O*, Overflow.

SECTION 7

HAND SORTING

1. Introduction

Hand sorting or HAND PICKING is manual removal of selected grades of material from a mass of broken ore. A concentrate of high-grade or SHIPPING ORE is one of the grades commonly selected. Such material may be worth more per ton, on account of its size, than mill concentrate, and, if taken thus early, is not subject to the danger of loss attendant on further treatment. Tailing or WASTE for rejection is likewise frequently made. The advantage of this practice is increased capacity from a given mill equipment, increase in the possible mining rate, and reduction in wear on mill equipment. Even if increased capacity is not utilized, there may be marked increase in efficiency because of the reduction in load on the mill. Sometimes both shipping ore and waste are picked at the same operation and a third class, *viz.:* MILLING ORE, requiring mechanical treatment, is the residue. High-grade complex ores may be picked into several classes; as many as sixteen have been made at one time at CLAUSTHAL (*4 SMQ 196*). Such close work requires breaking with hammers in addition to the actual sorting. Breaking with heavy long-handled hammers is called SLEDGING; further breaking of sledged material with light long-handled hammers is called SPALLING and final breaking with light short-handled chisel-peen hammers is called COBBING.

Sledging, spalling and cobbing are rarely practiced in the United States except around prospects and one-man mines, but they are established practice in many foreign countries. At certain CORNWALL mines picking and cobbing produce (*a*) rich copper pyrite, (*b*) coarse, rich galena, and (*c*) coarse rich blende, all of which are sent directly to the smelters; (*d*) fine tin oxide, (*e*) pyritiferous milling ore, (*f*) waste. At CLAUSTHAL (1880–4) six products were made from the original ore, *viz.:* (*a*) copper pyrite, nearly pure, (*b*) mixed copper-iron pyrite with copper predominating, (*c*) the same with iron predominating, (*d*) iron pyrite, nearly pure, (*e*) pyritiferous milling rock, (*f*) pyrite-chalcopyrite-galena-blende-gangue middling. The first four were sold to smelters, the fifth milled and the sixth further cobbed by expert workmen, producing: (*g*) galena, (*h*) mixed copper and iron pyrite (distributed between (*b*) and (*c*)), (*i*) intergrown pyrite and galena, (*j*) galena milling rock, (*k*) pyritiferous milling rock, (*l*) mixed pyrite and blende. Lots (*i*), (*j*) and (*k*) were accumulated and milled separately; (*l*) was further cobbed. These are extreme cases, but they result in salable products from an ore from which such products can be made only with extreme difficulty, if at all, by the most refined of modern concentrating methods.

When the ore-treatment process is chemical, sorting may be resorted to to remove deleterious substances, such as those that consume chemicals, hinder settling and filtration, absorb and carry valuable solutes into tailing, etc.

Apart from the necessity for removing refuse such as rope, wood, steel, dynamite, etc., from any mill feed and dangerously deleterious substances from feed to chemical ore-treatment processes, the decision as to the advisability and extent of sorting is purely economic. The cheaper the labor and the more inefficient and expensive the mechanical treatment, the further sorting can be carried and *vice versa*. Undoubtedly sorting could be introduced with

advantage in many plants, but just as undoubtedly it is being practiced in some places where economy demands its discard or curtailment. For investigation of the economics of sorting, see Sec. 22, Art. 19.

Sorting of some kind is a part of every mining and ore-treatment operation. In narrow ore bodies with distinct walls, much of the country rock that is unavoidably broken in mining is sorted out underground and left or used for filling. In certain mines in which the ore contains segregated masses of pure valuable mineral, as in some of the Lake Superior native-copper deposits, the valuable mineral is picked out underground and sent to the surface separately. In general, however, underground sorting is uneconomical on account of restricted working places, poor lighting, poor presentation of material and the obscuring effect of the fine dirt present. Some sorting to remove wood, rope ends, powder and tramp steel is done ahead of the primary crusher in practically all mills. This work is, however, incidental to crusher operation and is hardly to be considered as a part of the general problem.

When valuable mineral occurs in ore as coarse aggregates, or when considerable waste is mined with the ore and the mineral and gangue or ore and waste are readily distinguishable by eye, the economics of sorting should always be investigated.

2. Sorting surfaces

Sorting is performed on floors, stationary tables and grizzlies and on various sorts of moving surfaces such as revolving tables, conveyors of the pan or belt variety (see Sec. 20) and shaking surfaces such as shaking feeders or shaking screens or grizzlies. In modern practice in United States and on the Rand the material fed to sorting surfaces is prepared mechanically and there is little or no breaking during sorting, but in European and some Latin American mills considerable breaking (spalling and cobbing) of the ore is done during sorting.

Floors are used for sorting where labor is cheap or where spalling and cobbing are practiced. In its crudest form a sorting floor is any level space with a surface that can be swept up thoroughly, on which the ore to be sorted is dumped and picked over. The usual practice in such sorting yards is to let the work on contract, assigning to each contractor a certain space on the floor, delivery of feed and collection of products being made by the company. With coarsely-aggregated complex sulphide ores, such as the German and Austrian lead-zinc-iron ores, Cornish tin ores and certain Bolivian tin ores, it is possible by such methods to get out high-grade salable concentrates that could not be separated, or could be separated only at great expense, after grinding. The costs of such work are so variable and so dependent on the ore, the products required and on local custom as to be entirely unreliable for quotation. Sorting floors have been carried to their highest development at some of the Rand gold mines.

Fig. 1 shows one arrangement. The floor was covered with ½-in. steel plate.

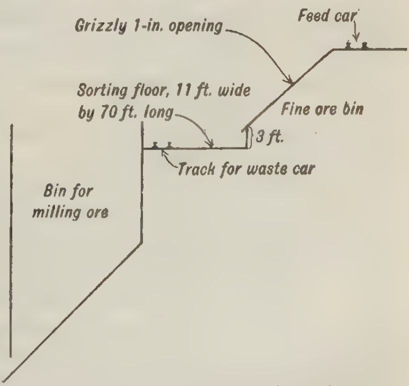


FIG. 1.—Arrangement of sorting floor at a South African cyanide plant.

About 550 tons per 24 hr. was fed over the grizzlies by two men per shift. Nine men per shift on the floor dragged the oversize out with heavy 2-pronged rakes, washed it, did some sledging, sorted out about 70 tons of waste per shift, and threw this into cars on the floor, while coarse milling ore was shoveled over into the coarse ore bin at a cost (1896) of about 35¢ per ton sorted. Waste was quartzite and some slate, the ore a cemented conglomerate.

The DISADVANTAGES of floors are that all material must be moved manually and that the sorters must work in a stooping position, which is tiring. The ADVANTAGE is thorough inspection, since every piece of material must be turned over and the pickers are not unduly hurried.

Table for sorting is shown in Fig. 2. Feed is delivered by cars or wheel barrows running on the floor at the left and is dumped onto a perforated plate submerged 1 or 2 in. in water in tank A. Shovelers standing on this plate work fines through the screen and wash the coarse oversize, then shovel it onto the inclined table B. Pickers sitting on plank C remove whichever separable component of the feed is present in smallest bulk and drop it below them into proper receptacles, finally scraping the residue through the opening into the tank below B.

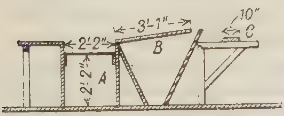


FIG. 2.—Hand-sorting table (*after Richards*).

For sorting on a small scale a punched-plate screen set horizontally on horses or the like is recommended as superior to a floor or table (*106 J 412*). It saves stooping, screens out, during the sorting operation, material too small to pick, and the reject is readily hoed off at one end.

Fixed chutes and grizzlies for sorting are of the usual types with the limitations, however, that their slope shall be near the sliding angle of the material (see Sec. 20, Art. 9) and that the width shall not exceed that readily inspected and worked, viz.: about 24 to 30 in. when worked from one side and 48 in. when worked from both sides. If the slope is less than the sliding angle, material is worked along with rake or hoe; if more, the flow is stopped as desired by a board, hoe or shovel inserted into the stream. Such sorting surfaces are not used when a large percentage of the total material is to be removed or when close sorting is desired. They serve principally when rope, wood, powder and steel are being taken out of the primary-crusher feed in order to obviate trouble in the mill. A grizzly makes selection easier than a chute because of removal of fines, but if the particles are tabular or wedge-shaped, the grizzly clogs badly at the low speeds at which material passes and it is, consequently, difficult to control the movement of material.

A sorting grizzly of adjustable slope used at the PORT HENRY IRON ORE CO. at Mineville, N. Y., is shown in Fig. 14, Sec. 5.

Moving surfaces for sorting include belt and pan conveyors, revolving tables and shaking chutes and screens. Such surfaces have the advantage that manual handling of rejected material is eliminated, but this rejected material is not turned over by or for the picker and material is therefore passed by oversight that should be removed. Further, all material passes at a given uniform rate, irrespective of the content of material that should be removed, with the result that pickers are, at certain times, hurried beyond their capacity and at other times are underworked, if the average speed of travel is right. Notwithstanding these drawbacks, most hand sorting at present is done on movable surfaces because of the advantage of mechanical transport of the reject.

Belt conveyors (see Sec. 20, Art. 1) are the most usual picking surfaces. Belts are commonly 24 to 30 in. wide for a single row of pickers and 48 in. wide

for a double row. Stations for operators are placed 3 to 6 ft. apart along the belt and chutes are provided at each station for receiving the material removed. These chutes are placed beside the picker or on the opposite side of the belt. It is probable that chutes on the opposite side are best for material up to 3- or 4-in. that can be thrown by a flick of the wrist, but pieces that need two hands are best drawn toward the picker and it is probably less tiring to draw one-hand pieces larger than 4-in. to the picker's side than to throw them away. Chute mouths should be so large that accurate throwing is not necessary and of such conformation that pieces will not tend to bound out. Speed of belts ranges between 10 and 80 ft. per min.; the average speed is between 30 and 40 ft. per min. The more difficult the job., *i.e.*, the smaller the pieces and the greater the amount to be removed the slower the speed and the longer the belt.

Tuttle (*17 SMQ 396*) states that coal-picking belts are usually 4 ft. wide and run at 30 to 60 ft. per min. For picking oversize of a 1.5-in. screen at 30 tons feed per hour he recommends a belt length of 15 ft. plus 10 ft. for each 3 per cent. of material removed. On 0.75- to 1.5-in. sizes he recommends 30 ft. per min. belt travel and, for a feed rate of 20 tons per hour, 15 ft. of belt for every 1.5 per cent. of material removed. For more than 4 to 6 per cent. of impurity he recommends washing rather than picking.

The belt should be troughed as little as possible to prevent heaping-up in the middle. In many cases a wide flat belt with feed coming on not nearer than 6 in. from the edges is used. A belt conveyor is suitable as a sorting surface for any size of material that can be handled by the pickers, but it will not stand any considerable amount of sledging, and wears excessively with feed coarser than 6- to 8-in. particularly when, as should be the case, the fine material has been screened out. It may be set on a slope not to exceed 20°.

Pan conveyor (see Sec. 20, Art. 2) is used for coarse material. The arrangement is similar to that described for belts. The speed is usually slower, both for mechanical reasons and because larger lumps are handled. A pan conveyor will stand sledging and is less subject than belts to wear from large lumps. The best form for sorting is one forming the bottom of a shallow trough, which has, therefore, stationary sides. This permits removal of large heavy lumps more readily than when articulated sides form part of the moving mechanism. At Shuler Coal Co. (*21 CA 1077*), a picking pan conveyor 72 in. wide is run at 50 ft. per min. and spreads run-of-mine bituminous coal about 8 in. deep.

Pan conveyors, if beaded or the equivalent may be set on slopes up to 30°, although material will pile up somewhat against the beads and not be so well presented as on flatter slopes.

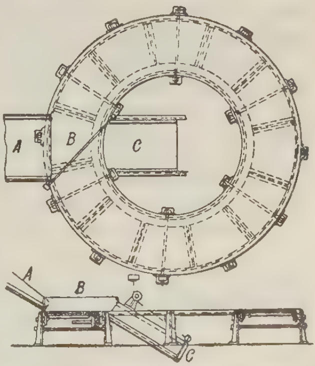


Fig. 3.—Revolving picking table.

Revolving table. One form is shown in Fig. 3. A metal picking surface (usually inclined toward one edge) is carried on a structural frame with a circular track running on suitably placed wheels on the supporting frame. The platform is revolved by means of gear and pinion. Feed is introduced through chute (A), pickers stand or sit at suitably placed stations around the outer and inner peripheries and throw material removed into appropriate chutes or other receptacles. Reject is removed by a scraper (B) and falls into chute (C). Some

tables are supported by ribs from a central spindle, which prevents picking stations on the inner periphery. The usual outside diameter ranges from 16 to 25 ft.; speed is usually between 20 and 40 ft. per min. Some tables are double-decked. The upper deck is about half the width of the lower and 6 in. higher; it receives the selected material, thus presenting both reject and selected material to the inspector.

Revolving tables have been largely used in So. Africa and in Europe, but installations in the United States are rare. German tables are 12 to 15 ft. diam. and 12 to 16 pickers work at each table The cost of sorting on revolving tables on the Rand, where the feed is washed oversize of 1.5- or 2-in. screen, averaging 3-in. size, is 18 to 50¢ per ton, pre-war. (Truscott, *Witwatersrand gold fields*, p. 413.)

The ADVANTAGES are compactness with consequent ease of supervision and collection of products; DISADVANTAGE, as compared to rectilinear conveyors, is the loss of elevation suffered by the reject.

Shaking surfaces have been used widely in collieries, but, except as primary-crusher feeders have not been used to any extent in metal-treatment plants. They are essentially chutes with perforate or imperforate bottoms, set on an incline of about 10° in the direction of flow and shaken at 100 to 250 @ 2- to 6-in. throws per min. by a simple eccentric. With the Ferraris method of suspension by means of short struts or hangers inclined backward about 15° from the vertical, or with a differential head motion (*e.g.*, Marcus screen) the picking surface may be horizontal. Limitations of width and the arrangement of receptacles are the same as previously discussed.

These are the least satisfactory of the movable picking surfaces for careful picking, but when screen bottoms are used, they serve the triple purpose of screens, conveyors and sorting surfaces and thus justify themselves.

3. Operation

Material picked should be that present in least amount in the feed, thus leaving the larger bulk unhandled. Wiard (*112 J 328*) recommends removing ore (one or two classes as the case may be) whenever the tonnages of ore and waste are approximately equal. His argument is that this puts a single and easily defined responsibility on the picker, *viz.:* to remove everything that looks like ore, and makes inspection of the products more simple.

Washing of feed is essential to rapid and accurate sorting. It is usually done in the trommels removing oversize, but may be done by hose or sprays on floors or moving sorting surfaces. Spray water run on to a troughed inclined belt just above the feed point will wash fines down the incline and over the tail pulley where they can be collected in a suitable receptacle. For the quantity of wash water required in trommel washing of non-clayey ores see Sec. 5, Art. 5; for clayey ores see Sec. 8, Art. 1.

Performance at various mills is given in Table 1.

At BRITANNIA M. AND S. Co. (*113 P 695*) the ore is cupriferous pyrite in chloritic schist assaying about 2.7 per cent. Cu, 8 per cent. Fe, 1.5 per cent. Zn, 6 per cent. S, 70 per cent. SiO_2, 25¢ Ag and a trace of gold. Material ranging from 1.5- to 3.5-in. is picked on a belt. Four men per shift at two belts pick shipping ore and hard pieces of country rock for tube-mill pebbles from the oversize from 600 tons per 24 hr. Shipping ore comprises about 10 per cent. of the total concentrate and assays 10 to 18 per cent. Cu. The concentrate shipped assays 15 to 16 per cent. Cu and is composed of the foregoing together with jig concentrate, 16 to 17 per cent. Cu; table concentrate, 14 to 15 per cent. and flotation concentrate, 14 to 15 per cent.

HANDY (*61 A 224*) states that the labor in a typical COEUR D'ALENE sorting plant, in which 800 tons per day, sorted at 1.5- to 4-in. size, yielded 50 tons shipping ore and 150 tons waste, consisted of 20 sorters with 5 bosses and repair men. The normal cost (1917)

Table 1. Performances in hand sorting

Plant	Type of picking surface	Width, inches	Length, feet	Slope, inches per foot
Copper Range	Sloping chute	......	....	6
Phelps-Dodge, Morenci	Belt	36	....	2⅝
Granitic zinc ore	Pan conveyor	......	....	3⅛
Federal Mining and Smelting Co., Morning	Belt	36	88	1.25
Witherbee-Sherman, No. 5	Flight conveyor	......	....	0
Elko Prince	Belt	30	8	......
Tonopah-Belmont	Steel belt	......	....	1.25
United Eastern	Pan conveyor	42	....	0
New Jersey Zinc Co., Franklin	Revolving table	......	....	0
New Jersey Zinc Co., Franklin	Belt	......	....	2⅞
New Jersey Zinc Co., Ogdensburg	Revolving table	......	....	0

Plant	Speed, feet per minute	Material picked	Size of material picked, inches	Number of pickers
Copper Range	......	Mass copper	6–12	1
Phelps-Dodge, Morenci	27	Smelting ore	0.75 to 3	4
Granitic zinc ore	30	Waste, wood and steel	3–12	1
Federal Mining and Smelting Co., Morning	43	High-grade ore and waste	1–6	10
Witherbee-Sherman, No. 5	20	Lump ore and waste	4–16	7
Elko Prince	20	Waste	2–7	1
Tonopah-Belmont	45	Waste	2–9	7
United Eastern	1.5	Waste	....	1
New Jersey Zinc Co., Franklin	30	Waste	3–18	6
New Jersey Zinc Co., Franklin	122	Refuse (wood, steel, etc.)	....	1
New Jersey Zinc Co., Ogdensburg	28.5	Waste	2.5–24	4

Plant	Spacing of pickers, feet	Distance picked material thrown, feet	Tons picked per man per hour	Percentage of total feed removed	Kind of ore
Copper Range	....	....	0.055	0.001	Copper
Phelps-Dodge, Morenci	6	6	0.33	0.7	Copper
Granitic zinc ore	....	1	0.62	1.0	Zinc
Federal Mining and Smelting Co., Morning	4	1.5	0.34	20	Lead
Witherbee-Sherman, No. 5	3	2	5	20	Iron
Elko Prince	....	1.5	0.16–0.25	10	Gold-silver
Tonopah-Belmont	4	1	1.5	15	"
United Eastern	....	5	0.75	1.75–2.25	"
New Jersey Zinc Co., Franklin	8	2	1.5	4	Zinc
New Jersey Zinc Co., Franklin	....	2	....	....	"
New Jersey Zinc Co., Ogdensburg	8	2	1.25	10	"

was 16¢ per ton of run-of-mine rock or 65¢ per ton sorted out. Picking coal on belts in English fields, with the percentage removed ranging from 2.1 to 17.5, the tons picked per worker per hour varied from 0.03 to 0.35, with the highest tonnage corresponding to the greatest percentage removed and general, though not exact, adherence to this rule.

(*Louis, p. 98.*) Huntoon (*93 J 53*) gives the COST of sorting 15 to 20 per cent. of waste from belts at TONOPAH BELMONT at $0.68 per ton of waste and at TONOPAH MINING CO., $0.80 per ton removed when this comprises 11 per cent. of the feed.

Amount removed by sorting varies according to the ore treated.

With native-copper ores of Lake Superior the amount is as small as 0.001 per cent. of the whole at COPPER RANGE. At FEDERAL MINING AND SMELTING CO. lead mines and at WITHERBEE SHERMAN magnetite mines 20 per cent. total high-grade ore and waste are removed. At ALASKA JUNEAU (*112 P 632*) 7.73 per cent. of the total mill feed was picked as milling ore assaying $5.06 per ton and 34.64 per cent., the balance of the 2-in. trommel oversize, rejected as 7-cent waste. On the RAND (*20 IMM 307*) material from 8-in. to 1.75-in. constitutes 50 to 70 per cent. of all rock hoisted. Waste picked out of this varies from 10 to 30 per cent. of the total hoisted, averaging about 16 per cent. This constitutes about 50 per cent. of the total waste hoisted. In IDAHO mills treating coarsely-disseminated lead ores, shipping ore picked out varies up to 60 per cent. of the total concentrate produced.

Labor is generally that unfit for any heavier work, boys, girls, women, or old or crippled men. Boys and girls are the quicker and, if properly supervised, most satisfactory.

Size of material sorted ranges on the average from 2.5- to 12-in.

Smelting ore as fine as 0.75-in. is picked at the Morenci plant of PHELPS DODGE and as coarse as 24-in. at the Ogdensburg plant of N. J. ZINC CO., but the number of moves necessary to make tonnage on the small sizes is so great that the capacity of pickers is low, and difficulty in deciding about and handling 24-in. lumps is likely also to slow the operation down below the rate on intermediate sizes. Wiard states (*112 P 327*) that the best size range for sorting is between 1- and 3-in. Richards (*TB 196*) estimates the maximum rate to be on 3- to 4-in. lumps, but Table 1 shows that maximum tonnages per man-hour correspond to feed averaging 6- to 12-in. Comparison between FEDERAL M. AND S. CO. and WITHERBEE SHERMAN (Table 1) is particularly instructive since at both mines 20 per cent. of the feed was picked as shipping ore and waste and the number of pickers indicates sorting to have been the sole responsibility of the workers. At WITHERBEE SHERMAN 5 tons per man-hour of 4- to 16-in. material was picked against 0.34 ton per man-hour of 1- to 6-in. material at the Federal plant.

Lighting. Daylight is best but ordinarily picking must go on on all shifts, hence artificial light must be used part of the time. Since luster and color are the principal guides in sorting, the light should be good and as uniform as possible. Light should be placed, if possible, to keep shadows off the material being picked. Ordinarily diffused or flood lighting is best, but incandescent lights placed directly over the feed stream and shaded from pickers' eyes may be best in some cases. Experiment is the best guide. Ore moist from recent washing is probably in the best condition for quick and ready selection.

Cost of hand picking bituminous coal at different sizes was thoroughly investigated by U. S. Coal and Coke Co. (*Am. I. & S. Inst., 1921*). With labor at $0.55 per hr., the costs per ton, including interest and depreciation on the investment in sizing and picking machinery, for picking at various sizes were as shown in Table 2. See also Performance, above.

Table 2. Cost of hand picking at different sizes. (*After O'Toole*)

Size, inches		Cost, dollars per ton of run-of-mine coal
Through	On	
....	4	0.22
4	1½	1.01
1½	1	4.55
1	¾	7.81
1	½	9.88
½	⅜	27.63
⅜	3/16	164.38

SECTION 8

WASHING

Introduction

Washers are of three classes, *viz.:* screening washers, classifying washers, streaming washers. Some of the members of each class are designed or arranged to effect disintegration of materials such as clays, soft shales and sandstones, as a preliminary to concentration, but this is an entirely extraneous function in so far as the separating principle is concerned. The usual methods of effecting disintegration are by high-pressure water jets, by tumbling, or by mechanical beating or stirring.

SCREENING WASHERS

Screening washers depend, for their effect, on the fact that the value of the fine and coarse particles of the material to be treated differs. Usually, as with wash iron ores, phosphate ore, bauxite ore and the like, the coarse material is the valuable constituent, but the reverse condition obtains with certain metallic ores, *e.g.*, Cripple Creek tellurides. Screening washers are used principally with ores of the first class.

1. Wash trommels

Trommel screen for washing do not differ from wet trommels in ordinary screening service (Sec. 5, Art. 5) except that when disintegration is necessary the machine is set on a flatter slope and powerful water jets play on the material in the interior. For such work screens supported on tires and rollers are better than those with a central shaft. Roller screens permit heavier construction, which is desirable when lump ore is being treated. The use of trommel screens for washing is usual in ore-sorting plants and on gold dredges.

Trommels on dredges may be 8- to 10-ft. diameter and 40 to 60 ft. long and are very heavy.

New Century wash trommel (Fig. 1) is designed especially for washing. The disintegrating surface is a cylindrical grid of heavy longitudinal bars (*a*) carried on two heavy spiders (*b*) on a central shaft. Screen cloth is stretched outside the grids and lifting buckets (*c*) are disposed on the outside of the screen. The whole is set in a shallow tank. A plow-shaped casting (*d*) fastened to the inside of the conical discharge-end lifts and discharges washed oversize. The heavier undersize is lifted from the tank by the buckets and thrown onto an apron (*e*). Suspended fines overflow the same apron.

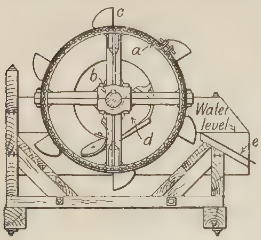


Fig. 1.—New Century wash trommel.

The diameter is 40 in., lengths from 4 to 8 ft. The manufacturer claims CAPACITIES of 75 to 300 tons per 24 hr. with consumption of 4 to 6 hp.

Grizzly and nozzle stream were used to disintegrate and concentrate in a simple and compactly arranged plant shown in Fig. 2 (*21 IMM 230*). The ore was delivered on an upper track, dumped into a chute (*A*) and brought to a washing platform (*B*) where it was disintegrated by means of a stream from a

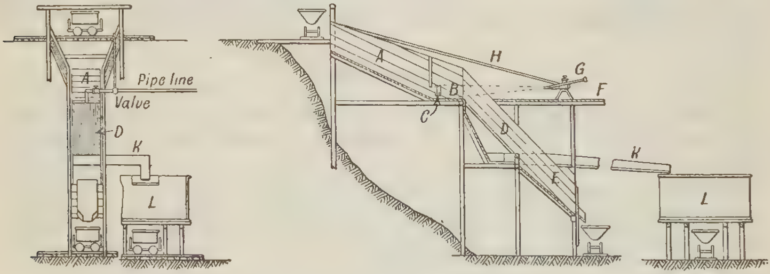


Fig. 2.—Monitor washing plant.

nozzle (*G*). Disintegrated material flowed over a grizzly (*D*), washed oversize was collected in a bin (*E*) and undersize flowed through launder (*K*) to a settling tank (*L*).

Nozzles delivering water under high pressure (50 lb. per sq. in. and upward), as above, are extremely effective in disintegrating loam, mud and light clays. They are used on sorting floors and on screens and grizzlies in sorting plants, in revolving screens on gold dredges and in gravel-washing plants, and in breaking down gravel banks in hydraulic mining. When disintegration is an important feature, the nozzle should be designed to deliver as nearly as possible a solid column of water at maximum velocity. When removal of thin, slightly-adherent layers of dirt is sought, as in preparing ores for hand sorting, delivery of the water in a thin sheet or a multiplicity of fine jets is desirable.

Dorr ore washer consists essentially of a standard Dorr classifier (Sec. 6, Art 6) and a trommel mounted above the settling pond in such a way that the lower part of the trommel is submerged. The discharge end of the trommel is fitted with a discharge scoop similar to that in a scoop-discharge tube mill.

In an exhaustive test at the Minnesota School of Mines (*Bul. 6, MSM*), the washer recovery on Lake Superior wash iron ores averaged above 90 per cent. against about 88 per cent. for a log washer and the concentrate was higher grade. Davis estimates the first cost of a Dorr plant less than that of a log-washer plant and the operating cost no higher.

CLASSIFYING WASHERS

These machines are essentially hydraulic or mechanical classifiers (Sec. 6) used as finishing machines because of the special amenability of certain ores to such one-step concentration.

2. Drum washers

Washing drum consists of an imperforate cylinder, 4- to 8-ft. diameter and 10 to 20 ft. long, set with the axis horizontal and driven at 250 to 300 ft. per min. peripheral speed. The ends are partially closed by an annular rim so that a certain amount of liquid pulp is maintained in the interior. Spirally-disposed lifting blades or ribbons are placed on the inner wall to tumble and move lumps from feed to discharge end. High-pressure water jets are set to play on the tumbling material. At the discharge end heavy solid particles are lifted by perforated buckets and discharged through a chute while water lifts the fines over the closing ring at the feed end.

Speed depends on the diameter of the drum and the character of the material treated; the higher the speed the greater the rate of progress and consequently the less the washing up to the point where centrifugal force causes material to cling to the walls. Power consumption, within the size range given, will vary between 5 and 10 hp. Water consumption is 1 to 4 tons per ton of feed. Capacity depends on the character of the feed: for fairly heavy, granular ores large machines with a long-pitch spiral run at maximum speed will wash 25 to 30 tons per hour; clayey material in a machine of the same size requires low pitch and speed and the capacity will not exceed 25 to 50 per cent. of the above.

Crickboom washer is designed for treatment of materials that are too tenacious for disintegration in the ordinary washing drum. In addition to the ordinary washing drum it has a horizontal shaft carrying beater arms which is independently driven at about 200 r.p.m. in the opposite direction to the drum moving at 10 to 15 r.p.m. The beater arms, therefore, strike and aid in disintegrating the rising material. The lifters on the drum are spaced about 0.75 in. from the shell, thus failing to lift fine material and water.

Richards (*TB 190*) gives the capacity of a 4 × 8-ft. machine as 5 to 5.5 tons of tough clayey ore per hour with a water consumption of 1.6 to 1.8 tons per ton of ore.

3. Puddling

Puddling is a method of washing in which material in a shallow cylindrical tank is stirred relatively slowly by means of sweep arms depending from an overhead sweep. In the form used in the Kimberley diamond fields feed is introduced at the periphery with only sufficient water to form a thick pulp. Sweeps revolve about 10 r.p.m. and the paddle arms are set at an angle that forces settled solid toward the outer rim while water containing suspended solids overflows the inner rim.

Operation is intermittent. When the operator judges that sufficient solid has collected in the bottom, feed is shut off, clear water is run through for a time, then settled material is swept through a bottom gate by means of a plow attached to the sweep.

Successful operation of a machine of this type depends on easy disintegration, a high percentage of clayey material and finely divided waste sand that is readily kept in suspension in the thick pulp.

4. Log washer

Description. The log washer (Fig. 3) consists essentially of a heavy log (a) carrying strong metal blades arranged spirally on its surface, the log being mounted on a slope of ¾ in. to 1½ in. per ft. in an inclined-bottom trough closed at the lower end and open at the upper. The log is so driven that the action of the blades transports material up the incline. The length and slope of the trough are such that the upper third to half of the length of the log is not submerged. The bottom of the trough is usually made to conform to the cylindrical surface generated by the blade tips with clearance between the tips and the bottom somewhat greater than the diameter of the largest lump of ore in the feed. Logs are built one or two to a trough. The usual length is between 20 and 30 ft. and the diameter, tip to tip of blades, between 24 and 30 in. Speed is from 12 to 24 r.p.m. Double logs revolve in opposite directions with the blades rising between them. Logs are made of wood or iron. Wooden logs are generally 12- to 18-in. sticks, and square, hexagonal or octagonal in section. They are shod full length with iron straps. Heavy cast-iron gudgeons (Fig. 4) are fitted to the ends, the lower gudgeon passing through a stuffing box and carried in a thrust bearing, the upper carrying the driving gear. The blades are usually chilled castings with a base for bolting to the log. Blades are sometimes made renewable in the bases, in which case the blade is special iron or steel and the base an ordinary casting.

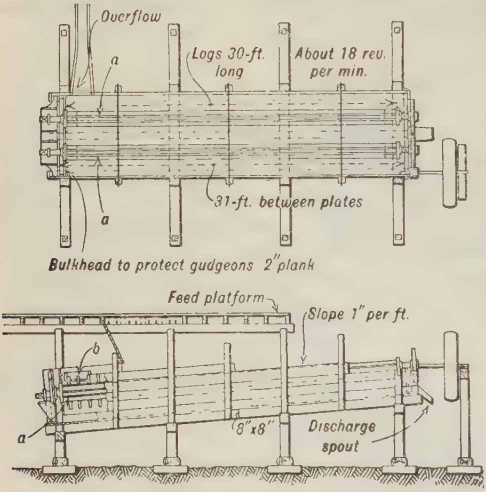


Fig. 3.—Two-log washer.

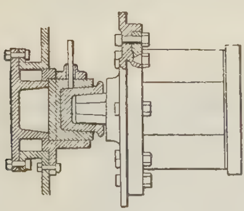


Fig. 4.—Gudgeon, chilled-iron thimble and step bearing for wooden log.

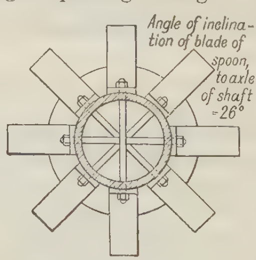


Fig. 5.—Cast-iron log.

Fig. 5 is a section of a log made of cast-iron pipe 11.5 in. diameter, 0.75 in. thick, flanged at both ends for attachment to the gudgeons. The blades are arranged 45° apart on two

spirals starting 180° apart on a 5-ft. pitch. The foot of each blade has two holes for 0.75-in. bolts. The arrangement of blades is such as to bring those in the two spirals at opposite ends of the same diameter every 5/8 ft. along the pipe and hence through bolts can be used, each pair holding two blades.

Structural-steel logs have been built in various forms. The principal difficulty is to effect secure attachment of the blades.

Feed is introduced on the upcoming side of single logs or between double logs and near the lower end of the trough, water is fed near the upper end; mud and fine sand are carried by the water over a weir (*b*) (Fig. 3) at the lower end while lump material and coarser sand are scraped up the trough and discharged at the upper end. Maximum size of feed is usually about 3-in.

Capacity varies with the size and speed of the log and the pitch of the spiral and is greatly affected by the character of feed. With easily washed ore of high specific gravity a double-log washer with 30-ft. × 7-ft. trough will handle about 500 tons per 24 hr. With difficult clayey ore a 20-ft. × 4-ft. trough may not handle over 50 tons of feed per 24 hr. Water consumption is from 4 to 8 tons of water per ton of solid. Power for double 25-ft. logs is about 25 hp. Treating manganese ores (*Bul. 734 USGS 99*) a double 25-ft. log making 12 to 15 r.p.m. treats 40 to 50 tons per 24 hr. with a consumption of 50 to 75 gal. water per min. and 20 to 25 hp.

Turbo washers are log washers with perforated false bottoms through which water is forced under pressure. They wash the lump material more thoroughly than the ordinary log washer does.

Mechanical classifiers (Sec. 6) are sometimes used as washers when the granular material is smaller than, say, 1-in, maximum size, and little or no disintegration is necessary.

5. Vertical-current washers

These machines are essentially hydraulic classifiers. They are satisfactory concentrators in the places in which they are used only because the service is relatively easy, such as the separation of clay and fine sand from lump iron ore; or because the amount of cleaning to be done is relatively small, as in the removal of slate from sized coal in the Draper washer; or because of repeated treatment, as in the Rheolaveur.

Wetherbee iron-ore concentrator (Fig. 6) combines mechanical agitation with free-settling hydraulic classification to effect separation of fine silica and clayey material from coarse and fine hematite in Mesabi wash ores. It is, therefore, a direct competitor of the log and turbo washers. Feed passing a ½-in. aperture is introduced into the revolving tub (*a*) and discharged by centrifugal force through the holes (*b*) into annular space (*c*) through which a current of water is rising. The theory of the machine is that the swirl in this annular space is sufficient to keep the lighter particles in suspension and that consequently a small rising current will lift them into the overflow, while the heavier particles will settle into the hopper-shaped bottom from which they may be removed by a drag or screw conveyor, or, if sufficiently fine, by a pipe-and-plug spigot.

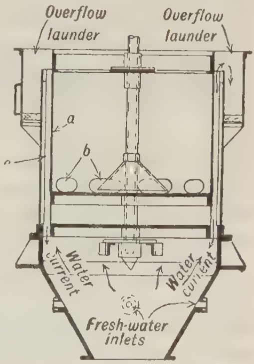


Fig. 6.—Wetherbee concentrator for wash iron ores.

Machines 3 ft. and 6 ft. in diameter have been used. Working on one particular ore, 28 r.p.m. for the 6-ft. machine and 36 r.p.m. for the 3-ft. machine were found to be the best speeds and hydraulic water consumption was from 80 to 150 gal. per min. The 6-ft.

machine readily treated 50 tons −½-in. feed per hr. and produced concentrate assaying 57 to 62 per cent. Fe from feeds carrying 39 to 54 per cent. Fe. Comparative results on two Mesabi mills, one using the Wetherbee machine and the other a turbo washer and tables to treat log-washer tailing, showed 81.4 per cent. recovery with a concentrate assaying 59.8 per cent. Fe for the first mill and 78.6 per cent. recovery with 58.8 per cent. Fe in the concentrate for the other *(103 J 301)*.

Draper tubular washer (Fig. 7) is essentially a hindered-settling classifier used for treating −⅜-in. sized coal. The usual size splits in British practice *(67 IME 502)* are −⅜ + 3⁄16-in., −3⁄16 + 1⁄16-in., −1⁄16 + 1⁄32-in. and −1⁄32-in. In one English plant the capacities at different sizes were −⅜ + 3⁄16-in., 6 tons per hr.; −3⁄16 + 1⁄16-in., 4 tons per hr.; −1⁄16 + 1⁄32-in., and −1⁄32-in., 1 ton per hr. Six machines required 200 gal. water per min. Performance is given in Table 1.

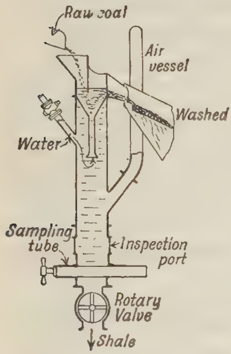


Fig. 7.—Draper coal washer.

Table 1. Performance of Draper washer

Size of feed, inches	Percentage ash		Free coal in refuse, per cent.
	Feed	Washed coal	
−⅜ + 3⁄16	16.8	3.8	0.25
−3⁄16 + 1⁄16	22.7	3.6	0.5
−1⁄16 + 1⁄32	22.0	4.7	1.6
−1⁄32	29.7	9.1	3.2
Average	....	4.7	....

Rheolaveur (Fig. 8) is essentially a shallow-pocket launder-type classifier (see Sec. 6, Arts. 2 and 3) applied to the separation of impurities from raw coal. At sizes below 3½-in. separation depends on three different phenomena, *viz.:* (*a*) stratification, by reason of difference in specific gravity, which takes place in a substantially horizontal rapidly-moving stream in the trough and results in a layer of pyrite and slate at the bottom, bone above and coal on top; (*b*) difference in rate of movement along the trough, due to difference in shape, difference in specific gravity and difference in stream velocity at different depths, resulting in relatively rapid movement of the rounded, light coal particles in the rapid upper portion of the stream and relatively slow movement of the flat, heavy slate particles at the bottom; (*c*) difference in settling velocity in water due to difference in size and specific gravity. Thus when a mixture of particles is first introduced into the classifying trough, stratification occurs; thereafter the lower layer of slate moves less rapidly than the upper layer of coal and when a discharge slot (Fig. 9) is reached and support is taken away from all the particles, the resultant path of the large slate particles, having the greater instantaneous vertical acceleration and the smaller horizontal component of velocity, fall through the slot, if everything is properly proportioned, while the particles of coal and finer particles of slate pass on.

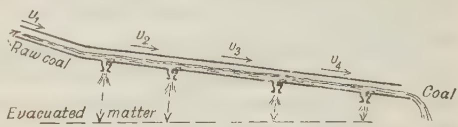


Fig. 8.—Rheolaveur.

Four different types of slate-discharge spouts are shown in Fig. 9. The form (A) is used for feeds coarser than ¼- or 5/16-in. The flap-valve (v) is arranged so as to open mechanically at short intervals, permitting slate to bank above it during the time that it is closed and remaining open only long enough to drop the accumulated slate without permitting coal to drop. An intermittent mechanically-controlled slate-discharge valve (Fig. 9, E) is attached to the bottom of the sorting pocket. Starting with chambers (A) and (B) full of water and valves (R_2), (V_1) and (V_2) closed, (valve (R_1) corresponds to valve (R_1) in Fig. 9, A), slate settles into chamber (A), displacing an equal volume of water upward through the sorting column, and collects for a predetermined time until (A) is partly full. Valve (V_1) is then opened, the slate falls into chamber (B), displacing water upward into (A) so that, again, no down current is induced in the sorting column. (V_1) is then closed, (V_2) opened, the slate discharged, and, after (V_2) closes, (R_2) opens and the water lost is replaced, a suitable vent for air being provided. A flap-valve slate discharge without rising current, discharging into a bucket-elevator boot tank (see Fig. 9, B) is used for later discharge ports. For fine material (−¼-in.) the continuously discharging spigot, Fig. 9, C, is used for the first slate discharges and the form, Fig. 9, D, without rising current, for the later pockets. The intermittent discharge (Fig. 9, E) may be attached to either of these, if the amount of water leaving with the slate is too great.

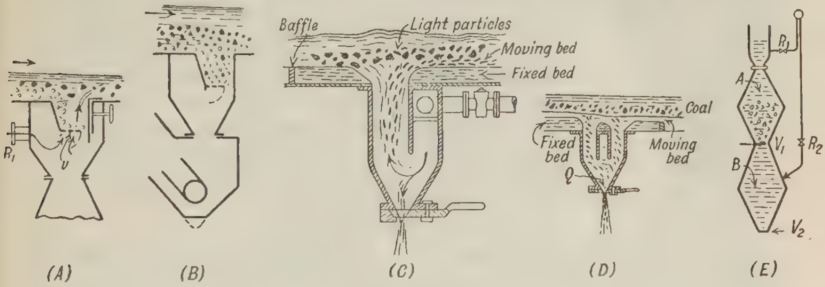


Fig. 9.—Discharge spigots, Rheolaveur.

Troughs are usually sloped 1 in 10 to 1 in 5 at the feed end, diminishing to 1 in 20 at the discharge end. A rising current may or may not be maintained in the sorting pockets. When feed is finer than 0.3- to 0.4-in. diameter, continuous discharge from sorting pockets is maintained, but for coarser feeds, up to 4-in., intermittent discharge, controlled by a mechanically-operated flap valve in the bottom of the trough, is used, with a constant supply of hydraulic water under the valve. When the valve is opened the upward rush of water prevents coal from falling.

As is true of all classifiers, clean separation on unsized feed is impossible. Hence in installing the process provision is made to take a coal-bearing middling from the sorting pockets of a primary machine, using low-velocity water currents, and clean coal from the later pockets and overflow end; then retreating the middling in a secondary machine with strong rising currents, making clean tailing and boney coal, the latter being returned to the head of the primary machine.

Considerable fine coal (−10- to 20-mesh) is necessarily lost with fine shale and clayey material that goes with the coarser coal and must be separated in the dewatering apparatus.

Capacity of 100 tons −3½-in. raw coal per hr. is claimed for a 2-spigot machine, with trough 1 ft. 8 in. wide by 16 ft. long, with a power consumption of 25 hp. for pumps, elevators and conveyors.

At Ormonde Colliery near Derby, England (*25 CA 455*), treating 100 tons per hr., the power consumed is 1 hp.-hr. per ton; the net water consumption 40 gal. per min. The refuse contains between 0.9 and 1.7 per cent. free coal while the ash content of the washed coal, including sludge, is in general not more than 2 per cent. in excess of the fixed-ash content. Sulphur is reduced from 1.6 to 1.1 per cent.

6. Tank washers

General. The principle of these machines is to agitate a mixture of minerals of different specific gravities in a tank containing a fluid which, either by reason of its density or the direction and velocity of its motion, causes the lighter mineral or minerals to float while the heavier sink. Tank washers such as the Robinson and Howe have been used for a long time in bituminous-coal washing, and the Chance washer is being used on both anthracite and bituminous coals. A considerable number of attempts have been made, as recorded in the patent office, to apply heavy-solution separation to southern hematite washing, but the high cost of solutions is a bar to commercial success.

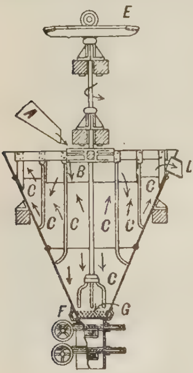


Fig. 10.—Robinson tub washer.

Robinson washer (Fig. 10) is essentially a hindered-settling hydraulic classifier (Sec. 6, Art. 3) with a stirring mechanism to prevent the material in the teeter chamber from packing and channeling. It is used to separate coal from slate and other heavy impurities. Hydraulic water enters through holes (G) from the annular pressure chamber (F) and rises with decreasing velocity through the cone to the overflow lip (D). Feed enters through the central ring (B), slate, pyrite, etc., settle against the rising current and are drawn off intermittently through the discharge valve at the bottom, while coal overflows.

A common size in bituminous washeries is 10½-ft. diameter by 10½ ft. deep with overflow lip 2 ft. long. The speed of the spindle is 20 to 25 r.p.m. Rated capacity on −3- or 3½-in. feed is 50 tons per hr. Water consumption is high and all fine dirt goes into the coal, but this can be separated by subsequent screening. Performance on different sizes of coal is shown in Table 2.

Howe washer is similar to the Robinson washer, except that the stirring arms are horizontal.

At Renton Coal Co., Renton, Wash. (*28 UW 104*), a Robinson washer treated −2½-in. raw coal containing 19.3 per cent. ash and made an overflow product containing 18 per cent. ash and refuse containing 61.5 per cent. Recovery of combustible was 97.3 per cent.; ash reduction, 6.7 per cent.

Hydro-separator (Fig. 11) is essentially a hindered-settling classifier. Feed entering behind the baffle (A)

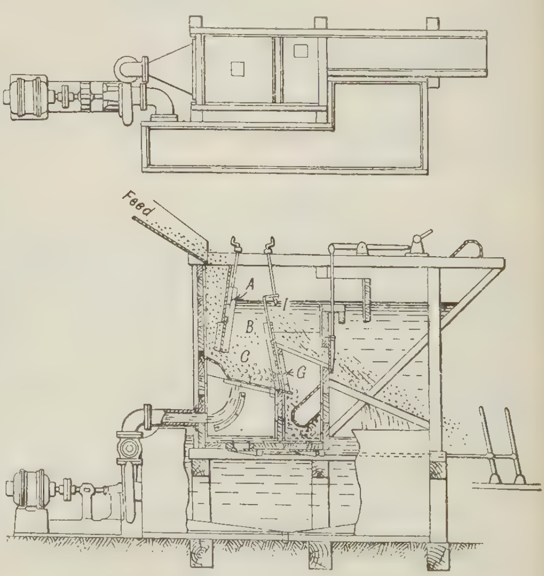


Fig. 11.—Hydro-separator for sized anthracite.

into the bottom of the sorting compartment (*B*) meets a rising current of water entering through perforations in (*C*) and the light material rises to the

Table 2. Performance of Robinson washer on British bituminous coals. (*After Drakeley, 54 IME 456*)

	Unsized coal, −1½-in.						
	Coal free from visible impurity	Raw coal		Washed coal		Refuse	
Ash, per cent	3.64	19.98		9.93		56.10	
S, per cent	0.47	0.63		0.51		0.97	
Calorific value, C. H. U.	7782	6431		7254		.....	
Float, per cent.(*a*)	....	72.02	......	88.73	......	21.67	......
Sink, per cent	....	......	27.98	....	11.27	....	78.33
Ash, per cent	....	5.16	58.17	4.35	53.89	7.76	69.47

	Unsized coal, − ½-in.						
	Coal free from visible impurity	Raw coal		Washed coal		Refuse	
Ash, per cent	2.63	9.13		4.73		55.03	
S, per cent	0.41	0.47		0.42		0.78	
Calorific value, C. H. U.	7992	7362		7801		.....	
Float, per cent.(*a*)	....	88.46	....	96.12	....	14.55	....
Sink, per cent	....	.....	11.54	....	3.88	....	85.45
Ash, per cent	....	2.88	56.89	2.78	53.20	8.21	62.80

	Sized coal								
	Coal free from visible impurity	Raw coal		Washed coal				Refuse	
				−3/16-in.		−2 + 3/16-in.			
Ash, per cent	3.12	13.71		11.35		4.47		76.23	
S, per cent	0.63	0.80		0.75		0.68		1.72	
Calorific value, C. H. U.	7985	7060		7172		7903		.....	
Float, per cent.(*a*)	....	82.00	.....	87.34	.....	97.92	.....	1.93	.....
Sink, per cent	....	.....	18.00	...	12.66	.....	2.08	.....	98.07
Ash, per cent	....	3.28	61.07	3.14	67.83	3.27	58.62	9.54	77.48

a Solution, 1.35 sp. gr.

top of the teeter column and overflows baffle (*I*). The heavy material is withdrawn through an adjustable slot (*G*). The feed must be closely sized.

At Pennsylvania Coal Co., Dunmore breaker, (*26 CA 188*), three of these machines with screens 18½ × 24 in. treat 15 tons per hr. each of No. 1 buckwheat, rejecting about 2¼ tons of refuse containing about 6.7 per cent. coal, of which about half is bone. The cleaned coal contains about 7 per cent. slate. Each machine is served by a centrifugal pump delivering 320 gal. of water per min. against a pressure head of about 2 lb. and driven by a 3-hp. motor.

7. Heavy-fluid washers

These machines embody classification carried to the limit where, by reason of the fact that the density of the separating medium is greater than that of one of the ore constituents, the downward component of velocity of that constituent is zero. The processes differ in the character of the separating fluid.

Chance washer. Chance (*patent* 1,224,138/1917), describes a method of treatment of crude coal and certain ores in which the minerals of different specific gravities are caused to sink or float by gravity or by centrifugal force in a fluid mass of intermediate specific gravity, obtained by suspending finely divided solid matter in a liquid, usually water, by agitation.

Normally the separating chamber is a conical tank 6 or 15 ft. diameter provided with stirring arms on a vertical spindle and supplied at the bottom with hydraulic water. The lighter material overflows the periphery or is skimmed off while the heavy material is removed from the bottom by a drag conveyor or bucket elevator or by gravity through an intermittent slush gate. The pulverized solid whose suspension raises the apparent specific gravity of the fluid mass should consist of particles smaller than the average size of the particles to be floated and of greater specific gravity. Maintenance of the predetermined specific gravity of the fluid mass is the important and difficult part of the operation, since the added solid is lost by overflow and the liquid becomes contaminated by fine particles of both the sink and float materials. In patent 1,392,401/1921, it is claimed that specific gravities of 1.50 to 1.70 can be maintained with ordinary quartz sea sand and water. The size of feed is relatively unimportant from the theoretical point of view, but practically fine material is difficult or impossible to separate.

The method has had considerable trial in coal separation but none in metal concentration. In coal separation, quartz sand is used as the separating solid and water as the suspending liquid.

In patents 1,392,399 and 1,392,400/1921, Chance describes a method of maintaining a mass of water above the mass of suspended sand, thus furnishing a means of overflowing or washing off floated coal and also slime and organic waste in the feed. The overlying body of liquor also lessens the submergence of the mass floating on the dense-fluid mass.

In patent 1,392,401/1921, Chance and Chance describe the application of the heavy fluid mass obtained by solid suspension to the operation of jigs, classifiers, trough washers and the like. It is pointed out that in a fluid mass of 1.25 specific gravity the relative weights of coal (sp. gr. 1.30), boney coal (sp. gr. 1.45) and slate (sp. gr. 2.00) are 6⅔, 25⅔ and 100 respectively while in water the corresponding weights are 30, 45 and 100. Separation by settling is correspondingly easier in the heavy fluid. In 1,462,881/1923, Chance describes the use of a mixture of comminuted solids of different specific gravities, maintained by agitation, in order to get liquid masses of closely-stepped differences in specific gravity with any desired interstitial volume. If a liquid mass is desired with specific gravity = 2.00 and with 60 per cent. interstitial volume, to be made of water and comminuted solids of 2.5 and 5.0 specific gravities, it can be made by using 24 per cent. of the light and 16 per cent. of the heavy solid by volume. $(0.60 \times 1.00 + 0.24 \times 2.5 + 0.16 \times 5.0 = 2.00.)$ To obtain a fluid mass of the same density using water and the heavy solid alone would require 75 per cent. interstitial volume and such dispersed solids are

hard to maintain uniformly suspended. The minimum efficient interstitial volume is about 55 to 60 per cent.

In patent 1,556,676/1925, Chance describes the use of a fluid mass sufficiently dense to float middling and an apparatus with overflows at two or more levels to remove products of different specific weights.

In patents 1,559,937 and 1,559,938/1925, are illustrated two forms of apparatus for practicing the Chance process.

In patent 1,561,909/1925, Chance suggests a method of increasing the specific gravity of the fluid mass without decreasing its fluidity by introducing a second fluid mass, containing finer solid in suspension, so that it will rise through the first. Thus, if a separating fluid mass consists of 40 per cent. quartz sand and 60 per cent. water, the specific gravity is $0.40 \times 2.65 + 0.60 \times 1.0 = 1.66$. This is substantially the practical limit of density with quartz sand. If, instead of water a fluid mass consisting of 20 per cent. fine quartz sand and 80 per cent. water be injected to maintain dispersion of the coarser sand (the density of the secondary fluid being $0.20 \times 2.65 + 0.80 \times 1.0 = 1.33$), the density of the resulting composite mass is $0.40 \times 2.65 + 0.60 \times 1.33 = 1.858$. Chance suggests the possibility of using, say, a 10 per cent. suspension of fine magnetite ($0.10 \times 5.0 + 0.90 \times 1.0 = 1.40$) as the secondary fluid, in which case the specific gravity of the mixed fluid is $0.40 \times 2.65 + 0.60 \times 1.40 = 1.90$.

Arrangement of a Chance washer with coal and refuse screens and sand-recovery system is shown in Fig. 12. Sand is maintained in suspension in the

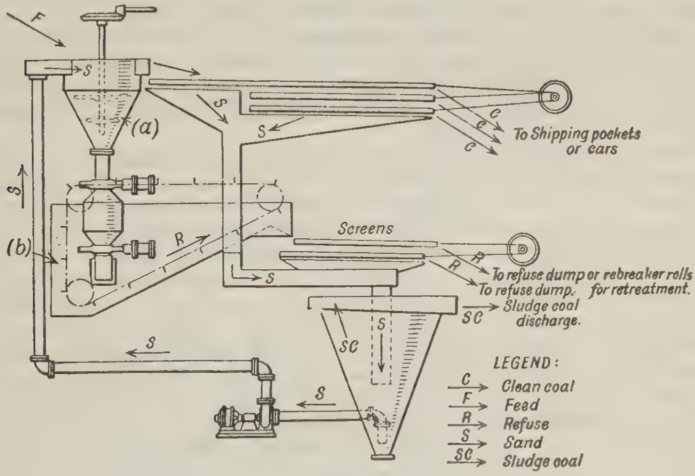


Fig. 12.—Arrangement of Chance washer.

washer (*a*) by means of the agitator and of water introduced at the apex. Raw coal is introduced at (*F*), the washed coal overflows with some sand onto a fine shaking screen which removes sand and this returns to the sand sump. Refuse discharges through the refuse valve into refuse tank (*b*) which likewise is discharged over a screen that sends sand to the sand sump. Fine coal and dirt overflow the sand sump and sand is returned to the washer.

The ratio of diameter of classifier column to diameter of the cone at the overflow level is determined by the density to be maintained and the sand used. The classifier column must be small enough so that, with sufficient water rising to maintain the desired density, the rising velocity in the column will prevent sand from settling. Hence the higher the density to be maintained and consequently, the less water introduced, the greater the necessary ratio between overflow and column diameters. If the large volume of water required for the low-density sand-fluid for bituminous-coal separation produces too great rising velocity in the classifier column to permit small slate particles to settle, part of the water

may be advantageously introduced through a manifold entering the sides of the cone at a number of points.

The Chance machine has been installed at several anthracite breakers. According to Chance (*70 A 743*) the capacity of a 7-ft. 8-in. (diam.) cone is 60 tons per hr. of fresh-mined coal or 30 tons of culm-bank material. The overflow capacity of this machine is about 50 tons per hr. (*21 CA 735*) hence this is the limiting factor on mine coal while slate-valve capacity is the limiting factor with culm. At the Nanticoke plant of Staples and Bell the average capacity on anthracite was 25 tons per hr. but the machine could be pushed to handle 60 tons per hr. The corresponding capacities of the 15-ft. cone are 200 and 150 tons feed per hr. The machine handles unsized feed from egg size down, although separation is poor below ⅛- to ¼-in., and this material should be removed in order to prevent contamination of the sand therewith.

Speed of agitator in a 7-ft. 8-in. cone treating anthracite is about 14 r.p.m. Power required in one installation (*21 CA 735*) was 11 to 16 hp. for agitator, refuse conveyor and screens and 20 hp. for the sand pump.

Sizing tests of feed and products of a machine treating run-of-mine and culm-bank anthracite are shown in Table 3.

For performance in bituminous washing see p. 63.

Conklin separator (*66 A 458*). The separating fluid consists of −200-mesh magnetite in water in the proportions, approximately, of 4.4 parts water to one of magnetite, which produces a fluid of about 1.9 specific gravity. The claimed advantage over the Chance process is that the magnetite fluid does not require agitation to maintain suspension and it is more readily separated from the products. The separating tank is rectangular, about 15 ft. long, and 8 to 10 ft. deep, while the width depends on the capacity desired, reckoning about 7 tons per hour per ft. Coal is removed by a chain drag while the slate is taken out by a screw conveyor.

Heavy solutions. The processes suggested may be classified as (1) pure gravitational settling in heavy solutions; (2) gravitational settling following treatment of the feed in such a way as to alter its effective specific gravity.

The basic requirements for a suitable heavy liquid are that it shall not react chemically with any of the constituents of the ore, which would result in irretrievable loss of solution; that it shall not, as a contaminant of the concentrate, interfere with subsequent use of the latter; and that it shall be cheap.

Most of the patented processes claim ability to treat ores in general, but economically, except in special cases such as flotation of wad or of coke, the various processes are applicable only to coal washing, where one of the constituents of the mixture to be treated has an exceptionally low specific gravity.

Pure gravitational settling in heavy solutions is an attractive idea, but fails commercially on account of the cost of solution and the impossibility or expense involved in so operating that there is little or no loss of solution with the products. There are a number of solutions capable of floating lighter minerals such as coal, *e.g.*, zinc chloride in water, but none that do not react with the coal are cheap enough. Various patented expedients suggested are:

Lurie (454,116/1891). Use of water and a liquid insoluble in and heavier than water, and also heavier than one mineral but lighter than the other, *e.g.*, (1) a liquid (sp. gr. nearly 4) made by distilling a mixture of bromine and alcohol, collecting the retort residue and decomposing with caustic alkali. (2) Tetrabromethane. (3) A liquid made from (1) by treating with pure acetylene (sp. gr. about 2). (4) Methyl-propyl-benzene or cymene in alcohol. Intermediate specific gravities may be obtained by mixing any two or more of these liquids.

DuPont (994,950/1911). A process for coal-slate separation in which after separation in a liquid of intermediate specific gravity (carbon tetrachloride) the floating material and separating fluid are then run into a liquid which has a higher boiling point than the separating fluid and is substantially immiscible therewith (water), and the separating fluid is volatilized and collected while the coal collects in the water. (1,002,865/1911) A method of separating limonite from silicious impurities, consisting in heating to change the iron to Fe_2O_3 and then separating in antimony bromide at 100° C. Addition of ammonium

Table 3. Performance of Chance process on anthracite (*after Ashmead*)

Run-of-mine coal

Size	Analyses, per cent. weight												
	Feed						Overflow		Refuse				
	Coal	Light bone	Coal-slate mid-dling (high-coal)	Coal-slate mid-dling (high-slate)	Slate	Ash	Coal	Ash	Coal	Light bone	Coal-slate mid-dling (high-coal)	Coal-slate mid-dling (high-slate)	Slate
Egg	14.8	0	0.7	0.5	9.0	....	100	....	0	3.2	3.3	2.2	43.0
Stove	22.5	0	0.9	0.5	8.1	....	100	....	0	2.5	2.8	2.6	27.6
Nut	14.8	0	0	0	4.0	....	100	....	0.2	0.2	0.2	0.1	7.7
Pea	7.5	0	0	0	1.7	....	100	....	0	0	0	0	1.9
No. 1 buckwheat	9.4	0	0	0	0	29.0	100	12.2	0	0	0	0	1.6
Rice and barley	5.6	0	0	0	0	26.4	100	14.0	0	0	0	0	0.9
Totals	74.6	0	1.6	1.0	22.8	....	....	....	0.2	5.9	6.3	4.9	82.7

Bank coal

Size	Analyses, per cent. weight											
	Feed				Overflow				Refuse			
	Coal	Coal-slate mid-dling (high-coal)	Coal-slate mid-dling (high-slate)	Slate and bone	Coal	Coal-slate mid-dling (high-coal)	Coal-slate mid-dling (high-slate)	Slate and bone	Coal	Coal-slate mid-dling (high-coal)	Coal-slate mid-dling (high-slate)	Slate and bone
Stove	4.3	0.5	0.5	11.0	86.0	9.0	4.5	0.5	0.5	1.0	0	....
Nut	11.0	1.5	1.0	32.0	91.7	5.0	3.0	0.25	1.0	0	0	....
Pea	7.0	0	0	14.0	95.0	....	4.0	1.0	2.0	0	0	....
No. 1 buckwheat	5.0	0	0	10.2	93.0	....	4.0	3.0	0.2	0	0	....
Rice and barley	2.0	0	0	0	....	....	....	....	0	0	0	....
Totals	29.3	2.0	1.5	67.2	....	....	....	....	3.7	1.0	0	95.2

chloride with antimony chloride prevents decomposition and insures complete volatilization (1,004,815/1911).

Stannous chloride (sp. gr. 2.27) liquefying at 114° C. may be used (1,064,459, 1,067,410/1913).

Nagelvoort (1,244,885/1917). The separating fluid may be removed by a volatile solvent and recovered by evaporation. Carbon disulphide is a suitable solvent for tin and arsenious bromides.

Gravity settling following change in density. The usual method for effecting density change is to heat. This may drive off water or otherwise increase the specific gravity of one of the constituents, *e.g.*, limonite, or it may render one of the minerals porous and thus decrease specific gravity, *e.g.*, pyrite. Another method is to coat the ore constituents, to a substantially like degree, with a substance, usually liquid, that is lighter than any of the constituents, thus rendering the lighter more easily floated while the heavier is affected less in proportion. The ideal condition would be to coat the light mineral preferentially.

Moxham (1,151,117/1915). Sulphuric and phosphoric acids are named as coating liquids and arsenic bromide, antimony bromide, bromoform, and tungsten solutions as separating mediums. (1,203,897/1916.) Calcining silicious iron ores in a neutral atmosphere to prevent reduction of the iron-bearing compound, since the reduced portion will react with bromides of antimony, tin or acetylene. (Sp. gr. of antimony bromide is given as 3.65 at 94° C.; tin bromide, 3.30 at 28° C.; acetylene bromide, 2.98 at ordinary temperatures.)

(1,294,519/1919.) Coating with a cheap liquid such as manganese bromide, zinc bromide or phosphoric acid before separating in an expensive liquid like arsenious bromide.

MISCELLANOUS SINK-AND-FLOAT PROCESSES

Separation of wad (1,277,144; 1,277,145/1918). Feed the pulverized ore to the center of a circular conical tank with a stirring apparatus that produces upward and outward currents. The porous manganese dioxide is floated over the periphery and the heavier material sinks.

Separation of metalliferous ores (386,504/1888). Use a liquid insoluble in and heavier than water but lighter than the rocky gangue of an ore, *e.g.*, carbon bisulphide, chloroform and oil of cloves on which, when submerged in water, the gangue minerals will float and in which "metals" will sink.

The principle involved here is not simple gravity settlement but a surface-tension phenomenon superimposed thereon, the effective forces causing flotation of the gangue being gravity of the liquid and resistance of the rocky material to wetting by the organic liquid.

STREAMING WASHERS

General. These washers are controlled applications of the typical phenomena of water flow on the earth's surface. The principles underlying these phenomena are: (*a*) The scouring or suspending action of a flowing stream on solid particles is in inverse relation to their weight, hence a stream of a given velocity will take light-weight particles of a given size in suspension and leave behind heavy particles of the same size. (*b*) The effect of a stream flowing down an inclined surface in moving particles not suspended along that surface is dependent upon the velocity of the stream. Velocity is least at the bottom and greatest at the top, hence the larger particles are acted upon most vigorously and, if they are just moved, the smaller unsuspended particles will not be moved. The resistance of the smaller particles to motion is enormously increased by roughness of the supporting surface, especially when the surface inequalities are the same size as the particles or larger. (*c*) In a mixture of particles of different sizes, sufficiently agitated to be semi-fluid, the smaller particles will work down through the interstices of the larger.

8. Pan

Description. The gold pan (Fig. 13) is used for prospecting gold-bearing detritus, for small-scale working of rich placer deposits, and for testing work in connection with gravity concentration. It is made of stiff sheet iron with the rim turned around a heavy iron wire for stiffness. Enameled-iron pans have the advantage of not rusting, but chip easily. Amalgamated-copper pans have been used for cleaning up black-sand concentrate. The usual size is 15 to 18 in. diameter at the top, 2 to 2½ in. deep, with the side inclined 30° to 80° to the bottom. Wilson recommends a 10-in. pan for prospecting. The weight is 1.5 to 2 lb. The inner surface should be smooth, and be kept free from grease and rust. A pan will save particles varying considerably in size, but for best work the heavy particles should average smaller than the lighter waste.

Manipulation. See Sec. 22, Art. 9.
When panning gold gravel as a concentrating as opposed to a testing operation, if there is much heavy sand present, it is best not to attempt complete separation with each pan, but to collect and later work up together the concentrate from many pans. In such work black sand (magnetite) may be removed (best dry) with a magnet and the gold, if amalgamable, may be collected with mercury, although this is not commonly done. Otherwise careful panning or blowing is necessary to make the final separation. In prospecting work when it is necessary to collect the gold from each pan separately, the quickest method of collection is to draw the colors into a head and then pick them out separately with a sharp-pointed metal or wooden tool. When there is a quantity of gold in the heavy sand, sizing in a series from 20- to 90-mesh before attempting final separation is helpful (*20 IMM 197*).

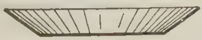


Fig. 13.—Gold pan.

Fig. 14.—Batea.

Capacity. The charge for a pan 16 to 18 in. diameter is from 15 to 30 lb. of ordinary gold gravel. Such material averages about 135 lb. per cu. ft; hence 243 @ 15-lb. pan loads = 1 cu. yd. A skillful worker can pan 100 charges in 10 hr.; gravel containing many boulders works more rapidly and fine or cemented gravel at about 75 per cent. of the rate stated.

Batea (Fig. 14) is an equivalent of the pan, used in Central and South America and in certain Asiatic countries. It is usually made of wood, less frequently of sheet iron. It is claimed that the wooden surface is superior to iron for catching and holding fine gold. *Bowie* states that Honduras mahogany is the best wood. The usual size for prospecting work is from 15 to 20 in. diameter with an angle of 150 to 155° at the apex of the cone, making the depth at the center from 1⅝ to 2½ in. Bateas 30 to 36 in. diameter are used in diamond washing in Brazil and in washing tin gravels in the Dutch East Indies.

9. Rocker

Rocker (Fig. 15) is used in the same kind of service as the pan, but has somewhat greater capacity. Rockers are of many designs and sizes but all consist essentially of a screen box (*a*) for rejecting coarse pebbles, an inclined apron (*b*) serving the double purpose of transporting undersize to the head of the rocker trough and of catching gold, and an inclined riffled trough (*c*) with flaring sides; all mounted on two rockers (*d*). The screen box may be provided with a handle (*e*) which serves both as an aid in lifting out and a grip for the operator in rocking, or the rocking stick may be longer and attached to the side of the trough near the head end. The SCREEN BOX ranges from about 12 × 12 in. in small prospecting rockers to 20 × 24 in. in large working rockers. It is sometimes made with a low side at the head end and the bottom inclined slightly toward the low side, in order to effect continuous discharge of oversize. The screen is occasionally a grizzly, but better perforated steel

plate, 10- to 18-gage, according to the service. The screen aperture depends to some extent on the character of the gravel; it varies between ¼- and ¾-in., normally is about ½-in. Percentage of opening should be as great as possible. The APRON is normally made of canvas tacked onto the frame in such a way as to leave a slight belly. Rubber sheeting backed with canvas (*93 J 1266; Peele 756*) and galvanized iron have been used. The slope of the apron is from 1.5 in per ft. upwards. The TROUGH proper is usually between 12 and 18 in. wide at the bottom and 3 to 6 ft. long, with slightly flaring sides from 1 to 2 ft. high at the deepest part. The bottom piece should be clear, soft lumber that will not shred or "rough-up" when wet under the action

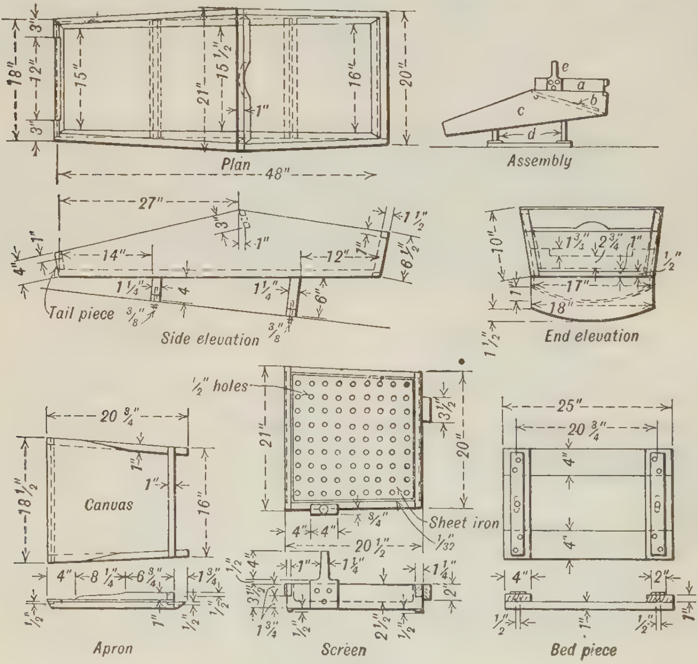


FIG. 15.—Rocker.

of the gravel. The slope of the trough ranges from 0.5 to 1.5 in. per ft. RIFFLE CLEATS of wood ½ × ½-in. to 1 × 1-in. are placed on the bottom of the trough and held down by strips nailed to the sides. The arrangement varies. Usually all are transverse but occasionally the upper half of the trough is riffled longitudinally. Canvas or blanket is sometimes used to cover the bottom and either may be covered with riffles made of expanded-metal lath. The TAIL PIECE at the lower end of the trough is usually between ¾ and 1¾ in. high at the center and from 1 to 3 in. higher at edges. It serves to hold a bed in the trough. The ROCKERS should have no more curve than is necessary to effect distinct agitation of the material in the machine; more makes the labor of rocking unduly fatiguing without any corresponding gain in concentrating efficiency. Dressed 1-in. lumber is heavy

enough. The joints should be tight. Warping may be prevented by light tie rods and the use of light-weight metal corner straps.

Manipulation. Material is shoveled into the screen hopper and washed by means of water poured in from a dipper. At the same time it is forked over and disintegrated and coarse clean boulders are thrown out. More water is then added slowly and the machine is rocked in such a way as to cause material to progress along the floor of the trough, to loosen the bed at each stroke, and to maintain even transverse distribution of the solids. *Wilson* states that one swing should be sharper than the other, to effect stratification, or even that a bumping block may be used, but either procedure will cause heaping of solids and concentrate at one side of the trough with consequent difficulty in manipulation and loss of values. When the deposit of gold has worked down to near the tail piece, or sooner if the exigencies of the operation demand, a clean-up is made. The material behind the tail piece is shoveled out, the screen is then carefully washed and removed, the apron lifted out, and the contents washed carefully into a pan for re-concentration. The riffles are taken out and carefully washed and the deposit in the trough is then washed on the plane bottom by streaming as described in Art. 11. Final clean-up is usually made in a pan.

The longitudinal progress of tailing depends upon slope, shake and wash water. Slope should be adjusted so that satisfactory longitudinal progress is effected with minimum consumption of water and energy and so that sand does not pack behind the tailboard and in the riffles. Slope must be steeper for a gravel containing much fine sand than for one of more pebbly character, likewise if much heavy sand (black sand) is present a steeper slope is necessary than otherwise. If gold is fine, the bed must be loose, hence a relatively steep slope is required.

Capacity varies with size. Purington (*Bul. 263, USGS*) gives 3 to 5 cu. yd. per 10-hr. day for 2 men working steadily, one feeding and removing boulders and tailing, the other rocking and washing. *Van Wagenen* gives 3 cu. yd. per 10 hr. per man in ordinary grave and 2 cu. yd. in cemented gravel. Capacity is less in prospecting work on account of more frequent clean-ups.

Water consumption varies widely, but from 50 to 100 gal. per cu. yd. treated is sufficient if the gravel is not too fine, slope sufficient, and water is used sparingly.

Applicability. A rocker catches coarse gold readily but will lose much fine gold. The loss will be greater with clayey or cemented gravel or if muddy water is used. In some cases quicksilver placed in the riffles will aid in catching fine gold, but since the loss occurs rather through failure to effect settlement of the gold than by washing it out of the riffles, this expedient is rarely successful. The rocker is rarely used except in sampling when a device of greater capacity than the pan is needed, and in cleaning up concentrate from sluices and gold tables.

North Carolina rocker consists of a semi-cylindrical trough, closed at both ends, with two longitudinal riffle cleats near the bottom. The charge with water is first rocked gently to effect stratification, then the apparatus is rocked differentially in such a way as to throw waste over the rim and leave the gold and heavy sands between the riffle cleats. Clean-up is made in a pan.

Mechanical rocker (*116 J 334*) driven by crank from a water wheel has been used to treat ordinary gravel at the rate of 5 cu. yd. per 8 hr. The trough was 14 in. × 12 ft.; the screen hopper sloped in the opposite direction from the trough and discharged oversize of a 0.5-in. screen automatically over a trifle.

10. Long tom

Description. The long tom (Fig. 16) has greater capacity than a rocker but requires more water. It consists of a sloping trough about 12 ft. long, 15 to 20 in. wide at the upper end, flaring to 24 or 30 in. at the lower end, with sides 8 to 12 in. high. A perforated plate with ⅜- or ½-in. holes, set level or slightly inclined is fitted at the lower end. The usual slope of the trough is 1 in. per ft. A wider and usually shorter trough with transversely riffled bottom, set on a flatter slope, follows the first and receives the undersize from the screen.

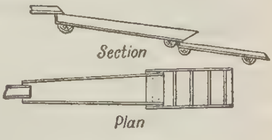


FIG. 16.—Long tom (*after Bowie*).

Manipulation. Gravel is shoveled into the upper trough and water is led in by a flume at the head end. The operator works the gravel over thoroughly, forks out the boulders

and works fine material through the screen. Coarse gold settles in the upper box and fine gold is caught behind the riffles in the lower trough. *Wilson* recommends that the slope of the riffled box should be so flat that fine mud will collect therein, thus insuring that fine gold is not scoured out, but this will result in sanding behind the riffle cleats to such an extent that their protective function for fine gold is defeated and the bottom will become merely a transversely undulating sandy surface.

Capacity varies with the kind of gravel and size of screen perforations and is limited by what a man can work through the screen. *Wilson* states it to be 6 cu. yd. of ordinary light gravel or 3 to 4 cu. yd. of cemented gravel per 10 hr. with 2 men working, one shoveling in and taking care of tailing and the other working the upper box. *Van Wagenen* gives 5 to 6 cu. yds. of ordinary gravel or 3 to 5 cu. yd. of cemented gravel per man per 10 hr., but this is undoubtedly too high.

Applicability. The long tom uses less water than a sluice (Art. 11) but at the expense of more labor. It saves a given amount of gold in a shorter linear distance. It has, therefore, been used in small-scale placer work where water and lumber were scarce, and has also been rather widely used on dredges for cleaning up black-sand concentrate. (*115 P 825, 113 J 251.*)

11. Sluice

Description. A sluice (Fig. 17) is a long, relatively narrow and deep inclined trough, usually with riffled bottom, through which gravelly material containing a small percentage of heavy valuable mineral is transported by water, with the result that the heavy material settles in the riffles and is held while the light is washed out at the lower end. Sluices have been used extensively in washing gold-bearing gravels in all parts of the world; much less extensively in working tin (cassiterite) placers, and occasionally for other service such as the recovery of metallic copper and brass from foundry refuse and the like.

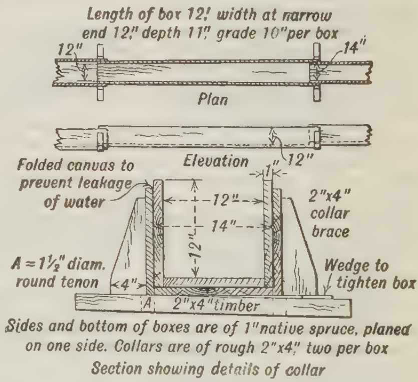


Fig. 17.—Sluice box.

The size and elaborateness depend on the scale of work. Sluices for prospecting and small-scale mining work are usually 12 in. wide and deep, made in 12-ft. sections, usually with sufficient flare to allow about 2 in. telescope in joining them together in strings. Some operators prefer butt joints with cover strips, on the ground that they cause less clogging than telescope joints. One-inch lumber is usually employed for sides and bottoms of small sluices and 2 × 3-in. or 2 × 4-in. scantlings for bracing (see Fig. 17). Such boxes will generally serve for a season (4 to 6 months), rarely for more than a year of continuous service. Sluices intended for permanent use are made with 1.5- to 3-in. bottom boards and bottom side boards and with 1- to 3-in. upper side boards; usually in 12-ft. lengths, the transverse dimensions depending upon the amount of solid and water to be transported, the slope or grade, and the character of the heavy mineral and gravel.

Water-carrying capacity. *Bowie* gives the following data: A sluice 6 ft. wide, 36 in. deep on 4 to 5 per cent. grade will run 3000 to 5250 cu. ft. of water per min.; 4 ft. wide and 30 in. deep, 1200 to 2250 cu. ft. per min. on a 2.1 per cent. grade and 3000 cu. ft. per min. on a 4 per cent. grade; 3 ft. wide, 30 in. deep, 900 to 1500 cu. ft. per min. on 1.5 per cent. grade. Assuming the wetted depth to be 50 per cent. of the total depth, which is usual,

these figures correspond to average water velocities in the sluice of from 4 to 10 ft. per sec. *Van Wagenen* states that the moving power of water in sluices varies with the velocity as follows: 16 ft. per min. begins to wear away fine clay, 30 ft. per min. just lifts fine sand, 39 ft. per min. lifts sand grains up to 1/16-in. diameter, 45 ft. per min. moves fine gravel, 120 ft. per min. moves 1-in. pebbles, 200 ft. per min. moves 2- to 3-in. pebbles, 320 ft. per min. moves boulders 3- to 4-in. in size, 400 ft. per min. boulders 6- to 8-in., 600 ft. per min., boulders 12- to 18-in. In all cases the depth of water must be sufficient to cover the largest particle to be moved. The bottom width should be 1.75 to 2.25 times the wetted depth for maximum water-carrying capacity.

The sluice boxes should be made thoroughly water-tight. The lumber should, therefore, be surfaced and sized, free from knots and cracks, but it need not necessarily be tongued and grooved. Sills and posts for large sluices are made of 4 × 6-in. or 6 × 6-in. lumber and spaced about 3 to 4 ft. apart. On every second or third set the posts should be braced to the sill with 1 × 6-in. or 1 × 8-in. angle braces and the sill should be about twice as long as the sluice width to give proper slope to the braces. The bottom and sides should be spiked to the sills and posts with 30d and 20d spikes respectively, spaced about 4 in. (*Bowie.*)

A large sluice used at La Grange mine in California is shown in Fig. 18 *F*. The sides, bottom and side liners are 3-in. plank. Rail riffles, set longitudinally in the head end of the sluice and transversely below, are used.

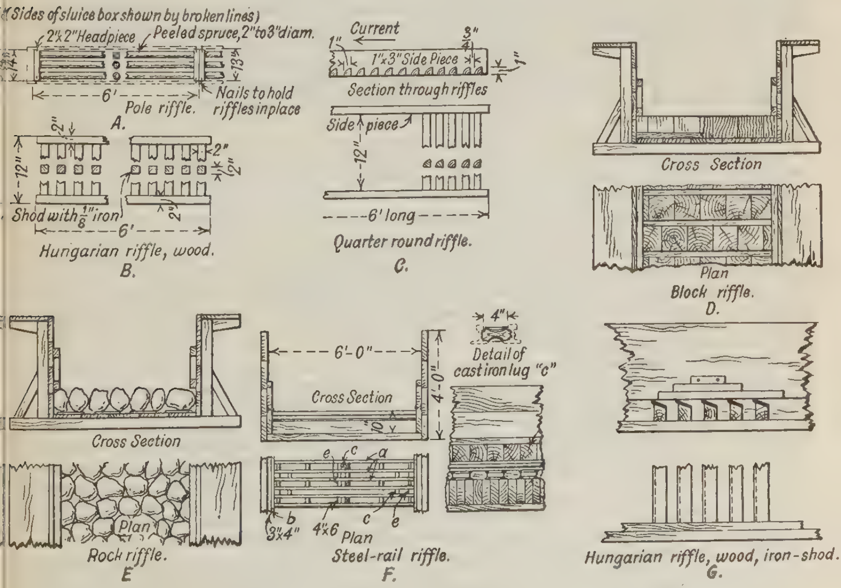


Fig. 18.—Riffles.

Australian practice differs (*17 MM 16*). Boxes for gold gravel are made 4 to 8 ft. wide, 12 to 15 in. deep, and 60 to 100 ft. long, in 12-ft. sections and are bolted together with flush joints and outside cover plates. Boxes for tin gravels are 10 ft. wide, 18 in. deep and 120 to 200 ft. long.

Riffles are placed in sluices to catch and hold concentrate and, to a minor extent, to disintegrate the gravel. They perform the first function by roughening the bottom surface of the sluice, thus decreasing the velocity of the lower

layer of water and permitting concentrate to settle readily. The interstices furnish protection to the settled material from the horizontal transporting effect of the stream. A secondary desideratum is a boiling action in the bed of settled material in the riffles, as a result of which the lighter material will be lifted out while the heavy particles remain. Disintegration is effected by tumbling of the solid as opposed to sliding.

Pole riffles (Fig. 18*A*) are probably the most common form for small sluices, when a maximum amount of material is to be moved with a minimum amount of water, gold is coarse and loss of a small amount of fine gold is relatively unimportant. They are made of 2- to 3-in. peeled poles or 3 × 3-in. squared strips spaced 1½ to 3 in. in the clear and made up in 6-ft. units by transverse straps nailed on at both ends. Pole riffles wear quickly but are inexpensive. Squared riffles are often protected by angle iron. The end straps are cut to fit snugly across the sluice box and are held in place by nails driven into the ends through the sides of the sluice. A somewhat similar riffle, made of 1.5 × 6-in. plank set on edge longitudinally, spaced by 1.5-in. blocks, and shod on the upper edge with an iron bar 1 in. × 1.5 in., is described by *Bowie*. Hutchins (*Bul. 263 USGS 200*) describes a pole riffle with 2 × 2 × 1/16-in. pieces of sheet iron driven into the poles cornerwise, long axis parallel to the poles, for use in disintegrating clayey material.

Rail riffles (Fig. 18*F*) are used in large sluices where resistance to wear is an important factor. They may be set lengthwise, in which case their action is similar to that of pole riffles; or they may be set transversely. Longitudinal-rail riffles are made up in 6-ft. lengths and the width of the sluice box. They are set bottom-side up and held together by special locking devices as shown. Bouery (*95 J 1055*) found that the life of rails set transversely was greater than with longitudinal setting, that between 4 and 6 in. center to center was the best spacing for transverse setting, considering both duty of water and gold saving; and that with longitudinal setting wear was more rapid with 8-in. spacing than with 5-in. He also found that when rails were set directly on the bottom of the sluice the gold-saving capacity was insufficient and therefore devised and used the combination riffling shown in Fig. 18*F*, in which the principal function of the rails is to protect the wooden riffles from wear and to increase the transporting duty of the water. 45-lb. rail riffles set longitudinally handled about 32,000,000 cu. yd. of water and 2,500,000 cu. yd. of gravel before replacement was necessary; the same weight rails set transversely handled 50 per cent. more material, with spacing in both cases 5 in. center to center.

Block riffles (Fig. 18*D*) are the commonest form in all large sluice lines. The blocks are usually about 8 to 13 in. long, either squared or round, 6 to 20 in. in diameter or on an edge. They are nailed to 1- or 1.25- × 2- or 3-in. cross strips before being put in place and these strips are nailed through the sides of the sluice. Holding-down strips nailed to the inner sides of the sluice are also used. Squared riffle blocks are usually spaced about 1 in. from face to face transversely (thereby differing from the figure) and the nailing strips produce a spacing of about 1 in. longitudinally. Longitudinal joints are preferably staggered in adjacent transverse rows. Round blocks are fitted as closely together as possible when fastening them to the nailing strip.

Block riffles produce a large amount of drag; they offer good protection to settled material and cause sufficient boiling to remove a considerable amount of sand from the concentrate. Their disintegrating effect is low, but, in compensation, they interfere but little with transport. *Bowie* advises the use of block riffles in the head boxes at all times. Their ADVANTAGES are: cheapness, efficiency in gold saving, ease of removal and clean-up; their DISADVANTAGE is rapid wear. At LA GRANGE, when handling 1000 cu. yd. of gravel per hr. in a 6-ft. sluice, the life of pine blocks was only 17 days (*95 J 1056*). *Bowie* states that nut- or pitch-pine or a wood that is long-grained and "brooms" readily is best and deprecates the use of oak or other hardwoods that wear smooth. Round blocks are more difficult to set than squared but may cost sufficiently less to be economical. Welton (*20 IMM 172*) criticizes block riffles on the ground that gold does not readily settle in them and that they absorb gold values. The first criticism is not in line with general experience. There is, undoubtedly, some catching of gold in interstices in the blocks themselves and old blocks should be and commonly are burned and the ashes panned.

Rock or stone riffles (Fig. 18*E*) are made by packing the bottom of the sluice with selected rocks, usually flattened, set on edge, inclined slightly with the run of the material and with the longer dimension transverse. They are prevented from shifting longitudinally by transverse poles nailed in at intervals. No devices for holding down are ordinarily necessary but with large quantities of coarse gravel they may become displaced. Rock riffles wear longer than block riffles and are more effective in breaking up cemented gravel but they are more expensive to set and to clean up. They also require more water and greater slope to handle a given quantity of a given gravel. Welton says that under conditions requiring 0.5 in. per ft. slope with block riffles, rock riffles will require 0.75 in. per ft. They are com-

monly used in California for a short distance at the head of the sluice line and also in the tail sluices following block riffles, where their principal function is as a chute liner and clean-ups are infrequent.

Rock for riffles should be hard and uniform in size and hardness. Quartz is probably the best material, if locally available. As between block and stone riffles, Welton states that the former will collect three times as much gold in a given length. He ascribes this to the fact that with the rocks there is no continuous bar formed across the sluice and that the gold, therefore, works along more readily and for longer distances.

Hungarian riffle. This name is used by different writers to denote different forms of riffles. Probably the nearest to a common characteristic in all riffles so named is an overhang on the down-stream side. Fig. 18*F* shows one form so made that the overhanging strap slopes upward slightly on the down-stream side. It is frequently claimed for this type of riffle that it is good for disintegrating gravel, but Perrett (*122 P 417*) rightly remarks that a sluice is not an efficient gravel disintegrator and that the use of obstructive riffles to effect disintegration results in undue loss of capacity. Riffles formed by gouging or boring out pockets in plank (PIT RIFFLES) are also called Hungarian riffles (*Wilson*) as are also transverse square riffles without overhang. (Fig. 18*B*, *100 J 995*.) PIT RIFFLES are used with mercury in the treatment of fine material. They are not suitable for coarse material on account of the fact that mercury and amalgam are splashed out of the depressions by impact of the large stones.

Miscellaneous riffles. A large number of other riffle forms have been used. Expanded metal over cocoa matting is common on dredge tables.

Moline (*17 MM 16*) describes several riffles used in Australian gold practice. A so-called Venetian riffle is used in certain parts of Russia (*15 MM 148*). The natural surface of bed rock or a bed-rock surface crudely riffled by cutting transverse gutters forms the first gold-catching surface in some hydraulic mines. The grade must be much greater (*Wilson* says twice) than that of a wooden sluice for the same material. Hutchins (*105 J 861*) describes the use of a riffle in a Siberian sluice, consisting of perforated plate with ¾-in. apertures, set 3 in. up from the bottom.

Blankets, corduroy, hides with the hair up, canvas, velvet, carpet, cocoa matting, and sod have all been used as gold-catching surfaces. For such surfaces the feed should not be coarser than 1- to 2-mm.

Comparison of riffles. It is impossible to classify riffles in other than most general terms with respect to their ability to perform their various functions. Riffles presenting a uniformly roughened upper surface of relatively fine texture have maximum retarding effect on the lower layer, but do not roughen the water surface, and roughening is desirable for catching fine gold. Transverse riffles forming a complete bar across the bottom of the sluice afford maximum protection to settled material and also cause considerable boiling, which enriches concentrate, but they obstruct the passage of gravel and consequently lower capacity. Longitudinal riffles cause the least drag on the lower layers of the stream and offer the least obstruction to the passage of gravel; they also offer minimum protection to settled material and are more subject to scouring action. The life of different types and the ease of removal and replacement are important considerations. In many cases the availability of a given riffle material or the prejudice of the operator is the deciding factor.

Slope of the sluice line depends on the character of the gravel, character of the gold, kind of riffles and quantity of water available. Flat and shingly gravel requires more slope than rounded gravel; coarse gravel requires greater slope than fine. Very fine gold may be carried in suspension in the water if the slope is steep and the velocity correspondingly high, especially if there is much clay or mud; on the other hand, moderately fine gold is best caught in "rough" water and this condition is obtained by steep slope. Obstructive and transverse riffles used for disintegrating clayey and cemented gravels require steeper slope than longitudinal or block riffles. Restricted water quantity demands steep slope, if the maximum quantity of gravel is to be handled. With a given amount of water narrow sluices require, of course, less slope than wide. As a general operating rule, the slope should be such that

the water used will transport the rocky material and prevent sand from packing in the riffles, but should not exceed this.

Bowie states that 6 to 6.5 in. per 12-ft. box is usual; 9 in. to 12 in. when much clay is present. Rounded gravel containing considerable clay or soil may be moved on a slope of about 2 in. per 12 ft. but it is better to have as much as 6.5 in. per 12 ft. to increase capacity and lessen the labor required for handling large boulders. Coarse gravel needs from 6 to 10 in. per 12 ft. and considerably more water than fine gravel. *Van Wagenen's* rule for slope for handling average gravel is: $F = V^2P/2A$ where F = fall in ft. per mile, V = velocity in ft per sec., P = wetted perimeter in ft. and A = area of stream in sq. ft. *Wilson* gives Chezy's formula for the velocity V in ft. per sec., necessary to transport boulders of average diameter a in ft. and specific gravity g as $V = 5.67\sqrt{ag}$.

Perrett (*122 P 417*) states that the shingly Siberian gravel, when unscreened, requires a slope of 18 in. per 12-ft. box. Sluices for treating tin gravels are usually set on flatter slopes, ranging from 4.5 to 6 in. per ft.

Length depends principally upon the character of the gold, less upon the dimensions and slope of the sluice and the kind of riffles. Coarse and granular gold settles quickly and is easily held in the riffles, fine and porous gold is carried long distances by the current.

The minimum length for relatively deep, narrow sluices is 500 to 600 ft.; the sluice line at some mines is several thousand feet long, but this great length is as often for transport of tailing to a suitable dumping ground as for saving fine gold.

Drops up to 12 in. in height in the line of boxes are of use in breaking up cemented and clayey gravel and in causing gold to settle in the riffles.

Curves lessen the velocity of the stream and cause deposition of solid matter with resultant clogging of the sluice.

Wilson recommends elevation of the outer edge 1 in. for every degree of curvature and an increase in slope for a short distance (2 or 3 box lengths) below the curve.

Mud box or **dump box** (Fig. 19) is frequently used at the head of a sluice line when material is fed in batches, as by horse scraper, drag-line or derrick

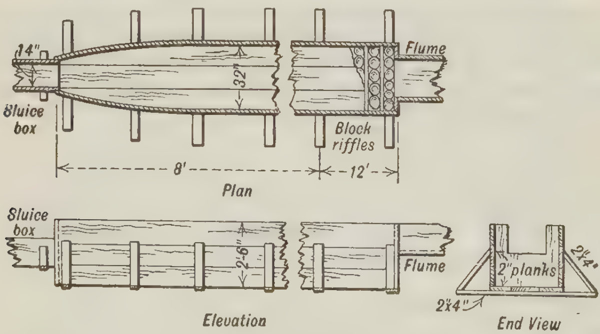


Fig. 19.—Mud box.

bucket or the like, and contains much clayey or cementing material, or when water is scarce. In it the material is worked over and disintegrated by means of a coarse-tined rake or fork and large boulders are removed. It may be riffled or not. If so, rock riffles are usual. The slope is steeper than in the main string, usually about 1 in. per ft. Much coarse gold is caught here and clean-ups should be frequent.

Undercurrent is a device used in connection with sluice lines to catch fine gold from the finer material fed to the sluice. In order to treat the mater-

ial in a thin film and thus get the turbulent current and small settling distance necessary for catching fine gold, the undercurrent is made wide; it must be made correspondingly steep in order that the water may have sufficient velocity to move the gravel. A perforated screen or a grizzly usually with ⅝-in. to 1-in. openings is set with its upper surface about 1 in. below the top of the riffles in the main sluice. The area and aperture of the grizzly must be such that the main sluice is not robbed of so much water that it will not carry the coarse tailing. Undersize is led off in a converging launder, with suitable distributing devices to spread the feed evenly across the width of the undercurrent. The latter is proportioned to the size of main sluice and the quantity of gravel carried.

The WIDTH is usually 5 to 20 times that of main sluice and the length varies from 20 to 50 ft. Welton (*loc. cit.*) recommends an undercurrent 20 ft. wide and 30 ft. long for a 4- to 5-ft. sluice. He also recommends dividing such an undercurrent by longitudinal walls into four sections, thus allowing three to be run while one is being cleaned upand re-blocked. This is a good arrangement, since the undercurrent must usually be cleaned up more frequently than the main sluice. The USUAL SIZE of undercurrent in Alaska for a 12-in. sluice line handling 400 to 500 cu. yd. per day is 20 ft. square. The SLOPE is usually about twice that of the main sluice; *Bowie* recommends 12 in. per 12-ft. box for longitudinal plank or pole riffles, 14 in. for blocks and 16 in. for stones. RIFFLES are placed both in the distributing launder and the undercurrent proper. They are of any of the varieties described. A common arrangement is to have blocks in the upper half and rocks below or longitudinal poles above and transverse poles below. TAILING is usually led back into the main sluice. The RETURN SLUICE should enter the main sluice at an acute angle and with little or no drop, in order not to interfere with the flow in the main sluice.

Recovery in undercurrents varies according to the character of the gold and the sluice treatment preceding. Welton says that an undercurrent following 500 ft. of sluice should make 10 to 15 per cent. of the total combined recovery. It frequently runs as low as 2 per cent.

Tables are short, wide sluices, usually set on a relatively steep slope, taking a finer feed than the ordinary sluice and usually operated with some mercury in the riffles. Their principal use is on gold dredges (see Sec. 2, Art. 19).

Hutchins (*105 J 861*) describes a table installation in Siberia where material through a 1-in. screen was treated on tables 10 ft. wide by 30 ft. long set on a slope of 2.25 in. per ft., the tailing after passing through a 0.5-in. screen passed over a similar set of tables on 2-in.-per-ft. slope, and this tailing after screening through a ¼-in. screen was sent over a similar set of tables on 1.5-in.-per-ft. slope. There were a few riffles on these tables, but sand packed behind the riffles and spoiled their usefulness. Very little gold was caught, most of it being caught in the sluice preceding the tables. It will be noted that this is an undercurrent installation.

Perrett (*122 P 417*) argues for tables as opposed to the ordinary sluice on the score of compactness, low water consumption and high gold saving. He cites a Russian plant with tables 38 ft. long on which 97.56 per cent. of the total recovery made was made on the first 10 ft., 1.95 per cent. on the following 14 ft. and 0.49 per cent. on the last 14 ft. The loss was 0.6 grain per cu. yd.

Water consumption is extremely variable.

Hutchins (*105 J 861*) states that ordinary cemented gravel from CALIFORNIA DRIFT MINES requires from 7 to 13 cu. ft. of water per cu. ft. of gravel and hard cemented gravel requires from 20 to 27 cu. ft. on the average. *Bowie's* figure for NORTH BLOOMFIELD, treating mostly top gravel in 6-ft. sluices with block and rock riffles on a slope of 0.54-in. per ft., is 18 cu. ft. of water per cu. ft. of gravel and at LA GRANGE, in a 4-ft. sluice with block riffles on a slope of 0.25 in. per ft., 56 cu. ft. Purington (*Bul. 263 USGS 142*) gives 40 cu. ft. of water per cu. ft. of gravel with block riffling on 0.33 in. per ft. slope under most favorable conditions in SOUTH-EAST ALASKA. In the ATLIN DISTRICT, B. C., with pole riffles on a slope of 0.67 in. per ft., 20 cu. ft. of water per cu ft. of gravel was used. At the Bowlder Creek mines of SOCIÉTÉ MINIÈRE DE LA COLOMBIE BRITANNIQUE a 24-in. sluice on a slope of 0.5 in. per ft., with block and rail riffles, required 80 cu. ft. of water per cu. ft. of gravel, of which 50 per cent. was boulders 18 to 30 in. diameter. A KLONDIKE bench-gravel sluice, 24 in. wide on a 1-in.-per-ft. slope, with block riffles, required 20 cu. ft. of water per cu. ft. of gravel. 12-in. sluices in ALASKA in shoveling-in work require 45 to 90

cu. ft. of water per min. Hutchins gives 80 cu. ft. of water per cu. ft. of gravel as an example of inefficient washing of clayey gravel in SIBERIA. Consumption on NEW ZEALAND GOLD DREDGES is given (*16 MM 81*) as 75 cu. ft., while on tin dredges about 20 cu. ft. is provided. Perrett (*122 P 417*) says that not over 33 to 40 per cent. of the amount of gravel can be washed in a sluice with a given amount of water that can be washed in a plant equipped for removal of oversize, with subsequent treatment on gold tables.

Operation consists in keeping gravel running through the sluice, undercurrent or table, until the riffles are so filled with concentrate that an excessive amount is going into the tailing, then cleaning up. The time between clean-ups depends upon the sluice and the character and quantity of gravel treated. Usually the head sluices are cleaned up at approximately regular intervals of one to four weeks, as dictated by experience, while the tail sluices are cleaned only once or twice per season and are depended upon to catch any gold that passes the head sluices by reason of too great an interval between clean-ups.

Cleaning up. The supply of gravel is stopped and clear water run until the tops of the riffles are clear of gravel. The riffles are then lifted out, beginning at the lower end of the first length of, say, 100 ft. of boxes, and washed carefully into the sluice. After all the riffles in a given section have been removed, a light flow of water, just sufficient to move the material slowly, is turned on and the concentrate is carefully and slowly turned over by means of wooden paddles 3 or 4 in. wide pushed upstream along the bottom. Rocks are thrown out by hand. Welton (*20 IMM 172*) describes a special clean-up shovel made of light sheet steel with flat bottom, sides turned up 1 in. and back sloping at 45° to the bottom. The shoveler faces down-stream to pick up a load and turns around and holds the shovel in the stream, when light material is washed over the back leaving rich concentrate on the shovel. As material works down-stream, a head of enriched material is left behind. This is carefully collected and further concentrated in a pan or rocker, with or without mercury, as circumstances dictate, or it may be brushed against a flat stone in the sluice, which serves as a planilla (Art. 13). (When the gold is amalgamated, iron or granite pails, not galvanized or tinned, should be used for collecting concentrate.) The next lower section is next treated in the same way, or, in some cases, concentrate from the lower sections is carried in pails to the head of the first section and there cleaned. Finally gold is collected from crevices by means of a pointed amalgamated-copper spoon. Considerable gold may get so far into crevices that it cannot be collected in this way. Such gold is recovered, when the sluice is worn out or the working is abandoned, by burning the sluices and panning the ashes. Ellis (*100 J 993*) states that some Alaskan boxes on burning have yielded as much as an ounce of gold.

The time required to clean up and re-block 100 ft. of 4-ft. sluice is about 4 hr. (*20 IMM 208*).

Heavy sand concentrate that banks up before the riffle at the lower end of the section being cleaned is collected separately and further concentrated in rocker or pan, or it may be sold or treated with mercury and cyanide in a tumbling barrel.

Distribution of gold in sluice. The bulk of the gold content of gravels settles to the bottom of the sluice with great rapidity and resists transportation with extraordinary tenacity.

Wright (*Gold Fields and Mineral Districts of Victoria, R. B. Smyth*) says that in a 12-in. sluice sloped 1 in 48, using 600 gal. per min., 95 per cent. of the gold was found within 3 ft. of the feed point, lying on a smooth board. *Bowie* states that at NORTH BLOOMFIELD 92 per cent. of the total yield was made in the first 1800 ft. of sluice, 3.75 per cent. in 6200 ft. of bed-rock sluice following, 2.5 per cent. in undercurrents. He gives considerable detailed data of similar import. Purington (*Bul. 263 USGS 142*) gives the following data: South Coast region of ALASKA, gold bright and rough though fine, mercury used in boxes, 75 per cent. saved in the first 600 ft. MCKEE CREEK, Atlin, B. C., slope 0.67 in. per ft., block riffles, gold coarse, 85 per cent. saved in first box, all that it pays to clean up saved in the first five boxes (60 ft.). At another mine in the district no gold is caught beyond the first 250 ft. of boxes. ELDORADO COUNTY, CALIFORNIA, slope 1.1 in. per ft., car-wheel riffles, mercury in the boxes; 75 per cent. of the total recovery was made in the first foot, 8.4 per cent. in the next 10 ft., 9.6 per cent. in the next 50 ft., and 2 per cent. in the next 60 ft.; slightly less than 5 per cent. of the total gold recovered was caught in 2380 ft. of rock-paved sluice following the first 1000 ft. BONANZA CREEK, KLONDIKE, gold fine, bright and smooth, 2-ft. boxes on 1-in.-per-ft. grade, block-riffled, 80 per cent. of total gold recovery was made in the first box, 85 per cent. in the first 180 ft., none in the last two boxes in the 500-ft. string.

Hutchins (*Bul. 263 USGS 190*) estimates that in small-scale Alaskan practice, shoveling-in to short strings (3 to 6) of 12-in. boxes set on about 0.5-in.-per-ft. slope, using pole riffles, about 80 to 90 per cent. of the gold fed is recovered but that most fine gold is lost. Bouery (*95 J 1056*) gives the data shown in Table 4 representing the distribution of gold in size and

Table 4. Distribution of gold according to size in La Grange sluice boxes. (*After Bouery*)

Box number	+ 10-mesh	− 10 + 50-mesh	− 50 + 100-mesh	− 100+150-mesh	150 + 200-mesh	− 200-mesh
5	45.80 oz.	50.70 oz.	1.38 oz.	0.36 oz.	0.31 oz.	1.45 oz.
6–16 incl.	18.00	83.30	2.33	1.00	0.31	0.83
22	1.73	20.22	3.08	0.70	0.25	0.62
48	0.18	2.18	1.06	0.12	0.05	0.16
88	0.018	0.12	0.47	0.008	0.026	0.005
136	None	0.053	0.027	0.043	0.011	0.01

quantity along the LaGrange sluice line. The table indicates remarkable horizontal travel of some of the coarse gold and probable lack of economy, from a gold-saving standpoint alone, in maintaining so long a string of boxes. It also indicates that only the more rounded grains of gold finer than 200-mesh are saved and that most of those grains too small to settle in the early boxes do not settle at all.

Capacity. A string of 12-in. Alaskan boxes rarely handles more than 150 cu. yd. per 10 hr., but this is done easily if sufficient water is available. (*100 J 994, 122 P 417.*) Purington (*Bul. 263 USGS 142*) gives the following figures on Alaskan practice: In the South-Coast region a sluice 4 ft. wide × 4 ft. 10 in. deep, slope 0.33 in. per ft., paved with block riffles handled 5000 cu. yd. of gravel and 200,000 cu. yd. of water per 24 hr. Atlin District (B. C.), 400 cu. yd. per 24 hr. of coarse heavy gravel were handled with 32,000 cu. yd. of water in a sluice 24 in. wide, 1400 ft. long, slope 0.5 in. per ft., with block and rail riffles. Klondike bench gravel, 1000 cu. yd. of gravel with 20,000 cu. yd. of water in a sluice 24 in. wide, 20 in. deep, slope 1 in. per ft., block-riffled.

At Y-Water in Australia sluices for treating tin gravels at the rate of 100 cu. yd. per hour are four in parallel, each 10 ft. wide by 3 ft. deep by 180 ft. long. The area of sluices on Australian tin dredges varies from 8 to 43 sq. ft. per cu. yd. treated per hour, all late installations ranging between 30 and the higher figure. The corresponding figure on gold dredges is about 23 sq. ft. in Australian practice and 37.5 sq. ft. in English practice (*16 MM 81*). Purington and Smith (*15 MM 148*) state that a Russian sluice 29 in. wide, 18 in. deep and 280 ft. long, sloped 14 in. in 12 ft., fitted with T-grate riffles and inverted rails set both longitudinally and transversely in different sections, treated 1100 cu. yd. (place measure) per 18 hr. and made 96 per cent. recovery, 88.3 per cent. being made in the first third. *Van Wagenen* gives figures of 100 to 200 cu. yd. of ordinary gravel or 60 to 80 cu. yd. of cemented gravel per man per 10 hr., but this is for sluice-box operators only and does not include delivery of material to the boxes. Hutchins (*Bul. 263 USGS 190*) gives about 5.5 cu. yd. per man shift when shoveling-in and Purington's figures for the same operation range from 2.75 cu. yd. per 10 hr., where large boulders interfered, to as high as 12 cu. yd. with 3-ft. bank and 5-ft. lift.

Use of mercury in sluices is local rather than general custom. It is relatively rare in Alaska and was relatively common in California. This probably followed from the character of the gold, which while perhaps as fine in Alaska deposits as in California, was granular, while the fine California gold was scaly and bright. Hence mercury was considered relatively unnecessary in Alaskan sluices while it was both necessary and highly effective in California. Hutchins (*Bul. 263 USGS 204*) however, brands the failure to use mercury in the Alaska sluices as a penny-wise-pound-foolish policy and recommends its use wherever fine gold that will amalgamate is present. Mercury is commonly used on undercurrents and tables. The finer feed is not so liable to splash it out of the riffles and flour it as the coarse gravel in main sluices and the short length of these devices demands the aid of mercury to keep down the values in the tailing.

Mercury is usually introduced after the riffles have sanded up. It should be put in in such a way as to prevent flouring and in an amount insufficient to flood the riffles. The endeavor should be to maintain as large and as clean a surface of mercury as possible without causing excessive travel along the sluice. In some plants mercury is not introduced until just before the clean-up, in which case its purpose is not so much the recovery of gold as aid in collecting it. A low-grade amalgam (mercury containing a small amount of gold) is more effective than clean mercury.

The quantity used, according to *Bowie*, is about 3 flasks (225 lb.) in the upper 200 to 300 ft. of a 6-ft. sluice. A 24-ft. undercurrent will require 80 to 90 lb.

Table 5. Performances of trough washer on British coals (*after Drakeley*)

	Unsized feed, −2-in.						
	Coal free from visible impurity	Raw Coal		Washed coal		Refuse	
Ash, per cent	4.13	16.74		11.46		62.53	
S, per cent	1.78	2.51		1.93		7.21	
Calorific value, C. H. U.	7738	6671		7129		...	
Float, per cent.(a)	...	81.35	...	89.45	...	11.13	...
Sink, per cent.(a)	...	...	18.65	...	10.55	...	88.87
Ash, per cent	...	5.88	64.10	5.84	59.09	8.36	69.31

	Unsized feed, −½-in.						
	Coal free from visible impurity	Raw coal		Washed coal		Refuse	
Ash, per cent	3.35	17.67		8.86		58.31	
S, per cent	0.94	1.65		1.09		4.13	
Calorific value, C. H. U.	7681	6448		7235		...	
Float, per cent.(a)	...	77.13	...	90.06	...	10.10	...
Sink, per cent.(a)	...	...	22.87	...	9.94	...	89.90
Ash, per cent	...	5.26	59.51	3.85	54.37	6.24	64.16

	Sized feed										
	Coal free from visible impurity	Raw coal		Washed coal						Refuse	
				−⅜-in.		−¾ + ⅜-in.		−1½ + ¾-in.			
Ash, per cent	4.36	12.48		10.49		7.70		7.19		66.52	
S, per cent	0.53	1.07		0.68		0.61		0.58		1.83	
Calorific value, C. H. U.	7801	6939		7251		7488		7536		...	
Float, per cent.(a)	...	89.55	...	90.96	...	96.19	...	97.75	...	15.17	...
Sink, per cent.(a)	...	...	10.45	...	9.04	...	3.81	...	2.25	...	84.83
Ash, per cent	...	5.65	69.15	5.41	61.63	5.72	60.18	5.99	59.47	11.41	76.38

a Solution 1.35 sp. gr.

Loss of mercury depends principally on the quantity used, the character of the gravel, the type of riffles, and the intervals between clean-ups. *Bowie* gives the loss at La Grange as 0.00024 lb. per cu. yd. He states that at North Bloomfield it was 11 to 25 per cent. of the total used and was greatest in the rock sluices, which were cleaned only at long intervals. At the same mine, with light gravel, low slope and little water the loss was from 4.4 to 6.1 per cent.

12. Coal sluices

Trough washer for coal is a form of sluice that has been used rather extensively in washing bituminous coal at English and Scottish collieries. The simplest form is a trough 1 to 2 feet wide and about three-quarters as deep, 50 to 250 ft. long, set on a slope of 1 in 20 to 1 in 40, with riffle cleats 2 to 4 inches high set at intervals of several feet along the bottom, the number depending on the amount of slate to be removed. Raw coal broken to 1½-in. or smaller is fed at the upper end with sufficient water to move it freely; the slate settles to the bottom and is caught and held in the riffles; the coal flows into a dewatering tank at the end, from which the coarse settlings are removed by a drag chain or bucket elevator while the fine material overflows and is sent to suitable reclaiming devices. When the riffles have filled with slate the flow of feed is stopped and the slate shoveled out. In order to maintain continuous operations, two or more troughs are installed in multiple with a common feed trough, thus allowing one to be cleaned while the other is working.

At Flimby Colliery, Maryport, Cumberland (*11 MIS 184*) two troughs 17 in. wide × 13 in. deep, and 150 ft. long, sloped 1 in 36, with two 2-in. riffles, one near the end, and the other about 20 ft. above, treated 10 tons per hr. with a water consumption of 400 gal. per min. (10 tons of water per ton of raw coal). The velocity of the stream was about 300 ft. per min. The refuse removed was about 18 per cent. of the feed.

Table 5 (*54 IME 455*) shows the results obtained by trough washers treating different sizes of raw coal at three British collieries.

Elliott washer is an improvement on the simple trough washer. It consists of a trapezoidal trough, 50 to 100 ft. long, inclined 1 : 12 to 1 : 18, up the bottom of which full-width flights about 3 in. high travel slowly. These act as riffles and at the same time drag the refuse to the upper end, while cleaned coal discharges at the lower end. Feed is introduced some 20 ft. from the upper end.

A trough 60 ft. long with 12-in. bottom width treats 5 to 10 tons of −1-in. raw coal per hr. with a water consumption of 8 to 10 tons per ton of coal. Table 6 shows performance at a British washery.

Table 6. Performances of Elliott washer. (*After Drakeley*)

	Coal free from visible impurity	Raw coal, −1-in.		Washed coal		Refuse	
Ash, per cent.	4.63	18.03		10.07		67.75	
S, per cent.	1.17	1.96		1.43		5.08	
Calorific value, C. H. U.	7872	6669		7443		.....	
Float, per cent.(*a*)	....	80.71	....	92.03	....	10.12	....
Sink, per cent.(*a*)	....	....	19.29	....	7.97	....	89.88
Ash, per cent.	....	5.33	71.18	5.27	65.52	8.78	74.01

a Solution 1.35 sp. gr.

13. Strake

Description. In principle, the strake is a shallow sluice designed for the treatment of fine material. The sluice riffles are replaced by blanket, plush, corduroy, carpet, canvas, silent felt, hides or like material. These serve the same purpose as riffles, but have shallower depressions, which are sufficient to hold concentrate against the wash of the pulp-carrying current but not deep enough to form dams behind which gangue particles collect. The name STRAKE is loosely used, some writers confining it narrowly to cover only blanket-lined machines used in gold milling, but properly the canvas table and all fixed, rectangular machines of the type, using slightly roughened surfaces in lieu of riffles for catching and holding concentrate, should be included.

The WIDTH is usually made that of the blanket or other bottom covering or some multiple thereof; the depth of the sides is merely sufficient to guard against splash, say 3 or 4 in.; the LENGTH depends upon the ore and upon the results demanded but varies in general between 15 and 50 ft. The SLOPE is usually between 1 and 2 in. per ft. and the feed pulp ordinarily carries from 5 to 15 per cent. solids. Pulp is distributed evenly over the width and allowed to flow for a definite time, predetermined by experience and dependent upon the time required for the concentrate layer to build down to the discharge end. The feed is then cut off and, with or without a preliminary washing, the blankets or other bottom covering are lifted out and washed in a tank to remove collected concentrate. They are then returned to the machine and feed is again started.

At PASSAGEM MINE, Brazil (*20 IMM 18*), the ore is gold-bearing quartz containing heavy constituents deleterious in cyanidation. Blanket strakes take −30-mesh stamp product in a pulp containing about 8 per cent. solids. Slope is 1.5 in. per ft. 200 tons of ore per 24 hr. are treated on 400 sq. ft. of blanket and each ton passes over 120 sq. ft. Recovery is about 50 per cent. and the ratio of concentration about 40 to 1. Concentrate assays about 10 oz. Au per ton. Blankets are 48 in. × 18 in. The life of a blanket is 3 weeks. The rough concentrate, amounting to 5 tons per 24 hr., is passed over 12 strakes, 18 in. wide × 20 ft. long, sloped 1.5 in. per ft., the first 8 and the last 4 ft. covered with blankets and the intermediate 8 ft. with canvas. Blankets are washed half-hourly. Concentrate assays about 160 oz. Au per ton and between 650 and 900 lb. per 24 hr. is produced from 5 tons of the rough concentrate. Tailing from this operation assays about 1.3 oz. Au per ton. The concentrate is further treated on two blanket strakes each 18 in. wide and 6 ft. long, producing between 65 and 90 lb. of final concentrate containing 48 to 64 oz. of Au. Tailing assays 16 to 19 oz. Au per ton. This final washing requires about 3 hours and about 16 gal. per min. of water per strake is used.

At OURO PRETO, Brazil (*20 IMM 30*), the ore is gold-bearing quartz containing considerable sulphides deleterious in cyanidation. Blanket strakes treat −30-mesh stamp product in a pulp containing not more than 3 per cent. solids. Slope, 1.5 in. per ft. Blankets are 18 in. wide × 4 ft. long and are changed two to three times per hour, or as soon as the nap becomes loaded with concentrate; 32 blankets can be changed and washed in 10 min. Concentrate assays 8 to 9.6 oz. Au per ton and contains 40 per cent. sulphide. The ratio of concentration is about 40 to 1. The rough concentrate, amounting to about 5 tons per 24 hr., is re-concentrated on 6 strakes each 18 in. wide × 15 ft. long, sloped 1.5 in. per ft. Concentrate assays about 230 oz. Au per ton and tailing about 0.8 oz., all included gold. The ratio of concentration is about 26 to 1. This concentrate is next treated on a table covered with short-nap blanket, 12 ft. long, sloped 2 in. per ft. Concentrate assays 2160 oz. Au per ton and tailing 16 oz.; ratio of concentration 10 to 1. This makes a final overall ratio of concentration of about 10,000 to 1. Water used on the re-concentrating strakes is a weak solution of the extract from the leaf of *Solanum paniculatum*, a local saponin-bearing plant. This solution lowers the surface tension of the water and prevents or lessens flotation of metallic particles.

Planilla is a Mexican device that had its counterpart in China and the East Indies. It consists of a concave sloping surface, a part of which is usally roughened in some way. The apparatus shown in Fig. 20 was built

of masonry and concrete and had a concrete concentrating floor. Brick, wood and sod turned root-side out have also been used for floors, the wood and sod for treating very fine material. The slope and curvature of the surface vary widely.

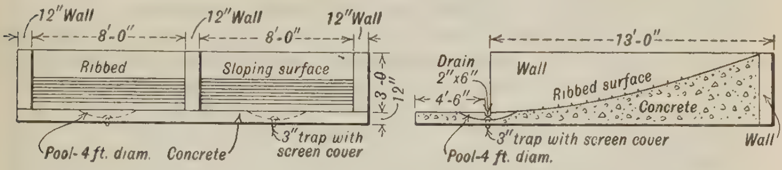


Fig. 20.—Planilla.

Baron (*30 M&M 377*) gives the materials in one planilla such as pictured as 1 bbl. cement, 1000 lb. lime, 5 tons crushed waste and sufficient large rocks for walls and foundations. The labor required was about 5 days for one native mason and two helpers.

Operation. A batch of crushed ore is shoveled onto the higher part, then drawn forward a small amount at a time and washed by throwing water on it or directing a stream against it from a hose. The rough concentrate remaining on the slope after this preliminary washing is turned over and over and coarse material raked up slope while any impoverished surface layers resulting from the turning and washing are carefully scraped off and thrown aside. Concentrate collects in a hard layer beginning at about the point of change of curvature; middling collects below the concentrate and down to the water level; tailing is washed into the sump. Middling is re-cleaned by another washing either with or without further crushing.

Performance. At the San Roberta mine, Zacatecas, Mex. (*30 M&M 377*) a silver ore with lime-silica gangue and small percentages of iron and copper sulphides, crushed and carefully sized through 5-mesh, was concentrated 5 into 1 at the rate of 1 to 2 tons of feed per 9-hr. shift by 1 operator and 2 boys. Concentrate assaying 32 to 40 oz. silver was made from 8-oz. ore and 48- to 60-oz. concentrate from 13-oz. ore, but this was exceptionally good work, obtained by careful sizing of the feed, and close supervision and management. The usual performance on such ore is from 0.5 to 1 ton per shift concentrated 3 or 3.5 into 1 with a recovery of 50 to 60 per cent.

A brick-floored device of the same variety for coarse ore and a board-floored form for cleaning fine middling from the first are described by Collins (*19 IMM 191*) as used in China for treating vein tin ore. Concentrate is re-cleaned in pans.

Film concentration

Concentration by washing on a relatively smooth, sloping surface in a thin film of water is a natural development of the sluice, undercurrent, planilla and strake, applied to treatment of fine material. Building buddles, frames, canvas tables, and revolving round tables represent successive steps in the development. The building buddle dates back to Agricola (1550) and is obsolete in modern plants, but is useful for preliminary work in isolated regions or with primitive labor. The stationary buddle is a useful machine in small, cheap plants. The 20-deck revolving round tables at Anaconda and the multiple-deck tilting slimers in the Missouri lead mills and the southwestern copper plants represent the highest development in film concentrators. These machines have been completely displaced by flotation for the treatment of sulphide ores, but they still are the most efficient devices known for saving extremely fine heavy mineral, not amenable to flotation, from slime pulps.

14. Buddle

The word buddle is limited by some to machines of the variety herein called building buddles; more generally it is made to include all machines for

the concentration of fine sands and slimes in which concentration is effected by flowing a stream of pulp in a relatively thin layer over a substantially smooth, slightly-inclined surface. Thus defined it is distinguished from sluices and undercurrents in which the carrying stream is relatively deep and the separating surface decidedly uneven, and from gold tables and strakes, which have separating surfaces that are intentionally roughened.

Principle of operation of buddles is to utilize the differences in velocity of the different layers in a thin film of liquid flowing over a substantially smooth surface to produce differences in the rate of movement of solid particles of different sizes and specific gravities along the surface. Two kinds of action of the solid particles are involved. The solid particles are more or less in suspension when first fed to the separating surface and they settle according to the laws of free and hindered settling (Sec. 6, Art. 1), the largest and heaviest nearest the feed point, the lightest and smallest farthest down slope or not at all. Friction between the film of flowing liquid and the solid surface, and between the different layers in the flowing film, causes the lowest layer of the film to move at a minimum rate and the upper layer to move at a maximum rate. Since the transporting effect of a current of water varies as a power of the velocity greater than one, the upper layers have the greater transporting effect. This has two results. A particle in suspension in the current is acted upon by a greater force near its upper than near its lower edge, an overturning motion whose resultant is toward the lower layer of the film is imparted, and the settling rate is accelerated. After the particles reach the separating surface, and it is only such particles that can be saved, the current causes them to move down slope by rolling, sliding, or by the movement of alternating suspension and deposition that Gilbert (*Prof. Pap. 86 USGS*) calls "leaping." Rolling and sliding can be considered as due to a substantially non-eddying stream; leaping is caused by eddies. In rolling and sliding, the larger particles, being the ones that project farthest upward from the table surface, are the ones that are acted upon by the most powerful currents, and they move most rapidly, notwithstanding their greater mass. Of two particles of the same size but of different specific gravities, the heavier moves more slowly, under the equal forces applied, by reason of its greater mass. The result, then of the rolling and sliding action of the film on the solid particles in contact with the separating surface, considered alone, is to cause a separation of fine from coarse and heavy from light, the finest heavy material remaining near the upper end and the largest light grains overflowing the lower edge. This is called FILM SIZING. If any of the grains is so large as to stand above the water surface, the transporting force on such a grain, although the maximum that can be exerted by the film, will have less effect than on the larger submerged grains, and such projecting grains will remain near the feed point. When, for any reason, the separating surface is rough, and this necessarily is the case, even on the smoothest bottom, as soon as solid matter begins to settle out, several new effects enter. The depressions in the separating surface act as riffles in which fine particles settle and are protected from the sweep of the current, eddying within the depressions causes concentration of heavy mineral, the rough surface prevents sliding and retards rolling, and eddies are produced in the main stream which sweep smaller particles upward into momentary suspension and cause them to move down slope with a velocity equal to that of larger particles. The net result is to dull the sharpness of the separation of heavy from light mineral.

The flowing stream advances in a series of waves. This fact aids in moving

the larger particles, either because the waves strike definite blows against the larger particles, while failing to strike the smaller, or because, when the crest of a wave is passing, large particles that projected in the trough are submerged.

Davis (*86 J 905*) developed experimentally the following empirical relation between slope (s), in degrees, of a ground-glass surface, water quantity (w) in lb. per min., and diameter in mm. (d) of the average grain of quartz to be washed from a quartz-galena pulp.

$$\sin s = \frac{1}{96}\left(\frac{275 - 50\sqrt{1/d}}{w} + \frac{10}{\sqrt[4]{1/d}}\right)$$

This equation gives no necessary clue to the real relation existing between these quantities and is useful only within the range of the experiments.

Separating surface has an important effect on the performance of a buddle. Truscott (*27 IMM 1*; *28 IMM 46*) believes that this effect is due both to the contour of the surface and the material of which it is made. The depth of the slowest-moving layer in the separating film is less, the smoother the surface, although probably with all surfaces the velocity of the liquid immediately in contact with the surface is zero with respect to that surface. Since any irregularity in the surface produces eddying and the effect of eddying is greatest on the finest particles, it follows that the smoother the surface the better it will hold very fine mineral. (It is not at all unlikely that surface forces, existing at the 3-phase contact between the separating surface, the deposited solid and the liquid, exert some effect, but the hypothesis is not sufficiently established to merit practical consideration.) Rough surfaces catch and hold both gangue and heavy mineral and, if clean concetrate is demanded, such large quantities of water are required to wash out the gangue that much mineral is carried off. Wood, slate, marble, glass, rubber and linoleum have been used to get smooth surfaces and canvas, cement, and fluted glass for roughened surfaces. Data showing the effect of various kinds of surfaces on revolving round tables are given in Art. 19.

Preparation of feed. Considered as film sizing alone, the action of the buddle would seem to demand classified feed and *Richards* so recommends. But as has been shown, the action of a working buddle is never pure film sizing; the part played by the other forces is large, and of such a nature as to defeat separation of classified products. Furthermore on the circular machines (Arts. 16, 18 and 19) the liquid film varies in thickness from center to periphery, and, if made deep enough in its shallowest part to submerge the largest feed particles, will exceed the depth for film sizing in its deepest portion, while if of the right depth in its deepest portion, it will leave the large particles stranded in the shallow areas. Therefore, since true film sizing actually plays but a small part in buddle separation, there is no reason for classified feed. The practical difficulty of classification of the fine sizes usually sent to buddles has also prevented such preparation. It is true that de-sliming a fine sand feed results in better recovery of the granular mineral, but this is due not to better film sizing of the classified material but rather to the fact that removal of the slime decreases the transporting power of the liquid and results in deposition of the fine granular mineral.

Duration of feed is one of the important factors in operation of a buddle. As soon as the stream of feed pulp loses its initial velocity, there is immediate deposition of most of the solid matter. Separation then starts and the solid particles form slowly-moving beds of gangue and mineral progressing toward the discharge lip, with the mineral lagging behind. Feeding must stop before the lower edge of the advancing mineral sheet reaches the discharge. Proper duration depends on the rate of travel of the mineral and the proportion of mineral in the feed. Rate of travel depends on slope, percentage of water in the pulp, feed rate, and retarding character of the surface. When the mineral blanket begins to discharge, there is, on a properly working machine, an immediate rise in assay value of the tailing to that of the feed, the same amount of mineral being added to the impoverished tailing per unit of time, that is abstracted at the upper end.

15. Building buddle

The simplest form is the BOX BUDDLE, a rectangular box, usually with slightly-sloping bottom, the lower or discharge end of adjustable height. Feed is distributed across the upper end by means of a distributor and flows through the box, the lighter material overflowing at the lower end. Solid particles deposit initially in order of size and weight, the heavier and larger particles near

the pulp inlet, ranging gradually to the finest and lightest near the discharge. The slope of the surface of the deposited solids increases to a maximum and with this increase the surface becomes smoother, due to deposition of fine heavy mineral, and the pulp stream becomes thinner. Under these circumstances less and less coarse gangue is able to remain near the head end, but is rolled down slope. When the slope of the settled solids becomes so steep that an excessive amount of concentrate is carried down, the height of overflow is raised, the slope thus lessened, and building again proceeds. If solids build up too rapidly at the head, the feed rate should be increased or feed pulp diluted to increase the velocity of the stream. After the overflow level has been raised to the maximum height feeding is stopped, the bed of solids is marked off into sections representing concentrate, middling, tailing and slime, and the respective sections are shoveled out separately. During the process of building, the surface of the depositing solids should be swept lightly with a cloth or broom to prevent the formation of banks and resulting channeling.

In a building buddle treating a feed containing relatively coarse sands, the action after the bed has begun to deposit is substantially the same as that on a canvas table. With very fine feed the surface of the deposited solids is substantially smooth and concentration takes place largely by film sizing.

Circular building buddles are a development from the foregoing in which the building box is a cylindrical tank usually 18 to 20 ft. diameter and about 18 in. deep with outlet pipes at various levels. A conical feed sole 5 to 6 ft. diameter at the center, and overhead arms revolving 5 to 12 times per min. carrying sweeps of rags or twigs for brushing the building surface, are provided. The products form concentric rings which are shoveled out separately.

Examples

Cornish tin (*24 IMM 422*). Convex pit 14-ft. diameter, with gently-sloping floor; distributing cone, about 5.5 ft. diameter. Overflow is taken through pipes along one element of the cylindrical wall and can be raised 1 in. at a time. Feed is all through 200-mesh. Products of a typical run are: concentrate; 1260 lb. assaying about 25 per cent. black tin; middling, 1420 lb.; tailing, 2870 lb.

Louis states that the capacity of a convex circular building buddle is from 1.5 to 3 cu. ft. of pulp per min., carrying 28 to 56 lb. of solids per cu. ft. Filling requires 3 to 10 hr. and the deposit in the machine is from 6 to 12 tons of solids.

Rumbo (*114 J 672*) is a form of circular building buddle locally developed in the Lenarés district in Spain for treating galena ores. It is a tank 12 ft. diameter and 2 to 3 ft. deep fed at the center with a pulp containing about 33 per cent. solids. Water and slime are worked over the rim by four slow-moving paddles carrying canvas sweeps. It can be filled, stopped and emptied in 5.5 hours and makes a clean concentrate and a tailing containing less than 1 per cent. Pb. Capacity is between 4 and 5 tons of solid per 8 hr.

16. Stationary buddles

These consist of smooth, slightly-inclined stationary surfaces down which finely-ground ore pulp is flowed in a thin film. Heavy mineral deposits near the feed sole, middling down to the discharge edge, and tailing is carried over the discharge edge.

Frame is a film-sizing device consisting of a slightly-inclined smooth-bottomed rectangular trough, open at the lower end to allow free egress of pulp and wash-water streams. The word is loosely but incorrectly applied to rectangular canvas tables and to circular stationary and revolving buddles.

Frames are from 3 to 5 ft. wide, 10 to 20 ft. long, with bottom of smoothly-planed boards, and a slope ranging from 1 in. per ft. for the finest material to 2.5 in. per ft. for fine-sand feeds. It is fed over a distributing board with a thin stream of pulp. Heavy mineral is retarded, as compared to light, and a thin bed of heavy material builds on the separating surface, gradually extend-

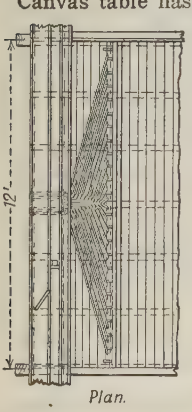
ing to the discharge end. The surface is sometimes worked over with wooden scrapers or brooms to prevent channeling. Just before the advancing sheet of deposited concentrate reaches the discharge end the feed is cut off, the deposit is washed with clean water and caused to discharge a middling product, then the remaining concentrate is removed with a jet of water, or is swept off. By building frames in pairs and providing for diversion of feed pulp to one or the other as desired, and by arranging them to tilt sidewise for washing, considerable time may be saved, one being fed while the other is being washed. Three frames, forming the sides of a triangular prism, mounted on a properly inclined axis, allow practically continuous operation. Water-tilting boxes lever-connected to diverting launders for the products have been used to make the work automatic. None of these devices compares for economy of operation with the revolving round table (Art. 19). The frame is, however, an easily constructed and highly useful slime concentrator for small preliminary plants.

Richards, after extensive experiments (*2 OD 707*; *27 A 76*) recommended a slope of about 0.6 in. per ft. and 12 lb. of water per ft. of width for sandy feed; slope between 1 and 1.25 in. per ft., 5 to 6 lb. of water per ft. of width for fine sand-slime feed; and 1.75 to 2 in. per ft. with 2 lb. of water per ft. of width for the finest slimes. Most writers and millmen, however, recommend a steeper slope as well as a larger water quantity for coarse feeds than for fine.

Capacity, according to *Rittinger*, is about 0.1 to 0.125 ton solid per ft. of width per 24 hr. in a slime pulp containing 8 to 10 per cent. solids and 1 to 1.25 tons per ft. per 24 hr. in a sandy pulp containing 15 to 20 per cent. solids.

Canvas table has been widely used for treating slimes in gold and base-metal concentration. It is better adapted for treating fine feeds than the blanket strake (Art. 13), is usually wider, and the length is less in proportion to the width. Operation is the same as that of the frame. Canvas is not removed for washing concentrate, as is done with the blankets of a strake, but feed is stopped, deposited mineral is washed with a stream of clean water, then swept off with a broom or flushed off with a flat-jet hose.

Wide use was made of canvas tables in California gold mills prior to the adoption of cyanidation, and in base-metal mills prior to the introduction of flotation. At present they are used, under the name of CANVAS FRAMES or RAGGING (ROUGHING) FRAMES in Cornish tin mills and, to a limited extent, supplementing cyanidation in gold mills and flotation in base-metal mills treating complex ores.



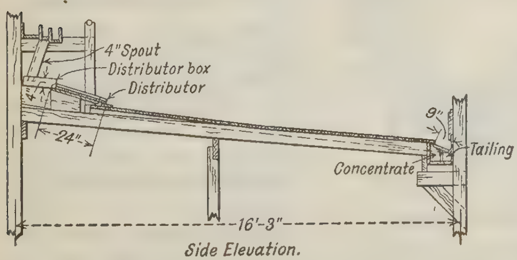


FIG. 21.—Canvas table.

A typical form is shown in Fig. 21. Width varies usually from 3 to 12 ft., length from 10 to 50 ft., slope from 0.5 to 3 in. per ft. Capacity is extremely variable and is dependent on the kind of ore, size of feed, slope, water quantity, and grade of concentrate and tailing demanded; average is about 0.01 to 0.15 ton per sq. ft. per 24 hr. Weight of canvas ranges from 8-oz. to 4-oz. duck. It is usually

stretched lengthwise, for simplicity of construction, although this requires heavier canvas for a given degree of resistance to flow than would be required if the warp of the duck were placed across the stream. The life of canvas is from 8 months to one year under average service, according to the weight used, when protected from the fall of the feed stream by a board, turned when worn too smooth on the upper side, and slipped longitudinally at intervals to bring new places over the joints of the underlying boards. Automatic alternation of feed pulp, wash and flushing water, with automatic throw of the concentrate-tailing splitter has been operated by means of levers from a water-tilting box (see Sec. 21, Art. 10).

At Tul-Mi-Chung (*119 P 808*) canvas tables were used for copper-gold-bismuth ore high in sulphide. The feed was tailing from Deister tables treating flotation tailing (about 0.2-mm. maximum size). Width, 7 ft.; length, 40 ft.; slope 1.25 to 1.75 in. per ft., the former best. One man attended 4 machines. Concrete strakes 4 ft. wide by 50 ft. long on 1.75-in.-per-ft. slope replaced canvas. Concrete was 3 to 6 in. thick tamped level, dressed to slope with a 0.75-in. layer of 1 : 1 cement mortar carefully troweled, which, when nearly dry was dusted with dry cement and scored lightly transversely with a wire comb with teeth at ⅛-in. intervals. Three days were allowed for setting. Twelve of these strakes treated 450 tons solid per 24 hr. (0.13 ton per sq. ft.) in a pulp containing about 12 per cent. solids and recovered about 10 cents in gold per ton treated.

At Suan mill (*119 P 920*) the ore was similar. The feed was tailing from cascade flotation, about 5 per cent. on 100-mesh. 21 strakes, 30 in. wide by 50 ft. long sloped 1 in. per ft., covered with 8-oz. duck, treated 250 to 280 tons solid per 24 hr. (0.11 ton per sq. ft.) and made a low-grade concentrate, but were overfed.

At the Combination mill, Goldfield, Nev. (*94 J 208*), milling gold-quartz ore, canvas tables treated 54 per cent. of the total mill feed and made from 7 to 13 per cent. of the total recovery. Feed contained 85.8 per cent. material through 200-mesh and 93.1 per cent. of the gold recovered came from this size. The cost per ton (1912) was divided as follows: Labor, 18.5¢; repairs, 3.0¢; power, 3.0¢ (including vanners for re-treatment of concentrate); acid for removing calcareous deposit from canvas, 0.5¢; sundry supplies, 5.0¢; total, 30¢.

At Federal Lead Co. in 1913 (*96 J 487*), the feed was galena-limestone ore, slime tailing from vanners, 66 per cent. through 200-mesh, assay 3 per cent. Pb. There were 48 tables, each 11 ft. 8 in. wide, 14 ft. long, slope 1$\frac{9}{16}$ in. per ft., covered with 18-oz. canvas stretched over 2-ply tar paper. The capacity of each table was about 19 to 23 tons solid per 24 hr. (0.14 ton per sq. ft.) in a pulp containing about 19 per cent. solid. One boy washed 36 tables per hr. when making rough concentrate and 24 tables when making clean concentrate. When making rough concentrate at this mill tables were washed every 20 min. Life of canvas was about 8 mo. Efficiency was greater when the canvas was new and fuzzy than later when worn smooth.

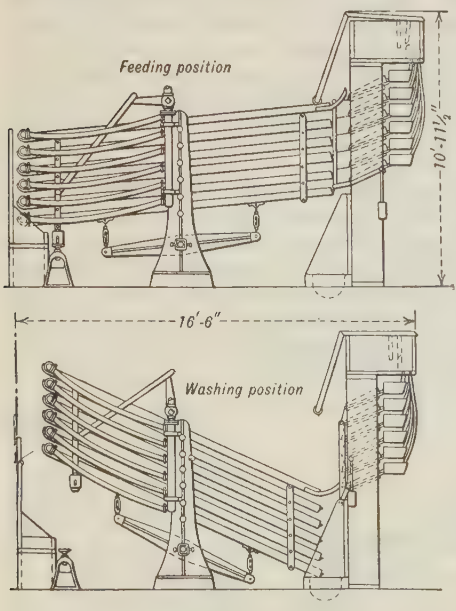


Fig. 22.—Deister tilting slimer.

17. Tilting canvas table

Several forms have been built. The Deister is shown in Fig. 22. It consists of a series of relatively small canvas tables mounted on a tilting framework actuated by a timing device. When in feeding position, pulp is flowed over the canvas decks from the multiple feed box, concentrate adheres to the deck and tailing passes into the tailing launder. After a time predetermined by experiment, the feed is

shut off, the decks tilt into the washing position and flushing water is introduced with sufficient velocity and for sufficient time to remove concentrate, when the decks again tilt to the feed position and feed is again turned on automatically. With a somewhat different mechanism, the BOYLAN (*116 J 766*) and PORTER slimers (*34 MW 453*) accomplish the same result. The feeding period varies from 10 to 30 min. and the washing period from 30 sec. to 1 min. The capacity per square foot of canvas surface is the same as for stationary canvas tables. The WILFLEY multiple-deck tilting slimer (Fig. 23) combines with the multiple tilting deck a simple eccentric shake to aid in stratification of the pulp and a preliminary water-washing period preceding the flushing period, during which wash the worst of the gangue is removed from the deposited concentrate.

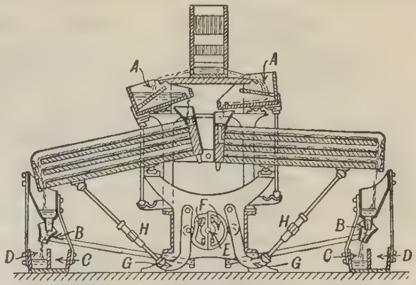


FIG. 23.—Wilfley multiple-deck tilting slimer.

The work of the Wilfley and Deister machines at MIAMI (*115 P 568*) was substantially the same, the machines making recoveries of about 45 per cent. with tailing assaying about 0.9 per cent. Cu from a slime feed assaying from 1.6 to 1.8 per cent Cu. Development of this type of concentrator was practically stopped by the introduction of flotation.

Rotary-tray canvas table is described by Martin (*32 M & M 428*) as having been used successfully in several Mother Lode mills. It consists essentially of a plurality of trays carried on a revolving framework and successively presented to fixed sources of feed and wash water, with final removal of concentrate by a strong, broom-shaped jet. Trays are 4 ft. wide by 3 ft. deep, 21 to 24 trays to a deck and 6 to 10 decks. Speed, one revolution in 15 to 20 min. Canvas, felt and sanded asphaltic paint are used for tray surfacing. Another machine with the same idea has the trays mounted on wheels running on an undulating track which acts to increase the slope of the tray when the discharge position is reached.

18. Circular stationary buddle

In this apparatus (Fig. 24) the separating surface is an obtuse cone, convex or concave, fixed in position, with a central revolving shaft carrying a suitably arranged feed-pulp distributor, wash-water sprays, and diverting launders. In the convex form pictured, fixed launders for concentrate, middling, sand tailing and slime are provided around the outer periphery of the annular concentrating surface. Fig. 25 shows a diagrammatic plan of such a device in operation. The arms carrying the moving parts are assumed to be traveling in a clockwise direction. Feed is introduced from

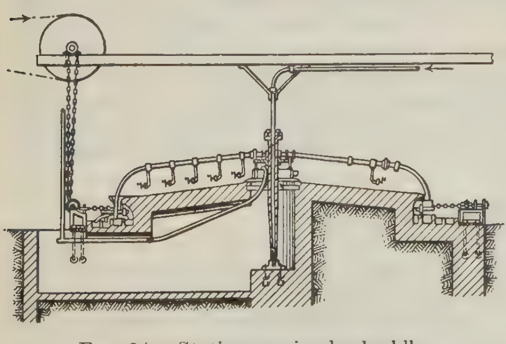


FIG. 24.—Stationary circular buddle.

trough (*a*) extending around one-third to one-half of the inner periphery. The rapidly-moving lighter particles quickly reach the outer periphery, where they are diverted by the movable tailing launder (*b*) to the fixed tailing launder (*c*) and thence to waste. The flowing tailing forms a spirally-bounded patch on the table top as indicated by radial hatching. Under the influence of wash water introduced around the balance of the inner periphery, the slower moving middling next reaches the outer periphery of the table and is diverted by a moving middling launder to fixed middling launder (*e*). The middling forms another spirally-bounded bed, behind the tailing, as marked. Concentrate, traveling most slowly under the influence of the wash water, reaches the outer periphery last and is diverted by the movable launder (*f*) to the fixed launder (*g*). A powerful jet (*h*), carried near the outer periphery and just ahead of the forward end of launder (*b*) removes the last of the concentrate from the path of the advancing tailing area.

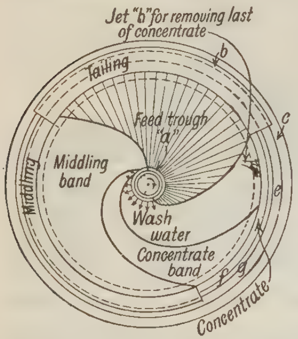


FIG. 25.—Diagrammatic plan of stationary convex buddle.

The concentrating surface is usually wood, iron or cement, but linoleum, glass, and similar surface coverings may be used.

Simons (*46 A 353*) states that these tables are built with as many as 3 decks, 19.5 to 32.75 ft. diameter and that the capacity of the largest size is from 24 to 26.8 tons per 24 hr. per deck. Speed is from 0.25 to 0.43 r.p.m.; power, 0.75 hp.; water consumption 48 to 55 gal. per min. per deck.

An advantage of this type over the revolving round table (Art. 19) is the fact that the heavy concentrating surface is stationary and, therefore, less power is consumed. Also, this surface, once trued remains true, which is not always the case with the revolving tables.

The velocity of the current on a circular convex buddle is greater at the center than at the periphery. This results in deposition of fine gangue near the periphery, but it insures relatively clean concentrate because of the high current velocity near the center, and relatively complete deposition of fine mineral near the periphery. In the concave form, on the other hand, with lowest velocity at the point of initial deposition, much gangue deposits, if substantially complete deposition of heavy mineral is effected. If the velocity at the periphery is made sufficient to prevent deposition of such gangue, fine mineral will be kept in suspension and, with velocity increasing toward the central discharge, never be able to deposit. This is the reason why the concave form has rarely been used.

19. Revolving round table

This is the modern form of buddle. It consists (Fig. 26), of an obtusely-conical surface, substantially smooth, carried on an umbrella-like frame, supported on a vertical shaft resting in a step bearing at the bottom and carrying at the upper end a worm gear by means of which the whole is caused to revolve slowly. A fixed feed distributor supplies pulp to from one-third to three-quarters of the upper surface. Fixed launders around the periphery receive tailing, middling and concentrate. Wash water is supplied over the remaining

surface at the center and there may also be a spirally-disposed spray pipe suspended over the surface along a line such as XX (Fig. 27) that will give a final wash to concentrate before its removal. A strong jet placed just ahead of the tailing launder insures removal of the last of the concentrate. In Fig. 27, consider three particles of the same size but different weights, say concentrate, middling and tailing respectively, starting at point (a). The motion of each with respect to the table surface is radial in direction, but the velocities will vary inversely as some function of the specific gravities. After a short interval of time, therefore, the radial components of travel will be as indicated by the vectors (c), (m) and (t), respectively. The angular travel will be the same for all three, say to the position of radius (r_1); the resulting positions will be (c_1), (m_1) and (t_1), respectively, and the paths the dotted spirals connecting (a) with the respective points. After a second increment of time during which the original radius has moved to the position (r_2), and during which also the radial travel of the particles will be less than in the preceding interval by reason of the lessened velocity of the pulp stream due to its increasing width and decreasing depth, the tailing particle will have left the table at position (t_f), while middling and concentrate particles will be at (m_2) and (c_2) respectively. They will finally leave the table at positions (m_f) and (c_f). Differences in rate of radial travel will be greater, if the concentrate and middling particles are smaller than the tailing particle.

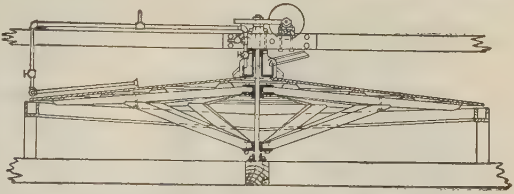


Fig. 26.—Revolving round table (single-deck).

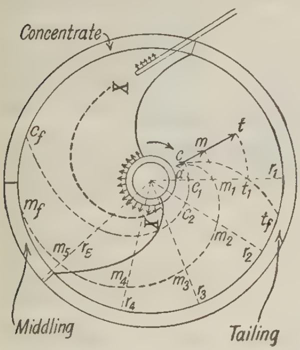


Fig. 27.—Sketch of action of revolving round table.

Evans revolving round table was furnished with a fixed conical center head with spiral periphery, so arranged that concentrate depositing on the upper portion of the table in the feed sector would pass under the head and be protected from further washing by feed pulp, but would be exposed when the wash sector was reached. This type of table has been extensively used in Lake Superior copper mills and in the early days at Anaconda.

Multiple-deck revolving round tables (Fig. 28) were devised to overcome the handicap of low capacity. The limit was reached in the 20-deck Anaconda table shown. The decks were 19 ft. diameter, built of concrete laid on sheet-steel support spaced 1 ft. apart vertically. The 20 decks were carried on eight steel columns around the outer periphery and the columns rested on a steel ring running on wheels on a circular track. Motion was transmitted through a pinion to a circular rack carried on the lower side of the supporting ring. A central shaft-way, 4 ft. in diameter, carried the distributing pipes for pulp and wash water and enclosed a 30-in. square ladderway.

Extensive tests were carried out at ANACONDA preceding the construction of the round-table plant (1912). The reports of these tests by Crowfoot (*49 A 417*) and Laist and Wiggin (*49 A 470*) are substantially a text book on the subject and form the principal basis for the following paragraphs.

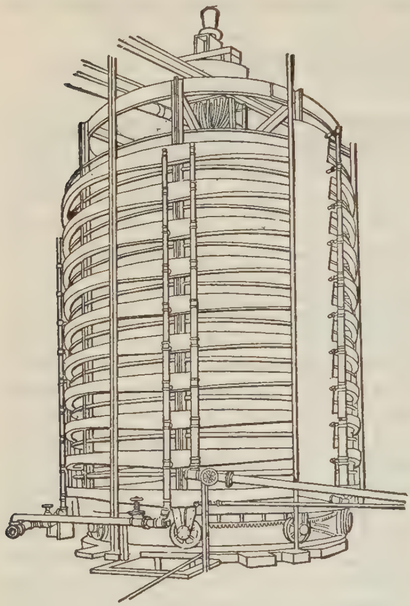


FIG. 28.—Multiple-deck revolving round table.

Surface. Planed wood, linoleum, medium-weight canvas and cement concrete were tested. The concrete mixture was one cement and two sand, laid on sheet steel, trowled to a finish approximating that of the medium-weight canvas. (At OHIO COPPER Co. concrete 2 in. thick was laid on reinforced expanded metal (*90 J 1107*)). Results are given in Tables 7 and 8. Slightly higher recovery was made on the wood than on the cement; the greater tonnage of tailing rejected by the cement deck is a distinct advantage in its favor. The canvas deck made a distinctly higher recovery than the cement, but the grade of tailing was the same and the grade of concentrate markedly better on the cement deck; it also had better wearing quality.

Comparison between cement decks with rough and smooth finish is shown in Table 9. The smooth-finished deck made somewhat better recovery with lower water consumption on the high and low tonnages, but the concentrate was lower grade. On medium tonnage, recovery, tailing and grade of concentrate were all better on the rough-finished tables, although water consumption was markedly higher.

Table 7. Comparison of wood, linoleum and cement deck surfaces on revolving round tables at Anaconda Copper Co.

	Kind of deck		
	Wood	Linoleum	Cement
Tons solid per 24 hr.	4.4	4.5	4.5
Per cent. solids in feed	7.6	7.5	7.5
Assay of feed, per cent. Cu	2.77	2.75	2.76
Assay of concentrate, per cent. Cu	5.29	5.92	7.03
Assay of tailing, per cent. Cu	1.55	1.65	1.61
Recovery, per cent.	62.1	55.3	53.8*a*
Ratio of concentration	3.1	3.9	4.7
Water, fresh, gallons per ton of feed	6022	4974	5424

a Results on cement decks were greatly improved in later work. See Table 9.

When using rough-surfaced cement and canvas-covered decks, solids built up as much as ⅜ in. deep. Yet the tests just described show that notwithstanding the building, the rough deck affords better protection for sulphide and holds it back against the wash-water.

Fluted- and corrugated-glass surfaces have been tested against wood on revolving round tables in the CORNISH TIN MINES (*14 MM 32, 89, 90, 154, 333*). Various forms of glass surface have been tried, but the best results were obtained with about 16 flutes to the inch,

1⁄32 to 1⁄16 in. deep, with crests either sharp or rounded. In one 16-hr. test on an 18-ft. glass-top, concave table with corrugations set at right angles to the slope, slope 1.5 in. per ft., when running against a wooden deck of the same diameter sloped 1.25 in. per ft., the glass top yielded 847 lb. of concentrate assaying 32.2 lb. Sn per ton and the wood 1557 lb. of 15-lb. concentrate. The tailing on both decks carried 6.5 lb. Sn per ton and the feed 9.2 lb. Sn per ton. The feed rate on the glass deck was 5.7 tons per 24 hr.; on the wood, 3.8 tons. The glass deck worked under the disadvantage that the deck was pyramidal rather than conical so that there was a rush of pulp in the valleys at the joints. Tailing assay at the center of a glass section was 5 lb. Sn per ton against 7.5 lb. at the joint. Trewartha-James (*14 MM 89*) says that the glass top is very sensitive to variations in size, pulp density and mineral content and that the pulp density must be closely correlated to the slope. The glass top is said to be better than the wood on rounded particles.

Table 8. Comparison of cement and canvas deck surfaces on revolving round tables at Anaconda Copper Co.

	Kind of deck	
	Cement	Canvas
Tons solid per 24 hr.	8.3	8.3
Per cent. solids in feed	9.3	9.8
Assay of feed, per cent. Cu	3.36	3.46
Assay of concentrate, per cent. Cu	7.84	5.79
Assay of tailing, per cent. Cu	0.93	0.94
Recovery	82.0	87.0
Ratio of concentration	2.9	1.9
Speed, minutes per revolution	19	19
Slope of deck, inches per foot	1.2	1.13
Water, fresh, gallons per ton of feed	1218	1517

Note—Crowfoot states that the extra cost of handling, transportation and smelting of the lower-grade canvas-deck concentrate would probably more than offset the increased recovery.

Table 9. Comparison between rough-finish and smooth-finish cement-deck round tables at Anaconda

Character of feed	Rough finish			Smooth finish		
	Medium	Fine	Fine	Medium	Fine	Fine
Tons solid per 24 hr.	31.5	12.3	4.8	20.2	14.0	6.3
Per cent. solids in feed	16.5	9.0	7.1	11.9	7.1	7.7
Assay of feed, per cent. Cu	4.25	3.37	4.23	4.86	3.23	2.63
Assay of concentrate, per cent. Cu	18.90	16.64	19.06	17.00	16.08	13.90
Assay of tailing, per cent. Cu	2.21	1.47	1.82	2.34	1.51	1.01
Recovery, per cent.	54.0	62.1	63.0	60.5	58.8	66.6
Ratio of concentration	8.2	8.0	7.2	5.8	8.4	8.0
Water, dressing, gallons per ton of feed	440	2035	3746	651	1424	3378
Water, concentrate removal, gallons per ton of feed	814	2480	6078	1094	1171	2491
Water in feed, pounds per minute per foot of total circumference	4.15	2.88	1.70	3.88	4.74	1.95
Water, dressing, pounds per minute per foot of total circumference	1.39	2.60	1.82	1.34	2.15	2.11
Water, concentrate removal, pounds per minute per foot of total circumference	2.86	3.32	3.18	2.40	1.79	1.69
Speed, r.p.hr.	2	2	2	2	2	2

Contour of surface. The length of the cross-section of the pulp or wash-water stream on a convex round table, cut along a line equidistant at all points from the center of the table, increases, obviously, as the distance from the center to the chosen section increases. Since the volume of water flowing down any given sector is substantially constant for any given position of the sector, the thickness of the film must necessarily decrease as the periphery is approached. This results in a decrease in velocity with resultant deposition of solid matter, principally gangue. Professor H. S. Munroe (School of Mines, Columbia Univ.) proposed a convex conoidal deck, to increase the water velocity near the periphery and thus

prevent deposition. Table 10 presents comparative performances of the conoidal and conical decks. The conoidal deck with chord sloping 1 in. per ft. substantially duplicated the performance of the conical deck sloping 1.25 in. per ft. With steeper slopes the conoidal deck made a higher grade of concentrate but lower recovery. The conoidal deck of a given slope required a thicker feed pulp than the conical deck and less wash water.

Table 10. Comparison of performances of conoidal (*b*) and conical (*c*) cement decks on revolving round tables at Anaconda

Contour of deck	Conoidal			Conical
Slope, inches per foot(*a*)	1.25	1.125	1.00	1.25
Tons of solid feed per 24 hr	6.5	6.9	6.9	7.0
Per cent. of solids in feed	9.9	9.9	9.9	9.8
Assay of feed, per cent. Cu	2.67	2.66	2.46	2.37
Assay of concentrate, per cent. Cu	12.15	8.89	5.92	5.92
Assay of tailing, per cent. Cu	1.11	0.97	0.94	0.87
Recovery, per cent	64.2	71.4	73.5	74.1
Ratio of concentration	7.1	4.7	3.3	3.4
Water, fresh, gallons per ton of feed	2659	2535	2711	2967

a On conoidal deck this is slope of chord of arc. *b* Diameter, 17 ft.; working radial length, 7 ft. 3 in.; speed, 1 revolution in 20.5 min. *c* Diameter, 18 ft.; working radial length, 7 ft. 9 in.; speed, 1 revolution in 16 min.

Cost of a conoidal cement deck on a 17-ft. table was as follows: Sand, 2890 lb. @ 17¢ per ton, \$0.25; cement, 14 sacks (@ 93 lb. per sack = 1302 lb.) @ 70¢ per sack, 9.80; labor, 4 man-days @ \$3.25, \$13.00; haulage, \$2.00; total, \$25.05.

On any given sector in position for concentrate removal the solid near the apex of the sector was coarser and richer than that near the rim, which shows the action of the round deck as a whole is not true film sizing but rather progressive deposition of the heaviest particles as the velocity decreases toward the rim after which film sizing removes the larger gangue particles. The deposit along the upper part of the slope was as high grade as could be made by re-treating the whole concentrate from the revolving tables on shaking tables, therefore the spray water was arranged so that the peripheral concentrate was first washed off as middling for further concentration and the rich upper streak was removed separately as finished concentrate.

Slope affects recovery, grade of concentrate and capacity. It is interdependent with quantity of wash water, a flat slope requiring more water to produce a given grade of concentrate than steep slope. On the other hand steep slope tends to lower recovery. Capacity is greater with steep slope.

The Anaconda tests showed that a slope of 1.25 in. per ft., for conical cement decks, was best for treating slimes containing 4 to 5 per cent. on 200-mesh. At less than 1.125 in. per ft. excessive gangue deposited while with more than 1.375 in. per ft. tailing loss was excessive. Linoleum decks also gave best results at 1.25 in. per ft. The best slope for the conoidal deck was 1.125 in. per ft. measured along the chord of the arc. Standard practice in Lake Superior copper mills is 1.5 in. per ft. with occasionally as low as 1.25 in. per ft. in treating very fine material. Anaconda tried a 2-slope convex-conical table with the central 5 ft. sloped 1 in. per ft. and the peripheral 30-in. sloped 1.25 in. per ft. and 1.25 in. drop at the line of change, but results were no better than on the conical deck with 1.25-in. slope.

Speed determines the horizontal component of velocity of particles and, therefore, fixes the radial velocity required to discharge products at the proper places on the periphery. High speed of revolution requires steep slope and much wash water; low speed, *vice versa*. In Lake Superior copper mills the speed on relatively coarse sandy feeds is 1 rev. per min. In some of the slime-testing work at Anaconda 1 rev. per 100 min. was tried, but channeling occurred. Increase to 1 rev. in 19 min. eliminated channeling. One rev. in 4 min. was the speed for the 20-deck machines when run to make finished concentrate and tailing and a circulating middling. At Ohio Copper Co. the speed of 20-ft. machines was 1 rev. in 72 sec.

Diameter affects the length of separating surface parallel to the flow of material and determines the allowable duration of feed. With small diameter the time required for the sheet of concentrate to travel from feed to tailing-discharge point is relatively short and duration of feed to a given section must be made correspondingly short by increasing speed of revolution. Greater diameter or less slope permits lower speed.

Feed. Early practice was to send a sandy feed, say all through 0.3-mm., to round

tables, either removing slime ahead of the table or counting on the table to make some small saving from it, discarding the bulk. Shaking tables largely displaced the round table in sand treatment because of greater capacity, higher recovery, ability to make finished tailing and concentrate at one operation and ability to separate sand from slime. The round table was then applied only to slime treatment. It makes cleaner concentrate when sand is eliminated than otherwise. Anaconda tests on concentrate showed that the part held on a 200-mesh screen assayed but 1.35 per cent. Cu while the undersize assayed 6.95 per cent. On the other hand, the presence of semi-colloidal matter in the feed causes sulphide to be carried into the tailing.

With slime feed the best pulp consistency at Anaconda was between 8 and 12 per cent. solids. Thicker pulp was so viscous that separation was difficult while thinner pulp had to be fed so slowly that capacity was unduly reduced. Pulp consistency had little effect on recovery at Ohio Copper Co. (*90 J 1107*).

For roughing, the feed can be introduced over all of the surface except a small sector devoted to removing concentrate. When clean concentrate is desired, feed is distributed on one-half to three-quarters of the total surface. On the 20-deck Anaconda tables the feed was run onto 70 per cent. of the deck, dressing water on 18 per cent. and 12 per cent. was devoted to concentrate removal.

Water consumption depends upon the size of feed particles, grade of concentrate desired and slope and character of deck surface. Water is supplied with the feed, as dressing water at or near the center, and as spray water for concentrate removal near the periphery. The 20-deck, cement-surface tables at Anaconda, treating 6 to 7 tons solid feed per deck per 24 hrs., required 3 gal. dressing water and 6 gal. spray water per min. The feed pulp contained 10 to 15 per cent. solids. Coarse feed requires the most water; more water is required to make high-grade concentrate than for low-grade. Canvas decks require more water than cement. Linoleum decks require less water than cement; wood require more. Conoidal decks require less water than conical. Richards (*TB 377*) gives the average water discharged per ft. at the periphery of 18-ft. tables in Lake Superior mills as 31.8 gal. per min. pulp water; 10.6, dressing water; and 9.5 spray water.

Power consumption is about 0.15 to 0.25 hp. per deck for tables 15- to 18-ft. diameter, the lower figure applying to the multiple-deck tables. The 20-deck tables at Anaconda each had a 5-hp. motor and actually consumed about 3 hp.

Attendance required is low, especially if the feed is regular. At Anaconda each operator took care of four 20-deck tables.

Capacity depends largely on the size of feed and slope of deck, being higher for relatively coarse feeds and steep slopes. At Lake Superior mills 10 to 20 tons of relatively sandy feed are treated per deck per 24 hr., the decks ranging from 16 to 20 ft. diameter. Final figures on Anaconda slimes were 6 to 7 tons per 24 hr. per 19-ft. deck or about 45 sq. ft. of deck area per ton of total feed, including 8 to 10 per cent. returned middling. At Ohio Copper Co. a 20-ft. deck treated 12 tons dry slime per 24 hr. and could treat 15 tons without crowding, making, however, only 20 per cent. recovery as against 54 per cent. at Anaconda.

Recovery. Probably 40 to 50 per cent. is the average range of recovery on slime feed. Some Lake Superior recoveries have run as high as 80 per cent., making a low-grade concentrate. The average of all Anaconda experimental work showed 54 per cent. net recovery of silver and copper in a concentrate assaying about 6.5 per cent. Cu from a slime feed assaying 2.25 per cent. Cu. Operation of 20-deck tables in April, 1913, yielded 51.6 per cent. recovery in the form of a concentrate assaying 7.02 per cent. Cu from a 1.95-per cent. feed. Tailing assayed 1.13 per cent. Cu.

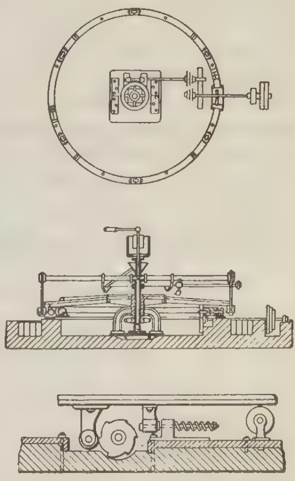


Fig. 29.—Bartsch round table.

Bartsch round table (Fig. 29) is a substantially convex stationary circular buddle with a differential tangential bump applied to the separating deck. Diameter is 13 ft.; speed of spindle, about 0.5 r.p.m.; 100 to 120 @ ⅜- to ¾-in. strokes per min.; power consumption, 0.25 to 0.5 hp.; water consumption, 12 to 35 gal. per min. according to the specific gravity of the gangue. (*46 A 358.*)

SECTION 9

JIGGING

Jigs are mechanical concentrators that utilize the differences in falling rate of grains of different specific gravities and sizes, in a semi-fluid mass of solid particles suspended in water, in order to effect separation of the heavy from the light. The maximum practical feed-size for heavy ores is about 1.5-in.; 3-in. material has been handled, but not very satisfactorily. The average upper size limit at the present time is about 0.5-in. and the minimum about 1-mm. (0.04-in.), but for sizes below 2-mm. (0.08-in.) shaking tables (see Sec. 10) are better machines. The maximum feed size in coal treatment is 4½-in., minimum ⅛-in.

Jigs consist essentially of a screen which supports an ore bed that is brought into partial suspension at regularly recurrent intervals by means equivalent to a current of water forced upward through the screen. Following suspension the bed comes again to rest on the screen, either by settlement under the action of gravity alone or aided by subsidence of the water. The water currents are brought about either by moving the sieve in a tank of water (MOVABLE-SIEVE JIGS) or by moving water through a fixed sieve (FIXED-SIEVE JIGS). After a few repetitions of the pulsation-subsidence cycle, distinct stratification of the ore particles on the screen is effected, grading from heaviest at the bottom to the lightest at the top, and, if the screen apertures are large enough with respect to the particles treated, with the small heavy particles in the tank below the screen. Final separation of the material on the screen is effected by skimming off the surface layers and collecting the lowest layer, either manually or automatically by means of suitable contrivances.

1. Principles of jigging

Separation of heavy from light particles takes place during both of the oppositely-directed motions occurring in each cycle of the jig movement, due to difference in rising rate of the particles under the upward impulse of the water currents and difference in falling rate in the still or downward-moving water during the remainder of the jig cycle. During the upward movement particles of heavy mineral of a given size lag behind particles of light mineral of the same size, both because the heavy mineral is slower to start on account of its greater inertia and because its terminal velocity under a given upward impulse is less than that of the light. (See Sec. 6, Art. 1.) During this same part of the cycle all small particles rise faster and farther than

their larger companions because their inertia is less, their falling rate less and their smaller size permits them to be freed more quickly from the interlocking effect of the settled bed. In their own size class, however, the tendency toward vertical separation is the same as that of the larger particles and the heavier lag behind. At the end of the upward movement the largest heavy particle in the mass has been lifted the least distance, the largest light-weight particle has been lifted a greater distance and the same distance as some smaller heavy particle; while the smallest particle of light mineral has been lifted farthest. When the upward impulse, called PULSION, ceases, it is succeeded by a period during which the suspended solid particles settle back into a mass on the screen. If this settlement is accompanied by downward movement of the water, the jig is said to be running with SUCTION; otherwise it is said to be running with pulsion only. At the beginning of the settlement period the heavy particles settle most rapidly because the settling rate during acceleration depends on specific gravity only. After constant velocities are reached, if there is time in the jig cycle for establishment of such a condition, the heavy particles of any given size settle more rapidly than the light particles of the same size and, so long as either free- or hindered-settling conditions obtain, there is equal settlement of particles of heavy and light mineral having diametral ratios in accord with the equation $d_L/d_H = (\delta_H - \delta')/(\delta_L - \delta')$ in which d_L and d_H are the diameters of light and heavy particles respectively, δ_L and δ_H are corresponding specific gravities, and δ' is the specific gravity of the suspending fluid, being 1.0 for free-settling and, in hindered-settling, the specific gravity of the bed itself in a quicksand state. But experience shows that particles of heavy mineral very much smaller than would be expected from the above equation can be separated from light particles of a given size on a jig bed. This is due to the fact that when the bed compacts to an extent that the quicksand conditions of hindered-settling no longer exist, the large particles of light mineral are interlocked and can no longer move downward, while small particles are still free to move down through the interstices between the larger and do so. This downward movement of the small particles is increased by suction, it may be to the extent that fine particles of light mineral are likewise drawn through the bed and screen.

Simons (*68 A 431*) develops mathematically equations that represent the facts during the time that the bed is loose enough to prevent interlocking of the grains, but his work does not include consideration of the interstitial movement of fine particles.

Starting with the fundamental equation $h = \frac{1}{2}gt^2$, where h = distance of fall due to gravity, g = acceleration due to gravity and t = time, he shows that for the first moments of fall in still water this equation becomes substantially $h = \frac{1}{2}g_1t^2$ where g_1 = acceleration in water. His assumption is that since the resistance of the water to the particle varies with the square of the velocity of the particle, the initial resistance, during the period of low particle velocity, is relatively small and can be neglected and that acceleration during this time will, therefore, be constant. From this it follows that the falling height,

$$h = \frac{gt^2}{2\delta}(\delta - 1), \qquad (1)$$

where δ = the specific gravity of the falling particle. Indicating particles of light and heavy minerals by the subscripts 1 and 2 respectively

$$h_2 - h_1 = \frac{1}{2}gt^2\left[\frac{1 - \frac{\delta_1}{\delta_2}}{\delta_1}\right], \qquad (2)$$

from which it appears that at the beginning of fall, which is the time that must be considered in a jig using short, rapid strokes, the ease of separation, measured by $h_2 - h_1$, increases with difference in density of the particles and that the size of the particles does not enter. From this same equation it follows that, with the same ratio $\frac{\delta_1}{\delta_2}$, ease of separation increases with decrease in the absolute values of δ_1 and δ_2.

For any pair of equal-falling particles whose diameters are d_1 (light) and d_2 (heavy) respectively, settling in still water, it follows from equation (1) that the respective falling heights during the first moments of fall are

$$h_1 = \tfrac{1}{2}gt^2\left[\frac{d_1(\delta_1 - 1)}{d_1\delta_1}\right],$$

and

$$h_2 = \tfrac{1}{2}gt^2\left[\frac{d_2(\delta_2 - 1)}{d_2\delta_2}\right].$$

From the equal-falling postulate

$$d_1(\delta_1 - 1) = d_2(\delta_2 - 1) \qquad (3)$$

and therefore

$$h_1d_1\delta_1 = h_2d_2\delta_2. \qquad (4)$$

From equation (3) it follows that $d_1 > d_2$. Hence $d_1 + d_1(\delta_1 - 1) > d_2 + d_2(\delta_2 - 1)$ and $d_1\delta_1 > d_2\delta_2$, therefore, $h_2 > h_1$. This means that of two particles equal-falling under free-settling conditions, the more dense falls farther in the first moments of fall.

The distance through which a spherical particle will fall in the first moments in a downward current of velocity, v_d, is

$$h = \tfrac{1}{2}gt^2\left[1 + \frac{3cv^2_d - 4gd}{4gd\delta}\right], \qquad (5)$$

c being an empirical coefficient. From this equation it appears that h depends on the relation between $3cv^2_d$ and $4gd$. If subscripts 1 and 2 represent particles of unequal density, 2 being more dense, then, for particles of equal size

$$h_2 - h_1 = \frac{t^2(\delta_2 - \delta_1)}{8d\delta_1\delta_2}(4gd - 3cv^2_d). \qquad (6)$$

The difference is a maximum when $v_d = 0$, which means that jigging of equal-sized particles is most readily performed with no suction.

When the particles are of the same density but different sizes, subscripts 1 and 2 referring to the smaller and larger particles respectively,

$$h_2 - h_1 = \frac{3cv^2_d t^2}{8\delta}\left(\frac{d_1 - d_2}{d_1d_2}\right), \qquad (7)$$

and $h_2 - h_1 = 0$ when $v_d = 0$. This indicates that there is no difference in the rate of fall of different-sized particles of the same density in the first moments of fall in still water. If there is any downward velocity of the water, $h_2 - h_1$ is negative, showing that the smaller particle falls more rapidly at first. When particles are equal-falling under free-settling conditions, the falling heights h_1 and h_2 of light and heavy particles respectively during the first moments of fall are given by the equations:

$$h_1 = \tfrac{1}{2}gt^2\left(\frac{d_1(\delta_1 - 1)}{d_1\delta_1} + \frac{3cv^2_d}{4gd_1\delta_1}\right), \qquad (8)$$

$$h_2 = \tfrac{1}{2}gt^2\left(\frac{d_2(\delta_2 - 1)}{d_2\delta_2} + \frac{3cv^2_d}{4gd_2\delta_2}\right). \qquad (9)$$

Under the conditions postulated

$$d_1(\delta_1 - 1) = d_2(\delta_2 - 1).$$

Multiplying both sides of equations (8) and (9) by $d_1\delta_1$ and $d_2\delta_2$ respectively, it appears that

$$h_1d_1\delta_1 = h_2d_2\delta_2.$$

Under the hypothesis $\delta_1 < \delta_2$, hence $d_1 > d_2$. By the method of proof followed in discussing equation (4), $d_1\delta_1 > d_2\delta_2$ and therefore $h_2 > h_1$. This indicates that with suction and rapid short strokes, separation by jigging can be effected between equal-settling particles obtained by hydraulic classification.

Rittinger, reasoning on the assumption that jigging is a free-settling phenomenon (see Sec. 6, Art. 1), set up the rule that the ratio of the diameter of the largest light grain to that of the smallest heavy grain that should be jigged with it should not exceed that given by the equation $d_L/d_H = (\delta_H - 1)/(\delta_L - 1)$ and continental jigging practice followed this rule closely.

Munroe (*17 A 637*) investigated American practice and concluded that its success in saving heavy particles much smaller than Rittinger's rule indicated was attributable to the fact that free-settling conditions exist for a very small part only of the jig cycle, if at all, and that during the greater part of the cycle the effective phenomenon is movement of water currents in the irregular passages existing in the bed of relatively quiescent coarse particles settled and supported in substantial contact on the jig sieve.

By experiments with spheres of different sizes falling in circular tubes Munroe developed the equation

$$V = K\left[1 - \left(\frac{d}{D}\right)^{\frac{3}{2}}\right]\sqrt{d(\delta - 1)} \qquad (10)$$

in which V is the velocity, following the period of acceleration, of a sphere whose diameter is d and specific gravity δ falling in water in a tube whose diameter is D. K is an empirical coefficient equal to 5.11 when d and D are taken in meters and V in meters per second. When the diameter of the sphere is less than one-tenth the diameter of the tube, its falling velocity is but little affected by the walls, but when d approaches D the falling velocity is much reduced. Maximum falling velocity in a given tube is attained by a sphere whose diameter is four-tenths that of the tube. Experiments with a mass of shot supported on a screen in a tube showed that these obeyed the same law as is expressed in equation (10), but that the falling velocity of the mass of spheres (*i.e.*, the velocity of the current required to just lift them) is less than one-sixth that for a single isolated sphere of the same material. For the mass of shot the average value of d/D determined by substituting observed experimental values for V in equation (10) was 0.892 and substituting this value in equation (10) reduces it to

$$V = 0.833\sqrt{d(\delta - 1)}. \qquad (11)$$

Similar experimental work established the equation

$$V = 0.307\sqrt{d(\delta - 1)}, \qquad (12)$$

as governing the fall of a large sphere when surrounded by a mass of much smaller spheres.

In the bed of a jig treating a mixture of large light and small heavy particles the large light particles will obey equation (12) and the small heavy particles, equation (11). The limiting case for separation exists when

$$V_H = 0.833\sqrt{d_H(\delta_H - 1)} = V_L = 0.307\sqrt{d_L(\delta_L - 1)},$$

from which $d_L/d_H = 30$ for galena and quartz. This compares with the ratio $d_L/d_H = 4$ under free-settling conditions. In a jig bed in which the interstitial channels were 1 mm. average diameter, the diameter of the most rapidly-moving quartz grain would be 0.4 mm. From equation (10), a 0.015-mm. grain of galena would have the same velocity. According to Munroe's assumption, the largest grain of quartz that can move in these channels is 0.89 mm., from which it would appear that with such fine material a ratio of $d_L/d_H = 0.89/0.015 = 60$ was allowable. Munroe further concludes that for fine jigging it is necessary to have this interstitial action and that it can be obtained only by the presence of larger material on the bed.

Richards (*24 A 409*) tested Munroe's work on interstitial currents by sorting tests in a hindered-settling glass classifier and concluded that while interstitial currents were effective in jigging, the ratio of diameters of quartz and galena particles that could be effectively treated together was 6, not 30 as determined by Munroe. The practical test of operations is, however, on Munroe's side.

Effect of suction. Tables 1 and 2 show: (1) The rapidity with which a separation by jigging can be made is greater the larger the differences in the specific gravities of the mater-

Table 1. Jigging tests on quartz and galena (Quartz 1.73-mm. (0.0683-in.) diameter in all cases). (*After Richards*)

Size of galena, mm.	Time and number of strokes required to effect . . . . per cent. recovery of galena								
	Much suction			Little suction			No suction		
	Time, seconds	Number of strokes	Per cent. recovery	Time, seconds	Number of strokes	Per cent. recovery	Time, seconds	Number of strokes	Per cent. recovery
1.73	40	257	100	15	95	100	4	18	100
1.09	45	302	100	60	384	100	10	50	100
0.67	135	748	98	30	153	98	10	62	98
0.50	60	337	99	40	210	99	60	368	95
0.24	30	190	100	30	153	100	60	368	60*a*
0.107	15	86	100	5	30	*b*	*c*	*c*	*c*

a Balance in equilibrium with quartz. *b* Coarse all through; fine in equilibrium with quartz. *c* Not tried as it was recognized from the preceding tests that galena would rise to the surface of the quartz bed.

Table 2. Jigging tests on quartz and sphalerite (Quartz diameter 1.73-mm. (0.0683-in.) in all cases). (*After Richards*)

Size of sphalerite, mm.	Time and number of strokes required to effect . . . . per cent. recovery of sphalerite								
	Much suction			Little suction			No suction		
	Time, seconds	Number of strokes	Per cent. recovery	Time, seconds	Number of strokes	Per cent. recovery	Time, seconds	Number of strokes	Per cent. recovery
1.73	420	2129	96	60	306	99	60	147	98
1.09	360	1676	95	180	838	99	60	202	95
0.67	360	1759	95	180	846	100	300	496	50*a*
0.50	120	603	100	300	1382	98	*b*	*b*	*b*
0.24	45	208	99	360	1729	97	*b*	*b*	*b*
0.107	60	288	99	30	84	99	*b*	*b*	*b*

a Reduced amount of water. *b* Failed with 0.67-mm. sphalerite, so not tried.

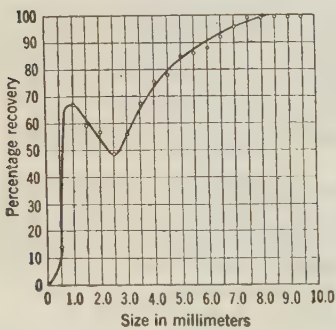


FIG. 1.—Efficiency curve of a single-jig mill (*after Roessler*).

ials jigged. (2) In jigging closely-sized feed, suction should be reduced to a minimum. This is particularly so when the difference in specific gravity is great. (3) Jigging without suction fails when the ratio of diameters of light to heavy mineral exceeds the free-settling ratio. (4) In jigging a feed in which the ratio of particle diameters exceeds the free-settling ratio the necessity fo strong suction is greatest when the difference in specific gravity is least. (5) The galena tests (Table 1) indicate that heavy-mineral particles which are nearly equal-settling with the largest quartz are particularly slow-settling with heavy suction, due, probably to the fact that they get little advantage in free-settling and are too large to get through the interstices during the suction period. The same phenomenon is shown in Fig. 1 (*95 J 785*). (6) The great changes in rapidity that occur in the heavy-suction tests in both series between the sizes

0.67- and 0.50-mm. indicate that the average diameter of interstitial passages in 1.73-mm. quartz lies in this range.

Capacity of jigs is almost entirely proportional to the transporting effect of the horizontal current and to the fluidity of the bed. The more fluid the bed, the greater the transporting effect of a given horizontal current. The limiting requirement is that the finest material that is to be saved shall settle and have a chance to pass down into the bed. The cross current must not be so great that the bed is churned up. Transportive effect is increased by increase in volume of cross water and by the increase in velocity gained by increasing the drop between screens. The latter is the better method because it saves water and lessens the danger of carrying valuable material in suspension. Increasing the plunger stroke or the quantity of hutch water increases the fluidity of bed. Long plunger stroke is not allowable on fine jigs because it produces boiling. Capacity is proportional to sieve area; it increases with increase in width but increase in length has not so great an effect. The usual range on heavy ores is from 0.5 to 2 tons per 24 hr. per sq. ft. of sieve area in fixed-sieve jigs and 4 to 9.5 in the movable-sieve type.

FIXED-SIEVE JIGS

2. Harz jig

Description. The Harz jig is shown in Fig. 2. It consists usually of a plurality of separate and independent rectangular hopper-shaped compartments, with the upper part subdivided by a shallow central partition into a screen compartment and a plunger compartment. The screen is fastened to grate (*K*), which is held in place on the lower, fixed liners by upper liners that are held in by wedges. Reciprocation of the plunger causes flow of water through the screen. Feed enters the first screen compartment through slot (*b*). The longest plunger stroke and consequently the most rapid pulsation are maintained in this compartment. The heaviest particles settle to the screen, the finer particles pass through into the lower part of the compartment, called the HUTCH; the coarse heavy particles collect on the screen until a bed is formed, when they pass under hood (*F*) and over the lip into spout (*J*); light material passes over tail-board (*C*) into the second compartment, where it is subjected to currents of lower velocity by means of which the heavier part of the remaining material is removed and the lighter goes on into the third compartment, etc.

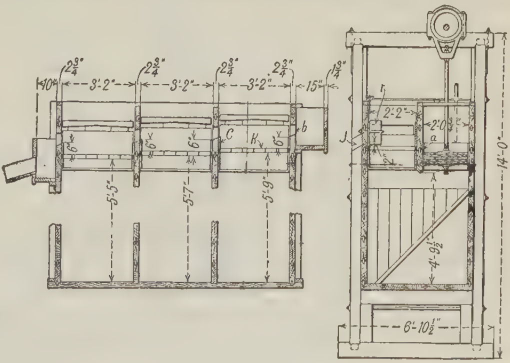


FIG. 2.—Harz jig.

Design of Harz jig. The important points are: size and number of screen compartments, drop between screens, length and number of plunger strokes, size and placing of concentrate draw, sizes of timbers, height of tail-

board, depth of longitudinal partition, design of plunger, kind of screen and grate, ease of changing screen and kind of hutch gate.

Number of compartments depends upon the duty demanded and upon the size of material treated. If the jig is required to make one finished concentrate only, a single-compartment jig will do the work. If two concentrates of different minerals are to be taken but finished tailing is not desired, a 3-compartment jig is required, making clean heavy mineral on the first compartment, clean mineral of intermediate specific gravity on the third, a middling product consisting principally of locked grains of the heavy and intermediate-weight mineral in the second, and a product over the tail board of the third compartment containing little or no free mineral but much middling. If clean tailing is likewise desired, from three to nine compartments will be necessary, depending upon whether one or two concentrates are to be taken and also upon the relative specific gravities of the gangue and valuable minerals, as well as upon the grade of tailing desired. For a one-mineral separation such as quartz-galena, three compartments will serve, if tailing requirements are not rigid, otherwise four; corresponding figures for blende-quartz are four and five. For two-mineral separation such as galena and blende from quartz, five compartments will be required to yield relatively high-assay tailing to nine for low-assay tailing with ores in which the mineral is intimately disseminated in the gangue. The number of compartments necessary increases with decreasing size of feed. At Clausthal (*100 J 426*), jigging closely-sized feeds and making lead and zinc concentrates and a tailing, a 4-compartment jig is used on the -11 $+2.8$-mm. size and a 5-compartment jig for the -2.8 $+1.4$-mm. material and also for the classified sands -1.4 mm.

Size of screen compartments depends upon the tonnage to be treated and is not capable of theoretical solution. The area of the screen determines the cross-section of the stream of ore grains passing over the jig, with a given tonnage. The length of the screen determines the time allowed for heavy grains to settle out of the horizontal carrying current, with a given forward velocity of this current. With a relatively short, wide screen surface the rate of travel is low, which compensates for the short horizontal travel, while with a long, narrow screen the rate of forward travel is high but time is afforded for settlement by reason of the correspondingly longer path. The depth of the bed, beyond a certain practical minimum, (p. 678) has no effect on the character of the compartment overflow, but will affect the grade of concentrate taken. The width of the screen is limited by the difficulty of obtaining an even distribution of water in the screen compartment. The practical limits lie between 24 and 36 in. With the width fixed between these limits, the required area is obtained by increasing length. It is better to divide the screen transversely into compartments whose length is from 1.2 to 1.5 times the width than to attempt to get the required area in one or two compartments, even where the exigencies of mineral separation would not require the greater number of compartments. Such division increases the number of draw gates with consequent lessening of the distance traveled by the concentrate to a discharge port and also gives greater flexibility of operation by allowing closer regulation of the grade of material drawn. Large area is particularly necessary for close separation such as between a concentrate and a high-grade middling or between a low-grade middling and a tailing, particularly in the latter case where the low settling rate of the particles makes necessary that they be given a long time to reach the screen.

Representative figures are given in Table 3. The limiting size of jig screen is reached when the jig members become so massive as to be unduly expensive and maintenance becomes difficult. The plunger rods on excessively large jigs break frequently and timber must be used extravagantly to prevent the sides from bulging and leaking.

Table 3. Capacities of fixed-sieve jigs

Plant	Ore	Size of feed, mm.	Tons per square foot per 24 hr.
Bunker Hill and Sullivan	Galena-siderite-quartz	30 to 10	2.4 to 3.2
Doe Run No. 3	Galena-limestone	2	2.5
Gennamari	Galena-limestone	15–4	2.25
Gennamari	Galena-limestone	4–1	2.0
Daly-Judge	Galena-blende-quartz	10–5	4 to 5
Clausthal	Galena-blende-quartz-limestone	8–2	1.8
Wisconsin	Blende-chert-dolomite	10–0	1.5–2

Drop between screens has a material effect on capacity by determining the fall of the horizontal carrying current in the jig and its corresponding velocity and carrying power.

A jig with large drop can handle occasional rushes of feed without choking. A large drop churns up and loosens the bed at the head end with some corresponding effect on the entire bed while a small drop has little or no such effect. In old practice the drop averaged about 1 in. and this figure is probably the best to-day for slow, close work on closely-sized feeds. For rapid work, on roughly sized or unsized feeds, from 2- to 3-in. drop is common.

Height of tail-board determines the depth of the ore bed. This depends, in turn, on the size of particles comprising the bed, the kind of work that is being attempted, the grade of feed, and the difference in specific gravity between valuable and waste mineral. When clean concentrate is desired the bed must be deep; when clean tailing is sought and everything of any value is to be taken in the gate and hutch discharges a relatively shallow bed must be carried. When there is much valuable mineral in the feed the bed may be relatively shallow; with heavy valuable mineral and light gangue a shallower bed may be carried to obtain clean concentrate than when mineral and gangue are more nearly of the same specific gravity. Depth of bed is properly reckoned in terms of number of grains, hence for a given kind of service the actual depth will be greater for coarse feeds than for fine, but the depth reckoned in number of grains will be greater on a jig treating fine feed. In any jig making a gate draw the layer of concentrate must be at least three grains deep, if the cup is set at the minimum height, in order to insure exclusion of middling grains from the pen. This necessary minimum depth will be relatively greater the smaller the particles. The overlying middling layer constitutes the testing zone in which concentrate is separated and must be deep enough to prevent penetration by lighter grains during local disturbances. The required depth will vary from that of the concentrate layer to twice that figure. The surface layer need not be more than one grain deep, but limitation to this depth will require such a high surface velocity in order to move the desired tonnages that it will be difficult to attain complete settlement and saving of the difficult grains. The minimum depth for coarse feeds is, say, seven times the diameter of the larger particles constituting the gate discharge; better twelve, if clean concentrate is to be taken; the depth may be less with the coarsest feeds, if middling only is to be roughed out. With fine feeds (2-mm.) the minimum depth is about 20 times the grain diameter. With much top or cross water the bed must be relatively shallower in order to get rapid removal of values in a soft bed to counterbalance the rapid horizontal flow.

Screen affects the action of the bed and determines the relative sizes of grains in gate and hutch discharges. The screen must be rigid, if boiling of the bed is to be prevented and a uniform thickness of concentrate layer is to be maintained. This requires that the screen be supported on a grate and tacked or wired thereto and that the grate bars be spaced with regard to the flexibility of the screen. The screen should have the maximum possible percentage of opening in order to obtain the highest fluidity of bed with minimum amount of water and also in order to obtain uniformity of water distribution and prevent boiling. The screen openings should be least at the upper surface, to reduce blinding. Punched plate has maximum rigidity and the most favorable shape of opening, it is easily cleaned by a scratching tool and is but little damaged thereby; but woven wire has the greater percentage of opening in the fine sizes. Slotted punching, 1/16 × 5/8-in. to 3/16 × 3/4-in., hit-and-miss endways, is preferred in Joplin. It is placed with the long dimension across the jig. It does not blind so readily as the round-hole, is more readily cleaned, and is stiffer for a given percentage of opening. Grates have been used to a considerable extent to replace screens in the mid-continent mills. Grates are usually made in sections, 6 in. wide by the length of the compartment, and with 1/16- to 3/16-in. spacing. Osage orange wood is used for wooden grate bars because of its hardness and the fact that it does not swell and close the openings between bars. Wooden grates clog less than iron, because they are more smooth and more flexible; they also resist acid water. Life of wooden grates is claimed to be 8 to 12 months under hard service in acid water against two weeks for the iron grates. Grates have less percentage of opening than cloth or punched plate, but they are more rigid, blind less readily and are more easily cleaned.

Size of aperture depends on the size of feed, the place that concentrate is to be taken and the grade of concentrate desired. Coarse feed requires and permits large apertures. Concentration through the screen requires larger apertures than concentration on the screen permits. If apertures are small, small particles will be kept on the screen, thus reducing the size of interstitial spaces in the concentrate bed. The result is to keep fine gangue out of the hutch concentrate and facilitate drawing down fine mineral. Hence a relatively smaller aperture is used when clean concentrate is desired than when the principal endeavor is for clean tailing. With closely-sized feeds the screen aperture is made slightly less than that of the sizing screen preceding except that the jig-screen aperture is rarely less than 2 or more than 5 mm. With unsized feeds or feeds with a large range between upper and lower sizes, the aperture is determined to a considerable extent by the character of work desired and the minerals in the feed. In Wisconsin a typical set of screens on an 8-cell rougher jig, treating 3/8-in. lead-zinc feed and making practically all concentrate from the hutch is 1/8, 3/16, 1/4, 1/4, 1/4, 3/16, 3/16, 3/16-in. The screens on the following 7-cell cleaner had 3/32, 1/8,

3/16, 3/16, 3/16, 1/8 and 3/16-in. apertures. When a jig is run to take all of the concentrate through the screen the aperture is made slightly larger than the largest particle in the feed and a bed of proper specific gravity, composed of particles larger than the apertures, is placed on the screen before jigging starts. In such practice at CLAUSTHAL, with a feed between 2.8- and 2-mm. the screen apertures were 3- to 4-mm.; with feed 2- to- 1.4-mm., the apertures were 2- to 3-mm.; four grades of sand finer than 1.4-mm. were jigged on and through 1-mm. aperture.

Grids for supporting the screens are made of wood or cast iron. The slats are placed from 1 to 6 in. apart according to the amount of support required, and should run at right angles to the length of the jig so that they will be parallel to and not cause eddies in the plunger-water currents. Wooden slats should be beveled to an edge at the top to lessen the area of dead space above them. Where there is much acid in the water wooden grates are practically essential. Grids and screens should be readily removable. This can be done by making the screen-compartment liners in four pieces and holding them in place with wedges, as shown in Fig. 2, and making the hood (gate) also readily removable. Then by having rings wired to the grate and a small grappling sling suspended from a tackle above the jig, a screen may be quickly removed with most of the bed in place and this can be subsequently dumped on the replacing screen, allowing the first to be cleaned or changed at leisure. Screens are frequently sloped 1 to 2 in. in their length toward the discharge end in order, by greater depth, to compensate for impoverishment and maintain uniform resistance to the water currents.

Concentrate draws. The best known is the gate-and-dam shown in Fig. 3. Discharge is prevented until there is a sufficient bed of heavy mineral formed to seal the opening between the lower edge of the gate and the screen. When the dam is lowered discharge begins and continues until a condition of equilibrium is reached, when the hydrostatic head of the column of heavy mineral (H) within the pen, when in partial suspension in the upward current and therefore free to flow, is just less than that of the composite column of heavy mineral, middling (M) and light mineral (L) without, friction losses due to flow of the bed of heavy material across the screen and under the gate constituting a deduction from the head of the bed outside the pen. With any given setting of the dam, increase in the amount of heavy mineral in the bed will cause corresponding increase in flow over the dam, decrease will cause cessation. This type of draw is usually placed at the front side of the screen compartment near the discharge end. Thus placed it requires that most of the concentrate shall travel at an angle more or less acute to the forward travel of the overlying bed as a whole, part of it at right angles and a certain part must actually travel against the stream, if it is to discharge. The result is necessarily some loss of free mineral over the tail-board of the compartment, but the most serious objection to this placing of the draw is met when concentrate must be removed rapidly. Under these circumstances there is a steep slope of the surface of the concentrate layer down to the bottom edge of the pen. This results in boiling at this point and consequent contamination of the concentrate by material from the overlying layers. This type of draw is sometimes placed at the discharge end of the compartment with the concentrate-discharge spout running in the compartment partitions. This placing is superior in so far as removal from the bed is concerned, but will give some trouble by clogging. Fig. 10 shows a pipe draw designed on the same principle. It makes the concentrate travel in a less favorable path than the side draw. The portion of the jig bed adjacent to any draw is coarsest. This produces a "live spot" in the bed that will permit gangue to pass down into the hutch, and wear on the screen is greatest at this point, which aggravates the trouble. The size of the pen in discharges of the gate-and-dam type and of all ore passages should be sufficient to allow free movement of impounded material, *i.e.*, at least three times the diameter of the largest particles, better more. There are a number of other draws whose fitness may be tested according to the method of analysis given.

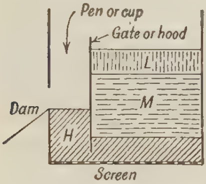


FIG. 3.—Gate-and-dam jig discharge.

Hutch draw must be capable of close adjustment in order that suction may be carefully controlled. Such adjustment results in frequent clogging, hence the draw should be of such variety that it can be opened wide to remove such obstruction. When the ordinary pipe-and-plug spigot is used, the working plugs can be fitted into the center or bottom of a 6-in. plug, which is readily removed when necessary. The ordinary molasses gate, 1½- to 3-in. sizes, is frequently used. Rotating slush gates (Fig. 4) are common when continuous hutch discharge is to be maintained.

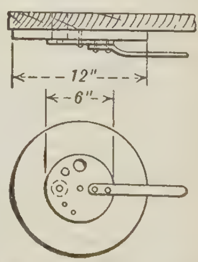


FIG. 4.—Slush gate for jig hutch.

Table 4. Distribution of back water over jig sieve. (*After Demond*)

Clean screen, no ore — Rising water							Bed of ore — Rising water						
Third nearest plunger		Center third		Third farthest from plunger		Total pounds. 100%	Third nearest plunger		Center third		Third farthest from plunger		Total pounds. 100%
Pounds per square foot	Per cent. total water	Pounds per square foot	Per cent. total water	Pounds per square foot	Per cent. total water		Pounds per square foot	Per cent. total water	Pounds per square foot	Per cent. total water	Pounds per square foot	Per cent. total water	
Jig A (see Fig. 5)													
8.4	97.7	0.2	2.3	0.0	0.0	8.6	....	....	....	....	....	....	....
30.3	64.6	14.2	30.3	2.4	5.1	46.9	22.4	71.8	8.8	28.2	0.0	0.0	31.2
43.1	58.7	23.0	31.3	7.3	9.9	73.4	43.9	59.3	28.2	38.1	1.9	2.6	74.0
50.2	54.4	31.1	33.6	11.1	12.0	92.4	58.0	57.9	35.6	35.6	6.5	6.5	100.1
54.6	52.3	35.6	34.2	14.1	13.5	104.3	69.5	59.5	40.6	34.7	6.8	5.8	116.9
60.0	53.7	36.3	32.5	15.5	13.8	111.8	66.1	50.5	56.1	42.8	8.8	6.7	113
Jig B (see Fig. 5)													
22.8	53.5	19.2	45.1	0.6	1.4	42.6	35.3	30.7	42.9	37.2	37.0	32.1	115.2
51.5	29.2	67.8	38.5	56.7	32.3	176.0	55.2	32.8	60.3	35.7	53.1	31.5	168.6
99.2	30.9	111.2	34.7	110.3	34.4	320.7	89.4	31.5	97.0	34.2	97.1	34.3	283.5
....	....	....	....	....	....	....	89.3	29.4	108.5	35.8	105.4	34.8	303.2
....	....	....	....	....	....	....	136.5	31.6	143.4	33.1	152.8	35.3	432.7
....	....	....	....	....	....	....	128.8	29.6	153.1	35.2	153.1	35.2	435.0

Plunger is ordinarily made with the same area as the screen. This gives symmetrical construction, minimum velocity changes in the water currents, and readily understood adjustment. There is some advantage in making the plunger larger than the screen, which allows shorter plunger stroke and larger throat, thus producing a free action in the beds and reducing power consumption. For most efficient mechanical operation the plunger should fit its compartment as snugly as possible without binding. The usual fit is between 1/16 and 3/16 in. clearance all around. Small plunger clearance creates strong suction and where this is undesirable a close fit must be compensated by plunger valves, otherwise excess back water must be used to cut down suction. A loose plunger requires a longer stroke than one that is tight-fitting. Clearance must be greater when water is fed above the plunger than when introduced below. Introduction of water below the sieve is likely to cause collection of air under the plunger with consequent uneven action. Plungers are built up of five courses of 1-in. tongue-and-groove stock, thoroughly wet with good white-lead or asphalt paint, laid with alternating courses at right angles and closely nailed. The outer courses are cut shorter than the center to allow for rocking of the eccentric. Courses both sides of the center course are shortened about 1/8 in. all around to provide water packing. Eccentrics should be of extra-heavy pattern with spherical sliding surfaces to prevent binding. They should be evenly spaced around the shaft. It is a convenience if they are graduated to show length of stroke. The bottom of the plunger should never rise above the sieve.

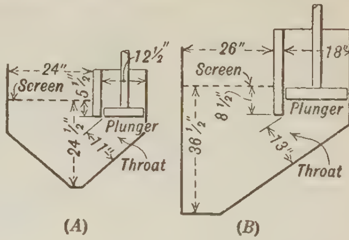


FIG. 5.—See Table 4.

Longitudinal partition. The position of the lower edge determines the changes in velocity of the water currents and thereby influences the distribution of water. The best position is with the distance from the lower edge to hutch walls equal to width of plunger and screen compartment. There is then minimum velocity change. The lower edge of the partition should extend at least 0.4 times the width of the screen compartment below the screen in coarse jigs and 0.33 times in fine jigs (*Richards*). Table 4 shows what can be done by attention to these details and the penalty for failure.

Jig box and frame. All dimensions are substantially fixed by the rules already set down. The hutch product cannot be drawn clean unless the bottom is hoppered both ways to the draw valve. The usual practice is to carry the front wall straight down. The walls should be not less than 3-in. surfaced plank, except for very small jigs. Even with this weight there will be some bulging with long strokes if the compartments are long and the span of the planks correspondingly great. All planks should be carefully tongued and grooved or grooved and joined with close-fitting feathers, and the joints should be set in white lead. Transverse partitions should be dapped into the walls and the center-board dapped into the transverse partitions. Fig. 6 shows a cheaper method of wall construction, using 2 × 3- or 2 × 4-in. scantlings set in paint and packed with wicking, then securely spiked. This is particularly suitable for bull jigs (Art. 5), which would require very heavy planking. Posts should be at least 6 × 6-in. better 6 × 8-in. in large jigs, with caps and sills the same width and one or two inches deeper. Mortise the posts into cap and sill, but provide for draining the sill joint. Transverse tie rods should be bored through the body of a transverse plank rather than let in in place of a feather, otherwise a difficult leak will occur. Strap bolts (Fig. 2) are easier to put in and just as satisfactory as through rods from cap to sill. Floor-space may be saved by placing jigs back-to-back, or face to face. Jig walls are sometimes built of reinforced concrete about 4½ in. thick. (*109 J 1115.*)

Transmission machinery. Shaft should be 2 7/16-in. minimum for fine jigs with compartments up to 36-in. length; 3 7/16 will carry a long-stroke plunger on a 48-in. compartment. Pulleys should be keyed and set-screwed, and of ample face. Tight-and-loose pulleys should always be provided. Common flat boxes will serve but adjustable ball-and-socket bearings will pay for themselves many times over in ease of maintaining alignment.

3. Operation of Harz jigs

The important operating factors are: length of stroke, number of strokes per min., screen aperture, depth of bed, size and character of bedding material, quantity of cross and back water, size and richness of feed, capacity and power consumption. These factors are closely dependent. Rate, size and richness of feed are fixed in any given case by the character of the ore and the

design of the mill; screen aperture, speed and depth of bed are parts of the jig design; the only variables within the control of the jig operator are length of stroke, character of bedding and quantity of water.

Jig operation requires more skillful labor than other forms of gravity concentration. Shut-downs should be as infrequent as possible, since it may easily require an hour or more to get back to good operating conditions after starting-up again.

Length of stroke is directly dependent upon the number of strokes per minute, unless the latter are so few that there is an appreciable rest period between pulsations. In any case the length of stroke must be sufficient to produce a certain amount of fluidity in the bed, *i.e.*, it must be sufficient to cause the bed to be lifted and the grains spread apart enough to allow interstitial settlement.

If the jig is running at relatively high speed, the length need not be so great as when low speed is used because of the greater velocity and hence greater lifting effect of the high-velocity current. For a similar reason length of stroke at a given speed need not be so great for light ores as for heavy ores. When the endeavor is to make clean concentrate the bed should be maintained as compact as is compatible with the desired recovery by using a short and rapid stroke and pulsion should be accented by using a large amount of back water; on the other hand, when clean tailing is the desideratum, the bed should be kept loose by long, slow strokes and strong suction obtained by the use of little back water. The effect of reduction in back water is shown in Table 5 (*17 A 637*). A relatively

Table 5. Effect of reduction of water on work of a fixed-sieve jig

Product	+0.25-mm.		0.25–0.125-mm.		0.125–0.05-mm.		−0.05-mm.	
	Normal water	Reduced water	Normal water	Reduced water	Normal water	Reduced water	Normal water	Reduced water
	Percentages of total weight							
Hutch conc.	26	39	68	94	67	93	60	83

short, rapid stroke with much back water is used for rich, heavy feed, while for low-grade feed, a long, slow stroke and little water are usual. Hence, a rougher jig requires a longer stroke, lower speed, and less back water than the corresponding cleaner. For separation of low-grade middling from tailing the usual practice is a long, slow stroke, shallow bed and a small amount of back water. This adjustment also has the result of overcoming the bad effect of the large volume of cross water on the later compartments. In the Wisconsin zinc field the operators test for the necessity of change in stroke by increasing the specific gravity of bed; if this raises the grade of concentrate, stroke length should be decreased.

With a given maximum size of particle, unsized feed requires a shorter stroke than sized feed because of the greater lifting effect of the water when the interstices of the bed are filled with fine material. Similarly a fine screen retains more fine material in the bed and shorter stroke can be used. High speed accompanies short stroke in order to maintain capacity. If a long stroke is used with unsized material, the bed is loosened so much that it boils and fine gangue is drawn down on the return stroke. If the bed becomes impoverished, there is a tendency to mat and choke the sieves, hence with low-grade feed, it is best to maintain a bed of coarse concentrate. Coarse feed requires a longer stroke than fine feed. Flat and thin pieces (floaters) require a short, rapid stroke, a thin, loose bed, and as little cross water as possible. Speed must be kept uniform or it will be impossible to hold a bed on the sieve. Excessive cross water must be avoided, if tailing loss is to be kept down and the jig beds are to be kept in good condition. Where large amounts of plunger water are required to keep down suction, there will be excessive cross water on the later compartments unless special means are taken to rectify the condition. Water may

be drawn off above the level of the bed. A long jig may be broken near the center and the feed dewatered between the two parts, but this involves lost head room and increased floor space.

Bedding, sometimes called RAGGING when not formed by the jig itself from the feed furnished, is the layer of heavy material lying at the bottom of the mass of grains on the jig screen. The word BED is used indiscriminately to describe this layer and also the whole mass of grains on the sieve; ordinarily the context will make the usage definitive. The important characteristics of the bedding layer are its thickness and the size and specific gravity of the grains.

The function of the bed is twofold: (1) to act in the fashion of a liquid seal for the concentrate cup, and (2) to serve as a screen when hutch concentrate is made. In the first case the desideratum is a semi-fluid whose specific gravity is greater than that of the grains to be excluded and not greater than that of the grains to be received. These requirements are best satisfied by a bed consisting of grains of the same material from which concentrate is being made, and of mixed sizes so that the lifting effect of the water will be a maximum, with consequent maximum fluidity. If the specific gravity of the bedding grains is greater than that of the concentrate to be made, excessive pulsion will be required to produce movement and this results in boiling of the overlying material and disturbance of the work of the jig. A similar result follows if the bedding is too coarse or of too great uniformity of size.

The thickness of the bed is determined in part by the size of concentrate grains and the consequent setting of the concentrate cup, in part by the requirements as to cleanliness of concentrate, and in part by the specific gravity of the bedding grains. The bed must be thick enough to effectually seal the cup against the entry of low-grade material and this means, practically, that the surface should be three or four grains higher than the bottom of the cup, except in the case of the coarsest feeds, and from 0.5 to 1 in. higher in the case of fine feed. To make clean concentrate requires a deeper bed than otherwise.

Beds of low specific gravity tend to bank up against the tail-board, hence, unless special precautions are taken such as inclining the screen against such flow or placing baffles across it at intervals, it will be necessary to use a thin bed with material of low specific gravity.

When jigging through the screen the interstices in the bed determine the size of grains that can pass and, in conjunction with other operating features, the strength of suction. Bedding grains about three times the diameter of the maximum grain it is desired to draw down are provided. Maximum suction is attained with maximum interstices, hence coarser and more uniform bedding is used on the later sieves, where clean tailing is sought and the grade of hutch product is of minor importance. If the valuable mineral abrades readily, grains of more durable material of the same specific gravity are frequently supplied for bedding; thus iron balls or punchings or lead shot will form a satisfactory bed for making galena concentrate, pyrite or magnetite for chalcopyrite or blende, and feldspar for slate in coal jigging. On the later compartments of a jig the bedding grains are usually middling grains of proper specific gravity, but artificial bedding is sometimes provided. At CALUMET AND HECLA middling grains were used to bed finishing jigs treating the hutch products of primary jigs that bedded themselves. If the bedding grains are much heavier than the concentrate, as, for instance, when iron slugs are used as bedding for pyrite or blende, the bedding is not lifted on the pulsion stroke and merely serves to decrease the sieve openings.

An ore that makes a small percentage of concentrate requires a thick bed; a high-grade ore gives satisfactory results with a thin bed and heavy suction.

Feeding. The principal requirements are that the feed rate be regular and that the material shall be spread evenly over the entire width of the screen. Automatic discharges for taking concentrate from above the screen respond slowly to changes in concentrate supply, with the result that a sudden increase in feed rate or metal content causes valuable mineral to pass over the tail-board, while sudden decrease will cause the discharge of low-grade material from the cup. When making hutch concentrate only, increase in feed rate will cause increased tailing loss, decrease will cause some lowering of the grade of the concentrate, but this will not be serious. Uneven distribution of feed puts the task of distribution on the jig with consequent decrease in capacity.

Power is dependent primarily upon sieve area, speed, length of stroke, depth and weight of bed. *Wiard* gives the formula, $HP = AD^{1/2}/5000$, where A is the sieve area in sq. in. and D is the diameter of feed in mm. For estimates, 0.1 to 0.15 hp. per sq. ft. of total sieve area is safe.

Water consumption varies with the size of feed, specific gravity of the minerals, depth of bed, size of screen aperture, number and length of stroke, and according to whether pulsion or suction is accented. *Wiard* gives the roughly approximate empirical formula, $G = (DA)^{1/2}/3.03$ where D is the diameter of feed in mm. and A is the area of screen in sq. in. and G = gal. per min.; the result tends to be low.

Three-compartment, 24 × 36-in. jigs treating 3-mesh lead-zinc ore at Daly Judge used about 11,000 gal. per 24 hr. treating 75 to 90 tons. Two-compartment, 24 × 36-in. jigs at Doe Run No. 3 mill treating classified feed used 30 gal. per min. each, treating 30 tons per 24 hr. At Gennamari a 4-compartment, 36 × 42-in. jig treating −15 + 4-mm. lead ore required 100 gal. per min. for 45 tons per shift. Jigging − ⅜-in. blende, a 5-compartment, 30 × 42-in. jig required 150 to 200 gal. per min.

Performance. Jig settings used for the treatment of closely-sized lead-zinc ores at Clausthal (*100 J 425*) are given in Table 6. Practice on a lead-silver ore at Silver

Table 6. Operating data on Clausthal jigs (Harz type)

Size of feed, mm.	Strokes per minute	Screen aperture, mm.	Length of stroke, mm.
22 −16	120	4*x*	46
16 −11	140	4*x*	30
11 − 8	160	2*x*	30
8 − 5.6	180	....	25
5.6− 4	200	....	20
4− 2.8	220	....	15
2.8− 2	240	3–4	13
2 − 1.4	260	2–3	8
Sand No. 1	280	1	5
Sand No. 2	300	1	5
Sand No. 3	300	1	5
Sand No. 4	300	1	5

x Concentrate taken from the screen.

King Coalition is shown in Table 7. At Bunker Hill and Sullivan material passing a 30-mm. screen is jigged in four sizes, *viz.:* −30 +15-mm., −15 +7-mm., −7 +3-mm.

Table 7. Operating adjustments on Harz jigs treating closely-sized feeds at Silver King Coalition (*116 J 370*)

Jig number	Size feed, inches	Screen aperture(*a*)	Revolutions per minute	Length of stroke, inch	Product(*b*)
1	½ to 7/16	5-mesh	165	1	Gate discharge
2	7/16 to 5/16	4	215	⅝	One gate, 1 hutch
3	5/16 to 3/16	4	240	½	2 hutch
4	3/16 to 3-mesh	5	270	⅜	2 hutch
5	− 3-mesh	5	280	5/16	2 hutch

a Hutch-making compartments bedded with coarse lead concentrate. *b* Average assay of concentrate from jigs Nos. 1 to 4: 1st compartment: 42.4 oz. Ag, 59.6 per cent. Pb; 2nd compartment: 34 oz. Ag, 32 per cent. Pb. Sand jig (No. 5): 1st compartment: 31 oz. Ag, 53 per cent. Pb; 2nd compartment: 11 oz. Ag, 15 per cent Pb, 26 per cent. Fe.

and − 3-mm. de-slimed. The bull jig (30-mm.) has four compartments, 25½ × 33½-in., fitted with 5-mm., 15-gage round-hole plate, life, 30 days. The jig makes 150 @ 1½-in. strokes per min. Treating feed carrying 8.3 per cent. lead and 50 per cent. moisture at the rate of 96 tons per 24 hr. it produces a middling assaying 18 per cent. lead from the

cups, and a tailing assaying 0.86 per cent. Pb. The beds are 6.5 in. deep. Water consumption, 850 gal. per min. Attendance, twelve machines per man. Lost time, 0.009 per cent., due to changing screens and liners. The (15-mm.) jig is 4-compartment, 25½ × 33½-in., fitted with 3-mm. round-hole 17-gage plate on all compartments; life of screens, 30 days; speed: 200 @ 1-in. strokes per min. with 5½-in. beds. The feed carries 50 per cent. moisture. Water consumption, 850 gal. per min. Feed rate, 48 tons per 24 hr. One man attends twelve machines. Assays, per cent. Pb: Feed, 14; tailing, 1.5; concentrate, 60; middling, 11.0. The (7-mm.) jig is 4-compartment with the first compartment 25½ × 37-in. and the others 25½ × 33½-in.; 6-mesh, No. 16 or No. 17 brass wire screen is used and lasts 180 days. Feed contains 50 per cent. water. Speed, 225 @ ¾-in. strokes per min. with 4½-in. bed. Feed rate, 72 tons per 24 hr. Water consumption, 624 gal. per min. Twelve machines per man. Assays, per cent. lead: Feed, 8; tailing, 1.2; concentrate, 70; middling, 10. The (3-mm.) jig is 4-compartment, same size as the preceding with 8- mesh No. 16 or No. 18 brass-wire screens which last 180 days. Feed is between 3-mm. and 20-mesh (0.833-mm.) and contains 50 per cent. water. Feed rate, 48 tons per 24 hrs. Water consumption, 624 gal. per min. Speed, 250 @ ¼-in. strokes per min. with 4½-in. bed. Assays, per cent. Pb: Feed, 7.5; tailing, 1.5; concentrate, 75; middling, 8. One man attends twelve machines and in case of all jigs controls the water supply and concentrate-discharge rate. The old practice at this mill (*2 MM 364*) was to make two sizes only for jigging, *i.e.*, −30 +10-mm. and −10-mm. A 4-compartment bull jig of the same size as at present treated 60 to 80 tons per 24 hr. with 155 @ 1⅝-in. strokes per min. and from a feed carrying 10 to 18 per cent. Pb made concentrate assaying 52 to 59 per cent. from the first cup, 40 per cent. from the second cup, middling assaying 8 per cent. from the third and fourth cups and tailing assaying 1.0 to 1.4 per cent. A 5-compartment classifier jig treated the 10-mm. undersize. It had 2-mm. plate on the first two compartments and 3-mm. plate on the others. Speed was 225 @ ⅝-in. strokes per min. Concentrate from the first cup assayed 65 to 73 per cent. lead; second cup, about 40 per cent.; third and fourth cups, about 10 per cent.; fifth cup, 5 per cent.; slime overflow from the first compartment, 14 per cent.; first hutch, 55 per cent.; second hutch, 30 per cent.; hutches 3 to 5 combined, 7 per cent.; tailing, 1.4 to 1.8 per cent. Table 8 is a sizing-assay test of the combined hutch product of this jig, showing a characteristic dip in assay values in the middle sizes. Table 9 shows the particle-size distribution in the various hutches.

Table 8. Sizing-assay test on combined hutch products of classifier jig at Bunker Hill & Sullivan mill. (*After Caetani*)

Screen, mesh	Per cent. weight	Assay, per cent. Pb
On 20	3.2	82.1
40	16.8	67.7
60	14.2	50.3
80	19.2	42.7
100	12.6	39.6
150	13.4	42.7
200	7.6	49.4
Through 200	13.0	58.8

Table 9. Sizing test of hutch products of classifying jig, Bunker Hill and Sullivan Mining Co. (*After Caetani*)

Hutch number	1	2	3	4	5
Size	Per cent. weight				
+20	6	27	31.5	51	91
+60	52	29	54	40	9
−60	42	44	14.5	9	......
Per cent. Pb, total	55*a*	32	14	8	5

a Can make 75 per cent. lead but causes heavy wear on bedding.

Table 10. Sizing-assay test of classifying-jig tailing, Bunker Hill and Sullivan Mining Co. (*After Caetani*)

Size	Per cent. weight	Assay, per cent. Pb	Cumulative per cent.
7-mm.	31.2	1.19	25.5
5	23.2	1.54	50.2
3	5.8	1.44	56.1
10-mesh	24.2	1.61	83.0
20	11.6	1.24	92.7
40	0.7	1.20	93.5
60	1.6	0.94	94.6
80	0.8	1.13	95.3
100	0.3	1.27	95.5
150	0.2	1.79	95.8
200	0.1	2.82	95.9
−200	0.3	17.22	4.1
Total......	....	1.45	

Classification of No. 1 hutch product in a free-settling classifier yielded a spigot product containing 77 per cent. lead and an overflow containing 49 per cent.; corresponding figures for the second hutch were 60 per cent. and 12 per cent. Table 10 is a sizing-assay test of the tailing from the same jig. At FEDERAL MINING AND SMELTING CO., Morning mill a feed all passing 1-in., 30.5 per cent. on 12-mm., 68.3 per cent. on 4-mesh and 1.2 per cent. on 6-mesh, containing 42 per cent. water, was treated on a 2-compartment 18 × 34-in. jig, fitted with 5-mm. punched plate. Speed was 158 @ 1.5-in. strokes per min. Bed, 6 in, thick. Tons per 24 hr., 200. Power, 1 hp. One man attended 6 jigs, 6 rolls, and the accompanying trommel equipment. He controlled stroke adjustment, water regulation and thickness of bedding. Lost time amounted to 0.05 per cent., principally due to screen changing. Life of screens was 90 days. Feed assayed 6.5 per cent. Pb; concentrate, 45.0; middling (no tailing), 4.5. Table 11 is typical of the lead-hutch concentrate to be expected

Table 11. Sizing-assay test on jig lead concentrate, Broken Hill—Central Mine (*28 IMM 19*)

Screen, mm.	Weight, per cent.	Assays		
		Ag, oz.	Pb, per cent.	Zn, per cent.
+40	43.8	37.6	68.4	7.3
60	13.4	35.2	70.0	4.0
80	15.0	32.4	66.8	4.3
100	7.5	26.6	60.8	6.0
120	3.4	27.2	58.2	5.8
150	5.9	24.8	51.4	7.4
200	3.1	27.2	55.0	7.9
−200	7.8	30.2	54.8	9.3
Total......	....	33.8	64.2	6.5

Note—Feed is de-slimed material through ⅛-in. screen.

in jigging a fine feed containing both lead and zinc. On this type of ore at VAN ROI MILL (*101 J 464*) jig feeds average 5 per cent. Pb, 7 to 15 per cent. Zn and 10 to 20 oz. Ag; jigs treating sized feeds at 16, 10, 7, 4 and 3-mm. make tailing averaging 0.5 to 1.0 per cent. Pb, 3 per cent. Zn and about 3 oz. Ag; lead concentrate assays 60 to 70 per cent. Pb, zinc concentrate about 40 to 45 per cent. Zn. About 70 per cent. of the tailing in this mill is made on the first four jigs and 20 per cent. on the finer jigs. Table 12 gives operating data on jigs treating complex lead-zinc ores at U. S. SMELTING, REFINING AND MINING CO., Midvale plant. At GENNAMARI, Sardinia (*100 J 794*) coarse jigs treat − 15 + 4-mm. material and fine, −4 +1-mm. The coarse jigs are 4-compartment, 36 × 48-in., making 1 ton of 55-per cent. Pb concentrate from the first cup and hutch, 20 tons of 5-per cent. middling from the last three compartments and 24 tons of tailing with a trace of lead, per shift. The fine jig is 5-compartment, 31 × 43-in., treating 40 tons, half primary and half re-ground middling, per shift, making 1 ton of 65-per cent. concentrate, 4 tons of 5-per cent. middling and 35 tons of tailing with a trace of lead. Table 13 shows highly efficient practice in jigging fine material but the high metal content in the finest size demonstrates one reason why tables and other concentrating processes have replaced jigs in treating fine feeds. Table 14 shows the results of re-treatment of the fine lead concentrate from the preceding operation. Treating a TUNGSTEN ORE consisting of ferberite in quartz and granite gangue (*101 J 717*), assaying 0.75 to 1.5 per cent. tungstic oxide, sized to − 0.5 + 0.25-in. and − 0.25-in., in a 2-compartment jig with 18 × 30-in. sieves, making 110 to 200 @ 1.25- to 1.5-in. strokes per min., with a 4-in. bed on 12-mesh screen, feeding at the rate of 6 to 10 tons per 8 hr., concentrate from the first screen assayed 50 to 55 per cent. tungstic oxide, second screen, 30 per cent. and hutch 1.5 to 3 per cent. Concentrate was skimmed by hand. The second concentrate was sometimes thrown back on the first screen. Water consumption was about 150 gal. per min. At CANANEA 17 @ 2-compartment double concrete jigs, 24 × 36-in., were used to treat a feed sized between $\frac{3}{16}$-in. and 100-mesh. The jigs were fitted with 5-mesh, No. 14-brass-wire screen (life 50 days), and carried 4-in. beds. Speed, 170 @ 1-in. strokes per min. One hp. ea. Feed rate, 70 tons per 24 hr. Water, 240 gal per min. One man attended three machines. Lost time, 0.25 per cent., due principally to changing screens. Assays, per cent. Cu: Feed, 1.60; tailing, 0.9; concentrate, 3.5. A one-compartment, 18 × 28-in. jig treating a DISSEMINATED COPPER ORE sizing 3.9 per cent. on 10-mesh, 5.4 on 14,

Table 12. Harz jig operations at U. S. Smelting, Refining and Mining Co., Midvale Plant

Jig number	1	2	3	4	5	6
Sieves, material	Woven-wire cloth bolted to cast-iron frame					
Aperture, in.:						
Compartment 1	0.252	0.145	0.095	0.087	0.087	0.087
Compartments 2 and 3	0.198	0.145	0.095	0.087	0.087	0.087
Life, days	105	89	72	69	74	76
Power consumed, hp	2.5	2	2	2	2	2
Speed, r.p.m.	190	217	248	278	252	264
Length of stroke, in.:						
Compartment 1	¾	9/16	7/16	5/16	¼	¼
Compartment 2	¾	9/16	7/16	¼	¼	¼
Compartment 3	1 1/16	9/16	7/16	¼	¼	¼
Tons solid per 24 hr	130	100	95	80	70	70
Attendance	One man attends all six					
Percentage of lost time	1.5	1.5	1.5	1.25	1.5	1.25
Principal cause of lost time	Repairs and cleaning screens					
Water consumption, gallons per minute	70	60	52	49	47	49
Thickness of bed, in.	6–7	5–6	4.5–5	4.5	5	4.5
Size of feed, in.	0.3–0.13	0.13–0.087	0.087–0.053	–0.053	0.053–0.036	0.053–0.036
Percentage of moisture in feed	33	30	34	38	40	43
Assay of feed, Pb(*a*)	8	8.5	8.5	12.0	12.0	12.0
Assay of feed, Zn(*a*)	7.5	7.5	7.5	8.5	8.5	8.5
Assay of concentrate, Pb(*a*)	22	22	22	26	26	26
Assay of concentrate, Zn(*a*)	5.6	5.6	5.6	4.5	4.5	4.5
Assay of middling, Pb(*a*)	3.5	5	4.5	8	8	8
Assay of middling, Zn(*a*)	9.9	9.5	10.0	10	10	10

Note—All jigs 3-compartment, about 23.5 × 35.5-in. sieves.

a Extremely variable; averages only given. Thickness of bed and regulation of hutch water left to operator.

Table 13. Distribution of galena on fixed-sieve jig, treating galena-limestone ore

Size, mm.	Per cent. weight	Assay, per cent. Pb	Hutch, per cent. lead	Bed concentrate, per cent. lead	Tailing, per cent. lead
All through 6-mm.					
On 1.0	41.1	6.32	....	16.20	1.06
0.25	29.6	9.10	74.0	7.97	0.96
0.125	9.3	13.81	19.2	....	0.71
0.083	1.5	12.93	14.8	....	1.09
0.05	2.2	7.84	8.8	....	1.74
–0.05	16.3	12.22	16.4	....	6.07
Total	100.0	8.93	22.3	16.54	1.53

6.8 on 20, 14.6 on 28, 14.5 on 35, 11.4 on 48, 6.8 on 65, 5.9 on 100, 3.1 on 150, 1.8 on 200 and 25.9 per cent. through 200-mesh, at the rate of 225 tons per 24 hr., made concentrate assaying 31.8 per cent. Cu and tailing, 2.78 per cent. from feed assaying 4.20 per cent. Speed was 245 @ ⅝-in. strokes per min. Water 42 gal. per min. in the hutch and 81.6 per cent. moisture in the feed. Two-inch bed. Four machines per man. At Chino Consolidated Copper Co. a 3-compartment 24 × 36-in. jig was used to treat middling

Table 14. Operation of Harz finishing jigs on roughing-jig concentrate, St. Joseph Lead Co. (*Munroe, 17 A 659*)

Size	Feed		Hutch, assay per cent. Pb			Tailing assay, per cent. Pb
	Weight, per cent.	Assay, per cent. Pb	1st	2nd	3rd	
+0.25-mm.	8.9	40.99	79.69	70.34	41.80	3.94
0.125	43.6	16.76				1.02
0.05	13.3	16.40				0.62
−0.05	34.2	32.58				11.97
Total.	. . . .	24.75	Average 3 hutches, 74.0			5.24

from Wilfley and Butchart tables; average size, 6 per cent. on 10-mesh. Jig screens were woven brass wire with 0.67-in. opening and lasted 60 days. Speed 190 @ ¾-, ⅝- and ½-in. strokes per min. 3.5-in. bed. 100 tons per 24 hr. Water, 75 gal. per min. in hutch and 75 per cent. moisture in feed. 3 hp. One man attended 10 machines. Assays, per cent. Cu: Feed, 6; tailing, 2; concentrate, 15.

4. Cooley jig

Description. This is a variant of the Harz type used extensively in the mid-continent zinc fields. The only essential difference from the ordinary Harz jig lies in the fact that the jig is almost always used to make hutch only and no screen discharge is provided. Coarse concentrate and middling are shoveled off intermittently as operation demands. Attempts to run with continuous draw of bed concentrate result in hard beds with impaired jigging action and the discharge of a large amount of low-grade material. If the amount of coarse concentrate and middling is large, it is usual, in the case of the middling at least, to use a coarse screen and jig through an artificial bed, using long stroke, low speed, shallow bed and high drop between screens.

Construction. The usual method of construction is cheap and highly efficient. The floor is built up of 3 layers of 1-in. T-and-G boards laid in white lead or asphalt paint and nailed flat; walls and partitions are 2 × 4-in studs, dressed, laid flat and nailed, with two strips of cotton wicking, saturated with white lead, laid between. The front of the hutch is stepped out 8 to 10 in., then carried up straight. The walls of the plunger compartment are carried up to give support to the journal boxes and to prevent splashing. The entire jig is lined with 1-in. boards and these are also used to form a sloping bottom for the hutch. The usual sizes of screen compartments are from 20 to 42 in. wide by 24 to 48 in. long with 6- to 9-in. tailboards on the first compartment ranging down to 5- or 6-in. on the last. Practice as to depth of bed varies. A deep bed requires much water, which sends fine free mineral to the later cells. These are made hard by the swift flow of excess top water and as a result the fine mineral goes into the tailing. Excess top water in the tail end of the jig may be eliminated by breaking the jig in two and de-watering before the last cells. A common drop in the 30 × 36-in. size is 1¾ in. In a 42 × 48-in. jig at the Media mill, a 3-in. drop was used. Rougher jigs are run at 90 to 125 @ ⅝- to as much as 2½-in. strokes per minute; cleaners at 160 to 200 @ ⅜- to ¾-in. strokes, and sand jigs at 150 to 190 @ ⅛- to ½-in. strokes. Some mills use special chat jigs of 4 to 5 cells run at higher speed and shorter stroke than the cleaner jigs. Typical adjustments are shown in Table 15. In some cases the drive shaft is in two sections and the head end is driven at 20 to 30 r.p.m. less than the tail end. Punched-plate screens are commonly used. The sieves are sloped

downward toward the tailing end, as much as 1 in. in their length, in order to compensate for the greater weight of concentrate at the head end and prevent uneven water distribution.

Number of compartments. Jigs used to make clean concentrate and tailing on ⅜-in or ⅝-in. screen undersize (one-jig mills) are 7- to 9-compartment. In such a jig, cell No. 1 is used for clean lead concentrate, cell No. 2 makes a lead-zinc middling that is either circulated or re-ground; cells 3 to 5 produce clean zinc concentrate and, in a 9-cell jig, cells 6 and 7 are also used to make clean zinc; the last two cells produce middling for re-grinding. The greater number of cells is used for high-grade feeds or with ores containing much lead; in the latter case cell No. 3 may also make middling and cell No. 4 may be the first zinc cell.

Table 15. Operating adjustments on Cooley jigs treating Wisconsin lead-zinc ores (*104 J 89*)

Cell number	Revolutions per minute	Screen aperture, inch	Stroke length, inches	Depth of bed, inches	Pitch of bridge, inches
8-cell rougher jig, 30 × 36-in.					
1	115	⅛	1¾	6	¾
2	115	3⁄16	1	6	¾
3	115	¼	1½	6	¾
4	115	¼	1¼	6	¾
5	125	¼	1½	6	¾
6	125	3⁄16	1⅛	6	¾
7	125	3⁄16	1⅛	6	¾
8	125	3⁄16	15⁄16	5	¾
7-cell cleaner, 30 × 36-in.					
1	220	3⁄32	7⁄16	6	¾
2	220	⅛	7⁄16	6	¾
3	220	3⁄16	5⁄16	6	¾
4	240	3⁄16	⅜	6	¾
5	240	3⁄16	⅜	6	¾
6	240	⅛	⅜	6	¾
7	240	3⁄16	7⁄16	7	¾

Single-jig treatment is suitable only for low-grade, coarsely-disseminated ores low in lead. If there is much lead it will appear in the zinc concentrate. With the 2-jig arrangement the combined hutch products of the first or rougher jig carrying 10 to 25 per cent. zinc, locally called SMITTEM, are sent to the cleaner. Tailing is discarded. Table 16 shows typical feed and tailing of a rougher jig. Some, but very little blende finer than 200-mesh is caught on these jigs and about 30 per cent. of the zinc in the tailing is this fine material. Finished concentrate, assaying 75 to 80 per cent. lead, is taken from the first hutch of the cleaner, the second hutch is sent back to the head of the cleaner, hutches 3 to 6 are clean zinc concentrate assaying 50 to 60 per cent. zinc, and hutch 7 is returned to the head of the cleaner or re-ground, depending upon whether the values are free or locked mineral. The latter material is locally called CHAT and assays from 4 to 8 per cent. zinc. Table 17 gives typical sizing-assay test on cleaner-jig zinc concentrate. The cleaner tailing is either thrown away or sent to a sand jig of 3 to 5 cells, usually of smaller grate area than the cleaners.

Operation. All jigs are run with strong suction, which is enhanced by leaving the hutch gates partly open. To make rich lead concentrate requires high-grade feed. There is an advantage in sending the first rougher-jig hutch to a small separate jig, otherwise it is necessary to build up the grade of the cleaner feed by circulation of the lead concentrate. Water consumption ranges from 800 to 1200 gal, per min. for a 7- to 9-cell jig. A common figure for estimate is 1500 to 2500 gal. per ton treated. The capacity of a rougher is from 8 to 12 tons per sq. ft. of screen surface per 24 hr.; for a cleaner 3 to 4; for a single-jig mill, 4 to 5 tons.

Performance. JOPLIN DISTRICT (*57 A 446*): Rougher jig on sheet-ground ore: Feed, 2.85 per cent. Zn; concentrate, 25.5 per cent.; tailing, 1.10 per cent. (dewatered 0.70 per cent.); recovery, 64.17 per cent. On lower-grade ore: Feed, 1.13 per cent. Zn; rough concentrate, 15 per cent.; tailing, 0.50 per cent. (dewatered, 0.38 per cent.); recovery,

57.67 per cent. Hard ground other than sheet: Feed, 1.20 per cent. Zn; rough concentrate, 9.57 per cent.; tailing, 0.55 per cent. (dewatered, 0.34 per cent.); recovery, 57.47 per cent. In general clean concentrate assays 50 to 60 per cent. Zn; the locked middling particles from rougher and cleaner jigs combined, 4 to 8 per cent. In the Wisconsin district (*111 J 1065*) the average recovery from 1.75-per cent. Zn feed is 55 to 60 per cent. in a 48-per cent. concentrate; tailing averages about 0.6 per cent. Zn, of which the part on 10-mesh runs about 0.4 per cent. Zn, the +40-mesh, 0.95 per cent., and slime, 8 per cent. On 6-per cent. feed through ⅜-in., jig mills in this district make 65 to 70 per cent. recovery in a 45- to 50-per cent. concentrate carrying less than 3 per cent. CaO ; about 75 to 80 per cent. of the concentrate is +20-mesh. At American Zinc, Lead and Smelting Co. mill at Mascot, Tenn, bull jigs, rougher and cleaner jigs and sand jigs are used. The bull jig is 3-compartment, 36 × 48-in., fitted with 10-gage steel-plate round-hole screens with 0.5-in. aperture (life, 185 days). Speed, 82 @ 2-, 1.75- and 1.5-in. strokes per min. Feed, 300 tons per 24 hr. 358 gal. wash water per min., 8- to 12-in. beds. Power consumption, 9 hp. The feed is tailing of a 6-cell rougher jig, all through 0.5-in., dewatered to 34 per cent. moisture on ⅛ × 1-in. Rek-tang screen. All hutches are run open. One operator attends 4 machines. Assays, per cent. Zn: Feed, 1.25; tailing, 0.8; middling (no concentrate), 3. The rougher jig is 6-compartment, 32 × 44-in., fitted with 14-gage punched-slot steel plate, apertures ⅛ × 1-in. (life, 92 days). Feed is all through 0.5-in. aperture and contains 37 per cent. water. Speed, 110 at 1⅝-, 1⅝-, 1½-, 1½-, 1⅜-, and 1⅜-in. strokes on the 6 compartments respectively. The beds are 8 to 10 in. deep. Wash water, 460 gal. per min.; power, 18 hp.; feed rate, 600 tons per 24 hr. Assays, per cent. Zn: Feed, 3.8; tailing, 1.25; middling (no concentrate), 7.0. One man attends 2 machines and regulates plunger water and thickness of bed. The cleaner jig takes the middling from the rougher. It is 7-cell, 28 × 42-in., fitted with cast-iron grates with 1⁄12- and 1⁄16-in. slots (life, 50 days). Speed of the first four plungers is 180 r.p.m. and of the last three, 200 r.p.m. Stroke lengths from head to tail end are ⅝, ½, ½, ½, ½, ⅜ and ⅜ in. Beds are 7.5 in. deep. Feed rate, 200 tons per 24 hr.; 68 per cent. moisture in feed; 552 gal. per min. of water added; power consumption, 21 hp. One man attends 2 machines. Assays, per cent. Zn: Feed, 15; tailing, 4; concentrate, 62. The

Table 16. Sizing-assay tests of feed and tailing of Cooley rougher jig treating zinc ore

Screen aperture, mm.	Feed			Tailing before passing the dewatering screen			Dewatered tailing		
	Weight of screen products, per cent.	Zinc content by assay, per cent.	Per cent. of total zinc	Weight of screen products, per cent.	Zinc content by assay, per cent.	Per cent. of total zinc	Weight of screen products, per cent.	Zinc content by assay, per cent.	Per cent. of total zinc
6.680	9.26	0.55	4.53	7.66	0.53	8.06	7.20	0.51	9.69
3.327	44.08	0.65	26.18	46.80	0.35	32.48	50.49	0.36	42.30
1.651	22.10	1.02	20.07	23.83	0.32	15.10	27.81	0.34	25.04
0.833	10.14	1.16	10.46	9.57	0.26	4.93	8.72	0.28	6.46
0.417	5.55	1.68	8.30	4.25	0.23	1.94	2.84	0.25	1.91
0.208	3.16	2.15	6.04	2.08	0.32	1.31	0.98	0.27	0.69
0.147	1.27	3.17	3.60	0.98	0.51	0.97	0.41	0.43	0.46
0.104	0.97	3.74	3.22	1.06	1.14	2.40	0.39	0.96	0.98
0.074	0.19	4.52	0.79	0.15	1.85	0.55	0.07	1.90	0.35
−0.074	3.28	5.77	16.81	3.62	4.50	32.26	1.09	4.20	12.12
Total........	100.00	1.13	100.00	100.00	0.50	100.00	100.00	0.38	100.00

3-compartment, 28 × 42-in. sand jig treats re-ground middling passing a ⅛-in. trommel. It is fitted with cast-iron grates, 1/10- and 1/12-in. slots (life, 60 days). Speed, 200 @ ⅜-, ⅜- and ½-in. strokes per min.; 7.5-in. bed; 150 tons per 24 hr.; 9 hp. Feed contains 85 per cent. water and new water is 391 gal. per min. One man operates 4 machines.

Table 17. Sizing-assay test of zinc concentrates from zinc compartments in a Cooley-type cleaner jig in a Joplin mill (*107 J 558*)

Mesh, Tyler	No. 1 cell			No. 2 cell		
	Weight, per cent.	Zn, per cent.	Pb, per cent.	Weight, per cent.	Zn, per cent.	Pb, per cent.
On 4	2.0	62.9	0.03	0.4	63.0	0.02
6	12.1	62.3	0.03	6.0	63.0	0.10
8	21.3	62.1	0.10	10.0	63.0	0.04
10	17.0	61.3	0.16	7.9	62.6	0.02
14	13.1	61.2	0.17	8.5	62.2	....
20	8.5	61.0	0.17	6.9	62.0	0.02
28	6.7	61.3	0.30	12.1	61.2	0.06
35	4.4	60.0	1.10	11.7	58.2	0.06
48	3.7	58.0	2.10	11.0	57.0	0.10
65	4.4	57.3	3.60	10.8	57.3	0.16
100	3.6	53.2	7.80	8.0	57.0	1.10
Through 100	3.2	34.8	24.20	6.3	45.1	12.00

Mesh, Tyler	No. 3 cell			No. 4 cell		
	Weight, per cent.	Zn, per cent.	Pb, per cent.	Weight, per cent.	Zn, per cent.	Pb, per cent.
On 4	....	....	....	....	....	....
6	2.7	62.9	0.03	0.03	57.0	0.55
8	0.1	62.7	0.03	1.7	56.0	0.42
10	0.6	62.7	0.06	7.3	57.0	0.32
14	10.6	62.0	0.06	12.8	59.0	0.14
20	1.2	62.1	0.06	10.7	60.0	0.05
28	7.7	63.0	0.04	20.6	60.0	0.02
35	3.0	62.3	0.02	19.4	61.5	....
48	9.6	60.3	0.02	10.7	61.4	....
65	9.0	60.0	0.02	8.2	60.3	0.02
100	5.4	59.4	0.20	5.3	59.0	0.06
Through 100	5.0	50.6	0.60	3.8	53.0	3.10

5. Other fixed-sieve jigs for treating metalliferous ores

Collom jig is of the fixed-sieve type with a special plunger mechanism that gives a quick downward stroke and retarded return. The mechanism has link-operated rocking hammers that strike the plunger rods down sharply against the pressure of springs which return the plunger more slowly on expanding. The plungers may be fitted more tightly than the eccentric-driven type, because of their non-rocking travel, but the mechanism is more complicated and less rugged than the simple eccentric. The jig is built double with sieve compartments on the outside of the plunger compartments, and these are about half the area of the sieve compartments and are placed side by side. This causes maximum water effect at the feed end of one compartment, which is not distinctly harmful, and at the tailing end of the other compartment, which is harmful.

Dee jig (Fig. 6) has a hopper-shaped valved plunger under the screen with the plunger rod passing through the center of the screen. The valved plunger decreases suction and gives uniform distribution of water currents, but at the expense of great inconvenience in operation and maintenance.

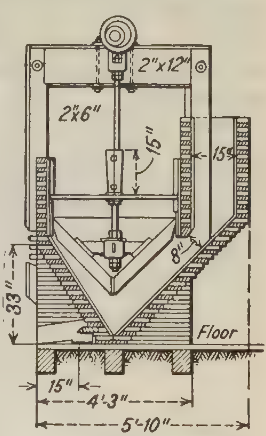


Fig. 6.—Dee jig.

Evans jig is of the Harz type and embodies no essential departures from the standard form except that the eccentric is designed to maintain the plunger in horizontal position. It has been largely used at Anaconda mills.

Exhaustive comparative tests with Harz jigs at this plant showed no conclusive difference in metallurgical results, but considerably higher tonnage treated per square foot of sieve area on the Harz jigs. Average results of one such comparative run are given in Table 18.

Hodge jig is of the Harz type but has a mechanism for attaining differential plunger motion similar to that on the Deister No. 3 slime table (Sec. 10, Art. 7). It has been used to a considerable extent in the Lake Superior copper mills. It is built in cast-iron double units which are set up in pyramid arrangement.

Table 18. Comparison of Harz and Evans jigs at Anaconda. (Treating de-slimed undersize of 4-mesh screen)

Jig	Number	Number of compartments	Size of sieves, inches	Tons per feed 24 hr.	Tons of feed per square foot per 24 hr.	Gallons of water per ton of feed	Assays, per cent. Cu		
							Feed	Tailing	Concentrate
Harz......	9	3	26×36	345.5	1.96	8976	1.17	0.64	7.23
Evans.....	11	3	26×41	231.6	0.95	8423	1.43	0.71	7.73

Performance. At Copper Range mills four double Hodge jigs with 24 × 36-in. compartments are used in series. They are fitted with double-crimped brass-wire screens of the following sizes: 8-mesh, No. 18 wire on No. 1 (life, 324 days); 10-mesh, No. 20 wire on No. 2 (life 405 days); 12-mesh, No. 21 wire on No. 3 (life, 260 days); 14-mesh, No. 22 wire on No. 4 (life 260 days). Speed of all jigs, 165 strokes per min., @ ⅞-, ¾-, ⅝- and ½-in. respectively, with 2.75- to 3-in. beds. Capacities: First sieve, 21 tons per 24 hr.; second, 16; third, 9; fourth, 6. Water consumption varies from 160 gal. per min. on No. 1 to 100 gal. per min. on No. 4. Power consumption, 0.52 hp. per sieve. Feed contains 80 to 85 per cent. water. Assays, per cent. Cu: Tailing, 0.5; concentrate, 65.

Slide jig has a Whitworth quick-return plunger mechanism arranged to give accelerated down stroke and retarded return, thus emphasizing pulsion and lessening suction. Otherwise it differs in no way from the typical Harz jig.

May jig (*27 IMM 338*) is of the fixed-sieve, plunger type with rocking-arm drive, arranged with plunger compartments between the sieve compartments. It is extensively used for treating the silver-lead-zinc ores of the Barrier district of Australia; for this service it has supplanted all other varieties after extensive testing. The usual practice is to treat the de-slimed undersize of a 3-mm. screen on three to five 30 × 42-in. compartments, the last being for tailing discharge, making all the concentrate from the hutch, bed-

ding the screens about 1.5 in. deep with iron punchings $\frac{5}{16}$- to $\frac{3}{8}$-in. diameter. Punched-plate or woven-wire screen with about 4-mm. apertures on the first compartment and 3- or 3.5-mm. apertures on the later compartments is used; brass grids 1.75 in. deep and with 2.5 × 5-in. pockets are used to hold the bed in place; speed varies from 180 to 240 r.p.m. The plunger area is about 40 per cent. that of the screen; plungers are fitted with board clack valves to lessen suction.

The ore of the district consists principally of galena, blende, rhodonite and quartz. Two methods of treatment are practiced; in the first a coarse galena concentrate only is sought and but two or at most three screen compartments are necessary; in the second a 5-compartment primary jig is followed by a 5-compartment secondary, the rougher makes clean galena concentrate in the first and second hutches, in the third, a middling containing free mineral which is returned to the head of the jig, true middling in the fourth for re-grinding and treatment in the secondary jig, and from the fifth tailing for re-grinding and fine concentration. The secondary jig makes a final quartz tailing. At the Zinc Corporation plant (*118 P 89*) a 3-compartment primary jig makes a 66-per cent. lead concentrate from the first two hutches and a 6-per cent. tailing from the third. At Broken Hill South (*27 IMM 337*) a double 3-compartment jig makes products as shown in Table 19. Capacity in this service varies from 12 to 14 tons per hr.

Table 19. Products of May jig at Broken Hill South mill

Hutch number	Product	Destination	Assay		
			Lead per cent.	Silver, oz.	Zinc, per cent.
1	Concentrate	Finished.................	67.2	22.4	7.1
2	Middling	Re-grind and re-jig........	38.3	15.3	18.4
3	Tailing	Re-grind and table........	7.6	4.3	14.6

Neill jig (Fig. 7) is of the fixed-sieve type. Water currents are produced by a swinging vertical paddle, actuated by a rocker arm. Hutch concentrate only is made and is discharged continuously through three $\frac{3}{8}$-in. spigots from each hutch. The jig was developed for service on gold dredges, to replace undercurrents in the sluice line.

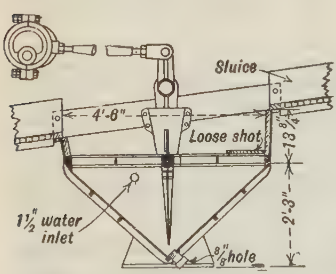


Fig. 7.—Neill jig.

Table 20. Sizing test of feed to Neill jig in treating tin gravel. (All material through $\frac{3}{8}$-in.)

	Tin oxide, per cent. weight	Gravel, per cent. weight
On 10 mesh	....	54.6
20	1.31	20.0
40	34.08	12.2
60	33.70	2.7
80	17.95	0.34
100	7.76	0.07
Through 100	5.20	0.08

On the Natomas No. 7 dredge (*101 J 207*) the jig screens were 2 ft. 5 in. by 3 ft. 8 in., 8-mesh Monel metal, bedded 1 in. deep with BB cast-steel shot. Each jig this size consumed 2.6 hp. at 172 r.p.m. Gold in the jig concentrate all passed 20-mesh and 43 per cent. passed 100-mesh. Jig concentrate contained 6 to 7 per cent. of all the gold caught on the dredge. This was ground in a conical mill and treated in a shaking amalgamator and on plates.

The following data refer to tests on artificial mixtures of cassiterite and stream gravel to determine the suitability of the jig in tin dredging (*106 J 78*). Feed contained 8 lb. tin oxide per cu. yd. Sizing tests of cassiterite and gravel are given in Table 20. Two jigs

were operated in series, the screen area of each about 8 sq. ft.; the sieves were Ton-cap screen with 0.093-in. aperture; bed of ⅛-in. cast-iron shot and gravel, 3.5 to 4 in. deep; 135 to 146 @ 3.5-in. strokes per min.; 50 to 60 gal. per min. hutch water for each jig. The best feed rate was under 4 cu. yd. per hr. Results are given in Table 21. A parallel test

Table 21. Results of Neill-jig tests on tin-gravel

	Test 1	Tests 2–5	Tests 6–11	Test 12
Assay of concentrate, per cent. Sn.	1.67	1.53	0.96	1.12
Ratio of concentration.	15.6	15.1	9.23	9.9
Recovery, per cent.	87.2	86.1	89.5	86.8

in a sluice gave 88 per cent. recovery. In another test on similar material (*112 J 644*) feeding 2 jigs in series, each with 9 sq. ft. screen area, at a rate of 5.8 cu. yd. per hr., the feed containing 0.27 per cent. to 0.53 per cent. cassiterite, the concentrate contained 4.3 to 6.4 per cent. cassiterite and average recovery in the first jig was 85.5 per cent. and in the second, 2.2 per cent. of the total tin in the feed.

Table 22. New Century jig operations at New Jersey Zinc Co., Franklin Furnace, N. J.

Jig number.	1–4	5–8	9–12	13–16
Material of screens.	*bw*	*bw*	*bw*	*sw*
Screen aperture, in.:				
Compartments Nos. 1, 2, 5 and 6.	0.0375	0.0460	0.0550	0.0650
Compartments Nos. 3 and 4.	0.0326	0.0395	0.0480	0.0580
Life of screen, days.	150	150	150	75
Speed, strokes per minute.	210	197	185	174
Length of stroke, 64ths in.:				
Compartments Nos. 1 and 2.	12	13	15	18
Compartments Nos. 3 and 4.	11	12	14	17
Compartments Nos. 5 and 6.	10	11	13	16
Tons per jig per hour.	0.77	0.84	0.91	0.98
Water, gallons per minute per jig.	52	59	67	76
Thickness of bed, in.:				
Compartment No. 1.	4	4¼	4½	4¾
Compartment No. 2.	4⅛	4⅜	4⅝	4⅞
Compartment No. 3.	4¼	4½	4¾	5
Compartment No. 4.	4⅜	4⅝	4⅞	5⅛
Compartment No. 5.	3	3¼	3½	3¾
Compartment No. 6.	3½	3¾	4	4¼
Jig number.	17–20	21–24	25–28	29–32
Material of screens.	*sw*	*sw*	*sw*	*sw*
Screen aperture, in.:				
Compartments Nos. 1, 2, 5 and 6.	0.0760	0.0840	0.0960	0.1130
Compartments Nos. 3 and 4.	0.0680	0.0790	0.0900	0.1020
Life of screen, days.	75	75	75	75
Speed, strokes per minute.	164	155	147	140
Length of stroke, 64ths in.:				
Compartments Nos. 1 and 2.	22	27	33	40
Compartments Nos. 3 and 4.	21	26	32	39
Compartments Nos. 5 and 6.	20	25	31	38
Tons per jig per hour.	1.04	1.10	1.16	1.24
Water, gallons per minute per jig.	86	97	109	122
Thickness of bed, in.:				
Compartment No. 1.	5	5¼	5½	5¾
Compartment No. 2.	5⅛	5⅜	5⅝	5⅞
Compartment No. 3.	5¼	5½	5¾	6
Compartment No. 4.	5⅜	5⅝	5⅞	6⅛
Compartment No. 5.	4	4¼	4½	4¾
Compartment No. 6.	4½	4¾	5	5¼

bw Brass, woven wire; *sw* Steel, woven wire. Sizing tests of feed given in Table 23,

New Century jig is of the Harz type but usually equipped with a differential motion giving accelerated down stroke to the plunger with corresponding accentuation of pulsion. Suction is decreased by using a rubber flap-valve around the plunger edge that closes on the down stroke and opens wide on the return. Differential motion is produced by raising the plunger against a spring by means of a cam acting against a roller, the plunger being forced down quickly by the spring when the cam releases.

Performance. NEW JERSEY ZINC Co. uses a most elaborate jig equipment in separating willemite and zincite from a calcareous gangue. There are 32 @ 6-compartment jigs with 24 × 36-in. sieves. Data concerning the installation at Franklin Furnace, N. J., are shown in Table 22. One man attends 4 machines. Lost time is given as 10 to 15 per cent., which is exceptionally high, the principal cause being repairs. 5 hp. are consumed per jig. Sizing tests of feeds are given in Table 23. Feed is prepared by dry screening in all cases. At the OGDENSBURG MILL of the same company, treating similar ore, capacity, speed and power consumption are higher than at the Franklin mill. Three men attend 10 machines. Lost time amounts to about 2 per cent., the principal cause being repairs.

Table 23. Sizing tests of jig feeds, New Jersey Zinc Co., Franklin mill

Jig number	1–4	5–8	9–12	13–16	17–20	21–24	25–28	29–32
Screen aperture, mm.	Per cent. of total weight							
3.327	....	....	....	....	....	....	....	0.60
2.362	0.10	....	....	....	0.02	0.40	0.38	10.50
1.651	1.40	....	....	1.38	2.75	11.50	23.10	50.90
1.168	2.00	0.55	0.50	1.50	23.00	38.75	54.50	31.80
0.833	2.05	3.90	9.05	36.70	43.10	36.10	19.20	5.70
0.589	8.88	31.80	49.55	40.85	26.20	11.58	2.00	0.40
0.417	39.00	48.50	32.82	18.25	3.80	1.05	0.25	0.10
0.295	29.35	11.90	4.35	0.55	0.25	0.15	0.12	....
0.208	10.40	2.20	1.20	0.20	0.25	0.10	0.10	....
0.147	3.30	0.55	0.80	0.12	0.20	0.05	0.10	....
0.104	0.90	0.10	0.35	0.10	0.12	0.05	0.08	....
0.074	1.20	0.10	0.60	0.18	0.12	0.05	0.10	....
−0.074	1.42	0.40	0.78	0.18	0.18	0.22	0.08	....

Richard's pulsator jig (Fig. 8) consists essentially of a compartmented jigging tank with a fixed screen, underneath which water is fed through a rotating valve. Concentrate is discharged from the screen through the usual gate-and-dam discharge. When operated with closed hutch the bed is subjected to pulsion impulses only and is kept remarkably loose. Speed for such jigging is about 200 r.p.m. Jigging with sized feeds is therefore very rapid and the tonnage per sq. ft. of screen area is many times that of a plunger jig. If suction is desired, the speed is lowered to 150 to 175 r.p.m. and the hutch gate opened. Due to the fact that an 8- to 10-in. bed can be maintained and due also to the relatively light suction, very clean concentrate can be made. Power consumption is much less than for plunger jigs. Water consumption is about the same.

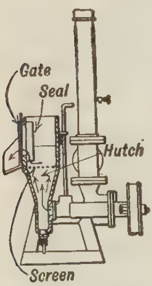


FIG. 8.—Richards pulsator jig.

This jig is most attractive on the basis of laboratory tests but has not worked out well in practice. It does not respond readily to changes in tonnage and richness of feed, probably

on account of the small size of the cells, yet it is limited to small cells to keep down water consumption. It should not be considered for hutch-making service, but is best suited to treatment of closely sized ores with large differences in specific gravity between mineral and gangue, and where low-grade tailing is not essential. In such service it will deliver a high-grade concentrate with small power consumption and not excessive water consumption, and, on account of its small size it can readily be placed high up in the mill where its reject can be spouted by gravity to re-treatment machines. Clean water must be supplied to prevent blinding of screens. At ANACONDA a No. 2 jig treated material sized between 8- and 2.5-mm. in a short test at the rate of 224 tons per 24 r. and made concentrate assaying 9.8 per cent. Cu and tailing about 2.2 per cent. from a feed containing 2.85 per cent. This result compares unfavorably on a metallurgical basis with the work of the Hancock, Harz, Evans and Woodbury jigs at these plants.

At PACIFIC COAST COAL Co., Issaquah, Wash. (*Bul. 28 UW 123*), a laboratory-type Richard's pulsator jig at 110 r.p.m., treating – 3/8 + 3/16-in. raw coal, containing 18.7 per cent. ash made 75.3 per cent. by weight of washed coal containing 13.7 per cent. ash and 24.7 per cent. refuse containing 37.4 per cent.

Shields and Thielmann jig (Fig. 9) is a plunger-type fixed-sieve jig used at the QUINCY MILL for treating unclassified native-copper ore through 5/8-in. grates from steam stamps. The unit is a 4-compartment cast-iron jig tank, 12 in. wide by 24 to 30 in. long, each hutch connected by a 4-in. pipe through the side wall with a vertical cylinder in which runs a piston, eccentrically operated from a common shaft. Eccentrics are individually adjustable and each hutch is fed by a separate water supply. Both gate and hutch discharge are provided.

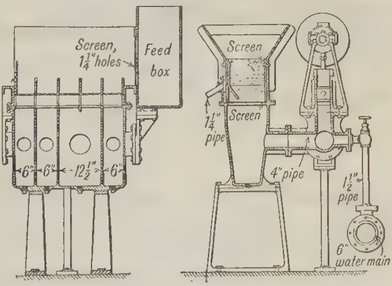


FIG. 9.—Shields and Thielmann jig.

A 4-section installation at the QUINCY mill treats 500 tons per 24 hr. Power consumption is between 3 and 4 hp. for the 16 compartments. The vertical screen between the sections causes size grading in successive discharges. Coarse copper, 3/8-in. to 5/8-in., discharges from gates 1 to 3. Clean concentrate is also taken from hutches 1 and 2. Gates 4 and 5 discharge coarse middling. Gates 6, 7 and 8 are kept closed to accumulate concentrate; gates 9 and 10 discharge middling for re-grinding. The remaining sections discharge middling products of various sizes from 1/8-in. to 40-mesh and of various grades from both gates and hutches. Slime (– 40-mesh) is overflowed from the last compartment.

Woodbury jig (Fig. 10) is a fixed-sieve quick-return plunger jig, with a plunger having a smaller area than the sieve compartment. Two plunger rods with separate eccentrics are used on each plunger. The jig is made in units that are erected end-to-end at different levels (PYRAMID SETTING), the feed enters over an apron above the plunger compartment. The jig is best known by reason of a slime-separating device that is used on the first unit of a series treating ungraded feed. This consists of a shield (*a*), that projects above the water-level and dips sufficiently into the sand layer on the screen to prevent slime from entering at the bottom. Slime is thus diverted around the shield and overflows a dam at the discharge side of the compartment. This same dam serves to hold back sand tailing, which rises within shield (*a*) and overflows lip (*b*). A concentrate cup (*c*), within shield (*a*), projects above the sand level and down into the concentrate layer on the screen, allowing, therefore, concentrate only to enter and overflow into pipe (*d*). This pipe is connected with pressure water to allow fine sand to be excluded from the cup concentrate by classification. De-slimed sand tailing (middling) passes to a second unit fitted with the usual gate-and-dam concentrate dis-

charge placed, however, at the tailing-discharge end of the compartment. When further units are used a dewatering trough is placed ahead of each subsequent unit and a similar device on the tailing-discharge box of the final unit.

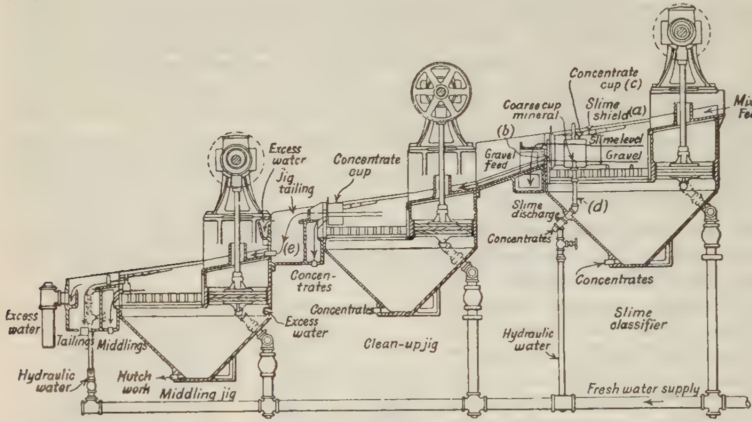


FIG. 10.—Three-unit Woodbury jig.

Performance. The Woodbury jig has had its largest use in treating native-copper ores. At CALUMET AND HECLA two 5-compartment jigs handle 300 to 375 tons per 24 hr. of $-\frac{3}{16}$-in. conglomerate rock from one steam stamp. Sieves are 30 × 48 in.; plungers, 15 × 48 in. Sieves on the first compartment are 10-mesh brass-wire, 12-mesh on the others (average life, 22 months). 180 strokes per min., 1- $\frac{7}{8}$-, $\frac{3}{4}$-, $\frac{1}{2}$- and $\frac{1}{2}$-in. length on compartments 1 to 5 respectively. 2.75-in. bed. One man attends 30 sieves and other work in addition. Lost time about 1 per cent., principally due to cleaning sieves. About 45 per cent. of the feed is separated as slime on the first compartments. The first two units discharge high-grade concentrate and the other three middling. Assays, per cent. Cu: Feed, 1.75; tailing, 0.6; concentrate, 80. On amygdaloid, three 5-compartment units handle 600 tons, of which 40 per cent. is separated as slime. At ANACONDA the Woodbury system was thoroughly tried but finally rejected. Feed was − 8-mm., ungraded. The classifier compartment when treating 2.7-per cent. Cu feed at the rate of 242 tons per 24 hr. rejected 39.3 tons of slime and fine sand (− 0.4-mm.) assaying 3.34 per cent., made 3.1 tons of cup concentrate assaying 14.4 per cent., 11.59 tons hutch concentrate at 9.11 per cent., 138.8 tons cup middling at 1.43 per cent. and 48.8 tons hutch middling at 3.88 per cent.; compartment 2, treating hutch middling assaying 3.09 per cent. Cu from the classifier jig, at the rate of 49.6 tons per 24 hr., made 2.2 tons of cup concentrate assaying 12.26 per cent. Cu, 6.4 tons hutch concentrate assaying 7.68 per cent., 14.6 tons hydraulic middling assaying 1.74 per cent., 8.8 tons hutch middling assaying 4.57 per cent. and a tailing assaying 0.62 per cent. Water consumption on the classifier jig was 859 gal. per ton of feed and on the regular jig 3431 gal. per ton. When treating de-slimed undersize of a 4-mesh screen, a classifier jig and three re-treatment jigs in series produced, from 127 tons per 24 hr. of feed containing 0.82 per cent. Cu, 15.5 tons of middling (concentrate) at 2.43 per cent., 6.8 tons hydraulic middling assaying 1.14 per cent., slime assaying 3.4 per cent. and tailing assaying 0.45 per cent. A 5-compartment system was tried on − 2.5-mm. material but failed to better the work of the Evans jig. At RAY CONSOLIDATED COPPER CO. the Woodbury system was tried on − 2.2-mm. ungraded material. Hutch concentrate only was made and this was cleaned on Wilfley tables. Slime was re-treated on vanners. On a feed containing 2.3 per cent. copper this flow-sheet made 25-per cent. concentrate and 0.83-per cent. tailing representing 64.9 per cent. recovery and 16.8 ratio of concentration. Competitive work on Garfield tables with the Garfield middling cleaned on Wilfley tables and slimes re-treated on vanners yielded 26.9 per cent. concentrate and 0.66 per cent. tailing, representing 73.1 per cent. recovery and 15.9 ratio of concentration. The failure of the Woodbury system was attributable to the fact that it cut out only 56 per cent. of the original feed, containing 35 per cent. of the copper as middling while the Garfield system put 80 per cent. of the original feed containing 50 per cent. of the total copper in the middling division. The

Woodbury system rejected 71 per cent. of the original feed containing 57 per cent. of the total copper as slime against 33 per cent. by weight and 24 per cent. of the copper content in Garfield slime. There was always much free mineral in the Woodbury-jig tailing. At Old Dominion three 3-compartment jigs were used, treating 350 tons each. Feed: 10 per cent. on 4-mesh, 59 per cent. moisture. No.1 jig compartments were 24 × 36 in. and Nos. 2 and 3, 30 × 50 in. Screens were woven brass wire, 0.187-in. aperture, on the first compartment of No. 1 and 0.120-in. on the second and third compartments; 0.120-in. on all compartments of the other jigs (life, about 6 months). Stroke on all compartments, 1.5-in. 4- to 5-in. beds. One man attended 5 jigs and the preceding screens. Change of water rate and feed distribution were left to the operator. Water consumption, 760 gal. per min. per jig. Assays, per cent. Cu: Feed, 4.10; tailing, 2.50; concentrate, 9.00. At the Ohio Brass Co. (*65 A 652*) a 2-compartment jig, first compartment 12 × 21 in. with 10-mesh screen and 1⅜-in. stroke, the second 18 × 24 in. with 6-mesh screen and ¾-in. stroke, 200 r.p.m., treated brass-foundry ashes sized between 3- and 10-mesh on heavy wire stationary screens set at 45°. Cup concentrate was almost pure metal (70 to 80 per cent. Cu) and represented 60 to 70 per cent. of the total metal in the feed; hutch concentrate assayed 65 to 70 per cent. Cu and contained about 20 per cent. of the total copper fed.

Bull jigs are jigs of any type, but usually of the fixed-sieve variety, used for treating the coarsest feeds. Heavy ores up to 3-in. diameter have been treated. Bull jigs differ from others of the same type only in that they are built heavier to withstand the longer stroke and greater wear, and discharge ports must be larger to prevent clogging. In a Joplin bull jig with 36 × 60-in. sieve compartments, two eccentrics are used on each plunger.

Performance. At Bunker Hill and Sullivan (*61 A 225*) bull jigs treat material sized between 1.25- and 0.5-in. and make middling and tailing only. This method of operation saves the heavy wear on soft galena concentrate that results if it is attempted to make a high-grade concentrate. The best grade that can be made on these jigs is 55 per cent. lead and to make this results in wearing the concentrate into rounded marbles. But by taking a 25 per cent. rough concentrate, crushing it, and re-jigging, a 65 per cent. concen-

Table 24. Competitive test between Woodbury and Harz bull jigs at Boston and Montana mill of Anaconda Copper Co. (Feed through 38-mm. on 8-mm.)

Legend	Woodbury	Harz system		
		1-comp. jigs	2-comp. jigs	Total
Number of jigs	1 @1-comp.	2	2	4
Number of jig screens	1	2	4	6
Net screen dimensions, in	57.5×39.5	41×21.5	36×23.5	......
Screen aperture, square hole	0.31 in.	0.31 in.	0.25 in.	......
Length of stroke, in	3.5	2.25–2.5	1.75–2.12	......
Strokes per minute	170	160	160	......
Height of tail board, in	6*a*	6.5	5	......
Height of conc.-discharge gate (min.), in	3.5	4	4.25	......
Bottom of cup above screen, in	2	2	1.75	......
Size of plunger compartment, in	24×60	20.5×42.75	17.5×37.5	......
Size of plunger, in	23.5×59.5	20×42.25	17×37	......
Ratio screen area to plunger area	1.62	1.04	1.35	......
Trommels used, 1.5-in rd.-hole	2	2	....	2
Trommels used, 0.87-in. rd.-hole	3	2	....	2
Trommels used, 8-mm. rd.-hole	2	....	2	2
Excess trommel area, Woodbury, sq. in	8144	....	....	......
Feed rate, tons per 24 hr	281	75	192	......
Assays, per cent. Cu: Feed	3.19	3.7	3.8	......
Cup concentrate	12.19	11.6	11.7 & 13.3	......
Hutch middling	4.86	7.5	9.4 & 6.1	......
Tailing	1.62	2.1	2.9	......
Water, gallons per ton of feed	1135	728	....	......

a Slime overflow, 10 in.

trate is made with but little wearing of the galena. These jigs will handle 150 tons per 24 hr. and reject 90 tons of tailing containing less than 0.4 per cent. lead. In the West mill five bull jigs treat 450 tons per 24 hr. at a cost per jig (1918) of $2. At other Coeur d'Alene mills the upper size of bull-jig feeds is as follows: HERCULES, 1.4-in.; MAMMOTH, 0.6-in.; MORNING, 0.9-in. At SLOCAN, B. C., bull jigs make a lead concentrate as coarse as 0.75-in. containing 100 to 175 oz. Ag. (*114 J 677.*) At OLD DOMINION two one-compartment Woodbury bull jigs, 30 × 50-in., treat 500 tons each per 24 hr.; feed, 51 per cent. on 3-mesh, 54 per cent. water. Brass-wire screen with 0.130-in. aperture lasts 8 months. Bed, 6 in. thick. Stroke, 2.25 in. Water, 240 gal. per min. Assays, per cent. Cu: Feed, 3.60; tailing, 2.70; concentrate, 11.00. One operator attends 11 sieves and the preceding screens and controls jig water and feed distribution. Table 24 gives data on a competitive test between a Woodbury bull or DE-WOODING JIG and the Harz bull-jig system at ANACONDA C. M. Co.

6. Fixed-sieve jigs used principally for coal

The jigs most widely used for anthracite are the Reading, Lehigh, Elmore, Wilmot (Simplex) and Delaware (Tench) which are all of the piston variety. Their use is explained by the fact that the feed in anthracite washeries is invariably sized and the piston jig is best suited for treating such feeds. For bituminous coal both pan (movable-sieve) and piston jigs are used, the former to a greater extent. The most widely used piston jigs are the Luhrig, Elmore, Foust, and Forrester.

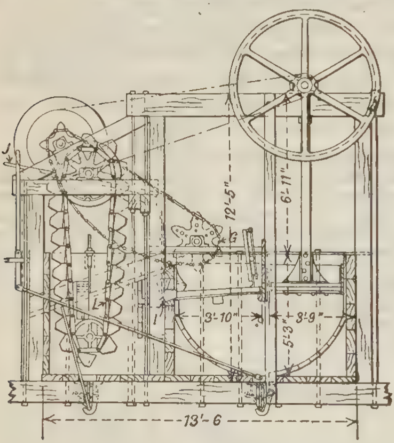


FIG. 11.—Reading jig.

Reading jig (Fig. 11) is of the typical piston type with coal discharge aided by the chain drag *G* and with intermittent slate discharge manually controlled by gate *I* and lever *J*. Discharged slate is removed by the bucket elevator *L*. The usual operating conditions are 140 @ 2-in. strokes per min. for pea and buckwheat sizes and 100 @ 4-in. strokes for larger sizes.

Lehigh jig (Fig. 12) is of the piston type with grate inclined about 1 in. to the foot toward the discharge side. Working adjustments are given in

Table 25. Operating adjustments of Lehigh jig working on anthracite

Size of feed	Strokes per minute	Length of stroke, inches	Height of feed gate, inches	Height of slate gate, inches	Size of openings (*a*)	
					Rear three-quarters	Front quarter
Buckwheat......	96–100	3 –4	1½–2	¾	5⁄16	¼
Pea............	96–100	3 –4	2½	⅞	⅜	5⁄16
Chestnut........	96–100	3½–4	2½–3	1½	⅝	½
Stove..........	96–100	3½–4	3	2½	¾	⅝
Egg............	96–100	3½–4½	4 –4½	4	⅞	⅝
Broken.........	96–100	3½–4½	7	7¼	⅞	¾

a Round or oblong holes, the latter set with long axis parallel to the run of the material.

Table 25. The hutch (slush) gate is opened intermittently for discharge only; normally the jig is run to give minimum suction. Slate discharge is regulated by the slate conveyor, which is run only when necessary to prevent slate from going with coal or when the bed becomes packed. PERFORMANCE is given in Table 26.

Elmore jig (Fig. 13) is a double-compartment machine of the piston type with overflow coal discharge and automatic slate discharge. According to Ashmead (*66 A 422*) it requires 700 gal. per min. of circulating water; 12 hp. to operate the jig mechanism and 18 hp. for the circulating pump. Capacity of the double unit is as high as 50 tons per hr. on the coarse sizes of anthracite. Two- and 3-

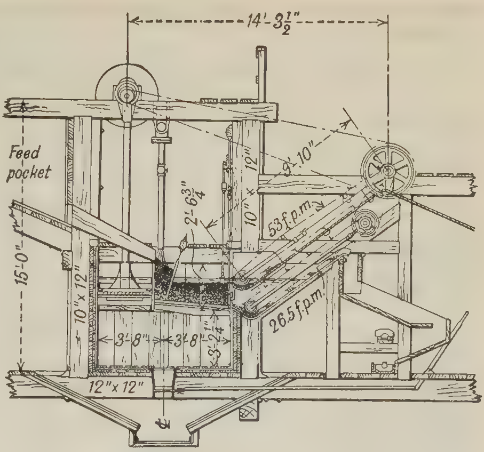


FIG. 12.—Lehigh jig.

Table 26. Performances of Lehigh jigs in upper and lower anthracite fields

Size of coal	Feed			Coal product			Slate product	
	Coal, per cent.	Slate, per cent.	Bone, per cent.	Coal, per cent.	Slate, per cent.	Bone, per cent.	Coal, per cent.	Slate, per cent.
Upper field								
Egg........	81	14	5	97	1	2	1	99
Stove......	80	16	4	94	2	4	1	99
Nut........	80	15	5	94.5	2.5	3	0.5	99.5
Pea........	84	12	4	91	6	3	0.5	99.5
Buckwheat..	83	12	5	84	8	8	1	99
Lower field								
Egg........	69	28	3	97	1	2	2	98
Stove......	70	27	3	94	3	3	3	97
Nut........	74	22	4	93.5	3.5	3	2	98
Pea........	76	20	4	90	7	3	4	96
Buckwheat..	76	20	4	87	10	3	4	96

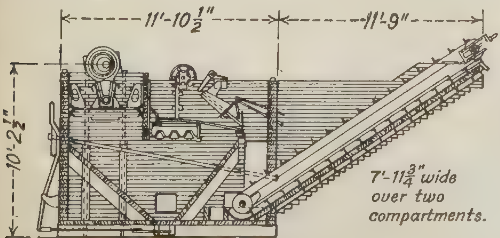


FIG. 13.—Elmore jig.

compartment forms of this jig are also widely used in washing coking coal in Alabama.

Table 27. Test on Elmore jig at Indianola mine. (*After Yancey*)

Test number	1			2		
Feed rate, tons per hour	49.5			42		
Speed, r.p.m.	114			75		
Stroke, in., comp. 1	1¼			2⅓		
Stroke, in., comp. 2	1⅛			1⅞		
Stroke, in., comp. 3	1⅛ and 7/16			1⅝		
	Weight, per cent.	Ash, per cent.	S, per cent.	Weight, per cent.	Ash, per cent.	S, per cent.
Raw coal	100.0	10.8	0.92	100.0	10.0	0.94
Washed coal(*a*, *b*)	83.7	8.1	0.75	82.4	7.3	0.77
First gate(*c*, *e*)	1.0	53.0	1.96	2.7	53.5	1.51
Second gate(*c*, *e*)	2.4	46.0	1.37	2.4	43.1	1.07
Third gate(*d*, *e*)	3.2	32.5	1.19	1.2	32.2	1.11
First hutch	1.2	38.2	6.82	1.8	31.5	5.28
Second hutch	1.5	23.5	2.99	2.0	17.0	1.65
Third hutch(*b*)	2.3	10.5	1.00	1.6	9.7	1.11
Sludge(*b*)	4.3	10.4	0.65	3.5	10.9	0.59
Shipped coal	90.3	8.3	....	87.5	7.5	....

Sizing—assay test	Test 1 Raw coal		Test 1 Washed coal		Test 2 Raw coal		Test 2 Washed coal	
	Weight, per cent.	Ash, per cent.	Weight, per cent.	Ash, per cent.	Weight, per cent.	Ash, per cent.	Weight, per cent.	Ash, per cent.
Original	100.0	10.8	100.0	8.1	100.0	10.0	100.0	7.3
+1	17.0	14.1	12.3	11.3	14.6	14.1	12.7	8.7
½	47.2	9.8	44.2	8.2	42.5	9.6	42.7	7.3
¼	18.3	10.2	22.1	7.8	19.8	9.5	22.6	7.0
⅛	8.0	10.0	11.2	7.2	9.8	9.2	11.9	6.7
1/16	4.0	10.1	4.9	5.7	5.3	9.5	5.2	5.0
1/32	2.4	9.9	2.6	4.8	3.5	9.3	2.7	4.1
1/64	0.5	10.8	0.7	4.8	0.7	10.0	0.5	4.2
−1/64	2.6	12.1	2.0	8.1	3.8	10.8	1.7	6.8

Proximate analysis	Test 1 Raw coal	Test 1 Washed coal	Test 2 Raw coal	Test 2 Washed coal
Volatile matter	34.0	34.6	34.0	35.0
Fixed carbon	55.5	57.3	56.1	57.8
Ash	10.5	8.1	9.9	7.2
Sulphur	0.92	0.75	0.94	0.77
B.t.u.	13510	13950	13660	14020

a Jig overflow. *b* Combined for shipment. *c* Valve operated almost continuously in both tests. *d* Valve operated intermittently, open about one-fifth of total time in second test. *e* In the first test the percentages of good coal in these products were 9.5, 15.1 and 36.3 respectively; in the second, 5.9, 9.1 and 19.7.

At Indianola mine of Inland Collieries Co. (*Bul. 16 CIT*) a 3-compartment unit was subjected to careful test. Each compartment was 6 ft. wide by 6¼ ft. long; the grates had ⅛-in. round holes; beds were 12½, 15 and 11½ in. deep in compartments 1, 2 and 3 respectively. Feed was bituminous coal crushed to about 1-in. and fed without sizing. Performances are shown in Table 27. The long, slow strokes of the second test improved the grade of shipped coal by improving the ash reduction in the coarse sizes. There was, however, a reduction in yield and capacity. The longer stroke gave a more fluid bed, permitting better stratification, as evidenced by the higher grade of coal in the coarse sizes and the larger quantity and lower grade of slate drawn. On the other hand, more fine coal was drawn down into the first two hutches and lost, on account of the increased suction. Fig. 14 shows the results of a sizing-specific-gravity analysis on the feed and products of the jig, reduced, in plotting, to efficiencies of removal of material of different specific gravities at different sizes. This shows that ease of removal decreases, of course, with decreasing specific gravity, that material of +1.80 sp. gr. is readily removed at all sizes down to 1⁄64-in., that −1.80 +1.70 material is also readily removed in the same size range, that the −1 +½-in. material is the most difficult of the +1⁄64-in. material to remove, due, no doubt, to the fact that the interstices in the coal bed are too small for its ready passage downward; that for the lighter materials maximum removal takes place in the size range from 1⁄64- to ⅛-in., which is accounted for by the fact that this material can pass down readily through the interstices of the bed and discharge either through the coarse-slate draw valve or into the hutch; finally that distinct ash reduction is effected even at 1⁄64-in. to 200-mesh size on a jig handling 1-in. lumps. Hancock (*6 CI 56*) found, on the other hand, that the ordinary single-compartment jig sends most of the −¼-in. impurities into the washed coal and that substantially no improvement in grade is effected at 20-mesh or finer sizes. Hancock's results are not entirely at variance with Yancey's, in that Yancey worked with a long, slow stroke and much suction, thereby making rather high-coal hutch products, but effecting noticeable reduction of fine ash, while Hancock refers to ordinary jig operation with suction minimized.

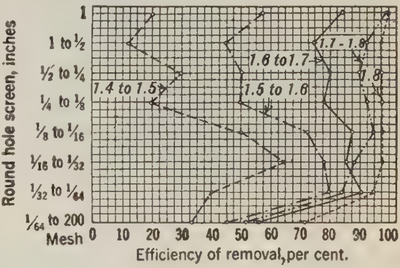


Fig. 14.—Relation between particle size, specific gravity and efficiency of removal in an Elmore coal jig (*after Yancey*).

Wilmot (Simplex) valve-plunger jig is of the pulsion-type with gate-and-dam discharge. The plunger is similar in position and arrangement to that of the Dee jig (Fig. 6) and is fitted with a plurality of small valves that open on the downstroke and thus, with free water inflow under the plunger, prevent any downward flow through the jig bed.

With a speed of 140 r.p.m. on buckwheat-size anthracite (−½ +¼-in. round hole) the capacity was 25 tons per hr. At this rate, with a feed containing 30 per cent. slate, the coal discharge contained 8 per cent. slate and the slate discharge 1 per cent. coal (*25 CA 674*). Performance at Drifton breaker (p. 48) is shown in Table 28.

Table 28. Capacities of Simplex jigs at Drifton breaker

Size	Tons per hour		Coal content of refuse, Oct., 1916
	Oct., 1916	Maximum	
Egg	5.5	7.4	1.00
Stove	7.7	7.5	1.12
Nut	6.8	11.7	1.50
Pea	5.4	11.0	2.90
Buckwheat	9.9	14.7	3.75

Delaware jig (Fig. 15) is of the piston type with grate inclined toward the discharge side. Feed enters through a chute under the piston compartment behind baffle A. Slate is discharged through and under adjustable baffle F, thence under another adjustable baffle and presses against the slate gate I which is connected by means of rods J and rope K to an adjustable counter-weight L, which, therefore, controls the rate of discharge into the boot of the bucket elevator. Coal discharges over a plate onto the lifting plunger O and is thereby discharged over chute R. The jig is usually built with two sieve compartments side-by-side, each about 4 ft. wide by 5 ft. 8 in. long. Depth of bed is about 21 in. at the discharge end. Performance is shown in Table 29 (*DHH*).

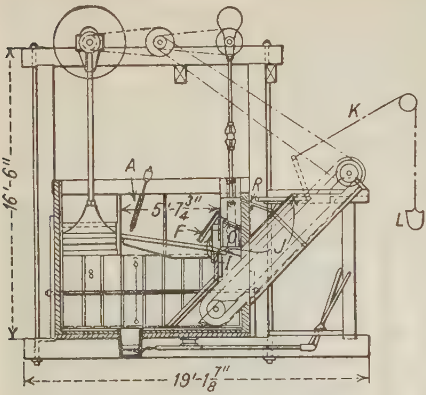


Fig. 15.—Delaware jig.

Table 29. Performance of Delaware jig

Size and assay	Material, weight, per cent.		
	Feed	Concentrate	Tailing
Stove, per cent of total	93.3	90.6	88.6
Stove, per cent. of coal	68.9	96.5	1.4
Stove, per cent. of bone	2.2	1.4	0.8
Stove, per cent. of slate	28.9	2.1	97.8
Nut, per cent. of weight	6.2	9.1	9.4
Nut, per cent. of coal	81.3	100.0	3.0
Nut, per cent. of slate	18.7	....	97.0
Pea, per cent. of weight	0.2	0.2	1.4
Pea, per cent. of coal	100.0	100.0	33.7
Pea, per cent. of slate	....	....	66.7
Smaller, per cent. of weight	0.3	0.1	0.6
Tons per 100 tons of feed	100.0	71.4	28.6
Assay, per cent. of coal	69.8	96.8	2.6
Recovery	....	98.9	....

Breakage of stove sizes 3.7 per cent.

Luhrig jig is a piston jig that is largely used for treating sized bituminous-coal feeds. It is made in two different forms according to whether the feed is fine or coarse. The coarse-coal jig (Fig. 16) has the grate slanting toward the discharge end, and is fitted with manually controlled slate-discharge gate and slush-discharge valve. Coal discharge is by overflow. The fine-coal jig is similar except the grate slopes slightly toward the feed end; there is no slate-discharge valve, but the jig is bedded with coarse feldspar and slate is discharged through the bed into the hutch. Coal is discharged by simple overflow. Grate sizes range from 33 × 35 to 42 × 54 in.

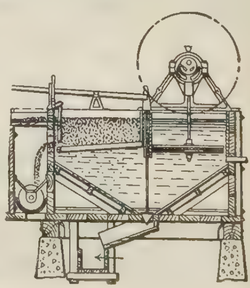


Fig. 16.—Luhrig coarse-coal jig.

Average speeds in Illinois practice (*Bul. 11 UI No. 9*) are 79 @ 3- to 7-in. strokes per min. for coarse-coal jigs, $-3\frac{1}{2}$ $+3$-in. to -1 $+\frac{3}{4}$-in. feeds, and 157 @ $\frac{3}{4}$- to 2-in. strokes per min. on fine ($-\frac{3}{4}$-in.) coal. Grates have $\frac{1}{4}$- to $\frac{3}{4}$-in. round holes in coarse jigs and $\frac{3}{4}$-in. or larger holes in fine jigs. Average capacity, all sizes, in twenty Illinois washeries was 0.6 ton per sq. ft. of grate surface per hour.

Foust jig (Fig. 17) is a multi-compartment piston jig for coal treatment, designed to jig refuse into the hutch through a bed made by the jig itself.

It has also had considerable success in treating lead-zinc ores in the Joplin district. It uses two plungers to each sieve, placed one on each side of the sieve, the combined area of the plungers being about 10 per cent. greater than that of the screen. The eccentric shafts are rigidly linked together to synchronize the plungers. More uniform water distribution is thus obtained than in the single-plunger construction, allowing a thinner and lighter bed with consequent reduction in water consumption. The construction is, however, necessarily more massive. A cup-type concentrate draw is used in metal concentration. This is designed to be self-regulating but is difficult to reach and liable to clog. The hutch draw must be placed under the center of the jig tank in a most inconvenient position. Partitions between screen and plunger compartments are extended 18 in. below the screen.

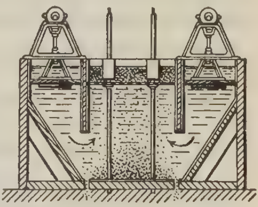


FIG. 17.—Foust jig.

A 2-compartment jig at ILLINOIS HOCKING WASHED COAL CO. (*Bul. 11 UI No. 9*) treated 45 tons per hr. of $-\frac{3}{4}$-in. bituminous coal, making a refuse containing 74 per cent. ash. This jig made 130 strokes per min., 1 in. long on the first compartment and $\frac{7}{8}$ in. on the second.

At PACIFIC COAL CO., Issaquah, Wash., (*28 UW 128*) a 2-compartment Foust jig treating -1-in. slack assaying 21 per cent. ash, at the rate of 13 tons per hr., made washed coal containing 13.3 per cent. ash and refuse assaying 46.1 per cent. ash. Recovery of combustible was 84 per cent.; yield, 77 per cent.; reduction of ash, 37 per cent.

Forrester jig (*Bul. 11 UI No. 9*) is of the plunger type; it is used in treating unsized coal. The plunger is carried by stirrups, actuated from a lever, which in turn is actuated by a crank wheel and connecting rod. Washed coal and refuse are discharged from the screen in the usual manner.

A 30 × 36-in. jig with $\frac{7}{16}$-in. screen perforations making 40 @ 6-in. strokes per min. treats 25 tons per hr. of -2.5-in. raw-coal screenings.

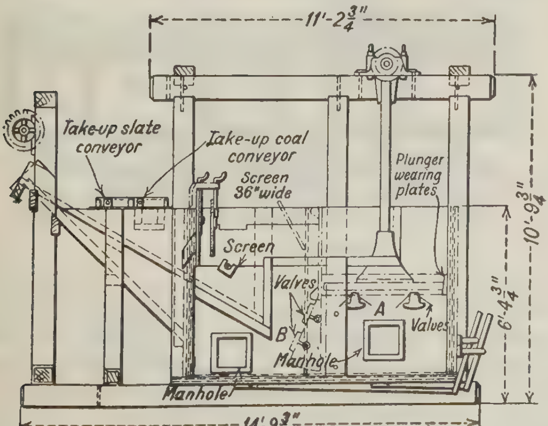


FIG. 18.—Ransom jig.

Ransom jig (Fig. 18) is a piston jig designed to eliminate suction and was particularly built for the treatment of fine sizes of anthracite. The characteristic element is the set of flap valves *B*, which open on

the pulsion (down) stroke of the plunger and close on the up stroke, thus causing a pulsating rising current with no downward currents. The valves *A* prevent the plunger from becoming airbound. Slate discharge is of the gate-and-dam type.

The following are the composite results of one 4-hr. and two 7-hr. tests on anthracite (*25 CA 674*): Feed, No. 1 buckwheat [20 to 30 per cent. ash]; slate in coal discharge, 3 per cent.; coal in slate discharge, 6 to 8 per cent, average 7⅓ per cent.

Baum jig is a fixed-sieve jig, built like the ordinary piston jig except that pulsation of the water is obtained by intermittent admission of air into and expulsion from a closed chamber which replaces the usual piston working zone.

A one-box machine (12 ft. long × 8 ft. wide) treats 20 tons per hr. of – ¼-in. bituminous coal at a British washery, making a clean coal containing 5.3 per cent. ash (*67 IME 497*).

New Century coal jig is similar to the Luhrig coarse-coal jig, except that the jig grate slopes toward the feed side and the piston mechanism gives a quick down stroke, controlled by means of a cam and spring.

Feeding coal jigs is a more difficult problem, even, than feeding jigs in metal-concentrating plants on account of the great variations from hour to hour, in normal operation, in the tonnage of different sizes coming from the screens. At the Pennsylvania breaker of the Susquehanna Collieries Co. (*19 CA 618*), a battery of jigs treating the same feed was arranged in steps and fed from the same feed chute. The latter was so designed that no material left the chute except by the lowermost outlet until the latter had filled with banked-up coal and the banking had extended back to the preceding opening, which then also drew. This banking repeated until all chute outlets, each feeding one jig, were filled, or until conditions stabilized at some lower feed rate. The individual chutes were so arranged that when coal backed up in them a belt shifter operated to start the jig, and when feed had all worked out of the chute the jig stopped. With this arrangement only the highest jig in the bank operated underloaded.

MOVABLE-SIEVE JIGS

7. Hancock jig

Description. The principal parts of the apparatus (Fig. 19) are a compartmented tank (*a*), movable sieve (*b*), and sieve-actauting mechanism (*c*, *h*,

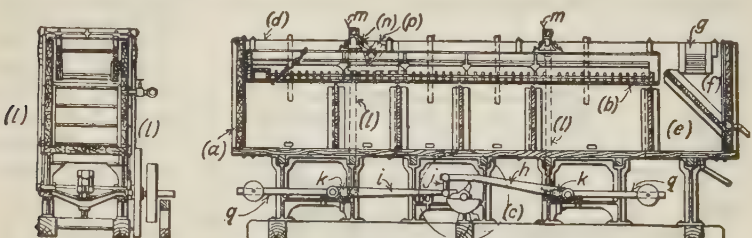


Fig. 19.—Hancock jig.

i, *j*, *k*, *l*, *m*, *n*, *p*). Feed is introduced onto the screen at (*d*), and caused to progress along the screen by a series of grasshopper-like jumps induced by the sieve-frame motion. Concentrate works down through the sieve into the head-end hutch compartments, middling is drawn down into the later compartments under the screen, coarse tailing falls into compartment (*e*), fine sand collects in (*f*), and slime and excess water overflow at (*g*). An extra, small compartment is often provided under a coarse screen between the last middling and the tailing compartments to catch any chance pieces

of concentrate or middling too coarse to pass the screen or any piece of the bedding material (RAGGING) that may carry over. Two standard sizes of jig are made: the larger with a 6-compartment tank 25 ft. long, 4 ft. 2 in. wide and 5 ft. 9 in. high; the smaller with a 5-compartment tank 18 ft. 6 in. × 4 ft. ft. 5 in. × 5 ft. The sieve of the larger jig is about 20 ft. × 2 ft. 8 in. in the clear, when made of wood; when made of steel the screening surface is 20 ft. 4 in. × 3 ft. The actuating mechanism, usually placed below the tank but occasionally above to escape splash and grit, is driven by a cam shaft with a 3-armed cam (*c*). The cam ears engage the end of lever (*i*). Lever (*i*) actuates lever (*h*) through the link (*j*) and the levers (*i*) and (*h*) actuate rocker-arms (*k*) carrying 4 upright rods (*l*) connected in pairs at the top by cross-bars (*m*) to which the sieve frame is attached. The head-end cross-bar is linked at both ends to the jig tank by inclined radius-links (*n*) adjustable as to their inclination with the horizontal by movement of the pivot pins in quadrants (*p*). A considerable part of the weight of sieve frame and load is counterbalanced by the lever arms (*q*). When lever (*i*) is raised by a cam ear, the sieve frame is raised by rods (*l*) and pulled forward by the radius links (*n*); when the cam releases the frame falls by gravity and at the same time is pushed backward by the radius links. The links work with an amount of lost motion controllable between 0 and ⅛ in. which produces a bump of greater or less intensity. The effect of the backward fall of the sieve-frame and the bump combined is to cause forward travel of the material over the sieve-frame, while the vertical motion produces the reciprocating water currents through the bed that cause stratification and separation of the minerals.

Construction. Complete jigs may be bought from various manufacturers, but usually only the iron-work is purchased and the jig tank and supports and sieve frame are built on the ground. Wooden tanks are invariably used. They are built of 4-in. tongue-and-groove clear plank strongly stayed both vertically and horizontally at the compartment partitions. Partitions may be placed at any desired position, the best places being determined by experimental work. They are usually so placed that the first concentrate and last middling compartment are considerably longer than the others. The sieve frame is made either of wood or steel. The wooden-frame has four sides made of 2½- or 3-in × 18-in. plank set on edge, and a grate (*e*) (Fig. 20) whose upper surface is about 6 in. above the bottom of the sides, for holding the screen. Top slats or battens (*d*) serve the double purpose of holding down the screen and of keeping the bed in place. One method of holding down the battens is shown in Fig. 20. Hardwood movable wedges (*a*) are driven between a fixed wedge (*b*) and a longitudinal bearing strip (*c*) placed against the side walls above the upper grid. The battens may be cambered about ⅛ in. at the center. Packing strips (*f*) on the outside of the sieve frame and in a corresponding position on the inside of the tank prevent excessive displacement of water on the down stroke with consequent loss of pulsion. Steel sieve frames are built up with light-weight plate girders for side bars and suitably cross-braced. A sectional grate of 1 × 3-in. oak cross slats, iron-shod, with 2 × 3-in. oak rails is bolted in position in the frame (*93 J 1178*). Steel is lighter than wood, hence consumes less power, and gives greater screen area, which increases capacity. Bearing boards and hard-pine wedges are used to hold down the upper slats and screen. Hutch discharges are of many types. Two forms are shown in Fig. 21. Gates should be capable of reasonably close regulation and of quick wide opening. Considerable water economy may be effected by discharging coarse tailing with the chain drag or a shovel wheel. The water level is maintained about 3 to 4 in. above the surface of the solids on the screen or about 12 in. above the screen itself. It is controlled by slats placed in guides in the overflow weir, provided that the water supply is sufficient to more than satisfy any hutch

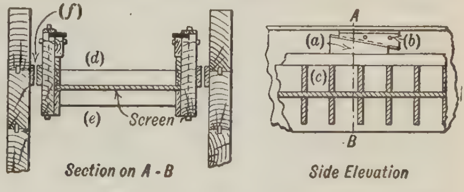


FIG. 20.—Wooden sieve frame for Hancock jig.

draws. To prevent fluctuation in water level at the Leadwood mill of the St. Joseph Lead Co. a float in the tailing compartment controlled a butterfly valve in the feed-water pipe. A solid foundation of concrete or masonry is usually provided but the Doe Run Lead Co. placed a 25-ft. jig on a steel trestle 18 ft. high with no serious vibration resulting.

Operation. The usual speed range is 180 to 195 strokes per min. corresponding to 60 to 65 revolutions of the drive shaft; a higher speed will increase tonnage but is very hard on the mechanism; at lower speeds the bed tends to pack. The length of the vertical stroke together with the amount of hutch water provided determines strength of pulsion and suction. The usual length is ⅜ to ¾ in. With a long stroke more hutch product is made and the operator varies this adjustment according to the tonnage and character of feed. Horizontal throw is usually about ¾-in. It should be as long as possible in order to cause quick travel across the bed and thus maintain a thinner bed for a given tonnage.

All of the concentrate and middling made on a Hancock jig is made through the screen. This necessitates the maintenance on the screen of a bed of grains of heavy material larger than the openings in the jig sieve. This may be coarser particles of the mineral that is being saved or artificial grains such as steel-plate punchings or iron or steel balls. The latter are used when the gangue is heavy. The bed is held in place by the grid on top of the screen and the maximum depth of bed is determined by the depth of the grid slats. The transverse slats in the grid correspond with the supporting slats and are spaced 3 to 6 in. in the clear. Frequently longitudinal slats are also used in order to permit close control of the thickness of the bed at different points. At Anaconda brass castings forming pockets 5 × 10 × 3 in. deep were used instead of wooden top slats. They were wedged down in the usual fashion. Before starting the machine each pocket is filled two-thirds to three-quarters full of ragging. If the bed is too loose, add more ragging and *vice versa*. The usual depth of the top grid is from 2 to 3 in. The screen aperture determines the size of particles held back and this, in turn, determines the size of interstitial passages, thus controlling the work of the jig. (See Art. 1.)

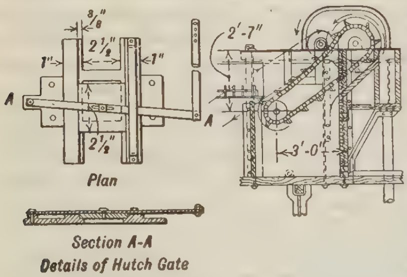


Fig. 21.—Discharge draws for Hancock jigs.

Screens are rarely of the same aperture for the full length of the sieve. The underlying principle controlling choice of screen aperture is that the bed should gradually decrease in specific gravity and increase in grain size from head to tail end. Openings are usually smallest over the first compartment in order to maintain a fairly tight bed and thus increase pulsion in comparison with suction and insure clean, fine-grained concentrate. The same opening may be maintained until near the end of the second compartment, when one to three rows of holes large enough to pass the largest particles of free mineral are introduced. These holes pass such particles while middling grains, both coarse and fine, are by this time stratified well above the large heavy grains of mineral and hence do not pass through with them. Smaller holes, although not usually so small as those over the first compartment, are placed above the third compartment and fine rich middling is drawn down. Openings generally increase in size toward the end of the screen, with or without intervening rows of larger holes. At the end of the screen large holes are again provided to pass the coarsest middling particles. It is, in general more difficult to make a clean separation of tailing from low-grade middling than to separate concentrate of suitable grade from high-grade middling, hence the latter operation is performed in as little space as possible and the balance of the jig is given over to the former work. In a particular operation on south-eastern-Missouri lead ores the concentrate was substantially clean galena assaying 82.3 per cent. Pb with a specific gravity of 7.00 and the rich middling assayed 60.4 per cent. Pb with a specific gravity of 5.20; difference in specific gravity, 1.80. The low-grade middling assayed 4.91 per cent. Pb, the tailing sought carried 0.14 per cent., the respective specific gravities were 2.95 and 2.82, difference 0.15. There being only a small amount of free galena in the feed, the first two hutches, used for making concentrate, were allowed only 80 in. out of a total length of 240 in.

The last hutch is usually run with very heavy suction in order to take out all possible value in the middling.

The screens are punched plate or woven wire. At Anaconda Ton-cap screen was most satisfactory; it outwore ordinary woven wire cloth and was easier to keep clean.

Satisfactory operation depends on a constant feed rate and metal content. Increase in the amount of valuable mineral from any cause produces heavy beds with consequent travel of concentrate and middling toward the tail end; decrease causes loss of bed with

consequent introduction of middling into the concentrate. The operator compensates for unavoidable fluctuation by shoveling off the bed when it becomes too heavy, shoveling on when the feed is light or plugging some of the screen holes or putting on finer screen. Fine gangue in concentrate may be due to leakage in the joints in the sieve tray. If the addition of ragging or increase in hutch water does not cure the condition, this possibility should be investigated.

Water consumption varies according to the size and character of feed, method of discharging tailing, tonnage treated and character of product desired. With chain-drag tailing discharge it may run as low as 150 gal. per min. when treating 350 to 400 tons per day of low-grade lead ore and about 850 gal. per min. with gate discharge of tailing and strong suction in the middling compartments induced by running with open gates.

Power consumption is from 4 to 6 hp. with normal load, stroke and speed. A test at ANACONDA showed 5.2 hp. for motor, countershaft and jig and 3.7 hp. for the jig alone.

Capacity depends on the character of work demanded but ranges from about 300 to 600 tons per day.

Applicability. The Hancock jig is particularly applicable to the treatment of low-grade ores, recovering a small amount of high-grade concentrate and rejecting tailing. It does this, however, at the expense of re-treatment, in circulation, of a large quantity of low-grade middling, much of which is true tailing. At many plants, also, it is necessary to re-treat the tailing of primary Hancock jigs in order to recover low-grade middling that has been carried along in the crowd for high tonnage. The jig is not suited to close separation of light middling grains from gangue and hence cannot be used where mineral is finely disseminated. It has had its principal use in this country in treatment of the mid-continent lead ores, but has also been used to treat copper ores at Anaconda and in the Lake Superior district. The best kind of feed is de-slimed but otherwise ungraded material ranging in size downward from about ⅜-in. Wiggin (*46 A 213*) determined that on ANACONDA ore the minimum economic size of free-mineral grain in the feed should be not less than 0.17-mm. On coarser grains the recovery was from 95 to 100 per cent., but at this size recovery dropped to 50 per cent. He found also, in treating de-slimed − 10-mm. material, that recoveries were improved by sizing the feed on a 4-mm. round-hole trommel and treating the sizes separately. Substitution of Hancock for Harz jigs in FEDERAL LEAD CO. NO. 3 MILL caused simplification of the flow-sheet by reduction in number of jigs and elimination of the screens and classifiers required for close grading of feed for Harz jigs; increase in capacity in the same mill space from 2600 to 4000 tons per day; and marked decrease in power and water consumption per ton milled. Experience at ANACONDA was similar, higher recoveries and saving in operating and repair labor were noted. DISADVANTAGES noted at DOE RUN NO. 3 MILL (*95 J 1283*) were the low recovery of mineral finer than 1.5-mm., no way to remove coarse galena concentrate from the screen until it wore down or went into middling, and inability to maintain even depth of bed with variable feed rate. Difficulty in separating low-grade middling from tailing has already been mentioned.

Performance. BONNE TERRE MILL, ST. JOSEPH LEAD CO. A most exhaustive study of the work of the jig on the galena-dolomite ore of this company is reported by Rabling (*57 A 309*). The jig was a standard 25-ft. size making concentrate from the first three hutches, middling from fourth and fifth and tailing from the sixth. Speed, 190 to 195 r.p.m.; vertical-stroke was varied by the operator between ⅜ and ¾ in.; horizontal, ¾ in.; depth of ragging, 3 in. The feed was the product between 9- and 2-mm. round-hole trommels. At the beginning of the testing work the results on 400 tons per day of original feed were as

Table 30. Hancock jig, Bonne Terre mill. (*After Rabling*)

Material	Before changes		After changes	
	Tons per 24 hr. total	Assay, per cent. Pb	Tons per 24 hr. total	Assay, per cent. Pb
Feed, original	400	....	423	....
Feed, total	750	4.06	505	4.74
Concentrate	16.5	70.00	7.7	75.00
Middling	475	3.50	195	8.16
Tailing	258.5	0.90	302.3	0.74

shown in Table 30. A sizing-sorting-assay test of the feed is given in Table 31. Improvement in results was effected principally by change in the system of screens used, with resulting change in the character of the bedding. The final screen system is shown in Table 32 and final results in Table 30. Sizing-sorting-assay test on a typical set of products is given

Table 31. Sizing-sorting-assay test on Hancock-jig feed, Bonne Terre mill.
(*After Rabling*)

Mesh	Per cent. weight	Assay, per cent. lead	Per cent. of total lead content	Per cent. of weight on each mesh			Per cent. of total weight		
				Free galena	Mid-dling	Free gangue	Free galena	Mid-dling	Free gangue
On 3.......	3.1	3.14	2.40	....	34.75	65.25	....	1.078	2.022
On 4.......	11.9	3.41	10.00	0.16	28.00	71.84	0.019	3.330	8.551
On 6.......	17.6	3.66	15.91	0.26	25.86	73.88	0.046	4.550	13.004
On 8.......	26.0	3.85	24.69	0.78	22.57	76.65	0.203	5.870	19.927
On 10......	24.7	4.73	28.84	1.02	20.69	78.29	0.252	5.110	19.338
On 14......	12.4	4.62	14.13	1.32	18.81	79.87	0.164	2.335	9.901
Through 14.	4.3	3.80	4.03	1.80	12.20	86.00	0.077	0.525	3.698
Total......	100.0	4.06	100.00	0.76	22.80	76.44	0.761	22.798	76.441

Mesh	Assay, per cent. of lead			Per cent. of lead on each mesh			Per cent. of total lead content		
	Free galena	Mid-dling	Free gangue	Free galena	Mid-dling	Free gangue	Free galena	Mid-dling	Free gangue
On 3.......	....	8.64	0.22	....	95.38	4.62	....	2.290	0.111
On 4.......	86.8	11.22	0.18	4.06	92.14	3.80	0.407	9.213	0.380
On 6.......	86.7	12.62	0.24	6.14	89.02	4.84	0.976	14.165	0.769
On 8.......	80.3	13.71	0.17	16.34	80.27	3.39	4.030	19.819	0.836
On 10......	82.0	18.09	0.20	17.80	78.90	3.30	5.140	22.745	0.955
On 14......	81.6	17.84	0.22	23.40	72.79	3.81	3.300	10.295	0.538
Through 14.	80.5	17.45	0.26	38.15	56.00	5.85	1.537	2.257	0.237
Total......	80.9	14.36	0.20	15.39	80.78	3.83	15.390	80.784	3.826

Table 32. System of screens used on Hancock jig at Bonne Terre mill

4-mm. round hole to rib...	3	3 ribs
5-mm. round hole to rib...	15	12 ribs
7-mm. round hole to rib...	16	1 rib
6-mm. round hole to rib...	22	6 ribs
7-mm. round hole to rib...	30	8 ribs
8-mm. round hole to rib...	31	1 rib
6-mm. round hole to rib...	41	10 ribs
9-mm. round hole to rib...	43	2 ribs
5-mm. round hole to rib...	End	3 ribs

in Table 33. In regular operations in this mill in 1919 the screens were as follows: sheet steel, 4-mm. aperture on compartment No. 1, 5-mm. on No. 2 with a row of 7-mm. holes at the tailing end, 6-mm. on No. 3, 7-mm. on No. 4 with one row of 8-mm. holes at the end, 9-mm. on No. 5 with 2 rows of 10-mm. holes 3 rows back from the discharge end. Life of screen was about 6 weeks for 4-mm., 7 weeks for 5-mm., 8 weeks for 6-mm., 9 weeks for 7-mm. and 12 weeks for 9-mm. A 5-in. bed was carried. Power consumption, 5 hp. at 195 strokes per min.; ¾-in. horizontal and ⅝-in. vertical throw. Capacity, 375 tons new ore per 24 hr. One man attended two machines. Lost time was less than 0.1 per cent., principally due to broken rocker-arm shafts. Water consumption, 500 gal. per min. Changes in length of stroke, character of bed and water quantity were left to the operator. Sizing test of feed as in Table 31. Feed contained 12 per cent. moisture. Assays, per cent. Pb; feed, 2.5; tailing, 0.75; concentrate from first hutch, 75.0; second hutch, 65; third and fourth combined, 20; fifth, 2. At the RIVERMINES MILL of the same company a 5-compartment jig.

Table 33. Sizing-sorting-assay test on Hancock-jig products, Bonne Terre mill

Mesh	Per cent. weight	Assay, per cent. of lead	Per cent. of total lead content	Per cent. of weight on each mesh		Per cent. of total weight		Assay, per cent. of lead		Per cent. of lead on each mesh		Per cent. of total lead content	
				Free galena	Middling	Free galena	Middling	Free galena	Middling	Free galena	Middling	Free galena	Middling
First hutch													
On 3	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
On 4	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
On 6	15.2	80.3	15.29	63.0	37.0	9.58	5.62	82.1	76.9	64.50	35.50	9.87	5.42
On 8	30.5	78.6	30.08	67.2	32.8	20.50	10.00	82.7	70.4	70.65	29.35	21.25	8.83
On 10	28.6	79.7	28.60	77.2	22.8	22.10	6.50	81.7	71.3	79.62	20.38	22.78	5.82
On 14	14.8	81.2	15.09	88.3	11.7	13.07	1.73	83.1	66.7	90.42	9.58	13.65	1.44
Through 14	10.9	79.8	10.94	87.8	12.2	9.57	1.33	82.0	65.1	90.00	10.00	9.85	1.09
Total	100.0	79.7	100.00	74.8	25.2	74.82	25.18	82.5	71.6	77.40	22.60	77.40	22.60
Second hutch													
On 3	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
On 4	8.0	68.2	7.60	12.0	88.0	0.96	7.04	79.0	66.8	13.90	86.10	1.06	6.54
On 6	16.1	68.9	15.45	26.0	74.0	4.18	11.92	80.2	65.1	30.15	69.85	4.63	10.82
On 8	14.9	71.3	14.80	41.0	59.0	6.11	8.79	80.6	65.0	46.30	53.70	6.86	7.94
On 10	18.9	69.5	18.30	51.3	48.7	9.70	9.20	81.7	56.8	60.20	39.80	11.02	7.28
On 14	20.9	74.7	21.75	77.2	22.8	16.15	4.75	80.9	53.4	83.75	16.25	18.22	3.53
Through 14	21.2	74.7	22.10	80.1	19.9	16.98	4.22	80.0	53.2	85.85	14.15	18.98	3.12
Total	100.0	71.8	100.00	54.1	45.9	54.08	45.92	80.7	61.3	60.77	39.23	60.77	39.23
Third hutch													
On 3	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
On 4	5.4	59.9	5.54	.....	100.0	.....	5.40	.....	59.9	.....	100.00	.....	5.54
On 6	10.0	58.4	9.98	.....	100.0	.....	10.00	.....	58.4	.....	100.00	.....	9.98
On 8	14.9	59.8	15.25	.....	100.0	.....	14.90	.....	59.8	.....	100.00	.....	15.25
On 10	23.4	59.2	23.68	2.3	97.7	0.54	22.86	80.1	58.7	3.12	96.88	0.74	22.94
On 14	23.7	63.3	25.65	29.8	70.2	7.06	16.64	79.3	56.5	37.30	62.70	9.57	16.08
Through 14	22.6	51.4	19.90	32.0	68.0	7.22	15.38	79.8	38.2	49.60	50.40	9.88	10.02
Total	100.0	58.5	100.00	14.8	85.2	14.82	85.18	79.7	54.8	20.19	79.81	20.19	79.81
Total conc.	100.0	74.4	.	64.4	35.6	.....	.....	82.3	60.4	.....	.....	71.1	28.9

Table 33. Sizing-sorting-assay test on Hancock-jig products, Bonne Terre mill—*Continued*

Mesh	Per cent. weight	Assay, per cent. of lead	Per cent. of total lead content	Per cent. of weight on each mesh		Per cent. of total weight		Assay, per cent. of lead		Per cent. of lead on each mesh		Per cent. of total lead content	
				Middling	Free gangue	Middling	Free gangue	Middling	Free gangue	Middling	Free gangue	Middling	Free gangue
Fourth hutch													
On 3	......	......	......	......	......	......	......	......	......	......	......	......	......
On 4	2.2	37.0	3.30	100.0	......	2.20	......	37.0	......	100.00	......	3.30	......
On 6	5.1	29.1	6.01	100.0	......	5.10	......	29.1	......	100.00	......	6.01	......
On 8	22.9	27.1	25.14	96.2	3.8	22.03	0.87	28.2	0.18	99.975	0.025	25.14	0.006
On 10	44.1	22.6	40.22	96.6	3.4	42.59	1.51	23.4	0.20	99.966	0.034	40.21	0.014
On 14	19.7	25.2	20.13	94.7	5.3	18.66	1.04	26.6	0.24	99.950	0.050	20.12	0.010
Through 14	6.0	21.4	5.20	89.0	11.0	5.34	0.66	24.0	0.36	99.815	0.185	5.19	0.009
Total	100.0	24.7	100.00	95.9	4.1	95.92	4.08	25.7	0.23	99.961	0.039	99.96	0.039
Fifth hutch													
On 3	2.0	1.15	1.16	46.0	54.0	0.92	1.08	2.32	0.15	93.05	6.95	1.08	0.08
On 4	12.2	1.63	9.95	36.1	63.9	4.40	7.80	4.07	0.24	90.49	9.51	9.01	0.94
On 6	15.8	1.56	12.38	30.5	69.5	4.80	11.00	4.66	0.20	91.07	8.93	11.27	1.11
On 8	27.6	2.20	30.47	26.0	74.0	7.15	20.45	7.96	0.18	93.94	6.06	28.62	1.85
On 10	24.4	2.36	28.90	27.4	72.6	6.67	17.73	7.97	0.25	92.37	7.63	26.68	2.22
On 14	13.1	2.04	13.42	22.7	77.3	2.96	10.14	8.25	0.22	91.66	8.34	12.30	1.12
Through 14	4.9	1.67	3.72	18.4	81.6	0.90	4.00	7.42	0.18	90.28	9.72	3.35	0.37
Total	100.0	2.00	100.00	27.8	72.2	27.80	72.20	6.60	0.21	92.31	7.69	92.31	7.69
Tailing													
On 3	4.6	1.04	7.26	32.8	67.2	1.510	3.090	2.92	0.13	91.67	8.33	6.665	0.605
On 4	18.6	0.67	18.90	19.3	80.7	3.585	15.015	3.07	0.10	88.00	12.00	16.630	2.270
On 6	21.4	0.60	20.70	14.8	85.2	3.165	18.235	3.84	0.08	89.20	10.80	18.490	2.210
On 8	24.4	0.63	23.23	10.9	89.1	2.660	21.740	5.12	0.08	88.67	11.33	20.625	2.605
On 10	17.4	0.63	16.74	8.8	91.2	1.530	15.870	6.60	0.06	91.40	8.60	15.300	1.440
On 14	9.3	0.69	9.70	6.6	93.4	0.615	8.685	6.24	0.30	59.30	40.70	5.750	3.950
Through 14	4.3	0.53	3.47	7.8	92.2	0.335	3.965	4.60	0.20	65.50	34.50	2.280	1.190
Total	100.0	0.66	100.00	13.4	86.6	13.400	86.600	4.22	0.11	85.73	14.27	85.730	14.270

Table 34. Performance of Hancock jigs, Morning mill

Tons per 24 hr.									
Date, April, 1917	10	11	12	13	14	16	17	18	Average
½ mill feed	614	611	582	519	618	655	621	591	601
Coarse-jig feed	275	262	231	255	281	348	349	309	289
Coarse-jig tailing	88	57	77	34	68	39	28	49	55
Coarse middling	172	190	139	206	198	294	306	245	219
Fine-jig feed	359	308	264	350	358	381	451	365	354
Fine-jig tailing	64	45	40	73	57	115	66	40	62
Fine middling	287	255	216	269	293	258	377	317	285
Combined tailing	152	102	117	107	125	154	94	89	117

Assays, per cent.																	
	Pb	Zn	Pb	Zn	Pb	Zn	Pb	Zn	Pb	Zn	Pb	Zn	Pb	Zn	Pb	Zn	
Mill feed	7.7	4.8	7.5	5.3	6.9	5.1	7.2	5.2	7.0	5.5	6.7	5.0	6.7	4.4	6.5	4.5	......
Coarse-jig feed	6.4	3.9	7.8	4.7	6.9	4.7	6.3	3.8	5.5	4.2	6.0	4.2	5.0	4.2	7.2	4.0	......
Coarse-jig tailing	1.8	1.8	1.0	0.8	1.0	1.2	1.0	0.9	1.0	1.0	0.8	0.7	0.9	0.8	1.0	1.0	......
Fine-jig feed	7.1	4.6	6.1	4.8	7.1	5.0	6.9	4.8	5.6	4.5	5.7	4.3	5.2	4.4	8.2	4.4	......
Fine-jig tailing	1.6	1.6	1.2	1.1	1.2	1.5	1.3	1.5	1.8	1.9	1.4	2.0	1.6	1.6	1.3	1.6	......
Coarse-jig hutch No. 1	70.2	3.4	71.2	3.2	68.0	3.2	66.5	3.8	66.2	3.7	62.0	3.5	68.0	3.6	66.5	3.4	......
Coarse-jig hutch No. 2	61.8	4.9	60.2	5.1	62.0	4.9	58.4	4.3	56.9	4.0	61.1	3.8	59.0	3.7	60.1	3.6	......
Coarse-jig hutch No. 3	40.3	8.1	40.2	9.0	40.0	7.9	37.2	8.2	37.6	8.4	32.8	8.1	31.8	8.4	33.1	9.1	......
Coarse-jig hutch No. 4	15.8	8.0	14.6	8.2	13.6	8.3	12.0	8.8	11.4	7.4	9.5	7.0	8.6	6.0	7.5	6.1	......
Coarse-jig hutch No. 5	4.6	3.9	3.5	2.5	4.8	4.6	3.2	1.8	3.2	3.4	3.7	2.3	1.8	1.5	2.9	2.0	......
Fine-jig hutch No. 1	72.8	3.1	74.6	2.5	70.6	2.8	71.0	3.0	72.4	2.9	73.9	2.7	67.6	2.9	69.2	2.6	......
Fine-jig hutch No. 2	40.3	8.1	74.0	3.0	71.2	3.4	74.0	3.3	73.4	3.2	70.4	3.2	63.8	3.2	64.3	3.0	......
Fine-jig hutch No. 3	66.4	4.0	68.0	3.7	57.0	5.8	56.2	5.6	57.1	6.0	57.9	6.2	50.4	6.6	48.7	6.9	......
Fine-jig hutch No. 4	37.6	9.4	48.2	7.2	39.6	9.0	33.6	9.1	34.0	10.8	27.0	9.7	19.6	10.2	12.1	10.0	......
Fine-jig hutch No. 5	5.8	5.4	6.4	5.4	4.9	5.5	4.6	4.1	5.9	7.2	4.5	5.0	3.6	3.0	3.5	3.6	......

3 × 25 ft., was fitted with punched soft-steel plates, 3 × 4 ft., with 5-mm. apertures on the first and second compartments, 6-mm. on the 3rd, 7-mm. on the 4th, and 8-mm. on the 5th. Life of screens was 30 days. Feed was all through 9-mm. and 10 per cent. through 10-mesh, and contained 40 per cent. moisture. 850 tons per day was treated including circulating load. Water consumption, 300 gal. per min. 3-in. bed. 190 strokes per min. 5 hp. Three machines per man. Lost time, due principally to changing screens, 2 per cent. Regulation of water and stroke were left to operator. Assays, per cent. Pb.: Feed 3.0; tailing, 0.5; concentrate, 70; middling, 4. At FEDERAL LEAD CO. MILL NO. 3, a 25-ft., 6-compartment jig with screen frame 32 in. wide was fitted with the following screens: First compartment, 3 ft. 9 in. long, 5-mesh brass wire; second and third, 3 ft. 1 in. each, 4-mesh brass wire; fourth, 3 ft. 6 in., 12-mm. punched-steel plate; fifth, 4 ft. 7 in., 9-mm. plate. Life of screens was 5 to 6 months. Feed: on 12-mm., 1.4 per cent.; 10-mm., 3.1 per cent.; 8-mm., 9.2 per cent.; 6-mm., 16.2 per cent.; 4-mm., 17.9 per cent.; 2-mm., 29.3 per cent; through 2-mm., 22.9 per cent., containing 43 per cent. moisture. Feed rate, 400 to 500 tons per day. 195 strokes per min., ¾-in. vertical and ⅝-in. horizontal throw. 4-in. bed. Water, 190 gal. per min. 5 hp. Two machines per man. Stroke regulation left to operator. 3 to 4 per cent. lost time, due to cleaning and changing screens and general repairs. Assays, per cent. Pb: Feed, 3; tailing, 0.65; concentrate, 70; middling, 3.5. At FEDERAL MINING AND SMELTING CO., Morning mill, standard 25-ft. Hancock jigs were used on both coarse and fine feeds. On the "coarse" jig the feed was all through 12-mm., 66 per cent. on 4-mesh, 26.6 per cent. on 6-mesh and 7.4 per cent. on 16-mesh, with 53 per cent. moisture. Screens were 5-, 7-, 9-, 12-, and 14-mm. round-hole punched-steel plates on compartments 1 to 5 respectively. Life of screens, 75 days. 6-in. bed was carried. 180 strokes per min., length variable. 5 hp. One man attended 2 jigs and 4 sets of rolls. Lost time, 0.05 per cent., due principally to changing screens. Length of stroke, quantity of water and character of bedding were regulated by operator. Results for April, 1917, are given in Table 34. "Fine" feed was all through 4-mesh; 9.6 per cent. on 6-mesh, 63 per cent. on 16-mesh and 27.4 per cent. through 16-mesh; 45 per cent. moisture. Screens, punched-steel plate, 3-, 3-, 3-, 5- and 7-mm. round holes on compartments 1 to 5 respectively, life about 75 days. 180 strokes per min. 5 hp. Attendance and lost time as above. See Table 34 for operations during April, 1917. ANACONDA ran extensive tests before replacing Evans jigs by Hancock jigs. The work was done on a 25-ft. jig with tray 20 ft. × 2 ft. 8 in. Speed, 190 to

Table 35. Hancock vs. Evans jigs, Anaconda Copper Co. (*46 A 217*)

	Natural feed, −8-mm. round-hole		Sized feed, −8-mm. +2.5-mm.	Classified feed, 11-mm. to 0.25-mm. quartz	Evans jig system (*c*)
	Low tons	High tons			
Tons of feed per 24 hr.:					
Average	420	865	545	480	430
Maximum	450	980	....	....	480
Minimum	400	750	....	....	420
Feed, per cent. Cu	3.31	3.43	2.71	2.95	3.38
Concentrate, per cent. Cu	9.00	9.15	13.90	9.58	10.5
Concentrate, per cent. insol. ($SiO_2 + Al_2O_3$)	30.7	25.8	18.7	23.8	12.3
Middling, per cent. Cu	1.56	2.04	1.46	1.10	1.70
Recovery, per cent.	59.2	49.6*a*	51.6*d*	58.7	48.7
Machines displaced by Hancock jig(*c*):					
Evans jig compartments	32*c*	64	64	72	....
Evans classifiers	2	4	0	3	....
Trommels, 3×6-ft.	4	8	4	4	....
Screen area, Hancock, square inches	6690	6690	6690	5500	....
Screen area, Evans retired, square inches	36,500	73,000	48,670	54,720	....
Water consumed, gallons per ton	1530*b*	....	350	515	3500

a Feed rate too great for efficient treatment. *b* High, on account of back water necessary to keep fine sand and slime out of concentrate. *c* 8 @ 2-compartment Evans jigs treating 8- to 5-mm. feed, same on 5- to 2.5-mm. feed; 2 @ 4-spigot classifiers on −2.5-mm. material; 8 @ 2-compartment Evans jigs on classifier spigot products; 2 @ 5-mm. and 2 @ 2.5-mm. trommels. *d* Lower than "natural" low tons on account of lower grade of feed.

Table 36. Comparison of Hancock, Evans and Woodbury jig systems at Anaconda. Feed all through 8-mm. screen

System	Number of tons averaged	Number of machines				Screen area, square feet	
		Jigs	Trommels			Jigs	Trommels
			8-mm.	5-mm.	2.5-mm.		
Evans	9	24	2	2	2	254	339
	11	12	1	1	1	127	170
	10	12	1	1	1	127	170
Hancock	6	1	2	0	0	46	113
	3	1	2	0	0	46	113
	9	1	2	0	0	46	113
	9	1	4	0	0	46	226
Woodbury	11	5	1	0	0	35	56
	10	5	1	0	0	35	56

System	Number of tons averaged	Tons of feed			Water, gallons	
		Per 24 hr.	Per square foot of jig screen per 24 hr.	Per square foot of floor space per 24 hr.	Per ton of ore	Per square foot of jig screen per 24 hr.
Evans	9	446	1.76	0.61	3457	6064
	11	190	1.50	0.52	....	....
	10	228	1.80	0.63	....	....
Hancock	6	411	8.94	3.70	884	7899
	3	414	9.00	3.73	979	8818
	9	408	8.87	3.68	919	8145
	9	756	16.43	6.81	569	9339
Woodbury	11	221	6.31	2.19	1502	9582
	10	223	6.37	2.20	1348	8599

System	Number of tons averaged	Assay, per cent. Cu				
		Feed	Concentrate	Middling	Slime	Tailing
Evans	9	3.27	10.50	1.70	3.49	0.97
	11	3.29	*a*	*a*	3.29	0.69
	10	2.89	*a*	*a*	3.18	0.79
Hancock	6	3.25	10.30	1.72	3.24	*b*
	3	3.40	8.10	1.62	3.55	*b*
	9	3.34	10.60	1.84	3.39	*b*
	9	3.58	9.15	2.49	3.63	*b*
Woodbury	11	3.01	9.69	1.65	3.53	0.76
	10	3.07	9.90	1.89	3.40	0.84

a No assays. *b* None made.

195 r.p.m., vertical stroke ⅜- to ¾-in.; horizontal stroke, variable. No tailing was made. Concentrate was drawn from the first three hutches, middling from the fourth hutch was circulated, middling from the fifth and sixth hutches was re-ground. Results are given in Table 35. Further data from ANACONDA presenting the results of competitive tests between Hancock, Evans and Woodbury jigs are given in Table 36.

Franz jig (*13 CME 599*) is of the movable-sieve type with a screen 2 ft. 3 in. by 10 ft. (long). For jigging coarse (+2-mesh) lead-zinc ores screen was round-hole punched plate with 5⁄16-in. holes in first 2 ft., ½-in. holes for next 3 ft., ⅝-in. holes from 5 to 7.5 ft. and ¾-in. holes to end. Fine jig (−2-mesh, deslimed) used equal lengths of 3⁄16-, ¼-, 5⁄16- and ⅜-in. plate. Both fine and coarse jigs were run at 240 r.p.m., the fine with ¾-in. strokes and the coarse 1- to 1¼-in. Capacities: 120 tons per day, coarse; 80 tons, fine. The first three hutches of the coarse jig averaged 30 per cent. Pb; the first hutch of the fine jig, 35 per cent.

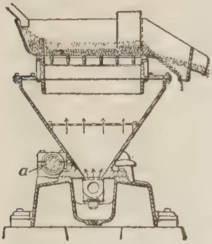


FIG. 22.—James jig.

James jig (Fig. 22), for metallic ores, consists of a movable sieve, usually less than 30 × 36 in., supported by means of a rubber diaphragm and two spring connecting rods on the drive shaft (*a*).

At NORTH RIVER GARNET CO. (*118 J 529*) the machines are run at 225 strokes per min.; the bed is about 3½ in. thick; screens are 6- to 30-mesh. Garnet is separated from hornblende and feldspar. Capacity is 10 to 50 tons per machine per 20 hr. depending on size of feed. About 2 hp. is consumed per sieve.

Hooper vanning jig (Fig. 23) consists of a shallow rectangular screen-bottomed tray open at one end, except for a shallow (1-in.) shoulder, suspended at the open end from a fixed shaft and at the closed end by an eccentric rod. The tray bottom slopes downward slightly toward the open end. A hopper-bottomed tank below the tray is filled with water to such a depth that the bed on the screen is submerged. The machine is run 240 to 330 @ 1- to 0.5-in. strokes, according to the fineness of the feed. Concentrate must be skimmed off the screen by hand. Hutch concentrate is discharged by spigot or by drag belt.

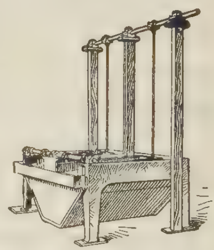


FIG. 23.—Hooper vanning jig.

This jig was developed at the NORTH RIVER GARNET CO. for separating garnet from hornblende and feldspar and has been successful in this service. (See Sec. 2.)

8. Movable-sieve (pan) jigs used in coal preparation

This type of jig is used widely in bituminous-coal cleaning but to a much less extent for anthracite. The best known are the Stewart, Shannon, Wilmot and James. They are used almost entirely for unsized feeds, 3½-in. or less maximum size, and, when the extent of overloading to which they are subjected is considered, do remarkable work. Hancock (*Iron making in Alabama, Geol. Surv. of Ala., 1912*) says that, in general, the pan-type jig is best for coarse bituminous-coal feeds and the piston type for fine.

Stewart jig (Fig. 24) is of the movable-sieve type with sieve basket suspended in a tank of water from two eccentrics. It has been used widely in coal treatment. An apron attached to the basket carries the upper or clean-

coal layer over the edge of the jig tank while the lower or refuse layer falls over the edge of the basket into the tank and is removed by mechanical means. The jig treats unsized feed. Water is supplied from a tank at the back of the jig through a flap valve that closes on the down stroke of the basket. The pan is always made 7 ft. long, width is variable.

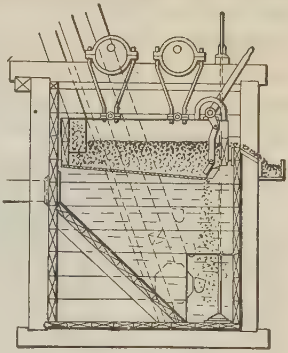


FIG. 24.—Stewart jig.

At twelve Illinois washeries reported by Lincoln (*Bul. 11 UI No. 9*) pan widths varied from 3½ to 6 ft. The feed rate on −3-in. coal ranged from 5 to 10 tons per ft. of width per hour and averaged 8 tons. On −1⅛-in. coal the capacity was about 7 tons per ft. of width per hour. The jigs were run 35 to 40 @ 4-in. strokes per min. At GRANBY CONSOLIDATED M. S. & P. Co. (*21 CA 367*) Cassidy, B. C., the jigs are 2-compartment, series type. Feed is 40 tons of −2-in. bituminous-coal screenings per hour. The bed in the first compartment is 12 to 14 in. deep and in the second, 10 to 12 in. Screens are 3/16-in. mesh. Hutch product is principally fine rock and clay. Performance is shown in Table 37.

Table 37. Performance of Stewart jig at Granby Cons. M. S. & P. Co. (*After Garman*)

Material	Ash, per cent.	Float		Sink	
		Per cent. weight	Per cent. ash	Per cent. weight	Per cent. ash
Feed, −2-in. raw coal........	22.80	56.0	9.38	44.0	43.43
Washed coal................	14.30	77.7	10.16	22.3	26.68
Refuse....................	67.05	2.5	16.00	97.5	68.16

Shannon jig is similar to the Stewart, but with the sides of the pan extended nearly to the bottom of the hutch in order to attain uniform rising current.

The average capacity of 4 × 6-ft. jigs at three Illinois washeries (*Bul. 11 UI No. 9*) was 47 tons of slack coal per hour, range 40 to 62½ tons. Speeds were 72 to 90 @ 3 to 3½-in. strokes per min. At PACIFIC COAL Co., Issaquah, Wash. (*28 UW 126*) the performance and the effect of change in length of stroke were studied. Results are given in Table 38. Determination of which of these results is best depends on the prices paid for coal of different ash contents. At this plant 15 per cent. ash was the standard aimed at, and on this basis test No. 3 would seem to be the best, since the slightly greater ash content of the cleaned coal as compared with test No. 4 is not enough to overcome the advantages of higher yield and higher recovery of combustible.

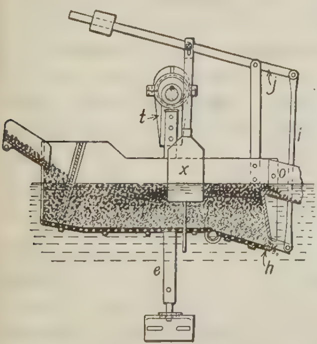


FIG. 25.—Wilmot jig.

American jig is similar to the Stewart, but the hutch water is pumped in and the water-inlet system is designed to reduce suction to a minimum. Baskets are usually 4 ft. wide and 6 to 7 ft. long. Speed in five Illinois washeries (*Bul. 11 UI No. 9*) treating unsized slack ranged from 35 to 50 @ 4-in. strokes per min., average 41. Capacity ranged from 9 to 12.5 tons of feed per foot of width per hr.

Wilmot pan jig (Fig. 25) consists of a sieve box suspended in a V-shaped

Table 38. Performances of Shannon jig at Pacific Coast Coal Co., treating −2½ +1-in. bituminous coal. (*After McMillan and Bird*)

Test number	1		2		3		4	
Stroke, in.	2⅜		2⅜		3⅜		3⅞	
Product	Weight, per cent.	Ash, per cent.	Weight, per cent.	Ash, per cent.	Weight, per cent.	Ash, per cent.	Weight, per cent.	Ash, per cent.
Feed	100.0	23.4	100.0	25.1	100.0	24.4	100.0	28.6
Float(*a*)	80.7	14.9	78.3	15.4	78.8	14.6	71.2	15.3
Sink	19.3	63.2	21.7	60.4	21.2	61.1	28.8	63.3
Washed coal	100.0	22.5	100.0	22.0	100.0	17.6	100.0	16.7
Float(*a*)	81.4	14.2	82.7	14.4	90.7	14.4	92.1	13.3
Sink	18.6	61.6	17.3	58.3	9.3	48.8	7.9	48.3
Refuse	100.0	34.1	100.0	35.2	100.0	57.1	100.0	59.6
Float(*a*)	61.1	16.0	60.6	15.6	22.0	16.3	24.8	21.3
Sink	38.9	64.1	39.4	65.0	78.0	68.7	75.2	68.6
Loss of "float" coal, tons per hour	0.6		3.0		0.9		2.3	
Recovery of combustible, per cent.	93.4		79.8		90.1		84.3	
Reduction in ash, per cent	3.8		12.3		27.8		41.6	
Yield, per cent	92.2		76.5		82.8		72.2	

a 1.55 sp. gr.

tank by straps (*e*) from two eccentrics (*t*), one at either side of the jig frame. Coal overflows at chute (*o*). The jig grate is hinged near the forward end and the hinged section (*h*) is connected by rods (*i*) and (*j*) to the float (*x*), which gives automatic control of slate discharge. The usual speed is 130 to 150 @ 1½- to 1¾-in. strokes per min. in treating anthracite. Grate apertures are 5/16 to ⅜ in. for pea and ½ to 9/16 in. for egg.

Performance on anthracite is shown in Table 39.

Table 39. Tests on Wilmot pan jig. (*After Ashmead*)

Breaker	Feed			Coal product			Refuse		
	Per cent. coal	Per cent. slate	Per cent. bone	Per cent. coal	Per cent. slate	Per cent. bone	Per cent. coal	Per cent. bone	Per cent. slate
Thomas	20	80	......	68–80	10–14	10–18	0.3–0.4	0.3–0.5	99.1–99.4
Enterprise (stove)	30	70	......	96.5	2	1.5	0.5	......	99.5
Enterprise (nut)	50	50	......	94	3	3	0.5	......	99.5
Enterprise (nut)	25	75	......	94	3	3	0.5	......	99.5
Lehigh Valley Coal Co.	50	44	6	95.25	2.25	2.5	3.75	......	96.25

James coal jig (Fig. 26) is a modification of the James ore jig. The sieve is moved by a crank mechanism. The gate-and-dam method is used for discharging slate; coal is discharged by overflow. Valves are provided under

the sieves to lessen suction. High water level is maintained and water consumption kept down by discharging into tanks made integral with the jig tank, from which discharged material is removed by scrapers. The gate-and-dam discharge limits use to fine sizes.

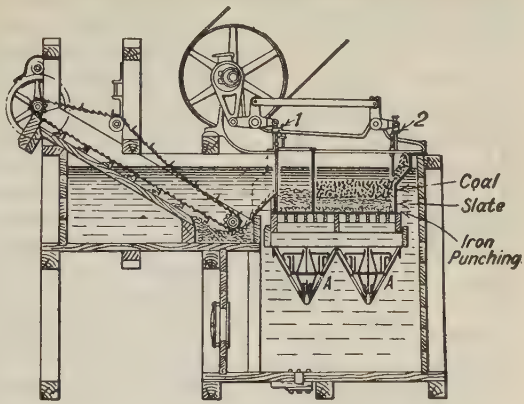


FIG. 26.—James coal jig.

Performance at LOCUST MOUNTAIN COAL Co. on barley-size anthracite ($-\frac{3}{16}$ $+\frac{1}{16}$-in. round hole) is reported by McNally (*18 CA 1089*). He found that the washed coal was cleanest (about 8 per cent. sink in 1.7-sp. gr. solution) at a feed rate of about 14 tons per hour while the refuse contained the least float at about 19 tons per hr. He also found that there was a straight-line increase in the ash content of the washed coal from 14 per cent. to 18 per cent. as the ash content of the feed increased from 16 per cent. to 37 per cent. During these tests the jig was run at 90 @ 2-in. strokes per min. Data on one series of tests are given in Table 40.

Table 40. Performance of James coal jig on barley-size anthracite. (*After Ashmead*)

Feed rate, tons per hour	Feed			Coal product			Refuse	
	Per cent. coal	Per cent. slate	Per cent. ash	Per cent. coal	Per cent. slate	Per cent. ash	Per cent. coal	Per cent. slate
13.3	78.5	21.5	20.1	90.5	9.5	13.0	8.0	92.0
14.0	80.5	19.5	20.4	91.5	9.0	13.5	7.5	92.5
19.3	72.5	27.5	24.0	83.0	17.0	17.1	4.0	96.0

Feldspar jig is any type of coal jig bedded with crushed, sized feldspar for the purpose of jigging refuse through the bed into the hutch. Maximum size of feed is usually between ½- and ¾-in. Apertures in jig screens usually run from ¾ in. upward and size of pieces of cleaved feldspar used for bedding is about four times that of the largest pieces of refuse to be removed. At one plant (*59 A 409*) a jig handling -10 $+6$-mm. coal had 12-mm. screen and 30-mm. feldspar.

9. Hand jigging

Hand jigging is an indispensable operation in preliminary field testing of ores, ranking with panning and vanning; it is frequently practiced in treating ores in prospecting and small-scale development work when it is endeavored to make the mine pay its way; and in certain districts, notably parts of the mid-continent lead-zinc field, it is established as the only method of concentration on many of the small properties.

Equipment and operation. For field testing any small sieve such as a testing sieve, a bucket or tub of water, and small metal scraper or skimmer, are

all that are necessary. The screen is filled one-half to three-quarters its depth with the material to be tested, then, after careful submergence and thorough wetting, is held firmly in two hands with bottom horizontal and top just not submerged and moved up and down in such a fashion as will bring the material into partial suspension on the down stroke and allow it to settle back on the up stroke. This requires an accelerated down stroke and retarded up stroke. The usual speed is 60 to 100 strokes per min. After a small number of strokes, depending on the size of particles and relative specific gravities of the components of the bed, the impoverished surface layer may be scraped off and new feed added and the process repeated until the top of the middling layer becomes so high as to leave insufficient room for new feed. This layer is then scraped off and set to one side and con-

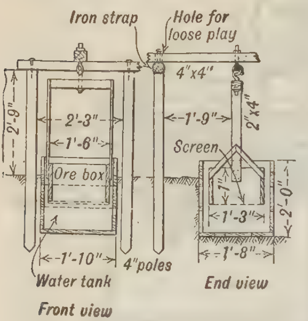


Fig. 27.—Simple hand jig.

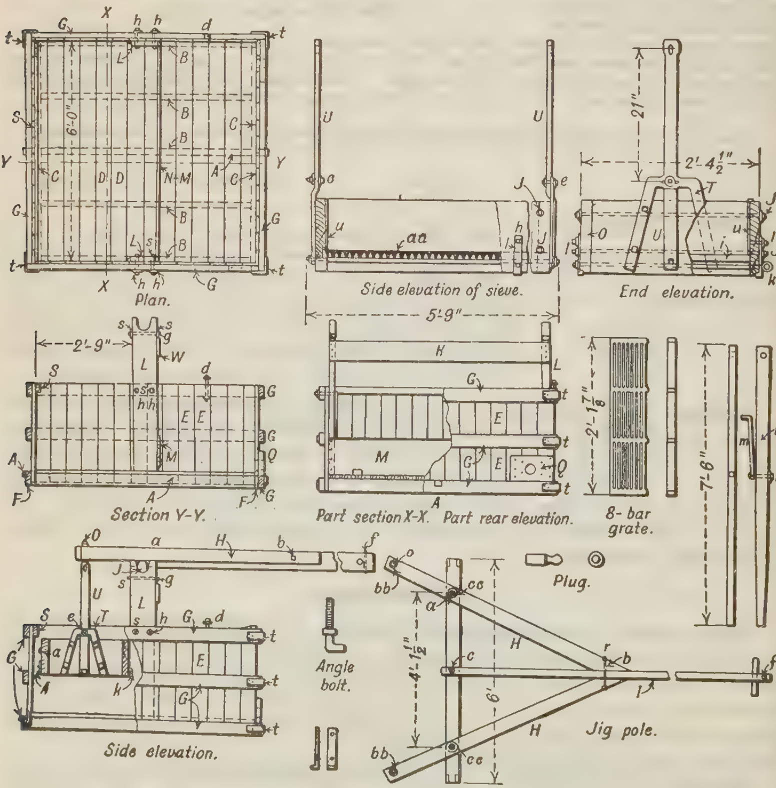


Fig. 28.—Joplin hand jig.

centrate is removed from the screen. Middling may then be put back to form the bed for further operations. The fine material that passes through the sieve and overflows the top is collected separately. If the jig screen is fine, this material is best cleaned up in a pan or plaque; if the screen is coarse, the fines may better first be re-jigged through a bed of coarser concentrate on a fine screen, in this work accenting the start of the up stroke and working with the top of the jig-sieve box always out of water. In this way considerable suction through the bed is induced and fine heavy mineral is thereby drawn through. Slime may be separated from this concentrate by decantation and be treated separately on a plaque.

A somewhat more elaborate testing jig may be made by suspending a jig sieve, say 12 × 12 in. and 6 to 8 in. deep by means of a rigid stirrup rod about 18 in. long, from a spiral spring of such strength that it is imperceptibly extended by the loaded jig sieve, yet extends readily under a downward pull by the operator and returns readily when the pull ceases.

When any extended operation is contemplated a more elaborate rig is needed. Figs. 27 and 28 show two forms that have been successfully used. The simpler form shown in Fig. 27 (*89 J 1265*) was used at an isolated copper mine in northern Transvaal; the form in Fig. 28 is typical of Joplin practice. This type is best used with a spring board, so arranged that the handle is about waist-high to the operator while jigging. The compartmented box is used to provide a quiet pool for slime settlement. Proportions are such that an 8-in. bed on the sieve, when submerged, just about balances the handle. Capacity of such a jig is from 1 to 3.5 tons feed per sq. ft. per 24 hr.

Elmore (*18 CA 1132*) says that a Joplin-type hand jig with 24 × 66-in. sieve in a 6-ft. × 6-ft × 33-in. tank can be built at not to exceed $60 (1920).

Notes to Fig. 28

Figure	Number of pieces	Material	Figure	Number of pieces	Material
A	1	2×4 in.×6 ft. 6 in.	e	2	½×1¾-in. bolts
B	4	2×4 in.×5 ft. 8½ in.	f	1	⅜×6-in. bolt
C	2	2×4 in.×6 ft.	g	2	⅜×8-in. bolts
D	14	⅞- ×5¼-in.×6-ft. 6-in.	h	4	⅜×4½-in. bolts
		flooring	h_1	10	⅜×2-in. bolts
E	56	⅞- ×5¼-in.×2-ft. 8-in.	i	1	⅝×5 ft. 9-in. rod
		flooring	j	4	⅝×3½-in. rods
F	4	⅞×1 in.×6 ft. 2⅞ in.	k	1	⅝×31½-in. eye-rod
G	12	2×4 in.×6 ft. 3¾ in.	l	2	1¼×⅜×6-in. irons
H	2	4×4 in.×7 ft. 1½ in.	m	1	½×19-in. eye-hook
I	1	2×6 in.×20 ft.	n	1	½×5-in. eye-bolt
J	1	4×6 in.×6 ft.	o	2	¾×2-in. angle bolts
K	1	2×4 in.×7 ft. 6 in.	p	5	⅝-in. cut washers
L	2	2×8 in.×4 ft. 2 in.	r	8	½-in. cut washers
M	1	1×12 in.×6 ft.	s	14	⅜-in. cut washers
N	1	1×6 in.×6 ft.	t	12	Corner irons
O	2	2×12 in.×5 ft. 6 in.	u	2	9½-in.×2-ft. 1-in. No.
P	2	2×12 in.×2 ft. 1 in.			16 sieve liners
Q	1	2×8 in.×14 in.	v	1	10½-in.×5-ft. 4-in. No.
R	1	4×4×10-in. plug			16 sieve liner
S	1	1×2×6 in.	w	1	9½-in.×5-ft. 4½-in. No.
T	2	Angle irons			16 sieve liner
U	2	Hanger irons	4a	8	½-in. cast washers
a	2	⅝×11-in. bolts	aa	10	Grates
b	1	⅝×9-in. bolt	bb	2	¾-in. cast washers
c	1	½×11-in. bolt	cc	4	⅝-in. cast washers
d	1	½×6½-in. bolt			

Performance. At SANTA BARBARA, Chihuahua (*87 J 910*), using a jig of the first type with a 3 × 3-ft. sieve, 0.25-in. aperture for coarse material and 0.12-in. for finer, the results shown in Table 41 were attained in treating 3000 tons of oxidized lead ore. One Mexican laborer made about 0.5 ton of combined screen and hutch concentrate per 9-hr. shift. Thirty jigs were arranged in two rows with a track for a tailing car down the center and room for ready access of wheelbarrows to all jigs. In the JOPLIN, Mo. district (*93 J 1079*) three men operating two rougher jigs and one cleaner jig of the type shown in Fig. 28 will treat 15 to 30 tons of coarsely-disseminated, non-clayey ore per 10-hr. shift. Feed is screened through 1-in. or 1.25-in. apertures; rougher-jig slots, $\frac{5}{8}$- to $\frac{3}{4}$-in., cleaner-jig, $\frac{1}{4}$- to $\frac{3}{8}$-in.; stroke, about 1 in. on rougher and $\frac{1}{2}$ in. on cleaner. If there is much clay in the ore it should be washed in a trough washer (Sec. 8, Art. 12) or the equivalent before jigging. Rougher jigs are run with no bed and each charge, after stratification, is divided by skimming into tailing, middling for sale to the mills, and concentrate. Hutch product collects in the jig box for re-treatment on the cleaner. On the cleaner the operator starts with a 2-in. bed and allows it to accumulate until about 6 in. thick, when it becomes too heavy to handle and is skimmed down to 2 in. again. The hutch product through the thin bed may have to be re-cleaned through the thick bed. Treating $-\frac{3}{4}$-in. WOLFRAMITE ORE with a quartz gangue on a jig such as shown in Fig. 28, using 10-mesh screen, the procedure was to charge 70 lb. of ore, jig for 1 min. (about 150 strokes) with an average stroke length of 1 in., longer at first and shorter at the end, shovel off tailing, then charge another 70-lb. lot and repeat. About 5000 lb. of ore carrying 5 per cent. tungstic acid was treated per day yielding 175 lb. of screen concentrate assaying 65 per cent. tungstic acid, and 200 lb. hutch product assaying 8 per cent. Tailing from the screen assayed 0.7 per cent. In another district, concentrating ferberite from quartz and granite (*101 J 717*); feed, $-\frac{3}{4}$-in., sieves, 2 to 4 sq. ft.; stroke, 0.5 to 3 in., speed about 40 strokes per min., and 2- to 3-in. bed, from 1 to 5 ton per 8-hr. shift was treated, including re-cleaning of concentrate to two grades, viz.: 50 to 63 per cent. tungstic oxide and 25 to 40 per cent. The hutch product was only slightly enriched. Water consumption per jig was about equal to the overflow through a 1-in. pipe. Table 42 shows the performance of a hand jig on BITUMINOUS COAL.

Table 41. Results of hand-jig operation on oxidized lead ore

	Ounces per ton		Per cent.		Value per ton
	Au	Ag	Pb	Cu	
Feed	0.12	8.0	11.6	......	$18.41
Screen conc.	0.17	24.5	34.0	2.0	55.94
Hutch conc.	0.42	12.0	29.0	0.6	43.52
Tailing	0.06	3.0	2.5	......	5.20

Table 42. Performance of a hand jig on Tennessee bituminous coal (*18 CA 1133*)

Product	Proximate analysis, per cent.				
	Ash	Volatile carbon	Fixed carbon	Sulphur	Moisture
No. 1					
Feed, raw coal	26.05	27.72	46.08	0.58	1.49
Washed coal	12.70	31.45	55.70	0.50	2.70
Middling	18.81	29.02	51.02	0.54	5.26
Refuse	59.32	17.25	23.28	0.58	2.12
No. 2					
Feed, raw coal	12.91	30.48	56.46	0.47	1.51
Washed coal	9.66	31.95	58.24	0.47	3.57
Middling	12.68	30.85	56.32	0.52	5.48
Refuse	53.04	21.14	25.67	0.38	3.09

SECTION 10

SHAKING TABLES

Modern shaking tables are concentrating devices that consist of substantially plane surfaces, inclined slightly from the horizontal, shaken with a differential movement in the direction of the long axis, and washed at right-angles to the direction of motion by a thin film of water. The earliest form of shaking table was shaken in the direction of the slope, causing the heavy material to climb against the flow of pulp. Salzburg, Schemnitz, Halley and Gilpin County tables for heavy ores were of this class. All are now obsolete. The simplest and earliest form of typical side-slope table is the Rittinger bumping table (Art. 2). The separating action of a side-slope table is shown diagrammatically in Fig. 1; (*A*) is the feed side, (*B*) the concentrate end, (*E*) the tailing side and (*D*) the head-motion end. The deck tilts as indicated by the arrow, is reciprocated at right-angles to this slope by means of pulley (*P*), eccentric (*X*) and the flexible connecting rod (*R*) and is stopped suddenly at the end of the forward stroke by the bumping block (*G*). Feed is introduced at (*F*), slime flows directly down slope and leaves at (*S*), granular material is moved toward (*C*) by the bumping action and is washed by water introduced at (*W*). Tailing discharges at (*T*), middling at (*M*) and concentrate at (*C*). In modern types of shaking tables longitudinal cleats are fastened to the upper surface of the table deck or riffles are cut into the surface, the resulting riffles in either case being arranged variously. Some makers build the deck in two or more planes slightly inclined to each other. Decks of different makes differ in outline from the rectangular form, and head motions and methods of deck suspension have been greatly improved over the original Rittinger model. With a smooth-surfaced table such as the Rittinger the maximum size of feed that can be efficiently treated is less than 1-mm. For years the maximum size of feed treated on riffled tables was 2.5-mm. and in most mills it was less than this. With some of the present methods of riffling, however, base-metal ores are treated as coarse as 0.25- to 0.37-in. and coal as coarse as 0.5-in. has been handled successfully.

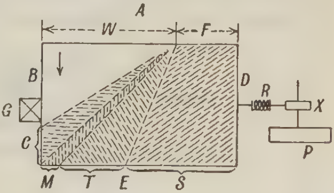


Fig. 1.—Sketch of action of a side-slope bumping table.

1. Principles of shaking-table action

Each particle on the plane inclined surface of a shaking table in operation is acted upon by two forces, *viz.:* (*a*) friction or adhesion between the deck and the particle and, (*b*) pressure of the moving water, and the particle moves in the direction of the resultant of these forces. In all tables the deck starts on its forward stroke at minimum velocity and adhesion between the deck and the particles in contact therewith is sufficient to overcome the inertia of the particles and cause them to move forward with the deck. Velocity accelerates to a maximum at the end of the forward stroke and the deck starts backward at this maximum velocity. Adhesion between particle and deck is insufficient to cause immediate reverse in direction of motion of the particles and they slide forward with the momentum gained on the forward stroke until this momentum is lost by friction against the deck surface. The particles then again move forward with the deck to the end of the forward stroke. The result is intermittent forward travel of each particle in contact with the deck from head-motion end toward concentrate end. If there is a layer of particles several grains deep on the deck, the deck motion is transmitted to the upper layers through the lower, but with some loss, so that the upper layers do not travel lengthwise so rapidly as the lower. If the particles in contact with the deck are of the same size but of different specific gravities, the respective momenta at the end of the forward stroke will be different, the momentum of the heavier being the greater, and the heavier will, therefore, travel further than the lighter before again coming to rest with respect to the table top. This conclusion, which is confirmed by experiment, assumes that the particles are substantially out of contact with the table top during this part of their motion. As soon as they resume contact retardation of the heavier particle is the greater in proportion to its weight (assuming equal coefficients of friction), just as its momentum at the instant of reverse was the greater, and the particles will thereafter be brought to the table velocity in the same time. Large particles of a given specific gravity similarly have more rapid forward travel than small particles of the same specific gravity. If, therefore, (P, Fig. 2), represents particles of heavy and light minerals of the same size, or particles of minerals of the same specific gravity but of different sizes, occupying at the same instinct substantially the same position on the deck of a shaking table, (H_H) may represent vectorially the forward velocity of the heavier or larger particle and (H_L) the lighter or smaller. The forces exerted by the wash water produce film sizing. (See Sec. 8, Art. 14.) If D_H and D_L are taken to represent vectorially the downstream velocities of the heavy and light particles of the same size or the small and large particles of the same specific gravity, respectively, the relative magnitudes of these velocities will be somewhat as shown. The resultant paths of the respective particles under the influences of the two sets of forces acting substantially at right-angles will be R_H and R_L respectively. By inserting some sort of division wall between the points where these paths cut the table edge, separation of particles P_H and P_L is made.

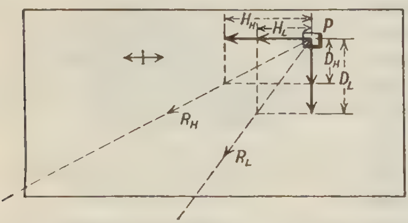


FIG. 2.—Analysis of forces in shaking-table concentration.

When the surface of the table is riffled the riffle grooves are so arranged that they grow shallower toward the concentrate end and in the best known form (Fig. 4) they taper off to nothing along a diagonal line running from the forward end of the feed box to a point near the corner formed by the concentrate end and tailing side. On a riffled table a further important phenomenon occurs that serves to increase capacity markedly above that of a non-riffled surface. As the solid particles carried along by the feed-water current flow out onto the surface of the riffled table, the velocity of the carrying current is lessened by reason of the increase in cross-section of the stream (Fig. 3, *a*) as between cross-section (*A-A*) and cross-section (*B-B*), and the bulk of the solid load settles out in the riffle. Here it is subjected to two sets of forces, both acting to produce segregation by stratification. The shaking motion of the table keeps the mass of particles in a loosened condition, free to move to a certain extent relative to each other, and under this condition the heavier particles and the small particles tend to work to the bottom of the mass. This is a familiar phenomenon that may be observed whenever a mass of grains of different sizes in a container is jarred repeatedly with sufficient force to cause movement of the individuals without sensibly lifting the mass as a whole. At the same time eddying, as indicated by the dotted arrows in Fig. 3, *a*, occurs in the riffles and since the smaller particles of the lighter mineral are more readily lifted by a water current of given velocity than are heavy-mineral particles of the same size, the light-mineral particles do not work down so far into the mass as the heavy. In a short time, therefore, the mass of grains is stratified in the riffles somewhat as is represented diagrammatically in Fig. 3, *b*, with both coarse and fine heavy mineral at the bottom, the fine, however, in the interstices of the coarse, and the light mineral above, more or less regularly arranged in layers with the coarsest on top. As this mass of grains is moved toward the concentrate side of the table the riffle groove in which it travels becomes progressively shallower. The result is the same as though the riffle cleats in Fig. 3, *b*, were successively sliced off at levels (*C-C*), (*D-D*), etc., thus exposing successively deeper layers of the stratified pulp to the cross wash of the water. This accentuates the separation that is effected on a smooth surface, as previously described; the riffles protect the heavy mineral from the action of the cross water, thus allowing a stronger cross-water current to be used, with consequent more rapid removal of waste and increase in tonnage handled. At the termination of the riffle cleats final washing out of the last of the fine gangue is done on a smooth surface, as previously discussed. The finest particles of both heavy mineral and gangue, that will not settle out of the feed-water current, pass directly across the table near the head-motion end and are discharged as slime (see Fig. 1). On an unriffled and to a considerable extent on a riffled table, the solid particles become arranged on the deck in a manner somewhat as indicated in diagrammatic sketch Fig. 3, *c*. Rows (*A*) to (*D*), inclusive, represent grains of con-

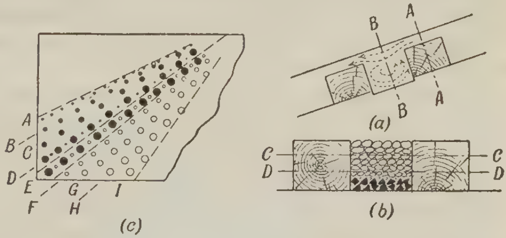


FIG. 3.—Behavior of particles on a diagonally riffled shaking table.

centrate. The grains in rows (*A*), (*B*) and (*C*) are substantially pure heavy mineral (indicated by full black circles); row (*D*) contains small grains of gangue (indicated by open circles) in addition to the large grains of heavy mineral. All very small grains of both heavy and light mineral go off in the slime. Row (*E*) contains the largest grains of pure heavy mineral and some small grains of pure gangue in addition to the large bulk of "mixed" grains, of intermediate specific gravity. Rows (*F*) to (*I*) inclusive contain the larger grains of pure gangue and the lighter grains of middling.

The ideal arrangement just illustrated is modified considerably by the character of the riffling, by irregularities in deck surface and water supply, differences in shape of grains of light- and heavy-mineral particles and of different grains of the same mineral, and by any irregularities in the motion imparted to the table deck.

Richards (*38 A 556*) found in treating "natural" feeds of quartz and galena on a Wilfley table that with −2-mm. feed there was marked retention of quartz between 0.7-mm. and 0.15-mm. sizes in the concentrate, while but little quartz of coarser and finer sizes was retained. With this same feed galena appeared in the tailing in amounts exceeding 1 per cent. at 0.2 mm. and the amount rose to 17.5 per cent. in the −0.08-mm. size. The slime from this run all passed 0.45-mm. and assayed 18.9 per cent. galena; there was no galena in the product on 0.36-mm., but 10 per cent. appeared in the 0.28-mm. size. Middling amounted to 21.5 per cent. of the total feed. The coarsest sizes assayed 75 per cent. galena and there was gradual diminution to a minimum of 3.68 per cent. galena at 0.20-mm. with a marked increase to 32.23 per cent. at 0.08 mm. and further increase to 90.5 per cent. in the −0.08-mm. size. In treating −1-mm., −0.5-mm. and −0.25-mm. feeds the results were similar except that the grade of total concentrate increased with increasing fineness, corresponding to decreased retention of intermediate-sized quartz; grade of tailing correspondingly increased, with accompanying decrease in grade of middling and slime. In every test maximum retention of quartz occurred at a size between one-quarter and one-third that of the largest grain in the feed, indicating that this retention is effected in the interstices of the concentrate layer on the table and is not the result of failure to move the material on a smooth plane. Richards' work likewise shows accordance with the ideal arrangement (Fig. 3, *c*) and the departure therefrom caused by using riffles. The particles in his concentrate are all distinctly smaller than those in his middling and the gangue in the middling is smaller than that in the tailing, but both middling and tailing contained much fine heavy mineral that on an unriffled table would have passed off in the slime and such mineral in the middling was coarser than that in the tailing. As the size of feed decreased and the amount of wash water necessary to remove coarse gangue from the concentrate likewise decreased, the amount of very fine mineral in the concentrate increased with correspondingly decreased tailing loss in the same sizes. The tests on "natural" feed indicate 0.15- to 0.2-mm. as a critical size at which galena losses in quartz tailing become excessive.

Feed. It follows from the foregoing discussion that in order to treat particles on a shaking table they must settle in water at a sufficiently rapid rate to come into contact with the table deck, or the mass of settled solids thereon; and they must have sufficient mass to travel forward, by reason of their inertia, for a short period of time after the table deck starts on its backward travel. In order to effect separation of one class of particles from another they must differ in size or in specific gravity or both. It would appear from Fig. 3, *c* that the most efficient separation of heavy from light grains can be made on a non-riffled surface when the heavy grains are smaller than the light, since small heavy grains and large light grains discharge from such a table the maximum distance apart. The limit of size difference is reached when the particle of heavy mineral is too small to remain in contact with the table top in the cross current required to wash down the large particles of light mineral. This limiting difference is not reached within the size range that it is possible to prepare by the most efficient hydraulic classification and hence the conclusion that classification is the best means of preparation of feed for unriffled shaking tables would seem to be justified. On a riffled table, on the

other hand, fine heavy mineral is protected by the riffle cleats from the cross-water current that removes the coarse gangue; 0.1-mm. galena is readily delivered at A (Fig. 3, c) while 1.0- to 1.5-mm. quartz is washed down to H, I.

2. Types of shaking tables

Tables are called sand or slime tables according to the size of material that they treat, and are classified as roughing or finishing tables according to the character of the service. Sand tables are always riffled with relatively deep riffles, over a majority of the surface at least, with the space between riffle cleats usually not more than ¾ in. to 1¼ in. wide; slime tables, on the other hand, are not riffled so deeply, and the space between riffle cleats is usually much wider than on sand tables in order to form pools of relatively quiescent pulp to induce settlement of solids. Slime tables always have a portion of the deck, at least, unriffled. But any sand table that has an unriffled portion may be used for slime and slime tables may be used for sands, if the settling pools are not too large. Roughing tables are usually riffled full length and the riffles are comparatively deep. These tables are thus enabled to treat large tonnages and yet save fine mineral with the coarse in the form of a low-grade concentrate, at the same time rejecting an impoverished tailing. Finishing tables, with a few exceptions, have an unriffled portion for cleaning out fine gangue from the concentrate streak; the riffles are shallower than on roughing tables, and, in general less resistance is offered to the cross travel of solids than on a rougher. While in some mills the same table with different operating adjustments is used for both roughing and cleaning, this is not, in general, good practice, and it is doubtful whether superior efficiency can be demonstrated for it in any case. Such practice has the advantage of lessening the stock of repair parts that must be carried, with consequent economy, but such saving will rarely compensate for the losses or reduced capacity incident to misuse of tables.

Rittinger bumping table was devised about the middle of the nineteenth century and was used to some extent in Germany. It was a plane-surfaced rectangular side-slope bumping table about 4 ft. long in the direction of motion and 6 to 8 ft. wide. The deck was suspended from overhanging supports by rods of adjustable length, and sloped 3° to 6° in the direction of the long axis. The deck was pushed in one direction by a cam acting against a spring, and, after release from the cam, was pushed smartly in the reverse direction by the spring until stopped by impact against a bumping block. The speed ranged from 70 to 100 @ 1-in. strokes per min. to upwards of 200. Feed was introduced along the short upper side near the end away from the bumping block and wash water was supplied along the remainder of the upper side. Division between concentrate, middling and tailing was made by means of adjustable fingers placed along the lower edge. In later forms two decks, side-by-side, were mounted on the same frame and shaken together. Capacity ranged from 0.75 ton of slime to 2.5 tons of sand feed per deck per 24 hr. Water consumption was excessive, ranging up to 30 tons per ton of feed, due to the fact that all movement of solids in the direction of discharge was effected by the water, the mechanical movement being utilized only to effect separation into bands at right angles to the direction of travel. The table was used as a finishing table for treating fine sand and slime. Low capacity, excessive water consumption and high repair costs combined to prevent its wide introduction and final adoption of the side-slope shaking table in the mills was deferred until about 1895, when the Wilfley table was first placed on the market.

3. Wilfley table

This table (Fig. 4) is a side-slope shaking table riffled over somewhat more than half the surface, the riffles terminating along a diagonal line extending from the forward end of the feed box to the corner formed by the tailing side and concentrate end.

Construction. The deck is built of redwood strips laid flat at 45° to the axes of four longitudinal beams; the whole is carried on two transverse steel trusses, each with two slipper bearings which rest in supporting bearings on the frame.

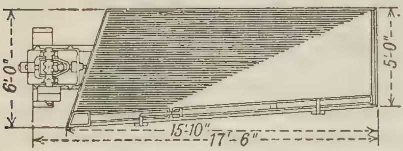


Fig. 4.—Wilfley table (right-hand).

Frame is built of steel or wood. The steel frame consists of a large, rigid through member composed of two 12-in. channels 15 ft. long, spaced 20 in. apart by cast-iron separators. The through member carries two circular machined seats in which rest cast-iron bolster yokes with suitably machined surfaces. The yokes carry cross members built up of lighter channels. In wood-frame tables, a 12 × 16-in. pine timber replaces the longitudinal channels and pressed steel is used for the cross members. The seats for the slipper bearings of the deck are bolted to the transverse deck-frame members. Transverse tilting is effected by power screws engaging bronze tilting nuts in the bolster yokes and actuated by suitable gearing from a hand wheel located at the feed side near the concentrate end. A permanent longitudinal tilt upward toward the concentrate end, ranging from ⅜ in. to 1¼ in. in the length of the table, is provided for; the normal tilt is between ⅜ and ⅝ in. When the longitudinal tilt is too steep, concentrate may refuse to climb and pack in the riffles. *Richards* cites an experience with two-mineral separation in which the lead content of zinc concentrate changed from 2 to 8 per cent. with change in longitudinal tilt from ⅝-in. to ⅜-in.

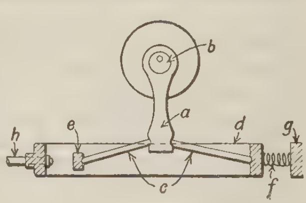


Fig. 5.—Diagram of Wilfley-table head motion.

Head motion is of the pitman-and-toggle type illustrated in outline in Fig. 5. The essential parts are the pitman (*a*), pulley-driven eccentric (*b*), toggles (*c*) and yoke (*d*). The forward toggle abuts against a fixed block (*e*) and lost motion is prevented by compression in spring (*f*) between the yoke and the fixed block (*g*). The yoke is attached to the table deck by rod (*h*). Horizontal movement of the outer end of the right-hand toggle for a given vertical displacement of the inner end is greatest when the divergence of the toggles from a straight line is a maximum, which is at the end of the forward stroke; it is least when the divergence is minimum, and this is at the end of the backward stroke. Hence with uniform rotation of the drive pulley the deck velocity increases from the beginning of the forward stroke to a maximum at the end of the stroke while the return stroke starts at maximum velocity and decelerates until the end. The backward stroke is effected entirely by the toggles, acting against the spring pressure and the inertia of the moving parts; the forward stroke is effected by the spring, but the spring tension is set to prevent lost motion so that the velocity of the forward stroke is controlled by the toggles. Graphical analysis of the motion is shown in Fig. 6, positions 1 to 8 inclusive being those occupied at eight equally-spaced moments of time in one revolution of the drive pulley. In the graph (Fig. 6, *c*) the slope of the tangent to the curve at any point is a measure of the velocity at that point and the curve clearly shows acceleration from 1 to 5, representing the forward stroke of the table, and de-

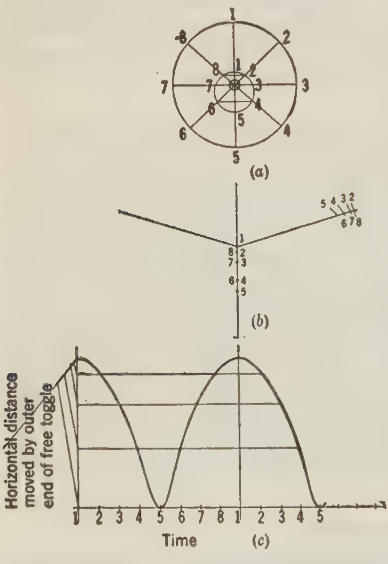


Fig. 6.—Analysis of motion of Wilfley table.

celeration from 5 to 1. To ADJUST THE SPRING properly, loosen until a slight knock is heard then tighten to just remove the knock. Further tightening increases wear and power consumption and may cause spring breakage. LENGTH OF STROKE and, to some extent, the SHARPNESS thereof, are controlled by vertical movement of the fixed toggle block, which is lowered to lengthen and sharpen the stroke and raised to shorten it. The usual range of STROKE LENGTH is from about 0.5 in. to 1.25 in.

DECK FRAME is covered with linoleum when delivered by the manufacturer and this is the usual covering, but canvas, rubber, pyroxylin-covered fabric (MINE-FAB), cement and glass have been used.

RIFFLES are usually formed on linoleum and similar surfaces by tacking on wooden cleats, but the riffle may be grooved into the linoleum or cast into glass or rubber. Softwood (sugar pine) attached riffles are usually used in finishing service, maple or oak in roughing service or with coarse abrasive feed. Riffles have been made by bending 16-gage galvanized iron into unequal-leg angle sections, using the short side as the riffle cleat and the long for the bottom of the riffle (*112 J 180*). These increased the capacity of a No. 9 Wilfley 25 per cent., the cost of laying was 20 per cent. less than for wooden cleats, and on the basis of the usual local average life of 5000 tons for soft-pine riffles the respective costs per ton of feed were $0.015 for wood and $0.008 for iron. STANDARD RIFFLING on a Wilfley table is formed as follows: A full-length cleat, ¼ in. wide, tapering from ½ in. deep at the head-motion end to a feather edge at the concentrate end is laid along the tailing side with the lower edge in the same plane as the outer face of the outer deck timber. The shortest riffle cleat, about 4 ft. long, ¼ in. wide, and tapering from ¼ in. deep at the motion end to a feather edge, is placed parallel to the first and partly under the feed box. Forty-four other riffles of the same width, spaced on about 1⅜-in. centers, are then placed between the first two. Their heights at the motion end are graduated between those of the first two cleats; they terminate along a diagonal line joining the thin ends of these cleats, and each is tapered gradually to a feather edge at this line. Brads penetrating at least ⅜ in. into the wood of the deck and spaced on about 6-in. centers should be used to hold down the cleats. In placing NEW LINOLEUM and riffles on a deck, first allow the linoleum to flatten out of its own accord in a warm, dry place, then set it on a clean, smooth deck with an overlap on the tailing side and concentrate end of about ¼ in. Tack lightly in place. Lay the longest riffle, mark the location of the shortest and of the diagonal line, then lay the remaining riffles, working upward from the tailing side, using a template to obtain the desired spacing. Finally tack down the edges of the linoleum and place quarter-round strips on the feed side to exclude water and pulp from the under side. Puffs will usually disappear after a few days' use and should not be tacked down.

Tables are built both right- and left-hand. RIGHT-HAND TABLES have the feed box and drive pulley at the right when the observer stands at the head-motion end and faces the table; LEFT-HAND, *vice versa.*

FEED AND WASH-WATER BOXES are open-topped troughs carried along the high side of the table deck. Feed box is attached to the deck and extends about one-quarter to one-third the length from the head-motion end. Wash-water box extends the remaining distance along the table and is supported on the table bed. Egress is by holes at the back (upper side) of the boxes and is regulated by fingers buttoned over the holes. The shaking motion keeps the heavy portion of the feed sufficiently in suspension to cause it to distribute and flow. Water is introduced by flexible hose through a hole in a cover board over the center of the water box, thus preventing splashing. An open water box is superior to perforated pipe as it permits ready cleaning of clogged exit holes.

Distribution of wash water along the concentrate end is effected on the usual form of table by a shallow cantilever wash-water trough along the end with drip holes near the lower corner. This is most unsatisfactory. The best method of keeping this end of the table washed is to cut the end back diagonally by shortening the tailing side 3 to 6 in., or water discharged from the feed-side water box may be diverted in part to the concentrate end by a proper arrangement of small cleats tacked to the unriffled surface.

SEPARATION OF CONCENTRATE FROM MIDDLING should always be made on the concentrate end where the movement of the discharging stream is parallel to the splitter edge. The tailing-middling split can be brought to the end also, with considerable advantage in some cases, by bringing the line of termination of riffle cleats up to about the quarter-point on the concentrate end. Fingers fastened to the deck are sometimes used to do away with constant tilting of the deck or shifting of the product pans. A finely-corrugated discharge lip on the concentrate end aids in effecting a clean cut between products.

Performance. Table 1 is a summary of operations of Wilfley tables on lead, zinc, copper and precious-metal ores. Table 3 shows the performance of tables on several different grades of classified feed at NEVADA CON. COP. Co. In most mills the Wilfley table is used on sandy feed, making clean

Table 1. Performances of Wilfley tables

Mill	Kind of ore	Size of feed, mm.	Covering	Riffle cleats	Speed, revolutions per minute	Length of stroke, inches	Tons per 24 hr.
Timber Butte(b)	Zinc(a)	−2.5–0	Linoleum	Special	243	......	100
Granitic zinc ore	Zinc(a)	c	Linoleum	Maple	257	7/8	25
American Zinc, Lead & Smelting Co.	Zinc	−20-mesh de-slimed	Linoleum	Poplar	275	3/4	14
Connecticut Zinc Co.	Zinc	Fine sand	......	......	......	......	21.5
Sunnyside M. & M. Co.	Zinc-lead	Flot'n conc.	Linoleum	Sugar pine	240	1/2 to 3/4	......
New Jersey Zinc, Ogdensburg	Zinc(f)	h	Linoleum	Sugar pine(g)	@ 260	3/4	6–8
New Jersey Zinc, Franklin	Zinc(f)	i	Linoleum	Sugar pine(g)	254	3/4	6–12
Federal Mining & Smelting, Morning	Lead	l	Linoleum and Fabrikoid	Pine	@ 240j	1/2 to 7/8	10–20
Federal Lead Co.	Lead	m	Linoleum	Wood	275	3/4 to 7/8	Av. 30
St. Joseph Lead Co., Rivermines	Lead	All −2-mm., 12% −48-mesh	Concrete	Oak	250	1	45
Calumet & Hecla	Native copper	−28-mesh, 40% −200-mesh	Linoleum	Wood	240	5/8–7/8	15–25
Chino Copper Co.	Chalcocite and chalcopyrite	n	Linoleum	Sugar pine	248	3/4–1	40–75
Porphyry copper ore	{ " "	......	......	......	245	3/4	45
Cananea Consolidated Copper Co.	"	5-mm.	......	......	......	......	172
Phelps-Dodge, Moctezuma	"	n	Linoleum	Pine	250	1 1/8	14
Phelps-Dodge, Moctezuma	"	n	C. I. plates and linoleum	C. I. plates and oak	250	1	200
Tonopah-Belmont (52 A 114)	Pyrite and gold in silicious gangue	p	Linoleum	Oak	250	7/8–3/4	20–50
Alaska Gastineau	"	r	Canvas	Wood	233	5/8	25–30
Liberty Bell	"	......	Linoleum	Sugar pine	254	7/8	30
Belmont-Surf Inlet	"	10- to 40-mesh	Linoleum	Wood	240	7/8–1/2	q
Melones Mining Co.	"	s	Linoleum	Wood	260	3/4	60g
U. S. S. R. & M. Co., Midvale	Complex	20-mesh	......	Pine	250	5/8	30
Tungsten Mines Co.	Tungsten	......	Linoleum	Maple	247	5/8–3/4	t
Tungsten Mines Co.	Tungsten	......	Linoleum, 1/8-in.	......	260	3/4	20
		......	Linoleum, 1/8-in.	......	230	1 3/16	30

For explanation of reference letters, see page 725.

Table 1. Performances of Wilfley table—*Continued*

Mill	Per cent. water in feed	Wash water, gallons per minute	Horse-power	Assays, per cent. Feed	Tailing	Concentrate	Middling	Attendance, machines per man	Recovery, per cent.
Timber Butte (*b*)	....	....	....	....	....	49–51	....	....	25–30
Granitic zinc ore	....	....	0.75	....	....	....	....	10	....
American Zinc, Lead & Smelting Co.	73	10	1(*d*)	5.5	0.5	59.5	8	44	....
Connecticut Zinc Co.	....	....	....	3.81	2.10	58.20	7.72	....	....
Sunnyside M. & M. Co.	....	5.5–7	1.31*e*	....	....	....	....	11	....
New Jersey Zinc, Ogdensburg	75	5.6	0.5	....	....	....	....	20	....
New Jersey Zinc, Franklin	75	10	0.5	....	....	....	....	12	....
Federal Mining & Smelting, Morning	70	8.3	0.75	6	1	55	4	30*k*	....
Federal Lead Co.	53	7–8	....	....	....	....	....	20	....
St. Joseph Lead Co., Rivermines	50	15	0.75	5	0.4	72	5	40	....
Calumet & Hecla	....	5.6	0.5–0.75	....	....	30–60	....	60–135	Av. 50
Chino Copper Co.	50	8	1	....	1.75	14	6.56	35	....
Porphyry copper ore	70	3.6	0.75	3.5 1.29	1.10 0.99	22 13.6	....	24	72.1 25.1
Cananea Consolidated Copper Co.	80	3.5	....	....	*o*	....	....	30	....
Phelps-Dodge, Moctezuma	50	40	1	3	1.6	10–11	....	7	....
Phelps-Dodge, Moctezuma	50–70	30	0.75	1.5	0.5	13–14	....	20	....
Tonopah-Belmont (*52 A 114*)	79	5	0.59	....	....	1.6 oz. Ag and Au 30 Fe, 31 insol.	....	16+	11.5
Alaska Gastineau	50.7	6	0.56	....	....	....	....	80	....
Liberty Bell	*q*	....	0.5	Au 0.8 Ag 27.1	0.6 24.2	1.6 250	....	33	....
Belmont-Surf Inlet	67	....	....	....	....	....	....	6	....
Melones Mining Co.	86	1.50	1.5	\$2	\$0.75	\$35	\$1.20	24	....
U. S. S. R. & M. Co., Midvale	60–80	9–19	0.75	....	*u*	....	....	20	....
Tungsten Mines Co.	78	13	0.75	0.5 WO_3	0.10–0.14	60	....	....	....
Tungsten Mines Co.	78	10.3	0.75	2.0	0.2	62	....	....	....

a Blende in granite. *b* Tables made a lead-zinc-iron-copper middling, silica-free, that was carefully finished on other tables; circulated a zinc-silica middling without re-grinding, and separated slime for flotation without dilution. The operation was performed at high capacity and low cost. *c* Garfield-table concentrate. *d* Includes transmission. *e* Installed. *f* Willemite and zincite from granitic rock. *g* 2-deck. *h* See Sec.

Notes continued on p. 726.

6, Table 15. *i* See Sec. 6, Table 15. *j* Varies with size of feed. *k* Also attends 4 drag and 7 hydraulic classifiers feeding tables. *l* See Sec. 6, Table 20. Practically all −30-mesh carefully de-slimed. *m* See Table 6. All −10-mesh. *n* See Table 2. *o* 4 sets of tables, assays as follows:

	No. 1	No. 2	No. 3	No. 4
Feed	1.18	0.72	0.32	1.28
Tailing	0.33	0.60	0.23	0.40
Concentrate	5.48	4.36	3.64	5.60
Middling	....	....	....	0.98

d 2-mm. to 150-mesh classified. See Sec. 6, Table 11. *q* 50 tons per 24 hr. on 4 : 1 feed and 40 on 5 : 1, both tube-mill sands from hydraulic classifier. 20 on 8 : 1 sands from cone treating hydraulic-classifier overflow and 40 and 10 tons respectively on tables treating coarse and fine middlings from other tables. *r* All through 80-mesh, 72 per cent. −200-mesh. *s* All through ⅛-in. trommel after ball mill. *t* Single-deck, 35 tons per 24 hr. of 10-mesh sands to 10 tons on 100-mesh sands; 2-deck, 50 tons on 10-mesh sands and 15 tons on 100-mesh.

u	Au, oz.	Ag, oz.	Cu, per cent.	Pb, per cent.	Insol., per cent.	Fe, per cent.	Zn, per cent.
Feed	0.12	3.3	0.4	4	48	8.9	9
Tailing	0.02	0.8	0.25	0.8	78	2.5	2.8
Concentrate No. 1	0.35	9.0	6	14	3	26	3
Concentrate No. 2	0.14	3.0	1	3.8	7	24	20

Table 2. (Supplement to Table 1). Screen tests of Wilfley-table feeds

	Percentages on screen		
Mill	Chino	Cananea	P. D. Moctezuma
Screen			
8-mesh	....	....	5.88
10-mesh	10.9	....	22.40
14-mesh	15.2	....	21.08
20-mesh	14.8	....	18.00
28-mesh	14.8	....	11.48
35-mesh	16.0	2.0	8.20
48-mesh	12.1	4.0	4.88
65-mesh	8.3	9.4	3.08
100-mesh	4.3	14.0	2.06
150-mesh	1.6	11.2	1.40
200-mesh	0.5	7.6	0.40
−200-mesh	1.5	31.8	0.60

Table 3. Performance of Wilfley tables at Nevada Consolidated

Tables	Assays, per cent.				Ratio of concentration	Recovery, per cent.	Tons per 24 hr.	Wash water, gallons per minute
	Feed, Cu	Concentrate		Middling + tailing + slime, Cu				
		Cu	Insoluble					
Gallery	3.44	17.5	26.4	2.02	11.5	48.7	......	......
Gallery (*d*)	3.9	14.5	41.6	2.18	7.2	51.3	9–25	......
Row No. 1 (*e*)	3.4	19.2	27.6	1.94	11.5	49.0	9–25	87
Row No. 2 (*a*)	3.32	19.2	29.7	1.72	10.9	53.0	7–15	10.8
Row No. 2 (*b*)	2.40	16.1	34.4	1.57	17.4	38.3	7–15	......
Row No. 2 (*c*)	2.26	22.3	19.3	1.46	26.0	38.0	7–15	......
Row No. 3	2.25	16.7	35.9	1.31	16.4	45.3	18	9.1
Row No. 4	2.12	18.6	28.6	1.38	23.2	37.7	20–22	10.7
Rows Nos. 5 and 6	1.60	18.0	35.3	1.28	52.0	21.6	12	8.1

a Feed, second spigot of classifier. *b* Feed, middling from gallery Wilfleys. *c* Feed, overflow of classifier. *d* Feed, first spigot of classifier. *e* Feed, second spigot of classifier.

concentrate and a tailing to be re-ground and re-treated. Fig. 7 (*4 CA 732*) shows results of a test run on bituminous coal.

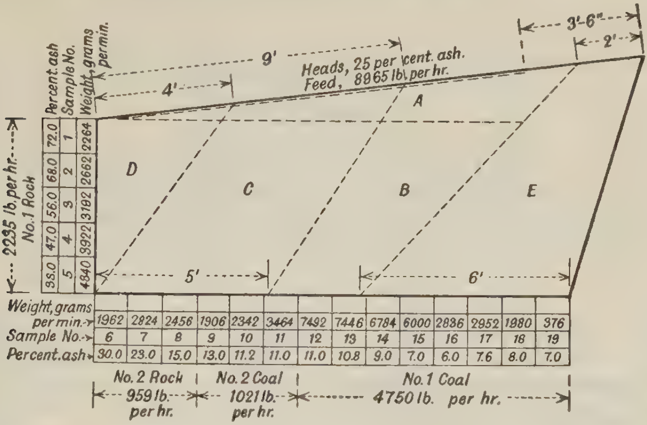


Fig. 7.—Performance of Wilfley table on $-\frac{3}{8}$-in. coking coal.

Capacity. On zinc-sulphide ores the tonnage treated ranges from 14 tons per 24 hr. on 0.8-mm. feed to 100 tons on 2.5-mm. feed; separating willemite, with closely classified feed, capacity varies from 6 tons with 0.4-mm. material to 12 tons at 2.3-mm. On lead ores, when both clean concentrate and finished tailing are sought, capacity varies from 10 tons on 0.5-mm. feed to 45 tons on 2-mm. size. Capacity treating native-copper ore all through 0.6-mm. and containing 40 per cent. slime is 15 to 25 tons per 24 hr. when making clean concentrate and finished tailing. Treating porphyry-copper ore, the capacity on material through 0.6-mm., when making finished concentrate and tailing, is 14 tons per 24 hr.; treating rougher-table concentrate capacity is 45 to 75 tons and when roughing -2.5-mm. feed, 200 tons. Separating auriferous pyrite from silicious gangue, capacity is about 25 to 30 tons per deck on 1-mm. sands, falling to 10 tons on 0.1-mm. sands and rising to 50 tons on 2-mm. material.

Size of feed rarely exceeds 2.5-mm. Wiggin (*46 A 232*) determined that 0.025-mm. was the smallest grain that could be saved with Anaconda ore. The coarser, sandy portion of 200-mesh slime is readily saved. Table 4 is a sizing-assay test of Anaconda table tailing

Table 4. Sizing-assay test of Wilfley-table tailing, Anaconda Copper Mining Co.

Size, mm.	Per cent. weight	Assay, per cent. Cu	Per cent. total Cu
+0.84	2.31	0.23	2.39
0.50	15.58	0.20	13.66
0.35	35.39	0.21	33.42
0.17	28.64	0.19	24.32
0.07	14.98	0.14	9.08
−0.07	3.10	1.25	17.13

when a roughly de-slimed feed was being treated. Table 5 gives similar results at Ray Cons. Cop. Co., except that the feed was not de-slimed. Here maximum loss of copper and maximum assay occur in the finest size; there is also a relatively high copper loss in the coarsest sizes.

Speed and stroke. Speed ranges from about 230 to 280 strokes per min., being close to 250 in the majority of mills. Length of stroke averages close to ¾ in. Within the range of practice presented, no distinct relation between stroke length and speed is established, but in general a short stroke corresponds to a high speed and *vice versa*.

Table 5. Sizing test of Wilfley-table tailing, Ray Consolidated Copper Co.

Screen, mesh	Per cent. weight		Assay, per cent. Cu		Per cent. original Cu	
	A	B	A	B	A	B
20	7.6	6.2	0.74	0.69	9.0	7.5
30	11.2	11.4	0.63	0.67	11.4	13.2
40	11.0	10.3	0.55	0.63	9.8	11.3
50	4.3	4.3	0.58	0.59	4.0	4.4
60	11.5	11.7	0.53	0.50	9.8	10.3
70	2.3	2.1	0.53	0.49	1.9	1.7
80	9.4	8.9	0.49	0.42	7.4	6.5
100	9.2	9.2	0.45	0.38	6.6	6.1
120	4.2	4.5	0.44	0.37	2.9	3.0
150	5.9	5.7	0.39	0.34	3.7	3.3
200	1.9	1.8	0.39	0.37	1.1	1.2
−200	21.3	23.8	0.95	0.76	32.4	31.5
Total.........	99.8	99.9	0.60	0.58	100.0	100.0

Wash-water consumption ranges from 1.5 to 40 gal. per min., the high figure being in roughing service and the low corresponding to a very dilute feed pulp. Average consumption for average service is about 8 gal. per min.

Slope varies with size of feed, specific gravity of minerals, character of products desired, and the place that the products are split, as well as with the quantity of wash water. The usual slope with fine feeds is from ¼ to ½ in. per ft. and ¾ to 1 in. with coarse feeds. When roughing the slope may be as much as 2 in. per ft.

Power. Actual power consumption under average load is close to 0.6 hp. with a usual installation of between 0.75 and 1 hp. per table.

Lost time is very small and is principally due to shut-downs for replacing deck covering and repairing or renewing riffles. Mechanism difficulties usually correspond to high tonnages and long stroke or to attempts to run double decks with the standard head motion (see Garfield-table head motion, Art. 9).

Operating adjustments available on t e standard table are length of stroke, tilt, wash water, location of product splitters. In small mills where irregularity in feed conditions is frequently the rule, control of all of these is necessarily left with the operator. The tendency in large mills is to limit operator's adjustments to tilt and wash water and in some mills tilt is not adjustable and the operator controls wash water only. In some plants splitters are fixed in position and products must be brought to the proper points by shifting their position on the table deck by changing tilt and wash water. It is much better to have movable splitters and to take care of all minor shifting of the product streams by moving the splitters, leaving water and tilt as nearly undisturbed as possible.

Comparison with other machines. For results of competitive tests with other tables see the other tables.

As a result of a competitive test with a fine jig at Bunker Hill and Sullivan, Caetani concluded (*3 MM 54*) that the Wilfley table had twice the capacity of the jig, made a higher recovery and required less skilled labor and less water, power, and repairs. The table made a slime separation that a jig fails to make and there was less fall of material in passage through the machine. Jig concentrates were richer and the middling contained less free mineral. Tailing assayed about the same in both cases but the coarse part of the jig tailing was lower grade while the fine part was of higher grade than the Wilfley.

Buss table is a side-inclined shaking table which has the deck frame supported on a number of lath-shaped ash or hickory strips, pivoted at the lower end and standing inclined toward the motion end about 15° from the vertical. The head motion is a simple eccentric. On the forward stroke particles in contact with the table deck move forward and upward with it; on the reverse stroke the particles continue to move forward but, by reason of the fall of the deck, are out of contact with the deck during a part of their forward travel. The result is that forward travel is much more active than on tables

with the Wilfley type of motion. Riffles are full-length and converge slightly toward the tailing side. Height and taper of riffles, speed, stroke length, power and water consumption and capacity are about the same as on the Wilfley table.

Ferraris table is closely similar, except that the number of spring supports is less and the riffling is not carried full length but is cut off at right-angles to the motion along a line about two-thirds the distance toward the concentrate end of the table.

4. Butchart table

This table (also called NATIONAL TABLE) is a rectangular-deck, full- or partly-riffled table, differing from those previously described in that the riffles are bent diagonally toward the feed side about along the line of the "Wilfley diagonal" and run in this direction for a few inches, where they terminate or are again bent back substantially parallel to the tailing side. Diag-

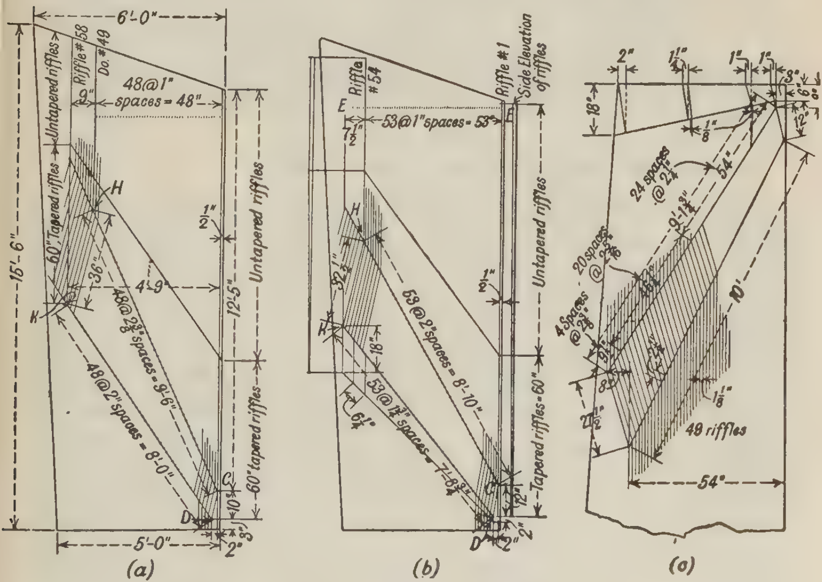


FIG. 8.—Butchart riffling.

onally-terminated riffling is used for cleaning; full-length riffling principally for roughing but also for cleaning. The "wave" in the riffles along the "diagonal" line is claimed to effect a rotary vanning motion that aids in bringing gangue to the surface of the beds in the riffles where it can be washed away. Fig. 8 shows plans of three forms of Butchart riffling. Either of types (*a*) and (*b*) may be changed to full-length by continuing the riffle cleats to the concentrate end in a direction parallel to the tailing side. Probably most Butchart riffling is mounted on Wilfley mechanisms, but W. A. Butchart manufactures a complete table with a head motion (Fig. 9) of the toggle type that is enclosed and runs in oil. This is a distinct advantage.

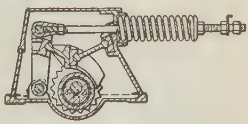


FIG. 9.—Butchart head motion.

Effect of full-length riffling is shown in Fig. 10. The coarsest heavy mineral discharges highest up on the concentrate end and there is a uniform decrease in size of heavy mineral in the concentrate as the tailing side of the table is approached. The gangue sizes, on the other hand, are distributed as on the Wilfley table. This arrangement of discharging grains is due to the fact that after stratification, which takes place as shown in Fig. 3, while the light minerals are washed away as on the Wilfley table, the heavy minerals are afforded the support of the riffles the full length of the deck, are not subjected to film-sizing and the only effective force acting to cause size segregation of these grains is eddying in the riffles. The eddying water lifts the finer particles more readily than the coarse and these particles are, therefore, carried over the riffle cleats and down slope, while the coarse particles travel along to the end of the table. The result is to spread the product bands at the discharge edges of the table and permit much closer and cleaner splitting than is possible with diagonally-terminated riffles. This is particularly true of the concentrate-middling split. On the other hand, the full-length riffles support fine gangue and carry it toward the concentrate end, with the result that more water must be used to get clean concentrate than on a table with partly plane surface, and there is more fine clean gangue in the middling.

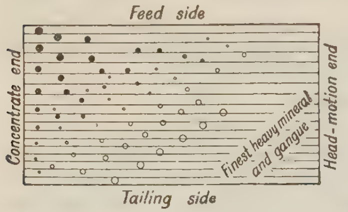


FIG. 10.—Distribution of minerals at discharge end of a shaking table rifled full length.

Performances at several mills are given in Table 6.

Maximum size of feed that can be handled depends upon the type of riffling. In roughing service, with tables riffled full-length, the maximum size of feed is probably about ⅜-in., but with such coarse feed, while a clean coarse concentrate can be taken, it is difficult, if not impossible, to make a satisfactory separation of middling from tailing. In finishing service, with diagonally-terminated riffles, the maximum size that can be satisfactorily treated is about the same as on a Wilfley table, *viz.*: 2- to 2.5-mm. Minimum size is the smallest that will settle in the cross-water current and will therefore be smaller the smaller the maximum-sized particle of feed. At the Bonne Terre mill of ST. JOSEPH LEAD CO., general mill concentrate was de-slimed at 100-mesh and the fines thickened and treated on a Butchart table. The concentrate assayed 80 per cent. Pb.; 95 per cent. passed a 200-mesh screen; tailing assayed 6 per cent. Pb. When coarse feed is treated it is necessary to have fine sand present. Attempts to enrich jig lead concentrate on the table failed until sandy table concentrate was added.

Riffling is varied according to the character of service and size of feed. Full-length riffling is used principally in roughing, but may be used also for finishing, especially when the ratio of concentration is low; diagonally-terminated riffles are used for cleaning. Riffle cleats are usually ⅛ in. high at the concentrate-discharge end, tapering from about ½ in., but when the feed is coarse, ratio of concentration low, or tonnage exceptionally high, the taper may be from 1 in. to ⅜ in., in order to provide concentrate-carrying capacity. With such deep riffles at the concentrate end, they are usually given a slight curve downhill near the end in order to keep sufficient water along this end to prevent banking. The average RISE OF RIFFLE CLEATS in the "wave" section is 1 in. in 4 in. of run parallel to the tailing side. The steeper the rise the cleaner the concentrate, other conditions being constant. Riffle cleats in roughing service or with coarse feed are usually of hard wood, oak or maple preferred; in cleaning, sugar-pine cleats are satisfactory. Life of oak riffles is three times that of pine.

Linoleum is the usual DECK COVERING, but soft rubber (old vanner belts) has been used at the Morenci branch of PHELPS DODGE and in the SOUTH-EAST MISSOURI lead district. Concrete has been used at Bonne Terre mill of ST. JOSEPH LEAD CO. Concrete mixture is 2 of sand (2-mm.) to 1 cement; it is laid on linoleum flush with the tops of riffle-nailing cleats 3⁄16 to ¼ in. high and is given a steel-trowel finish. Life of linoleum covering is 6 months to 3 years, depending upon the size of feed; rubber has a longer life, but new covers cost three to four times as much as linoleum. Concrete deck at ST. JOSEPH LEAD CO. has given very satisfactory service. It has the disadvantage of increasing the weight of the rapidly-reciprocating deck, with consequent increase in power consumption and wear.

Capacity on 2- to 3-mm. feed in roughing service is from 100 to 200 tons per 24 hr. which is about the same as that of the Garfield table (see Art. 9). In finishing work on 1.5- or 2-mm. feed the capacity on lead ores is from 30 to 65 tons per 24 hr. and in cleaning rough copper concentrate, through 8-mesh, ranges from 40 to 75 tons. Cole (*51 A 405*) reports 80

Table 6. Performance of Butchart table

Mill	Kind of ore	Size of feed	Speed, revolutions per minute	Length of stroke, inches	Tons per 24 hr.	Wash water, gallons per minute
Chino Consolidated Copper Co.(*c*)	Copper	*b*	248	1 1/2	40–75	12.5
Phelps-Dodge, Morenci(*d*)	Copper	−3 1/2-mesh	256	7/8	175	...
Phelps-Dodge, Morenci(*c, f*)	Copper	*b*	256	3/4	45	10
Phelps-Dodge, Morenci(*w*)	Copper	*v*	...	...	43.6	...
Braden Copper Co.	Copper	*e*	250	...	214	...
Braden Copper Co.	Copper	*g*	240	...	116	...
Phelps-Dodge, Burro Mountain(*y*)	Copper	−4-mesh	275	3/4	150–200	...
Phelps-Dodge, Burro Mountain(*u*)	Copper	*t*	...	...	42.5	...
Phelps-Dodge, Burro Mountain(*x*)	Copper	14-mesh	...	...	144.3	...
St. Joseph Lead Co., Bonne Terre(*c, m*)	Lead	*b*	270	7/8	55	12
St. Joseph Lead Co., Bonne Terre(*c, m*)	Lead	*b*	280	1	65	10
Federal Lead Co., No. 4	Lead	*b*	275	3/4–7/8	30	7–8
Shattuck-Arizona(*i*)	Lead carbonate	*j*	267	5/8	200	25*s*
Shattuck-Arizona(*i*)	Lead carbonate	*k*	295	7/8	100	25*s*
Belmont-Shawmut(*q*)	Auriferous pyrite	−10-mesh	245	1 1/8	57	...
Real Compañia Asturiana de Minas(*p, r*)	Lead-zinc	*b*	...	...	45	...

Mill	Water in feed, per cent.	Horse-power	Assays, per cent.				Attendance, machines per man
			Feed	Conc.	Tailing	Middling	
Chino Consolidated Copper Co.(*c*.)	50	1	*a*	14.89	1.64	4.41	35
Phelps-Dodge, Morenci(*d*)	...	0.5	1.8	11.5	1.05	...	23
Phelps-Dodge, Morenci(*c, f*)	...	0.5	1.15	10.5	0.58	0.75	23
Phelps-Dodge, Morenci(*w*)	78	...	0.39	4.49	0.34	...	...
Braden Copper Co.	37.5	...	2.80	17.8	2.16	...	...
Braden Copper Co.	55	2*h*	2.15–1.56	16.2–18.4	1.03–1.83	...	...
Phelps-Dodge, Burro Mountain(*y*)	65	0.7	1.9	15.6	1.2	...	12
Phelps-Dodge, Burro Mountain(*u*)	...	...	0.646	10.56	0.484	...	...
Phelps-Dodge, Burro Mountain(*x*)	55	...	2.17	14.71	1.33	...	...
St. Joseph Lead Co., Bonne Terre(*c, m*)	50	0.75	8	77	0.45	8*n*	18
St. Joseph Lead Co., Bonne Terre(*c, m*)	50	7/8	8	78	0.45	8*n*	18
Federal Lead Co., No. 4	53	1	5	70	0.4	4	20
Shattuck-Arizona(*i*)	60	1*h*	...	...	...	...	...
Shattuck-Arizona(*i*)	65	1*h*	*l*	*l*	*l*	*l*	...
Belmont-Shawmut(*q*)	...	...	$5	$63	...	...	...
Real Compañia Asturiana de Minas(*p, r*)	68	...	...	42–51 Zn(*o*)	...	...	...

For explanation of reference letters, see page 732.

Notes to Table 6

a Garfield-table concentrate, about 5–6 per cent. Cu. *b* For screen test, see Table 6*a*. *c* Standard full-length Butchart riffling on No. 5 Wilfley deck. *d* Riffles terminated on diagonal at end of curve. Wilfley deck cut 2 ft. short. *e* 1 per cent. on 8-mesh, 35 per cent. through 65-mesh. *f* Tables set 10 ft. center-to-center. *g* 5 per cent. on 14-mesh, 19 per cent. through 65-mesh. *h* Installed. *i* See Fig. 8, *c* for sketch of riffling. *j* All −4-mesh, 15 per cent. −200-mesh. *k* 10 per cent. on 48-mesh, 16 per cent. −200-mesh.

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	Au, oz.	Ag, oz.	Pb %	Fe %	Insol. %
Feed	0.04	5.8	2.9	8.0	82.0
Concentrate	0.22	21.0	15.6	20.4	47.8
Tailing	0.02	3.1	0.6	....	....

m Riffles 7⁄16 in. deep at head end and 1⁄8 in. at concentrate end; slope in curve zone 1 in 4. *n* About 10 per cent. of feed by weight. *o* Contains 2 to 4 per cent. Pb and about 9 per cent. dolomite. *p* Recovery about 70 per cent. of lead and 65 per cent. of zinc. *q 121 P 660*. *r 115 J 395*. *s* Estimated. *t* Flotation tailing de-slimed in Allen cones. *u* Oxide Cu in feed, 0.156 per cent.; tailing, 0.140 per cent.; insoluble in concentrate, 31.9 per cent. *v* Flotation tailing, all through 48-mesh, 70 per cent. through 200-mesh. *w* Oxide Cu in feed, 0.15 per cent.; tailing, 0.15 per cent.; insoluble in concentrate, 31.7 per cent.; iron, 29.1 per cent. *x* Oxide Cu in feed, 0.25 per cent.; tailing, 0.227 per cent.; insoluble in concentrate, 15.0 per cent. *y* Rougher.

Table 6*a*. (Supplement to Table 6.) Sizing tests of feeds to Butchart tables

Screen aperture, mm.	Per cent. weight on screen				
	Chino Consolidated Copper Co.	Phelps-Dodge, Morenci	St. Joseph Lead Co., Bonne Terre	Federal Lead Co. No. 4	Real Compañia Asturiana de Minas
3.327	....	....	....	....	....
2.362	....	....	....	....	....
1.651	4.3	0.10	....	....	....
1.168	6.9	0.74	6.5	....	19
0.833	8.5	4.47	16.2	15.2	....
0.589	12.7	18.71	20.0	....	17
0.417	20.4	26.70	14.0	29.3	....
0.295	19.7	19.91	12.3	17.5	....
0.208	14.0	12.32	9.2	12.4	....
0.147	6.8	6.45	8.6	7.9	45
0.104	2.9	3.85	8.8	9.5	12
0.074	1.1	0.94	2.2	4.4	5
−0.074	2.7	5.81	2.2	3.8	2

tons per day treating de-slimed Hancock-jig concentrate, −3⁄8-in., at Morenci, lowering the percentage of insoluble from 30 to 15.

Speed and length of stroke. The usual range in speeds is from 240 to 280 strokes per min. and average length is close to 1 in. The examples cited do not show any distinct relationship between character of feed and the stroke, nor between speed and stroke length, but in general the rule applies that coarse feeds require longer and slower strokes than fine feeds.

Wash-water consumption averages close to 10 gal. per min.

Lost time is estimated as close to 1 per cent. on the average and is due principally to replacement of deck covering and riffling.

Attendance. Operators control tilt and wash water in all mills reporting and at Morenci also control feed rate to a certain extent. The relatively low figures under "machines per man" in Table 6 are probably due to the fact that no more tables were installed calling for attention. Butchart riffling, particularly full-length, spreads the concentrate and middling bands out into wide fans which change position only slightly with changes in feed rate, hence control is easy and attendance should be small.

Butchart vs. Wilfley. Cole (*51 A 405*) reports the following 4-day test at Morenci.

Feed, de-slimed rougher tailing after re-grinding to pass 1.5-mm. Results as in Table 7, test A. Test B in the same tabulation shows the results of treating roughly de-slimed 20-

Table 7. Butchart *vs.* Wilfley table at Morenci plant, Phelps-Dodge Co.

Test	Table	Feed		Concentrate		Middling, per cent. Cu
		Tons per 24 hr.	Per cent. Cu	Per cent. Cu	Per cent. insoluble	
A	Butchart........	101	1.66	12.46	11.3	0.91
	Wilfley.........	35.5	1.47	13.21	10.9	2.26
B	Butchart........	96.8	1.57	14.32	17.4	1.0
	Wilfley.........	23.2	1.39	15.2	16.8	1.55

Test	Table	Tailing, per cent. Cu	Middling +tailing, per cent. Cu	Ratio of concentration	Recovery, per cent.	Water	
						Gallons per minute	Gallons per ton
A	Butchart........	0.55	0.61	11.3	66.5	16	256
	Wilfley.........	0.59	0.76	17.5	51.2	12	867
B	Butchart........	0.74	0.79	17.3	52.6	....	....
	Wilfley.........	0.68	0.81	24.8	44.1	....	....

mesh feed on a Butchart table with special deep riffles. On the basis of metallurgical results alone the advantage, if any, in these tests is with the Wilfley table on account of the fact that tailing is too high grade to discard in both cases and the operations must, therefore be judged on the concentrate taken out; but capacity and water consumption and necessarily, power, labor and maintenance are all in favor of the Butchart. Results of similar competitive work in treating lead ores in south-eastern Missouri were the same, the Butchart table yielding the same tailing and concentrate when treating 50 to 60 tons per 24 hr. of de-slimed 2-mm. feed as the Wilfley table yielded when treating 20 to 25 tons. (*57 A 350, 429*). In analyses of operation in both these fields the Butchart table is credited with elimination of all classification other than rough de-sliming, but this saving in treatment is rather due to the introduction of flotation, which removed from the tables the burden of making finished tailing.

Butchart table *vs.* **jigs.** Table 8 presents the results of a competitive run between a Butchart table and a Harz jig at the Detroit Copper Co. plant on primary feed passing

Table 8. Butchart table *vs.* Harz jig at Detroit Copper Co.

Machine	Feed		Concentrate		Tailing, per cent. Cu	Ratio of concentration	Recovery, per cent.
	Tons per 24 hr.	Per cent. Cu	Per cent. Cu	Per cent. insol.			
Butchart table.......	160	3.15	16.65	9.8	1.42	8.8	60.04
Harz jig............	35	3.19	17.43	11.9	1.63	10.1	53.95

a 7-mm. screen, without de-sliming. (*51 A 405.*) At St. Joseph Lead Co., Bonne Terre, Hancock jigs gave better separation of middling from tailing than the table on -9 $+2$-mm. feed and tables were rejected for treating material coarser than 2-mm. (*57 A 429.*)

5. Card table

This table differs from the Wilfley table principally in that the riffles are cut into the linoleum and are triangular rather than rectangular in cross-section (Fig. 11). The cross-section of the riffles increases from the head-motion end to a maximum along a diagonal corresponding to the termination of Wilfley riffles, then decreases to nothing at the concentrate end. On tables for fine feed the riffles contract from maximum section at head-motion end to disappearance at the diagonal line. Riffles for coarse feed are about 2.5 in. apart, ⅝-in. maximum depth and $\frac{1}{16}$-in. minimum.

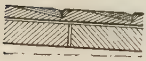


Fig. 11.—Cross-section of riffling on Card table.

Head motion is shown in Fig. 12. (*A*) and (*C*) are fixed pins and (*B*) a fixed toggle block. The motion of crank-shaft (*G*) is transmitted through the pitman and toggles to lever arm (*D*), thence by connecting arm (*F*) to lever (*H*), whose working arm (*E*) draws the table deck back against spring (*S*). The forward stroke is impelled by spring (*S*) but controlled by the mechanism chain. Length of stroke is varied by changing the position of pin (*P*) in lever (*D*). The deck motion is differential, of the same general character as the Wilfley, but Caetani (*3 MM 50*), discussing the choice of Card tables after competitive tests at Bunker Hill and Sullivan, says that the Card mechanism gave a sharper return stroke than the Wilfley and consequently greater capacity. The deck is roughly 5½ × 16 ft. There is a line of flexure in the deck along the diagonal from the feed corner, which permits the slime corner to be raised or lowered out of plane with the other half of the deck. When the deck is tilted up, crowding occurs along the diagonal of separation. The deck is usually set horizontally lengthwise.

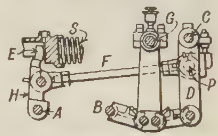


Fig. 12.—Head motion, Card table.

Performance. At Broken Hill South mill (*28 MM 8*) the table had a wood deck, sloped 1.75 in. in the width, and made 297 @ 0.75-in. strokes per min. Results on coarse unclassified feed are given in Table 9. A sizing test of a concentrate of similar assay, made

Table 9. Assays of Card-table products at Broken Hill South mill

	Product	Assays: Lead, per cent.	Silver, oz.	Zinc, per cent.
−16-mesh feed	Lead concentrate	63.8	23	7.8
	Lead-zinc middling	14	7.6	23
	Zinc tailing	2.8	3.0	16.5
	Quartz tailing	2.0	2.0	8.4
	Slime tailing	9.7	7.0	14.0
−24-mesh feed	Lead concentrate	52.8	20.0	12.7
	Lead-zinc middling	8.7	8.0	27.2
	Zinc tailing	2.9	2.5	14.0
	Slime tailing	8.8	6.6	13.0

on Card tables at the Central Mine, Broken Hill, is shown in Table 10 (*28 IMM 14*). This concentrate represents a recovery of about 20 per cent. of the silver, 34 per cent. of the lead and 3 per cent. of the zinc in the table feed. In planning the Bunker Hill and Sullivan, West Mill, extensive competitive tests were run, as a result of which the Card table was chosen. Caetani (*3 MM 50*) reports that the differences in performance were small but that the Card consistently recovered more silver than the other tables, probably because the wider concentrate streak, which is the result of full-length riffling, allowed a cut further down into the middling without too greatly lowering the grade of concentrate. The main factors in the choice were, however, mechanical. Great operating stability, by

Table 10. Sizing-assay test of Card-table concentrate, Central Mine, Broken Hill

Screen mesh, I.M.M.std.	Weight on screen, per cent.	Assays		
		Silver, oz.	Lead, per cent.	Zinc, per cent.
40	0.5	....	30.6	14.0
60	3.2	56.2	48.6	8.3
80	6.4	39.6	56.6	5.5
100	10.3	35.0	59.2	4.7
120	7.6	32.0	59.4	4.4
150	9.2	30.8	59.4	4.6
200	17.9	30.6	61.2	4.8
−200	44.9	32.4	69.6	4.0
Whole....	100.0	33.4	63.8	4.6

reason of the broad concentrate streak, was noted. The table stood up to 20 per cent. overload without important loss in efficiency, but overloading caused sand tailing to discharge with the slime. The feed was classified. Capacity on different-sized feeds was as given in Table 11. Speed, 250 @ ¾- to ⅞-in. strokes per min.; hp., 0.5; wash water, 7.5 to 9

Table 11. Capacity of Card tables at Bunker Hill and Sullivan, West mill

Classifier-spigot product, number	Coarsest material		Per cent. through 100-mesh	Tons per 24 hr.
	Mesh	Per cent. retained		
1	40	16	13	18
2	40	2	22	10
3	60	2	65	5
4	80	2	91	3

gal. per min. On coarse feed assaying 15 per cent. lead, tailing assayed 2 per cent., concentrate 72 and middling 14; on finer feed assaying 13 per cent., tailing was 1.5 per cent., concentrate 70 and middling 12.5.

Table 10 shows one of the results of full-length riffling in the high zinc content of the coarse sizes of concentrate. On a table riffled full length the coarsest particles of the heavy minerals ride highest, reversing the ideal Wilfley-table arrangement. On the Central-mill tables the coarse blende was at the upper edge of the middling streak and was cut into the concentrate. The large percentage of fine lead in this concentrate is the result of the V-shaped riffles. The eddying characteristic of rectangular riffles (Fig. 3) is largely absent in these, with the result that fine heavy mineral is retained higher on the table and goes, in greater quantities, into the concentrate.

Table 12 shows tailing made on classifier-spigot products at Mary Murphy mine (*12 CME 21*).

Table 12. Tailing made on base and precious metals at different sizes on Card tables, Mary Murphy mine

Tables	Classifier spigot	Tailing assays			
		Gold, oz. per ton	Silver, oz. per ton	Lead, per cent.	Zinc, per cent.
2 Card	1	0.15	1.4	0.7	2.8
1 Card	2	0.11	1.0	0.5	2.1
1 Card	3	0.04	0.7	0.4	2.2
3 Deister	4	0.04	0.7	0.7	2.3
1 Card	Overflow	0.03	1.0	0.7	3.8

6. Deister sand table

This was the first serious competitor of the Wilfley. Sketch of deck and riffling is shown in Fig. 13. The deck motion is parallel to the riffles, which, therefore, terminate on a line diagonal to the motion. On this ground the table was subservient to the Wilfley patent (now expired). The deck rises about ⅜ in. beginning about 7 in. back from the concentrate-discharge end thus causing the concentrate to climb against the flow of wash water. The deck is supported on extended ball-and-socket bearings. Tilt is effected by sliding wedges located under the seats of the bearings on the feed and tailing sides and actuated synchronously by a hand wheel or lever. The table is built both single-and double-deck.

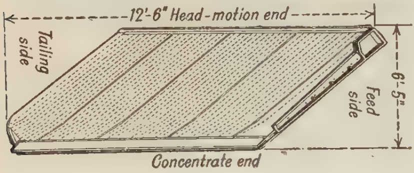


FIG. 13.—Deck of Deister No. 2 sand table.

Table 13. Summary of tests on Deister No. 2 sand table, Nevada Consolidated Copper Co., 1910

Test Number	Feed	Assays, per cent. Feed, Cu	Concentrate Cu	Concentrate Insoluble	Slime, Cu	Middling, Cu	Tailing, Cu	Ratio of concentration	Recovery, per cent.	Tons per 24 hr.
1	Not classified........	2.20	24.2	19.8	1.00	1.80	0.75	20.0	57	24.7
2	Middling............	1.95	16.1	23.6	0.80	0.95	0.50	15.0	62	11.9
3	Middling............	1.85	16.7	30.0	1.20	1.65	1.15	24.0	38	18.1
4	"Unclassified slimes, 60 per cent. —200-mesh."	1.35	10.4	50.3	0.55	0.70	0.40	12.0	52	8.2
5		1.50	13.6	39.4	0.55	0.45	0.40	14.0	66	7.7
6		1.50	17.0	34.4	0.80	1.15	0.60	23.0	49	12.5
7		1.60	16.2	41.0	0.65	1.10	0.65	16.0	62	12.0
8		1.35	14.2	34.2	0.80	1.80	0.55	24.0	59	18.4
9	"Middling and sands from 5th and 6th Wilfley tables."	1.00	11.6	54.6	0.80	1.95	0.65	54.0	21	33.3
10		0.80	11.1	57.0	0.65	1.25	0.65	42.0	30	24.4
11		0.95	10.8	68.8	0.80	1.60	0.65	67.0	17	33.4
12		0.80	12.9	49.4	0.55	1.20	0.65	50.0	33	.....
13		0.80	7.5	66.0	0.60	1.05	0.45	35.0	27	.....
14		0.35	4.7	67.8	0.25	0.55	0.25	45.0	30	.....
15		0.70	10.6	50.8	0.55	1.20	0.35	67.0	23	.....
16	Coarsest re-ground	0.75	7.8	12.6	0.55	1.55	0.45	36.0	29	.....
17	material.	1.40	12.6	26.6	0.55	0.95	0.85	15.0	63	.....

Head motion for single-deck tables is a combination of a quick-return slide-crank mechanism and a bell lever. (See Fig. 14.) The fly-wheel driving pulley (*a*) is mounted eccentrically to crank shaft (*b*) which carries a slide crank (*c*) engaging a pin (*d*) that is attached to pulley (*a*). Roller (*e*) is eccentrically mounted and free to revolve on shaft (*b*) and actuates the long arm of the bell lever (*f*) which is pivoted at (*g*) and carries on the short arm the yoke (*h*) which attaches to the drawbar of the table deck. The action of the slide-crank mechanism is shown in Fig. 15. Starting with a given point on the surface of the drive pulley in position 1, the time intervals required for this point to move from

1 to 2, 2 to 3, etc., are equal. But in the same time intervals the shaft (*s*), actuated by slide crank and pin from the driving pulley, moves through different angular distances and the total angular movement of this shaft during the time that the chosen point on the drive pulley moves from 1 to 5 is less than that moved through while the same point moves from 5 to 1. With a uniform rate of motion of the drive pulley the rate of revolution of the crank will, therefore, be non-uniform. Eccentricity of the drive-pulley center with respect to the center of the crank shaft is varied by means of slotted arm (*i*) (Fig. 14) by which the sleeve bearing of pulley (*a*) may be so rotated as to cause the center to approach or recede from the center of shaft (*b*). Sharpness of differentiation between forward and back strokes increases with the distance between the two centers. The relation of the eccentricity of roller (*e*) to the position of pin (*d*) is such that when the roller is depressed the table deck is thereby started back at the instant of maximum angular velocity of shaft (*s*) (Fig. 15).

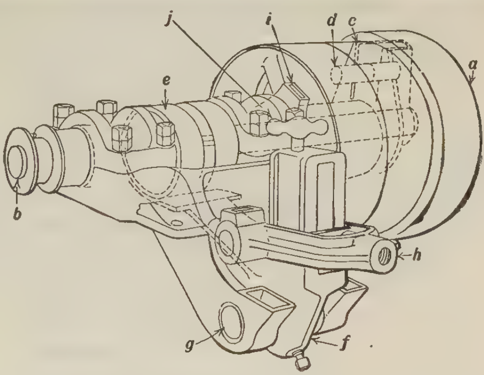


Fig. 14.—Deister head motion.

Length of stroke is varied by moving yoke (*h*) (Fig. 14) along the short arm of the lever (*f*) by means of hand wheel (*j*). This change is made without changing the sharpness of the return stroke except in so far as increasing length without changing speed increases velocity. The action of this head motion is strong and positive and causes material to progress rapidly. A serious disadvantage is the exposure to grit. On double-deck tables the heavy head motion described in Art. 9 is used.

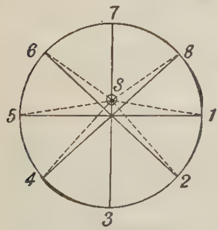


Fig. 15.—Analysis of Deister head motion.

Riffling varies somewhat with the character of the feed. The typical riffling shown in Fig. 13 has ¼-in. cleats spaced 1 in. center to center. When treating unclassified feed and fine sands the lowest riffle cleat, the 9th, 18th and 27th, etc., therefrom, and a cleat near the feed box are made higher than the others (*e.g.*, 0.5 in. against 0.25 in.) in order to form pools to aid settlement of fine material. The riffle cleats are not tapered, but the upper surface is set level longitudinally and an effect of taper is gained by raising the plane of the deck near the discharge end, as described. Pool riffling may be used, although it is unnecessary, on closely classified relatively coarse feeds. Tables are rated as right-hand when the feed box is to the right of an observer facing the table at the head-motion end. The drive pulley is on the side opposite the feed box.

Performance. At Miami Copper Co. No. 2 sand tables running at 240 to 256 @ 1⅛- to 1¼-in. strokes per min. and treating the sand from the first spigot of a hydraulic classifier taking − 20-mesh feed made 38.6-per cent. (Cu) concentrate, 1.21-per cent. middling and 0.64-per cent. tailing from a feed assaying 3.96 per cent. At Old Dominion 30 tons per 24 hr. of − 0.6-mm. feed assaying 2.78 per cent. Cu was treated per table; concentrate assayed 10.5 per cent., middling 2.5 and tailing 0.7. Table 13 shows performance on different classes of feed at Nevada Cons. Cop. Co. Table 14 gives performances in finishing service on a classified feed and compares the performance of standard riffling with a Schwarz glass top. Sizing-assay tests of tailing show that the glass top retained the coarser mineral better than does the standard riffling, but lost a greater part of the fine mineral.

Capacity ranges from 10 to 100 tons per deck, depending on the size and character of feed and whether in finishing or roughing service. The usual range of feed size is from 2-mm. to 0.15-mm. sands. Wash-water consumption is 3 to 10 gal. per min. per deck. Power consumption is about

0.75 hp. for single-deck and about 1 hp. for double-deck machines. The average SPEED recommended by the manufacturer is 215 @ 1- to 1.25-in. strokes per min.

Table 14. Performance of Deister No. 2 sand table with linoleum top and standard riffling and the same table with Schwarz glass top, both treating classified feed, all through 0.833-mm.

Test number	Standard riffling						
	Feed		Concentrate		Middling		Tailing
	Tons per 24 hr.	Assay, per cent. Cu	Tons per 24 hr.	Assay, per cent. Cu	Tons per 24 hr.	Assay, per cent. Cu	Assay, per cent. Cu
1	17.70	2.17	0.47	48.97	0.96	4.51	0.67
2	13.50	2.37	0.59	36.87	2.06	1.27	0.70

Test number	Glass top						
	Feed		Concentrate		Middling		Tailing
	Tons per 24 hr.	Assay, per cent. Cu	Tons per 24 hr.	Assay, per cent. Cu	Tons per 24 hr.	Assay, per cent. Cu	Assay, per cent. Cu
1	17.20	2.01	0.56	42.22	2.14	0.89	0.63
2	11.40	2.18	0.41	45.06	2.66	0.75	0.53

Note—In both tests the assay of −100-mesh tailing was higher on the glass top as was also the percentage of total copper contained in this size.

7. Deister slime table

The No. 3 table is shown in Fig. 16. The deck tilts toward the tailing side and also toward the head-motion end. The deck surface is in two distinct planes, that on the feed side of riffle (*a*) being higher than that on the tailing side and less-steeply sloped. Riffle cleat (*a*) and the cleat along the tailing-discharge side are ½ in. high while the other riffles are but $\frac{1}{16}$ to $\frac{1}{32}$ in. The deck covering is carried up along the back board on the head-motion end. The result is that relatively quiescent pools of pulp are formed at (*b*) and (*c*) in which the fine solids are given a chance to settle after which they progress up a grade of about ⅛ in. per ft. toward the concentrate end. The low riffles afford needed support for the finest concentrate and middling against the force of the wash water while tailing is carried in the current over the lowest riffle. The table is supported on extended ball-and-socket bearings.

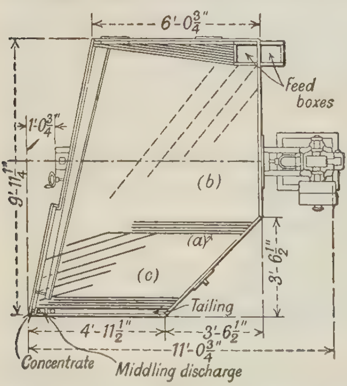


FIG. 16.—Deister simplex slime table.

The bearing seats along the feed and tailing sides are supported on sliding wedges, oppositely faced on the two sides and connected by suitable levers to a hand wheel, movement of which causes tilting. The head-motion is of the variety shown in Fig. 14, for single-deck tables, or of a heavier variety for double-deck tables. The table is rated RIGHT-HAND when the feed box is on the right of an observer facing the head-motion end and *vice versa*. The drive pulley is usually on the same side as the feed box.

Performance. Copper. At OHIO COPPER CO. a table treating primary-slime feed assaying 1.31 per cent. Cu, 90 per cent. −200-mesh, recovered 42 per cent. of the copper in a concentrate assaying 19.2 per cent. The feed rate was 5.6 tons per 24 hr.; speed, 265 @ 7⁄16-in. strokes per min.; deck, enameled linoleum. At BURRO MOUNTAIN concentrator of Phelps Dodge Co. flotation tailing was treated at the rate of 29 tons per 24 hr. making concentrate assaying 11 per cent. Cu and 16.5 per cent. insoluble and tailing assaying 0.47 per cent. total Cu, 0.26 per cent. oxide from feed containing 0.78 per cent. total Cu, 0.26 per cent. oxide. Table 15 shows performances in tests at NEVADA CONSOL. COPPER Co. Vanner recoveries corresponding to the second and third tests were 30 and 50 per cent. and the concentrates contained 41 and 28 per cent. insoluble respectively. **Lead.** At the Morning mill of the FEDERAL MINING AND SMELTING Co. this table treated a feed assaying 10.8 per cent. Pb at the rate of 4.3 tons per 24 hr. Speed, 275 to 295 @ 5⁄8- to 1⁄2-in. strokes respectively per min. Concentrate averaged 55 per cent. Pb and recovery was about 72 per cent. **Complex lead-zinc-silver ore.** At Midvale (Utah) plant of U. S. S. R. & M. Co. the table treated 4.5 tons per 24 hr. per deck of material all passing 200-mesh, in a pulp containing 75 to 80 per cent. water. Speed, 258 @ 3⁄8- to 1⁄2-in. strokes per min. Assays of feed and products are given in Table 16. Power for double-deck table, 1 hp. Water consumption, 2.4 to 3.5 gal. per min. per deck. One man attended 10 machines and controlled operations by varying tilt and wash water. **Gold-pyrite.** Daily performances in a 13-day test on slime feed at the GOLDFIELD CONSOLIDATED COMBINATION mill are given in Table 17. There is no distinct relation in this series of tests between the operating variables and metallurgical results. At LIBERTY BELL a table roughing out concentrate treated 10 to 14 tons of 100-mesh feed per 24 hr., running at 240 @ 1⁄2-in. strokes per min. and consuming 24 gal. of water per min. One man attended 33 machines, controlling opeartions by means of tilt and wash water. At HEDLEY GOLD MINING Co. 50 tons of −200-

Table 15. Tests on No. 3 Deister slimer at Nevada Consolidated Copper Co.

Days run	Tons per 24 hr.	Character of feed	Assays, per cent.						Ratio of concentration	Recovery, per cent.
			Feed, Cu	Concentrate		Middling, Cu	Tailing, Cu	Slime, Cu		
				Cu	Insol.					
1	6.7	Finest slime..........	1.70	9.3	67.7	1.70	1.48	1.10	13.5	39
3	9.2	Slime-vanner feed.....	1.02	7.6	67.8	0.93	0.62	0.70	23.3	32
6	18.5	Smooth sand-vanner feed...............	0.89	10.5	58.3	2.03	0.46	0.65	47.7	29
2	.....	Wilfley middling......	1.13	13.5	54.3	2.15	0.85	0.63	28.5	46

Table 16. Performance of Deister No. 3 slimer at U. S. S. R. & M. Co., Midvale plant

Product	Au, ounce	Ag, ounces	Pb, per cent.	Zn, per cent.	Fe, per cent.	Cu, per cent.	Insoluble, per cent.
Feed...............	0.10	3.5	6.5	11.0	8.5	0.6	....
Lead concentrate.....	0.20	12.0	16.0	11.0	20.0	0.4	12
Zinc concentrate.....	0.10	4.0	6.0	26.0	14.0	1.0	17.0
Tailing.............	0.03	2.6	3.0	5.9	4.0	0.35	58.0

Table 17. Performance of Deister No. 3 slimer at Combination mill of Goldfield Consolidated Mining Co.

Day	1	2	3	4	5	6	7
Tons of solid feed per 24 hr.	4.9	4.7	7.2	9.5	8.3	8.8	8.2
Moisture in feed, per cent	83	75	76	79	83	81	81
Assay of feed, oz. Au per ton	1.06	0.895	1.025	1.09	0.90	1.30	1.06
Assay of conc., oz. Au per ton	17.02	12.52	16.53	27.00	19.27	22.80	19.90
Assay of tailing, oz. Au per ton	0.60	0.52	0.565	0.66	0.555	0.755	0.66
Per cent. extraction by assays	45.0	41.8	46.5	40.3	39.4	43.4	39.0
Water, gallons per minute	1.1	1.4	1.7	1.2	1.1	1.3	1.2
Number of strokes per minute	290	290	290	290	290	290	290
Length of stroke, in.	7/16	½	½	½	½	½	½
Longitudinal tilt, inches in length of deck	2.5	3	3	3	3.5	4	2.5

Day	8	9	10	11	12	13	Arith. aver.
Tons of solid feed per 24 hr.	11.4	8.4	8.6	9.6	10.4	15.6	8.9
Moisture in feed, per cent	82	85	78	80	74	74	78
Assay of feed, oz. Au per ton	1.08	0.83	0.96	1.14	0.65	0.85	0.987
Assay of conc., oz. Au per ton	16.46	11.45	12.85	11.80	6.11	13.35	15.9
Assay of tailing, oz. Au per ton	0.68	0.545	0.63	0.68	0.435	0.53	0.60
Per cent. extraction by assays	38.6	36.1	36.1	42.7	35.7	39.2	40.7
Water, gallons per minute	....	....	....	....	....	....	1.3
Number of strokes per minute	290	288	290	290	295	310	....
Length of stroke, in.	½	½	½	½	7/16	7/16	....
Longitudinal tilt, inches in length of deck	2.5	2.5	2.5	2.5	2.5	2.5	....

mesh slime was treated per 24 hr. on 12 machines run at 190 r.p.m. Power, 0.8 hp. per machine; water consumption, 1.7 gal. per min. One man attended 12 tables and 24 vanners. **Tungsten.** Tungsten Mines Co. treated 10 tons per deck per 24 hr. of a mixed feed consisting in part of cone-spigot discharge from original slimes and in part of the overflow of a hydraulic classifier grading tailing and middling from other tables and vanners. Feed contained 90 per cent. water; wash-water consumption was 8.3 gal. per min. for two decks; power consumption, 1.5 hp.; speed, 280 @ ⅜-in. strokes per min. One man attended 10 machines, controlling wash water only. Assays, per cent. WO_3: Feed, 0.5 to 1.0; concentrate, 40 to 60; tailing, 0.3.

Feed-pulp consistency. The effect of pulp consistency on capacity and saving of fine mineral was investigated by Bland (*107 J 1116*) with the result given in Table 18. In the

Table 18. Deister No. 3 slimer: Effect of feed-pulp consistency. (Feed all through 200-mesh)

Feed-pulp consistency, per cent solids.	Per cent. of concentrate passing 0.0125-mm. aperture	Assay of concentrate, per cent. WO_3	Feed-rate, tons per 24 hr.
29	40	61	2.4
25	34	59	3.9
20	21	62	5.1
17	18	58	7.2
14	6.4	63	9.6
11	7	64	4.8
9	4.5	61	9.6
8	0	63	4.8
6	0	59	12.0

tests with thick pulp, only a small portion of the deck was used, hence Bland recommends working with thick pulp with resulting small capacity per deck, cutting off the unused

portion of the deck and multiplying decks in order to save power and floor space and thus compensate for decreased capacity. He also recommends close control of pulp dilution in order to insure regular operation and advocates dry sizing of feed to effect this, citing practice at New Jersey Zinc Co. (See Sec. 2).

8. Deister-Overstrom diagonal-deck table

This table has a roughly rhomboidal deck, shaken substantially in the line of the short diagonal, and riffled parallel to the direction of motion. The deck surface is in one plane. Two sizes of deck are used (Fig. 17), a long and narrow one for roughing service and a shorter and broader deck for finishing. The shape is varied on the theory that on the coarse sizes the longitudinal component of travel is proportionally greater and greater length must, therefore, be afforded to permit sufficient washing, while on fine material the transverse travel is proportionally most rapid and width of deck must be furnished to allow the reciprocating motion time to move concentrate to the concentrate-discharge end. The long table is made narrow in order to afford prompt removal of tailing while permitting the table to be run on a flat slope. The concentrate end of all tables is cut off on a backward bevel so that wash water from the water box will flow over the concentrate end and do away with a spray pipe along this end. The deck is inclined against the travel of concentrate, 0.75 to 1.25 in. in its length for sand feeds and 0.25 to 0.5 in. for slimes. Tapered riffle cleats of the usual dimensions (0.25 in. wide ranging up to 1 in. in depth at the head-motion end, according to the service) are used on roughing- and sand-table decks; pool riffling, effected by the addition of a number of broad V-topped riffle cleats, are used on the slime table. In south-eastern Missouri grooving has been substituted for riffle cleats in some places; grooves tapering from ⅛ to $\frac{1}{32}$ in. on the coarse-sand tables, ⅛ in. to nothing on the intermediate tables and $\frac{1}{16}$ in. to nothing on slime tables.

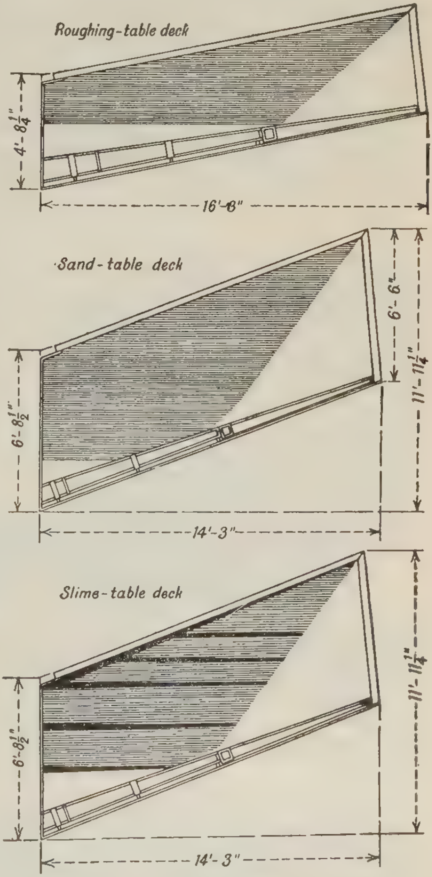


Fig. 17.—Deister-Overstrom table.

It is endeavored to make the depth of grooves at the discharge end equal to the diameter of the gangue particles to be separated.

The deck is supported on slide bearings carried on a tilting frame of light structural steel which, in turn, rests on a heavy, built-up through member that carries the head-motion. Transverse tilting is effected by sliding wedges actuated by a hand-wheel. The head-motion is usually the same as shown in Fig. 14. As a result of testing work at Bunker Hill and Sullivan, Caetani (*3 MM 50*) brands this as the best of all the head-motions tested and Robie (*116 J 403*), writing about the tables at the Northern Ore Co. also commends it.

The shape of the table deck results in increase in the cleaning area along the diagonal separating line and longer time of treatment of partially enriched material. Long narrow decks require long feed boxes.

In south-eastern Missouri practice they are made one-third the total length of the deck on coarse-sand tables, one-half on intermediate-sand and three-fifths on slime.

Performance at several mills is shown in Table 19.

Table 19. Performance of Deister-Overstrom tables

Mill	Ore	Type of deck	Tons feed per 24 hr.	Size of feed	Revolutions per minute	Length of stroke, inches
Phelps-Dodge, Morenci	Copper	*R*-1	45	*a*	256	¾
Chino Consolidated Copper Co.	Copper	*S*-1	25–35	......	224	1⅛
American Zinc, Lead & Smelting Co.	Zinc	*R*-1	14	*c*	295	⅞
S. E. Missouri	Lead	*R*-1	......	*e*	210	¾–1
S. E. Missouri	Lead	*S*-1	......	......	240–245	⅝–¾
S. E. Missouri	Lead	*Sl*-1	10–12	......	285	½
U. S. S. R. & M. Co., Midvale	Complex	*S*-1	*g*	......	243	⅝
Tungsten Mines Co.	Tungsten	*S*-1	85	*h*	240	1

Mill	Wash water, gallons per minute	Horsepower	Attendance, machines per man	Assays, per cent.			
				Feed	Concentrate	Tailing	Middling
Phelps-Dodge, Morenci	10	0.5	23	1.15	11	0.57	0.8
Chino Consolidated Copper Co.	10	1.0	35	......	......	......	......
American Zinc, Lead & Smelting Co.	8	1.0*d*	44	......	......	......	......
S. E. Missouri	......	......	......	......	......	......	......
S. E. Missouri	......	......	......	......	......	......	......
S. E. Missouri	......	......	......	......	73	......	......
U. S. S. R. & M. Co., Midvale	9–20	0.75	20	......	*f*	......	......
Tungsten Mines Co.	5.5	2	10	0.3	58	0.1–0.14	2

a +1.168-mm., 0.15 per cent.; +0.833, 6.34 per cent.; +0.417, 14.08; +0.295, 24.20; +0.208, 28.05; +0.147, 15.56; +0.104, 7.25; +0.074, 1.20; −0.074, 3.17. *c* Through 20-mesh, de-slimed. *d* Installed. *e* First spigot of hydraulic classifier treating feed through 3-mm. screen. *f* Same as Wilfley. See Table 1. *g* 35 tons 10-mesh sands to 10 tons 100-mesh sands. *h* 20-mesh middling from roughing table. *R*-1 Single-deck, rougher. *S*-1 Single-deck, sand. *Sl*-1 Single-deck, slime.

Feed-pulp consistency ranges from 60 to 80 per cent. water. In south-eastern Missouri work it was found (*96 J 57*) that pulp consistency was an important factor in obtaining a proper bed on the table, and the best consistencies were 10 : 1 (by volume) for coarse feed, 12 or 15 : 1 for medium sands, and 15 or 18 : 1 for slime. Results of test work at Connecticut Zinc Corporation are given in Table 20. It is interesting to note

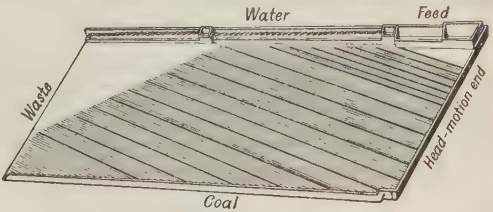
Table 20. Performance of Deister-Overstrom table at Connecticut Zinc Co., Oronogo, Mo.

Test number	Tons per 24 hr.	Feed		Concentrate		Tailing		Middling	
		Per cent. Zn	Per cent. Pb	Per cent. Zn	Per cent. Pb	Per cent. Zn	Per cent. Pb	Per cent. Zn	Per cent. Pb
1	25.8	3.83	0.094	57.70	0.85	2.39	0.065	7.74	0.071
2	29	3.75	0.061	57.60	0.33	1.16	0	2.70	0
3	16.3	3.23	0.055	58.10	0.43	1.20	0	5.24	0.090
4	40.7	3.83	0.105	58.80	0.42	2.16	0.021	4.04	0.050
5	48	3.64	0.065	59.40	0.21	1.64	0	8.82	0.064

Notes—Test 1, Feed ranged from "very coarse" to slime. Test 2, Feed coarse. Test 3, Typical feed, mixed coarse and fine. −60-mesh material in feed assayed 5.64 per cent. Zn, tailing 2.46 per cent., concentrate 58.10 per cent. Lead concentrate in this test assayed 61.30 per cent. Pb, 9.29 per cent. Fe and 6.20 per cent. Zn. Adjustment difficult, concentrate streak wandered badly. Test 5, −60-mesh in feed assayed 5.10 per cent. Zn, in tailing 2.84 per cent. and in concentrate 60.60 per cent. Wash water, 6.25 gallons per min. Continuous shifting of concentrate streak, requiring much adjustment. Tonnage of middling return per 24 hr., 21; length of middling cut, 73 in.

in connection with the assays of the −60-mesh material that the fine size in the concentrate was considerably richer than the coarse while the reverse was true on a competing table, which, however, used but half the quantity of wash water and made a lower-grade tailing. The use of the relatively large amount of wash water to clean fine gangue out of the concentrate resulted in also sweeping fine blende into middling and tailing.

Deister-Overstrom coal-washing table is similar to the ore-concentrating table, but has an extra-large deck, 16 ft. 10 in. long by 8 ft. 4 in. wide, riffled as shown in Fig. 18. It treats screenings as coarse as −0.5-in. and as fine as −⅛-in. The deck is tilted longitudinally and is 2 to 2.5 in. higher at the waste end for coarse feeds, ⅞ in. for fine. The average transverse tilt is about ⅞-in. per ft. with coarse feed and ⅝-in. with fine. Capacity is claimed to be 8.5 to 10 tons of feed per hour with a water consumption of 1.8 tons per ton of feed. Speed, 245 to 260 @ 1- to 0.75-in. strokes per min. Power consumption, 1.5 hp. This table, a large-size Wilfley table (MASSCO) and a large-size Plat-O table, are being used to an increasing extent for washing the finer sizes in both anthracite and bituminous-coal washeries.



The high cleats (black) in the upper portion of the deck are ¾ in. deep, tapering to 0 in. The other high cleats are ⅝ in. at the head-motion end and taper to 0 in. All low cleats are carried ¼ in. until the plane of the upper surfaces intersects the plane of the tops of the ⅝-in. cleats, when all taper alike.

FIG. 18.—Riffling of Deister-Overstrom deck for coal washing.

Griffin (*26 CA 217*) gives Table 21 as representative of the capacity of the table on anthracite. The lesser figures in each case correspond to an operation in which maximum ash removal is attempted. In one breaker the capacity of a table on barley size was 50 to 60 tons per day (8 hr.) when producing cleaned coal containing 10 to 11 per cent. ash; when the feed rate was increased to 85 tons per day the ash content of the cleaned coal ran up to 12 to 14 per cent. Table 22 shows the performance of the table when treating dif-

Table 21. Capacity of Deister-Overstrom table on anthracite

Size of coal	Tons of feed per hour per table
No. 1 buckwheat	15–20
No. 2 buckwheat (rice)	12–18
No. 3 buckwheat (barley)	6–14
No. 4 buckwheat	4– 6
Slush	2– 5

ferent sizes of anthracite in the same breaker. At another breaker six tables treating No. 4 buckwheat produced 200 tons per day of cleaned coal containing 13 to 14 per cent. ash from a feed containing 25 to 30 per cent. ash. Another table treating two tons per hour of −3/64-in. +20-mesh (No. 4 buckwheat) containing 29 per cent. ash produced clean coal that averaged 9.5 per cent. ash for one month's operations. Treating anthracite at Hudson Coal Co. the tables have a capacity (*66 A 422*) of 10 to 12 tons per hr. on No. 1 buckwheat down to 3 to 4 tons an hour on slush (−3/64-in.) The tables are run at about 275 @ 3/4-in. strokes per min. Table 23 shows average results when treating de-slimed material passing 3/64-in. round-hole screens and sizing 88 per cent. on 60-mesh, 8 per cent. on 100-mesh, 3 per cent. on 200- and 1 per cent. −200-mesh. At one of the Madeira-hill breakers six tables treating rice and barley coal handled 400 tons per 8 hr., made a coal product containing 12.5 to 13 per cent. slate, and a slate product containing 9.25 per cent. coal. The respective calorific values were 12,663 and 3636 B.t.u. per lb. Water consumption was two tons per ton of coal. Ashmead estimates labor and power cost at $0.04 per ton with power at $0.015 per kw.hr. and labor at $0.50 per hr., and total cost at $0.10 per ton. At Granby Consolidated M. S. & P. Co. (*21 CA 368*) Deister-Overstrom tables treated 4 to 6 tons per hr. of −3/16-in. slack from jig-washed bituminous coal and 6 to 7 tons per hr. of washed-coal conveyor drainings. Performance is shown in Table 24. Table 25 shows the results of a number of tests on a quarter-size laboratory table treating Washington bituminous coal and coal-washery products (*Bul. 28 UW*). The tests on −3/8-in. raw coal are all unpromising. The tests on −3/16-in. raw-coal screenings are better, but whether because the table treats this size better or because the ash was more completely freed is not apparent. Probably both facts aided. Some of the tests on refuse showed some promise for the production of mine fuel, but not for salable coal. At the Renton Coal Co., Renton, Wash. (*19 CA 741*) five Deister-Overstrom coal-washing tables treating −3/16-in. reclaimed subbituminous sludge handled approximately 40 tons of feed per hour and reduced the ash from 28 or 30 per cent. to 16 or 18 per cent. with high recovery. Reduction to 10 per cent. is possible by putting more coal into the refuse.

Table 22. Performances of Deister-Overstrom table on different sizes of anthracite

Size of coal	Pea	No. 1 buckwheat	Rice	Barley
Feed rate, tons per hour	20.2	19.3	18.0	14.4
Clean coal produced, tons per hour	13.2	11.2	12.1	10.5
Ash in feed, per cent.	38.5*a*	43.3	35.8	33.5
Ash in cleaned coal, per cent.	5.7*a*	15.4	15.0	17.5
Ash in refuse, per cent.	....	80.6	78.2	74.7
Coal in refuse, per cent.	2.0	....	....	....
Recovery of combustible, per cent.	99 −	85.4	88.8	89.3

a Refuse.

Table 23. Performance of Deister-Overstrom No. 7 table at Hudson Coal Co.

Material	Tons per hour	Ash, per cent.	S, per cent.	Distribution, per cent.			
				Solids	Combustible	Ash	S
Feed	3.41	28.0	1.71	100.0	100.0	100.0	100.0
Washed coal	2.43	13.0	0.79	71.3	86.1	33.2	32.8
Slate product	0.93	65.0	1.91	27.2	13.2	66.2	33.2
Pyrite	0.05	70.0	42.00	1.5	0.7	3.6	36.8

Ash reduction, 53.5 per cent.; sulphur reduction, 53.8 per cent.

Table 24. Performance of Deister-Overstrom tables treating bituminous coal at Granby Consolidated M. S. & P. Co. (*After Garman*)

Material	Ash, per cent.	Float		Sink	
		Per cent. weight	Per cent. ash	Per cent. weight	Per cent. ash
Tables treating washed jig slack					
Feed	....	59.00	10.00	41.00	40.00
Washed coal	....	71.00	9.50	29.00	31.32
Refuse *a*	....	4.00	14.65	96.00	46.60
Tables treating drainings from washed-coal conveyor					
Feed	21	69.0	12.50	31.0	41.49
Washed coal	14	70.0	11.10	30.0	35.20
Refuse	45	1.5	20.67	98.5	53.73

a Equivalent to 5.4 per cent. of feed.

Shaking tables will treat coal as coarse as $-\frac{3}{4}$-in. but performance is better on $-\frac{3}{16}$-in. or smaller. Anthracite feeds must, in general, be smaller than bituminous. For treating fine sizes, power and water consumption per ton treated are both markedly less than that of jigs.

Empire table is a rectangular-deck diagonally-riffled table with 4 × 10-ft. deck suspended by means of 3-ft. rods from an overhanging frame. The rods are inclined from the vertical as in the Ferraris screen and the head-motion is a simple eccentric. The stroke is exceptionally long, ranging from 2 to 5 in. and correspondingly slow, 135 to 160 per min. The long, slow stroke keeps the bed loose and gives high capacity. The long stroke and coarse feed cause tremendous wear on riffle cleats; Wiard (*112 J 417*) says that ordinary linoleum and soft-wood cleats last only one to two weeks in such service. Gross (*102 J 428*) states that the table has been operated to give high capacity on 0.5-in. feed; it will not handle slime (*107 J 453*). At St. Joseph Lead Co., Bonne Terre mill, a table making 168 @ 3.5-in. strokes per min. treated 65 tons per day of de-slimed − 10-mesh feed (6.5 per cent. on 14-mesh, 13.2 per cent. through 100-mesh) containing 50 per cent. water. Power consumption, 0.5 hp.; wash water, 10.5 gal. per min. Assays; per cent. Pb: Feed, 8.0; tailing, 0.4; concentrate, 78.0; middling, 9.0. Gross also describes the treatment of 30-mesh gold-silver ore at the rate of 37 tons per 24 hr. on a 4 × 10-ft. deck making 135 @ 5-in. strokes per min. Power consumption, 0.5 hp.; wash water, 10 gal. per min. Assays: Feed, 0.2 oz. Au, 8.4 oz. Ag; tailing 0.03 oz. Au, 1.9 oz. Ag; concentrate, 0.54 oz. Au, 21.4 oz. Ag.

9. Garfield table

This is essentially a Wilfley table with riffles carried straight across from head-motion to concentrate end over the full surface of the table. It is used exclusively for roughing service.

Construction. When specially built and not merely a Wilfley table with modified riffling, the deck is made rectangular, 4 ft. × 12 ft., and the head-motion, although of the same type as the Wilfley, is heavier in order to handle the heavier pulp loads. The table is frequently double-decked, the additional deck carried about 8 in. above the regular deck on heavy cast-iron stanchions attached to the lower deck. Head-motion for double-deck tables is twice the size and several times heavier and more rugged than the Wilfley motion. The riffles are deeper and wider than the Wilfley. At Utah Copper Co. (*112 J 415*) riffle cleats were 1.5 in. face to face, $\frac{3}{4}$ in. deep at the head end tapering to $\frac{1}{8}$ in. at the line corresponding to the Wilfley diagonal, and extended thence to the concentrate side at uniform depth. The bottom face of the riffle cleats is beveled to allow the side faces to stand vertically at the steep inclination of the table. Riffle cleats are usually hardwood, maple or oak. Linoleum is the commonest deck covering but wears rapidly and in some mills cast-iron plates with riffles cast on are let in near the feed corner to take the excessive wear at this point.

Table 25. Performance of laboratory Deister-Overstrom table on bituminous coal and coal-washery products

Mine	Feed		Washed coal, per cent.		Middling, per cent.		Refuse, per cent.		Recovery of combustible, per cent.	Ash, reduction, per cent.	Stroke	
	Character	Per cent. ash	Weight	Ash	Weight	Ash	Weight	Ash			Length, inches	Number per minute
Renton	– 3/8-in. raw-coal screenings	22.5	57.3	16.9	.....	.....	42.7	30.0	61.4	21.3	5/8	300
Renton	– 3/8-in. raw-coal screenings	27.4	66.8	16.9	14.1	32.3	19.1	60.5	75.4a	38.3a	5/8	300
Renton	– 3/16-in. raw-coal screenings	27.1	50.4	12.4	.....	.....	49.6	44.0	64.3	54.2	3/16	292
Renton	– 3/16-in. raw-coal screenings	27.1	45.1	12.5	20.8	26.5	34.1	46.9	54.4a	53.9a	5/8	292
Renton	– 1/16-in. from sludge dump	27.1	45.8	11.4	16.7	26.4	37.5	46.6	55.7a	57.9a	1/2	292
Issaquah	– 3/8-in. raw-coal screenings	36.3	31.0	17.2	30.1	32.6	38.9	54.3	40.3a	52.6a	1/2	292
Issaquah	– 3/16-in. raw-coal screenings	19.3	48.6	9.9	48.6	25.5	2.8	62.5	54.3a	48.7a	5/8	293
Issaquah	– 1/8-in. washed-coal screenings	19.5	53.7	8.7	.....	.....	46.3	20.5	9.6	55.4	5/8	293
Grand Ridge	– 1/4-in. jig refuse	15.1	92.1	11.7	.....	.....	7.9	52.5	95.3	22.5	5/8	293
Pocahontas	– 3/16-in. raw-coal screenings	47.0	38.2	17.6	4.8	33.2	57.0	65.2	59.4a	62.5a	5/8	293
Wilkeson	– 1/8-in. refuse screenings	23.3	66.1	12.1	.....	.....	33.9	43.7	74.0	48.0	5/8	293
Wilkeson	– 3/8-in. raw-coal screenings	68.3	13.8	29.6	.....	.....	86.2	74.5	30.6	56.6	.....	.....
Wilkeson	– 3/16-in. raw-coal screenings	23.2	67.4	11.5	22.4	27.0	10.2	62.3	77.6a	50.4a	.....	.....
Wilkeson	– 3/8-in. raw-coal screenings crushed through 3/16-in.	22.5	74.1	11.6	.....	.....	25.9	49.3	81.1	48.5	.....	.....
Wilkeson	– 3/8-in. raw-coal screenings crushed through 1/8-in.	23.2	72.0	11.5	13.2	26.2	14.8	65.6	83.0a	50.4a	.....	.....
Wilkeson	– 3/16-in. crushed refuse	23.2	77.9	12.0	2.8	24.1	19.3	55.9	89.3a	48.3a	.....	.....
Wilkeson	– 3/16-in. crushed refuse	55.8	28.7	23.7	.....	.....	71.3	68.8	49.8	57.5	.....	.....
Mendota	– 3/16-in. raw-coal screenings	44.6	53.8	27.9	.....	.....	46.2	66.3	73.5	37.4	.....	.....
		20.2	73.3	12.1	10.0	22.2	16.7	54.5	80.7a	40.1a	5/8	300

a Middling reckoned as tailing.

Performances are given in Table 26. Average capacity on −2-mm. feed is close to 125 tons per deck per 24 hr. with 245 to 250 @ 1-in. strokes per min. Table 27 shows the effect

Table 26. Performance of Garfield tables

Mill	Kind of ore	Number of decks	Size of feed	Riffle cleats
Butte & Superior	Zinc	2	*a*	Maple
Chino Cons. Copper Co.	Copper	1	*a*	....
Ray Cons. Copper Co.(*b*)	Copper	1	*a*	Sugar pine
Ray Cons. Copper Co.(*c*)	Copper	1	*a*	Sugar pine
Ray Cons. Copper Co.(*d*)	Copper	1	....	Sugar pine
Utah Copper Co.	Copper	2	12-mesh	Hardwood
Alaska Gastineau	Auriferous pyrite	2	*a*	Fir

Mill	Speed, revolutions per minute	Length of stroke, inches	Tons per 24 hr. per deck	Wash water, gallons per minute	Horsepower	Per cent. water in feed
Butte & Superior	256	7/8	125	....	1.5	....
Chino Cons. Copper Co.	260	1 1/8	200	3.4	1	60
Ray Cons. Copper Co.(*b*)	245	1	125	6.2	1	50
Ray Cons. Copper Co.(*c*)	245	1	125	10.5	1	50
Ray Cons. Copper Co.(*d*)	....	....	65	3.5	0.75	60
Utah Copper Co.	....	....	100	....	....	....
Alaska Gastineau	251	1	150	12	1	58

Mill	Assays, per cent.				Recovery, per cent.	Attendance machines per man
	Feed	Conc.	Tailing	Middling		
Butte & Superior	15.1	41.4	6.6	32	....	48
Chino Cons. Copper Co.	1.88	6.44*e*	1.32	....	37.4	35
Ray Cons. Copper Co.(*b*)	1.26	4.20	0.84	....	41.7	40
Ray Cons. Copper Co.(*c*)	0.97	5.0	0.53	....	50.7	40
Ray Cons. Copper Co.(*d*)	0.72	6.0	0.45	....	40.8	....
Utah Copper Co.	....	....	....	....	....	....
Alaska Gastineau	$1.25	$13.15	$0.347	....	73.8	80

a See Table 26*a*. *b* Primary. *c* "Mill." *d* Secondary.

Table 26*a*. Sizing tests of feed to Garfield tables in Table 26

Screen aperture, mm.	Per cent. weight on screen				
	Butte & Superior	Chino Consolidated Copper Co.	Ray Consolidated Copper Co., "Primary"	Ray Consolidated Copper Co., "Mill"	Alaska Gastineau
2.362	....	....	2.15	....	....
1.651	6.79	3.8	12.58	....	7.08
1.168	19.86	7.3	11.54	....	....
0.833	17.70	8.3	9.95	2.92	30.94
0.589	12.61	8.3	11.23	13.91	11.70
0.417	11.21	7.9	9.80	11.33	4.84
0.295	7.49	5.5	5.89	6.87	5.56
0.208	5.13	5.8	5.10	4.64	5.60
0.147	4.08	6.4	5.17	6.44	5.58
0.104	3.22	5.6	2.87	3.43	1.88
0.074	1.15	3.4	2.71	3.60	5.16
−0.074	10.76	37.7	21.01	46.86	21.68

Table 27. Effect of tonnage on Garfield-table performance, Ray Consolidated Copper Co.

Size, mesh	Feed								
	Weight, per cent.			Assay, per cent. Cu			Per cent. total Cu		
	1	2	3	1	2	3	1	2	3
20	37.3	42.3	43.7	1.68	1.75	1.86	28.9	33.7	34.1
30	18.0	16.5	14.7	1.96	2.10	2.26	16.3	15.6	14.0
40	8.0	7.7	5.5	2.38	2.51	2.62	8.8	7.9	6.1
50	5.4	4.4	6.4	2.77	2.80	2.97	6.9	5.5	8.0
60	1.9	1.9	1.9	2.79	2.88	3.26	2.4	2.5	2.6
70	3.3	2.8	3.1	3.04	3.26	3.41	4.6	4.1	4.4
80	0.2	0.2	0.4	3.00	2.78	3.53	0.3	0.3	0.6
100	2.9	2.5	2.6	3.22	3.51	3.87	4.3	4.0	4.3
120	1.3	1.1	1.1	3.66	3.71	3.69	2.2	1.8	1.7
150	2.5	2.4	2.4	3.63	3.78	4.04	4.2	4.1	4.1
200	0.8	1.1	1.1	3.03	3.30	3.07	1.1	1.6	1.4
−200	18.4	17.1	17.1	2.35	2.45	2.61	20.0	18.9	18.7
Totals.......	100.0	100.0	100.0	2.10	2.28	2.38	100.0	100.0	100.0

Size, mesh	Concentrate								
	Weight, per cent.			Weight, per cent. Cu			Per cent. total Cu		
	1	2	3	1	2	3	1	2	3
20	25.5	25.6	24.4	10.00	7.70	8.98	23.7	25.8	27.9
30	18.4	23.0	20.7	8.46	5.64	5.75	14.5	17.1	15.1
40	12.1	13.7	14.3	8.19	5.34	5.30	9.2	9.6	9.7
50	11.0	11.8	12.1	8.84	6.00	5.63	9.0	9.3	8.7
60	3.5	3.0	3.5	9.81	6.78	6.22	3.2	2.6	2.8
70	8.4	7.7	8.5	10.41	7.83	7.07	8.2	7.9	7.7
80	0.7	0.8	0.8	10.41	9.10	6.90	0.7	0.9	0.7
100	6.8	5.8	6.4	12.53	10.17	8.83	8.0	7.8	7.3
120	0.9	0.9	1.4	15.98	10.23	10.81	1.3	1.2	1.9
150	2.2	1.3	1.0	14.72	13.08	11.33	3.0	2.2	1.4
200	4.5	3.0	3.4	17.13	15.90	15.41	7.2	6.3	6.7
−200	6.0	3.4	3.5	21.30	20.72	22.24	12.0	9.3	10.1
Totals.......	100.0	100.0	100.0	10.80	7.85	7.72	100.0	100.0	100.0
Per cent. of original....	8.8	12.7	13.7	....	....	....	....	....	....

Size, mesh	Tailing									Recovery, per cent.		
	Weight, per cent.			Assay, per cent. Cu			Per cent. total Cu					
	1	2	3	1	2	3	1	2	3	1	2	3
20	42.2	47.5	47.8	1.21	1.17	1.32	34.8	41.0	46.2	35.8	33.8	37.0
30	16.1	14.7	14.2	1.40	1.27	1.27	15.4	13.8	13.2	38.8	47.6	49.1
40	5.5	4.7	5.6	1.68	1.25	1.23	6.3	4.4	5.0	45.7	48.1	72.0
50	5.8	4.8	3.5	1.87	1.23	1.15	7.4	4.4	2.9	57.2	72.9	49.1
60	1.6	1.3	1.4	1.51	1.13	1.04	1.6	1.1	1.0	57.1	47.1	48.2
70	2.6	2.0	2.2	1.51	1.06	0.97	2.7	1.6	1.5	76.7	84.0	77.7
80	0.4	0.3	0.1	1.69	1.01	0.76	0.5	0.2	0.1	.....	.....	.....
100	2.3	1.8	2.1	1.50	1.06	0.85	2.4	1.4	1.3	80.2	85.4	76.8
120	0.9	0.7	1.1	1.58	1.05	0.85	1.0	0.5	0.7	26.6	} 25.3	.....
150	2.2	2.1	2.3	1.63	1.10	0.91	2.5	1.7	1.5	31.4		16.0
200	0.9	0.8	0.8	1.47	1.05	0.87	0.9	0.7	0.5	28.0	16.7	21.3
−200	19.5	19.3	18.9	1.84	2.02	1.89	24.5	29.2	26.1	26.1	21.4	24.0
Totals......	100.0	100.0	100.0	1.48	1.37	1.33	100.0	100.0	100.0	45.3	43.7	44.4

Test 1, 129 tons per 24 hr. Test 2, 91 tons per 24 hr. Test 3, 65 tons per 24 hr.

of tonnage on performance. This test shows maximum recovery of grains between 0.15- and 0.2-mm. at all tonnages with a sharp drop in recovery in the sizes below 0.15-mm. Low recovery in the coarser sizes is apparently due to included grains. The efficiencies of the respective concentrating operations, based on copper contained in free mineral, were 73.3 per cent., 79.5 per cent. and 82.8 per cent. By increasing speed and length of stroke, CHINO raised capacity to 200 tons per deck without marked loss in recovery as compared with RAY results. Since tailing is to be re-ground in any case, such overcrowding of the table would seem justifiable. SIDE TILT in roughing is usually held constant and variations in feed conditions are taken care of by varying the WASH WATER. Consumption of wash water ranges from 3.5 to 12 gal. per min. Dirty water is usually used. POWER CONSUMPTION is slightly higher than for the Wilfley table on account of the heavier bed of solids on the table; power consumed by double-deck tables is 25 to 33 per cent. in excess of that for single deck.

10. Overstrom Universal table

This is a one-plane table, roughly rectangular in plan, about 18 ft. 6 in. by 5 or 6 ft., with diagonal riffling. The deck is supported on leaf springs of hickory set so that the long transverse axes center at a point on the center line of the head-motion shaft extended beyond the feed side, thus giving the deck a horizontal twisting motion. These springs are inclined backward slightly, as in Ferraris and Buss tables. The riffle cleats, however, are curved in such a manner that they are substantially tangent at any given point to the motion of the deck at that point.

Head-motion. Differential reciprocating motion is obtained by mounting an eccentrically-loaded pulley loosely on a shaft carried on extended deck sills. Motion resulting from revolution of the pulley is constrained to the horizontal by gravity and the spring supports; horizontal movement is stopped by a bumper at the forward end of the stroke and controlled by a spring on the back stroke. Length of stroke is controlled by a canvas pad of variable thickness inserted between a bumping strut on the deck frame and the bumping post. Sharpness is controlled to some extent by varying the spring tension. SIDE TILT is adjusted by a screw-controlled multiple wedge under the feed side. POWER CONSUMPTION varies between about 0.25 hp. light and 0.75 hp. loaded, average is about 0.5 hp. CAPACITY is claimed to range from 20 to 300 tons per 24 hr. according to the character of feed and kind of service.

Performance. Results of a test at NEVADA CONSOLIDATED COPPER CO. are given in Table 28 and of another at RAY CONSOLIDATED COPPER CO. in Table 29. At the latter

Table 28. Performance of Overstrom Universal table at Nevada Consolidated Copper Co.

Test number	Feed (*a*)			Concentrate, per cent. Cu	Tailing, per cent. Cu	Recovery, per cent.
	Tons per 24 hr.	Per cent. solids	Per cent. Cu.			
1	39	18	2.92	11.10	1.31	62.5
2	36	17	2.47	9.22	1.05	64.8
3	82	21	2.37	7.31	1.06	64.7

a Middling from Wilfley tables treating primary rougher concentrate from 10-mesh feed.

plant the table was clearly overloaded when the feed rate exceeded 200 tons per 24 hr. Table 30 presents performance on classified feed at UTAH COPPER CO. Classifier feed all passed 28-mesh. At CHINO COPPER CO., in two 19-day tests, treating Garfield-table rough concentrate assaying 4.49 per cent. Cu and 3.95 per cent. Cu respectively in the two periods, the table made concentrates averaging 11.78 per cent. Cu, 34.7 per cent. Fe, 34.7 per cent. insoluble and 21.86 per cent. Cu, 29.28 per cent. Fe and 16.6 per cent. insoluble respectively; tailing 1.91 per cent. and 1.98 per cent. Cu, representing recoveries of 55.7 per cent. and 49.6 per cent. At the same mill, in similar test periods, treating the combined products of the fourth and fifth spigots of a Richards-Janney classifier, assaying, for the two periods 1.42 and 1.51 per cent. Cu respectively, the table made corresponding concentrate assaying 9.81 per cent. Cu, 29.16 per cent. Fe, 31.8 per cent. insoluble and

Table 29. Performance of Overstrom Universal table at Ray Consolidated Copper Co.

Test number	Tons per 24 hr.	Assays, per cent. Cu			Recovery, per cent.
		Feed	Concentrate	Tailing	
1	155	1.17	12.9	0.89	27.7
2	169	1.26	12.7	0.94	27.4
3	172	1.25	13.7	0.97	24.1
4	199	1.16	12.9	0.95	19.5
5	210	1.39	15.3	1.23	12.5
6	217	1.37	14.7	1.17	15.8
7	319	1.45	14.4	1.14	23.2

Note—Feed is − 5-mm. impact screens.

Table 30. Performance of Overstrom Universal table on classified feed at Arthur plant, U. C. Co. (1917)

Legend	Spigot number					
	1	2	3	4	5	Average
Average tons per day	21	22	17	18	34	22
Feed, per cent. Cu	1.00	1.20	0.99	1.29	1.28	1.17
Feed, per cent. Fe	2.46	2.06	1.88	1.86	1.39	1.87
Tailing, per cent. Cu	0.35	0.34	0.30	0.28	0.25	0.30
Middling, tons	3	3	4	4	6	4
Middling, per cent. Cu	0.84	1.12	2.00	0.73	0.70	1.08
Concentrate, per cent. Cu	14.63	13.02	17.30	25.38	24.47	18.96
Concentrate, per cent. Fe	29.39	24.39	26.97	24.18	21.66	25.32
Concentrate, per cent. insoluble	15.80	27.76	18.72	15.29	22.22	19.96
Ratio of concentration	21.87	14.78	24.85	24.90	23.40	21.43
Recovery, per cent	66.83	73.54	70.54	79.15	81.56	75.70

12.57 per cent. Cu, 22.5 per cent. Fe and 39.9 per cent. insoluble; and tailing assaying 0.57 per cent. Cu and 0.65 per cent. respectively, representing recoveries of 56.8 and 54 per cent. At Detroit Copper Co. (Phelps-Dodge, Morenci), treating de-slimed sand (0.2 per cent. on 8-mesh, 5.69 per cent. through 65-mesh) at an average rate of 70 tons per 24 hr. the table made concentrate assaying 9.06 per cent. Cu, 23.9 per cent. insoluble and tailing carrying 0.51 per cent. Cu from feed assaying 1.02 per cent. Cu. On finer sand (0.14 per cent on 20-mesh, 13.1 per cent. through 100-mesh) average tonnage per 24 hr. was 47; average assays: Feed, 0.87 per cent. Cu; middling 0.74 per cent.; tailing, 0.41 per cent.; concentrate, 11.6 per cent. Cu, 27.2 per cent. insoluble. At Old Dominion Copper Co., treating flotation tailing at an average rate of 39 tons per 24 hr. in a pulp containing 20 per cent. solids, assays were as follows: Feed, 0.35 per cent. Cu; tailing, 0.32 per cent.; middling, 0.43 per cent.; concentrate, 4.65 per cent. Cu; 23.2 per cent. insoluble. This table was operated at 265 @ 11⁄16-in. strokes per min. and used 3.8 gal. wash water per min.

11. Plat-O table

This is a substantially rectangular diagonally-riffled table with deck about 14 ft. long by 6 ft. wide at the head-motion end and 5 ft. wide at the concentrate end. The deck is in two planes joined by a sloping surface along the separating diagonal, the higher plane being at the concentrate end. For some conditions three planes or plateaus are used (Fig. 19). The methods of riffling are various, according to the character of the feed, but in general the riffle cleats are wide with relatively narrow spaces. They are carried at full depth from

the head-motion end to the plateau, and taper in riffle depth is obtained by the rise in deck surface at this point. On the concentrate side of the rise in deck surface riffle cleats are omitted on slime and fine-sand tables, wafer cleats are carried to the end of the coarse-sand table and higher cleats on the roughing table. When the concentrate plateau is unriffled the upper surface

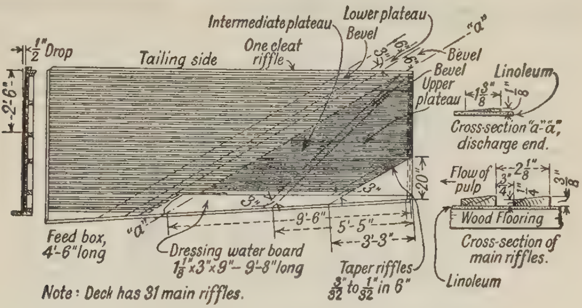


FIG. 19.—Plateaus and riffling on a Plat-O table.

of the riffle cleats and the surface of the plateau are in the same plane. This gives the effect of a one-plane table with shallow grooved riffles widely spaced. The amount of rise at the plateau and the slope vary; a typical figure is $\frac{3}{8}$-in. rise in 12 in. measured along the line of the riffles. Linoleum is the usual deck covering, but rubber may be supplied on roughing tables treating coarse feed at high tonnage rates. The head-motion is the same as illustrated in Fig. 20 and is carried in a covered tank-like housing that allows it to be run nearly submerged in oil and completely protected from splash and grit. The deck is supported on self-oiling dust- and grit-proof slipper bearings and is tilted by means of an adjustable multiple wedge under the feed-side slippers. A special table for coal washing is rectangular, 7 ft. wide by 14 ft. long. The manufacturer recommends 290 @ $1\frac{1}{16}$- to $1\frac{3}{16}$-in. strokes per min. for slime treatment, 290 @ $\frac{3}{4}$- to $1\frac{3}{16}$-in. for fine sand, 275 @ $\frac{7}{8}$- to 1-in. for roughing 8-mesh feed, 240 @ $1\frac{1}{8}$- to $1\frac{1}{4}$-in. for roughing $\frac{1}{4}$-in. feed, and 285 @ $\frac{7}{8}$-in. for $-\frac{3}{4}$-in. coal.

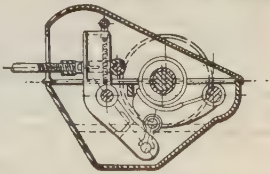


FIG. 20.—Head motion for Plat-O table.

Performance. Copper. COPPER RANGE CONSOLIDATED treats 15 tons per 24 hr. of −35-mesh feed assaying 0.5 per cent. Cu and containing 80 per cent. moisture. Concentrate assays 45 per cent. and tailing 0.22 per cent. The table is a standard Wilfley with plateau and Plat-O riffling, linoleum deck cover and pine riffle cleats. Speed 245 @ $\frac{3}{4}$- to $\frac{7}{8}$-in. strokes; water, 5.6 gal. per min.; 0.3 hp. One man attends 106 machines. At RAY CONSOLIDATED COPPER CO. a Plat-O rougher treating −5-mm. primary feed gave results shown in Table 31. Judgment of this result from the viewpoint of mill economics requires consideration of the relative value of table and flotation concentrate (the latter obtained by treatment of re-ground table tailing), which is out of place here. The tabulation shows a consistent decrease in recovery and relative decrease in tonnage of concentrate with increase in feed tonnage. Grade of tailing is dependent rather on tonnage than on grade of feed while grade of concentrate is rather dependent on grade of feed and independent of tonnage treated. At the BURRO MOUNTAIN concentrator of Phelps Dodge Co. a Plat-O table treated flotation tailing de-slimed in Allen cones at the rate of 39 tons per 24 hr. in a pulp containing 22 per cent. solids. Feed assayed 0.667 per cent. total Cu, 0.150 oxide; concentrate, 11.28

per cent. total Cu, 35 per cent. insoluble; tailing, 0.446 per cent. total Cu, 0.143 per cent. oxide. Recovery was 34.5 per cent. based on total Cu and approximately 42 per cent., based on sulphide. At the same plant a roughing table treated 147 tons per 24 hr. of 14-mesh feed in a pulp containing 45 per cent. solids. Feed assayed 2.136 per cent. total Cu, 0.261 per cent. oxide; concentrate, 14.658 per cent. Cu, 15.86 per cent. insoluble; tailing, 1.200 per cent. total Cu, 0.228 per cent. oxide. Recovery of total Cu was 47.8 per cent.,

Table 31. Performance of Plat-O rougher at Ray Consolidated Copper Co.

Tons of feed per 24 hr.	Assays, per cent. Cu			Recovery, per cent.	Ratio of concentration
	Feed	Concentrate	Tailing		
137	1.39	14.4	0.82	43.5	23.8
190	1.16	13.9	0.78	34.7	34.5
210	1.17	12.7	0.82	32.0	34.0
214	1.26	12.8	0.89	31.5	32.2
243	1.25	12.7	0.90	30.2	33.7
287	1.29	13.9	0.93	29.9	36.1
312	1.29	13.6	0.96	27.5	38.3

sulphide about 52 per cent. At the Bisbee test mill of Phelps Dodge Co., a Plat-O table treating flotation tailing (70 per cent. through 200-mesh) in a pulp containing 22 per cent. solids handled 53 tons per 24 hr. Feed assayed 0.26 per cent. total Cu, 0.05 per cent. oxide; concentrate, 5.80 per cent. total Cu, 27.2 per cent. Fe and 36.8 per cent. insoluble; tailing, 0.19 per cent. total Cu, 0.05 per cent. oxide. This represents a recovery of 27.8 per cent. of total copper and 34.1 per cent. sulphide. At the Morenci plant of Phelps Dodge a single-deck linoleum-covered table with maple riffles, running at 280 @ ¾-in. strokes per min., treated 60 tons per 24 hr. of flotation tailing assaying 0.45 per cent. Cu and made tailing assaying 0.38 per cent. and 11-per cent. concentrate. The table consumed 0.5 hp. One man attended 23 tables. At Old Dominion Copper Co. a Plat-O slime table treating flotation tailing at the rate of 41 tons per 24 hr. in a pulp containing 20 per cent. water made concentrate assaying 3.56 per cent. Cu and 32.2 per cent. insoluble and tailing assaying 0.33 per cent. Cu from feed containing 0.38 per cent. Cu. The table was run at 283 @ ¾-in. strokes per min. and used 4.8 gal. wash water per min. Results of eight weekly test periods at Anaconda roughing 8-mesh feed are given in Table 32. Increasing speed in this test and placing cap riffles on top of the main riffles cut down the grade of tailing while increase

Table 32. Performance of Plat-O table at Anaconda

Test period	Feed, per cent. Cu	Concentrate, per cent.		Tailing, per cent. Cu	Recovery, per cent.	Revolutions per minute	Length of stroke, inches
		Cu	Insoluble				
1	2.62	6.03	38.0	1.56	54.6	240	¾
2	2.64	6.65	31.3	1.22	64.9	240	¾
3	2.38	6.16	33.4	1.09	65.9	280	⅞
4	2.17	6.71	26.5	1.02	62.5	280	⅞
5	2.38	7.00	23.4	1.00	67.6	280	⅞
6	2.49	6.98	26.4	1.10	66.3	280	⅞
7	2.41	7.17	18.4	1.13	63.0	280	⅞
8	2.23	6.50	24.9	1.00	63.4	280	⅞

Notes—Feed rate 80 tons per 24 hr. of 8-mesh feed. Periods 1 and 2: Deck with 3 plateaus, riffled as shown in Fig. 20, no longitudinal slope, transverse slope ¾ in. per foot. Period 3. Cap riffles ⅛-in. high on top of main riffles for 4 ft. back of first rise, 1/16-in. cap riffles on intermediate plateau. No longitudinal slope, ⅞-in. per foot transverse. Periods 4 to 8, incl. Upper plateau removed. Riffles extended from second rise 3/16 in. high to concentrate-discharge end, ⅛-in. cap riffles as above. Feed box 5 ft. 6 in. long. No longitudinal tilt, transverse 1 ⅛-in. per foot.

Table 33. Performance of bowl classifiers and Plat-O tables at Miami Copper Co.

Material	Bowl-classifier feed			Classifier sand			Classifier overflow			Table tailing			Concentrate		
Tons per 24 hr.	1061			233*a*			828*b*			231			1.7		
Assay, per cent. of total Cu	0.294			0.372			0.255			0.234			18.62		
Assay, per cent. of oxide Cu	0.141			0.072			0.156			0.054			.....		
Screen, mesh	Per cent. weight	Per cent. Cu	Per cent. total Cu	Per cent. weight	Per cent. Cu	Per cent. total Cu	Per cent. weight	Per cent. Cu	Per cent. total Cu	Per cent. weight	Per cent. Cu	Per cent. total Cu	Per cent. weight	Per cent. Cu	Per cent. total Cu
48	0.9	0.30	4.4	2.2	0.36	2.0	.....	.....	.....	2.6	0.33	3.8	0.8	19.05	2.8
65	3.4			19.5	0.27	13.6	0.3	0.13	2.3	19.3	0.26	21.9	1.9		
100	13.8	0.25	12.1	45.0	0.29	33.7	4.2			46.9	0.21	42.9	14.0	28.80	21.6
150	8.9	0.26	8.1	15.9	0.43	17.7	7.3	0.11	3.1	14.6	0.22	14.0	22.4	23.13	27.7
200	15.0	0.23	12.2	10.8	0.71	19.8	15.1	0.12	7.0	10.2	0.24	10.7	34.8	17.26	32.1
−200	58.1	0.31	63.2	6.6	0.77	13.2	73.1	0.31	87.6	6.4	0.24	6.7	26.1	11.36	15.8

a 70 per cent. solids. *b* 23.1 per cent. solids.

in side slope reduced the amount of insoluble in concentrate. At MIAMI COPPER Co. two 13-ft. bowl classifiers received flotation tailing, overflowed slime and sent sand to 8 Plat-O sand tables. Results are shown in Table 33.

Zinc. Performance of a sand table in finishing service at the CONNECTICUT ZINC CORPORATION plant at Oronogo, Mo. is presented in Table 34. This table had the single

Table 34. Performance of Plat-O table at Connecticut Zinc Corporation, Oronogo, Mo.

Test number	Feed			Concentrate, per cent.		Tailing, per cent.		Middling, per cent.	
	Tons per 24 hr.	Per cent. Zn	Per cent. Pb	Zn	Pb	Zn	Pb	Zn	Pb
1	18.2	3.85	0.12	55.10	1.30	1.48	0.051	3.08	0.09
2	31	3.35	0.064	58.40	0.82	0.84	0.020	4.74	Tr.
3	18.2	3.42	0.065	58.30	0.67	0.83	Tr.	4.50	0.044
4	22.5	4.12	0.052	57.50	3.30	1.65	0.027	2.60	0.036
5	22	3.45	0.065	57.40	0.32	1.23	0.010	2.94	0.020

Notes—Test 1, Feed varied from very coarse to fine slime. Test 2, Feed very coarse. Test 3, Feed coarse and fine, typical −60-mesh material in feed assayed 5.42 per cent. Zn; in tailing, 2.07 per cent. and in concentrate, 58 per cent. Lead concentrate in this test assayed 64.50 per cent. Pb, 5.60 per cent. Fe and 9.40 per cent. Zn. Test 5, −60-mesh material in feed assayed 5.23 per cent. Zn, in tailing 2.77 per cent. and in concentrate 55.50. Wash water 3.25 gallons per minute. Tonnage of middling return, 12.5 per 24 hr.; length of middling discharge 42 in.

plateau with thin riffle cleats extended to the concentrate-discharge end. Commenting on the test work the company metallurgist stated that the table was very stable in operation, handling fine material as well as coarse at a given setting and requiring but little attention; that the enclosed head motion was a great advantage and that oil consumption was much lower than with an exposed head motion. Robie makes similar comment (*116 J 403*) on the Plat-O rougher at the NORTHERN ORE Co., and Mette (*109 J 1314*) in writing of the NEW CORNELIA test mill. He ascribes stability to height of rise to plateau, noting greater stability with ⅜-in. rise than with ¼-in. Stability is an important item in the choice of tables in that it lessens the necessary attendance and thereby lowers the labor charge against table operation.

Coal. The manufacturer states the capacity of the coal-washing table to be 5 to 8 tons per hr. on feed through ¼-in. screen, 7 to 10 tons with ⅜-in. screen, 10 to 12 tons with ½-in., 12 to 14 tons with ⅝-in. and 14 to 16 tons with ¾-in. and claims removal of 90 per cent. of the free refuse.

12. Rotary shaking table

Many forms have been proposed but none has made its way into the mills. The PINDER CONCENTRATOR is one of the soundest attempts. It is substantially a side-inclined end-shake diagonally-riffled table curled around a vertical axis. A plan of the deck is shown in Fig. 21. The deck is supported on extended ball-and-socket bearings spaced around the periphery and is shaken with a differential reciprocating motion around the central axis by means of a typical toggle head-motion with drawbar attached to the deck periphery. Lost motion is controlled by a spring. The deck is adjustably concave, 10 to 12 ft. diameter. Speed and stroke length are the same as for rectangular tables. Feed pulp is introduced at (*a*) and is distributed by feed box (*b*) along the upper side of the full-riffled section, *viz.:* about 10 ft. In the riffles stratification takes place in the usual fashion, due to the shaking action. Gangue is carried down slope to the tailing discharge at the center by cross-wash water fed around the periphery. Centrifugal force is claimed to be effective to throw the

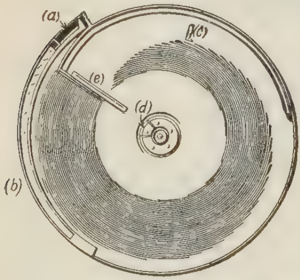


FIG. 21.—Pinder circular-deck shaking table.

heavier particles of concentrate further toward the upper side than they normally appear on a diagonally-riffled table. Concentrate is cut out through an opening in the deck at (c). Middling travels to the end of the longer riffles where it is washed down to pen (d) by spray water from pipe (e).

13. Campbell bumping table

This table (Fig. 22) is used in bituminous-coal washeries to separate coal and the heavier impurities. It consists of a suspended movable trough or deck about 2½ to 3 ft. wide by 9 to 11 ft. long, sloped from a minimum of 7° at the lower end to a maximum of about 12° at the upper end. The bottom of the trough is transversely riffled. The deck is shaken about 75 times per min. by an eccentric, the length of stroke being about 5 in. for coarse coal (−3¾ + 1¾-in.), 2½ in. for medium (−1¾ + ⅜-in.) and 1¼ in. for fine coal (−⅜-in.). A bumping block (b) causes the backward stroke to end suddenly, thus impelling material on the deck to travel toward the back (upper end). Feed is introduced at (c) and wash water at (d). Slate works up slope against the flow of water and discharges at (e) while coal is washed over the lower end into trough (f).

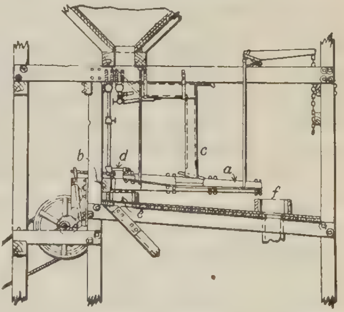


Fig. 22.—Campbell bumping table.

At one Illinois washery (*11 Bul. UI No. 9*) seven tables treated 40 tons per hr., roughly sized as above. Capacity of 3 × 11-ft. tables at Cambria Steel Co. (*23 CA 289*) is 5.75 net tons of raw coal per hr. with a water consumption of about 2200 gal. per hr. One 15-hp. motor drives 12 tables and 12 screw feeders. Performance at this plant is shown in five sets of typical analyses in Table 35.

Table 35. Performance of Campbell bumping table at Cambria Steel Co.

Run number	Raw coal, per cent.			Washed coal, per cent.			Refuse, per cent.			Recovery of combustible, per cent.	Reduction, per cent.	
	Volatile	Ash	S	Volatile	Ash	S	Volatile	Ash	S		S	Ash
1	16.4	10.2	2.98	16.6	7.3	1.40	16.0	61.0	24.8	97.7	53	28
2	16.5	10.3	3.02	16.6	6.8	1.37	16.0	59.8	24.4	97.1	55	34
3	16.6	11.3	2.60	16.7	7.9	1.29	15.5	62.8	21.9	97.4	50	30
4	16.6	12.0	2.12	16.6	8.4	1.21	15.8	64.3	18.7	97.4	43	30
5	17.1	12.5	2.11	17.2	8.8	1.12	16.4	61.3	15.1	96.9	47	30

14. Operation of shaking tables

Applicability. A separation can be made on shaking tables between any two minerals or substances that differ in specific gravity to an appreciable extent, but unless the difference is greater than 1.0, separation by reason of specific gravity alone will be relatively crude and imperfect. Completeness and ease of separation increase with difference in specific gravity; a difference of 3 or 4 is sufficient for rapid treatment and substantially complete recovery. When there is a decided difference in shape of particles as, for instance, exists in the case of coal and slate, the heavier particles in this

case being tabular while the light are rounded, separation is aided thereby and can be successfully performed even though the difference in specific gravities does not greatly exceed 1.0. Particles of the same specific gravity but of different sizes, all very fine, can be separated to a certain extent, thus removing accidental grit from finely-ground abrasives or washing small quantities of slime from granular products.

Size of feed. It is essential that particles settle to the table deck in order to be collected as concentrate, hence the lower size limit is determined by the velocity of the cross water and by the movement of the table. The necessary water velocity depends on the size of the particles that are to be washed down the slope. No established mathematical relation exists between the smallest size of concentrate particle and largest size of tailing particle that can be treated together. It depends, of course, on the relative specific gravities and shapes of particles, the nature of the table surface, character of feed and other more or less indefinite factors.

Wiggin determined that chalcoyprite smaller than 0.025-mm. could not be economically treated on the shaking tables at ANACONDA. Bland found that in treating tungsten slime, 40 per cent. of the concentrate would pass a 0.0125-mm. screen when feeding pulp containing 29 per cent. solids at 2.4 tons per 24 hr. while all of this material was lost when a pulp containing 8 per cent. solids was treated at 4.8 tons per 24 hr. (Table 18.) Beringer (*24 IMM 411*) investigated the effect of shake on the settling rate of cassiterite by fastening to a Buss table 2-oz. phials containing cassiterite grains of various sizes suspended in water and noting the time required for the grains to settle. With the table making 270 @ ¾-in. strokes per min. 0.045-mm. particles settled in 60 sec., 0.035-mm. in 90 sec., 0.030-mm. in 120 sec., and 0.025-mm. in 150 sec. Corresponding sizes with the tables at rest were 0.025-, 0.020-, 0.015- and 0.010-mm. The average minimum size of grain in Buss-table concentrate was 0.05-mm. Johnson and Heinz (*107 J 558*) report sizing tests on blende tailing in the JOPLIN DISTRICT, showing 2.7 per cent. zinc in the sand between 0.833- and 0.295-mm., 2.2 per cent. in that between 0.295- and 0.147-mm. and 9.4 per cent. in the material passing a 0.147-mm. screen. Similar results reported by Wright (*TP 41 USBM*) are given in Table 36. As a general rule governing this subject, it may be stated

Table 36. Sizing-assay test on Joplin table tailing.

Screen aperture	Weight, per cent.	Assay, per cent. Zn	Total zinc, per cent.
On ⅜-in.	15.22	0.77	12.25
3-mm.	47.56	0.96	47.61
1.5-mm.	23.96	0.76	18.95
0.46-mm.	10.41	0.71	7.68
Through 0.46-mm.	2.84	4.55	13.51
Totals	99.99	0.96	100.00

that the granular or sandy portion of material passing a 200-mesh (0.074-mm.) screen is readily settled to a shaking-table deck and moved along by the table motion and that good recoveries can be made on such material if the accompanying gangue is not so coarse as to require excessive wash water or excessive tilt to remove it. Watt (*57 A 371*) states that reciprocating (shaking) slime tables do excellent work saving galena coarser than 300-mesh and make good recovery of finer galena. In a test on a feed in which 94 per cent. of the lead passed a 200-mesh screen the concentrate assayed 76 per cent. lead and represented a recovery of 65 per cent.; 89 per cent. of the lead in the concentrate would pass 200-mesh; 98.5 per cent. of the lead in the tailing would pass 300-mesh. Caetani (3 *MM 50*) concluded from testing work at BUNKER HILL AND SULLIVAN that on such material reciprocating tables will do better work than vanners.

The upper size limit is just as indefinite as the lower. On tables that pass concentrate across a smooth cleaning plane it is practically impossible to make clean concentrate and finished tailing when the particles are larger than 2-mm., and difficult when they are larger than 1-mm. Full-riffled roughing tables will treat unsized material passing a ⅜-in. aper-

Table 37. Performance of Wilfley table on "natural," sized and classified feeds

Run number	Method of preparation	Size of feed, mm.	Feed rate, tons per 24 hr. (*a*)	Concentrate, per cent.		Middling, per cent. weight
				Weight	Assay, PbS	
1	Natural	2 –0	22	4.1	90.1	21.5
2	Natural	1 –0	22	3.0	91.5	20.7
3	Natural	0. 5–0	11	4.7	97.6	26.0
4	Natural	0.25–0	11	5.3	97.7	21.0
5	Natural	2 –0	11	2.8	95.3	45.5
6	Sized	2 –1.4	22	6.5	99.2	1.7
7	Sized	1.4 –1.0	16.6	9.7	99.2	1.9
8	Sized	1.0 –0.75	11	12.1	99.0	1.7
9	Sized	0.75–0.50	11	13.2	97.5	1.6
10	Sized	0.50–0.36	11	12.2	99.3	5.3
11	Sized	0.36–0.28	11	15.1	97.9	3.2
12	Classified: Spig. 1; 105 mm. per sec.	2.0 –0	11	49.7	99.3	5.5
13	Spig. 2; 85 mm. per sec.	2.0 –0	11	4.0	98.6	1.3
14	Spig. 3, 4; 55.4 mm. per sec.	2.0 –0	16.6	3.4	98.8	0.7
15	Spig. 5, 6; 36.3 mm. per sec.	2.0 –0	11	4.9	98.4	0.8
16	Spig. 7, 8, 9; 19.1 mm. per sec.	1.4 –0	11	5.7	98.8	1.0
17	Spig. 10, 11, 12; 10 mm. per sec.	1.6 –0	11	5.0	99.6	4.3

Run number	Method of preparation	Middling, assay, per cent. PbS	Tailing, per cent.		Slime, per cent.	
			Weight	Assay, PbS	Weight	Assay, PbS
1	Natural	23.6	72.9	·0.5	1.5	18.9
2	Natural	26.6	72.9	0.6	3.4	9.7
3	Natural	15.9	62.8	0.9	6.5	12.2
4	Natural	12.8	60.5	1.6	13.1	10.0
5	Natural	13.7	50.7	0.3	1.0	14.6
6	Sized	59.7	91.8	0.04	....	....
7	Sized	23.1	88.4	0.00	....	....
8	Sized	17.0	86.2	0.09	....	....
9	Sized	22.6	85.2	0.35	....	....
10	Sized	20.0	82.5	0.24	....	....
11	Sized	15.4	81.7	0.44	....	....
12	Classified: Spig. 1; 105 mm. per sec.	13.8	44.8	0.29	....	....
13	Spig. 2; 85 mm. per sec.	34.0	94.7	0.36	....	....
14	Spig. 3, 4; 55.4 mm. per sec.	34.4	95.9	0.20	....	....
15	Spig. 5, 6; 36.3 mm. per sec.	28.9	94.2	0.55	....	....
16	Spig. 7, 8, 9; 19.1 mm. per sec.	16.2	93.2	0.46	....	....
17	Spig. 10, 11, 12; 10 mm. per sec.	16.3	90.7	0.66	....	....

a Proportioned to full-size table on basis of comparative area of small table and tonnage treated thereon.

ture and deliver a small band of clean coarse concentrate along the upper riffles and a considerably impoverished tailing between the forward edge of the slime streak and the corner of the table deck. The Empire table is reported to have made clean concentrate on ½-in. base-metal ore, and coal this size can be treated on any of the shaking tables, if properly riffled, but both of these operations are of roughing character. In the great majority of plants treating base-metal ores, roughing-table feed has passed a 2- or 2.5-mm. screen.

Preparation of feed for shaking tables. There has been much discussion on this point and the general opinion at present probably is that classified feed is best, "natural" feed is poorest and sized feed occupies an intermediate position. This conclusion is based upon Richards' interpretation of the data presented in Table 37 (*38 A 556*), which shows the results of a series of runs with a laboratory Wilfley table on an artificial mixture of quartz and galena, prepared variously, as indicated, for table treatment. (A similar series with cupriferous pyrite and quartz gave similar results.) Professor Richards' conclusion was as follows:

> "While the sized-product feed appears to have done better work than the classifier-product feed, if we give full weight to the great performance of Run No. 12, we can agree that this has fully off-set the slight falling-off of runs Nos. 13 to 17, and that the classifier-feed work is fully up to the sized-feed work on the Wilfley table, and with a perfect classifier the work will be better done than with screens."

But if Run No. 12 is properly weighted with the other classified-feed runs and compared with the sized-feed runs similarly weighted, it appears that the average assay of tailing from classified feed is 0.43 per cent. PbS and that from sized feed 0.19 per cent.; that the respective percentages of middling are 13.6 and 15.4, and that the grades of concentrate are substantially the same. It would appear, therefore, on the face of these tests, that preparation of a given lot of finely crushed material by sizing would result in more efficient table treatment of the products than preparation by hydraulic classification. This conclusion has been reached by other experimenters from data similar to Richards; one of these (Bland, *107 J 1112*) even going so far as to advocate dry grinding in order to effect the careful sizing necessary. Hancock (*24 MM 87*) citing Cox, Porter and Gibbon (*14 CMI 490*) says that treatment of a "natural" feed yielded 64.4 per cent. recovery, the mean recovery on the same ore sized into six lots and separately treated was 63.9 and on classified feed was 66.7 per cent. Ellis (*7 MMt 156*) compared the treatment of de-slimed natural feed, classified feed and sized feed, prepared from Coeur d'Alene ore. The results are shown in Table 38. They would justify very little expense for any preparation other than de-sliming.

Table 38. Comparison of classified, sized and natural feeds for shaking tables. (*After Ellis*)

Feed	Classified		Screen-sized		Natural	
Tons per 24 hr.	4.15		4.46		4.22	
Product	Pb, per cent.	Ag, ounces	Pb, per cent.	Ag, ounces	Pb, per cent.	Ag, ounces
Feed	21.4	7.0	21.6	7.2	21.2	7.1
Concentrate	81.8	24.5	74.7	24.0	73.2	23.5
Middling	16.0	6.9	13.9	5.0	13.5	4.9
Tailing	1.4	0.7	1.6	0.9	1.9	0.9
Recovery	95.1	92.7	94.6	90.9	93.4	90.8

As a practical matter in the mills, classification is superior to sizing as a means of preparation because close fine sizing, as was practiced in Richards' tests, cannot be done in the mills as cheaply as classification; cannot, in fact, be done economically at all. Hence most practice has followed Richards' conclusion.

In Richards' tests while slimes are charged against the runs on natural feed, they do not appear, although, of course, present, as charges against the other runs. Supplementary treatment of these products is contemplated in all cases.

Since Richards' work was done, the growth of roughing-table practice has brought treatment of natural feeds on shaking tables into prominence again. Elaborate test work at Miami and Inspiration in 1912 and 1913 showed that careful treatment of natural feed on roughing tables with subsequent enrichment of the rough concentrate after classification yielded better metallurgical results and would probably yield better economic results, than most careful and elaborate preparation of the whole feed by classification prior to tabling. With ores adapted to concentration by flotation, there can be no doubt that classification of the original table feed is wholly unnecessary from a metallurgical standpoint and is uneconomic. Limiting the upper size of roughing-table feed by means of a screen or a mechanical classifier is all that is necessary.

Capacity of full-riffled tables such as the Garfield, Butchart, Deister No. 2, and Plat-O in roughing service is 100 to 200 tons per 24 hr. on feed through 2- or 2.5-mm. screens. It may be pushed to 300 tons with a feed as coarse as 4-mm. maximum, but at considerable sacrifice of recovery. Cleaning rough concentrate from full-riffled tables, practically any of the usual sand tables (Butchart, Card, Deister, Deister-Overstrom, Plat-O) will treat from 40 to 75 tons per 24 hr. When feed is coarse, riffles extended to the concentrate-discharge end will aid in keeping up capacity. Treating classified or de-slimed feed and making both clean concentrate and tailing, capacities range from about 10 tons per 24 hr. on 0.5-mm. material to 40 to 45 tons on 1.5- to 2-mm. feed. In general, the capacity on lead ores will be higher than on copper or zinc. These same tonnages will apply to recovery of auriferous pyrite from quartz sands, but this is essentially roughing service, as re-treatment of tailing by cyanidation is always contemplated and frequently the concentrate is also cyanided. Slime tables in finishing service treat from 3 to 6 tons each per 24 hr.; roughing auriferous pyrite from quartz the capacity rises to from 10 to 15 tons, and treating flotation tailing (also essentially roughing service) from 30 to 60 tons per 24 hr. per deck is handled in some mills. Results of tests on Joplin zinc ores *(57 A 456)* are shown in Table 39.

Table 39. Effect of size of feed on capacity, recovery and grade of concentrate, Joplin ore

Table number	Size of feed			Assays, per cent. Zn			Recovery, per cent.	Operating data	
	All pass, . . mm.	Per cent. on following screen	Per cent. through 0.074-mm.	Feed	Concentrate	Tailing		Speed, revolutions per minute	Length of stroke, inches
1	2.36	0.40	2.48	4.83	58.2	0.88	83.1	239	1
2	0.59	36.55	2.08	4.70	61.8	0.66	86.9	242	1·
3	0.59	15.73	3.96	5.0	58.78	0.78	85.8	242	1⅛
4	0.59	4.21	5.98	4.78	53.92	0.57	89.0	243	⅞
5	0.59	1.0	8.30	5.0	58.0	1.03	80.8	245	1
6	0.29	5.04	17.02	5.22	54.05	1.50	73.3	245	¾
7	0.59	0.16	13.18	5.01	55.44	1.16	78.5	270	9/16
8	0.59	0.45	14.75	5.07	45.16	1.37	75.0	270	9/16
9	0.59	1.04	8.71	5.17	49.6	1.66	70.2	274	⅝
10	0.29	0.70	26.29	5.55	50.82	1.90	68.3	276	¾

Note—In all cases but No. 9 the tailing was noticeably coarser than the feed. In 1, 5, 8, 9, 10, more than 30 per cent. of the total zinc loss was in −200-mesh material. Feeds were successive spigot products and final overflow of a series of 9 hydraulic spitzkasten. Tonnage per table ranged from about 20 per 24 hr. on table 1 to 2 tons per 24 hr. on Table 10.

Speed and stroke are properly related, a low speed and long stroke being suitable for coarse feeds and the reverse for fine, but in the examples cited

in the preceding pages, which comprise a fairly representative cross-section from practice, this relation is decidedly obscure. On the other hand Wright (*TP 41 USBM*) gives results shown in Table 40 to illustrate the effect of wrong *vs.* right stroke-length and speed. The average in roughing service is: Garfield, 256 @ 1-in. strokes per min.; Butchart, 261 @ ⅞-in.; in sand-

Table 40. Tests on effect of speed and stroke-length on zinc ores

Test number	Revolutions per minute	Stroke, inches	Feed, per cent. Zn	Tailing, per cent. Zn
1	220	¾	9.25	0.9
	175	1¼	9.25	3.05
2	244	⅞	7.75	1.25
	224	½	7.75	3.20

finishing service: Butchart, 274 @ ⅞-in.; Wilfley, sands coarser than 1-mm. maximum, 255 @ ⅞-in.; sands finer than 1-mm. maximum, 249 @ ¾-in.: in slime service (Deister slimer only included), 273 @ ½-in. Wright gives practice in Joplin district as follows: Coarse feed (1.5- to 2-mm. maximum), 220 to 240 @ ¾-in. to ⅞-in. strokes; medium sands, 240 to 260 @ ⅝- to ¾-in.; fines, 250 to 280 @ ½- to ⅝-in.; ungraded feed through 1.5- or 2-mm. screen, 230 to 250 @ ¾- to ⅞-in., depending on rate of feed. He states that if stroke length or speed is too low, galena packs in the riffles at the head-motion end of the tables and gradually works down into the tailing. Study of details in the various examples of operation will, however, show that variations from the averages are in all cases so great as to throw individuals of any one class into another class. The stroke for slime tables must be much sharper than for sand on account of the greater relative tendency for fine particles, once in contact with the deck surface, to stick. Stroke should, therefore, be correspondingly short in order to prevent agitation of the mass of pulp on the table, thereby preventing settlement.

Water consumption in roughing service ranges from about 50 to 350 gal. per ton treated; in finishing service on sands it is usually between 300 and 400 gal. per ton, on slimes from 600 to 3000. Water and transverse slope are interdependent and both are dependent on the size of feed. The requirements are that solids shall settle in the riffles, that the pulp shall be sufficiently fluid to allow stratification, that there shall be sufficient velocity of cross flow to carry off the upper strata as the riffle support is withdrawn and that there shall be sufficient agitation in the riffles to keep fine gangue from settling as readily or as far as the fine mineral. Granular pulps containing 25 per cent. solids and slime pulps containing as high as 30 per cent. are sufficiently fluid to allow stratification and are sufficiently lively in the riffles, hence feed pulps may be of these consistencies. Wash water must be supplied in sufficient quantity to form a freely moving film on the deck deep enough to cover the largest particles. Beyond this point transport of material may be gained either by increasing volume at the same slope or increasing velocity by increasing slope. To increase transporting power by increasing slope is economical of water, but it narrows the bands of the various products at the concentrate end and makes accurate splitting difficult. This is allowable in roughing practice, which employs steep slope and minimum water, but when clean products are desired, as in finishing practice, more water is used. Water consumption in slime treatment is exceptionally high on account of the fact

that the tonnage of slime per table is low, a certain minimum amount of water is necessary to keep a uniformly moving film flowing and finally fine particles adhere tenaciously to smooth deck surfaces and require prolonged washing to remove them.

Power consumed averages close to 0.6 hp. per single-deck table and that installed between 0.75 and 1 hp., except that the Garfield table under heavy loads consumes close to 1 hp. Double-deck tables require from 50 to 75 per cent. more power than single-deck.

Attendance averages about 30 tables per man in finishing service and 50 in roughing service but as many as 135 tables per man in finishing service are reported from one mill (Calumet & Hecla) and 106 at another. The number of tables that one man can run depends, of course, on the difficulty of the job. The most difficult service is making finished tailing and concentrate on sand tables with fluctuating feed. The operator in such service must continually change transverse tilt, wash water and (if possible) the position of the product splitters. In roughing service with full-riffled tables the tilt may be fixed, if the feed supply is steady, wash water is rarely changed, and control is effected almost entirely by shifting the product splitters. In this case one man can attend almost any number of tables, the practical limit being the number that he can keep properly lubricated and running.

Lost time practically never exceeds 1 per cent. of possible running time. The principal cause is renewal of riffling and deck covering. In heavy roughing service soft-wood riffle cleats may last only one or two weeks and linoleum the same number of months but lost time is cut down by use of hardwood or metal-protected riffles and rubber or concrete decks or metal plates inserted near the feed box, at the point of greatest wear. In ordinary service soft-wood riffle cleats will last from six months to a year or upward and a linoleum deck for two to four years. If head motions are of proper size and protected from grit they will last almost indefinitely with occasional re-babbiting of bearings and replacement of renewable wearing parts.

Riffling. The principles underlying correct riffling are as follows: Riffles must be deep enough at the head end to hold all of the solid particles in the feed that can settle out of suspension in the cross flow of feed water, in order to give opportunity for stratification of this material. They must decrease gradually in depth toward the concentrate-discharge end to permit gradual shearing off of the impoverished upper layers by the cross flow of wash water. If the surface is unriffled at the concentrate end the termination of the riffles should lie along a diagonal line extending from near the corner formed by the tailing side and concentrate end to a point on the feed side one-fourth to one-half the distance toward the concentrate end. Such diagonal termination of the riffles results in catching and moving toward the concentrate end such particles of concentrate as are unable to withstand the full flow of wash water on the unriffled surface. If the feed to such a table contains coarse grains of free mineral that tend to go into the middling, extension of occasional thin riffle cleats to the concentrate end, particularly near the feed side of the deck will result in removal of these particles without holding back much fine gangue. Wright (*100 J 642*) recommends termination of riffles along a curve convex to the concentrate side instead of along the usual straight diagonal, in order to hold concentrate higher on the deck and to spread the concentrate band wider and thus make the cut between concentrate and middling sharper. Garber (*111 J 788*) recommends carrying broken extensions of the regular riffling to a second advanced diagonal for the treatment of complex ores. If

the lower end of the diagonal is brought to the concentrate end 2 to 4 in. above the lower corner, the concentrate-middling split can be made on the end of the table where the waggle of the discharging stream is parallel to the cutting edge, with corresponding increase in sharpness of cut as compared with corner cutting. If the middling streak is narrow the middling-tailing cut can also be made on this end. Such riffling, however, requires that the transverse tilt be less and more wash water must, therefore, be used. If the concentrate end is cut back diagonally to the feed side, say 6 in. in the width of the deck, the same end is served without the necessity of flattening the table when running and, in addition, the concentrate end is kept wet by wash water supplied at the feed side. When high tonnages or coarse feed are treated, shallow riffles should be extended from the separating diagonal to the concentrate end for the full width of the deck in order to hold up concentrate with the steep slope necessary to carry off the tailing. This will result in somewhat lower-grade concentrate than otherwise, but increases recovery. The WIDTH OF RIFFLES is determined in part by the size of the largest grains of feed and in part by the demand as to grade of tailing, other conditions remaining the same. Width must be sufficient to prevent jamming of coarse particles with consequent clogging of a riffle and this means that it should be more than three times the diameter of the largest grain. Agitation due to swirling is a maximum in narrow deep riffles, hence these will result in clean concentrate, but at the expense of higher-grade tailing. Steep-sided riffles produce more agitation than those with slanting sides. Hence riffles for slimes should be shallow, relatively widely spaced and have slanting sides. Richards calls attention to the fact that riffle cleats fastened onto a plane surface cause the plane of the roughing surface and that of the cleaning surface to intersect at an angle along the diagonal of riffle termination, thus forming a valley in which the grains pile up and hinder separation. He recommends grooving the riffles into a plane surface to overcome this effect and this has been done in some tables (Card) and its equivalent has been effected in the plateau arrangement of the Deister-sand and Plat-O tables.

Cost of tabling. HOLLINGER (*51 A 124*) 5¢ per ton of feed; ALASKA GASTINEAU (1917) two treatments on Garfield tables plus two on Wilfleys, total $0.051 per ton of feed; at GOLDFIELD CONSOLIDATED (*89 J 1230*), $0.054 per ton. The principal element of cost is labor and where, as in a flotation mill, table attendance can be made part of the flotation operators' duties, the cost of tabling is substantially negligible.

SECTION 11

VANNERS

1. Principles of vanner concentration

Vanners are concentrating machines adapted to the treatment of fine sands. They consist essentially of an endless belt with upper surface horizontal transversely and inclined longitudinally, all carried on a frame that oscillates in the plane of the belt. The belt is usually made of rubber and the upper surface travels slowly uphill. Feed is introduced about one-quarter of the distance from upper to lower pulley, heavy mineral is discharged as concentrate over the upper pulley and light mineral is washed over the lower pulley. The simple, non-differential shaking motion effects stratification of the solids in the semi-fluid pulp, the heavier and smaller particles working down to the bottom of the mass. The uphill movement of the supporting surface transports the settled solids upstream against the downward flow of the overlying layer of impoverished pulp and wash water.

Apart from the stratification induced by shaking (see Sec. 10, Art. 1) the effectiveness of the vanner depends on the adhesive force between finely divided solid matter and the smooth wet supporting surface. It is not proved quantitatively that this force is greater in the case of sulphide particles than in the case of gangue minerals, but qualitative proof all points in this direction. If this hypothesis is true, difference in the surface energy at the wetted contacts between sulphide-rubber (*e.g.*) and gangue-rubber is far more responsible than difference in specific gravities between the two classes of minerals in effecting the adhesion that results in concentration. It is worthy of note in this connection that more than one of the early patents that led up to modern flotation practice specified rubber as a substance showing preferential adherence to sulphide particles over gangue, (Sec. 12, Art. 3), and also that several oil-selection processes depend upon increasing the selective adhesion of sulphides to vanner belts by coating either belt or sulphide with oils. (Elmore, Luckenback, Schwarz.)

TYPES OF VANNERS

There are four types, characterized by the direction and character of shake and direction of slope, viz.: (*a*) oscillating side-shake, end-slope; (*b*) oscillating end-shake, end-slope; (*c*) differential end-shake, side-slope; (*d*) gyrating, end-slope. The first are the oldest and most used, the Frue, Johnson and Isbell being typical representatives; the Embrey, Craven and Triumph are of the second class; the Luhrig, Weir-Meredith and Monell are of the third class and the Senn the fourth.

Table 1. Performances

Type of vanner	Kind of ore	Name of plant	Width of belt, ft.	Revolutions per minute	Length of shake, inches
Frue......	Lead.......	Bunker Hill & Sullivan(*c*)........	6	200	1⅛
Frue(*e*)...	Lead.......	Federal Lead Co., Mill 3.........	6	190-200	¾
Frue(*j*)....	Copper(*l*)...	Phelps-Dodge, Morenci..........	6	210	1
Frue......	Gold.......	Ouro Preto(*m*).....................		178	....
Frue......	Gold.......	Alaska Treadwell(*o*).............	6	196	....
Frue......	Gold.......	Hedley G. M. Co................	6	185	1
Isbell.....	Copper.....	Utah Copper Co.(*u*).............	6	176	1
Isbell.....	Copper.....	Ray Consolidated Copper Co.....	6	180	1
Isbell.....	Copper.....	Chino Consolidated Copper Co....	6	160	1
Isbell.....	Copper.....	Chino Consolidated Copper Co....	6	180	1
Isbell.....	Gold.......	Alaska Gastineau...............	6	177	1.25
Isbell.....	Tungsten...	Tungsten Mines Co.............	6	190	0.5
Johnson...	Copper.....	Phelps-Dodge, Moctezuma(*ag*)...	6	120	⅞-1¼
Senn......	Copper.....	Phelps-Dodge, Burro Mountain...	6	140	0.75
Senn......	Copper.....	Phelps-Dodge, Morenci(*am*)......	6	....	....
Senn......	Copper.....	Phelps-Dodge, Morenci(*am*)......	6	....	....
Senn......	Copper.....	Phelps-Dodge, Morenci(*am*)......	6	....	....
Senn......	Copper.....	Old Dominion(*am*).............	6	....	....
Senn......	Copper.....	Old Dominion(*am*).............	6	....	....
Senn......	Gold.......	Mother Lode Mill(*am*)..........	6	....	....
Embrey...	Gold.......	Anaconda Copper Mining Co.(*as*).	....	....	....
Luhrig....	Complex....	Broken Hill Junction North(*au*)..	4	240	0.75
Luhrig....	Complex....	Broken Hill Junction North(*au*)..	5	220	0.75

Type of vanner	Kind of ore	Name of plant	Moisture in feed, per cent.	Size of feed	Assays, per cent.
					Feed
Frue......	Lead.......	Bunker Hill & Sullivan(*c*)........	77	*a*	12
Frue(*e*)...	Lead.......	Federal Lead Co., Mill 3.........	85	*d*	3.7
Frue(*j*)....	Copper(*l*)...	Phelps-Dodge, Morenci..........	84	*i*	0.45*k*
Frue......	Gold.......	Ouro Preto(*m*)	....	....	*n*
Frue......	Gold.......	Alaska Treadwell(*o*)............	....	*q*	*p*
Frue......	Gold.......	Hedley G. M. Co................	....	*t*	....
Isbell.....	Copper.....	Utah Copper Co.(*u*).............	....	*v*	....
Isbell.....	Copper.....	Ray Consolidated Copper Co.....	75	....	....
Isbell.....	Copper.....	Chino Consolidated Copper Co....	66-80	*y*	1.20
Isbell.....	Copper.....	Chino Consolidated Copper Co....	66-80	*ab*	1.15
Isbell.....	Gold.......	Alaska Gastineau...............	93	*ae*	$2.08
Isbell.....	Tungsten...	Tungsten Mines Co.............	85-90	....	....
Johnson...	Copper.....	Phelps-Dodge, Moctezuma(*ag*)...	85-88	*ah*	3.0-3.6
Senn......	Copper.....	Phelps-Dodge, Burro Mountain...	80	....	0.65
Senn......	Copper.....	Phelps-Dodge, Morenci(*am*)......	66	*al*	1.56
Senn......	Copper.....	Phelps-Dodge, Morenci(*am*)......	74	*an*	1.33
Senn......	Copper.....	Phelps-Dodge, Morenci(*am*)......	60	*ao*	1.61
Senn......	Copper.....	Old Dominion(*am*).............	62	*aq*	4.72
Senn......	Copper.....	Old Dominion(*am*).............	62	*ar*	2.93
Senn......	Gold.......	Mother Lode Mill(*am*)..........	70	*at*	$2.52
Embrey...	Gold.......	Anaconda Copper Mining Co.(*as*).	....	....	....
Luhrig....	Complex....	Broken Hill Junction North(*au*)..	....	*av*	....
Luhrig....	Complex....	Broken Hill Junction North(*au*)..	....	*ax*	....

of vanners

Speed of belt, inches per minute	Slope, inches per foot	Horsepower	Wash water, gallons per minute	Tons of feed per 24 hr.
36	5/16–7/8	0.5	3	5*c*
3	1/2	1	4	6
100	1	0.5	*f*	15*g*
....	....	....	....	....
60	0.4	....	....	12.5
4.5 to 6	0.25	0.5	2	5–15
60–120	0.15	....	....	....
60–72	0.7	0.55	1.3	13
64	0.55	0.5	0.7–1.4*aa*	18
72	0.7	0.5	0.7–1.4*aa*	25
41	0.6	0.5	4*ac*	5
54	11/32	0.75	4	6*ai*
130	2.75–3	0.25	5–5.5	10–12
....	0.25	0.7	....	30–35
....	....	....	2.3	111
....	....	....	....	54
....	....	....	2.1	33
....	....	....	....	21
....	....	....	4	10.2
....	....	....	....	....
....	....	....	....	....
180	....	....	....	12
180	*ay*	0.5	....	17

Assays per cent.		Belt		Attendance, machines per man
Conc.	Tailing	Kind	Life	
67	7*b*	Smooth	6 yr.	....
60	2.40	Smooth	3 yr.	*h*
7.00	0.38	Smooth	804 da.	13
1.12–1.25 oz.	1.7–1.9 gm.	Smooth	3–5 yr.	....
r	$0.165	....	....	....
....	....	Smooth & Corr.	4 yr.	*s*
x	....	Corr.	*w*	....
....	....	Smooth	2.5 yr.	95
10.4	0.65	Smooth	2+ yr.	80
4.5	0.58	Corr.	1+ yr.	80
af	$1.22	Smooth	....	*ad*
2–10*ak*	0.15–0.2*aj*	Smooth	1.5+ yr.	....
12–14	1.2–1.8	Smooth	4–5 yr.	40
10	0.55	Smooth	2.2 yr.	30
10.17	1.03	....	....	....
9.18	0.78	....	....	....
9.98	0.51	....	....	....
ap	1.31	....	....	....
7.67	0.45	....	....	....
$145.00	$0.88	....	....	....
....	....	....	....	....
68Pb	*aw*	....	....	....
68Pb	*aw*	....	....	....

a See Table 1*a*. *b* See Table 2 for sizing-assay test of tailing in this mill. *c* See also Table 3 for metallurgical results at different tonnages. *d* All through 80-mesh. *e* Similar performance on Isbell and Johnson vanners. *f* 130 gal. per ton of feed. *g* Runs up to 30 tons per 24 hr. *h* 6 vanners plus 20 tables. *i* See Table 1*a*. *j* To be replaced by Plat-O tables. *k* Flotation tailing. *l* See also Table 4 for performance of corrugated-belt machine at Anaconda Copper Mining Co. *m* *20 IMM 34*. *n* Blanket tailing. *o* *114 P 412*. *p* Battery pulp after amalgamation. *q* See Table 1*a*. *r* Carries 35 per cent. sand. *s* 24 vanners and 12 Deister slimers per man. *t* Through 100-mesh, de-slimed. *u* *117 J 224*. *v* Fourth spigot of primary classifier taking −2-mm. feed re-classified in 5-spigot Richards-Janney classifier; first, second and fifth spigots to 8 Isbell vanners, third and fourth to 8 Johnson. *w* Corrugations still sharp after 2 years' use. *x* Made low-grade concentrate which was re-classified and re-concentrated by Wilfley tables and flotation. Vanners now abandoned. *y* See Table 1*a*. *aa* 2 to 3 gal. per minute additional for washing concentrate off belt. *ab* See Table 1*a*. *ac* Plus 5.4 gal. per minute to wash concentrate off belt. *ad* 3 men for 6 vanners, 48 Wilfleys, 2 tube mills, 4 elevators, 4 classifiers and 4 cones. *ae* See Table 1*a*. *af* 2.5 oz. Au, 25 oz. Ag, 25 per cent. Pb. *ag* Machines eliminated in new flow-sheet. *ah* See Table 1*a*. *ai* Not up to capacity. *aj* Per cent. WO_3. *ak* On rougher. 40 to 60 per cent. WO_3 on finisher. *al* See Table 1*a*. *am* Data furnished by manufacturer. *an* See Table 1*a*. *ao* See Table 1*a*. *ap* Cu, 9.22 per cent.; Fe, 26.8 per cent.; Insoluble, 27.2 per cent. *aq* See Table 1*a*. *ar* See Table 1*a*. *as* See Table 4 for summary of tests. *at* See Table 1*a*. *au* *22 IMM 496*. *av* Wilfley middling sized on 30-mesh, undersize de-slimed and sent to vanners. *aw* Make also middling and tailing. *ax* Thickened slime from operation described in note *av*. *ay* Side slope 0.75 in. per foot. Head end raised 6 in.

Table 1*a*. (Supplement to Table 1). Sizing tests of feed to vanners in Table 1

Screen aperture, mm.	Per cent. weight on screen						
	a Bunker Hill & Sullivan	*i* Phelps-Dodge-Morenci	*q* Alaska Tread-well	*y* Chino Cons. Copper Co.	*ab* Chino Cons. Copper Co.	*ae* Alaska Gas-tineau	*ah* Phelps-Dodge, Mocte-zuma
2.362	...	...	...	...	...	...	...
1.651	...	...	...	0.10	0.20	...	...
1.168	...	...	...	0.10	0.40	...	...
0.833	...	...	6.25	0.30	2.51	...	...
0.589	...	0.79	...	1.60	8.73	...	...
0.417	...	7.46	25.00	3.30	15.35	...	...
0.295	...	18.55	12.50	4.30	10.83	...	1.20
0.208	...	25.08	12.50	6.80	8.83	...	2.35
0.147	...	18.37	8.35	8.70	7.32	...	4.60
0.104	...	13.42	12.50	9.80	4.81	4.7	7.35
0.074	5	2.61	6.25	2.10	1.00	6.9	5.70
0.074	95	13.72	16.65	63.70	40.02	88.4	78.80

Screen aperture, mm.	Per cent. weight on screen					
	al Phelps-Dodge, Morenci	*an* Phelps-Dodge, Morenci	*ao* Phelps-Dodge, Morenci	*aq* Old Dominion	*ar* Old Dominion	*at* Mother Lode
2.362	0.14	...	...	...	...	...
1.651	3.05	0.55	...	...	...	...
1.168	4.77	1.25	...	0.29	...	...
0.833	5.78	3.00	...	0.97	...	...
0.589	7.58	6.95	1.40	4.44	...	...
0.417	8.23	9.80	8.20	9.12	...	...
0.295	8.18	11.50	20.50	16.67	...	...
0.208	7.03	10.05	24.30	17.67	0.80	...
0.147	7.28	10.05	21.10	20.63	9.39	5.70
0.104	5.12	6.90	10.10	14.22	31.90	12.93
0.074	3.50	4.45	4.50	5.70	23.97	14.00
0.074	39.34	35.50	9.90	10.29	33.94	65.63

2. Side-shake vanners

Frue vanner (Fig. 1) has an endless rubber belt (*a*), usually 6 ft. wide but sometimes 4 ft., mounted to pass around four large rollers or pulleys *b*), (*c*), (*d*), and (*e*) known respectively as head roller, tail roller, dipping roller and tightening roller, all carried on a shaking framework (*f*) which is in turn supported on a plurality of lath-like hickory or steel springs (*g*) suitably seated on the framework (*f*) and main frame (*h*). Intermediate small-diameter rollers (*i*) (best made of brass) prevent the upper run of the belt from sagging. The plane of the upper surface of the tail and intermediate rollers below the feed box cuts the head roller about 0.5 in. below the top, thus making the slope of the belt between the feed box and head roller steeper than below the feed box. The intermediate rollers above the feed box conform to the steeper slope. The framework (*f*) is shaken by means of three straps (*j*) actuated by simple eccentrics synchronously mounted on shaft (*k*) which is carried on the main frame.

A cone pulley (*l*) on this same shaft drives a shifting pulley on the flexibly-mounted shaft (*m*), on the forward end of which is a worm (*n*) engaging a worm wheel that is flexibly connected by means of a spiral spring with the shaft of the head roller, thus effecting upslope travel of the upper run of the main belt. The rate of travel is changed by moving belt (*p*) by means of hand wheel (*q*). The feed box (*r*) with a suitable pulp-distributing sole is carried on standards attached to the shaking framework (*f*). Wash water is applied from box (*s*) and a suitable spray for removal of concentrate is mounted under the head roller. Tailing is discharged over the tail roller. The belt is caused to travel straight on the head and tail rollers by swinging the tightening roller forward on the side toward which the belt is desired to travel, by means of hand wheels (*t*). Adjusting bolts on the tail-roller boxes may also be used to guide the belt. Longitudinal tilt is adjusted by means of bolts (*u*) but the machine must be stopped to make this adjustment accurately.

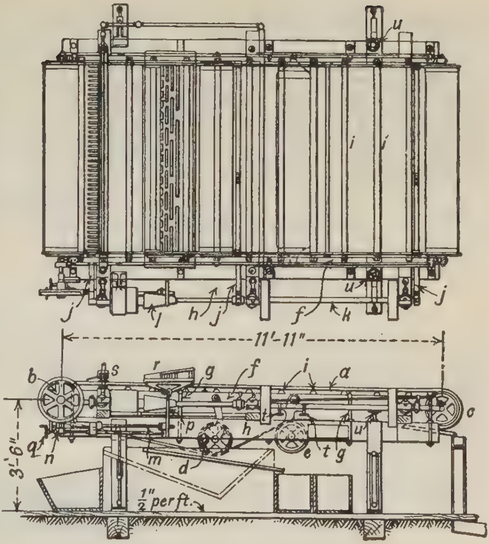


Fig. 1.—Frue vanner.

Performances are given in Tables 1 to 4.

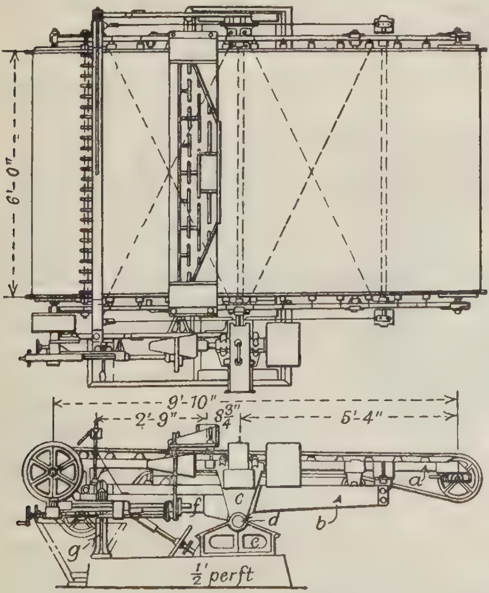


Fig. 2.—Isbell vanner.

Isbell vanner (Fig. 2) is a mechanical improvement on the Frue vanner. The shaking framework (*a*), which is of steel, is supported at the ends of two longitudinal leaf springs (*b*) which are bolted at their centers to

brackets (*c*) clamped to the transverse supporting shaft (*d*). This shaft rests, in turn, on a main transverse framework (*e*). An arm (*f*), likewise clamped to shaft (*d*) is attached to a power screw operated in floor stand (*g*), thus permitting ready variation in longitudinal tilt as one of the operating adjustments. Side shake is effected by means of a single eccentric, which permits ready adjustment of stroke length. Belt travel is effected in a manner similar to that of the Frue except that the flexible connection between worm wheel and head-pulley shaft is obtained by means of flexibly-connected cranks on the respective shafts. The method of mounting the shaking frame produces rectilinear motion of points on the belt surface, as compared to the curved path of the Frue.

Performances are given in Table 1.

Table 2. Elutriation-assay analysis of Frue-vanner tailing, Bunker Hill and Sullivan Mining Co. (*3 MM 54*)

Material	Assays	
	Pb, per cent.	Ag, oz.
Passing 0.074-mm. and settling in 6-mm.-per-second current	1.10	2.96
Rising in 6-mm. current and settling in 3-mm. current	8.10	4.58
Rising in 3-mm. current and settling in 0.3-mm. current	13.55	7.06
Rising in 0.3-mm. current	18.05	8.80
Feed to vanner	18.00	9.00

Table 3. Performances of 6-ft. Frue vanners at Bunker Hill and Sullivan Mining Co. (*3 MM 54*)

Character	Feed				Concentrate		Tailing	
	Tons per 24 hr.	Assays		+200	Per cent. Pb	Oz. Ag	Per cent. Pb	Oz. Ag
		Per cent. Pb	Oz. Ag					
Crushed middling, through 80-mesh Callow screen	6	12	5	32	......	......	7.0	3.0
Average slime	2	18	9	0.1	65	29	9.6	5.7
Very finest slime	1	15	8	0.1	70	26	13.5	7.4
Slime(*a*)	1.2	20	10	0	65	29	10.4	6.8
Slime(*b*)	1.2	20	10	0	44	20	9.3	6.2
Vanner tailing(*c*)	1.5	9.6	5.7	0.1	45	22	7.2	4.0
Vanner tailing, North mill(*d*)	2.0	9.6	5.7	0.1	40	16	6.6	3.8

a Making high-grade concentrate. *b* Making low-grade concentrate. This was the more profitable operation in 1909-10. *c* Experimental. *d* Regular mill run.

Table 4. Tests on Frue vanner with corrugated belt at Anaconda C. M. Co. (*49 A 426*)

Test number	1	2	3	4	5	6
Belt speed, ft. per min	5	4	2	6	9	10
Slope of belt, inches per foot	0.5	0.58	0.75	0.42	0.69	0.69
Tons solid per 24 hr	1.89	2.27	2.04	2.40	2.54	2.74
Per cent. of water in feed	91	......	90	91	90	89
Assays, per cent. Cu; feed	3.3	2.7	2.5	3.1	2.2	2.4
Assays, per cent. Cu; concentrate	8.9	7.2	6.7	7.2	5.1	6.5
Assays, per cent. Cu; tailing	2.7	2.2	2.0	2.4	1.6	1.8
Recovery, per cent	24.4	30.7	29.9	33.5	35.7	33.3
Ratio of concentration	11.1	8.6	9.4	6.8	6.2	8.7

Note—Speed in all tests: 196 @ 1-in. shakes per minute. Feed, 96 per cent. through 0.07-mm. screen.

Johnston vanner (Fig. 3) differs from the Frue and Isbell principally in the method of support of the shaking frame, which is hung by means of inwardly-inclined adjustable suspension links (*a*) from standards (*b*). This method of suspension produces an undulatory motion of the belt which, taken with the usually slower speed and longer stroke, prevents the formation of sand banks along the flanges.

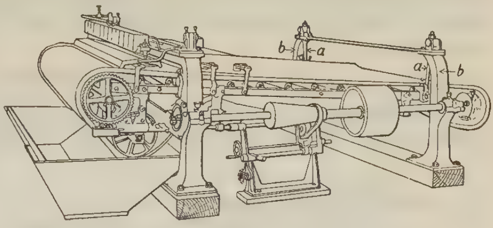


Fig. 3.—Johnston vanner.

Performances are given in Table 1.

3. End-shake vanners

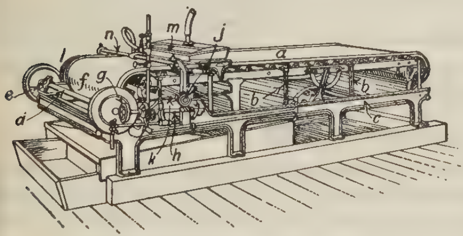


Fig. 4.—Triumph vanner.

Triumph vanner (Fig. 4) is of the oscillating end-shake end-slope variety. The shaking frame (*a*) is carried on lath springs (*b*) which, in turn, rest on main frame (*c*). Shake is transmitted through rods from eccentrics (*e*) mounted on drive shaft (*d*). The belt (*f*) is driven through a power chain composed of the friction disk (*g*), shaft (*h*), worm and gear (*j*), and shaft (*k*), carrying at the other end a pinion that drives gear (*l*). Feed sole (*m*) is mounted on the shaking frame and wash-water spray (*n*) on the main frame. The **Embrey vanner** is similar.

Performances are given in Table 1.

4. Differential end-shake, side-slope vanners

Luhrig vanner, Weir-Meredith and Monell vanners and the Bilharz-Stein shaking tables are all of the general type shown in Fig. 5, consisting of a shaking frame carrying an unflanged endless traveling belt 3 ft. 6 in. to 5 ft. wide by 12 ft. from center to center of head and tail rollers, all supported by means of rods from a tilting mechanism carried on the main frame. Differential end shake is transmitted from a suitable head-motion (see descriptions of shaking tables, Sec. 10) mounted on the main frame. Belt travel is effected by suitable connection of the head roller with the drive shaft. Pulp is fed from a distributing box near the tail roller and along the upper edge of the belt and suitable wash-water sprays are provided along the remaining length to the head roller. Tailing discharges along the lower edge opposite

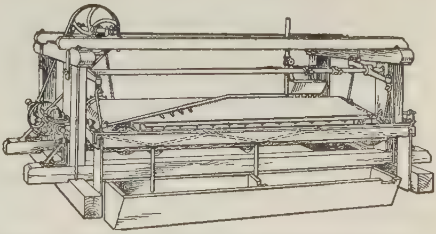


Fig. 5.—Differential, end-shake, side-slope vanner.

and somewhat forward of the feed box, middling along the same edge forward of the tailing, and concentrate at the lower edge near the head roller. This class of vanner makes three products as opposed to the two usually made on vanners.

5. Gyrating vanner

Senn vanner (Fig. 6) is of the end-slope variety but the shaking frame (*a*) oscillates both endwise and sidewise on ball-bearing supports (*b*). Sidewise shaking motion is transmitted from drive shaft (*c*) through eccentric (*d*); end shake is effected through eccentric (*e*), cranks (*f*) and (*g*) and rod (*h*). Belt motion is transmitted from the drive shaft through friction cones (*i*) and worm (*j*) to gear (*k*) on the head roller. Gyratory movement is depended upon to cause rapid stratification in a thick pulp and allow higher capacities than are usual with vanners. (See Table 1.)

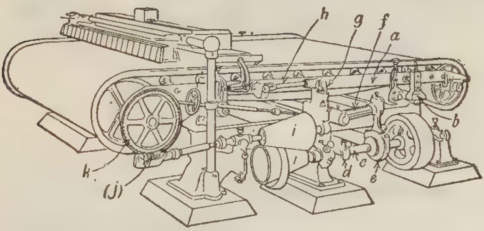


FIG. 6.—Senn vanner.

6. Operation of vanners

Removal of concentrate from the belt is normally effected by a spray directed at the face of the belt about half-way between the head and dipping rollers. Water consumption here is upward of half of the total for the vanner.

A rotary brush placed in the same position, driven from the vanner mechanism and followed by a padded roller is said (*106 J 713*) to have improved recovery as well as removed concentrate with less consumption of water. Explanation of increased recovery would lie in the maintenance of a clean rubber surface in place of a surface made "slimy" by clayey and colloidal materials in the feed pulp. Use of a 3-in. wood roller, placed so as to just touch the belt and a short distance below the head roller, is described (*106 J 1040*) as causing discharge of the bulk of the concentrate at a higher elevation than usual, allowing collection of most of the concentrate from a battery of vanners at a central point by means of launders running just below the rollers. In most mills concentrates are hoed out of boxes intermittently but some use a continuous spigot discharge. With such discharge a device similar to the Shackleford washer shown in Fig. 7 is necessary, if excessive dilution of concentrate is to be avoided. This washer is merely a ratchet-shaped rake (*a*) attached by means of rigid straps (*b*) underneath the dipping roller. The side shake of the vanner effects movement of solids to one side of the concentrate box, where they are discharged through spigot (*c*). Water level is maintained constant in the box by ball valve (*d*) and overflow (*e*).

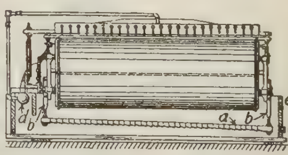


FIG. 7.—Shackleford vanner-box cleaner.

Important factors affecting performance are: Feed, shake, slope, belt travel, kind of belt and amount of wash water.

Size of feed. The maximum rarely exceeds 1.5-mm., the average maximum is between 0.6- and 0.8-mm. It is probable that the vanner should never be fed with material coarse enough to be treated on shaking tables, since the latter will treat sands with an efficiency equal to that of vanners and at much greater capacity.

Caetani (*3 MM 54*), after thorough testing of vanners, shaking tables, round tables and frames at Bunker Hill and Sullivan concluded that reciprocating (shaking) tables were superior to vanners for fine sand but that the latter were superior in every way on 200-mesh slime. Nowadays flotation will treat such material from sulphide ores much more efficiently than either machine.

A vanner will save finer mineral than can be saved on shaking tables because mass is not an essential in the moving of particles toward the concentrate discharge on vanners while it is on tables. The essential requirement, if a particle is to be caught on a vanner, is that it settle to the belt surface. Once there it adheres with relatively greater force the less its size and it is safe to say that a vanner will save any mineral particle that comes into contact with the belt, provided that very fine particles are not admixed with so much coarse sand that the amount of wash-water necessary to keep back the sand is sufficient to wash away the fine mineral.

The ability of a vanner to save mineral lost by a table is indicated in Table 5 (*106 J 711*) showing the performance of a Senn vanner treating Wilfley-table tailing. The feed to both

Table 5. Wilfley table *vs.* Senn vanner treating Mother Lode gold ore

Screen aperture, mesh	Table feed, weight per cent.	Table concentrate			Vanner concentrate		
		Weight, per cent.	Assay, ounces Au	Assay, ounces Ag	Weight, per cent.	Assay, ounces Au	Assay, ounces Ag
40	16.2	8.7			3.7		
60	24.1	22.6			1.8		
80	11.3	5.4	1.10	146.42	0.6	0.62	122.22
100	10.3	12.1			1.6		
150	5.7	7.9			0.9		
200	9.5	14.4			8.4		
Through 200	22.1	26.1	6.20	1688.5	79.1	1.02	147.94
Totals......	99.2	97.2	1.56	284.36	96.1	0.86	143.82
Tons.........	....	0.44	....	....	0.60	....	....

machines was of practically the same size, but substantially 80 per cent. of the vanner concentrate passes a 200-mesh screen as compared to about 25 per cent. of the table concentrate. The fine table concentrate was very rich, meaning very heavy, and therefore of sufficient mass to settle on and be moved along the table deck, while the large amount of slime value caught on the vanner would not settle and move with the concentrate on the Wilfley.

The size of grains affects the vanner adjustments. The usual practice is to treat fine feed with flat slope, slow belt travel and little wash water, and coarse feed *vice versa*.

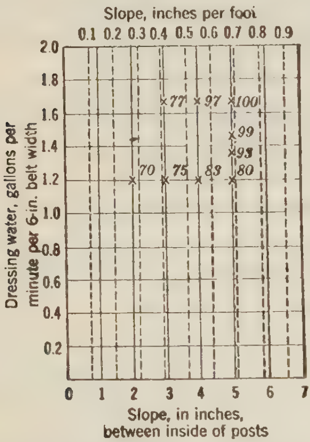
Richards (*TB 361*) recommends steep slope, rapid belt travel and little water for fine feed and gentle slope, much water and a long, slow shake for coarse. Gahl's work at Detroit Copper Co. (*40 A 517*) indicated steep slope, rapid belt travel and more than the usual amount of wash water to be best for slime treatment on smooth-belt vanners. Results are indicated graphically in Fig. 8.

Pulp consistency is usually between 80 and 90 per cent. water except on the gyrating vanner where it varies from 60 to 80 per cent.

Caetani recommends (*3 MM 48*) as high as 95 per cent. water in slime pulps, stating that thicker pulps increase capacity at the expense of efficiency. Gahl, on the other hand, found

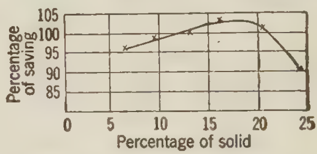
that maximum saving was obtained with between 15 and 20 per cent. solids in the feed, recovery increasing slowly to the maximum, then falling off rapidly with increased pulp thickness. His results are shown graphically in Fig. 9. The tests at the COMBINATION MILL on fine sand pulp (Table 6, tests 2 and 3) show that increase in percentage solids from 15.4 to 36.8 per cent., with increase in slope (and probably in belt speed) raises recovery, but at the expense of a decrease in grade of concentrate.

The pulp consistency should be such that the bed of pulp on the belt is sufficiently fluid to allow fine particles to settle readily. If the feed is too thick, the bed FELTS, *i.e.*, compacts into a semi-solid mass, if there is much clayey slime present, after which excessive wash water must be used to keep sand out of the concentrate, and practically no fine mineral can get through to the belt surface. The only remedy is to loosen the bed with a broom or scraper, sacrificing most of the mineral contained, and start afresh with thinner feed, slower belt travel or steeper slope. Felting is most common on side-shake vanners, less so with Johnston than with Frue and Isbell; it is less frequent on end-shake and least frequent on gyrating. Consequently end-shake vanners can be run with thicker feeds than side-shake, while gyrating can be run with very thick feed on account of the relatively perfect fluidity of the bed.



Figures at co-ordinate points are recoveries expressed in percentages of the maximum recovery attained in the series of tests. Belt speed, 120 in. per min. 220 @ 1-in. strokes per min. Feed: 13 per cent. solids; 90 per cent. −200-mesh; 1.40 per cent. Cu; 9.5 tons dry per 24 hr.

FIG. 8.—Relation of slope, wash water and recovery on smooth-belt vanners.



Saving is expressed in percentage of saving made with a pulp containing 12 per cent. solids. Slope, 0.54 in. per ft. 209 @ 1-in. strokes per min. 2.2 gal. wash water per min. Feed: 91 per cent. −200-mesh; 1.48 per cent. Cu; 7.93 tons solid per 24 hr.

FIG. 9.—Relation between pulp consistency and recovery on smooth-belt vanners.

Mineral of high specific gravity can be saved with steep slope, slow travel and much water as compared to mineral of low specific gravity; gangue of high specific gravity will require steeper slope, slower travel and more water than gangue of low specific gravity. If the ratio of concentration is low, concentrate will come over in a thick sheet with entangled gangue unless the machine is run with steep slope, rapid belt travel and much water. An ore with a high ratio of concentration, on the other hand, needs slow, careful treatment to save the small amount of mineral and discard the large amount of gangue, hence the machine is run at low slope and slow belt travel with little water.

Capacity varies with size of feed and character of ore, as well as with character of service, *i.e.*, roughing or finishing.

Table 6. Performance of Frue vanner, Combination mill, Goldfield, 1908.
(*Deister Machine Co.*)

Test number	Tons per 24 hr.	Per cent. of solids in feed	Slope, inch per foot	Revolutions per minute	Length of shake, inches	Wash water, gallons per minute
1	4.83	24.9	0.42	174	1	1.93
2	5.04	15.4	0.42	174	1	1.16
3	5.04	36.8	0.58	180	1	....
4	6.37	21.1	0.42	174	1	1.47
5	7.89	18.6	0.38	125	2	1.36
6	8.23	28.6	0.52	180	1	....
7	8.49	18.7	0.38	125	2	1.50
8	8.77	26.1	0.52	180	1	....
9	8.83	18.5	0.38	125	2	1.31
10	9.79	20.4	0.38	120	2	1.30
11	10.52	29.1	0.52	180	1	....
12	14.24	30.5	0.38	125	2	....
13	17.28	21.2	0.38	125	2	....
14	20.06	21.1	0.38	125	2	....

Test number	Assays, oz. Ag per ton			Ratio of concentration	Recovery, per cent.
	Feed	Concentrate	Tailing		
1	0.90	18.97	0.58	57.5	36.8
2	1.06	20.91	0.68	53.2	37.0
3	0.84	11.45	0.46	28.9	47.1
4	1.02	15.08	0.68	42.3	34.9
5	0.90	22.37	0.56	64.2	38.7
6	0.72	9.90	0.50	42.7	32.1
7	1.06	20.12	0.66	48.7	39.0
8	0.88	12.40	0.52	33.0	42.7
9	1.30	25.05	0.76	45.0	42.7
10	1.09	29.90	0.70	74.9	36.7
11	0.66	10.74	0.48	57.0	28.6
12	0.85	11.40	0.56	37.4	35.8
13	1.28	26.85	0.90	68.3	30.5
14	0.85	14.05	0.65	67.0	24.7

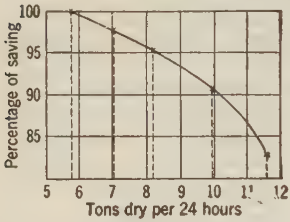
Note—Feed, fine sand underflow from Callow tanks.

Table 7. Tests on Embrey vanner at Anaconda C. M. Co. (*49 A 425*)

Test number	1	2	3	4	5	6	7
Revolutions per minute	225	225	225	225	225	248	247
Length of stroke, in.	1	1	1	1	1	0.62	0.62
Belt speed, inches per minute	80	114	120	121	192	192	216
Slope of belt, inches per foot	0.33	0.5	0.79	0.96	1.5	1.5	1.5
Tons solid per 24 hr.	2.55	1.70	1.96	1.95	1.91	2.60	2.71
Moisture in feed, per cent.	86	92	88	89	90	90	88
Assays, per cent. Cu: feed	2.4	2.7	3.2	2.5	2.5	2.3	2.5
Assays, per cent. Cu: concentrate	9.5	6.8	10.0	7.0	7.5	7.2	7.7
Assays, per cent. Cu: tailing	1.7	2.1	2.5	1.9	2.0	1.7	1.8
Recovery, per cent.	33.0	30.5	32.5	36.8	37.3	37.8	31.0
Ratio of concentration	11.7	8.3	9.8	7.6	7.2	9.4	10.8

Note—Feed, 96 per cent. through 0.07-mm. screen.

Table 3 gives capacities of a Frue vanner in finishing service on galena ore at Bunker Hill and Sullivan Mining Co. Capacity of 1 to 2 tons per hr. on slime is in accord with results at Anaconda, shown in Tables 4 and 7. Five to six tons per 24 hr. is close to the average on fine sands containing slimes. A general rule on such material is 1 ton per 24 hr. per ft. of belt width. Caetani recommends 0.6 ton per 24 hr. per ft. as maximum. In roughing service, making finished tailing only, the capacity is 10 to 20 tons. The Senn vanner at Phelps Dodge Co., Morenci plant, treated as high as 150 tons per 24 hr. of feed through 3-mm. Chilian-mill screens in roughing service but it is probable that shaking tables would be a better installation for such work in general. Fig. 10 (Gahl, *ibid.*) shows variation in recovery on a corrugated-belt vanner with variation in feed rate.



Saving is expressed in percentage of the saving made on a similar vanner with the same adjustments at 5.77 tons solid per 24 hr. Feed: 14.7 per cent. solid; 90 per cent. − 200-mesh; 1.31 per cent. Cu. 210 @ 1-in. strokes per min. Slope, 0.82 in. per ft. Belt travel, 120-in. per min.

Fig. 10.—Relation between recovery and feed rate on a corrugated-belt vanner.

Preparation of feed. Controversy has raged concerning whether the feed to vanners should be classified or unclassified. Proponents of the first method argue that the vanner is a film sizer (see Sec. 8, Art. 14) and that it will, therefore, do its best work when the particles of valuable mineral are smaller than the gangue particles, but not so much smaller that the mineral is washed away in the current necessary to move the gangue.

Richards (*TB 363*) plots the results of a sizing-sorting test on the tailing of a well-run vanner treating −0.75-mm. quartz-pyrite ore, and shows the average ratio of size of quartz to that of pyrite to be about 10 to 1. He concludes that the finer pyrite should have been removed from the feed by classification. Hancock (*24 MM 80*) holds that since film sizing results in putting together in the tailing fine heavy mineral and coarse light mineral it is wrong to feed material that has this composition to a vanner, hence he advises against classification. He cites Gilbert's experiments (*PP 86 USGS*; see Sec. 20, Art. 1C) showing the relation between size transported and stream velocity over a smooth surface to show that fine mineral is transported in suspension in a current necessary to roll coarse gangue. He thinks that with a vanner operated under a given set of conditions the size of mineral in concentrate will be close to a definite mean figure, that tailing will contain both coarser and finer particles, and that these will be saved by another vanner, differently run, treating the tailing of the first. Beringer's results (*24 IMM 411*) showing that cassiterite in a vanner concentrate made by re-treating vanner tailing fell into two definite size groups, viz.: from 0.050- to 0.075-mm. and from 0.010- to 0.015-mm., confirm this view. McDermott (*24 IMM 447*) maintains that suspended fine material flows, with the water, over and through the interstices between coarse grains in the bed of pulp on the vanner and that retardation of the flow by coarse grains effects settlement. Removal of coarse would, in his opinion, cause the fine pulp to progress in waves and at higher velocity, with less settling time available and more disturbance of settled particles due to the impact of waves. He notes, also, that the capacity of a vanner is increased three to four times over that as a slime vanner, when coarse material is present. The weight of practice has been to treat unclassified feeds. In California gold mills 40-mesh stamp product was sent to vanners after amalgamation on plates; in Cornish tin practice vanner feed comes directly from stamps or is the tailing of tables taking feed direct from stamps; in American base-metal practice vanner feed has almost always been hydraulic classifier overflow, which, while it contains grains of gangue coarser than the largest grains of mineral, is not a true classified product (see Sec. 6, Art. 1).

Richards' and Beringer's results seem to point rather to series treatment of unclassified pulp on different machines than to classification with accompanying dilution of products. All of the work shows that one machine cannot be expected to make clean concentrate and substantially complete recovery on unclassified feed, notwithstanding the upper size limit, and that the difficulty will increase markedly with increase in this size.

Shake is dependent on both length of stroke and number per min. Side-shake vanners are usually run at between 180 to 200 @ 1-in. strokes per min.,

except that the Johnston is run more slowly (120 to 140) with a stroke between 1 and 2 in. End-shake vanners usually make from 200 to 240 @ 1-in. strokes per min. Side-slope vanners run at 160 to 200 @ ¼- to ¾-in. strokes per min. and gyrating at 140 to 160 @ ½-in. to ¾-in. strokes.

The amount of shake has an important bearing on the minimum size of grain that will settle on a belt.

Beringer (*24 IMM 411*) investigated the effect of shake on rate of settlement of cassiterite grains. He found that on a Frue vanner run at 180 strokes per min. 0.035-mm. particles required 60 sec. to settle in a 2-oz. phial while 0.025-mm. particles settled the same distance in the same time when the phial was at rest. Complete results are given in Table 8. Corresponding sizes on a shaking table were 0.005- to 0.010-mm. larger (see Sec. 10, Art. 14). He concluded that cassiterite between 0.030- and 0.060-mm. would be saved on a vanner without difficulty. Study of a vanner concentrate made in a CORNISH mill showed that most of the mineral lay between 0.015-mm. and 0.050-mm. Coarser mineral had already been taken out by shaking tables. Another vanner, treating unclassified feed directly from stamps saved mineral ranging from 0.010- to 0.120-mm., but there was a very small proportion below 0.010-mm. On the other hand, the smaller the particle the stronger the adhesion between it and the belt surface. Therefore the important matter in saving fine material is to get it down to the belt and the shake should be the minimum that will maintain the bed in a fluid condition. With the vanner adjusted to a given shake, decrease in speed with consequent decrease in violence of shake will result in carrying sand into the concentrate.

Table 8. Rate of settlement of cassiterite grains on a Frue vanner. (180 strokes per minute)

Time, seconds	Size of particles that settle, mm.	
	Phial at rest	Phial attached to vanner
60	0.025*a*	0.035
90	0.020	0.030
120	0.015	0.025*b*
150	0.010	0.025*a*

a, b The particles marked 0.025*a* are slightly smaller than those marked 0.025*b*.

The COMBINATION MILL tests (Table 6, tests 4, 10, 11 and 12) indicate that a long, slow shake as opposed to a short, rapid one results in increased capacity and higher grade of concentrate with the same recovery and that, if grade of concentrate is held constant, capacity is increased or recovery can be increased, if the feed rate is held constant.

Slope is closely dependent upon speed of belt, steep slope corresponding to high belt speed and *vice versa.* With side-shake vanners slope ranges from 0.15 to 1 in. per ft. and is usually steeper with coarse feed or where clean concentrate is desired than with slime feed or in roughing service. Richards' average for all service (*Peele* 1684) is from 0.28 to 0.31 in. per ft.

Prof. Richards has consistently recommended steep slope and high speed for slime feed, probably on the theory that with the correspondingly thin bed of pulp, mineral particles had a short and relatively unimpeded path to the belt, and Gahl's results shown in Fig. 8, confirm this view. Tables 4 and 7, presenting ANACONDA test work also illustrate this practice. The COMBINATION MILL tests (Table 6, tests 1, 4 and 8) show that increase in slope permits increase in tonnage treated. End-shake vanners have been used frequently to clean up concentrate from side-shake and, therefore, show a steeper average slope than side-shake. Richard's figures (*Peele,* 1684) are 0.6 in. per ft. Side-slope vanners are set at ½ to ¾ in. per ft. side slope and from level to ½ in. per ft. rise toward the head-roller end. The Senn vanner is recommended to be tried at ⅝ in. per ft. slope to allow high belt speed and thick feed pulp. If, under these conditions, fine mineral goes into the tailing, the slope must be flattened. Corrugated belts are usually set on a steeper slope than smooth belts in order to cut down the amount of wash water necessary.

Belt speed determines the rate at which settled concentrate is removed and also the amount of wash water that must be used to produce clean concentrate. Belt speed and wash water are the two adjustments most depended upon by vanner operators for controlling results. High belt speed tends to keep fine mineral out of the tailing but also drags coarse gangue into the concentrate unless considerable wash water is used and this increase in water currents flowing down the belt may defeat the purpose of high belt speed by

keeping fine mineral in suspension. The average speed for side-shake vanners is between 36 and 48 in. per min., with higher speed when a rough concentrate is sought and lower when particularly clean concentrate is desired. Corrugated belts may be run at lower speeds than smooth on account of the fact that fine mineral settles in the corrugations and is protected from the wash-water; they must be run slowly when coarse sand is present on account of the difficulty of washing this down-slope. Speed of belt on end-shake vanners averages 60 in. per min. according to Richards (*Peele* 1684) and in the Anaconda test work (Table 7) ranged from 80 to 216 in. per min. It is worthy of note, however, that in this work a speed of 80 in. per min. at 0.33 in. per ft. slope produced results substantially equal to those obtained at 216 in. per min. with a slope of 1.5-in. per ft.

Luhrig vanner at BROKEN HILL was run at 180 ft. per min. belt speed, this high speed being necessary in order to move the concentrate through the required horizontal distance before it was washed across the relatively narrow smooth belt into tailing or middling launders.

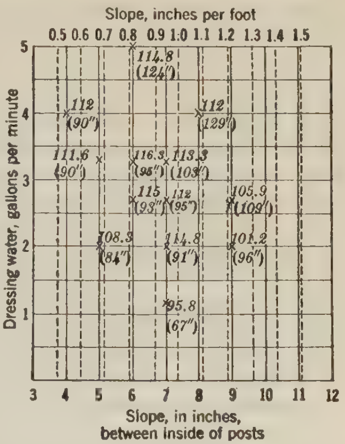
Wash-water consumption per machine is low, ranging from about 1 to 5 gal. per min., including both dressing water (added on top of belt) and water added to remove concentrate from the belt. The maximum water consumption per ton corresponds to attempts to make high-grade concentrate from finely ground low-grade feed. Side-slope vanners use more water per minute than end-slope. The LUHRIG requires from 5 to 10 gals. Wash-water consumption per ton of ore treated is less on vanners than on shaking tables in the same service. The SENN vanner uses much less water per ton of feed than side- and end-shake on account of its large capacity.

Depth of bed is a resultant of all adjustments. Flat slope, high belt speed, coarse feed, thick pulp and small amount of wash water all tend to produce a thick bed of pulp on the belt and *vice versa*. Richards recommends 0.25 in. maximum thickness (*Peele* 1685). Beds up to 0.75 in. thick are used when crowding vanners in roughing service but are too thick for finishing. Channeling of bed, either by reason of side banks or center banks will cause dirty concentrate or rich tailing or both.

Side banks. Frue and Isbell vanners are peculiarly liable to formation of hard banks of sand along the flanges. These creep up into the concentrate and also increase tailing loss. They are formed by any knock or irregularity in the shaking motion such as by a loose or worn eccentric or connecting rod or end play in small supporting rollers or in end rollers. A bank on one side may be formed because the belt is not level transversely or because the supports of the shaking frame are not parallel. Parallel longitudinal ridges will form if the bed is too thick. The REMEDIES are to increase water, decrease belt travel or increase slope. The Johnston vanner, on account of the more gentle and tilting side motion, is comparatively free from side banking and end-shake vanners are, of course, entirely free.

Belt is usually smooth rubber with two plies of canvas and a flanged edge 1 in. to 1.25 in. high. Belts with transverse 60° corrugations spaced 8 to 32 to the inch have had considerable vogue in treating relatively coarse feeds. Fig. 11 (*40 A 517*) shows comparative results with corrugated- and smooth-belt vanners on slime feed. Corrugated belts permit low belt speed but require large quantities of wash water to hold back gangue. This difficulty increases with the size of the corrugations. Tailing is generally lower than on smooth belts but concentrate is correspondingly lower grade. Table 9 gives comparative results on a silver ore. *Richards* reports running a corrugated belt on a Johnston vanner at 5 in. per min. without excessive tailing loss. Canvas belt, usually painted with some water-proof compound, has been used for fine slimes but it is difficult to wash out fine gangue from the uneven surface without also removing fine mineral. Belts should be kept sufficiently tight to prevent sagging of the upper surface between the support-

ing rollers. End rollers are adjustable lengthwise to permit tightening. An egg-shell surface, pitted with shallow depressions about 0.6 by 0.1 in. has been made. The life of rubber belts is from 2 to 6 years, if protected from abuse. Belts are frequently injured by hoes used in digging concentrate out of collecting boxes. This may be obviated by placing a light guard in front of the belt or by using some kind of automatic discharge. Old vanner belts make good covers for shaking tables or liners for chutes and launders.



Saving is expressed in percentage of the saving made on a standard vanner with smooth belt: 229 @ 1-in. strokes per min., 5-in. slope between posts, belt travel 123 in. per min. Corrugated belt, 190 @ 1-in. strokes per min.; feed: 12.64 per cent. solids, 91 per cent. −200-mesh; 1.4 per cent. Cu.; 8.43 tons dry per 24 hr. () Figures in parenthesis are belt travel in inches per minute.

Fig. 11.—Comparison of smooth- and corrugated-belt vanners on slime feed.

Attendance necessary varies with service and regularity of feed and wash-water supply. In finishing service one man can attend to 30 to 40 machines, if feed and water supply are regular, but when dirty reclaimed water is used and the wash-water supply consequently clogs, or if the feed is irregular, one man will have difficulty with 20 machines. In roughing service a man can handle 100 machines without difficulty. The vanner is a complicated machine, when judged by other concentrators; it requires skilled operatives and repairs are frequent.

Efficiency of vanners is always low. Recovery rarely exceeds 50 per cent., even on rich galena slimes; it ranges between 20 and 35 per cent. in finishing service on copper slimes, and rises to 60 to 75 per cent. when making rough copper concentrate.

Table 9. Comparison of corrugated-belt and smooth-belt vanners on silver ore (*21 A 280*)

Test number	Smooth belt				Corrugated belt			
	Concentrate			Tailing, assay, oz. Ag	Concentrate			Tailing assay, oz. Ag
	Pounds	Assay, oz. Ag	Silver content, oz.		Pounds	Assay, oz. Ag	Silver content, oz.	
1	705	142.5	63.3	5.1	1005	147.5	72.5	3.3
2	982	235.5	120.5	13.3	1401	199.2	145.5	8.8
3	1540	328	251.8	10.9	2275	291	332.5	7.7
Average	1079	286.8	145.2	8.1	1563	212.6	183.5	6.6
Percentages								
1	100	100	100	100	143.8	85.8	118.8	63.4
2	....	....	....	....	147.8	84.3	125.2	68.0
3	....	....	....	....	147.8	88.2	132	70.7
Average	100	100	100	100	146.3	86.2	125.3	67.4

End-shake *vs.* side-shake vanners. Theoretically the concentrate from an end-shake vanner should be of lower grade and the tailing of higher grade than the corresponding products of a side-shake machine on account of the fact that feed and wash water are brought to the belt in streams and these have greater transverse distribution on a side-shake than an end-shake machine, hence ridges will form on an end-shake machine up which sand will travel between the wash-water streams while fine mineral will be carried into the tailing in the streams rushing down the valleys between these ridges. Exact comparative data are not available but Tables 4 and 7, giving results on the same kind of feed, show that as between a Frue vanner with corrugated belt and an end-shake machine, the result is the reverse of that to be expected and the end-shake machine produced a higher average grade of concentrate and lower grade of tailing than the side-shake.

Frue *vs.* Johnston vanners. Table 10 presents a comparison between these types of vanners in the old mill of Nevada Cons. Cop. Co. The Frue corrugated vanner is distinctly superior to the Johnston in grade of concentrate and grade of tailing but the capacity is markedly less; there is little to choose between the smooth-belt machines, such advantage as there is lying with the Johnston. All smooth-belt machines were much overloaded.

Table 10. Comparison of Frue and Johnston vanners at Nevada Consolidated Copper Co., 1909–10

Row number	Frue vanners, Sections 1 and 2							
	Assays, per cent.				Recovery, per cent.	Tons per 24 hr.	Water consumption, gallons per minute	
	Feed, Cu	Concentrate		Tailing, Cu			Above belt	Below belt
		Cu	Insoluble					
1	1.28	19.2	34	0.43	68	8	3.4	4.0
2	0.97	18.8	30	0.53	47	12	2.7	3.1
3	0.92	16.1	38	0.70	25	9	1.8	2.6

Row number	Johnston vanners, Sections 3 to 8, incl.					
	Assays, per cent.				Recovery, per cent.	Tons per 24 hr.
	Feed, Cu	Concentrate		Tailing, Cu		
		Cu	Insoluble			
1	1.58	17.9	36	0.56	67	11
2	1.00	16.5	34	0.52	50	12
3	0.93	16.4	36	0.67	29	11

Notes—Speed: Frue, 192 @ 1-in. strokes per minute; Johnston, 120 @ 2-in. First row, all sections, corrugated belts; other rows smooth. Feed to first row was middling from Wilfley tables treating products from first three spigots of hydraulic classifiers treating reground middlings; second row took middling from Wilfleys treating primary slimes; third row took thickened slime from these same Wilfleys and slime from the re-grinding circuit. Assays are averaged from shift samples; each figure represents 25,000 cuts on Frues and 58,000 on Johnstons, 3900 and 3000 copper determinations respectively and 430 and 1000 determinations of insoluble.

SECTION 12

FLOTATION

1. Introduction

Definition. Flotation is a method of wet concentration of ores in which separation of mineral from gangue is effected by causing the mineral to float at or above the surface of a body of liquid pulp while the gangue becomes or remains submerged. The method operates, in general, only on particles smaller than 0.5- or 0.3-mm. diameter. The minimum recoverable size probably includes the finest particles produced in grinding ore. Minerals that float readily are those of metallic, resinous or adamantine luster, such as the base-metal sulphides; those that readily sink are the vitreous or earthy gangue minerals; but separation by flotation is possible between minerals of the first class, and some of the minerals of the second class can be floated away from others of that class. Thus galena can be separated from blende; fluorite, calcite and apatite can be separated from quartz and most other silicates; coal can be separated from slate, and mica from other silicates.

Types of processes. Flotation processes are of three different varieties, namely, (*a*) film flotation, (*b*) oil-buoyancy flotation, (*c*) froth flotation. In film flotation the separation is made at the upper or air surface of a body of water; in oil flotation, at the interface between a mass of oil and a mass of water; in froth flotation, by means of a froth floating on the surface of a body of pulp.

Only froth flotation is of commercial importance, but understanding of the phenomena of film flotation and oil flotation is necessary to a proper understanding of froth flotation.

2. Film flotation (Skin flotation)

If a sulphide ore is crushed dry and sized through, say, 0.3- or 0.2-mm. and brought gently onto the surface of a body of still water, a certain number of the particles will float. The floating particles will usually contain a materially higher proportion of sulphide mineral than was present in the original ore. Each floating particle or group of particles appears to rest in a dimple in the water surface. The actual condition is the same as that of the floating needle in the familiar parlor trick and is illustrated in Fig. 1*c*. A clean glass needle of equal length and weight will not, ordinarily, float. This difference in behavior between the two substances is specific and is due to a difference in surface characteristics that can be described as RESISTANCE TO WETTING. Metallic surfaces are generally more resistant to wetting than non-metallic. Metallic substances, therefore, if treated as above, generally float; non-metallic become wet and sink. If the crushed ore described above is wetted and then drained, as, for instance on a vanning plaque, the sulphide minerals dry more rapidly than the gangue, and if a film of water is thereafter flowed gently over the drained mass, a part of the solids, sulphides predominating, will float on the water surface.

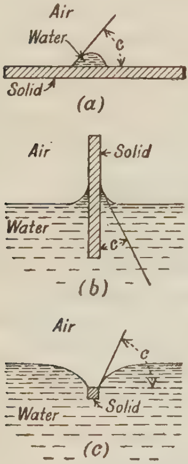


FIG. 1.—Contact angles, gas-liquid-solid.

Theory of film flotation. When a drop of a liquid, such as water, is placed upon a solid surface, *e.g.*, a piece of metal, the drop assumes a position shown in cross-section in Fig. 1*a*. The angle (*c*) between the solid surface and the tangent to the water surface at the apparent meeting point of water and solid, in a plane passing through the center of the water drop and at right-angles to the solid surface, is called the ANGLE OF CONTACT. This angle is specific within certain limits for the contacts of different liquids and solids. For water against solid surfaces of metallic luster it is generally larger than against solid surfaces of vitreous or earthy luster; it is generally larger for water than for oil against any given solid; and the difference in the angle for water against two different solids, as say, quartz and galena, may be affected by small amounts of solutes in the water. Other examples of the apparent contact angle formed when the three phases, solid, liquid and gas, meet are shown in Figs. *1b* and *1c*.

Existence of a contact angle is challenged by some physicists, or, at least, a zero angle is claimed to exist in all cases of a three-phase contact such as is pictured in Fig. 1. It is claimed that when the condition pictured exists a fourth phase consisting of a film on the solid surface intervenes between it and the two fluids. Experiment indicates this to be a fact. If the solid is platinum sheet, cleaned carefully in alkali and strong sulphuric acid and burned to remove the last trace of grease, a drop of water will not stand on its surface; if inserted into water as in Fig. 1*b* and then withdrawn, it is uniformly wet; the adhering water does not draw together in drops; and the platinum will not float as pictured in Fig. 1*c*. The same is true of clean glass or quartz. It is difficult to similarly clean metals other than platinum, or metallic sulphides, and at the same time insure against surface oxidation, but mineral surfaces produced from the center of lumps of pure mineral, with every precaution against surface contamination, behave more like clean platinum than like the solid in Fig. 1. Dean and White (*124 P 410*) found that the air-water contact angle against sulphides cleaned with sodium hydroxide and pyrogallol was zero immediately after the cleaning, but that after a period varying from a few minutes to several days, according to the specimen, a definite contact angle was again found. Calcite and garnet behaved similarly, but recovery (contamination) was slower and more erratic. From the practical standpoint, however, contact angles between water and various minerals, ranging from a few degrees to 90° or more, do exist, and the differences are sufficiently definite to permit commercial utilization.

Contact angle is the result of equilibrium of a system of forces, mutually exerted by the molecules of the three phases in contact, whose effective

resultants lie in the contact surfaces. Fig. 2 is a vector diagram of these forces at any point (O) on the periphery of the drop pictured in Fig. 1a. The actual directions of the forces OA and OD are not known, but they must act in opposite directions. The equation of equilibrium is $OA = OD + OB \cos C$, from which it follows that $OA > OD$. If OA and OD are assumed reversed in direction, the equation becomes $OD = OA + OB \cos C$, and $OD > OA$. Neither conclusion can be confirmed, because of inability to measure the interfacial force or surface tension of solids against other phases. Surface tension of liquids against gases and against other liquids can, however, be measured. The surface tension of a liquid against a second liquid, whose tension against a gas (air) is less than that of the first, is generally less than that of the first against the gas. Further, the surface tension of molten solids, such as salts and metals, is much higher than that of substances liquid at ordinary temperatures. Hence, it is not unreasonable to suppose $OA > OD$ and that the directions of the vectors in Fig. 2 are correct. (For an exhaustive discussion with many experimental data see Coghill and Anderson, *TP 262, USBM*.)

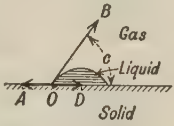


FIG. 2.—Forces at 3-phase contact.

Experiment shows that the relative magnitudes of OA and OD are different for water against different minerals. Table 1 presents determinations on several common constituents of ores. Values under the caption "Minimum value = θ" represent the angle maintained on the higher edge of a drop rolling down an inclined surface of the mineral and those under caption "Maximum value = θ'" are those on the lower or advancing edge. Sulman applies the term HYSTERESIS OF CONTACT-ANGLE to the difference between the two values.

Table 1. Contact angles of ore minerals with plain water. (*After Sulman*)

Mineral	Contact angle		Difference or hysteresis
	Minimum value = θ	Maximum value = θ'	
	degrees	degrees	degrees
Chalcopyrite	37.0	87.0	50.0
Stibnite	24.0	62.8	38.3
Rosin blende	47.0	81.0	34.0
Magnetite	21.7	80.8	69.1
Marcasite	56.5	83.5	28.0
Iron pyrite	25.5	87.0	61.5
Quartz	19.5	58.5	39.0
Calcite	39.6	85.5	45.9
Garnet	58.2	94.5	36.3
Galena No. 1	35.0	73.0	38.0
Galena No. 2	41.6	70.0	28.4
Molybdenite	12.6	62.5	49.9
Chalybite	45.2	90.0	44.8
Glass	33.0	39.5	6.5

Effect of differences in contact angle on flotability of two pieces of the same size, shape and weight is shown in Fig. 3. Fig. 3a shows a particle against which the liquid makes a contact angle nearly 90°. The vertical component of (T), available to counteract (W) (weight of solid in liquid), is nearly as great as (T) itself. In Fig. 3b, the contact angle is small and, although (T) is the same as before, the effective component is much less.

Hence the first particle may float and the second sink. This was for many years taken as the complete explanation of the separation possible by film flotation. But reference to Table 1 shows that on the basis of magnitude of contact angles alone, the advancing angle being the effective one, the order of floatability would be garnet, chalybite, chalcopyrite and pyrite, calcite, marcasite, rosin blende, magnetite, galena, stibnite, molybdenite, quartz, glass. This order is distinctly not in accord with experience, as is shown by Table 2. From this table the order of floatability, at the most favorable size for comparison (−1.168 +0.833-mm.) is sphalerite, galena, chalcocite, calcite, dolomite, quartz, siderite.

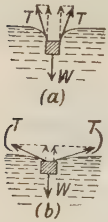


FIG. 3.—Forces in film flotation.

The order of magnitude of the hysteresis of the contact angle is claimed by Sulman (*29 IMM 44*) to gage the order of floatability more correctly. The minerals in Table 1, arranged on this basis are: magnetite, iron pyrite, chalcopyrite, molybdenite, calcite, chalybite, quartz, stibnite, galena, garnet, rosin blende, marcasite, glass. While this arrangement likewise does not agree with actual experience as to floatability, as presented in Table 2, it approaches it more nearly. Sulman's argument is plausible. He says that hysteresis of the contact angle increases the range of equilibrium of the forces causing flotation of the particles, and thus allows the system to adjust to changes in direction and magnitude of the forces caused by disturbances of the water surface. An unchangeable contact angle, he properly maintains, would give a system incapable of such adjustment.

Table 2. Flotation of dry, unoiled minerals at a clean water surface. (*School of Mines, Columbia University*)

Mineral	Sp. gr.	Percentage of different sizes (*a*) floating (*b*) Mm.				
		−4.699 +3.327	−3.327 +2.362	−2.362 +1.651	−1.651 +1.168	−1.168 +0.833
Calcite	2.7	0	19	19–31	19–25	67
Quartz	2.7	. . . .	0	5–32	15–17	30–44
Dolomite	2.8	. . . .	0–4	0–29	21–34	37–55
Siderite	3.8	. . . .	2	7	15	30
Galena	7.5	. . . .	0	0	1	85–90
Chalcocite	5.7	. . . .	6–8	18	42	74
Sphalerite	4.0	. . . .	5	46	95	100

Mineral	Sp. gr.	Percentage of different sizes (*a*) floating (*b*) Mm.				
		−0.833 +0.589	−0.589 +0.417	−0.417 +0.295	−0.295 +0.208	−0.208 +0.147
Calcite	2.7	79–85	75+	95+	. . . .	. . . .
Quartz	2.7	65–79	76–92	71–76	±80	±90
Dolomite	2.8	86–96	65–83	88–93	. . . .	. . . .
Siderite	3.8	56	70–77	70+	±80	±90
Galena	7.5	100	. . . .	. . . .	. . . .	. . . .
Chalcocite	5.7	95	. . . .	. . . .	. . . .	. . . .
Sphalerite	4.0	. . . .	. . . .	. . . .	. . . .	. . . .

a Sizes in mm. *b* Flotation was obtained by placing the pieces on top of a glass dome, 1 in. diameter, projecting 5⁄16 in. above the water surface, and allowing them to slide down onto the surface. Percentages were obtained by counting the number of pieces that floated out of 100 tried.

Effect of dissolved substances on contact angles is shown in Table 3. Minerals in the order of magnitude of advancing angles are: chalcopyrite, rosin blende, marcasite, galena, iron pyrite, molybdenite, stibnite, garnet, quartz, magnetite and glass. In order of hysteresis range they are: Molybdenite, iron pyrite, galena, chalcopyrite, stibnite, marcasite, rosin blende, garnet, magnetite, quartz and glass. Both of these orders are in consistent accord with general experience as to the greater floatability, on acidified water, of sulphides than gangue, and the second probably approaches the truth more closely in detail than the first. The remarkable things to note, however, in comparing Tables 1 and 3, are the marked and consistent drop in all contact angles produced by the acid in solution, and the fact that the drop averages 19.2° for the sulphide minerals as against 54.8°, nearly three times as much, for the gangues. There is a similar proportionate drop in the hysteresis. The possible benefit of dissolved substances on film flotation is referred to by Macquisten (U. S. patent 865,194).

Table 3. Contact angles of ore minerals with acidified water (0.7 per cent. H_2SO_4). (*After Sulman*)

Unoiled mineral	θ, degrees	θ', degrees	Hysteresis range, degrees
Chalcopyrite...	44.0	72.0	28.0
Stibnite........	17.0	44.0	27.0
Rosin blende...	40.6	64.7	24.1
Magnetite.....	4.0	4.0	0.0
Marcasite......	32.5	59.4	26.9
Iron pyrite.....	14.4	52.0	37.6
Quartz........	9.25	9.25	0.0
Garnet........	16.0	37.0	21.0
Galena No. 1...	19.0	53.0	34.0
Galena No. 2...	21.4	57.8	36.4
Molybdenite...	11.7	50.5	38.8
Glass..........	4.0	4.0	0.0

Oiling of mineral particles changes both contact angles and hysteresis. Oiling is a phenomenon similar to water-wetting, when performed in the presence of air (i.e., on dry pulp), but differing therefrom in the fact that while all contact angles are low, those on particles of metallic luster are lower than those on gangue particles. Hence when oil is incorporated with a dry pulp there is some tendency for the sulphide particles to become wetted more completely than the gangue. But the tendency is not sufficiently sharp and definite to be useful. When, however, oil and water are brought simultaneously to the surfaces of mineral and gangue particles, out of the presence of air or other gas, a relatively sharp differentiation in behavior occurs, as illustrated in Fig. 4. The oil makes a small contact angle with sulphide mineral and displaces water from the sulphide surface, while the reverse is true at the gangue surface and the latter becomes or remains water-wet. When dry-oiled particles are brought into contact with water a similar tendency exists and the water to some extent displaces oil from the gangue surfaces and wets them, while it is repelled from sulphide surfaces to an even greater extent than when these are unoiled.

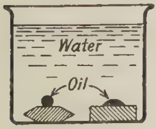


FIG. 4.— Behavior of oil and water at gangue and mineral surfaces.

Sulman and Picard (*The theory of concentration processes involving surface tension, Exhibit Minerals Separation North American Corp.* vs. *Magma Copper Co.*) gave the results presented in Table 4 for contact angles of oiled minerals with acidified water. These are to be compared with those for unoiled minerals in Table 3. The order of decreasing floatability based on advancing contact angle is: chalcopyrite, stibnite, rosin blende, specular iron, marcasite, galena, molybdenite, garnet, quartz, magnetite. Based on hysteresis the order is: Chalcopyrite, galena, marcasite, specular iron, molybdenite, stibnite, rosin blende, garnet, quartz, magnetite. Excluding the specular iron, these results accord fairly with experience. Table 5 presents results with oiled sulphides comparable to those with unoiled minerals in Table 2.

Wood dry-feed film-flotation machine (1,088,050/1914) is typical of the dry-feed machines (Fig. 5). It consists of two tanks (A) and (P), filled with water. A roller (C), covered

Table 4. Contact angles of oiled minerals with acidified water

Mineral	Contact angles made between minerals coated with film of paraffin wax, and 0.7 per cent. H_2SO_4 water		
	θ, degrees	θ', degrees	Degrees of hysteresis range
Chalcopyrite	54.5	166.5	112.0
Chalcopyrite composite face	58.0	96.0	38.0
Specular iron	41.5	141.5	100.0
Stibnite	73.0	154.0	81.0
Rosin blende No. 1	73.0	153.4	80.4
Rosin blende No. 2	44.0	112.5	68.5
Magnetite	8.5	29.3	20.8
Marcasite	30.0	135.0	105.0
Iron pyrite	55.0	....	....
Quartz	18.0	76.3	58.5
Calcite	....	....	....
Garnet	29	86 to 110	57 to 81
Galena No. 1	23	134	111
Galena No. 2	25	132	107
Molybdenite	38–48	123–139	85–91
Chalybite	....	....	....
Glass	....	....	....

Table 5. Flotation of dry, oiled sulphides at clean water surface. (*School of Mines, Columbia University*)

Mineral	Sp. gr.	Percentages of different sizes (*a*) floating (*b*) Mm.				
		−3.327 +2.362	−2.362 +1.651	−1.651 +1.168	−1.168 +0.833	−0.833 +0.589
Galena	7.5	....	....	6	77–85	100
Chalcocite	5.7	2	14	72–80	100	....
Sphalerite	4.0	3	42	80–92	100	....

a Sizes in mm. *b* Method of effecting flotation is described in note *b*, Table 2. Mineral was oiled with about 2 per cent. oleic acid by weight, applied in benzol solution, from which the benzol evaporated leaving oleic acid as a film-coating on the solid.

with corrugated rubber belting and submerged with its center well below the surface of water in the tank (*A*), rotates in the direction indicated. As the roller emerges from the body of liquid, it carries with it, covering its upper surface, a thin layer of water. Dry ore is fed onto the surface of the revolving roller, in a thin sheet by means of the shaking feeder (*G*), the gangue minerals tend to wet and sink into the grooves, while the minerals of metallic luster tend to float. When the floating and submerged minerals are carried over to the point where the surface of liquid in the tank intersects the surface of the roller, the floating mineral rides out onto the water surface because of the fact that the surface film is continuous over the tank and the roller. The gangue minerals remain submerged and finally fall off and settle to the bottom of the tank. At the opposite side of tank (*A*) is another roller (*I*). An endless rubber belt (*K*) passes over this roller, thence in turn over pulley (*R*), and guide roller (*M*). A gentle current is maintained from

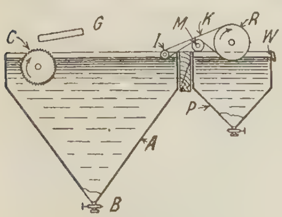


FIG. 5.—Wood skin-flotation machine.

(C) toward (I) by reason of the constant addition to the surface film at (C) and constant removal at (I) by the traveling belt (K). The surface film is continuous from the liquid in tank (A) to the liquid in tank (P) over the surface of belt (K). Due to the disturbance at the point where belt (K) passes below the surface of the liquid in the tank (P), the less tightly held material in the film is shaken out and settles to the bottom of tank (P). This material constitutes the middling of the process and is re-treated on gravity-concentration apparatus. Tailing is discharged through valve (B). Floating concentrate overflows lip (W), the level of liquid in tank (P) being maintained so as to overflow a thin sheet of liquid at this point.

In an earlier patent, 984,633/1911, Wood states that in certain instances a small quantity of oil will increase selection.

The machine as built had a feed roller 3 ft. wide, required about 0.25 hp. and is said (*44 A 684*) to have had a capacity of from 1000 to 2000 lb. per hour.

Other dry-feed film-flotation machines differ in no way in principle from the Wood machine.

Nibelius (486,495/1892). Vertical gravity feed. Radial skimming current induced by rising current of water at the center of the separating box. Re-treatment of float concentrate in successive shallow troughs with side overflow and short vertical drops between.

Brumell (678,860/1901). Fixed inclined feed. Lateral skimming current induced by a special submerged fitting on the water-supply pipe.

Allen (688,279/1901). Vertical gravity feed onto water in an inclined launder, discharging, so as to cause ripples, into a spitzkasten. The spitzkasten discharges tailing and a rough concentrate, the latter by inclined launder onto the surface of water in a shallow box with several submerged weirs, where middling is shaken out by ripples caused by the weirs.

Wheelock (734,641/1903). Fixed inclined feed by means of special chutes. Lateral skimming current induced by perforated pipe at the surface near the back of the separating spitzkasten. Re-treatment of concentrate by vertical fall along the lip of the first spitzkasten onto the surface of the second.

Davis (816,303/1906). Vertical gravity feed onto the surface of a rising stream. Compartmented trough with special knife-edge skimmers at each compartment to cut out suspended matter just below the surface.

Behrend (973,467/1910). Feed sifted onto the surface of water in a cone with a radial skimming current induced by a radially-discharging pipe at the center.

Behrend (979,820/1910). Vertical gravity feed by shaking tray onto a horizontal current in a compartmented trough with film-thinning boards over the partitions.

Wood (984,633/1911). Vertical gravity feed onto water on a slightly-inclined plate with provision for discharge of sunken material. The film from the plate flows onto the surface of water in a spitzkasten. Overflow passes over a steeply-inclined screen for final cleaning of concentrate.

Wood (987,209/1911). The film in a tray receiving the feed flows tangentially, at an acute angle, onto the surface of pulp in a cone fitted with curved-screen skimmer opposed to the surface current in a way to lead the film into a central overflow.

Smith (1,014,977/1912). Shaking feed onto a steeply-inclined plate, thence onto a film in a slightly-inclined launder leading to a spitzkasten. Overflow by launder to a further spitzkasten. Repeated re-treatment of concentrate. Provision for varying the slope of connecting launders, resulting in variation in height of fall onto the surface of the pulp in successive spitzkasten.

Jeffrey (1,052,061/1913). Feed distributed at center of cone by sliding from a spherical segment onto a radially traveling film on a flat conical surface. Radial current guided by underside of spherical segment.

Smith (1,136,622/1915). Vertical gravity feed through screens of increasing aperture placed above a launder or launders carrying the separating film. Inclination of feed screen opposed to that of separating launder.

Rowand (1,159,713/1915). Dry feed to traveling oiled belt on which preferential oiling of sulphides in air is expected. Subsequent presentation of solids at a water surface by submergence of the belt. Belt continuously oiled by dipping into a tank containing oil.

Munroe (1,230,081/1917). Inclined fixed feed onto lateral current in spitzkasten, induced by making upper portion of back of spitzkasten circular in vertical section.

Macquisten wet-feed film-flotation machine (865,194, 865,195, 865,260/1907) is one of the best known types of wet-feed machines.

Fig. 6 shows the essential features, *viz.*: a rotating rifled tube (b) 1 × 6-ft., set horizontally, with central opening at the inlet end and full opening at the discharge end into a pointed box

(*d*), with overflow lip. Water level is maintained in box and tube about 3 in. above the bottom of the tube so that a film of water about 1/32 in. deep overflows the discharge lip. Liquid pulp is introduced through the feed trough (*a*). The tube makes 30 r.p.m. The solids in the pulp are thus raised above the surface of pulp in the tube, water drains away, most rapidly and completely from particles of metallic luster, and, as the material slides back, the minerals of metallic luster float, in part, while the gangue minerals submerge. This operation is repeated many times as the pulp passes through the machine. Slime tends, in large part, to pass through in suspension and thus gets no chance to separate. A gentle surface current (about 10 ft. per min.) is maintained from feed to discharge end. When the submerged solids reach the tank they sink and are withdrawn as tailing.

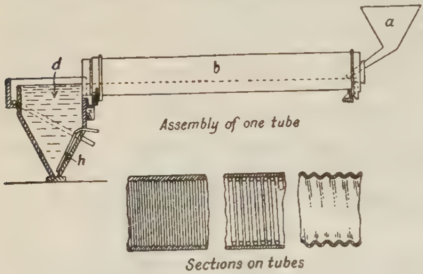


FIG. 6.—Macquisten skin-flotation machine.

Floating concentrate passes over the lip. Tailing is re-treated in another tube. At the MORNING MILL (*43 A 692*) 175 to 200 lb. of zinc concentrate assaying 48 per cent. zinc were floated per tube per 24 hr., representing a recovery as high as 85 per cent. Three tons of solid per 24 hr. was passed through four tubes in series. The feed sized about 9 per cent. on 40-mesh and 11 per cent. through 200-mesh. The feed pulp carried from 14 to 20 per cent. solids.

Macquisten states in patent 865,194 that acid, alkali or soluble salt or other substance like petroleum may be used to modify the surface tension of the separating liquid or to alter the surface condition of the particles.

DeBavay wet-feed film-flotation process (864,597/1907; 912,783/1909). The apparatus used in Australia is described by *Hoover* and differed from that shown in the patents.

The process described in the patents consists in first digesting pulverized ore with a solution of alkaline carbonates or bi-carbonates or carbonic acid or any other reagent in order to remove surface coatings from the sulphide particles, or it may be removed by trituration. The pulp is then de-slimed, suspended in new water and flowed in a thin film over an inclined plane onto the surface of a body of water in a tank or trough where the mineral floats. The feed should be between 40- and 80-mesh. The strength of the washing solution is from 0.5 to 10 per cent. when alkaline carbonates are used.

In the process as described by *Hoover*, tailing from gravity concentration, crushed to pass about 40-mesh, was first de-slimed, then fed into a mixing tank and agitated for a considerable time with cold sulphuric-acid solution, about 0.2 per cent. strength, in a pulp containing 15 to 20 per cent. solids, the acid solution was decanted, the settled solid washed twice with fresh water, then thoroughly agitated with water containing about 0.02 per cent. chlorine and from 2 to 3 lb. per ton of ore of a mixture of 1 part of castor oil and 4 parts of low-grade kerosene. The oiled pulp was elevated by a monteju onto a series of separating cones of the type shown in Fig. 7. Concentrate floated from (*A*) to (*B*), gangue sank and discharged through (*D*). The floating material was principally froth. This is not true of the process described in the patent.

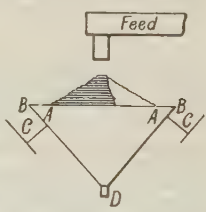


FIG. 7.—DeBavay separating cone.

Other wet-feed film-flotation machines comprise a variety of methods of bringing the solids in a wet pulp to the surface in order that the sulphides may preferentially drain and then float.

Sulman and Picard (793,808/1905). Oiled pulp is sprayed by means of jets, revolving disks, or otherwise onto the surface of water.

Sulman, Picard and Ballot (879,985/1908). Three methods of bringing solids to the surface are described: (*a*) specially riffled reciprocating table, the exposure taking place as the

solids climb the riffles; (*b*) air jets applied to material on the surface of a reciprocating table; (*c*) concave vanning belt, which drains on alternate sides at each reciprocation.

H. L. and E. A. Sulman (902,018/1908). Fixed buddle with two scraper arms carrying rubber scrapers to "dry" the solid, followed by perforated arms to distribute a liquid, as acidified water, onto the dried surface. Thick feed.

Brown (1,081,360/1913). Pulp, with or without "a substance to enhance the flotation of the mineral particles" is successively raised above the surface of water in a tank and then submerged, by means of a traveling belt.

Stevens (1,116,642/1914). Oiling with a small quantity of a mineral oil in a warm, acid pulp, presenting the pulp to air and then to the surface of a liquid by any method.

Livingston (1,147,633 1915). Oiled, acidified pulp flowed through the bottom of a ribbed revolving drum. Internal spray pipes to furnish a liquid film for flotation of the drained mineral and a longitudinal concentrate-overflow launder submerged in the pulp along the center line of the drum. Several variations.

Stone (1,156,041/1915). Oiled, acidified pulp fed intermittently at an acute angle, with as little agitation as possible, to a liquid (water) surface by means of a plurality of small movable mechanically-operated trays. The trays are mounted on a carriage so that they alternately submerge with a load of feed and emerge empty on the return trip for another load.

Haultain (1,218,400/1917). Pulp, with or without oil or inorganic agents, fed successively by pluralities of inclined, fan-shaped trays onto water surfaces in settling tanks. Air jets to set up skimming currents.

Spearman (1,377,937/1921). Grinding in the presence of water and a small quantity of a non-frothing oil followed by separation on an inclined screen on which floating mineral discharges as oversize and sunken material as undersize. Specifically for graphite.

Bonnell (1,399,539/1921). Oiled pulp raised to a smooth or riffled apron by air-lift and flowed onto the surface of water in a spitzkasten. Succession of apparatus with both series and circulating treatment. Some froth flotation here also, depending upon oil and ore.

Spearman (1,491,110; 1,491,111; 1,497,804; 1,509,266/1924). Freely-flowing oiled pulp is sent in a thin layer over an inclined porous membrane discharging gas, thence onto the surface of water in a spitzkasten from which a skin float is withdrawn by suction while tailing sinks. The process is claimed to be particularly suitable for separating graphite from mica in refining graphite concentrate.

Use of film flotation has not been great. It has been extensively experimented with in the treatment of graphite ores and was used for floating sphalerite in Broken Hill and at one mill in the Coeur d'Alenes.

3. Oil flotation

Bulk-oil flotation and OIL-BUOYANCY FLOTATION are other names for the same method. The process depends upon the interfacial forces at the contact of a mass of oil and water for selecting and holding sulphide mineral, and upon the buoyancy of the oil in water for the levitation of the selected sulphide. Operation of the process consists in mixing relatively large quantities of oil with finely-ground ore, either before or after admixture with water, then allowing the mixture of ore, oil and water to stratify. Following such procedure, if the oil is of lower specific gravity than water (or pulp), a load of solid, predominately metallic mineral, is carried at the interface between the oil and water, and by overflowing the oil layer with a thin layer of the underlying water, this mineral load can be separated from the settled gangue.

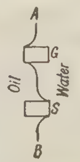


FIG. 8.—Selection at oil-water interface.

Theory of oil flotation. The fundamental phenomenon of selection is illustrated in Fig. 4. The conditions existing in a freely-flowing pulp into which a mass of oil is being mixed are illustrated in Fig. 8. *AB* represents an oil-water interface and (*G*) and (*S*) gangue and sulphide particles respectively. When a gangue particle is presented at the interface, the contact angle of water against a gangue surface in the presence of oil being small, the oil-water interface assumes the position shown and the force of surface tension acts to throw

the gangue particles back into the water. On the other hand, when a sulphide particle is similarly presented, it is the oil-sulphide contact angle that is small and the direction of the forces at the oil-water interface in the neighborhood of the sulphide particle is, therefore, such that the sulphide is held or moves toward the oil. *Bancroft* says (p. 83):

"The solid particles tend to go into the water phase if they adsorb water to the practical exclusion of the other liquid; they tend to go into the other liquid phase, if they tend to adsorb the other liquid to the practical exclusion of the water; while the particles tend to go into the dineric interface in case the adsorption of the two liquids is sufficiently intense to increase the miscibility of the two liquids very considerably at the surface between solid and liquid."

Levitation of the selected solid occurs when the specific weight of the system composed of selected solid and oil is less than that of the water or pulp in which it is submerged. The amount of oil theoretically necessary depends, therefore, on the specific gravities of the oil and pulp and the quantity of sulphide mineral to be lifted.

Assuming an ore containing 4 per cent. of mineral having a specific gravity of 5, the amount of oil having a specific gravity of 0.9 theoretically necessary to just float the mineral is 575 lb., if the specific gravity of the pulp is taken as 1.0, *i.e.*, the same as water. The reasoning follows: 4 per cent. of mineral to be floated equals 80 lb. The weight of this mineral in water is $80 - (80 \times 62.5)/(5 \times 62.5) = 64$ lb. or 29,030 gm. Each cubic centimeter of the oil assumed weighs 0.9 gm. and can buoy just less than 0.1 gm. submerged weight of mineral. It requires, then, to lift 29,030 gm. of mineral $29{,}030 \times 10$ or 290,300 cc. of oil, which is 76.7 gal. or 575 lb.

Practically, the oil is not completely utilized in levitation, some gangue is raised, some mineral is lost, and finally air is entrained in the process of mixing the oil and this entrained air aids levitation markedly. The net result is that less than the theoretical quantity of oil is necessary.

Oil-flotation processes may be grouped as dry-oiling and wet-oiling, depending upon whether the oil is added to dry or moist ore or to an aqueous pulp; and further with regard to whether any special means is employed to increase the viscosity of the oil.

Elmore wet-oiling oil-flotation process (643,340; 676,679 and 689,070/1901; 692,643/1902) is the best known of the oil-flotation processes. The operation consists in first producing a freely-flowing pulp by mixing pulverized ore with water in the proportions of 6 to 10 of water to 1 of ore, by weight; adding thereto a relatively large quantity of oil, up to more than a ton of oil per ton of solids; adding also sulphuric acid; mixing the ingredients in a trough provided with stirring blades on a horizontal shaft; then passing the mixture to a spitzkasten. The pulp level in the spitzkasten is kept at such a height that a slight overflow of pulp liquor is maintained. Under these circumstances the oil layer on the surface of the pulp should be one-half inch or less in thickness. The mixing should be so limited in violence as not to break the oil into minute globules. Oil is recovered from concentrate by washing with solvents, filtration, centrifugation, etc. The actual oil loss per ton of ore is said to have been between 10 and 20 lb. per ton of ore treated.

Other wet-oiling oil-flotation processes using non-thickened oils are described in the following patents:

Glogner (736,381/1903). Treatment of gravity graphite concentrate. Pulp containing 25 to 33 per cent. solids is thoroughly stirred in a closed vessel with petroleum in an amount equivalent to 50 per cent. of the graphite by weight. The mixture is then sprayed with water, allowed to stand for settlement, and finally skimmed to remove the floating graphite.

Orr (758,464/1904). Pulp mixed with oil by agitation of a water wheel and by flow in a pipe is discharged upwardly by a submerged pipe in a spitzkasten. Overflowing oil with concentrate is filtered through charcoal or similar medium, the oil being returned to the system.

Schwarz (766,289/1904). Pulverized ore mixed with water or saline solution is introduced gently onto the surface of oil floating on further saline solution in a tank. Gangue passes through the oil-solution interface and sulphide remains in the oil or at the interface.

Kendall (771,075/1904). Freely-flowing pulp is mixed with thin oil by causing the two to flow under pressure through perforated plates. The mixture is discharged as a thin sheet at considerable velocity beneath the surface of water in a spitzkasten. Oil with mineral overflows and gangue is discharged at the bottom.

Tunbridge (777,159/1904). Passage of aqueous pulp through a layer of oil and subsequently through a perforated septum which removes oil-mineral aggregates and passes unoiled gangue and water.

Orr and Finley (790,913/1905). Vertical passage of aqueous pulp, subdivided into drops by falling through a screen, through a horizontally-moving layer of oil floating on a body of pulp. Removal of mineral from the oil by filtration through a blanket and charcoal.

Finley (822,515/1906). Passage of aqueous pulp through a layer of oil floating on a body of pulp in a spitzkasten. Separation of concentrate from the oil by filtration.

Kirby (809,959/1906). Agitation of freely-flowing pulp with a solution of bitumen in a thin distillable hydrocarbon liquid such as kerosene, in such a way as to break the oil into small globules and bring them into contact with the mineral particles; followed by settlement, aided by gentle agitation with sub-surface injection of gas and fine streams of oil; skimming of oil with concentrate; finally, distillation of the oil from the solid. Gas undoubtedly aids levitation.

Elmore, A. S. (865,334/1907). Mixing aqueous pulp with oil in a centrifugal pump and circulation through the pump and a tank that is oil-covered to prevent oxidation of the metallic minerals.

Dunstone (956,800/1910). Aqueous pulp agitated violently with acid (about 1 per cent. on the solid) then with a light mineral oil (4 times as much in bulk as the solid) until an emulsion containing the metallic particles is formed. This is skimmed and settled to drop the metal.

Reed (1,262,984/1918). Aqueous pulp flowed against the under side of a stratum of oil held in place by baffles in a shallow trough.

Rideout (1,562,125/1925) describes the use of a kerosene emulsion made with water, sodium bi-carbonate and potassium carbonate for use in separating molybdenum.

Wet-oiling oil-flotation processes using thickened oils are described in the following patents.

Scammell (770,659/1904). Aqueous pulp agitated with oil (heavy petroleum, animal, vegetable or fish oil) that has been treated with chloride of sulphur. The extent of thickening should be increased with increase in mineral content of the ore.

Wolf (787,814/1905). Like the preceding with the addition that enrichment of oily concentrate is effected by passing the same through warm water. A method of recovering oil from waste pulps by blowing air through them is mentioned.

Dry-oiling oil-flotation processes with non-thickened oil, including those processes in which the ore is more or less moistened but not in the form of a freely-flowing pulp before oiling, are described in the following patents.

Robson (575,669/1897). Moistened ore in soft and plastic state, *i.e.*, with say, 25 to 35 per cent. water, is agitated with an oily liquid, the mixture is allowed to stratify in a separating vessel, the oily concentrate is drawn off and the solid separated from the oil.

Wolfe (725,609/1903). Dry ore mixed with oil is fed onto a layer of oil floating on brine thickened by clay or other earth. Metallic mineral adheres at the oil-brine interface in the separating vessel and gangue sinks. Clayey brine increases the buoyancy of the concentrate.

Darling (795,823/1905). Dry ore is mixed with oil (petroleum), the mixture is suspended in water by agitation and flowed to a screen, which separates the buttery mass of mineral and oil from non-agglomerated gangue and water.

Latimer (851,599 and 851,600/1907). Dry ore is mixed with oil, the mixture suspended in water by rotary agitation in a cylindrical vessel sufficient to cause centrifugal forces that will bring about some segregation of solid from liquid, and the suspended mixture is then subjected to hydraulic classification in the agitation vessel. The oiled mineral rises on the periphery while gangue sinks.

Seinsche (1,335,612/1920). Dry ore is fed onto a belt conveyor previously fed with oil. The oiled ore is discharged by the belt onto the surface of acidified water. Oiled mineral floats, gangue sinks. There is considerable skin flotation here.

Dry-oiling oil-flotation processes with thickened oil including those processes in which the ore is more or less moistened before oiling but not

brought to the state of a freely-flowing pulp, are described in the following patents.

Everson (348,157/1886). Ore is dry-mixed with a gelatinous, probably collodial, mixture of water with a sulphonated fatty oil. The mixture is subsequently worked, by slow churning or squeezing in acidified water. This causes the gangue to be dropped and leaves buttery masses of oily matter and mineral of specific gravity roughly between one and two. These may be separated from the pulp by any gravity method. A second method described utilizes untreated oil and considerable agitation. In the second method gas undoubtedly aids levitation.

Schwarz (771,277/1904; 807,501 to 807,506 incl./1905; 825,080/1906; 842,255/1907). A series of patents describing the mixing of oils, temporarily thin, such as melted hydrocarbons that are solid at normal temperatures, with moist or dry pulps by agitation; subsequently adding water, with further agitation and with or without the injection of air or steam; collecting a concentrate as a floating, more or less solidified and more or less gasified mass, and separating the same from the balance of the pulp by skimming.

FROTH FLOTATION

Froth flotation comprises two entirely different types of processes which resemble each other only in the fact that in both the concentrate is removed in the form of a froth or foam composed of gas, liquid, and solid matter which is preponderantly one of the minerals or classes of minerals. In the early days the mineral that was floated was mineral of metallic luster but greater knowledge of controlling conditions has now made it possible to float other kinds of mineral. The processes differ fundamentally both in the place in which concentration is done and in the mechanism of the selection of sulphide from gangue. On the basis of the first difference the processes may be classified as pulp-body processes and bubble-column processes.

PULP-BODY PROCESSES

These processes may be subdivided, on the basis of the method of introducing the bubble-making gas, into four types: (1) chemical-generation, (2) pressure-reduction, (3) boiling, and (4) agitation. All four types depend upon the fact that in a pulp, the liquid part of which is saturated with a gas, preferential precipitation of the gas onto particles of one particular class can be brought about by so changing the conditions that the liquid is supersaturated with respect to the surfaces of particles of this class. Preferential precipitation of gas from the supersaturated liquid is enhanced, if the mineral particles are coated with a film of some organic substance, and the presence of such a coating also makes the force of adherence between the precipitated bubbles and sulphide particles greater. As a result of preferential precipitation of gas on certain particles in the pulp, and its adhesion thereto, agglomerates consisting of one or more gas bubbles with mineral particles firmly attached to them are formed in the body of the pulp. These agglomerates later rise to the surface in the form of a froth which is separated as concentrate.

4. Chemical-generation process

This process depends upon chemical action of some kind to generate gas in the pulp. In some methods, as, for instance, the electrolytic, a part of the gas, at least, generates at the surface of the metallic minerals. In others, as the Froment, it generates at the surface of non-metallic minerals, goes into solution, and later precipitates preferentially at the surface of particles of sulphide mineral.

Potter-Delprat process (735,071/1903; 763,662/1904; 768,035/1904; to Delprat and 776,145/1904 to Potter) is the first commercially successful froth-flotation process applied to sulphide ores.

Dewatered pulp is fed into an apparatus of the type shown in Fig. 9. Hot sulphuric-acid solution of 1- to 10-per cent. strength or acid salt-cake solution of 1.3 to 1.4 density is introduced through the pipes (*a*) to the bottom of the vat. A layer of solids 2 ft. or more in height, depending upon the depth of the vat, is maintained in teeter above the spigot. Gas is precipitated as bubbles onto the sulphides and the bubbles rise to the surface with a load of sulphide, forming there a coherent froth, which overflows. The tailing is drawn off as a thickened product from the spigot at the bottom of the box. The compartment without a spigot is for the purpose of collecting any coarse particles that would tend to clog the spigot. With ores that contain carbonates, as do most of the ores at Broken Hill where this process was invented and practiced, it is probable that most of the effective gas is carbon dioxide. Air and water vapor will, however, precipitate in sufficient quantity, under the conditions of the practice of the process, to float the sulphide effectively in the form of a coherent froth, when treating ores containing no carbonates. No oil is added. On theoretical grounds it is questionable whether the process will work on an otherwise suitable ore if there is no organic material present, but there is no doubt that the lubricating oil introduced in ordinary mining operations, amounting in general to 0.05 to 0.1 lb. per ton of ore, is sufficient to cause successful practice of the process.

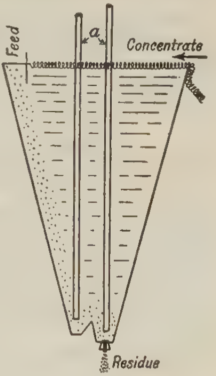


Fig. 9.—Delprat frothing box.

Performance of Potter-Delprat process (*Deposition of Leslie Bradford, M.S.N.A. Corp. and M.S., Ltd., vs. Magma Copper Co.*; *117 P 375*). The principal use of this process was at the Broken Hill Proprietary mine, N.S.W., from 1903 to 1917. The early work was in a shallow, sloping-bottom pan heated on the bottom by an open flame. About one long ton per hour was treated in a pan 3 × 6 ft. in area and about 2 ft. 6 in. deep. Feed was de-slimed tailing from the gravity-concentrating mill, practically all of which would pass a 40-mesh screen and from 20 to 25 per cent. remain on 60-mesh. The gangue was principally quartz, rhodonite and garnet with some gneiss from the walls and small amounts of rhodocrosite, siderite and calcite, totaling about 0.5 per cent. CO_2. Acid salt-cake ($NaHSO_4$) solution of about 1.3 specific gravity, heated to about 190° F. in stock-solution tanks by the use of superheated steam was mixed with the moist ore (6 to 9 per cent. H_2O) in the tank. Feed assayed about 17 per cent. Zn, 3 per cent. Pb and 4 oz. Ag per long ton; concentrate, 40 to 42 per cent. Zn, representing about 80 per cent. recovery. By 1904 apparatus similar to that shown in Fig. 9 was in use. The first boxes were 4 ft. × 4 ft. 6 in. in surface area and about 6 ft. deep, made of wood lined with sheet lead. Solution pipes were cast iron, tipped for 2 ft. 6 in. with 2-in. lead pipe drawn down at the end to ½-in. and perforated near the end with ¼-in. holes. Capacity was 10 long tons per hour. Other operating conditions and the results were approximately the same as above. The final installation used larger boxes of cast iron, 8 ft. × 9 ft. surface area, 2 ft. × 1 ft. at the bottom, and 15 ft. deep; 1-per cent. sulphuric-acid solution was substituted for the salt cake. The capacity of the larger boxes was 10 to 20 long tons per hour. Recovery occasionally got up to 90 per cent. but the general average was 80 per cent. Grade of concentrate was worked up to 48 to 49 per cent. Zn, 6 per cent. Pb and 10 to 12 oz. Ag. Concentrate was about 8 per cent. on 40-mesh and 17 per cent. −200-mesh. Tailing assayed 3 per cent. Zn, 2 per cent. Pb, 2 oz. Ag per long ton. Three to 3.5 tons of solution were added per ton of ore, entering under a pressure of approximately 15 lb. per sq. in.; acid consumption, including 1 to 2 lb. per ton of original feed discarded with the tailing, was 25 to 30 lb. per ton; strength of solution returning to stock tanks, 0.6 per cent. acid. Temperature of the incoming solution was 190° F., of solution in the frothing boxes, 165° F.; and of solution drained from concentrate and returned to the stock tanks, 145° F.

The purpose of de-sliming was to get high-grade zinc concentrate. Slimes assayed 17 per cent. Zn, 14 per cent. Pb and 14 oz. Ag as compared with the sand assay given above and concentrate from treatment of mixed sand and slime assayed 32 to 33 per cent. Zn, 22 to 23 per cent. Pb and about 30 oz. Ag. This concentrate could not be sold to advantage. Lead slimes were, therefore, dried in the open, heap-roasted, and smelted direct. Elimination of slimes made possible the high capacity of the frothing boxes, since slime treatment by this process requires long digestion of the pulp with hot acid solution in order to

float the slime mineral and, in fact, before high recoveries of the coarser material can be effected.

At this same plant slime-feed was concentrated with heated (140° F.) acid solution (0.3 per cent. H_2SO_4) without oil by sending the pulp through a series of four centrifugal pumps and separating boxes in alternate series. Concentrate assayed 34 per cent. Zn, 25 per cent. Pb and 40 oz. Ag; tailing, 4 per cent. Zn, 9 per cent. total Pb (8 per cent. oxide) and 4 oz. Ag. Plant capacity was 3000 tons per week.

Other apparatus designed for the practice of this process is described in patents 763,749/1904 and 784,999/1905, to **Goyder and Laughton**; and 778,747/1904, and 780,281/1905, to **Gillies.**

Froment process was patented in 1902 in Italy and Great Britain but was not patented in the United States.

Finely-pulverized ore with 1 to 2 per cent. of limestone is fed into a mixer in a pulp containing about 30 per cent. solids; animal or vegetable oil in an amount equivalent to from 1 to 1.5 per cent. on the ore is added and the mixture agitated at the rate of 1000 to 1500 ft. per min. peripheral speed. The agitated mixture is next run into a shallow vat provided with a slow-moving rake at the bottom. Sulphuric acid of about 30 per cent. strength in an amount sufficient to react with the limestone present is slowly added through a perforated coil suspended just above the rakes. Gas precipitates preferentially on the oiled-sulphide surfaces and forms aggregates of gas bubbles and mineral that rise to the surface and are removed in the form of a coherent froth. De-sliming of the feed aids in making clean concentrate, but is unnecessary with certain oils or if a small amount of acid is added in the mixing vat.

The Froment process never went into commercial operation. Results of tests in a laboratory apparatus are given in Table 6.

Table 6. Results of laboratory operation of Froment process

Ore	Oil		Metallurgical results				
	Name	Pounds per ton	Assays, per cent.			Ratio of concentration	Recovery, per cent.
			Feed	Tailing	Concentrate		
Copper..	Coal-tar creosote..........	18.4	1.96	1.18	9.19	10.2	45.9
Copper..	Coal-tar creosote...........	19.2	1.94	1.03	9.14	8.9	52.8
Copper..	Coal-tar creosote...........	9.4	1.86	1.14	10.40	12.9	43.4
Zinc....	Lubricating oil............	16.8	11.88	1.47	43.70	4.1	90.7
Zinc....	Pine oil.................	15.6	12.55	1.02	40.62	3.4	94.2
Zinc....	Lubricating oil............	16.4	12.32	0.72	38.88	3.3	95.9
Zinc....	Oleic acid.................	16.6	11.60	0.65	35.50	3.2	96.2
Zinc....	Kerosene.................	15.6	12.16	0.74	37.72	3.2	95.8
Zinc....	Amyl acetate..............	15.2	12.24	0.74	38.20	3.3	95.8
Zinc....	Rosin oil..................	16.4	12.42	0.60	36.08	3.0	96.8
Zinc....	Cotton-seed oil...........	21.2	12.15	0.80	41.60	3.6	95.3

Sanders (805,382/1905 and 911,077/1908). Pulverized ore is agitated at a rate insufficient to beat in air in a non-acid solution, preferably heated, having a specific gravity of 1.15 to 1.25 and capable of reacting with the ore with evolution of gas. Aluminum sulphate and ferric sulphate are named for making the solution. In 988,737/1911, Sanders describes removal of calcite from calcitic ores prior to flotation with non-acid and with acid solutions.

Greenway (1,045,970/1912) describes a process that is essentially the Potter-Delprat except that dry or moist ore is oiled before addition to the flotation machine. This treatment stiffens the froth.

Leuschner process (*93 J 924*) as practiced at the Friedrichssegen and Ludwigseck mines in Germany was an operation of the Froment process in which oiled pulp containing carbonates was treated in a sulphuric-acid bath of 1 to 2° Bé. at 60 to 80° C. At Friedrichssegen the feed was – 1-mm. and contained 10 to 15 per cent. Zn. Concentrate contained 47 to 50 per cent. Zn and tailing 0.3 per cent. Oil consumption was 4 to 6 kg. per ton.

Electrolytic chemical-generation process was first described by **F. E. Elmore** in British patent 13,579/1904. In this process the water of a freely-flowing oiled pulp is decomposed by an electric current, gas precipitates preferentially on the sulphides and the mineral-coated bubbles rise and form the typical coherent froth.

Lockwood (1,329,127/1920). Ores whose metalliferous particles are capable of conducting electricity are passed through, over or between cathode and anode surfaces closely spaced so that the particles become temporarily anodes and cathodes on which hydrogen or oxygen or both from the decomposed water precipitate. Oxidized metallic particles are to be coated with oil containing ground conductors, such as sulphides, before treatment. Various forms of apparatus are illustrated.

Theory of the chemical-generation process. The actual phenomena involved may be seen clearly in the following experiment.

De-slime about 20 gm. of a silicious ore of fairly high mineral content (say 15 per cent. zinc ore) drain, mix in about 1 gm. of similarly sized limestone, add 1 or 2 drops of oleic acid and mix thoroughly. Make a small heap of the oiled ore in a shallow dish, cover with water, place a low-power microscope for observation from above, then add slowly, at a short distance from the ore, a few drops of concentrated sulphuric acid. As soon as the acid diffuses to the solid, gas bubbles may be seen to form and grow at the surfaces of both carbonate and sulphide particles. Those forming at carbonate surfaces detach and rise to the surface without any solid load. Those that form at carbonate surfaces down in the heap can be seen to push through slowly, throwing aside indiscriminately the gangue and sulphide particles with which they come in contact, and rising to the surface unloaded.

The phenomena producing the attachment of gas to sulphides are: (*a*) generation of gas at the carbonate surfaces; (*b*) solution of some of the gas in the water; (*c*) travel of this solution by diffusion and by reason of the stirring by rising bubbles until a sulphide surface is reached, and (*d*) precipitation there by reason of the fact that the solution is supersaturated with respect to such surfaces.

The same phenomena can be observed by placing individual pieces of, say, quartz, galena and calcite in a small cell under a projecting microscope magnifying 200 to 300 diameters, spacing the pieces at the points of a roughly equilateral triangle whose sides are several times the diameter of the pieces. Upon addition of acid, bubbles can be seen rising from the calcite, but none of these approach the other minerals. Shortly, however, numerous gas bubbles begin to grow on the galena while only rarely is a bubble to be seen on the quartz. In this experiment rapidity of precipitation and tenacity of adherence of bubbles are enhanced by previous oiling of the solids, but oiling is not ordinarily essential to the behavior described.

Formation of heavily-coated bubbles takes place by coalescence of lightly-loaded bubbles. Such coalescence results in reduction of bubble surface and consequent more complete covering of the surface of the resulting bubble by the double load.

5. Pressure-reduction process

Gas precipitation is brought about by reducing the external pressure upon a pulp that is saturated with gas. Processes utilizing this phenomenon are of two types, involving respectively pressures in excess of atmospheric (PLUS-PRESSURE) and less than atmospheric (VACUUM).

Plus-pressure process

Sulman, Picard and Ballot (835,479/1906). Pulp, oiled or otherwise, is pumped into a pressure tank and there subjected to a gas pressure of from 50 to 100 lb. per sq. in. for a sufficient time to allow saturation with gas. Pressure on the pulp is then released, when gas from the now supersaturated pulp precipitates preferentially on the metallic-mineral sur-

faces. The mineral-coated bubbles rise and form the characteristic coherent froth at the surface of the pulp. (See also 835,120/1906.)

Norris (864,856 and 873,586/1907; 1,167,835/1916). Water saturated with gas at a pressure well above atmospheric is discharged into an oiled pulp at atmospheric pressure. The result is the same as in the preceding process.

Vacuum process

F. E. Elmore (826,411/1906). Pulp, mixed with a small quantity of oil, and acid or alkali if necessary, is passed into a vacuum apparatus. Under the reduced pressure the water is supersaturated and gas precipitates preferentially on the oiled sulphide minerals and raises them to the surface as a coherent froth.

As practiced in the mills, the process was operated in a plant such as is shown in Fig. 10. Pulp ground to at least 0.5-mm. maximum size and containing about 50 per cent. solids was mixed in a tank (d) with oil in an amount generally less than 0.5 per cent. on the ore, with or without acid, and thence was discharged into the feed pipe (a) of the separating apparatus. Here water was added to bring the pulp to a consistency of 15 to 25 per cent. solids. The separating apparatus was a closed conical chamber fitted with a slowly revolving rake at the bottom, a tailing-discharge pipe (b) at the periphery and a concentrate-discharge pipe (c) from near the apex. The separating chamber was attached to a vacuum pump. The lower ends of pipes (b) and (c) were sealed by causing them to discharge below the surface of liquid in tanks as shown. The vertical lift in the pipe (a) was about 25 ft. and the vertical length of the pipes (b) and (c) was somewhat over 30 ft. A vacuum of 20 to 27 in. of mercury was maintained. Under the influence of this vacuum the pulp fed into pipe (a) passed up into the separating chamber. Here air was precipitated at the surfaces of the sulphides and the bubbles raised the particles through the liquid in the separator to the apex where they overflowed into an annular launder and passed down pipe (c). At the same time the tailing was slowly scraped to the periphery of the floor and passed down pipe (b). The rate of flow in pipes (a) and (b) was so regulated that the pulp level was maintained slightly below the overflow lip.

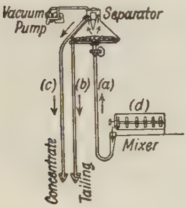


Fig. 10.—Elmore vacuum plant.

The capacity of a 5-ft. separator was from 25 to 50 tons of ore per day. Re-treatment of concentrate was not ordinarily necessary. Power consumption per pan was well under 5 hp. for mixer and vacuum pump together.

Table 7. Results of laboratory tests with Elmore vacuum process

Ore	Reagents: Name	Reagents: Pounds per ton	Metallurgical results: Assays, per cent.: Feed	Tailing	Concentrate	Ratio of concentration	Recovery, per cent.
Zinc	Rosin oil	7.3	13.45	1.47	41.90	3.4	92.3
	H_2SO_4	27.2					
Copper	Coal-tar creosote	6.2	2.12	1.19	32.46	33.9	45.4
Copper	Rosin oil	3.0	2.49	1.16	39.38	28.7	55.0
Zinc	Oleic acid	8.9					
	H_2SO_4	18.8	13.40	1.43	41.38	3.35	92.5
Zinc	Amyl acetate	5.4	13.62	6.37	47.90	5.7	61.4
Zinc	Lubricating oil	7.6	14.43	2.57	53.00	4.3	86.4
	H_2SO_4, 0.18 per cent. solution						
Zinc	Cotton-seed oil	5.6	11.95	0.98	42.90	3.8	93.9
	H_2SO_4, 0.18 per cent. solution						
Zinc	Cotton-seed oil	1.5	12.80	1.80	43.55	3.8	89.7
	H_2SO_4, 0.18 per cent. solution						

Development of the vacuum process was stopped by the introduction of the agitation-froth process but it is probable that if the same amount of work had been expended in attempts to make the vacuum process highly efficient, as was spent in bringing the agitation-froth process to its present degree of efficiency, the results wouldhave been equally favorable. Slimes are easily treated, if sufficient agitation with proper agents precedes the vacuum treatment, but granular sulphide is most easily treated out of the presence of slime. Table 7 presents results attained in the laboratory. The copper ore was of much lower sulphide content than the zinc, which to some extent explains the poorer recovery. Table 8 is a summary of parts of mill operations extending over several months. Mixing was performed in the apparatus shown in Fig. 10 at about 50 per cent. solids. Sizing-assay tests on composites of the feed and products of the run shown on the last line but one of Table 8 are given in Table 9. Improvement in metallurgical result should follow finer grinding, although the small amount of total copper present in the coarser sizes might not pay the increased grinding cost.

At Zinc Corporation (*88 J 205*) the feed to vacuum machines contained 20 per cent. Zn, 5.75 per cent. Pb and 8 oz. Ag per long ton. Concentrate assayed 43 per cent. Zn, 11 per cent. Pb and 17 oz. Ag.; tailing 3.5 per cent. Zn, 2.2 per cent. Pb, and 2.2 oz. Ag. Acid consumption varied from 10 to 20 lb. per long ton; Texas fuel oil, 6 to 8 lb. per ton. Cost of flotation alone (1909) was \$0.55 to \$0.60 per ton. Claudet (*103 J 786*) says that a standard unit will treat 35 tons of − 10- or − 20-mesh molybdenum ore per 24 hr., using a small quantity of coal oil (kerosene) and that molybdenite floats preferentially to pyrite and pyrrhotite. Slime is overflowed before flotation. At Kvina mines, Norway (*19 CMI 127*) feed assaying 0.8 to 1.0 per cent. MoS_2 produced concentrate containing 75 to 85 per cent. MoS_2, representing 80 per cent. recovery.

Schiechel (1,212,566/1917) recommends a number of vacuum machines in series, with vacuum increasing from first to last.

Callow (1,176,428/1916). Use

Table 8. Results of mill operations with Elmore vacuum machine on copper ore

Tons solid per 24 hr.	Per cent. solid in machine	Reagents: name,(x) pounds per ton			Assays, per cent. Cu			Ratio of concentration	Recovery	Length of run, days
					Feed	Tailing	Concentrate			
31.1	18.4	*R*-19/0.46	*K*/0.74	*P*/0.10	1.88	0.65	40.45	32.4	66.5	6
30.3	18.9	*R*-19/0.52	*K*/0.79	*P*/0.09	1.64	0.67	35.91	36.3	60.0	5
29.7	18.9	*R*-19/0.55	*K*/0.64	*C*/0.04	1.43	0.61	36.37	43.6	58.2	4
30.2	19.7	*L*/0.68	*K*/0.63	*C*/0.05	1.61	0.84	38.66	49.2	49.1	4.33
30.5	18.3	*L*/0.64	*C*/0.13	. . . .	1.64	0.86	36.08	45.2	48.4	4.33
28.8	20.1	*L*/0.70	*K*/0.53	*C*/0.05	1.64	0.94	33.57	46.6	44.0	5
25.0	17.7	*B*-635/1.08	*C*/0.64	. . . .	1.78	0.73	39.97	37.4	59.8	2
24.7	16.0	*B*-635/1.07	*C*/0.65	. . . .	1.88	0.78	34.36	30.5	59.8	4
24.7	16.4	*B*-635/1.14	*C*/0.66	. . . .	1.83	0.70	30.72	26.5	63.5	3
28.5	17.2	*B*-65/0.93	*C*/0.54	. . . .	1.66	0.54	40.26	35.5	68.2	3
30.8	18.0	*BF*/0.96	. . . .	. . . .	1.77	0.64	41.17	35.9	64.7	5
30.0	17.0	*BF*/1.18	. . . .	. . . .	1.74	0.59	39.69	34.0	66.8	5
31.8	16.9	*BF*/1.11	. . . .	. . . .	1.69	0.60	39.78	36.0	65.4	5
31.1	16.3	*BF*/1.20	. . . .	. . . .	1.69	0.60	34.53	31.1	65.5	4
30.0	17.2	*BF*/1.34	. . . .	. . . .	1.95	0.70	41.62	32.7	65.4	5
30.2	17.7	*BF*/1.47	. . . .	. . . .	1.75	0.50	41.31	33.3	72.1	5

x Abbreviations: *R*-19, Reilly No. 19, coal-tar creosote. *K*, Crude kerosene. *P*, Steam-distilled pine oil. *C*, Chatham tar, coal tar. *L*, Lewis tar, coal tar. *B*-635, Barrett No. 635, coal-tar creosote, low tar-acid content. *BF*, Blast-furnace oil.

Table 9. Sizing-assay test of feed and products, Elmore vacuum operation

Tyler, mesh	Feed, per cent.			Tailing, per cent.		
	Weight	Assay, Cu	Total Cu	Weight	Assay, Cu	Total Cu
28	1.7	0.83	0.7	1.5	0.81	1.6
35	3.7	0.92	1.8	3.5	0.88	3.9
48	6.0	1.04	3.3	5.9	0.96	7.2
65	9.7	1.26	6.4	10.0	1.03	13.1
100	13.0	1.81	12.3	14.5	0.77	14.2
150	8.9	2.21	10.3	9.2	0.46	5.4
200	9.8	2.53	13.0	12.5	0.39	6.2
−200	47.2	2.11	52.2	42.9	0.89	48.4

Tyler, mesh	Concentrate, per cent.			Recovery in concentrate per cent.	
	Weight	Assay, Cu	Total Cu	Individual	Total
28	0.1	8.02	0.2	2.74	0.14
35	0.2				
48	0.9				
65	4.0	23.88	2.4	21.28	1.42
100	17.1	37.46	15.9	62.59	9.51
150	12.3	43.39	13.2	78.62	7.92
200	19.8	47.41	23.2	84.88	13.93
−200	45.6	39.93	45.1	58.17	27.02

of a partial vacuum over a flotation pulp, but the flotation accomplished is not the work of the reduced pressure and the process is not pulp-body type.

Colburn and Colburn (1,226,062/1917; re-issue 14,497/1918; 1,226,063/1917; 1,415,314/1922). In an agitation-froth process, agitation of the pulp in a partial vacuum in the chamber of a centrifugal pump. This is nothing more than a statement of the known conditions existing in centrifugal pumps and is no departure in principle from other agitation-froth machines, as will be seen later.

Eldred and Graham (1,515,942/1924). A vacuum apparatus for use with unoiled carbonaceous pulps, especially graphite, so arranged that the floated particles, upon reaching the pulp surface and losing their bubbles sink again and are caught in a concentrate-receiving compartment that discharges through a barometric leg.

6. Boiling process

This process uses heat to cause supersaturation and selective precipitation of gas from the water of a pulp, with the usual coherent-froth formation. The phenomenon is effective both with and without added oil. Actual boiling is not essential.

Potter-Delprat process (Art. 4) is strictly a boiling process, although as operated in Australia it was a chemical-generation process.

Sulman (835,143/1906). Pulp, oiled by agitation, and with or without acid, is heated to a high temperature. The process works best with de-slimed, highly-mineralized ores. It is not economical and has not found commercial application.

7. Agitation-froth process

This process depends upon local supersaturation of the water of a pulp with air by the mechanical action of a swiftly revolving beater, and the simultaneous preferential precipitation of the air in the form of bubbles on the metalliferous particles. Agitation-froth machines consist essentially of an agitation chamber in which a stirrer mounted on a vertical spindle rotates

at high speed, and a froth-separating compartment in which the pulp is allowed to come to rest and the bubbles carrying the metalliferous mineral rise to the surface to form a froth which is skimmed off. The pulp in the agitating compartment, under the influence of the rotating agitator, is thrown from the center toward the side of the chamber. The result is that the surface of the pulp assumes the shape of an inverted cone. When the cone becomes so pronounced that the tip reaches down to the revolving beater arms, the tip is cut off and a large bubble of air is entrained. This bubble is immediately broken up into a large number of small bubbles by the direct impact of the impeller arms and by the violent swirling of the pulp. These bubbles, due to their minute size, are in the most favorable state to be taken into solution, and many of them are, at the time of their formation, subjected to considerably more than atmospheric pressure, due to the impact of the impeller blades. They have, also, on account of their small size, but slight vertical motion relative to the pulp mass, and are, therefore, kept for a comparatively long time in contact with the water and subject to the impact of the impeller blades. As a result, some of the air goes into solution in the water. At the same time there exists behind each impeller blade a volume of pulp on which the pressure is reduced by reason of the forward movement of the blade and the inertia of the pulp mass. Here air comes out of solution at the surfaces of the sulphide particles in the form of bubbles. The excess bubbles that never go through the solution stage, in this, as in the other pulp-body concentration processes, in part coalesce with the bubbles already formed on sulphide surfaces; in part pass with the pulp into the froth-separating chamber and there, rising, add buoyancy to the froth; in large part, however, they rise to the surface of the pulp in the agitating compartment and are lost to the process.

Theory of the agitation-froth process

Experiments illustrating the various steps in the preceding statement follow.

Gas precipitation by agitation. If a square-glass-jar agitating machine is provided with horizontal baffles fitting as closely as possible around the shaft and against the walls and placed just below the water level, when agitation is started no vortex forms and there is no introduction of air from the surface. If, however, the rate of revolution of the impeller is gradually increased, a speed is finally reached at which the liquid in the vessel becomes cloudy, showing the presence of finely-divided gas bubbles. When agitation ceases, air bubbles collect under the baffle plates. If these bubbles are released by tilting the plates and the impeller is again started, a higher speed must be reached before gas again precipitates, provided the temperature of the water has remained the same. Increase in temperature from 12° C. to 30° C., the heating being accomplished on a water bath without stirring, lowered the critical speed of gas precipitation with a 3-in. impeller from 1535 to 395 r.p.m. When heating to the same temperature was done by an electric heater, on which much gas precipitated during heating, the critical speed was 940 r.p.m. Adding pine oil to the water (0.006 per cent.) lowered the critical speed at 12° C. to 980 r.p.m.

Vacuum produced by agitation. In the apparatus described in the preceding paragraph an opening in the back of one of the beater blades, near the tip, was connected to a vacuum gage by a conduit within the blade and by a hollow shaft with a stuffing box at the top. Pressures were read at different speeds with the results given in Table 10. At the higher speeds the vacua recorded were well below those actually existing, because of the fact that air coming out of solution behind the blades broke the vacuum by passing into the hole in the beater arm.

Effect of agitation on froth formation is shown in Table 11. The critical speed for the laboratory Gabbett apparatus at ± 30° C. lies between 310 and 380 r.p.m., but at this speed the rate of precipitation of gas was so slow that 75 min. was required for frothing to start. At 380 to 420 r.p.m. fair concentration was attainable in 70 min., but at 40 min. it had not started. Compare these results with those in tests 10 to 13, representing the normal speed range for this machine. The speed at which froth production begins corresponds

with that at which gas precipitation begins in the square-glass-jar machine of corresponding size.

Attachment of air bubbles to mineral particles in a pulp. No appreciable part of the froth formed in the agitation-froth process is made by attachment of coursing air bubbles in the pulp to mineral particles therein, but substantially all air-mineral attachment is effected by precipitation of gas from solution onto sulphide surfaces.

Table 10. Suction behind blades of the impeller of an agitation-froth machine

Revolutions per minute of impeller	Vacuum measured, inches mercury	Revolutions per minute of impeller	Vacuum measured, inches mercury
370	0.22*	1380	4.4
475	0.29*	1490	5.2
580	0.43*	1595	5.9
685	0.68*	1700	6.8
740	0.9	1810	7.5
850	1.3	1865	7.8
955	1.6	2070	8.7
1060	2.3	2300	9.6
1170	2.9	2610	10.9
1275	3.6		

* Calculated from observations on water gage. Temperature ±20° C.

To prove that lack of froth production in tests 1 to 3, Table 11, was not due to lack of air bubbles introduced at the lower speeds, these tests were repeated with air introduced in very fine bubbles at the bottom of the machine through a carborundum thimble, agitation proceeding as before. No difference in result was observable. Yet with no pulp present the water in the machine was clouded with a myriad of minute air bubbles.

The same failure of coursing bubbles in a pulp to adhere to sulphide particles is shown by another experiment. In a Gabbett mixer, the baffles were held in place by a metal ring fitting against the inside of the glass wall. At places there was sufficient space left for a bubble to catch between the ring and the wall. When this occurred large numbers of par-

Table 11. Effect of agitation on froth formation in agitation-froth process

Test number	Impeller, revolutions per minute	Duration, minutes	Temperature, degrees C.	Froth, inches	Segregation of sulphide(*a*)
1	170– 235	75	34–30	0	0
2	265– 320	75	32–29.5	0	Very slight
3	310– 320	15	31–30	0	Slight
3 continued	310– 370	30	31–29	0	Slight
3 continued	310– 370	45	31–28.5	0	Slightly increased
3 continued	310– 370	60	31–28	0	Slightly increased
3 continued	310– 380	75	31–27.5	1/4	Partial
4	380– 390	25	31–30.5	0	Very slight
4 continued	380– 390	40	31–30	0	Very slight
4 continued	380– 400	55	30–29.5	3/16	Partial
4 continued	380– 420	70	29.5–29	1/2	Fair
5	455– 480	10	28	0	Slight
5 continued	455– 500	20	28	1/16 scum	Partial
5 continued	455– 500	30	28	3/8	Fair
6	540– 590	25	28	1/4	Fair
7	680– 710	20	27.5	1/2	Fair
8	700– 720	18	29	1/2	Fair
9	835– 850	11	28	1/2	Fair
10	900– 940	7	28.5	1/2	Fair
11	1040–1100	6	28	5/8	Fair
12	1270–1300	3	28	1/2	Good
13	1650–1660	2.5	28	3/4	Good

Laboratory Gabbett-mixer (see Fig. 11). Zinc ore, 18.7 per cent. Zn. 18.8 per cent. solids. Oleic acid, 3.0 lb. per ton; H_2SO_4, 36.6 lb. per ton. (*a*) Degree indicated by change in color of settled solids from gray to white and actual appearance of agglomerates.

ticles of both gangue and sulphide could be seen to shower against the stationary bubble (this was particularly easy of observation with a de-slimed ore), but no particles adhered on a direct hit. Occasionally particles were caught in the solid angles formed between the bubble and the walls of the machine, and some of these eventually were pressed sufficiently hard against the bubble by the weight of the particles above, and held there for sufficient time, to effect attachment, so that when they eventually slid around the bubble they adhered to the bottom. But such particles formed an infinitesimal minority of the particles presented at the bubble surface by the swirl of the pulp.

The froths produced in pulp-body concentration processes are small-bubble, coherent and persistent, and characteristic. The volume of gas effectively utilized in floating the mineral is of the order of 20 to 50 cu. ft. per cu. ft. of solid floated. There is marked concentration of oil in the froth.

At Utah Copper Co. oil analyses of the feed and products of a Janney mechanical machine gave results shown in Table 12.

Table 12. Distribution of oil in products in agitation-froth process

Oil (*a*) added per ton, pounds	Oil analyses, pounds per ton			
	Feed (*b*)	Concentrate	Middling	Tailing
1.65	0.71	11.67	....	1.40
3.23	1.28	13.36	....	1.72
6.56	1.30	23.38	....	3.96
12.94	0.96	28.40	....	4.36
21.00	1.48	64.00	12.60	1.96

a 89 per cent. fuel oil, 10 per cent. coal-tar creosote, 1 per cent. pine oil. *b* Sampled ahead of point of oil addition. Contained oil is from circulation of middling.

8. Apparatus for agitation-froth process

The agitation-froth process was first introduced commercially in 1906, in Australia, but the results there obtained were little, if any, superior to those attained by the Potter-Delprat, DeBavay and Elmore vacuum processes, working on the same ores, and it was not until after the invention of the Hoover apparatus (953,746/1910) that the process became in any way dependable or generally useful. Patents have been issued covering a great variety of apparatus and reagents for the practice of the process. The principal apparatus are arranged below. For reagents recommended see Art. 16.

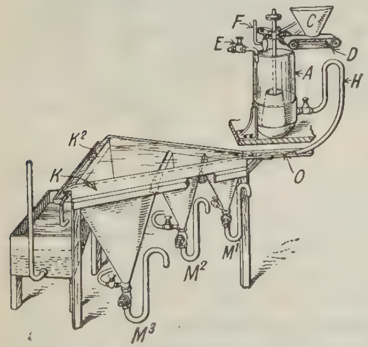


Fig. 11.—Apparatus for process of patent 835,120 (from patent drawing).

Sulman, Picard and Ballot (835,120/1906). Pulp with oil in an amount less than 1 per cent. on the ore, with or without acid and with or without added heat, is agitated with such intensity and for such a period of time as will result in passing through solution in the water and precipitating preferentially on the metalliferous mineral a sufficient volume of gas to float the major portion of the mineral. If the agitation is insufficient in intensity to cause gas precipitation, although sufficient to entrain air, no duration of agitation will be sufficient to cause flotation. The oil used must be to some extent soluble in the water of the pulp in order to spread at air-water interfaces and reduce surface tension. If it does not spread or reduce surface tension (the alternative expressions are equivalent in this connection) no frothing and hence no concentration result.

Apparatus described by the patent is shown in Fig. 11. Dry ore was fed into the agitat-

ing vessel (A) by belt (D) from hopper (C) together with oil from pipe (F) and water and acid from pipe (E). Following agitation, the aerated pulp was discharged through pipe (H) and passed in a thin sheet over apron (O) into spitzkasten (K) fitted for introduction of "hydraulic" water in the several compartments. (See Sec. 6, Art. 1.) Froth overflowed the lip (K^2) and tailing of increasing fineness was discharged from goose-necks (M^1), (M^2) and (M^3), in order. This apparatus is effective only on highly mineralized ores with oils that form heavily-loaded bubbles and the stiffest kind of coherent froth. It fails utterly to produce more than film flotation under other circumstances, because of the fact that the froth is broken up by surface tension at the large free-water surface on the apron and spitzkasten. For commercial apparatus see Fig. 14 and the descriptions therewith.

Hoover (953,746/1910). Fig. 12, taken from the patent, is sufficiently explanatory of the principle of the apparatus. (D) and (E) are oil and acid cans respectively. The stirrers operate at 1200 to 1800 ft. per min. peripheral speed. The close coupling of the agitation chamber with the spitzkasten and feeding of the agitated and aerated pulp below the surface of the pulp in the spitzkasten were the secrets of the success of this apparatus.

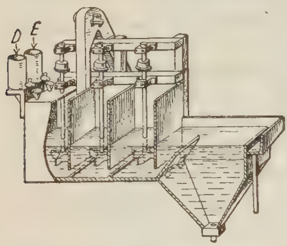


FIG. 12.—Hoover apparatus for agitation-froth process.

Hoover (979,857/1910) described an apparatus for the necessary re-treatment of tailing from the first flotation operation. This apparatus is shown in Fig. 13. Unfinished tailing or middling from the first spitzkasten (O') is pumped through pipe (Q') into agitating compartment (L^3) and, after agitation, passes through the slot shown into another agitating compartment (L^4) and thence into spitzkasten (O^2) for further froth separation. This retreatment is multiplied as many times as desired by multiplication of agitating compartments and spitzkasten. Pulp is maintained at a constant level in all the spitzkasten by the pipes (R), equalizing tank (S) and overflow pipe (S'). Any deficit of water is made up from faucet (T).

This patent has all of the essential elements of the present standard Minerals Separation machine except the mechanical froth skimmers which do away with the necessity for maintaining a pulp overflow from the spitzkasten. Many variations in the baffle over the inlet from agitation chambers to spitzkasten have been used and many forms of impeller. These are illustrated in part in the following figures.

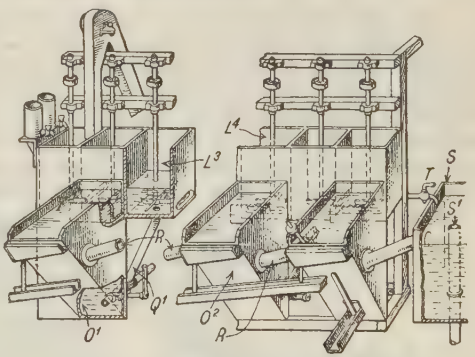


FIG. 13.—Hoover apparatus for repeated treatment.

Minerals Separation machine is shown in Fig. 14. The machine is installed as shown in the figure with agitators and froth-separating boxes on the same level. It consists essentially of the agitating compartment (a) and froth-separating compartment (b). The agitator, which is of the four-armed cross-type, with blades inclined 45 degrees, is placed close to the bottom of the agitating compartment and is carried on a vertical spindle (c), driven through enclosed bevel gears from a horizontal line shaft. End thrust is lessened by opposing the bevel gears on the shaft. Noise and power consumption are decreased by using bronze gears. Feed pulp is introduced into the first agitating compartment, or may be passed first through one or more agitating compartments, without froth-separating boxes, corresponding to the emulsifiers of the Janney machine. The pulp, after agitation and aeration, is thrown out through the

slot (*d*) into the froth-separating compartment, entering at a point about six inches below the pulp level. The tailing from the froth-separating compartment passes through pipe (*e*) into the bottom of the next agitating compartment, under the influence of the pumping effect of the agitator therein. The rate of flow is regulated by means of valve (*f*) actuated by hand wheel (*g*). Froth is removed by means of the revolving scraper (*j*). From 6 to 20 agitating compartments in series, each with one froth-separating compartment, comprise a unit.

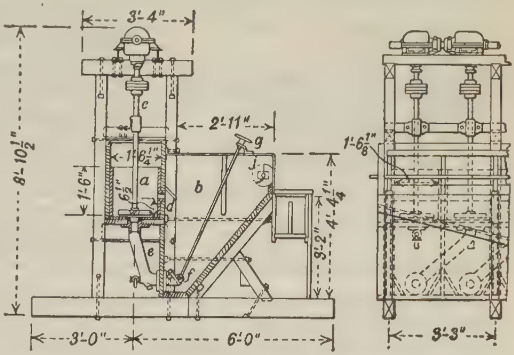


Fig. 14.—Minerals Separation 12-in. standard machine.

The size of a unit is indicated by the distance tip to tip of the impeller blades and the number of compartments in series. The usual impeller sizes are 12-in., 18-in., and 24-in. These machines have rated capacities of 50 tons, 300 tons and 600 tons respectively per 24 hours on silicious ores in pulps containing 25 per cent. solids. Average actual capacities are about 2, 12, and 25 tons per cell per 24 hours on such a pulp, with a diminution in capacity with decrease in percentage of solids about proportional to the increase in volume of pulp. The power consumption per agitator is 1.5 to 3 hp., 2.5 to 5 hp., and 3.5 to 10 hp., for the three sizes respectively, depending upon the speed of the impeller and the tonnage of pulp passed through. The usual peripheral speed is from 1500 to 1800 ft. per minute. The so-called low-level machines, which carry a relatively low pulp level, consume about 60 to 70 per cent. as much power as the standard machine.

Impellers are made of cast iron, cast steel, manganese steel or, in cases of acid copper pulps, of brass or bronze. Life ordinarily is from six months to two years. The agitating compartment is lined with hardwood or cast iron. Life of liners is usually between one and three years.

The minimum of attendance necessary is illustrated by practice at Anaconda and Inspiration. At the former mill four men handle eight M.S. standard machines treating 3000 tons copper ore per 24 hr. At Inspiration one man could handle two 800-ton sections containing two 10-compartment and four 6-compartment machines. (*56 A 677.*)

Performance. At Reocin mines, Spain (*115 J 398*) a 12-in. 12-compartment standard machine at 500 r.p.m. consumed 17 hp. when treating 200 tons per 24 hr. This is 11.8 tons per hp. or 1.52 kw.-hr. per ton. At St. Joseph Lead Co., Leadwood mill (*57 A 433*), one 23-cell 24-in. standard machine at 335 r.p.m. consumed 83 hp. and treated 540 tons per 24 hr., making 6.5 tons per hp. or 2.7 kw.-hr. per ton. At Federal Lead Co. (*57 A 374*) a local form of M.S. machine with 24-in. agitation compartment and 18-in. agitators consumed 4 to 4.5 hp. per spindle at 280 r.p.m. Increase in speed increased extraction somewhat but increased power consumption out of all economical proportion. Twelve to 24 compartments in series were used. At Mount Morgan, Australia (*102 J 755*) a 6-compartment 24-in. standard machine drew 55 to 60 hp. at 295 r.p.m. when treating 100 tons per 24 hr. of −48-mesh pyritic copper-gold ore or 10.8 kw.-hr. per ton. At Calumet and Hecla (*108 J 9*) a 16-cell 24-in. standard machine handles 400 tons of thickened mill slime per 24 hr. At Suan Concession, Korea (*119 P 843*) 250 tons per 24 hr. was sent through one 8-compartment 24-in. primary rougher, the first compartment having no spitzkasten. Stirrers were 21 in. tip-to-tip, driven at 285 r.p.m. by a 60-hp. motor, or 4.3 kw.-hr. per ton. A froth paddle of 6-in. radius made 24 r.p.m. It required 18 to 25 min. for pulp to pass through the machine. Crowding boards were used on the last three spitzkasten. Froth from the first six boxes went to an 8-compartment 18-in. cleaner with 12-in. stirrers driven at 390 r.p.m. and consuming 15.7 hp. The time required for pulp to pass this machine was 18 to 30 min. Total power consumption in flotation alone (1917) was 6.537 kw.-hr per ton. At Catemu (*123 P 928*) a 12-compartment 18-in. machine drew 30 hp. when treating 75

tons of slime per 24 hr. or 7.2 kw.-hr. per ton. At Swansea, Ariz., a 16-cell, 18-in. low-level machine required 3.4 hp. per spindle.

Smith (1,056,952/1913). Launder devices for regulating feed and discharge of spitzkasten.

Smith (1,058,111/1913). Agitator on horizontal shaft in enclosed chamber. See Fig. 15.

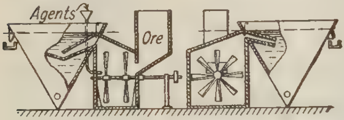


Fig. 15.—Horizontal-shaft agitation-froth machine (*Smith*, U. S. pat. 1,058,111, 1913).

Hebbard (1,064,209/1913). Multiple, staggered-spitzkasten, single-level, non-circulating, vertical-spindle machine.

Broadbridge and Howard (1,084,096/1914). Multiple agitators, two to a spitzkasten, single-level, non-circulating, vertical-spindle machine. Partitions between spitzkasten are submerged beneath the pulp surface allowing a continuous froth layer over all spitzkasten. Not used commercially.

Howard (1,084,210/1914). A variety of forms of vertical-spindle impellers and of methods of baffling in the agitation compartment to increase turbulence. No commercial forms.

Janney (1,167,076/1916). Multiple, pyramid, circulating, vertical-spindle machine. This is the best of the agitation-froth machines. Fig. 16 shows it in the form introduced in the mills. It consists essentially of an agitating compartment (*a*) with two froth-separating compartments (*b*). In the usual and best form the agitator shaft is an extension of the spindle of a 6-hp. vertical motor, 570 r.p.m. It carries two four-armed impellers with blades set at 45°. The agitating compartment is circular and contains four baffles extending slightly more than one-half the distance from the bottom toward the top. The arms of the lower impeller are shorter than those of the upper in order to clear these baffles. Feed is introduced through the side of the agitating compartment, near the bottom, by means of a pipe; is thrown out through the channels at the top on each side of the agitator compartment, and is introduced, by means of the submersion blades (*d*) slightly below the level of the pulp in the froth-separating compartments. In general, several machines are installed in series. Tailing leaving the first compartment passes through an opening (*e*), regulated by means of the valve rod (*f*), into the froth-separating compartment in the succeeding machine. The froth-separating compartments are divided to within about a foot of the overflow lip by means of a wall (*g*). From the bottom of the two compartments thus formed, pipes (*h*) lead back to the agitating compartment. Pulp, entering the first or upper of these two sub-divisions in a given froth-separating compartment, is drawn upward into the agitating compartment, thrown over the top into the froth-separating compartment and falls back, a part on each side of divider (*g*). Practically all of that which falls back on the upper side of the divider is again drawn up through the pipe from that compartment into the agitating compartment. A part of that which falls on the down-stream side of the divider (*g*) is also drawn back into the agitating compartment. Thus a part of the pulp is circulated in each machine and subjected to agita-

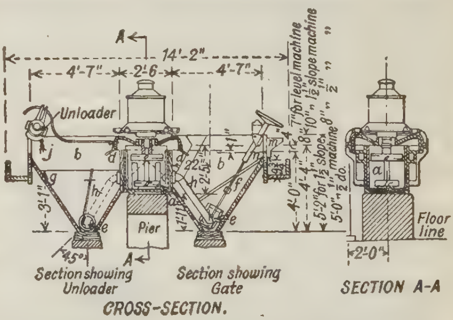


Fig. 16.—24-in. Janney mechanical machine.

tion and aeration more times than would be the case if the flow were alternately through successive agitating and froth-separating compartments. Froth is skimmed by means of the eccentrically driven unloader (*j*) which is operated by means of a small independent motor. The series of machines is sometimes preceded by one, two, or three agitating compartments built without froth-separating compartments, called EMULSIFIERS. Flotation agents are added to the pulp entering the emulsifiers. Additional flotation agents are added, if necessary, through the oil funnel (*m*) and pass through the oil pipe (*n*) into the circulating pipes (*h*). A multiple arrangement of feed, *i.e.*, with incoming pulp stream split between the first three or more cells is ordinarily used where it is desired to make a finished concentrate on the early cells and to circulate the froth from the later cells back to the head of the series. A series arrangement with all of the feed stream entering the first cell is used where the froth taken from the various froth-separating compartments is cleaned on other machines.

From five to fifteen agitators, each with two froth-separating boxes and preceded by one or two emulsifiers, comprise a unit. The size of a unit is indicated by the diameter of the agitation chamber and the number of single machines in series. A 24-inch, 16-compartment unit consists of one or two 24-in. emulsifiers followed by 16 agitating compartments with cross-armed impellers, the upper impeller 20 in. and the lower 14½ in. tip-to-tip, respectively, and 32 froth-separating boxes. The **capacity** of such a machine depends upon the percentage of solids in the pulp, the number and diameter of impellers, and the character of the ore.

On a silicious, sandy copper ore in pulps carrying from 10 to 28 per cent. solids, a 13-compartment, 24-in. machine at UTAH COPPER Co. had a capacity of from 150 to 550 tons per 24 hr., the relation between tonnage and percentage of solids being represented by a straight line.

Decrease in the number of cells in series will mean a corresponding and almost proportionate decrease in capacity, if the same grade of concentrate and percentage of recovery are to be maintained. There will also be a decrease in capacity if a slimy feed replaces a sandy feed. This is due principally to the fact that such a feed must be treated in a pulp containing a lower percentage of solids and that consequently the bulk to be passed through the machine for a given tonnage of solid is increased. The decrease in capacity is about in proportion to such increase in bulk.

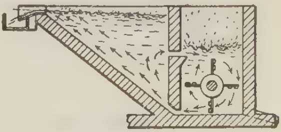


FIG. 17.—Horizontal-spindle agitation-froth machine.

On $-$80-mesh lead ore, an 8-cell 24-in. machine at FEDERAL LEAD Co. handled 500 tons per 24 hr. in a pulp containing 20 per cent. solids. At AMERICAN ZINC, LEAD AND SMELTING Co. mill at Mascot, Tenn., a 7-cell 24-in. machine treated 170 tons per day of $-$35-mesh zinc ore in a pulp containing 33 per cent. solids. The power consumption of a 24-in. machine is approximately 6 hp. per agitator. The machine is usually run at a peripheral speed of 3600 ft. per min.

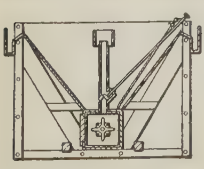


FIG. 18.—Mishler flotation machine.

Eberenz and Brown (1,187,822/1916). Horizontal-spindle, circulating machine. See Fig. 17. The impeller arms carry several spaced blades for the purpose of increasing agitation.

Mishler (1,197,843/1916). Several impellers on a horizontal shaft, each working in a closed agitation chamber at the apex of a spitzkasten. Compressed air supplied to pulp in the agitation compartments. Discharge into a spitzkasten by overflow of a standpipe above the agitation compartments. (Fig. 18.)

Janney (1,201,053/1916). Multiple, vertical-spindle, single-

level, circulating machine, differing from that described in patent 1,167,076, in that successive units are on the same level.

Brittain (1,224,066/1917). Multiple, vertical-spindle, single-level, non-circulating. Agitation and froth separation in the same chamber. Vertical disturbance of the surface is prevented by a feed conduit with bottom discharge placed directly above the impeller (see Fig. 19). Entering pulp flows successively through (8), (4), (10) and (11) to the impeller and, these passages being only partly filled, air likewise enters and is beaten in by the impeller in the usual fashion. Aerated pulp, discharged at the periphery of the impeller rises in (1), froth overflows lips (15) into concentrate and middling launders (19) and (20). Tailing discharges through openings (5) into ducts (8) leading to adjacent compartments for retreatment.

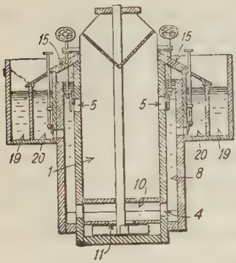


FIG. 19.—Brittain flotation machine.

Colburn and Colburn (1,226,062, 1,226,063/1917; re-issue 14,497/1918; 1,415,314/1922). Special centrifugal pump as agitator with regulated air inlet into a submerged feed pipe and discharge below the surface of pulp in a spitzkasten. Agitation in a vacuum is claimed, but the vacuum effect is probably no greater than in the usual open type of agitation-froth machine. See Fig. 20.

Norvell (1,243,093/1917). Vertical-spindle agitator. Agitation compartment suspended on the center line near the top of a V-box froth-separating compartment with discharge of pulp thereinto below the surface of pulp in the box.

Saffold (1,256,263/1918). Multiple, cross-bladed, vertical-spindle agitator mounted in a closed, compartmented vertical cylinder, the discharge ports for agitated pulp opening below the surface of the pulp in a conical or pyramidal spitzkasten. Also aeration through a perforated pipe at the bottom of a cylinder at the center of a spitzkasten with discharge of aerated pulp through ports in the side of the cylinder below the overflow level of the spitzkasten.

Pearce (1,277,750/1918). Multiple unit: vertical-spindle, cross-bladed agitator in square agitation compartment with a horizontal partition directly above the agitator blades and central opening around the shaft for pulp inlet. Direct centrifugal discharge from the agitating compartment into the apex of the spitzkasten with froth overflow at the side farthest from the pulp inlet and tailing discharge by overflow directly above the pulp inlet. (See Fig. 21.)

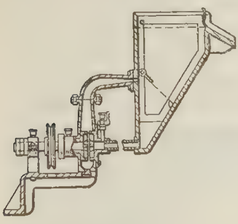


FIG. 20.—Colburn flotation machine.

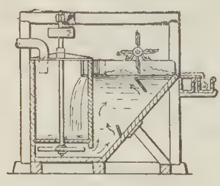


FIG. 21.—Pearce flotation machine.

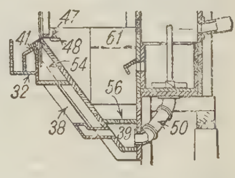


FIG. 22.—Lyons and Haff froth cutter.

De Mier (1,308,049/1919). Multiple unit: series flow. Vertical-spindle machine of substantially the standard Minerals Separation single-level type, but with two spitzkaste.. to each agitator. Cribbed agitating compartments are suggested.

Boggs (1,324,791/1919). Horizontal spindle with multiple, cross-bladed, 45-degr impellers mounted in the bottom of a long trough with feed at one end and tailing disch. at the other end of the trough.

Lyster (1,352,072/1920). In a series of centrifugal-pump agitators and spitzkasten, wi. air-valve control of the feed rate to the pump from the apex of the preceding spitzkaste a float in the spitzkasten automatically regulating the valve on the air-supply pipe.

Lyons and Haff (1,389,674/1921). In an agitation-froth machine of the Minerals Separation type, (Fig. 22), a floating froth cutter (47) regulated by float (48) to send the lower layer of overflowing froth to a middling launder (54) and thence by pipe (38) to chamber (39) under a false bottom in a preceding spitzkasten, there joining the tailing from this spitzkasten and thence by oblique pipe (50) to agitation chambers of the adjacent succeed-

ing spitzkasten. Any excess middling from chamber (54) flows through slot (41) to middling launder (32), there joining similar products from other spitzkasten and being returned by a bucket elevator or the like to the head of the machine. Coarse tailing from any spitzkasten discharges by sand-hole (56) into chamber (39); fine tailing overflows weir (51) at the tailing side of the spitzkasten into a channel leading to chamber (39).

Shimmin and Bushnell (1,402,099/1921). A modification of Janney (1,167,076/1916 and 1,312,115/1920), in which an open-ended prism is so supported in the agitation chamber as to form an annular space and the direction of rotation of the impeller is such as to force pulp down through the inner space and up through the annular space and thence into the froth-separating compartments.

Eberenz (1,505,324/1924) describes a type of horizontal-spindle machine, pictured with submerged blades, but, of course, not so operating. The agitating compartment is sealed against the atmosphere and injection of a gas such as hydrogen sulphide with an inert gas such as carbon dioxide, nitrogen, and the like is contemplated.

Kleinbentink (1,539,746, 1,557,369/1925) describes a circular machine in which a central agitating compartment in the shape of an inverted conical frustum, containing a pump-bladed impeller on a vertical axis, is surrounded by a flaring-sided froth-separating compartment with peripheral overflow, bottom-fed from the agitating compartment and feeding back thereinto through ports above the impeller and above the outlet ports. The apparatus is described principally for coal treatment but the claim names ores also.

Hydrotator flotation machine (Fig. 23) is essentially an agitation-froth machine using the centrifugal pump (*a*) with air inlet (*b*) and oil inlet (*c*) for agitation. Feed enters at (*d*), pulp is withdrawn through pipe (*e*), passes through the pump and thence into the rotating-sprinkler type of mechanism and, through jets (*g*), into the tank. Air-mineral agglomerates, formed in the pump, rise and overflow as froth.

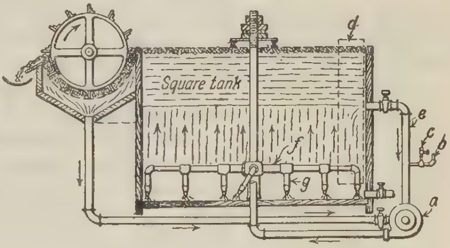


Fig. 23.—Hydrotator flotation machine.

Some tests on bituminous coal (− 20-mesh jig-overflow water) showed 60 to 70 per cent. ash reduction and 85 to 95 per cent. recovery of carbon content. In a test on −3/32-in. raw bituminous carrying 21.6 per cent. ash, the float analyzed 2.2 per cent. ash against 9.2 per cent. in jigged coal. Screenings (−3/32-in. rd. hole) from washery refuse assaying 50 per cent. ash yielded float containing 9.5 per cent. ash and representing 55 per cent. recovery. (*These data furnished by the Hydrotator Co.*)

9. Bubble-column process

General. In this process substantially all of the concentration is performed in a column of bubbles above and floating on the surface of the body of pulp. The volume of gas effectively used to produce concentration is 20 to 100 times greater than in pulp-body processes, being of the order of 1000 to 2000 cu. ft. per cu. ft. of solid floated. The result is that the froth is fragile and evanescent and strikingly different from that characteristic of the other class of froth processes. Further investigation of the process, by observation of the operation in glass-sided machines, shows that: (1) The bubbles are much larger than in pulp-body processes; (2) they are more numerous; (3) they rise through the pulp more rapidly; (4) they arrive at the surface of the pulp with a solid load composed of sulphide and gangue in substantially the same proportions that these exist in the pulp through which they have passed; (5) concentration begins at the bottom of the bubble column (*i.e.*, the upper surface of the pulp body) and progresses upward. The actual mechanism of concentration can be studied with a hand-glass. There is a differential draining of the gangue and sulphide particles in the bubble walls; the average downward velocity of the sulphide particles is less than the average upward velocity of the bubbles; the average downward

velocity of the gangue is greater than the average upward velocity of the bubbles; and as a result the sulphides are lifted up and away from the gangue. The sulphide particles in the bubble column are nowhere firmly adherent to bubbles as they are in the pulp-body processes. These various phenomena are generalized in Fig. 24. Segregation of sulphide from gangue in the water-filled space between the bubbles brings about a distribution of such particles as is idealized in the sketch. All particles are moving downward with respect to the bubbles, but there is a momentary retardation of sulphide particles at the bottom of the bubbles, indicated on the sketch by the crowding of particles at the bottom of the bubbles and the dependent tip. Bubbles near the top of the column carry more attached particles at any given instant than those lower down. The falling path of any given sulphide particle, following, as it does, the bubble surfaces, is longer than that of a given gangue particle, which falls as directly as possible through the liquid channels forming the bubble walls.

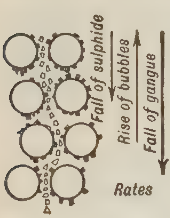


Fig. 24.—Action in bubble column.

Theory. Superficially bubble-column concentration is similar to hydraulic classification, with the essential differences, however, that a rising column of bubbles replaces the rising current of water, and that the buoyant effect of the rising current on the solid particles is modified and aided, in the case of the metallic minerals, by adhesion between the bubbles and these particles. But in both processes the essential feature is the presentation of a mass of grains of unequal settling power, under the conditions of treatment, to a rising current whose velocity lies between the average settling rates of the two classes of grains it is desired to separate. In the ordinary practice of bubble-column concentration the pulp is fed to the bottom of the bubble column, but it may be fed at the top, as in hydraulic classification. Selection of mineral particles by gas in the body of pulp, which is the essential basis of pulp-body processes, is not an element of bubble-column concentration, and consequently heat, chemical action, agitation, and external pressure change are unnecessary. All that is necessary in bubble-column concentration is to form and maintain a slowly-rising column of bubbles, and to present to it solid particles prepared by treatment with some selective agent, so that preferential adhesion of sulphide particles to the bubble walls will be more marked. Maintenance of a bubble column requires addition to the pulp of some substance that changes (usually lowers) the surface tension of the water. The selective agent is one that either from solution or from minute mechanical dispersion in the pulp, preferentially adheres to metallic-mineral surfaces. All successful mixtures of flotation agents must have both of these properties.

Quantitative experimental evidence of the difference between the bubble-column and agitation-froth processes is presented in Fig. 25. A pneumatic bubble-column machine and the spitzkasten of a Minerals Separation standard machine were fitted with ⅛-in. sample pipes projecting inward 6 in. from the inner surface of the walls, arranged in vertical rows running from bottom to top of the respective apparatus. Simultaneous samples from all pipes were drawn from each machine, assayed, and the assays plotted against position in the machine. Plot B shows that bubbles on emergence into the bubble column in the pneumatic cell are carrying a load of pulp; that no concentration at the bubble surface has taken place in the pulp body. Plot D shows that the solid load on the bubbles emerging from the pulp in the agitation-froth process is a concentrate of substantially the grade of the finished concentrate from the cell. The tests recorded in Fig. 25 were made in regular mill operation. Similar comparisons have been made in a 24-in. M.S. standard machine, a Janney mechan-

ical machine, a standard Callow machine, two special large-size Callow machines, an Inspiration-type pneumatic machine and a Cascade machine. These tests all show that plots *B* and *D*, Fig. 25, are typical of the behavior of bubble-column and agitation-froth machines respectively.

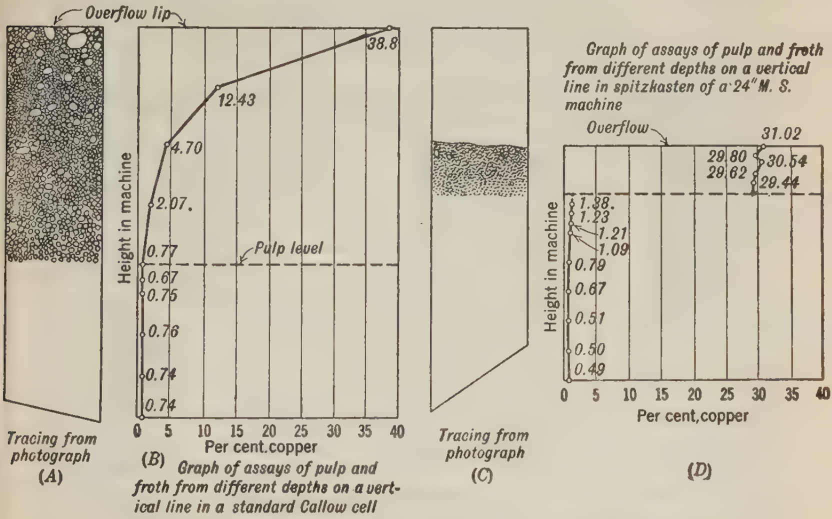


Fig. 25.—Charts showing place of concentration in standard Callow (bubble-column) and 24-in. M. S. (agitation-froth) machines.

It logically follows from the conclusions reached in the preceding paragraph that the body of pulp in the bubble-column type of machine can be eliminated without affecting the concentration.

A cell built to eliminate the pulp body, 10 ft. long, 14 in. wide inside and 12 in. from blanket to overflow lip, with bottom horizontal (see Fig. 26), was run side-by-side with a regular mill cell in the Miami Copper Co. plant. This shallow cell recovered the same weight of copper per square foot of blanket area that was recovered in the standard cells adjacent, and made the same grade of concentrate.

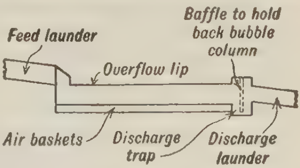


Fig. 26.—Shallow pneumatic cell.

Attachment of mineral particles to bubbles. Whether the actual method of adhesion of solid particles to bubbles is different in the bubble-column and pulp-body processes is not yet known definitely. It can be said without fear of contradiction that in pulp-body processes solid particles are held to bubbles in the same way that particles are held in film flotation (Fig. 3). There is, of course, a more or less complete film of the selective substance between the air in the bubble and the actual solid surface, but this is, in most cases, of monomolecular dimensions. That such is the mode of attachment follows from the way in which attachment is effected. The initial precipitation of gas is from solution in contact with the filmed particle and takes place at the interface between particle and solution, *i.e.*, in contact with the particle. Subsequent precipitation, causing growth of the bubble, may be at the liquid-solid interface or at the liquid-gas interface; probably it is at both. When the bubble is small with respect to the particle, particularly if it is growing on a substantially plane surface, the shape is that shown in Fig 27*a* and further growth causes it to take the shape shown in Fig. 27*b*. This non-spherical shape, indicative of definite contact angles, can only occur when there are three-phase contact points. Since the only visible phases present are gas, solid and pulp liquid, the relation must be that shown. This kind of attachment results in

substantially rigid systems of air, liquid and solid, evidenced generally by stiff, coherent froths and showing under the microscope little or no movement of solids in the bubble walls. The appearance of individual bubbles composing a pulp-body froth is of rough, solid-armored spheres packed together more or less closely in substantially quiescent water. This is entirely different from the unstable and temporary adhesion between bubbles and metalliferous particles in a bubble column and leads to the belief that in the bubble column another mode of attachment prevails.

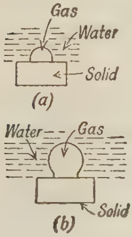


FIG. 27.—Relation of phases in pulp-body flotation.

Mode of attachment in bubble column. To a test tube two-thirds full of water add a few drops of an oil such as oleic acid or pine oil and a few hundred particles of finely-divided granular sulphide (about 0.1-mm. diam.) cork the test-tube and turn it end for end a few times, then examine with a low-power microscope. Aggregates of oil and mineral such as are illustrated in Fig. 28*a*, may be seen. Shake the tube gently so as to introduce some air, but do not shake enough to cause any substantial pressure differences. Aggregates such as are shown in Fig. 28*b*, may be observed. In both cases the rigidity of attachment of mineral particles to oil globules is slight. Continued shaking results in increase of air-oil-mineral aggregates with successive thinning of the oil film on the air bubbles, but with no change in the fragile nature of the bond between solid and oil. A limit is finally reached when, on many of the bubbles, the oil film is invisible, but one still exists and, barring such agitation as will produce gaseous supersaturation, the mode of attachment of solid particles thereto must be that shown in Fig. 28*c*, in which the thickness of the oil film is greatly exaggerated. The surface tension of an oil-water interface is from one-third to one-half that of the contaminated water-gas interface and the three-phase contact angle oil-water-solid is much smaller than the water-gas-solid angle, which would explain the more feeble adhesion observed in the bubble column as compared with the froth in pulp-body processes. The same conclusion as to necessary difference in character of attachment is reached by study of Tables 22 and 23.

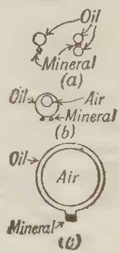


FIG. 28.—Probable method of adhesion of mineral particles to gas bubbles in bubble-column flotation.

The machines in which the bubble-column process is practiced may be classified, on the basis of the method of introducing air, as pneumatic machines, plunging-stream or cascade machines, and centrifugal machines.

10. Pneumatic machines

Pneumatic machines pump air directly into an oiled pulp through a pipe or pipes or the equivalent. In the best-known type the air for making the bubble column is introduced through a porous medium such as canvas, cotton twill, blanket, carborundum, or concrete. Machines described in the patents, as also some modifications introduced in the mills, are described below.

Sulman and Picard (793,808/1905) first described a bubble-column apparatus. In Fig. 29, (B) is a rectangular separating tank with sloping bottom, above which is suspended a perforated spiral coil (B'), revolved by gear (B^3). Air is supplied at (C) and oil from tank (C') may be atomized into the entering air or the pulp may be pre-mixed with oil. Feed enters through pipe (A') concentrate is overflowed into launder (B^4) and tailing discharged through valve (B^5). The utility of the process disclosed was not recognized by the inventors nor by the art, and the pneumatic process as at present practiced is not clearly claimed, if claimed at all, in the patent. The principle was not known to the patentees as late as 1907 as they say in a privately distributed publication of that date:

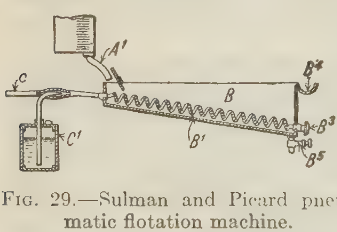


FIG. 29.—Sulman and Picard pneumatic flotation machine.

" . . . mineral flotation is difficult to achieve by simply passing a *current* of gas bubbles through a pulp. The explanation is that under such conditions supersaturation takes

long to establish. If on the other hand such a pulp be shaken vigorously with air, or air be whipped into it by mechanical agitation (*835,120*) flotation is just as readily ensured as if the gas were generated in the pulp itself by chemical means (*Froment*) ".

Towne and Flinn (1,295,817/1919) first recognized and described the action in the cell. Their apparatus (Fig. 30) shows a circular tank (1) which is fed with oiled pulp and supplied with low-pressure air through porous medium (5). The bubble column overflows into annular launder (22) and tailing discharges through pipe (12) and automatic valve (15). Other forms of apparatus by the same inventors are described in 1,317,244/1919; 1,367,322/1921; 1,378,920/1921; and 1,410,-781/1922. Flinn, 1,314,316/1919, describes a similar apparatus.

None of the forms of apparatus described by these inventors has had any commercial use. An installation of 24-in. cells was tried in the INSPIRATION TEST PLANT (*55 A 576*), but was rejected in favor of the Callow machine. The 24-in. cells were also tried at CANANEA and ARIZONA COPPER CO. but not adopted, although recoveries were good. One great fault was settlement of sand on the bottom and consequent boiling.

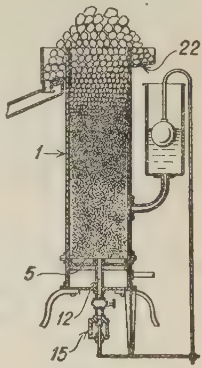


FIG. 30.—Towne and Flinn pneumatic flotation machine.

Callow pneumatic cell, as used in the mills, is shown in Fig. 31. It consists of a rectangular box with sloping porous bottom. The usual dimensions of the box are 8 to 9 ft. long, 2 ft. to 2 ft. 6 in. wide, about 18 in. deep at the feed end and 4 ft. deep at the tailing-discharge end. The porous bottom consists of three or four layers of medium-weight canvas or palma twill, supported on a screen or grid on top of an air box. The usual cell has the air box divided into eight compartments, each with an independent connection to a header, the purpose being to allow independent regulation of the air pressure on the under side of the blanket at different points in the length of the cell, in order to balance the different hydrostatic heads and prevent eddy currents due to unequal air distribution. Pulp is fed into the cell behind the baffle (*c*), froth overflows the sides, and tailing is discharged through the pipe (*d*). The rate of discharge is regulated by means of the adjustable float valve (*e*). The capacity of a single unit such as illustrated is from 35 to 80 tons per 24 hours, the lower figure corresponding to a slimy ore in a pulp containing a low percentage of solids, the higher figure corresponding to a silicious, rather sandy ore in a pulp containing in the neighborhood of 25 per cent. solids.

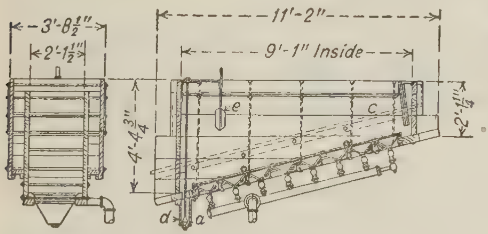


FIG. 31.—Standard Callow pneumatic cell.

In the MISSOURI lead mills, where pneumatic machines are used as scavengers, tonnages per machine run as high as 150 per 24 hr. At the Morenci mill of PHELPS DODGE (*69 A 180*) the feed rate to standard Callow cells was 2.5 tons per sq. ft. of bottom per 24 hr.

The air consumption is decidedly variable, ranging in different plants from about 6 to 12 cu. ft. of free air per min. per sq. ft. of porous bottom, at pressures of from 3 to 5 lb. per sq. in. on the supply side of the regulating valves. A good average figure is probably in the neighborhood of 9 cu. ft. per min. per sq. ft. of porous bottom. This air requirement corresponds to

a power consumption of between 3.5 and 4 hp. per cell. Large, shallow cells require about 0.3 hp. per sq. ft. of porous bottom.

The following detailed figures are given by Callow (*54 A 14*): NATIONAL COPPER CO., 500 tons per day treated in eight standard roughers and two cleaners; 950 cu. ft. of free air per min. at 4 lb. pressure required; blower power, 35 hp., 3.5 hp. per cell, 12.53 tons per hp. or 1.25 kw.-hr. per ton. At another plant treating 2400 tons per day in 48 roughers and 12 cleaners, 9600 cu. ft. of free air per min. at 5 lb. pressure was furnished with 210 hp., 3.5 hp. per cell, 11.45 tons per hp. or 1.56 kw.-hr. per ton. In the INSPIRATION experimental plant 200 tons per day was handled in 4 roughers and one half-size cleaner, requiring 950 cu. ft. of free air per min., at 5 lb.; blower power was 18 hp. making 4 hp. per cell, 10 tons per hp. or 1.79 kw.-hr. per ton. Laist (*113 P 634*) is quoted as giving comparative power consumptions for flotation alone as 0.15 hp. per ton of daily capacity for pneumatic at INSPIRATION against 0.25 hp. in M.S. machines at ANACONDA. Minimum attendance requirement is illustrated by INSPIRATION practice, where one man handles four 800-ton sections, and MIAMI, where two men handled 60 standard cells treating 3200 tons per 24 hr. (*56 A 577*).

Callow cells are usually run in parallel. General experience is to the effect that, whether run in parallel or in series, the capacity per sq. ft. of porous bottom, in machines producing concentrate of the same grade and recovering approximately the same amount of mineral, is the same irrespective of the method of operation.

A modified form of Callow cell designed to economize floor space is shown in Fig. 32. It consists of a rectangular box about 20 ft. long, 7 ft. wide and an average depth of 3 ft., set on a slope of about 2 in. per ft. The box is divided into eight compartments, 3 ft. 5 in. wide by 4 ft. 6 in. long, by means of a central longitudinal wall which extends to the bottom

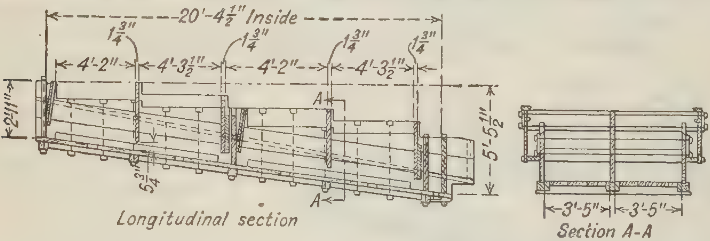


FIG. 32.—Miami-type pneumatic cell.

of the box, and three transverse walls. The first and third transverse walls extend to within 5 or 6 in. of the bottom of the box, the second or center wall is so arranged as to cause the pulp to overflow a weir in passing from the second to the third compartment on both sides of the cell. The purpose of this weir is to maintain the desired pulp level in the first two compartments on each side. Practically the partitions serve to divide the box into four cells. Air baskets consisting of shallow boxes with a porous top, of such size that they fit loosely into a compartment, are placed with the porous side up in the bottom of the compartment. The capacity of one of these machines is between 400 and 800 tons per 24 hr. treating a silicious ore in a pulp containing from 20 to 25 per cent solids.

Tests on air and power consumption of Miami-type pneumatic cells at MIAMI COPPER CO. showed that air consumption per square foot of porous bottom in the rougher cells ranged, during a period of 7⅓ days, from 8.1 to 12.1 cu. ft. of free air (12.82 lb. absolute pressure and 70° F.) per min.; average 9.6 cu. ft. Figures based on two- and three-day tests on individual bottoms showed a range from 7.4 to 14.5 cu. ft. per sq. ft. per min. for No. 1 bottom, average 9.5; 6.7 to 11.7 for No. 2, average 8.5; 8.7 to 13.0 for No. 3 average 10.4; and 7.2 to 10.6 for No. 4, average 9.3. The cleaner cells showed a range, on 4-day test, of 2.6 to 6.4 cu. ft. per sq. ft. per min., average 4.7. Power consumption per section containing 32 rougher and 8 cleaner bottoms, to which the average air supply based on the above figures was 4141 cu. ft. of free air per min. at 3.8 lb. per sq. in., ranged, on 15-day test, from 94.4 to 114.5 hp., average 106 hp. for the period. This is roughly 40 cu. ft. of free air per min. per hp. or 0.22 hp. per sq. ft. of blanket surface. One of these sections handles 1300 to 1500 tons per 24 hr., making power consumption 1.7 to 1.9 hp.-hr. per ton. Further test

showed that the power draft for the section with the cells empty was 39.3 hp. and the pressure 1.5 lb. per sq. in. With the cells full of water the power draft was 59 hp. and the pressure 2.5 lb. per sq. in. The increase above this latter figure with pulp in the cell was probably due largely to deposition of sand on the blankets, as the difference in density between pulp and water is insufficient to account for the increase in pressure and power consumption.

At the new mill of Morenci Branch of Phelps Dodge (*69 A 176*) a shallow cell, 4 × 40 ft., 15 in. deep at the feed end, bottom slope ⅛-in. per ft., stepped up to a depth of 15 in. at mid-length, was installed with an allowance of 1.25 sq. ft. of blanket area per ton treated per 24 hr. This figure was based on thorough experiment with the low-grade sulphide ore to be treated. Use of the shallow cell reduced the required air pressure from 4.75 to 3.5 lb. per sq. in. At Moctezuma Copper Co., Nacozari, Mex. (*118 J 446*) the blanket allowance was 1.5 sq. ft. per ton per 24 hr.

The principal advantage of the large cells lies in the compactness of the installation. A comparison of the standard cell with a large shallow cell at Morenci (*109 J 1352*) indicated little or no advantage in performance when the reduced feed rate in the large cells is considered.

Most of the recovery in a pneumatic machine is effected in the first few feet. Table 13 shows results of a test on a Miami-type cell. In one test on a

Table 13. Recoveries in successive compartments of a 4-compartment pneumatic cell at Miami Copper Co.

Date, 1921	Mar. 14	Mar. 18	Mar. 26	Apr. 4	Average	Recovery, cumulative per cent.
Feed, per cent. Cu(*a*)	1.90	1.50	1.39	1.55	1.58	....
Tailing, per cent. Cu:(*a*)						
Compartment No. 1	0.39	0.30	0.26	0.34	0.32	79.9
Compartment No. 2	0.17	0.14	0.12	0.20	0.16	90.5
Compartment No. 3	0.14	0.07	0.10	0.16	0.12	93.3
Compartment No. 4	0.12	0.08	0.11	0.13	0.11	93.2
Total(*b*)	0.16	0.15	0.16	0.17	0.16	....
Concentrate, combined	25.72	33.41	24.59	29.44	28.29	....
Tons per 24 hr	....	....	....	....	123	....

a As sulphide. *b* Automatic sample. Compartment samples were cut by hand under considerable difficulty.

standard Callow cell in the Miami mill over 80 per cent. of the total recovery was made in the first 16 in. of length.

Table 14 shows assays of individual concentrates from the various compartments of a Miami-type cell. This is typical of the multi-compartment cells and also of discharges from corresponding positions along the side of the standard Callow and other trough-type machines.

Table 14. Assays of concentrate from different compartments of a 4-compartment pneumatic cell at Miami Copper Co.

Compartment number	Assays, per cent. Cu		
	May, 1920	Oct., 1920	Feb., 1921
1	34.48	37.20	35.96
2	18.90	11.60	20.47
3	11.77	4.90	11.87
4	11.01	4.30	9.35
Total	27.21	23.55	23.00

Patents issued to Callow covering various forms of porous-bottom bubble-column apparatus are: (1,104,755/1914). A circular flat-bottomed tank with slow-speed stirrer to keep coarse sand in suspension. (1,124,853/1915) The same with a high-speed stirrer, giving a combination of pneumatic bubble-column and agitation-froth flotation. (1,124,855 and 1,124,856/1915) A circular tank with slow-speed rotating porous arms. (1,141,377/1915) A circular tank with horizontal porous bottom, no stirrer, and perforated grids to lessen

pulp movement and disturbance of the bubble column. (1,176,428/1916) A rectangular tank with sloping compartmented bottom and a vacuum chamber above for removing and disintegrating froth. It is not likely that any substantial vacuum-type pulp-body concentration is effected in this cell, although it is not impossible. (1,182,748/1916) A rectangular box with horizontal porous bottom and a mechanical scraper for moving sands from feed to tailing ends. (1,201,934/1916) Substantially the modern form of cell as described above, but without the automatic tailing-discharge valve. (1,329,335/1920) Substantially the modern form with inclined bottom but with the tank compartmented with vertical transverse baffles extending from above the overflow lip downward to within a few inches of the bottom, in order to prevent sand deposition by creating a scouring action. (1,366,766/1921) A rectangular tank with horizontal porous bottom, adjustable transverse baffles to prevent sedimentation, and a weir overflow. (1,366,767/1921) In an apparatus similar to a Dorr thickener, a porous bottom, a peripheral feed into an annular space behind a cylindrical baffle extending nearly to the bottom, a float-controlled tailing-discharge valve, and revolving suction pipes for removing froth.

MacDonald (1,134,690/1915). In a circular tank, a plurality of air-lifts disposed near the periphery and discharging tangentially a short distance below the pulp surface, thus setting up a circular motion of the pulp; a skimmer or skimmers projecting from the side wall toward the center of the tank for leading froth to the periphery; a central cylinder extending above and below the pulp surface to confine the bubble column to an annular space; submerged feed of pulp; oil fed with compressed air in the air-lifts.

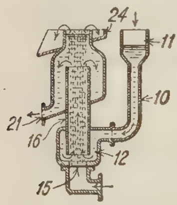


FIG. 33. — Wagner pneumatic machine.

Crerar (1,232,772/1917). An inclined porous bottom, the lower end only submerged. Pulp fed at the upper end, overflow of froth at the lower end. A porous surface similar to a 2-deck circular buddle with upper deck convex and lower concave is illustrated. The principle of this machine is correct in that no pulp body is necessary in a pneumatic machine and elimination of pulp body results in saving of power.

Wagner (1,235,083/1917, Fig. 33). A porous-bottom machine in which the flow of pulp in the separating chamber is upward with the rising air bubbes. This is interesting in that it recognizes the necessity of bringing all of the pulp into the bubble column, which is essential to recovery of mineral in bubble-column processes. In the figure, feed enters hopper (11) and passes through pipe (10) into chamber (12), thence upward through pipe (16) with air introduced through porous bottom (15). Tailing that falls out of the bubble column discharges through pipe (21). Concentrate overflows into launder (24).

Cole (1,243,814/1917; 1,375,211/1921, Fig. 34). The essential feature is the use of a grid of porous pipes (22) placed some distance above the bottom in the separating tanks (14), (15), (16) in order to prevent clogging of the air inlet passages by sedimentation of coarse sand. In operation feed enters at (19), meets the rising air bubbles and is lifted into the bubble column. Tailing from the bubble column passes downward through the grid of porous pipes, is roughly classified at (34), coarse sand being discharged from spigot (36) and fine sand and slime passing through pipe (23) to a second similar unit, etc. Hand wheels (28), controlling valves, regulate the flow and the pulp level. See also Flinn, (1,314,-316/1919.)

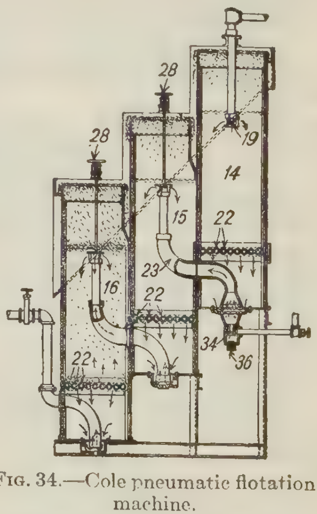


FIG. 34.—Cole pneumatic flotation machine.

At CANANEA CONSOLIDATED COPPER Co. four roughers and four cleaners treated 800 tons per 24 hr. of −28-mesh feed with a consumption of 132 hp. Air pressure was 5 lb. per sq. in. Tubes were covered with silencer cloth, which lasted 30 to 45 days. This apparatus has been tried out thoroughly at INSPIRATION (*55 A 576*) and ARIZONA COPPER Co. (*55 A 656*), but was not adopted at Inspiration and has been replaced by Callow cells at Arizona Copper Co. (*66 A 724*).

Peterson cell used at TIDEWATER COPPER Co. (*122 P 423*) is of this type. Five roughers and one cleaner required 2.8 hp.-hr. at the blower per ton of flotation feed.

Greenawalt (1,250,303/1917) similarly guards against interference with air introduction

due to clogging by the use of a grid of porous pipes above the bottom of the separating tank, but goes a step further in suspending the pipes and moving them slowly backward and forward by means of a suitable mechanism

Armstrong (1,269,150/1918). Restriction of froth overflow in pneumatic or other cells, for the purpose of deepening the layer of froth and thus obtaining cleaner concentrate, is described. The patent illustration shows a standard Callow cell with sides built up except for a short distance near the discharge end so that all froth must travel to this point to overflow. The effect is to increase the time for drainage of gangue from the concentrate in the column of bubbles. As this draining phenomenon does not occur, or occurs to but the slightest extent, in properly operating pulp-body machines, the method is applicable only to bubble-column processes. The method has been used in the mills. Overflow has been placed at the discharge end and counter flow of the bubble-column to side overflows arranged near the feed end, as well as to the tailing end, has been tried.

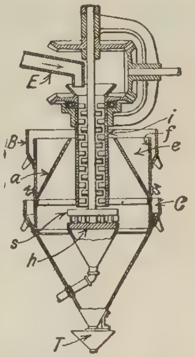


Fig. 35.—Rowand pneumatic flotation machine.

Rowand (1,312,754/1919, Fig. 35). Oiled feed pulp enters through pipe (*E*) and mixing chamber (*i*) directly onto the porous bottom (*h*) which is kept from sanding by scraper (*s*). A bubble column with constricted surface is formed in conical frustum (*a*) and overflows the upper rim of this frustum onto a body of water maintained in (*e*). Middling drains out as the bubbles pass over to the lip (*f*) and are discharged through suitable openings into launder (*C*). Concentrate overflows into launder (*B*); tailing is discharged at (*T*). The patent mentions the use of heat, which is both unusual and an unnecessary expense in bubble-column operation.

Wilson (1,341,770/1920, Fig. 36). Feed entering through pipe (27) is mixed with oil from tank (31) in the closed-top Pachuca tank (*F*), and passes into tank (*A*) where it is subjected to the action of air bubbles introduced through the porous medium (24) and rising in the tortuous passage (*B*). Material dropped in the passage of the mass of bubbles toward the overflow lip (9) returns by the passages (C) to be treated again. Concentrate overflows into launder (10) and tailing discharges through pipe (13). Enrichment of concentrate by draining during a passage over non-aerated pulp, as described in this and the preceding patent, is not so good as a second "cleaning" operation, in a bubble column maintained over aerated pulp, as is the usual practice.

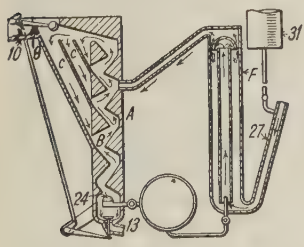


Fig. 36.—Wilson pneumatic machine.

Inspiration pneumatic cell is described in patents to Gahl (1,346,817 and 1,346,818/1920, and 1,401,598/1921).

One form used in the mills is shown in Fig. 37. It consists essentially of a launder about 3 ft. wide and 4 ft. 6 in. deep, with a slope of about ½ in. per ft. It is provided with a removable segmented porous bottom and is divided into compartments by partitions spaced about 3 ft. center to center along its length. The usual number of compartments in a roughing machine ranges from 15 to 20. The partitions have slots nearly 3 ft. wide and 6 to

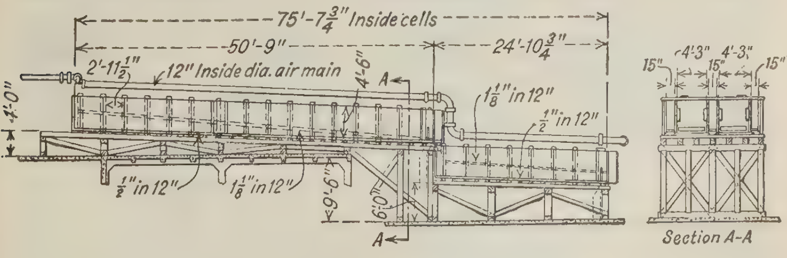
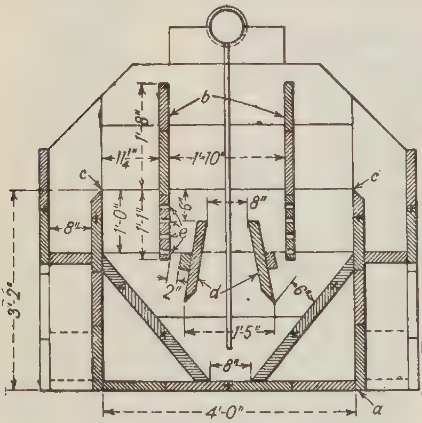


Fig. 37.—Inspiration pneumatic cell.

8 in. high, cut about 3 in. above the bottom of the launder. The size of the opening between compartments is regulated by gates operated by a threaded rod from a hand wheel and lug supported on timbers placed across the top of the launder. Air baskets, consisting of shallow boxes of such dimensions that they make a loose fit in a compartment and with the air-supply pipe coming down from the top, are placed in the bottom of the launder. In general, two such launders set side-by-side, with a common central froth launder, constitute a roughing unit. The rougher froth is cleaned in a similar smaller machine, fed by gravity from the roughing machine. The capacity of such a double unit with 16 roughing compartments each side and six cleaning compartments each side, is from 600 to 1200 tons per 24 hr., the lower figure on a slimy low-grade copper ore in a pulp containing 12 to 15 per cent. solids, the higher figure on a silicious, rather sandy low-grade copper ore in a pulp containing 20 to 25 per cent. solids. These figures reduce to 1.2 to 2.4 tons per 24 hr. per sq. ft. of blanket area.

At INSPIRATION, with 48-mesh pulp containing about 15 to 20 per cent. primary slime, 1.6 tons tons per sq. ft. is the regular feed rate and 3.2 tons has been treated. Power consumption in Aug., 1916 (*102 J 679*) was 2.8 kw.-hr. per ton. At AJO TEST MILL (*109 J 1314*) the feed rate was 1.88 tons per sq. ft. of rougher surface (2.07 tons per sq. ft. including cleaner tailing) and 0.85 ton per sq. ft. of cleaner. Air consumption was from 10 to 12 cu. ft. per sq. ft. of blanket surface, at from 4 to 5 lb. pressure on the supply side of the regulating valves. This means a power consumption of from 0.3 to 0.35 hp. per sq. ft. of blanket. At the GOLD HUNTER mill (*100 J 1044*; *102 J 16*) air was supplied at 8 lb. per sq. in. and the power requirement per cell was 9.6 hp. This is exceptionally high, due in part to the high pressure, but probably also to an inefficient blower installation. At INSPIRATION one operator and two Mexican helpers to wash bottoms take care of four roughers and four cleaners.



Cross-section of rougher

FIG. 37*a*.—Forrester flotation machine.

Forrester machine (Fig. 37*a*) consists of a hopper-bottomed trough (*a*), compartmented longitudinally by partitions (*b*), which are perforated below the pulp level. Length varies from 14 to 24 ft. Feed enters at one end above the pulp level, tailing discharges over a weir at the other end, and froth overflows lips (*c*). Air is introduced through ¾-in. pipes on 4-in. centers depending from a 6-in. header. Longitudinal partitions (*d*), arranged as shown or similarly, serve to form the foot-piece for an air-lift to raise pulp from the bottom of the hopper to a level between walls (*b*) higher than that of the pulp outside and to maintain the pulp therein in such a state of turbulence that a part of the air is broken up into sufficiently small bubbles to remain in the pulp in its passage through the perforations (*e*) and to form an effective bubble column in the quiet zones on the outside of walls (*b*).

Performance. This cell has been remarkably successful. The metallurgical results on porphyry copper ores have been about the same as in Callow cells while power and labor costs have been much lower. At one plant comparative performances of an 18-ft. Forrester and a 20-ft. Callow cell were as follows, the figures for the Forrester being given first: Daily tonnage, 350, 270; air pressure, lb. per sq. in., 1.6, 4.0; air consumption, cu. ft. per min., 702, 534; per ton of feed, 2888, 2848; power consumption, kw.-hr. per cell, 131, 224; per ton, 0.37, 0.83; cost, cents per ton: power, 0.448, 1.005; operating labor, 0.498, 0.783; repair labor, 0.095, 0.213. The reason for the great difference in the last item is the elimination of blanket scrubbing in the Forrester cell; this, with the almost universal use of lime with the chemical collecting agents is an important item in all plants.

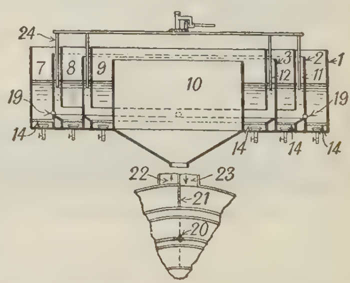


FIG. 38.—Myers pneumatic flotation machine.

Allingham (1,357,921/1920). A movable porous bottom suspended in a separating tank. See also Greenawalt (1,250,303/1917).

Myers (1,323,373/1919, Fig. 38). In a circular tank (1), a central cylindrical compartment (10) and a series of annular compartments (7), (8), (9), (11) and (12) formed by cylindrical walls of varying heights; and porous bottoms (14) arranged as shown. The outer annular compartment (7) is divided from top to bottom by partition (21). Oiled feed enters this compartment through inlet (23), is subjected to bubble-column action therein and divided into a rough concentrate that overflows the lip (2), and a tailing that is discharged at (22). The rough concentrate is re-treated in compartment (8) making a froth that overflows lip (3) and a middling that passes by check valves (19) back into compartment (7). The once-cleaned concentrate is re-treated in (9), overflowing finished froth into compartment (10) and discharging middling back into compartment (8) through check valves (20), spaced 90° to valves (19). Froth in the feed compartments of the various annular cells is broken down by revolving arms (24) or similarly driven sprays. Various types of porous media are described. One is a layer of fine lead shot, 0.5 to 2 in. deep, resting on canvas or muslin.

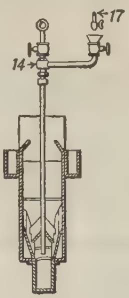


Fig. 39. — Waterhouse pneumatic flotation machine.

Waterhouse (1,346,286/1920). Fig. 39 is a cross-section of a sloping-bottom tank similar in shape to the standard Callow cell. (See Fig. 31.) Pulp is introduced into the tank at the shallow end and air is introduced with high-pressure water through pipes depending from injectors (14). Oil may be introduced at (17) and carried into the pulp with the aerated water. There is suggestion of both plus-pressure and cascade action in this description.

Terry (1,362,370/1920). A V-shaped tank with sides of a porous medium, feed troughs along the sides, concentrate overflow into a trough suspended along the center line, and tailing discharge at the bottom.

Riser (1,391,078/1921, Fig. 40). A covered rectangular tank (1) with sloping bottom (2), air mats (3) and sloping discharge end (4). Feed enters at (5), tailing discharges from the bottom by overflow of pipe (6). Froth formed by bubble-column action is forced to travel over unaerated pulp from a point over the lowest air basket to the end-discharge lip, thus allowing some drainage of gangue. (See remark on this practice under Wilson, 1,341,770.)

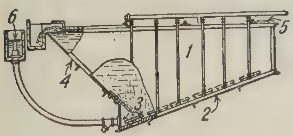


Fig. 40.—Riser pneumatic machine.

Borcherdt (1,440,129/1922). A long, flat-bottomed, compartmented pneumatic cell with weir overflow from compartment to compartment.

Conners (1,441,560/1923). A pneumatic cell with plunging feed behind a baffle and oil fed into the stream at the pulp level; also a horizontal screen just below the pulp surface, extending over the entire cell area.

Simpson (1,518,010/1924, Fig. 41). A spitzkasten with an air-lift to circulate pulp, a blanket placed above the cell bottom to prevent sanding, and a baffle to divide off a non-aerated pulp surface over which some draining of the bubble-column may be effected.

Dolbear (1,478,703/1923; 1,480,884/1924) describes a method and apparatus for feeding pulp on or into a bubble column, with the idea that selection will take place therein and the gangue will sink into the body of pulp beneath. The idea is attractive, but resolves itself into ordinary pneumatic operation for the reason that, while part of the mineral never reaches the pulp body, much does and is thereafter treated in the fashion usual in pneumatic machines. Failure to catch and hold the mineral is in part due to the fact that feeding tends to break down the bubble column at the feed point. The patent also describes methods for discharge from different depths in the bubble column by means of V-shaped troughs emerging through the sides of the cell, spaced so as to permit rise of bubbles between them, but designed to catch and discharge material dropped above them.

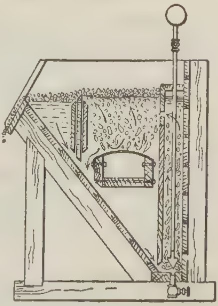


Fig. 41.—Simpson pneumatic cell.

Malmros (1,526,997/1925) describes a pneumatic cell with level bottom, flaring sides and longitudinal vertical baffle boards that register with the edges of the air basket but are so supported as to permit clearance to form pulp-circu-

lating passages between their lower edges and the cell bottom and of such height that the upper edge is below the bottom of the bubble column. In overflowing, the froth passes over a quiescent space between the upper edge of the baffles and the overflow lips and here additional draining takes place, which results in higher-grade concentrate.

Allen and Reid (1,547,548/1925) suggest a long, shallow, flat-bottom pneumatic cell with the side walls converging up to a height about three-quarters of the cell depth above the bottom, then flaring sharply to the overflow lips. The purpose is to provide an unaerated area over which accelerated draining of the bubble column may take place.

Mackintosh (1,608,896/1926) describes a slowly-rotating porous cylinder placed horizontally in the bottom of a V-shaped trough. Pulp is fed at one end of the trough, tailing discharges over a weir at the other and concentrate overflows the sides. The rotor has 2.3 sq. ft. of area per foot of length. At one plant eighteen 10-ft. (length) rougher cells and six 10-ft. cleaners (double cleaning) treat 3000 tons of copper ore per day. Air pressure varies from 1.5 to 3 lb. per sq. in. and quantity from 5 to 7 cu. ft. per min. per sq. ft. of porous bottom. Power-requirement is 0.6 to 0.8 kw.-hr. per ton. Blanket consumption is less than with standard pneumatic cells. Economies in power, blanket renewal and labor follow from the rotation of the porous medium which results in automatic removal of settled sand, thus decreasing air pressure and eliminating pounding of blankets. (*Data furnished by General Engineering Co.*)

Porous bottoms are usually made of three or four thicknesses of canvas or palma twill, stitched together with crossed seams spaced ½ to 2 in. Gahl (*55 A 576*) found that incrustation and loss of porosity were least near the center of the blankets where bellying and flexure were greatest and therefore concluded that rigid bottoms, such as carborundum stone, filtros, cement and the like would become clogged so quickly as to compare unfavorably with canvas. On the other hand the rigid bottoms are attractive in their promise of greater strength and longer life and much work has been done in attempting to develop them.

McCrae (*109 J 837*) describes the method used in making cement bottoms for Inspiration-type cells at Ray Consolidated Copper Co. In the early work the sides and bottom of the air basket were made of dense concrete poured in place in the cells and a porous top was then cast and cemented into place. Later practice was to make sides and bottom a sheet-iron pan, 7 in. deep, of such size as to fit readily into the 35½ × 51½-in. cell compartments. The pan was divided into three compartments and each of these into two sections. One-inch angle irons were riveted 3 in. from the top of the side walls and partitions, to serve as rests for the porous top blocks. These were made of sand-table tailing screened first through 18-mesh cloth held horizontally, then over the same screen at 45°. The concrete mixture was 5 parts sand to 1 part cement, mixed with only enough water to cause the cement to coat the sand and harden. The blocks were 3 × 10¼ × 22¾ in. reinforced with two ¼-in. rods 23 in. long and three 10½ in. long, wired securely at the crossings. The blocks were sprayed each day for 10 days, sparingly at first, to retard setting. These blocks were set into the pans with neat cement. Life at Ray was upward of 2½ years. The upper surface was scraped every 60 days. At Miami it was found that less scraping was necessary after the first time.

When canvas blankets are used it is necessary to tie them down at frequent intervals to prevent bellying, which would result in irregular distribution of air. Fig. 42 shows one method of reinforcing.

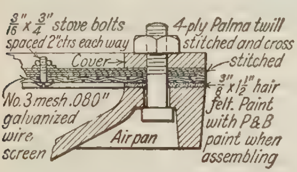


Fig. 42.—Air-basket joint and reinforcing.

Air pressure. The pressure required to force air through blankets is a function rather of incrustation and sand on the blanket than of the number of plies. In a test at Miami with 1-ply and 3-ply blankets working side-by-side it was found that as soon as the single blanket reached the normal incrusted and sanded condition (about seven days in the test quoted), the resistance to passage of air was substantially the same as that of the 3-ply blanket and that no power saving was, therefore, to be expected from the use of the thinner blanket. This reasoning is extensible to any porous medium. Multi-ply blankets have longer life in cells in which the bottoms must be pounded to prevent sanding, hence are generally used.

Useful life of canvas blankets is ordinarily between two and six months. It depends upon the amount of wear from scouring of the pulp and pounding to lessen clogging, upon the extent of chemical incrustation and upon how much dust is in the air furnished. Actual wear is normally not very great with find feed (65-mesh or smaller) and the life of blankets may be one or two years, but if there is considerable chemical incrustation which it is sought to relieve by pounding, or much coarse material, the blankets may last only a few weeks. Incrustation is usually removed at intervals by scrubbing with dilute hydrochloric acid. When lime is used to render pulp alkaline the blankets may need to be scrubbed every few days and wear will be high. Clogging by dust is lessened by filtering or even scrubbing the air entering the blower.

At Bunker Hill and Sullivan the use of quilted mats with rubber-impregnated canvas for the upper layer increased life from 3 months to 4 months. Increase was due to the fact that rusting (incrustation due to siderite) and felting with wood pulp were lessened (*106 J 257*). Life of cloth bottoms at Inspiration was about 6 months.

11. Cascade machines

Cascade machines effect aeration by causing a stream of water or pulp to plunge into a body of pulp. Air is entrained by pushing it into the body of pulp ahead of the masses or droplets into which the entering stream is broken. Any stream of pulp that falls through a distance greater than a few times its diameter breaks up into distinct masses, the breaking being greater the greater the velocity of the stream and the further its free fall. Each mass of entering pulp, if moving at sufficiently high velocity, pushes ahead of it a small volume of air in precisely the same way as an air bubble is pushed into water by a pebble thrown therein. The air thus introduced is broken up into smaller bubbles by reason of the swirl and movement caused by the entering stream and, rising to the surface of the pulp, forms thereon a column of bubbles, in which concentration takes place as previously described. The efficiency of the operation depends on the effectiveness of the apparatus in forming and maintaining an undisturbed, highly-aerated bubble column.

Ohrn (1,187,772/1916, Fig. 43). In a tank, a gas-and-pulp injector formed by hopper (*B*) and the steam or compressed-air pipe (*D*). Oil is fed with the air or steam through funnel (*F*). Gas introduced rises as bubbles (*L*) and forms at the surface a bubble column that overflows into launder (*J*) as concentrate. Tailing discharges at (*K*).

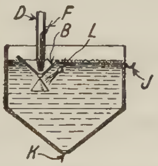


Fig. 43.—Ohrn pneumatic machine.

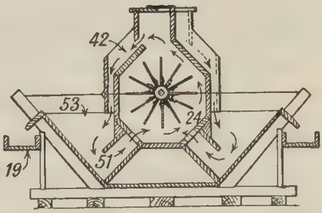


Fig. 44.—Rork cascade flotation machine.

Rork (1,136,485/1915, Fig. 44). Pulp with reagents is fed into chamber (24) and is lifted by the paddle wheel and thrown in the direction of the arrows through conduit (42) into spitzkasten (53), wherein a bubble column is formed that overflows into launder (19). Tailing follows the path of the arrows through conduit (51) back into the paddle-wheel chamber. Alternately pulp flows similarly through the spitzkasten on the other side. Feed enters at one end of chamber (24) and after passing through a plurality of spitzkasten is discharged from the last.

Arzinger (1,282,730/1918). A body of pulp in a trough is aerated by discharging water containing oil thereinto through small nozzles placed close to the pulp surface. High velocity of the jet is insured by high pressure in the water line. The patent reports 96 per cent. recovery on a graphite ore in the form of 80 to 88-per cent. graphite concentrate.

Blomfield (1,310,051/1919). By means of an air-lift placed in a tank containing oiled pulp, a part of the pulp is lifted above the general level and discharged against an umbrella which causes it to fall back into the body of pulp and thus entrain the air needed to set up bubble-column action therein.

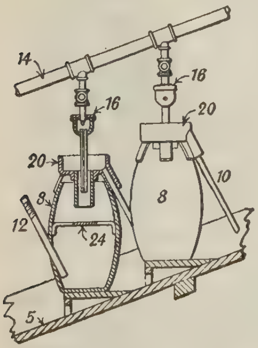


Fig. 45.—Emerson cascade flotation machine.

Emerson (1,311,882/1919, Fig. 45). A series of barrels (8) is set in an inclined trough (5) forming a concentrate launder. Oiled pulp enters the barrels by inclined pipes and (10) funnel-shaped feed boxes (20). Aeration is effected by means of water injectors (16) attached to a pressure pipe (14). Froth overflows the barrels and tailing passes

on through pipes (12) to succeeding barrels. Baffles (24) are placed in the barrels to prevent air from passing down into the tailing discharge.

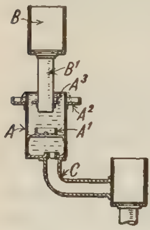


FIG. 46.—Seale and Shellshear cascade machine.

Seale and Shellshear (1,311,919 and 1,311,920/1919, Fig. 46). Oiled pulp falls from box (*B*) through pipe (*B*′) into a body of pulp maintained at the proper level in tank (*A*). Pipe (*B*′) is not filled with falling pulp, hence the stream breaks and the falling masses introduce air in the usual fashion. Baffle (*A*′) prevents passage of air into outlet pipe (*C*). Baffle (A^3) tends to prevent disturbance of the bubble column. Froth overflows into launder (A^2). Water is fed into (*B*) as necessary to maintain the proper pulp level in (*A*). The apparatus is operated with several pots in series with a pulp elevator in the flow where necessary. Another form is described in the second patent.

At JUNCTION NORTH MILL, Broken Hill, N.S.W. (*9 Min. & Eng. Rev. 296*) ten 16-in. (diam.) × 24-in. pots in series, five for lead flotation and five for zinc treated 22 tons per hr. Fairchild (*104 J 392*) reports failure of cascade machines on low-grade copper ore at RAY CONSOLIDATED, the apparent fault being lack of sufficient aeration.

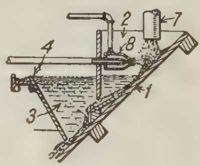


FIG. 47.—Ross cascade machine.

Ross (1,328,456/1920, Fig. 47). Oiled pulp is discharged through pipe (7) onto inclined apron (1) forming the back of feed-box (2). The falling stream is met by a stream of water from injector (8) and then passes through feed box (2) into spitzkasten (3), where a bubble column forms and overflows lip (4). Tailing is re-treated in subsequent similar boxes.

Appelquist and Tyden (1,367,223/1921). Oil, gasified or liquid, is fed with ore or pulp into a small hopper and there mixed and the mixture aerated by the cascade action of a jet of water. The oiled and partly aerated pulp discharges with more or less fall into a spitzkasten where flotation takes place by bubble-column action in the usual fashion.

Luckenback (1,397,815/1921) describes a machine similar to Seale and Shellshear (1,311,919; 1,311,920.)

Bonnell (1,399,539/1921, Fig. 48). Oiled feed pulp entering machine through pipe (17) is elevated by air-lift (24) to apron (21) and discharged onto the pulp in spitzkasten (12). Some mineral may float by skin flotation, the majority is raised and concentrated by bubble-

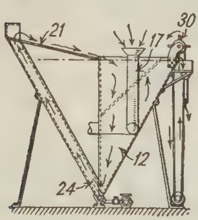


FIG. 48.—Bonnell cascade machine.

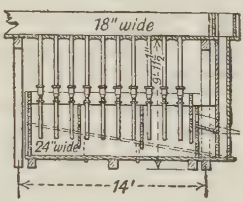


FIG. 49.—Donaldson cascade cell.

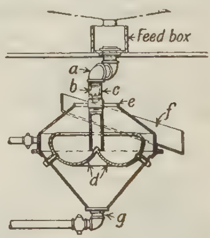


FIG. 50.—Court cascade machine.

column action in the spitzkasten and overflows or is scraped off by scraper (30). Tailing enters an adjoining spitzkasten for similar treatment. Some, but very little, of the air introduced in the air-lift remains in the pulp through its passage down the apron.

Donaldson cell (Fig. 49) was used for a time as a scavenger at the ARIZONA COPPER CO. mill (*103 J 665*). During 1915–1916 three boxes in series recovered about 1800 lb. of copper per 24 hr. in an 11 per cent. concentrate.

Court (1,470,350/1923) describes a cascade apparatus (Fig. 50) consisting of a double-conical shell containing a deflecting plate (*d*), fed by a nozzle (*b*) within an enclosing pipe,

the latter projecting below the pulp level in the machine. The bubble column is crowded by the upper walls of the cell and by a crowding plate and overflows into launder (*e*). An equalizing pipe permits regulation of pulp level by by-pass to the succeeding unit. Tailing discharges by gravity from the apex of the lower cone. The inventor recommends a number of units in series.

Experiments at MIAMI COPPER Co. showed that the pipe enclosing the nozzle had no other function than to prevent mechanical destruction of the bubble column by splash of the entering stream. This and similar devices have been tested at several mills, *e.g.*, OLD DOMINION, BRADEN, MIAMI, and have been found of no particular worth except to scavenge a minute amount of mineral from tailing.

Christensen (1,521,277/1924) describes a horizontal revolving cylinder whose lower surface just touches the upper surface of the pulp in a chamber that connects by suitable conduits with the bottom of two spitzkasten set back-to-back. Revolution of the cylinder throws pulp against a curved hood whence it falls into the spitzkasten. This is simple cascade action.

Hynes (1,394,306/1921, Fig. 51). A plurality of impellers (36) formed of perforated disks, set close together on a horizontal shaft and revolving somewhat less than half submerged. Discharge of agitated and aerated pulp through slot (19) into spitzkasten (17). The machines used at the ROSEBERRY MILL, Sandon, B. C. (*114 J 679*) had 35, 50 and 100 disks. The tank of the 50-disk machine was 6 ft. long, 5½ ft. wide and 4 ft. deep. Disks were 30 in. diameter, of 12-gage steel, with ½-in. holes on ⅞-in. centers and were placed 1¼ in. apart. Speed was 90 r.p.m. Performance at this mill was superior to that of the M.S. standard machine. There may be some agitation-froth action in this machine, in addition to the cascade action, but the speed is relatively slow and the appearance in the spitzkasten is distinctly that of a bubble-column machine.

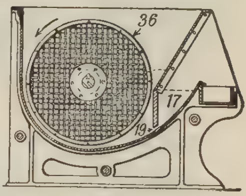


FIG. 51.—Hynes flotation machine.

12. Centrifugal bubble-column machines

These machines are sometimes known as SUB-AERATION MACHINES. They introduce air into the pulp by reason of the centrifugal force induced by a mechanism rapidly revolving in the pulp. Some of these machines have enjoyed considerable use in the mills, although such use has been small by comparison with the agitation-froth and pneumatic machines. Various forms of the apparatus are listed below.

Higgins and Stenning (1,155,815/1915, Fig. 52). Oiled pulp is fed through slot (*J*) into compartment (*A*) where it is aerated by air drawn in through pipe (*E*) by reason of the centrifugal pumping action of impeller (*B*). Aerated pulp passes up and under baffle (*L*) into the upper part of chamber (*A*) where typical bubble-column action is set up in and above a body of pulp protected by the inclined partition from the agitation caused by the impeller beneath. Froth overflows lip (*N*) while tailing passes on through a valve-controlled slot (*F*) for further similar treatment. The machine is inefficient on its face, by reason of the obstruction offered to the rise of air bubbles after introduction and has had little or no mill use. As in some of the other centrifugal machines later described, there is a small amount of pulp-body concentration (agitation-froth type) in this machine, the proportion with regard to the whole action increasing as the amount of air inflowing through pipe (*E*) is decreased.

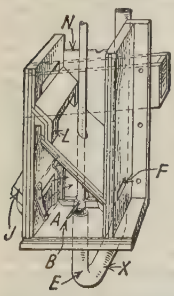


FIG. 52.—Higgins and Stenning sub-aeration machine.

Higgins (1,155,816/1915). Other forms of apparatus similar to that just described.

Owen (1,155,836/1915, Fig. 53) describes a practical form of apparatus consisting of a circular tank (*J*) with radial baffles (*E'*) to prevent vortical action of pulp therein, a pump impeller (*D*), feed inlet (*T*), tailing outlet (*M*), and constricted concentrate overflow rim (*P*). Air, pumped in through pipe (*N*) and regulating valve (*O*) by the action of impeller (*D*) is dispersed through the pulp by the action of the impeller and, rising through the pulp, forms a bubble column at the top, in which concentration takes place.

Fagergren and Green (1,195,453/1916, Fig. 54). A square pyramidal box (6) contains an inner box (17), perforated plates (30), (31) and (32) and a cross-shaped impeller (26) on a vertical spindle (29). Pulp enters through pipe (19). Air, introduced through pipe (20) with the feed pulp, is dispersed therethrough by the impeller and rises to form a bubble column at the top, causing concentrate to overflow into launder (9), as indicated by the arrows. Tailing, falling, is deflected over the lip (18) of box (17) and discharged through pipe (23). The perforated plates (30,) (31) and (32) prevent the turbulence set up by the impeller from reaching and disturbing the bubble column.

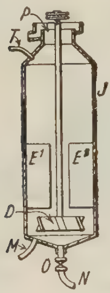


Fig. 53.—Owen sub-aeration machine.

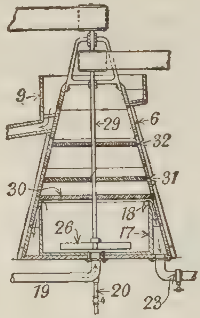


Fig. 54.—Fagergren and Green sub-aeration machine.

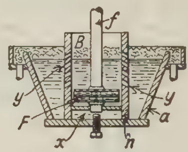


Fig. 55.—Groch sub-aeration machine.

Groch (1,276,753/1918 and 1,413,723/1922, Fig. 55). Air, drawn down the hollow impeller shaft (*f*) by reason of the rotation of the specially designed impeller (*F*) at the bottom, is dispersed through pulp in the box (*B*). Aerated pulp flows by gravity through the ports (*y*) into the spitzkasteñ (*a*), in which separation takes place by bubble-column action. Ports (*n*) allow circulation of pulp from the spitzkasten back, by means of the pumping action of the impeller, to the aerating chamber of the same compartment, and other ports (not shown) pass pulp from the spitzkasten into a chamber corresponding to (*X*) below the impeller in the following compartment.

Groch and Simpson (*21 CMI 145*) describe a commercial machine 14 ft. long × 5 ft. wide containing 6 impellers, having a capacity of 35 to 75 tons per day and consuming 7.5 hp. They state that at 200 r p.m. the vacuum in the hollow shaft becomes measurable, at 450 r.p.m. it is equivalent to 2.75 in. of mercury and at 750 r.p.m. to 6 in.

Ruth machine (1,445,042, 1,463,405 1923, Fig. 56) is similar to the Groch apparatus. It consists of a box (*a*) divided by partition (*b*) into an aerating compartment (*c*) and a spitzkasten (*d*). The aerating compartment is fitted with a grid (*g*) which prevents the creation of a vortex in the upper part of the chamber. A hollow vertical shaft (*e*), open at the upper end, extends into the aerating compartment and carries at its lower end a disk (*f*) for circulating pulp and introducing air. The revolution of the disk in the direction shown by the arrow produces a vacuum behind the shields (*Y*) over the air passages and air passes in through the hollow shaft to fill these spaces. The rotation of the disk also causes pulp to be drawn up through the passages (*X*) from the chamber (*h*) which opens into the lower part of the spitzkasten. Separation takes place in the bubble column above the compartments (*c*) and (*d*) and concentrate in the form of froth is overflowed at the lip (*i*).

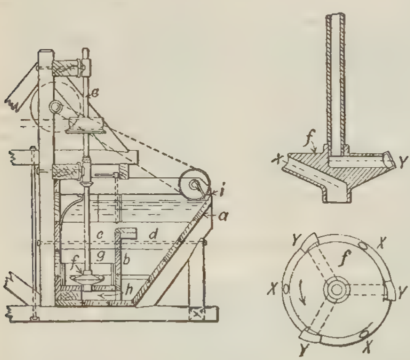


Fig. 56.—Ruth machine.

In the standard machine the disk is made 14 in. diameter and the hollow shaft is 1.1 in. inside diameter. The machine is driven at from 270 to 300 r.p.m. A capacity of 150 to

200 tons per day with a power input of about 1 hp. per spindle is claimed for an 8-cell machine. It is also claimed that 20-mesh material can be treated. These claims seem decidedly optimistic and probably a considerably lower tonnage of pulp ground to the usual flotation size (65-mesh) must be handled in order to get good results. At ZINC CORPORATION MINE, Broken Hill, Australia (*118 P 88*) impellers 10 in. diameter × 2 in. deep at 700 r.p.m. required 2.1 hp. per spindle.

Kraut (1,322,909/1919, Fig. 57). Pulp fed in at (*a*) is pumped by the revolving slotted cylinder (*b*) through the ports (*c*) into the shallow separating chamber (*d*) and there maintained in its flow toward the tailing-discharge box (*e*) by the rotating cylinder. The revolution of the cylinder also causes air to be drawn in at the ends thereof (*f*) and to pass through the ports (*g*) into the pulp and therewith into the separating chamber. Herein the air forms a bubble column with the oiled pulp causing flow of froth concentrate over the lips (*h*).

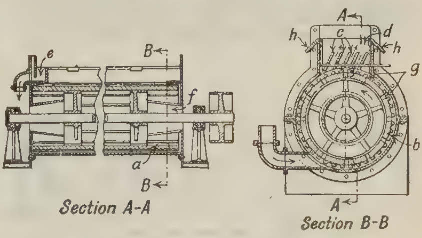


FIG. 57.—Kraut sub-aeration machine.

Hebbard sub-aeration machine is shown in Fig. 58. It consists of a trough (*A*) partially sub-divided into compartments by partitions (*t*), in which compartments are rotated vertical spindles (*a*) carrying at their lower ends disks (*b*) with radial arms on the lower face. The machine pictured is known as a 24-in. machine, so denominated by the diameter of the disks. The trough (*A*) is 3 ft. wide, 24 ft. long, and 5 ft. deep, allowing 3 × 3 ft. cross-sectional area for each 24-in. disk. Feed is introduced into the machine under the first disk, through a feed pipe (*e*) from a pressure box (*d*), or it may be introduced by means of an ordinary feed box through a slot in the end wall. Air under from 2 to 5 lb. pressure per sq. in. is supplied through the pipes (*g*) directly under each disk except the first. In machines fed through the end of the trough, air is also supplied under the first disk. Froth overflows the sides of the trough. Tailing is discharged through the slot (*l*) into the box (*m*). Pulp level is regulated by means of the valve in the discharge pipe (*o*).

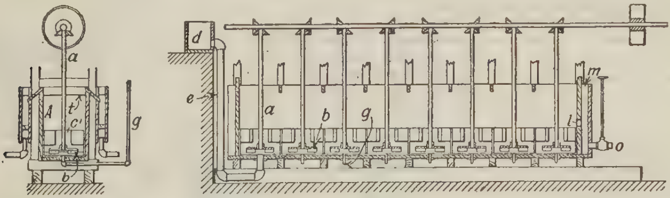


FIG. 58.—Hebbard machine.

The capacity of a 24-in. machine is about 80 tons of sandy feed per compartment per 24 hr. in a pulp containing 20 to 25 per cent. solids, with a power consumption of 10 hp. per spindle. An 18-in. machine treats about 40 tons per cell with a power consumption of 6.5 to 7 hp. per spindle. At INSPIRATION (*102 J 619*) the Hebbard machine drew 4.9 kw.-hr. per ton treated, including both spindles and compressed air furnished below the spindles. The air consumption is about 0.5 cu. ft. per min. per ton of daily capacity at a pressure of 5 lb. per sq. in. The machine has been almost uniformly unsuccessful in the mills.

13. Combination bubble-column machines

Many of the bubble-column flotation machines employ two or more of the methods listed for introducing the air necessary for bubble-column formation.

K. and K. machine (Kohlberg and Kraut, 1,174,737/1916, Fig. 59) utilizes the principles of both the centrifugal-type and the cascade-type bubble-column machines. The essential parts are an aerating compartment (*1*) and a froth-separating compartment (*3*). Aeration is accomplished by rapid revolution of the cylinder (*17*), which is about 30 in. diameter and 9 ft. long.

Pulp is introduced at one end of the aerating compartment at a point somewhat above the shaft (*16*) and discharges through a pipe at the other end of the settling compartment. The pulp level is maintained well below the center of the shaft in the aerating compartment. Circulation of pulp within the machine is accomplished through the ports (*11*). Aeration is accomplished differently according to which of the cylinder surfaces (*A*), (*B*), (*C*), (*D*) is used. When (*B*) or (*C*) is used, air is carried into the pulp on the down-coming side in the spaces between the cleats. This is essentially the action of the plunging stream in the cascade-type machine. When type (*D*) is used there is, in addition to air thus carried into the pulp, a creation of vacua in the ports (*21*) into which air passes from the center of the cylinder. A similar combination of phenomena occurs in the use of (*A*). The direction of rotation is as indicated by the arrow. Partially aerated pulp is thrown through the port (*7*) onto the body of pulp in (*3*) and flows under hood (*8*) and over the baffle (*9*) thus introducing air into the pulp in (*3*). Froth overflows the lip of compartment (*3*). Air is allowed to enter through pipes (*23*) and water can be added through pipes (*26*).

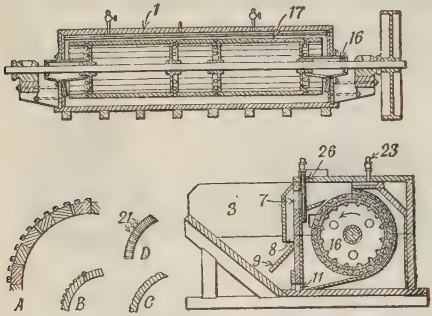


Fig. 59.—K. and K. machine.

The K. and K. machine is ordinarily driven at from 160 to 200 r.p.m. The capacity varies from 30 to 100 tons per 24 hr. with a power consumption of from 10 to 15 hp., according to the volume of pulp passed. Hardwood impeller slats last 6 weeks to 3 months. Watt (*57 A 376*) gives the following data concerning K. and K. machines in SOUTH-EAST MISSOURI. Cylinder housing, 30 in. diameter by 10 ft. long; cylinder shaft 3 7/16 in. diameter; drum lagged with 16 strips spaced 12 in. edge to edge, each strip carrying four hardwood riffles; minimum clearance between riffles and housing, 5/8 in.; speed, 180 to 200 r.p.m. Floor space required is 4 × 14 ft. and headroom 3 ft. Capacity, 50 to 80 tons per 24 hr. Power consumption, 10 to 15 hp. Weight, wood type, 2500 lb.; steel, 3500 lb.

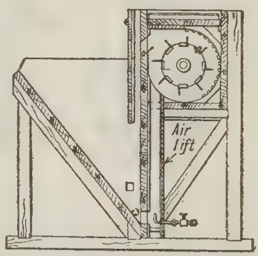


Fig. 60.—Ziegler machine.

Ziegler (1,324,139/1919). The usual form is shown in Fig. 60. It is similar to the K. and K. machine except that the rotating cylinder is placed higher with respect to the spitzkasten and air-lifts are used to aid pulp circulation. Cylinder speed is about 175 r.p.m. About 30 to 50 cu. ft. of air per min. is required for the air-lifts in a 5-cell machine and 5 to 6 hp. to drive the cylinder. Capacity on ordinary pulps is 75 to 100 tons per 24 hr. (*105 J 707*).

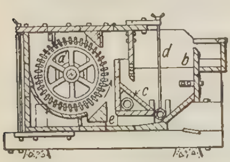


Fig. 61.—Rork and Sandberg machine.

Rork and Sandberg machine (1,454,656/1923, Fig. 61) is a combination of the pneumatic and cascade types. The essential parts are a long slatted cylinder (*a*), about 30 in. diameter by 9 ft. long, revolving in the direction indicated by the arrow, a transversely compartmented spitzkasten (*b*), provided with air baskets (*c*) and valves (*d*) for regulating circulation of pulp.

Feed is introduced at one end of chamber (*e*), lifted by the paddle wheel, and thrown into the first spitzkasten, thus aerating the pulp therein by cascade action. Further aeration is effected by air introduced through the porous medium (*c*) and a bubble column is built up and overflows. Tailing from the first spitzkasten passes back into chamber (*e*) and is thrown into the adjoining spitzkasten and thence in similar fashion until it discharges from the bottom of the last spitzkasten. Three to five is the usual number of compartments.

A modification of this type of machine has been used in various plants of the Phelps-Dodge Corp. At Morenci a single-cell machine treated 35 to 45 tons per day and a 3-cell machine 60 to 90 tons with a power consumption of 13.5 hp. for the single cell and 25 hp. for the 3-cell machine. The pulp contained 20 per cent. solids. Paddle speeds were 160 and 200 r.p.m. respectively. Tailing and concentrate on the same feed were substantially the same on both types of cell. At Burro Mountain the roughing cells were 3- and 5-compartment and the cleaners 2-compartment. The smaller roughers handled 85 tons per 24 hr. and the larger 130 tons, in pulps containing 18 to 20 per cent. solids. Power consumption averaged 27 hp. per cell at 200 r.p.m. Life of hardwood paddles was 60 to 100 days on feed containing 4 per cent. +48-mesh.

Parker (1,492,933/1924) describes a combination centrifugal and plunging-stream bubble-column machine, similar to the K. and K. except that the settling compartment is not divided off from the cylinder.

This machine was used at Bunker Hill and Sullivan. Three machines in series treated 75 tons per 24 hr. of 7.0 per cent. lead ore, 95 per cent. −200-mesh, in a pulp containing 33 per cent. solids. The oil mixture was 80 per cent. Barrett No. 4 and 20 per cent. steam-distilled pine oil; total added, 0.28-lb. per ton. Each machine drew 7 hp. at 78 r.p.m. Concentrate from the three machines, combined, assayed 60 per cent. Pb. Tailing went to a Callow cell which made a 1.5-per cent. tailing and 12-per cent. overflow, which was returned to the head of the Parker cells.

Dunn (1,219,089/1917, Fig. 62). Pneumatic and cascade. Pulp introduced into spitzkasten (*A*) is projected with gas (air) and oil into chamber (*B*) by means of injectors (17) until the pulp level in (*B*) builds up sufficiently to overflow (15) and fall back onto the surface of the pulp in the spitzkasten, when a bubble column builds up in (*A*) and overflows as concentrate at (*K*). Tailing discharge is at the opposite end of the spitzkasten from the feed inlet.

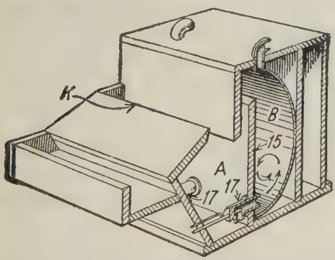


Fig. 62.—Dunn flotation machine.

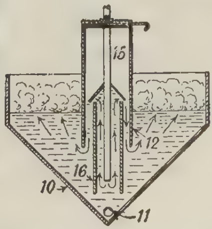


Fig. 63.—Welsch flotation machine.

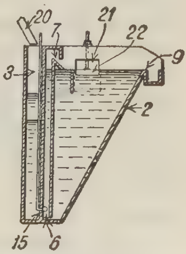


Fig. 64.—Brown flotation machine.

Welsch (1,253,653/1918, Fig. 63). In a spitzkasten (10) one or more air-lifts (16) discharging in a closed pulp-sealed chamber (15). Pulp in the spitzkasten is aerated by the escape of air under the walls (12) and also by the air introduced by the cascade action of the air-lift discharge, resulting in the formation of a bubble-column and an overflow of concentrate. Feed enters by a pipe (11) at one end of the spitzkasten and tailing discharges by a similarly placed pipe at the other end.

Terry (1,254,173/1918). This is a patent for differential flotation (see p. 877) but it incidentally discloses an operation in which the flotation reagents are mixed with pulp in closed agitation chambers so fed that no gas is present during the agitation, then discharged into an air-lift and thence with a free fall onto the surface of pulp in a spitzkasten.

Brown (1,351,155/1920, Fig. 64). In a spitzkasten (2) an air-lift (3), (15), (6), (7) discharging onto the surface of pulp in the spitzkasten. Pulp is fed through pipe (20) in the

down-going leg of the air-lift, aerated in the uprising leg (5) and further aerated in the fall into the spitzkasten. The bubble column overflows lip (9) and tailing goes over a weir (21), (22) into an adjoining feed compartment or to waste. There is also circulation of pulp within each compartment by a connection between the spitzkasten and corresponding air-lift.

Otsuka (1,393,821/1921). A modification of the K. and K. machine in which pre-mixing chambers are provided and air under positive blower pressure is introduced into the main aerating chamber.

Robbins (1,398,394/1921). In a Dorr thickener or similar apparatus, a grid of porous pipes suspended above the rakes for gas (air) introduction, and an air-lift or similar device centrally located for lifting settled solids to revolving distributing arms above the pulp surface. Aeration is both pneumatic and by cascade action of the circulating pulp. The bubble column overflows peripherally and tailing is disposed of by the central bottom discharge usual in Dorr thickeners.

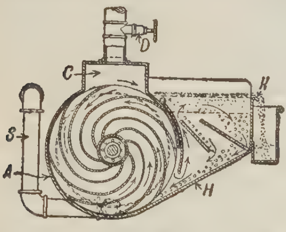


FIG. 65. — Gross, Akins and Bucher flotation machine.

Gross, Akins and Bucher (1,401,535/1921, Fig. 65). Pulp introduced through (*S*) into the bottom of chamber (*A*) is caught up by the spiral sand pump which, on further revolution, engulfs air from chamber (*C*). The mixture of air and pulp discharges from the spiral pump centrally and laterally into a squirrel cage surrounded by screen cloth, and thence is thrown out through the screen into the spitzkasten (*H*) and thus introduces more air by cascade action. There may be, also, some agitation-froth action by reason of the revolving parts in the body of pulp, but this is small on account of the relatively low speed. The bubble column formed in the spitzkasten overflows lip (*K*), middling is circulated, as indicated by the arrows in the spitzkasten, tailing discharges through a pipe at the opposite end from the pulp-entry. Some regulation of the extent of aeration is afforded by valve (*D*).

14. Atomizing

Atomizing processes differ from ordinary bubble-column operations only in that the oil is introduced in the form of a gas or vapor or a very fine spray with the air. The only essential difference between these processes and those in which liquid oil is added to the pulp is that, in atomizing, the oil, when introduced, is already finely divided, and dispersion throughout the pulp is, therefore, more rapid than in the case of liquid addition. Most of the patents state that this method of oiling lessens the amount of oil required, but this has not been the case in practice. All of the air introduced may be oiled, or only a part, in which latter case the oiled air is introduced first, as through the first compartments of the air basket in a pneumatic cell. Atomizing is first disclosed in patent 793,808 (see p. 808). Subsequent patents are described below.

Gröndal (1,202,512/1916) describes " . . . distributing the oil in a streaming elastic fluid under pressure such as steam or air and afterward mixing the steam or air under pressure with its contents of oil with a large quantity of an elastic fluid such as air or any other gas and then pressing the mixture into water holding the ore suspended."

Dunn (1,219,089/1917, see p. 823) describes also addition of oil by atomizing.

Schwarz (1,237,961/1917) describes the use for flotation in a bubble-column machine (since the use of an agitation-froth machine is practically debarred by the exigencies of the operation) of a gas other than air together with an oil; the whole apparatus being sealed from the atmosphere and the gas, with oil vapors contained, being re-used.

Scott (1,246,665/1917) describes " . . . using a volatile oil, impregnating the air with the gasified volatile oil, and introducing the gaseous mixture of air and oil into the pulp. . . ." In 1,508,478/1924 he describes the use of atomized oil in the agitation-froth process.

In 1,261,303/1918, Scott claims " . . . that it is not necessary to use any oils whatever in the [flotation] process, that is, it is not necessary to use any of the liquid substances heretofore employed in the flotation process, but that the concentration of the ore can be affected by the use of an admixture of air with a permanent gas . . . [*i.e.*], a substance that remains gaseous under ordinary conditions as opposed to vapors which condense at

ordinary temperatures . . . of the atmosphere and of the pulp. . . ." Acetylene is given as an example.

In 1,365,281/1921, Scott says: " . . . it has heretofore been proposed to dissolve a soap in the pulp and to decompose the soap by means of a mineral acid also mixed with the pulp, thus freeing the fatty acid contained in the soap. . . . I dissolve the soap in the pulp as formerly . . . but introduce the mineral acid in gaseous form in admixture with the air . . . the gaseous mineral acid within the bubble reacts with the soap in solution in the pulp . . . with the result that the fatty acid . . . is freed at the precise place where its properties are to be utilized, namely in the interfacial film." Any mineral acid that can be maintained in the gaseous condition while being introduced with the bubble-forming gas may be used, and under some conditions the mineral acid may be introduced in a finely-divided liquid or solid form with the gas. Any substance that is capable of reacting with a substance in the pulp to produce a third substance that will effect flotation, may be introduced with the gas.

In 1,369,045/1921, Scott describes the addition of acid with the bubble-forming gas to effect economy in acid consumption in the case of ores containing a constituent that reacts with and consumes acid.

Dosenbach (1,257,329/1918) states that "it is unnecessary that the modifying agent [oil] mixed with the air be in the gaseous or vaporized form, [referring to a preceding patent] but efficient results are obtained if the modifying agent in the liquid form is minutely subdivided, so . . . as to be carried with the stream of air in the form colloquially referred to as a 'fog' . . . the modifying agent is not injected into the pulp by means of an atomizer or injector, but a mixture of air with a modifying agent so finely divided as to remain suspended in the air is prepared in advance and a mixture of air and modifying agent so prepared is then . . . introduced into the ore pulp through a porous medium."

In patent 1,350,364/1920, Dosenbach describes the removal from an oil of the water-soluble and/or condensable portions and the introduction of the balance in gaseous form with air. In patent 1,354,031/1920, he describes apparatus for practicing atomizing and vaporizing methods in flotation operations.

Moffatt (1,400,308/1921) describes a method and illustrates apparatus for atomizing that was used on a large scale for treatment of porphyry copper ores. (Fig. 66.) The description applies to flotation in a pneumatic cell of the Inspiration type (see p. 813). The agent is any flotation oil such as oil, tar, creosote, etc. The porous mat or section of the cell floor through which the mixture of flotation agent and air is admitted is preferably at the feed end of the cell. Porous mats are preferably made of molded material such as porous cement made by bonding rock screenings with Portland cement, 6- to 8-mesh size being used for the more porous mats through which agent is to be introduced and 20-mesh size for the less porous mats for air alone. In the diagrammatic sketch *(28)* is a low-pressure blower, *(29)* an air heater, *(30)* a receiver with pipes *(32)* leading to the oiling compartments of the flotation cells. *(36)* is a tank containing flotation agent in liquid form, with oil-supply pipe *(45)*, and a pipe *(42)* for supplying high-pressure air from receiver *(41)* and compressor *(40)* for atomizing. *(38)* is an atomizer for dispersing the oil in the current of heated air in *(31')*. Condensed oil is drawn by pipe *(50)* into tank *(52)*. *(57)* is the unoiled-air main for supplying air to the later air baskets of the machine.

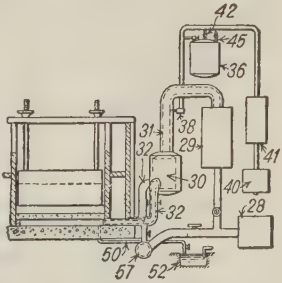


Fig. 66.—Moffatt apparatus for atomizing.

At Ray Consolidated Copper Co. 5000 to 6000 tons per day was treated in four 15-cell and ten 16-cell Inspiration-type rougher machines (3 × 4-ft. compartments) with oiled air introduced through the first six bottoms in each machine. The oil mixture consisted of one part of steam-distilled pine oil to 6 to 10 parts of coal-tar creosote, the total averaging 1.34 lb. per ton of ore. Rougher tailing was sent to waste. Rougher concentrate went to fourteen 9-compartment Inspiration-type cleaners, the last three compartments run counter-

current and discharging from cell No. 7. Cleaner concentrate was finished and cleaner tailing sent to waste. With a feed averaging 0.626 per cent. Cu, average tailing for one year's operation was 0.341 per cent., concentrate 16.93 per cent. and recovery 46.6 per cent. Oxidized copper in feed and tailing ranged from 0.25 to 0.40 per cent.

15. Combination pulp-body and bubble-column processes

A number of the processes described in flotation patents are combinations of one of the methods of pulp-body concentration, usually the agitation-froth, with one or more of the bubble-column methods. These combination processes sometimes give a better metallurgical result than can be achieved by either type of process alone, but the improvement is oftentimes made at an expense for additional power and equipment that results in a poorer commercial result.

Machines utilizing agitation-froth and pneumatic treatment are described in the following patents:

Callow (1,124,853/1915). See p. 811.

Clawson (1,240,824/1917). Pulp with reagents is fed into the feed hopper of a centrifugal pump which is fitted with an extension casing containing agitator blades mounted on an extension of the pump shaft. The pump discharges through an injector and thence through a zigzag pipe into a spitzkasten. Froth overflows and tailing discharges from the apex. The spitzkasten is closed over and the closed space connected by pipe with the injector, in order to re-use any gaseous frothing agent.

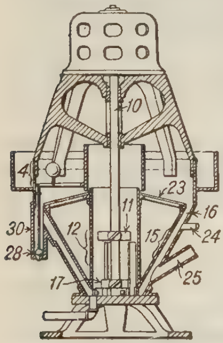


FIG. 67.—Jones-Belmont flotation machine.

Jones (1,326,453/1919, Fig. 67). Vertical spindle (10) carrying two cross-bladed, 45-degree impellers (11) and (17) is suspended in the cylindrical baffled agitating compartment (12), open at the top to the atmosphere and at bottom to the annular conical space (16). An annular sloping porous medium (23) on top of the air chamber formed by the walls (12) and (15) is supplied with air under pressure through pipe (24) and the air passes into the pulp through this medium. Feed enters by pipe (25) and passes into the agitation compartment. Agitated and aerated pulp is discharged from this compartment over the porous medium (23). Froth overflows rim (4). Tailing discharge through pipe (30) is controlled by the automatic ball valve (28). This is a combination of agitation-froth and bubble-column action similar to the Janney mechanical-air machine.

Janney mechanical-air machine (1,342,115/1920; 1,457,077/1923) utilizes pulp-body concentration by the agitation-froth method and bubble-column action by pneumatic and cascade means. The machine as installed in the mills is shown in Fig. 68. The combination of processes is effected by placing air baskets in the froth-separating compartments of a Janney mechanical cell and effecting therein further aeration of the pulp discharged from the agitating compartment, by reason of the cascade action of the incoming spitzkasten feed and by air introduced through the porous bottom. The agitating compartment is the same as that in the Janney mechanical machine, and has an individual vertical motor. The three-compartmented air baskets in each froth-separating compartment are supplied with air at from 4 to 5 lb. pressure on the supply side of the regulating valves. The machines are set up end-for-end as indicated in the drawing. The first machine in series is usually preceded by a Janney emulsifier Pulp passes from the emulsifier discharge box, which may be taken as represented by box (*a*) in the drawing, through a pipe by gravity into the agitator compartment of the machine and is thrown up over the top of the agitator compartment onto the air baskets. Froth overflows the lips of the air-basket compartments. The tailing is in part circulated

through the pipes (*b*) and finally passes through the adjustable overflow slots (*c*) into the tailing-discharge launders (*d*) and thence to the following machine. By thus utilizing both methods of froth concentration it is possible to make a good recovery in a relatively small number (5 or 6) of the machines in series.

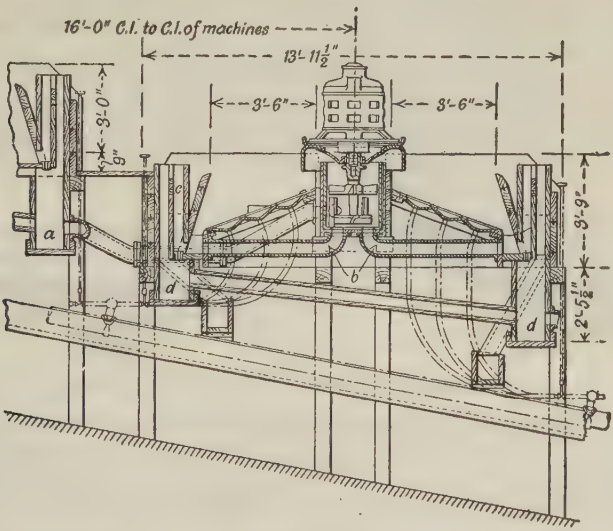


Fig. 68.—Janney mechanical-air machine.

The capacity of a 24-in., 5-compartment machine on silicious ore in a pulp containing 20 to 25 per cent. solids, is 150 to 200 tons per 24 hr. The power consumption per agitator is 6 to 7 hp. The air consumption per square foot of air-basket is from 5 to 10 cu. ft. of free air per min. at 4 to 5 lb. pressure, corresponding to an additional power of 5 to 8 hp. per machine.

The froth in this machine, as in others of its type, is not to be distinguished from a typical bubble-column froth from a cursory observation. It is probable that a large part of the increased metallurgical recovery is due to the fact that the buoyancy of the air-mineral agglomerates formed in the agitation chamber results in the presentation of a greater proportion of the mineral to the lower layer of the bubble column for treatment, but that the final effective concentration in the machine is due wholly to bubble-column action.

The flotation section of the Arthur plant of the Utah Copper Co. has 13 sections, each containing 55 of these machines, built, however, of concrete instead of wood and using vitrified tile instead of iron pipe for the pulp-circulating pipes.

Shimmin and Bushnell (1,402,099/1922). See p. 827.

Machines utililizing agitation-froth and cascade action are described in the following patents:

Janney (1,167,076/1916, see p. 802). In this machine, although the bulk of the flotation is of the agitation-froth type, nevertheless a considerable amount of air that is effective in making the froth buoyant is introduced by cascade action in the discharge of the pulp from the agitation chamber into the spitzkasten.

Piersol (1,335,600/1920, Fig. 69). Pulp with oil is introduced into chamber (5) and is agitated and aerated therein and then thrown through conduit (13) into the pulp in the spitzkasten. Pulp circulates from the spitzkasten back into the agitation compartment through conduit (18) and tailing discharges through pipe (32). The proportion of the recovery due to agitation-froth treatment will depend upon the speed of the impeller; if that is low, most of the recovery will be due to cascade action and *vice versa.*

Haley (1,357,556/1920). This is an apparatus consisting of a spitzkasten fed with oiled pulp pumped by means of a centrifugal pump with air admitted to the suction pipe through a nozzle discharging above the surface of the pulp in the spitzkasten.

Boggs (1,390,080/1921, Fig. 70). Ore is fed into chamber (A) with oil and is agitated by the slatted wheel carried on shaft (9) and thrown over into box (C) where it is further agitated by a similar wheel and finally thrown through slot (27) onto the surface of the pulp in spitzkasten (D). Concentrate froth overflows the spitzkasten; tailing discharges through pipe (32) into conduit (E), fitted with a spiral for elevation of sand tailing. Pulp may circulate through slot (F) back into the agitation chamber (A).

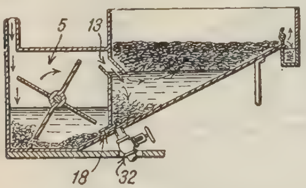


Fig. 69.—Piersol flotation machine.

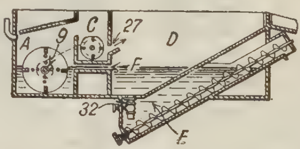


Fig. 70.—Boggs flotation machine.

Daman (1,556,083/1925) describes an apparatus in which the agitating compartment is placed so far below the spitzkasten that an air-lift must be used to return the agitated pulp to the spitzkasten. He thus obtains cascading of the return into the agitating compartment (which is wasted and useless) and also of agitated pulp into the spitzkasten, which is of some, though little effect. His apparatus also attempts, by suitable valve arrangement, to pass sandy tailing directly from compartment to compartment while slime is circulated several times in each compartment, without agitation, however. This lack largely vitiates any useful idea involved in slime circulation.

Machines utilizing agitation-froth and centrifugal or sub-aeration bubble-column action are described in the following patents:

Seale and Shellshear (1,341,024/1920, Fig. 71). Feed pulp admitted at (B) is pumped by impeller (F) through chamber (G) into separating compartment (A) and at the same time impregnated with air introduced through pipe (K'), which is curved and perforated at the lower end as shown at (K^2). In compartment (A) a bubble column forms on the surface of the pulp. Crowding boards (J) deflect the bubble column toward the overflow lips. Pulp from the first compartment flows toward the right where, in chamber (G^2) it meets two impellers, the first directing it back toward compartment (A'), the second pumping it onward toward compartment (A^2). Here it is further impregnated with air introduced through another pipe (K') and again passes into a separating chamber. Similarly in succeeding units. Tailing is discharged over a weir into box (C^3), thus maintaining the desired pulp level in the cell. Coarse sand is discharged at (C^2) as necessary.

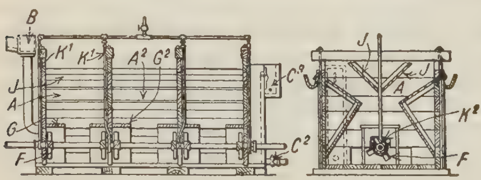


Fig. 71.—Seale and Shellshear flotation machine.

Kraut (1,549,492/1925). A downwardly-converging perforated conical rotor, closed except for a few holes at the top, revolving on a vertical spindle in a similarly-shaped chamber of slightly larger size which is, in turn, centrally mounted in fixed position in the center of a square pyramidal spitzkasten. The enclosing chamber is open at the bottom and has ports at about the pulp level. Revolution of the impeller causes aeration of the pulp both by agitation and by centrifugal action. Aerated pulp is discharged at the top of the fixed enclosing chamber and enters it at the bottom.

Lemmon-Hebbard machine (Fig. 72) was developed at Tul Mi Chung (*33 IMM 3*) to answer a demand for more vigorous frothing. It consists of a standard high-level M. S. machine with air introduced under the impeller, a grid baffle placed in the beater box just above the outlet to the spitzkasten and bubble-column overflow provided by cutting down the back of the beater box. The capacity of the machine was twice that of the standard machine and power consumption was reduced about 6.5 per cent. per spindle (to 7.1 hp.).

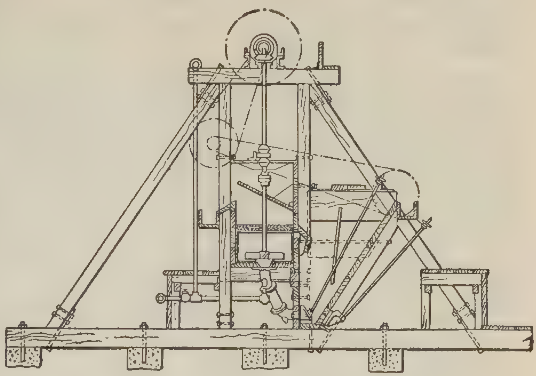


Fig. 72.—Lemmon-Hebbard machine.

Miscellaneous processes and apparatus, none of which has had any commercial application nor carries on its face any promise of success, are described in the following patents.

Nutter and Hoover (1,093,463/1914). This is an apparatus designed to cause "the froth as it is formed [by agitation-froth action] to fall over a trap baffle or wall below the level of the free air surface whereby the froth is collected and removed from the pulp before the bubbles can burst." The idea is impractical.

Wood (1,155,861/1915). This is a sub-aeration machine to be used without a frothing agent. It is expected that air bubbles attached to mineral particles will rise to the pulp surface and there break and drop their load onto a tray which discharges through the side of the box. The usual baffles are provided to prevent vortical action at the pulp surface. But this is a bubble-column machine; air bubbles do not arise carrying a load predominately mineral but pushing ahead of them a load of pulp. Concentration does not take place unless a bubble column is formed and an effective bubble column cannot be formed without a frothing agent, *i.e.*, an agent that modifies the surface tension of water.

Dolbear (1,343,313/1920) describes an apparatus that is essentially a pneumatic bubble-column machine in which the bubble column is supposed to be maintained below the general surface of the pulp underneath a transverse inclined baffle, and to discharge through valved openings in the side wall. The idea is ingenious but unworkable as the pressure of the head of pulp above the mass of bubbles would cause them to collapse, allowing the air to escape through the ports and the solid to drop back into the pulp.

Scott (1,375,233/1921). This is a porous-bottom tank arranged with a suction filter, such as the Oliver (see p. 1003), mounted above it in such a way that the filter drum dips below the surface of the liquid in the tank. It is stated that air bubbles carrying mineral will rise through the pulp and come into contact with the filtering surface, and that the air will be abstracted leaving the mineral adhering to the surface. As stated above (Wood, 1,155,861), the rising bubbles do not carry a mineral load, hence no such action can take place.

Dosenbach and Scott (1,401,055/1921). This is an apparatus built on the same theory as the preceding and failing for the same reason. It seeks to introduce air into an ore pulp through a porous medium and, by various means, to prevent the formation of a bubble column, by breaking the bubbles as they reach the pulp surface and collecting, as concentrate, the solid load that they drop. But, since the rising bubbles in the pulp do not carry concentrate, they cannot drop it

Ellis (1,488,745/1924) describes electrically grounding the flotation machine to prevent formation of static charges. Considering the substantially complete impossibility of obtaining or maintaining insulation of an operating flotation machine, grounding would seem to be superfluous.

Ellis (1,555,915/1925) recommends the use of oxygen or ozonised air as superior to atmospheric air for difficultly-floatable minerals, *e.g.*, sphalerite. He ascribes the improved results that he claims to electrical effects.

It is true that in the agitation-froth process, the more soluble the gas employed, the quicker the flotation effected. Thus carbon dioxide and nitrous oxide, with given flotation agents, yield considerably more rapid flotation than air. On the same basis oxygen should be somewhat more effective, but not sufficiently so to pay for its use.

16. Flotation agents

The term FLOTATION AGENT is used in this article to describe all substances added to an ore pulp for the purpose of aiding the separation therefrom, by flotation, of the valuable part.

Oil, according to dictionary and ordinary text-book definition, is an organic liquid having a greasy, slippery or unctuous feel, chemically neutral, inflammable, and immiscible in bulk with water. Oils are called animal, vegetable or mineral according to their source. Oil chemistry makes further subdivisions, only one of which is important here, *viz.:* the subdivision of vegetable oils into fixed oils and essential oils. FIXED OILS are those that cannot be distilled, either alone or with steam, without undergoing chemical decomposition; those that can be so distilled are called VOLATILE OR ESSENTIAL OILS. In flotation terminology the word oil has a much wider meaning than that above given. In its widest use it means any organic liquid added to the flotation pulp and common mill usage narrows this, if at all, only to the extent of requiring some oily feel or appearance. The word, therefore, includes substances such as oleic and other fatty acids which are not chemically neutral nor distinctly inflammable; kerosene and gasoline, which, while neutral, have not a distinctly oily feel; synthetic compounds such as the ester, amyl acetate; and many others. The purpose of addition of flotation agents is different in different processes, but in all processes one of the purposes is to effect, or aid in effecting, selection of valuable mineral from gangue. Other purposes are: (*a*) in film flotation, to accentuate the resistance to wetting offered by the mineral to be floated, and thus to aid in the support of this mineral at the air-pulp interface: (*b*) in oil flotation, to act directly as the buoyant medium; (*c*) in froth flotation, to make possible the formation of a froth, and thus, indirectly, to aid in buoying the mineral to be floated.

Alphabetical list of flotation agents described in U. S. patents up to Dec. 30, 1925, follows:

Acacia, 1,446,375, 1,446,376, 1,454,838.
Acetic acid, 348,157, 1,452,662, 1,457,680.
Acetic-phenyl-hydrazid, 1,364,306.
Acetone, 962,678, 1,425,327, 1,448,929.
Acetone phenyl hydrazone, 1,364,306.
Acetylene, 1,261,303, 1,541,292, 1,541,293, 1,548,351.
Acids (see also specific acids), 793,808, 807,501, 807,503, 807,504, 807,505, 807,506, 826,411, 835,120, 835,143, 842,255, 865,194, 865,260, 879,985, 938,732, 955,012, 956,773, 962,678, 1,045,970, 1,101,506, 1,104,755, 1,102,873, 1,126,965, 1,147,633, 1,156,041, 1,159,713, 1,176,441, 1,182,890, 1,228,183, 1,228,184, 1,236,933, 1,236,934, 1,240,591, 1,240,596, 1,240,824, 1,260,668, 1,269,157, 1,274,505, 1,286,922, 1,288,350, 1,302,966, 1,317,945, 1,322,816, 1,329,127, 1,329,493, 1,335,612, 1,369,054, 1,394,640, 1,394,958, 1,398,989, 1,398,990, 1,401,435, 1,417,261, 1,425,185, 1,425,186, 1,429,544, 1,438,435, 1,446,375, 1,448,929; 1,452,662, 1,457,680, 1,467,354, 1,469,042, 1,473,192, 1,490,736, 1,541,292, 1,548,351, 1,549,316, 1,560,170.
Acid sludge, 1,170,665, 1,452,662.
Acridine, 1,438,435.
Acridine derivatives, 1,438,435.
Acridine salts, 1,438,435
Agar agar, 1,499,872.
Albumen, 1,499,872.
Alcohol (see also specific alcohols), 955,012, 1,094,760, 1,170,637 (sulphonated), 1,246,665, 1,257,329, 1,364,304, 1,364,305, 1,364,308, 1,370,366, 1,386,716, 1,417,261, 1,417,262, 1,417,263, 1,425,327, 1,448,929, 1,452,662, 1,549,316, 1,552,197.
Aldehyde, aliphatic, 1,549,316.
Aldehyde condensation product, 1,378,562.
Aldehyde fatty acids, 1,549,316.
Aldol, 1,378,562, 1,478,697.
Aldol condensation products, 1,378,562.
Alkali, 521,899, 807,501, 807,505, 807,506, 826,411, 835,143, 865,194, 865,260, 870,985, 1,094,760, 1,176,441, 1,182,890, 1,203,341, 1,203,372, 1,203,373, 1,203,374, 1,203,375, 1,208,171, 1,208,334, 1,228,183, 1,228,184, 1,236,933, 1,236,934, 1,240,597, 1,240,598, 1,261,810, 1,301,551, 1,302,966, 1,317,945, 1,322,816, 1,329,127, 1,329,493, 1,364,304, 1,364,305, 1,364,306, 1,364,307, 1,364,308, 1,364,858, 1,364,859, 1,370,357, 1,370,366, 1,370,367, 1,370,843, 1,378,562, 1,394,640, 1,394,958, 1,401,435, 1,417,261, 1,417,262, 1,417,263, 1,421,585, 1,425,185, 1,425,186, 1,427,235, 1,429,544, 1,438,435, 1,446,314,

Classification of flotation agents may be made on several grounds. On the basis of chemical composition, they may be classified under the major

chemical subdivisions as organic or inorganic; on a physico-chemical basis, as soluble or insoluble in water; on the basis of their usual origin in the three great kingdoms of matter, as animal, vegetable, or mineral; on the basis of their supposed or apparent function in flotation, as collectors, frothers, depressants, etc. The difficulty is that no one of these classifications is rigid and consequently the assignment of certain agents to a given class or classes is a matter of individual opinion or experience. Chemistry ordinarily classes all substances containing the element carbon as organic and all other substances as inorganic, but in many text-books metal carbonates and the oxides of carbon are classed as inorganic. Solubility is a difficult ground because the concentrations in flotation practice are so small that published data are unreliable and original experimental determination difficult. Assignment to classes on the basis of origin causes similar, although not important, difficulty, because many flotation agents may be derived from substances originating in two or three of the kingdoms; thus phenol would be classed as mineral when derived from coal tar and as vegetable when derived from wood tar. Finally a substance that one flotation operator classes and uses as a frother may be used by another as a collector and by yet another as the sole agent, serving both ends.

Soluble and insoluble agents. This classification has had an artificial importance because of the legal effect of U. S. patent 962,678. Notwithstanding the claims of the patentees, no substance has ever been effective as a sole froth-flotation agent that was not to some extent soluble in the water of the pulp, and with the possible exception of a few petroleum products, no substantially insoluble flotation agent has ever been used in practice. It is not to be understood that there is not undissolved oil present in many flotation operations, but some soluble flotation agent must be present in every froth-flotation operation.

Solubility figures published in the chemical handbooks and elsewhere are useful with respect to the more soluble individual chemical compounds used in flotation, but are worthless in the case of very-slightly soluble substances, like the higher fatty acids. No figures are, of course, available for substances such as pine oil, tars, creosotes, and the like, which are not definite compounds but mixtures of a great many compounds of varying solubility. All of this class of substances are immiscible in bulk with water, but when mixed with water in the proportions usual in flotation operations, *viz.:* 1 : 4,000 to 1 : 80,000, all of them dissolve to a considerable extent. Table 15 gives the results of solubility determinations on a number of such agents.

Table 15. Solubilities of certain flotation agents in water when agitated therewith in the proportions 1 : 4000

Substance	Percentage dissolved
Kerosene	0.5
Olive oil	3.0
Fuel oil	4.0
Linseed oil	5.0
Oleic acid (crude)	5.5
Coal tar	6.5
Oleic acid (U. S. P.)	8.0
Coal-tar oil	12.5
Crude carbolic acid	15.0
Coal tar	29.0
Coal-tar oil	35.0
Blast-furnace oil	45.0
Eucalyptus oil	68.0
Pine oil	79.0
Oil of wintergreen	89.0
Cresol	92.0
X-cake	96.0
Phenol (U. S. P.)	100.0
Alkaline xanthates	100.0

Classification on the basis of duty in flotation. The phenomena that are vital in froth flotation and that, at the same time, are determined or fundamentally affected by the flotation agents, are: (*a*) the degree of water-wetting of the solid particles, and (*b*) the amount and character of the froth. The degree of water-wetting determines the possibility and ease of gas precipitation at particle surfaces or the behavior of particles at an oil-water interface or its equivalent. The amount and character of the froth determine its carrying and holding power and thus, to a considerable extent, the recovery and grade of concentrate, although both of these results are likewise dependent upon the degree of water wetting. It is necessary that the particles which it is desired to keep down in any froth-flotation operation be thoroughly and persistently wetted by water or aqueous solution, while those that it is desired to float shall be actually wetted (filmed) in whole or in part by an organic substance, substantially immiscible with the water in the state in which it is functioning, or shall tend strongly to become wetted with such a substance in the presence of water or aqueous solution.

The things that flotation agents do to affect and control these two fundamental phenomena may be grouped into five general classes; the groups of agents, named according to their function, are: (1) FROTHING AGENTS; (2) COLLECTING AGENTS; (3) DEPRESSING AGENTS; (4) DISPERSION AGENTS; (5) CONSERVING AGENTS.

A frothing agent is one that is primarily effective in froth formation. It may incidentally have more or less selective property. The froth formed should have sufficient stability and volume to maintain a regular and plentiful overflow when the machine is properly operated, but it should be sufficiently fragile to break down readily and the agent should not be so active that excessive frothing occurs in the ordinary movement of the pulp through the mill.

To effect frothing of an ore pulp a substance must affect the surface tension of water. Most frothing agents lower the surface tension. Change in surface tension requires solution, hence frothing agents must dissolve to some extent in the pulp water. The most effective agents are those whose effect on surface tension is greatest in solutions of a given strength. In any homologous series of substances the most effective are those of highest molecular weight and these are also the least soluble. Examples are: methyl and amyl alcohol; phenol and cresol; acetic and valerianic acid. In each case the second member is higher in an homologous series, *i.e.*, of higher molecular weight; it is the less soluble; the surface tension-concentration curve is the more concave; and the frothing effect is the greater.

Change in the surface tension of a liquid by a solute is the result of adsorption of the solute at the gas-liquid interface. In saturated solutions the concentration of a positively-adsorbed substance at the gas-liquid interface is that of a complete monomolecular film. The concentration of solute and the surface tension of the solution bear a linear relation in dilute solutions; they vary according to a logarithmic law in concentrated solutions; in solutions of intermediate concentration, the law of variation is variable and intermediate between the other two.

The rate of adsorption of solute increases with the concentration at a decreasing rate, and is greater the less the solubility of the solute. If a substance is insoluble in water and present alone therein, it will not adsorb at a water-air interface. This explains the non-effect of insoluble oils on surface tension. Distinct concentration of soluble oil takes place in the froth in a

flotation machine, with corresponding impoverishment of the water in the machine. The depression of surface tension due to the simultaneous addition of two or more solutes that are members of an homologous series is, in the main, the sum of the depressions caused by the individuals when present separately. This additive rule does not, necessarily, apply when the added substances are not chemical homologs. The simultaneous addition to water of a substance (*e.g.*, oil) insoluble therein and of another substance soluble in the water but not soluble in the first substance, produces only the effect on the surface tension of the water that would have been produced by the addition of the second substance alone. The simultaneous addition to water of a substance (*e.g.*, oil) insoluble therein and of another substance soluble both in the water and in the first substance, produces an effect on the surface tension of the water less than that which would have been produced by the addition of the second substance alone. Surface tension-concentration curves for this type of mixtures change with concentration from convex to concave, indicating progressive increase in extraction of solute by the water.

Oleic acid has but slight effect on the surface tension of water until the amount present is sufficient to and does form a mono-molecular film. Beyond this point, surface tension drops rapidly until an amount of acid equivalent to a film about one-and-a-half molecules deep is reached, when another change occurs and further addition produces again but slight effect. Oleic acid is a limiting case in the series of soluble substances affecting surface tension.

Maximum frothing occurs upon the addition of amounts of solute about one-half that required to saturate the solution. The point of maximum frothing is that at which the difference between the static and dynamic surface tensions of the solution is greatest. Emulsions in water, in which no water solute is present, do not froth. Emulsions of oil in water, consisting in part of mechanically dispersed oil and in part of dissolved oil, froth less than does a solution containing an equal amount of solute alone. The undissolved oil of an emulsion acts as a reservoir for soluble oil so that the frothing power of the emulsion is not diminished as greatly by dilution with water as would be expected were the solute present alone.

The larger the surface-tension depression at the beginning of a flotation operation, the greater the recovery and the poorer the grade of concentrate. The higher the recovery, the greater the possible change in surface-tension depression without resulting change in recovery. The finer the ore, the less the necessary depression in surface tension to effect a given recovery.

Experimental data underlying these statements are recorded in *68 A 479*.

The following list contains the best-known frothing agents:

Acid sludge (=kerosene acid sludge), aldol, coal-tar creosotes of high tar-acid content, cresols, essential oils (pine oil, eucalyptus oil, turpentine), terpineol, wood creosote, xylidin. In probably 90 per cent. of all flotation operations pine oil or a creosote is the frothing agent.

Collecting agents are organic substances that wet certain minerals, *e.g.*, sulphides, in the presence of water in preference to other minerals, *e.g.*, earthy or rocky gangues, and cause the wetted minerals to precipitate gas preferentially from supersaturated aqueous solutions of gas, or cause them to cling at the interface between an aqueous solution and a layer of frothing agent at a bubble surface.

The mechanism of the action is not the same in all cases. With oleic acid, definite preferential coating of the sulphide-mineral particles occurs, following contact of the particles with globules of the oil. Both sulphide-mineral and

gangue particles adsorb phenol molecules from solution, but the adsorption is greater at the mineral-particle surfaces. The same thing is true of xanthates, thiocarbanilid, and other non-frothing collecting agents; the differential adsorption is more marked the less the amount of reagent necessary to get the effect.

Collecting agents are frequently called COATING AGENTS, but since coatings of many different kinds function in flotation in many different ways and are markedly affected both by reagents in no way to be classed as collectors and by other conditions of the operation, the name is vague and misleading and should be dropped.

Collecting agents may be either solid or liquid. Most of them are more or less soluble in water but solubility is not so essential as it is in the case of frothing agents.

Oil-filming by incompletely-soluble liquid collecting agents is largely a mechanical process, effected by bringing the mineral particles into contact with the undissolved substance by stirring. With soluble agents the phenomenon is physico-chemical and the material is adsorbed from water solution. Typical examples of INCOMPLETELY-SOLUBLE COLLECTING AGENTS are: oleic and other higher fatty acids; animal and vegetable oils such as fish oil, castor oil, linseed oil, etc.; petroleum oils; the less-soluble parts of essential oils, creosotes, tars and the like. Typical ADSORBING COLLECTING AGENTS are alpha-naphthylamine, thiocarbanilid, various metal xanthates and a number of other soluble chemicals, such as pyridine, quinoline, etc. Some of the members of each class have frothing as well as collecting properties, *e.g.*, oleic acid and alpha-naphthylamine.

The action of incompletely-soluble collecting agents is pictured in Fig. 4 and described in the text therewith. Owing to the fact that the droplets of oil, after coming into contact with the mineral particles, do not spread to a mono-molecular film; that, in pulp-body processes at least, the film is effective only at its thinnest part; and that the degree of dispersion is relatively coarse, the amount of such agents necessary to effect good recovery is of the order of 1 or 2 pounds per ton of ore and upward.

The action of adsorbing collecting agents is preferential adsorption at certain of the solid surfaces in amounts of the order of mono-molecular films. The adsorption is, in general, greater by two to three times at sulphide surfaces than at gangue surfaces. This adsorption is an irreversible phenomenon in a given pulp, but a part of the adsorbed film can frequently be removed from the sulphide by aqueous solutions of smaller concentration of the adsorbed substance or by other solvents for the adsorbed substance. The amount of adsorption of different solutes at a given surface varies with the solute. The molecules in the adsorbed films are definitely oriented with respect to the adsorbing surface and the orientation in films that aid collection is such that the water-repelling end of the molecule is away from the solid. If the reagents also adsorb at air-water interfaces, the water-repelling end is likewise away from the water. The difference in behavior of filmed and non-filmed solid particles toward air takes place by reason of the orientation of the molecules in the adsorbed film. The difference in adsorbing power of sulphide and gangue minerals is one of degree only and varies with different solutes; hence by proper regulation of conditions, variation in difference of floatability can be controlled.

Owing to the fact that the film is mono-molecular and generally not complete and to the further fact that the degree of dispersion is substantially molecular, the amount of adsorbing collecting agent necessary to effect a

satisfactory recovery with most ores is of the general order of 0.1 to 0.3 lb. per ton of ore or about one-tenth that of the mechanically-applied collecting agents, provided the reagent is impervious to or is protected against the action of the gangue or of soluble salts in the pulp water.

Dispersion agents are substances added to an ore pulp which affect the state of dispersion of the gangue particles and at the same time change the extent to which these particles adsorb at sulphide-particle surfaces. When a finely-ground ore pulp is stirred with water of ordinary purity in the proportions usually employed in flotation there is invariably a certain amount of adsorption of fine non-metallic particles at the surface of the metallic particles; if the gangue is clean, unaltered silicate, the adsorption is usually slight and the essential nature of the sulphide surfaces is unchanged; on the other hand, if the gangue is decomposed and contains kaolin, sericite, iron oxides and like substances, adsorption at sulphide surfaces is immediate and so heavy that these surfaces are essentially changed. Sulphides thus coated differ little in floatability from the gangue minerals. The sulphide mineral in flotation froths has relatively little adsorbed gangue on its surface, less in rich concentrate than in low grade, and less the smaller the amounts of frothing and collecting agents used. On the other hand, the sulphide in the tailing from an unsuccessful float is usually heavily coated with gangue. It follows that successful flotation of a given sulphide mineral involves reduction of gangue adsorption to substantial nullity.

Solid-solid adsorption is in part, at least, an electrical phenomenon dependent upon the electrical charges carried by the adsorbing surface and the dispersed particles. In this respect it is, therefore, dependent upon the same properties as those that control dispersion of the slime (see Sec. 16, Art. 3) and any change in the pulp that affects the dispersion of the gangue is likely also to affect gangue adsorption. Ordinarily gangue adsorption is greatest when gangue dispersion is maximum and any treatment of the pulp such as heating, agitation, or the addition of a colloid or electrolyte that produces flocculation will decrease adsorption, but there are cases where the reverse situation holds and increase in the degree of dispersion accompanies decrease in gangue adsorption.

Thus the slime from Anaconda and from Utah ores in pulps made with ordinary potable water are highly dispersed and adsorb heavily at the surfaces of sulphide minerals in the ore while a slime, almost equally fine and highly dispersed, made by grinding a certain sample of barren quartz, adsorbs practically not at all on the same sulphide minerals immersed therein. Addition of one part of pine oil in 50,000 parts of water had no apparent effect on the dispersion of Anaconda slime and made no change in the gangue adsorption. Larger amounts of pine oil (1 in 16,000 and 1 in 4000) together with agitation produced incipien flocculation and materially decreased gangue adsorption. With Utah slime the addition of pine oil in the proportions of 1 in 50,000, 1 in 16,000, 1 in 4000 and 1 in 2000 of water had little or no effect on flocculation and correspondingly small effect on gangue adsorption. With Anaconda slime, pine oil (1 in 50,000) and lime (1 in 2000), the slime is distinctly flocculated and gangue adsorption is markedly less than with no reagents; Utah slime with the same reagents is slightly flocculated but gangue deposition is greater than with no reagent. On the other hand, the dispersion of Anaconda slime is increased when pine oil (1 in 8000) and sulphuric acid (1 in 800) are added, and gangue adsorption is increased, while with Utah slime similar dispersion produced by sulphuric acid and pine oil is accompanied by decrease in gangue adsorption. Increased recovery in flotation follows decreas in gangue adsorption in every case.

Common dispersion agents are sulphuric acid, lime, copper sulphate soda ash, caustic soda and sodium silicate. Certain organic agents of this class, *e.g.*, argol, glue, albumen, etc., have been proposed but have had littl or no commercial use.

The usual apparent physical effect of successful dispersion agents is to cause flocculatic of the ore pulp. Moses states (*104 J 749*) that the use of copper sulphate with zinc ore w. accompanied in one case by flocculation of both gangue and sulphide slimes, and in anothe case by flocculation of the sulphide slimes alone.

Flocculation of very fine oil-filmed sulphide particles is a great aid to gas precipitation thereon, if it is not a necessary condition precedent thereto. This may be proved readily by filming, say, 0.1-mm. and 0.01-mm. particles of galena with oleic acid (by immersing them in a dilute benzol solution and allowing them to dry), then placing them in water and heating. If the finer particles remain as individuals, gas precipitates on them reluctantly, if at all, although it precipitates freely on the larger particles. If now the small particles are caused to flocculate, as may be done by adding sulphuric acid and stirring, gas precipitates readily on the floccules when heating is continued.

Borcherdt (U. S. patents 1,445,989, 1,446,375 to 1,446,378 incl., 1,448,514, 1,448,515, 1,454,838), sets forth the hypothesis that the dispersion agents produce de-flocculation of gangue-slime particles in the pulp, thus opening previously-locked sulphides to the action of gas and lessening mechanical entanglement of flocculated gangue slimes in the froth.

Some oils have marked effect on dispersion, making the use of inorganic agents unnecessary; some ores show no need for dispersion agents by reason of the small amount of primary slime present.

Loth (*108 J 950*) states that the wood creosote used for collective flotation of lead and zinc at Picher, Okla., is added in the froth-overflow launder as the concentrate goes to a shaking table for separation. The table slime and tailing return to the feed-thickening tank and the overflow contains much semi-colloidal injurious slime that will not similarly overflow when oil is not entering.

Until within the last few years a majority of flotation operations were performed in acid pulps. At present, however, nearly all flotation is carried on in slightly alkaline pulps, as these are best for the chemical collecting agents. The change has effected large operating economies, since acid pulps, with copper ores particularly, were destructive of all iron with which they come into contact.

Depressing agents is the name given to substances that are used to lessen the floatability of one or more of the minerals of the ordinarily-floatable class in a mixture of such minerals, *e.g.*, to depress sphalerite when it occurs with galena, or pyrite when associated with chalcopyrite, and thus make it possible to float the galena and chalcopyrite respectively in concentrates relatively free of sphalerite and pyrite. The art of differential flotation is built on the judicious use of depressing agents.

The agents of this class do not all act in the same way. Most of them fall also in the dispersion-agent class; a few react chemically with some sulphides and not with others; and there is some evidence of a third type of action involving preferential deposition of a compound formed by the added agent and some constituent of the ore pulp other than the mineral upon which the salt deposits.

Depressing agents that are also dispersion agents include lime, sulphuric acid, sodium carbonate, sodium bicarbonate, sodium silicate, alkaline cyanides, zinc sulphate, copper sulphate, alum (potassium), and like inorganic substances. Their action is to effect differential adsorption of gangue at the sulphide surfaces.

With a certain lead-zinc ore having a clean quartzitic gangue both lead and zinc floated in neutral (salt-free) solution and the sulphide-particle surfaces were substantially gangue-free. Addition of 1 lb. LIME per ton of ore caused differential gangue adsorption, heavier on the galena, after which, by intensifying the flotation effect, differential flotation of sphalerite could be effected. Another lead-zinc ore having a slimy gangue showed heavy gangue adsorption with pine oil alone; the addition of 4 lb. LIME per ton of ore produced marked lessening of the adsorption at the sphalerite surface, although the differential effect persisted for only a short time (less than 5 min.). It was, therefore, possible to float blende preferentially for a short time after the addition of the lime, but thereafter both blende and galena were depressed so vigorously that flotation was impossible. POTASSIUM CYANIDE and ZINC SULPHATE used together in combining proportions in the presence of SODIUM CARBONATE or SODIUM BICARBONATE and Barrett No. 4 creosote substantially prevent gangue deposition on galena in some pulps while blende and pyrite in the same pulps are heavily slime-coated, hence galena floats while blende and pyrite remain submerged. COPPER SULPHATE added to some pulps after flotation of the lead as above removes the coating

from the sphalerite while causing an intense coating on the galena and intensifying the coating on the pyrite, hence blende may be floated subsequently substantially free of the other minerals. ALUM in the presence of lime and pine oil may prevent gangue deposition on galena while that on sphalerite is relatively heavy, hence the galena can be floated preferentially.

Not all of the effects above noted are equally strong and persistent; the cyanide-zinc sulphate and the copper-sulphate actions are the most distinct.

Depressing agents that react chemically with one of the sulphides and thus alter its surface are typified by the alkaline dichromates.

With PINE OIL (1 in 8000), SODIUM CARBONATE (1 in 4000) and SODIUM BICHROMATE (1 in 1000) in water alone, at 140° F., galena is markedly attacked and covered with a yellowish deposit that completely obscures the characteristic luster. Sphalerite is unaffected in the same solution. The same reagents in the same quantities in a non-slimy pulp produce the same effect and differential flotation of blende can be effected. In a slimy pulp, on the other hand, both galena and sphalerite are coated with adsorbed gangue and the galena surface is not attacked. Similarly the addition of 1 part in 20,000 of ALBUMEN to the water mixture first described inhibits attack on the galena. But the same amount of albumen added to the slime-pulp along with the other reagents prevents gangue adsorption and permits attack of the chromate, with the result that the galena surface is dulled and preferential flotation of blende made possible. POTASSIUM PERMANGANATE is said to act like the dichromate (Australian patent No. 9508 of 1913). GLUE acts like albumen and both are dispersion agents rather than depressing agents, used merely to prevent gangue adsorption from interfering with the dichromate (or permanganate) attack.

TANNIC and PYROGALLIC ACIDS are similar to the dichromates in that they apparently react chemically with galena in slime-free water and do not attack blende. In the presence of one slime tested there was such heavy gangue deposition with both reagents on both minerals that no flotation was possible. Addition of ferrous sulphate with both reagents with this slime markedly decreased slime deposition, permitting attack on the galena and rendering it unfloatable while the blende surface remained bright. There was, of course, distinct reaction between the tannic acid and ferrous sulphate, resulting in a fine black precipitate, so that the action of the two cannot be segregated.

Preferential agents that deposit salts formed by reaction of the agent with some constituent of the pulp are indicated by the work of Gates and Jacobsen (*Bul. 16 UU No. 4*) but their work could not be duplicated at Columbia University. They show photomicrographs of coatings produced on galena, sphalerite and pyrite in solutions containing 0.0375 per cent. of lime; the coating was least on the lime and greatest on the pyrite. Their description indicates identification of the coating as calcium carbonate. In light of the fact, however, that the weights of coating pictured are not in agreement with the relative floatabilities under similar conditions presented elsewhere in the paper and of the further fact that surface effects are distinctly different in fine pulps and in water, it is doubtful whether action of this sort plays any part in flotation.

Conserving agents are substances added to ore pulps to protect the other flotation agents from attack by substances present in the ore pulp. They are of no particular chemical character and may, in other pulps, play some other part in the flotation operation.

In one pulp in which alpha-naphthylamine was used as the frothing and collecting agent there was large consumption of the reagent by ferric iron. Sodium hydroxide added in an amount sufficient to precipitate the iron as insoluble ferric hydroxide reduced the consumption of naphthylamine markedly. Lime used with ores containing free acid, when an alkaline xanthate is used as a collecting agent, is, in part at least, acting as a conserving agent.

Trade names of flotation reagents are almost invariably used around the mills and frequently used without explanation in flotation literature. Table 16 names a number of the commoner oils and gives many physical properties useful for identification and specification. X-cake is alpha-naphthylamine; X-Y reagent is, ordinarily, 4 parts of X-cake dissolved in 6 parts of xylidin. T-T is a solution of thiocarbanilid in orthotoluidin, ordinarily 10 parts of the former to 90 of the latter.

Reconstruction of oils by distilling them with a small amount of sulphur in a reflux condenser causes a chemical change that results in solution of the sulphur, probably in the form of a $C = S$ compound with one of the constituents of the oil. With some oils and ores reconstruction consistently causes improved flotation results, both more active frothing and sharper selection accompanied by higher grade of concentrate and better recovery. With other oils frothing is d creased somewhat, although selection may be bettered.

Patent 1,593,232/1926, to Whitworth describes reconstruction with phosphorus and sulphur or with a phosphorus-sulphur compound, *e.g.*, P_2S_5. Whitworth claims that more sulphur combines than when sulphur is used alone thus forming more $H - S$ compounds. He claims best results by treating cresol with phosphorus pentasulphide but states that almost any substance containing a hydroxyl (OH) group is suitable; substances containing carboxyl (COOH), aldehyde (CHO), or ketone (CO) groups may also be used. The patent recites results with untreated cresylic acid, cresylic acid reconstructed with sulphur and reconstructed with phosphorus penta-sulphide. These results show progressive lowering of tailing by the reagents in the order named.

Kind of flotation agents used depends primarily upon the kind of ore and water, the result sought, the price, and the dependability of the supply. It is possible to compensate for certain variations in grinding and in pulp consistency by corresponding changes of agents, but it is better to eliminate the variations than to attempt compensation.

Character of ore to be treated is to be considered from the point of view of both the sulphide and gangue minerals. Certain agents appear to film certain sulphides more quickly or more completely than they do others, *e.g.*, pine oil and eucalyptus oil will produce good concentrate with most lead and zinc ores but do not ordinarily effect clean concentration of copper ores; coal tar and X-cake are usually highly selective with chalcocite and bornite but not so effective with galena and sphalerite. No general rule can be set down to the effect that for certain sulphides certain reagents are to be used and for certain other sulphides certain other agents. Thiocarbanilid and the alkaline xanthates come as near being universal agents as can be expected and pine oil, in the minute quantities necessary with these powerful collectors, is an almost universal frother. Coal- and wood-tar creosote are useful in conjunction with the chemical collectors in the case of most ores, and are also extremely effective with many when used without the chemical agents. Table 17, compiled by J. M. Callow, gives data concerning agents used and results obtained at many of the principal U. S. plants as of Jan., 1925. The following examples from practice at different mills, culled at random from periodical literature, show older practice and are useful in indicating reagents alternative or supplementary to the chemical reagents.

Copper ores

INSPIRATION (*54 A 11*): chalcocite and chalcopyrite in schist; 80 parts crude coal tar and 20 parts coal-tar creosote in neutral pulp. CONSOLIDATED ARIZONA SMELTING CO. (*104 J 72*): chalcopyrite and pyrite in quartzitic gangue; 70 parts stove oil, 25 parts wood creosote, 5 parts acid sludge, total, 1.31 lb. per ton. OLD DOMINION (*102 J 754*): chalcocite, chalcopyrite and pyrite in quartzitic gangue; 85 parts coal tar, 8 parts coal-tar creosote, 4 turpentine and 3 of pine oil in neutral pulp. This mixture produced concentrate containing only 8 per cent. Cu with 31 per cent. Fe and 21 per cent. insoluble. UTAH LEASING CO. (*105 J 535*), 85 parts coal tar and 15 parts coal-tar creosote with crude soda ash. NEW CORNELIA COPPER CO., Ajo test mill (*109 J 1314*): chalcopyrite in monzonite porphyry; 70 parts coal tar and 30 parts crude pine-wood oil (destructively-distilled). Fuel oil in the mixture brought up too much gangue. ARIZONA COPPER CO. (*109 J 1350*) used coal-tar, coal-tar creosote and pine oil in many of the experimental flow-sheets but found X-cake and xylidin with lime superior. SWANSEA LEASE (*109 J 845*): chalcopyrite with hematite in gneiss; petroleum oil, pine oil, pine-tar oil and coal-tar creosote. ARIZONA HERCULES (*109 J 1115*): coal-tar creosote and pine oil reconstructed with sulphur for an ore containing sulphides, oxides and metallic copper. MOUNTAIN COPPER (*119 P 333*): acid sludge, crude turpentine, tar oil and light fuel oil. MIDDLEMARCH (*118 P 182*): coal tar, pine oil, coal-tar creosote. MT. LYELL (*123 P 90*): eucalyptus oil and coal tar. ENGELS

Table 16. Trade names and

Name	Authority	Class	Specific gravity		Index of refraction	
			Value	Temperature, degrees C.	Value	Temperature, degrees C.
American Creosote No. 2(*v*)	*u*	Coal-tar creosote	1.0456	15	......	....
Atlantic No. 1	*u*	Crude turpentine(*dd*)	0.9268	15	1.4970	20
Barrett No. 1	*u*	Coal-tar creosote	1.0326	15	......	....
Barrett No. 2	*u*	Coal-tar creosote	1.0203	15	......	....
Barrett No. 4(*d*)	*t*	Coal-tar creosote	1.032	20	......	....
Barrett No. 609	*rc*	Coal-tar oil	1.04	....	......	....
Barrett No. 635	*t*	Coal tar	1.132	20	......	....
Blast-furnace oil(*e*)	*t*	Coal-tar creosote	0.987	20	......	....
California fuel oil	*u*	Petroleum residuum	0.9702	15	......	....
Chesapeake pine oil	*u*	Crude pine oil(*g*)	0.8452	15	......	....
Cleveland Cliffs No. 1	*t*	Wood creosote	1.0427	20	1.5132	22.0
Cleveland Cliffs No. 2	*u*	Wood creosote	1.0783	15	Very high	20
Cleveland Cliffs Refined	*t*	Wood creosote	1.046	20	......	....
Coal-tar light oil	*u*	....	1.0021	15	......	....
Crude coal tar	*t*	....	1.1428	20	......	....
Crude turpentine, G. N. S.	*u*	....	0.9886	15	......	....
Flotco No. 22N	*u*	Coal-tar creosote	1.0193	15	......	....
Flotco No. 23(*z*)	*u*	Coal-tar creosote	1.0216	15	......	....
Flotco No. 24	*u*	Coal-tar creosote	1.0266	15	......	....
Flotco No. 25	*u*	Coal-tar creosote	1.0223	15	......	....
Fuel oil, California	*rc*	Petroleum residuum	0.9–1.0	....	......	....
Fuel oil, Greybull	*t*	Petroleum residuum	0.893	20	......	....
Fuel oil, Utah Copper Co.	*t*	Petroleum residuum	0.9396	20	......	....
G. N. S. No. 4	*t*	Pine oil(*dd*)	0.9155	20	1.4928	20.5
G. N. S. No. 5	*t*	Pine oil(*sd*)	0.942	20	1.4820	21.0
G. N. S. No. 8	*t*	Pine-tar oil	1.0347	20	1.5388	21.0
G. N. S. No. 15(*a*)	*t*	Pine-wood creosote	1.0125	20	1.5519	22.5
G. N. S. No. 16	*u*	Pine-tar distillate	0.9644	15	......	....
G. N. S. No. 30	*t*	Crude turpentine	0.903	20	1.4852	22.0
G. N. S. No. 70	*rc*	Pine-tar oil	0.99	....	......	....
International creosote	*t*	Coal-tar creosote	1.075	20	......	....
Junior red engine oil	*t*	Petroleum lubricant	0.905	20	......	....
Lewis No. 4(*w*)	*u*	Coal-tar creosote	1.0251	15	......	....
Lewis tar	*t*	Coal tar	1.111	20	......	....
Lewis tar acid	*rc*	Coal-tar creosote	1.03	....	......	....
Oil tar	*u*	....	1.1098	15	......	....
P. T. & T. No. 17	*u*	Pine distillate(*dd*)	0.9304	15	......	....
P. T. & T. No. 70	*t*	Heavy pine oil	1.029	20	1.5539	21.0
P. T. & T. No. 75	*t*	Crude wood turpentine	0.883	20	1.4830	20.5
P. T. & T. No. 80	*t*	Crude pine oil	0.913	20	1.4913	21.0
P. T. & T. No. 90	*u*	(*m*)	1.0109	15	Very high	20
P. T. & T. No. 200(*c*)	*t*	Pine-wood creosote	0.989	20	1.5110	21.5
P. T. & T. No. 350	*t*	Crude pine-wood oil	1.017	20	1.5340	21.0
P. T. & T. No. 400	*u*	Wood creosote	1.0210	15	......	....
P. T. & T. No. 750	*t*	Pine-tar oil	1.0471	20	1.5592	22.5
Reilly No. 19	*t*	Coal-tar creosote	1.052	20	......	....
Reilly No. 20	*t*	Coal-tar	1.108	20	......	....
Stove oil	*t*	Petroleum distillate	0.818	20	......	....
Wood-tar oil, U. N. S(*l*)	*u*	Wood creosote	1.1266	15	Very high	20
Yaryan No. 66	*u*	Pine oil(*sd*)	0.9253	15	1.4805	20
Yaryan pine(*h*)	*u*	Pine oil	0.9436	15	......	....
Yaryan pine oil	*u*	Pine oil	0.9236	15	1.4760	20

a General Naval Stores Co. *b* 37.8 per cent. water by volume. *c* Pensacola Tar and Turpentine Co. *d* Barrett Manufacturing Co. *dd* Destructively distilled. *e* From Scotch coal-fired iron blast furnaces. *f* Includes 2 per cent. water. *g* Low specific gravity and high non-polymerizable residue indicate adulteration with mineral oil. *h* Yaryan Rosin and Turpentine Co. *i* Includes 1.25 per cent. water. *k* Includes 5 per cent. water. *l* United Naval Stores. *m* Apparently a mixture of about 15 per cent. pine oil and 85

properties of flotation oils

Specific viscosity		Non-polymerizable, per cent.	Per cent. that distills, degrees C.								Residue by difference	Highest temperature in distillation	Tar acid, per cent.
Value	Temperature, degrees C.		Below 150	150–200	200–250	250–270	270–300	250–300	300–350	350–400			
......	....	.....	3.0	5.5	50.5	....	......	15.5	18.5	....	7.0	.....	7.0
......	....	0.5	3.0	63.0	11.8	....	......	......	......	....	22.2	....	....
......	....	.....	1.0	6.5	74.0*r*	....	18.5*s*	......	......	....	...	.....	13.5
......	....	.....	2.2	18.5	56.0	10.5	12.8*q*	......	......	....	...	.....	15.5
1.46	20.0	.....	2.7	24.0	55.4	8.0	3.4	......	2.0	....	4.5	314	27.0
......	....	.....	0.2	8.0	35.5	13.0	8.2	......	......	....	35.0	.....	11.7
1.53	72.0	.....	0	1.0	22.0	7.0	8.0	......	17.0	....	45.0	350	2.4
2.7	23.0	.....	1.4	3.0	46.0	14.0	16.0	......	17.0	....	2.6	347	28.0
High	....	.....	0	0	2.5	....	......	9.2	36.0	....	52.3	.....	....
......	....	64.0	2.0	31.0	29.0	....	......	......	......	....	38.0	.....	....
2.01	21.0	0	8.6	4.4	61.0	....	......	23.0	......	....	3.0	291	....
......	....	2.0	10.0*k*	6.0	37.5	....	......	......	......	....	46.5	.....	....
1.54	33.0	0	8.8	19.2	51.4	....	......	16.0	......	....	4.6	263	....
......	....	.....	4.0	41.5	40.5	4.2	6.5*y*	......	......	....	3.3	.....	14.0
7.0	92.0	.....	0	1.0	13.0	3.0	7.0	......	11.0	....	65.0	323	4.0
......	....	10.0	6.5	34.5	22.5	....	......	......	......	....	36.5	.....	....
......	....	.....	0.5	10.0	48.5	14.5	24.0*y*	......	......	....	2.5	.....	8.0
......	....	.....	1.5	26.5	36.0	15.5	17.5*y*	......	......	....	3.0	.....	17.0
......	....	.....	1.2	11.5	52.0	12.8	20.0*q*	......	......	....	2.5	.....	22.0
......	....	.....	2.0	12.0	49.5	12.5	22.5*y*	......	......	....	1.5	.....	18.0
......	....	.....	......	7.3	5.5	....	......	12.3	22.0*za*	....	52.9	.....	....
1.51	93.0	....	0	1.0	4.0	8.0	12.0	......	23.0	40.0	12.0	351	....
2.82	83.0	.....	0	2.0	9.0	6.0	10.0	......	58.0	....	15.0	347	....
1.46	22.0	4.0	1.4	44.2	53.8	....	......	......	......	....	0.6	247	....
2.50	22.0	6.0	5.4	10.0	84.0	....	......	......	......	....	0.6	237	....
3.45	44.5	2.0	6.6	24.2	14.6	....	......	10.4	23.2	....	21.0	323	....
3.52	40.0	0	42.6*b*	1.8	15.0	....	......	6.6	2.7	....	31.3	345	....
......	....	20.0	2.5	14.0	13.2	....	......	8.5	36.0	....	25.8	.....	....
1.13	19.5	3.0	24.0	46.2	23.8	....	......	......	......	....	6.0	250	....
High	....	.....	4.4*zb*	9.4*zc*	25.7*zd*	....	......	52.4*ze*	......	....	8.1	.....	....
1.9	20.5	.....	0	3.0	44.0	10.0	25.0	......	6.0	....	12.0	347	7.0
1.84	82.0	.....	0	2.0	0.7	1.0	0.7	......	10.0	15.0	70.7	383	....
......	....	.....	7.0	26.2	47.5	....	......	14.5	......	....	4.8	.....	34.0
1.65	82.0	.....	1.6	9.0	20.0	7.4	4.0	......	17.0	....	41.0	349	12.6
Mobile	....	.....	4.8	40.5	39.0	7.5	5.5*zf*	......	......	....	2.7	.....	30.0
x	....	.....	1.5	5.0	31.0	7.5	30.0*y*	......	......	....	25.0	.....	2.0
......	....	1.0	58.0	19.5	18.0	....	......	......	......	....	4.5	.....	....
3.29	22.0	1.3	2.4	4.8	33.0	....	......	22.0	10.4	....	27.4	330	....
1.07	20.0	4.0	6.0	84.8	8.2	....	......	......	......	....	1.0	247	....
1.54	21.5	8.0	3.0	66.8	22.0	....	......	4.4	......	....	3.8	269	....
......	....	0	1.0	3.0	14.5	....	......	......	......	....	81.5	.....	....
1.98	20.0	8.0	14.4	14.4	44.0	....	......	16.2	4.2	....	6.8	310	....
4.70	20.5	3.0	4.8	32.8	10.6	....	......	4.8	42.0	....	5.0	347	....
......	....	1.0	4.0*f*	3.2	58.5	....	......	......	......	....	34.3	.....	....
4.42	69.0	0	1.0	3.8	15.2	....	......	23.0	55.0	....	2.0	343	....
1.45	23.5	.....	2.7	20.0	46.0	8.0	7.0	......	8.0	....	8.3	370	16.0
3.43	21.0	.....	1.0	5.0	25.0	6 0	7.0	......	18.0	....	38.0	395	7.2
1.14	18.0	.....	2.0	72.0	23.0	2.0	......	......	......	....	1.0	270	....
......	....	2.0	23.0	5.5	9.5	....	......	......	......	....	62.0	.....	....
......	....	4.0	3.2	45.0	47.0	....	......	......	......	....	4.8	.....	....
......	....	.....	3.5	3.0	91.5	....	......	......	......	....	2.0	.....	....
......	....	10.0	4.0*i*	22.0	73.5	....	......	......	......	....	0.5	.....	....

per cent. wood-tar creosote. *q* 270 to 340° C. *r* 200 to 270° C. *rc* Ray Consolidated Copper Co. *s* 270 to 345° C. *sd* Steam distilled. *t* A. F. Taggart. *u* Utah Copper Co. *v* Solidifies at 8° C. *w* Lewis Mfg. Co. *x* High but more mobile than coal tar. *y* 270 to 350° C. *z* Flotation Oil and Chemical Co. *za* 300 to 330° C. *zb* 85 to 180° C. *zc* 180 to 220° C. *zd* 220 to 280° C. *ze* 280 to 360° C. *zf* 270 to 315° C.

Table 17. Reagents and performances at U. S. flotation plants. (*After Callow*)

Plant	Machine	Reagents	Tons per 24 hr.	Assays, per cent. Feed	Concentrate	Tailing
		Copper				
Anaconda Copper Mining Co.	*MS, P*	Alkaline xanthate and lime	9,000	4.2	8-9	0.18
Chino Consolidated Copper Co.	*JMA*	Alkaline xanthate and lime	7,000	1.5	27.0	0.33
Ducktown S., C. and I. Co.	*MS*	Pine oil, coal tar and lime	600	1.68	17.5	0.16
Inspiration Consolidated Copper Co.	*P*	Coal tar	17,000	1.1	30.0	0.30
Iron Cap Copper Co.	*P*	Pine oil, reconstructed Barrett No. 4	450	3.8	26.4	0.32
Magma Copper Co.	*P*	Thiocarbanilid, orthotoluidin and lime	600	2.8	16.0	0.20
Miami Copper Co.	*P*	Alkaline xanthate and lime	7,000	1.8	45.0	0.24
Nevada Consolidated Copper Co.	*P*	Alkaline xanthate and lime	12,000	1.2	16.0	0.20
New Cornelia Copper Co.	*P*	Coal-tar creosote	5,000	1.7	38.0	0.51
Phelps-Dodge, Arizona Copper Co.	*P*	Alkaline xanthate and lime	3,000	2.3	21.5	0.46
Phelps-Dodge, Copper Queen	*P*	Alkaline xanthate and lime	5,000	1.45	5.3	0.21
Phelps-Dodge, Old Dominion	*P, KK*	Alkaline xanthate and lime	1,000	3.5	20.0	0.40
Ray Consolidated Copper Co.	*P*	Alkaline xanthate and lime	8,000	1.5	15.0	0.30
Tennessee Copper Co.	*M*	Pine oil, Barrett No. 635, Portland cement	500	1.55	15.0	0.15
Utah Copper Co.	*JMA*	Reconstructed Reilly No. 19	40,000	1.1	20.0	0.15
Walker Mining Co.	*P*	Alkaline xanthate and lime	500	3.5	27.0	0.14
		Lead				
Desloge Consolidated Lead Co.	*P, KK*	Hardwood creosote	400	3.5	50–55	0.3
Federal Mining and Smelting Co., Mo.	*JM*	Hardwood creosote	900	3.5	55	0.35
National Lead Co.	*P*	Hardwood creosote	600	3.0	55	0.35
St. Joseph Lead Co.	*MS, P*	Hardwood creosote	5,000	3.5–5.0	60	0.2–0.35
Utah Apex Mining Co.	*P*	Alkaline xanthate and lime	400	6.5	55	0.15
		Zinc				
American Zinc Co.	*JM*	Thiocarbanilid	1,000	3.7	60.5	0.08
Anaconda Copper Mining Co.	*MS, P*	Thiocarbanilid, orthotoluidin and lime	1,500	12.0	48.0	0.80

JM Janney mechanical. *JMA* Janney mechanical-air. *MS* Minerals separation. *P* Pneumatic. *KK* Kraut and Kohlberg.

Copper Co. (*123 P 190*): crude turpentine and petroleum oils. Nevada Consolidated Copper Co. (*123 P 328*): X-cake and xylidin with lime. Consolidated Coppermines (*64 A 823*): coal tar and coal-tar creosote with pine oil; later X-cake and xylidin with lime. Catemu (*123 P 921*): chalcocite and oxidized copper minerals in a clayey gangue; blast-furnace oil and Mexican fuel oil. Sodium polysulphide was used to de-flocculate the gangue and frothing was kept down in order to keep from carrying over the de-flocculated clay mechanically. It was found here that pine oil, pine tar and wood oils generally floated calcite and clay and thus lowered the grade of concentrate. Calumet and Hecla (*117 J 284*): native copper; coal-tar products were most effective, *e.g.*, a mixture of coal tar, two coal-tar creosotes, wood creosote and a little pine oil as needed, in a neutral pulp.

Lead ores

Missouri (*57 A 337*): wood creosote in all lead plants. Bunker Hill and Sullivan (*102 J 36*): pine oil. Silver King Coalition (*116 J 372*): pine oil. Shattuck Arizona (*110 J 761*): oxidized ore; coal-tar creosote, hardwood creosote, oil tar, pine oil.

Zinc ores

Butte and Superior: pine oil. Timber Butte: pine oil. Anaconda: pine oil. The latter two plants are now practicing differential flotation of lead and zinc. Butte and Superior has also used coal-tar creosote, various petroleum products, and wood creosotes, both with and without pine oil, but the latter alone is most satisfactory. Copper sulphate is necessary for best results with all these ores. Thornberry (*113 J 54*) found X-cake and orthotoluidin with copper sulphate best for Tri-state zinc ores.

Lead-zinc ores

Joplin (*105 J 733*): hardwood creosote, coal tar and pine oil in acid pulps. Picher, Okla. (*108 J 950*): hardwood creosote with copper sulphate. Reocin (*115 J 397*): pine oil, coal-tar creosote and copper sulphate. If cresol is used to replace pine oil, more copper sulphate is necessary. Daly-Judge (*54 A 11*): 40 parts crude coal tar, 40 parts creosote and 20 pine oil. See also "Differential Flotation."

Precious-metal ores

Gold King (*100 J 634*): pyritic gold ore; coal-tar creosote and pine oil in acid circuit. Belmont Shawmut (*121 P 661*): pyritic gold ore; 3 parts wood creosote and 4 parts California fuel oil with sodium sulphide. The latter is essential to flotation of the gold; it causes a reduction in tailing with a 3-oz. ore from 0.045 oz. to 0.01 oz. California Rand (*124 P 15*): pyritic gold ore; 10 parts water-gas tar, 7 parts stove oil, 2 parts hardwood creosote and 1 part pine oil with sodium sulphide or sodium carbonate. Suan concession (*119 P 844*): complex gold-silver-copper ore; eucalyptus oil, brown camphor oil and thin coal tar with lime. Campbell (*103 J 920*) reports the successful use of a mixture of pine oil and wood creosote with or without small quantities of gas- or coal-tar oil for treating a complex gold-silver ore containing chalcopyrite, pyrite, antimonides of silver and tellurides; also small amounts of sericite, kaolin and calcite. Acid was necessary to clean the froth. Parsons and Gilmore (*20 CMI 73*) report conclusions from an extensive series of tests on Canadian silver ores to the effect that crude hardwood creosotes or pine oils were satisfactory frothers; coal-tar creosote was indispensable as a collector, and that sodium hydroxide "gave an almost incredible improvement in extraction and grade of concentrate." Cobalt (*105 J 785*): native silver and arsenic and antimony compounds of silver in quartzitic gangue; coal tar (10), coal-tar creosote (75) and pine oil (15) or a similar mixture was used in most plants.

Oils vs. chemicals for flotation agents. Chemical agents are preferable on operating grounds. Their outstanding advantages are: (1) they can be more accurately specified for purchase and the seller can more closely approach specifications; (2) they are, in general, more readily dispersed in the pulp; (3) the concentrate made with them is usually of higher grade and more readily broken down in the concentrate thickeners than that made with oil; (4) the concentrate formed with chemicals filters more rapidly and completely.

At Consolidated Copper Mines (*64 A 825*) substitution of X-cake for creosote oils gave a higher-grade concentrate and made it possible to filter one day's concentrate in one shift with one or two filters while three shifts with three filters had been required with oil.

The cake was drier, also, with the X-cake. The DISADVANTAGES of the chemical reagents are: higher cost, the fact that they are rarely locally available, but must be shipped long distances, and an unsettled patent situation (Mar. 1927).

Inorganic agents. The practice in the addition of inorganic flotation agents is no more subject to generalization with regard to different types of ores than is the question of organic agents. The only general statement that seems to be founded in experience is that a small amount of copper sulphate solution, say such an amount as will introduce into the pulp between 0.1 and 0.3 lb. of copper per ton of solids, usually improves results with sphalerite ores. Results at SLOCAN, B. C., showed that addition of copper sulphate had no effect on grade of tailing nor did it have any differential action with respect to pyrite, but the grade of concentrate was raised 10 points by exclusion of gangue slime (*114 J 680*). Freeman (*120 P 833*) says that at BROKEN HILL addition of 0.5 lb. $CuSO_4 \cdot 5H_2O$ per ton of ore reduced the acid required in floating blende from 60 or 70 lb. per ton to 25 lb. Sulphuric acid has been widely employed with blende ores, when oils were used as collectors, but with the chemical collecting agents alkalis have been substituted in many plants. Sodium silicate is sometimes used where preferential separation of galena from sphalerite is practiced. A majority of the tonnage of low-grade chalcocite ores is probably being treated in neutral or slightly alkaline circuit. This is almost invariably the case when thiocarbanalid or xanthate is used. Acid salt cake was used in place of sulphuric acid in some instances, notably where it was cheaper or where there was a considerable amount of carbonate in the ore and treatment in an acid pulp seemed necessary.

The most extensive work reported on the effect of inorganic agents is that of Thornberry and Mann (*Effect of addition agents on flotation, Bulletins Missouri School of Mines, Vol. 4, No. 2 and Vol. 5, No. 2*). The work comprised several hundred tests on Missouri lead ores with various addition agents including sulphates, hydroxides, nitrates, chlorides, bromides, acetates, oxalates, chromates, carbonates, tartrates, phosphates, nitrites and permanganates and a number of miscellaneous substances as follows: sodium sulphite, tannic acid, dextrose, lactose, gum arabic, gum tragacanth, boric acid, carbolic acid (phenol), picric acid, and potassium chlorate. The tests with the inorganic agents were of distinctly negative character except on two or three points. None of the reagents produced any distinct improvement on the results of the standard tests with creosote or pine-tar oil. Chromates were distinctly harmful, lowering the grade of concentrate to between 20 and 30 per cent. Pb and the recovery to generally less than 20 per cent. Salts of certain metals, notably tin, uranium and cadmium were markedly harmful, especially in the higher concentrations. The higher concentrations of tartaric acid, potassium tartrate, phosphoric acid, ammonium phosphate, sodium sulphite, tannic acid, gum arabic, gum tragacanth, and phenol substantially destroyed flotation.

Unfortunately this elaborate record omits all of the essential observations required for interpretation. Chemical attack of the galena is, of course, indicated in the case of the chromates, and heavy gangue adsorption with colloids such as tannic acid and the gums, but further than this no generalization is warranted other than that this particular ore could be floated as well or better with certain oils alone than with certain specific organic and inorganic agents present in addition.

Ralston and Yundt (*104 J 749*) state that sulphuric acid was harmful with INSPIRATION ore and ascribe the bad effect to copper sulphate formed by reaction with the oxidized copper mineral, but this cannot be taken as a safe generalization, since UTAH COPPER Co. used sulphuric acid successfully for several years with an ore that contained sufficient oxidized copper to cause rapid replacement of all iron with which the pulp came in contact.

An anonymous writer (*117 P 931*) records the results of an investigation of the effect of the soluble components of a silicious silver ore. This showed that ferrous, magnesium, manganese and alkaline sulphates were present; that the ferrous sulphate was the only harmful solute, and that its bad effect could be overcome by the use of sodium hydroxide, sodium carbonate or lime (which, of course, precipitated the iron salts). In laboratory experiments it was found that the effect of the soluble salts was cumulative with re-use of water and that even when neutralizing reagents were used it was necessary to add considerable fresh water.

At the Mountain Top Mining Co. (*107 J 1130*) it was found that a small amount of spent calcium carbide would cause froth to form when frothing had failed with the usual oils.

Fineness of the ore has some effect on the kind and amount of frothing oil required. The froth must be sufficiently stable to permit removal from the machine. The more closely particles of solid are packed together in the bubble walls and the thinner the layer of liquid between adjacent pieces of solid, the stronger the bubble film, and with finely-ground ore little or no aid in stabilizing is needed from the oil. If the ore is coarse a froth-stiffening agent must be used. Tars, wood creosotes and some petroleum oils add stiffness or toughness.

Percentage of solids. When the percentage of solids is low, bubbles arrive at the surface less heavily loaded than in a thick pulp and are, therefore, less stable. If more stability is desired, an agent that stiffens or toughens the froth must be used.

If a given tonnage of solid matter is to be passed through a given machine in a given interval of time, say 24 hours, a greater volume must pass through in each unit of time with a thin than with a thick pulp and less time is afforded for dispersion of the agent through each unit of volume of thin pulp. With the degree of agitation and the rate of solid feed fixed, it follows that a more mobile or more easily soluble agent, or both, is necessary with a thin than with a thick pulp.

Kind of froth formed by various agents. Essential oils produce small-bubble, effervescent and brittle froths that drop mineral freely unless a large amount of oil is used. Excess oil makes the froth dirty and very liquid, but does not cause noticeable increase in size of bubble. Wood-creosote froth is distinctly irregular in size of bubbles, very tough, hard to break down and hard to clean. An excess causes the froth to carry much gangue, which is practically impossible to remove in cleaning. Coal-tar creosote forms a froth similar in texture to that formed by wood creosote, but the small bubbles are not generally so small and the large are more frequent and not so large. Neither is the froth so tough as that made with wood creosote nor so hard to clean when excess oil has been added. Coal-tar creosote and pine oil may form a very tough froth. Aldol, terpineol, turpentine, and cresol are intense frothers. They produce small-bubble, rather fragile froths, with the exception of turpentine which produces a tough froth in which the bubbles are initially small, but quickly grow by coalescence to considerable size. None of these substances has any useful selective qualities. Sodium oleate and kerosene sludge acid produce froths with relatively large bubbles, lightly loaded and glassy in appearance, and tending to carry considerable fine gangue. Xylidin froths are reasonably uniform in texture, with medium-sized bubbles. They break down readily, but are not so effervescent as the froths formed with essential oils. Alpha-naphthylamine is not ordinarily used as a frothing agent, but in a few mills where it has been used alone it forms fragile, rather coarse, irregular froth with strong carrying power, yet breaking down readily in the cleaner cells and thickening tanks. Petroleum products and tars are not frothing agents, but they add considerable toughness and stability. Tars are likely to bring in considerable silica; froths produced with petroleum products present are frequently of higher grade than when the petroleum is absent.

The best kind of froth, from the point of view of mill operation, is one that is fairly tough, with medium-sized bubbles and not too voluminous. Such a froth requires little attention to maintain steady overflow from the flotation machines, it does not carry coarse gangue mechanically, and breaks down readily in the cleaner machine and in the thickening tanks. These conditions

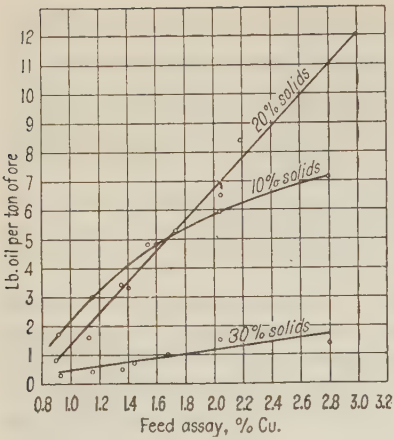


FIG. 73.—Relation between grade of feed, pulp consistency and quantity of oil.

are ideally satisfied when chemical agents are used as collectors and a mimimum of pine oil as a frother. This mixture eliminates the relatively insoluble, so-called neutral oil, which causes stiff, dirty froths.

Quantity of flotation agent necessary depends upon (*a*) the kind; (*b*) the recoverable mineral content; (*c*) the percentage of solids; (*d*) fineness of grinding and, less directly, on (*e*) the treatment of pulp subsequent to oil addition; and (*f*) the mineralogical composition of the ore.

Mobile and highly soluble agents can be employed in minimum quantities because of ready dispersion and the minute state of subdivision.

In the agitation-froth process, the recoverable mineral content of an ore, the amount of given frothing and collecting agents necessary, the percentage of solids in the pulp treated, the grade of concentrate and the recovery attained are distinctly related.

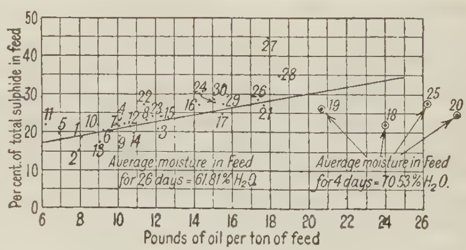


FIG. 74.—Chino Copper Co. Relation between total sulphide in feed and quantity of oil required.

In order to recover a given percentage of the recoverable mineral in an ore in the form of a concentrate of given grade, if the percentage of solids is fixed, the amount of a given collecting agent necessary is in direct proportion to the amount of recoverable mineral in the feed.

In order to recover a given percentage of the recoverable mineral in a concentrate of given grade, if the grade of the feed is kept constant, the amount of reagent necessary is in almost direct proportion to the percentage of moisture in the pulp.

The results of laboratory data leading to this conclusion are shown in Table 18, presented graphically in Fig. 73. The ore tested was a porphyry-copper ore assaying about 0.9 per cent. copper and 1.5 per cent. iron, enriched, in order to get the higher-grade feeds, by gravity concentrate made from the same ore. Flotation tests to the number of about two hundred, made by the same operator, identical as to speed, temperature, reagent, and manipulation of solids and quantity of agent, are the basis of the curves. The reason for the departure, with the richer feeds, of the curve for "10 per cent. solids," from the law indicated by the tests of low-grade feeds, is not apparent.

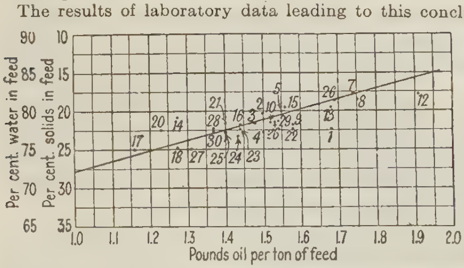


FIG. 75.—Butte and Superior Mining Co. Relation between pulp dilution and quantity of oil required.

Figs. 74 and 75 show these relationships in operating plants. Payne (*116 J 1106*) states that at Magistral-Ameca the quantity of flotation agent was increased as the feed increased in richness. Barcena y Diaz (*115 J 396*) says that at Reocin, Spain, the amount of oil needed when the feed ran 10 per cent. Zn was 2.2 lb. per ton while 4.4 lb. was necessary for a feed assaying 20 per cent. Zn.

Table 18. Relation between feed assay and required oil quantity

Feed assay, per cent.		Oil, pounds per ton	Recovery of Cu, per cent.	Actual percentage of solids, average
Cu	Fe			
Nominal percentage of solids = 10				
0.92	1.55	1.7	90	10.9
1.15	1.65	3.0	91	11.6
1.36	2.00	3.4	94	11.2
1.44	2.35	3.5	92	11.2
1.68	2.70	4.8	96	10.9
2.04	3.20	5.9	95	10.9
2.09	3.55	5.1	96	11.3
2.80	4.85	7.1	97	11.3
Nominal percentage of solids = 20				
0.90	1.28	0.8	94	19.0
1.13	1.62	1.6	94	18.8
1.40	2.00	3.3	94	19.5
1.54	2.31	4.8	94	20.3
1.74	2.88	5.3	95	19.4
2.06	2.96	6.5	94	20.5
2.19	3.30	8.4	94	19.4
2.99	4.80	12.0	94	20.8
Nominal percentage of solids = 30				
0.92	1.55	0.3	70	32.9
1.15	1.65	0.4	90	31.8
1.36	2.00	0.5	89	31.8
1.44	2.35	0.7	90	32.0
1.68	2.70	1.0	90	32.2
2.04	3.20	1.5	85	31.4
2.09	3.55	1.0	85	34.0
2.80	4.85	1.4	60	33.3

Excess of oil generally results in a tough, low-grade concentrate that is hard to clean; usually, also in high tailing content; a deficiency produces a tender froth and high tailing.

When water is reclaimed the frothing element comes back in considerable quantities and the amount of new frothing agent can be cut down, at times to the vanishing point.

Quantity of flotation agent that can be used was of artificially great importance for several years prior to November, 1923, because of the limitation of the basic agitation-froth patent (835,120) to an amount of oil less than 1 per cent. on the weight of the ore. The experience of the patentees had led them to conclude that their process was dependent upon such restricted use of oil and they had so limited themselves in their claims. On account of the exorbitance of the early royalty demands, it was in many cases cheaper to use more than 20 lb. of cheap oil per ton than to meet the license contract offered, and the result was that methods of operation with more than 1 per cent. of oil were quickly devised. The patentees

then clamorously denied that the part of the oil in excess of 1 per cent. was active in the process, and claimed infringement. The fact is that no critical point in oil quantity exists at 1 per cent., and that the large quantity of oil plays precisely the same role in agitation-froth flotation as the small quantity. When the oil quantity is increased beyond a certain maximum, continued agitation may cause the oil-sulphide sheaths of the air bubbles to become dislodged and roll up into shot-like granules as described in the Cattermole patents (Sec. 14, Art. 4). The change from flotation to granulation takes place, however, over a wide range of oil quantity, dependent upon other factors of the operation. It may be accomplished with less than 1 per cent. of certain oils, and it may be deferred until the amount of oil is upwards of 25 or 50 per cent. Table 19 presents the results of a series of tests run in a

Table 19. Effect of oil quantity on flotation performance

Oil mixture				Assays, per cent. Cu			Recovery, per cent.
Kind	Per cent.	Kind	Per cent.	Feed	Concentrate	Tailing	
Tests using approximately 1.25 lb. oil per ton (*a*)							
SFG	89	***YP***	11	0.940	2.575	0.670	38.8
SFG	93	***YP***	7	0.940	30.400	0.675	28.8
SFG	83	***YP***	17	0.955	31.300	0.270	72.4
Tests using approximately 20 lb. oil per ton (*a*)							
SFG	95	***X***	5	0.980	25.000	0.115	88.4
SFG	98	***YP***	2	1.050	39.300	0.130	87.9
SFG	98	***YPR***	2	1.030	25.050	0.040	96.3
Tests using 100 to 170 lb. oil per ton (*a*)							
SFG	99.75	***YP***	0.25	1.050	33.100	0.383	64.3
SFG	99.81	***YP***	0.19	1.050	26.650	0.175	83.9
Tests using 218 to 240 lb. oil per ton (*a*)							
SFG	100	....	....	0.935	3.300	0.433	61.8
SFG	99.87	***YP***	0.13	0.955	17.400	0.200	80.0
SFG	99.93	***YP***	0.07	0.935	18.600	0.133	86.4
Tests using approximately 470 lb. oil per ton (*a*)							
SFG	100	....	....	0.935	8.400	0.180	73.6
SFG	99.98	***YP***	0.02	0.935	14.800	0.100	89.9

a About 7 lb. of sulphuric acid per ton in all tests. ***SFG*** A mixture composed of 30 per cent. stove oil, 30 per cent. petroleum residium and 40 per cent. gilsonite. ***YP*** Yaryan pine oil. ***X*** mixture of 5 per cent. Yaryan pine oil and 95 per cent. Barrett No. 4. ***YPR*** Yaryan pine reconstructed with 7 per cent. sulphur.

Janney laboratory machine on regular flotation pulp at the Utah Copper Co. mill. Tests in the operating mill confirmed these. Mill operation is easier with a smaller amount of oil, on account of the difficulty of cleaning and dewatering high-oil froths; and the use of large quantities of oil would have been uneconomical except under the peculiar conditions that existed entirely unconnected with the technology of the process. It is also true that with certain oils, notably oleic acid, it is difficult to produce froth with large quantities of oil, but there is substantially no oil mixture that cannot be used in excessive proportions, if other elements of operation are regulated to correspond.

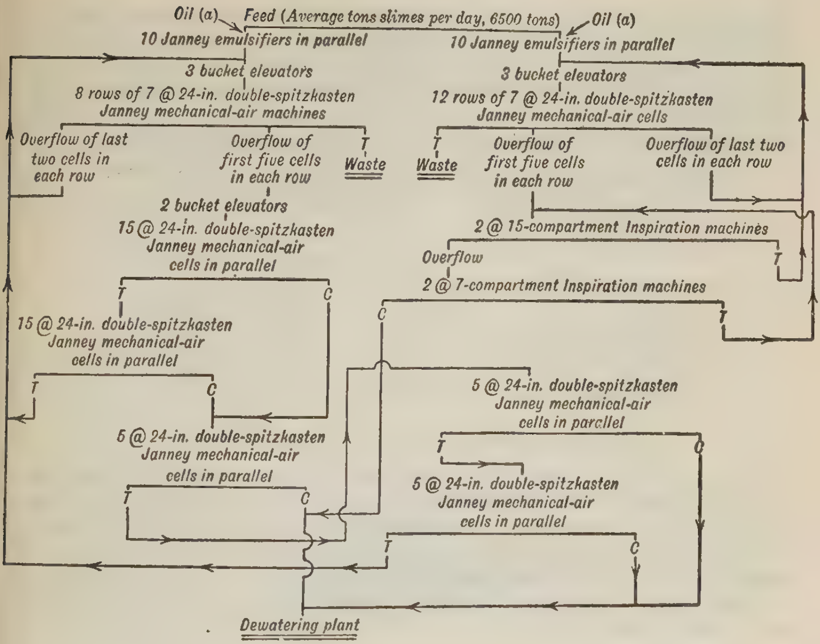


Fig. 76.—Flow-sheet for operations with more than 20 lb. of oil per ton at Ray Consolidated Copper Co.

At Ray Consolidated Copper Co. slimes were treated at the rate of 6500 tons per day according to the flow-sheet shown in Fig. 76. Average results are shown in Table 20. Comparative results at the same mill with a flow-sheet in which the oil was thoroughly emulsified in water with a Penberthy steam injector, then mixed with the slime feed in a Pachuca tank and treated in one 16-compartment Inspiration rougher (3 × 4-ft. compartments) and one 6-cell cleaner of the same type are given in Table 21. In another arrangement of the same machine, adding fuel oil to the boot of a bucket elevator returning cleaner tailing, and wood creosote and coal-tar creosote to a distributing box ahead of the rougher cells, 13.8 per cent. concentrate and 0.372 per cent. tailing were made from 0.705 per cent. feed when adding 20.2 lb. oil per ton; a recovery of 48.6 per cent. Table 21 also shows the marked improvement in grade of concentrate that is typical of the use of chemical agents with certain ores.

Table 20. Comparison of flotation results using less and more than 1 per cent. of oil on the ore, at Ray Consolidated Copper Co.

Legend	Less than 1 per cent.	More than 1 per cent.
Feed, per cent. Cu.......	0.746	0.755
Concentrate, per cent. Cu.	21.80	12.78
Tailing, per cent. Cu.....	0.440	0.554
Recovery, per cent.......	41.9	27.8
Pounds of oil per ton.....	1.4	20.8

Cost and availability of reagents. Price of oils ranges from a few cents per gallon for petroleum fuel oils to about 50 cents per gallon for the best grade of steam-distilled pine oil (1926). The organic-chemical reagents ordinarily cost more than oils per unit of weight, but, on account of the smaller consumption

and greater ease of operation, as well as greater recovery and higher grade of concentrate they frequently show sufficient economy to stand the greater price and the royalty payment. When considerable quantities of inorganic agents are necessary, these will ordinarily constitute the greatest item of cost, notwithstanding the usually lower unit price. Market prices of reagents are quoted at short intervals in Eng. and Min. Jour. and Chem. and Metl. Eng.

Table 21. Comparative results with less than and more than 1 per cent. of oil and with X-cake at Ray Consolidated Copper Co.

Legend	+1 per cent. of oil	−1 per cent. of oil	X-cake
Feed, per cent. Cu	0.663	0.753	0.739
Concentrate, per cent. Cu	10.94	9.78	29.36
Tailing, per cent. Cu	0.399	0.383	0.372
Recovery, per cent	41.4	51.2	50.3
Pounds reagent per ton	20.4*a*	6.12*a*	0.39

a Coal tar, coal-tar creosote, fuel oil in various mixtures.

It is rarely that an ore will respond to one reagent or reagent mixture only and the difference in cost of reagents per ton treated is ordinarily small. Such being the case, the question of dependability of supply is much more important than the price per pound. Other things being equal, locally-produced reagents are superior to those manufactured at a distance, but it is rarely that a suitable local supply can be found. Irregularity in transportation can be taken care of readily by generous provision for storage at the plant.

17. Operation

General. Operation of froth-flotation processes requires a higher class of labor and more careful and intelligent supervision than does any other operation in a concentrating mill. This is due to the fact that successful operation involves harmonizing many different elements and it is substantially impossible to formulate simple rules that set forth the proper things to do under the multifarious conditions that arise. Even in the laboratory where conditions are under the best possible control, skilled and unskilled operators working from the same set of instructions and with the same materials will obtain results differing as much as 30 or 40 per cent. in recovery and proportionately as much in concentrate assay, and yet the differences in the things they do are very slight and are differences of degree rather than of kind. The early results obtained in any plant are almost invariably poorer than those attained at a later date, notwithstanding that the elements of the operation have not, in so far as they are capable of definition, substantially changed. It is not, of course, to be understood that any mystery is involved, but rather that control of the magnitude or intensity of essential operating factors and judgment of results are matters that must be gained by intelligent observation and experience. All of which may be summarized in the statement that in the present state of knowledge froth flotation is an art rather than a science, to perhaps a greater extent than any other operation in concentrating practice.

The definable elements of operation are: (1) size of feed, (2) character of feed, (3) kind of machine, (4) kind and quantity of flotation agent, (5) pulp density, (6) feed rate, (7) intensity and duration of agitation and aeration, (8) temperature, (9) character of water supply.

Size of feed. Apart from the fact that low-grade tailing and high-grade concentrate are not to be expected until grinding has freed substantially all of the valuable mineral, froth flotation will not, in general, recover mineral grains larger than 0.25-mm. and will not make a high recovery of grains coarser than 0.15- to 0.175-mm. Maximum recovery is made of grains between 0.03- or 0.04- and 0.10- or 0.12-mm.; there is a considerable drop in recovery of the finest sizes. The agitation-froth process will save coarser mineral than the bubble-column process and the latter is more efficient in saving very fine material. (See Tables 22 and 23.)

Table 22. Sizing-assay test of feed and products of agitation-froth machine, Braden Copper Co. (*After Broadbridge, 22 IMM 37*)

Screen, mesh	Feed, per cent.		Concentrate, per cent.		Tailing, per cent.	
	Weight	Cu	Weight	Cu	Weight	Cu
40	6.4	0.8	....	....	3.6	0.60
60	18.3	1.0	5.8	15.0	26.1	0.65
80	11.5	1.8	9.3	14.4	8.6	0.45
100	14.9	2.9	18.0	17.3	11.7	0.32
150	8.1	3.6	25.6	22.0	12.7	0.38
−150	40.8	3.8	41.3	24.7	37.3	0.75

Table 23. Sizing test of feed and products and sizing-assay test of tailing of pneumatic cells at Miami Copper Co.

Screen, mesh	Sizing test			Sizing-assay test of tailing			
					Assays, per cent. Cu		
	Feed	Tailing	Concentrate	Weight, per cent.	Total	Oxide	Sulphide
28	....	....	....	1.4	0.61	0.18	0.43
35	....	....	....	4.3	0.58	0.14	0.44
48	0.7	0.7	....	8.1	0.52	0.14	0.38
65	4.6	4.9	....	9.7	0.45	0.15	0.30
100	11.8	12.9	....	12.9	0.38	0.14	0.24
150	17.3	16.3	....	12.1	0.40	0.15	0.25
200	5.6	4.6	0.8	5.9	0.40	0.23	0.17
−200	60.0	60.0	99.2	45.6	0.83	0.68	0.15

Fig. 77 (*55 A 576*) shows the relation at Inspiration between fineness of grinding and grade of flotation tailing.

There has been a gradual decrease in size of flotation feed ever since flotation was first introduced into the mills, so that while in early practice 28- or 35-mesh (0.59- or 0.42-mm.) was the limiting size, the present average maximum lies between 65- and 80-mesh (0.21- and 0.18-mm.). This trend is in part due to study of losses in tailing and in part to the steadily increasing adoption of pneumatic machines.

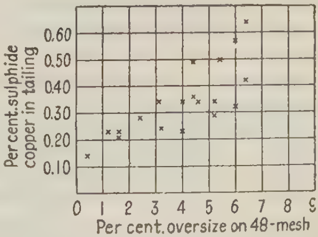


Fig. 77.—Relation between size of feed and assay of tailing (pneumatic machine)

The reasons for the failure to recover large and small particles are different. Large particles are lost because of the inability of the bubbles to carry them. In the agitation-froth process many of the bubbles carrying large particles do not have sufficient buoyancy to lift them to the top of the pulp in the spitzkasten, as may be seen by removing some of the pulp from the spitzkasten in a test-tube and examining it before a microscope. Also in many cases the large particles are jarred loose from the bubbles that precipitated upon them, because of the relatively great inertia of these large particles. In pneumatic machines, it is difficult to lift the large particles into the bubble column, on account of their greater settling velocities, and it is, of course, more difficult to lift them in the bubble column, for the same reason. As between the two types of processes, the agitation-froth process is the better fitted to hold large particles in the froth, once they have been raised to that point, both because of the firmer bond between the particles and the bubble walls and because the fine-textured froth acts mechanically as a screen and prevents large particles from falling back, even though they become detached from the bubbles.

The reason for loss of small particles is not definitely known. It is not improbable, however, that it is in part due to the locking of these particles in slime floccules and in part to the lack of flocculation of the sulphide particles themselves. Both of these hypotheses accord with the observed facts that the losses in the finest sizes are greatest in the agitation-froth type of machine, since the agitation of this machine in itself tends to produce flocculation (see Sec. 16, Art. 4), and flocculation of the slime-sulphide particles is essential to gas precipitation, which is no part of the pneumatic process.

Coghill (*119 P 404*) states that at the PRIMOS CHEMICAL Co. molybdenum plant recovery was raised from 60 to 83 per cent. by floating coarse pulp and then re-grinding and re-floating the tailing and ascribes the improvement to the fact that first grinding all of the pulp to ultimate flotation size overground some of the mineral.

Character of feed. High-grade feed makes for easy operation on account of the heavy loading of the bubbles and the consequent great stability of the froth. It is easier to make high-grade concentrate with high-grade feed, probably on account of the tendency of the mineral to crowd gangue from the bubble surfaces mechanically. On the other hand, the tailing from the treatment of high-grade feed will almost invariably be higher than that obtained from a low-grade feed of equal general floatability. It may happen, however, as is the case with free-milling non-sulphide gold ores, that the amount of metal present is insufficient to stabilize a selective froth, in which case, of course, flotation is inapplicable.

There is no great difference in floatability of the sulphides of different metals. The principal differences of importance are in the gangue minerals. An ore containing a large amount of clayey material, *i.e.*, a large amount of primary slime, will invariably be more difficult to float than an ore with a clean silicious gangue, and different methods of treatment may have to be employed. (See pages 844–858.)

Attempts have been made in some mills to solve the problem of primary slimes by separating them from the granular material by classification and treating the granular and slime materials separately, but these attempts have not been successful. In several instances it has been found, particularly in pneumatic machines, that the recovery from the granular material was not so high when it was treated alone as it was when treated in the presence of the slime, and that the recovery of the slime was not improved sufficiently to make up for this loss or to pay for the more complicated treatment.

Gahl (*55 A 576*) found at INSPIRATION that the granular material could be treated with ease but that the slime portion would not respond to flotation at all. He found, however, that if the percentage of primary slime in the flotation feed was kept below 20, Inspiration ore could be treated without greater tailing loss than was experienced when treating the granular material alone, although the grade of concentrate went down as the percentage of slime in the feed increased.

Control of the feed to keep the primary-slime content within proper limits is now the practice at all mills where such control is possible, and fairly elaborate methods of mixing in the mill bins is practiced to attain this end.

Kind of machine is not an operating variable, but is, of course, within control of an experimenter in preliminary testing. What has been said under "Size of feed" indicates that the pneumatic machine is preferable to the agitation-froth type for the reason that it is better fitted to save slime mineral, while its lesser ability to save coarse mineral can be taken care of by finer grinding. The pneumatic machine is easier to operate and control, once its operation is learned, than the mechanical machine, takes less power, and is cheaper to install and maintain.

Kind and quantity of flotation agent. Over-oiling is a frequent cause of poor operation. It is usually indicated by decrease in grade and increased mobility and overflow of froth. Aggravated over-oiling may kill frothing completely unless other conditions are adjusted to this condition. Excess of inorganic agents usually lowers both grade of concentrate and recovery. Where large amounts of inorganic salts are present in the mill water, however, it is usually possible to change other operating conditions, so as to overcome the bad effect.

Ralston (*105 J 735*) relates that at OHIO COPPER CO. the bad effect of the presence of copper and iron sulphates in the mine water used for flotation was rectified by addition of potassium cyanide.

Small-bubble froths frequently carry considerable coarse gangue. This condition can usually be rectified by changing oils in such a way as to produce coarser froth. The effect is to increase the inter-bubble space in the froth and allow the coarse material to drain back. The effect is particularly noticeable in machines of the combination agitation-froth and bubble-column types.

Place that agents are added makes considerable difference in their performance. Viscous oils and slow-dissolving chemicals should be added at the intake of the grinding mills. On the other hand, chemicals that alter quickly and, when altered, lose their effect, should be added directly to the flotation machine. Reagents that depend for their effect on chemical reaction with a constituent of the ore should be added far enough ahead of the cells to allow time for the reaction, *e.g.*, sodium sulphide with mixed sulphide-oxide ores.

Benitez (*123 P 922*) records that at CATEMU sodium sulphide added to the ball mills upset classifier performance and also lost its effectiveness by the time the pulp reached the flotation cells and that addition of freshly formed saturated solution near the tail end of the machine, where it would not interfere with sulphide flotation, gave the best results. Ralston and Yundt (*115 P 547*) state that at MIAMI lime added to the grinding mills increased recovery, but if added at the head of the flotation cells, was harmful.

Methods of adding reagents. The fundamental requirement is that the addition shall be regular and proportioned to the quantity of feed. No satisfactory method of automatically changing the addition rate with change in pulp-feed rate has been perfected. Solutions and mobile oils are readily

fed through needle valves, but it is essential, if the feed rate is to remain constant, that the head on the discharge shall not vary.

Rice (*106 J 1022*) describes a can feeder in which the oil is displaced upward and overflowed by means of water dripped into a pipe that feeds into the bottom of the oil container. This method is, of course, applicable only with oils and mixtures that are lighter than water and substantially immiscible therewith in bulk and has the disadvantage that the water abstracts a part of the water-soluble portion of the oil. With heavy oils a salt solution may be used to float the oil.

For viscous liquids various mechanical methods have been perfected, such as a small piston pump whose stroke length can be varied as desired: a miniature bucket elevator, of which either the speed; pitch, size, number or inclination of buckets; or the height of liquid in the boot can be varied at will: or a mechanism in which a revolving pulley dips into a tank of oil and the oil is removed from the face of the pulley by a scraper, the width of the scraper cut determining the amount removed (Braun oil feeder). Solid reagents are best added in solution, where possible. Failing this, the best method of addition is to pulverize them and then feed over a miniature of one of the various types of ore feeder, such as a traveling belt or chain, roller, vibrating or shaking plate or the like.

Ordinarily the reagents are dropped onto the surface of the pulp, but Barker (1,447,006/1923) describes an apparatus consisting of a horizontal perforated pipe surrounded loosely by a screen (about 18-mesh), attached by a tee to the lower end of a feed standpipe. Screen and perforated pipe are submerged in a moving stream of pulp and the screen acts as a permeable cylinder, emitting the reagent in minute droplets and films into the passing pulp, This apparatus, while not so successful in effecting dispersion as grinding the agent with the pulp, is nevertheless remarkably effective with mobile oils. At RAY CONSOLIDATED COPPER Co., using the same flow sheet as in atomizing (see Fig. 76) but adding a mixture of one part wood creosote to three or four parts of coal-tar creosote at the rate of 1.0 lb. per ton through a perforated pipe, the average tailing was 0.435 per cent. Cu and concentrate 16.03 per cent. Cu from feed averaging 0.682 per cent. Cu. This is a recovery of 37.2 per cent. Oxide-copper assays in feed and tailing respectively were 0.25 and 0.40 per cent.

In the modern use of chemical reagents such as alpha-naphthylamine, thiocarbanilid, xanthates and the like with slightly alkaline pulps, regulation of the degree of alkalinity in the pulp is an important factor. It is usual to effect this by frequent titration.

McLeod (*114 J 991*) describes an apparatus that indicates continuously the condition of the pulp as regards alkalinity. It consists of a filter that is placed in the pulp near the head of the cell, connecting by means of a flexible tube and a glass tube coated on the inside with fused phenolphthalein, with a vacuum bottle. The redness of the liquid dropping into the bottle is an indication of the degree of alkalinity of the pulp. Arrangement is made for periodical flushing of the system with water or compressed air.

Pulp density. Apart from the effect on the kind and quantity of flotation agents necessary and on the capacity of the machine (see descriptions of various machines), pulp density has a marked effect on recovery and grade of concentrate. The usual range is between 18 and 25 per cent. solids in rougher machines for pulps with average slime content. Density of slimy pulps ranges between 10 and 15 per cent. solids, in general. High pulp density, compared to the above figures, ordinarily causes low recovery and low-grade concentrate, with a sluggish, gangue-colored small-bubble froth similar to that produced by over-oiling, although this is not necessarily the case with sandy pulps. Low density causes production of a lightly-loaded, watery froth, a decrease in recovery and, usually, a drop in the grade of concentrate in the rougher-cell. This is because of the more intense aeration required with dilute pulps to maintain froth overflow, and the consequent overflow of gangue.

Low density is the usual condition in a cleaner cell, but here, although the recovery is generally low as compared with the rougher, the grade of feed is so high that overflow can be maintained with moderate to slight aeration and gangue is not, therefore, carried over mechanically.

Nevett (*121 P 349*) cites experiments at the JUNCTION NORTH mine, Broken Hill, to the effect that with 56 per cent. solids in the feed the machines treated 8 tons per hr. with poor results, while with 35 per cent. solids the same plant treated 24 tons per hr. with much improved results. The lower figure is much higher than is usual in the United States.

Maintenance of constant pulp density is just as important as determination of the right density. Ordinarily other conditions can be so adjusted that any pulp density within a considerable range will yield the same results, but once adjusted, results will fall off, if density varies. When mechanical classifiers prepare flotation feed, pulp density can be readily watched either by a hydrometer or by noting the height to which water rises in a small glass tube (about ⅛-in. bore) supported near the overflow end so as to project about 6 in. below the overflow lip. At the SULLIVAN MILL (*119 J 170*) flotation pulp is run through a small spitzkasten (18 in. square by 30 to 36 in. deep) suitably baffled to give uniform flow; a ¾-in. pipe with a bell fitting on the bottom carrying an 8-oz. canvas diaphragm is supported with the diaphragm 7 in. from the bottom of the spitzkasten; the upper end of the pipe is connected with an inclined glass tube against a calibrated scale and the whole is filled with colored water. At several mills hydrometer-controlled valves are installed on the line supplying water to the classifier and pulp density is kept substantially constant. This also keeps the size of flotation feed substantially constant. Where automatic or hydrometer control is not practiced, the usual procedure is to weigh samples of known volume and read density corresponding to the result from a chart or table.

Feed rate. There is a maximum feed rate for any cell under given conditions, but there is no minimum rate other than that imposed by economic considerations in design. That is to say, recovery in a given cell under given conditions is a maximum at the minimum feed rate, but the fall in recovery with increasing feed rate is very slow until the overload point is reached, when recovery falls rapidly with further increase. Many cells are fed far below the maximum rate.

Gahl says that at INSPIRATION doubling the section feed caused only a slight diminution in flotation recovery.

This condition of underload is more evident in series installations, such as the standard multi-compartment agitation machines and pneumatic machines of the Inspiration type, than with single-cell machines of the standard Callow type. In the multi-compartment machines the later compartments are acting as scavengers only, under normal operating conditions, and the recovery that they effect is small. When the feed rate is increased, more work is put on these later, normally underloaded, compartments. On the other hand, in the parallel-type installation, the machines are usually worked much more nearly to ultimate capacity, and when increased load comes on, some of the additional mineral goes into the tailing.

The ultimate capacity per cell is the same in both series and parallel installations. This was conclusively proved by careful experiment, both with standard Minerals Separation machines and with Callow machines, at the MIAMI COPPER CO.

Constant feed rate is important, for the reason that other variables, particularly agent addition, are based on a given rate, and change in rate changes the proportions and usually results in poorer performance.

Agitation. Intensity of agitation is not an operating variable, but recovery can frequently be increased, at the expense of increase in power consumption, by increasing the speed of impellers in agitation machines. This is particularly necessary when changing over from oils to chemical collecting agents. Agitation is not a function of pneumatic machines.

Attempts have been made to utilize the first cell as an agitator to mix flotation agents with the pulp, but they have been almost uniformly unsuccessful.

Duration of agitation may be increased by adding more compartments to a series machine, or decreasing the rate of feed. The effects of both of these changes are discussed above.

Aeration can be varied in agitation machines only by varying the intensity of agitation. In pneumatic machines variation in intensity of aeration is one of the principal operating variables. If there is too little aeration, the bubble column may break down at the surface nearly as fast as it is added to at the bottom, and recovery will suffer although grade of concentrate will increase. If there is too much aeration, boiling of the pulp may be sufficient to prevent a bubble column from forming, when, of course, there will be no concentration. If a bubble column forms under conditions of too much aeration, the froth will be low-grade on account of excessive return of gangue to the bubble column.

Temperature of the pulp has great effect in pulp-body flotation processes on account of its effect on the solubility of air in the water. The intensity of effect of all other flotation conditions is increased by increase in temperature of the pulp, minerals that are difficult to float in cold pulps are readily floated in hot pulps, and readily-floated minerals may be floated with less agitation or with less oil. Rise in temperature has a certain beneficial effect in pneumatic flotation with oil, but the effect is apparently only that it aids dispersion of the oil, since the same effect is not found when soluble, readily-dispersed frothing and collecting agents are used.

At Mt. Lyell (*123 P 90*) the winter temperature of the pulp is 47° F. and summer, 74°. Results in agitation-froth machines consistently improve in summer. Nevett (*121 P 349*) says that in the differential plant at Junction North mine, Broken Hill, a rise in temperature of even 1° F. above 90°, which was the desired temperature for lead flotation, caused zinc to commence to float, and that in the zinc-flotation machines, which were operated at 135° F., drop to 130° caused increased zinc losses. Ralston and Cameron (*99 J 937*) state that on practically any flotation pulp heating aids selective action and betters grade of concentrate.

Water supply is not normally an operating variable, except in those cases where reclaimed and fresh water are at the disposal of the operator for alternative use.

In the early work at Broken Hill it was found that when the amount of inorganic salts in solution in the flotation-pulp water exceeded a certain amount, flotation was harmed, hence sufficient fresh water was added to the circuit to keep the supply below this harmful concentration. As late as 1920, Nevett stated (*121 P 349*) that at the Junction North mine results fell off, if the concentration of salts in the mill water fell below 1400 or rose above 2600 grains per gal. In Arizona, where much of the mill water may be reclaimed from a tailing dam, the water frequently becomes so concentrated in certain soluble substances, due to evaporation, as to have a harmful effect on performance; on the other hand, a heavy rain which dilutes the water in the tailing pond may also have a harmful effect, particularly where the return water is depended upon for some of the flotation agents. In other words, the situation is that after the mill operation has been adjusted to a certain amount of contamination in the water as a regular operating condition anything that disturbs the regular condition is harmful. At Tul Mi Chung the spring floods bring large quantities of clay-like solid into the water supply and so long as this material is present flotation is uncertain and unsatisfactory. Bates (*109 J 552*) notes similar interference with flotation in a Mexican mill. Water from bogs and swamps usually contains tannin and vegetable acids which are ordinarily deleterious.

Stage flotation. One form is described by Gayford and Crerar (1,176,441/1916), consisting in grinding in two or more stages with intermediate flotation to remove concentrate.

At MIAMI COPPER Co. recent practice is to grind to 3 to 5 per cent. +48-mesh (instead of 65-mesh, as formerly), float, classify primary flotation tailing, sending slime to secondary flotation and sand to re-grinding mills and thence back to the primary flotation machines. This practice has greatly increased capacity, although it involves 33 per cent. additional grinding equipment.

Typical flotation flow-sheets may be classified into two general types on the basis of the part that flotation plays in the mill-treatment scheme and each of these classes may be again subdivided on the basis of the method of routing the pulp through the flotation machines. On the first basis, a flow-sheet is of the PRIMARY type when flotation is the primary or principal means of concentration employed and the bulk of the concentrate is recovered thereby; it is SECONDARY type when flotation is an accessory or subordinate process and some other means of concentration, usually gravity concentration, is the principal method of treatment. On the basis of pulp routing, a flow-sheet is of the CONCENTRATE-MIDDLING type when the flotation feed pulp passes through a set of machines in series, and these machines deliver finished froth concentrate off the early cells, a clean tailing as the underflow or spigot product of the last cell; and a low-grade froth or middling as the overflow of the later cells; this middling being returned to the head of the machine. A flow-sheet is of the ROUGHER-CLEANER type when two machines, not in series, comprise the flotation installation; the first (ROUGHER) makes a finished tailing and low-grade concentrate, which is sent to a second machine for cleaning; the second (CLEANER) makes finished high-grade concentrate and an underflow or spigot product constituting a middling, which is returned to the rougher cell. Combinations of these two methods of routing are also met with and may be classed as COMBINATION methods. Many combination routings are possible.

The bases for the differences in methods of flotation treatment are (1) the differences in the mode of occurrence of the valuable minerals in an ore and (2) the inability inherent in flotation processes to make a finished concentrate and a finished tailing with no material of intermediate value, in one treatment on one and the same machine. If the sulphide mineral in an ore occurs in coarse aggregates, a considerable proportion can be saved by gravity concentration at a less cost than by flotation, and ordinarily in the form of a concentrate that is more valuable than the concentrate made by flotation; assuming, of course, that the specific gravity of the gangue is sufficiently different from that of the sulphide to make gravity concentration efficient. In such case, flotation will probably form a subordinate part of the flow-sheet. On the other hand, if the sulphide mineral is disseminated through the ore in fine grains and the difference in specific gravity between the sulphide mineral and the gangue is not great, gravity concentration can recover only a small part of the valuable mineral and flotation should form the principal part of the treatment scheme. The choice as to the method of routing depends, to a large extent, on the percentage of floatable minerals present in the ore and on the grade of concentrate desired. A rougher-cleaner routing is usually used where the percentage of mineral in the flotation feed is low, and the concentrate-middling routing or combination routing is used when the percentage of mineral is high. The rougher-cleaner routing is best adapted to making a high-grade concentrate.

Cost. The approximate cost of pneumatic flotation (1926) as estimated by J. M. Callow (*PC*), follows:

		Cents per ton	
		Simple, one-metal ores (Cu)	Complex, two-metal ores (Pb-Zn)
1. Power @ 1¢ per kw.-hr.		3.0– 4.0	6.0– 8.0
2. Repairs and renewals		1.5– 2.0	3.0– 4.0
3. Reagents		4.5– 9.0	20.0–30.0
Total		9.0–15.0	29.0–42.0
4. Labor	100-ton scale	12–15	15–20
	500-ton scale	6–7	7–9
	1000-ton scale	4–5	5–6
Total costs, items 1, 2, 3, 4	100-ton scale	21–30	44–62
	500-ton scale	15–22	36–51
	1000-ton scale	13–20	34–48

(The above estimate includes only those items directly chargeable to flotation.)

For agitation-froth flotation add 1¢ to 2¢ per ton for additional power. The following actual costs at Utah Apex, grinding to 60-mesh and making a lead-zinc separation verify the estimate: Power, 6.2¢; repairs, 1.8¢; reagents, 21.9¢; labor, 4.0¢; total, 33.9¢ per ton of flotation feed. Total cost was \$1.60 per ton of crude ore, hence flotation was 21 per cent. of the total.

Total milling costs in a number of flotation plants follow (*J. M. Callow*):

Plant	Year	Tons per 24 hr.	Cost, dollars per ton
Utah Copper Co.(*b*)	1920	30,000	0.61
Inspiration(*b*)	1916	15,000	0.53
Nevada Cons. Cop. Co.(*b*)	1918	12,000	0.93
Miami(*b*)	1923	6,000	0.64
Porphyry Copper (*a*),(*b*)	1926	4,200	0.60
Porphyry Copper(*a*).(*b*)	1926	3,500	0.55
Utah Cons. Copper Co.(*c*)	1920	1,000	1.10
Magma Copper Co.(*b*)	1916	400	1.04
Bluestone(*b*)	1918	500	1.23
Utah Apex(*d*)	1924	400	1.36
Silver Dyke(*e*)	1923	400	1.36
Eustis Copper Co.(*c*)	1925	175	1.54

Notes: ***a*** Name withheld. ***b*** Straight sulphide copper. ***c*** Copper-iron differential. ***d*** Lead-zinc differential. ***e*** Lead-zinc-iron; tables and flotation.

18. Flotation of gold and silver ores

When precious metals are intimately associated with base-metal sulphides, or the precious-metal value is a sulphide, or in the metallic form and sufficiently finely divided and present in sufficient quantity or with a sufficient quantity of base-metal sulphides to stabilize a froth, flotation may be employed and good recovery made. Low-grade non-sulphide precious-metal ores cannot be treated by flotation because there is not enough metallic substance present to stabilize froth.

Kaanta (1,539,120/1925) describes a process for treating precious-metal ores mixed with sulphides, consisting of pre-treatment by grinding and agitation in a dilute solution of alkaline cyanide (about 3 lb. per ton) and a collecting agent, *e.g.*, coal tar, in order to brighten and coat the precious metals, followed by agitation with a precipitant such as aluminum or

zinc dust, sodium sulphide or charcoal, for any dissolved precious metals, after which a frothing agent is to be added and flotation carried forward in the usual fashion. He states that the process is also applicable to oxidized ores.

At Vindicator Consolidated Gold Mining Co. mill (*114 P 202*) results on a 300-ton basis showed $40.25 concentrate and $0.375 tailing from $5.40 feed, which is a recovery of 93 per cent. Del Mar (*123 P 497*) states that at a mill treating a silver ore containing native silver and polybasite a K. and K. machine with crude petroleum and steam-distilled pine oil averaged 1189-oz. concentrate and 1.5-oz. tailing from feed containing 11.3 oz. silver. At a gold mill treating oxidized ore containing lead carbonate, the same type of machine with the same oil mixture made 22.2-oz. concentrate and 0.14-oz. tailing from a feed assaying 0.87-oz. gold.

Flotation *vs.* amalgamation and cyanidation. The following summary accords generally with experience: (1) If amalgamation will recover any considerable portion of the precious-metal values, this part will be recovered more cheaply than by either cyanidation or flotation; the amalgamation treatment will not interfere with subsequent practice of either of the other processes, and should be installed. (2) Cyanidation requires a more expensive plant than flotation. (See Sec. 23.) (3) Operation of the cyanide process is, in general, more expensive than operation of the flotation process. (4) Clean-up and refining of cyanide precipitate is cheaper, per ounce of metal in the original ore, than the recovery of metal from flotation concentrate. (5) If the gold remaining after amalgamation is free, *i.e.*, not associated with or included in sulphides, cyanidation will almost surely make a considerably lower-grade tailing than flotation; if the precious metals are associated with sulphides, flotation will probably make the better recovery and will probably, also, not require the pulp to be ground so fine as for cyanidation. This is because it is not necessary for the gold and silver in a sulphide particle to be exposed to the pulp liquid in order that they may pass into flotation concentrate, but it is necessary that they be exposed, if they are to be dissolved in cyanide solution. (6) The base-metal sulphides may be extremely deleterious in cyanidation, and will certainly be largely wasted in a cyanide mill, while in a flotation mill they may add value to the concentrate.

Rickard (*115 P 265*) has analyzed comparisons at several plants. At North Star, 80 @ 1050-lb. stamps crushed 110,000 tons per year to pass 20-mesh. The product was amalgamated, tabled, classified, and sand and slime cyanided separately. Table concentrate was re-ground to 200-mesh and cyanided. Amalgamation recovered $8 per ton, cyanidation $2. Tailing assayed $0.25 to $0.35 per ton; recovery was 97 per cent. The cost, including tailing loss, was $1.14 per ton in 1915, which could be reduced to $0.98 per ton. To treat the residue from amalgamation by flotation required re-grinding to pass 80-mesh; flotation tailing assayed $0.25 to $0.30 per ton. Ratio of concentration was 30 : 1 and concentrate assayed $70 to $90 per ton. Concentrate handling and smelting cost $17 per ton of concentrate or, roughly, $0.50 per ton of ore. Cyanidation of flotation concentrate required re-grinding to 200-mesh, consumed 6 lb. of cyanide per ton of concentrate, and the tailing assayed $6 per ton. Assuming $0.25 per ton for re-grinding, $0.30 per lb. for cyanide, and $2 per ton of concentrate for precipitating and refining of bullion makes the cost per ton of concentrate, including losses, substantially $10. Flotation operation and concentrate treatment (excluding loss) were estimated at $0.63 per ton, tailing loss was taken at $0.25 per ton and concentrate-treatment loss would amount to $0.20 per ton; total, $1.08 per ton. At a Mexican mill cyanidation recovered 77 per cent. of the gold-silver content and gravity concentration 14 per cent., milling cost was $1.55 per ton and tailing loss $1 per ton. Flotation tests indicated recoveries between 70 and 83 per cent. on $10.50 ore at a cost about as much below that of tabling and cyanidation as is equivalent to the decreased recovery. Flotation concentrate required to be shipped to a United States smelter which cost more per ton of original feed than treatment of cyanide-plant product. At Melones Mining Co., pyritic-quartz gold ore assaying $3.65 per ton was treated by amalgamation, tabling, classification and cyanidation at a cost of $0.50 per ton, recovery 86 per cent. Re-grinding and flotation of amalgamation tailing was estimated at $0.47 per ton and recovery at 90 per cent. The total estimated difference in return on this basis was $0.146 per ton, assuming similar cyanide treatment of gravity and of flotation concentrate.

At Dutch-App, stamping, amalgamation and gravity concentration cost $0.38 per ton, concentrate handling, treatment and loss cost $0.36 per ton of mill feed and tailing loss was $0.90 per ton, total $1.64 per ton. Flotation treatment in the remodeled mill cost $0.35 per ton for stamp crushing and ball milling, $0.15 for flotation, $0.38 for concentrate handling, treatment and loss, and $0.35 tailing loss; total $1.23 per ton. At the Argo custom mill, the usual ore contains gold and silver associated with pyrite, chalcopyrite and tetrahedrite. The first step is table treatment to remove coarse gold. Flotation made slightly better recovery on table tailing than cyanidation and cost less; it was sufficiently flexible to handle ores ranging from $1.25 to $80 per ton and copper was not a hindrance but rather an advantage in that it added value to the concentrate. At Portland, a reverse result was shown, resulting in change from flotation back to cyanidation. Treating $2.25 dump material, flotation cost $0.10 per ton more than cyanidation for grinding, as flotation required 48-mesh grinding against 20-mesh for cyanidation for substantially equal recoveries. Flotation had to bear also a royalty charge. Flotation concentrate was highly silicious and more expensive to treat than cyanide precipitate and high-grade table concentrate. The smelters paid $20 per oz. for gold in concentrate and nothing for silver; the mint paid $20.67 for gold and 95 per cent. of New York quotation for silver. Wartenweiler (*17 JCM 87*) reports a comparative test on a Transvaal gold ore containing considerable arsenopyrite. The feed contained 0.7 oz. Au per ton. Recovery by amalgamation was negligible. Grinding through 150-mesh and cyaniding gave a recovery of 45 per cent. Flotation of −90-mesh pulp gave a concentrate assaying 3.35 oz. Au and representing 71 per cent. recovery. Tailing and middling, assaying 0.20 and 0.435 oz. respectively, yielded a final tailing of 0.089 oz. after cyanidation, the interfering substances having been thrown into the concentrate by the flotation operation.

DIFFERENTIAL FLOTATION

19. Introduction

The term differential flotation is used in contradistinction to the phrase collective flotation. The latter indicates an operation in which all of the minerals of one general class, *e.g.*, sulphides, are separated by flotation from all minerals of the non-sulphide class; the first term describes separation effected between two or more minerals of the same class, *e.g.*, lead sulphide from zinc sulphide, or one non-metallic mineral from another. Typical cases are: galena from blende, pyrite from blende, chalcopyrite from pyrite or pyrrhotite, chalcopyrite from blende; carbonates of copper from rock-forming silicates, mica from rock-forming silicates, and coking from non-coking bituminous coal. Fluorspar may be separated from quartz, apatite and tri-calcium phosphate from rock-forming silicates, cassiterite from quartz, cerussite from cassiterite, etc. Separation is never sharp. Differential galena concentrate, for example, will contain some gangue and considerable zinc, the blende concentrate will contain considerable gangue and considerable lead, and the tailing will contain more or less of both sulphides.

It is possible, by proper choice of the method of treatment, to float the whole of the solid matter of a given pulp and leave behind clear water; at the other extreme a frothing operation can be conducted on the same pulp to yield nothing but watery bubbles. In the first case all of the factors tending toward the formation of solid-carrying froths have been over-emphasized, in the latter, these factors are missing or weakened to such a degree as to have become ineffective. Differential flotation is performed by careful control of flotation conditions so that a particular degree of flotation between these extremes is effected.

The factors whose control is most important are: (*a*) intensity of frothing, (*b*) sulphide filming, (*c*) adsorption of gangue at sulphide surfaces.

Intensity of frothing is controlled by varying the kind and quantity of frothing agent and the intensity of gasification.

Frothing agents that make fragile, effervescent froths yet which, when present in extremely minute quantities, have considerable frothing effect, are

most desirable for differential flotation. The best known of these are pine oil, eucalyptus oil, cresylic acid and certain coal-tar and wood-tar creosotes of high phenol content. These agents are all relatively soluble in water, small quantities have large effect on the liquid-gas surface tension, they are readily dispersed with a minimum of agitation, and the amount of frothing is substantially proportional to the amount of agent added.

Gasification comprises introduction of gas into the pulp and, in pulp-body processes, precipitation onto the desired mineral particles. Intensity of gasification involves volume alone in bubble-column processes; in the agitation-froth process, additionally violence of agitation and heat. If the intensity of gasification is low, *e.g.*, if the volume of gas is small, in a bubble-column machine, only the most easily floated mineral is raised to the surface of the bubble column.

Shellshear points out (*115 P 613*) that the upper part of the bubble column in a subaeration machine in lead-zinc flotation is much richer in lead than the lower part, while the lower part is richer in blende.

The mineral raised may be the most easily floated because it has been previously preferentially filmed with collecting agent; or because, though equally filmed, it is inherently easier to float; or, what is usually the case, because the other sulphide is more heavily coated with adsorbed gangue (see page 844). In any case, intense gasification may be sufficient to float the less-floatable mineral, when, of course, the more-floatable is raised also and no differential action occurs.

Agitation. In the agitation-froth machine volume of gas and passage through solution in the water of the pulp both increase with increasing violence of agitation, hence differential work requires reduced speed for the first flotation.

Del Mar (*117 P 691*) says that galena floats under given conditions in a Minerals Separation machine at 200 r.p.m. while blende requires 600 r.p.m. under similarly favorable conditions.

Heat decreases the solubility of gas in the liquids of the pulp and hence increases the rate at which it precipitates, all other things being equal. Hence heating a pulp increases intensity of gasification. It also affects gangue coating.

Machines. It is an extension of the same reasoning that explains the fact that bubble-column and agitation-froth machines are unequally applicable to differential processes. Gasification is so intense in agitation machines that they are unsuitable for delicate differential work and can be used for the first frothing only in those cases where the differentiating effect due to the reagents is great.

Shellshear (*115 P 616*) notes this and says that the agitation-froth machine may be used in the Bradford salt process because of the intense differentiating effect of the solution but that the Lyster process, with only slight differentiation effected by the chemical character of the solution, works best in a bubble-column machine. On the other hand, the agitation-froth machine may give much the better recovery of the second sulphide, which, having been intentionally deadened before the first flotation operation, may be difficult to raise in the second.

Size of particles is of greater importance in differential flotation than in collective because of the fact that the first float is made with feeble flotation conditions, *i.e.*, the buoyant effect of the gas is lessened as much as is possible, consistent with any flotation at all, and as a consequence any desired particle that is unduly heavy is likely to go into the tailing. When galena is floated

from blende it is frequently the finest of the galena only that floats. For this reason the tailing from gravity concentration is particularly suitable for lead-zinc differential flotation since the coarse lead has already been removed and hence is not lost in the lead-flotation tailing.

Pulp density is usually above normal, *e.g.*, 25 to 50 per cent. solids during lead flotation and drops to 20 to 25 per cent. solids in zinc flotation. Maintenance of constant conditions is particularly important in the first flotation.

Filming is of paramount importance. Within certain limits, the mineral or minerals that will be filmed by a given collecting agent in a given mixture of minerals is dependent on the quantity of agent present; *i.e.*, if too little reagent is present for filming all of the sulphide minerals in the pulp, one of the sulphides may be filmed in preference to the others; if, on the other hand, the quantity of reagent is in excess of that required to float the sulphides, gangue minerals will usually begin to float. Thus one method of producing differential flotation is to add such a small quantity of filming agent that it will all be appropriated by one mineral. This method is known as the STARVATION METHOD.

The same filming agents that are used in collective flotation of a given mineral are frequently effective in differential flotation.

Gangue adsorption on sulphide surfaces prevents filming by collecting agents and thus prevents flotation. Control of gangue adsorption is the most important operation in differential flotation. Control is effected by the use of agents, mostly inorganic, of which the principal ones are lime, sodium carbonate, sodium cyanide, sodium hydroxide, sodium silicate, sodium sulphide, copper sulphate, sulphur dioxide and zinc sulphate. The active phenomena are described at page 845.

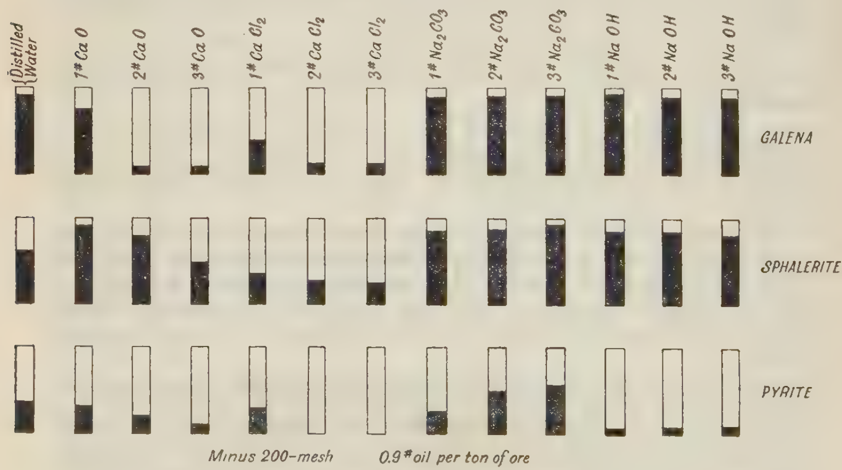


FIG. 78.—Relative floatabilities.

Soda ash aids in producing a high-grade lead concentrate with "starvation" quantities of oil on some lead-zinc ores and shows differential effect in certain copper-iron separations (*205 Bul. USBM 23, 53; 36 Aa 98*). Sodium silicate and sodium carbonate have been used to aid lead flotation with coal-tar creosotes (*120 P 459*). Sodium sulphide is used to keep down blende in the presence both of galena and chalcopyrite (*114 J 629*) but Fahrenwald (*4 UId 16*) says that blende tends to float in preference to galena if the solution is

alkaline with sodium hydroxide or sodium sulphide. Kirtley says (*104 J 64*) that when more than 5 lb. per ton of sodium sulphide is used it raises iron but keeps down zinc, while with less than 1 lb. per ton the zinc is lifted; an excess lifts gangue to the exclusion of sulphides and forms a watery froth. Sodium sulphide may be used to keep down both chalcopyrite and blende in the presence of molybdenite or to keep down chalcopyrite in the presence of galena (*114 J 629*). Gates and Jacobsen (*16 UU 38*) investigated the effect of various agents on the floatability of galena, sphalerite and pyrite. Their results are summarized in Fig. 78. See also p. 846.

Tucker and Head (*73 A 354*) made an ingenious investigation of the effects of cyanide, zinc sulphate and lime. A summary of the results of their tests is presented in Table 24. The tests show that cyanide plus zinc sulphate lessen the floatability of zinc

Table 24. Effect of cyanide on floatability of lead, zinc and iron sulphides. (*After Tucker and Head*)

Test number	Reagents, 0.92 lb. Barrett No. 4, Yarmour, 0.25 lb. plus						Floatability		
	CaO	Na_2CO_3	$NaCN$	$ZnSO_4$	$CuSO_4$		Galena	Sphalerite	Pyrite
1	....	....	....	....	....		92	62	33
2	1.0	....	....	....	....		76	74	30
3	2.0	....	....	....	....		6	56	18
4	3.0	....	....	....	....		8	36	7
5	....	1.0	....	....	....		92	70	28
6	....	2.0	....	....	....		93	70	48
7	....	3.0	....	....	....		92	68	55
8	....	....	....	....	....		92	87	32
9	....	....	1.0	....	....		96	82	21
10	1.0	....	1.0	....	....		93	83	8
11	2.0	....	1.0	....	....		43	82	15
12	3.0	....	1.0	....	....		32	83	18
13	....	1.0	1.0	....	....		96	88	35
14	....	2.0	1.0	....	....		97	87	36
15	....	3.0	1.0	....	....		98	85	35
16	....	....	....	....	....		92	62	30
17	....	....	1.0	2.0	....		93	5	27
18	1.0	....	1.0	2.0	....		94	4	25
19	2.0	....	1.0	2.0	....		26	4	15
20	3.0	....	1.0	2.0	....	Distilled	15	5	8
21	....	1.0	1.0	2.0	....	water...	94	7	34
22	....	2.0	1.0	2.0	....		94	5	21
23	....	3.0	1.0	2.0	....		96	2	12
24	....	....	....	....	....		.....	62	32
25	....	....	....	....	2.0		.....	65	38
26	....	....	1.0	2.0	2.0		.....	51	30
27	1.0	....	1.0	2.0	2.0		.....	42	16
28	2.0	....	1.0	2.0	2.0		.....	14	21
29	3.0	....	1.0	2.0	2.0		.....	10	14
30	....	1.0	1.0	2.0	2.0		.....	34	24
31	....	2.0	1.0	2.0	2.0		.....	31	16
32	....	3.0	1.0	2.0	2.0		.....	17	16
33	....	....	....	....	....		93	62	35
34	....	....	....	2.0	....		90	20	45
35	1.0	....	....	2.0	....		30	21	33
36	2.0	....	....	2.0	....		7	19	19
37	3.0	....	....	2.0	....		5	16	13
38	....	1.0	....	2.0	....		77	23	49
39	....	2.0	....	2.0	....		75	24	39
40	....	3.0	....	2.0	....		73	26	38

and iron markedly but do not substantially affect the floatability of lead; that this depression is not entirely due to either the cyanide or the sulphate alone; that lime depresses lead and hence cannot be used for insuring alkalinity, but that sodium carbonate can be used;

and that copper sulphate added to a pulp in which blende and pyrite have been sickened by cyanide and zinc sulphate, permits the blende to be floated away from the pyrite.

The most enlightening part of the work consisted in microscopic study of the surface effects of the solutions. Individual grains of the various sulphides, half-coated with paraffin, were fastened to the shaft of an agitation-froth machine and this was run for 20 min. with a solution of zinc sulphate and sodium cyanide. The sulphide particles were then removed, washed free of paraffin and microphotographed. The photographs showed substantially complete loss of luster by the pyrite and sphalerite surfaces, while the galena surface was unaffected.

This work may explain the differential action of these reagents when the gangue is one that does not adsorb at the sulphide surfaces under the conditions of the flotation operation, but it was impossible to duplicate the tests at Columbia University. On the other hand, the Columbia work showed that with a typical clayey slime which adsorbed heavily on galena, sphalerite, pyrite and chalcopyrite in the presence of pine oil and sodium carbonate or bicarbonate, addition of potassium cyanide and zinc sulphate substantially prevented adsorption at the galena or chalcopyrite surfaces while adsorption on sphalerite and pyrite was, if anything, increased.

Sulphur dioxide is a powerful retarder of blende flotation. It is particularly useful when the galena is relatively difficult to float and flotation conditions must, therefore, be intensified, or when a galena-blende collective float is to be separated.

Shellshear (*115 P 616*) points out that it is very difficult to secure differential flotation of a collective float. This is because a part of the differential phenomenon is selective prevention of filming of the sulphide minerals by collective agents. After all sulphides are filmed as in the case in a collective float, one of the means of differentiation has been removed and more of the burden is consequently placed on the others, resulting in added difficulties. Acid is frequently an aid to differential flotation of lead in the presence of SO_2. On the other hand it is more usual to find that lead flotation is best effected in neutral or alkaline pulp with acid added for zinc flotation, if at all.

Control. The difference in floatability between different sulphides is much less than the difference between sulphides as a class and the rock-forming minerals as a class, hence it follows that differential flotation is a much more delicate operation than collective flotation. Close control of the effective flotation factors is essential to success and regularity of operation is important.

List of agents for differential flotation patented in the United States prior to Jan. 1, 1926.

Acacia, 1,446,376.
Acid, 1,126,965, 1,182,890, 1,260,668, 1,269,157, 1,446,376, 1,469,042.
Albumin, 1,499,872.
Alkali, 956,381, 1,142,821, 1,157,176, 1,203,372, 1,203,373, 1,203,374, 1,257,990, 1,261,810, 1,317,945, 1,446,376, 1,469,042, 1,478,697.
Alkaline carbonate, 1,301,551.
Alkaline salt, 1,421,585, 1,427,235.
Alkaline sulphite, 1,486,297.
Ammonia, 1,254,173.
Barium chloride, 1,203,372, 1,203,374.
Barium sulphate, 1,446,376.
Bicarbonate, 1,203,372 (see also Sodium ——).
Bichromate, 1,102,738, 1,142,821, 1,257,990 (see also Potassium ——).
Bi-sulphites, 1,274,505.
Bleaching powder, 1,300,516.
Calcium chloride, 1,203,372, 1,203,373, 1,203,374.
  hydrate, 1,203,373, 1,203,374, 1,254,173.
  nitrate, 1,203,372, 1,203,373.
  sulphate, 1,203,372, 1,446,376.
  sulphite, 1,486,297.
Carbonaceous material, 1,261,810.
Carbonate, 1,203,372, 1,486,297.
  of an alkali, 1,486,297.
  of an alkali metal, 1,486,297.
Charcoal, 1,261,810.
Chloride of an alkali metal, 1,182,890, 1,203,372, 1,203,373, 1,203,374.
  alkaline earth, 1,182,890, 1,203,372, 1,203,373.
  a metal, 1,203,372, 1,203,374 (see also Specific metals).

Starch, 1,499,872.
Sulphates, 1,203,372 (see also Specific metals).
Sulphide of an alkali, 1,469,042 (see also Sodium ——).
  alkaline earth, 1,469,042.
Sulphites, 1,182,890, 1,274,505, 1,478,697, 1,486,297.
Sulphur, 1,401,435.
  acids (lower), 1,182,890.
  dioxide, 1,274,505, 1,486,297.
Sulphuric acid (see also Acid), 967,671, 1,020,353, 1,269,157, 1,274,505, 1,401,435, 1,486,297.
Sulphurous acid, 1,274,505.
Tannin, 1,499,872.
Thiosulphites, 1,182,890, 1,274,505.
Tri-sodium phosphate, 1,446,376.
Zinc sulphate, 1,203,372, 1,421,585, 1,427,235.

Parsons (*Bull. 617, Mines branch, Canada Dept. of Mines, 1923*) gives the following summary of flotation agents found useful for differential work at the laboratory of the Bureau. For floating galena in the presence of zinc and iron sulphides: A mixture of coal tar and coal-tar creosote; cresylic acid; thiocarbanilid; steam-distilled pine oil; light hardwood-creosote oil; lime (but see Gates and Jacobsen, p. 871); sodium sulphite. For floating blende from iron sulphides after lead or copper sulphide has been removed; K. and K. oil No. 2, Southwestern Engineering Co.; Barrett No. 634 coal-tar creosote; sodium creosote from hardwood; thiocarbanilid and xylidin (General Engineering Co. Y-Z mixture); thiocarbanilid and orthotoluidin (Gen'l Engineering Co. T-T mixture); fuel oil, 34° Bé., G. W. Oil Co.; Barrett water-gas tar; copper sulphate; soda ash. For floating copper sulphides from zinc and iron sulphides: a mixture of coal tar and coal-tar creosote; xylidin and alpha-naphthylamine (Gen'l Engineering Co. X-Y mixture); neutral fractions of hardwood oils; lime. For floating copper sulphides from iron sulphides; Barrett No. 634; X-Y mixture; T-T mixture; Y-Z mixture; Thio-fizzan (Gen'l Engineering Co.); sodium resinate; paraffin-base fuel oils; lime; soda ash. For floating copper-nickel sulphides from iron sulphides: Thio-fizzan; T-T mixture; X-Y mixture; paraffin-base fuel oil; mixtures of coal-tar and coal-tar creosote; soda ash. For floating molybdenite and graphite from iron sulphides: kerosene; lime. For frothing agents: steam-distilled pine oil; Fumol (Canadian Electro Products Co.); Ketone oil (Standard Chemical Co.).

PROCESSES

Processes of differential separation may be divided into three groups as follows:

1. Processes in which the surfaces of the particles of one of the normally floatable minerals are permanently changed by chemical action so as to render the particles non-floatable. The chemical action utilized may be either pyro-chemical or hydro-chemical.

2. Processes in which the surfaces of the particles of one of the normally floatable minerals are temporarily changed so as to render these particles non-floatable, but in which the deadening effect may be subsequently eliminated or overcome by suitable treatment.

3. Processes in which the activity of one or more of the physical phenomena that combine to effect flotation is limited or controlled to such an extent that at first only the most readily floated of the floatable constituents is raised.

Some of the patented processes do not fall easily into any of the three groups.

20. Processes involving permanent change of one of the floatable minerals

These are not properly differential-flotation processes, for the reason that before flotation is attempted particles of an originally floatable class have been permanently changed into the non-floatable class by an irreversible chemical reaction and subsequent flotation differentiates only between readily-floatable and non-floatable particles rather than between two minerals of the readily-floatable class.

The dry-chemical patents in this class follow.

Fractional roasting

Wentworth (938,732/1909) points out that different compounds of the same base metal differ widely in their behavior in skin flotation, their susceptibility to oiling and their ability to precipitate gas from solution onto their surfaces and that these differences can be utilized to effect separation by flotation. He points out further the differences in the roasting temperatures of different metallic sulphides and that, if mixed sulphides are roasted at a temperature below the oxidizing temperature of one of them but above that of the others, the latter are converted into non-floatable material while the former retains its floatability. Iron and lead sulphides are converted into oxides and sulphates by roasting at a dull red heat, while zinc is unaffected at this temperature, hence flotation treatment subsequent to a low-temperature roast of a mixture of the three sulphides results in flotation of the zinc sulphide away from the oxidized and non-floatable iron and lead compounds. Wentworth classes chalcopyrite among the easily-roasted sulphides.

Ramage (949,002/1910) describes a process similar in general principle to Wentworth, but states that chalcopyrite is more resistant to oxidation than blende. He recommends roasting a mixture of pyrite, blende and chalcopyrite at 600° C. to decompose pyrite, floating blende and chalcopyrite as a collective concentrate, roasting this at 700° C. to decompose blende and dissolving the oxidized blende from the roasted residue with sulphuric acid. Elsewhere in the same patent he recommends a roasting temperature of 800° C. to decompose cobaltite, niccolite and sulphides and sulpharsenides of silver, leaving chalcopyrite unchanged and floatable. Ralston (*110 P 980*) questions these statements in so far as they set up such relatively great refractoriness for chalcopyrite.

Horwood (1,020,353/1912) describes slow roasting of a collective sulphide concentrate, *e.g.*, galena-pyrite-blende, at say 400° to 500° C. to sulphatize the galena and oxidize the pyrite, and subsequent flotation of the unaffected blende. He also describes a wet chemical method of effecting the same result. In a later patent (1,108,440/1914) Horwood describes an improvement consisting in a water wash prior to roasting in order to remove soluble salts and thus lessen the oxidation of zinc in the roasting operation. He claimed also that the water wash causes silver to be deadened with the lead and that such roasted material could be subsequently floated at 120° F. instead of at 180° as formerly.

Roasting is the difficult part of this process. The extent of sulphatization necessary depends upon the character of the ore and the fineness of grinding. Horwood (*16 Aa 229*) says that with Broken Hill slimes, 75 per cent. sulphatization is necessary while with granular material of the same composition 25 per cent. is sufficient, and that with some ores, even when slimed, 10 per cent. is sufficient. Clark (*89 J 460*) states that in roasting a pyrite-galena-blende concentrate the pyrite oxidizes first at 300 to 400° C., then the galena, and that about three hours is required for the roast. At the Zinc Corporation (*Pro. Austr. I.M.E. (1913) No. 12*) roaster feed assayed 39 per cent. Zn, 16 per cent. Pb and 18 oz. Ag; rough zinc concentrate, 49.5 per cent. Zn, 6 per cent. Pb and 11 oz. Ag; lead residue, 9 per cent. Zn, 47 per cent. Pb and 45 oz. Ag. The roaster was an Edwards duplex-type, 14 × 102-ft. with 12 panels and 48 rabbles. Feed contained 10 to 12 per cent. water. Fuel consumption was 4.8 per cent. of the ore charged. It was necessary to sulphatize about 30 per cent. of the lead to effect good results in the furnace. Rain (*113 P 529*) says that for proper roasting the ore must be crushed to 80-mesh. The roast should start at a low temperature and be gradually raised to 500° C. and the charge rabbled freely. Only the surface of the particles need be sulphatized. Control is effected by determining the necessary degree of roasting by a flotation test, calibrating this chemically, then using chemical control. Heller (*119 P 151*) describes an elaborate trial of the process on a copper-zinc ore at the Afterthought mine on a scale of 275 to 400 tons original feed per 24 hr. The feed was ground wet to 48-mesh, a collective float, recovering 90 per cent. of the copper and 85 per cent. of the zinc, was made with stove oil and Pensacola No. 80. The best assay for the collective float was 30 per cent. Zn, 6 per cent. Cu, 18 per cent Fe, 10 per cent. insoluble, 6 per cent. alumina and 4 per cent. lime. If there was less silica, the roast was too quick; if the concentrate was too low-grade, the copper residue was likewise low-grade. Roasting was done in a 25-ft. 9-hearth Wedge-type Skinner furnace with the five lower hearths arranged for rapid cooling by the removal of the brick lining and quick withdrawal of hot gases. The best temperature range was 920° F. maximum and 400° F. or lower on the bottom hearth. Slow cooling was particularly important; quick cooling killed selective action in the float. The zinc froth assayed 2.6 per cent Cu, 47.1 per cent. Zn, 4.2 per cent. Fe, 3.3 per cent. $BaSO_4$, 4.4 per cent. CaO, 3.8 per cent. Al_2O_3, 3.2 per cent. SiO_2; copper residue, 7.2 per cent. Cu, 12.3 per cent. Zn, 23.4 per cent Fe, 8.2 per cent. $BaSO_4$, 12.7 per cent. CaO, 10.5 per cent. Al_2O_3, 10.4 per cent. SiO_2. There was about 8 oz. Ag per ton in each concentrate. The costs were: collective flotation, 10¢ per ton; flotation of roasted material, 15¢; roasting and cooling, 30¢; royalty, 50¢; total milling cost, $2.09.

Wood (1,263,503/1918) describes the production of a collective float of molybdenite and pyrrhotite followed by high-temperature drying, *i.e.*, substantially flash roasting, to very slightly oxidize the iron, with subsequent flotation of the molybdenite.

The flotation processes described in the specifications of these various patents are the old-time processes of skin flotation and chemical-generation (see Art. 2 and 4) but the method of the patents is equally applicable to both agitation-froth and bubble-column processes. No extensive application of any of the processes has ever been made, and no practice exists at present.

The wet-chemical processes of this class, like the dry-chemical, all depend on reactions with the lead and iron minerals to change the surfaces of these while zinc and copper sulphides are left unaffected. A distinct disadvantage is the fact that silver sulphides are not attacked and therefore go with the zinc.

Ramage (967,671/1910) treated a finely-ground, un-oiled ore containing a mixture of lead, iron and zinc sulphides with a 10 per cent. solution of sodium nitrate or sodium chloride and sulphuric acid at 140° F., forming a pulp containing 25 to 30 per cent. solids. As a result blende rose in the form of a heavy, spongy froth. Subsequently raising the temperature to 180° F. caused iron sulphides to be raised while the lead was permanently oxidized or chloridized and stayed behind with the gangue.

Wentworth (970,002 and 980,035/1910) proposes treating moist ground ore with chlorine (or another halogen) in order to chemically change and deaden the surfaces of some of the floatable minerals. He states that at blende surfaces an oily-appearing substance is formed which may be chloride of sulphur and which accentuates the floatability of the blende, while at pyrite surfaces a water-soluble compound is formed which induces wetting and sinking of the affected particles. Chalcopyrite is placed in the non-affected class.

Mac Gregor (972,459/1910) has the same idea.

Horwood (1,020,353/1912) describes digestion in nitrated sulphuric acid solution to sulphatize galena and oxidize pyrite, leaving blende substantially unchanged.

Greenway and Lowry (1,102,738/1914) describe digestion in a solution containing a salt of chromium, *e.g.*, sodium or potassium bichromate, in order to render galena and iron unfloatable. After digestion the chromate solution is decanted and the residue floated with organic reagents in the usual manner. They cite the following EXAMPLES: (1) Chalcopyrite-pyrite ore crushed to 100-mesh and digested 30 min. in a hot 1 per cent. solution of sodium bichromate; solution decanted and the residue floated by the agitation-froth process using 1 lb. of eucalyptus oil per ton of ore. Assays: Feed, 6.5 per cent. Cu and 35 per cent. Fe; concentrate, 19.0 per cent. Cu and 30.2. per cent. Fe; tailing, 0.7 per cent. Cu and 36.2 per cent. Fe. (2) A galena-blende collective concentrate similarly treated. Assays: Feed 18.6 per cent. Pb and 32.2 per cent. Zn; concentrate, 47.2 per cent. Zn and 6.3 per cent. Pb; tailing, 31.6 per cent. Pb and 16.3 per cent. Zn. (3) A molybdenite-pyrite ore floated with 1 lb. of eucalyptus oil per ton at 120° F. in a 0.25 per cent. solution of sodium bichromate. The pulp contained 20 per cent. solids. Assays: Feed, 15 per cent. molybdenite and 25 per cent. pyrite; concentrate, 93 per cent. molybdenite and 4.9 per cent. pyrite.

Lavers (1,142,821/1915) describes the use of chromate in a purposely alkaline solution *e.g.*, 1 per cent. sodium carbonate, preferably at 120° to 150° F. EXAMPLES: (1) Lead-zinc-iron ore treated by agitation-froth flotation in a pulp containing 20 per cent. solids at 130° F. adding 22 lb. of sodium carbonate, 6 lb. of sodium bichromate and 0.5 lb. each of eucalyptus oil and kerosene per ton of ore. Assays: Feed, 9 per cent. Pb, 28.2 per cent. Zn and 14.2 per cent. Fe; concentrate, 50.1 per cent. Zn, 4.25 per cent. Pb and 8.3 per cent. Fe; tailing not given, and probably, therefore, not worth giving. (2) A lead-zinc ore, agitation-froth flotation, 20 per cent. solids, 130° F., 24 lb. sodium carbonate and 1 lb. eucalyptus oil per ton. From a feed assaying 11.6 per cent. Pb and 13.4 per cent. Zn a collective concentrate assaying 22.2 per cent. Pb and 27.4 per cent. Zn was obtained. This concentrate was re-floated at 130° F. with 24 lb. sodium carbonate, 6 lb. sodium bichromate, 0.12 lb. eucalyptus oil and 0.75 lb. kerosene per ton of ore producing a float concentrate assaying 48.6 per cent. Zn and 7.5 per cent. Pb and a leady residue assaying 55.9 per cent. Pb and 8.9 per cent. Zn.

Coghill and Anderson (*TP 283, USBM*) report an extensive series of tests on complex lead-zinc-iron ores from Colorado mines, using slightly-acid dichromate solutions with oil. They found that, in the laboratory, at least, the following procedure was applicable to all the ores tested: (1) Crush to about 50-mesh in such a way as to produce as little fine lead as possible, separate − 120-mesh material and table the oversize to recover lead, (2) Re-grind table tailing to 120-mesh and join with the original slime. (3) Float with coal tar, cresylic acid (or steam-distilled pine oil or xylidin) and Na_2S to obtain a differential lead concentrate; add more of the frothing agents and make a second froth for tabling; finally add Cleveland Cliffs No. 2 and float the balance of the mixed sulphides and leave a final tailing. (4) Table the second overflow above and obtain a lead concentrate and tailing. Re-grind and re-float the tailing with further coal tar, cresylic acid and sodium sulphide; collect lead concentrate and send tailing to join the froth obtained with Cleveland

Cliffs oil. (5) Table the Cleveland Cliffs-oil froth (together with the re-floated-middling underflow) and make lead concentrate and tailing. (6) Re-float this tailing with acid dichromate solution (1 lb. of dichromate per 4 per cent. of zinc and 0.1 lb. H_2SO_4 per lb. of dichromate) and a minute amount of the Cleveland Cliffs oil or No. 350 Pensacola Tar and Turpentine Co. oil; clean the froth one or more times as necessary to grade up the zinc concentrate, returning the middling to the dichromate rougher cell. The underflow of the dichromate cell is an iron-rich product. By this method they obtained results shown in Table 25. They found that while the acid dichromate would hold back coarse lead, it would not hold back fine lead and that its principal useful function was in holding down iron. It was necessary to grind to 120-mesh to float the zinc in the presence of dichromate.

Table 25. Results of treatment of Colorado complex ore. (*After Coghill and Anderson*)

Product	Assays					
	Per cent. Pb	Per cent. Zn	Per cent. Fe	Per cent. Cu	Au, ounces	Ag, ounces
Feed	6.83	6.37	5.2	0.44	0.02	10.92
Lead concentrate	33.9	6.51	18.2	1.57	0.08	45.78
Zinc concentrate	2.4	49.1	7.4	1.20	0.04	18.68
Zinc middling	3.2	14.0	13.8	0.34	0.02	11.28
Tailing	0.33	0.58	0.88	0.05	....	0.68
Totals	......	....	......	....	....	....

Product	Recoveries					
	Pb	Zn	Fe	Cu	Au	Ag
Feed	....	....	....	....	....	....
Lead concentrate	92.00	19.00	64.88	66.34	76.18	77.70
Zinc concentrate	3.16	68.20	12.59	24.10	17.70	15.10
Zinc middling	1.40	6.64	8.01	2.33	3.02	3.12
Tailing	3.34	6.24	11.65	7.62	2.60	4.22
Totals	99.90	100.08	97.13	100.39	99.50	100.14

Terry (1,254,173/1918) states that ammonia in solution oxidizes iron sufficiently to prevent oil coating. On the other hand it dissolves oxide from partially-oxidized lead, zinc and copper sulphides. He describes mixing a pulp first with ammonia, then with oil in a closed vessel to exclude air, then floating by the cascade method. Prior treatment of the ore in a ball mill with calcium hydrate and sodium thiosulphate are also recommended. Ammonia solution of 0.04 to 0.06 per cent. strength is required and the pulp should be digested for some time prior to flotation. Treating an ore containing zinc, iron and copper sulphides, it was found necessary to remove ammonia before flotation or copper floated with zinc. On this ore neutral pine oil and neutralized wood-creosote oil were best, coal-tar creosote would not give differential action. Laboratory tests showed concentrate carrying 48 to 60 per cent. zinc and 2 to 15 per cent. iron from feeds carrying 4 to 25 per cent. each iron and zinc; recoveries, 60 to 90 per cent. The cost of the process would be high on account of the high price of ammonia (*19 CME 319*).

Pellegrini (1,233,398/1917) proposes differential separation of mixed base-metal oxide ores by preferential sulphidizing and says that the process is particularly suitable for separation of lead vanadanate from other lead salts, particularly the molybdate. He arranges the lead minerals in decreasing order of rate of sulphidizing as follows: Carbonate, oxide, molybdate and vanadanate. Hydrogen sulphide, sodium sulphide and the like, in limited amounts, are recommended for sulphidizing.

Smith (1,452,662/1923) states that in an ore containing oxidized minerals of lead, zinc and manganese, the lead may first be sulphidized by the usual soluble sulphides in alkaline solution in a thick pulp and subsequently diluted and floated and that thereafter the tailing should be thickened and acidified, after which zinc and manganese minerals may be sulphidized.

21. Processes in which organic or inorganic chemicals are used to retard flotation of one of the normally-floatable minerals

Under this heading are included all those processes in which some substance, usually a salt, is added to a flotation pulp for the purpose of rendering one of the ordinarily-floatable minerals temporarily non-floatable. The action of the reagents is, usually, to affect the degree of adsorption of gangue at the mineral surfaces, so that the particles of one sulphide are gangue-coated and hence readily water-wetted just like gangue particles themselves, while the particles of the other sulphide present normal lustrous surfaces to the action of the collecting agent.

Owen (1,157,176/1915) adds a small amount of an alkaline permanganate or a somewhat greater amount of manganese dioxide in a neutral or alkaline (*e.g.*, NaOH) solution to effect retardation of blende in the presence of lead, silver and copper sulphides. Heating to 120° F is recommended when the ore pulp reduces the manganese compound rapidly. Subsequent flotation with acid raises blende but acid prevents differential flotation. Eucalyptus oil is recommended. The process is claimed to be particularly applicable to weathered material and weathering for a few hours is suggested as advantageous with some current mill pulps. Weathered Broken Hill slime assaying 16 per cent. Pb, 13.5 per cent. Zn and 17 oz. Ag per ton was floated in a pulp containing 20 to 25 per cent. solid with 2.5 lb. potassium permanganate and 0.3 lb. eucalyptus oil per ton of ore. Concentrate assayed 60.5 per cent. Pb, 11.8 per cent. Zn and 54 oz. Ag per ton; 15 lb. of sulphuric acid per ton of ore was then added to the residue and flotation continued. Concentrate assayed 43.2 per cent. Zn, 6.2 per cent. Pb and 11.2 oz. Ag per ton; tailing, 2.0 per cent Pb, 1.6 per cent. Zn and 3.0 oz. Ag per ton. Fahrenwald (*4 UId 19*) reports a test using about 0.25 lb. per ton of permanganate on a Cœur d'Alene ore containing organic matter and therefore difficult. Without permanganate the concentrate assayed 36.6 per cent. Pb and 22 per cent. Zn and represented 62 per cent. recovery of the lead; with permanganate the corresponding figures were 49.8 per cent. Pb, 14.3 per cent Zn and 65.5 per cent. recovery. Shellshear says that a small quantity of permanganate will keep down zinc and produce a high-grade lead float. An excess will keep down both sulphides unless heat and acid are used, when the lead can be floated. The process is economically impracticable on account of the high cost of permanganate.

Bradford (1,182,890/1916) states that with oleic acid and a temperature of 120° to 160° F. blende will float away from galena and pyrite in a solution, preferably weakly acid, of one or more chlorides of the alkaline metals or alkaline earths. The best strength of acid solution is between 0.1 and 0.2 per cent.; 1 per cent. solution causes galena and pyrite to float. The quantity of salt may vary widely, but 10 per cent. is suitable. Sodium chloride is preferred on account of cheapness. The lower sulphur acids and their salts (sulphites and thiosulphates) are stated to further retard galena and pyrite. With crude ores the oil may sometimes be omitted.

Shellshear (*115 P 616*) says that lead chloride is found in solution, but the fact that the lead can be subsequently floated by use of potassium permanganate or copper sulphate indicates that any surface change is exceedingly minute. The process yields clean zinc concentrate and the recovery of zinc is high.

Lyster (1,203,374/1916) states that at normal temperatures in a neutral or alkaline solution of a chloride of a metal, *e.g.*, NaCl, KCl, $BaCl_2$, $CaCl_2$, $MgCl_2$, with or without an organic frothing agent as the case may be, galena floats in preference to blende. Solution strengths range from 300 to 800 grains of chloride per gal. plus 1.8 to 18 grains $Ca(OH)_2$. Two pounds of eucalpytus oil per ton of ore is stated as a suitable kind and quantity of organic agent.

Lyster (1,203,372/1916) states that flotation at temperatures below 100° F. with, say 2 to 5 lb. per ton of eucalyptus oil in a neutral or alkaline, but not acid, solution of a sulphate, chloride or nitrate of calcium, magnesium, sodium or potassium; or a sulphate of manganese, zinc, or iron; or barium chloride; or a carbonate or bicarbonate of sodium or potassium; or any mixtures of these salts will cause galena to float in preference to zinc. The solution strengths recommended are 160 to 800 grains per gal. In patent 1,203,373/1916, the same patentee states that flotation at a temperature below 100° F. with eucalyptus oil or other frothing agent in alkaline solution, causes galena to rise in preference to blende. Examples of solution composition are: Sodium carbonate, 500 grains per gal.; sodium bicarbonate, 600 gr.; calcium chloride, 300 gr. with 300 gr. calcium hydrate, calcium nitrate 300 ft. with 18 gr. calcium hydrate. He warns that with certain ores, as the calcitic ores of Broken Hill, excessive alkalinity retards galena flotation. Patent 1,203,375/1916, to the same patentee adds nothing new.

Freeman (*36 Aa 95*) says that Lyster's process is workable with ores that contain blende and galena with large differences in floatability, but that the differentiating action of the salts is not nearly so intense as that of sulphur dioxide (Bradford) and that the latter is used when blende and galena are of varieties close together in floatability. Ralston (*110 P 980*) points out that these patents contemplate a minute amount of oil, excessive agitation and a crowded spitzkasten, all of which conditions, irrespective of the kind and quantity of salts present, would tend toward differential flotation.

Faul and Lavers (1,257,990/1918) describe flotation of blende from galena effected by adding metallic mercury, mercury amalgam or a mercury salt to the pulp. Retardation of galena may be aided by a bichromate in an alkaline medium (see also Pat 1,142,821). EXAMPLE. Feed containing 9 per cent. galena, 45 per cent. blende and 30 per cent. pyrite was made into a pulp with potassium bichromate, 0.4 per cent. on the ore; sodium silicate, 0.2 per cent. on the ore and 0.5 lb. per ton of cresol and floated at 140° F. in a wooden agitation machine containing metallic mercury. A froth concentrate was separated assaying 1.7 per cent. Pb, 51.5 per cent. Zn and 8.6 per cent. Fe. It is stated that similar results were obtained when sodium carbonate alone or sodium carbonate and hydrogen sulphide replaced the potassium bichromate and sodium silicate of the example.

The results stated in this patent differ so little from those described by one of the same patentees in 1,142,821 (p. 876) as to lead to the suspicion that the presence of mercury had little or no effect. See also Freeman, pat. 1,301,551.

Callow (1,269,157/1918) prescribes the use of sodium acid sulphate ($NaHSO_4$) or salt cake or niter cake and an acid, preferably sulphuric, to retard flotation of one of the sulphides present in a complex ore. The process may be applied to crude ore or to a collective concentrate. EXAMPLE. A feed containing 14 per cent. Pb, 18 per cent. Fe and 16.5 per cent. Zn was ground with a small amount of an organic frothing agent. Niter cake in an amount equivalent to 100 lb. per ton of ore and about 2 lb. per ton of sulphuric acid were added in the flotation cell. The concentrate assayed 5 per cent. Pb, 45 per cent. Zn, 7 per cent. Fe and 3 per cent. insoluble. The residue was treated on a shaking table and yielded a concentrate assaying 30 per cent. Pb, 6 per cent. Zn, 25 per cent. Fe and 1 per cent. insoluble and a tailing containing 3 per cent. Pb and 3 per cent. Zn.

The quantity of niter cake used would indicate that the surface of the lead sulphide was permanently deadened by a lead sulphate film.

Hebbard (1,261,810/1918) states that the addition to an alkaline or neutral, but not acid, pulp of 1 to 2 lb. per ton of powdered charcoal, graphite or like carbonaceous material will cause certain sulphides such as galena to float in preference to certain others such as blende. Flotation of blende can then be effected by ordinary methods in an acid pulp. EXAMPLE. Feed containing 8.2 oz. Ag per ton, 4.2 per cent. Pb and 17.3 per cent. Zn was floated in reclaimed water (containing a minute amount of organic flotation agent) with 1.7 lb. per ton of powdered coke. The first froth assayed 53.2 oz. Ag per ton, 58.6 per cent. Pb and 13.8 per cent. Zn. Tailing and recovery are not stated.

Bradford (1,274,505/1918). In general terms this patent describes pretreatment of a pulp with a reducing gas more or less soluble in the water of the pulp to prevent blende from floating with galena and pyrite. Bradford names sulphur dioxide and hydrogen sulphide as examples of such gases. Practically sulphur dioxide only has been useful. This gas may be externally generated and introduced into the pulp, or generated in the pulp by the action of acid on sulphites, bisulphites or thiosulphates. Blende may be subsequently floated by de-gassing the solution or decanting and adding sulphuric acid and heating. The amount of reducing agent varies, but 8 oz. to 8 lb. per ton of ore is suggested.

Performance. In the United States the Bradford process was given a thorough trial at MIDVALE, UTAH (*121 P 455*) treating 500 tons per 24 hr. of dump and current tailing from the U. S. S. R. AND M. Co. plant. Combined current and dump tailing was thickened to 25 per cent. solids, a rough collective float made in Janney mechanical-air machines, tailing was sent to waste and the concentrate cleaned and re-cleaned in pneumatic cells. Clean concentrate was thickened to 20 per cent. solids then sent through four gasifying tanks in series; SO_2 was pumped into the first two, the other two gave sufficient storage to allow three-hours' contact with the gas before flotation. The gasified pulp was floated with additional oil as necessary in Fagergren bubble-column machines, yielding lead float and a leady zinc tailing. The latter was de-gassed with heat and sulphuric acid, then treated in Fagergren cells for zinc. Results are shown in Table 26.

Riddell (*19 CME 823*) summarizes operating conditions at BROKEN HILL plants as follows: Organic agents were not used at JUNCTION NORTH; SO_2 only in an amount equivalent

to 1.5 lb. sulphur per ton of slimes was added. Pulp density in lead flotation should not exceed 27 per cent. solids or the grade of the lead concentrate falls off. Temperature in lead flotation must be kept below 90° F. or zinc floats also. Acidity must be kept below 0.03 per cent. free acid or zinc floats; on the other hand lead does not float well in a neutral pulp. The supply of SO_2 must be regular. Typical lead concentrate assays 63 per cent. Pb; typical zinc concentrate, 50 per cent. Zn. Shellshear (*115 P 614*) warns that excess of SO_2 will temporarily render all sulphides unfloatable, zinc, iron and lead being prevented in the order named. The action of the sulphur dioxide is unaffected by the nature and amount of salts in solution, within reasonable limits, *i.e.*, any likely to be met with when there is no plant addition. At Broken Hill Proprietary (*117 P 407*) 20 to 25 tons per hr. were treated in machines with special centrifugal pumps for agitators, discharging the aerated pulp into cylindro-conical froth-separating boxes. Pulp contained 23 to 26 per cent. solids, acidity was between 0.025 and 0.03 per cent., temperature 85 to 90° F.; 1.5 to 2 lb. sodium hyposulphite per ton of solid was added to the pulp before agitation and air and sulphur dioxide were admitted at the pumps. Lead was floated in 9 cells in series. Tailing was diluted to 20 per cent. solids, temperature raised to 125° F., acidity raised to 0.3 per cent. and zinc floated. Results are given in Table 27. See also flow-sheet of Central mine.

Table 26. Operation of Bradford process at U. S. S. R. and M. Co., Midvale plant. (Aug. 7 to Sept. 8, 1920)

	Pb, per cent.	Zn, per cent.	Insoluble, per cent.
Collective plant:			
Mill feed	2.3	4.5	....
Janney-machine feed	3.1	5.6	....
Mill tailing	0.45	2.1	....
Cleaner concentrate	17.4	30.6	....
Lead section:			
Feed	14.2	26.5	....
Concentrate	41.8	14.5	....
Tailing	6.6	30.2	....
Zinc section:			
Feed	7.8	30.7	....
Concentrate	7.9	40.2	4.5
Tailing	5.5	2.8	....

Table 27. Performance of Bradford process at Broken Hill Proprietary

Product	Assays			
	Zn, per cent.	Ag, ounces	Total Pb, per cent.	Oxidized Pb, per cent.
Feed	16.5	15.5	12.8	4.7
Lead concentrate	8	84	63	....
Zinc concentrate	50	13	4	....
Tailing	2	4.2	8	7

Williams (1,300,516/1919) describes the addition of bleaching powder ± a persulphate, *e.g.*, ammonium persulphate, to retard flotation of blende. Examples. (1) Lead-zinc ore, 25 lb. per ton each of bleaching powder and ammonium persulphate with no organic frothing agent. (2) Bleaching powder, 17.5 lb. per ton; eucalyptus oil, 0.25 lb. per ton. (3) Bleaching powder, 7 lb. per ton; iron-ammonium alum, 10 lb. per ton; no organic frothing agent. (4) Bleaching powder, 17.5 lb. per ton; iron-ammonium alum, 3.5 lb. per ton; eucalyptus oil, 0.25 lb. per ton. The feed carried 10 to 20 per cent. Pb, 16 to 24 per cent. Zn and about 7 oz. Ag per ton; concentrates assayed 50 to 60 per cent. Pb, 12 to 15 per cent. Zn and 35 to 50 oz. Ag. The best recovery was 85 per cent. of the lead.

Freeman (1,301,551/1919) develops a theory to explain conflicting results obtained in differential flotation of galena from blende in alkaline carbonate solutions. He states that when copper, silver, mercury or their salts or any metal or the salt of any metal electronegative to copper is present in or in contact with an alkaline or acid pulp containing blende, the blende becomes "metallized," (*i.e.*, coated with the metal) and then floats as readily as galena, thereby preventing differential flotation. Hence to effect differential separation, exclude such metals and their salts. He further states that both oil and acid aid blende flotation, the latter particularly if flotation is performed in a copper vessel. He recom-

mends treatment in a 1 to 10 per cent. solution of alkaline carbonate, without frothing agent, in an iron or wooden vessel. Heat aids the separation. The pulp should carry 20 to 25 per cent. solids. The best temperature is about 60° C. Freeman claims that copperizing will cause blende to float away from iron sulphides, if essential oil is used. Results of tests are given in Table 28.

Table 28. Tests recorded in Freeman patent, No. 1,301,551

Test number	Type of ore	Flotation vessel	Reagent	Temperature, ° C.	Duration of flotation min. (*a*)	Assays, per cent.					
						Feed		Rough concentrate		Cleaned concentrate	
						Pb	Zn	Pb	Zn	Pb	Zn
1	Calcitic	*wi*	*b*	50	4	15.0	9.7	63.2	8.6	73.0	6.2
2	Calcitic	*Cu*	*b*	50	4	15.0	9.7	39.2	27.8	....	....
3	Rhodonitic	*wi*	*b*	60	3	14.4	14.6	65.8	7.1	74.6	4.8
4	Rhodonitic	*Cu*	*b*	60	3	14.4	14.6	23.5	35.8	21.4	39.0
5	Silicious	*wi*	*b*	59	4	12.5	10.4	60.0	10.2	....	....
6	Silicious	*Cu*	*b*	59	4	12.5	10.4	33.4	29.0	....	....
7	Collective conc.(*c*)	*wi*	*b*	57	4*d*	27.5	31.4	62.2	11.6	....	....
8	Collective conc.	*Cu*	*b*	57	4*d*	27.5	31.4	15.8	43.2	....	....
9	Collective conc.(*e*)	*Cu*	*f*	60	5*g*	27.5	30.9	16.0	42.4	....	....
10	Collective conc.(*h*)	Wood	*i*	52	3*j*	38.4	23.4	62.8	12.4	....	....

a All tests by agitation-froth process. *b* 2.5 per cent. sodium carbonate solution. *c* Made by boiling with 4 per cent. sulphuric acid solution. *d* Collective float first washed and aerated with a small quantity of sodium sulphide. *e* Made with acid and eucalyptus oil in a copper machine. Drained, soaked 30 minutes in hydrogen-sulphide water, again drained, agitated in water to expel H_2S. *f* 3 per cent. sodium carbonate solution; 0.1 lb. eucalyptus oil per ton of ore. *g* Preceded by 1-min. contact. *h* Made as in test No. 9, except that a wooden vessel was used. *i* 3 per cent. sodium carbonate solution. *j* Preceded by 2-min. contact. *wi* wrought-iron.

Edser and Sulman (1,337,548/1920) state that differential separation of lead from zinc can be effected by a flotation operation with a soluble soap in the presence of a proper amount of sodium silicate. The quantity of sodium silicate is important, *e.g.*, 5 lb. per ton of ore is sufficient to produce the differential effect while 3 lb. permits a collective float. The silicate is preferably added as a 40 per cent. solution made by adding the commercial substance (140° Twaddell) to water. Sodium palmitate, stearate, oleate or resinate are mentioned as suitable soaps. The amount varies with the ore but 4 to 5 lb. per ton is usually sufficient. Soft water should be used to prevent formation of insoluble soaps. Examples. (1) 500 gm. lead-zinc ore freshly ground to 80-mesh; 1380 cc. soft water; sodium silicate, 16 lb. per ton and sodium oleate, 4 lb. per ton were used to float the lead. Further soft water to make a total of 2200 cc. was added and a second froth taken. (2) A similar ore, in a 20 per cent. pulp made by adding soft water containing 0.25 per cent. commercial sodium silicate with sodium oleate, 5 lb. per ton of ore, was floated in a sub-aeration machine. (3) The same ore and other conditions as in (2) except that the solution contained 0.125 per cent. sodium silicate. Results are given in Table 29. Comparison of tests (2) and (3) shows the effect of reduction in the amount of sodium silicate, the froth in test (3) being a collective concentrate, rather than a differential float.

Faul and Lavers (1,375,087/1921). This patent is substantially the same as 1,257,990, except that the specific metal is copper instead of mercury. Operation in an alkaline pulp is specified and the addition of a bichromate is recommended. Examples. (1) Feed: 250 lb. lead-zinc sulphide ore assaying 31.5 per cent. Zn, 13.5 per cent. Fe and 7.8 per cent. Pb floated at 140° F. with 0.5 lb. per ton of eucalyptus oil in a solution containing 0.1 per cent. potassium bichromate. The flotation machine was made of wood and lined with copper. Pulp contained 20 per cent. solids. Froth concentrate assayed 50 per cent. Zn, 7.2 per cent. Fe and 6.9 per cent. Pb and tailing, 7.3 per cent. Zn, 20.9 per cent. Fe and 8.1 per cent. Pb. (2) A feed assaying 32 per cent. Zn, 15 per cent. Fe, and 8 per cent. Pb was floated at 130° F., with 0.5 lb. of cresol and 20 lb. carbonate of copper per ton of ore in a 0.1-per cent. solution of potassium bichromate. The pulp contained 20 per cent. solids.

Concentrate assayed 53 per cent. Zn, 6 per cent. Fe and 4 per cent. Pb. Other experiments are cited in pulps containing sodium carbonate with or without sodium hydrogen sulphate in a copper-lined machine.

Table 29. Results of tests with sodium oleate and sodium silicate

Material	Weight, gm.	Assays, per cent.		Recovery, per cent.	
		Pb	Zn	Pb	Zn
Test No. 1					
Feed	500	19.54	33.4	....	....
Concentrate No. 1	140	51.8	16.2	74.3	13.6
Concentrate No. 2	268	7.4	45.4	20.3	72.5
Test No. 2					
Feed	500	18.2	33.5	....	....
Concentrate	147	54.6	15.5	88.2	13.6
Tailing	353	3.6	41.0	....	....
Test No. 3					
Feed	500	18.2	33.5	....	....
Froth	421	20.7	35.3	95.6	88.7
Tailing	79	3.4	24.0	....	....

The results cited in these tests are not sufficiently superior to those in 1,102,738 and 1,142,821, presumably without copper, to warrant the claim of the present patent that copper is an essential factor in the differential effect.

Dosenbach (1,377,189/1921) is for the use of deterrent agents in atomizing flotation. (Art. 14.)

Palmer, Seale and Nevett (1,401,435/1921). The essence of this patent, in so far as it deals with differential flotation, is that the presence of elemental sulphur in the pulp, preferably in solution, causes one sulphide, *e.g.*, blende, to become temporarily less susceptible to flotation. The sulphur may be merely mechanically intermixed. The quantity necessary will ordinarily be less than 10 lb. per ton of ore. The quantity of oil and acid should be small and the temperature kept low. Several methods are stated to be available for getting the sulphur into solution, but one recommended is to boil powdered sulphur in a weak aqueous solution of sulphuric acid to which has been added a small amount of coal tar.

The sulphur solution is made with oil, whenever the method is indicated, and probably in all cases. It is quite likely that the differential effect is largely due to the small quantity of oil. Tests at Columbia University indicate that in most cases the same effect is obtained without as with sulphur present.

The process is said to have been used successfully at the Junction North Mine, Broken Hill, and to be cheaper than the Bradford sulphur-dioxide process, as it requires only heating to 120° F. to float blende after galena has been removed (*36 Aa 97*).

Sheridan and Griswold (1,421,585/1922). The essence of this patent is flotation of galena from associated iron and/or zinc sulphides by pre-treatment of the pulp with a cyanide and an alkaline salt. The cyanide may be alkaline, alkaline-earth or metallic and the quantity will usually range from a fraction of a pound to 4 lb. per ton of ore. Sodium carbonate is a suitable alkaline salt, in quantities ranging from 1 to 10 lb. per ton of ore. Sodium bicarbonate may replace the carbonate. Zinc sulphate in quantity ranging from a fraction of a pound to 4 lb. per ton of ore may be a useful addition to keep down blende.

The pulp may be heated to between 120° F. and boiling during pre-treatment; the necessary time is from 2 min. to several hours, and pre-treatment is usually best accomplished in a thick pulp. Flotation of galena is effected at a temperature of about 130° F. by diluting to about 20 per cent. solids, adding a minute amount of frothing agent and agitating or aerating. After lead is floated, add 1 to 5 lb. per ton of an appropriate alkali, *e.g.*, sodium hyroxide, and a frothing agent, when blende will float but iron stay down.

Examples. (1) In 65-mesh material, 50 per cent. solids, 1 hr. pre-treatment at 160° F. with 10 lb. sodium carbonate and 1 lb. sodium cyanide per ton of ore, diluted to 20 per cent. solids, oil added and the lead floated. (2) 65-mesh feed, 50 per cent. solids, 3-min. pre-agitation in a flotation machine with 5 lb. sodium carbonate, 2 lb. zinc sulphate and 1 lb. sodium cyanide per ton of ore; then diluted to 20 per cent. solids, oil added and the lead floated. After taking off the lead concentrate, 2 lb. of sodium hydroxide was added to the residue in the machine and flotation continued, yielding a zinc concentrate and a final tailing. Results are given in Table 30. For mill performances, see pp. 166 and 894.

Table 30. Results of differential-flotation tests on lead-zinc ore with cyanide

Test number	Assays, per cent.											
	Feed			Lead concentrate			Zinc concentrate			Tailing		
	Pb	Zn	Fe	Pb	Zn	Fe	Pb	Zn	Fe	Pb	Zn	Fe
1	7.0	15.0	8.0	56.0	7.5	2.0	......	......	.....	1.55	15.6	8.7
2	7.2	14.5	8.9	55.5	12.9	2.8	5.54	53.1	1.4	1.00	0.9	12.1

Table 31 shows the results of a test of Park City ore by this process. The coarsely-ground sample was first de-slimed, the sand was ground in a thick pulp in a pebble mill to the desired fineness, then the slimes together with 1.2 lb. sodium cyanide, 2.4 lb. zinc sulphate, 12 lb. sodium carbonate, and (probably) about 0.4 lb. Barrett No. 4 per ton of ore were floated at 60° F. in a Janney laboratory machine in a pulp containing 20 per cent. solids. After removal of lead concentrate 2 lb. copper sulphate, 0.4 lb. xanthate and 0.1 lb. pine oil were added and zinc was floated. The zinc concentrate was cleaned twice. Two lb. sodium sulphide and about 0.3 lb. Barrett No. 4 per ton were then added and a concentrate containing iron and oxidized lead taken. In another test the ore was first ground to 20-mesh and tabled, the tailing de-slimed, the sand ground in a pebble mill to substantially 100-mesh, and the slimes together with 0.4 lb. sodium cyanide, 0.8 lb. zinc sulphate, 8 lb. sodium carbonate and 0.5 lb. Barrett No. 4 per ton added and ground 10 min. longer, then floated at 60° F. in a 20-per cent.-solid pulp in a Janney laboratory machine to make a lead concentrate which was cleaned. The lead tailing was re-treated with 1 lb. copper sulphate, 0.32 lb. T-T mixture and 0.12 lb. pine oil per ton, making a zinc concentrate which was cleaned. The zinc tailing was re-floated with 1 lb. sodium sulphide and 0.3 lb. Barrett No. 4 per ton of ore to get the oxidized lead.

The tabling test yielded by far the better lead tailing. Zinc tailing is too high in both tests and the reagent cost, estimated at $0.50 per ton is also high.

The patentee's theory is that the cyanide cleans the galena surfaces and deadens the zinc and iron surfaces by forming films of the respective metal cyanides and that the alkaline salts react with the iron cyanide to form hydrated oxides that permanently deaden the iron. The cyanide is most active in neutral solution, hence it may be best to defer addition of the alkaline salt. The caustic soda in the zinc-flotation period removes the zinc cyanide and exposes a cleaned sulphide surface. This treatment is not always necessary. The zinc sulphate may help to form a more complex and less soluble film on the zinc blende or it may remove hydroxylion. But see p. 844.

Sheridan and Griswold (1,427,235/1922). In this patent the same patentees describe treatment of copper-iron ores by the same methods.

Examples. (1) 65-mesh feed, pre-agitation for 2 min. at 160° F. with 5 lb. sodium bicarbonate and 1 lb. potassium cyanide in a pulp containing 50 per cent. solids. Dilute, add

Table 31. Tests of Sheridan and Griswold process on Park City ore. (*After Thompson and Varley*)

Products	Weight, per cent.	Assays				
		Ag, ounces	Pb, per cent.	Zn, per cent.	Fe, per cent.	Insol., per cent.
All flotation						
Feed	100.0	....	18.8	25.2	4.38	22.74
Lead concentrate	18.1	....	67.74	11.0	0.51	2.6
Iron concentrate	10.6	....	28.77	9.3	15.53	11.7
Lead-iron concentrate	28.7	....	53.1	10.4	6.05	5.97
Zinc concentrate	31.4	....	4.18	59.2	2.53	1.2
Zinc cleaner tailing	5.4	....	8.59	20.9	2.53	40.2
Rougher tailing	31.4	....	2.32	5.80	4.05	59.7
Totals	96.9	....	18.4	25.4	4.02	24.8
Tabling and flotation						
Feed	100.0	8.96	18.8	25.2	4.38	22.74
Table lead concentrate	7.25	27.36	59.2	8.2	5.56	1.72
Flotation lead concentrate	12.0	36.36	68.9	10.4	0.34	1.42
Lead-iron concentrate	5.96	8.06	48.8	3.2	8.77	9.66
Total lead concentrate	25.21	27.10	61.2	8.08	3.84	3.46
Zinc concentrate	31.0	2.72	2.67	60.7	2.53	0.62
Lead cleaner tailing	1.65	24.24	39.2	20.8	1.69	12.54
Zinc cleaner tailing	5.43	4.96	5.92	24.2	8.61	26.62
Rougher tailing	36.60	2.52	0.41	5.3	5.40	57.96
Totals	99.89	9.24	17.39	24.5	4.22	23.8

Products	Weight, per cent.	Distribution, per cent. of original ore content				
		Ag	Pb	Zn	Fe	Insoluble
All flotation						
Feed	100.0	....	100.0	100.0	100.0	100.0
Lead concentrate	18.1	....	69.2	8.15	2.36	2.04
Iron concentrate	10.6	....	17.0	4.0	42.10	5.4
Lead-iron concentrate	28.7	....	86.2	12.15	44.46	7.44
Zinc concentrate	31.4	....	7.35	75.50	19.50	1.63
Zinc cleaner tailing	5.4	....	2.61	4.60	3.49	9.45
Rougher tailing	31.4	....	4.08	7.40	32.60	81.70
Totals	96.9	....	100.24	99.65	100.45	100.02
Tabling and flotation						
Feed	100.0	100.0	100.0	100.0	100.0	100.0
Table lead concentrate	7.25	21.5	24.2	2.42	9.55	0.52
Flotation lead concentrate	12.0	47.3	47.5	5.06	0.97	0.72
Lead-iron concentrate	5.96	5.3	16.8	0.78	12.40	2.42
Total lead concentrate	25.21	74.1	88.5	8.26	22.92	3.66
Zinc concentrate	31.0	9.15	4.8	77.0	18.70	0.81
Lead cleaner tailing	1.65	4.16	3.72	1.4	0.65	0.87
Zinc cleaner tailing	5.43	2.90	1.84	5.33	11.0	5.85
Rougher tailing	36.60	10.0	0.87	7.95	47.0	89.2
Totals	99.89	101.31	99.73	99.98	100.27	100.39

oil and float. (2) As above except that the temperature was 150° F. and 2 lb. zinc sulphate was added in addition to the other reagents in the pre-agitation period. After the first froth, a lead concentrate, was taken, 2 lb. sodium hydroxide was added and the pulp agitated for 2 min. then more oil and a rough zinc concentrate was removed, which was later cleaned. (3) Ore was floated without pre-treatment at 113° F. using 3 lb. sodium bicarbonate and 1 lb. cyanide. (With oil.) (4) 80-mesh feed no pre-treatment, flotation at 55° F. with 2 lb. sodium hydroxide, 1 lb. sodium cyanide and oil. Results are given in Table 32.

Table 32. Results of differential-flotation tests on copper-iron ore, with cyanide

Test number	Assays, per cent.																Recovery, per cent. Cu
	Feed				First concentrate				Second concentrate				Tailing				
	Cu	Fe	Zn	Pb	Cu	Fe	Zn	Pb	Cu	Fe	Zn	Pb	Cu	Fe	Zn	Pb	
1	3.44	13.9	4.1	....	15.64	12.6	20.2	....	....	6.7	47.1	....	0.38	14.2	Tr.	0.00	91.2
2	1.86	15.2	7.1	12.5	5.86	6.6	6.6	50.5	3.52	6.7	43.1	9.0*a*	0.1	18.1	0.1	0.62	*b*
3	6.22	12.3	...	....	27.0	16.7	....	....	....	...	....	....	0.56	11.2	...	....	94.7
4	6.22	12.3	...	....	27.12	14.9	....	....	....	...	....	....	0.66	11.7	...	....	90.7

a After cleaning. Cleaner tailing assayed 3.28 per cent. Cu, 17.6 per cent. Fe, 13.2 per cent. Zn and 6.77 per cent. Pb. *b* In first concentrate: 85.6 per cent. of the lead and 65.4 per cent. of copper; in zinc cleaner concentrate, 67 per cent. of the zinc.

Stevens (1,429,544/1922) prescribes an agent for effecting differential flotation that is a solvent for the sulphide to be retarded but not for the one to be floated, or, if not initially a differential solvent, is saturated with the sulphide to be floated but not with the one to be retarded. Cyanide is mentioned as a typical solvent.

The principle that appears to underlie this patent is that solution of a substance involves wetting of that substance by the solution. It follows that a substance thus wetted will not be wetted (*i.e.*, filmed) by an organic flotation agent, and will not, therefore, be in any way affected by gas introduced into or generated in the pulp and hence cannot be floated. Stevens' instructions for lead-zinc separation direct a procedure that is opposed to mill performances.

Hellstrand (1,469,042/1923) describes the use of a sulphide or polysulphide of an alkali or an alkaline earth in pre-treatment of a thick pulp in order to prevent zinc from floating while sulphides of lead, copper and iron float. Later the zinc can be floated by adding more oil together with an acid or alkali.

Examples. (1) Feed: Lead-zinc-copper-iron ore ground to 65-mesh, 4 lb. commercial sodium sulphide (62 per cent. Na_2S) and 0.1 lb. oil (90 per cent. coal-tar creosote and 10 per cent. pine oil) per ton of ore, mixed with the ore in a thick pulp. Lead concentrate floated in a pulp containing 25 per cent. solids upon the addition of 0.15 lb. of the same oil mixture. Tailing was acidified, heated to 95° F. and re-frothed with additional oil to raise zinc. (2) Similar ore pulp, 7 lb. commercial sodium sulphide and 0.1 lb. coal-tar creosote per ton of ore in pre-treatment. Lead concentrate floated in a pulp containing 20 per cent. solids with addition of 0.1 lb. pine oil per ton. Further addition of 2 lb. sodium carbonate and 2 lb. of a mixture of coal-tar oil, coal-tar creosote and pine oil raised the zinc. Results are given in Table 33.

Table 34 shows the results obtained in a test by this process treating Park City ore (*15 UU 55*). The sample was de-slimed, the sands were then ground in a pebble mill to the desired fineness, after which the slimes together with 10 lb. per ton of sodium sulphide, 0.4 lb. of thiocarbanilid and 0.5 lb. of Barrett No. 4, were added and lead was floated in a Janney laboratory machine in a pulp containing 20 per cent. solids, at 60° F. The concentrate was cleaned once, the tailing was re-floated after adding 0.12 lb. of T-T mixture (thiocarbanilid and orthotoluidin), 0.9 lb. yellow pine oil and 1.5 lb. of copper sulphate per ton of ore. Pulp consistency and temperature were the same as in the first flotation. Concentrate was cleaned twice. Weights and assays of all products are shown in the table. Table 34 shows the results of another test on the same ore in which a lead concentrate was made on tables prior to flotation. The procedure was the same as in the preceding

test except that the reagents used for lead flotation were 12 lb. sodium sulphide, 0.4 lb. thiocarbanilid, 0.16 lb. Barrett No. 4 and 0.16 lb. straw-colored cresylic acid per ton of ore; and in zinc flotation 4 lb. of T-T mixture and 2 lb. of copper sulphate.

Table 33. Results of differential-flotation with sodium sulphide

Material	Weight, per cent.	Assays						Recovery, per cent			
		Cu, per cent.	Ag, oz. per ton	Insoluble, per cent.	Fe, per cent.	Zn, per cent.	Pb, per cent.	Ag	Fe	Zn	Pb
Test No. 1											
Feed	100	0.18	6.7	66.4	1.7	16.2	2.02	.....	.....	.....	.....
Lead concentrate	7.1	1.13	32.8	8.6	19.6	13.1	22.30	34.7	82.8	5.8	78.4
Zinc concentrate	23.3	0.45	16.5	5.5	0.4	60.2	1.34	57.4	5.5	86.8	15.4
Zinc middling	6.7	.....	5.1	.....	0.6	14.3	0.76	5.1	2.4	5.9	2.5
Tailing	62.9	.....	0.3	.....	0.25	0.4	0.12	2.8	9.3	1.5	3.7
Test No. 2											
Feed	100	.....	12.6	65.3	3.3	14.7	1.34	.....	.....	.....	.....
Lead concentrate	10.3	.....	37.1	10.8	30.3	6.0	12.40	30.3	94.8	4.2	95.4
Zinc concentrate	21.9	.....	37.9	3.8	0.4	60.0	0.27	65.8	2.8	89.2	4.4
Zinc middling	3.4	.....	7.1	75.2	0.6	11.5	0.05	1.9	0.6	2.7	0.2
Tailing	64.4	.....	0.4	.....	0.1	0.9	Tr.	2.0	1.8	3.9	.....

Comparison of the tests shows much lower lead tailing by the combination treatment. The zinc tailing is too high in both cases and the consumption of reagents is excessive.

At Timber Butte (see p. 165) (*120 J 687*) 6.4 lb. of commercial sodium sulphide and 0.21 lb. Barrett No. 4 per ton were added in the primary grinding mills and a small amount of frothing agent (cresylic acid, 0.05 lb.) at the lead cells. Concentrate was about 8.5 per cent. of the feed and assayed 53 oz. Ag, 43.5 per cent. Pb, 12.0 per cent. Zn, 13.7 per cent. Fe and 5.0 per cent. insol.; 1.8 lb. sodium carbonate, 0.85 lb. copper sulphate, 1.2 lb. Barrett No. 634 and 0.15 lb. steam-distilled pine oil per ton were added to the lead tailing and this was floated in an M. S. machine and the concentrate cleaned three times in Callow cells. Concentrate represented 17 per cent. of the original flotation feed and assayed 13.0 oz. Ag., 56.1 per cent. Zn, 6.9 per cent. Pb, 2.4 per cent. Fe; 3.5 per cent. insol. Tailing assayed 1.7 oz. Ag.; 0.7 per cent. Pb, 1.1 per cent. Zn and 2.7 per cent. Fe. On an easier ore at the same mine (Elm Orlu), adding 5.7 lb. sodium sulphide and 0.25 lb. Barrett No. 4 at the grinding mill and 0.08 lb. pine oil at the lead machine; 40 lb. sulphuric acid, 0.65 lb. Barrett No. 4 and 0.80 lb. pine oil to the zinc-machine feed, with temperatures of 54° F. in the lead cells and 100° F. in the zinc cells, the results were as shown in Table 35.

Bragg (1,478,697/1923) describes the flotation of galena from blende in a slightly alkaline solution by the use of a small amount of a soluble sulphite, *e.g.*, sodium sulphite. He recommends lime for producing alkalinity but mentions also sodium sulphide, sodium carbonate, soda ash, and caustic soda. As flotation agents he mentions thiocarbanilid with slightly acidulated aldol. After flotation of galena the pulp should be dewatered or otherwise treated to decrease the sulphite concentration. To float zinc, add further alkali, *e.g.*, sodium carbonate and copper sulphate together with more thiocarbanilid and, say xylidin. The quantities recommended in the lead-flotation operation are 1 lb. sulphite, 1 lb. lime and about 0.5 lb. thiocarbanilid per ton of ore. These should be added in the grinding mill. In zinc flotation use about 5 lb. sodium carbonate, 1 lb. copper sulphate, and 0.4 lb. of a mixture of 85 parts xylidin and 15 parts thiocarbanilid, per ton of ore.

Pallanch (1,486,297/1924) describes the use of an alkaline sulphite, *e.g.*, sodium or calcium, and recommends sulphuric acid to destroy the sulphite. He also recommends simultaneous addition of sulphur dioxide gas and a hydrate or carbonate of an alkali or alkaline metal in order to form the desired sulphite *in situ*. The quantities indicated are 2 lb. calcium sulphite per ton for lead flotation and 5 lb. per ton of sulphuric acid to destroy the sulphite. The claims are specific to calcium sulphite.

Table 34. Tests of Hellstrand process on Park City ore. (*After Thompson and Varley*)

Products	Weight, per cent.	Assays				
		Ag, ounces	Pb, per cent.	Zn, per cent.	Fe, per cent.	Insol. per cent.
All flotation						
Feed	100.0	....	18.8	25.2	4.38	22.74
Lead concentrate	22.8	....	60.67	7.6	7.26	1.1
Zinc concentrate	32.2	....	6.15	59.2	1.86	1.3
Lead cleaner tailing	2.4	....	28.76	13.0	6.58	22.6
Zinc cleaner tailing	8.0	....	12.58	18.9	5.06	33.8
Rougher tailing	32.7	....	1.91	6.1	8.44	58.7
Totals	98.1	....	19.8	25.1	5.7	23.6
Tabling and flotation						
Feed	100.0	8.96	18.8	25.2	4.38	22.74
Table lead concentrate	6.95	31.12	64.03	7.1	5.23	....
Flotation lead concentrate	18.45	22.40	50.92	12.9	8.19	0.96
Total lead concentrate	25.40	24.80	57.47	9.5	6.71	....
Zinc concentrate	28.50	2.96	4.06	59.3	1.10	1.21
Lead cleaner tailing	6.34	15.12	35.26	15.0	9.25	13.60
Zinc cleaner tailing	5.20	4.23	4.01	29.5	4.97	32.80
Rougher tailing	34.80	1.12	0.35	5.5	4.21	67.40
Totals	100.24	8.13	18.0	26.3	4.48	26.98

Products	Weight, per cent.	Distribution, per cent. of original ore content				
		Ag	Pb	Zn	Fe	Insoluble
All flotation						
Feed	100.0	....	100.0	100.0	100.0	100.0
Lead concentrate	22.8	....	78.1	7.04	29.6	1.08
Zinc concentrate	32.2	....	10.2	77.50	10.75	1.81
Lead cleaner tailing	2.4	....	3.41	6.15	2.83	2.35
Zinc cleaner tailing	8.0	....	5.22	1.27	7.35	11.70
Rougher tailing	32.7	....	3.21	8.09	49.50	83.20
Totals	98.1	....	100.14	100.05	100.03	100.14
Tabling and flotation						
Feed	100.0	100.0	100.0	100.0	100.0	100.0
Table lead concentrate	6.95	26.6	25.2	1.98	7.95	....
Flotation lead concentrate	18.45	50.5	53.6	9.6	33.40	....
Total lead concentrate	25.40	77.1	78.8	11.58	41.35	....
Zinc concentrate	28.50	10.4	6.74	71.00	7.05	1.30
Lead cleaner tailing	6.34	4.93	12.70	3.84	13.10	3.23
Zinc cleaner tailing	5.20	2.69	1.19	6.20	5.75	6.41
Rougher tailing	34.80	4.78	0.68	7.74	32.60	88.00
Totals	100.24	99.90	100.12	100.36	99.85	99.00

Table 35. Performance of Hellstrand process at Timber Butte

Material	Weight, per cent.	Assays					Recovery, per cent.			
		Ag, oz.	Pb, per cent.	Zn, per cent.	Fe, per cent.	Insoluble, per cent.	Ag	Pb	Zn	Fe
Feed	100	4.8	1.0	14.0	1.8	.....	.....	.....	.....	.....
Ag-Pb concentrate	4.7	34.5	13.2	13.5	24.9	6.9	33.7	62.0	4.5	65.0
Zinc concentrate	21.5	13.25	1.44	59.1	1.7	3.4	59.4	31.0	90.8	20.6
Tailing	73.8	0.45	0.1	0.9	0.35	.....	6.9	7.0	4.7	14.4

Feed, 2 per cent. +65-mesh, 20 per cent. solids.

Tucker and Edser (1,497,310/1924) describe the use of a "product obtained by adding to a solution of a soap, such as sodium oleate, a solution of an alkaline sulphide or sulphydrate; or by treating the solution of an alkaline sulphide with a fatty acid; or by mixing or combining materials or ingredients necessary to produce soluble soaps and alkaline sulphides, as for example by treating an alkaline hydrate or carbonate with sulphur and adding a fatty acid; or by treating a solution of an alkaline sulphide containing sulphur in excess of the normal sulphide with a fatty acid; or by adding a soluble soap to an aqueous solution of sulphuretted hydrogen." They cite a concentrate containing 60 per cent. Pb and 12 per cent. Zn made by use of such a reagent from a feed containing 17.6 per cent. lead and 34 per cent. Zn. They claim also that the reagent is useful to float oxides and give an example of 88 per cent. recovery of wolframite in a concentrate assaying 16.8 per cent. WO_3 from a feed containing 4.1 per cent. WO_3.

Price (1,499,872/1924) describes a process for differential flotation of different varieties of bituminous coal by the use of organic colloids, *e.g.*, starch granulose, tannic acid, gelatines (glue), albumens, caramel, dextrine, etc. The effect is to retard flotation of all of the varieties of coal, but if the amount added is limited, the result is claimed to be to keep down the high-ash non-coking fusain variety, while the balance floats. Many varieties of starch may be used, *e.g.*, rice, wheat, potato, maize. "Soluble starch," tapioca and maize flour are also applicable. Several varieties of glue are named; both egg and blood albumen; casein in alkaline solution, animal glutin, and agar-agar. In the examples given cresol and kerosene were used as flotation agents.

McArthur (1,552,936/1925) describes the use of a cyanide alone or with a hydroxide to effect differential separation of lead, silver and copper from zinc and iron in a collective-flotation concentrate. The operation consists in thickening the collective concentrate, treating for some time, say 30 min., with the cyanide and hydroxide, then floating.

Examples. A concentrate assaying 14.8 oz. Ag, 4.6 per cent. Pb, 56.8 per cent. Zn, and 2.3 per cent. Fe was thickened to 35 per cent. solids, agitated for 30 min. with 5 lb. per ton of sodium cyanide, then floated in a Callow cell. Overflow assayed 40.0 oz. Ag, 44.1 per cent. Pb, 26.4 per cent. Zn and 3.6 per cent. Fe; residue, 12 oz. Ag, 1 per cent. Pb, 60 per cent. Zn and 2.1 per cent. Fe. Similar treatment of a similar product, using 7.5 lb. per ton of sodium hydroxide and 2.2 lb. of sodium cyanide gave a float assaying 45 oz. Ag, 60 per cent. Pb, 14 per cent. Zn and 2.4 per cent. Fe and a residue containing 12 oz. Ag, 0.5 per cent. Pb, 60 per cent. Zn and 3 per cent. Fe.

McArthur (1,552,937/1925) describes a similar process in which a halogen is used with the cyanide. This method he applies both to collective concentrate and to primary ore. He states that the iron may be floated with the lead or left in the final tailing as desired by varying the relative proportions of halogen and cyanide.

Examples. 1500 gm. of −10-mesh ore with 750 cc. of water, 0.8 lb. per ton of bromine and 0.4 lb. of sodium cyanide were ground in a ball mill until the sulphides were free and of flotation size, then floated with Barrett No. 4 making a silver-lead float. One lb. of copper

sulphate, 4 lb. sodium carbonate and 0.2 lb. Barrett No. 634 per ton were then added and a zinc float made. Assays are given in Table 36, line 1. The halogen-cyanide ratio was large

Table 36. Performances of McArthur process

Line	Feed				Lead concentrate			
	Ag, ounces	Pb, per cent.	Zn, per cent.	Fe, per cent.	Ag, ounces	Pb, per cent.	Zn, per cent.	Fe, per cent.
1	5.1	5.0	10.3	7.4	30.0	40.1	14.5	11.4
2	11.3	7.7	9.7	5.1	71.2	68.1	9.5	2.6
3	16.2	4.0	55.5	2.2	24.4	44.5	16.2	11.0
4	17.1	5.9	56.0	1.8	27.4	55.0	18.0	2.4

Line	Zinc concentrate				Tailing			
	Ag, ounces	Pb, per cent.	Zn, per cent.	Fe, per cent.	Ag, ounces	Pb, per cent.	Zn, per cent.	Fe, per cent.
1	3.6	1.9	57.6	3.0	1.0	0.1	0.5	6.4
2	13.4	2.8	52.0	4.6	1.4	0.1	0.7	5.6
3	15.4	0.7	60.1	1.0	....	....	....	....
4	14.6	0.3	60.4	1.6	....	....	....	....

and iron came up with the lead. In a similar test using 0.4 lb. per ton of bromine, 1.0 lb. of sodium cyanide and 0.3 lb. of Barrett No. 4 for the first float and 1 lb. of copper sulphate, 3 lb. sodium carbonate and 0.3 lb. Barrett No. 634 for the second, the iron was kept down. See line 2. A collective concentrate was thickened to 35 per cent. solids and treated 30 min. with 1 lb. bromine and 2 lb. sodium cyanide per ton of solids, then floated in a Callow cell. Assays are given in line 3. A relatively large amount of bromine was used and iron came up with the lead. In a similar test using 0.3 lb. bromine and 2.5 lb. of sodium cyanide flotation of iron was inhibited. See line 4.

22. Control processes

Nutter and Lavers (1,067,485/1913). This patent is an early attempt to explain and generalize the then-known physical principles underlying differential flotation. It states that the process depends on difference in the inherent floatability of minerals and that by careful control and regulation of the factors in a flotation operation that go to produce flotation, the net effect can be so adjusted as to raise the most floatable mineral while the others remain submerged. The patentees name the following flotation factors over which control is to be exercised: amount and character of agitation and aeration, chemical constitution of the solution employed, the percentage of solids in the pulp, the pulp temperature and the amount and nature of the organic flotation agent.

They also noted that in a collective froth, particularly one made with a minute amount of organic reagent, the particles of one of the sulphides are generally much smaller than those of the other and they point out that such material is readily susceptible to gravity treatment. As an example of control they state that if a copper-zinc sulphide ore is floated in a cold neutral pulp with a minute amount of cresol or eucalyptus oil, the froth is relatively rich in copper, but contains some fine zinc and some yet finer gangue. By heating the remaining pulp, adding acid and floating, a froth relatively high in zinc is raised. The zinc sulphide is coarser than that in the first concentrate, but the copper sulphide is still coarser. By adding oleic acid the coarsest of the zinc and copper sulphide particles may be raised.

Higgins (1,236,933/1917) makes a specific application of this idea by suggesting preparation of flotation feed by hydraulic classification. He reasons that by treating together grains of

minerals of different floatability that are equal-settling in water, the gravitational pull that resists flotation will be the same on all grains and differences in floatability will have full play. He states that the arrangement pyrite, galena, blende, gangue is one of decreasing floatability. In his experiments he used generally from 0.06 to 0.12 lb. eucalyptus oil per ton of ore and floated in a 0.1 to 0.3 per cent. solution of sodium carbonate. He cites a test on classified feed in 0.088 per cent. sodium carbonate solution. After adding 0.19 lb. eucalyptus oil, he frothed in a sub-aeration machine and collected separately the froths overflowing in the intervals 0 to 3 min., 4 to 8 min. and 9 to 12 min. inclusive, respectively. He then added 0.19 lb. more of eucalyptus oil and collected froth for six minutes. The assays of the products are shown in Table 37. Higgins (1,236,934/1917) develops another variant

Table 37. Differential flotation of classified feed

Material	Weight per cent.	Assays, per cent.			Recovery, per cent.	
		Zn	Pb	Insoluble	Zn	Pb
Feed	100	17.35	9.74	....	....	....
Concentrate:						
0 to 3 min.	5	7.9	68.8	2.6	2	35.6
4 to 8 min.	11.3	26.8	41.4	1.0	18	48.1
9 to 12 min.	20.8	46.6	4.4	2.4	55.7	9.4
13 to 18 min.	8.2	41.3	2.2	9.6	19.5	1.8
Residue	54.7	1.6	0.9	....	....	....

of the same idea in prescribing close sizing of the feed in preparation for differential flotation with the idea that the resulting difference in settling rate of the particles in water should affect the net effective floatability, enhancing the inherent differences in some cases, reversing them in others. Sizing should be as close as possible; between 80- and 100-mesh, 100- and 120-, and 120- and 150-mesh are recommended.

Examples. (1) A lead-zinc ore sized between 120- and 150-mesh was floated in a solution containing 0.11 per cent. dry sodium carbonate. Froths were collected for four separate intervals following oil additions as shown in Table 38. Assay results are given in the

Table 38. Differential flotation of sized feed

Material	Duration of frothing, minutes	Oil added		Weight, per cent.	Assay, per cent.			Recovery per cent.	
		Kind	Pounds per ton		Zn	Pb	Insoluble	Zn	Pb
Feed	......	....	....	100	11.31	11.76	.....	....	....
Concentrate No. 1	4	Eucalyptus	0.17	4.5	42.3	6.4	1.8	17.0	2.4
Concentrate No. 2	4	Eucalyptus	0.17	9.5	44.8	2.6	0.5	37.8	2.1
Concentrate No. 3	8	Eucalyptus	0.34	6.7	42.9	9.4	1.6	25.5	5.4
Concentrate No. 4	3.5	Texas oil and wood-tar oil	0.19	15.2	6.2	67.4	2.2	8.3	86.8
Tailing	......	....	....	58.4	2.0	0.6	.....	....	....

same table. A sub-aeration machine is recommended for the bubble-column action as distinguished from the permanent and stable bubble attachment in the agitation-froth machine.

These experiments are interesting but impracticable, on account of the impossibility of accurate commercial sizing and classification at these fine sizes.

Hebbard and Harvey (1,260,668/1918) describe another variant on the theme of Nutter and Lavers. They point out that flotation with a limited amount of an emulsifying agent (meaning an organic frothing agent) and a limited amount of acid will float galena in the presence of blende and that the addition of more of the frothing agent and further agitation and aeration will raise the blende. They state that if water reclaimed from a previous flotation operation (containing, therefore, dissolved frothing agents) is used, no organic agent need be added in the first frothing. Using such reclaimed water and 22 lb. of acid per ton of solid they obtained on the first of a string of 12 flotation cells a concentrate assaying 66

per cent. Pb and 9.8 per cent. Zn from a feed containing 19 per cent. Pb and 21 per cent. Zn. The combined concentrate from the 12 boxes assayed 50.5 per cent. Pb and 21.2 per cent. Zn.

Schwarz (1,317,945/1919) states that a phenol plus a small amount of a resin soap used in neutral or alkaline pulp will float galena and molybdenite away from blende. He says that the separation is better in a pneumatic cell; the zinc also tends to float in an agitation machine. Resin soap may be made by boiling together two parts of resin, one of caustic soda and 97 parts water, until the resin goes into solution. After the lead and molybdenum have floated, the zinc may be raised by adding further soap and acid. Quantities mentioned are 2 lb. carbolic acid and 0.5 lb. per ton of the soap for the lead float and 10 lb. additional soap and 5 to 10 lb. per ton of acid for the zinc. Pulp consistency should be 16 to 25 per cent. solids.

Borcherdt (1,445,989/1923) sets up the theory that colloidal matter in the ore itself inhibits flotation and that the inhibitory action has a differential incidence on different minerals. He directs step-removal of the colloidal material to permit successive removal of different sulphides. In treating a lead-iron-zinc ore the direction is to add first about 0.5 lb. per ton of rosin soap and float, which will remove some of the inhibitory colloidal material; then add, *e.g.*, wood-creosote oil and float galena; float off further colloid after addition of 0.25 lb. per ton more of rosin soap; next add a small amount of pine oil and crude petroleum in emulsion and float blende; add another 0.25 lb. of rosin soap and float off more colloid and finally some more of pine oil-petroleum emulsion and about 0.5 lb. per ton of sulphuric acid in order to raise pyrite.

This is very close to Schwarz (1,317,945) differing, apparently, only in that the so-called colloid froth is collected separately.

Borcherdt 1,446,376/1923) enlarges on the theoretical matter of the previous patent and describes a variation in operation involving first complete removal of the ore colloid and later partial return of the same or addition of other colloids in order to utilize, under control, the inhibitory effect. He states that the ore colloids are derived from clay, shale and chert. Other colloids that may be used are: sodium silicate, acacia, and colloidal precipitates of barium or calcium sulphate, ferric oxide or hydrate, and magnesium hydrate. The guiding principle to be followed in the choice of a colloidal inhibitory agent is that colloids adsorb on surfaces of opposite polarity and hence a colloid of opposite sign to that of the mineral to be inhibited should be chosen. For dispersing the ore colloids prior to removal, sodium silicate, alkaline or acid agents, salts, foundry molasses, tri-sodium phosphate and gum arabic are mentioned. If colloid is to be removed by flotation, soaps (especially rosin soaps), oils having a high pitch content and ordinary flotation oils in excess are recommended and preliminary emulsification of those agents is said to be desirable. Emulsified creosote oil may be used for a differential galena float: pine, crude oil and copper sulphate to float zinc after removal of the inhibiting colloid. An illustrative flow-sheet shows "de-colloiding" followed by rough flotation, return of part of the colloid to the froth concentrate and differential flotation. In (1,454,838/1923) the same patentee describes step removal of colloids to effect differential separation, *e.g.*, a small amount of sodium silicate is added, the pulp thickened by decantation of part of the slime, the thick product is diluted and the lead floated with creosote oil; more silicate is added to the tailing, more slime decanted, the thick pulp is diluted with fresh water and zinc floated with petroleum and pine oil and copper sulphate.

23. Miscellaneous processes

The following patents are not to be grouped with any of the foregoing nor do they form in themselves a rational group:

Haultain (1,226,330/1917) and **Sundberg** (1,326,545/1919) depend upon complicated schemes for re-treatment of flotation concentrates in order to utilize the difference in floatability, under given conditions, that is inherent in minerals. Thus, galena being more easily floated than blende under given conditions, the first concentrate of the primary machine will contain a higher proportion of lead to zinc than was present in the original feed. By re-treating this concentrate under similar conditions a further increase in the lead-zinc ratio will be effected. The increase in the lead-zinc ratio is set forth by Sundberg as being of the nature of a geometrical progression, hence a small number of re-treatments is expected to produce large differentiation. Actually the ratio does not increase rapidly and there is a large amount of middling formed that cannot be graded up practically and must be joined with concentrate or tailing, thus ruining results.

Peck (1,420,138 and 1,420,139/1922) describes an impracticable extension of the principle set up in Higgins (1,236,934). He proposes to mount a couple of pneumatic cells in a vertical-spindle centrifugal machine, placing the canvas "bottoms" parallel to the spindle, and by controlling the centrifugal force, increasing the difference in the resistance offered by heavy and light sulphide particles to gas-bubble buoying. He would thus float blende from galena.

Lockwood and Samuel (956,381/1910) claim that preliminary agitation of a ground ore in a solution of a substance such as caustic alkali or sodium or potassium silicate results in cracking apart attached sulphides such as blende and galena, thus freeing them for separation by any means, *e.g.*, table concentration.

Lockwood (1,043,850/1912) claims that the silicate of an alkali metal in a pulp being oiled with the magnetic paint of the Murex process (see Sec. 13, Art. 18) will cause differential oiling of sulphides, or oiling of oxides or carbonates in preference to gangue or oiling of gangue in preference to sulphide. Differences in the amount of silicate and in the oils used for the paint produce the differences in behavior. The patent gives several examples. Any differential oiling that occurs here is a result of differential gangue adsorption.

Emerson (1,126,965/1915) describes treatment of a table concentrate containing blende and other sulphides by immersion in a 10-per cent. acid bath, which is said to result in preferential agglomeration of the blende by gas bubbles. The agglomerates are buoyant enough to rise slightly above the settled mass and are to be skimmed away by a submerged bucket conveyor.

Bacon (1,312,668/1919) describes selective separation of the oxidized minerals of iron, copper, lead and zinc by differential sulphidization. See p. 897.

Thornhill (1,338,264/1920) points out that in ordinary flotation concentration, using certain oils such as coal oil (kerosene), certain minerals, notably molybdenite, flocculate more completely than others, *e.g.*, iron or copper pyrite, and that by screening the flotation concentrate the molybdenite floccules will remain on the screen.

24. Mill performances of differential flotation

Lead-zinc ores

Zinc Corporation, Broken Hill. Feed: Zinc middling assaying 9.2 per cent. Zn, 14.3 per cent. Pb and 2.6 oz. Ag per ton. Treated in a pulp containing 33 per cent. solid in a 10-compartment 24-in. Hebbard (sub-aeration) machine with spindles running at 700 r.p.m. Oil: 0.2 lb. per ton of 1-to-1 mixture of eucalyptus oil and tar. Lead concentrate removed from all boxes. Tailing, with 0.3 lb. of eucalyptus oil and 0.6 lb. copper sulphate per ton of solid, sent to a 14-compartment machine. Rough zinc concentrate assayed 39 per cent. Zn and was cleaned to 42–43 per cent. Zn. Rougher tailing, 2 per cent. Zn, 0.5 per cent. Pb and 0.5 oz. Ag per ton. Recoveries: 85 per cent. Pb, 68.2 per cent. Ag, 50 per cent. Zn.

Broken Hill South (*28 MM 9*). Current slime was treated in nine 3.5 × 7-ft. circular under-driven sub-aeration machines with 18-in. disk impellers, in series. Capacity, 9 tons solid per hr.; 200 kw. for the whole plant. Each impeller drew in 2200 cu. ft. of air per hr. The best pulp consistency was 45 per cent. solids. Oil, 0.05 lb. eucalyptus per ton. Lead concentrate assayed 61 per cent. Pb, 47 oz. Ag and 8.4 per cent. Zn; tailing, 2.5 per cent. Pb, 1.5 oz. Ag and 13.9 per cent. Zn; recovery of lead, 80 per cent., silver 84 per cent. Flotation of the tailing for blende has been discontinued and this material is being impounded. Over 99 per cent. of the float galena passed a 0.063-mm. screen. The silver was apparently in an easily floated mineral free from the lead and zinc at this size. The circuit water contained 370 grains of dissolved salts per gallon. Detailed costs are given in Table 39. Central Mine, Broken Hill (*118 P 149*). Dump slime and current leady

Table 39. Cost of differential flotation at Broken Hill South (1922)

Item	Labor	Supplies	Power	Total
Superintendence	\$0.016	....	....	\$0.016
Flotation	0.308	\$0.226	\$1.074	1.608
Tailing disposal	0.128	0.040	0.046	0.214
Filtration	0.082	0.130	0.016	0.228
Concentrate handling	0.024	....	....	0.024
Sundry	0.036	0.118	0.036	0.190
Total	\$0.594	\$0.514	\$1.172	\$2.280

tailing from the gravity plant were treated in 10-compartment sub-aeration machines. Galena float was taken off in the first three boxes; acid but no oil was added; the water, however, was reclaimed from zinc-flotation products and therefore contained oil. Blende was floated from the tailing of the lead machines on addition of acid and eucalyptus oil and heating to 130° F. Results are given in Table 40. The lead concentrate was much finer than the zinc as is shown in Table 41. Also the coarser sizes were higher in zinc in both concentrates. Cœur d'Alene, 1918 practice (*105 J 741*). Ziegler states that "starva-

Table 40. Metallurgical results of differential flotation at Central Mine

Product	Assays			Recovery, per cent.		
	Ag, oz. per ton	Pb, per cent.	Zn, per cent.	Ag	Pb	Zn
Feed, dump slime	15.6	17	19.1	....	....	....
Lead concentrate	53.8	49.9	21.4	35.5	30.1	11.6
Zinc concentrate	17.5	15.2	41.7	33.9	26.9	65.9
Tailing	5.9	8.6	6.7	....	....	....
Feed, current	....	4.2	18.0	....	....	....
Lead concentrate	....	50.0	....	....	....	....
Tailing from lead cells	....	3.0	18.5	....	....	....
Zinc concentrate	....	6.0	47.5	....	....	....
Tailing	....	1.0	2.0	....	....	....

Table 41. Sizing-assay tests of lead and zinc concentrates from differential flotation at Central Mine

Screen, I.M.M. mesh	Zinc concentrate				Lead concentrate			
	Weight, per cent.	Assays			Weight per cent.	Assays		
		Ag, ounces	Pb, per cent.	Zn, per cent.		Ag, ounces	Pb, per cent.	Zn, per cent.
On 40	6.7	8.8	3.0	50.2	....	....	....	....
60	22.0	11.4	4.2	48.4	....	....	....	....
80	20.0	11.4	4.5	47.6	....	....	....	....
100	11.6	12.0	5.0	46.6	....	....	....	....
120	4.2	11.2	3.8	47.0	1.2	....	48.8	16.6
150	6.2	11.2	3.5	47.2	1.2	....	61.4	12.4
200	5.0	11.8	4.7	47.4	3.6	63.4	61.0	12.2
Through 200	24.2	14.2	6.0	45.4	94.0	56.6	60.4	13.6
Totals....	100.0	12.2	4.6	47.3	100.0	59.1	60.0	13.3

tion" quantities of pine oil or wood creosote were the usual reagents for lead flotation. Also, that a light coal-tar distillate made soluble in strong caustic soda solution floated lead almost entirely free from zinc. A small amount of pine oil dissolved in a large amount of wood alcohol also raised lead, but with it considerable fine blende. Sodium chloride or sodium carbonate aided in galena flotation at times, but the action was erratic. Additions of copper sulphate and pine oil were made to raise zinc. At the tailing plant at WALLACE (*106 J 528*) a feed sizing 10 per cent. on 48-mesh and 60 per cent. through 200-mesh was treated in a pulp containing 13.3 per cent. solids in a K. and K. machine with about 0.5 lb. per ton of pine oil. The bubble column was carried 14 in. deep and the top 3 in. scraped as lead concentrate, which was tabled, yielding a lead streak assaying 70 to 75 per cent. Pb and 3 per cent. Zn; middling 40 per cent. Zn and 10 per cent. Pb and slime, 30 per cent. Pb and 10 per cent. Zn. Tailing of the first machine was treated in a second of the same type with 0.5 lb. each of Pensacola Tar and Turpentine Co. No. 350 (wood creosote) and coal-tar oil. The concentrate was tabled yielding a lead streak assaying 60 per cent. Pb and 3 per cent. Zn and a zinc residue carrying 35 per cent. Zn and 12 per cent. Pb. Parker (*114 J 629*) describes the use of sodium sulphide at BUNKER HILL AND SULLIVAN. It was added at the ball mill in order to get about 15 min. contact with the ore before lead flotation was attempted. Oil was added at the flotation cell. Galena concentrate was cleaned with the addition of more sulphide. The best results in zinc flotation were obtained by grinding lead-section tailing with oil. SLOCAN, B. C. (*114 J 679*). Early practice at the SURPRISE mill was to float lead in neutral pulp with 0.1 lb. cresylic acid per ton in a Hebbard machine, then add 5 lb. of water-gas tar and 0.6 lb. per ton of Cleveland Cliffs hardwood creosote and float zinc in an M. S. machine. The original feed assayed 1.5 to 2.5 per cent. Pb, 10 to 15 per cent. Zn and 25 to 30 oz. Ag per ton; lead concentrate, 40 to 45 per cent. Pb, 25 to 30 per cent. Zn and

200 to 300 oz. Ag, but the recovery was low and the operation delicate. Tailing from the M. S. machine assayed 6 oz. Ag, 5 per cent. Zn and a trace of lead. Re-treatment of tailing in a Hynes machine with 0.3 lb. water-gas tar, 0.02 lb. of Pensacola No. 350 pine oil yielded a concentrate assaying 10 per cent. Zn, which was returned to the M. S. machine, and a tailing containing 2 oz. Ag and 1.7 per cent. Zn was made. Later, at the ROSEBERRY mill in the same district, the same procedure was used for lead, but lead-machine tailing, sizing 60 to 75 per cent. through 150-mesh, was thickened to 30 to 35 per cent. solid and sent through four Hynes machines in series, the first three making concentrate assaying 40 to 45 per cent. Zn and 45 to 95 oz. Ag and the last a middling containing 10 to 12 per cent. Zn. Tailing assayed 0.6 to 3.0 per cent. Zn and 0.5 to 3.7 oz. Ag, depending on the ore. The oils were water-gas tar and Cleveland Cliffs hardwood creosote ranging from 0.75 to 1.65 and 0.21 to 0.55 lb. per ton of each respectively. CONSOLIDATED MINING AND SMELTING CO. OF CANADA (*Bull. 617, Canada Dep't of Mines, Mines Branch, 1923*). Ore from the Sullivan mine of this company was ground to pass 65-mesh in a ball mill. A mixture of coal tar and coal-tar creosote together with 5 lb. per ton of soda ash was fed to the ball mill. A little cresylic acid and 0.1 lb. sodium cyanide per ton were added at the head of the lead cells. Tailing from lead flotation was sent to a mixer and Barrett No. 634, T-T mixture and 1 lb. per ton of copper sulphate were added and the pulp was sent to the zinc cells. Results are shown in Table 42. (For performance at the KIMBERLEY mill of this company see

Table 42. Laboratory test on Sullivan ore of Consolidated Mining and Smelting Co. of Canada

	Assays						Recovery, per cent.	
	Pb, per cent.	Zn, per cent.	Cu, per cent.	Au, ounce	Ag, ounces	Insoluble, per cent.	Pb	Zn
Feed	1.96	5.52	....	0.02	3.23	....	....	....
Lead concentrate	51.09	4.17	2.60	0.30	66.20	16.13	89.4	....
Zinc concentrate	1.09	46.19	....	....	....	....	....	94.8
Tailing	0.15	0.32	....	....	....	....	....	....

p. 166.) In the zinc mill at Anaconda the feed contains 10 to 11 per cent. Zn, 2.25 per cent. Pb and considerable pyrite. It is floated in agitation-froth and pneumatic machines with 2.5 lb. CaO per ton and about 0.6 lb. of T-T mixture to make a collective concentrate of lead and zinc and drop iron. Concentrate assays 48–49 per cent. Zn, 9 10 per cent. Pb, 4 per cent. FeO and 4 per cent. insol.; tailing 0.7 per cent. Zn. The concentrate is thickened to 65 per cent. solids, agitated in M. S. beater boxes for 15 min. at 50 per cent. solids and 140° F. with about 2 lb. per ton of a mixture of potassium cyanide and zinc sulphate (in theoretical combining proportions), then discharged into Callow cells, with a small amount of additional frothing agent. The overflow is cleaned once in a pneumatic cell; final overflow assays 72 to 75 per cent. Pb, 5 to 7 per cent. Zn and 0.5 per cent. insol., representing 50 to 55 per cent. recovery of lead. Tailing from the rougher and cleaner machines combined assays 53 per cent. Zn, 4 to 5.5 per cent. Pb, 4 per cent. FeO and 4.5 to 5 per cent. insol., making about 92 per cent. recovery of zinc.

At UTAH APEX thiocarbanilid (dry), 0.06 lb. per ton; soda ash, 2 to 4 lb.; xanthate, 0.10 lb.; cyanide, 0.10 lb.; and zinc sulphate, 0.30 lb. are fed to the ball mills; pine oil, 0.05 lb., to a centrifugal pump ahead of the pneumatic flotation cells, and lead is floated. Alkalinity is maintained constant by half-hourly titrations and pulp density by hourly determinations. The tailing of the lead cells is pumped to a contact tank together with soda ash, 1 to 2 lb.; copper sulphate, 1.5 lb.; T and T, 0.10; cyanide, 0.10; zinc sulphate, 0.20; pine oil, 0.40 lb. per ton. From the contact tank the pulp passes to a pneumatic rougher, the early froth to a cleaner, the later froth and the cleaner tailing back to the head of the rougher. Zinc is floated and iron depressed. For flotation results see Table 17.

For performance at SUNNYSIDE MINING AND MILLING CO. see p. 180; at TIMBER BUTTE, see p. 165 and Table 35.

Copper-iron ores

At Rossland, B. C., a pyrrhotite-chalcopyrite-gold concentrate from coarse jigs was ground and floated to drop pyrrhotite. (*115 J 568.*) Mill operation yielded a concentrate assaying 0.91 oz. Au, 4.66 per cent. Cu and 28.5 per cent. Fe as sulphide when treating Wilfley-table tailing carrying 0.074 oz. Ag, 0.30 per cent. Cu and 6.3 per cent. Fe. Tailing assayed 0.043 oz. Au, 0.10 per cent. Cu and 5.2 per cent. Fe. The pulp was ground to 5 per cent. +60-mesh and 86 per cent. −120-mesh; it carried 20 per cent. solids. The Bureau of Mines reports some differential effect on chalcopyrite, pyrrhotite and pyrite with soda ash and sodium hydroxide in the presence of coal tar and creosote. (*Bul. 205 USBM 53.*) On an oxidized feed carrying 0.04 oz. Au and 0.35 per cent. Cu a concentrate carrying 0.34 oz. Au and 8.03 per cent. Cu and tailing, 0.01 oz. Au and 0.13 per cent. Cu were made. At Cia. de Cobre Magistral-Ameca (*116 J 1106*) an ore containing chalcopyrite, bornite, chalcocite and pyrite in a quartz gangue was treated in a K. and K. machine with an oil mixture consisting of a solution of 0.3 lb. heavy Mexican asphalt-base fuel oil in 0.1 lb. per ton of gasoline for the differential effect and 0.1 lb. per ton of steam-distilled pine oil for frothing. Daily recoveries were 93 to 95 per cent. in a concentrate assaying 26 to 28 per cent. Cu. Coghill (*TP 182 USBM*) reports tests on chalcopyrite-pyrrhotite ores of Oregon, using starvation quantities (0.1 lb. per ton) of Flotco No. 1 (Flotation Oil and Chemical Co., N. Y.). The feed assayed 1.3 to 3.2 per cent. Cu, tailing 0.12 to 0.90 per cent. and cleaner concentrate 25 to 30 per cent. The ore contained about $2 per ton in gold in metallic state which was freed by crushing and was not recovered in the froth. The tests indicated that the pulp should not be ground in the presence of "insoluble" oil, nor in a disk pulverizer and that iron in the pulp from grinding machines was probably harmful. An anonymous writer (*116 P 167*) describes flotation of chalcopyrite from pyrite and pyrrhotite using a salt solution of substantially the composition of sea water and no oil and also with sea water alone. With the latter, from a feed assaying 1.75 per cent. Cu, concentrate was made assaying 21.6 per cent. Cu and tailing assaying 0.09 per cent. Cu. Middling contained 0.71 per cent. Cu but was small in bulk and could be expected to disappear in circulation. Tests on other ores indicated that cranky operation was to be expected. At Moctezuma Copper Co. (*118 J 445*) 90 per cent. of the pyritic iron is rejected and a chalcopyrite concentrate made assaying 28 per cent. Cu, 29 per cent. Fe and 8 per cent. insoluble. The sulphide content is practically pure chalcopyrite. The feed is ground to 12 per cent. on 48-mesh, 52 per cent. through 200-mesh and is floated in Callow cells. The tailing assays 0.31 per cent. Cu. Concentrate is cleaned three times. The lower rougher cells are run counter-current. Originally the oil mixture consisted of gas-works coal tar and steam-distilled pine oil and it was found that the best quantity was the minimum possible, viz.: 0.35 lb. per ton. Later T-T mixture (20 per cent. thiocarbanilid and 80 per cent. orthotoluidin) with lime and pine oil were found to give the best results. Lime is added at the ball mills, the other reagents to classifier overflow. Consumption is 0.28 lb. per ton of T-T mixture, 0.14 lb. per ton of pine oil, and 5 to 6 lb. of lime, the latter sufficient to show 0.15 to 0.35 lb. of CaO content per ton of tailing water. Lower alkalinity causes higher tailing loss; higher makes froth hard to clean. See p. 102 for work at Phelps Dodge Morenci branch. At Cananea (*121 J 507*) a high-iron chalcopyrite-pyrite ore with small amounts of chalcocite, bornite, sphalerite and galena in a quartz-feldspar gangue is floated differentially, using lime to depress the iron. Present results compared to those using gravity concentration and collective flotation follows:

Item	Last Half 1923			First Half 1925		
Cost per ton (concentrating	$0.83			$0.90		
Ratio of concentration	2.575			10.18		
Recovery, Cu	87.40			91.17		
Recovery, Ag	88.21			90.18		
Recovery, Au	95.80			88.82		
Assays, per cent.	Cu	Fe	Insol.	Cu	Fe	Insol.
Feed	1.95	16.7	....	1.91	15.7	......
Concentrate	4.38	36.0	15.20	17.69	30.2	7.3
Tailing	0.297	5.2	......	0.17	12.1	......
Power consumed, kw.-hr. per ton	15.8			18.20		
Coal tar, pounds per ton	1.243			....		
Fuel oil, pounds per ton	1.137			....		
Pine oil, pounds per ton	0.608			0.227		
Xanthate (Z-3), pounds per ton	....			0.090		
Lime, pounds per ton	....			11.18		
Moisture in concentrate	11.4 *a*			9.1		

a Average of gravity concentrate at 7.76 per cent. and flotation concentrate at 18.67 per cent.

Results with T-T mixture were the same as with xanthate. An enormous economy in smelting charge per pound of copper results from the change. The mill water comes from the mine and contains large quantities of iron salts; it is added to the tailing thickeners and there neutralized with lime, the gelatinous precipitate thus produced is dragged down rapidly by the settling pulp solids.

CANADA DEP'T OF MINES (*Bul. 617, Mines Branch, 1923*) reported tests on ore which contained 3 per cent. copper as chalcopyrite, 39 per cent. Fe as pyrite and 8 per cent. insoluble and required to be ground to 100-mesh to free the minerals. Ten pounds lime per ton of ore was added to the ball mill and by maintaining the pulp between 25 and 28 per cent. solids a 22-per cent. Cu concentrate representing 95 per cent. recovery was obtained.

Zinc-iron ores

Most present-day practice uses sodium carbonate or bicarbonate with or without cyanide and zinc sulphate or copper sulphate to hold down iron. At MAGMA COPPER Co. blende was floated from pyrite using a mixture of fuel oil, 34° Bé. and G. N. S. No. 17 together with 0.1 lb. copper sulphate per ton of ore (*4 UId 18*). The concentrate was later tabled to separate galena from blende. The cost of the flotation operation alone was $0.34 per ton (1918); filtering concentrate cost 7¢ per ton on the basis of 185 tons feed per day. The total milling cost was $1.10 per ton including $0.22 for sorting, (*114 P 662*). In the Canada Dept. of Mines laboratory (*Bul. 617, Mines Branch, 1923*) it was found that in a pulp made alkaline with soda ash, iron was usually held down. If not, lime was added to the ball mill and the pulp then de-watered and made distinctly alkaline with soda ash. Copper sulphate frequently increased the recovery of zinc.

Copper-zinc-iron ore

CANADA DEP'T OF MINES, (*Bul. 617, Mines Branch, 1923*). Ore contained 7 per cent. chalcopyrite, 7 per cent. blende, 73 per cent. iron sulphides and 13 per cent. non-metallic minerals. It required to be ground to 200-mesh to free the minerals. An excess of lime above that required for high-grade copper concentrate was added in the grinding mill and prevented pyrite and blende from floating. After floating the chalcopyrite, the pulp was dewatered to remove part of the lime and fresh water and soda ash were added. Results are shown in Table 43.

Table 43. Laboratory test of copper-zinc-iron ore

Product	Assay				Distribution, per cent.		
	Cu, per cent.	Zn, per cent.	Fe, per cent.	Au, ounces	Cu	Zn	Au
Feed (see text)	....	....	....	....	100.0	100.0	100.0
Copper concentrate	16.7	4.4	24.3	0.10	87.9	10.6	18.2
Copper middling	1.8	8.7	28.3	0.10	1.9	4.2	3.6
Zinc concentrate	0.45	40.7	17.9	0.11	1.3	52.5	10.8
Zinc middling	0.5	8.9	33.7	0.07	1.3	10.9	6.6
Tailing	0.3	1.9	35.4	0.07	7.6	21.8	60.8

FLOTATION OF NON-METALLIC MINERALS

As early as 1885 EVERSON had the idea of separating oxidized minerals of copper and lead from the ordinary gangue minerals by the selective action of organic compounds in the presence of water, but her process of separation, even as applied to sulphides, was not practical, and the conception languished. In 1905 SCHWARZ conceived the idea of producing a sulphide film on the surface of the oxidized minerals of the base metals and thereafter floating them as sulphides. This conception, in many variations, appears in a number of subsequent patented proposals, but, with a few notable exceptions, *e.g.*, the flotation of lead carbonate at SHATTUCK-ARIZONA (see p. 188), has not been commercially effected. It has shown promise with carbonates of copper (azurite and malachite), but has failed with the silicate, and the

latter forms so important a part of most oxidized-copper deposits that, unless it responds, the process is a commercial failure. An obvious departure from sulphide filming, particularly for copper ores, was complete solution of the oxidized compounds and subsequent precipitation in floatable form, *e.g.*, as sulphide or metallic copper. A large amount of experimentation on both small and large scales has been done on this method, but with the exception of Chino Copper Co., no particularly close approach to success has been made. The difficulties have been numerous. Precipitated copper sulphide is difficult to float. Copper sulphides present in the ore do not dissolve and the presence of the precipitated sulphide or of some of the reagents or reaction compounds incident to solution or precipitation inhibits their flotation. The same is true, in a measure, when metallic copper is precipitated. Precipitation of metallic copper in floatable form has proved difficult and expensive and regeneration and re-use of solutions has been complicated by fouling. Precipitation by sponge iron, as developed at Chino, has been the most promising method. A large part of the problem of successful operation was the economical production of the sponge. More recently various patentees have reverted to the Everson conception and proposed direct flotation of the oxidized base-metal minerals by the use of certain reagents, principally of a class including soluble soaps, whose action is speculated to be a chemical reaction with the surface layer of the mineral to be floated, resulting in the formation of an organic-film coating that will act similarly to an oiled-sulphide surface.

In the course of investigations along the latter line, experimenters have found that there are differences in floatability of non-metallic minerals. Coal has been found to be easy to float away from slate and other non-carbonaceous impurities, and differential flotation of coking and non-coking varieties of bituminous coal has been described. (See p. 888.) Phosphate minerals have been separated from quartz and other silicates, hornblende from feldspar and quartz; calcined alumina from silicate impurities, and calcite and fluorite are quite readily separated from quartz.

25. Flotation of oxidized base-metal minerals

The proposed methods may be classified into three general groups, *viz.*: (1) sulphide filming; (2) solution and precipitation as sulphide or metal; (3) selective oxide-flotation. None of the methods has had any general success, and it is safe to say that the problem has not, as yet, been solved. Each ore appears to be a separate problem, amenable to a partial solution by any one of the three methods, but only in rare cases responding satisfactorily.

Sulphide-filming processes are described in the following patents.

Schwarz (807,501/1905) describes treating oxide ores with a soluble sulphide, *e.g.*, sodium or potassium sulphides or polysulphides, in solution, in order to convert the oxidized surfaces to sulphide surfaces prior to separation from the rocky gangue by reason of the selective action of oils.

Terry (1,094,760/1914) describes the use of hydrogen sulphide in water for sulphide filming of base-metal oxides and carbonates, after which froth flotation is effected. He notes also that hydrogen sulphide will precipitate sulphides from base-metal solutions and claims that such precipitated sulphides aid flotation of original sulphides and sulphide-filmed particles, but this is not general experience. A base-metal solution, *e.g.*, copper sulphate, may be added to produce sulphide precipitate. The process may be carried on in neutral, acid or alkaline solutions, with or without heat.

Bacon (1,140,866/1915) recommends neutralizing the excess of soluble sulphide over that used for filming and simultaneously making the solution faintly acid. Sulphur dioxide is suggested for neutralization and acidification. In some cases flotation is better in neutral or

slightly alkaline solutions, but it is necessary in such cases to destroy the excess of soluble sulphide or render it innocuous before flotation.

Bacon (1,180,816/1916) states that filming by soluble sulphides is much more efficiently performed with the pulp under pressure. He cites an example in which hydrogen sulphide is used and the pressure in the filming chamber is 20 lb. per sq. in.

Martin (1,236,856/1917) recommends monosulphides, polysulphides or hydrosulphides of sodium, potassium and calcium to effect direct selection of oxidized particles without filming. This last qualification is apparently an effort to escape the incidence of previous patents covering filming by soluble sulphides. Martin cites in defense of his claim that copper carbonate particles thus floated are green and hence, according to him, non-sulphidized, but this fact is, of course, no proof of his contention. He recommends making the polysulphide solutions by boiling for ten minutes the following mixtures: (1) 52 parts sodium hydroxide, 48 parts flowers of sulphur, 6000 parts water; (2) 47 parts lime, 53 parts flowers of sulphur, 6000 parts water; (3) 44.6 parts lime, 50.4 parts flowers of sulphur, 1 part sodium hydroxide, 6000 parts water; (4) fused sodium sulphide of commerce (sodium polysulphide) may be used.

Example. 500 gm. copper sulphide-carbonate ore, 1500 cc. water at normal temperature, 10 cc. of solution (1) above, 1 lb. per ton of a mixture of 90 per cent. gas-tar creosote and 10 per cent. yellow pine oil. Solution (3) was used with reconstructed oils (see p. 847) on slimes carrying 0.9 per cent. Cu of which 0.3 per cent. was oxidized. According to Martin, the concentrate assayed 10 per cent. copper and tailing about 0.1 per cent.

Bacon (1,312,668/1919) describes making soluble sulphides by heating alkaline or alkaline-earth materials, *e.g.*, lime or the hydroxides, carbonates or borates of alkalies or alkaline-earths with metallic sulphides or concentrate, and subsequent separation of metallic oxides from ores and of the metallic oxides produced, by using the soluble sulphide for filming and flotation. He notes also that those native oxidized minerals whose sulphides are insoluble in acids may be separated from those whose sulphides are soluble by sulphidizing and floating in an acid pulp, and that subsequently the second mineral may be sulphidized by rendering the pulp alkaline. Thus copper minerals may be sulphidized in the presence of iron, or lead in the presence of zinc.

Thompson (1,334,721/1920) recommends treatment of a sulphidized pulp by blowing with air prior to flotation. In case hydrogen sulphide has been used as the sulphidizing agent this treatment removes the residual gas, which, in certain instances, appears to interfere with subsequent flotation. Thompson describes a porous-bottom launder as suitable for the blowing treatment.

Callow, Thompson and Terry (1,334,733/1920) recommend simultaneous sulphidizing and oiling of the pulp, preferably during grinding. They also mention that further subsequent sulphidization may, in some cases, be beneficial and that in some cases blowing before flotation, as described by Thompson (1,334,721), is advantageous. In patent 1,334,734, the same patentees describe oiling of the pulp prior to sulphidization as preferable in certain cases.

Palmer, Seale and Nevett (1,401,435/1921) claim that "elemental sulphur" or sulphur in solution, prepared as described on p. 882 will float weathered sulphides and will accelerate the action of sulphidizing agents on oxidized ores.

Ellis (1,425,185; 1,425,186; 1,425,187/1922) claims that the use of salts containing ions of high valence, such as sodium pyrophosphate, titanium chloride, etc., has beneficial effect in flotation of oxides, carbonates or hydroxides either with or without sulphidizing.

Borcherdt (1,446,375/1923) recommends removal of colloidal constituents of the ore prior to sulphidizing and flotation. If the colloids are de-flocculated, removal is effected by simple overflow (decantation); otherwise dispersion must first be effected by adding suitable reagents, *e.g.*, sodium silicate, gum arabic, foundry molasses, sodium tri-phosphate, etc. Acidity should be neutralized before dispersing by means of lime, caustic soda and the like. If an excessive amount of flocculating salts, such as magnesium sulphate, is present, they should be removed by washing prior to de-flocculation.

Smith (1,452,662/1923) describes a process in several steps, as follows: (1) Grind to suitable size, preferably -30-mesh. (2) Separate the ground product into sands and slimes. (3) Sulphidize each portion separately in the same manner preferably in a thick, slightly-alkaline pulp at a temperature of about 50° C., using a soluble sulphide. (4) Thicken as much as possible, then oil the thickened pulp with a "filler oil," *e.g.*, petroleum sludge. The quantity of oil used should be sufficient to fill about 20 per cent. of the voids in the mass of solid material. (5) Add a flotation oil such as crude turpentine. (6) Discharge the oiled pulp into a flotation cell containing four or five times its volume of clean, cold water and agitate slightly in a way equivalent to shaking a few times in a test-tube.

Smith (1,459,167/1923) describes sulphidizing in the presence of a so-called preparation oil in a thick pulp, removing the water, adding a comparatively large quantity of water uncontaminated by preparation oil, and producing a froth concentrate by any of the customary methods. Alternatively dilution alone may replace decantation of the preparation

oil and dilution. The laboratory examples of the patent show about 70 to 80 per cent. recovery of silver and 85 or 90 per cent. recovery of lead.

Eberenz (1,505,323/1924) describes flotation of oxidized minerals (together with sulphides) by the use of hydrogen sulphide in an agitation-froth machine with air excluded or with some "other reducing or non-oxygenous reagent," *e.g.*, an alkaline sulphide, and adding an inert gas such as nitrogen or carbon dioxide for levitation. The pulp is preferably neutral but may be either slightly acid or slightly alkaline. It is stated that a small amount of resin may be added to improve frothing. An agitation-froth machine with sealed agitation chamber is pictured for effecting the operation.

Smith (1,551,588/1925) describes a method for oxidized or mixed sulphide and oxide copper ores, consisting in pre-treatment by slow agitation in a thick pulp with a mixture of gases, principally H_2S, CO_2 and H, and subsequently floating in a more dilute pulp by the usual methods. The patent contains also a description of a method for making the hydrogen sulphide free of various deleterious sulphur compounds.

Nokes (1,444,552/1923) recommends preparing an oxidized ore for flotation by incorporating with it, as by grinding, a reagent made by melting together a normally solid hydrocarbon, such as paraffin, and an alkaline sulphide; thereafter floating with a suitable frothing agent, such as pine oil.

Example. 500 gm. of oxidized silver ore with 500 gm. water was ground to 80 per cent. −80-mesh in a ball mill with a mixture consisting of 2.5 gm. sodium sulphide and 1.75 gm. paraffin, fused as above, and 0.12 gm. yellow pine oil. Excess water was decanted from the ground pulp, and it was floated with 1200 cc. water, 0.25 gm. reconstructed pine oil and 0.75 gm. of a mixture of 50 per cent. benzol, 20 per cent. reconstructed pine oil and 30 per cent. petroleum. From a feed assaying 0.24 oz. Au, 33.36 oz. Ag, and 4.35 per cent. Mn a concentrate was made assaying 3.50 oz. Au, 374 oz. Ag and 4.50 per cent. Mn and the tailing assayed 0.04 oz. Au and 4.96 oz. Ag.

This patent contemplates simultaneous sulphidizing and oiling because of the immediate proximity of the paraffin and sulphide when the latter is freed in the water by grinding.

Solution-and-precipitation methods have been less frequently described in the patent office than either of the other classes, but more large-scale experimental work has been done on them. The method is practically restricted in application to copper and zinc ores. Lead is excluded because of the difficulty of rendering it water soluble. With completely oxidized copper ores and with zinc ores the method comes into competition with highly efficient hydrometallurgical processes and so far has failed utterly in the competition. This practically restricts its application to copper ores containing mixed sulphides and oxides, the former under ordinary circumstances readily amenable to flotation concentration and not readily soluble in commercial-leaching solvents. Most of the large porphyry-copper companies have experimented with the patented methods and with unpatented variants, but with the exception of Chino Copper Co., as previously mentioned, none has approached success. At Chino the process involved solution in dilute sulphuric acid, precipitation of metallic copper in the pulp by means of sponge iron, and flotation of the sulphide and metallic copper at one operation.

Sulman and Picard (1,333,688/1920) describe what they claim to be an improved method for precipitating copper as sulphide for flotation purposes. This consists in mixing with the ore pulp containing dissolved copper a powdered mixture of calcium and ferrous sulphides. This mixture is made by heating together pyrite or copper pyrite and an equivalent of lime at a low red heat and cooling out of contact with air or, in another method, by heating together with the above ingredients an amount of carbon equivalent to the oxygen present. In precipitation the solution should be slightly to strongly acid, according to the copper content. If the solution contains more than 0.2 to 0.4 per cent. copper, acid in an amount equivalent to 3 to 5 per cent. is necessary, if rapid precipitation is desired.

The patentees also state, like Terry (1,094,760), that flotation of the sulphides present in the original ore is enhanced by the presence of the precipitated sulphide.

Terry (1,541,292/1925) describes a method especially adapted to copper ore, consisting in dissolving the oxidized copper compound in sulphuric acid solution, precipitating with lime, calcium carbonate, an alkali-metal carbonate or sponge iron, then treating the pulp with acetylene to form a copper acetylide and floating this by the usual methods. It is desirable that these reactions be confined to the surface of the mineral. He states that the same method is applicable also to silver and mercury minerals. **Terry** (1,541,293/1925),

describes the use of an ammoniacal solution containing a small amount of copper, followed by acetylene to produce the copper-acetylide coating. This patent is applicable also to sulphide ores.

Cremer (1.550,512/1925) recommends solution and precipitation of the oxidized mineral as sulphide, using the precipitated colloidal or semi-colloidal metallic sulphide as an agent to float primary sulphide of the same metal also present in an ore pulp.

Nevill (1,551,605/1925) describes a process for floating cement copper in a pulp containing ferrous sulphate by using a neutral or reducing gas such as coal gas instead of air as the levitating agent, thus preventing oxidation of the cement-copper surface or formation of ferric sulphate and solution of copper thereby.

Selective-oxide flotation. The methods proposed comprise two varieties, *viz.:* (1) Methods in which chemical reaction probably occurs between the surface of the oxidized metal-bearing mineral and one of the reagents used to effect flotation, resulting in coating of the mineral particles with the reagent. (2) Methods in which differences in surface are depended upon in precisely the same way as in ordinary sulphide flotation. The first may be called CHEMICAL-COATING METHODS; the second, OXIDE-FLOTATION METHODS.

Chemical-coating methods

It is not impossible that Nokes (1,444,552, p. 899) is in this class.

Whitaker (1,457,680/1923) recommends coating oxide particles with insoluble or slightly soluble soaps, which soap surfaces then act like sulphides in flotation processes. Thus an oxidized copper ore may be treated with a dilute solution of acetic, sulphurous, sulphuric or hydrochloric acid, forming at the surface of the copper particles the corresponding copper salt, which will then react with certain saturated or unsaturated fatty acids or their glycerides, or vegetable or animal oils containing the acids or glycerides. Oleic and stearic acids are named.

Sulman and Edser (1,492,904/1924) describe a non-sulphidizing process for oxidized ores comprising the addition of a soluble soap, *e.g.*, sodium oleate, stearate, palmitate or resinate, in an amount that will leave a small amount of free soap in solution, usually varying between 0.1 and 1 per cent.; also in some instances, in addition, about 1 per cent. or less on the ore of certain oils, *e.g.*, kerosene, gas oil or other hydrocarbon oils; or certain soluble salts of alkaline reaction, such as sodium silicate or sodium phosphate; or silicic acid sol; and, in some cases, caustic alkali. The pulp water should be free of substances such as calcium or similar salts or acids that will decompose the soap. Some ores may require a frothing agent in addition to the soap.

The patentees claim that the method is applicable not only to the usual oxidized ores but also to sulphides, sulphur, carbonaceous materials and for separation of hornblende or hematite from feldspar and quartz.

Clark (1,548,351/1925) describes a method for oxidized lead ores in which the ore is treated with a chromate, preferably of copper or in the presence of a dissolved copper salt, or by some other substance which reacts with the oxidized lead and forms at its surface a coating with oxidizing powers; subsequently adding an organic agent capable of being oxidized by the substance formed at the lead-mineral surface, thereby coating the lead mineral with an organic oxidation product, and thereafter floating by the usual methods. Copper bichromate or sodium bichromate and copper sulphate are recommended for the initial treatment in neutral or slightly acid pulps and aniline oil or an acid salt of aniline, linseed oil, turpentine, pine oil or the like for the organic coating compound. Calcium hypochlorite (bleaching powder), hydrogen peroxide or ferric cyanide together with the above organic substances are recommended for alkaline pulps. With some ores preliminary treatment in alkaline or neutral solutions with copper sulphate or potassium-copper cyanide followed by coating treatment with acetylene or other oxidizable organic substances is recommended.

Oxide flotation

Everson (348,157/1886) (p. 790) states that her process is applicable to "oxides and carbonates of copper and the carbonate of lead," but there is no evidence in the patent that any different treatment was contemplated for these minerals from that designed for minerals of metallic luster.

Lockwood (1,043,851/1912) states that metallic carbonates, oxides and silicates may be oiled and thereafter separated by froth flotation, by agitating the oil and ore together in a thick aqueous pulp containing a small amount of an alkaline carbonate alone or together with a small amount of a silicate of an alkaline metal, *e.g.*, sodium silicate. In the examples given he used in one case 6.2 lb. soda ash per ton of water in a 50 per cent. pulp; in another,

the proportions were 1.25 lb. soda ash and 40 lb. sodium silicate per ton of water in a pulp containing 48 per cent. solids.

Wiser (1,288,350/1918) describes the use of sodium resinate with ordinary frothing agents in order to float mixed sulphides and oxide minerals. Resinate solution may be made by boiling together rosin and a solution containing an excess above the combining proportion of sodium carbonate or sodium hydroxide, *e.g.*, 1 part soda with 6 parts rosin and enough water to make a 5-per cent. solution, is suitable. The amount of sodium resinate recommended is 0.25 to 10 lb. per ton, conveniently added as a solution of 5 to 10 per cent. strength. The resinate may, in some cases be used without other frothing agent. The pulp should generally be strongly alkaline, but may be near neutrality. In some cases sodium sulphide in addition to the resinate improves recovery of both oxides and sulphides, and oils containing sulphur may also be useful.

Luckenback (1,386,716/1921) claims that oxidized minerals may be floated by the use of reagents made by dissolving resinous substances, particularly copal, shellac, or rosin or mixtures thereof in a solvent such as a water solution of caustic soda, borax, or ammonia, or in alcohol. Oleic acid, grease or tallow or other oily substances capable of reacting with the above mixture or some of its constituents in such a way that the "oil constituents [are] neutralized" may be added.

Luckenback (1,417,262/1922) describes the use of "a reagent comprising a liquefied resin or combination of resins" for treatment of carbonates, oxides and silicates of metals, as well as for minerals of metallic luster. He states that the resin "may be liquefied by means of an alkali, or alcohol, and may be combined with the reaction product of an alkaline metal and a fatty acid" (soap). The patented reagent may be used in conjunction with a normally unsaponifiable oil or oily substance, if desired. He recommends lac in alcohol, oleic acid and caustic soda or ammonia. The examples cited do not give much promise of success of the process.

Luckenback (1,417,263/1922) describes a reagent made by mixing the reaction product of a resin and an alkali other than caustic soda (alkaline resinate), a normally unsaponifiable resinous substance, *e.g.*, copal, mastic, congo, caoutchouc and the reaction product of an alkali and a fatty acid, *e.g.*, sodium oleate. Sodium oleate may replace the resinate with certain ores. The example of the patent does not indicate commercial utility.

Christensen (1,467,354/1923) describes a non-sulphidizing process for concentrating oxidized ores and, more generally, for separating non-silicate non-metallic minerals from silicates by froth flotation. Essentially the treatment comprises the agitation-froth process with the following limitations; (1) the ore should be ground and oiling of the pulp carried on out of the presence of metals, *i.e.*, grinding in a pebble mill and oiling in an agitation machine with non-metal walls and a non-metal, or at least, not an iron, impeller. (2) The oil should be one containing a goodly proportion of a fatty or resin acid or a similar compound. Typical oils of this class are fish oils, such as menhaden or salmon oil; liver oils such as cod-liver oil; blubber oils such as whale oil; tallow dissolved in other oils, sperm oil, linseed oil, castor oil, olive oil, cocoanut oil and palm oil; oleic and stearic acids; rosin dissolved in kerosene or coal-tar oil. (3) The quantity of oil should be relatively large, *e.g.*, 10 to 15 per cent. of the weight of the minerals to be concentrated. Experiments are cited to show that the silicate mineral in the concentrate decreases with increase in oil quantity up to 4.5 per cent. by weight on a given lead ore and 6 per cent. on a given manganite ore. (4) The water of the pulp should be free of electrolytes. The patentee states that with a manganite-porphyry ore he made a 60-per cent. Mn concentrate and 94 per cent. recovery from a 30-per cent. feed; with a copper-carbonate sandstone ore, 46-per cent. concentrate and 98 per cent. recovery from 12-per cent. feed, also 38-per cent. concentrate and 94 per cent. recovery from a 2-per cent. feed; and with a lead-silver-gold ore in silicified limestone assaying 0.25 oz. Au, 17.2 oz. Ag and 11.8 per cent. Pb he recovered 95 per cent. of the gold, 91 per cent. of the silver and 96 per cent. of the lead in a concentrate assaying 0.07 oz. Au, 45 oz. Ag and 32.6 per cent. Pb.

Tucker and Edser (1,497,310/1924) describe a selective-oxide process both for differential separation and for floating oxidized ores (see p. 888).

26. Flotation of coal

Flotation of coal from the associated rocky impurities (see Sec. 2, Art. 12) is an operation similar to the flotation of sulphide minerals from the usual gangues and the underlying principles are the same in both operations. Some differences in procedure follow from the differences between run-of-mine coal and metalliferous ores. Thus, on account of the lower specific gravity of coal, it is unnecessary to grind it as fine as metalliferous mineral must be ground; 16- to 30-mesh is ordinarily sufficiently fine provided the coal is freed from

waste at these sizes. Peterson (*Bul. 117 CMI 62*) found that − 10-mesh coal was readily lifted. For the same reason, a less coherent froth may be employed. The oil that goes with the concentrate (coal) has fuel value, hence, if oil is cheap enough, considerable quantities may be used and will aid in the subsequent dewatering. The proportion of valuable mineral to waste in the feed is much greater in coal than in metal separation, the valuable material is less valuable and cleanliness of concentrate not so important, hence the operation need not be so careful, cleaning is generally unnecessary and the volume of material treated in a machine may be pushed up to a point that largely compensates for the low specific gravity. Almost any oil that will froth may be used but coal-tar oils and wood-tar oils are the most usual. The reagent generally adopted in England, where there has been some actual washery use of the process for bituminous coal, is two parts of cresylic acid to one part paraffin (kerosene) or gas oil (gasolene). The froth is preferably voluminous and tender and pulp density should be lower than in ore flotation,

Table 44. Performance of agitation-froth flotation process on English coals. (*After Tupholme, 23 CA 177*)

Kind of coal	Feed, per cent. ash	Washed coal, per cent.						Refuse, per cent.		Recovery, combustible, per cent.
		First froth		Second froth		Total				
		Weight	Ash	Weight	Ash	Weight	Ash	Weight	Ash	
Washery waste(*a*)	70.4*b*	....	....	....	....	15	15.6*c*	....	.....	.....
Washery waste(*a*)	67.6*d*	....	....	....	....	21	10.5*e*	....	.....	.....
Washery waste(*a*)	48.5*f*	....	....	....	....	30	10.5*g*	....	.....	.....
Coking coal......	24.0	62.7	3.4	13.2	24.5	75.9	7.1	24.1	78.5	90+
Non-coking coal..	21.8	41.0	6.0	28.6	9.1	69.6	7.3	30.2	74.4	85−
Washery waste...	61.2	....	....	....	....	33.2	10.1	66.8	.....	75
Welsh coal......	10.1	87.2	3.2	2.4	19.2	89.6	3.6	10.4	72.1	95+
Derbyshire coal..	29.5*h*	....	....	....	....	75.1	9.9*i*	24.9	82.4*j*	93
Coking coal......	12.4	....	....	....	....	87.8	3.8	12.2	72.4	93
Coking coal......	24.2	....	....	....	....	75.9	5.2	24.1	78.5	93
Coking coal......	15.8	....	....	....	....	83.2	5.4	16.8	76.0	96
Non-coking coal..	25.5	....	....	....	....	73.8	8.9	26.2	84.5	96
Non-coking coal..	27.0	....	....	....	....	68.3	9.0	31.6	76.0	91
Non-coking coal..	21.8	....	....	....	....	69.8	7.3	30.1	74.4	93
Non-coking coal..	28.2	....	....	....	....	66.7	9.4	27.5	78.8	92
Slack............	30.0	....	....	....	....	68.9	12.2	31.1	80.0	92
Slack............	30.5	....	....	....	....	71.0	9.6	29.0	86.5	95
Silt.............	35.5	....	....	....	....	60.2	8.1	39.8	74.0	83
Silt.............	33.8	....	....	....	....	48.5	11.7	51.5	78.0	89
Silt.............	21.5	....	....	....	....	83.8	9.9	16.2	81.5	96
Silt.............	45.2	....	....	....	....	59.0	12.5	40.5	82.8	85
Washery wastes and dumps....	74.0	....	....	....	....	16.8	13.0	83.2	86.6	57
"	40.3	....	....	....	....	53.0	7.9	47.0	75.8	81
"	61.2	....	....	....	....	30.2	10.1	67.8	86.1	76
"	76.0	....	....	....	....	14.5	13.5	83.0	87.6	56
"	62.2	....	....	....	....	24.3	9.6	70.7	84.8	72
"	62.0	....	....	....	....	22.8	9.2	71.0	82.7	67
"	75.0	....	....	....	....	16.2	14.0	83.8	88.5	62
"	63.8	....	....	....	....	28.7	10.8	71.3	86.5	74

a Powell-Duffryn Coal Co., Aberamau tipple. *b* 2280 B.t.u. per pound. *c* 12,450 B.t.u. per pound. *d* 5230 B.t.u. per pound. *e* 12,280 B.t.u. per pound. *f* 4390 B.t.u. per pound. *g* 12,680 B.t.u. per pound. *h* 48.2 per cent. fixed carbon and 22.35 per cent. volatile. *i* 60.24 per cent. fixed carbon and 29.9 per cent. volatile. *j* 2.75 per cent. fixed carbon and 14.8 per cent. volatile.

particularly if much clay is present, otherwise, without cleaning, the froth will be of low grade. When pyrite is present, it is best to remove it by gravity concentration prior to flotation for the reason that it floats with the coal.

Bacon (1,329,493/1920) states that bituminous coal, if properly crushed and mixed with a small amount of oil or oil mixture or equivalent froth-forming agent can be separated from slate and other non-carbonaceous impurities by ordinary flotation processes. He recommends grinding to 48-mesh in such a way as to produce sharp-edged rather than rounded grains, *e.g.*, in a ball mill rather than in a crusher of the type of a disk pulverizer. As useful flotation agents, he names coke-oven light oil, pine oil, "cold-weather mixture," and petroleum. The pulp may be either acid or alkaline.

Bates (1,552,197/1925) states that pulverizing with oil in a thick pulp at a temperature well above atmospheric is best for subsequent flotation, especially when the surfaces of the coal have become oxidized, as in coal from dumps, caved workings and the like.

Price (1,499,872/1924) describes differential flotation of different grades of bituminous coal, claiming that colloids such as tannin, starch, albumin and glue depress the already low floating tendency of the non-coking constituent.

Performance. Tables 44 and 45 show the results obtained by flotation on various kinds of coals and washery products. For flow-sheet of an operation in a British washery see p. 70.

Applicability. The DISADVANTAGES of froth-flotation in coal cleaning are numerous. The outstanding one is the difficulty of utilizing the product. Dewatering concentrate is difficult and expensive. The fine coal can be readily coked on a small scale, but the possibility of coking on a large scale in present-day furnaces is not clearly established. Transportation would entail some loss and unloading, especially in freezing weather, would be difficult and expensive. Storage and handling of the dry product would be dangerous. Briquetting is a possible outlet for anthracite, but requires education of domestic consumers. The process tends to increase the sulphur content of the washed coal above that of the feed. Another possible outlet is in the form of Trent amalgam (Sec. 14, Art. 4), and it may easily turn out cheaper

Table 45. Results of agitation-froth flotation on coals of Pacific Northwest. (*f*) (*After Ralston, 22 CA 912*)

Coal	Oil: Kind	Oil: Pounds per ton	Feed, per cent. ash	Concentrate, per cent.: First froth Weight	First froth Ash	Second froth Weight	Second froth Ash	Total Weight	Total Ash	Tailing, per cent.: Weight	Ash	Recovery of combustible, per cent.
a	Kerosene (4), pine (1)	6	15.26	25.9	12.24	53.9	13.50	79.8	13.15	20.2	25.28	81.7
a	Kerosene (5), hardwood creosote (1)	6	15.26	95.5	14.38	. . .	. . .	95.5	14.38	4.5	66.18	97.1
b	Crude pine creosote	4	20.81	75.0	15.52	. . .	. . .	75.0	15.52	25.0	36.82	80.0
b	Crude pine creosote	2	20.81	49.7	16.65	42.1	16.29	91.8	16.50	8.2	68.55	76.0
c	Kerosene	4	26.46	82.3	18.41	. . .	. . .	82.3	18.40	17.7	63.90	90.0
c	Pine oil	1.5	26.46	64.9	15.81	17.7	35.90	82.6	19.68	17.4	60.00	89.5
d	X-cake and xylidin	3	24.75	79.0	15.46	10.9	42.45	89.9	18.81	10.1	79.22	96.4
e	Pine oil	2.5	13.70	47.7	6.33	48.7	16.63	96.5	11.51	3.5	67.23	98.4
e	Kerosene	2	13.70	92.6	10.08	. . .	. . .	92.6	10.02	7.4	59.87	95.1

a Low-grade, sub-bituminous. *b* Good sub-bituminous. *c* Bituminous, non-coking. *d* Bituminous, coking. *e* Semi-anthracite. *f* Tests in M. S. type laboratory machine. Feed ground through 65-mesh.

to prepare the latter by concentrating by froth flotation and then forming the amalgam than to concentrate and make the amalgam in the same step.

As the situation stands at present, the process is available, at a cost of between $0.05 and $0.10 per ton of raw fines fed, to treat low-sulphur washery fines and will make a satisfactory low-ash coal. Dewatering may easily more than double the cost per ton of washed coal.

Dewatering coal-flotation concentrate. Tupholme (*24 CA 277*) describes a method in which the froth concentrate in a pulp with water is agitated with 3 to 10 per cent. of oil, tars or other hydrocarbon or carbonaceous liquids, reckoned by weight on the solid coal, as a result of which irregular agglomerates are formed that may be separated from the water by draining, filtration, or pressure. This is, of course, the old Cattermole process (see Sec. 14, Art. 4), applied to coal. It is claimed further that reduction in ash may be made at the same time, the unagglomerated refuse passing off with the water. No commercial application is cited.

27. Flotation of miscellaneous non-metallic mineral substances

Alumina

Hornsey (1,255,749/1918) describes flotation of alumina from associated impurities by means of a small amount (2 lb. per ton) of oleic acid or other oil or frothing agent. According to the patent, the method of procedure is the same as that followed with sulphide ores.

Phosphates

Broadbridge and Edser (1,547,732/1925) describe flotation of phosphates such as phosphorite, apatite and the like away from silicates, calcium carbonate, ferric oxides, aluminates, etc., by means of a frothing agent such as oleic acid or soap and a dispersing agent such as sodium silicate. They also suggest pre-emulsification of the frothing agent by agitation in water in the presence of a small amount of sodium carbonate, sodium resinate or sodium oleate. Example. — 80-mesh dry-ground feed containing 20.5 per cent. P_2O_5 was floated with 2-lb. sodium silicate (60° Bé.) and 2.3 lb. per ton of oleic acid. The froth assayed 35.4 per cent. P_2O_5, and 10.3 per cent. insoluble and represented 94.5 per cent. recovery. Tailing assayed 1.05 per cent. P_2O_5. In a similar test on — 60-mesh material assaying 52 per cent. calcium phosphate, using 1 lb. of oleic acid and 1 lb. of sodium silicate, concentrate assayed 79.3 per cent. calcium phosphate and tailing 1.8 per cent. Recovery was 98.2 per cent.

Cassiterite

Edser describes (*29 IMM 263*) flotation of a mixture of Nigerian cassiterite and quartz gangue, ground to pass 80-mesh, in a 4 : 1 pulp with soft water and a small amount of sodium oleate solution and alkali. He states that several Cornish ores have shown recoveries of 80 to 90 per cent. in a concentrate assaying 20 to 25 per cent. metallic tin.

Sulphur

The fact that sulphur could be floated has been recognized since early in flotation history. Hyde (*Minerals investigation series, USBM, No. 15*) found that under favorable circumstances 80 per cent. recovery in a concentrate assaying upwards of 80 per cent. S could be made. The oils that he found could be used were pine oils, wood and coal-tar creosotes, cresylic acid, coal tar, kerosene. Carbolic creosote or a mixture of equal parts kerosene and Pensacola T. and T. Co. No. 80 pine oil gave the best results. Sherman (*14 UU 82*) reports an extensive series of tests on ores containing native sulphur in a gangue of quarzite and feldspar. The lots tested assayed 15 and 32 per cent. S respectively. The work showed that the feed should be ground to pass a 200-mesh screen in order to free the sulphur and produce high-grade concentrat . A pulp containing 20 per cent. solids was best in a centrifugal bubble-column machine (Fahrenwald) and either acid or alkaline pulp could be used. The best tests gave about 80 per cent. recovery in a concentrate containing about 90 per cent. S. In this test about 1 lb. per ton of Genasco flotation oil was used in an alkaline pulp. The best of the numerous reagents used were the Genasco; T–T and X–Y mixtures, 0.25 lb. per ton; Barrett No. 4, 1 lb.; a gas-works distillate, 2 to 3 lb.; amyl alcohol, 0.25 lb.

Miscellaneous

See Christensen, 1,467,354; Sulman and Edser, 1,492,904.

SECTION 13

MAGNETIC SEPARATION

1. Introduction

Magnetic separation depends upon the fact that minerals differ in permeability and, therefore, in their behavior in magnetic fields. Some minerals, *e.g.*, magnetite and franklinite, are readily attracted to the poles of a magnet in fields of relatively low intensity, while others, such as quartz and the acid silicates, are apparently unaffected by the most powerful fields. Certain other substances, of which metallic bismuth is, perhaps, the best-known example, are actually repelled from the magnet poles in very powerful fields. Experiment shows that all substances are permeable to some extent and that attraction and repulsion depend on the relative permeabilities of the substance and the medium in which it is tested. Physical terminology classes substances that are attracted to the poles in a magnetic field in air as PARAMAGNETIC and those that are repelled as DIAMAGNETIC. For ore-dressing purposes minerals can be better classified as strongly magnetic (the physicist's FERROMAGNETIC), weakly magnetic, and non-magnetic. Such classification is shown in Table 1. Any mineral in the strongly-magnetic class can be separated readily from any in the classes that follow it, and any pure mineral in the weakly-magnetic class from any pure minerals in the non-magnetic class; but the values given in the table are not to be depended upon to determine the possibilities of separation in any given case, without some confirming experiment, for the reason that small amounts of impurity, mechanically or chemically held, may change markedly the position of a given mineral in the list. Thus certain sphalerites containing small percentages of iron are attracted in strong fields and readily separated from pyrite and quartz; iron-bearing garnet is sufficiently magnetic to be separated from quartz and other acid silicates and corundum; magnetic galena occurs at one of the Cœur d'Alene mines, and similar examples of divergence from the tabular order are not infrequent.

Elementary sulphur is diamagnetic while oxygen is paramagnetic and, probably dependent upon these facts, no pure sulphide, except pyrrhotite, is found in the list of magnetic substances while many of these substances are high in oxygen content. On the other hand, research makes it appear probable that permeability is due to molecular structure rather than to chemical compo-

sition. Heat affects permeability, usually decreasing it, but a ferruginous blende is cited that was readily separable (magnetic) when warm but separable with difficulty when cold.

Table 1. Relative magnetic attractability of minerals. (*After Davis, Bul. 7, MSM*)

	Substance	Relative attractability
Strongly magnetic..	Iron (taken as standard)...	100.00
	Magnetite....	40.18
	Franklinite...	35.38
	Ilmenite......	24.70
Weakly magnetic..	Pyrrhotite....	6.69
	Siderite......	1.82
	Hematite....	1.32
	Zircon.......	1.01
	Limonite.....	0.84
	Corundum....	0.83
	Pyrolusite....	0.71
	Manganite....	0.52
	Calamine.....	0.51
Non-magnetic..	Garnet.......	0.40
	Quartz.......	0.37
	Rutile........	0.37
	Cerussite.....	0.30
	Cerargyrite...	0.28
	Argentite.....	0.27
	Orpiment.....	0.24
	Pyrite........	0.23
	Sphalerite....	0.23
	Molybdenite..	0.23
	Dolomite.....	0.22
	Bornite.......	0.22
	Apatite.......	0.21
	Willemite.....	0.21
	Tetrahedrite..	0.21
	Talc.........	0.15
	Arsenopyrite..	0.15
	Magnesite....	0.15
	Chalcopyrite..	0.14
	Gypsum......	0.12
	Fluorite......	0.11
	Zincite.......	0.10
	Celestite......	0.10
	Cinnabar.....	0.10
	Chalcocite....	0.09
	Cuprite......	0.08
	Smithsonite...	0.07
	Orthoclase....	0.05
	Stibnite......	0.05
	Cryolite......	0.05
	Enargite......	0.05
	Senarmontite.	0.05
	Galena.......	0.04
	Niccolite.....	0.04
	Calcite......	0.03
	Witherite.....	0.02

The most important separations in concentration practice are: Tramp iron from crude ores; magnetite in ores from associated gangue minerals such as quartz, feldspars, hornblende, garnet, and apatite; iron from zinc-iron gravity concentrate, *e.g.*, roasted pyrite from unaltered blende, pyrrhotite from blende, ferruginous blende from pyrite, siderite from blende, roasted siderite from blende; iron from copper-iron gravity concentrate, *e.g.*, siderite from chalcopyrite, pyrrhotite from chalcopyrite; franklinite from willemite, zincite and calcite; wolframite from cassiterite; and magnetite and ilmenite from monazite sands. Other examples of separations made by magnetic means are, the magnetic material being in each case named first: hematite from silica; siderite from cryolite; rhodonite and ferruginous garnet from blende; some copper carbonates from silicious gangue; garnets and basic silicates from diamondiferous sands; burnt magnesite from lime and alumina; hornblende from apatite; rutile from apatite; garnet from metamorphic silicates; garnet from corundum; biotite from metamorphic silicates; leucite from lava; chromite from silicates; iron introduced in mining and grinding from ceramics; limonite from silicious and aluminous gangue; roasted hematite and limonite from gangue minerals.

2. Theory

Definitions. A MAGNET is a body possessing certain electrical properties that cause it, when freely suspended, to align itself in the magnetic meridian (roughly north-south in middle latitudes). It has the ability to draw to itself through short distances small pieces of iron and steel and certain other sub-

stances, all of which are known as magnetic substances, although they are generally not magnets themselves. This attractive force exerted by magnets is due to a field of force surrounding them which, both for purposes of illustration and numerical calculation is conceived to be made up of LINES OF FORCE as shown in Fig. 1.

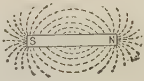


FIG. 1.—Lines of force surrounding a bar magnet.

If such a magnet is placed under a sheet of cardboard or glass and iron filings are dusted above it and agitated by tapping the sheet in order to give them freedom of movement, they will arrange themselves in chains, end-to-end, in a pattern similar to that of the dotted lines. If the same magnet is so suspended as to be free to move horizontally, then, if not near a mass of highly magnetic material or another magnet, it will swing around until it finally comes to rest in an approximately north-south direction, and one particular end of any given magnet will always point north. This is called the north-seeking or north pole, and, from the fact that it was usually marked with a cross, is now called the (+) or POSITIVE POLE; the other is called the (−) or NEGATIVE POLE. Lines of magnetic force are conceived to be directed from north pole to south pole outside the magnet and from south pole to north pole inside the magnet and to close, i.e., to form closed circuits.

Lines of force outside the magnet, in passing through different media, diverge or converge according to the relative permeability of the media. A highly permeable body is one that concentrates or draws together the lines of force in its vicinity and the PERMEABILITY of a body may be thought of as the ratio of the concentration or crowding of the lines in a given area through the body in a given field to the spacing in air at the same position, the body being absent. Thus Fig. 2 represents the effect of a piece of magnetite, with relatively high permeability, in concentrating the lines of force in the magnetic field around a bar magnet.

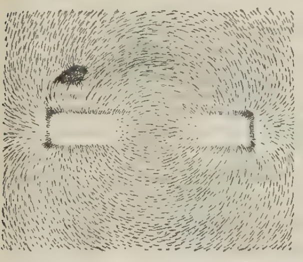


FIG. 2.—Magnetic field containing a large particle of magnetite.

Electromagnetism. When an electric current flows through any conductor, a magnetic field is set up in the medium surrounding the conductor. If the conductor is wound in a helical coil a magnetic circuit is set up in which the lines of force are approximately axial through the coil and curved without in a manner entirely similar to the field of a magnet of the same shape and size as the coil, in fact the coil itself may be regarded as a magnet. If the length of the coil is great in proportion to its diameter, the strength of the magnetic field in the interior of the coil is given by the equation $H = 4\pi AN/10l$, where H is conceived of as the number of lines of magnetic force per square centimeter of the medium, A is the current flowing in the coil, expressed in amperes; N is the number of turns or windings, and l is the length of the coil in centimeters. If the diameter of the coil is relatively great with respect to the length, $H = 2\pi NA/10r$, where r is the radius of the coil in centimeters. The product AN is called the AMPERE TURNS of the magnet coil. If a soft-iron core is inserted in the coil, the number of lines of force within the coil is enormously increased, with corresponding increase in the strength of the magnetic field, and the combination becomes a typical electromagnet. In such a magnet the magnetic field continues as long as current flows through the coil; its strength depends on the strength of the coil field and the permeability of the core; and the field ceases

Table 2. Magnetic susceptibility of low order

Substance	Volume susceptibility, k_v
Minerals	
Magnetite	3.07 to 0.12
New Zealand iron sand	0.19
Magnetite, very impure	0.056
Iron sulphide (artificial)	0.022
Pyrrhotite	0.00575 0.000337
Dolerite	0.0045
Franklinite	0.0025 0.0037
Ilmenite	0.0015
Specular hematite	0.0011
Granite	0.0010
Chrome iron ore	0.00050 0.00032
Chalybite, massive	0.00056
Spathic ore	0.00048
Green serpentine	0.00035
Red serpentine	0.000041
Hornblende	0.00033
Wolfram ore	0.00030
Ironstone	0.00030
Nickel sulphide, native	0.000183
Red hematite	0.0002
Brown hematite	0.00011
Monazite	0.00010
Zinc-blende	0.000022 0.000012
Iron pyrite, cubic	0.00015 0.0000015
Iron pyrite, rhombic	0.00002
Copper pyrite	0.000027
Fergusonite	0.000072
Arsenical pyrite	0.000011
Tinstone crystal	+0.000011 −0.000001
Tinstone crystals in matrix	+0.000017
Mica, Bengal ruby-clear	0.000012 0.000008
Mica, Bengal ruby-spotted	0.0147 0.000287
Tourmaline	0.00013 0.000086
Limestone	0.000006
Cobaltite	0.0000058
Sapphire, Al_2O_3	0.0000057
Glasses (various)	+0.0000025 −0.00000015
Antimony sulphide, native	−0.00000085
Feldspar	0.0000016
Ruby, Al_2O_3	0.0000019
Zircon	0.0000005
Corundum crystal, Al_2O_3	0.00000136
Apatite	−0.00000072
Metals	
Manganese iron (13 per cent. Mn)	0.0004(*a*) 0.530(*b*)
Chromium	0.000031
Tungsten	0.000006
Platinum	0.000029
Manganese	0.000084
Aluminum	0.0000018
Tin	0.00000019
Silver	−0.0000021
Copper	−0.0000073
Antimony	−0.0000063
Bismuth	−0.000014
Fluids	
Manganese sulphate(*c*)	0.000017
Ferrous sulphate	0.000015
Water	0.00000075

a Before heat treatment. *b* After heat treatment. *c* Molecular normal solution.

except, perhaps, for a weak residual permanent field, when the current stops. The driving force in a magnetic circuit, conceived to cause the lines of force to penetrate the different media in the circuit, is called the MAGNETO MOTIVE FORCE, M. It is expressed numerically as

$$M = Hl = 4\pi AN/10 = \text{(approximately) } 1.25AN.$$

This force is conceived to create a magnetic flow or MAGNETIC FLUX in the magnetic circuit, which varies directly as the magneto motive force and inversely as a quality of the circuit, analogous to the resistance of an electrical conductor, known as RELUCTANCE, R. The unit of reluctance is RELUCTIVITY, ρ, and is defined as the reluctance of a magnetic circuit 1 cm. long and 1 sq. cm. in transverse section. The total reluctance of a homogeneous circuit l cm. long and S cm. in transverse section is: $R = \rho l/S$. The relation between reluctivity ρ and PERMEABILITY μ is expressed in the equation $\rho = 1/\mu$. PERMEABILITY is expressed numerically in terms of field strength and flux density as $\mu = B/H$. Hence $\rho = H/B$.

SUSCEPTIBILITY K, which may be considered as the increase in the number of lines per square centimeter at a given point in a given magnetic field, caused by the introduction of a given substance, over the number that would be present at the same point in air alone, is related to permeability by the equation $\mu = 1 + 4\pi K$. It may also be considered as the ratio of the intensity of magnetization, I, to the magnetizing force H, or $K = I/H$. Since the mechanical effect on a particle of given size and shape at a given point in a given magnetic field is proportional to the intensity of magnetization of the particle, it follows that susceptibility is a direct measure of the relative response to be expected from different substances

when presented to a given magnetic field. Table 2 (Wilson, *59 Trans. IEE 319*) represents the latest published figures on susceptibility of minerals and is apparently the result of careful work, but the warning given in connection with Table 1 is to be remembered.

The total MAGNETIC FLUX, Z, in any circuit is given by the equation $Z = BS$ from which $M = B\rho l = Z\rho l/S$, and $AN/l = 0.8Z/\mu S$. This is the equation for determining the number of ampere turns required to produce a given total flux density in an electromagnetic field. But permeability varies with flux density, hence experimentally determined permeability curves showing the relation between flux density and permeability for the substances composing the magnetic circuit are required before the formula can be used. Also, since the magnetic circuit in an electromagnet is non-homogeneous, the reluctance of the different parts of the circuit must be calculated separately and summed to obtain the total reluctance.

EXAMPLE. Given an annealed Swedish-iron ring, 24 in. mean diameter and 3-in. cross-section, cut transversely and leaving an air gap of 0.75 in.; required NA to give a total flux $Z = 400{,}000$ C. G. S. units. The length of the iron circuit = 59.1 cm. $B = 400{,}000/39.82 = 10{,}050$. μ (from Table 3) corresponding to 10,050 (B) for iron is 1330. $R(\text{iron}) = \frac{l}{\mu S} = \frac{59.1}{1330 \times 39.82} = 0.0011$. $R(\text{air}) = \frac{l}{\mu S} = \frac{1.9}{1 \times 39.82} = 0.0478$. $R(\text{total}) = 0.0489$. $AN = 0.8Z\frac{l}{\mu S} = 0.8ZR\ (\text{total}) = 0.8\ (400{,}000)\ 0.0478 = 15{,}300$.

Tractive force. If a bar magnet is cut transversely and the cut ends are then placed in contact, the force F in dynes holding them together can be expressed by the equation $F = B^2S/8\pi$; but this equation is in no way applicable to the attractive force exerted by a magnet on a body separated from the poles by an air gap. The attractive force exerted on a small sphere in a magnetic field is $F = \frac{vK}{1 + \frac{4}{3}\pi K} \cdot \frac{dH_0^2}{dx}$, where K is the susceptibility $\left(= \frac{\mu - 1}{4\pi}\right)$, v = volume of sphere and H_0 is the magnetic force or field strength in the direction of a displacement, X. It follows from the above equation that when H_0 is constant, $F = 0$. Thus the sphere in the magnetic field between the flat-faced poles of the magnet shown in Fig. 3, *a* will not move toward either pole because of the fact that the field strength is the same across every section of the field adjacent to the sphere. On the other hand, in the case shown in Fig. 3, *b*, where one of the poles is wedge-shaped, the field increases in strength from the flat-faced pole to the wedge-shaped pole, which means that H_0 increases, and the sphere will tend to move toward the stronger field. If the sphere were diamagnetic instead of paramagnetic as illustrated, the coefficient $K/(1 + \frac{4}{3}\pi K)$ would be negative, indicating a tendency to move from a stronger to a weaker part of the field.

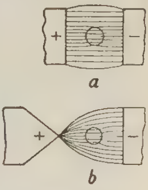


FIG. 3.

Design. It is impossible to proceed with theoretical analysis to the point of design of electromagnets for separation of ores. The last equation tells that the field must be of variable strength, and for this reason wedge-shaped pole pieces are used on the collecting poles, or other devices such as sharp projections on pole pieces or armatures, laminated pole pieces, armatures of alternate disks of magnetic and non-magnetic material, or parallel cylinders are employed as pole pieces for producing converging fields. The air gap between the collecting pole and the particle should be as small as possible, in order to

bring the particle to be collected into the stronger and more rapidly changing part of the field. But for determination of proper field strength and number of ampere turns to effect a given separation, it is necessary to resort to experiment with a magnet of known construction, and from the data thus obtained to draw the specifications for the desired magnet.

It is clear from the equation $AN = 0.8lZ/\mu S$, that for a given total flux density Z and with a given cross-section S, the number of ampere turns per unit of length (AN/l) is inversely proportional to the permeability of the core. Since iron and steel have the greatest permeability of any known substances they are used for magnet cores. The permeability of different kinds of iron and steel with different flux densities are given in Table 3. Mild

Table 3. Permeability and flux density of various materials for electromagnet cores

Flux density (B), C. G. S. units	Permeability (μ)			
	Cast iron	Annealed Swedish iron	Steel casting	Forged ingot iron
2,000	400	....	....	....
4,000	460	....	1620	....
6,000	300	1180	1880	....
8,000	180	1330	1960	....
10,000	100	1330	1700	....
12,000	40	1200	1320	....
14,000	20	900	900	1900
16,000	....	550	450	900
18,000	....	220	180	240

steel is now generally used for magnet cores, except where the highest possible permeability throughout a wide range of flux densities is required, when forged ingot iron is employed. At New Jersey Zinc Co. Swedish cast steel for cores, pole pieces and pole points is specified to contain not more than 0.06 per cent. C, 0.025 per cent. P, 0.025 per cent. S, 0.30 per cent. Mn, 0.30 per cent. Si, and to be thoroughly annealed.

The same equation shows that a given magnetic flux is dependent upon the product of the current flowing and the number of turns of wire around the core per unit of length. Hence by increasing the number of turns the current can be decreased and *vice versa*. On the other hand, as the diameter of the coil increases with respect to the length, the magnetic effect of a given number of ampere turns decreases. Ordinary design makes the length of the coil 5 to 10 times the diameter. The amount of current that can be safely carried through a given wire is limited by the heating; a general rule in well-ventilated core windings is that the current shall not exceed 2.5 amp. per sq. mm. of copper wire. Enclosed magnets must carry less current. Asbestos-covered wire should be used in high-intensity magnets to prevent break-down of insulation and lessen fire risk, and careful waterproofing is also essential. Winding specifications for Wetherill-Rowand high-intensity magnets at New Jersey Zinc Co. call for the wound spool to stand 10,000 volts; temperature not to exceed 70° F. above the surrounding air with 125-volt direct current. One spool answering this specification was wound with No. 13 wire, 37 layers, 233 turns per layer, 55.56 ohms resistance per spool. Protective rope winding, ¼-in. manila, free from moisture before winding.

Effect of size of particle on strength of field required is dependent on the method of presentation. Attractive force is proportional to the product of magnetic masses and inversely proportional to the square of the distance between them. But the magnetic masses in a given field, while proportional to the volumes of the particles, act as though concentrated at the extremities of the particles and with a large particle the distance to the like pole, which is repelled by the magnet pole, is relatively greater with respect to the distance to the unlike pole than is the case with a small particle. Hence the counter-repulsive force due to this pole is relatively less, and the net attractive force relatively greater. Therefore, although gravity, acting against the magnetic

attraction, is likewise proportional to the respective volumes of the particles, and, considering volume alone, the net attractive force per unit of volume is the same for all particles of the same mineral at the same distance from the poles, irrespective of size, nevertheless, where concentration of magnetic mass at the poles is taken into consideration, the greater net force is exerted on the larger particle. Moreover, if the material to be treated is presented resting on a supporting surface below the magnet pole, so that the particles must be lifted to the pole, then the unlike induced pole in the larger particle will be nearer the magnet pole than that of the smaller particle and this, in addition to the fact that the like induced pole will be relatively further away, results in a net effective force on the larger particle considerably greater than on the smaller, or, conversely, the larger particle can be separated in a weaker field. Further, the smaller particle may be so deeply buried in the mass of material as to be less free to move than the larger particle, and for this reason, also, requires a stronger current. When the magnet is under the presenting surface, a somewhat reverse condition holds, and a weaker field will suffice for the smaller particle. It follows, therefore, that if the particles are not pure, but are composed of a mixture of magnetic and non-magnetic minerals, they must be closely sized, if clean products are desired. Also, if a field of high intensity is necessary, the air gap must be small and close sizing is essential in order that the large pieces may not touch the magnet poles when the gap is sufficiently small with respect to the small particles. With highly-magnetic feeds the tendency for the larger magnetic particles to entrain gangue and for the larger middling particles to attract and hold small particles of magnetic mineral also calls for sizing of the feed.

Time required to induce magnetism. Evidence is conflicting as to whether or not there is a distinct time element involved in the induction of maximum magnetization in a given particle in a given magnetic field. Thompson (*Electricity and Magnetism, p. 384*) cites the fact that a large electromagnet at the Royal Institution (London) requires about two seconds to reach maximum strength. *Truscott*, on the other hand, states that with 600,000 alternations per second the limit to the speed of magnetization of a piece of laminated iron was not reached. When, however, the question is one of the speed of passage of minerals through a magnetic field, the result is the same as though time were an essential element.

When a mixture of magnetite, rhodonite and ferruginous blende, ground to pass a 0.75-mm. screen, was presented at different speeds to a given field, it was found that at a speed of 330 ft. per min. magnetite only was removed, at 230 ft. per min. some of the rhodonite but none of the blende came off, at 165 ft. per min. all of the rhodonite and some of the blende were lifted, at 130 ft. per min. more of the blende and at 100 ft. per min. all of the blende was recovered. It is true that the determining factor here may well have been the relation between final magnetic attraction and momentum of the particles and that the effective result of lowering belt speed was to lessen momentum rather than to allow time for greater magnetization. In so far as separator design and operation are concerned, however, the conclusion is clear that the allowable speed of passage of particles through the magnetic field is closely related to the permeability of the magnetic constituents; the lower the permeability the lower the allowable speed.

Forces employed to oppose magnetic attraction in separators are gravity, centrifugal force, friction, momentum of the particle, and the blow of a blast of air or water. When centrifugal force, momentum, or the force of fluid currents are the opposing forces, the magnitude and direction of the resultant force can be varied by varying either the magnetic force or the opposing force. When gravity or friction is the opposing force, control lies with the magnetic force alone.

Current. The energizing current is normally direct-current at 110 to 250 volts. Alternating-current has been used experimentally for energizing magnets for mineral separation, but the practice has not passed the experimental stage.

3. Types of magnetic separators

Separators are built to treat coarse or fine material, wet or dry, and the design differs somewhat according to the service. The design also differs according to whether the magnetic material is strongly or weakly magnetic, a field of high intensity being required for weakly-magnetic materials while one of low intensity will serve for strongly magnetic. In both types the working magnet may be either directly energized by a coil or may be energized by induction. Further classification may be made on the basis of the method of presentation of the material to the magnet as on a belt, pulley, drum, shaking tray, or by a free fall through air, etc.

The fundamental elements of every successful magnetic separator are (*a*) a magnetic field of sufficient intensity to cause a marked difference in the magnetic forces acting on the magnetic and non-magnetic particles in the mass to be separated; (*b*) a means for bringing the material to be separated to the magnetic field in such a way that the particles have sufficient freedom of movement under the influence of the magnetic forces to permit the magnetic particles to move away from the non-magnetic or *vice versa;* (*c*) a force or forces such as gravity, centrifugal force, or the friction of a solid or fluid acting against the magnetic force and serving to intensify the difference in direction of movement of the magnetic and non-magnetic particles; (*d*) a means for removing both magnetic and non-magnetic particles from the magnetic field after separation.

The underlying ideas of magnetic separation are old and are disclosed in a large number of patents, now expired. Existing patents are all for apparatus, covering various mechanical and electrical details, but not applying to basic principles.

Low-intensity machines are used principally for separating magnetite from non-magnetic gangues; magnetic oxides of iron, formed by roasting sulphides, from associated unaltered and non-magnetic sulphides; and for removing metallic iron from various non-metallic admixtures, as for instance, tramp iron from crude non-magnetic ores, flake iron from ground ceramics and the like, nails from brass-foundry refuse, etc. As a general rule it may be stated that the permeability must be greater than 20 when iron is taken as 100 in order to make a substance separable on a low-intensity machine. Typical machines of this class are described below.

Edison machine is the simplest in principle of the continuous machines. A stream of crushed highly-magnetic ore is allowed to fall freely in air past the pole piece of a primary electromagnet. As a result the magnetic material is deflected toward the magnet and falls into a suitable receptacle, while non-magnetic material falls vertically into an adjacent box. Although simple in principle, and not subject to the disadvantage of entrainment of gangue by concentrate, this machine has the disadvantage that the speed of presentation is high and the air gap is necessarily great, greatly different for different particles and hard to control. As a result the machine has had but little use in the mills.

4. Pulley separator

This type (Fig. 4) consists essentially of a belt conveyor with a magnetic head pulley, a device to distribute feed in a thin layer on the carrying portion of the belt, and a divided receiving chute to take the two discharge products.

The magnetic head pulley consists of an iron or steel pulley with a battery of electromagnets inside, mounted on the pulley shaft and revolving, therefore, with the pulley, current being supplied through slip rings on the pulley shaft. As ore passes over the head pulley, non-magnetic material is thrown free as in the ordinary belt-conveyor discharge, while magnetic material clings until separated by the divergence of the return belt from the under side of the magnetic pulley.

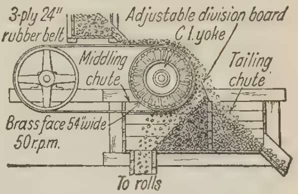


FIG. 4.—Pulley separator.

Size of HEAD PULLEY ranges from 12 × 12 in. to 48 × 60 in. SPEED is from 100 to 400 ft. per min. CAPACITY on feed passing 1.5- or 2-in. ring is from 15 to 25 tons per hour per ft. of pulley width. A machine with 24-in. belt at MT. HOPE (*99 J 562*) takes 12.5 amp. at 250 v.; at MOOSE MOUNTAIN (*99 J 974*) 21 amp. at 120 v.; at WITHERBEE, SHERMAN AND CO., (*48 A 254*) 25 amp. at 125 v. and at RICHARD mine (*115 J 973*) 31 amp. at 125 v. The magnets are usually so wound that the alternate poles are of opposite polarity, thus producing concentrated magnetic fields at and near the belt surface.

This type of machine, with the strong current usually employed, will hold everything that contains an appreciable amount of magnetic material and reject clean tailing. The concentrate, therefore, is usually a middling that must be further treated. In mills treating magnetite ores, pulley machines treat the reject from drum machines (Art. 5) and make tailing carrying from 8 to 12 per cent. Fe (a large part of this being non-magnetic iron). They thus materially reduce the load on the re-grinding machines.

Performance. At WITHERBEE, SHERMAN CO. (*90 J 959*) 20-in. pulley machines treat crude ore sized between 2- and 0.75-in. at the rate of 300 tons per machine per day and also treat the sized middling from drum machines. Each pulley carries 25 magnets and draws about 25 amp. at 125 volts. Feed is practically dry. Pulley speed, 40 to 50 r.p.m., giving a belt speed of 250 to 320 ft. per min. Feed assays 35 per cent. Fe and tailing 6 to 8 per cent. Rough concentrate is sent to drum machines. Pulley machines are also used to rough out tailing on the sizes ¾- to ⅜-in. and ⅜- to ¼-in. One man attends 17 separators of the drum and pulley types.

5. Ball-Norton drum separator

This machine (Fig. 5) consists of a closed drum of non-magnetic material, such as brass or bronze, revolving around a segment of a stationary electromagnet which covers between ½ and ⅔ of the face of the drum as shown. These magnets are wound so that adjacent poles are of opposite polarity. Electrical connection is made through a hollow shaft. When ore, properly crushed to free the magnetic mineral, is fed onto the upper part of the drum, in the direction of rotation, the non-magnetic portion rebounds or slides off while the magnetic material is drawn down to the face of the drum by the magnetic force. As any given magnetic particle is carried forward, however, from the field of one magnet into the field of the next, then, because of the fact that the change in position of the particle is more rapid than the change in polarity of the induced magnetism, the particle turns end for end in order

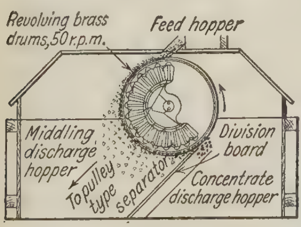


FIG. 5.—Ball-Norton drum-type separator.

to present the proper pole to the succeeding magnet. In so doing it leaves the surface of the drum and thereby frees any non-magnetic particle that may have been mechanically held by it in position on the drum. This winnowing effect is repeated from pole to pole until, by the time the bottom of the drum is reached, most of the non-magnetic and weakly-magnetic particles have been shaken out. This agitation of material by means of magnetic circuits of alternate polarity is one of the outstanding advances that has been made in the art of magnetic separation. At the point on the periphery where the segmental battery of magnets terminates the magnetic particles leave the drum by centrifugal force and gravity and fall into the concentrate hopper.

Drums are usually made 16 to 24 in. face, 24 to 36 in. diameter and are run at 350 to 400 ft. per min. peripheral speed. CURRENT varies with size of feed; at MT. HOPE (*99 J 562*) 6 amp. at 250 v. is used with 1.5- to 2-in. material, 4.5 amp. with 0.5- to 1.5-in. material and 3.75 amp. with 0.75- to 0.25-in. material. CAPACITY on magnetite ores is from 20 to 50 tons per hr. on coarse sizes (0.5- to 2-in.) and considerably less on finer feeds.

At MESABI IRON Co. a 30 × 30-in. drum drawing 7½ amp. at 110 volts was fed with de-slimed −4-mesh sand (58.2 per cent. water) at the rate of 8.7 tons per hr. Assays (magnetic Fe, per cent.) were: Feed, 36.7; concentrate, 48.2; tailing, 9.5.

At MT. HOPE manganese-steel shells were substituted for brass on the drum. These shells, ⅛-in. thick, lasted four times as long as the brass drums and the cost per ton was one-quarter as much. Roche substituted pure rubber bands for manganese steel at the RICHARD mine and found that the life of 3/16-in. rubber was three times that of the steel, with a further saving in installation time and power consumption. Roche used four bands on a 24-in. drum and found that the bands could be readily snapped on drums by merely lifting one end of the shaft out of its bearings.

The drum separator is essentially a roughing machine for making a clean concentrate and a middling for re-treatment. This middling is ordinarily sent to pulley machines (Art. 4).

Ball-Norton double-drum separator (Fig. 6) is an attempt to make finished concentrate on a drum-type machine. The first drum is run slowly with a strong field in an attempt to make a low-grade concentrate and clean tailing. The second drum (*G*) is fed by centrifugal force with the rough concentrate from the first. It is run at higher speed and with a weaker field; as a result low-grade material is dropped into middling compartment (*M*) and only high-grade concentrate is carried around to be thrown over into hopper (*C*). This machine is intended for finer feed than the single-drum machine, the size recommended by the makers being below 2.5-mm. On such material a CAPACITY of 15 to 20 tons of magnetite ore per hour is claimed on drums with 24-in. face, running the first drum at 40 r.p.m. and the second at 50, drawing 10.5 amp. and 13 amp. respectively. POWER required for driving is 0.5 to 0.75 hp.

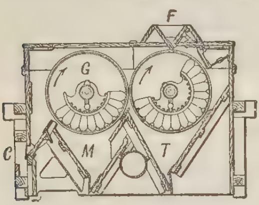


FIG. 6.—Ball-Norton double-drum separator.

Performance. Single-drum separators at WITHERBEE, SHERMAN Co., (*96 J 959*) are 30-in. diameter by 18-in. face, made of brass, rubber covered. Each contains 16 magnets. They run at 50 to 56 r.p.m., are used to treat sized feed − ¾ + ⅜-in. and − ⅜ + ¼-in. In one mill treating coarsely crystalline ore, material −2 +0.75-in. is also treated on drum machines. They make concentrate assaying about 60 to 65 per cent. Fe and a middling assaying about 18 per cent. from feeds ranging between 25 and 45 per cent. Fe. About 4 amp. at 125 v. are used on the coarse feeds, 7 amp. on finer feeds. One man attends 17 separators of the drum and pulley types. At REPLOGLE STEEL Co. 2-drum machines with drums 36-in. diam. by 28-in. face with 14 magnets per drum are run at 49 r.p.m.; feed rate is 14.9 tons per hr. Sizing test of feed and products is given in Table 4. Average assays: Feed, 33.4 per cent. Fe; concentrate, 62 per cent. Fe; tailing, 22.4 per cent. Fe. Iron in tailing is mostly hematite. Similar drum machines are used to treat re-ground middling from belt separators. Sizing test of feed is about the same as given in Table 4. Average assays: Feed, 26.8 per cent. Fe; concentrate, 57.4 per cent. Fe; tailing, 20.6 per cent. Fe (mostly hematite, goes to gravity mill). One of the separators in re-treatment service has 30 × 42-in. drums with 27 magnets in the upper drum and 45 in the lower. Average tonnage on re-treatment machines, 17 each.

Ball-Norton drum-pulley machine (Fig. 7) is a combination of the two machines just described, designed to economize floor space and head room and deliver, from one machine, finished concentrate and tailing and middling for re-treatment. Construction and operation of each part are the same as in the independent machines.

Table 4. Sizing tests of feed and products of Ball-Norton drum-type separators, Replogle Steel Co.

Screen, mesh	Weights, per cent.		
	Feed	Concentrate	Tailing
20	14.6	15.5	10.2
40	37.5	50.1	37.4
60	30.4	14.5	23.7
80	7.6	3.8	5.1
100	3.1	1.4	3.4
Through 100	6.8	14.8	20.1

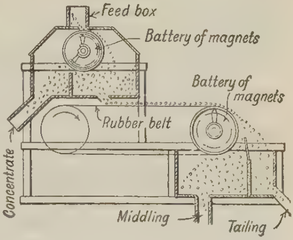


Fig. 7.—Ball-Norton combined drum and pulley machine.

Wenstrom separator is a drum-type machine for coarse feeds, used in Swedish mills in the same position as the single-drum Ball-Norton. The flanged drum is built up of alternate lamellæ of wood and soft iron. The electro-magnet is circular in cross-section but has annular grooves containing the winding and is wound in such a way that alternate poles have opposite polarity. It is placed eccentrically within the drum. Projections from the inner face of the iron drum lamellæ are so arranged that those of alternate lamellæ approach opposite poles of the magnet during their downward travel and are thus magnetized by induction; during their upward travel they are out of the effective field of the primary magnet and lose their induced magnetism. The effect is somewhat the same as in the Ball-Norton machine, except that the magnetic agitation of pulp on the drum is not so active in the Wenstrom machine.

This machine with 30 × 24-in. drum is said to require about 15 amp. at 110 volts and to treat from 5 to 10 tons of 1- to 2-in. magnetite ore per hour at 30 r.p.m.

6. Ball-Norton belt separator

This machine (Fig. 8) is adapted to the treatment of finely-ground dry ores containing a strongly magnetic mineral. Feed is introduced by means of a suitable feed device (*a*) onto feed belt (*b*), which carries the feed forward until, near the head pulley, it comes into the field of the battery of electromagnets (*c*). These magnets, of which there are 8 to 12, are wound so that alternate poles are of opposite polarity. Magnetic material is lifted to the under side of belt (*d*), which runs at a higher speed than belt (*b*), and is carried forward, clinging to the belt. By reason of the alternating polarity of the magnets the material cling-

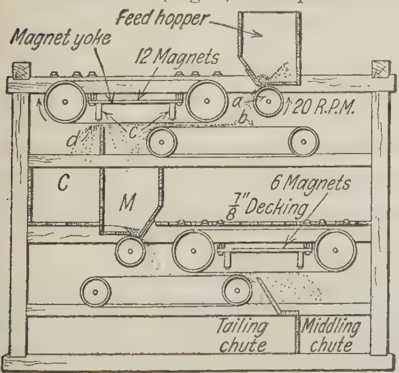


Fig. 8.—Ball-Norton belt separator, series type.

ing to the upper belt is turned over and over, forming depending loops in the air in passing from one field to the next, and in this manner weakly-magnetic middling and entangled non-magnetic material are winnowed out and fall in the hopper *M*. Strongly-magnetic material is carried forward until it passes out of the magnetic fields and falls into hopper (*C*) while non-magnetic material remains on the feed belt until it reaches the head pulley and is discharged therefrom, usually into the same hopper as the middling.

Usual belt speeds are 100 to 250 ft. per min. for the feed belt and 200 to 400 ft. per min. for the take-off belt, the lower speeds corresponding to the more weakly-magnetic material, such, for instance, as magnetite middling, and coarser sizes. In treating magnetite ore passing 0.25-in. screen, the current requirement for a 12-pole machine is from 4 amp. at 250 volts when lifting concentrate, to 8 to 10 amp. when lifting middling. The machine shown in Fig. 8 is a 2-deck machine of the series type in which middling from the first belt is re-treated on a second belt that makes a true middling, and a tailing that can be rejected.

In the 2-deck parallel machine, used for fine material that does not contain sufficient middling to justify re-grinding and re-treatment, clean concentrate and finished tailing are made on each deck and the use of multiple decks is resorted to only in order to increase capacity per unit of floor space. Where both series and parallel types are used in a mill, as at WITHERBEE, SHERMAN Co., the series type is used on coarser feeds, say +1/16-in. and the parallel type on the −1/16-in. material. Usual BELT WIDTHS are between 18 and 30 in. CAPACITY of a 24-in. machine on −1/4-in. crude magnetite ore is 20 to 25 tons per hr.; ROCHE (*115 J 972*) gives the capacity on magnetite ores crushed to pass 2.5-mm. as 1 long ton per ft. of belt width per hour but this is undoubtedly low. Capacity varies nearly in proportion to diameter of feed within the range of sizes treatable. Capacity is increased and grade of products is bettered, if feed is closely sized. At MT. HOPE (*99 J 563*) the sizes treated are: −5/16 +3/16-in., −3/16 +1/8-in. −1/8 +1/16-in. and −1/16-in. At REPLOGLE STEEL Co. double-deck separators are of the parallel-type, with 26-in. 2-ply belts and 10 magnets per deck. Speed of feed belts is 209 ft. per min.; magnet belts, 335 ft. per min. Average feed rate, 15 tons per hr. Sizing tests of feed and products are given in Table 5. Average assays: Feed, 31.4 per cent. Fe; concentrate, 61.1 per cent. Fe; tailing (middling) 27.0 per cent. Fe. Iron in tailing is mostly hematite.

Table 5. Sizing tests of feed and products of Ball-Norton belt-type separators at Replogle Steel Co.

Screen	Coarse feed, weight, per cent.			Fine feed, weight, per cent.		
	Feed	Concentrate	Tailing	Feed	Concentrate	Tailing
0.125-in.	10.3	....	....	....	....	....
10-mesh	8.0	25.6	33.4	....	....	....
20	42.4	31.8	44.9	....	....	....
40	20.3	28.0	11.8	33.0	26.6	32.9
60	8.1	6.8	5.5	27.8	48.2	40.0
80	3.3	2.0	1.1	10.7	4.4	8.1
100	1.8	1.5	0.5	5.9	4.4	5.8
Through 100	5.8	4.4	2.7	22.6	16.4	13.2

The same-sized separators at the same belt speeds are used to treat −20-mesh product at the rate of 10 tons per hr. Sizing tests of feed and products are given in Table 5. Average assays: Feed, 32.7 per cent. Fe; concentrate, 59.1 per cent. Fe; tailing, 19.9 per cent. Fe (hematite; to gravity mill).

7. Cleveland-Knowles separator

This machine (Fig. 9) is a relatively low-intensity machine that has had considerable vogue in the treatment of roasted blende-pyrite middling in the Wisconsin zinc district. It differs from the machines thus far described in that the magnetic material comes directly in contact with the primary-magnet poles and is removed mechanically by scraping, both of which practices are bad.

Feed is introduced over a suitable feeder from hopper (*a*) onto feed belt (*b*) and carried under and into the magnetic field of magnet (*c*). The essential parts of this magnet are shown in Fig. 9, *b*. A cylindrical cast-iron core (*d*) with beveled flange constituting the pole piece carries the winding (*e*) and a flanged shell (*f*) supported by the circular bridge plate (*g*), the shell forming the other pole. The annular gap, ½ in. wide, between the beveled pole pieces is filled with cast metallic zinc, which, being diamagnetic, crowds the external field into the air space below. Magnet (*c*) revolves at about 40 r.p.m. and is of greater diameter than the width of the feed belt. Magnetic material lifted from the belt by the field of this first magnet bridges across the gap and is carried out to the sides and there scraped off into hopper (*h*). Material remaining on the belt passes to the second magnet (*i*), which is made of magnet steel, is more heavily wound than the first, and has a stronger field. The less-magnetic material is removed here in a similar manner. Nonmagnetic tailing passes over the head pulley of the feed belt. Usual speed of feed belt is about 100 ft. per min. Current on first magnet ranges from 0.5 to 2 amp. and on second from 3 to 10. Machine is made in two belt widths, 12-in. and 21-in. Capacity on roasted pyrite-blende concentrate, −3⁄16-in., is about 1 ton per hour per foot of belt width.

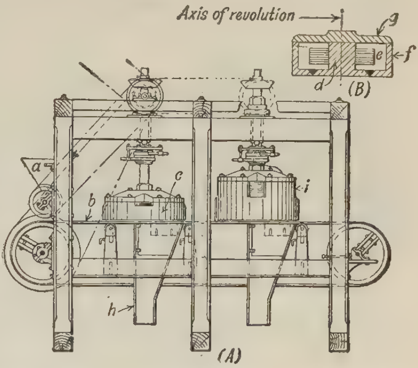


Fig. 9.—Cleveland-Knowles low-intensity separator.

Performances on roasted zinc-iron ores from various camps, compiled from figures furnished by the manufacturer, are shown in Table 6.

Table 6. Performances of Cleveland-Knowles separators on roasted pyrite-blende products. (*United Iron Works Co.*)

Source of material	Assays, per cent.					
	Feed		Concentrate		Tailing	
	Zn	Fe	Zn	Fe	Zn	Fe
Joplin, Mo.	30.50	20.96	64.28	0.93	1.23	49.33
Galena, Kan.	46.00	11.90	62.10	0.90	7.8	58.10
Meeker's Grove, Wis.	44.30	12.36	58.75	0.99	5.40	51.00
Rico, Colo.	39.40	16.60	56.80	1.40	5.50	48.60
Sandon, B. C.	44.45	10.10	54.10	4.81	3.80	34.55
Park City, Utah	43.97	9.45	55.19	1.24	15.80	40.28
Leadville, Colo.	27.55	26.33	48.99	5.94	4.36	53.96

8. Dings tray-type separator

Dings reversing-field induced-magnet type separator is shown in Fig. 10. This is a low-intensity machine for treating highly-magnetic materials. The essential parts are the two fixed primary electro-magnets (*a*) with pole pieces (*b*) suspended above the feed tray or belt and shaped as circular arcs with chords equal to the tray width. Cast-aluminum, brass or bronze circular plates (*c*) about 26-in. diameter, carried on gear-driven inclined shafts (*d*), carry on their periphery Y-shaped laminated-steel segments (*e*) 1 to 2 in. long and spaced a similar distance edge-to-edge. The upper part of these segments closely engages the arc-shaped poles of the primary magnets while the lower

ends barely clear the bed of feed in the shaking tray (*f*). A steel bed-plate (*g*) underlies the tray and the magnetic circuit is substantially that indicated by the dotted lines (*h*) passing through core (*a*) and pole pieces (*b*), segments (*e*), air gaps between (*b*) and (*e*) and between (*e*) and (*g*), and completed through bed-plate (*g*). The segments at any given time above the tray are, therefore, magnets and lift magnetic material from the mass on the tray. As plate (*c*) revolves, any given segment passes out of the field of a given pole as it approaches the edge of the tray or belt and as it does so loses more and more its induced magnetism until it reaches the diameter perpendicular to the tray axis when its magnetic flux becomes zero. Further revolution causes it to approach the opposite pole of the primary magnet where induced magnetism of opposite polarity is effected. As a segment carrying magnetic material leaves a primary pole the least magnetic material is dropped out, due to the decreasing strength of field, and at the neutral point the most highly magnetic material is dropped. Since the diameter of the segment circle is 8 in. greater than the width of the tray, the magnetic concentrate drops clear of the tray into chutes (*i*) provided to receive it. The second magnet usually has more ampere-turns than the first, thus allowing it to pick up the magnetic material that the first passed, and the air gap of the second is adjusted to a smaller distance than the first. This adjustment is effected by changing the length of the tray-suspension rods (*j*), by means of hand wheels (*k*). Thus concentrate is delivered into hopper (i_1), middling into hopper (i_2) and tailing from the lower end of the tray.

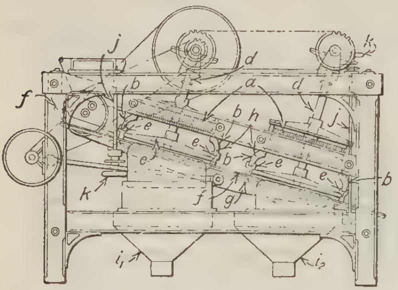


FIG. 10.—Dings tray-type low-intensity separator.

In another form of the same machine, the shaking tray is replaced by a rubber conveyor belt. The tray type is superior where hot roasted ores are being handled or where agitation of the feed is necessary to prevent entrainment of low-grade material in the concentrate; the belt is quieter and gives less trouble mechanically.

The standard width of belt or tray is 18 in. The tray is vibrated 450 to 500 times per min. and slopes about 4 in. per ft.; the belt is horizontal and runs between 50 and 100 ft. per min. CURRENT drawn is about 2 amp. on the first magnet and 3 on the second. CAPACITY on roasted zinc-iron concentrate passing 3⁄16-in. screen is about 1 to 2.5 per tons per hour.

Performance. *Roasted zinc-iron concentrate,* WISCONSIN (*107 J 1110*). Material was passed over two machines in series, the first a 2-disk shaking-tray rougher, and the second a 1-disk belt-type finisher. Shaking tray was made of maple with ¼-in. pressed asbestos liner, sloped 4 in. per ft. and vibrated 450 times per min. Magnets over first rougher disk drew 2 amp. at 225 volts when heated to 100° F.; over second disk, 3 amp. at the same temperature. Tips of secondary poles were spaced ⅜ to ½ in. above the top of the tray liner. Finisher had two 12-in. feed belts running at 70 ft. per min. and discharging at the center of the machine under an armature of solid steel 24 in. diameter and 6 in. thick, revolving on a vertical shaft. Primary-magnet poles were ¾ to ⅞ in. below the lower face of the revolving armature. These magnets had 6-in. cores and carried a current ranging from 9 amp. minimum cold and 7.5 amp. hot to 19 and 15 amp. maximum respectively. Capacity of the two machines was 40 to 45 tons per 24 hr. Cost of roasting and separating (1919) was $1.28 per ton, excluding overhead.

9. Knowles magnetic-belt separator

This machine (Fig. 11) uses the inductive principle to produce magnetism, during a part of their path, in soft-steel studs fastened to an endless belt. Referring to the figure (*a*) is a hopper feeding the electrically-reciprocated tray (*b*). The endless belt (*c*) is studded with about 200 soft-annealed steel rivets per square foot of area, heads on the pulley side, each rivet carrying a saucer-shaped washer with serrated edges. The belt passes around the pulleys (*d*) and around the adjustable directing roller (*e*) and travels so that the downwardly-sloping part goes with the feed stream. The primary electromagnet (*f*) is mounted on the frame so that the studded belt passes through the air gap. The upper pole only is wound. Both poles are chamfered to concentrate the magnetic flux and the upper is cut out to accommodate the directing roller (*e*). The studs in that part of the belt between the poles become powerful magnets by induction and attract magnetic particles from the feed while the non-magnetic particles slide down the tray into the tailing compartment (*g*). Intensity of magnetization in the studs is varied by varying the primary current and by changing the air gap between belt and upper pole by means of roller (*e*). As the magnetized studs pass to the left of the upper pole the induced magnetism weakens and they come more under the influence of the lower pole, the cylindrical pole-piece of which projects forward as shown. As a result their polarity is reversed. During the early part of this period of decreasing magnetism and changing polarity the least magnetic material leaves the studs and falls into compartment (*h*), constituting a middling. At the time of reversal the most magnetic material is dropped into compartment (*i*) as concentrate. By adjustment of splitters (*j*) considerable variation in the character of the products taken with a given electrical adjustment is possible.

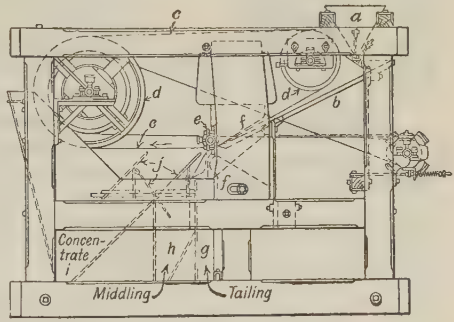


FIG. 11.—Knowles magnetic-belt separator.

Usual belt SPEED is 200 to 250 ft. per min. Machine is made in belt widths from 6 to 36 in. and CAPACITY on roasted blende-pyrite concentrate, $-\frac{3}{16}$-in., is slightly more than 1 ton per in. of belt width per 24 hr. CURRENT requirement at 110 volts ranges from 3 to 20 amp., depending upon the size of machine and the ore treated.

High-intensity machines are used for separating minerals that fall in the weakly magnetic class in Table 1. These machines must have very powerful magnets and a small air gap. As a result of this last requirement the feed must be relatively fine, it must be spread in a thin layer, best not more than one grain deep on the feeding surface, and, consequently the capacity of the machines is low in comparison to that of those treating strongly-magnetic material.

10. Wetherill separator

The Wetherill separator (Fig. 12) consists essentially of a flat-pole magnet (*a*) below and a wedge-pole magnet (*b*) above a flat conveyor belt (*c*) carrying the

feed stream. The lines of magnetic force converge sharply from the flat lower pole toward the lower edge of the upper pole so that magnetic particles travel toward the upper pole despite their greater proximity, when on the feed belt, to the lower pole. Cross belts (*d*) run directly under the upper poles in a direction at right angles to the feed belt and when magnetic particles rise from the feed belt toward the upper pole they come against the under side of these belts and friction causes them to be carried off to one side where they discharge into proper receptacles as concentrate. Removal is facilitated by a horn-shaped piece on the front side of the upper pole, by means of which the intensity of the field at this point is lessened. Non-magnetic tailing discharges over the head pulley of the feed conveyor.

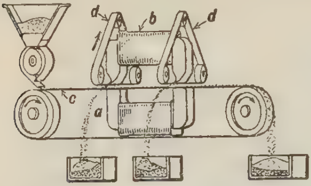


Fig. 12.—Wetherill magnetic separator (high-intensity type).

The machine is made with two, four or six poles and with feed-belt widths of 6 to 18 in. Each pair of magnets, *i.e.*, a 2-pole upper and a 2-pole lower, is wound in series to bring opposite poles together, and is provided with a separate ammeter and rheostat with small increments. The lower magnets are adjustable in order to bring the pole pieces level under the feed belt, the upper have an operating vertical adjustment by means of a hand wheel. Speeds of feed and take-off belts are adjustable by means of cone or step pulleys. Table 7 gives manufacturer's data on the various mill sizes. Width of poles is 18 in. in all machines.

Table 7. Sizes of Wetherill separators

Number of poles	Magnet wound for ampere turns			Maximum amperes at 110 volts direct current	Shipping weight, pounds
	First magnet	Second magnet	Third magnet		
2	30,000	....	....	6	14,000
2	60,000	....	....	14	15,000
2	100,000	....	....	30	16,000
4	30,000	60,000	....	20	22,000
4	30,000	100,000	....	36	23,000
4	60,000	100,000	....	44	24,000
6	30,000	60,000	100,000	50	30,000

Effect of adjustments. Variation in CURRENT strength varies the strength of the magnetic field and consequently, if the pole distance remains constant, the character and amount of products lifted from the feed belt. Too great current strength causes concentrate to cling under the upper poles and fail to discharge uniformly, with the result that clusters of concentrate build up and are scraped off by the feed belt or crowd material off the feed belt. The current drawn varies from about 5 amp. on the first magnet of a 6-pole (3-magnet) machine to 35 amp. on the last magnet. Variation in POLE DISTANCE is an important adjustment. If the pole distance is too great, current strength must be increased in order to make the desired recovery and difficulty in discharging concentrate results, as explained above. With too small pole distance the cross stream of concentrate brushes gangue off the feed belt into the concentrate receiver and thereby lowers the grade of concentrate. Pole distance depends upon the permeability of the concentrate and upon the size of feed particles. It may be as great as 1 in. plus the thickness of the feed belt (usually ¼ in.) on coarse material of relatively high permeability while it may

need to be as small as ⅛ in., if the permeability is low and the feed fine. Thickness of feed belt is preferably less than ¼ in., if the concentrate is very feebly magnetic; ⅛-in. is, however, the practical minimum. The take-off belt should be and is made as thin as possible, usually about $\frac{1}{32}$ in. thick. SPEED OF BELTS affects capacity and recovery. The feed belt is run as rapidly

Table 8. Adjustments of Wetherill-Rowand separators at New Jersey Zinc Co., Franklin mill

Magnet separator number.	1	2	3	4	5	6	7	8	9	10
Number of machines for each size	2	2	2	2	2	2	4	4	4	4
Capacity in tons per hour	2.25	0.75	2.30	3.15	3.75	4.30	1.88	0.63	1.93	2.63
Size of feed, in., through	.010	.011	.013	.016	.020	.025	.010	.011	.013	.016
Size of feed, inches, on	.0058	.010	.011	.013	.016	.020	.0058	.010	.011	.013
Speed conv. belt, f.p.m.	290	290	272	272	253	253	160	160	144	144
Speed take-off, f.p.m.:										
Pole 1	1045	1045	996	996	962	962	764	764	683	683
Pole 2	718	718	696	696	672	672	683	683	656	656
Pole 3	...	...	...	...	...	...	656	656	569	569
Pole 4	...	...	...	...	...	...	569	569	547	547
Pole 5	...	...	...	...	...	...	...	...	...	...
Pole 6	...	...	...	...	...	...	...	...	...	...
Air gap, 64ths inches:										
Pole 1	20	22	24	26	28	30	20	22	24	26
Pole 2	16	18	20	22	24	26	16	18	20	22
Pole 3	...	...	...	...	...	...	12	14	16	18
Pole 4	...	...	...	...	...	...	8	10	12	14
Pole 5	...	...	...	...	...	...	...	...	...	...
Pole 6	...	...	...	...	...	...	...	...	...	...
Energizing voltage	125	125	125	125	125	125	125	125	125	125
Current, amperes:										
Magnet No. 1	0.75	0.75	1.25	...	0.75	...	0.75	...	...	...
Magnet No. 2	2.25	1.75	2.0	...	2.25	...	2.25	...	...	...
Magnet No. 3	4.0	6.25	4.0	...	6.25	...	6.75	...	...	...
Power consumed, hp.	1	1	1	1	1	1	1½	1½	1½	1½
Attendance, machines per man	6	6	6	6	6	6	6	6	6	6
Per cent. of lost time	8	8	8	8	8	8	8	8	8	8

Magnet separator number.	11	12	13	14	15	16	17	18	19	20
Number of machines for each size	4	4	4	4	4	4	4	4	4	4
Capacity in tons per hour	3.13	3.58	1.63	1.78	1.93	2.08	2.20	2.33	2.45	2.63
Size of feed, in., through	.020	.025	.031	.038	.046	.055	.065	.076	.088	.101
Size of feed, inches, on	.016	.020	.025	.031	.038	.046	.055	.065	.076	.088
Speed conv. belt, f.p.m.	128	128	118	115	113	110	108	105	103	100
Speed take-off, f.p.m.:										
Pole 1	656	656	500	510	520	530	540	550	560	570
Pole 2	569	569	500	510	520	530	540	550	560	570
Pole 3	547	547	438	446	455	464	473	481	490	500
Pole 4	456	456	438	446	455	464	473	481	490	500
Pole 5	...	...	375	383	390	398	405	413	420	428
Pole 6	...	...	375	383	390	398	405	413	420	428
Air gap, 64ths inches:										
Pole 1	28	30	32	34	36	38	40	42	44	46
Pole 2	24	26	28	30	32	34	36	38	40	42
Pole 3	20	22	24	26	28	30	32	34	36	38
Pole 4	16	18	20	22	24	26	28	30	32	34
Pole 5	...	...	16	18	20	22	24	26	28	30
Pole 6	...	...	12	14	16	18	20	22	24	26
Energizing voltage	125	125	125	125	125	125	125	125	125	125
Current, amperes:										
Magnet No. 1	0.5	...	0.5	...	0.5	...	0.75	...	0.5	...
Magnet No. 2	2.0	...	2.25	...	2.0	...	2.0	...	1.25	...
Magnet No. 3	10.25	...	9.75	...	8.75	...	10.0	...	10.0	...
Power consumed, hp.	1½	1½	1½	1½	1½	1½	1½	1½	1½	1½
Attendance, machines per per man	6	6	8	8	8	8	8	8	8	8
Per cent. of lost time	8	8	8	8	8	8	8	8	8	8

as possible, in order to make capacity as large as possible, but if ore is fine, dusting limits the speed to between 200 and 300 ft. per min., while on coarser material a limit is imposed by the inability of the magnetic field to overcome the momentum of the particle. From this point of view, of course, the speed for feebly magnetic materials must be least and feed must be fairly uniform in size, while greater speed and less uniformity in feed size are allowable with more permeable substances. Effect of size of feed on speed is shown in Table 8. The usual speed of the feed belt is from 50 to 100 ft. per min. on feebly magnetic materials and from 100 to 300 ft. per min. with material of medium permeability. WIDTH OF FEED BELT also affects capacity but is practically limited to 18 in. because of the fact that with greater width the mass of material on the take-off belt is so great that it scrapes gangue off the feed belt unless the pole distance is made impossibly great. Ratio of concentration affects width of feed belt, a low ratio with the correspondingly great bulk of concentrate requiring a narrower belt than a high ratio. SPEED OF TAKE-OFF BELT is limited by the kind of material it is removing and the size of particles. It may be higher with strongly- than with weakly-magnetic material and higher with fine than coarse particles.

Richards gives 1000 ft. per min. allowable with strongly-magnetic concentrate and 200-ft. per min. for feebly magnetic. Removing franklinite at NEW JERSEY ZINC CO., the speed ranges from 570 ft. per min. on 2.5-mm. material to 1045 ft. per min. at 0.25-mm. (See Table 8.) The take-off belts on the early poles may run more rapidly than those on the later, as is likewise shown in Table 8. Speed of take-off belt is the limiting factor in the capacity of a machine when the ratio of concentration is low, while speed of feed belt limits when ratio is high. Young (*106 J 868*) recommends not more than 50 to 100 lb. per hr. per cross belt for a 6-in. machine. Capacity on wolframite-cassiterite ore is as low as 0.75 ton per 24 hr. on account of fineness and necessity for clean products (*120 P 379*). The feed must be perfectly dry in order to permit free movement between particles and not too fine or slime will stick to the feed belt.

Performance. At NEW JERSEY ZINC CO., Franklin mill, 6-pole 18-in. Wetherill-Rowand machines (Fig. 13) treat an ore containing franklinite, willemite, zincite and calcite and make

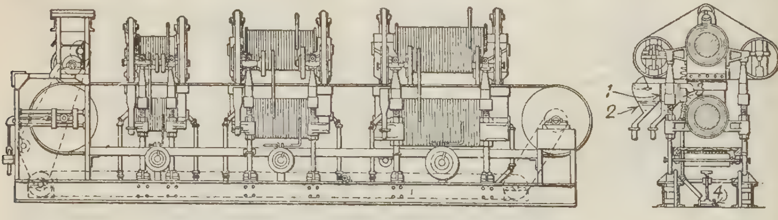


FIG. 13.—Wetherill-Rowand separator.

franklinite as concentrate. Two or more products are taken from each cross belt by means of the hoppers (1) and (2), etc., the most magnetic material being thrown farthest. Gangue swept off falls into (3) and thence by conveyor (4) back to the feed end. Adjustments are shown in Table 8. An approximate mineralogical analysis of the products of one separator is shown in Table 9. At ALGONQUIN MINE, Philipsburg, Mont. (*116 J 181*) pyrolusite and psilomelane occur in a silicious gangue. A Wetherill 6-pole machine (see Table 7) takes feed between 5- and 10-mesh. Speed of feed belt, 95 ft. per min.; take-off belt, 250 ft. per min. Concentrate from the first pole contained 10 per cent. Fe and was rejected; other poles delivered concentrate assaying 75 to 80 per cent. MnO_2; tailing assayed about 10 per cent. MnO_2. A second machine of the same type treating material $-10 + 100$-mesh made similar products except that the product of the first pole contained about 20 per cent. Fe. S. AND M. SYNDICATE (*120 P 379*). Feed: gravity concentrate containing wolframite, bismuthinite and cassiterite; 4-pole separator; weak current on first two poles removed magnetite and pyrrhotite; later poles with strong current removed a wolframite concentrate assaying 73 per cent. WO_3, 0.7 per cent. Sn and a trace of Bi; non-magnetic product

Table 9. Mineralogical analysis of products of a Wetherill-Rowand separator at the Franklin mill, New Jersey Zinc Co.

Mineral	Products										
	Feed	First magnet			Second magnet				Third magnet		Tailing
		First pole	Second pole		First pole		Second pole		First pole	Second pole	
			Inside	Outside	Inside	Outside	Inside	Outside			
Magnetite	2.4	47.1	.....	0.1	.....	.....	.....	.....	.....	.....	.....
Franklinite	28.6	42.3	60.2	81.0	26.7	80.9	29.9	63.2	50.4	26.2	2.9
Spinel	0.5	.....	0.6	0.1	1.0	0.1	2.6	1.0	0.5	.....	.....
Rhodonite	9.3	.....	3.4	1.1	8.9	2.6	12.8	6.3	8.0	7.4	2.6
Rhodochrosite	1.8	0.3	0.7	0.2	5.3	0.5	6.0	2.2	6.0	4.6	1.4
Garnet	3.5	.....	0.1	0.1	3.4	0.2	9.4	3.0	12.4	22.5	3.7
Micas	1.9	.....	.....	.....	1.5	.....	2.4	.....	0.5	0.6	1.4
Chlorite	0.1	.....	.....	.....	0.3	.....	.....	.....	.....	.....	.....
Tremolite	.....	.....	0.2	.....	.....	.....	.....	.....	.....	.....	.....
Zincite	0.5	.....	0.2	.....	0.2	.....	.....	0.2	.....	1.8	4.4
Willemite	9.1	.....	0.1	.....	0.3	0.1	0.2	0.2	0.3	2.9	27.8
Calcite	27.7	0.1	3.0	0.5	6.7	0.2	4.8	2.6	6.6	12.7	48.6
Sulphide, slate	.....	.....	.....	.....	.....	.....	0.3	.....	.....	.....	0.6
Quartz	.....	.....	.....	.....	.....	0.2	.....	.....	0.4	0.1	2.1
Middling	14.0	10.2	31.5	16.9	34.6	15.0	31.6	21.4	15.1	21.3	4.4

contained 63 per cent. Sn, 4 per cent. Bi and upward of 2 per cent. WO_3 depending on the amount of scheelite (non-magnetic) present. Recoveries were about 97 per cent. WO_3 and 99 per cent Sn. At SAKIET-SIDI-YOUSSEF, Algeria (*97 J 899*) a modification of the Wetherill machine (Fig. 14) was used to treat roasted blende-pyrite concentrate, assaying about 30 per cent. Zn before roasting. Belt (*a*) (14-in.) traveling at 175 ft. per min. carried the feed stream between two sets of powerful primary electromagnets (*b*) (*c*), with wedge-shaped pole pieces, the upper pole of (*b*) and both poles of (*c*) being enclosed in metallic drums. The first magnet drew 7 to 9 amp. at 40 volts and the second 10 to 12 amp. at the same voltage. Magnetic material drawn up against the first drum was thrown onto cross-belt (*d*) and thence discharged. Action at the second magnet was similar to that of a Mechernich separator (Art. 11). Two sizes were treated on separate machines, viz.: $-3 +1$-mm. and -1-mm. Each separator made four products: (*1*) highly-magnetic iron product assaying 17 to 18 per cent. Zn, (*2*) a less highly-magnetic iron product carrying 19 to 20 per cent Zn, (*3*) a zinc-iron middling assaying 27 to 28 per cent. Zn and (*4*) blende concentrate, 42 per cent. Zn. The last two products were mixed to yield a salable product assaying about 40 per cent. Zn and representing about 90 per cent. of the zinc in the original feed. ROASTING was done in a 5-hearth McDougall-type furnace, and cooling in a multi-tube rotary cooler, each tube individually water-jacketed; tubes were about 2.75 in. diameter; slope, 7 per cent.; 8 r.p.m.

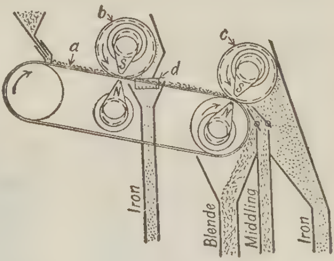


FIG. 14.—Wetherill-Mechernich separator.

11. Roller-type high-intensity separators

In these separators the feed is brought directly in contact with a pole or poles of the separating magnet, separation of the non-magnetic material is effected by gravity in the strongest part of the magnetic field and the concentrate is thrown off the roller by gravity and centrifugal force when it reaches the weak or neutral portion of the field.

Mechernich separator consists of two drums, the upper revolving and the lower stationary, set with axes horizontal and lying in a plane slightly inclined from the vertical. These form the pole-pieces of a powerful electromagnet. The external magnetic flux is concentrated in the plane of the axes. Feed is brought by a properly designed chute to the underside of the upper roller, the non-magnetic material immediately drops away while the magnetic clings until it reaches a weaker part of the field, where it leaves the roller under the combined forces of gravity and centrifugal action.

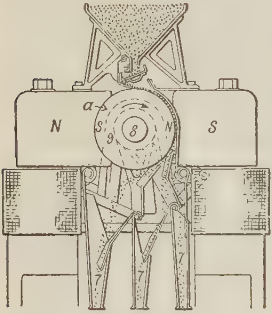


Fig. 15.—International separator.

This separator has had considerable use in Europe and has been used to separate rhodonite from blende at Broken Hill.

International separator (Fig. 15) places a revolving roller (a), built up of alternate radial projecting laminae of magnet steel and depressed laminae of non-magnetic material, between the poles of a powerful electromagnet. Induction causes each steel lamina to become a secondary magnet with field converging from the primary-magnet pole toward the projecting lamina; this field is most intense when the lamina is horizontal and becomes zero when the polarity of the strip reverses in passing from one pole to the other. Feed is introduced near the top of the roller in the direction of rotation, non-magnetic tailing discharges by gravity at or near the horizontal axial plane, while magnetic concentrate carries around on the under side until gravity and centrifugal force dislodge it near the bottom or lower neutral point of the roller.

Speed of roller is 75 to 100 r.p.m.; *Richards* gives the capacity of a machine with 24-in. roller as 2 to 4 tons per hr.

Motor separator is similar in general build and operation to the International except that the separating roller or armature is composed of a copper or brass shell with inner core of alternate disks of magnet steel and non-magnetic filler suitably wound to cause it to revolve when current is passed as well as to set up induced magnetic fields on the surface. No mechanical drive is necessary. The roller is about 30 in. long. This separator has been used in Europe to separate siderite from blende and at Broken Hill to make rhodonite as the highly-magnetic product, blende as intermediate, and a galena-quartz tailing.

Ullrich separator (Fig. 16). The essential parts are one or more concentric rings of wedge-shaped

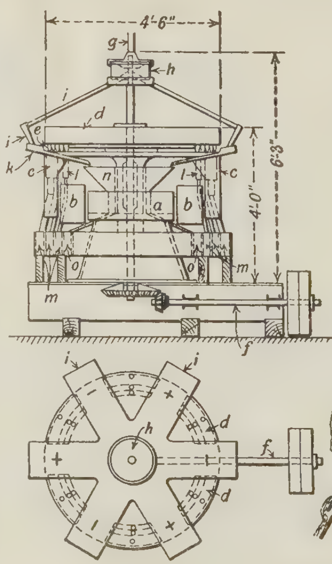


Fig. 16.—Ullrich separator.

pieces of soft iron (*e*) carried depending from a revolving table (*d*); powerful fixed magnets (*a*), radially disposed, with windings (*b*) and flat pole pieces (*c*) for energizing the suspended wedge pieces by induction; feed devices (*k*) that lead the feed in a thin uniform stream between the primary and secondary poles; suitably disposed receiving spouts (*l*) and (*m*), alternating with the fixed primary-magnet poles, for magnetic material, and a common central hopper (*n*) for tailing. As the wedges come over a pole and feed chute in the course of their revolution a strong magnetic field converging on the wedge is formed and magnetic material is drawn from the feed sole to the induced pole. On passing to the next radially disposed pole, which is of opposite polarity, the induced magnetism reverses sign and magnetic material is dropped or removed by a scraper. By varying the air gap between successive rings of secondary poles and the poles of the primary magnet from a maximum at the outer ring to a minimum at the inner, fields of increasing intensity are traversed by the radial feed streams, thus permitting the production of several classes of product in a very small space. The separator is made for either dry or wet service. In dry service the feed channels are shaking trays or traveling belts about 6 to 8 in. wide; in wet, they are shallow launders. In wet work the revolving table (*d*) is filled with water and discharges through pipes to form water sheaths around the wedges. These sheaths are continuous with the liquid in the feed launder (by reason of the small pole distance and the high surface tension of water) so that the movement of magnetic material to the poles is made under water.

Performance. Kranafeldt, writing as the distributors' representative (*34 CMJ*

Table 10. Performances of Ullrich separators

Kind of ore	Assays, per cent.																		Recovery per cent.			
	Feed						Magnetic product						Non-magnetic product									
	Fe	Mn	Zn	Cu	P	Ins.	Fe	Mn	Zn	Cu	P	Ins.	Fe	Mn	Zn	Cu	P	Ins.	Fe	Mn	Zn	Cu
Blende-siderite	15.9	3.8	27.8	...	...	...	30.1	8.2	2.5	...	...	...	...	...	49	...	...	...	...	75.2	92	...
Blende-siderite	...	...	22.0	...	...	...	...	...	...	...	...	...	...	...	40.6	...	...	...	...	...	98.8	...
Blende-siderite	...	...	33.1	...	...	...	...	...	...	...	...	...	...	...	52.9	...	...	...	...	...	97.5	...
Blende-siderite	...	...	38.9	...	...	...	...	...	...	...	...	...	...	...	55.9	...	...	...	...	...	92.8	...
Chalcopyrite-siderite	38.9	...	...	1.01	...	...	39.8	...	...	0.13	...	...	...	...	...	3.97	...	...	68	...	...	88
Magnetite-hematite	37.3	...	...	...	0.238	...	65.32	...	...	...	0.0255	...	...	...	...	...	...	...	80	...	...	...
Swedish hematite	34.6	8.9	...	...	0.349	33	48.3	11.1	...	...	0.0276	12	...	...	...	...	...	...	79.1	70.9	...	...
Bohemian hematite	56.5	...	...	...	...	...	68.6	...	...	...	...	...	...	...	...	...	...	...	93.3	...	...	...
Roasted siderite	37.2	6.9	...	...	...	...	48.1	8.5	...	...	...	...	...	...	...	...	...	...	93.5	89.5	...	...
Swedish magnetite	34.6	...	...	...	0.329	...	51.6	8.5	...	...	0.021	...	...	...	...	...	...	...	88.5	...	...	...
Swedish magnetite-hematite	50.2	...	...	...	0.106	...	61.6	...	...	...	0.029	...	...	...	...	...	...	...	86.4	...	...	...
Italian magnetite	54.5	...	...	...	...	...	65	...	...	...	...	...	...	...	...	...	...	...	90	...	...	...

703), states that the capacity on strongly magnetic material at 2-in. size is 7 tons per hr. while with fine material of the same character 4 tons per hr. can be treated. Capacity on feebly magnetic material passing a ⅜-in. screen is said to be 0.5 to 3 tons per hr. The data on performance in Table 10 are from the same source.

Wet machines, treating blende-siderite ore in SARDINIA (*100 J 911*) were fed material sized between 7- and 4-mm., 4- and 2-mm. and through 2-mm., each size on a separate machine. Feed pulp contained about 20 per cent. solids. Combined capacity of the three separators was 2.5 to 3 tons per hr. Power, 1.5 hp. for driving and 10 amp. at 125 volts for excitation of each machine. Assays: Feed, 34 per cent. Zn and 38 per cent Fe; concentrate, 53 per cent. Zn and 5 per cent. Fe; tailing, 2.5 per cent. Zn and 48 per cent. Fe. Recovery 96 per cent. Zn; 90 per cent. of siderite eliminated.

Rapid separator (Fig. 17) is another high-intensity machine in which powerful fields are induced in sharp-edged secondary poles overhanging the flat-pole primary magnets (*a*) placed below the upper run of an endless-belt conveyor (*b*). The secondary magnets are saucer-shaped rotating disks (*c*), with diameter several inches greater than the width of the belt. Any given point on a disk, when over the belt is also over the pole piece of a permanent magnet and therefore becomes magnetized and lifts magnetic material from the belt. As the disk revolves, this point travels to one side, induced magnetism is lessened, finally passes through zero in reversing polarity, and its load of magnetic material is discharged under the combined influence of gravity and centrifugal force. By tilting the axes of the disk shafts as shown, the effective strength of the secondary magnetic fields may be varied with the result that successively less-magnetic products are discharged as indicated in the figure.

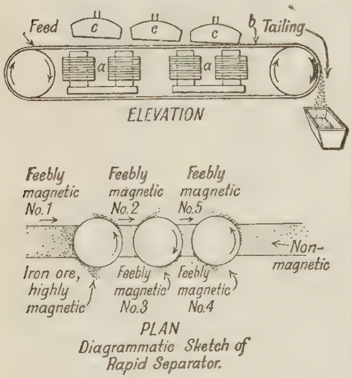


FIG. 17.—Rapid separator.

Laboratory machine has a feed belt 6 in. wide, commercial sizes are 12 and 15 in. Usual CURRENT requirement is from 2 to 10 amperes. Belt SPEED depends on magnetic character of the feed, but will range from 50 to 200 ft. per min. CAPACITY may be estimated from belt speed and size of feed.

Performance. At STOREY'S CREEK, Australia (*18 MM 266*), wolfram-tin separation is made. The feed contains 50 to 66 per cent. WO_3, magnetic concentrate assays 70 to 74 per cent. WO_3 and non-magnetic product 65 to 68 per cent. metallic tin. Capacity of separator is 1.5 tons per 24 hr.

12. Wet magnetic separation

Wet magnetic separation has lagged behind dry on account of the greater mechanical difficulties in presenting feed to the magnets and in cleaning concentrate. However, when the magnetic mineral is very finely disseminated, as is the case with many of the Swedish magnetites, certain of the Adirondack and Pennsylvania magnetite deposits, and the low-grade Mesabi magnetite ores, wet separation is essential. This is because with finely-divided feeds practically absolute dryness is necessary to permit freedom of movement between the particles on the feed sole, and with such dryness dusting is excessive. Furthermore powdered apatite clings with extreme tenacity to magnetite particles, so that when this common constituent of magnetite ores is present the concentrate will be high in phosphorus.

At Cornwall, Pa., it was found (*56 A 901*) that talc clung to the dry concentrate particles to such an extent that it was impossible to raise the iron content above 52 per cent., while by wet separation 58-per cent. concentrate was readily obtained.

With dry feeds finer than 30-mesh it is necessary to remove dust before concentration, or low-grade concentrate results. Many operators state that when it is necessary to grind to ⅛-in. or less in order to free the mineral, wet separation is preferable to dry.

In general, a stronger field is required to concentrate a given ore wet than dry. The speed of presentation must be slow and carefully regulated. If the feed is in a swiftly flowing stream, it is practically impossible to regulate the field properly and either the whole stream will pass by or practically the whole stream will be lifted. Many machines have been devised and patented for such service, but the number of successful machines is small. The best known are the Ullrich (Art. 11), Gröndal, Heberle, Dings-Roche, and Dings cross-belt machines and Davis magnetic log washer. The first two have been used to a considerable extent in Europe, the Dings-Roche in New Jersey and the Davis machine at the Mesabi Iron Co. All these have been used on highly-magnetic iron ores; there are but few instances of the successful use of high-intensity machines on wet pulps, the Weatherby machine at Northern Ore Co. (p. 169) being one.

In most wet magnetic separators the medium in which separation takes place is water instead of air. The permeability of water is, however, substantially the same as air so that no difficulty is met with on this score. It is advisable, however, that the separating surface dip into the water in order to eliminate the complication arising from the presence of a gas-liquid interface through which either tailing or concentrate or both must pass. Such an interface is the seat of forces that oppose its rupture (see Sec. 12, Art. 2) and when the particle is small, these forces are so great relatively to the other forces acting on the particle (gravity and magnetism, which depend on volume) that it is difficult to draw magnetic particles through such a surface film and substantially impossible to cause gangue to fall away from a magnetic-separating surface that is covered with a film of water surrounded by air. For this reason, in most wet machines, the separating surface is submerged during the time that magnetic material is being drawn to it and gangue is being washed out, and emerges from the body of water, if at all, only after washing of the concentrate is complete.

Heberle wet separator (Fig. 18) has had considerable use in Europe. It consists of deep tank in which a water-tight box of magnets m is suspended. An endless belt B about 30 in. wide, runs around pulleys in such direction that the downward run is close to the pole pieces of the magnets. Feed ground to at least 30-mesh, in the form of a freely-flowing pulp with water, is fed against the down-coming belt at A, magnetic material is drawn against the belt while non-magnetic settles in the body of water. Final separation is effected at the upper edge of a dividing baffle at the tail pulley; tailing passes out through compartment G, concentrate falls into hopper E and is discharged through standpipe F, slime overflows at H. Rated capacity is 35 tons per 24 hr.

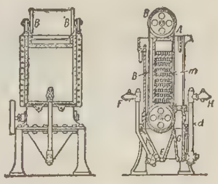


Fig. 18.—Heberle wet separator.

13. Gröndal separator

Gröndal drum-type wet magnetic separator (Fig. 19) consists essentially of the ordinary drum-type separator (a) 24- to 30-in. diameter and 17- to 32-in. face with alternating-pole magnets, mounted above a spitzkasten (b) in such a

way that the lower face of the drum just touches or just fails to touch the water surface. Magnets are made adjustable in angular position and the drum as a whole is adjustable vertically and horizontally. Feed is kept in suspension by rising water currents andall solid matter is forced to flow through the effective part of the magnetic field by means of properly disposed baffles (*c*). Sand tailing sinks to the bottom of the spitzkasten and is drawn off through appropriate ports; slime overflows with excess water as shown; concentrate adheres to the drum surface until it has been lifted sufficiently to be discharged over the side of the spitzkasten. At this point the bank of magnets ends and concentrate is washed off by a spray into a concentrate launder. When two drums are run in series, a strong current is carried in the first to lift all magnetic material as a rough concentrate for re-treatment on the second machine which, in turn, produces finished concentrate and a middling for re-grinding, re-treatment, or discard as tailing, according to the exigencies of the problem.

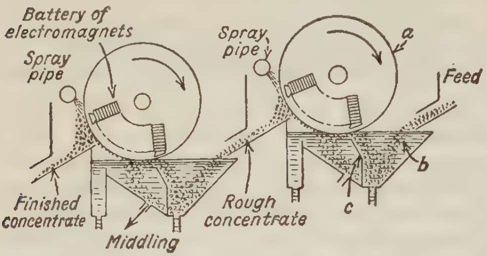


FIG. 19.—Gröndal drum-type wet magnetic separator.

In another form of this separator used in Europe, the drum surface is composed of alternating radial strips of non-magnetic material and iron. The primary magnet has a pointed pole piece directed vertically downward and mounted directly above the baffle in the spitzkasten. Hence the induced magnetism in the iron strips is greatest when the strips are submerged and decreases as they emerge, becoming sufficiently small by the time they reach the horizontal diameter of the drum to permit concentrate to be washed off.

Usual drum SPEED is from 30 to 50 r.p.m. Suitable FEED SIZES range from 8-mesh to 100-mesh. CAPACITY on magnetite ores is from 2 to 4 tons per foot of width of drum face per hour. CURRENT requirement is from 6 to 15 amp. at 110 volts and from 0.25 to 0.5 hp. is required to drive the drums. Water consumption is about 10 gal. per min. per ft. of drum width.

Performance. At BENSON MINES. N. Y., two machines were run in series, the second taking rough concentrate from the first; tailing from both was sent to waste. Two such pairs handled 700 to 750 tons per 24 hr. of 20-mesh feed assaying 30 per cent. Fe and about 0.35 per cent P, but the machines were somewhat overloaded and a third pair was installed. Titanium in the feed ran 3.5 per cent. and in the concentrate, 1 per cent. Table 11 gives sizing-assay test of concentrate. Tailing assayed about 8.5 per cent. Fe, not all magnetic. HANOVER BESSEMER IRON AND COPPER Co., Fierro, N. M., treated 400 tons per day of 30-mesh feed assaying 48 per cent. Fe, 0.6 per cent. Cu and 0.02 per cent. P on three sets of two Gröndal machines in series. Concentrate assayed 60 per cent. Fe, 0.2 per cent. Cu and 0.007 per cent. P. At BETHLEHEM STEEL Co., Lebanon, Pa., the ore consists of magnetite, pyrite and chalcopyrite in talc and ferro-magnesian silicates;

Table 11. Sizing-assay test of concentrate of Gröndal separators, Benson Mines Co., N. Y.

Screen, mm.	Weight, per cent.	Assays, per cent.	
		Iron	Phosphorus
0.833	3.0	47.65	0.060
0.589	11.2	53.72	0.057
0.417	20.8	57.48	0.055
0.295	15.5	60.18	0.056
0.208	16.0	60.86	0.051
0.147	9.0	61.04	0.049
0.104	9.2	61.76	0.044
0.074	3.8	62.10	0.045
−0.074	11.5	62.88	0.042

assay, 43 per cent. Fe, 1.8 per cent. S and 0.4 per cent. Cu. 3000 tons per day crushed to 30-mesh went to 20 Gröndal machines in sets of two in series. Concentrate assayed 60 per cent. Fe. 0.9 per cent. S, and 0.2 per cent. Cu. Sintering reduced S to less than 0.1 per cent.

Gröndal slime separator utilizes the fact that magnetized particles when free to move will bunch together, with unlike poles in contact so far as possible, the net effect being to reduce the external field to a minimum. A primary magnet with wedge-shaped pole piece is mounted above a spitzkasten through which the slime feed is run. Under the influence of the magnetic field the magnetic particles become magnetized, are drawn near together just below the water surface and there form agglomerates that sink and are drawn off from the bottom of the tank. In another form a strong horizontal surface current skims the enriched surface layer of slime from the tank before sufficient agglomeration has occurred to cause the magnetic material to sink.

Wiser-Chino machine designed and built at the plant of the CHINO CONSOLIDATED COPPER Co., is an exception to the general rule that magnetic separation of fine material should not take place through a two-phase interface. It consists of an ordinary drum-type separator mounted above the discharge end of a shaking deck actuated by an ordinary Wilfley-table head motion. The function of the table is merely to move the feed pulp in a thin film through the separating zone. At the Chino plant the machine treats copper concentrate containing some magnetite, the object being to recover clean magnetite for use in making sponge iron. Feed analyses 3 per cent. on 10-mesh and 15 per cent. −200-mesh. Feed rate, 150 tons per 24 hr. in a pulp containing 12 per cent. solids. Magnets draw 3 to 4 amp. at 250 volts. About 1 hp. is consumed in driving. Peripheral speed of drum is about 200 ft. per min. One man attends 8 machines. Typical assays are as follows: Feed, 11.41 per cent. Cu and 38 per cent. Fe; non-magnetic product, 13 37 per cent. Cu and 32.7 per cent. Fe; magnetic product, 0.61 per cent. Cu and 60.0 per cent. Fe.

Dings-Roche wet belt machine (Fig. 20) consists of an endless rubber vanner belt (*a*), 30 in. wide, 2-ply with 2-in. flanges, set on a slope of from 10 to 80°, usually 30°; a battery of 20 electromagnets 30 in. long, spaced 3 in. center to center, having alternating poles of opposite polarity, enclosed in a properly ventilated, waterproof copper box (*b*), all set underneath the upper run of the belt as shown. Feed is introduced at (*f*), concentrate is carried up-slope against a stream of wash water supplied through spray pipes (*c*), tailing flows down slope and discharges over the tail roller. Washing of concentrate is aided by the winnowing motion caused by the alternating polarity of the magnet poles. Concentrate is discharged by immersion of the belt in water box (*d*) aided by spray (*e*).

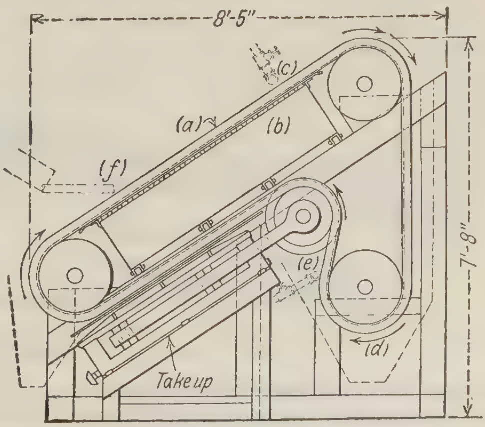


FIG. 20.—Dings-Roche wet belt separator.

Maximum SIZE OF FEED is 0.25-in. but the best results are obtained with material passing ⅛-in. or 1/16-in. screen. Current supply should be up to 20 amp. at 125 volts. One hp. is required for running a 2-belt unit. Belt SPEED is 200 ft. per min. Roche (*115 J 971*) states

the CAPACITY on magnetite ore to be 3 tons per hr. per ft. of belt width with a total water consumption of 2 gal. per min. per in. of belt width. CURRENT draft is 5 to 10 amp. at 125 volts. Table 12 gives comparative results obtained by this machine and a Ball-Norton belt-type machine treating ore from the RICHARD MINE.

Table 12. Comparison of concentrate made by Dings-Roche wet separator and Ball-Norton dry separator (*a*)

Screen size, mesh	Ball-Norton separator, per cent.		Dings-Roche separator, per cent.	
	Weight	Fe	Weight	Fe
20	34	54.06	37	55.16
30	22	57.03	22	69.03
60	20	62.61	20	68.31
80	14	59.01	20	67.22
100	7	51.03	0.5	66.41
200	2	43.21	0.25	66.03
300	1	41.16	0.25	61.09
Total.......	100	56.56	100.00	63.25

a Feed to both machines all through 8-mesh (0.084-in.) screen and on 300-mesh; assay, 48 per cent. magnetic Fe. Ball-Norton machine: tailing, 17.93 per cent. magnetic Fe; ratio of concentration, 1.28; recovery, 91.8 per cent. Dings-Roche machine: tailing, 2.19 per cent. magnetic iron; ratio of concentration, 1.33; recovery, 98.9 per cent.

Dings cross-belt wet separator is similar to the Wetherill machine (Art. 10) except that the feed belt has 0.25- to 0.75-in. side flanges to prevent slop. Sizes, determined by width of feed belt, are from 18- to 60-in. Feed pulp should contain about 20 per cent. solids.

The manufacturers claim that the machine will treat pulps containing 90 per cent. of −200-mesh material. Capacity of a 48-in. machine on 100-mesh magnetite is claimed to be 100 tons per 24 hr. (*121 P 218*). A current of 15 amp. was used to remove partly-roasted pyrrhotite from galena and blende.

14. Magnetic log washer

This machine takes its name from the fact that as originally designed it was similar to the ordinary two-log washer (Sec. 8, Art. 4) but had, additionally, a bank of primary magnets underneath the tank as shown in Fig. 21. In later

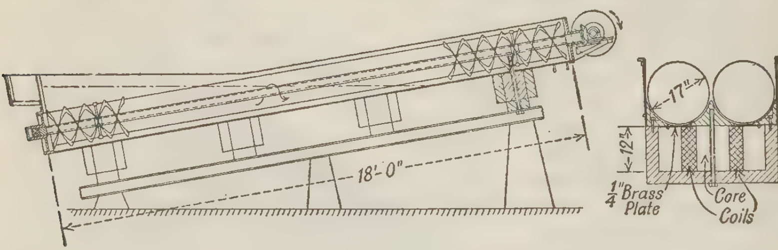


FIG. 21.—Magnetic log washer.

designs the log was replaced by a ribbon spiral and the action, apart from that of the magnets, is rather that of an Akins classifier (Sec. 6, Art. 6). The machine is essentially a slime machine, not suitable for treating anything coarser than 48-mesh feed and increasing in efficiency of treatment as the feed becomes finer. With coarser feed all +48-mesh material goes into the concentrate with resulting lowering of grade.

The action consists in drawing magnetic material to the bottom of the tank by means of the magnetic field, after which the deposited material is conveyed up slope against a stream of wash water, finally discharging over the upper lip while the gangue particles in suspension pass over the overflow lip.

Machines installed by Mesabi Iron Co., 14 ft. long and 2 ft. wide, with two spirals per tank, treated − 150-mesh feed containing about 35 per cent. magnetic Fe at the rate of 50 to 75 tons per 24 hr. and produced 65-per cent. concentrate and a tailing containing from 1 to 2 per cent. magnetic iron. The machines for the final installation at Mesabi are 18 ft. long and 6 ft. wide equipped with 4 spirals and will produce from 50 to 125 tons of concentrate per 24 hr., the amount depending on the difficulty experienced in washing concentrate. W. G. Swart states that this machine requires about 0.25 hp. for excitation and the same for mechanical operation and that it will recover 95 to 98 per cent. of the magnetic iron fed. Character of concentrate depends upon the extent to which mineral is freed, fineness of grinding and amount of washing. With grinding fine enough to permit gangue to be kept in suspension and to free the mineral with substantial completeness, concentrate assaying 65 to 70 per cent. Fe as magnetite is readily made. Wear with fine feed is negligible.

15. Weatherby separator

This is a high-intensity wet machine working on a different principle from any of the other machines described. It consists (Fig. 22) essentially of a small double side-inclined shaking table with Wilfley head motion and aluminum deck, 4 × 4 ft., with steel riffle cleats, all suspended by means of hickory hangers between the poles of a powerful primary electromagnet in such a position that the riffles are nearer the upper than the lower pole of the magnet. As a result the riffle cleats become secondary magnets with the field converging most sharply at their upper surface and magnetic material is therefore drawn to the top of the riffles and washed down slope while non-magnetic material progresses to the end.

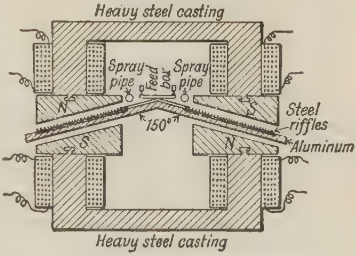


Fig. 22.—Weatherby separator.

Performance. At Northern Ore Co., Edwards, N. Y. (*115 J 1053*, *116 J 404*). The feed is jig and table concentrate passing 4.5-mesh and assaying 30 per cent. Zn (the blende contains 5 per cent. Fe and is magnetic). The feed pulp contains about 30 per cent. water. Concentrate assays 45 to 50 per cent. Zn; tailing, about 6 per cent. Zn. Concentrate assaying 56 per cent. Zn, which is practically pure ferruginous blende, can be made, but this is not economical procedure. Speed of table, 250 @ 1-in. strokes per min. Current draft, 15 amp. at 110 volts. Capacity, 22 tons per 24 hr. per table. The machine has been in successful operation at this mill for several years, but has not been installed elsewhere.

16. Guard magnets

Guard magnets are used in crushing plants to remove tramp iron and steel from the feed before it goes to a crusher incapable of handling it.

As an example of the economy of such an installation, it is stated (*112 J 728*) that at the Melones Mining Co. a magnetic pulley placed ahead of the primary crusher saved $1400 per month in delays and repairs.

Guard magnets are placed above or at the side of a chute or conveyor feeding the crusher. They are usually of the type employed for lifting scrap iron in handling plants, with manual control of current. They must be powerful in order to lift pieces of steel from rapidly-moving masses of coarse ore, and, as the load builds down, to hold it against blows of the material moving on the belt. Fig. 23 shows a typical installation over a conveyor belt. The magnet is hung on a sufficiently long suspension so that it can be swung aside for discharging. When there is danger that the load will build down sufficiently to interfere with material on the belt, it is well to guy the suspension rod with

wires that will complete a bell or light circuit and thus attract attention when the magnet is knocked forward.

Magnetic chute. Fig. 24 shows a method of applying a guard magnet to a chute. Magnets (*a*) are so arranged as to produce a strong field in the chute and catch and hold tramp iron until it is removed by an attendant. Just below

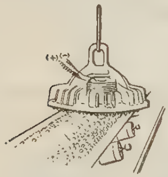


FIG. 23.—Mushroom magnet over belt conveyor.

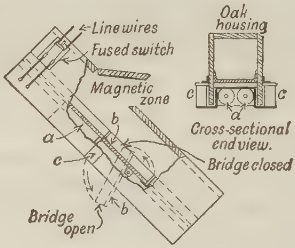


FIG. 24.—Dings magnetic chute.

the magnetic zone a tilting section (*b*) is placed in the chute bottom. This is held in closed position by the magnets (*c*) so long as there is current flowing through the guard magnet but if current is cut off, accidentally or otherwise, the tilting section is released and spills the released iron together with the feed stream onto the floor until such time as the feed is cut off or the tilting section closed and current flowing.

Pulley magnets (Art. 4) are frequently used as guard magnets.

Guard magnets cannot be used in plants treating magnetite ore, on account of the magnetic character of the ore itself. Also, guard magnets will not lift manganese steel, because of its low permeability. At the REPLOGLE STEEL CO. plant, vertical disk crushers set to ¼-in. were guarded by a screen with ¾-in. aperture sending oversize in closed circuit to a set of rolls. These were rugged enough to break up the steel fine enough to pass the disk machines without injury to them. At the UTAH COPPER CO., where manganese-steel dipper teeth are used on the steam shovels, close watch is kept on the shovels and when a tooth is missing a telephone message is immediately sent to the mill (18 miles distant) and particularly close watch is kept as the ore is unloaded into the mill bins and run over the unloading conveyors to the primary crusher, until the tooth is found and removed. These examples are typical of the expedients that must be adopted when guard magnets cannot be used.

Magnets in grinding-mill circuits are used in cyanide plants to remove iron introduced by abrasion from the crushing and grinding machines.

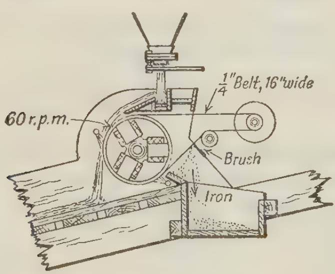


FIG. 25.—Pulley-type separator in tube-mill circuit at Simmer and Jack mill.

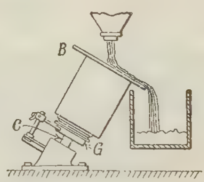


FIG. 26.—Brown electromagnet for tube-mill circuit.

Unless removed this iron, being difficult to grind, builds up to such an extent that it may amount to as much as 15 per cent. of the total circulating load (ALASKA-TREADWELL GOLD

Mining Co., *96 J 452*). Schmidt (*RMP*), states that as much as 900 lb. of metallic iron per day has been removed from a Rand tube-mill circuit and that prior to the introduction of a magnet at the Simmer and Jack mill it was necessary to open the tube-mill circuit for 15 min. each day, and pass the whole pulp to the cyanide plant without re-grinding, in order to clear out the accumulation of iron. Fig. 25 shows an application of a pulley-type separator (Art. 4) at this mill. Fig. 26 shows another type of separator in similar service (*90 J 445*). A flat disk-shaped cast-iron pole-piece (*B*) of an electromagnet with cast-iron core is mounted to revolve on shaft (*C*) which is inclined 35° from the vertical. Current enters through slip-ring *G*; the other end of the coil is grounded. Winding is encased water-tight by a zinc sheath soldered to the iron bobbin of the winding. The magnet is driven at 6 r.p.m. by a 0.5-hp. motor; exciting current is 2.5 amp. at 100 volts. Iron is caught on the rim of the plate and is scraped off by a piece of canvas belting.

A magnet used at Alaska-Treadwell is shown in Fig. 27. The magnet poles dip into the tube-mill discharge launder leading to a Dorr classifier. Current on a given pole is broken by a commutator when the pole-piece reaches a position above the apron. Fig. 28 shows a pair of oscillating magnets with curved pole-pieces that swing in a semi-cylindrical section of a main pulp launder at the Liberty Bell G. M. Co. Water jets wash the poles as they emerge from the pulp and current is cut off when the magnet reaches the highest point of the swing at which time it overhangs the edge of the launder. Each pole has 800 turns of wire

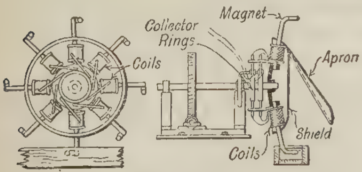


Fig. 27.—Magnet for tube-mill circuit at Alaska Treadwell.

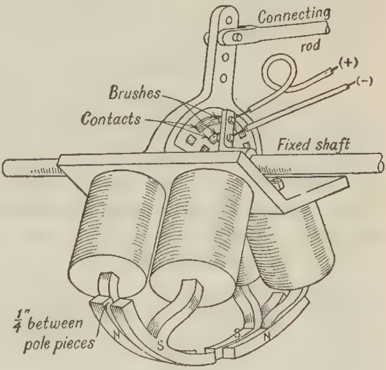


Fig. 28.—Pulp magnet at Liberty Bell.

and draws 2.0 to 4.0 amp. at 110 volts. The machine makes 4 to 5 oscillations per min. and requires about 0.1 hp. for driving. Feed rate is 100 tons of − 16-mesh material per 24 hr. in a pulp carrying about 10 per cent. solids. Removal of metallic iron is practically complete.

17. De-magnetizer

This consists of a conical coil carrying alternating current, surrounding an iron pipe carrying magnetized pulp. When an ore that has been over a magnetic separator is finely ground, there is sufficient residual magnetism in the magnetic particles to cause them to agglomerate and, in classification, to sink with the unground sands rather than overflow with the slimes. Effective closed-circuit grinding is thus prevented. Passage through a de-magnetizer remedies this difficulty.

The device is used on the discharge of ball mills feeding bowl classifiers at Mesabi Iron Co.

18. Murex process

The Murex process is a combination of magnetic separation with the differential-oiling phenomenon that is utilized in flotation processes (Sec. 12). It is applicable to the separation of sulphides and other minerals of metallic luster from minerals of non-metallic luster. Minerals of metallic luster become coated with oil when mixed with oil in an aqueous pulp, while minerals of non-metallic luster remain water-wet. In the Murex process the oil is first loaded with

powdered magnetite, then mixed with the aqueous pulp and the whole is presented in a thin film under the poles of a magnet, when the oil-coated minerals are attracted to the magnet because of the magnetite in the oily coating, while the non-coated gangue particles pass on. One oil mixture that has been used was composed of fuel oil, tar and resin. This had the power to hold the magnetite as well as to coat sulphides preferentially.

At CLAUSTHAL (*100 J 429*) the process was used, 3- to 4-mm. material being ground in a pebble mill in a thin pulp with the above oil mixture to effect coating, the product passed through a screen with a 2 × 4-mm. openings then spread out on a shaking table under an overhanging magnet. Galena was thus separated from barite and silicious gangue minerals. At DARWIN LEAD AND SILVER MINES (*104 J 58*) a 50-ton mill treated a partly oxidized lead-silver ore. A recovery of 80 to 85 per cent. of the mineral content was claimed at a cost of $1.78 per ton. Oil consumption was 15 lb. petroleum residuum and 0.8 lb. oleic acid; magnetite, 17 lb. per ton.

19. Roasting to increase permeability

Roasting is practiced in the case of many iron-bearing products in order to convert the iron-bearing mineral into a magnetic state. Fe_3O_4 and Fe_7S_8 are highly-magnetic compounds of iron while Fe_2O_3 (hematite), $2Fe_2O_3 \cdot 3H_2O$ (limonite), $FeCO_3$ (siderite), FeS_2 (pyrite) are feebly- or non-magnetic. These latter may, however, be converted to the magnetic state by proper roasting. Chalcopyrite, bornite and other complex sulphides containing iron are similarly rendered magnetic.

Hematite is changed to Fe_3O_4 by abstraction of oxygen. This can be effected by strong heating in air, less heat in a reducing atmosphere and a still lower heat when there is intimate admixture of a reducing substance such as hydrogen or carbon. Reducing roasting has had some commercial application in Europe and much experimentation has been done in this country. Theoretically only about 1 per cent. by weight of carbon or equivalent fuel would be necessary to effect the conversion of pure Fe_2O_3, if there were no heat losses, but the Minnesota School of Mines Experiment Station has determined that on a typical low-grade hematite ore from 5 to 10 per cent. by weight is necessary. Their conclusions as to operating conditions follow: (1) For efficient and rapid roasting the temperature should exceed 400° C. (2) Comparatively small increases in temperature effect considerable increases in efficiency. (3) Efficiency generally increases with increase in duration of roast. (4) Efficiency increases with decrease in size of particles, but this factor is less important than temperature and duration. (5) Simple heating in air produces little change in magnetic character. (6) The reducing agent should contain as little methane as possible, as this gas has low efficiency.

Siderite is converted to the magnetic oxide by calcining with restricted access of air. Over-roasting produces the feebly-magnetic ferric oxide, Fe_2O_3.

Pyrite may be converted by roasting either to Fe_7S_8 or to Fe_3O_4, both of which are highly magnetic, according to the character of the roast. When heated in the presence of air, pyrite (and marcasite) decrepitate at 60° C., and as the temperature is raised lose sulphur more and more rapidly until at about 400° C. the magnetic sulphide, Fe_7S_8 is formed, the sulphur igniting and passing off as SO_2 gas. If the period of subjection to heat is short (FLASH ROASTING), only the surface of the pyrite is altered, and Fe_7S_8 is formed, but this surface shell of magnetic material is sufficient to cause the particles to be lifted by the magnet in a field of high intensity. Further roasting at 500 to 600° C. with limited air and an atmosphere of CO, H and SO_2 results in conversion to the highly magnetic Fe_3O_4, more or less complete according to the duration of the roast and the size of the particles. Roasting to form the magnetic sulphide is hard to control, some of the more strongly magnetic oxide is sure to form, especially from the smaller particles, while the larger particles contain mixtures of the original material and the various possible products. The product is, therefore, difficult to separate on account of the varying permeability of the constituents, and for this reason it is frequently better to carry the roast to magnetic oxide. Over-roasting, especially if accompanied by access of air, causes conversion of Fe_3O_4 to the feebly-magnetic Fe_2O_3. Where conversion must be closely regulated the ore must be closely sized before roasting and, if complete conversion to oxide is desired, it should be ground to −2-mm. On the other hand, if the feed is too fine, it is difficult to obtain proper exposure to the furnace gases, with the result that roasting is again uneven.

Furnaces. Revolving kilns and multiple-hearth furnaces of the McDougall type are ordinarily used. The latter type has had extensive use in the Wisconsin zinc district in roasting marcasite-blende concentrate to convert the marcasite into magnetic sulphide. CAPACITY of a 6- or 7-hearth, 22-ft. furnace in this service is 100 to 125 tons per day on material passing a 3/8-in. screen.

Cooling. Roasted material must be cooled before passing it through the separators, on account of the destructive effect on the machines and difficulty in handling. Cooling must be done out of the presence of air in order to prevent re-oxidation. Temperature of material leaving the cooler should be less than 100° C. Revolving cylinders, usually with water sprays on the outside of the shell are frequently used.

At TRAIL (*120 P 864*) it was found that the effect of a roast to increase the magnetic quality of pyrrhotite without complete conversion to Fe_3O_4 was lost if immediate quenching or slow cooling was practiced, but retained with rapid cooling in a rotary cooler.

Performance. Table 13 shows the results of a series of experiments made at the Minnesota School of Mines Experiment Station on roasting HEMATITE ore to the magnetic oxide. The work was done in a rotating cylindrical externally-heated gas-fired furnace, using illuminating gas as a reducing agent. PYRITE-BLENDE CONCENTRATE, Wisconsin. (*107 J 1107*). Self-roasting furnaces of the Wedge and Skinner types are used with rotary coolers. Characteristic roaster feeds range from 21.6 to 45.9 per cent. Zn, 12.8 to 27.4 per cent. Fe, 0.22 to 0.82 per cent. Pb, 1.50 to 2.15 per cent. CaO and 35.7 to 41.6 per cent. S. One plant uses an 8-hearth Wedge furnace, 22 ft. 6 in. diameter by 24 ft. high. Ore is heated gradually by the heat of its own combustion to a maximum temperature of 900 to

Table 13. Effects of temperature, duration of roast and size of particles on magnetic roasting of hematite

Temperature, degrees C.	Duration, minutes	Percentage completeness of conversion to magnetic oxide			
		Size			
		−3/4 +1/2-in.	−1/2-in. +8-m.	−8 +300-m.	−300-m.
300	15	10.2	27.5	34.3	39.1
	30	25.5	57.0	62.4	65.0
	45	49.5	75.0	80.0	82.8
	60	73.4	87.0	90.0	93.5
	120	86.4	95.0	95.8	99.0
350	15	17.0	52.0	63.0	69.0
	30	43.7	74.5	84.8	86.5
	45	68.0	86.0	90.5	93.0
	60	87.2	93.0	95.0	97.0
	120	94.3	97.0	97.7	99.3
400	15	23.2	67.5	85.4	91.0
	45	80.9	92.0	95.3	97.0
	120	97.0	97.5	98.0	99.3
450	15	30.2	77.0	91.5	94.5
	45	89.0	95.5	96.3	97.2
	120	98.0	98.1	98.4	99.3
500	15	37.5	83.2	94.0	95.5
	45	93.1	96.5	96.8	97.5
	120	98.5	98.5	98.6	99.3
550	15	42.8	86.5	95.0	96.0
	45	94.0	96.7	96.9	97.8
	120	98.7	98.8	98.8	99.3
600	15	45.0	89.5	96.0	96.8
	120	99.0	99.1	99.2	99.4

1000° F. on the lowest hearth, at which temperature marcasite is strongly oxidized but blende is unaffected. Gases contain 4 to 5 per cent. SO_2. Decrepitation and abrasion increase the percentage of −40-mesh material from 9 per cent. in the entering feed to 30 per cent. in the discharge. Four rotary coolers are used, 2 × 26-ft., 6 r.p.m., with outside spray (30 gal. per min. each). The magnetic quality of the roasted ore is greater when slightly warm than cold. Dings tray-type separators make rough concentrate assaying 56 to 58 per cent. Zn; this is raised to 61.5 per cent. Zn by a high-intensity cleaner, and a tailing containing 4 to 5 per cent. Zn and 25 per cent. S, which is suitable for sulphuric-acid manufacture. At another plant in the same district (*99 J 977*) the zinc-iron gravity concentrate is given a slight, positively-controlled roast in an oil-fired revolving kiln, cooled in a revolving cooler and separated on a Campbell separator (tray-type with cross belts for concentrate removal). The raw concentrate assays 21 to 34 per cent. Zn and finished concentrate 49 to 61 per cent. with recoveries of 92 to 95 per cent. reported. Shrinkage in weight during roasting runs from 50 to 70 per cent. The iron product is suitable for acid manufacture, if not over-roasted. CASSITERITE-PYRITE SEPARATION, Llallagua (*100 J 513*). The furnaces are of the McDougall type, 5-hearth; ore is self-roasting (no carbonaceous fuel added); feed is all −1-mm. and assays about 25 per cent. sulphur as pyrite and 15 per cent. Sn; furnace feed contains about 5 per cent. moisture. Best results are obtained on Stern-type wet separators. (A star-shaped pole piece revolving 10 to 20 r.p.m. around a horizontal axis in a tank of pulp picks up magnetic material and carries it above the surface of the pulp where it is washed off the pole arms and over the sides of the tank by a strong jet of water. Current draft is about 6 amp. at 110 volts. Capacity: 18 tons per 24 hr. was handled when the roasted product contained 10 to 12 per cent. S. Under these conditions the magnetic product contained 50 per cent. Fe and 22 per cent. S and the non-magnetic 2.5 per cent. Fe and 1.5 to 2 per cent. S.) Properly-roasted product was black with metallic luster; over-roasting was indicated by a reddish product on which the separators did poor work and discharged a tin product high in iron. Capacity of roasting furnace was about 0.04 ton per sq. ft. of hearth area per 24 hr. Amount of flue dust was less than 5 per cent. of the furnace feed and tin content of dust was about 5 per cent. on a 20 per cent. feed, due in part to the fact that cassiterite does not decrepitate. There was never more than 2 per cent. loss of tin in dust. COPPER-IRON, Calumet and Arizona smelter, Douglas, Ariz. (*Bul. 9, MSM 38*). Davis states that magnetic roasting is being incidentally performed at this plant in connection with drying and de-sulphurization prior to smelting. The ore, crushed to pass a 1-in. screen is roasted in a 6-hearth McDougall-type oil-fired furnace 21 ft. 6 in. diameter at the rate of 80 to 100 tons per 24 hr. with conversion of about 80 per cent. of the iron to magnetic oxide at a total operating cost in 1919–20 of $0.31 per ton.

SECTION 14

MISCELLANEOUS PROCESSES OF CONCENTRATION

1. Mechanical pickers

Anthracite pickers are built to take advantage of the difference in shape between particles of coal and of slate. The simplest and earliest form, typified by the ZIEGLER, consists of a sloping steel chute with a transverse slot in the bottom through which the flat, sliding, slow-traveling slate falls while the rounded, rapidly-rolling coal particles "jump" over and pass on to the end. The principal modification of this type is a slate-bottomed chute, which retards slate in the coal more than a steel bottom and causes more rolling of the coal. Another modification of the same device was introduced by shaking the separating chute, which, of course, makes it possible to set the separating surface on a flatter slope and thus save head room. Another modification is the roller picker, which has a series of transverse spaced rollers across the bottom of the separating chute, the spaces corresponding to the slots in the jump-type pickers. AYERS PICKER consists of a flat apron conveyor traveling up-slope at such an inclination that rounded coal particles roll down-hill while flat slate particles are carried up and over the head roller.

At PHOENIX COAL CO. (*18 CA 796*), treating culm, an Ayers chestnut picker runs 150 to 200 ft. per min.

Spiral picker is the most widely used of the mechanical picking devices. One form is illustrated in Fig. 1. In sliding down the spiral chutes the coal, being rounded, travels faster than the flat slate, works to the outer periphery, and falls off into the larger spirals, which discharge separately from the smaller. Spiral pickers will not work on wet coal. They are adjustable for temperature. Each type and size of coal requires a different setting. Spirals are of little value for fine coal. Certain types of coal are not amenable to spiral treatment. According to Sinnat and Mitton (*67 IME 491*) the coal must be sized between the following limits: $-6 +4$-in., $-4 + 2\frac{1}{2}$-in., $-2\frac{1}{2} +1\frac{1}{2}$-in., $-1\frac{1}{2} +\frac{3}{4}$-in., $-\frac{3}{4} +\frac{1}{4}$-in. Spirals have had their greatest success in treating anthracite, because the slaty impurities are tabular while the coal is rounded. Ashmead (*66 A 422*) gives the data in Table 1 as the results of a test. The usual CAPACITY is 8 to 12 tons per hr., depending upon the size of feed.

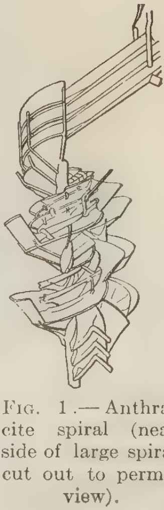


FIG. 1.—Anthracite spiral (near side of large spiral cut out to permit view).

Table 1. Performance of anthracite spiral

	Per cent. of coal	Per cent. of slate
Feed	84.60	15.40
Coal discharge	98.93	1.07
Slate discharge	5.88	94.12
Recovery	99	94

Disadvantage of mechanical pickers is the fact that the coal product carries considerable slate and *vice versa*. Therefore it is always necessary to concentrate the products further, either by hand-picking or by jigging. For this reason such pickers are being superseded by jigs except for grate-size coal, which is too coarse for efficient discharge from jigs The principal advantages of the pickers are their high capacity and low operating cost.

2. Pneumatic concentration

Pneumatic table for treating bituminous coal is a development of a table used for cleaning seeds, cereals, nuts, etc.; it has been tried spasmodically without much success in metal concentration. The usual form is shown in Fig. 2 (*19 CA 811*). It consists essentially of a reciprocating riffled deck (*a*),

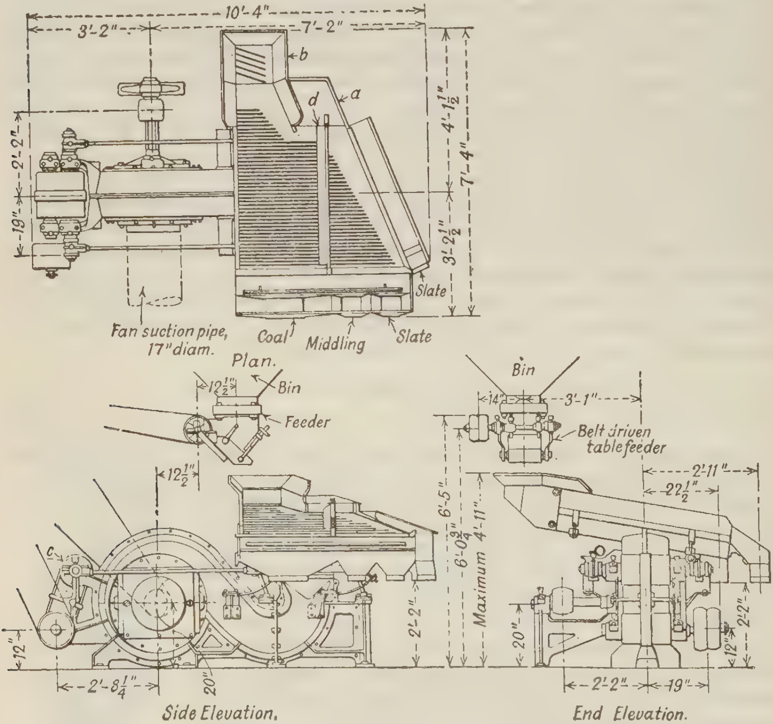


Fig. 2.—Pneumatic shaking table.

about 4 × 4 ft., driven by an eccentric but mounted like the Ferraris or rocker screen (Sec. 5, Art. 6). The deck has an adjustable inclination, averaging about 5° transversely to the riffles, away from feed box (*b*), and has a fixed inclination parallel to the riffles of about 1° upward from the mechanism end. It is driven at from 270 to 330 @ ⅝-in. strokes per min. through the cone

pulleys (*c*). The deck surface is made of cloth or woven wire. Riffles are spaced about 2 in. and taper from about ½ in. at the head end to nothing at the discharge end. For coarse material, the deck is covered with ⅜-in.-mesh wire cloth to keep the bed from sliding; for fine material, tapering paper strips terminating one-half to two-thirds of the distance to the slate-discharge end are pasted under the cover between the wood supporting strips (Fig. 3) to give a quiet space for fine impurity to travel in. (*22 CA 876.*) Air is supplied to a pressure box underneath by means of a 40-in. steel-plate centrifugal suction blower at 1600 r.p.m. An air gate is placed in the table base to regulate the amount of air admitted at just enough to produce stratification without boiling.

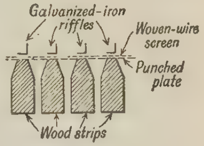


FIG. 3.—Cross-section of deck of pneumatic table.

Operation depends upon the fact that the heavier particles in a mixture of particles of different specific gravities on the deck of the table offer the greater resistance to lifting by the issuing air and that the combination of the lifting force of the air and the agitating effect of the table motion imparts sufficient fluidity to the mass to permit the lighter (coal) particles to rise to the surface of the mass, where they roll down slope, while the heavier (shale and pyrite) particles at the bottom are constrained by the riffles to move to the end away from the head motion. Bone and some slate also work to the lower edge and are collected as a middling product as indicated in the figure.

Bituminous coal as coarse as 3-in. is being treated. The feed should be closely sized. The usual screens for −3-in. material are 1½-, 1-, ½-, ¼-, ⅛- and ¹⁄₁₆-in. Coal finer than ¹⁄₁₆-in. cannot be economically treated on the regular table on account of the small capacity and the consequent high cost. A different form of table, known as the Y-TABLE, from the shape of the deck (Fig. 4), has been used at McCOMAS for ⅛-in. to 40-mesh material (*MCJ, Jan., 1926*). It showed a capacity about twice that of the older type.

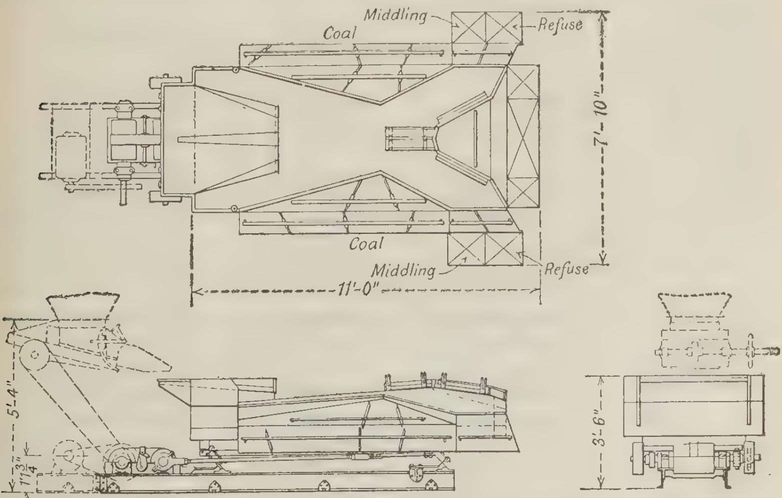


FIG. 4.—Pneumatic coal separator, Y-deck.

COARSE FEEDS require relatively large volumes of air at low velocity and are, therefore, treated on a deck with relatively coarse apertures (woven-wire screen cloth); fine material

Table 2. Tests on performance of pneumatic table in cleaning bituminous coal. (*After O'Toole*)

Size, inches		Raw coal, per cent.				Cleaned coal, per cent.				Refuse, per cent.				Dust, per cent.			
		Weight	Prox. anal.			Weight	Prox. anal.			Weight	Prox. anal.			Weight	Prox. anal.		
Through	On		Ash	S	P		Ash	S	P		Ash	S	P		Ash	S	P
							Test No. 1(*a*)										
1½	1	0.11*b*	44.20	....	....	....	....	....	....	....	....	....	....	....	....	....	....
1	¾	3.35	21.76	0.45	0.012	2.91	5.26	0.50	0.004	0.33	63.90	0.25	0.024	0.11	8.27	0.52	0.003
¾	½	8.74	14.23	0.45	0.008	7.69	5.58	0.50	0.003	0.95	68.11	0.23	0.030	0.10	6.64	0.49	0.006
½	⅜	11.17	12.67	0.47	0.009	10.16	6.75	0.64	0.003	0.95	66.22	0.23	0.026	0.06	8.62	0.54	0.008
⅜	¼	6.94	8.26	0.49	0.007	6.26	5.00	0.53	0.006	0.62	73.85	0.19	0.031	0.06	8.70	0.49	0.007
¼	⅛	23.74	9.41	0.50	0.008	22.12	3.95	0.53	0.003	1.50	71.94	0.21	0.031	0.12	10.80	0.48	0.008
⅛	1/16	16.39	6.50	0.57	0 008	15.57	3.14	0.55	0.004	0.70	79.24	0.20	0.033	0.12	12.00	0.46	0.006
1/16	26-m.	6.65	9.80	0.53	0.007	6.25	4.28	0.56	0.003	0.31	77.37	0.22	0.032	0.09	11.71	0.52	0.005
26-m.	....	22.91	9.85	0.53	0.005	....	....	....	....	....	....	....	....	....	....	....	....
Totals	....	100.00	....	....	....	70.96	....	....	....	5.36	....	....	....	0.66	....	....	....
							Test No. 2(*c*)										
¾	½	24.89	13.13	2.98	0.018	24.00	7.96	2.46	0.009	0.81	66.60	16.94	....	0.08	....	....	....
½	⅜	21.54	11.46	3.50	0.007	20.68	7.48	2.23	0.005	0.79	64.72	20.02	....	0.07	....	....	....
⅜	¼	12.06	13.53	3.05	0.008	11.29	7.94	2.31	0.004	0.63	62.25	18.66	....	0.14	....	....	....
¼	⅛	25.91	11.63	2.99	0.007	24.13	8.32	2.18	0.004	1.54	62.00	13.91	....	0.24	....	....	....
⅛	1/16	10.52	15.00	3.11	0.007	9.55	9.00	2.26	0.005	0.88	75.11	11.49	....	0.09	....	....	....
1/16	26-m.	5.08	15.50	3.15	0.008*d*	4.64	10.35	2.36	0.005	0.39	71.38	12.40	....	0.05	....	....	....
Totals	....	100.00	14.42	2.98	....	94.29	....	....	....	5.04	....	....	....	0.67	....	....	....
							Test No. 3(*e*)										
¾	½	17.53	10.20	2.31	0.006	16.59	7.31	1.65	0.003	0.87	65.00	8.92	....	0.07	....	....	....
½	⅜	19.23	13.83	3.05	0.009	18.09	7.50	1.64	0.005	1.08	67.38	7.90	....	0.06	....	....	....
⅜	¼	11.04	13.24	2.27	0.007	10.27	7.22	1.54	0.003	0.73	72.00	8.62	....	0.04	....	....	....
¼	⅛	23.71	11.91	2.22	0.006	21.89	6.62	1.57	0.003	1.73	72.00	8.78	....	0 09	....	....	....
⅛	1/16	10.91	14.18	2.68	0.010	10.13	7.00	1.53	0.004	0.77	71.41	12.94	....	0.01	....	....	....
1/16	26-m.	5.33	15.61	2.64	0.016	4.77	8.87	1.54	0.005	0.52	67.16	14.42	....	0.04	....	....	....
26-m.	....	12.25	17.39	2.90	0.015	....	....	....	....	....	....	....	....	....	....	....	....
Totals	....	100 00	....	....	....	81.74	....	....	....	5.70	....	....	....	0.31	....	....	....

a Pocahontas, No. 3-seam coal. *b* Weight (13 lb.) too small to treat. *c* Indiana, No. 5-seam coal. *d* − 26-mesh raw coal assayed 16.56 per cent. ash, 3.52 per cent. S and 0.008 per cent. P. It was not treated. *e* Illinois coal. *m* mesh.

requires a smaller volume at higher velocity. Skinner satin is a satisfactory material for the finest sizes.

Performances in three large-scale tests on −1½-in. Pocahontas coal (*20 CA 53*) are given in Table 2. The +1-in. material was thrown directly into the refuse and the −26-mesh material into clean coal.

At American Coal Co., McComas, W. Va. (*MCJ, Jan., 1926*; *22 CA 875*) the −3-in. screenings contain approximately 83 per cent. of coal (−1.35 sp. gr.) assaying 4.30 per cent. ash; 5 per cent. low-ash bone (1.35 to 1.50 sp. gr.) with 15 per cent. ash; 2 per cent. high-ash bone (1.50 to 1.70 sp. gr.) with 30 per cent. ash; and 10 per cent. of slate and fireclay with 86 per cent. ash, a total for the raw screenings of 13.5 per cent. ash. The bone is found principally in the +½-in. material, which contains 13 to 17 per cent. ash. The finer sizes contain 9 to 11 per cent. ash. The −1⁄16-in. material contains about 11 per cent. ash. The present dry-cleaning plant for treating 240 tons per hr. of −3-in. +1⁄16-in. bituminous coal contains 8 tables, one table for each of the sizes −3 +1½-in., −1½ +1-in., −1 +½-in., −½ +¼-in., and two each for the −¼ +⅛-in. and −⅛ +1⁄16-in. sizes. Tests have shown that on the coarser sizes the capacity of a machine is 25 tons per hr. while on the finer sizes it is 12 to 14 tons. The entire plant consisting of 8 tables, 36 Hum-mer screens and 3 fans (100,000 cu. ft. per min. of free air) with necessary elevators and conveyors requires about 2.35 kw.-hr. per ton of coal treated. Air consumption on coarse-feed tables is about twice that on fine-feed tables. The average labor force is 4½ men per shift. The cleaned product contains about 92 per cent. −1.35 sp. gr.; 5.5 per cent, 1.35–1.50; 1.2 per cent., 1.50–1.70, and 1.5 per cent. +1.70 sp. gr.; total ash, 6.4 per cent. The +½-in. cleaned coal averages 6.5 to 7 per cent. ash and the smaller sizes 4.5 to 5 per cent. (*25 CA 761*). The refuse is about 8 per cent. of the total feed and averages more than 80 per cent. ash. Average cost of cleaning (1924) was $0.20 per ton, including power, labor, supplies, repairs, depreciation and interest. At St. Louis, Rocky Mtn. and Pacific Co., Raton, N. M. (*23 CA 791*) the tables are run at about 350 @ ⅝-in. strokes per min. Fan speeds range from 900 r.p.m. on the finest size to 1040 r.p.m. on the coarsest. About 6000 cu. ft. of free air per min. comes up through the table deck. Capacity on different-sized feeds is shown in Table 3. From auxiliary tests it is shown that the tables

Table 3. Performance of S. S. and S. (American Coal Co.) pneumatic table on bituminous coal at St. L., R. M. and P. Co. (*After Young*)

Size of feed, inches	Tons of feed per hour	Analyses, per cent. ash			Recovery of combustible, per cent.
		Raw coal	Cleaned coal (*a*)	Refuse	
1 to ¾	12	17	11	63	94.9
¾ to ½	10	17	10.5	65	94.9
½ to ⅜	8	16	9.5	70	96.2
⅜ to 3⁄16	6	18	10	66	94.1
3⁄16 to 1⁄16	4½	22	14	70	94.4
1⁄16 to 60-mesh	3	25	19	70	95.3
Totals........	....	16.8	11.1	64.6	95.5

a Ash in picked samples of clean coal runs 8 to 9 per cent.

remove about 72 per cent. of the free impurity. The waste rejected is about 10 per cent. of the raw coal by weight. At West Canadian Collieries, Ltd., Blairmore, Alberta, Canada (*MCJ, Jan., 1926*) the capacity on −¼ + ⅛-in. coal is 18 tons per hr.; −7⁄16 + ¼-in., 24 tons; −¾ + 7⁄16-in., 30 tons; −1⅝ + ¾-in., 38 tons per hr. Cost of plant to treat 110 tons per hr., using Hum-mer screens for sizing was $1360 per ton of hourly capacity.

Cost of a dry-cleaning plant for bituminous coal is estimated by Tams (*MCJ, Jan., 1926*) at from $500 to $660 per ton of hourly capacity, and the installed power from 1.33 to 1.67 kw.-hr. per ton. The power consumed will be less than this.

Advantages of dry cleaning are: (*a*) No water is required, which is an important consideration at many mines; (*b*) no moisture is added to the coal, on the contrary a certain amount of drying is effected. This latter fact is important when the coal is to be shipped

long distances or in cold weather; or when it is to be used for coking in which case common specifications call for less than 5 per cent. moisture as received; or when the coal is to be pulverized, when, unless it is dry, the drying equipment may be half of the total equipment and the drying cost more than half of the total cost.

Hooper pneumatic jig (Fig. 5) consists of a stationary adjustably-inclined deck (*a*) covered with broadcloth (or other porous material) supported on slats (*b*). Above the broadcloth are carried two sets of metal riffles (*c*), the lower set, ¼ in. high spaced 1 in. running toward one side while the upper set, 3½ in. high spaced ¾ in. run toward the other side. A leather diaphragm (*d*) with flap valves opening upward is actuated by eccentrics on the pulley-driven shaft (*e*), thus causing air to pulsate upward through the broadcloth. Feed is introduced into hopper (*f*), stratified according to the specific gravity of the particles by means of the pulsating air current and travels generally down slope toward the discharge chute the lower and upper strata being guided toward opposite sides by the riffles. Splitters are provided at the discharge end to separate concentrate, middling and tailing.

This machine was first developed to concentrate graphite, but has been displaced in that service by froth flotation. At the NORTH RIVER GARNET Co. (Sec. 2) it is used to separate garnet from feldspar and hornblende at 30- to 68-mesh sizes.

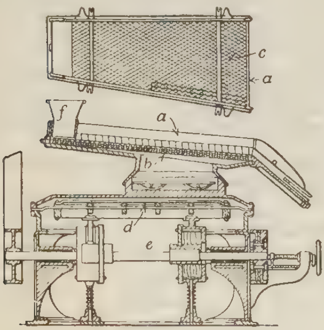


FIG. 5.—Hooper pneumatic jig.

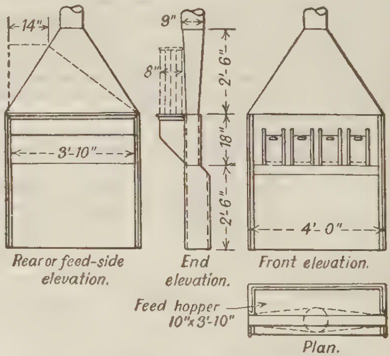


FIG. 6.—Sutton, Steele and Steele aspirator.

Air classification. ASPIRATOR (Fig. 6) (made by Sutton, Steele and Steele) used by ST. LOUIS ROCKY MOUNTAIN AND PACIFIC Co. (*23 CA 794*) to de-dust −1/16-in. bituminous coal consists of an enclosed chute, 1 in. deep by 30 in. wide, sloping 50° to the junction with a vertical chute which narrows down at the junction to the same dimensions but widens above to form a 10-in. circular pipe leading to a suction fan, and below to the discharge. The air velocity at the junction is about 7000 ft. per min. and nearly all dust smaller than 60-mesh is removed.

Gayco separator (Fig. 7) is fed at (*a*) onto the rotating plate (*b*) which distributes the solid in a thin roughly horizontal sheet across the annular space (*c*). A current of air, circulated by the fan (*d*) enters through the vanes (*e*) and rises through space (*c*) at such a rate that all but the coarsest material is lifted; the coarsest drops and passes out at (*f*). The rising current is given a definite swirling motion by the rotating vanes (*g*) and, due to centrifugal action, the coarser solids are thrown against the walls (*h*) and pass out through (*f*); the finer rise up into the main fan zone where a stronger whirl throws most of the solid against the walls (*i*), down which it settles and passes out

through (j). The usual speed of the spindle carrying (b), (d) and (g) is 160 r.p.m. Sizes run from 30-in. outside diameter (laboratory machine) to 14-ft. Power requirement (maker's rating) is ¼+ hp. for the 30-in. size; 6 hp. for the 8-ft., and 12 hp. for the 14-ft.

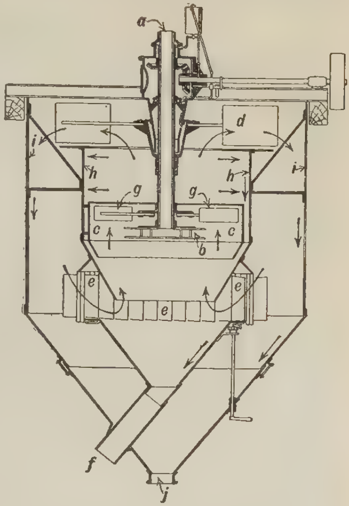


Fig. 7.—Gayco separator.

At Eastern Magnesia Talc Co. the capacity of a 14-ft. machine sizing tripoli is 1600 to 2000 lb. of feed per hr. when the fine product is 99.5 per cent. −325-mesh; 2000 to 2400 lb. for 97 per cent. −325-mesh; 2400 to 3000 for 90 to 92 per cent. −325-mesh; 4000 to 5200 for 99.7 per cent. −140-mesh and 80 to 85 per cent. −325-mesh. This machine had to be speeded up above the recommended speed in order to get the desired grade and capacity; the grade drops off with decreased speed and the product is non-uniform with light feeding. At W. H. Loomis Talc Corp., Gouverneur, N. Y., a 14-ft. machine produces about one ton per hr. of 99 per cent. −325-mesh short-fiber talc; feed runs upward of 50 per cent. of −325-mesh material; the performance is affected to some extent by the weather. The manufacturer's rating for limestone, cement, etc., separating at 85 per cent. −200-mesh is 15 to 20 tons per hr. on the 14-ft. machine; the 10-ft. machine has about half of the capacity of the 14-ft., and the 8-ft. about one-third.

Air-sand process for coal separation (U. S. pat. 1,534,846; *PP 1561-F A*) is a sink-and-float method of separation in which the separating medium is fine sand, supported on a porous medium and maintained in semi-fluid state by means of air forced through the bed. Using −20-mesh silicious river sand and sufficient air to make the mass fluid without "boiling," the density of the sand-mass is about 1.45, as tested by a spindle hydrometer. The air pressure required for this density is 1½ to 3 in. of mercury, according to the porosity of the permeable diaphragm and the thickness of the sand layer. A testing apparatus used by the Bureau of Mines is shown in Fig. 8. The small pneumatic tube (Sec. 22, Fig. 21) may be used for smaller tests. Fig. 9 shows diagrammatically a layout suggested for continuous operation. Table 4 shows results of a test on −3 + 2-in. Illinois bituminous

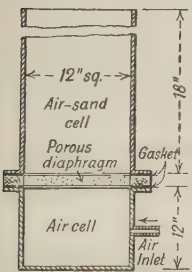


Fig. 8. — Laboratory apparatus for air-sand concentration (*after Fraser and Yancey*).

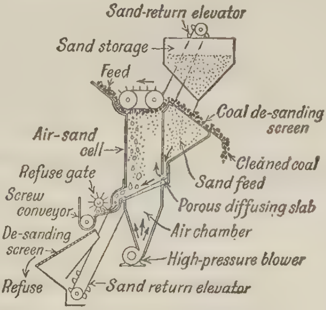


Fig. 9.—Proposed lay-out for continuous air-sand separation (*Fraser and Yancey*).

Table 4. Results of air-sand separation. (*After Fraser and Yancey*)

Product	Weight, pounds	Yield, per cent.	Ash, per cent.	Sulphur, per cent.
Raw coal.......	62.3	100.0	14.3	6.09
Cleaned coal....	54.4	87.3	9.9	4.57
Refuse.........	7.9	12.7	44.9	16.56

coal. A 2-compartment jig treating the same coal made a yield of 82.6 per cent. washed coal assaying 10.3 per cent. ash. The sink-and-float yield on the feed was 90.4 per cent. assaying 9.9 per cent. ash.

3. Greased-surface concentrators

Greased-surface concentrators apply the principle of selective wetting of metalliferous-mineral and gangue particles at greased surfaces, usually in the presence of water. The principle is similar to that of precious-metal amalgamation. (Sec. 15, Art. 6.) The important elements of such apparatus are the mechanical means of presentation of the ore pulp to and removal from the surface, and the character of the grease.

Elmore (703,905/1902) described flowing a thin pulp against an oiled-canvas belt to which oil globules and oiled particles were expected to adhere. The tailing launder was lined with blanket or matting and blanket or matting were floated on the pulp in the launder, the idea being that these would become oiled in time and thereafter catch oiled mineral particles that came in contact with them. Tar and heavy residuum oils and the like and acids were recommended. Foote (744,322/1903) described coating a fixed inclined surface with an oily soap or grease made by mixing petroleum and slaked lime or lime water, and then flowing pulp thereover. Schwarz (807,505/1905) suggested coating an inclined surface with a grease made by mixing a hydrocarbon and sulphur. The hydrocarbons named were paraffin, ozocerite, resin, pitch, asphaltum or any animal, vegetable or mineral oil. A specific mixture cited was paraffin, resin and sulphur. Alkaline, acid or neutral pulp was to be flowed over this surface. Wolf (899,149/1908; 899,478/1908) described coating an endless belt with a viscous mixture made by treating an oil or grease with chloride of sulphur. The same patentee (1,310,492/1919) described a special collecting belt of "woven-hard" duck, soaked with one oil, *e.g.*, a mineral oil, and lightly sprayed with another oil, *e.g.*, a vegetable drying oil, or *vice versa*. Luckenback (1,370,601/1921, 1,448,927; 1,448,928/1923) describes passing pulp over a belt and through a tangled mass of filamentous material coated with a mixture of oil or grease and a resin such as caoutchouc, mastic, shellac, congo, rosin, asphaltum, pitch, etc. Petroleum grease or liquid rubber or a mixture of the two is recommended as a covering compound. The same patentee (1,478,237/1923) recommends an oil or grease and an alkali for surface coating, *e.g.*, petroleum grease, an animal fat or oil and sodium silicate or a mixture of sodium silicate and sodium hydroxide. Addition of liquid rubber makes the coating more adhesive. Dolbear, (1,458,467/1923) suggests coating the mineral by rinsing with water and oil and then flowing the pulp over a metallic surface to collect the mineral. Tin and zinc are given as examples and 10 lb. per ton of pine-tar or coal-tar oil as suitable oils and quantities. Revolving drums are recommended as separating apparatus. Alston (1,534,481/1925), describes a revolving drum with inner surface coated with a fatty, oily or greasy collecting substance.

The only instance of successful commercial application of greased-surface concentrators is in diamond milling (see Sec. 2, Art. 15).

4. Granulation

This term describes a phenomenon that may be observed when certain finely-crushed minerals, *e.g.*, sulphides or coal, are stirred together with a relatively large amount of oil in the presence of water. Under such circumstances the minerals form with the oil a coherent mass or masses whose consistency depends upon the kind of oil, the relative quantities of oil and mineral, and the character of the stirring. The ordinary rock-forming minerals do not similarly granulate, hence if these are present, separation may be effected after granulation by screening or by ordinary gravity-concentration methods.

Tunbridge (228,004/1880) was the first to mention this property of metallic substances. He applied it to the separation of metals such as gold, silver and alloys in suspension in water, by means of soaps which were caused to coagulate and collect the metals, upon agitation and the addition of a salt such as sulphate of lime, or an acid. The coagulum was thereafter separated by filtration through wood shavings, sawdust, straw or the like. Everson, (348,157/1886) described dry mixing of sulphide (and oxide) ores with any one of a large variety of oils or compounds of oils or fats to form a mass of doughy consistency, and thereafter kneading or stirring this mass in acidulated water, as a result of which operation the

rocky minerals were expelled from the oily mass and a coherent mixture of oil and mineral remained. If the kneading has been slow, the oil-mineral substance is in relatively large masses readily separable from the dispersed rocky particles by screening or settling in water. If vigorous stirring has been used, the oil-mineral matter will have considerable air in it, and will either float or, on a Wilfley table, come off below the sands. CATTERMOLE (763,259; 763,260; 777,273; 777,274/1904), came nearer to success than either of his predecessors. His process consists essentially in violent agitation of a freely-flowing pulp of finely-pulverized sulphide ore to which has been added an amount of oil usually between 20 and 100 lb. per ton of solid. Acid in an amount equivalent to 0.2 to 1 per cent. on the water and heat both aid the operation. An ore containing upwards of 10 per cent. mineral is necessary for the best performance. Granulation ordinarily takes place in from 3 to 10 minutes and gentle rolling of the pulp thereafter causes increase in size and firmness of the granules. Under the best conditions these granules are 1/16- to 1/4-in. diameter and sufficiently firm and dry to be readily handled. Separation of granules from the gangue may be effected on a screen or shaking table or in a hydraulic classifier. The Cattermole process was tried for the treatment of zinciferous dump tailing at BROKEN HILL, Australia, but was discarded for the reason that in the attempt to cut down on oil consumption a practical method of operating the agitation-froth process was discovered.

The physical phenomena underlying granulation are the same as those acting in agitation-froth flotation, *viz.*: preferential coating of the mineral particles with oil followed by gas precipitation on the oiled particles. Subsequently excess oil meets and spreads on the mineral-coated bubbles, and when the oil coating becomes sufficiently thick, most of it, together with the mineral, which has been displaced to the oil-water interface, slips off the bubble like a sheath. This sheath is relatively viscous by reason of the solid coating and rolls up, under the swirling action of the pulp, into the characteristic granule. The loaded air bubbles and viscous sheaths may be observed readily by taking test-tube samples during the course of a Cattermole test and examining before a binocular microscope.

DARLING (763,859/1904) pointed out that carbonaceous materials mix with oil to "form a buttery and homogeneous mass, with which the earthy or other materials occurring in the mixture do not associate" and that separation may be made by mixing the oiled mass with water and screening. He described application of his process to a graphitic ore, using crude petroleum, and to finely-powdered coal, coke or charcoal. SCHWARZ (807,501 to 807,503; 807,505; 807,506/1905) described granulation of sulphides with normally-solid hydrocarbons, the mixing being effected with the hydrocarbon in melted condition and hardening brought about by cooling.

Trent (1,420,164/1922) describes the granulation process applied to coal. He recommends the use of liquid hydrocarbons, specifically fuel oil, crude oil (petroleum), gasoline and benzol in amounts up to 50 per cent. of the carbonaceous content of the solid feed. The feed should generally be ground to pass 100- to 200-mesh. This process differs from that of Darling in that mixing of the oil and carbonaceous material is done in the presence of water while Darling mixed dry.

A small amount of oleic acid greatly improves the operation of the Trent process.

Performance. Table 5 shows the results of an exhaustive series of tests made by the Bur. of Mines (*20 CA 172*).

Feed was crushed to 65-mesh, then ground in a 50-per cent. pulp in a laboratory ball mill for 6 hours and then agitated with the oil in a 25-per cent. pulp. The oil used was a Navy fuel oil (petroleum residuum), 0.875 sp. gr. (30° Bé.), running 125 sec. at 25° C. in a Saybolt viscosimeter. The table shows excellent recovery of combustible, and in most cases, a cleaned coal low in ash. In general the sulphur reduction is negligible. For the 17 tests it averages an actual reduction of 0.17 per cent. and a percentage reduction of only 8.8 per cent., although in three cases the percentage reduction lies between 30 and 48 per cent. Supplementary tests on a high-sulphur coal (*j*) to study the behavior of the sulphur showed that of the total sulphur in the coal, amounting to 4.75 per cent., 3.01 per cent. (on the coal) was pyritic, 0.36 per cent. sulphate and 1.37 per cent. organic. The cleaned coal contained

Table 5. Performance of Trent process. (*After Perrott and Kinney*)

Kind of coal	Oil used, pounds per ton of raw coal	Time for initial granulation, min.	Per cent. of moisture in raw coal	Total time for granulation and rolling, min.	Raw coal, ash			Cleaned coal, per cent.			Refuse				Efficiency, per cent.		
					Per cent.	Corrected, per cent. (*a*)	S, per cent.	Weight	Ash	S	Weight, per cent	Ash, Per cent.	Ash, Corrected, per cent.	S, per cent.	Ash reduction	S reduction	Recovery of combustible
b	65	2	2.15	30	27.7	30.4	1.00	74.0	7.0	0.70	26.0	87.0	95.0	1.99	74.7	30	97.8
c	65	....	....	30	31.4	34.8	1.63	69.0	6.5	0.85	31.0	87.0	95.6	3.05	79.2	48	98.0
d	75	....	....	120	21.7	23.8	0.85	82.0	6.7	0.83	18.0	90.7	98.3	0.95	69.2	......	99.5
e	80	½	1.53	60	12.5	14.2	1.27	92.0	6.0	1.34	8.0	88.0	95.2	0.40	52.0	......	99.5
f	80	2	1.50	120	9.3	11.2	2.28	96.5	6.7	2.34	3.5	87.6	94.8	0.60	28.0	......	99.7
g	80	....	....	60	21.7	23.9	0.93	88.0	12.5	0.80	12.0	88.7	96.9	2.08	42.3	14	99.4
h	80	30	10.93	180	16.6	20.7	5.33	85.0	7.4	5.28	15.0	69.7	76.6	2.25	55.4	1	89.8
i	80	1	1.75	30	9.9	13.0	4.38	96.4	6.3	4.27	3.6	86.2	93.5	0.80	36.4	3	99.8
j	80	5	5.10	120	19.5	23.6	4.74	69.0	5.7	3.08	31.0	50.5	59.0	8.50	70.8	35	83.5
k	80	....	....	30	22.6	24.7	0.49	87.5	13.6	0.50	12.5	85.0	92.1	0.50	39.8	......	98.7
l	60	....	....	120	54.7	59.3	0.55	45.0	22.9	0.86	55.0	80.6	87.3	0.29	58.1	......	82.8
m	50	....	....	120	63.5	69.4	1.64	31.0	20.6	1.48	69.0	82.7	90.2	1.65	67.7	10	77.8
n	80	....	....	60	23.5	26.2	1.60	80.5	6.6	1.76	19.5	92.8	100.7	0.90	72.0	......	100
o	80	....	....	180	19.3	21.1	0.48	87.0	10.0	0.50	13.0	80.0	86.7	0.45	48.4	......	97.8
p	80	....	....	120	35.1	39.3	1.77	81.5	25.7	1.56	18.5	75.9	83.2	2.30	26.8	12	95.0
q	80	....	....	120	33.5	36.9	1.41	79.7	18.1	1.42	20.3	91.2	102.4	1.25	46.0	1	100
r	60	....	....	240	35.6	39.7	2.47	66.0	9.4	2.32	34.0	86.0	94.4	2.71	73.6	6	97.0

a Corrected per cent. ash $= 1.08A - \frac{21}{40}S$ where A and S are percentages of ash and sulphur respectively, as determined by proximate analysis. Parr (*5 IEC 523*) suggests:

$$\text{Corrected per cent. ash} = A_w - 3C + \frac{5S}{8} + 0.08\left[A_w - \left(\frac{34C}{3} + \frac{10S}{8}\right)\right],$$

where A_w = percentage of ash as weighed after ignition at 750° C. with addition of H_2SO_4; C = percentage of carbon appearing as carbonate in the unburned coal, and S = percentage of sulphur in unburned coal. *b* Anthracite culm. *c* Anthracite culm from Hudson Coal Co. *d* Anthracite, Rhode Island. *e* Pulverized bituminous coal. *f* Run-of-mine bituminous coal. Allegheny Steel Co., Avenue mine, Pa. *g* Bone-coal refuse from same washery. *h* Illinois bituminous, run-of-mine, Superior Coal Co. *i* Indiana bituminous, run-of-mine, Vandalia Coal Co. *j* Oklahoma bituminous, run-of-mine, Folsom Morris Coal Mining Co. *k* Washington bituminous, run-of-mine, Wilkeson Coal and Coke Co. *l* Bituminous-washery refuse, Phelps-Dodge Co., New Mex. *m* Bituminous-washery refuse, Durham Coal and Iron Co., Tenn. *n* Bituminous-washery refuse, Black Creek Coal Co., Ala. *o* Washington sub-bituminous, Pacific Coast Coal Co. *p* California lignite, McKissick Cattle Co. carbonized at 500° C. *q* Texas lignite, Bertetti Coal Co. carbonized at 500° C. *r* Brazil bituminous.

2.10 per cent. pyritic sulphur, representing 47 per cent. of that in the feed. Sulphate sulphur in the cleaned coal was 0.15 per cent. while organic sulphur increased to 1.50 per cent. Similar tests on coal (*c*) showed only 5.5 per cent. of the original pyritic sulphur in the cleaned coal, but the sulphate sulphur as well as the organic sulphur in this product increased. The normal tendency is, of course, for pyritic sulphur to go with the oil. If it does not do so, it is probably because it is too coarse or because it is tarnished. Sulphate sulphur should ordinarily decrease and organic sulphur, being, probably, molecularly mingled with the coal structure, naturally goes with the coal. Some tests on artificial mixtures of coal and pyrite indicate maximum sulphur reduction at 200-mesh maximum size with smaller reduction on − 100-mesh and on − 600-mesh feeds.

Oils. A series of tests with carbon disulphide, benzine, carbon tetrachloride, gas oil, fuel oil, and two cylinder oils indicated that the results with the first four oils, whose viscosity was 40 sec. Saybolt and less, were cleaner coal and better ash reduction than with the oils of higher viscosity, but more combustible is lost with the former oils, and considerably more oil is necessary. Details of the results of tests on several coals with residuum and gasoline are given in Table 6 (*22 CA 913*). Table 7 shows comparative results with several oils on the same coal.

Table 6. Comparative results of Trent process with viscous and mobile oils. (*f*)
(*After Ralston*)

Coal sample	Residuum oil, per cent.		Gasoline, per cent.	
	Granules, ash	Coal recovery	Granules, ash	Coal recovery
a	9.0	98.9	8.1	97.8
b	10.6	98.5	8.0	89.6
c	12.2	97.4	8.0	91.8
d	15.8	98.4	12.8	98.7
e	7.5	99.4	6.1	99.1

a, b, c, d, e See the same letters, Sec. 12, Table 45. ***f*** Feed ground to 300-mesh.

Table 7. Performance of Trent process with several oils. (*After Ralston*)

Oil	Relative viscosity	Ash, per cent.		Coal recovery, per cent.
		Feed	Granules	
Gasoline	0.95	25.9	8.0	91.8
Crude benzol	0.97	25.9	9.4	90.9
Kerosene	1.0	25.9	9.5	93.1
Stove oil (30°)	1.3	25.9	9.6	95.9
Crude oil (25°)	5.0	25.9	11.8	96.7
Residuum (18°)	50.0	25.9	12.2	97.4
Asphaltum (10°)	Very high	25.9	13.5	98.2

The best amount of oil to be used was found by Perrott and Kinney to be from 0.3 to 0.4 lb. per pound of clean dry coal, the higher figure corresponding to the finest feeds. The amount necessary to effect good granulation can be reduced to between 3 and 5 per cent. on the feed by adding a small amount of oleic acid or pine oil to the petroleum.

Moisture in cleaned coal. According to Perrott and Kinney a cleaned coal consisting of ⅛-in. granules will retain 10 to 15 per cent. of moisture after draining; if the granules are very fine, the moisture content will run up to 30 or 40 per cent.

Size of granules depends upon the amount and kind of oil, the size of feed, the intensity and duration of agitation and the kind of coal. It can also be controlled to some extent by varying the acidity of the pulp water and by rolling or slow stirring after the granules have first formed.

Time of agitation was found to be proportional to moisture content. The time necessary for initial segregation and granulation of the coal varied from 0.5 to 30 min. with variations in moisture content of raw coal from 1.50 to 10.93 (see Table 5) while the lignites with

15 and 27 per cent. moisture could not be granulated until after carbonization; drying at 110° C. was ineffective. In general, coals containing more than 3 or 4 per cent. of hygroscopic moisture were difficult to treat.

Size of feed. An exhaustive series of tests with feeds ground to 65-, 200- and 600-mesh indicated that the percentage of ash reduction was, in general, greatest with the finest grinding and that with low-ash bituminous feeds it was necessary to grind to 600-mesh to affect any useful ash reduction. With high-ash bituminous feeds 65-mesh was fine enough in some cases, while with others and with the anthracites it was necessary to grind to 200-mesh but not to 600-mesh. Table 8 shows incidentally the effect of difference in feed size on ash content of granules.

Table 8. Comparison of performances of Trent process and agitation-froth flotation on Pacific Northwest coals.(*f*) (*After Ralston*)

Coal	Trent process, per cent.				Flotation, per cent.	
	−300-mesh feed(*f*)		−65-mesh feed(*f*)		−65-mesh feed(*f*)	
	Granules, ash	Coal recovery	Granules, ash	Coal recovery	Froth, ash	Coal recovery
a	9.0	99.5	13.5	99.0	13.8	90.4
b	10.6	98.0	13.5	98.0	15.5	80.0
c	12.2	98.0	16.3	96.0	18.4	90.0
d	15.8	99.5	18.0	98.5	18.8	96.0
e	7.7	99.5	10.8	99.0	10.7	97.0

a, b, c, d, e See same letters, Sec. 12, Table 45. *f* For analyses of feed, see Sec. 12, Table 45.

Trent process *vs.* agitation-froth flotation. Table 8 shows comparative results of a series of tests by different investigators by the two processes on the same coals. At 65-mesh, which was as fine feed as the agitation-froth process could successfully treat, the ash contents of the concentrates from the two processes were not greatly different, but when the feed was ground down to 300-mesh and more ash was freed, the Trent process rejected it and obtained better concentrate. Neither process is, as yet, worked out generally with respect to coal washing. The fact that Trent granules may be substantially dewatered by simple screening and draining while flotation concentrate requires thickening, filtering and partial drying or else granulation similar to the Trent treatment is, of course, in favor of the former. On the other hand, the Trent product is not easy to store, ship or handle under boilers when cheap oil is used and left with the coal and, if expensive oil is used, it must be removed by low-temperature distillation, which is, of course, a charge against the process. Direct comparisons are not available.

Trent (1,421,862/1922) suggests using pulverized coal and hydrocarbon liquid together in treating ores by granulation, claiming thereby to get cleaner concentrate than is possible by froth flotation and further that the fuel for smelting will be incorporated in the concentration process. He suggests, also, using the process to clean flotation concentrate and for the separation of the metallic content of iron- and steel-mill flue dusts.

5. Electrostatic concentration

Electrostatic concentration employs the difference in electrical conductivity of minerals to effect separation. Fig. 10 illustrates two methods of application; in (*a*) the good conductors immediately upon making contact with the roll take on like charges and are repelled, the poor conductors require a longer time to become charged and therefore, in the short time that they are

in the electrical field, are substantially unaffected and fall directly downward. In (*b*) a stream of charged air particles goes from the charged wire toward the grounded roll and charges particles on the face of the roll. In the case of conducting particles the charge is transmitted immediately to the roll and ground; they are therefore unaffected and fall normally; poor conductors become charged by the stream on the face toward the wire, this charge induces a charge of unlike sign on the adjacent face of the roll, the particle is attracted and held until the charges dissipate by conduction or until the clinging particles are mechanically swept away. Fig. 10 (*a*) shows the principle of the early Blake-Morscher machines which were impractical and unreliable; the Huff machines utilize the principle shown in (*b*). All sulphides except pure blende, all native metals and most metallic oxides class as good conductors; all other common minerals as poor conductors. The forces acting are those between electrical charges, and their magnitude is a function of the surface expanse of the individual particles, hence surface must be large in proportion to weight, *i.e.*, the particles must be small, say, −10-mesh; close sizing is necessary for clean separation.

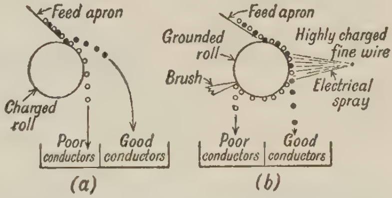


Fig. 10.—Principles of electrostatic separation.

Application. The process has found but little use in the mills. The best known installation was that at the Midvale plant of U. S. S. R. & M. Co. (see p. 177) for separating pyrite from blende; it has now been replaced by differential flotation.

Dielectric separation is proposed by Hatfield. The underlying principle is that particles whose dilectric constant (specific inductive capacity) is greater than that of the medium by which they are surrounded are attracted toward electrically charged points in the medium and *vice versa*, in a way entirely analogous to the action of paramagnetic and diamagnetic particles respectively in a magnetic field (Sec. 13, Art. 1). The dielectric constant of good conductors *e.g.*, the ordinary metallic minerals, is high and that of poor conductors, such as the ordinary gangues, is generally less than 10; by using a medium such as a mixture of nitro-benzene (dielectric constant, 36.4) and kerosene (2.4) any desired dielectric constant between the two can be obtained (in proportion to the respective parts of each in the mixture) so that with a given mixture of minerals such a medium can usually be made with an intermediate dielectric constant in which finely pulverized particles can be separated, the particles with high constant collecting on charged points. Single-phase alternating current of about 200 volts and 100 frequency is suitable for charging the collecting plates.

The process is at present nothing more than a laboratory expedient; it gives little practical promise on account of the high expense of the medium. For details of laboratory procedure see *33 IMM 335 and 343.*

SECTION 15

HYDROMETALLURGY

BY

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1. Introduction

Hydrometallurgy is the art of recovering metals from ores by first effecting solution of the metal or metals in the form of a salt, separating the solution from the impoverished solid, then decomposing the metallic salt in such a way as to cause precipitation of the metal in a state of comparative purity. The solvent is usually regenerated in the precipitating reaction. The LIXIVIANT (leaching solution) is invariably an aqueous solution.

Hydrometallurgy is substantially never an economic alternative to either concentration or smelting; rather it is applicable to ores of too low grade to be smelted economically and of too small metal content or with the metal in such mineralogical combination as to render it not amenable to concentration.

Gold, silver and copper are the metals whose ores are most commonly treated; zinc concentrate is treated in fairly large amounts at two plants; lead has been the subject of much experimentation but of little or no economically successful operation. This is due, not to lack of amenability of lead ores but rather to the fact that lead ore has peculiar value in smelting other ores, and hence gets favorable rates from the smelters, while gold or copper ores of the types suitable for hydrometallurgical treatment are costly to smelt.

In substantially all cases it is necessary that the metal before dissolving be in an oxidized or metallic state, either naturally, or so rendered by roasting or the use of oxidizing solutions, the possible exceptions being precious-metal tellurides and selenides and silver sulphides. The ores usually treated are those containing metallic gold and silver; carbonate, oxide or metallic copper and, less frequently, copper sulphide or copper silicate; zinc sulphide (this must first be roasted); lead carbonate, chloride and oxide.

The fundamental operations are the same in all hydrometallurgical processes, irrespective of the metal or solvent used, and may be classified as (1) PREPARATION for solution, including crushing and, if necessary, oxidation or reduction to a soluble state; (2) solution in a suitable, generally dilute, lixiviant (LIXIVIATION); (3) separation of pregnant solution from leached solid (WASHING); (4) PRECIPITATION of the metal; this may require preliminary PURIFICATION of the solution.

2. Preparation

Crushing is necessary in order to permit access of leaching solution to the metallic-mineral particles, and, in the case of ores requiring roasting, to facilitate this operation. The SIZE necessary for successful leaching varies with different ores; caved copper ore in the mine is leached at OHIO COPPER Co. and few copper ores are crushed smaller than ⅛-in.; gold and silver ores must, on the other hand, substantially always be crushed to 40- to 60-mesh and in many cases to 200-mesh. In most cases recovery increases with increased fineness of grinding and the rate of solution likewise increases. Decision as to proper size is, therefore, a balance between increased crushing cost on the one hand and increased recovery and saving in leaching-plant equipment and operation on the other.

Crushing and grinding machinery is described in Secs. 3 and 4.

Cost of crushing to ¼-in. should not exceed \$0.06 to \$0.10 per ton in large plants nor double these amounts in smaller plants. Crushing and grinding to, say, 48-mesh costs from \$0.30 to \$0.50 per ton in large plants and \$0.40 to \$0.75 in small; to 200-mesh the costs will exceed these by 50 to 100 per cent.

Roasting is rarely practiced except on zinc ores, in which case a high-temperature oxidizing roast is employed. Certain high-grade sulphide and sulpho-telluride gold ores (CRIPPLE CREEK) are roasted but the same extraction would be possible with finer grinding *(14 CME 205)*. An oxidizing roast for copper sulphides and a reducing roast for silicates is promising theoretically but, in general, is not commercially practicable on account of the large bulk of barren rock that must be heated. Chloriziding roasting has a limited application to lead-silver ores prior to brine leaching.

Oxidizing roast. The reaction starts at about 350° C. with pyrite according to the following simplified equation: $2FeS_2 + 11\ O = Fe_2O_3 + 4SO_2$. Lead, copper and zinc oxidize, with increasing temperatures, in the order named, thus: $PbS + 4\ O = PbSO_4$ or $6\ PbS + 23\ O = 6PbO + 5\ SO_3 + SO_2$; $CuS_2 + 2\ O = CuS + SO_2$; $CuS + 3\ O = CuO + SO_2$; $ZnS + 3\ O = ZnO + SO_2$. The final temperature for a zinc roast should run up to from 700 to 780° C.; for copper about 590° C. should be the maximum.

Whether the end product is a metallic oxide or sulphate depends to some extent on the concentration of SO_2 in the interstices of the charge; high concentration with gentle rabbling produces more sulphate, vigorous rabbling and strong draft, more oxide.

Chloridizing roast is carried on at moderate temperature in a neutral atmosphere in the presence of a chloride, usually of an alkali or an alkaline earth. The end sought is conversion of sulphides of copper, lead and silver into chlorides; at the same time the metal of the original chloride is changed into sulphate. Loss of lead and precious metals by volatilization is serious unless the temperature is kept low. Experiments are at present under way on low-grade oxidized lead ore at CERRO DE PASCO involving low-temperature batch roasting in Holt-Dern furnaces with a chloride and recovery of the lead by electrolytic precipitation. This process aims to eliminate volatilization losses by low temperature in the initial roast and elimination of smelting.

Reducing roast is one performed at low to moderate temperatures in an atmosphere of carbon monoxide. The object is to reduce the desired metal (copper, in so far as present hydrometallurgical processes go) to the metallic state. The reducing roast has a most important application at present in the preparation of oxidized and copper-silicate ores for ammonium carbonate leaching. At the BWANA M'KUBWA pre-heated ore is treated in a gas-locked cylindrical reducing furnace, fitted with an internal spiral to regulate time of contact *(119 J 838)*. Producer gas flows counter-current to the ore and is not ignited. The ore enters the furnace at a temperature of 540° C. *(120 J 852)*.

Roasting furnaces. For descriptions of various furnaces see: *The metallurgy of non-ferrous metals*, Wm. Gowland, Griffin, Lond., 1921. *General metallurgy*, H. O. Hofman, McGraw-Hill, 1921. Multiple-hearth furnaces of the MacDougall type, *e.g.*, the Herreshoff, Wedge and Skinner, are the principal ones used for hydrometallurgical work in North America. CAPACITY of roasting furnaces in hydrometallurgical practice is not yet generally established but known capacities correspond sufficiently with those in roasting for smelting, to justify use of the latter standards, which follow *(92 J 1271)*:

Method of roasting	Lb. per sq. ft. of hearth area per 24 hr.
Roast heaps and stalls	5 to 20
Reverberatory roasters:	
(1) Hand rabbled	24 to 35
(2) Mechanical, average	33 to 75
(3) Mechanical, special conditions	150
(4) Revolving cylinders	128
Blast-roasting pots	500 to 900
Average	600
Blast roast in layers:	
(1) Intermittent down draft	1000 to 2000
(2) Continuous	2200 to 3000
McDougall (7-hearth, 20-ft.) average	60
At Great Falls	37

Removal of impurities prior to leaching is rarely practiced; such practice as exists has been in cyanidation. Substances which consume cyanide (CYANICIDES), particularly oxidized copper minerals; partly oxidized sulphides of zinc, iron and copper; tellurides, arsenides and antimonides; mineral acids and their basic salts and organic material will, if present in any considerable quantity, render an ore unamenable economically to cyanidation. Sulphides can be removed by concentration; small quantities of free or latent acid can be neutralized by lime; larger amounts of acid may require a preliminary wash with water or alkaline solution; oxidized copper minerals require an acid wash. Graphite causes re-precipitation of gold; it has been successfully removed by flotation.

3. Lixiviation

Lixiviation is the operation of effecting contact between the ore to be leached and the leaching solution. Two general methods are used, *viz.*: (1) circulation of solution through a stationary ore mass (PERCOLATION); (2) stirring of a mixture of ore and solution (AGITATION). The second method is clearly applicable only to ores in finely-pulverized state; the first may be practiced on either coarse or fine ore, according to the method employed.

Percolation is applied to caved ore standing in the mine (LEACHING IN PLACE; OHIO COPPER CO.), or to run-of-mine ore piled in heaps at the surface (HEAP LEACHING; RIO TINTO; ARIZONA COPPER CO.) or to crushed granular ore ranging in maximum size from 3/8-in. (NEW CORNELIA) to 80-mesh for oxidized ore and 100-mesh for unoxidized ore at HOMESTAKE (TANK OR SAND LEACHING). When the nature of the ore is such that percolation is applicable, *i.e.*, when an economic amount of valuable mineral is exposed and soluble at sizes permitting relatively free circulation of leach liquor and air through the standing mass, this is the cheapest method of operation, since it eliminates the cost of fine grinding and the power required for agitation.

Rate of percolation. Two and one-half to 3 in. per hour is good; 1½ to 2 in. is fair; ¾ to ½ in. indicates a condition unsuitable for percolation. Vacuum may be used to hasten the rate.

Filter leaching is a form of percolation applicable either when the ore is very readily leachable or when pressure in excess of atmospheric is desirable for solution. A cake of finely pulverized ore is formed on a filter leaf (see Sec. 17) and leach solution is passed through (usually alternating with air or steam) until the desired solution is effected.

Agitation is the usual method of leaching finely-pulverized ore. The purposes of pulp movement are: (1) to effect continuously changing contact of mineral and solution and (2) to entrain oxygen. Agitation in crushing and in the flow of pulp through launders, pumps, elevators and the like is highly

effective, hence leach solution is frequently added to the grinding mills. Special agitating devices include mechanical and pneumatic means.

Mechanical agitators include tanks with reciprocating or rotary stirring mechanisms, revolving drums containing a multitude of small tumbling bodies, centrifugal pumps and mechanical classifiers. The important feature of such devices from the solution standpoint is that they cause thorough and continuous splashing of the pulp, which effects aeration. PNEUMATIC AGITATORS have been most widely used. The PACHUCA TANK is typical. It consists essentially of a vertical cylindrical tank with cone bottom, 35 to 55 ft. high, 7½ to 13 ft. diameter, with an air-lift (see Sec. 20, Art. 15) about one-tenth the diameter of the tank, extending from about 18 in. above the bottom of the cone to within 18 in. of the top of the tank. The lift itself aerates the pulp and the lifted pulp splashes against an umbrella above the lift pipe and falls back onto the pulp surface outside, thus entraining more air. Common operating limits are: 4 to 88 cu. ft. of free air per min., 22 to 33 lb. pressure, ⅛ to 1¾ hp. and 15 to 110 tons daily capacity. (*97 P 424*.) The PARRAL TANK is a flat-bottomed tank with a plurality of air-lift columns which may be readily installed in any flat-bottom tank (*42 A 819*). Combined mechanical and pneumatic agitators are typified by the DORR. Diagonally-placed slow-moving ploughs on horizontal rotating arms move settled slime to an axial conduit which acts as an air-lift column, and delivers the pulp onto rotating distributing launders. In the HENDRIX agitator the pulp is raised through a central column by propeller blades, assisted by an air-lift.

Leaching solution depends, of course, on the metal (or mineral) to be dissolved. CYANIDE of potassium or sodium is used for gold and silver (other solvents such as CHLORINE for gold and SODIUM HYPOCHLORITE for silver have had some little vogue but are not used at present); SULPHURIC ACID is used for copper carbonates, sulphates, oxides and the like and for roasted zinc concentrate; FERRIC SULPHATE, for copper sulphides and oxides; AMMONIA (actually ammoniacal cupric carbonate) for native copper; and brine (usually acidified) for lead and silver.

Strength of solution is dependent upon the ore and solvent chemical and is also related to the time available, fineness of grinding, temperature, and a number of other minor elements of plant operation. It was early thought that very weak CYANIDE solutions dissolved gold and silver selectively. While this is not actually so, there is lessened efficiency in solutions exceeding 0.25 per cent. KCN and marked increase in cyanide consumption. MacFarren (*Practical stamp milling and amalgamation*, Min. and Sci. Pr., San Francisco, 1914) points out that the normal oxygen content of cyanide solution decreases as the strength increases, but that if oxygen is supplied, strong solutions dissolve faster than weak. At HOMESTAKE the essential factor seems to be the passage of a definite volume of solution through the slime cake, rather than treatment with a solution of definite strength or for definite time (*52 A 15*). With SULPHURIC ACID increase in solution strength causes increase in rate of solution but the gain may be more than offset by increased solution of alumina, iron, etc., which do not regenerate in precipitation and therefore waste acid. These dissolved metals may also affect electrodeposition unfavorably, as in the case of copper. In the high-acid process for roasted ZINC ores, the increased solution of silica and iron facilitates filtration and keeps the anode free from incrustation during electrodeposition. With acid ferric sulphate much excess of ferric iron over 1 per cent. decreases current efficiency in electrodeposition.

Strength is expressed in per cent. or in pounds per ton of lixiviant. Usual strengths of various lixiviants are as follows: CYANIDE (generally expressed in terms of KCN): for gold, sand leach, 0.1 per cent.; agitation leach, 0.05 per cent.; for silver, sand leach, 0.1 to 0.3 per cent.; agitation leach, 0.05 to 0.15 per cent. In certain cases much greater strength is used, *e.g.*, REAL DEL MONTE (0.45 per cent.) and in ONTARIO. Leaching concentrate requires solutions up to 0.75 per cent.

SULPHURIC ACID for copper ores, 2 to 10 per cent.; for zinc, 5 to 15 per cent. Tainton (*U. S. Pat.* 1,891,498) proposes 28 to 30 per cent. for zinc. ACID FERRIC SULPHATE: On

INSPIRATION ORES; copper sulphate, 2.5 to 3.5 per cent.; total iron, 1.5 to 2.5 per cent. (ferric up to and somewhat higher than 1.0 per cent.); free (sulphuric) acid, 3.5 to 7.5 per cent. (*73 A 58*).

AMMONIA (NH_3) for copper, sand leaching, 1.25 to 5.0 per cent. with about the same amount of CO_2 (*70 A 395*; *120 J 849*). In the ammonia process constant strength is maintained by removal of copper (*70 A 602*).

The necessary time of contact with lixiviant is affected by a number of factors; principally the degree of pulverization, strength of lixiviant, degree of solubility of the mineral or the form in which it occurs, and the effectiveness of contact. In any given case it must be determined by experiment.

In CYANIDING from four hours to six days is required; the majority of cases are between 36 and 72 hours. Dilute SULPHURIC ACID with copper ores: CHILE COPPER CO., 24 hours; AJO, 7 days (*40 A 30*); these are both tank leaching. ACID FERRIC SULPHATE, tank leaching, INSPIRATION, probably 9 days maximum. In heap leaching and leaching IN SITU acid ferric sulphate leach extends over many months or even into years. In the AMMONIA process a solution of proper strength becomes practically saturated in one passage through fresh sand (*70 A 595*).

Heating is unusual with sulphuric acid or cyanide solutions. Tainton recommends heating to 60° C. in leaching zinc calcines with strong acid solution. He estimates coal consumption at 1 ton per 20 tons of zinc produced, if the return electrolyte is at 30° C. (*PP 1301-M A*). The rate of solution of metals in cyanide solutions increases with rise of temperature. Heated solutions are used on silver ores at TONOPAH, but in most cases the increased cyanide consumption more than offsets the increased rate of extraction. Ferric sulphate and ferric chloride solutions are usually applied hot but at the INSPIRATION experimental plant commercial results have been obtained at atmospheric temperature. Brine for lead leaching is heated to at least 75° C., when precipitating on dense iron, but 30° C. is sufficient for precipitation on sponge iron (*PP 1281-M A*).

Aeration is an essential element in leaching precious-metal ores with cyanide or copper-sulphide ores with ferric salts. In percolation it is most effectively performed by forcing air through the charge; in agitation it is a concomitant of the splash, as previously stated. In cyanidation, oxygen, although not actually entering into the solution reaction, is essential to its consummation; in copper-sulphide leaching the oxygen is an essential part of the reaction, $Cu_2S + 5\ O + H_2O = CuSO_4 + Cu(OH)_2$, which precedes solution of the copper.

Addition agents have been used principally in cyanidation. LIME is used to "protect" the cyanide by neutralizing free or latent acid, consumption ranges from two to about six pounds per ton; sodium peroxide and bromocyanogen have been used as oxidizers in treating telluride ores and ores containing mispickel; lead acetate or litharge is frequently used with silver ores containing sulphides.

Regeneration of solutions. See Art. 5.

4. Washing

After the economic degree of solution is effected it is necessary to separate the pregnant solution from the leached solid in order that the metal, subsequently precipitated, may be collected in a state of comparative purity. Simple draining off in percolation, or filtering or decantation, following agitation, is insufficient separation on account of the valuable liquid that remains with the solid. This liquid must be displaced and collected or so diluted as to constitute a reasonable loss. The operation is called WASHING. The method of accomplishment differs according to the method of lixiviation.

In percolation a succession of batches of solutions of decreasing metal content and eventually barren water are passed through the batch of leached ore. These washes displace and dilute the interstitial remnant from the preceding liquid so that eventually the interstices are filled with a solution containing little more metal than the barren water. If regular advance of solutions through the charge were effected, then in the most unfavorable case, where diffusion and convection kept pace with the advancing wash, the value of the liquor remaining after washing would be $(ab + cd)/(b + d)$ where a and c are the assays of the remnant liquor after the preceding drain and of the wash liquor respectively and b and d are the respective volumes. (Interstitial volume is readily estimated by determining the dry weight of a given volume of drained material and the mean specific gravity of the solid.) If diffusion and convection do not keep pace with the advancing wash, the assay of the interstitial solution after washing will be less than that given by the equation. Usually the advance of solution is uneven and the wash remnant assays higher than calculation indicates.

The volume of barren-water wash is usually limited to the amount of water lost by leakage and evaporation plus that discharged with tailing; otherwise solution accumulates until the tank capacity of the plant is exceeded. In certain cases, *e.g.*, where acid (Chile Copper Co.) or ferric salts build up excessively, a certain amount of solution is discarded periodically after precipitation, in which case, of course, more wash water can be added.

Decantation is the method of washing usually applied to slimes, following lixiviation by agitation. It consists of a succession of the operations of dilution and thickening of the pulp, each successive dilution with a solution of lower metal content, finishing with barren water. The underlying principle is the same as that in washing following percolation, but thorough mixture of solutions is assured. On the other hand a greater proportion of solution to solid is finally discarded unless the tailing is filtered.

Counter-current decantation is a method of continuous operation of decantation washing. It is performed in a plurality of continuous thickeners so arranged that thickened pulp moves in order from the first to the last, while overflowing solution progresses in the opposite direction. The feed to the first thickener is a pulp of strong, rich solution and finely pulverized ore that has been thoroughly leached in the preceding operations; the overflow is pregnant solution ready for precipitation; the underflow is solid plus rich solution; it goes to the second tank. Here it meets and is mixed with solution that has overflowed from the third tank, and which is much lower in metal content than that contained in the thickened pulp. The overflow of the second tank goes to the first and the underflow to the third. Fresh water is added to the last tank, thus effecting the maximum possible dilution of solution discharged with the tailing and furnishing the progressive washes in the preceding tanks.

5. Precipitation

Metal is recovered from pregnant solutions by precipitation. The principal methods are chemical reaction and electrolysis; decomposition by heat is used in the ammonia process, ammonia and carbon dioxide being driven off in a special still and re-absorbed for further use. With one or two important exceptions, the leaching chemical is regenerated by the precipitating reaction.

Chemical precipitation without regeneration is typified by precipitation of copper from sulphuric-acid solutions by means of metallic iron. The reaction

is: $CuSO_4 + Fe = FeSO_4 + Cu$. Theoretically 0.875 lb. of iron is consumed per pound of copper precipitated; actually the consumption is about pound per pound. Subsequent oxidation of ferrous to ferric sulphate produces a copper solvent but not, of course, the original acid. The earliest practice was to use SCRAP IRON, but the consumption would exceed the supply for large-scale operations. PIG IRON may be used. GRANULATED IRON gives a larger surface but is not completely consumed, which is wasteful and contaminates the product. SPONGE IRON, made by reducing oxidized iron compounds (magnetite, hematite, calcined pyrite) with 15 to 50 per cent. of coal, coke, charcoal or other reducing agent in special reverberatory, muffle or retort furnaces at about 1700° F., has been much experimented with. About 1.5 to 1.75 lb. is consumed per pound of copper.

Precipitation on scrap iron is usually done in launders; tanks are more efficient in terms of pounds precipitated per unit volume of container. Precipitated copper is usually loosened by shaking or washing off the scrap; At AJO precipitation is performed in a long rectangular tank and precipitated copper is loosened by agitation of the liquid in the tank by compressed air.

The important factors in iron precipitation are strength of solution in copper and acid, temperature, contact time and velocity of flow of the solution. RATE OF PRECIPITATION decreases with decrease in copper content but at OHIO COPPER CO. solution containing only 0.204 per cent. copper yields 97 per cent. in 40 min. contact. HEAT and HIGH ACID CONTENT prevent precipitation of insoluble salts on the iron with consequent interference with copper precipitation, but since both are costly (the acid is consumed by the iron) it is cheaper to reject foul solution from time to time. The VELOCITY of flow of the solution should be sufficient to prevent precipitation of foreign material on the surface of the iron.

Cost of precipitation of copper on iron ranges from 2½ to 4½ cents per lb. of copper.

Iron is used to precipitate lead from brine solutions. Sponge iron is the best, since it will precipitate at 20 to 35° C. while with dense iron the solution temperature must be at least 75° C.

Chemical precipitation with regeneration. Precipitation of gold and silver from cyanide solution by means of zinc or aluminum results in regeneration of cyanide, probably not in the form of alkali cyanide as originally added, but in a form in which an equivalent amount of effective cyanide ion is present. The metal is added in the form of shavings of the general structure of excelsior, or as dust; the latter is the modern development. When solutions contain the usual amount of dissolved air (oxygen) it is necessary that the cyanide strength be high in order to prevent formation of "white precipitate" (a mixture principally of oxide and ferrocyanide of zinc) on the zinc, which stops the action. The CROWE PROCESS involves de-aeration of the solution before precipitation; this prevents formation of "white precipitate," makes it unnecessary to add cyanide, and greatly increases the efficiency of the zinc. Metal dust is the more convenient form for use in the Crowe process.

Precipitation with zinc shavings is performed in compartmented boxes; the shavings, after dipping in lead acetate (and being thus coated with lead) are packed into alternate compartments and solution is circulated so that it rises through the zinc-filled compartments and flows downward in the others. The boxes are 12 to 24 ft. long, 1½ to 3 ft. wide and 1½ to 3 ft. deep; the zinc compartments are substantially square, the intermediate compartments narrow. CAPACITY is 5 to 15 tons of solution per cubic foot of zinc-compartment space per 24 hr. ZINC CONSUMPTION on the RAND is 0.12 to 0.26 lb. per ton of ore milled.

Metal dust (90 per cent. − 200-mesh, containing, in the case of zinc, upward of 95 per cent. free metal) is mixed with solution in a plunger pump and forced into a plate-and-frame filter press. A small amount of lead acetate aids precipitation. ZINC CONSUMPTION on the Rand is 0.10 to 0.14 lb. per ton milled. In the Crowe process the consumption of metal is about half the above figures (*111 J 624*).

Recovery of cyanide from foul solutions by liberating hydrocyanic acid and re-absorbing the gas in fresh alkaline solution is an old idea. Some recent patents (*U.S.* 1,387,289; 1,486,137) promise successful operation. Both patents specify an acid stronger than hydrocyanic for decomposition of cyanides and both use sulphurous acid; the first (MILLS-CROWE) recommends blowing air through the acidified solution at normal temperature to remove hydrocyanic acid; the second (HALVORSEN), boiling under high vacuum. With such recovery possible much stronger cyanide solutions can be used than formerly with consequent economy and the harmful effect of cyanicides is distinctly minimized.

Sodium sulphide (Fig. 3) and **charcoal** have been used in cyanide precipitation but in isolated cases only.

Precipitation of copper from sulphate solutions by means of iron generates ferrous sulphate which oxidizes to ferric sulphate and thus regenerates the solution.

Electrolytic deposition is the standard method in copper and zinc precipitation; it is used for lead at BUNKER HILL AND SULLIVAN and is being tried for the same metal at CERRO DE PASCO; it has been many times proposed and experimented with for cyanide, and this latter application is the subject of a recent patent of much promise (*121 J 113*). Electrolytic precipitation is invariably regenerative.

Typical general procedure is to clarify the solution, usually by sedimentation; condition it, if necessary, which is usually the case for zinc, so that it is of the proper chemical composition; then flow it through electrolytic cells, between the poles of which suitable current is passing. The desired metal precipitates, and is collected either as an adherent deposit on the cathode (copper and zinc) or as a collection of flakes or crystals (lead and precious metals).

The important elements in operation are current strength and voltage, temperature, concentration of the desired metal and of other substances in the solution, rate of solution flow and the character of anodes and cathodes.

Current. The fundamental consideration is the fact that a given amount of current passing through a solution of a given metal will precipitate a definite amount of that metal. The theoretical maximum amount of a given metal that it is possible to deposit with a given current, at 100 per cent. current efficiency, is $W = 0.0104Aw/v$ mg. where A = the current in amperes, w = the atomic weight of the metal to be deposited and v is the valence of the metal. CURRENT EFFICIENCY is the ratio of the weight of metal deposited to this theoretical maximum, usually expressed in percentage. The lack of effectiveness that such a statement of efficiency may indicate does not reside in the current itself but arises from the solvent action of the electrolyte upon the cathode metal; this dissolving action may counteract the precipitating action of the current to any extent. CURRENT DENSITY is limited, in copper and zinc deposition, by the fact that a smooth, adherent deposit of metal is desired and that excessive current density causes flocculent, spongy or tree-like deposits. (For other factors influencing character of deposit see p. 958.) In copper deposition with the usual solutions a current density of 8 to 10 amp. per sq. ft. of cathode surface has proved best; in zinc cells, with 3.5 to 7.5 per cent. acid content, 20 to 31 amp. According to a recent patent (Tainton, 1,491,498), with 22 to 27 per cent. sulphuric acid present in zinc sulphate solution, and a temperature of 60° C., it is advantageous to use a current density as great as 100 amp. per sq. ft.

Voltage is determined by the composition of the solution and the spacing and

composition of anodes and cathodes. In copper cells with solutions containing about 2 per cent. copper as sulphate, 0.5 to 2 per cent. ferrous iron and about 3 per cent. free acid, which may be looked upon as an average condition, with lead anodes and copper cathodes spaced about 2 in. face-to-face, and with temperatures not exceeding 60° C., the normal voltage drop from anode to succeeding anode is two volts. At TRAIL in zinc cells with 6 to 8 per cent. zinc-sulphate solution at 60° C. and lead anode and aluminum cathode spaced about 2 in. face-to-face, the normal current density for a smooth, adherent deposit is 20 to 31 amp. per sq. ft. and the voltage 3.9.

Cell. The transverse section is determined by the convenient size of cathode; length by the reasonable size of the bus bars. Table 2 shows some recent practice.

Table 2. Size of electrolytic cells

Plant	Length		Width		Depth		Number of	
	ft.	in.	ft.	in.	ft.	in.	Anodes	Cathodes
Katanga	55	7	3	2½	4	3	115*a*	110*a*
New Cornelia	29	7	4	9	4	3	84	83
Chile Copper Co.	18	8	3	5	5	10	56	55
Anaconda	10	3	2	10	5	0	28	27
Cons. Min. and Sm. Co.	6	8	2	3	3	6	17	16

a In five groups.

Cells are usually made of wood lined with sheet lead; reinforced concrete lined with a mixture of sand and sulphur is used at TRAIL (*34 IMM 2*); reinforced concrete and asphalt lining at Chuquicamata (*45 AES 381*).

Anodes are insoluble. Carbon is theoretically ideal but practically, in a non-diaphragm cell with sulphate electrolyte, when the cell becomes polarized and the voltage rises oxidation of iron decreases, gas collects, and the anode disintegrates.

Van Arsdale (U. S. Pat. 1,553,415) proposes a composite anode with exactly the proportion of carbon area, in conjunction with lead or other suitable insoluble anode material, required to oxidize a certain definite amount of ferrous sulphate and leave no destructive excess of anodic oxygen. Lead anodes are satisfactory in sulphate electrolytes free from chlorides and nitrates (*115 J 20*) and are used at NEW CORNELIA, ANACONDA and CONS. MIN. AND SM. CO., but cannot be used at CHILE COPPER CO., owing to the presence of chlorides. Hollow cast magnetite anodes proved too brittle at the latter plant; DURIRON and many other alloys were tried; "CHILEX," essentially an alloy of copper, silicon, iron and lead with a small amount of tin and about 4 per cent. manganese is now used (*45 AES 381*).

Cathodes for copper are copper sheets; for zinc, aluminum, from which the zinc deposit is subsequently peeled off.

Anodes and cathodes are usually hung transversely in the cell, but are placed longitudinally at NEW CORNELIA. (This arrangement was first used by the writer in the KEYSTONE experimental plant in 1913.) SPACING must not be so close that short circuiting can occur because of "trees" or buckling. Glazed porcelain insulators are used at AJO (*114 J 188*). Spacing depends upon the thickness of anode and cathode; with ½-in. anodes, spacing ranges from 5 in. to 4 in. or less. POWER and CAPACITY are affected by the spacing. At CHILE COPPER CO., with ferro-silicon anodes, current efficiency decreased with reduction in spacing from 5- to 4-in.; with CHILEX anodes, increased capacity and proportionately increased power efficiency followed reduction in spacing (*45 AES 381*).

Character of solution. Certain impurities (substances other than the desired salts of the sought-for metal and, in most cases, a residue of the leaching

chemical) have decidedly harmful effect in electrodeposition; *e.g.*, ferric sulphate causes re-solution of precipitated copper; arsenic (one part in 100,000), antimony and cobalt lower current efficiency in zinc deposition (*26 CME 596*); excess of iron prevents smooth deposition of copper; and iron, vanadium, cobalt, nickel, arsenic and antimony must be practically excluded from zinc cells for the same reason; any metal of lower decomposition voltage than the metal to be deposited will deposit and in addition to its effect on purity (which may be negligible), decrease current efficiency and may harmfully affect the character of the deposit; solid particles in suspension cause irregular and non-adherent deposition. Certain other impurities are helpful; thus in copper deposition ferrous salts combine with anodic oxygen and prevent voltage increase from polarization; sulphur dioxide acts similarly and likewise prevents formation of ferric sulphate, which would re-dissolve copper; manganese prevents attack on the anodes in zinc cells; certain colloids, *e.g.*, glue, retard or overcome the harmful effect on deposition of certain impurities (*26 CME 601*) and Tainton (*PP 1301 A*) says that glue, about 3 lb. per ton of zinc precipitated, or some similar colloid, is essential for smooth zinc deposits with high current density.

Removal of impurities. A neutral wash is used for ZINC. It consists in neutralizing the acid in the pregnant solution by addition of roasted ore with either calcium carbonate, zinc oxide or zinc powder, in the presence of manganese dioxide or other oxidizing agent, ferric sulphate and copper sulphate. This operation is usually carried out in a series of Pachuca tanks at 40 to 60° C. The precipitate is separated by decantation and filtration (see Fig. 7). COPPER SOLUTIONS are usually purified by discarding a portion of the solution over scrap iron, as necessary. If the amount of such discard is great, the cost of the operation is seriously increased on account of the expense of refining the cement copper produced.

Oxidized copper agitated with heated cupric sulphate solution at 75 to 80° C. will precipitate salts of iron, arsenic, antimony, bismuth, cobalt, nickel, etc. (*8 MI 192*). Chlorine is reduced to less than 0.5 gm. per liter at CHILE COPPER CO. by agitation of the copper-bearing solution with cement copper; this reduces cupric chloride to cuprous chloride, which is insoluble and is separated by decantation. The cuprous chloride is then dissolved in ferrous chloride and cement copper is recovered by passing the solution over scrap iron (M. D. Thompson, *Applied electrochemistry*, Macmillan, 1914).

Precipitation is the most difficult part of hydrometallurgy; the foregoing text is meant only to give a general idea of the art. For details see *Principles and applications of electrochemistry*, Creighton & Fink, Vol. II, and the recent files of Amer. Inst. of Min. and Met. Engineers, Amer. Electrochem. Soc., and Chem. and Met. Eng.

6. Amalgamation

Amalgamation is the process of collecting metallic gold and silver with metallic mercury. The mixture formed, called AMALGAM, is not completely understood, it is variously called an alloy or compound. Practically the phenomenon of amalgamation may be looked upon as wetting of metals by mercury, as a result of which the metals are drawn into the body of the mercury. The latter can be driven off by heat. Most gold ores yield a certain recovery readily; the tellurides are an exception. If high recovery is obtainable by amalgamation the ore is called FREE-MILLING.

Plates. Amalgamation is usually performed in broad, shallow, sloping troughs in the bottom of which AMALGAMATED (mercury-coated) copper

plates are placed. These are called APRON PLATES. They are usually but not necessarily placed directly after the fine-grinding machines. BATTERY PLATES are those within a stamp mortar. They are rarely used on account of the scour of the pulp and the fact that they necessitate wide mortars with consequent reduction of capacity.

Apron plates are usually 1/8 in. thick; thin plates buckle and cause uneven flow of pulp; if there is much scour, as on battery plates, 1/4-in. plates are used. Usual WIDTHS are 4 to 5 ft. (corresponding to the length of a battery mortar); LENGTH, 8 to 12 ft. The AREA ALLOWANCE, when amalgamation is principally relied upon for recovery, is 3 to 4 sq. ft. per ton of ore per 24 hr. SLOPE is 1½ to 2 in. per ft.

Surfacing plates. Copper is usually plated with silver (1½ to 3 oz. per sq. ft.) and then dressed with mercury or, better, an amalgam of mercury with silver. Sodium amalgam (3 parts sodium to 97 parts mercury) and certain other amalgams such as zinc and cadmium resist surface contamination and are sometimes used for this purpose. Amalgam does not penetrate the plates, hence the kind of copper used makes no difference (*A. J. Clark*, *PC*).

Fundamental requirements in plate amalgamation are: (1) that the precious metals shall be free and have clean metallic surfaces; (2) that they shall be flowed over the plate in a pulp sufficiently fluid to permit metallic particles to sink readily; (3) that the velocity of flow be sufficiently low that the precious-metal particles can sink to the plate surface and yet high enough that other constituents of the pulp cannot remain permanently at rest thereon; (4) that the plate surface be clean and bright; and (5) that the amalgam be sufficiently soft (fluid) to permit it to spread over the metal particles. Bright metallic surfaces are usually obtained in grinding. Pulps normally contain 10 to 20 per cent solids; any higher solid content decreases the mobility of fine particles materially. The velocity and thickness of the pulp stream should be such that the pulp progresses in a series of waves; contact of solid particles with the plate may be increased by drops of an inch or more; amalgam builds up at each such drop. Plate surfaces are best kept bright by keeping grease away, preventing banking, and by brushing, scraping with a rubber, wood or steel scraper, and by scrubbing. If these expedients are insufficient it may be necessary to use special amalgams or chemicals to counteract the particular coating influence. Soluble sulphates, such as zinc and iron, cause crusting on the plate surface with consequent lowering of extraction. Certain minerals such as pyrolusite, sulph-arsenides, and sulph-antimonides, and grease or oil prevent coalescence of mercury globules; the mercury is said to be SICKENED or FLOURED; it is no longer a good collector nor will it remain on the plates, but passes on and must be caught by some method of gravity concentration, *e.g.*, an amalgam trap (settler) or a shaking table or vanner.

Amalgam on plates hardens with loading up of the mercury with precious or other metals and then fails to catch and hold further metal. This condition is overcome by adding more mercury, or by scraping, in case the hardening is a surface effect only.

Clean-up to recover metal is made periodically. The procedure is to first wash off all ore, then heat the plates, preferably by live steam under a covering of planks, and then to remove all amalgam that comes off readily with a wood or steel scraper. The plates are then dressed again with mercury or dilute amalgam. The period between clean-ups is determined principally by the metal content of the ore. The collected amalgam is squeezed in chamois skin to express as much free mercury as possible, then retorted (see Art. 7).

Loss of mercury averages 0.07 to 0.15 troy oz. per ton crushed (*116 P 483*).

Intensive amalgamation consists in the use of aids to contact or union of the precious metals and the mercury, *e.g.*, passing a small low-voltage electric current through the plate; placing iron bars on the plates and milling in salt water (sea water); placing plates on the walls of a centrifugal machine; grinding in the presence of mercury, etc. Except for the last expedient, the results do not justify the expense.

Blankets are sometimes used when very coarse gold or an excess of sulphides is present. See Sec. 2, Art. 19.

Amalgamation vs. cyanidation. Amalgamation is cheaper, less complicated and recovers coarse metal more readily; on the other hand it is rarely that the recovery is sufficiently high to justify the use of amalgamation alone, so that the practical choice always is between amalgamation plus cyanidation and cyanidation alone. This question is ordinarily resolved by the coarseness of the precious metal; if coarse and amalgamable, use amalgamation on account of the reduced time required for cyanide leaching after removal of the larger metal particles, and the probable greater recovery; if the precious metal is readily amenable to cyanidation, then introduction of amalgamation must be justified on the basis that it makes part of the metal more quickly available; that, due to the cheaper recovery of a part of the metal by amalgamation an overall saving is accomplished; or that sending impoverished feed to the cyanide plant permits less careful and therefore sufficiently cheaper operation there to more than make up for the added expense and complexity of milling with amalgamation in the flow-sheet. Amalgamation, on the other hand, precludes milling in cyanide solution and has indirectly been a most potent factor in the complacent use of the stamp battery.

7. Melting precipitate

Precipitate and sponge gold or silver from amalgamation are melted down into bars, slabs or ingots for marketing. Ordinarily some refining is done at the same time. The scale of the work varies, according to the quantity treated, from an assay-office operation to large-scale copper refining.

Amalgam. Free mercury is first squeezed out of the amalgam, the residue is charged into a cast-iron retort and the latter is gradually brought up to bright red heat, when the last traces of mercury are driven off. The remaining SPONGE is melted with borax or other suitable flux in graphite crucibles, and poured into heated cast-iron moulds, coated with graphite and usually also with paraffin or tallow.

Cyanide precipitate. If from zinc boxes, the precipitate is washed and screened on 20- to 60-mesh screens, to remove undecomposed metallic zinc, which is returned to the zinc-boxes. Undersize is collected under water in settling tanks and dried in flat pans. Sometimes a hot dilute sulphuric acid wash is given to remove fine zinc. Dried precipitate is melted in graphite crucibles with suitable flux, *e.g.*, borax, or a mixture such as 15 parts borax, 8 parts soda and 4 parts silica sand. With zinc-dust precipitation in filter presses, the precipitate is washed and dried by a current of air in the filter press, then melted in large crucibles (400-lb. charges), or special carborundum-lined, egg-shaped furnaces heated by oil through the trunnion; or in small blast or cupola furnaces with litharge and a flux, usually borax. The resulting bullion is cupelled.

Copper cathodes are not treated locally. For melting and refining practice see *Copper refining*, L. Addicks (McGraw-Hill, 1921). Cathodes from chloride solutions are subject to large refining losses.

Zinc. An ordinary coal-fired reverberatory is used. On account of rapid oxidation, the cathodes must be got beneath the surface of a molten bath of

zinc as quickly as possible. Fortunately the demand for zinc oxide in neutralization of solutions before precipitation will care for any reasonable amount of oxide produced. The dross and oxide may represent 20 per cent. of the weight melted.

At Cons. Min. and Sm. Co. a melting temperature of 1250° F. is maintained by pyrometric control. Only large slabs are charged into the reverberatory; scrap zinc is melted down in graphite crucibles protected from oxidation.

8. Flow-sheets

Homestake Mining Co., Fig. 1. (*114 J 757; 52 A 3; 23 IMM 68; A. J. Clark, P. C.*)

Location: Lead, S. D.
Ore: Gold and silver with a small amount of pyrite in quartz.
Assays: Feed, $3.50 to $4.00 per ton.
Capacity: 4500 tons per 24 hr.
Extraction: 72 per cent. by amalgamation; 22 per cent. by cyanidation; 94 per cent. total.
Cost: $0.22 per ton (1925).

Summary. Amalgamation and cyanidation.

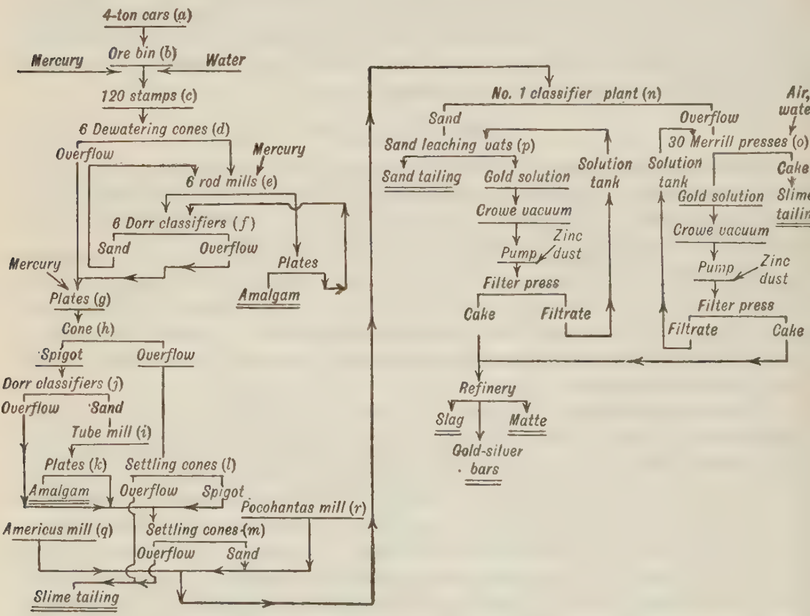


Fig. 1.—Homestake Mining Co.

a, Ore crushed at mine to 2-in. (see Sec. 3, Fig. 16). 4-ton cars, mine to mill. *b,* V-shaped, 7200-ton capacity. *c,* 1550-lb. stamps, 100 @ 8-in. drops per min.; order, 1–3–5–2–4; ½-in. screen; battery amalgamation; Challenge feeder; short trommel ahead of stamps to by pass fines. *d,* 6 @ 7 ft. (diam.) × 7 ft. *e,* 6 @ 5 × 10-ft. 22 r.p.m.; 100 hp., 29,000-lb. charge; 200 tons per day each. *f,* 6 @ 4-ft. 6-in. × 21-ft. 4-in. Dorr duplex classifiers; 2¼-in. per ft. slope; 26 strokes per min.; overflow, −80-mesh. *g,* 24 @ 9 × 12-ft. amalgamated copper plates, ⅛ in. thick. 2 oz. silver per sq. ft.; 21.6 sq. ft. of plate per stamp; each stream flows over two plates. *h,* 4 @ 7-ft. dewatering cones. *i,* One 5 × 14-ft. tube

mill. *j*, Dorr classifier. *k*, Two 9 × 12-ft. amalgamating plates. *l*, Eight 10-ft. settling cones. *m*, Eight 7-ft. settling cones. *n*, No. 1 dewatering plant; eight 10-ft. and eight 7-ft. cones; three 32-ft. Dorr double-tray thickeners; plant preceded by two banks of cones. *o*, 30 Merrill filter presses; each with 90 frames, 4 ft. × 6 ft. 4 in. One press holds 26 tons of slime. Daily capacity, 70 tons; 33 lb. pressure. (Increased from 16-lb. pressure.) Treatment time about 15 min. *p*, 24 sand-leaching vats, 44 ft. (diam.) × 9 ft. 610 tons sand capacity. Canvas-covered false bottom. Ore drained and air forced through the charge at a pressure that will not disturb the bed. About 75 cu. ft. of oxygen absorbed per ton of ore. *q*, Amicus mill. 240 @ 1000-lb. stamps, 5-tube mills, 5200 sq. ft. of amalgamating plates, 8 cones. *r*, Pocahontas mill. 160 @ 900-lb. stamps, 2900 sq. ft. amalgamating plates, 4 cones.

United Eastern Mining Co. Fig. 2. (*119 J 436, Q.*)

Location: Oatman, Ariz.

Ore: Gold and silver in very finely divided state in quartz-calcite gangue with some andesite.

Assay of feed: $19.80 per ton (aver. 1917–1924, incl.).

Capacity: 325 tons per 24 hr.

Extraction: 96.6 per cent.

Cost: 1917–1924 incl., average. See adjacent table.

Cyanide consumption: Aero brand, 0.713 lb. per ton.

Item	$ per ton
General expense	0.161
Lighting	0.005
Water	0.108
Coarse crushing	0.055
Coarse grinding	0.356
Fine grinding	0.469
Cyaniding	0.523
Tailing disposal	0.025
Classification	0.033
Precipitation	0.095
Refining	0.074
Sampling	0.003
Assaying	0.031
Re-treating skimmings	0.014
Mill heating	0.031
Solution-recovery system	0.010
Experimental work	0.001
Total	$1.994

Summary. All-slime cyanide.

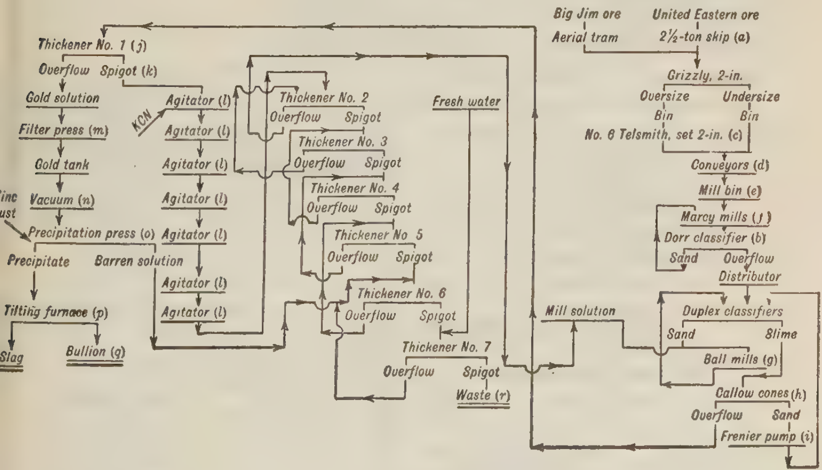


Fig. 2.—United Eastern mill.

a, Mill at collar of United Eastern shaft. Big Jim ore brought in by Riblet aerial tram. *b*, Simplex: slope, 3-in. per ft., 34 strokes per min. *c*, 35 tons per hr. *d*, 18-in. + 20°, 250 ft. per min. 50 tons per hr. Merrick weightometer on conveyor. *e*, 24 × 24-ft. flat-bottom tank, feeding onto 18-in. pan conveyor. *f*, Two 6 × 4½-ft. Marcy ball mills, 26 r.p.m., 67 kw.; in closed circuit. *g*, Three 5 × 6-ft. ball mills, 28 r.p.m., 48 kw., lined with 35-lb. T-rails. Each in circuit with one 54-in. duplex Dorr classifier. Slope 2½ in. per ft., 18 strokes per min., overflow maintained at 5 solution to 1 of solid. *h*, Two 8 ft. Overflow: United Eastern ore, 85 per cent. −200-mesh; Big Jim, 92 per cent. −200-mesh. *i*, Two No. 3 Abbe-Frenier pumps. *j*, One 40 × 12-ft. Dorr thickener. Spigot, 50 per cent.

solids. 3.87 sq. ft. per ton of solid. *k*, Duplex, Campbell-Kelly, No. 4 diaphragm pump. *l*, Seven 24-ft. (diam.) × 14- and 16-ft. (deep) Dorr agitators. Duration, 52 hr.; total contact, 146 hr. *m*, Merrill central-sluicing clarifying filter, with 28 frames 42 in. square. Sluiced once or twice every 8-hr. shift; acid treated with 1-per cent. hydrochloric acid every 3 days; sluicing time, 10 min.; acid treatment, about 1 hr. *n*, Crowe vacuum. 4 × 10-ft. receiver, 6-in. inlet at center of top. 23.4 ft. above the solution level in the steady-head tank. Solution poured down over perforated trays. Depth in receiver, 30 in.; regulated by float butterfly valve in intake pipe. Solution drawn from bottom of receiver by 7 × 8-in. triplex pump, glands 32.8 ft. below solution level in receiver; the column is under vacuum, insuring slight pressure at the pump glands, thus preventing admission of air. Zinc dust is fed by a screw feeder directly to a cone from which it is drawn by the precipitation pump, and mixed with vacuum-treated solution, then pumped to the precipitation press. Strength of solution 1.8 lb. KCN per ton, 1.0 lb. CaO, 0.27 to 0.44 oz. gold; silver, ½ oz. per oz. gold. *o*, Two Merrill presses each with 32 triangular 36-in. plates, dressed with four thicknesses of sheeting; inner sheet is burned on removal. Precipitate contains 45 per cent. water. *p*, Flux: 12 per cent. borax glass, 12 per cent. sodium bicarbonate, 6 per cent. manganese dioxide, 3 per cent. bottle glass, 10 per cent. or more of slag from previous melts; put in No. 8 paper bags. The charge is fed into a No. 150 Case tilting furnace, using No. 125 long-lipped graphite crucibles containing 600 to 1000 oz. bullion and 40 to 50 lb. slag each. *q*, Bullion buttons re-melted and cast into bars; average weight, 1800 oz. Ground slag is concentrated on a small Deister table. *r*, Tailing is distributed to various ponds through 6-in. slip-joint pipes; minimum grade of 3°. Tailing is deposited in benches 8 to 12 ft. high. After standing for several months, the accumulated layer of surface salts is of sufficient value to gather. Three "crops" of salts are collected and fed to the re-grinding mills.

Nipissing Mining Co. Fig. 3. (*31 Ont. Dep. Mines 280.*)

Location: Cobalt, Ont.
Ore: Native silver and arsenides of cobalt and nickel in calcite gangue.
Assay of feed: 44.78 oz. Ag (1923).
Capacity: 230 tons per 24 hr.
Extraction: 94 per cent.
Cost: \$4.60 to \$5.39 per ton (1923).

Summary. Concentration and all-slime cyanidation.

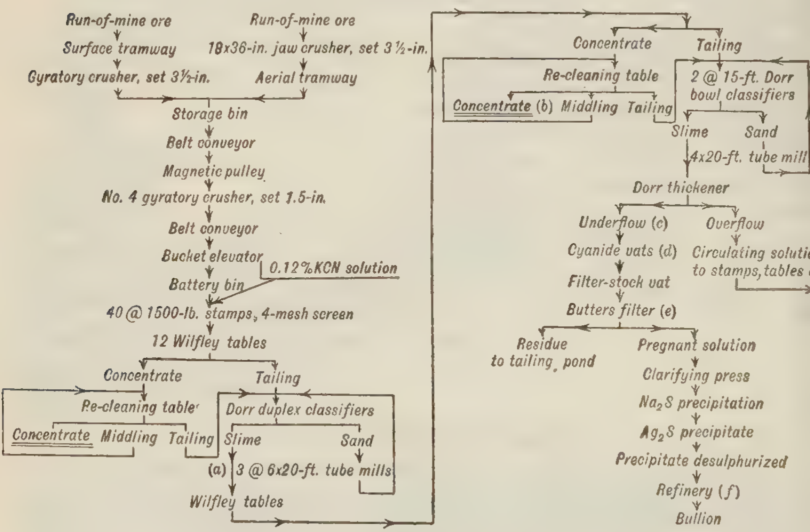


FIG. 3.—Nipissing low-grade mill.

a, 6 to 10 per cent. + 200-mesh. *b*, Treated further in high-grade mill. *c*, 29 per cent. solids. *d*, Cyanided by charge system. Agitated mechanically for 50 hr. in solution main-

tained at 0.25 per cent. KCN. Aerated by 6-in. air lift during agitation. Pulp then settled 8 hr., pregnant solution decanted, and the residue again agitated with new solution. *e,* 72 leaves, 40 tons solid per 3¾ hr. *f,* Sulphide precipitate reduced to 999-fine silver by treating in solution of caustic soda with aluminum ingots.

New Cornelia Copper Co., Fig. 4. (*60 A 22; 6 MMt 524; 119 J 285.*)

Location: Ajo, Ariz.

Ore: Malachite and chrysocolla in silicified monzonite porphyry. Some chalcopyrite and bornite.

Assay of feed: 1.5 to 1.6 per cent. Cu.

Capacity: 5000 tons per 24 hr.

Extraction: 78 per cent.

Cost: Crushing, \$0.157; leaching, \$0.275; precipitation, \$0.0217 per ton of ore (Power, \$0.0075 per kw.-hr.)

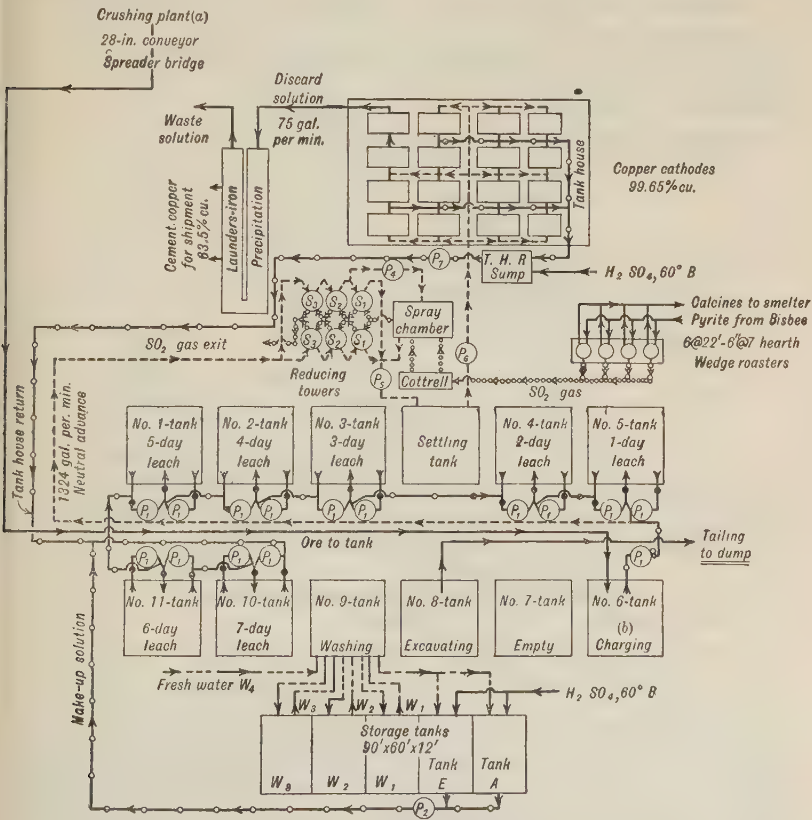


FIG. 4.—New Cornelia Copper Co.

a, See Sec. 3, Fig. 5. *b,* The flow lines with respect to the variously-numbered tanks are representative of the status on any given day. On the day following the one represented, tank No. 7 would be charging; No. 9, excavating; No. 8, empty; No. 10, washing; No. 6, 1-day leach, No. 5, 2-day leach, etc.

Summary. Sulphuric-acid leaching. The operation consists of five steps: (1) Crushing from steam-shovel size to $-\frac{3}{8}$-in.

(2) Charging the lixiviation tanks with dry ore. This step includes submerging the ore with solution, and circulating solution until it is clear.

(3) Leaching proper, which, although the ore remains fixed, is, in effect, a counter-current operation in which the newest ore has the weakest acid (0.3 per cent H_2SO_4 on first day increasing up to 2.3 per cent. on the eighth day).

(4) Washing, fresh water being added as the final wash; each preceding wash stepped up until that richest in copper advances to the stock-solution tank.

(5) Discard of tailing.

Inspiration Cons. Copper Co., Fig. 5. *(73 A 58)*.

Location: Miami, Ariz.
Ore: Oxidized copper minerals in silicified schist and decomposed granite.
Assay of feed: 1.23 per cent. Cu.
Capacity: 4500 tons per 24 hr.
Extraction: 85 per cent. (est.).

Summary. Ferric-sulphate leaching.

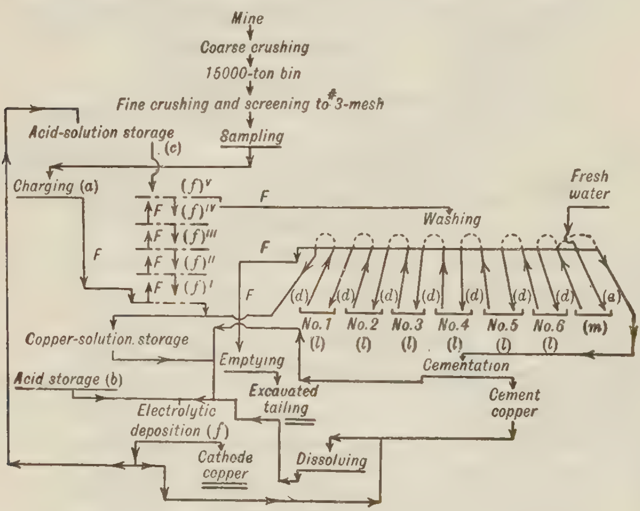


Fig. 5.—Inspiration Copper Co., leaching plant.

a, Charging by conveyor belt and spreader bridge requires two days for 9000 tons. *b,* Acid solution, 25.6 lb. 60° Bé. acid per ton of solution. *c,* 2250 gal. per min. *d,* Washed 3 days; 6 unit washes of 40 gal. per ton of ore; each wash advanced after each cycle, the strongest going to the tank house for electrodeposition. A circulating wash is finally pumped through until sufficient soluble copper is removed. A portion is then advanced to No. 6 wash-storage tank and is replaced by the same volume of fresh water, which may, if necessary, be passed through the leached ore to recover iron as needed. There is no discard of solution. *e,* Advance from fresh water. *f,* Anodes have exactly that proportion of carbon area in conjunction with lead or other suitable insoluble anode as is necessary to oxidize a certain definite amount of ferrous sulphate and leave no destructive excess of anodic oxygen. Solution, 2.5 to 3 per cent. Cu; 1.5 to 2.5 per cent. total Fe with ferric iron up to 1 per cent.; acid 3.5 to 7.5 per cent. Current density is sufficiently high to counteract, to a commercial degree, the solvent action of ferric iron upon the cathode; about 15 amp. per sq. ft. Current efficiency is about 60 per cent. or ⅔ lb. copper per kw.-hr. $F, f^{I}, f^{II}, f^{III}, f^{IV}, f^{V}$: The flow lines marked F from the tank being charged to the leaching circuit (f^{I}, etc.) and to the wash-

ing circuit and to "emptying" do not represent actual "flow" of ore, but merely indicate sequence of steps. The lines marked f^1, etc. represent different stages of leach solution. *l* Advancing-wash storage tanks, cap. 500,000 gal. each. *m* Circulating wash storage tank, cap. 500,000 gal.

Calumet and Hecla, Fig. 6. (*117 J 277; 70 A 595.*)

Location: Torch Lake, Mich.
Ore: Current mill tailing. See Sec. 2, Fig. 44.
Assays: Feed, 0.51 per cent. Cu; tailing, 0.10 per cent. Cu.
Extraction: 79.7 per cent. Cu.
Cost: \$0.32 per ton of feed; \$0.04 per lb. of copper.

Summary. Ammonia leaching.

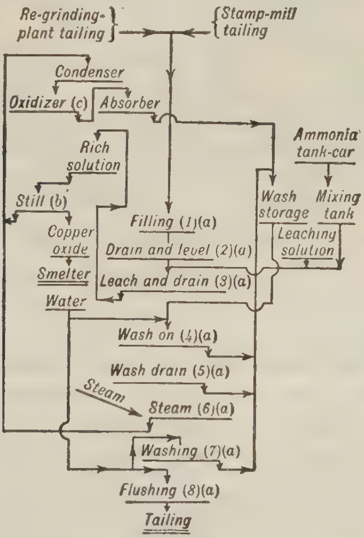


Fig. 6.—Calumet and Hecla leaching plant.

a, 16 steel leaching tanks, 54 ft. (diam.) × 12 ft. Water-sealed during leaching. Filled by portable distributor, Butters and Mein type. Numbers 1 to 8 represent successive operations in the same leaching vat as follows:

1. Filling vat: requires about 6 hr. 2. Draining and leveling, 1 hr. 3. Leaching, 10 to 12 hr. Leaching solution 30 gm. Cu, 60 gm. NH_3, and 40 gm. CO_2 per liter. Ammonia-copper ratio, 3 : 1. 622 cu. meters of solution per hour. 4. Wash 5 to 6 hr. with ammonia water to remove all copper. About 200 cu. meters of solution followed by 75 cu. meters of fresh water. 5. Draining, about 5 hr. 6. Steaming. A partial vacuum is maintained beneath the filter bottom of the tank; steam and ammonia pass downward as the charge becomes heated. Steam is admitted at atmospheric pressure, 12 to 13 kg. per hr., decreasing to about 4 kg. per hr. at the end of the 5-hr. period. 7. Washing with fresh water. 8. Flushing tailing with water, about 6 hr.

b, Solution distilled contains 35 gm. Cu, 35 gm. NH_3 and 30 gm. CO_2 per liter. *c*, Copper changed from cuprous to cupric. This is essential to maintain the dissolving power of the solution.

Consolidated Mining and Smelting Co. of Canada, Trail electrolytic zinc plant, Fig. 7. (*34 IMM 2; 23 CME 227.*)

Location: Trail, B. C.
Ore: Zinc concentrate.
Assays: Feed after roasting: Ag, 2 oz. per ton; Pb, 11 per cent.; Zn, 27 per cent; Fe, 25.8 per cent.

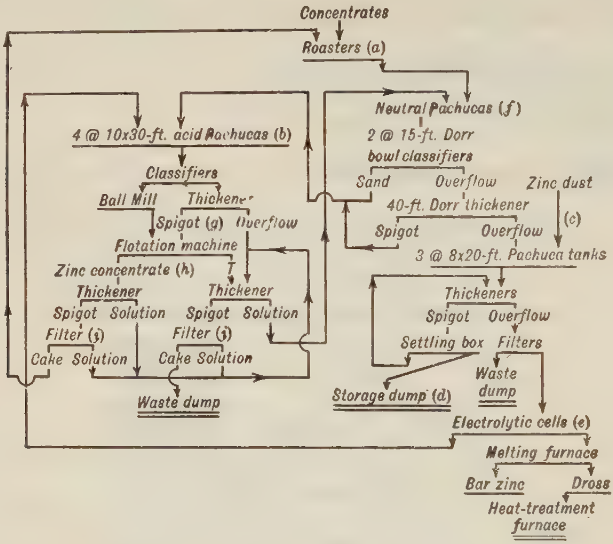


FIG. 7.—Consolidated Mining and Smelting Co. of Canada, Trail electrolytic plant.

a, 13 @ 25-ft. Wedge roasters. Temperature on sixth hearth, 1350° F. Calcine about 2.5 per cent. sulphate sulphur and 0.7 per cent. sulphide sulphur. *b,* 0.5 to 1 per cent. H_2SO_4. *c,* An amount sufficient to precipitate copper and cadmium and the remaining traces of arsenic and antimony. *d,* Contains cadmium and zinc. *e,* Two plants 12 and 14 electrical units each. Each unit 18 double-cell tanks of reinforced concrete, ¾- to 1½-in. lining of sand and sulphur; inside dimensions 27 × 80 × 42 in. (deep). 17 cast electrolytic-lead anodes and 16 sheet-aluminum cathodes. Cathodes 24 × 36 in., spaced 4 in.; anodes, 23½ × 35 in. Cathodes 1/10 in. thick, supported by 1-in. copper rod at one side and ½ × 1-in. strap iron on the other side. *f,* Solution alkaline. *g,* 40 per cent. solids. *h,* About 35 tons per day, 41 per cent. Zn. *i,* American, 8 @ 6-ft. disks.

SECTION 16

DEWATERING

DEWATERING

Dewatering consists in separating a mixture of solid and water into two parts, one of which is relatively solid-free, the other liquid-free, with respect to the original mixture. Substantially complete separation is attainable only by evaporating the water (drying). A solid-free liquid fraction and a solid fraction containing considerably less than 20 per cent. water can be obtained by bringing the mixture of solid and water to one side of a porous membrane at a pressure greater than that existing on the other side of the membrane (filtration). When less complete separation is sufficient, two methods are available. If the solid matter is in such coarse particles that solid and liquid are visually distinct, draining is effective. If the solid matter is so fine that slight movement maintains it in suspension in the water, separation is effected by bringing the water substantially to rest and allowing the solid to settle under the influence of gravity (thickening).

1. Draining

Draining is a crude form of filtration. The simplest case is that of a pile of wet material on a floor; the liquid settles to the lowest layers of the heap and, if the supporting surface is inclined slightly, runs away. Some residual liquid is held by adhesion at the surface of the solid particles and by capillarity between adjacent solid surfaces. The volume of water thus retained depends primarily on the size of the particles of solid; the finer the grains, the greater the particle area and hence the greater the volume of retained water. The moisture content of the drained solid is higher the lower the specific gravity of the solid, all other things being equal. Bins may replace floors. In some cases a screen forms the draining surface, permitting continuous as opposed to batch operation. Rectangular or circular tanks are frequently used for dewatering fine table concentrate.

At Santa Barbara (*112 J 1056*) lead-carbonate concentrate at the rate of 74 tons per 24 hr. is sent alternately to two concrete tanks 10 ft. wide by 60 ft. long by 4 ft. deep. The moisture content is reduced from 84 per cent. to 14 per cent. At Le Roi No. 2 mill (*114 J 1121*) about 30 tons per day of flotation concentrate is fed at one end of a series of two tanks each 1 ft. deep, 2 ft. wide and 22 ft. long with baffles 6 in. high at intervals along the bottom and double burlap screens at the outflow end. As concentrate accumulates it is tamped down for two or three minutes every hour or so. The material shoveled out when the tank is full contains only 10 to 15 per cent. moisture. The froth is efficiently broken down by flowing across the semi-dry surface of the settled solid. This method is not so cheap as thickener-filter operation but is immeasurably cheaper in first cost.

2. Scraper dewaterers

Scraper dewaterers are used principally for dewatering coarse sand products, and, less frequently, to effect a rough sand-slime separation.

Shovel wheel used at Tigre Milling Co. (*97 J 227*) is shown in Fig. 1. Feed enters through a chute, sand settles to the apex of the V-shaped trough, is scraped up-slope by the wheel, revolving counter-clockwise, and discharged through a slot, water or slime overflows a lip on the opposite side of the box at a level about 4 in. below the sand discharge. The V-box in Fig. 1 is 42 in. long and 52 in. wide, the wheel revolves 8 r.p.m. and treats 125 to 135 lb. solid per min. Performance is affected by the speed and inclinaton of the shaft and the character of the feed pulp. At the Santa Barbara mill of the Amer. Smelters Securities Co. (*112 J 1055*) two 48-in. wheels, revolving in opposite directions so as to raise sand along the transverse center-line of the box are set in a V-box 9 ft. 6 in. long by 4 ft. wide. The feed is the oversize from a 0.5-in. trommel fed at the rate of 430 tons per 24 hr.; it contains 42.5 per cent. solids; the sand discharge, 70 per cent. solids.

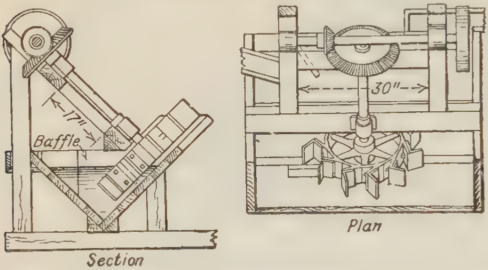


Fig. 1.—Shovel wheel.

Sand wheel (Fig. 2) consists of a wheel (*A*) carrying convex, perforated

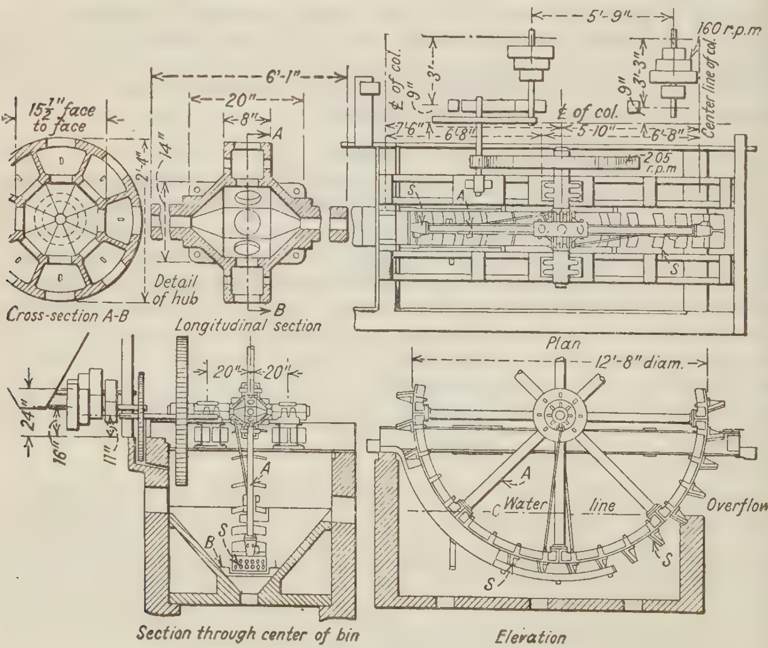


Fig. 2.—Sand wheel.

scrapers (S) at the periphery, revolving in a narrow trough (B) supported in a V-box. The wheel in Fig. 2 is 14-ft. diameter to the tips of the scraper blades.

The most efficient speed for dewatering combined jig and table tailing (−9-mm.) at Doe Run Lead Co. was 2 r.p.m. The 14-ft. machine handled 1500 tons of such material per 24 hr. The sand discharge carried about 15 per cent. moisture. The power requirement was less than 2 hp. A more elaborate form lifts the sand in buckets carried on a large wheel dipping into the pulp, decants water as the wheel revolves and finally discharges dewatered sand at the top of the revolution by automatically tilting the buckets. (*111 J 291.*)

Mechanical classifiers. For use in de-sliming see Sec. 6, Art. 6. For use in dewatering concentrate see Sec. 2, Fig. 60. Table 1 shows the performance of Dorr classifiers dewatering de-slimed coal.

Table 1. Performance of Dorr hydro-separator (26-ft. diam.) and three Dorr classifiers treating separator-spigot discharge at Loree anthracite breaker (*20 CA 608*)

Material	Feed	Separator		Classifier	
		Spigot	Overflow	Rake	Overflow
Total solids, tons per hour	49.8			22.7	
Plus 60-mesh, cumulative per cent	44.5			89.0	
Plus 100-mesh, cumulative per cent	57.6			97.0	
Plus 200-mesh, cumulative per cent	72.1			99.5	
Ash, per cent	32.9			24.8	
Total solids, per cent	100.0	60.9	39.1	45.6	15.3
Total + 60-mesh material recovered, per cent				91.0	
Total +100-mesh material recovered, per cent				76.7	
Total +200-mesh material recovered, per cent				62.9	
Total combustible, per cent				51.2	
Total water eliminated, per cent				98.4	

Cost (1921) was $0.053 per ton of recovered material for labor, power, supplies and repairs and $0.088 for interest, insurance, taxes and depreciation, being 20 per cent. on an investment of $17,500 for building and equipment.

Drag conveyors and elevators of various forms (see Sec. 20) are used to dewater coarse and fine granular coal. The conveyor runs in a tight or in a perforated trough. In metal concentrators drags, when used, are ordinarily applied to jig and table concentrate.

At Britannia (*113 P 696*) flotation concentrate is sent to a drag elevator and thence to bins. The drag discharge contains 20 per cent. moisture and the bin discharge 8 per cent. moisture. This method of treatment is applicable only to a very quick-settling concentrate.

A drag dewaterer for fine anthracite is described by Griffen (*61 A 514*). The tank was 2 ft. 8 in. wide, a flight conveyor with 6 × 18-in. flights, spaced 18 in. apart, dragged 60 ft. horizontally, then raised on a short incline to discharge the sand. Speed was 50 ft. per min. Feed rate was 800 gal. per min. of a pulp containing 13.5 per cent. solids, all through 1/16-in. round hole. Overflow contained 1.2 per cent. solids, which was 91 per cent. of the feed. Pulp was fed near the tail pulley and overflow was at the sides of the tank near the solid-discharge end. The dewatered material usually contained all of the +100-mesh coal.

The cost (1920-21) of a machine with 4 × 4 × 100-ft. tank and 8 × 18-in. flights together with a 65-ft. stacking conveyor was $10,250. Another machine to dewater 2000 gal. per min. and stack the dewatered product cost $9000.

THICKENING

THICKENING is the process of concentrating a relatively dilute slime pulp into a THICK PULP, *i.e.*, one containing a low percentage of moisture, by rejecting liquid that is substantially solid-free. SETTLING is another name for the same operation.

3. Principles

Slime, is the term used in milling practice to describe a suspension, in water, of the finely-divided fraction of pulverized ore.

The terminology is not precise, *e.g.*, the overflow of a mechanical classifier or cone guarding the discharge of a grinding mill is called slime as distinguished from the coarser SAND, even though the separation be made at upwards of 0.5-mm. size; the overflow of a hydraulic classifier is called slime, more or less irrespective of the size of the coarsest grains. Some writers (*41 A 398*, *42 A 752*) define slime as crushed rock in water when the rock is of such fineness that it will pass a 150- or 200-mesh (0.1- to 0.075-mm.) screen.

The solid particles in mill slimes are rock or mineral fragments formed by crushing operations, and secondary minerals such as steatite, talc, and clayey substances that have been disintegrated and dispersed by wetting. These latter substances are often called TRUE SLIMES.

A slime product separated in the early stages of reduction, which contains, therefore, an exceptionally high percentage of true slime, is called PRIMARY SLIME. The product formed by fine-grinding the crystalline part of the ore contains but little true slime.

Every slime consists initially of individual solid particles separated from other solid particles by a layer of water. Irrespective of whether the largest particles are 0.5-mm. (20-mesh) or 0.07-mm. (200-mesh) diameter, they are enormously larger than the smallest particles, large numbers of which are in the size range below 0.001-mm. The difference in settling behavior of these two classes is much greater than that between the coarse and fine particles met in hydraulic classification. The granular particles settle slowly under the influence of gravity, at definite rates approximated by Stokes' equation (Sec. 6, Art. 1). The fine particles may not settle at all, or they settle at a rate so slow as to be of no practical benefit in thickening. The viscosity term in Stokes' equation is relatively very large for these fine particles, and molecular effects such as Brownian movement and electrical repulsion between particles may effectively counterbalance any residual tendency toward downward motion due to gravity. These particles are in colloidal or semi-colloidal state.

Colloids. The word COLLOIDAL describes a state of matter. A COLLOID is not a certain specific kind of matter but any matter in a certain specific state. The COLLOIDAL STATE is sub-division into molecular aggregates of such minuteness that the aggregates are capable of maintaining their state of uniform dispersion in a given medium, without the necessity for expenditure of power on the mixture. The dispersed substance, the COLLOID, may be in gaseous, liquid or solid form and the dispersing medium may, likewise, be in any of these three states except that gas is never at the same time the disperse and dispersing phase. The mixture is a COLLOIDAL SOLUTION. Such solutions differ from the more familiar types of solution in all of the known solution characteristics except that of maintaining uniformity without requiring work to be done; they are not ionized, their boiling and freezing points do not differ from those of the dispersing medium, they do not dialyze nor exhibit osmotic pressure; finally, the dispersed particles are visible, with ultramicroscopic equipment, and separable by means of filters. SEMI-COLLOIDAL particles have, in a con-

siderable degree, colloidal properties. The dispersed particles in a colloidal solution are said to be PEPTIZED or DE-FLOCCULATED. The converse condition, resulting from aggregation of the dispersed particles, is called FLOCCULATION, and the aggregates are called FLOCCULES or FLOCS.

The effective mechanism underlying colloidal solution (DISPERSION) is adsorption of ions by the dispersed particles, resulting in electrical charges at the particle surfaces. The charges cause the particles to mutually repel each other, thus preventing coalescence and settlement. Ion adsorption is preferential and specific to the particular colloidal substance. Thus certain colloidal substances will adsorb negative ions from solutions of certain electrolytes and positive ions from solutions of other electrolytes and may, therefore, be negatively charged in one instance and positively charged in another. Beans and Eastlack (*37 ACS 2667*) show that only those ions are adsorbed that will, under proper conditions, enter into chemical combinations with the dispersed substance, and that the amount or intensity of adsorption is greatest for those ions that form with the substance chemical compounds of greatest stability.

Colloidal solutions are destroyed and the colloidal substance is precipitated by any means that will neutralize the charges on the dispersed particles or separate the adsorbed ions. The commonest means of destruction is to add an electrolyte from which the colloidal particles will adsorb ions of opposite sign to those that produced stabilization. An amount of electrolyte just sufficient to neutralize the charged particles produces maximum flocculation; a small excess will usually reverse the sign of the charge and tend to again produce and stabilize dispersion. (Burton, *Physical properties of colloidal solutions*, 1916.) Further excess decreases peptization until finally a point of absolute excess is reached beyond which peptization cannot be effected.

Coalescence of uncharged particles with formation of floccules is movement in accord with the second law of thermodynamics. The potential energy of a colloidal system is relatively large on account of the large area of INTERNAL SURFACE, *i.e.*, interface between the dispersed particles and the dispersing medium. Coalescence decreases this surface and thereby lessens the potential energy of the system.

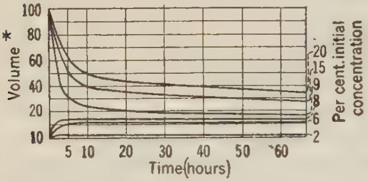
Primary slime. Most ores and rocks as mined contain both earthy secondary minerals that disintegrate in water to colloidal sizes, and suitable electrolytes to furnish the ions necessary to stabilize dispersion of the fine particles. The viscosity and density of this disperse system are such that coarser particles, which would settle in water, are suspended, if any turbulence at all exists. Thus primary slime is formed. Its satisfactory settlement is one of the difficult problems of milling.

Sedimentation. When a de-flocculated slime pulp is brought to rest there is gradual clearing of the upper part of the liquid, accompanied by collection of the coarser particles in the bottom of the container. This is called SUBSIDENCE SETTLING (Free, *101 J 681*) or SEDIMENTATION. If an actual colloidal suspension exists, complete settlement cannot be effected, but the overlying liquid will remain turbid.

The particles that deposit by sedimentation settle more rapidly the more dilute the suspension, because particles in a dilute suspension are more widely spaced and the effects of their electrical charges on their neighbors are least. Dilution also lessens both the viscosity and density of the colloidal solution and these changes likewise aid settling, as may be seen from Stokes' equation. Shellshear (*8 Aa 5*) and Nichols (*17 IMM 328*) confirm this conclusion experimentally.

Heating increases the settling rate in de-flocculated suspensions containing semi-colloidal material, probably by decreasing the viscosity of the suspending medium.

Economy forbids attempts to thicken de-flocculated pulps, even when there is a considerable amount of semi-colloidal and granular material present, both on account of the retardation of the settlement of the coarser material by the colloid and on account of the loss or inconvenience due to the colloidal solid in the liquid fraction.



* Ordinate is the height of the upper surface of the settling material, divided by the total height of the water surface, expressed as percentage.

Fig. 3.—Relation between settling rate and percentage of solids in a colloidal clay suspension (*after Free*).

Consolidation settling is the name suggested by Free for settlement of flocculated pulps. When such pulps settle the solids fall *en masse*, clear liquid appears at the surface immediately, and the liquid layer increases in thickness through addition of water displaced upward by consolidation of the settling solids. In Fig. 3, the three lower curves illustrate sedimentation, the three upper, consolidation settling. The slopes of the curves, irrespective of sign, measure settling rates.

Stokes' equation cannot be practically applied to the settling of flocculated pulps, because the flocs, which consist of unstable aggregates of solid particles holding considerable water, have indeterminate specific gravities; their surface is highly irregular and their shape nonuniform; finally they are in contact with their neighbors at many points, at least in all but the earlier stages of settling, and this condition violates one of the hypotheses upon which the equation was founded.

Compacting settling. Ralston (101 *J* 768), working with short (10-in.) settling columns, noted that when free-settling slimes reached a consistency by consolidation settling of about 40 per cent. solids the settling rate fell off markedly (see Fig. 4). This phenomenon was not noticeable in 36-in. columns. He describes such settling a compacting settling. It may be important in shallow-tray or baffle thickeners (Art. 7), but is not in thickeners of usual dimensions.

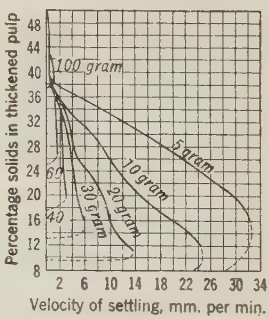


Fig. 4.—Effect of dilution on consolidation of slimes.

4. Flocculation

Consolidation settling yields clear overflow and a thickened product substantially as dense as that obtained by sedimentation. Since clear overflow is practically a *sine qua non* of pulp thickening and flocculation is essential to its accomplishment, means of flocculating slime pulps are of prime importance. Four means are known, *viz.:* (1) by addition of suitable electrolytes, (2) by addition of suitable colloids, (3) by guarding against production of too dilute feed pulps, (4) by heat. In so far as each of these methods is

effective, it must act by affecting the adsorbed ions that stabilize the slime colloid.

Flocculation by electrolytes is the most effective way to produce consolidation settling. Fig. 5, compared with Fig. 3, shows the effect of adding 0.25 gm. of NaCl per 100 cc. of water to clay suspensions. The point of change from sedimentation to consolidation settling comes between 2 and 3 per cent. initial solid concentration when the electrolyte is present, instead of between 8 and 9 per cent. without the added electrolyte, and the rate of settling is increased even at those concentrations at which flocculation already existed.

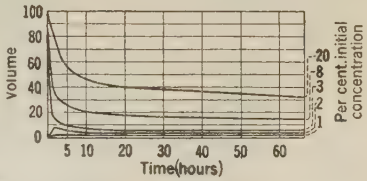


Fig. 5.—Settling curves of kaolin suspensions flocculated with sodium chloride (*after Free*).

The explanation of the effect of electrolytes is two-fold. Beans and Eastlack (*loc. cit.*) found that in making colloidal gold by the Bredig arc method, the amount of gold suspended varied with the quantity of electrolyte after the fashion of Fig. 6. If it is assumed that in a given slime the amount of colloid dispersed corresponds for that slime to the peak of the curve and that the sign of the colloid (*i.e.*, sign of the adsorbed ion) is negative, then when a suitable electrolyte is added *i.e.*, one of which the positive ion is adsorbed, the effect is to move down the left leg of the curve until, when substantially all of the initial ion charge is neutralized, complete flocculation exists. Ellis (*9 Trans. Faraday Soc. 14*), working with positive hydrate-of-iron colloid and negative cylinder-oil emulsions found that the resultant of the charges on the particles varied from negative through zero to positive with increasing addition of the iron hydroxide and that maximum instability occurred when the voltages were between -0.04 and $+0.035$ volt. Addition of further quantities of electrolyte may again produce dispersion. This means that the system is moving back to the right along the curve, this time with opposite sign. Large excess of added electrolyte again produces complete flocculation, corresponding to the field to the right of the point where the right leg of the curve cuts the X-axis.

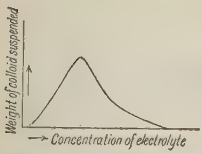


Fig. 6.—Relation between colloid dispersion and concentration of electrolyte.

Electrolytes containing polyvalent ions are, when the polyvalent ion is adsorbed, more effective both in flocculating and dispersing than those containing monovalent ions.

Ralston (*101 J 894*) states that a negatively-charged quartz colloid was equally flocculated by sodium, calcium and aluminum chlorides in amounts that varied as $1 : \frac{1}{40} : \frac{1}{1600}$. This is a well-recognized phenomenon of colloid behavior (Taylor, *General properties of colloids, p. 98*). It indicates that the amounts of different ions adsorbed from a solution of given concentration, as well as the neutralizing power of the individual ions, increases with the valence of the ion and that the resultant increase in effect is of the general order $1 : x : x^2$. (Whetham, *Theory of solution, 396.*) The generally accepted ratio for metallic ions is 1 : 35 : 1023. Ralston (*101 J 894*) also cites an experiment in which a given slime was equally flocculated by 1 per cent. of NaCl, 0.1 per cent. of H_2SO_4 and 0.01 per cent. of $Fe_2^{III}(SO4)_3$. As between the sodium chloride and ferric sulphate, this may be looked upon as another instance of the greater potency of the polyvalent ion, if the colloid was negative, but this assumption does not explain the position of the sulphuric acid. On the other hand, if the colloid was positive, the sodium chloride and sulphuric acid are in line, but the effect of the ferric sulphate is not explained. The probability is that the colloid was negative and the greater potency of the sulphuric acid was due to the commonly observed fact that hydrogen and hydroxyl ions are much more readily adsorbed than the other univalent ions (Taylor, *loc. cit. p. 102*).

Cataphoresis is the migration of colloidal particles in an electric field. The direction of migration is dependent upon the sign of the electrical charge on the particle. Taylor summarizes the behavior of SUSPENSOID (solid-particle) COLLOIDS in the statement that most such are negatively charged, *e.g.*, the metals, metallic sulphides, and most other

Table 2. Effect of added substances on settling rates. (*After Ralston*)

Ore (*v*)	Horn Silver, (*a*) per cent.		Grand Central, (*a*) per cent.		Bullion Coalition, (*a*) per cent.		Ophir Hills, (*a*) per cent.	
Added substance	Amount added	Increase in settling rate (*c*)	Amount added	Increase in settling rate (*c*)	Amount added	Increase in settling rate (*c*)	Amount added	Increase in settling rate (*c*)
Oxalic acid	0.08*b*	−40	0.04	0				
Carbolic acid		0						
Citric acid	0.0125	−31	0.01	*e*	0.0125	−80		
	0.05	−33						
	0.10*d*	−22						
Hydrochloric acid	0.025	−16*f*	0.025	−5				
			0.204	−15				
Sulphuric acid	0.04	−11		*g*				
	0.16	+6					0.0516	−10
Sodium hydroxide	0.025	+41*h*	0.0516	*i*	0.0258	*j*	0.0516	+37
			0.4	*w*				
Sodium carbonate	0.05	+46*k*	0.026	*i*				
			0.050	*l*				
Sodium silicate	0.1	*m*	0.02	−5	0.0516	*p*		
	0.4	*n*	0.05	*j*				
Ferrous sulphate	0.82	−30*b*		*q*				
Alum	0.025	−23	0.0258	*g*				
Sodium chloride	1.0	*r*	No effect	No effect	No effect	No effect	0.02	−7
Calcium hydroxide	0.00625	+18						
Tannin	0.025	−60	0.0516	*j, s*	0.026	*t*		
Gelatin	0.0013	+90*u*	0.0006	+50	0.006	+5		
			0.0012	+90				
Saponin	0.005	−21	0.005	−12				
Egg albumen	Very small effect			*g*				
Glue	0.0001	+32	0.0001	+50				
Soap	0.005	+48	0.001	+80				
Ox-blood	0.5	*l*	0.00516	+62				

a See Table 2*a* for analysis. *b* Increasing amounts below this maximum gave increasing retardation. *c* Negative sign indicates retardation. *d* Subsequent addition of NaOH increased dispersion. *e* De-flocculates pulp, especially the iron. *f* Final pulp density unaffected. *g* Very slight de-flocculation. *h* Final pulp, 30 per cent. solids. *i* De-flocculates iron. *j* De-flocculates ore. *k* Final pulp, 35 per cent. solids. *l* De-flocculates ore completely. *m* Slight retardation, 32 per cent. final solids. *n* Slight retardation, 24 per cent. final solids. *p* De-flocculates ore, sulphuric acid subsequently added flocculates and increases settling rate. *q* Effect small but change from sedimentation to consolidation settling occurs higher in tank. *r* No effect up to this amount. *s* 0.10 per cent. alum in addition flocculates. *t* De-flocculates and sands settle. *u* Final pulp, 32 per cent. solids. *v* All pulps 9.4 per cent. solid. *w* Speed increasing.

metallic compounds except oxides and hydroxides; the sulphur group of elements; silicic and stannic acids; and a variety of other substances such as mastic, gamboge, fuchsin, eosin, indigo, aniline blue, etc. The basic hydroxide sols are positively charged.

Effect of various substances on settling rate of ore slimes, as determined by Ralston (*101 J 991*) is given in Table 2.

Table 2*a*. Analyses of ores used in Ralston settling tests, Table 2.

Ore from...	Horn Silver, per cent.	Ophir Hill, per cent.	Grand Central, per cent.	Bullion Coalition, per cent.
Insoluble...	53.6	51.5	69.0	12.5
CaO.......	2.2	4.97	2.45	1.1
Fe........	4.4	4.3	13.05	15.0
Al_2O_3......	*a*	*a*	2.8	8.6
MgO......	Tr.	*a*	Tr.	8.7
Zn........	6.3	3.9	0.58	29.9
Pb........	7.4	3.83	0.05	0.21
Cu........	0.25	1.16	1.04	0.13
S..........	6.13	4.69	*a*	0.92
As........	*a*	0.25	*a*	*a*
CO_2.......	1.37	*a*	2.60	19.04

a Undetermined.

With the ores tested the organic acids were strong de-flocculants as were also the inorganic acids in low concentrations. The reversal in the action of sulphuric acid on Horn Silver ore with increasing concentration is in accord with Fig. 6. The powerful flocculating effect of the inorganic bases indicates that the general statement, frequently made, that ore slimes are usually negatively charged is not to be accepted without question. The experiments with sodium hydroxide indicate that the two sulphide-ore slimes, Horn Silver and Ophir Hill, were positively charged while the remaining two, which were oxidized ores, were negatively charged. The action of the inorganic salts indicates that the metallic ions were strongly adsorbed on the Horn Silver slime while the acid ions were rather feebly adsorbed on the Grand Central. The tri-valent Al ion is shown to have been much more effective than the bi-valent Fe.

Shellshear (*8 Aa 12*), working with 2-per cent. suspensions of extreme fineness and of many minerals (see Table 3) found that sulphuric acid increased the settling rate of all

Table 3. Settling rates of various mineral powders. (*After Shellshear*)

Rapid, 1½ to 3 hr.		Intermediate, 4 to 7 hr.		Slow, 7 to 168 hr.	
Mineral	Time, hours	Mineral	Time, hours	Mineral	Time, hours
Blende...........	3–4	Chalcopyrite.....	4–5	Tin oxide........	96–168
Calcite...........	2–3	Bornite..........	4–5	Wolframite......	96–168
Garnet...........	2–3	Molybdenite.....	4–5	Feldspar.........	72– 96
Pyrite............	3–4	Quartz..........	7–9	Steatite..........	20– 30
Magnetite........	2–3	....	....	Mica schist......	30– 50
Stibnite..........	3.5	....	....	Rhodonite.......	48– 96
Galena...........	3	....	....	Scheelite.........	72– 96
Fluorite..........	2–3	....	....	....	....

classes; calcium chloride was particularly effective on silicious material; lime was not so potent as calcium chloride but, like it, was most effective with silicious material; sodium hydroxide in small quantities dispersed silicious material, but accelerated settlement in greater concentrations; alum and sodium bicarbonate, added together, produced a gelatinous precipitate that swept down the suspension; potassium permanganate accelerated settlement greatly. Bruhl (*107 J 1089*) notes that slaked lime is 30 per cent. more soluble than unslaked and produces a more marked settling effect. Sulman (*17 IMM 312*) says that ammonium chloride and other ammonia salts have marked coagulative effect on slimes.

Nichols (*17 IMM 322*) found no marked difference in effect between $BaCl_2$ (83 mg. per 200 cc.), H_2SO_4 (17 mg. per 200 cc.), lime (56 mg. per 200 cc.) and NaCl (167 mg. per 200 cc.) in a 2-per cent. pulp, but on the basis of weights of electrolyte added, H_2SO_4 is the most potent. At ANACONDA (*49 A 478*), settling a 2.5-per cent. slime pulp to 10 per cent. in Callow cones, 1 lb. per ton of ferrous sulphate increased settling rate (capacity) 30 per cent.; 2 lb., 40 per cent.; and 5 lb., 50 per cent. Sodium chloride, 0.25, 0.5 and 1 lb. per ton, respectively, increased the rate 3 per cent. to 4 per cent. only. A mixture of glue and ferrous sulphate, 0.25 lb. of each per ton, was remarkably effective; it increased the capacity of a 28 × 10-ft. Dorr thickener from 195,000 to 311,000 gal. of slime pulp per 24 hr., and on round-table concentrate caused the same settlement in 1.25 min. that required 11 min. with untreated pulp. Nicolai (*103 J 1064*) found that $MgCl_2$ in the end liquor from German potash plants, added in the proportion of 1 cc. of solution per liter of slime, increased settling 67 to 95 per cent.

The effect of electrolytes on settling rate is greater, the more dilute the feed pulp.

Table 4 (*101 J 991*) shows that with 5 per cent. solids in the feed an increase of 35 per cent. in settling velocity is effected by addition of 0.025 per cent. lime while with 20 per cent. solids the same lime addition produced only 4 per cent. increase in velocity over that with no lime. Addition of lime also caused the change from sedimentation to consolidation settling to take place much earlier in dilute pulps than it occurs without lime, while no such change occurred with thick feed pulps.

Table 4. Effect of electrolytes on pulps of different densities. (*After Ralston*)

Solids in feed, per cent.		CaO added, per cent.				
		0	0.0031	0.0062	0.0125	0.025
5	Increase in settling velocity, per cent.	0	17	39	39	35
10		0	1	18	7	20
20		0	0	3	3	4
5	Height of surface of settling solids when change from sedimentation to consolidation settling occurred	23	34	60	70	96
10		67	....	160	190	180
20		197	197	197	198	198

Flocculation by colloids is a special case of flocculation by electrolytes. The difference is that the neutralizing ion is added attached to a colloidal particle. The principal use of colloids as flocculators involves varieties that form jelly-like precipitates. Their usefulness lies probably as much in their sweeping action, when they themselves are flocculated by the other colloid, as in their neutralizing function.

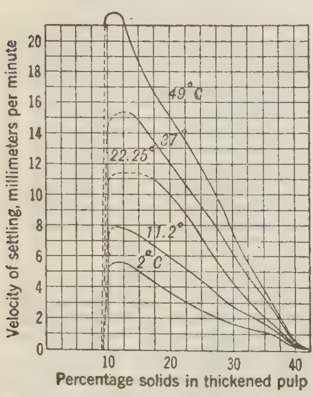


FIG. 7.—Effect of dilution on settling rate (*after Ralston*).

The effect of several organic colloids on ore slimes is shown in Table 2. Some of these, notably gelatin, exhibit both positive and negative charges in the same solution (Taylor, p. 83), which may explain the apparent anomaly of the equal effectiveness of gelatine as a flocculant on both the positively-charged HORN SILVER and the negatively-charged GRAND CENTRAL ores.

Flocculation by increase in solid concentration is illustrated by Fig. 7.

Pulps containing 8 per cent. or less of clay were so little flocculated that they could not be

cleared by settling, those containing 9 per cent. or more were flocculated and settled readily. The essential point here, however, is not fundamentally the concentration of solid but the concentration of electrolyte which was unintentionally introduced with and by the solid. Its presence is proven by the fact that dispersion was effected when solid concentration was low, since electrolyte is necessary to peptization. The behavior with an excess is that to be expected from Fig. 6.

This method of increasing flocculation is important in mill practice, since to reduce the amount of water added to a slime pulp before it reaches the thickeners means a saving due to lower water consumption or less return water to be handled. It is not always a possible solution, but the possibilities should always be closely investigated. Removal of primary slime ahead of hydraulic classifiers or jigs and sending it directly to the thickener will frequently accomplish the desired result.

Dilution of feed has an important effect on settling rate apart from its effect on flocculation.

Fig. 8 (*94 J 646*), presenting the results of laboratory experiment, shows roughly direct increase in settling rate with increase in feed-pulp dilution. Table 5 (*ibid.*) shows that the same relation holds in the mill and that laboratory settling rates are substantially the same as mill performances. Fig. 4 (*101 J 893*) shows that the difference in settling rate is not merely the one to be expected by reason of the greater freedom of the falling particles in dilute pulps, but that at any given pulp density in the consolidating layer the settling rate (velocity of subsidence of the surface of the consolidating layer) is greater the more dilute the feed pulp.

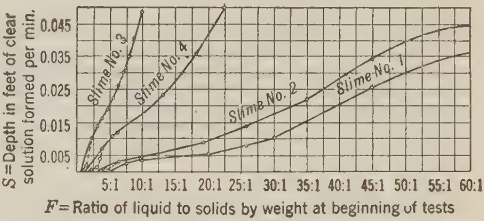


FIG. 8.—Effect of dilution on settling rate (*after Mishler*).

Table 5. Effect of feed-pulp dilution on settling rates in laboratory and mill. (*After Mishler*)

Mill						Laboratory
Number of days averaged	*F*	*D*	*T*	Tons of overflow per 24 hr.	*S*	Depth in feet of clear solution formed per minute
6	3.9	2.1	155	279	0.014	0.016
3	4.2	1.9	136	313	0.015	0.017
3	4.5	2.0	162	405	0.020	0.018
3	4.8	2.7	161	338	0.017	0.020
1	5.1	2.1	161	483	0.024	0.021
1	5.3	1.9	165	561	0.028	0.022
5	5.5	2.6	150	435	0.021	0.023
4	6.0	2.7	134	442	0.022	0.024
2	6.5	2.8	150	555	0.027	0.027
7	7.0	3.2	115	437	0.021	0.030
2	8.0	3.1	128	627	0.031	0.035

F Liquid : solid ratio in feed, by weight. *D* Liquid : solid ratio in discharge, by weight. *T* Tons dry slime per 24 hr. $S = 0.00004912\ T(F\text{-}D)$. Mill data are results of daily samples over 49-day period from a 24 × 12-ft. Dorr thickener. Laboratory tests were of 10-min. duration and were made in glass cylinders 0.1 ft. diam. and 2 ft. high.

Flocculation by heating. This phenomenon is quite different from that involved in the common practice of employing heat as a temporary expedient

to increase capacity of thickening tanks. Heating may increase flocculation, if the electrolyte that is stabilizing peptization is less soluble in cold than in warm water, and is present in excess of its soluble proportions. Under such circumstances heating will increase the amount of electrolyte dissolved and ionized and flocculation will result from excess of electrolyte according to Fig. 6.

This is undoubtedly the explanation of the case cited by Ralston (*101 J 891*) of a Mexican cyanide plant at which the slimes suddenly ceased to settle during a cold spell and normal working was restored by a few steam jets introduced into the solution.

Heating, however, does not always produce flocculation or aid settlement but may actually increase dispersion.

Nichols (*17 IMM 293*) reports a case in which high temperature (200° F.) with accompanying violent agitation resulted in de-flocculating a pulp and thereby markedly decreasing its settling rate. If initial flocculation was due to a position for the system to the left of the peak in Fig. 6, yet undissolved electrolyte was present, heating, if it caused more electrolyte to dissolve, would cause further peptization. If, on the other hand, initial flocculation indicated a state for the system to the right of the peak in Fig. 6, then unless there was an absolute excess of electrolyte present, the increased activity of the water molecules caused by heating would, by increasing the molecular bombardment producing Brownian movement, increase dispersion.

Effect of heat on slime settlement is ordinarily to increase the rate. Richards (*2 OD 1147*) calls attention to this fact and Dorr (*49 A 224*) states that heating may prove an economical method of adding 10 to 20 per cent. to the capacity of a thickener.

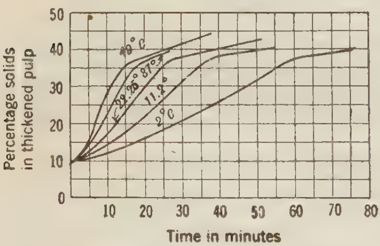


FIG. 9.—Effect of heat on slime settlement (*after Ralston*).

Nicolai (*103 J 1064*) states that the thickener overflow of a German lead-zinc mill carried from 0.02 to 0.1 gm. solid per liter on warm summer days and 0.2 to 0.4 gm. on cloudy or windy days.

The effect may be due to increase in flocculation but is more frequently due to decrease in the viscosity of the water. (See Sec. 25, Table 2.)

Figs. 7 and 9 summarize experiments by Ralston (*101 J 890*) on the effect of heat on settlement. He assigns all of the increase in settling rate with increased temperature to decrease in viscosity and shows that, for the data represented, the quotient obtained by dividing the time of settling to a given pulp thickness by the absolute viscosity is a constant, which confirms his conclusion for this particular set of data. Nichols (*17 IMM 321*) found that the rate of settlement of a 2.33-per cent. slime increased on heating from 60° F. to 190° F. and decreased at the same rate on cooling, which is substantially conclusive that the effect was due to change in viscosity.

Results of tests at ANACONDA (*49 A 479*) are given in Table 6. Laist and Wiggin call attention to the fact, as a matter of practical design, that while the average settling rate at the average summer pulp temperature of 50° is greater

Table 6. Effect of pulp temperature on settling rate of Anaconda slime. (*After Laist and Wiggin*)

Pulp temperature, degrees F.	Time to settle from depth of 17.5 in. to 3.5 in., minutes		
	Average	Maximum	Minimum
35	41.7	57.0	29.5
40	38.0	50.7	27.3
45	35.0	45.2	25.0
50	31.7	40.2	23.0
55	28.5	36.0	20.5
60	26.0	33.0	18.3
65	23.3	29.8	16.0
70	21.2	26.7	13.8
75	19.5	25.0	12.0

than that at the average winter temperature of 38°, yet other factors may enter to make the minimum rate at 50° less than the maximum, or even the average rate at 38°, and they conclude that pulp temperature is, therefore, a relatively unimportant factor in thickener-plant design.

Flocculation is a reversible phenomenon. Reversion may occur on account of changes in solid concentration when effective electrolytes are brought in with the solid, if the total amount of electrolyte present constitutes only a slight excess. When flocculation is effected by an added electrolyte, small variations either side of the iso-electric point, due to small deficiency or excess of electrolyte, will cause reversion. Changes in temperature may also be effective. Change in character of ore is another possible cause of reversion. Agitation will not usually cause reversion and even when such is the apparent effect, it is probable that some change in electrolyte concentration is the real cause. Any reversion in mill practice is likely to produce cloudy overflow from thickeners and decrease their capacity.

Depth of settling tank has no effect on settling rate with dilute feeds, *i.e.*, the rate of subsidence of the surface of the consolidating solids is the same irrespective of the depth of the tank, for any given pulp density, provided that the initial feed pulps are of the same density.

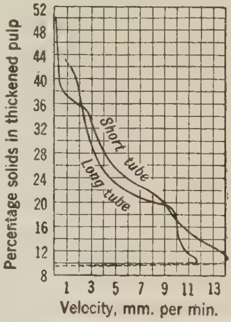


FIG. 10.—Effect of depth of tank on pulp thickening (*after Ralston*).

Fig. 10 presents a summary of Ralston's results (*101 J 892*) and Table 7 gives Mishler's data (*94 J 643*).

Table 7. Effect of depth of tank on rate of settlement in dilute feed pulps.(*a*)
(*After Mishler*)

Height of settling column in feet	Duration of test in minutes	L : S ratio		Depth of clear solution at end of test, feet	Depth in feet of clear solution formed per minute
		Beginning of test	End of test		
0.66	10	10 : 1	9.2 : 1	0.05	0.005
1.33	20	10 : 1	9.2 : 1	0.11	0.005
2.00	30	10 : 1	9.2 : 1	0.16	0.005
0.66	10	30 : 1	24.9 : 1	0.11	0.011
1.33	20	30 : 1	24.8 : 1	0.23	0.011
2.00	30	30 : 1	25.2 : 1	0.37	0.012
0.66	10	60 : 1	24.0 : 1	0.39	0.039
1.33	20	60 : 1	24.7 : 1	0.76	0.038
2.00	30	60 : 1	26.7 : 1	1.09	0.036

a Quiet settling in glass cylinder 2.3 ft. high by 0.1 ft. diameter. *L* Liquid. *S* Solid.

When the feed pulp is thick, and the tank shallow, the settling rate increases with the depth of tank, as is shown by Table 8. This table shows also that the transition between "thick" and "dilute" feed pulps comes at different densities, according to the depth of the tank; the greater the tank depth the thicker the pulp that settles at the dilute-pulp rate. Mishler points out, how-

ever, that the tanks used in practice are all "deep" compared to those investigated, and that in such deep tanks the settling rate for thick pulps, also, is independent of the depth of the tank.

Table 8. Effect of depth of tank on rate of settlement in thick feed pulps.(*a*)
(*After Mishler*)

Slime number	Initial $L:S$ ratio	Final $L:S$ ratio	Depth in feet of clear solution formed per minute		
			Depth of column ½ ft.	Depth of column 1 ft.	Depth of column 2 ft.
4	1.9 : 1	1.8 : 1	0.0001	0.0001	0.0002
4	2.1 : 1	2.0 : 1	0.0002	0.0002	0.0004
4	2.7 : 1	2.6 : 1	0.0002	....	0.0009
4	3.2 : 1	3.0 : 1	0.0003	0.0004	0.0012
4	3.6 : 1	3.4 : 1	0.0004	....	0.0045
4	4.5 : 1	4.3 : 1	0.0009	0.0031	0.0072
4	4.7 : 1	4.5 : 1	0.0013	....	0.0073
....	....	....	....	0.0067	....
4	5.0 : 1	4.7 : 1	0.0014	0.0090	0.0087
4	6.5 : 1	6.2 : 1	0.0092	0.0092	0.0090
1	6.0 : 1	5.7 : 1	0.0001	0.0001	0.0001
1	8.0 : 1	7.6 : 1	0.0002	0.0003	0.0004
1	9.0 : 1	8.5 : 1	0.0007	0.0021	0.0042
1	12.0 : 1	11.4 : 1	0.0009	0.0087	0.0092
1	14.0 : 1	13.3 : 1	0.0032	0.0092	0.0094
1	17.0 : 1	16.1 : 1	0.0067	0.0095	0.0092

a Quiet settling in glass cylinder 0.1 ft. diam. Each test made by noting time required to form a volume of clear solution equal to 1/20 part of the original volume of pulp. Below the broken lines the slime begins to follow the principles of settling for thin slime. *L* Liquid. *S* Solid.

For the effect of tank depth on density of discharge see p. 992.

Effect of kind of mineral on settling rate is not, apparently, related to the specific gravity.

Shellshear (*8 Aa 7*) has tested various minerals in 2-per cent. suspensions in pure water with the results shown in Table 3. He found that mixing rapid-settling and slow-settling pulps resulted in flocculation and great increase in settling rates. The extent of flocculation depended upon the proportions of the mixture.

5. Thickeners

Thickeners are tanks of such capacity that the time required to fill them at the rate of flow of the stream to be thickened is sufficient to permit the upper surface of the solid matter to settle a safe distance below the overflow level and the lower stratum of settled solid to consolidate to the desired consistency. They are provided with an overflow weir and one or more bottom-discharge openings so located as to draw off the settled solids uniformly. They are built either for intermittent or for continuous operation. The latter type may be discharged by gravity or mechanically. Intermittent thickeners have found their greatest use in the Rand gold mills; continuous thickeners, first of the gravity-discharge type but latterly of the mechanical type, are almost universal in the United States.

Intermittent thickeners are usually rectangular tanks or elongated ponds fed at one end and overflowed at substantially the same level at the other end,

Feeding continues until the overflow contains more than an allowable load of solid, when the feed stream is diverted to another tank. After the solids have settled the supernatant clear liquid is drawn off through pipes variously arranged for adjusting the draw-off level. The thickened solid is finally flowed or pumped out from a sump and the tank is ready for another charge. The draw-off level for liquid is varied by means of pipes placed at various heights in the walls, or by means of a jointed stand-pipe from the top of which short lengths are easily removed; most frequently by means of an inclined pipe joined, by means of a 90° ell, with a horizontal pipe in the bottom of the tank. (See Fig. 11.)

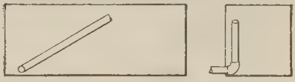


Fig. 11.—Adjustable draw-off for intermittent settling tank.

The horizontal cross-section of intermittent settling tanks should be such that the rate of rise of liquid therein is less than the settling rate of the surface of the solids in consolidation settling.

Thomas and Osterloh (*17 JCM 214*) state that two slime ponds with a total circumference of 6000 to 7000 yd. are sufficient to handle 30,000 tons of thick Rand slime per month.

Continuous thickeners of the gravity-discharge type are typified by the cone; the Dorr thickener is typical of the mechanically-discharged type. In these machines there is continuous feed of pulp, usually at the center of the tank, continuous peripheral overflow of substantially solid-free liquid and continuous discharge of thickened pulp from the bottom.

Cones are typified by the Callow cone (see Sec. 6, Art. 5).

At the Great Falls plant of Anaconda Copper Co. (*49 A 423*) 8-ft. cones treating −0.074-mm. slime were fed with slime pulp carrying 1.5 to 2 per cent. solids at the rate of 25 to 30 gal. per min. or 0.582 to 0.698 gal. per min. per sq. ft. of settling area. The overflow contained about 2 to 2.5 per cent. of the total solids fed and 90 per cent. of the total water. The spigot product ran about 8.75 per cent. solids and was discharged through a ⅜-in. gooseneck spigot under 24-in. head. Thirty 8-ft. tanks handled 1,000,000 gal. of slime per 24 hr. carrying 80 tons solid. At the Anaconda plant (*49 A 473*), the capacity of an 8-ft tank treating −0.074-mm. slime from a different ore, was only 12.3 gal. per tank per min. or 0.286 gal. per sq. ft. of settling area per min. The spigot discharge contained 9 to 14 per cent. solids representing 90 to 95 per cent. of the total solids fed (*46 A 253*).

Table 9. Effect of overfeeding a cone thickener

Rate of feed, gallons per minute per square foot	Solids recovered, per cent. of total feed
0.194	100.0
0.213	95.0
0.233	90.0
0.272	85.0
0.301	80.0
0.330	75.0
0.369	70.0

The effect of overfeeding is shown in Table 9, presenting results of a test on a rectangular settling tank 10 ft. by 9 ft. 6 in. by 8 ft. (deep), center fed, treating Anaconda slime with 2.5 per cent solids.

Lead ores have been treated at the rate of 30 to 35 gal. per min. of feed containing 1 per cent. solids, giving a spigot product with 25 to 40 per cent. solids and an overflow carrying 5 gm. per gal.

At Alaska-Gastineau 8-ft. 6-in. tanks were used for dewatering a feed containing 88.5 per cent. −200-mesh at the rate of 20 tons solid per 24 hr. in a pulp containing 96 per cent. moisture. The spigot product contained 91.2 per cent. moisture and 88.4 per cent. −200-mesh, the overflow, 98.3 per cent. moisture.

At Homestake (*99 J 412*) 26-ft. conical-bottom tanks, 23 ft. 9 in. deep, treated 406 tons of slime pulp (sp. gr. 1.036) per 24 hr. making 84 tons underflow containing 33 tons solid and 322 tons clear overflow. This pulp contained no reagent and water was at winter temperature. Increase in the feed rate to 505 tons pulp per 24 hr. increased the density of discharge to 33 tons solid in 86 tons pulp, but caused cloudy overflow.

Intermittent discharge lessens the difficulty in maintaining thick spigot

product from gravity cones. Fig. 12 shows a typical form. Speed depends upon amount of solids fed to the cone, the greater the amount the greater the number of openings necessary per unit of time.

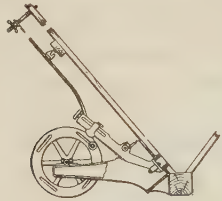


FIG. 12.—Ayton intermittent pulp-discharge valve.

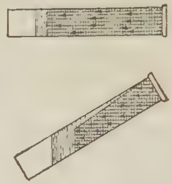


FIG. 13.—Effect of baffles on slime settlement (*after Dorfman*).

Baffles set at an incline in a thickening tank increase the settling capacity. Fig. 13 illustrates the action. The thickness of the layer of consolidating solid through which water must be expelled is lessened by inclining the container, and the number of surface pores available for egress of water is increased, both of which changes make for an increased rate of expulsion. Table 10 is a summary of the results of tests in a rectangular tank 3 ft. wide by 3 ft. deep by 9 ft. long at ANACONDA (*46 A 250*), with baffles of corrugated iron set at 45°, tops 4 to 6 in. below the overflow level and the lower ends about 9 in. from the tank bottom. The tank was fed at one end and overflowed the other; baffles were arranged in various positions without causing marked differences in result.

When high removal of solids is desired, there is little to choose between the open and baffled tanks, but with less complete solid removal the capacity of the baffled tank is markedly greater than that of the open tank.

Table 10. Comparison of baffled and open settling tanks at Anaconda. (*After Hayden*)

Feed rate, gallons per foot of width per 24 hr.	Per cent. of solids settled	
	In baffle tank	In open tank
1883*a*	100.0	100.0
2000*a*	99.9	94.8
3000	97.5	67.8
4000	94.0	53.5
5000	90.3	44.8
6000	86.8	38.0
7000	82.0	32.6
8000	75.9	28.0
9000	67.8	23.8

a Extrapolated.

6. Dorr thickener

Description. The Dorr thickener (Fig. 14) consists of a relatively shallow tank (*a*) of large horizontal area and with flat or slightly conical bottom, a central feed well (*b*), a peripheral overflow launder (*c*), finally, a slow-moving mechanism (*d*), suitably supported, carrying rake arms near the bottom of the tanks that serve, by their revolution, to move settled solids to a central bottom-discharge opening. Thickened pulp may be discharged by gravity alone through a suitable pipe-and-plug spigot, but is best handled by means of a diaphragm pump set slightly above the overflow level. Tanks are made of wood staves, sheet steel or concrete; in rare cases they are merely pits dug in the ground. Rake arms are set on a slope. When the feed is

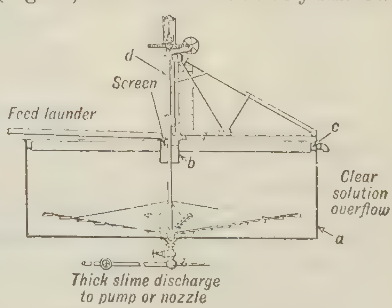


FIG. 14.—Dorr thickener.

low-grade, the solid is allowed to build up in flat-bottomed tanks to the slope of the rakes, but if the feed pulp contains much value, as for instance, flotation concentrate, the bottom is frequently filled in with cement concrete by pouring in a relatively stiff slow-setting mixture of sand and cement with rakes operating in the lowest position. When concrete tanks or concrete bottoms are used these are built to the desired slope initially.

The driving mechanism (Fig. 15) is fitted with a power screw (*a*) for changing the vertical position of shaft and rakes while operating, and a resistance indicator and overload alarm to call attention to the necessity for such change. Block (*b*) abuts against the end of shaft (*c*) which carries the driving worm. If the resistance of the rakes, transmitted through the worm gear, is great, block (*b*) is pushed back against the pressure of spring (*d*) and pointer (*e*) is moved toward the left. Dangerous resistance causes the pointer to move over until a copper plate (*f*) closes an electrical circuit and an alarm bell is rung. The remedy is to raise the rakes by means of (*a*) and to decrease the feed rate or increase the discharge rate or both. Increasing the speed of the arms will, in many cases, lessen resistance by decreasing the thickness of the layer of solid in motion toward the center.

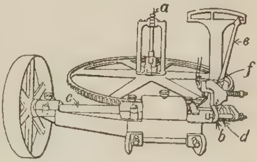


Fig. 15.—Driving mechanism and overload alarm for Dorr thickener.

Discharging thickened pulp. The old method was by means of a pipe-and-plug spigot. To discharge at some distance from this fitting, 4-in. pipe was extended to the discharge point and then bushed. With this arrangement thick discharges cause trouble due to deposition of solid in the 4-in. line.

Hanson (*109 J 853*) recommends replacing the 4-in. line by 1½-in. line to increase pulp velocity and prevent sedimentation.

The Dorr Co. now recommends a diaphragm pump mounted at or a few feet above overflow level with the suction pipe attached to the tank-discharge opening. The pump prevents clogging and permits close and positive regulation of rate and density of discharge by means of a pet-cock tapped into the pump chamber below the diaphragm, which regulates pump suction.

Cole (*100 J 131*) installed eight mechanically-operated 3-in. ball valves, on the 130-ft. thickener at Arizona Copper Co., and reports satisfactory operation. At Bunker Hill and Sullivan (*100 J 639*) a rotating cock, opened 15 to 25 times per min. eliminated clogging of a gravity discharge.

Performance at various mills is shown in Tables 11, 12, 13, 14 and 17.

Table 11. Performance of Dorr thickener on cyanide pulps

Mill	Elko Prince	Nevada Packard	Pittsburg Dolores	Cia. Real del Monte	Liberty Bell	United Eastern
Size, diameter × depth, ft.	26×16 *T*	28×10	30×5	30×10	33×10	40×12
Speed, min. per rev.	5.3	7	....	7	8	10
Feed, per cent. through 200-mesh	....	72	80	75	64	82
Feed, per cent. solid	12	20	40	7	9	20
Spigot product, per cent. solid	40–45	50	50	42	67	50
Overflow, per cent. solid	*c*	*c*	*c*	*c*	*c*	*c*
Tank area, square feet per ton of solid feed per 24 hr.	19.3	4.9	11.8	4.7	15.5	4.4
Tank area per gallon of liquid overflowed per minute	19.2	9.4	153	2.4	10	8.7
Rising current, mm. per second	0.03	0.06	0.004	0.25	0.06	0.07
Recovery of liquid, per cent.	82	75	30.8	89.6	95.2	75

T Single-tray. *c* Substantially clear.

Table 12. Performance of Dorr thickener on flotation-feed pulps

Mill	Chino Consolidated Copper Co.	Calumet & Hecla	Anaconda (*a*)	Anaconda (*a*)	Anaconda (*a*)	American Zinc, Lead & Smelting Co.
Size, diameter × depth, ft.	20×9	25×12	28×3	28×10	28×10	30×—
Power installed, hp.	2	...	0.12*b*	...	...	2
Speed, min. per revolution	5.5	...	...	...	...	4.5
Speed, peripheral, feet per minute	11.4	...	...	...	...	21
Feed, tons solid per 24 hr.	300	100	13.5	16	31.8	75
Feed, per cent. through 200-mesh	61	...	...	...	...	70.5
Feed, per cent. solid	12–25	5	2	2	3	3
Spigot product, per cent. solid	20–34	28	15	...	50	30
Overflow, per cent. solid	10	0.3	...	...	*c*	*c*
Tank area, square feet per ton of solid feed per 24 hr.	1.0	4.9	45.6	28.4	19.4	9.4
Tank area per gallon of liquid overflowed per minute, sq. ft.	3.5	1.8	6.2	...	3.7	1.8
Rising current, mm. per second	0.17	0.34	...	...	...	0.34
Recovery of liquid, per cent.	52.3	87.1	...	...	97	98.6

Mill	Timber Butte	St. Joseph Lead, Rivermines	St. Joseph Lead, Bonne Terre	Ray Consolidated Copper Co.	Bunker Hill & Sullivan	Calumet & Hecla
Size, diameter × depth, ft.	36×16*T*	40×8	40×8	40×10.5	40×12	40×16 (3*T*)
Power installed, hp.	...	1.25	1.25	5	...	...
Speed, min. per revolution	10	7.5	6	10	...	...
Speed, peripheral, feet per minute	11.3	16.8	20.9	12.6	...	...
Feed, tons solid per 24 hr.	350	200	125	750	90	250
Feed, per cent. through 200-mesh	...	65	...	98.2	95	97
Feed, per cent. solid	15	5	2.5	12	3.7	2
Spigot product, per cent. solid	35	40	32	22	50	23
Overflow, per cent. solid	1	*c*	*c*	9	*c*	0.5
Tank area, square feet per ton of solid feed per 24 hr.	5.8	6.3	10.1	1.7	14.0	20.0
Tank area per gallon of liquid overflowed per minute, sq. ft.	9.0	2.2	1.6	1.7	3.4	2.6
Rising current, mm. per second	0.07	0.28	0.38	0.36	0.18	0.23
Recovery of liquid, per cent.	68.5	92.1	94.6	79.6	96.2	94.7

Table 12. Performance of Dorr thickener on flotation-feed pulps—*Continued*

Mill	Miami Copper Co.	Chino Consolidated Copper Co.	Federal Lead Co. No. 4 Mill	Santa Barbara (*f*)	Santa Barbara (*f*)	Anaconda (*a*)
Size, diameter × depth, ft.	46 × 10	48 × 9	50 × —	50 × —	50 × —	50 × 12
Power installed, hp.	2	2	5	.....	.....	1
Speed, min. per revolution	.....	13	15	.....	.....	.....
Speed, peripheral, feet per minute	.....	11.6	10.5	.....	.....	.....
Feed, tons solid per 24 hr.	200	400	175	99	185	74
Feed, per cent, through 200-mesh	93.3	80.6	85*e*	.....	.....	*g*
Feed, per cent. solid	8	3-8	4	3.3	9.6	2.5
Spigot product, per cent. solid	33	30–40	20–25	20	40	50
Overflow, per cent. solid	0.3	*c*	*c*	.....	.....	*c*
Tank area, square feet per ton of solid feed per 24 hr.	8.3	4.5	11.2	19.8	10.6	26.5
Tank area per gallon of liquid overflowed per minute, sq. ft.	4.2	1.6	4.9	4.8	8.0	3.9
Rising current, mm. per second	.....	0.38	0.12	0.07	0.05	.....
Recovery of liquid, per cent.	83	90.3	86.1	86	84	97.5

Mill	Federal Mining & Smelting, Morning	U. S. Fuel Co. (*i*)	Chino Consolidated Copper Co.	Inspiration	Arizona Copper Co. (*j*)	Inspiration
Size, diameter × depth, ft.	50 × 12	70 × —	75 × 20	100 × 10	130 × 8.5	200 × 17
Power installed, hp.	5	.....	2	.....	5	2.5*b*
Speed, min. per revolution	14	.....	26	.....	30	.....
Speed, peripheral, feet per minute	11.2	.....	9.1	26.2	13.6	14.0
Feed, tons solid per 24 hr.	150	120	600	1150	700*k*	3500
Feed, per cent. through 200-mesh	75*h*	.....	94	89	.....	89
Feed, per cent. solid	1.8	2	8–10	18	6	16
Spigot product, per cent. solid	40	53	17–25	37.6	31	24
Overflow, per cent. solid	*c*	0.3	3	*c*	*c*	*c*
Tank area, square feet per ton of solid feed per 24 hr.	13.1	32.0	7.4	20.56	18.6	9.0
Tank area per gallon of liquid overflowed per minute, sq. ft.	1.5	4.0	6.1	14.0	7.6	30.2
Rising current, mm. per second	0.41	.....	0.10	.....	.....	.....
Recovery of liquid, per cent.	97.3	39.7	71	63.6	86	39.7

a *49 A 470*. *b* Actual. *c* Substantially clear. *e* Estimated. 100 per cent. −80-mesh. *f* *112 J 1056*. *g* Round-table concentrate from slime feed. *h* Estimated. 63 per cent. −240-mesh. *i* *63 A 686*. Coal sludge. *j* *100 J 131*. *k* This machine has handled 1200 tons per 24 hr. and Cole thinks it could handle 2000 tons of −40-mesh tailing (*13 CME 92*). *T* Single-tray. *3T* 3-tray.

Table 13. Performance of Dorr thickener on flotation concentrate

Mill	Cananea Consolidated Copper Co.	Federal M. & S., Morning	Braden
Size, diameter × depth, ft.	30×6	30×12	*b*
Power installed, hp.	2	5	......
Speed, min. per revolution	3	14	......
Speed, peripheral, feet per minute	31.4	6.7	11.1*d*
Feed, tons solid per 24 hr.	38–44	80	772
Feed, per cent. through 200-mesh	64	98*a*	95.
Feed, per cent. solid	40	2.8	12.8
Spigot product, per cent. solid	60	58	49.2
Overflow, per cent. solid	0.2	*c*	7.3
Tank area, square feet per ton of solid feed per 24 hr.	17.2	8.8	18.1
Tank area per gallon of liquid overflowed per minute, sq. ft.	166	1.6	17.2
Rising current, mm. per second	0.004	0.38	0.04
Recovery of liquid, per cent.	41.7	98	92.4

Mill	Belmont-Surf Inlet	Phelps-Dodge, Morenci	Consolidated Arizona Smelting Co.
Size, diameter × depth, ft.	36×12	39×10	40×10
Power installed, hp.	......	1.25	5
Speed, min. per revolution	8	3	5
Speed, peripheral, feet per minute	14.1	40.8	25
Feed, tons solid per 24 hr.	45	30	125
Feed, per cent. through 200-mesh	......	90.5	......
Feed, per cent solid	14	4	15
Spigot product, per cent solid	60–70	31	60–65
Overflow, per cent. solid	*c*	*c*	*c*
Tank area, square feet per ton of solid feed per 24 hr.	22.6	39.8*e*	10.0
Tank area per gallon of liquid overflowed per minute, sq. ft.	24.2	11.0	11.9
Rising current, mm. per second	0.03	0.06	0.05
Recovery of liquid, per cent.	91.2	90.7	89.7

Mill	Bunker Hill & Sullivan	Chino Consolidated Copper Co.	Federal Lead Co., No. 3 Mill
Size, diameter × depth, ft.	40×12	48×20	50×10
Power installed, hp.	......	2	5
Speed, min. per revolution	......	11	15
Speed, peripheral, feet per minute	......	13.7	10.5
Feed, tons solid per 24 hr.	17	150	13
Feed, per cent. through 200-mesh	......	91.3	......
Feed, per cent. solid	......	5–10	......
Spigot product, per cent. solid	35	50	......
Overflow, per cent. solid	*c*	1	......
Tank area, square feet per ton of solid feed per 24 hr.	74	12.1	151
Tank area per gallon of liquid overflowed per minute, sq. ft.	......	6.3	......
Rising current, mm. per second	......	0.10	......
Recovery of liquid, per cent.	......	92.9	......

a Estimated. 96.1 per cent. through 240-mesh. *b* 7 tanks as follows: 1 @ 30 × 10.25, 1 @ 34.75 × 8.6, 1 @ 34.75 × 10.25, 4 @ 60 × 13. *c* Substantially clear. *d* Maximum. *e* 80 sq. ft. per ton required to thicken to 50 per cent. solids (*69 A 178*).

Speed of rake arms depends to some extent on the character of the feed, *e.g.*, sandy feeds and those containing solids of high specific gravity require higher speeds than ordinary slime pulps. The usual peripheral speed of rake-arm tips on slime pulps is 10 to 15 ft. per min.; on flotation concentrate, 15 to 20 ft. per min.. Speed should not be sufficient to stir up the solid. With flocculent material slow speed, with consequent high resistance, gives thicker discharge, apparently because of consolidation under the rake pressure.

Table 14. Performance of Dorr thickener at Middlefork washery of U. S. Fuel Co.
(*After Campbell, 63 A 686*)

	Feed	Overflow	Spigot
Water, per cent.	98	99.7	47.2
Specific gravity	1.0052	1.0008	1.1580
Tons of pulp per hour(*a*)	500	485	15

a This represents the wash water from about 1200 tons of washed coal per 8 hr. Two @ 70-ft. tanks are used.

Power consumption is very small, probably not over 0.1 hp. for thickeners under 50 ft. diameter, but practice is to install larger motors, particularly with individual drive, in order to save stalling when, for any reason, the load of solid builds up, and also to furnish excess power in starting up after a shut-down.

Labor is a negligible item in operation. Except where an exceptionally large number of tanks is in use, operators on adjacent machines can take care of the tanks. At ANACONDA 1 man, full time, takes care of 40 @ 28-ft. tanks. At CHINO COPPER CO. 1 man attends 21 tanks of 20 to 75-ft. diameter.

Repairs are substantially *nil*, barring accidents due to attempts to start after a shut-down when the rakes are imbedded in solid. Such accidents may be avoided by raising the rakes before shutting down and lowering them slowly on starting. Wooden tanks rot in course of time and should not be used, if long life is desired, except where metal cannot be used on account of the chemical character of the liquid and concrete is not justified.

Capacity depends upon the kind of feed and the density of discharge pulp desired. It is usually reckoned in terms of square feet of tank area per ton of solid per 24 hr. or per gallon of liquid overflowed per minute. With relatively dense feed pulps the tonnage of solid is the limiting factor while with dilute pulps the volume of overflow governs. On cyanide pulps running 75 to 90 per cent. through 200-mesh and containing from 10 to 20 per cent. solids the usual allowance is 4.5 to 5 sq. ft. of settling area per ton of solid per 24 hr., up to 15 or 20 sq. ft. when considerable clayey matter is present. The area allowed with cyanide pulps per gallon of overflow per minute ranges from 2.4 to 19.2 sq. ft., average 9.7 sq. ft., but this is not the determining factor in this service. The rising velocity in tanks fed with the more dilute pulps was as high as 0.25 mm. per sec. Velocities with thicker feeds were far below this, which indicates that considerably more water could have been added to these feeds without making it necessary to reduce the solid tonnage.

In thickening flotation-feed pulps the area allowance ranged from 1 to 20 sq. ft. per ton of solid feed per 24 hr., average 8.4 sq. ft.; and from 1.5 to 9.0 sq. ft. per gal. of water overflowed per min., average 3.2. These feeds contained, in general, less than 5 per cent. solid and volume of overflow was the determining factor.

Ramsey (*57 A 412*) recommends 6 to 7 sq. ft. per ton to discharge at 20 per cent. solids. At TUL MI CHUNG mill (*114 P 362*) addition of 2 lb. caustic soda per ton (calcitic ore, 40 per cent. limestone) decreased settling rate to such an extent that where formerly the allowance was 6 sq. ft. per ton per 24 hr. for clear overflow, 36 sq. ft. was necessary after the addition, the feed containing 25 per cent. solids.

Rising velocities with flotation-feed pulps ranged from 0.10 to 0.41 mm. per sec., average 0.19 mm.

In thickening flotation concentrate before filtering, the area allowance ranged from 8.8 to 39.8 sq. ft. per ton of solid per 24 hr., average 18.4; and from 1.6 to 24.2 sq. ft. per gal. of overflow per minute, average 12.0.

Ramsay (*56 A 715*) states that INSPIRATION, ANACONDA, RAY and CHINO are thickening copper concentrate to about 60 per cent. solids on a tank allowance averaging 40 sq. ft. per ton of solid feed and that UTAH COPPER thickens to the same consistency in a 75-ft. tank at

the rate of 200 tons per day, or 22 sq. ft. of tank area per ton. Zinc concentrate is easier to settle. TIMBER BUTTE, BUTTE AND SUPERIOR and ANACONDA allow 12 to 15 sq. ft. per ton per 24 hr. when thickening to 60 to 70 per cent. solids. At AFTERTHOUGHT (*119 J 154*) by taking an overflow containing 2 per cent. solid, a mixed copper-zinc concentrate was thickened from 25 per cent. to 75 per cent. solids at the rate of 150 tons per 24 hr. in two 21-ft. tanks, or 1.6 sq. ft. per ton per 24 hr. Area requirement for lead concentrate is about the same as for zinc.

This is the most difficult service on account of the extreme fineness of the solid, lack of flocculation, difficulty of breaking up the gas-mineral aggregates, and thick spigot discharge required. Rising velocities allowed in most cases were of the order of 0.05 mm. per sec.

Breaking flotation froth is one of the principal difficulties in thickening this class of material. It is necessary either (*a*) to subject bubble walls to exterior mechanical strains sufficient to cause rupture or (*b*) to so disturb the surface-tension equilibrium that they rupture by reason of the disturbance.

Water jets and sprays of various kinds are the commonest of the mechanical methods of breaking. It is necessary to strike practically every bubble that is to be broken, hence the water must be projected at the froth as a multitude of small masses (spray) or as a number of small streams (multiple jet) or as a sheet. The water must be under some pressure in order to be moving with sufficient velocity to break the bubbles, but the velocity should not be so high as to produce froth by cascade action. (See Sec. 12, Art. 11.) The sprays or water jets are applied in the launders leaving the flotation machines or are directed at the stream as it enters the thickener.

Pumps and bucket elevators have been used at several plants but neither is particularly satisfactory. If the froth is persistent and voluminous it will not enter these machines, but overflows the sumps and water must be used to break it down. The cascade action at the discharge will cause considerable froth to form.

At CIA COROCORO DE BOLIVIA (*112 J 983*) a very tough froth was broken by running it into the intake of a 4-ft. × 6-in. ventilating fan running at 250 r.p.m. The fan handled more than 50 tons of froth per 24 hr. Blades lasted 4 to 6 months. Only the foam was sent to the fan.

Soluble reagents that lower surface tension greatly in relatively low concentrations, such as pine oil, wood creosotes and the like have been mixed with the spray water at some plants. Water containing such reagents lowers the surface tension of the bubble films that it strikes to such an extent and so suddenly that the adjacent film contracts sufficiently to break the bubble wall at the point of low tension.

The use of Wilfley tables as froth breakers is an application of the method of surface-tension change.

When the reagent that is added is suitable for flotation in the plant, thickener overflows and clear water from the filter plant pumped back to the flotation machines carry much of the oil value with them.

Lime has been added at Cœur d'Alene plants to break down froth (*105 J 716*) but the probability is rather that it promotes settling, since it has not sufficient effect on surface tension to act from that angle.

Frequently small-bubble persistent froth collects on the surface of concentrate thickeners and, if not confined, overflows. If it can be confined by a ring placed a few feet in from the overflow rim and extending down for several feet into the tank and up a foot or more above the overflow rim, it gradually dries out and compacts and equilibrium is reached with settlement of the densest portion from the bottom near the periphery balancing accession at the center. At MIAMI this end was attained by using several tanks more or less intermittently and thus allowing sufficient time in each for the necessary compacting. At GOLD HUNTER (*107 J 839*) floating froth was scraped off into a special launder by arms connected to the revolving mechanism, and joined with the thickened product ahead of the filter plant.

Density of discharge pulp depends primarily upon the specific gravity of solids and the size of particles. In thickening flotation feed a spigot discharge containing from 20 to 30 per cent. solids is usually all that is desired. On the other hand, in thickening concentrate a spigot discharge containing 50 to 70 per cent. solids is wanted and capacity is correspondingly reduced. In cyanide work, also, a thick spigot discharge is sought, but capacity is high on account of the flocculating effect of the electrolytes present. In general, silicious ores can be thickened to greater density, with a given feed rate, than can clayey and highly oxidized feeds.

Dorfman (*20 CMI 103*) reports, as the results of experiments on four different varieties of PORCUPINE ore, that a quartz pulp will settle to 35 per cent. moisture in 3½ hr.; a gray schist,

in the same time, to 51 per cent.; quartz porphyry to 60 per cent. and sericite schist to 64 per cent. The latter still contains 47 per cent. moisture after 18 hr. settling.

Coarse feed yields thicker discharge than fine, with a given feed rate. The limit of economical thickening of fine-ground clayey material, even with flocculating agents present, is probably between 40 and 50 per cent. solids while clean silicious ore, equally ground, can be readily thickened to 60 or, in some cases, 70 per cent. solids. Thick feed produces thicker discharge than dilute feed.

Ralston (*101 J 991*) notes that when flocculation is produced by electrolytes, the discharge is increasingly moist with increased flocculation of the pulp. He states that heating to accelerate settlement does not have the same bad effect.

Distribution of material in an operating thickener is shown by Figs. 16 to 18 incl. The deep clear-water layer in Fig. 16 leaves a safe working margin for feed fluctuation while the shallow layer in Fig. 17 will quickly disappear with slight increase in solid content of the feed or decrease in settling rate. The segregation of sizes shown in Fig. 18 is to be expected, the larger and heavier solids settling near the center. The act of raking to the central dis-

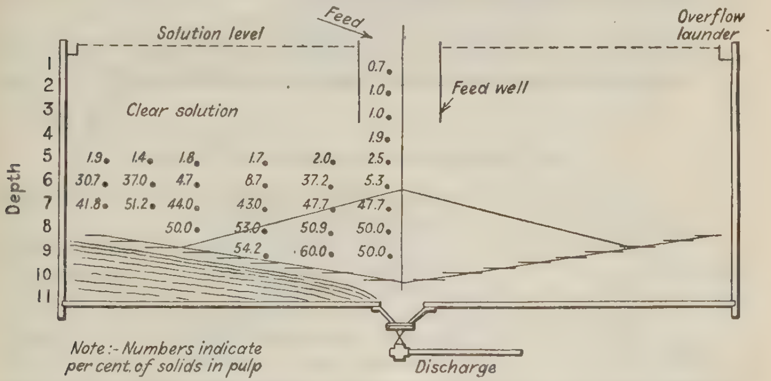


FIG. 16.—Pulp densities in a Dorr thickener, normal feed rate (*after Dorfman*).

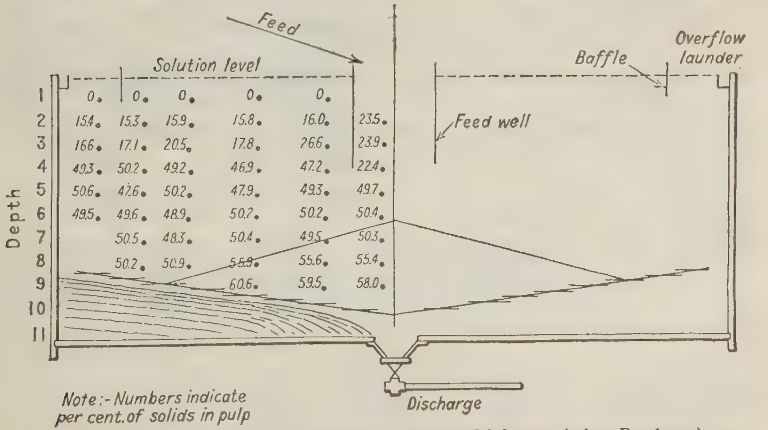


FIG. 17.—Pulp densities in an overfed Dorr thickener (*after Dorfman*).

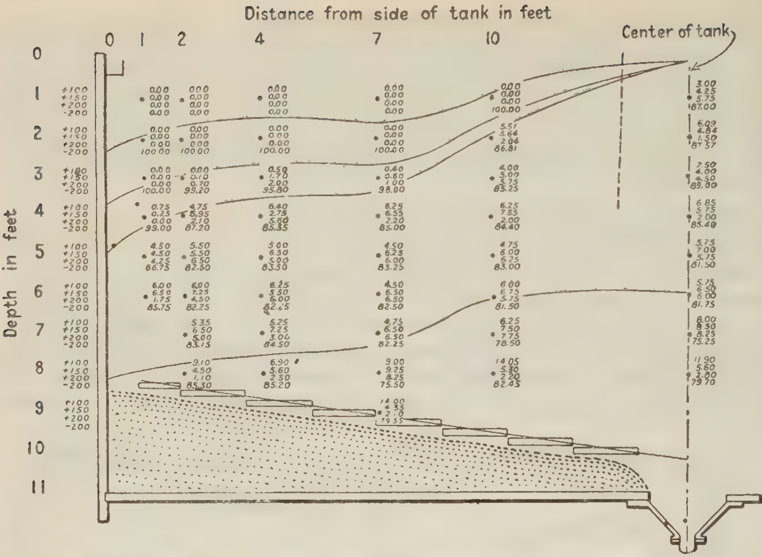


FIG. 18.—Size distribution in a Dorr thickener (*after Dorfman*).

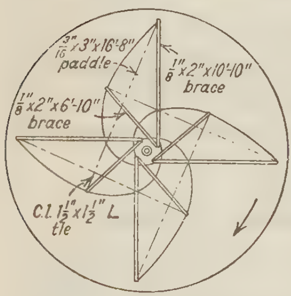


FIG. 19.—Rake for El Tigre thickener.

charge, of course, restores the size distribution to that of the original feed.

Effect of depth of tank on density of spigot discharge is small. The time during which consolidated pulp is compacting is the important element in determining discharge density. Shallow tanks ordinarily show slightly thicker discharges for a given time of settling, on account of the fact that compacting starts earlier therein. Table 15 is typical.

El Tigre thickener (*92 J 691*) is a Dorr tank with spiral paddles (see Fig. 19) replacing the rakes. The face of the spiral blade is approximately 45° to the tangent to its circular path at any point. The object of this form of rake is to prevent eddying.

Table 15. Effect of depth on density of spigot discharge. (*After Coe and Clevenger, 55 A 356*)

Time of settling, hours	Depth of tank: 114 in.		45 in.		11 in.	
	Depth of clear liquor, inches	Liquid-solid ratio in settled pulp	Depth of clear liquor, inches	Liquid-solid ratio in settled pulp	Depth of clear liquor, inches	Liquid-solid ratio in settled pulp
0	....	3.26–1	....	3.26–1	....	3.26–1
1	1.5	....	1.5	....	1.56	2.75–1
5	9.4	2.95–1	9.75	2.47–1	3.92	1.97–1
23	45.5	1.81–1	19.5	1.69–1	5.25	1.53–1
29	51.5	1.62–1	21.0	1.56–1	5.39	1.49–1

7. Tray thickeners

Description. These are essentially shallow thickeners superimposed, with a common shaft carrying independent rakes for each tank. Fig. 20 shows the type in use at ANACONDA with separate tanks, known as the SUPERPOSED TYPE; Fig. 21 shows the SUBMERGED TYPE with one or more trays in the same deep tank. The first type is more accessible, the second more compact and has all overflows at the same level.

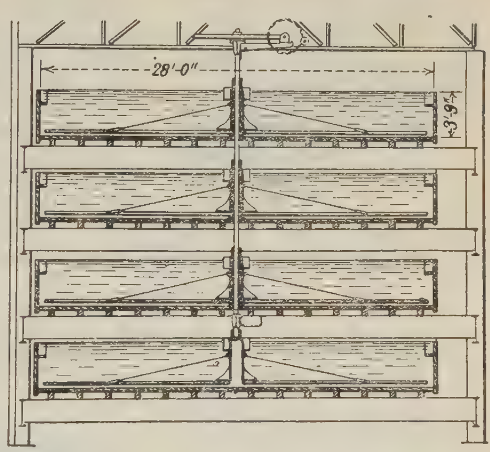


FIG. 20.—Dorr tray thickener, superposed type.

The principle of the tray thickener is illustrated by an experiment reported by C. H. Jakin (*94 J 872*) and analyzed by Mishler (*94 J 1114*). Jakin found that one slime with which he worked settled from 59 per cent. to 46 per cent. moisture, yielding clear overflow, in 45 min. in a tank 1.25 ft deep and required 420 min. to attain the same thickness in a tank 12 ft. deep. With another slime, thickening from 90 per cent. to 81 per cent. moisture, 120 min. was required in a tank 1.25 ft. deep and 1020 min. in a tank 10 ft. deep. Mishler shows that the settling rates were 0.0092 and 0.0089 ft. per min. in the deep and shallow tanks respectively in the first experiment and 0.0050 and 0.0052 ft. per min. in the second, *i.e.*, substantially the same, as his previous experiments had shown to be the case, and since there was sufficient time to produce thick pulps of the desired consistency in the shallow tanks, both shallow and deep thickened at the same rates. Actually, the time required to effect the desired compacting of discharge pulp is the essential factor in determining minimum depth of tank, while, since the settling rate is independent of depth, the amount of clear overflow produced is dependent only upon tank area.

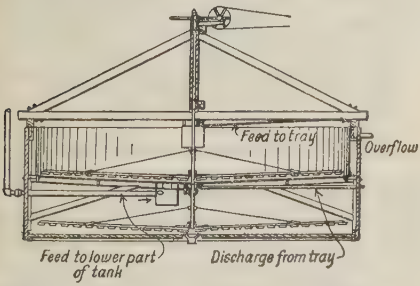


FIG. 21.—Dorr tray thickener, submerged type.

Performance. Laist and Wiggin (*49 A 474*) report that 28 × 3-ft. tanks at ANACONDA had 85 per cent. of the capacity in thickening flotation feed, of 28 × 9-ft. tanks and that the capacity of 28 × 2-ft. tanks was 85 per cent. that of the 3-ft. tanks. At LIBERTY BELL (*49 A 221*) a tray in a 33 × 10-ft. tank doubled capacity. At TONOPAH BELMONT addition of trays to 30 × 12-ft. tanks increased capacity about 75 per cent. (*52 A 104*). In general, with quick-settling ores, when discharging relatively thin pulps, trays will increase capacity upwards of 90 per cent., somewhat less with clayey ores or thicker discharges. OLD DOMINION (*56 A 715*) is using a 40-ft. single-tray thickener for 600 tons per day of flotation concentrate in a pulp containing 8 per cent. solids. The spigot product contains 63 per cent. solids and the overflow is clear.

Trays are not suitable when there are great changes in character or quantity of feed; the excess volume of a deep tank is then necessary to take care of the fluctuations in depth of clear-water layer dependent on the feed changes.

Hydrotator thickener (Fig. 22) may be used either as a dewaterer or a de-slimer. The machine superimposes on the ordinary continuous-thickener principle the further operation of taking suspended solid matter from the top of the settling tank, as at (*D*) or (*E*) and pumping it, by pump (*G*) into the rotating sprinkler mechanism (*B*) thereby placing such solid nearer the discharge point. Settled solid is drawn off from any of the pipes (*F*), as desired.

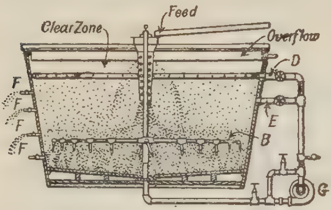


FIG. 22.—Hydrotator thickener.

The machine is new; it has been tested principally on fine coal. In one test – 3/32-in. anthracite slush (15 per cent. + 20-mesh; 35 per cent. – 100-mesh) yielded clear overflow at the rate of 1.3 gal. per sq. ft. per min. and overflow containing 1 per cent. solids (all – 100-mesh) at the rate of 2.2 gal. per sq. ft. per min. A shallower machine is used as a de-sliming classifier. Another form has been used for froth flotation (see p. 805).

8. Filter thickeners

Hardinge super-thickener (Fig. 23) is a combination of the usual settling-type dewaterer with mechanical removal of settled solids (Dorr thickener), and a sand filter. Its underlying principle is to increase the rate and extent of compacting (see p. 974) of settled solid by making the direction of the currents of expelled water downward instead of upward. It does this by maintaining a head of solution on the sand-filter bottom, either by suitable difference in elevation between the level of overflow and filtrate discharge or, if this is insufficient, by discharging filtrate by means of a vacuum pump. In Fig. 23 (*a*) is the thickener tank with overflow launder (*b*), spigot discharge (*c*) through a diaphragm pump, scraping mechanism (*d*) for moving settled solids toward the discharge spigot, a sand bed (*e*) through which filtrate is to be drawn, supported on the usual framework of studs, slats, cocoa-matting and burlap (*f*) (see Sec. 17). A pipe (*g*), connected to the space under the filter bottom, discharges filtrate either into the atmosphere or into a vacuum tank, according to the filtering duty required. The essential element of successful operation is the continuous removal of a thin skin from the top surface of the filter bed, thus maintaining this at full filtering efficiency (see p. 1000). This is effected by means of a thread (*h*) on the upper end of the drive shaft and a ratchet-driven feed mechanism (*i*) which lowers the scraping mechanism at the rate of 1/64 in. to as much as 8 in. per 24 hr., depending upon the amount of material present that tends to clog the filter. The thread (*h*) is 5 ft. long. The sand should be fine, granular and of such a mixture of sizes as will give maximum porosity and yet permit minimum penetration of solid, as in Table

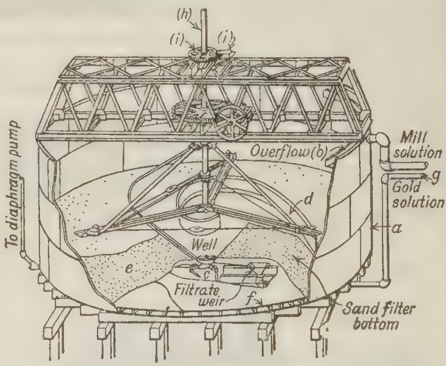


FIG. 23.—Hardinge super-thickener.

16. Power requirement for the scraper and diaphragm pump is about the same as in the Dorr thickener. If suction is used, more power, in accord with the suction requirement, will be needed.

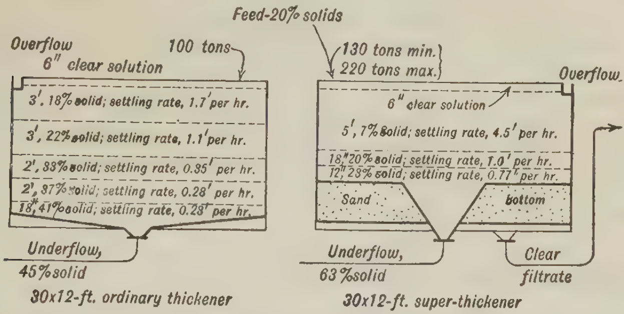


FIG. 24.—Comparison of densities and settling rates in super-thickener and ordinary thickener at Golden Cycle.

The effect of the filter bottom on density of pulp at various depths in the tank and consequently on settling rate of the solids is shown in Fig. 24.

The underflow of the super-thickener at GOLDEN CYCLE could be maintained at a much higher density than was possible in the usual type. When feeding cyanide pulp containing 20 per cent. solids, ground so that 95 to 98 per cent. was − 200-mesh, the super-thickener would discharge at 70 per cent. solids, working under a static head of 20 in. of water, but the diaphragm pump could not handle pulp of this thickness, so that density was lowered to 64 per cent. solids. This is to be compared with maximum operating thickness of 45 per cent. solids on the same pulp in the ordinary machine. (*Hardinge Co.*)

Applicability. The super-thickener is particularly adapted to clarification and to production of exceptionally thick spigot products. For thickening service that is within the capacity of the ordinary Dorr thickener the latter is simpler to operate and probably cheaper, notwithstanding a somewhat lower capacity.

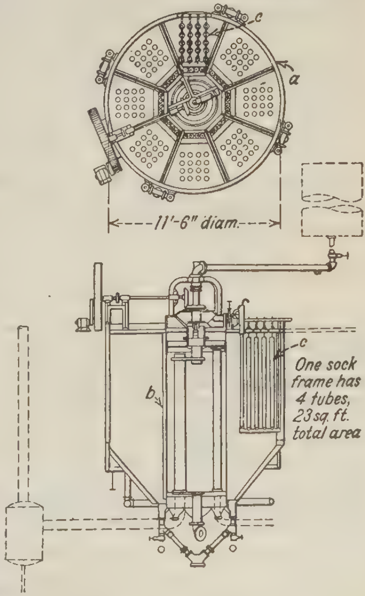


FIG. 25.—Genter thickener.

Genter thickener (Fig. 25) is essentially a vacuum filter, useful for dilute, difficult slimes. It consists of a compartmented annular tank (*a*) for receiving feed pulp, a central vacuum chamber (*b*) and a series of tubular filtering elements (*c*) suspended in the compartmented tank. Clear liquor is drawn to the interior of the filtering elements and a layer of thickened slime is deposited on the outside. This slime is dropped off by reversing the flow of liquid through the filter cloth momentarily. The thickened slime falls to the bottom and is discharged by means of a diaphragm pump. The filtering tubes have 5.75 sq. ft. filtering area each.

Table 16. Screen tests of super-thickener sand-filter bottom at Golden Cycle.

Mesh	April 26, 1923		October 1, 1923		May 23, 1924	
	Actual, per cent.	Cumulative per cent.	Actual, per cent.	Cumulative per cent.	Actual, per cent.	Cumulative per cent.
+ 20	1.7	1.7	18.0	18.0	....	....
+ 30	14.1	15.8	22.0	40.0	....	....
+ 40	18.9	34.7	14.0	54.0	0.5	0.5
+ 60	31.9	66.6	23.0	77.0	14.9	15.4
+100	11.3	77.9	10.0	87.0	18.9	34.3
+150	....	....	6.0	93.0	35.7	70.0
+200	3.5	92.5	3.0	96.0	18.1	88.1
−200	4.0	100.0	4.0	100.0	11.9	100.0

Performance is shown in Tables 17 and 18.

Table 17. Tests of Genter thickener on flotation concentrates at U. S. S. R. & M. Co., Midvale plant (*Gen'l Eng'g Co.*)

Materials treated	Lead	Zinc	Iron
Feed, per cent. solids	20.3	19.2	14.5
Spigot discharge, per cent. solids	65–70	65–70	65–70
Vacuum used, in.	23	23	23
Gallons per square foot of filter area per minute	0.17	0.1	0.127
Dry tons per 1000 sq. ft. of filter(*a*)	327	172	152
Filtrate	Clear	Clear	Clear
Square foot of tank area per ton of dry feed	0.37	0.71	0.80
Number of experiments (*b*) averaged	9	22	17

a This area corresponds to an operating tank 12 ft. 6 in. diameter × 10 ft. deep.
b Experimental tank had 20 sq. ft. of filter area; treated regular mill products.

Table 18. Operation of Genter thickeners at Consolidated Mining & Smelting Co. of Canada, Trail, B. C., 1925 (*Gen'l Eng'g Co.*)

Material treated	Lead-flotation concentrate	Flotation tailing	Leached zinc concentrate
Number of machines	2	3	4
Filter area per machine, sq. ft.	672	1008	1254
Tank, diameter × depth, ft.	12×10	18×10	18×10
Feed, dry tons per 24 hr. per machine	480	535	120
Feed, per cent. −200-mesh	95	55	83.6
Feed, per cent. solids	43	25	7.3
Thickened discharge, per cent. solids	50–60	50–60	50–60
Filtrate	Clear	Clear	Clear
Filtrate, gallons per minute per square foot of filter	0.0627	0.16	0.12–0.15
Square feet of tank area per ton of dry feed	0.24	0.48	2.12

9. Centrifugal dewaterers

These have been extensively developed and widely used in the chemical industry. They consist essentially of a feed container rapidly rotated on a vertical axis whereby centrifugal force amounting, in some cases, to as much as 40,000 times the force of gravity is applied. The basket or EXTRACTOR TYPE

has a rotating member with porous walls which retain solid matter but pass the suspending liquid. This is substantially pressure filtration (see Sec. 17). Usual sizes of basket are 30 to 72 in. diam. and 24 in. or less in depth. They develop centrifugal force 200 to 600 times as great as gravity. Operation is intermittent. The machine is charged while running at low speed, extracted at full speed, then stopped and the basket cleaned of extracted solid. Use is limited to solid-liquid separation. For best operation the solids should be granular, with little variation in size and free of gelatinous matter. The percentage of solids in feed should be high (40 to 50 per cent. or more) on account of the necessity for discharging all liquid through the bed of solids. The BULK CENTRIFUGAL has a solid basket or cup and discharges liquid by overflow. The basket of the Gee machine is 36 in. diam. by 54 in. deep and is driven at 1000 r.p.m. Solid is deposited on plates hung on the inner surface of the basket. The Resine machine has horizontal baffles in the basket to lessen flow of solids to the liquid discharge. Most bulk centrifugals must be stopped to discharge solid. This type is suitable for treating mixtures containing low percentages of solid, but yields a cloudy liquid and a solid fraction containing much more water than is obtained from the extractor type. The bulk centrifugal has been used for purifying and grading clays and for recovery of coal from waste waters. The HIGH-SPEED CENTRIFUGAL and SUPER-CENTRIFUGE are of the solid-basket type with baskets of small diameter to withstand the great forces developed. The high-speed machines are run at 5000 to 15,000 r.p.m. and the super-centrifuge up to 40,000. Capacity is so small and power consumption so high that the high-speed machines can be used economically only for treating valuable products of small bulk.

A successful centrifugal thickener for ore-dressing must discharge both clear liquor and thickened solids continuously and must have sufficiently high capacity to overcome its relatively great capital and power costs. Many machines have been developed in the attempt to fulfill these conditions, but none has as yet been successful commercially.

ADVANTAGES of a successful machine will be small floor space and positive control of spigot density. DISADVANTAGES of the present machines are high repair and power costs, dirty overflow, and, with most, intermittent operation.

COMPARISON OF THICKENERS

Continuous thickeners have greater capacity per square foot of settling surface than intermittent because the settling rate in the latter is slowed down, when they are nearly full, by the approach of thick pulp to the surface. Convenience in operation is, of course, all in favor of the continuous machines. Gravity-discharge continuous thickeners have the same capacity per square foot of settling area as the mechanically-discharged machines, so far as settling alone is concerned, but on account of the difficulties in maintaining uniform discharge of thickened material by gravity, the practical effect is to cut down the capacity. Boss (*103 P 326*) says that slow rotary motion of a settling pulp aids settlement, but there is no conclusive evidence on this score in favor of the mechanical thickeners. It does appear, however, that the slow raking of the thick-pulp layer to the discharge opening aids and accelerates final compacting. Filter thickeners are superior to gravity thickeners for clarification, for treating very dilute pulps, and for making very thick spigot products. Centrifugal thickeners are not yet practical in ore dressing.

10. Design of settling tanks

Coe and Clevenger (*55 A 356*) describe a method based on laboratory tests made in glass cylinders 2 in. or more in diameter and 12 in. to 18 in. deep. Two series of tests should be made made, the first to determine the area required to obtain clear overflow, the second to determine the tank volume necessary to obtain a spigot discharge of the required density. In both series all conditions must be the same as will be encountered in practice, *e.g.*, the time that the ore has been in contact with water or solution; the kind and quantity of electrolyte or other dispersing agent present, if any; the method of grinding and the temperature must be watched particularly.

Tests for area require from six to a dozen samples of pulp of varying consistency ranging from that of the original feed pulp to that of the thickest free-settling pulp (normally between 20 and 30 per cent. solids). The samples are best taken from a large batch which has been thickened to the desired consistency by decantation, then thoroughly stirred and sampled. Each sample, diluted to the desired consistency with decanted liquor, is allowed to stand until the upper surface of the solids has settled about ⅛ in., then a reading is taken and the rate of subsidence for a period ranging from 2 or 3 min. for thin pulps to from 6 to 10 min. for thick pulps determined. With thick pulps intermediate readings should be taken to insure that the rate of subsidence is uniform. A decrease in rate indicates departure from free-settling conditions. The observed rates should be converted into feet per hour and tabulated as shown in the first three columns of Table 19. Column No. 4 of the table is computed from the equation

$$A = \frac{2000}{24\left(\dfrac{62.35R}{F - D}\right)},$$

in which A = area required per ton of solid feed per 24 hr. in sq. ft.; R = rate of subsidence of top of solids in feet per hour; F = the parts of fluid per part of solids in the feed pulp, and D = parts of fluid per part of solids in the pulp to be discharged from the projected settling tank. This is taken as 1.12 in the computations for this table. If D is unknown from other sources spigot-density tests must precede the calculation of column 4.

Table 19. Results of tests to determine area of settling tank. (*After Coe and Clevenger*)

Test number	Feed consistency, parts fluid to one part of solid	Rate of subsidence of surface of settling solids, feet per hour	Area required per ton of solid feed per 24 hr., square feet
1	6.00	2.180	3.00
2	4.94	1.190	4.29
3	4.00	0.893	4.31
4	3.51	0.758	4.22
5	3.00	0.600	4.05

Table 20. Results of spigot-density test. (*After Coe and Clevenger*)

Time of settling, hours	Parts of fluid per part of solid in settled mass
2	1.70
4	1.59
9	1.35
14	1.20
19	1.12

Spigot-density test is made on the thickest free-settling pulp by reading the level of the surface of th subsiding pulp in the same cylinder as above used a intervals of several hours until subsidence ceases or unti the required pulp consistency is reached. The overlyin clear liquor should be removed often enough to keep th water surface near that of the subsiding solid. Table 2 shows the results of such a test.

From Table 19 it appears that the maximum are required to be provided per ton of solids per 24 hr. i 4.31 sq. ft. If less area is provided the layer of pulp o this density will increase in thickness and solid matte will eventually overflow. Table 20 shows that pulp mus be retained in the thickening zone 19 hr. in order t attain the required discharge density of 1.12 : 1 Tank volume must, therefore, be provided fo

$\frac{19}{24} \times \frac{2000}{4.31} = 367$ lb. of solid per sq. ft. That part of the solids that has been in the tank for 14 to 19 hr. will have settled to an average density of 1.16 : 1. The weight of solids in this volume will be $5/19 \times 367 = 96.6$ lb. which is 5 hr. supply to 1 sq. ft. If the sp. gr. of the solid is 3.5, each cu. ft. of pulp whose consistency is 1.16 : 1 will contain 43.2 lb. of solid ($W = 62.4S/(1 + CS)$, where W = lb. solid per cu. ft. of pulp, S = sp. gr. of solid, and C = parts water per part of solid in pulp by weight). The depth of pulp of 1.16 : 1 consistency will, therefore, be $96.6 \div 43.2 = 2.23$ ft. Similarly the following 5-hr. supply will have an average consistency of 1.275 : 1, will contain 37.6 lb. solid per cu. ft. and will require 2.57 ft. depth in the tank. Corresponding figures for the next 5-hr. supply are 1.47 : 1, 33.7 lb. and 2.87 ft.; and for the last 4 hr., 1.7 : 1, 30 lb. and 2.58 ft. Add to these figures 1 to 2 ft. of clear solution for a margin of operating safety, and in a Dorr thickener, an additional allowance for the pitch of the rakes. The total is the depth required with no allowance for storage, or variation in tonnage, character of ore or solution, temperature, etc. If the tank thus calculated is too deep, the necessary volume may be obtained by increasing diameter. Coe and Clevenger recommend a minimum area of 6 sq. ft. per ton per 24 hr. for pulps composed of fine granular material with a considerable proportion of colloidal material which may vary in character.

Table 21 compares capacities computed by the tests outlined with actual capacities at a number of plants. The great discrepancies at PORTLAND and PRESIDIO correspond to underloaded tanks, as shown by the excessive depth of clear solution. The NIPISSING excess of actual over calculated capacity appears to correspond to overloading. All of the pulps in Table 21 are cyanide pulps containing lime, which is an aid to settlement.

Table 21. Comparison of computed and actual capacities of Dorr thickeners. (*After Coe and Clevenger*)

Pulp from	Computed capacity, pounds per square foot per hour	Actual capacity, pounds per square foot per hour	Depth of clear solution in tank, feet	Ratio of water to solids in	
				Feed pulp	Discharge pulp
Liberty Bell	4.9	5.9	1.25	10	2.00
Belmont	14.1	14.8	1.5	7	2.11
Portland	8.3	6.0	6.0	15.1	1.66
Nipissing	8.2	11.8	....	11	1.50
Presidio	33.0	17.6	6.0	5.6	1.58
Hollinger	19.7	18.0	2.0	5.6	1.00
West End	15.2	12.0	5–6	6.1	2.02
Homestake	7.8	7.0	....	33	2.18
Homestake	8.9	8 6	....	17.5	1.50
Golden Cycle (*a*)	19.3	19.1	0	7.7	1.00

a Roasted ore.

SECTION 17

FILTRATION

Filtration is employed in milling to separate liquid from solid more completely than is possible by settling alone. The principal applications are in thickening flotation concentrate and, in the cyanide process, in separating pregnant solution from leached solid and in collecting precipitate. (See Sec. 15.) A FILTER is a permeable septum so mounted that the material to be filtered can be brought to one side at a pressure higher than exists on the other side. Under such circumstances, if the pores in the filtering medium are of suitable size, the solid particles are held back while liquid passes through. After the first short period of filtering the effective medium is the layer of solid deposited on the original septum.

1. Principles of filtration

Sperry (*15 CME 198, 17 CME 161*) has determined experimentally that the velocity of flow of liquid through a filtering medium varies directly with the pressure and inversely with the thickness of the cake and that the character of the flow is, therefore, the same as that in capillary tubes under low pressure, which is expressed in the equation $v = \pi p r^4/8l\mu$, where p = difference in pressures at ends of tube, r = internal radius and l = length of opening, and μ = viscosity of liquid. The terms l and r in this equation cannot be determined for any practical filtering problem, but Sperry has expressed them in terms of a unit of resistance R, defined as the resistance of a filtering medium of such permeability that 1 sq. ft. 1 in. thick will pass 1 gal. of water per hour at 68° F. under a pressure difference on the two sides of 1 lb. per sq. in.; and a unit of deposition or cake-forming ability K, possessed by the solid being filtered, which he defines by saying that a substance has a unit rate of deposition when, under standard temperature conditions a 1-per cent. mixture of the substance with water produces a flow of 1 gal. per hr. over an area of 1 sq. ft. with a pressure difference of 1 lb. per sq. in. With these terms he has developed from the preceding equation the relation

$$Q = \left[\sqrt{\frac{2PKT}{RS} + \left(\frac{KR_m}{RS}\right)^2} - \frac{KR_m}{RS}\right]\frac{N_S(1 + at_1 + bt_1^2)}{N_0},$$

where Q = quantity of liquid passing the filter, P = difference in pressure on two sides of septum, R = resistance of filter cake to passage of liquid, R_m = resistance of septum, K = rate of deposition of cake, T = time, S = percentage of solids in feed pulp, N_S = coefficient of viscosity of liquid at standard

conditions and $\frac{N_0}{1 + at_1 + bt_1^2}$ is Poiseuille's statement for viscosity at temperature t_1 compared with the viscosity at t_0, a and b being constants dependent upon the liquid. Sperry has verified this relation experimentally. The equation is not practically useful for purposes of design of filters but it shows that filtering rate increases when pressure and temperature increase and decreases with resistance of the filter and increase in thickness of the cake. Hatschek (*27 SCI 538*) has shown experimentally that resistance to filtration increases rapidly with decrease in size of particle forming the cake. Hence the equation of capillary flow can be taken as indicative of the effect of variation in the several factors involved in filtration. Young (*42 A 752*), working with vacuum filters and ore slimes confirms these conclusions. Fig. 1, *A* shows the effect

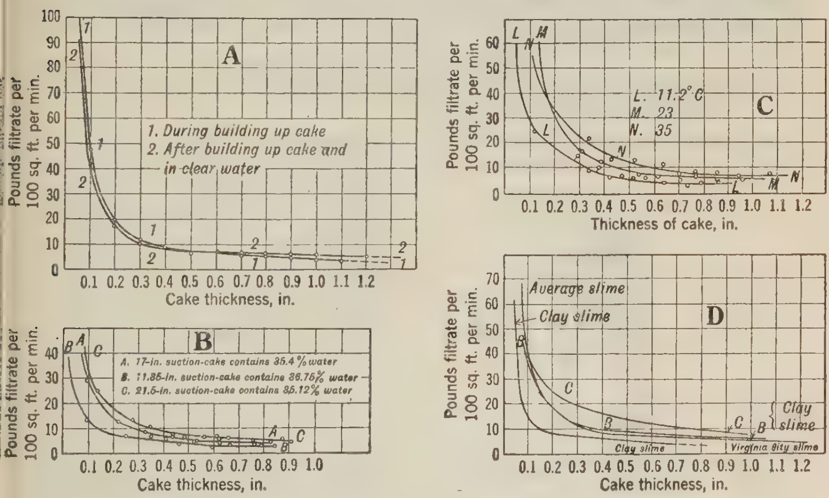


FIG. 1.

of cake thickness on filtering rate; Fig. 1, *B*, the effect of variation in vacuum; Fig. 1, *C*, of pulp temperature; and Fig. 1, *D*, of the character of the pulp, all taken from Young's results. He concludes from other work that small changes in the amount of clay in a pulp or in a cake have more effect on the filtering rate than much larger changes in the amount of sand.

2. Filter medium

The filtering medium is composed of the porous septum itself plus the cake of filtered material held thereon. The septum may be a bed of solid grains merely piled together and, therefore, incapable of use in any position other than horizontal; it may be a porous slab, such as the naturally-occurring tripoli or an artificial material like Filtros; most frequently cotton cloth is used, rarely a woolen, mineral-fiber or metallic cloth. In milling work a COTTON FABRIC is almost invariably employed. It has the ADVANTAGES of great tensile strength, flexibility, durability, small weight, ease of handling and cleaning, and, compared to porous slabs, low first cost.

The size and spacing of the pores in the filter septum have great effect on filtering rate, not so much on account of the resistance to passage of liquid as in their determining effect on the porosity of the first layers of cake deposited. The first layer of particles will tend to deposit over the mouths of the pores in the septum. If these are so spaced that the deposited particles are out of contact or are just in contact on all sides with adjacent particles, there will be maximum reduction of pore area. If the mean diameter of the particles is greater than the mean pore spacing, crowding will prevent all holes in the septum from being covered and increased porosity of the septum will result. On the other hand, the reduction in size of pores that accompanies decreased spacing may increase resistance of the septum itself, but this is rarely as important a factor as that of spacing. CHOICE OF A CLOTH is usually a balance between the desiderata of large pores and close spacing, which make for quick filtration but low durability, and the higher durability of heavier and less porous cloths.

Hixson, Work and Odell (*73 A 225*) have found that distinct bridges are formed over the filter pores by the solid particles, as shown in Fig. 2, that the relation between maximum particle size and maximum filter opening is as set forth in Fig. 3, that this relation holds for feed-pulp densities of 20 to 60 per cent. solids, but that for more dilute pulps smaller ratios must be used, and that the thicker the feed pulp the more porous the cake formed.

FIG. 2.—Bridging of solids over a filter pore.

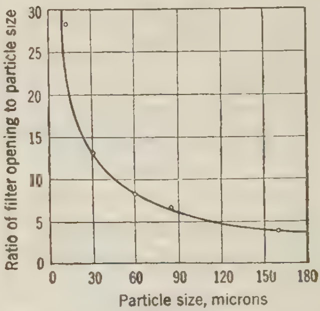


FIG. 3.—Relation between filter opening and size of largest particle in pulp to be filtered (*after Hixson et al.*).

TYPES OF FILTERS

The cyanide process produced a large number of different kinds of filters that may be classified fundamentally as (1) vacuum and (2) pressure, depending upon the means employed for effecting the required pressure difference on the two sides of the porous septum. Vacuum filters may be further classified as (*a*) continuous and (*b*) intermittent. Pressure filters are invariably intermittent.

3. Continuous vacuum filters

These filters are of the drum type, including the Oliver and Portland; the disk type, such as the American and Robacher; the horizontal revolving-leaf type, of which the Ridgway is best known; and the table type, such as the Caldecott.

Oliver filter (Fig. 4) is typical of the drum machines.

Drum (*a*), mounted on horizontal trunnions, is faced with selected wooden staves (*b*) forming a tight shell. At suitable intervals longitudinal wooden partitions strips (*c*) are

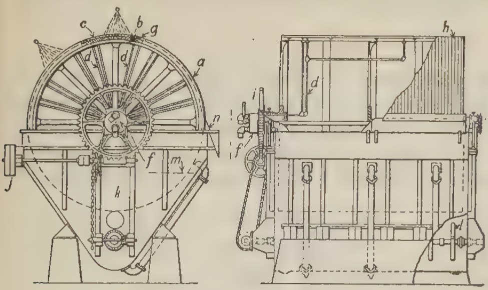


FIG. 4.—Oliver continuous filter.

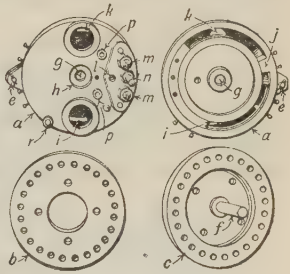


FIG. 5.—Valve for Oliver continuous filter.

fastened to the face of the shell, dividing it into a number of shallow troughs which connect by pipes (*d*) with valve (*f*) and thence with vacuum pump and compressor. The troughs on the face of the drum are filled with suitable backing to support the filter cloth (*g*), which covers the entire drum. Wire (*h*) is wound on the outside of the canvas to keep it tightly in place and protect it. The drum is driven by a worm gear (*i*) from pulley (*j*) and revolves in a clockwise direction with the lower segment immersed in pulp in tank (*k*). Feed pulp is kept in suspension by a paddle agitator (*l*) or by some other form of mechanical or air agitation. VALVE MECHANISM is illustrated in Fig. 5. It consists of a port plate (*a*), valve seat (*b*), and wear plate (*c*). The port plate is held stationary by adjusting-rod pin (*e*) and a rod therefrom attached to the frame of the filter. Pipes from the filtering compartments on the drum face are tapped into the valve seat and the wear plate (*c*) is bolted through the valve seat to the drum frame in such a way that the peripheral holes in the two plates register. The valve stem (*f*) passes through hole (*g*) in port plate (*a*) and the latter is held tightly against (*c*) by means of a coiled spring on the stem, which bears between a nut on the end of the valve stem and the ground surface (*h*) on the port plate. In simple filtering, a pipe from port (*i*) runs to the vacuum pump, bridge (*j*) is removed and port (*k*) is plugged. Under such circumstances that part of the drum surface extending from just below the pulp level on the down-coming side, at about point (*m*) (Fig. 4), to a point to the right of the top of the drum, is in suction all the time. The three small ports in plate (*a*) (Fig. 5), provide for blowing the cake prior to its removal by scraper (*n*) (Fig. 4). Air connection is made at (*l*) (Fig. 5) and the number of compartments under pressure is regulated by stops (*m*) and (*n*). Stop (*n*) is hollow, thus providing for a steam connection, if desired, and its position may be changed so as to give first a steam blow followed by one

Table 1. Sizes of Oliver filters

Drum		
Diameter, feet	Length, feet	Area, square feet
3.0	0.5	4
3.0	1.0	9
3.0	2.0	18
3.0	4.0	36
4.0	2.0	25
4.0	4.0	50
4.0	6.0	75
6.0	4.0	70
6.0	6.0	105
6.0	8.0	140
6.0	10.0	175
6.0	12.0	210
8.0	6.0	150
8.0	8.0	200
8.0	10.0	250
8.0	12.0	300
11.5	8.0	288
11.5	10.0	360
11.5	12.0	432
11.5	14.0	504
11.5	16.0	576
11.5	18.0	648
11.5	20.0	720
14.0	14.0	616
14.0	16.0	704
14.0	18.0	792
14.0	20.0	880
14.0	24.0	1056

or two air blows, or a steam blow between two air blows as shown in the figure, or a steam blow following two air blows. Holes (p) are for gage connections. When WASHING is to be done, or if two different vacuum pressures are desired, bridge (j) is inserted and port (k) is connected to a second vacuum pump. Wash solution is then withdrawn through the latter port. A grease cup at (r) supplies grease to the adjacent ground faces of plates (a) and (c).

Sizes of Oliver filters are given in Table 1.

Vacuum production in flotation work is usually DRY, *i.e.*, a vacuum receiver is connected between the filter and the vacuum pump, liquid separates from air in the receiver and therefore does not pass through the pump. Liquid is removed from the receiver by means of a centrifugal pump or through a stand pipe terminating in a sump at a distance below the receiver correspond-

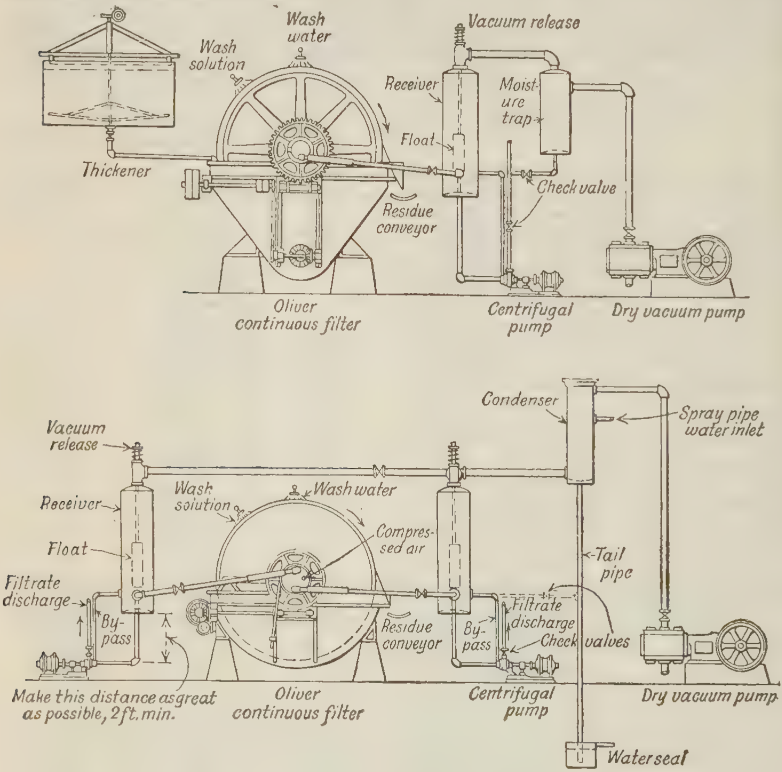


FIG. 6.—Layout for Oliver filter.

ing to barometric pressure. The CENTRIFUGAL EXHAUSTING PUMP should be set below the bottom of the vacuum tank so that liquid flows freely into the pump inlet. The pump should be self-priming or a check valve be placed at the high point on the discharge line and an equalizing pipe should be run back to the receiver from the suction line. BAROMETRIC LEGS should be sealed by extending into a sump, the capacity of which is greater than the volume of the drain pipe. The VACUUM TANK should be provided with a safety seal at the

outlet to the dry-vacuum pump to prevent liquid passing over into this pump, which is designed with small clearance and would be injured by liquid. For handling hot solutions a condenser tank should be inserted between the receiver and the vacuum pump. Lay-out as recommended by Oliver Filter Co. is shown in Fig. 6.

A reciprocating air compressor with the inlet port connected with the vacuum tank is commonly used to produce vacuum. The compressor provided should ordinarily have free-air capacity ranging from 0.5 to 1.5 cu. ft. per min. per sq. ft. of filter surface. For very porous cakes as high as 5 cu. ft. per sq. ft. per min. may be necessary. The vacuum-pump displacement necessary per sq. ft. of filter surface may be obtained from equation $V_2 = P_1V_1/P_2$, where P_1 and P_2 are barometric pressure and barometric minus vacuum-gage pressures respectively and V_1 is displacement volume per square foot per minute desired.

Rotary blowers, with suction end attached to vacuum tank are sometimes used in place of a reciprocating compressor for dry-vacuum production.

Wet-vacuum production, in which the filtrate passes through the pump, is often used in cyanide work where the volume of filtrate is relatively larger than in concentrate dewatering and filtrate must be transported for further treatment, so that the same pump can serve both ends. Reciprocating or centrifugal pumps and rotary exhausters are commonly used in this service. Grit that passes the filter causes considerable wear with corresponding decrease in efficiency.

Power requirement for dry-vacuum pumps is computed from the same equation as that for air compression, with appropriate change of symbols. For a pump working adiabatically,

$$Hp. = \frac{144}{33{,}000} \cdot \frac{n}{n-1} \cdot P_2V_2\left[\left(\frac{P_1}{P_2}\right)^{\frac{n-1}{n}} - 1\right],$$

in which P_2 = absolute pressure in vacuum produced in lb. per sq. in., V_2 = gas volume corresponding to P_2 in cu. ft. per min., P_1 = atmospheric pressure, and n is a constant ranging from a theoretical value of 1.406, down, probably, to 1.15 or 1.2 for ordinary vacuum-pump practice. Table 2 shows the estimates by Ingersoll Rand of the power required for their Type ER–1 dry-vacuum pump, which is frequently used. By investigation of the power equation throughout the range of the theoretically possible vacuum production for any given conditions, it is found that the power requirement per cubic foot of gas exhausted from a given vacuum passes through a maximum, as shown in Fig. 7.

Table 2. Power requirement of Ingersoll-Rand Type ER-1, 22 × 8-in. dry-vacuum pump

Inches of vacuum	Horsepower required
16	34
18	33
22	29
24	26
27	17
28	12

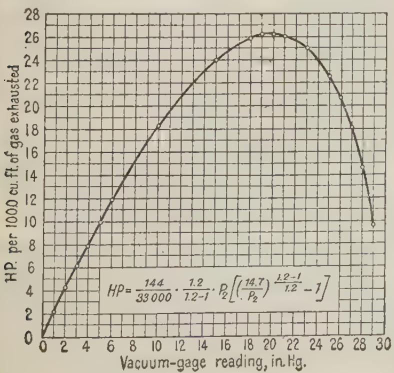


Fig. 7.

For blowing, compressed air is furnished at 10 to 15 lb. pressure; the usual provision is for 0.15 to 0.25 cu. ft. of free air per minute per square foot of filter area.

Operation. The factors that influence results are diameter of drum, speed, submergence, size of feed particles, percentage of solids in feed, reagents present, kind and condition of filter cloth, temperature of pulp, homogeneity of pulp in tank, vacuum, blowing, and uniformity of conditions.

Drum diameter, speed and submergence are dependent in that they determine the time during which cake is formed. Four minutes is about the maximum time that can be economically permitted for cake formation; the usual time is about 3 min. In general, low pulp level gives thin, dry cake and low capacity, high pulp level *vice versa*. The more granular material in the feed the more porous the cake and consequently the greater the capacity. Addition of fine granular gravity concentrate to flotation concentrate may, by increasing porosity of the cake, lower the moisture content and also prevent cracking, but it makes it more difficult to maintain the pulp in suspension in the tank.

Feed pulp should be as thick as possible, if high capacity and dry cake are desired.

Trauerman (*104 J 87*) reports a test on an 11.5 × 16-ft. machine on which 35 dry tons of cake containing 20 per cent. water were made in 8 hr. from a feed pulp containing 35 per cent. solids and 51 tons containing 12 per cent. moisture were made in the same time from a pulp containing 55 per cent. solids. At Sunnyside, with feed containing 30 to 35 per cent. water, cake was 1¼ in. thick and contained 8 per cent. water, while with feed containing 40 to 45 per cent. moisture, cake was ½ in. or less thick and contained 10 per cent. moisture.

Thickening in Dorr tanks costs less than 5¢ per ton of solids and filtration above 25¢, hence the more preliminary thickening, the cheaper the whole operation. The filter tank may be shallower with thick feed than with thin, which gives more time for drying cake.

Lime added to filter feed increases filtering rate, decreases moisture in the cake and, in case of acid pulps, increases life of cloth and solution passages. On the other hand, it increases clogging.

At Miami it prevented water from settling out in cars on the way to the smelter (*115 P 573*).

Young (*loc. cit.*) calls attention to the fact that thick cakes made from a given slime pulp contain more water than thin and that the increase in moisture content of the cake with increasing distances from the filter cloth is greater the greater the slime content of the feed pulp.

Filter cloth must be sufficiently "dense," *i.e.*, close-woven, to hold back the bulk of solid material and thus form, from the retained material, the filter wall proper, and it must be sufficiently heavy and strong not to break or deform materially under the pressures applied. It is desirable that all of the material retained be held at the surface of the cloth, since material that penetrates is not removed with the cake nor by scrubbing and therefore permanently decreases porosity. Cloth with considerable nap is harder to discharge than smooth, hard cloth on account of the binding effect of nap threads. Cotton cloth is almost invariably used in milling practice. The strongest is the close-woven canvas duck;. soft twilled canvas is more porous but less durable.

Woolley (*104 J 875*) recommends twill for fine concentrate that makes compact cake, and duck for more granular product for which the cloth itself must do much of the filtering.

The fiber of cotton cloths swells materially when the cloth is wet and fabrics that appear too porous when new may give entirely satisfactory service. Eight- to 10-oz. duck and 12- to 16-oz. twilled canvas are the commonest coverings in milling work. The best covering can be determined only by experiment.

Life of the above coverings on drum filters in flotation service is two to six months with the average between three and four months. Lighter weaves backed by coarse heavy material are sometimes used. Cloths should be scrubbed whenever the filtering rate falls off due to clogging of the filter medium. Scrubbing should be done with a brush stiff enough to work into the fabric sufficiently to loosen particles held therein. Weak hydrochloric acid solution (0.25 to 1.0 per cent. strength) should be used when carbonate precipitates. At some plants the filter is scrubbed every day, in most the period between scrubbings is less than a week.

Re-covering, including removal of old canvas and re-winding wire requires 16 to 36 hr. with a crew of 3 or 4 men.

Heating lowers the viscosity of the liquid and also tends to cause flocculation of fine solids. On both scores it increases filtering rate and it may also decrease the moisture in the cake. Heating to 100 to 120° F. is a common procedure in treating flotation concentrate but heating of cyanide pumps is not usual.

At Afterthought, heating pulp increased tonnage 17 per cent. to 750 lb. per sq. ft. per 24 hr. in a cake containing 15 per cent. water. Watt (*57 A 379*) stated that in treating lead concentrate in south-east Missouri, heating to 120° F. increased capacity 20 per cent. and decreased moisture in cake 2 per cent. (to 15 per cent. final). At Utah Copper Co. pulp is both heated and agitated by live steam injected into the filter tank. Experience shows that the hotter the pulp the dryer the cake (*117 P 752*).

The amount of vacuum to be carried depends on the porosity of the bed and the capacity required. With readily filterable pulp or low duty, the vacuum may be as low as 15 in.; the usual range is 20 to 25 in. High vacuum will frequently cause the cake to crack and will also clog cloths. Different pressures may be maintained on the submerged and unsubmerged parts of the drum by independent vacuum connections provided in the valve. Cracking may be lessened by speeding up the drum and making a thinner cake.

A mechanical method for preventing cake cracking is the application of a canvas belt and rollers; the belt is applied on the upcoming side just before cracking starts and removed before the vacuum is released. With cracking eliminated, a dryer cake can be made and less vacuum-pump capacity need be provided.

Blow-off pressure should be kept as low as possible; the usual pressures are from 7 to 15 lb. per sq. in. The higher pressures keep the filter cloth cleaner but excessive pressure is likely to cause the canvas to split under the wires, especially when acid cleaning is used and the acid is not completely neutralized before operation starts again.

Agitation. Mechanical agitation is generally superior to air agitation. The latter tends to make thin spots in the cake and also, under favorable conditions, to produce heavy carbonate deposits. Furthermore, if heavy granular sulphide is present, air is insufficient to keep it in suspension; the power consumed in air agitation is greater than in mechanical and much of it is wasted on account of excessive air consumption.

Bradley (*106 J 207*) reports the use of discarded rubber pump diaphragms for packing in the stuffing boxes of the rotating agitator shown in Fig. 4. Leakage here is not uncommon, it is mussy in any case and expensive in cyaniding. Rice (*106 J 899*) recommends driving the agitator from an independent shaft so that it can be kept running, if necessary, when the filter is stopped.

When the material to be filtered is difficult to keep in suspension, the difficulty may be lessened or overcome by feeding the pulp by gravity from a surge tank through a manifold in the bottom of the tank at a sufficient rate to maintain suspension and pumping the overflow back to the surge tank. The surge tank should have about twice the capacity of the filter tanks.

When a plant makes both table and flotation concentrate, a good scheme for dewatering is to send the table concentrate to a mechanical classifier, join the overflow with the flotation concentrate and thicken and join the granular discharge from the classifier with the spigot product from the classifier in a surge tank feeding the filters. The thickened slime concentrate keeps the granular material in suspension readily. The granular table concentrate will probably increase the capacity of the filter on the slimy flotation concentrate.

Scraper blade should be kept smooth on the upper edge by filing or grinding as necessary; otherwise removal of cake is not thorough and the filter cloth is likely to be torn. Adams (*105 J 724*) recommends the use of hard 15- or 16-gage wire instead of the usual 10- or 12-gage for winding since this allows the scraper to come closer to the cloth.

Shimmin filter is of the rotating-drum type but has the filter medium on the inside of a drum which is supported at one end by a trunnion bearing and at the other end by rollers. The roller end is open for feeding, removal of cake, and inspection. The drum itself acts as the pulp container. The size segregation in the pulp in the drum assists filtration because the coarser particles settle onto the filter cloth and form a relatively porous filter bed. The cake is carried up to a point at or beyond the top of the circular path. Here the filter cloth is caused to expand and contract suddenly by alternate subjection to pressure and vacuum. The valve is of the usual revolving-filter type. Cake drops into a hopper from which it is removed by belt or screw conveyor.

The machine can be used on coarser material than usual because the natural settlement of the solids brings them to the filtering surface.

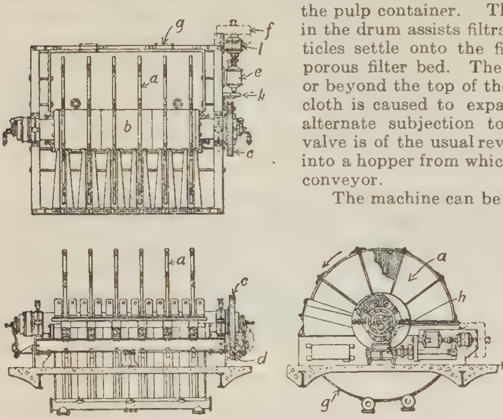


Fig. 8.—American filter.

American Filter (Fig. 8) consists of a plurality of disk-shaped filter leaves (*a*) mounted on a heavy hollow shaft (*b*), caused to rotate by means of the worm gear (*c*) and worm driven through gear (*k*) either by a motor-driven gear speed-reducer (*e*) and motor (*l*) or by pulley (*f*).

The lower parts of the revolving disks dip into a tank (*g*) which is partitioned on the front side of the central shaft in order to allow for discharge of cake. Each filter disk consists of several sector-shaped units which connect by means of conduits in shaft (*b*) with a special rotary plug valve (Fig. 9). This valve ordinarily has three outlet ports. During the time that any given sector is submerged and for a time after its emergence it is connected through a conduit in the shaft and a corresponding channel in the journal with the filtrate outlet port. On further emergence it connects with the wash-water outlet port, and finally, just before it reaches the scraper it connects with the compressed air port and the cake is loosened.

The standard disk diameters are 4 ft., 6 ft. and 8 ft. 6 in. The 4-ft. size is made with one to four disks, as desired; the 6-ft. and 8-ft. 6-in. with two to six disks.

Robacher filter is a continuous disk filter employing Filtros, a porous silica slab, as a filtering medium. It has not been used in metallurgical work.

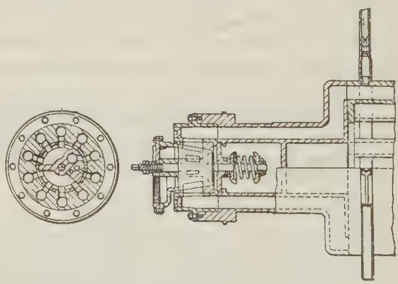


Fig. 9.—American filter valve.

Ridgway filter consists of a number of horizontal filter trays carried on the ends of radial arms extending from a central revolving spindle. The trays travel over a series of annular tanks, a mechanism being provided to lift each tray over the partitions between the tanks. In cyanide work the first tank contains leached slime; the second, weak solution; a third, wash water; and the remaining 15 to 20° of arc constitute the discharge hopper. The valve is similar in principle to the drum valve. The use of filter has been principally restricted to So. African and Australian cyanide mills.

Portland filter is similar to the Oliver.

Performance of Oliver and Portland filters treating flotation concentrates is given in Table 3.

Flotation concentrate. On zinc concentrate an Oliver machine at Butte and Superior Min. Co. handled 890 lb. per sq. ft. per 24 hr. from 55 per cent. water in feed to 10 per cent. in cake, and at Timber Butte corresponding figures were 1300 lb., 50 per cent. and 10 per cent. Round-table concentrate was thickened from 50 per cent. to 15 per cent. moisture at Anaconda at the rate of 850 lb. per sq. ft. per 24 hr. At Inspiration (*115 P 680*) six 12 × 12-ft. drum filters handled 700 tons flotation concentrate per 24 hr. from 40 per cent. moisture to 17 per cent. moisture (moisture content of cake varied with percentage of insoluble). This is 540 lb. solid per sq. ft. per 24 hr.

At the Arthur plant of Utah Copper Co., the filter installation for 1000 to 1200 tons per 24 hr. of mixed table and flotation concentrate, consisted of twelve 14 × 14-ft. Oliver filters and two 4-leaf 14-ft. 6-in. American filters. The table concentrate was all −10-mesh and constituted 20 to 40 per cent. of the total. The flotation concentrate was 90 per cent. −200-mesh and upwards of 10 lb. of oil per ton was used in flotation. The pulp was heated to 160 to 180° F. Filters were run at 3 to 4 rev. per hr. The cover on the Oliver drums was 4-mesh screen, one thickness of burlap and one of palma twill. Drums were wound with No. 6 galvanized-iron wire, coils spaced 2 in. American leaves were covered with two layers of palma twill. Vacuum displacement for the whole plant (13,354 sq. ft. of filtering surface) was 1.394 cu. ft. per min. per sq. ft. Dry-vacuum installation was one 36 × 20-in. duplex Imperial Type *XB* pump at 150-r.p.m. (7058 cu. ft. displacement) and two 27 × 14-in. Imperial *XB* at 170 r.p.m. (displacement 3143 cu. ft. per min. each). Vacuum produced was 21.5 in. Two 5 × 8-ft. receivers served the whole plant. Compressed air was received from the central plant and used at ½ to ¾ lb. per sq. in. Unit capacity of the entire plant was 230 to 280 lb. per sq. ft per 24 hr. from 50 per cent. to 15 or 20 per cent. water. Cake was about ¼ in. thick. Operating force consisted of one foreman, one operator and three repair men, total $66.75 per 24 hr. (1921). Life of cloth was 80 to 120 days. Live steam at 15 to 20 lb. pressure was used for heating and agitation in the filter tanks.

At Afterthought the installation for filtering 40 tons per day each of roasted zinc and copper flotation concentrate from the Horwood process was two 4-ft. 6-leaf American machines, one 14 × 8-in. Doak dry-vacuum pump at 250 r.p.m. producing an average vacuum of 14 to 16 in., one 7½ × 4-in. compressor at 400 r.p.m. furnishing air at 12 lb. pressure, and a barometric leg on the receiver. The filters were run at one revolution in 5 min. 10 sec. The zinc concentrate was all −40-mesh and 60 per cent. −200-mesh, the copper all −40-mesh and 50 per cent. −200-mesh. The feeds varied in moisture content from 45 per cent. to 85 per cent., on account of insufficient thickener capacity. Zinc cake averaged 11 to 12 per cent. moisture, copper 13 to 14 per cent. Average capacity was 600 lb. solids per square foot per 24 hr. Life of bags in the zinc filter was only six days and in the copper filter three days on account of the high sulphate content. The thickness of the cake on new bags on the copper filter was 1⅛ in. but by the third day had decreased to ¼ in. At Ducktown Copper and Iron Co. an American filter handled 500 lb. of copper concentrate per square foot per 24 hr.

At Doe Run Lead Co. one 11½ × 12-ft. Oliver filtered 65 to 70 tons of −100-mesh (70 per cent. −200-mesh) lead concentrate per 12 hr. making ⅜-in. cake containing 13 per cent. moisture. Temperature, 100 to 120° F.; 740 lb. per sq. ft. per 24 hr. At St. Louis S. and R. Co. one 12 × 12-ft. machine handled 30 dry tons per day (140 lb. per sq. ft. per 24 hr.) making a ⅜-in. cake containing 18 per cent. moisture. Feed contained 55 to 60 per cent. solids. Speed, 1 rev. in 10 min. Doak 14 × 8-in. dry-vacuum pump, 800 cu. ft. per min. displacement at 300 r.p.m. Receiver 30 in. × 5 ft.; trap, 12 in. × 2 ft. Centrifugal pump, 1½ in. Life of palma-twill covering, 3 months.

In handling flotation concentrate the duty per square foot of filtering surface is so greatly dependent upon the character of the ore and kind of frothing agent used that capacity ranges between wide limits. At Chino Copper Co. (1918), treating high-oil flotation concentrate, four 14 × 14-ft. Oliver filters averaged 100 lb. per sq. ft. per 24 hr. to 27 per cent. of water. At one copper company change from an oil mixture of coal tar, coal-tar creosote and wood creosote to xanthate and pine oil increased filter capacity fivefold.

Coal. Capacity on coal-flotation concentrate is 26 to 28 tons per hour on an 8 × 8-ft. drum when taking a feed containing 50 per cent. moisture and making a product containing 18 per cent. moisture (*67 IME 513*). This is at the rate of 6500 lb. per sq. ft. per 24 hr.

Iron.—At Mesabi Iron Co. a 5½ × 10-ft. Oliver filter making 1 rev. in 40 sec. was fed with 300 to 325 long tons per 24 hr. of a mixture of 94 per cent. of −100-mesh magnetite concentrate (78 per cent. −325-mesh) and 6 per cent. −14-mesh anthracite in a pulp containing 75 per cent. solids. Cake was ⅜ in. thick, and contained 10.5 per cent. water. Submergence was 3 to 6 in. Vacuum of 2 to 3 in. was used to pick up cake and 18 in. to dry. Life of 12-oz. duck was 60 days. Dry vacuum capacity was 1655 cu. ft. displacement for two filters; this was sufficient with old cloth but insufficient with new.

Roasted zinc concentrate. In filtering solutions in the electrolytic zinc plant at Anaconda the capacity of a 12 × 12 drum-type filter was 300 lb. dry residue and 0.063 ton of solution per sq. ft. per 24 hr. and the cake contained 30 per cent. moisture. At the Consol.

Table 3. Performance of rotary drum

Mill	Sunnyside	Federal M. & S. Co.	Utah Leasing Co.
Size, diameter × length of drum, ft.	3.5×12	6×8	6×25
Filtering surface, area, square feet	132	140	437
Material	Canvas	*a*	Soft canvas
Life, months	5–8	4	3.25
Tons of solid feed per 24 hr.	80	20	15–20
Pounds solid per square foot per 24 hr.	1200	280*s*	68–92
Size of feed, per cent. −200-mesh	94	98	*c*
Moisture in feed, per cent.	35–45	50	55–60
Moisture in cake, per cent.	6–8	6.2	18–22*e*
Speed of drum, min. per revolution	11	4	18
Vacuum, in.	20	22	18–20
Power consumption; filter, pump and compressor, hp.	....	15	10
Thickness of cake, in.	½–1½	⅜	⅜–½
Temperature, degrees F.	50–60	*b*	*d*

Mill	Moctezuma Copper Co.	Cananea Consolidated Copper Co.	Engels
Size, diameter × length of drum, ft.	11.5×12	11.5×12	12×8
Filtering surface, area, square feet	432	432	288
Material	No. 7E canvas	*n*	*o*
Life, months	2.5–3	3	5
Tons of solid feed per 24 hr.	20–30	....	50
Pounds solid per square foot per 24 hr.	100–150*m*	....	350
Size of feed, per cent. −200-mesh	97	53	75
Moisture in feed, per cent.	65	40	50
Moisture in cake, per cent.	30	10	12.5
Speed of drum, min. per revolution	12	14	14.5
Vacuum, in.	22	15	20.5
Power consumption; filter, pump and compressor, hp.	7.5–10	16–18	12
Thickness of cake, in.	3/16–1/4	¼–½	¾
Temperature, degrees F.	*b*	*b*	90

a 17-oz. canvas twill. *b* Normal. *c* All −65-mesh. *d* Normal heat increased capacity but did not reduce moisture in cake. *e* 10 lb. unslaked lime per ton of solid added at elevator ahead of filter. *f* Filter Fabrics Co., No. 31. *g* In good condition only 30 da. A hard iron scale forms on both surfaces of cloth. *h* All −100-mesh. *i* Less than 24 hr. *j* For filter only. *k* 6-oz. single twill. *l* Estimated. 150 tons to this and a 14 × 14-ft. *m* Low duty accredited to low percentage of solids in feed. Cannot be improved, even with intermittent settling using lime, caustic soda or soda ash. Cold weather affects adversely. *n* Oliver filter cloth. *o* No. 12 M. S. cloth. *p* Installed. *q* Heavy cheese-

MIN. AND SM. Co., Trail, B. C. a disk-type filter handled 600 lb. dry solid and 0.097 ton solution per sq. ft. per 24 hr. with 25 per cent. moisture in the final cake.

Cyanide. There is great difference in behavior of cyanide pulps. At ARGONAUT mine, Dane, Ontario, two 6-ft. 4-disk machines filtered and washed cyanide pulp at the rate of 200 lb. dry solid per sq. ft. per 24 hr. Feed pulp contained 48 per cent. moisture and cake 20 per cent. moisture. Speed, 4 min. 45 sec. per rev. At HOLLINGER seven 14 × 16-ft. drum-type machines filter and wash 550 tons dry solid (7 per cent. + 150-mesh, 70 per cent. −300-mesh) per 24 hr. or 225 lb. per sq. ft. Feed varies from 1.0 to 0.65 ton solution per ton of solid; cake contains 21 per cent. moisture. COST of plant complete, including building, was $152,500, of which $95,000 was for the filters, vacuum and other pumps, tanks, receivers, etc. Total cost of filtering (1923) was $0.045 per ton, about half of which was labor.

filters on flotation concentrate

Bunker Hill & Sullivan	Shattuck-Arizona	Calumet & Hecla	Sunnyside	Belmont-Surf Inlet	St. Joseph Lead, Rivermines	Ray Consolidated Copper Co.
6×25	8×6	8×6	9×12	11.5×8	11.5×12	11.5×12
437	150	150	340	288	432	432
Canvas	*f*	Soft canvas	Canvas	Twill	Twill *w*	*k*
6	2*g*	4	5–8	6	4	3
21	54–75	30	120	25*i*	70	75*l*
100	720–1000	400	700	....	320	286*l*
....	96	*h*	89	....	....	90
35	40	70	35–45	50	35–40	45
12	16–18	22.5	7–9	11	15	21
8	8	10	7.5	10	10	6.5
20	18–20	....	20	25	22	23
10	....	....	....	2*j*	35	3–3.5*j*
3/16	5/8	3/16–1/4	1/2–1 1/2	1/16–3/8	1/4–1/2	3/8
65	Hot	*b*	50–60	....	90	120

Consolidated Arizona Smelting Co.	Federal Lead Co.	Chino	St. Joseph Lead Co., Bonne Terre	Phelps-Dodge, Morenci	Phelps-Dodge, Burro Mountain	Mt. Lyell *u*
12×9	12×12	12×15	12×37.5	14×14	14×14	8×8
330	432	504	1400	616	616	200
Canvas	Canvas	8-oz. duck	Canvas	*q*	*r*	Canvas *v*
3	3	3–6	4	4	6–9	8
125	50	58	50–100	30*t*	60	....
740	230	230	70–140	100	190	600
....	90	91	100	94	95	77–150
40	34–36	52	40	69	65	52
12	15	26	15	31*t*	28	11.5
4.5	9	9	8.5	7.7	7.5	8
15	23	18	25	22	18	22
25*p*	23*p*	7.5	12	13.5	....	....
1/4–1/2	1/4	1/4–1/2	3/16	1/16–1/8	....	1/4 2
80	140	140	130	....	....	Normal

cloth and twill. *r* Light canvas. *s* This filter is much underloaded. Another performance shows 2700 lb. per square foot per 24 hr. dried from 50 per cent. moisture in feed to 10.8 per cent. in cake. *t* Have run up to 70 tons per 24 hr. and dried to 17 per cent. water. *u 112 J 217.* Submergence varies. The greater the submergence (up to 3 ft. 6 in.) the greater the capacity but the wetter the cake. *v* Canvas over hessian. Wound with No. 14 copper-covered steel wire at 5/8-in. centers. 16 hr. to re-canvas and re-wire. *w* No. 3 Oakdale.

Capacity of continuous-vacuum filters varies greatly as may be seen from the foregoing figures. For normal lead and copper flotation concentrate made with 1 to 2 lb. of oil, 400 to 600 lb. dry solid per sq. ft. of filter surface per 24 hr. is a fair figure for estimate. If for any reason, as *e.g.*, the use of more oil or a de-flocculating agent, the concentrate is slow-settling so that the feed to the filter is unduly dilute, the filter duty may run down to 100 lb. or less per sq. ft. while admixture of granular table concentrate may increase duty to 700 or 800 lb. Duty on zinc-flotation concentrate with low oil goes as high as 1300 lb. at Timber Butte. Granular products filter very rapidly, *e.g.*, coal-flotation concentrate, 6500 lb. per sq. ft. per 24 hr. and the mixture of coal and

magnetite at MESABI (3600 lb.). Cyanide pulps, which require washing in addition to filtration average from 200 to 400 lb. per sq. ft. per 24 hr., the lower figure being the safer for estimate.

Cost of continuous-filter installation. Table 4 gives an estimate for an American filter installation as of 1924.

Table 4. Estimated cost of an American-filter installation

1 @ 6-ft. 3-disk machine	$2600
Extra for agitating device	60
Extra for variable-speed motor drive	375
1 @ 12 × 8-in. Ingersoll-Rand dry-vacuum pump, Type ER-1, 311 cu. ft. per minute displacement at 300 r.p.m. 10 hp	776
Extra for short-belt drive	104
1 @ 10-hp., 3-phase, 60-cycle, 2200-volt, 1900-r.p.m. motor	195
2 @ 18 × 60-in. vacuum tanks	96
2 @ 1½-in. centrifugal pumps	295
2 @ 3-hp. motors, 1900 r.p.m.	240
1 No. 3 roots positive blower, 850 r.p.m.	84
1 @ 1-hp. motor	79
1 @ 12 × 30-in. moisture trap	40
1 @ 12 × 36-in. air receiver	22
	$4966

Cost of continuous filtration (1922) ranges from 25 to 45¢ per ton of concentrate, of which labor is 35 to 50 per cent., supplies 30 to 35 per cent., and power 20 to 30 per cent. At one plant using 65 hp., 6 per cent. was for driving drum, 19 per cent. for air-lifts for keeping pulp suspended in the filter tank, 12 per cent. for compressed air for blowing, 40 per cent. for dry vacuum pump, 7 per cent. for pump exhausting filtrate from receiver, and 16 per cent. for agitation of pulp in the feed tank.

Cost of re-covering a 13 × 37½-ft. drum filter in the electrolytic zinc plant at ANACONDA was as shown in Table 5. Eight blankets handled 40,000 tons of solid at a cost of $0.0325 per ton for blankets.

Table 5. Cost of re-covering a 13 × 37½-ft. drum filter at Anaconda

Blanket (wool), 57 sq. yd.		$48.35
Burlap, 57 sq. yd.		15 40
Copper wire, 242 lb.	$31.80	
Less salvage, old wire	5.50	
		26.25
Labor		18.00
		$108.00

4. Sand filters

Caldecott sand table is a revolving annulus about 25 ft. outside diameter by 3 ft. wide with horizontal porous top over suitable vacuum compartments. It is run at about 20 ft. per min. peripheral speed with a vacuum of 5 to 10 in. Thickened sand pulp from diaphragm cones or mechanical classifiers is fed at any given point in the revolution and removed, after filtration, by a diagonal plow just ahead of the feed point. About 0.5 cu. ft. per min. vacuum-pump displacement is required per square foot of filter area.

At SIMMER AND JACK PROPRIETARY MINES, LTD. one such table handled 500 tons quartz sand per 24 hr. and reduced moisture from 30 per cent. to 15 per cent. (*RMP*). At NEW JERSEY ZINC Co. 6-ft. tables with 21 sq. ft. of filtering area (made by Oliver Filter Co.) treat 7, 2 and 6 tons per hr. respectively of shaking-table concentrate, middling and tailing, all of which passes 8-mesh screen and contains about 4 per cent. −200-mesh. Feed contains 30 per cent. water and product about 8 per cent. The layer of solid on the filter is about

4 in. thick. Speed is 1 rev. in 3 min.; vacuum, 6 in. Hg. The canvas used for filtering medium lasts 30 days.

Rotary hopper dewaterer is essentially a drum-vacuum filter with the sides of the filtering compartments extended beyond the filtering surface to form V-shaped hoppers with radial sides in which granular materials can be piled and retained on the drum. The hoppers are fed by overhead chutes at a point slightly ahead of the zenith and discharge by gravity, aided by compressed air, if necessary. Rock sands will discharge with 10 to 15 per cent. water, if feed contains not more than 30 to 40 per cent. and if substantially free from slimes. Capacities are not well enough established to be accepted without experiment, but in filtering granular slate ranging from about 0.3 mm. to 1.5 mm. and substantially slime-free, 2 tons per sq. ft. per 24 hr. has been handled.

Filter tanks are rectangular or cylindrical in shape, fitted with porous false bottoms, through which liquid passes by gravity, aided by vacuum toward the end of the draining period, if desired. They are frequently used for dewatering granular concentrate. The filter bottom must be supported on a grid-like frame sufficiently close-spaced to prevent harmful deformation of the filtering medium, which usually consists of a filter cloth laid over cocoa-matting or the like. A second grid is usually placed above the filter cloth to protect it when the filtered charge is shoveled out. Shoveling may be manual or a mechanical shovel of the orange-peel or clam-shell type may be used.

At SHATTUCK ARIZONA (*110 J 761*) gravity concentrate containing 21 per cent. water is drained in concrete tanks with sand-filter bottom to a product containing 10 per cent. water.

In the BLAISDELL SYSTEM the tank is circular with a central bottom-discharge opening about 12 in. diameter under which a belt conveyer runs. The discharge hole is closed during filling by a downward-tapering hollow steel-plate plug extending to the top of the tank. When this is removed it leaves a steeply-conical hole through the charge to which the filtered material is scraped by a revolving plow arm carrying a plurality of disk plows. A series of tanks is served by one plow mechanism.

5. Pressure filters

These are of two general types: (*a*) plate and frame presses, and (*b*) pressure tanks. The first are widely used in the chemical industry, but have had only a restricted use in milling; the second have been used to a considerable extent in cyanide work and slightly in filtering flotation concentrate.

Plate-and-frame filter. A typical form is shown in Fig. 10. The essential parts are a framework consisting of two uprights (*a*) and two parallel tie-rods (*b*), a fixed head (*c*), a movable head (*d*), and a plurality of plates (*e*) alternating with frames (*f*). Plates are of cast iron, suitably perforated with ports (*g*) and slightly recessed at the center as shown. The recessed portions are grooved to allow passages for solution behind the filter cloth. Frames are similar to plates with the recessed portion removed. Both are practically square in the section at right angles to the plane of the drawing. Plates and frames have outside lugs which rest loosely on rods (*b*) when the press is open and they may be lifted out separately. When assembling the press for operation, filter

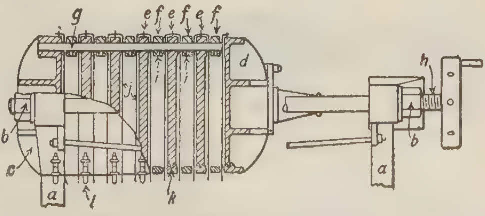


FIG. 10.—Plate-and-frame filter.

cloths are draped over the plates, perforated to correspond with the opening (g), are set on rods (b) alternating with the frames, and, when the desired number are in place the movable head is pushed forward by power screw (h) until all joints are made tight by reason of the canvas packing between the machined faces of plates and frames. Holes (g) in plates and frames, now registering, form a passage for feed pulp which is pumped in under pressure and passes through openings (i) into the space between the plates. Liquid passes through the cloth and along the faces of the plates to passages (k) which discharge through cocks (l). When the chambers are filled or nearly filled with cake, wash water may be sent through the press to wash the cakes. finally compressed air or steam to dry them, after which the movable head is run back and cake is dropped out of the chambers one at a time. The MERRILL PRESS, which is the only one that has had any considerable use for pulp treatment in cyanide work has an interior passage along the bottom for discharge of cake and a high-pressure water pipe with jets into the individual compartments for washing spent cake into the discharge passage. Hence it may be discharged without opening.

Plate-and-frame presses are rated by the size and number of plates. The usual sizes are 18 × 18-in., 24 × 24-in., 30 × 30-in., and 36 × 36-in. with from 25 to 50 plates per press. Pressures depend on the filtering character of the material. They usually start at a low figure, say 5 to 10 lb. per sq. in. and finish at 50 to 60 lb. High initial pressure clogs the cloths and makes the initial layer of solid too compact; high final pressures are necessary to force liquid through the thick cakes.

Design of plates and frames is highly various. Complexity and number of ports depends on the number and kind of washes, whether blowing is to be practiced, and whether both air and steam are to be used. For considerable further detail as to construction and operation of presses see D. R. Sperry, vols. 18 and 19, *CME*.

Centrifugal pumps are the best type for forcing feed pulp into plate-and-frame presses, because pressure is low at the beginning and gradually increases as resistance builds up, and also there is no pulsation to compact cake as by tamping, which effect is distinctly noticeable with reciprocating pumps.

Pressure-tank filters are of several varieties of which the Kelly and Burt are best known in milling work. The former has had considerable use in cyanide work and some in concentrate filtration, the latter in cyanide work alone.

Kelly pressure filter (Fig. 11) consists of a basket (a) of filter leaves, carried on a frame (b) that can be slid in or out of a pressure tank (c). The tank is inclined so that gravity aids egress of the loaded basket. One end of the movable frame forms the closing head (d) for the pressure tank. Each filter leaf consists of a bag or slip carried on a rectangular pipe frame. Collapse of the covers under pressure is prevented by spacers of wood, wire cloth or coarse fiber held between the walls. The lower pipes of the frame are perforated within the slips and are extended outside the slip covers through the movable head and terminate in cocks (e). When the tank is closed, pulp is introduced under pressure, filtrate passes through the canvas and out through cocks (e) while a cake of retained solid builds up on the leaves. When sufficient cake has built up, judged by the pressure on the gage and the rate of flow of filtrate, air is introduced to displace the remaining pulp through drain pipe (g) and to maintain pressure within the tank and hold the cakes in place, then water or

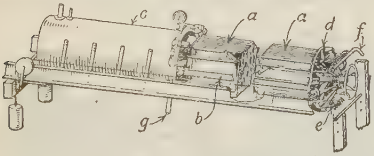


FIG. 11.—Kelly filter.

dilute solution may be pumped through for washing or air or steam for drying. Finally the leaves are run out and cake forced off by steam or compressed air introduced within the covers through pipes connecting with header (*f*). Gravity pressure may be used instead of pumps, if sufficient head is available.

Performance on flotation concentrate:

At TIMBER BUTTE a 5 × 14-ft. tank with 850 sq. ft. of filtering surface treated 200 tons (max.) per day of zinc concentrate (470 lb. per sq. ft. per 24 hr.), 15 per cent. + 200-mesh, reducing moisture content from 45 per cent. to 10 per cent. Cake was 1.5 to 2.5 in. thick. Cycle was 10 min. for charging, 10 for filtering and 20 for discharging. Pulp was unheated. At FEDERAL MINING AND SMELTING Co., Morning mill, a Kelly press with 240 sq. ft. of filter surface treated 18 tons per 24 hr. (150 lb. per sq. ft. per 24 hr.) of lead concentrate, 99 per cent. −200-mesh, and reduced the moisture content from 50 per cent. to 8 per cent. Pulp temperature was normal. The cycle was 5 min. charging, 30 min. filtering, and 7 min. discharging. Oliver filters treating the same material made a cake containing 6 to 6.5 per cent. water at the rate of 280 lb. of solid per sq. ft. per 24 hr. At BRADEN, Kelly filters made a cake containing 18 per cent. water as against 23 per cent. in Oliver filter cake, the feed to both machines containing 35 per cent. water.

Sweetland filter is similar to the Kelly except that the leaves are transverse and stationary, the tank is jointed along a cylindrical element, and the bottom swung open for discharging cake.

Burt leaf filter is similar to the Kelly. Leaves are transverse to the axis of the cylinder and hang vertically with the cylinder set at 45°. The leaves are fixed. Cake is blown off the leaves by compressed air or steam and flushed out of the tank with enough water to cause solid to flow on the steep angle.

Burt revolving filter (Fig. 12) consists of a steel-plate cylinder (*a*) similar to a tube mill or rotary kiln, one end supported by a hollow trunnion (*b*), the other on a tire and rollers (*c*). The cylinder is closed by hydraulically operated toggles (*d*). The interior of the shell is lined with filter mats bolted on. The whole is revolved about 15 r.p.m. by means of gear (*e*). Feed is introduced through feed valve (*f*) until a proper charge has entered, when the air pressure is turned on. Filtrate is forced through filter mats and out through holes in the shell. Wash liquid is then introduced, as desired. Finally the closing head is withdrawn and the charge is tumbled out, sluicing water being used, if necessary.

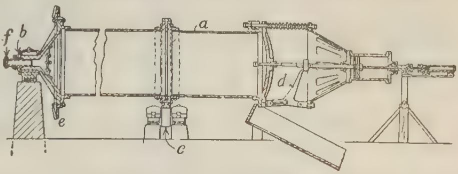


FIG. 12.—Burt revolving filter.

Ordinary PRESSURES are from 25 to 45 lb. The usual SIZES are 36 to 48 in. diameter and 20, 40 or 60 ft. long. CAPACITY of 700 to 800 lb. solid per sq. ft. per 24 hr. on quartzitic slimes are claimed with cake thickness of 4 to 4½ in. Average cycle at EL ORO MIN. & RY. Co. (*Chalmers & Williams catalog*) was: charging pulp, 3.8 min.; forming cake, 25.5 min.; adding wash water, 7.4 min.; washing 41.7 min.; discharging, 2.9 min. Average POWER consumption for running a 42-in. by 40-ft. filter was 4 hp, with a peak of 20 hp. at the time of discharging. (For detailed operations, see Rhodes and Myers, *97 J 1185*.)

6. Vacuum-leaf filters

Moore filter (Fig. 13) consists of a basket (*a*) formed by a plurality of individual filter leaves (*b*) (shown in detail in Fig. 14) all suspended from a traveling crane over a compartmented tank (*c*). LEAF (Fig. 14) consists of a canvas bag (*a*) slipped over a rectangular pipe frame (*b*) and stitched between wooden separating strips (*c*). The bottom pipe of the frame is perforated and is connected to header (*d*) and thence to a vacuum pump. In operation one of the tank compartments is filled with pulp and the filter basket is lowered

therein and allowed to stay, with vacuum on, until a cake of the desired thickness, usually 1 to 2 in., has built up. The basket is then lifted to an adjoining compartment and immersed in wash liquid and finally, after washing in one or

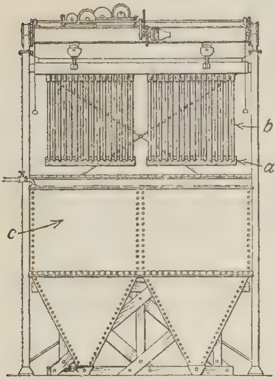


FIG. 13.—Moore filter.

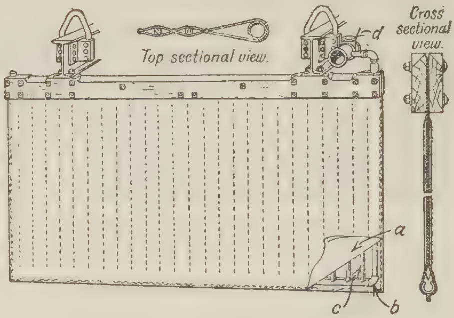


FIG. 14.—Detail of Moore filter leaf.

more compartments, as desired, is transferred over a discharging hopper where the cake is blown off by compressed air.

Butter's filter is similar except that the filter basket is stationary in one tank, into which are pumped in succession feed pulp and wash liquids. Finally the cake is blown off into the same tank and flushed away. Cake in this type of filter must be at least 0.5 in. thick to insure clean discharge.

Performance of Butters and Moore filters on cyanide pulps is given in Table 6. This type of filter is not used in filtering flotation concentrate.

Table 6. Performance of Butters and Moore filters handling cyanide pulps

Filter	Butters	Butters	Moore
Plant	Cobalt Reduction	Nipissing	Hollinger
Size, mesh	All −200, 97%–200	All −100, 80%–200	All −100, 96.5%–200
Feed $L:S$	2 : 1	1.5 : 1	2 : 1
Per cent. of water in cake	25	25–28	28–29
Solution in feed, per ton of solution	5 oz. Ag	5 oz. Ag	$4.50 Au
Barren wash, per ton of solid	2 : 1	2 : 1	1.25 : 1
Assay of solution in cake after wash	0.18 oz. Ag	0.1 oz. Ag	$0.009 Au
Water wash per ton of solid	*Nil*	5 : 1	4 : 1
Total filter area, square feet	4300	4300	24,000
Tons of dry slime per square foot per 24 hr.	116	116	60
Cost per ton for power	....	$0.03	$0.0083
Labor	....	.08	.0584
Supplies	....	.04	.032
Miscellaneous items	....	.01	....
Total	$0.15	$0.16	$0.0987
Cycle min., Building cake	120	60–75	40
Barren wash	90	90	30
Water wash	....	20	10
Discharge	45	45	30–40
Total	255	215–230	110–120

7. Centrifugal filters

Centrifugal filters are essentially the familiar basket centrifuge of the chemical laboratory adapted to large-scale continuous operation. They are used to dewater granular materials only, *e.g.*, to dry fine granular bituminous coal as an alternative to gravity draining.

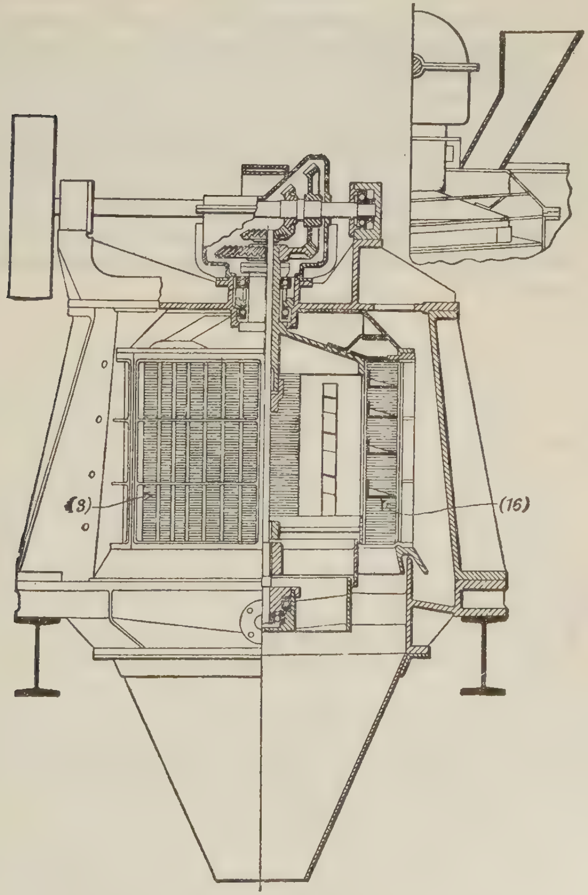


FIG. 15.—Hoyle centrifugal coal dryer.

Hoyle dryer (Fig. 15) is typical of a continuous machine. The essential parts are the screen basket (*3*) revolving at 600 r.p.m., and the spiral scraper (*16*), revolving slightly slower, which scrapes the deposit downward. Air is forced upward inside the basket by means of a fan to aid in drying.

Cost of centrifugal coal drying is said to be less than $0.02 per ton (*23 CA 784*).

8. Comparison of filters

Continuous filters have the surpassing advantage of continuous operation with consequent low attendance charge and non-necessity for provision for storage of thickened feed pulp or alternative intermittent operation of thickeners. They are and should be used whenever a cake of ¼ in. upward in thickness can be built in 3 to 4 min. under the vacuum available (this time being the economic maximum time allowable) and when sufficient washing can be effected. Intermittent filters of both vacuum and pressure types have the advantage of complete control of cake-making and washing periods, they wash more thoroughly and with less water than the continuous machines, and have greater filtering area per unit of floor space. Disadvantages are the intermittent operation. Pressure filters are better than vacuum when the cake has low porosity, but in them porosity of cake is less, cake is less uniform and consequently not so uniformly washed, wear on filter cloth is greater than in vacuum filters, and in those pressure filters in which pulp is forced in by pump the high wear on the pump is a charge against filtration. Porous-bottom tanks, filter tables and rotary dewaterers can be used only on highly permeable granular pulps. Centrifugal filters are used only in drying granular materials, notably bituminous coal.

SECTION 18

DRYING

Drying, in milling, is the process of evaporating, from ores and mill products, moisture that cannot be removed by mechanical means such as draining, decantation and the like. The process enters into mill operations at two places, *viz.:* (*a*) when wet ore or an intermediate wet-mill product is to be treated by a dry process, *e.g.*, dry screening, magnetic or electrostatic concentration or pneumatic separation; (*b*) when a mill product, *e.g.*, concentrate, is to be shipped a considerable distance and freight on water is more costly than its removal.

1. Principles

The fundamental principle in all drying operations is to maintain the partial pressure of the water vapor in contact with the liquid water below the vapor pressure of the liquid water.

If a liquid (*e.g.*, water) is introduced into a vacuum, it evaporates quickly and, if the amount introduced is sufficient so that not all will evaporate a condition of equilibrium is soon reached at which vapor exerts a steady pressure on the walls of the container. This pressure is independent of the volume of the container but varies with the temperature. It is called the VAPOR PRESSURE. When it has reached a constant value the space is said to be SATURATED with vapor and the numerical value of the pressure is the same as the VAPOR PRESSURE (OR VAPOR TENSION) OF THE LIQUID. At this point any further evaporation of liquid is accompanied by condensation of an equal weight of vapor. Numerical values of the vapor pressure of water at different temperatures are given in Sec. 25, Table 11, Col. 2.

If the liquid, instead of being introduced into a vacuum, is introduced into a space containing air or another gas not the vapor of the liquid, evaporation takes place, although more slowly than in the preceding case, until a constant state obtains, when the amount of vapor in a given space will be practically the same as though no other gas were present, and the partial pressure due to the vapor will be the same as the total pressure before and numerically equal to the vapor pressure of the liquid. Now the air or gas is said to be saturated with the vapor of the liquid. Since the amount of vapor in the gas-and-vapor-filled space is greater the higher the temperature, it follows that the amount of vapor in saturated air likewise increases with increase in temperature. The relation is given in Table 11, Sec. 25.

If, instead of being placed in a container, the liquid is open to the atmosphere, evaporation similarly takes place and continues until the adjacent atmosphere is saturated, *i.e.*, until the partial pressure of the water vapor in

contact with the liquid water is equal to the vapor pressure of the liquid water. The rate of evaporation increases with increase in the difference between the vapor pressure of the liquid and that of the adjacent vapor, hence it increases with increase in temperature of the liquid and with the vapor-absorbing capacity of the surroundings. Since evaporation into a vacuum is more rapid than into a gas-filled space, the vacuum gives maximum vapor-absorbing capacity. Since heated air can carry more vapor than cold air, it accelerates evaporation, and since the partial pressure of dry air is less than that of moist air, continuous replacement of the gas in contact with the liquid will likewise accelerate evaporation.

Heat required. It is fundamental, of course, to the discussion, that heat is the cause of evaporation. The molecular explanation is that heating increases the velocity of the liquid molecules sufficiently so that some of these that leave the liquid surface travel far enough not to return, and, in the presence of the elevated temperature, have sufficient molecular velocity to maintain the gaseous spacing.

The heat necessary to cause evaporation may be transmitted directly to the liquid through the walls of the container or may be carried by the air or other gas that is brought into contact with the liquid surface. Both methods of heating are employed in the dryers used in milling practice. In both cases heat must also be supplied to raise the temperature of the apparatus to the working temperature and to maintain it there in the face of heat loss by radiation and conduction, to raise the temperature of the solid from the inlet to the discharge value, to similarly raise the temperature of all of the entering water, to evaporate the water, and to insure that the discharged gases are above the temperature at which condensation of water vapor will take place in the exit flues.

Rate of evaporation of water from a mixture with discrete solid particles depends, during the first part of the drying operation, principally upon the difference maintained between the vapor pressures of the liquid and adjacent gaseous medium, but after the moisture has been lowered to the point that the material can be caked by pressure, the rate of diffusion of liquid to the surface of the cakes controls. At this time the evaporation rate can be greatly increased by agitation that will break up lumps and shorten the distance through which liquid must diffuse. The drying rate from the point where liquid is visible to a moisture content below the caking point varies with the character and size of the solid particles. Porous and finely-divided materials reach the caking point more quickly than dense and coarse materials, hence their average drying rate over the above range is slower. Unsized material dries more slowly than sized. There is no method of theoretical analysis of the problem of rate, but Atwater and Borkland (*29 CME 226*) report tests on several typical substances using an indirect-heat steam dryer. Their results are summarized in the curves in Fig. 1.

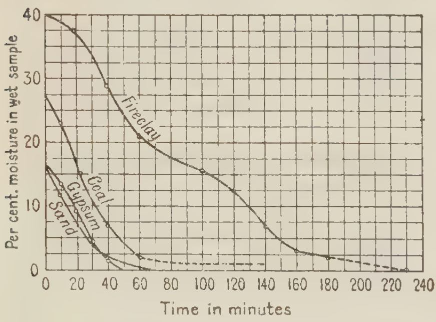


FIG. 1.—Rate of drying various materials.

The low drying rate at the beginning of the operation on coal, gypsum and fire-clay represents time during which the material and apparatus were being brought up to efficient evaporating temperature. The flattening at the lower end of all the curves begins at the point where caking becomes important. The average drying rates for lump gypsum and for quartz sand were substantially the same throughout the drying range and remarkably alike above the caking point. The rate for slack bituminous coal was less, due to its greater porosity and mixed sizes while that for fire-clay was minimum, due to the fineness of grain and high caking point (18 per cent. water). Apart from these facts the noteworthy feature in all of the curves is the disproportionate time required to drive off the last of the moisture. Thus bituminous coal dried from 27 to 2.5 per cent. moisture in, roughly, 55 min. or 0.45 per cent. reduction in moisture per minute, while to dry from 2.5 per cent. to 1 per cent. required 80 min. or 0.014 per cent. per min., roughly one-thirtieth as fast. Gypsum dried from 17 to 2.5 per cent. moisture in 36 min., 0.4 per cent. per min.; and from 2.5 per cent. to substantial bone-dryness in 26 min. or 0.1 per cent. per min., say one-quarter as fast. Sand behaved substantially the same as gypsum. Fire-clay required 150 min. to dry from 40 to 2.5 per cent. or 0.25 per cent. per min., and 80 min. more to reach substantial dryness, equivalent to 0.031 per cent. per min., say one-eighth as fast. While these curves cannot be read directly to give drying time for similar materials in other types of dryers, the relative times required to dry down to and below 2.5 per cent. moisture with similar materials may be used in lieu of more direct experimental information in any given case.

TYPES OF DRYERS

2. Drying floor

The simplest drying operation consists in spreading material on floors, protected from the elements but open to free circulation of air. Under these circumstances drying takes place even with freezing temperature, although very slowly. The rate is increased by turning material over from time to time. This type of dryer is used in certain small and crude non-metallic milling plants, *e.g.*, tripoli, bauxite, etc. It is applicable only where labor is the cheapest commodity entering into the treatment process, tonnage is small, and time is not an important element.

Heated floor is used at Tul Mi Chung mill (*119 P 814*) for drying flotation concentrate. The floor is of concrete (1 of cement, 2 of washed sand, 4 of ¾-in. gravel), 2½ in. thick, reinforced with barbed wire laid in 3-in. squares at ¾-in. from the bottom of the slabs. Checker-work flues underlie the floor. The slope toward the stack end, which is also the loading end, is 1 in. per foot for the last 3 ft. and ¼ in. per ft. for the rest of the length. Concentrate is brought in cars from the thickeners, dumped at the stack end and allowed to drain. Material is then raked by hand toward the fire end, where it arrives dry. Six floors, each 15 × 40 ft. are needed for 30 to 35 tons per day of concentrate screening 97 per cent. −200-mesh. Fuel consumption ranges from 0.1 to 0.2 cord of wood per ton dried, summer and winter respectively, and averages about 0.14 cord. Cost of drying, sampling, sacking, and weighing is $0.90 per ton with wood at $3.60 per cord and labor at $0.25 per day of 10 hr. At Tonopah Mining Co. (*91 J 1214*) fine gravity concentrate was drained in an 8-in. layer on sloping (⅜-in. per ft.) plank floors to 15 per cent. moisture in 20 hr., then dried to 3.4 per cent. on a steel-plate floor. With coal at $13.50 per ton and wood at $9.50 per cord the total cost of drying 4 tons per day was $0.84 per ton.

3. Tower dryer

Tower dryer consists essentially of a furnace with high vertical brick stack down which the material to be dried falls. Fall is impeded by grids or baffles set at right angles across the stack or by a series of shelves projecting from the walls.

Fig. 2 shows two different forms in use at the Franklin mill of New Jersey Zinc Co. The stack is usually laid up of three or four courses of common red brick, bound both ways at about 3-ft. intervals with tie-rods laid in the brick work. or with buck-stays and tie rods. The sides and arch of the fire-box and the first few feet of the stack are lined with one course of fire brick. Racks for T-bar grids are carried on the inner walls or, as at the Empire Iron and Steel Co. mill (*99 J 560*), carried on independent hollow sectional cast-iron columns, built up like tile pipe, set in the corners of the stack. If shelf baffles are used, they are

usually carried on the stack walls and may be made of adjustable slope. At Empire Iron & Steel Co.. the stack is 5 ft. square and 45 ft. 4 in. high and is set on a concrete block 10 × 10 × 10 ft. The baffle bars are arranged in ten sections or groups. The lowest seven sections consist of five tiers of five bars each, the bars in alternate tiers at right angles. The upper surface of the bars (the cross-bar of the T) is 6 in. wide and ¾ in. thick. The bars are set to give 6 in. clear horizontally and 8 in. vertically. In section No. 8 there are 4 tiers of 4 bars per tier, each bar 9 in. wide and 1 in. thick. This leaves 7 in. clear between bars horizontally and 10 in. vertically. In section 9 there are three tiers of 3 and 4 @ 9-in. bars per tier, leaving 10- and 10½-in. spaces horizontally and 12 in. vertically. In the top or No. 10 section there are two tiers with 3 and 4 @ 9-in. bars per tier, leaving the same hori-

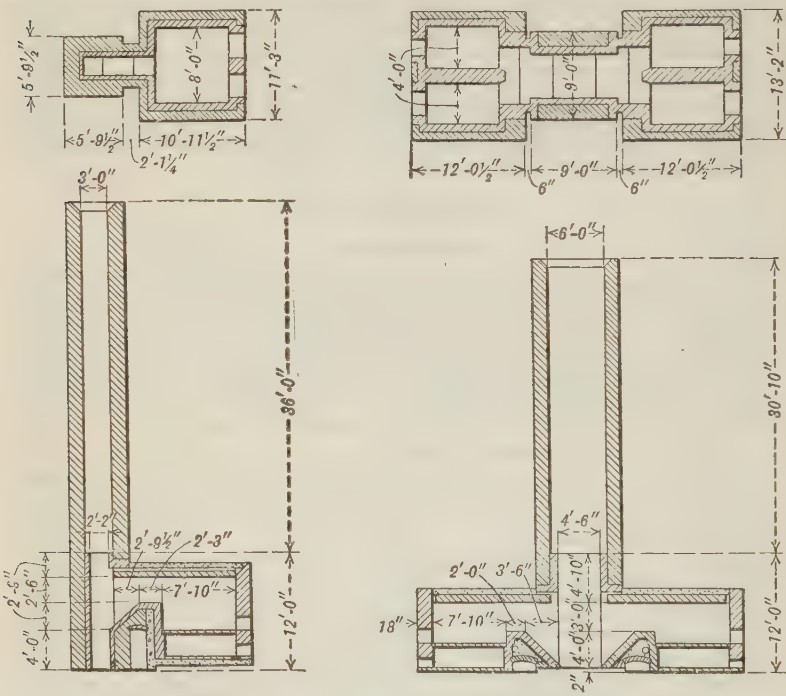


Fig. 2.—Tower driers at New Jersey Zinc Co.

zontal spacing as in No. 9 but 15-in. vertical spacing. The web of the bars fits loosely into the supporting racks and doors are provided at each section for ease in changing. The firebox is 6 ft. 9 in. × 7 ft. 6 in. with grate for burning No. 2 buckwheat coal under forced draft. Additional air is furnished by a No. 80 American Blower Co.'s steel-plate exhaust fan. This air is pre-heated by drawing it over the arch of the furnace; it then passes under the arch, where it joins the gases of combustion in passing up the stack. A 30-in. (diam.) by 20-ft. stack is provided and a damper arrangement to cut in the blower or the stack for draft, as necessary. Feed is through a 1⅛ × 2½-in. slot and carries an average of 14 per cent. moisture. Average feed rate is 25 tons per hr., maximum 35 tons. Product is substantially dry. At New Jersey Zinc Co., Franklin mill, the double-furnace dryer shown in Fig. 2 dries 100 tons of −1-in. feed per hour from 3 per cent. moisture to apparent dryness. Fuel consumption is 700 lb. anthracite buckwheat per hour. The air used amounts to 20,000 cu. ft. per min. and the blower draws 30 hp. The single-furnace dryer is fed 6 tons per hr. of −10-mesh material containing 8 per cent. moisture; discharge is substantially dry. Coal consumption is 250 lb. per hr. (anthracite). The blower supplies 5500 cu. ft. of free air per min. with a consumption of 10 hp. A similar dryer at the same plant with a stack 3 ft. 6 in. square and

32 ft. high handles 11 tons per hr. of −6 + 35-mesh feed carrying 10 per cent. water and brings it to substantial dryness with a coal consumption of 170 lb. per hr., air consumption of 3200 cu. ft. per min. and power draft of 15 hp. At the OGDENSBURG mill of the same company 20 tons of −1-in. ore per hr. is dried in a stack 3 ft. 6 in. square and 40 ft. high from 10 per cent. water to apparent dryness. Fuel consumption (anthracite) is 450 lb. per hr., air consumption 10,000 cu. ft. per min., requiring 10 hp. at the blower motor. In the same mill a double-stack dryer, each stack 3 ft. square and 38 ft. high dries 3 tons per hr. of −8 + 35-mesh feed from 15 per cent. moisture to apparent dryness. Fuel consumption is 200 lb. per hr. of powdered bituminous coal, air consumption is 8000 cu. ft. per min. and power consumption about 10 hp. In all of these NEW JERSEY ZINC CO. dryers the attempt is to keep the exit gases down to 200° F. At WITHERBEE-SHERMAN MILL No. 3, −¾-in. roll product (magnetite gneiss) is dried from 3 per cent. to 0.2 per cent. moisture at the rate of 50 tons per hr. in a stack 4 ft. 9 in. square × 50 ft. high. Coal consumption is given as 1 ton of anthracite (birdseye) per 24 hr., but this is undoubtedly below the actual consumption for the reduction in moisture stated. Average evaporation in tower dryers is between 4 and 6 lb. of water per pound of 13,000-B.t.u. coal.

The DISADVANTAGE of the tower dryer is the difficulty of regulating the rate of fall and, consequently, the amount of drying. In dryers with adjustable-shelf baffles this may be done by changing the inclination of the shelves, but not much is to be gained here because, if the shelves are made flat enough to materially retard flow, the wet material sticks and piles up and substantially all drying effect of the shelf surface is lost. Another disadvantage is the large loss in head room. The ADVANTAGES are simplicity of design and operation.

4. Rotary dryers

The rotary dryer is the commonest form of mechanical dryer. It consists essentially of a long cylinder mounted on a slight incline (acute cones and horizontal cylinders with internal spirals are also used), revolved slowly while the material to be dried and the heated gases pass therethrough. Lifting angles on the interior of the shell lift the drying material and drop it through the current of gas, thereby both spreading the material out in a manner favorable to evaporation of the contained moisture and aiding its progress through the shell. Several types are manufactured.

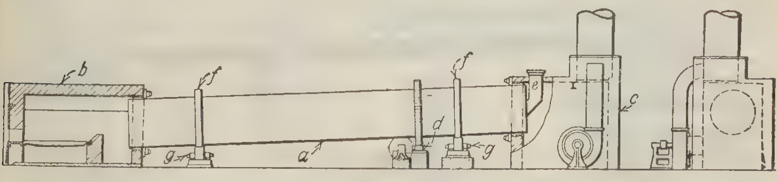


FIG. 3.—Direct-heat cylindrical drier.

Direct-heat cylindrical dryer (Fig. 3) is the simplest form. The essential parts are the drying cylinder (*a*), furnace (*b*), exhaust chamber (*c*) and driving mechanism (*d*). Feed enters through spout (*e*) and is discharged into a suitable receptacle or conveying mechanism at the other end of the shell. The shell is made of heavy steel plate with riveted joints. Longitudinal lifting angles are riveted to the inner surface. Heavy steel tires (*f*) bolted to the shell, ride on hardened-steel rollers (*g*).

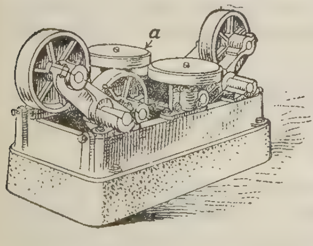


FIG. 4.—Roller bearing for rotary dryer.

Various methods are employed to resist end thrust. Fig. 3 shows flanged rollers used at the head end. Fig. 4 shows a more efficient, although more expensive form with horizontal rollers (*a*) that bear against the sides of the tire.

A gear drive is usual although chain-and-sprocket drive is used on some small, light machines. Coal is the usual fuel, but any fuel may be used. The use of a blower in addition to a stack is wise, since it gives positive control of draft and increases thermal efficiency.

It is, of course, possible to place the furnace at either end of the shell, as desired. The arrangement shown in Fig. 3 is the usual one, as combustion is not checked by contact of the gases with cold wet material and dust losses are less because the sheet of wet solid at the feed end serves as a sort of filter for the dust-laden gases.

Usual cylinder sizes are 4-, 5-, and 6-ft. diameter with lengths ranging from 25 to 35 ft. for the 4-ft. shell and 35 to 60 ft. for the 6-ft. shell. Usual SPEEDS are 3 to 6 r.p.m. POWER required for dryer cylinder and fan averages about 10 hp. for a 3 × 25-ft. machine; 15 hp. for a 4 × 30-ft., 25 hp. for a 5 × 35-ft., 25 hp. for 6 × 40-ft., 50 hp. for 7 × 50-ft., and 70 hp. for 8 × 60-ft. (*C. O. Bartlett and Snow.*) FUEL consumption averages about 1 lb. of 13,000-B.t.u. coal per 5 lb. of moisture evaporated.

Direct-indirect-heat cylindrical dryer is shown in Fig. 5. The elements of the shell are the same as those in the dryer just described but the circulation of hot gases is first around the outside of the shell, then through the shell.

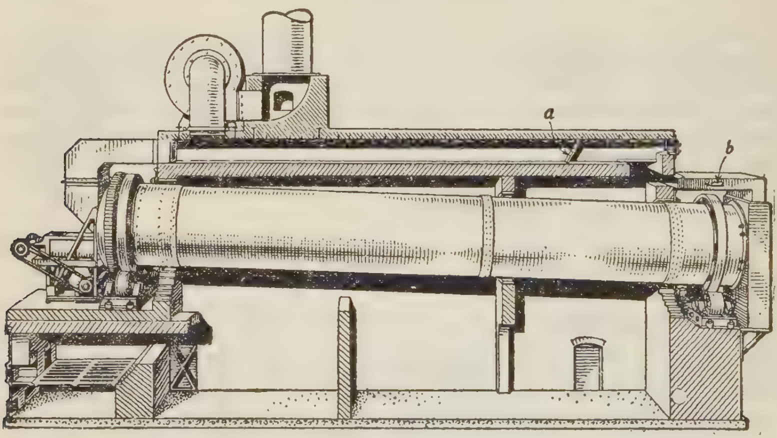


FIG. 5.—Direct-indirect-heat cylindrical dryer.

Dampers (*a*) and (*b*) determine the proportion of gas passing through the shell interior. By closing (*b*) and opening (*a*) the machine becomes an indirect-heat dryer. The usual cylinder sizes are the same as those of the preceding machine.

This machine utilizes heat more completely and efficiently than the direct-heat machine, both on account of the enclosure of the shell, which decreases radiation losses, and the longer time that the gases are in contact with surfaces to be heated. FUEL consumption is about 1 lb. of 13,000-B.t.u. coal to 6 or 6½ lb. of water evaporated.

Another form of direct-indirect-heat cylindrical dryer is shown in Fig. 6. Hot gas directly from the furnace passes through the inner shell and returns between the two shells, as indicated by the arrows. Feed enters at the furnace end of the outer shell, is lifted by the ribs of the outer shell, showered through the stream of hot gas onto the ribbed outer surface of the inner shell,

held for a while in contact with this hot surface, and then showered again through the gas stream onto the outer shell. This cycle is repeated many times in the passage to the discharge-end head where the material is finally picked up by buckets on the inner surface of the outer shell, dropped onto the fluted discharge cone (*a*), and carried out through the discharge trunnion.

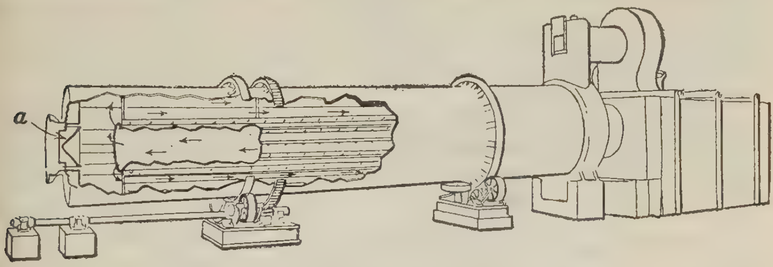


FIG. 6.—Ruggles-Coles double-shell dryer.

Cylindrical dryers in concentrating mills are usually of the direct or direct-indirect type, since discoloration and overheating are of no importance. Their use is ordinarily limited to granular materials, however, on account of the high dust losses with finer feeds.

Performance of double-shell dryers on different classes of material are indicated by Table 1.

Table 1. Tests on Ruggles-Coles dryers. (*Ruggles-Coles Eng. Co.*)

Size of dryer, diameter, in. × length, ft.	60×30	80×45	70×35	70 × 35	90×55	60×30*a*
Material being dried	Sand	Clay	Cement rock	Coal	Iron ore	China clay
Temperature of external air, deg. F.	40	52	65	45	60	50
Temperature of exhaust from fan, deg. F.	98	140	102	180	123	166
Temperature of material entering dryer, deg. F.	32	40	60	60	48	58
Temperature of material discharged from dryer, deg. F.	210	350	142	238	188	227
Calorific value of fuel used, B.t.u.	12,100	*14,000	*13,600	13,500	13,544	12,405
Fuel consumed per hour, lb.	266	857	346.58	338	767.4	280
Amount of moisture in material fed, per cent	5.67	29.92	4.52	9.8	17.6	27.3
Amount of moisture in material discharged, per cent	0.27	2.3	0.56	4.3	7.1	0.7
Water evaporated per hour, lb.	2187	7319	2034	2463	6328	1414
Water evaporated per pound of fuel, lb.	5.49	8.54	6.46	7.3	8.25	5.05
Dried material per hour, lb.	24,307	18,539	48,150	23,400	46,144	3915
Fuel per ton of dried material, lb.	21.8	92.4	14.4	28.9	33.3	142
Heat lost by exhaust air, per cent	7.3	8.7	5.9	14.2	7.4	26.6
Heat lost by radiation and other causes, per cent	11.6	11.8	12.2	8.2	8.8	22.9
Heat used to raise temperature of material, per cent	28.6	9.8	32.6	16.2	14.2	3.8
Heat used to evaporate water, per cent.	52.5	69.7	49.3	60.7	69.6	46.7
Power required for fan	2.5	5	3	3	7	3.5
Power required for drum	11	27	17	18	47	12
Total thermal efficiency	81.1	79.5	81.9	76.9	83.8	50.5

* Estimated. *a* Completely indirect.

In PHOSPHATE WASHING (*50 A 929*) a 5 × 40-ft. machine dries 135 to 165 tons of − ¼-in. slime-free gravel per 24 hr. from 20 per cent. to 2 or 3 per cent. moisture with a consumption of 75 lb. bituminous coal per dry ton and 15 to 20 hp. for driving the shell and fan. At the U. S. SMELTING, REFINING AND MINING CO., Midvale plant, a 5 × 30-ft. Ruggles-Coles (double-shell) machine dried 90 tons of −16-mesh table middling per 24 hr. from 14.5 per cent. moisture to bone-dryness with a consumption of 3 tons of bituminous coal per 24 hr. Temperature of exit gases was 350° F. Speed, 5 r.p.m. Power consumption, driving shell, 6 hp.; fan, 4 hp. About 5.5 per cent. possible running time was lost due to shell repairs. At AFTERTHOUGHT COPPER CO. (*119 P 154*) a 5 × 30-ft. oil-fired machine was used to dry about 150 tons flotation concentrate per 24 hr. from 15 per cent. to 5 per cent. moisture. About 10 tons of flue dust was collected monthly. Some caking occurred near the feed end when concentrate was sticky.

Multiple-tube dryer is a form of cylindrical dryer with the shell replaced by a number (usually 3 to 5) of parallel tubes aggregating in outside diameter the usual 3 to 8 ft. of the single-shell machines. Various arrangements are shown diagrammatically in Fig. 7. Sufficient space is maintained between the tubes to allow free circulation of hot gases. The machines may be run either indirect-heat or direct-indirect.

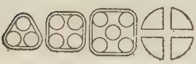


FIG. 7.—Arrangements of tubes in multi-tube revolving dryers.

ADVANTAGES are greater heating surface for indirect heating and reduction in power consumption due to distribution of the ore stream around the axis of revolution. POWER CONSUMPTION is from 70 to 80 per cent. of that required for a single-tube dryer. C. O. Bartlett & Snow claim an average of 8 lb. of water evaporated per pound of 13,000-B.t.u.-coal in their multi-tube dryer operated with both direct and indirect heat.

Dust losses may be an important factor in drying in rotary and tower dryers. Dietz and Keedy (*43 A 342*) give the data shown in Table 2 for operation of a 4½ × 24-ft. rotary

Table 2. Dust loss in a rotary dryer. (*After Dietz and Keedy*)

Products	Weight, per cent.	Assay		Percentage of total	
		Pb, per cent.	Ag, ounces	Pb	Ag
Total feed	100	10.35	9.68	100	100
Dry discharge	96.2	9.77	9.34	90.7	95.40
Settled in chamber	1.77	35.30	10.30	6.04	1.94
Settled in smoke-stack	0.76	18.50	8.60	1.36	0.69
Lost in gases	1.27	14.30	15.20	1.75	2.04

dryer running at 8 r.p.m. with light draft, the feed carrying 5 to 8 per cent. moisture. At a dry-crushing cyanide plant, drying −2-in. material in rotary dryers that needed strong draft, from 5 to 6 per cent. of the feed, which assayed 0.31 oz. Au and 0.15 oz. Ag, was lost as −150-mesh dust that assayed 0.48 oz. Au and 3.48 oz. Ag. In an elaborate test with a 4 × 30-ft. rotary dryer, sloped ¼ in. per ft., they found that in drying −2-in. zinc ore, about 12 per cent. of the feed was reduced from +16-mesh to −16-mesh by the tumbling action in the dryer, and that this 12 per cent. of the feed carried 19 per cent. of the zinc. In a dust-collecting system in the same plant substantially 16 per cent. of normal dryer feed was collected carrying 19 per cent of the total zinc and the more-gritty part of the collected product had the higher assay.

5. Rabbled-hearth dryer

Rabbled-hearth dryer is of the indirect-heat type. It consists essentially of a long bottom-heated trough through which the material to be dried is slowly moved and at the same time turned over by suitable rabble arms attached to a reciprocating or chain-drag mechanism. Inclination of the rabble blades is made adjustable to permit variation in the speed of travel of material. In the reciprocating machines, length of stroke and height of

lift are also adjustable. The firing box is usually placed at the feed end and the hot gases pass through a checker-work under the cast-iron plate bottom. Common sizes are 3 × 30-ft., 3 × 40-ft., 4 × 40, 4 × 50, 5 × 50 and 5 × 60-ft. The Lowden dryer (Fig. 8) is the best-known of the hearth machines.

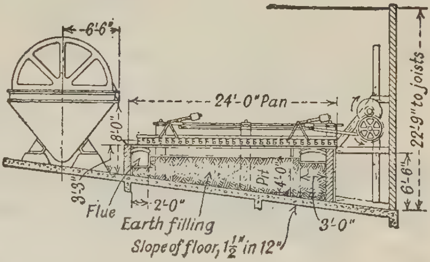


FIG. 8.—Lowden dryer and Oliver filter.

Performance of rabble driers. According to Watt (*57 A 385*), Lowden dryers require 70 to 120 lb. of coal per ton of lead-flotation concentrate to dry from about 14 per cent. to about 6 per cent. moisture and the usual size allowance is 10 sq. ft. of hearth area per ton of concentrate to be dried per 24 hr. At FEDERAL LEAD CO. MILL No. 4 one 12 × 24-ft. machine dried 50 tons per 24 hr. of −80-mesh flotation-lead concentrate from 15 or 16 per cent. moisture to 4 to 6 per cent. with a consumption of two tons of soft coal. The machine was run at 2½ strokes per min. and consumed about 6 hp. Lost time for tightening rakes, replacing crank pins and repairing furnace hearth and arch amounted to about 2 per cent. A common set-up, with gravity feed from filter and gravity discharge to a car-loading hopper is shown in Fig. 8. At HAYES MILLING CO. (*111 J 909*) a 5 × 17-ft. Lowden dryer, oil-heated, dried 175 tons per month from 35 to 15 per cent. water at a cost for fuel of $6.90 per ton of water evaporated. Results of efficiency tests on two Lowden driers are given in Table 3. Reynolds (*116 P 745*) gives data on FUEL CONSUMPTION in drying flotation con-

Table 3. Efficiency tests on Lowden dryer. (*Colorado Iron Works Co.*)

Mill	St. Joseph Lead Co.	Vindicator
Material	Flotation concentrate	Flotation concentrate
Size of hearth, ft	12×28	6×40
Fuel	Bit. coal	Lignite
Feed, wet weight, pounds per hour	11,959	3638
Moisture in feed, per cent	14.0	29
Moisture in product, per cent	6.5	13
Water evaporated, pounds per hour	959	673
Fuel burned, pounds per hour	275	166
B.t.u. per pound	10,500	10,000
Fuel per ton of dry solids	53.5	128.5
Water evaporated per pound of fuel, lb	3.49	4.05
Temperature of feed, degrees F	52	52
Temperature of product, degrees F	212	212
Heat used to raise temperature of solids,(*a*) B.t.u	411,400	103,320
Heat used to raise temperature of total water,(*b*) B.t.u	267,840	168,800
Heat used to evaporate water,(*c*) B.t.u	926,394	650,118
Total heat usefully applied, B.t.u	1,605,634	922,238
Total heat developed, B.t.u	2,887,500	1,660,000
Efficiency	55.6	55.5

a 10,285 (weight of dry solids, lb.) × 160 (temperature rise) × 0.25 (specific heat of solid, B.t.u.) = 411,400. *b* 1674 (weight of water, lb.) × 160 (temperature rise) × specific heat of water, (B.t.u.) = 267,840. *c* 959 (weight of water evaporated) × 966 (latent heat of evaporation) = 926,394.

centrate at three different plants as follows: (1) 56 tons (dry weight) per 24 hr. from 15 or 16 per cent. to 4 per cent. moisture with 3½ tons of coal; (2) 30 tons (dry weight) per 24 hr. from 15 to 16 per cent. to 5 per cent. moisture with 1¾ tons of coal; (3) 31 tons (dry weight) per 24 hr. from 29 per cent. to 13 per cent. moisture with 2 tons of coal. At LIBERTY BELL two steam-heated rabble-type hearth dryers with 4 × 10-ft. drying surface, in series, are used to dry 12 tons of 300-mesh flotation concentrate per 24 hr. from 20 per cent. to 10 per

cent. moisture. The company estimates a fuel consumption of 0.5 ton of coal per 24 hr. for the two dryers.

A hearth dryer with chain-drawn rabbles is shown in Fig. 9. This form is likely to give trouble by dragging sticky concentrate in a thick sheet along the hearth, resulting in high-power draft and wet discharge. C. O. Bartlett & Snow estimate the CAPACITY at between 2 and 3 lb. of moisture per square foot of heating surface per hour with a feed containing 25 per cent. moisture, and a FUEL CONSUMPTION on the same material of 1 pound of 13,000-B.t.u. coal per 3 lb. of moisture evaporated.

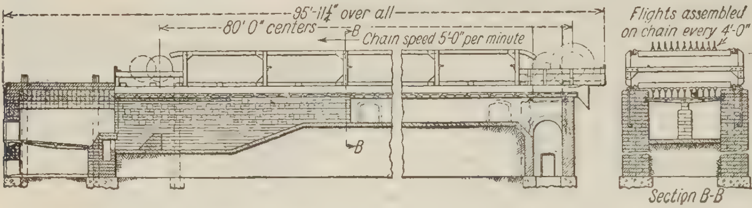


FIG. 9.—Chain-drawn rabbled-hearth dryer.

Applicability. The rabble dryer is particularly useful for drying sticky material that would ball up in a revolving dryer and for fine material such as flotation concentrate that shows excessive dust loss in such a machine.

6. Design of cylindrical dryers

Cylindrical dryers are designed by a method of cut-and-try, using in part accurate thermodynamic data and in part empirical data, finally checking the result, if possible, against actual performance under similar conditions. The items to be determined are, (*a*) the dimensions (volume or area) of the drying zone, (*b*) the amount of gas that must be handled, (*c*) the amount of heat that must be furnished, (*d*) the amount of power that will be consumed. The data required are, (1) the amount of solid to be handled per unit of time, (2) the inlet moisture content, (3) the exit moisture content, (4) the character of the material, its size, specific gravity, and whether it will suffer from overheating or discoloration, (5) the type of fuel.

Type of dryer. If material is not sticky and breakage and dust loss are not serious items, a tower dryer or a rotary dryer may be employed, preferably the latter. This is the case with most mill products other than fine concentrate; with such material a dryer of the rabbled-hearth type should be used.

Size of dryer depends upon the amount of drying that is to be done, the type of dryer and the character of the material. These factors combine to give the DRYING TIME, which, if determinate, forms the basis of the calculation. Published data on drying time are very scanty (see Art. 1). An approximation may be made by laboratory experiment that reproduces the method of drying as closely as possible. Thus heating in a thin-bottomed metal pan over a naked flame, with intermittent stirring and a gentle draft blowing over the surface of the material will give drying time for a layer of material of given depth. Similar heating of a thin bed with constant stirring and a current of hot air passing across the surface of the material will give a maximum figure for drying time in a tower or rotary-shell dryer. Unless the necessary reduction in moisture is very small and the final moisture content required is upwards of 3 or 4 per cent. a tower dryer should not be considered, and if considered, can probably be most safely designed by following the particulars of such a device successfully working under similar conditions as to size and kind of solid, and initial and final moisture content.

If a reliable figure for drying time can be obtained, the proper size of cylinder for a rotary dryer can be determined as follows: If the drying time is between 20 and 30 min., it is usual to make the volume of the shell (V) five times the volume of material in it at any one time, or

$$V = 5tTc/24, \qquad (1)$$

in which t = drying time in hours, T = tons feed per 24 hr. and c = cu. ft. of feed per ton. The diameter (d) and length (l) of the shell may be found from V through the relation, ordinarily prevailing, that $l = 7.5d$, from which

$$V = 6d^3 \text{ (approx.)}. \qquad (2)$$

Usual commercial cylinder diameters are 3, 4, 5, 6, 7 and 8 ft. and the usual corresponding lengths are the nearest 5-ft. to 7.5-times the diameter.

Slope and speed (r.p.m.) may next be settled from the relation

$$l = tnsd[d/2 + \sin(\alpha - 90)], \qquad (3)$$

where l, t and d have the significance already assigned, n = r.p.m., s = slope in ft. per ft., and α = the average angle, reckoned in the direction of revolution, from the bottom of the shell to the point where the lifters discharge. With radial lifters, α may be taken as 112°, with U-shaped lifters at 135°. The USUAL SPEEDS range from 3 or 4 r.p.m. for 8-ft. diameter to 5 to 8 r.p.m. for 3-ft. diameter. The usual slope is 1 : 24. With α fixed, setting either n or s obviously sets the other.

Temperature. When no limits are set by the requirements of the material itself, the attempt should be to make the temperature drop of the gases in passing through the drying zone as great as possible. The temperature limit is then imposed by the amount of heat that the dryer shell itself can stand. The strength of steel plate reaches a maximum at about 600° F. and falls very rapidly at higher temperatures until at about 660° it is the same as at ordinary temperatures. At 1000° the strength is only 40 per cent. of maximum, at about 1200° F. the plate becomes dull red and hereabout the susceptibility to corrosion increases greatly. Hence the temperature of the incoming gas should never be as high as this unless the gas-inlet end of the shell is protected by very moist, cold ore, and even with this protection the shell should not be called upon to support any great load at the heated portion.

High inlet temperatures with direct-heat dryers almost invariably result in high exit temperatures, hence if large temperature drop is to be attempted the dryer should be of the indirect-direct type with the hot gas entering under the feed-inlet end, passing around the shell to the discharge end and passing through the shell counter to the flow of ore.

The lower temperature limit may be anything desired, except that it must be borne in mind that the volume of gas necessary to carry the heat to effect evaporation will be very great when the inlet temperature is low, that as a result the heat loss in the outlet gases may be greater than when a smaller volume of gas at higher temperature is used, that the power required to move the larger volume of gas will be greater and that dust loss will be increased. On the contrary, radiation loss will be less and the life of the dryer shell will be longer with low inlet temperature.

Best exit temperature is stated by different writers at between 120 and 170° F. With lower temperatures there is danger that the gas will become supersaturated and deposit moisture while with higher temperatures heat loss is unnecessarily high.

Heat required is that needed (1) to raise the solid from inlet to outlet temperature, (2) to raise all of the water in the feed through the same temperature

range, (3) to evaporate the water, (4) to supply radiation loss and (5) to supply the loss through gas leaks.

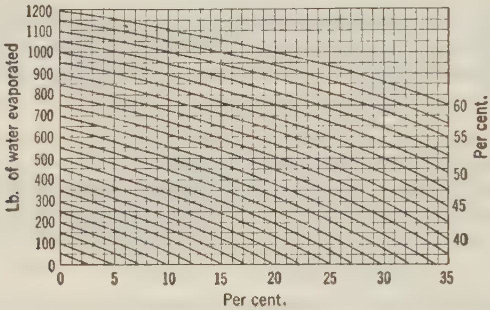
Heat required to raise the temperature of the solid is given by the equation

$$H_S = W_S h_S (t_e - t_i), \qquad (4)$$

where H_S is the required number of B.t.u., W_S is the weight of solid heated, in pounds; h_S is the specific heat of the solid in B.t.u. per lb. per deg. F.; t_e is the exit and t_i the inlet temperature in deg. F. The specific heats of common materials are given in Sec. 25, Table 7.

Heat required to raise the temperature of the water is obtained by the equation

$$H_W = W_W (t_e - t), \qquad (5)$$



Enter chart on diagonal representing percentage of moisture in feed, follow diagonal to intersection with ordinate representing moisture in product and read on left-hand scale pounds of water to be evaporated per ton of wet feed.

FIG. 10.—Weight of water evaporated per ton of wet feed from a given inlet to a given outlet moisture percentage (*after Ruggles-Coles Eng. Co.*).

where H_W is the required quantity of heat in B.t.u., and W_W = total weight of water entering. The specific heat of water is practically 1.0, so does not need to appear in the equation.

Heat required for evaporation (H_e) is the product of the weight of water evaporated (W_e) and the latent heat of evaporation of water at the outlet temperature (h_l). See Sec. 25, Table 11.

$$H_e = W_e h_l. \qquad (6)$$

Fig. 10 gives a graphical solution of the amount of water to be evaporated in terms of moist weight of feed and feed and discharge moisture percentages.

Heat lost by radiation depends upon the construction of the dryer and upon the inside temperature. Heat loss in terms of B.t.u. per sq. ft. per hr. per degree F. difference in temperature inside and outside the dryer walls is given in Table 4 for various kinds of dryer enclosures.

Heat lost by leakage of gas forms a considerable part of the total heat supplied. An allowance of 10 to 20 per cent. of the sum of the above four items should be made to cover.

All heat to perform the above computed heating duty must be supplied by gas passing around and through the heating zone. The heat balance is shown in the equation

$$W_G h_g (t_I - t_E) = H_s + H_w + H_e + H_{\text{radiated}} + H_{\text{lost}}, \qquad (7)$$

in which W_G = total weight of gas required in lb., h_g = specific heat of the gas (0.23, for air, is sufficiently accurate, since air predominates), and t_I and t_E are inlet and exit temperatures of the gases respectively. With this air there will pass through the exit passages of the dryer the vapor of the evaporated water. The DEGREE OF SATURATION may be determined by adding to the evap-

orated water the water entering with the inlet air. Compare the ratio of total pounds of water in exit gas to total pounds of dry air therein with the column headed "Ratio, vapor/air" in Sec. 25, Table 11. If the corresponding temperature in the table is well below the exit temperature no precipitation need be feared.

Table 4. Heat loss by radiation from dryer walls. (*After Buck, 29 CME 626*)

Material	Heat loss, B.t.u. per square foot per hour per degree F. temperature difference
Asbestos, Johns-Manville special, Type A, built-up	0.13
Asbestos, Johns-Manville special, Type B, built-up	0.20
Asbestos, Johns-Manville special, Type C, built-up	0.28
Asbestos sheet, 2-in., fine, refractory-coated	0.27
Asbestos, ¼-in. Transite, 2-in. impregnated Asbestocel sheets, sheet steel	0.25
Brick, 4-in. wall	0.77
Concrete, 4-in. wall	0.86
Steel, 50° temperature difference	1.95
Steel, 100° temperature difference	2.15
Steel, 150° temperature difference	2.40
Steel, 200° temperature difference	2.67
Sheet steel, 2 plates separated by ¾-in. air space	0.63
Sheet steel, 2 plates separated by 1-in. impregnated Asbestocel sheet	0.44
Sheet steel, 2 plates separated by 1½-in. impregnated Asbestocel sheet	0.32
Sheet steel, 2 plates separated by 2-in. impregnated Asbestocel sheet	0.25
Wood, ⅞-in. T. and G. sheathing	0.73
Wood, ⅞-in. T. and G. sheathing, and sheet steel	0.72
Wood, ⅞-in. T. and G. sheathing, building paper, 4-in. air space, building paper, ¼-in. Transite, built up on 2 × 4-in. studs	0.41

Volume of gas. To determine the volume of gas to be handled by the blower, take the reciprocal of the corresponding value in the column headed "Weight in lb. per cu. ft. of saturated mixture" and multiply by the ratio $(t_E + 491)/(t_t + 491)$ where t_t is the tabular temperature corresponding to saturation, as above.

Velocity of air leaving the dryer shell should not exceed 300 to 400 ft. per min., if dust loss is an important consideration. If a dryer, calculated as above, shows greater velocity than this, and dust loss must be prevented, higher initial and lower final temperatures should be investigated. If such change in conditions does not reduce velocity sufficiently, a dust collector is probably the cheapest method of overcoming the difficulty.

Amount of fuel required is determined by dividing the total heat requirement $(H_s + H_w + H_e + \ . \ . \ .)$ by the heating value per pound of the fuel that is to be used. See Table 5.

Table 5. Approximate heating value of various fuels

Fuel	B.t.u. per pound
Anthracite	12,000 to 14,000
Bituminous	10,000 to 15,000
Lignite	6,000 to 12,000
Air-dried peat	5,000 to 9,000
Dry wood	8,000 to 9,000
Fuel oil	18,000 to 19,000

C. O. Bartlett & Snow estimate the consumption of 13,000-B.t.u. coal as 1 lb. per 5 lb. of moisture evaporated in an exposed-shell direct-heat revolving dryer, 1 lb. per 6.5 lb. of moisture in an enclosed-shell indirect-direct heat machine and 1 lb. per 8 lb. of moisture in the enclosed-shell, indirect-direct heat, multi-compartment machine, when the final moisture content required is not below 3 or 4 per cent.

Size of grate necessary for coal-burning dryers may be determined by reference to Fig. 11.

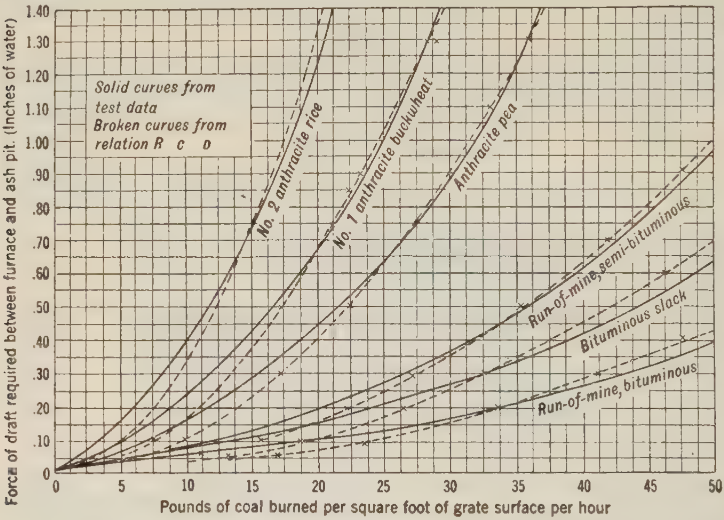


Fig. 11.—Size of grate required for different kinds of fuels (*after Lucke, Engineering thermodynamics*).

Centrifugal dryers. See centrifugal filters, Sec. 17, Art. 7.

Electrical dryers.—At Gold Hunter (*111 J 909*) eight coils of high-resistance wire wound on 1-in. iron-pipe supports were spaced at 1-ft. intervals across and near the face of an Oliver filter. Each unit consumed 3.95 kw. The filter handled 30 tons mixed table and flotation concentrate per 24 hr. The cake without the dryer averaged 11 per cent. water; with the dryer, 8.7 per cent. The cost per ton of water evaporated was $6.86. At Bunker Hill and Sullivan (*ibid.*) a rotary dryer consisting of a piece of 12-in. iron pipe 10 ft. long, covered with ⅜-in. asbestos was wound with 205 turns of bare No. 4 hard-drawn copper wire (90 lb.) terminating on collector rings. A current of 82 amp. at 282 volts was supplied and heat generated by hysteresis and eddy-current losses in the pipe walls. The pipe rotated 7 r.p.m. spiral rabbles inside, turning 1.4 r.p.m., moved the ore through. Pre-heated air was forced through counter-current. The capacity was 17 tons of flotation concentrate per 24 hr. from 11.4 to 6 per cent. moisture; cost of power, $5.66 per ton of water evaporated.

SECTION 19

STORAGE

1. Introduction

Necessity for storage arises from the fact that different parts of the operation of mining and milling ores are performed at different rates, some are intermittent and others continuous, some are subject to frequent interruption for repairs and others are essentially batch operations, so that unless reservoirs for material are provided between the succeeding different steps the whole operation is rendered spasmodic and, consequently, uneconomical. Storage bins are the best means of distributing the product of a crusher of high capacity to several crushers of low capacity, as for instance, jaw-crusher product to stamps. Large bins are desirable because they allow mill feed to be well mixed and tend toward uniformity. When a bin is fed with undersize from run-of-mine ore and crushed oversize, these products should be delivered at the same point in the bin or segregation of sizes, and probably, of values, will be unavoidable in drawing.

Amount of storage necessary is ordinarily considered and expressed in terms of the daily tonnage capacity of the plant. It depends on the equipment of the plant as a whole, its method of operation, and the frequency and duration of regular and unexpected shut-downs of individual units. At many mines ore can be taken out during a part of each day only on account of the necessity for getting supplies into the mine through the same opening; on the other hand concentrating mills are most efficient when continuously operated, due to unavoidable tailing losses in shutting down and starting up the machinery. Mine operations are more subject to unexpected interruption than mill operations and coarse-crushing machines are more subject to clogging and breakage than fine crushers and concentrating machinery. Consequently both the mine and the coarse-crushing plant should have greater hourly capacity than the fine-crushing, and concentrating units and storage reservoirs should be provided between them and the mill proper. Ordinary mine shut-downs, expected or unexpected, will not generally exceed 24 hours duration and ordinary coarse-crushing plant repairs can be made within an equal period, if a good supply of repair parts is kept on hand. Therefore, if a 24-hour supply of ore that has passed the coarse-crushing plant is kept in reserve ahead of the mill proper, the mill can be kept running independently of shut-downs of less than 24 hours' duration in mine and coarse-crushing plant. It is wise to provide for a similar shut-down of the mill. In order to do this the reservoir between coarse-crushing plant and mill must contain at all times unfilled space capable of holding a day's tonnage from the mine. Ore-storage reservoirs ordinarily are not of such shape that they can be filled or emptied completely without shoveling and, since to incur the expense of shoveling would to a large

extent defeat the purpose of storage, additional storage room must be provided so that a day's tonnage may be *drawn* into the mill proper, in case the ore supply stops for a day, and the same tonnage may be discharged into the partly-filled bin in the usual way, in case the mill ceases to draw. The amount of space that must be provided for un-drawable material and the additional amount that cannot be loaded without shoveling depend upon the shape of the bin, the method of drawing, and the method of loading. In ordinary rectangular bins it will amount to something more than that necessary to hold a day's supply of ore. Hence a bin to fulfill the conditions set down must have a volume equivalent to something more than three days ore supply and should be run about half full to provide against unexpected shut-downs at either end of the plant. Similar methods of analysis are applicable at other points in the path of the ore. The average of present-day practice, derived from data from 23 mills chosen at random is three-days' storage capacity; individual figures range from 0.9 to 10 days. These figures in many cases comprise both coarse- and fine-ore storage, *i.e.*, between mine and coarse-crushing plant and between coarse-crushing plant and mill. Peculiar conditions, such as long and uncertain haul from mine to mill, great variety in character of ores treated in a given mill, and the like, give rise to special storage problems, resulting in the provision of storage in excess of the average.

Methods of storage. Considerable storage is usually provided in the mine itself, in stopes and skip pockets, thus allowing actual breakage of ground and some underground haulage to proceed even after available surface storage space has been exhausted. When for any reason an extended mill shut-down is incurred and it is particularly desirable that underground breaking be carried forward in excess of the underground storage capacity, stockpiles may be built on the surface in a location convenient both for building and subsequent excavation. This may be the solution when extensive exploration work is carried forward during the building of a mill. STOCKPILING is resorted to when winter conditions are such as to prohibit excavation in open-cut mines and the year's mill supply must be taken out in the open months; or when transportation is stopped during the winter, though mining can be carried forward, as is the case in the Lake Superior iron ranges; or when the product of a milling operation is bulky, as in the case of coal or crushed stone and the demand is seasonal and in excess of mine or mill capacity. RAILROAD CARS may form a considerable part of the storage system; particularly when the haul from mine to mill is long. BINS are the commonest type of ore-storage reservoir. They are built in several different shapes, of masonry, concrete, steel and wood. Shape is determined primarily by the service the bin is to render, secondarily by the materials of construction. Wooden bins are ordinarily cheapest in first cost and easiest to construct. Masonry and concrete bins have the longest life. Steel bins are lightest and occupy the least space for a given capacity. They can be built in shapes that better withstand the pressure of the filling material and they maintain their original rigidity much better than wooden bins. Fire hazard is great in wooden bins, and, if the ore is wet, the bottom rots rapidly unless well ventilated. If bins are loaded by trains they are subjected to much vibration and heavy racking strains. Under these conditions the tension rods in wooden bins must be continually watched and tightened and the bins will, in time, become so rickety as to be dangerous. The large sticks of timber needed in high and wide wooden bins are becoming so scarce and expensive that the cost of such bins approaches more and more closely that of the other types.

2. Shape of bin

Common shapes are shown in Fig. 1.

Flat-bottom bin (*a*) is suited to timber, steel or concrete construction. It is the cheapest type of all to build and in it the ore forms its own bottom so that there is no wear on the bottom of the bin itself in drawing.

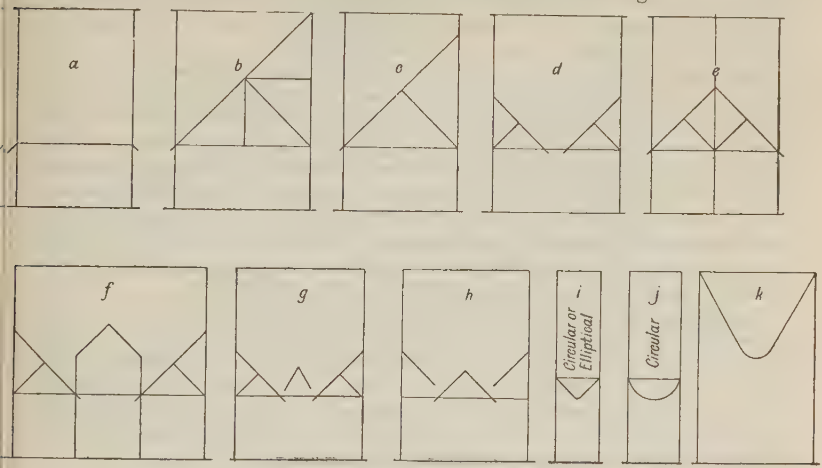


FIG. 1.—Typical bin shapes.

DISADVANTAGES are (1) that the bin cannot be emptied without shoveling, and (2) that the angle of repose material in the dead part of the bin may be as great as 60° when the ore contains a large amount of fines and an appreciable percentage of moisture, so that the free-running bin capacity is much less than in a sloping-bottom bin of the same horizontal cross-section. A flat-bottom bin is not suitable for service where material must be treated in lots, as in custom plants. Flat bottoms are frequently filled with waste, particularly if the ore is high-grade, so that interest will not be lost on high-grade ore locked up in bottom filling. But if the mill is working to capacity in any case, treatment of material that would otherwise form bin filling leaves a corresponding amount "locked-up" in the mine. The loss with high-grade ore by passage of fine material down into the waste filling may well exceed an actual interest loss. If the dead space in a flat bin is filled with ore, this ore forms a reserve if the emergency justifies shoveling, but after the bin has been thus emptied no ore can be drawn therefrom until an amount substantially equal to that shoveled out has been replaced. Anthracite coal should not be handled through flat-bottom bins because of breakage of coal in the dead space by pressure of the coal above.

Slanting-bottom bins. Type (*b*) is typical of small bins that it is desired to empty frequently, such, for instance as shipping bins and concentrate bins. Type (*c*) is perhaps the commonest around milling plants. It is suitable for distributing to and feeding gravity stamps, rolls, ball mills, or other multiple units. Completeness of discharge depends upon the spacing of gates lengthwise of the bin and the bottom slope. The minimum slope upon which material will slide depends upon the kind of material, size of pieces, percentage of fines, percentage of moisture and surface of bin bottom. Minimum sliding angles for dry materials containing only a small percentage of fines, on steel-lined bottom, are given in Table 3. The slope of the bin bottom should be at least 2°, better 5° greater than these figures. Type (*e*) is a modification of (*c*) rarely used on coarse feeds, but sometimes used for fine-ore bins and shipping bins. It saves timber for a given capacity, and is useful when delivery on two sides can be utilized.

Hopper-bottom bin (*d*) gives the highest percentage of gravity delivery of any of the first five types. Discharge must ordinarily be collected and transported to the following machines by a conveyor or cars. This type is particularly suitable for long bins of large capacity. Types (*f*), (*g*) and (*h*) are typical coal bins. They give a relatively high percentage of discharge by gravity; are shallow and do not cause the coal to crush, yet are of good capacity per foot of length. All of these types, while best adapted to timber construction, may be built of steel or reinforced concrete.

Steel bins. Forms (*i*), (*j*) and (*k*) are particularly adapted to steel construction, although both (*i*) and (*j*) have been built with reinforced concrete. Wood-stave circular tanks with flat bottoms have been used for both coarse-ore and concentrate-storage bins, but the gravity discharge percentage is low. Circular and elliptical shapes are particularly suitable for deep-bins, since the metal of the walls is under tensile rather than bending stresses and the pressure on the bottom is relatively small. Such bins with conical bottoms will draw practically clean, if the ore is not too moist. Stress in the hemispherical bottom (*j*) is less for a given loading than in a conical-bottom, hence lighter material can be used, but the gravity draw-off is not so complete.

Suspension bunker (*k*) puts all of the wall in tension and can be extended indefinitely at right angles to the section shown without any increase in weight of section, and without the loss in storage capacity that is attendant upon placing circular or elliptical bins side by side. It has been used extensively in fuel storage and handling plants, but not much in milling work. Its merits warrant investigation, however, when the amount of storage needed per foot of length of mill bin is within the limits of its capacity.

3. Design of bins

This problem involves the usual elements of determination of loads and design of members to bear them. Wear due to passage of material through the bin must be anticipated and structural members guarded against blows and abrasion that would weaken them. Determination of loads is by the same methods that are employed in the design of retaining walls (see Sec. 27, Art. 14). Many formulas and graphical methods have been proposed. Those by Rankine, Coulomb and Cain, summarized by Ketchum (*Design of walls, bins and grain elevators*), are best known and should be investigated in the case of large and elaborate structures. The graphical method given below, based on Coulomb's theory of "maximum wedge thrust" is satisfactory for most cases.

Preliminary considerations are: (*a*) required capacity in tons; (*b*) corresponding cubical content based on weight of filling (Table 1), space that cannot be loaded and space that cannot be emptied without shoveling; (*c*) location of bin and effect of topography on shape; (*d*) purpose of bin and effect of delivery requirements on shape, *e.g.*, the length of a bin to feed a battery of secondary crushers is determined to a considerable extent by the overall length of the battery; (*e*) material of bin, *e.g.*, height and breadth of a timber bin are limited to a great extent by maximum sizes of stock timber, deep steel bins are more economically loaded and drawn than shallow ones, etc.; (*f*) angle of repose of material to be stored (Table 2); (*g*) angle of friction of material on bin walls (Table 3).

Graphical method of determining pressures on bin walls. If $ABCD$ (Fig. 2) represents a transverse section of a flat-bottomed bin, broken rock, when poured therein, along the center line, will come to rest with inclined surfaces BE and EC. Angle EBC is known as the ANGLE OF REPOSE (ϕ). Its magnitude depends on the friction between the particles of

Table 1. Weights of bin-filling materials

Material	State	Weight per cubic foot, pounds
Ashes, coal	Dry, packed	40–45
Barite	Broken, loose	180
Cement	Clinker, broken	95
Cement	Ground, loose	50–56
Clay	Moist	120
Coal	Any size, broken, loose	45–60
Coke	Loose	23–32
Earth	Slightly moist	70–90
Earth	Soft, flowing mud	104-120
Gravel	Dry	120
Iron ore (hematite)	Loose	150
Lime, quick	Ground, shaken	64
Phosphate, rock	Ground, loose	75
Quartz	Pulverized, loose	90
Quartz	Pulverized, well shaken	105
Sand	Dry, loose, even sizes	90–106
Sand	Dry, loose, uneven-sized grains	117
Sand	Even sizes, voids full of water	118-120
Slag	Granulated	53-60
Slag	Bank, crushed	80
Stone, average	Crushed	95–100
Stone, heavy (trap, greenstone)	Crushed, loose	107

Note.—Volume of broken rock in dumps and bins is from 1.6 to 1.9 times volume in place.

Table 2. Angle of repose of bin fillings = ϕ

Material	State	Angle ϕ, degrees
Anthracite	Broken, loose	27
Ashes	Dry	40–45
Cement	Dry	40
Cement	95 per cent. through 20-mesh screen	37.5
Cement	96 per cent. through 100-mesh screen, piled	*a*
Cement, clinker	Through 1½-in. screen	33
Cement, material, raw	90 per cent. through 20-mesh screen	33
Cement material, raw	89 per cent. through 100-mesh screen, piled	*b*
Clay	Damp	27–45
Clay	Wet	16–17
Coal, bituminous	63 per cent. through 10-mesh screen	34.5
Coal, bituminous	98 per cent. through 100-mesh screen	16
Coal, bituminous	Broken, loose	35–45
Coal, bituminous	Slack	37.5–45
Earth	Dry	29
Earth	Moist	45
Earth	Mud	17
Gravel	Dry	37–48
Gravel	Sandy	26
Iron ore, soft	Broken	35
Ore	Broken	45
Sand	Fine, dry	31-37
Sand	Wet	26
Sand	Very wet	32
Slag	Small sizes	45
Stone	Crushed, fines screened out	37 (average)

a Slope concave. Radius of curvature 78 ft. Angle 6° at 40 ft. horizontally from apex and 38.1° at apex. (*Ketchum.*) *b* Slope concave. Radius of curvature 7 ft. 9¾ in. Angle 5° at 5 ft. horizontally from apex and 48.3° at apex. (*Ketchum.*)

Table 3. Friction of bin fillings against walls = θ

Material	State	Angle of friction, θ, degrees (*b*)		
		Bin lining		
		Cribbed wood	Steel plate	Concrete
Anthracite	Screened	25	16–20	27
Ashes	Dry	40	31	40
Coal, bituminous	Broken	35	18–30	35
Coke	Mixed sizes	40	25–30	40
Gravel	Dry	....	40	....
Ore	Crushed and screened	40	30	40
Ore	Run-of-mine	45	35–40	45
Sand	Dry to moist	30–45	18–40	40–45
Stone	Broken	22*a*	16.7–40	40

a With grain. *b* Angles will be increased by presence of large amount of fine, damp material. Range of figures, where given, represents limits of authorities.

material and is different with different substances, sizes, mixtures of sizes, and moisture content. (See Table 2, compiled from various sources.) A heap of ore with cross-section *BEC* will exert no pressure on walls *AB* and *DC*. Side pressures are all taken up within the pile of material itself. Herein lies one of the differences between such materials, known as SEMI-FLUIDS, and true fluids. If bin *ABCD* is charged level full, the mass of material will tend to rupture by sliding of the upper part over the lower along some plane such as *BF*, lying between *AB* and *BE*. The thrust of wedge *AFB* against wall *AB* is the pressure against the bin front, according to Coulomb's theory. This thrust is a maximum when *BF* bisects angle *ABE*. If wall *AB* is considered displaced a small distance to the left, wedge *ABF* will slide a small amount along *BF* and in coming to rest will exert a force *P* on *AB* that is the resultant of a force *N*, normal to the wall and another *F*, acting vertically downward in the plane of the wall. This latter force is due to friction between the material and the wall. The ratio F_1/N_1 is equal to the coefficient of friction of the filling on the bin wall. (See Table 3.) To determine *P*, consider a wedge of material having the section *AFB* and 1 ft. in thickness along the length of the bin, in equilibrium under the action of forces exerted by the wall *AB* and the material lying to the right of *BF*. Lay off *AN* proportional to the weight of block *AFB*. The resultant of the reaction of the material to the right of *BF* on *ABF* acts in a

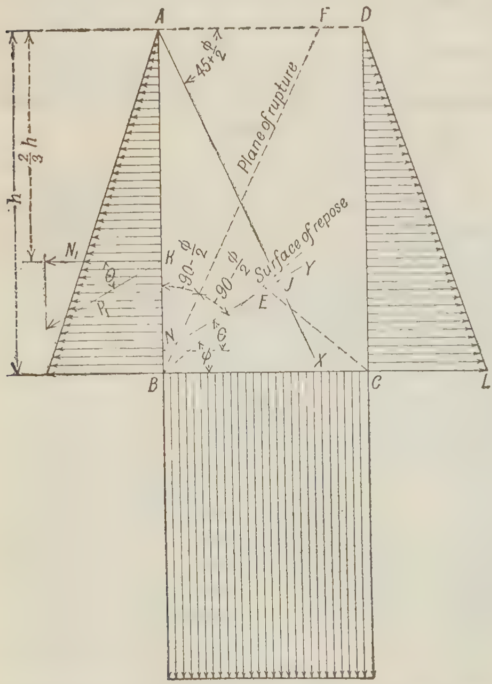


FIG. 2.—Solution of stresses in flat-bottom bin, based on Coulomb's theory.

direction inclined ϕ below the normal to BF. This will be $\left(45 + \frac{\phi}{2}\right)$ deg. below AF. Draw AX making angle FAX equal $\left(45 + \frac{\phi}{2}\right)$ deg. Through N draw NY so that angle θ is equal to the angle of friction ($\tan\theta = \mu$ = coefficient of friction) between the material and the bin wall. (See Table 3.) NJ is the total pressure in 1 ft. length of wall AB, in magnitude and direction. The point of application is K located $2h/3$ downward from A. Normal pressure N_1 is the horizontal component of P_1 ($= NJ$). Pressures on CD are the same. Total pressure on BC is the weight of block $ABCD$ and is uniformly distributed along BC. The distribution of pressure along wall $DC(= AB)$ is determined by placing $N_1 = \frac{1}{2}\overline{CL} \times h$, solving for CL and completing the triangle as shown. Unit pressure at any point along DC can then be read off directly.

In a slanting-bottom bin (Fig. 3) take $\phi = 40°$, $h' = 8$ ft., breadth of bin = 12 ft , weight of bin filling = 100 lb. per cu. ft., and $\theta = 30°$. Solve for P_1 as described in the preceding paragraph. To determine P_2, draw $CN = RQ$, determine area DCN and lay off N_2 through the point of application, $2h'/3$ down from D. Through this point draw the line of action of P_2 making an angle $\theta(=30°)$ with N_2, then complete the triangle of forces. Consider block $ABCD$, weighing W, in equilibrium under the action of the forces P_1, P_2, W and P_3, P_3 being the total reaction of the bottom to the load, unknown in magnitude and direction. W acts through the center of gravity of $ABCD$, located at T. (See Sec. 26, Art. 13.) To solve for the magnitude and direction of P_3, lay off TS to scale, equal and parallel to W; SV equal and parallel to P_1, and VU equal and parallel to P_2; UT, closing the force polygon, determines P_3 in magnitude and direction. The line of action of P_3 is determined as follows: Choose any point O so that angles between lines OT, OS, OU and OV are as large as conveniently possible. Choose any point such as b on the line of action of P_1 and draw lines vo and so parallel to OV and OS, respectively. Produce so to intersect TS, produced if necessary. Through this intersection draw to parallel to OT. Produce vo to meet line of action of P_2. Through the intersection draw uo parallel to OU. Produce uo and to to intersection. Through this intersection draw ut parallel to UT; ut is the line of action of P_3. To determine N_3 = (normal component of P_3) lay off P_3 along line ut from the point where ut cuts the bin bottom. Project this length onto a normal to the bin bottom through the same point and scale off N_3. To find the distribution of N_3 along the bin bottom, lay off the pressure trapezoid $BCaZ$ so that the area = N_3. Now $\frac{1}{2}(BZ + Ca) \times BC = N_3$. **(1)** If BC and Za were produced they must meet at Y on AD produced, then $Ca/BZ = YC/YB$. **(2)** Scale YC and YB and solve for Ca in equation (2). Substitute in equation (1) and solve for BZ. Lay off $\overline{BZ}$ and ΔYBZ. Lay off Ca parallel to BZ and scale off or solve by the preceding equations.

Ketchum gives another method for this solution in which friction between the walls and material is disregarded. In bin $ABCD$ (Fig. 4) the total normal pressure on wall AB is $P_1 = \frac{1}{2}wh^2 \frac{1 - \sin\phi}{1 + \sin\phi}$, acting through a point $2h/3$ below A, where w = weight of material in lb. per cu. ft. Similarly $P_2 = \frac{1}{2}wh'^2 \frac{1 - \sin\phi}{1 + \sin\phi}$. To find total pressure on BC, produce BC and AD to intersect at J. Calculate weight of wedge $ABJ = \frac{1}{2}\,\overline{AB} \times \overline{AJ} \times w = W$. This acts vertically downward through the center of gravity of triangle ABJ. Produce the line of action P_1 to intersect the line of action of W at K. Lay off KL to scale equal to W. Lay off LM equal and parallel to P. $MK = P_3$. Construct the normal component of $P_3 = N_3$. Construct ΔJBO so that area = N_3. CQ = unit pressure at point C. Area $BCQO$ = total normal pressure on the bottom per foot of length of bin. Lay off $CH = FG$. Complete the pressure triangle DCH. Area DCH = total normal pressure on CD. Note that the assumption of smooth walls results in larger values for the pressures on walls and bottom than does the preceding solution.

Surcharged bin. Pressures on the walls of a surcharged bin are much greater than in one level full. Fig. 5 gives the pressure analysis for such a bin assumed surcharged at an angle equal to the angle of repose of the material ($= \phi$). Weight of filling = 100 lb. per cu. ft., dimensions are shown; $\phi = 40°$. The resultant pressure on the bin wall for this condition is a maximum and is inclined parallel to the surface of the ore.

To determine the pressure against AB: Lay off BX parallel to AM. Draw AE perpendicular to BX and lay off AF so that angle $EAF = \theta$ = angle of friction between wall and filling. From F as a center with radius FA strike arc AG and draw the straight line AG. $P_1 = \Delta AGF \times w$. The point of application is $2h/3$ down from A. Lay off P_1

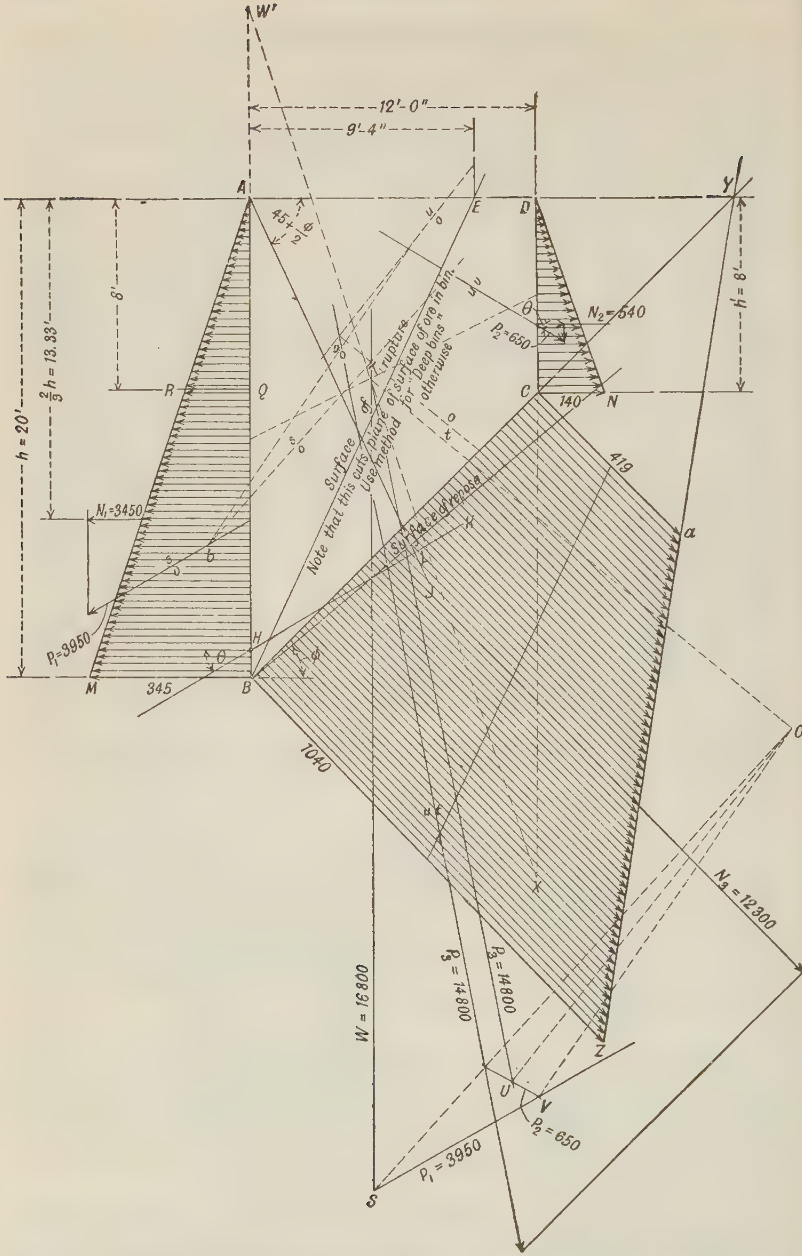


FIG. 3.—Solution of stresses in slanting-bottom bin, based on Coulomb's theory.

and the normal component N_1 in the usual fashion. Lay off the pressure triangle $ABH = N_1$. Lay off $CL = JK$ and complete the pressure diagram DCL. Area $\triangle DCL = N_2$. P_2 acts parallel to MD and is determined from N_2 as shown in Fig. 5. Total pressure on bottom

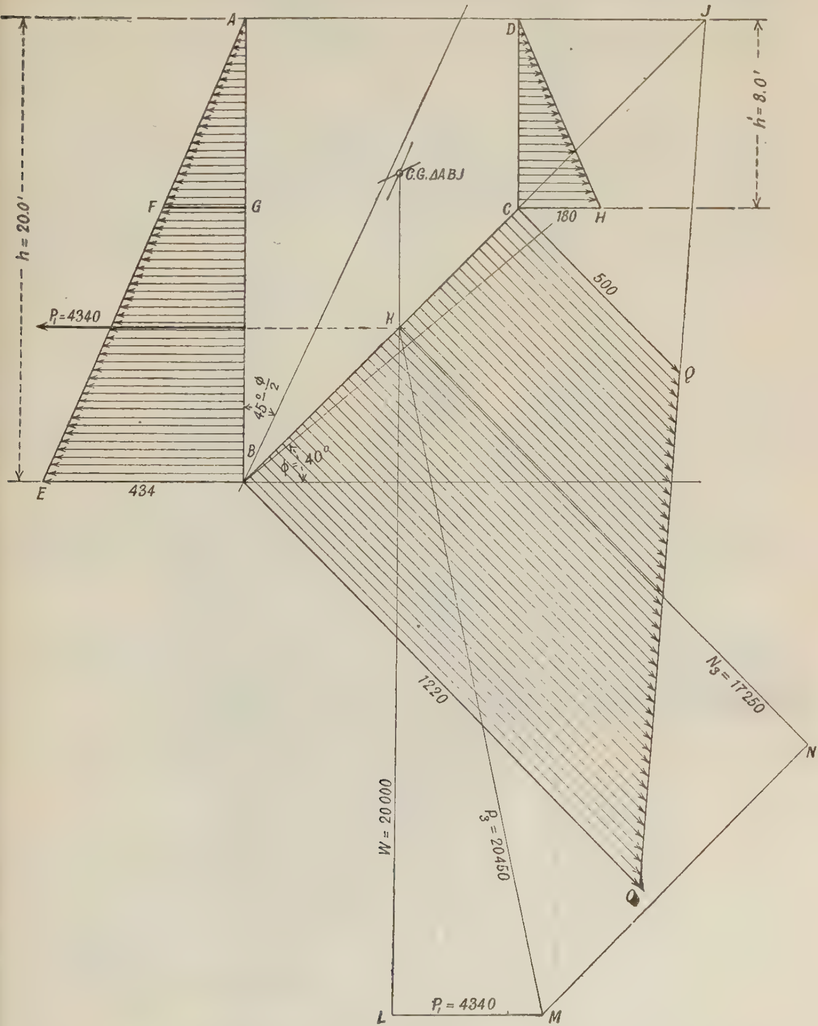


Fig. 4.—Graphical solution for slanting-bottom bin, neglecting friction against walls.

may be calculated by assuming block $ABCDM$ as a rigid body in equilibrium under the action of the non-concurrent forces $-P_1$, $-P_2$, $W (= \textit{area } ABCDM \times w)$ and P_3. The method of solution is as described for Fig. 3 and is shown in Fig. 5, taking the center of force polygon $SUQR$ at T. This assumption results in a value for P_3, in a bin of a given size, less than the value for P_3 in a bin level full, when calculated as in Fig. 4. For maximum pres-

SURES ON BOTTOM, assume block $ABCDM$ in equilibrium under the non-concurrent forces N_1, N_2, W and P_3 and solve for P'_3 and N'_3, as in Fig. 3. (To determine the center of

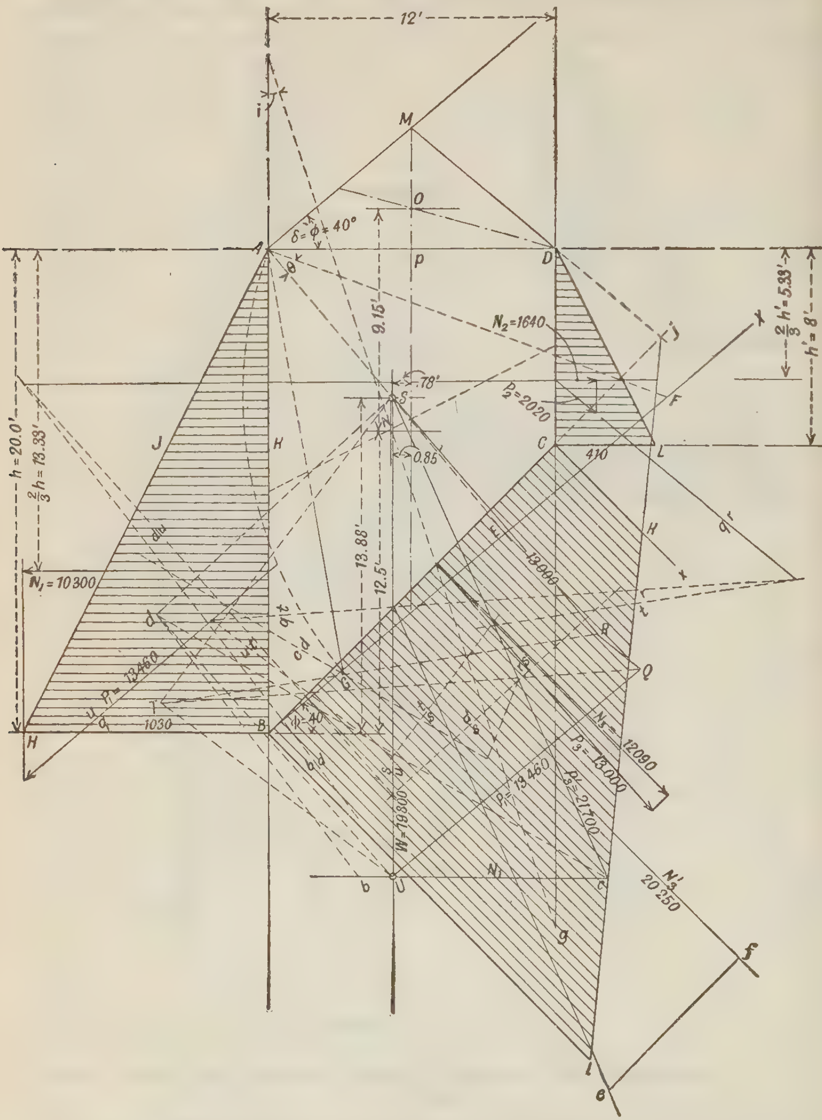


FIG. 5.—Graphical solution for surcharged bin.

gravity of block $ABCDM$, determine the centers of gravity of $ABCD$ (see Fig. 3) and ADM separately. Take moments around axes at right angles, for instance, lines HB and DC, and solve for the resultant moment arms.) TO DETERMINE DISTRIBUTION OF NORMAL PRES-

SURE ON BOTTOM, produce MD and BC to intersect at j. Lay off the trapezoid $BCkl$ to equal N'_3 as follows: $Bl = \dfrac{2N'_3}{BC\left(\dfrac{jC}{jB}+1\right)}$. Lay off Bl to scale, draw lj. Point k is determined by the intersection of Cx (parallel to Bl) with lj. The bottom pressure may also be determined by assuming a block $ABjM$ in equilibrium under action of forces W'' (= weight of this block) N_1 and P''_3, as in Fig. 4 and solving for P''_3, N''_3, and the pressure trapezoid as in that figure.

Deep bins. The preceding methods apply only to shallow bins, *i.e.*, those in which the plane of rupture cuts the surface of the filling material. In deep bins a large part of the vertical pressure is taken by the walls of the bin, which must, therefore, be designed to withstand the resulting compression. Either of two methods of solution is commonly used; both accord well with experimental results.

Janssen's method arrives at the equations:

$$V = \frac{wR}{k\mu'}\left(1 - e^{\frac{-k\mu' h}{R}}\right),$$

$$L = \frac{wR}{\mu'}\left(1 - e^{\frac{-k\mu' h}{R}}\right),$$

where V = vertical pressure in lb. per sq. ft., and L = lateral pressure in lb. per sq. ft. at any point on the bin walls; w = weight of filling material in lb. per cu. ft.; R = hydraulic radius of bin = area in sq. ft. ÷ perimeter in ft.; k = an experimental constant that can be approximated by the formula $k = \dfrac{1 - \sin\phi'}{1 + \sin\phi'}$ where ϕ = the angle of repose of the bin filling; $\mu' = \tan\theta$ = coefficient of friction of filling against bin wall (Table 3); h = depth in feet of the point investigated below the surface of the filling; e is the Napierian base (Sec. 24, Art. 14).

Airy's method (*Pro. Inst. Civ. Engrs., vol.* 131 [1897]) gives

$$P = \frac{wd(2h - d\tan x)(\tan x - \mu)}{2(1 - \mu\mu' + (\mu + \mu')\tan x)},$$

where P = total pressure on bin wall, d = breadth of bin, $\mu = \tan\phi$ = coefficient of internal friction of filling material, μ = coeficient of friction of filling material on the bin wall, and x = angle that plane of rupture makes with horizontal $= 45 - \dfrac{\phi}{2}$. P is a maximum when $\tan x = \sqrt{\left(\dfrac{2h}{d} + \dfrac{1-\mu\mu'}{\mu+\mu'}\right)\left(\dfrac{1+\mu^2}{\mu+\mu'}\right)} - \dfrac{1-\mu\mu'}{\mu+\mu'}$, hence the equation for $P_{max.}$ is

$$P_{max.} = \frac{wd^2}{2}\left[\frac{\sqrt{\dfrac{2h}{d}(\mu+\mu') + 1 - \mu\mu'} - \sqrt{1+\mu^2}}{\mu+\mu'}\right]^2$$

and

$$L \text{ at any depth} = \frac{wd}{\mu+\mu'}\left[1 - \frac{\sqrt{1+\mu^2}}{\sqrt{\dfrac{2h}{d}(\mu+\mu') + 1 - \mu\mu'}}\right]$$

V at any point $= L/k$.

Compression in bin walls at any depth y is given by *Ketchum* as

$$F = wR\left[y - \frac{R}{k\mu'}\left(1 - e^{\frac{-ku'y}{R}}\right)\right],$$

where F is the total compressive load in the plane of the wall per foot of length of bin. When the height of the bin is more than twice the diameter (or breadth), this equation may be written without serious error,

$$F = wR\left(y - \frac{R}{ku'}\right).$$

Bins with conical bottom. In Fig. 6 to determine unit stresses T parallel to an element of the conical bottom across any section such as $x - x$, determine first the total vertical load W' acting at this section. In a shallow bin W' = entire weight of right cylinder of material whose base is the section $x - x$, plus the weight of the part of the bin below $x - x$ and the contained material. In a deep bin $W' = \pi r^2 V$ plus the weight of bin and material below $x - x$. In either case $T = W' \csc s/2\pi r'$. In the deep bin the weight of bin material and filling below $x - x$ may be so small in relation to the total pressure that $T = Vr' \csc s/2$ is a satisfactory approximation. Unit stress T' in the ring $x - x$ in a shallow bin is $T' = r'V \left\{ \dfrac{\sin^2 (s/\phi)}{\sin^4 s \left(1 + \dfrac{\sin \phi}{\sin s}\right)^2} \right\}$; in a deep bin $T' = Lr'$. (*Ketchum.*)

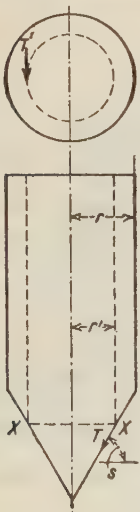


Fig. 6.—Conical-botton bin.

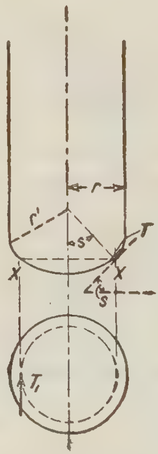


Fig. 7.—Spherical-bottom bin.

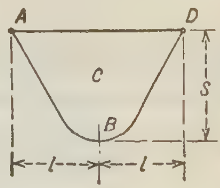


Fig. 8.—Suspension bunker.

Bins with spherical bottom. Stresses in the bottom are calculated as in conical-bottom bins. Referring to bin in Fig. 7, unit stress T tangent to the bin bottom at x and in a plane through the bin axis is $T = W'/2\pi r' \sin^2 s$ and for a deep bin this may be approximated as $T = \frac{1}{2}Vr'$. Tension in the ring cut by section $x - x$ is $T' = \frac{1}{2}Vr'$. (*Ketchum.*)

Suspension bunkers. In Fig. 8 let S = sag in feet, l = ½ of span in feet, C = capacity in cu. ft. per ft. of length, P = maximum pressure, located at B, w = weight of filling in lb. per cu. ft., V = vertical component of force exerted by the bunker on supports at A and D, H = horizontal component of the same force, T = lb. maximum tension in plate per ft. of length of bin, L = length of curve AB. When the bin is level full, $P = 1.25Sw$; $H = Cwl/3S$; $V = 5Slw/8$; $T = Cw\sqrt{\frac{1}{4} + l^2/9S^2}$;

$$L = \frac{1}{60l^2}\left[y_0 + y_{10} + 4(y_1 + y_3 + y_5 + y_7 + y_9) + 2(y_2 + y_4 + y_6 + y_8)\right],$$

where $y = 4l^6 + 9S^2(2xl - x^2)^2$. Substitute for x values 0, $l/10$, $2l/10$, $3l/10$, etc., to obtain values for y_0, y_1, y_2, y_3, etc., respectively. Table 4 gives values of L for values of the ratio s/l ranging from ⅓ to ³⁄₂.

Table 4. Length of one-half curve of suspension bunkers = L. (*From Ketchum, "Walls, bins and grain elevators"*)

$\frac{S}{l}$	L
⅓	1.06378l
½	1.13686l
⅔	1.22992l
¾	1.28307l
⅞	1.36651l
1	1.45722l
⁶⁄₅	1.61131l
⁴⁄₃	1.71906l
³⁄₂	1.85815l

Notes on design. With loads known, the design and spacing of members to carry them is done by the usual methods. LINING must be provided wherever moving ore comes in contact with the bin structure. Walls and slanting bottoms are lined with hard pine or hardwood planking set usually with the length parallel to the run of the ore, or with steel plate or old rails. Hardest wear comes just around the gates and special, easily removable wearing surfaces should be provided at these points. Rails and old crusher-jaw plates are commonly used. TENSION RODS running through timber bins should be placed just above or just below a timber. Timbers directly under loading points may be protected by iron straps bent over the top and spiked on with a space of 1 to 2 in. between straps. Steel members may be protected by riveting to their upper surfaces steel plates 1 to 3 ft. wide on which ore will collect and form a cushion. When ore is allowed to form the bottom in flat-bottom bins, the discharge-chute liner plate should be projected 2 to 3 ft. into the bin to make discharge easier. It is well to make the dimensions of a bin such that stock lengths of planking will not have to be cut on the ground. A change of a few inches one way or another in dimensions will ordinarily accomplish this. Where possible lay end and partition PLANKING so that it forms a tie between front and rear posts of bent, in order to gain added stiffness.

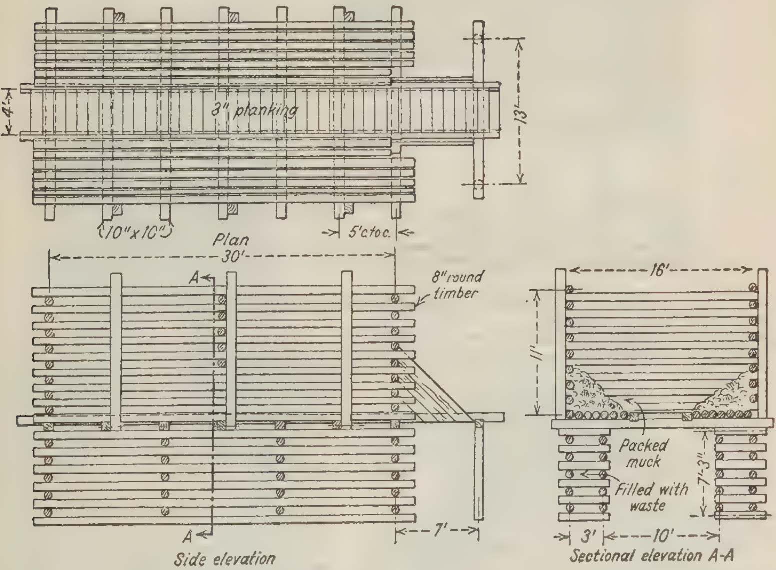


FIG. 9.—Cribbed bin made of round poles (*98 J 739*).

Examples of timber bins. In flat-bottom timber bins the bents are spaced 3-ft. to 6-ft. centers, depending upon the height of the bin and weight of filling; 2-in. and 3-in. plank are usual for front and back; 6-in. plank or 2 × 6 laid on edge are usual on the bottom. The flat-bottom bin lends itself to CRIBBED CONSTRUCTION, similar to that shown in Sec. 9,

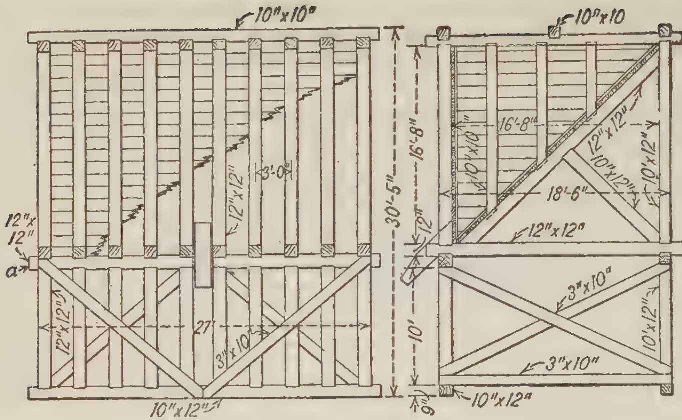


FIG. 10.—150-ton slanting-bottom timber bin (*99 J 195*).

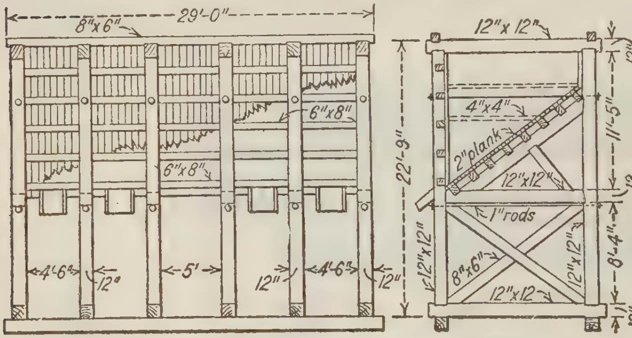


FIG. 11.—100-ton slanting-bottom timber bin (*99 J 195*).

Fig. 6, which is much cheaper than the framed type on account of the fact that framing and expensive carpenter labor are eliminated, and that the smaller sizes of lumber are cheaper per MBM than big sticks. Fig. 9 represents a 220-ton bin built of 8-in. round timbers notched at the corners, with the crevices chinked with 3-in. and 4-in. round poles. This particular bin had no gates but was unloaded by moving the 3-in. bottom planks along with a pick. With a full bin ore piles up in the chute; a car is run under the platform at this point and planks are moved along toward the end of the platform until the edge of the pile is reached when, upon moving the next plank ore will fall into the car. With the bin only part full, a man enters the bin through the chute and proceeds as before, first removing the plank nearest the pile of ore. Figs. 10 and 11 are typical slanting-bottom timber bins. The wide spacing of bents in Fig. 11 is made possible by the small height of the bin; vertical

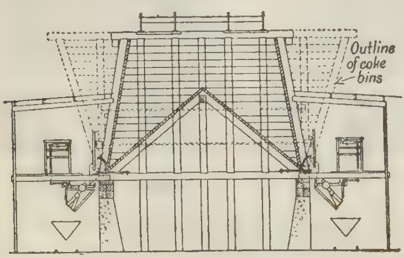


FIG. 12.—Bins and weighing hoppers, Old Dominion smelter.

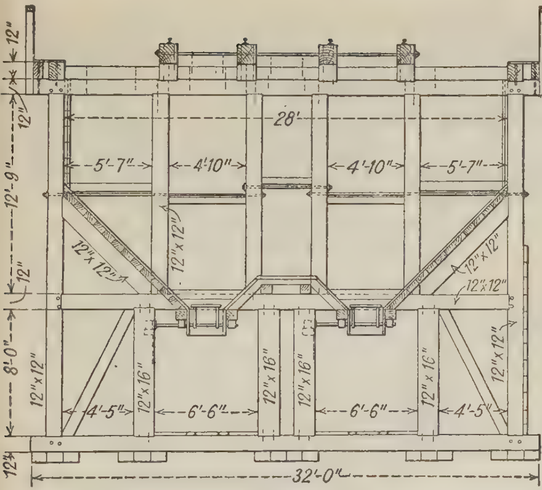


FIG. 13.—Double-hopper timber bin at Tennessee Copper Co.

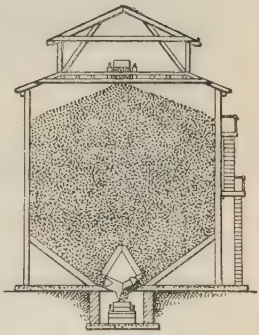


FIG. 14.—Hopper-bottom coal bin.

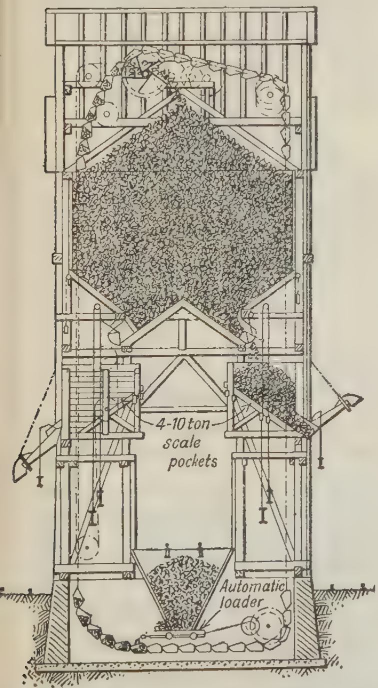


FIG. 15.—Hopper-bottom coal-storage pocket, with bucket carrier.

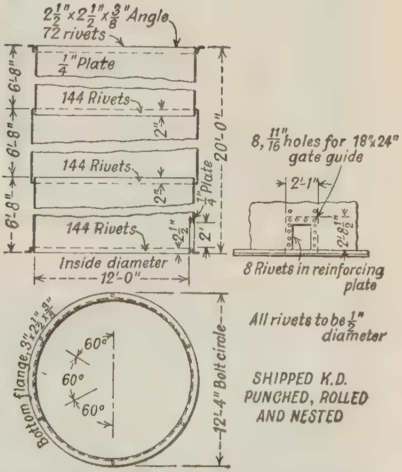


FIG. 16.—100-ton flat-bottom cylindrical steel bin (*99 J 195*).

timbers are carried through to the sills. In Fig. 10 the greater depth requires closer spacing of bents and the greater over-all height requires the use of the intermediate sill (*a*), if special sticks of timber are to be avoided. Fig. 12 shows the double-hopper type of slanting-bottom bin with variation in size for heavy ore and light coke. The batter of the walls is to give stability when the empty bin acts as a trestle. Bin walls are 20 ft. high above the foundations; the width of the bins is 30 ft. at the bottom; ore bins are 26 ft. and coke bins 44 ft. wide at the top. Fig. 13 is the cross-section of a bin at TENNESSEE COPPER Co. This bin is 168 ft. long with bents spaced 6-ft. centers. Figs. 14 and 15 show two types of coal bins.

Examples of steel bins. Fig. 16 shows a small circular steel bin with flat bottom and side discharge. Central bottom discharge is more usual. Flat-bottom circular steel bins in the Lake Superior copper-mine rock houses are 40 ft. diameter by 45 to 50 ft. high, built of 5/16- and 3/8-in. plates. Fig. 17 is a small rectangular bin with flat bottom. At TIMBER BUTTE (*116 J 549*) steel bins 25 ft. diameter and 32 ft. high are set on a V-shaped concrete bottom of two slabs sloping 40°.

Examples of concrete bins. Fig. 18 shows a circular reinforced-concrete bin built by WITHERBEE, SHERMAN AND Co. at Mineville, N. Y. for crude-ore storage. The free-running capacity is about 1000

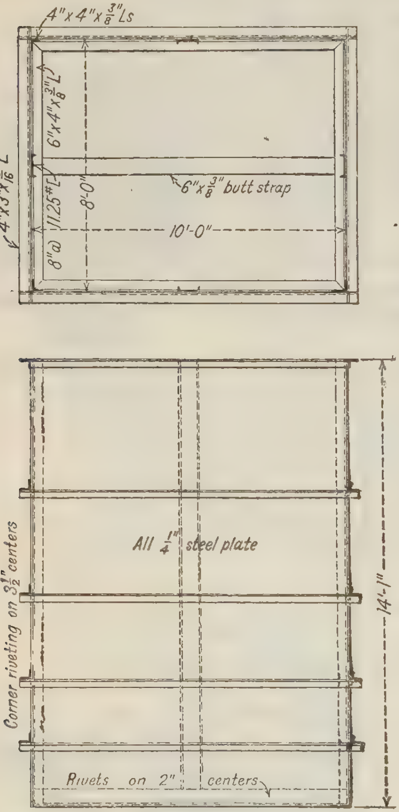


FIG. 17.—50-ton flat-bottom rectangular steel bin.

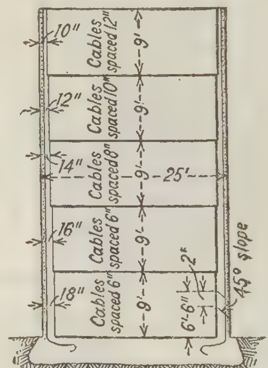


FIG. 18.—Circular reinforced-concrete bin, Witherbee, Sherman Co. (*91 J 704*).

tons. The foundation block is of coarse stone bonded with a 1 : 10 mixture of cement and fine tailing; the wall mixture is 1 : 4 cement and fine tailing. Reinforcement, placed 4 in. from the outside of the walls, consists of horizontal hoops of worn 1 1/8-in. hoisting cable, spaced as shown, with vertical cables at 4-ft. centers wired to the hoops. The bin was filled to the discharge point with barren rock before ore was run in. Fig. 19 (*71 EN 1234*) shows another bin of the same type but with central bottom discharge. Vertical reinforcement consisted of ½-in. round rods spaced 18 in.; horizontal, of worn ¾-in. cable spaced 4¼-in. for the bottom 5 ft., 6 in. for the next 10 ft. and 9 in. to the top. Cables were lapped 3 ft. at the joints, raveled and clipped; vertical rods were lapped 2 ft. at the joints. The concrete mixture was 1 : 3 : 6 for foundation and 1 : 2 : 4 for walls; the maximum size of stone used was 2½ in. Fig. 20 (*96 J 305*) shows a rectangular hopper-bottom bin reinforced with "corrugated bar" and complicated interlocking hooks. The mixture was 1 cement, 2 sand and 4 mill tailing about ⅓-in. size. Shallow cylindrical bins

for fine materials may be cheaply made by setting up a frame of expanded metal lath, properly reinforced, and plastering inside and out with sand-cement plaster. Such bins cannot, however, be made to bear any heavy superstructure.

Examples of suspension bunkers. Fig. 21 (*118 J 566*) shows two recent typical steel suspension bunkers for ore. The Inspiration bin is 330 ft. long with a nominal capacityof 40 tons per running foot. Columns are spaced 16 ft. 8 in. longitudinally and draw gates are spaced at the halfway points between columns. The New Cornelia bin is 330 ft. long; nominal capacity is 33 tons per running foot, columns are spaced 20 ft. longitudinally and gates on 10-ft. centers. The dotted lines in the drawings show the positions assumed by the loaded bins. At New Cornelia the distortion strained the hopper fastenings badly. Fig. 22

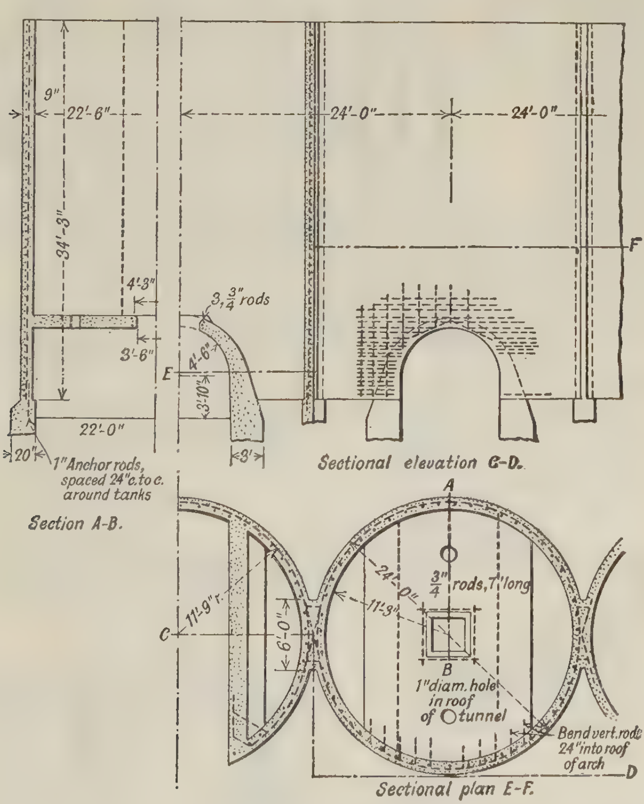


Fig. 19.—Concrete bins at Croton Iron mine.

(*97 J 908*) is a modification of the ordinary suspension bunker, in which a rigid bottom is carried on a series of transverse triangular frames hung by suspension rods from the main supports; the walls are supported against these rods. The cut shows wood, reinforced-concrete and steel sides.

Steel *vs.* wooden bins. An instructive comparison of construction costs of small steel and wooden bins is given by Barbour (*99 J 195*). Bins are shown in Figs. 10, 11, 16 and 23. Comparative cost data are given in Table 5. Labor in timber framing and erecting was found to be equal to the cost of the timber itself ($30 per MBM) in other classes of work, and was thus estimated.

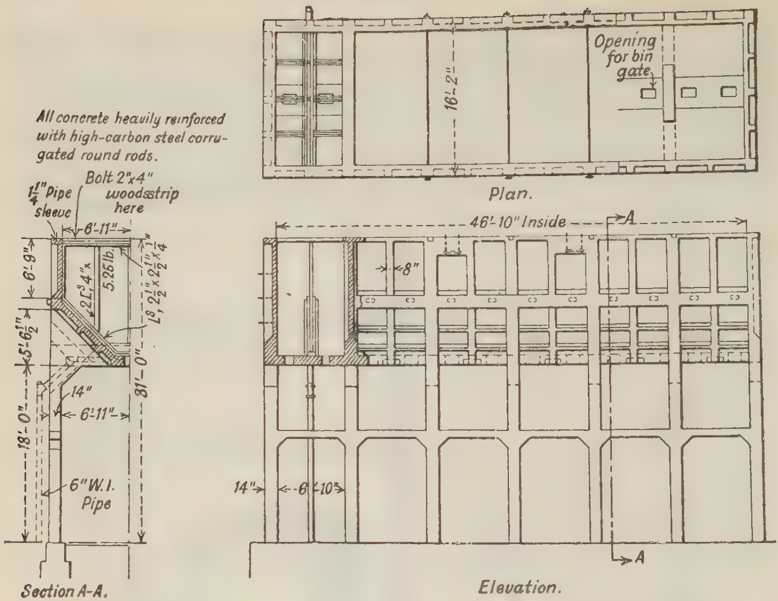


FIG. 20.—Reinforced-concrete tailing bin, St. Joseph Lead Co.

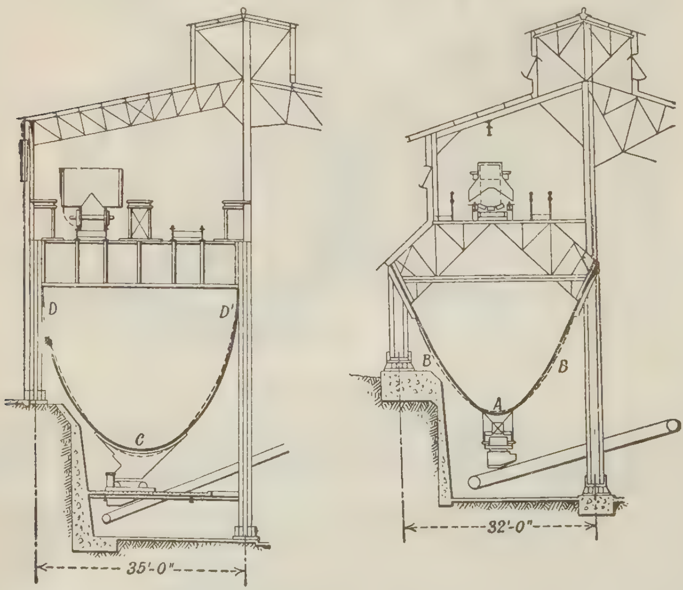


FIG. 21.—Steel suspension bunkers.

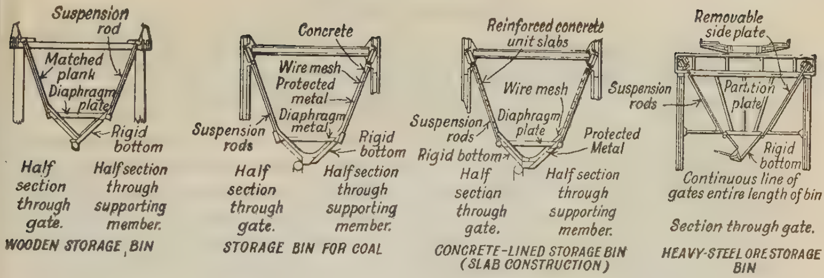


FIG. 22.—Baker types of suspended bins.

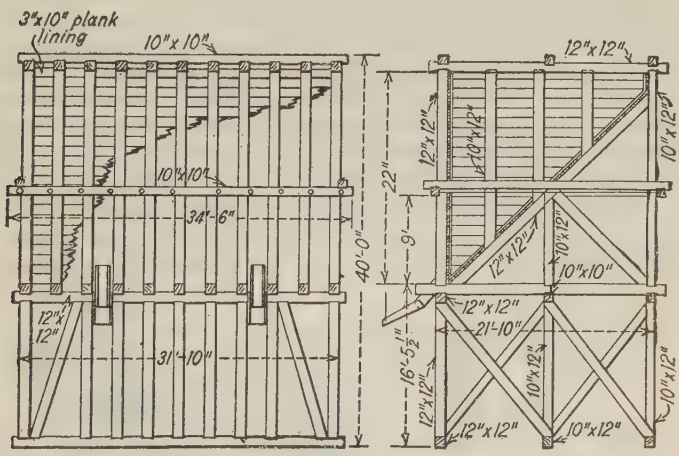


FIG. 23.—350-ton slanting bottom timber bin (*99 J 195*).

The bin shown in Fig. 11 is the cheapest form of wooden bin, due to the wide spacing of bents, but this method could not be applied to the larger bins without using excessively large and expensive posts.

Table 5. Comparative costs of small steel and timber bins

	Cylindrical steel bin	Slanting-bottom timber bins		
Figure number	16	11	10	23
Nominal capacity, tons	100	100	150	350
Capacity of bin, cu. ft.	2262	2389	3594	7565
Feet lumber and timber	10,000*a*	13,000	23,000	47,000
Cost, dollars	549.87*b*	794*c*	1380	2850
Cost per cubic feet, dollars	0.243	0.332	0.384	0.377
Cost per ton capacity, dollars	5.50	7.94	9.20	8.14

a Pounds of steel. *b* Actual. *c* Estimated.

4. Discharge from bins

Bins are discharged through openings placed in the bottom or the side walls. The size of opening is made adjustable by means of gates. Rate of discharge may be regulated by a feeder. The minimum size of discharge opening should be not less than three times the maximum dimension of particles passing through, if the material is sized; nor less than twice the maximum dimension when material is a mixture of coarse and fine. These minimum dimensions will usually be greatly exceeded in fine-ore bins, but with run-of-mine rock it is difficult to control the flow of material through such large openings and there is a tendency to cut down the dimensions, with resulting clogging and lost time.

Gates. The type used depends on the size of material, destination, and regularity of discharge. When material contains large lumps, a gate that readily cuts completely across the stream is wanted; if the destination of the discharged material is a car, a quick, wide-opening gate is desirable; if discharge is highly intermittent, a gate that opens and closes readily and quickly is necessary, while for regular discharge a slow-opening gate may be used and it is not so important that it be easily moved. Usual types are: (*a*) Sliding gates that move in grooves set into bin walls or bottom. These gates are hard to open when the pressure of the filling against them is great and under such conditions must be geared down so much that they are slow. (*b*) Cut-off gates, that present a portion of a cylindrical surface to the stream of ore, usually in a chute. Either the convex or the concave surface may be presented. (*c*) Finger gates, for coarse ores. (*d*) Lifting chute or apron gates.

Slide gates. The LEVER SLIDE GATE, Fig. 24, is the simplest of this type. Ordinary SIZES are 12 × 15 and 12 × 18 in. It is not suitable for heavy pres-

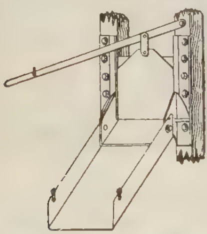


FIG. 24.—Lever slide gate.

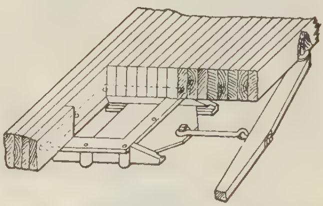


FIG. 25.—Standard bottom-draught bin gate.

sures and is difficult to close tightly when the ore contains both coarse lumps and fine material, as a lump caught under one side will stop the gate and hold it open, allowing fine material to run until larger pieces bridge the opening. For heavier service, up to 36 × 36-in. size, the slides are made with rollers against which the gate works. Fig. 25 shows a similar gate for bottom draught. Usual sizes are 12 × 12 and 12 × 18-in. A bottom-draw valve for fine concentrate may be made of ordinary 6-in. pipe and fittings and strap iron (*90 J 704*).) Fig. 26 shows two forms of upward-closing slide gates. These have the advantage over the downward-closing gate that they will not jam against lumps in closing. They have the disadvantage, however, that fine material causes wedging in the slot through which they work and that wear on this slot is excessive. RACK-AND-PINION GATE, Fig. 27, is perhaps the most widely used of all types. While slow and sometimes difficult to move, it is easy to regulate closely and a pawl on the pinion will hold it in any position desired. The

usual size range is 18 × 18 to 30 × 36 in.; corresponding weights are 250 to 450 lb. A double-rack is used for very heavy service; some have roller slides and chain-controlled hand wheel. Fig. 28 illustrates the use of air for operating sliding gates; these gates are quick-opening even under heavy pressures.

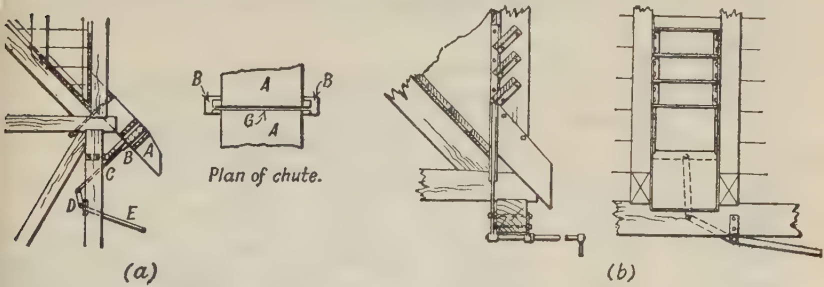


FIG. 26.—Upward-closing slide gates.

At WITHERBEE, SHERMAN AND CO. the gate for a lump-ore (6-in. to 2-ft.) bin was 4 ft. 8 in. square, made of 2-in. plank faced both sides with ½-in. plate, weighed 1300 lb., operated under 60-lb. pressure and was capable of crushing large lumps that wedged under it. It was run on rollers (*89 J 809*).

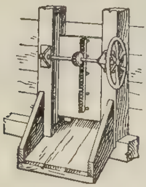


FIG. 27.—Rack-and-pinion gate.

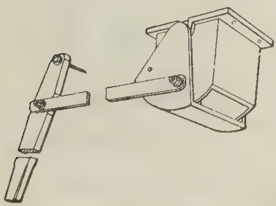


FIG. 29.—Standard bottom-draught arc cut-off gate.

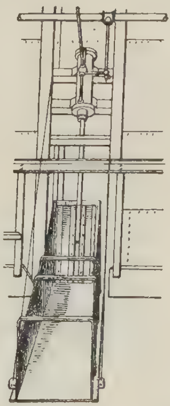


FIG. 28.—Air-operated slide gate.

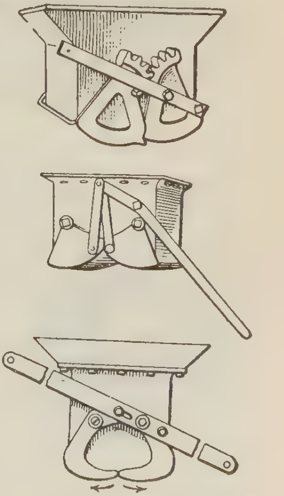


FIG. 30.—Duplex bottom-draught arc cut-off gates.

Arc cut-off gates are used both for bottom draught and side draught. Fig. 29 shows a standard bottom-draught type made in 10 × 10, 15 × 15 and 20 × 20-in. sizes. This gate is not hard to open, and is easy to close, but may catch against a large lump and therefore not cut off quickly. This same gate is made for side draught. Fig. 30 shows three types of duplex bottom-draught arc gates. These are made in sizes from 12 × 12 to 26 × 26 in.

The gear type is the least rugged of the three. Fig. 31 shows details of a particularly rugged type of arc gate for side draught, used at Cananea Cons. Copper Co. (*92 J 933*). These gates should be so set as not to back into the ore in opening yet to run with the stream to some extent in closing. With such an arrangement they are readily and quickly opened and closed and are

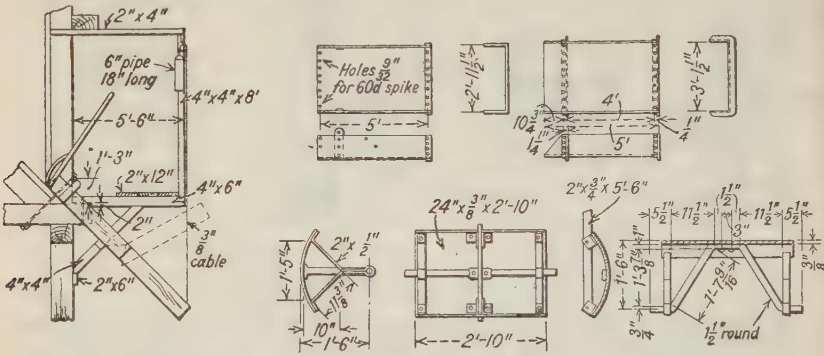


Fig. 31.—Side-draught arc gate, Cananea Cons. Copper Co.

the best kind for intermittent service such as loading cars from a bin front. Fig. 32 shows one of these gates arranged at the lower end of the bin chute, instead of at the bin wall. There is less pressure against the gate in this position, and hence it will be more readily opened. When open, one side of the sector forms part of the chute bottom. This type of gate cannot jam in closing,

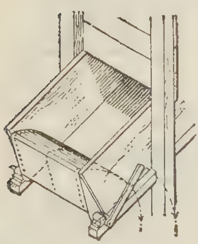


Fig. 32.—Convex arc gate in chute (*92 J 740*).

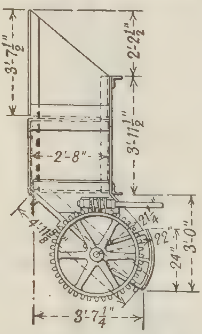


Fig. 33.—Geared undercut concave cut-off valve, side-draught type.

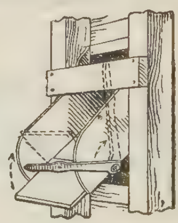


Fig. 34.—Flap-valve bin gate.

but is likely to cause material to jump out of the chute when it begins to close. The gate may be air-operated. Fig. 33 (*94 J 64*) is a geared concave form for coarse ore, designed to open downward. It does not jam in closing but will cause fast running lumps to jump as it begins to close. By failing to open full, a layer of ore is kept in the bottom of the chute, thus protecting it from wear.

Flap-valve gates. Fig. 34 shows the simplest form; it opens and closes readily and quickly, but is subject to wear by discharging material, particularly at the crack in the chute bottom. An air-operated form is used at Quincy Mining Co. (*98 J 827*).

Finger gates are particularly useful in discharging coarse ore. Fig. 35 (*97 J 856*) shows an air-operated form with all fingers operated together. Fingers are sometimes arranged for independent operation; this allows closer regulation of coarse material than is possible with any other type of gate.

Jamming at bin discharge occurs either by reason of bridging of large lumps or by compacting of moist material under the pressure of the overlying filling. The usual cures are barring through the open gate, sledging the outside face of the bin around the gate opening, and dynamite. Much trouble of this character can be saved by means of openings in the bin walls that give access to the material immediately around the gates. Fig. 36 (*122 P 593*) shows the method applied to a hopper bottom; it may be similarly applied to a vertical wall. In either case the angle θ should be less than the angle of repose of the filling. At some Lake Superior rock houses bins for coarse ore have two gates in the same chute, the lower one so placed that it will stop flow through the upper; this permits opening the other wide in case it clogs; the arrangement also allows stopping the stream for picking.

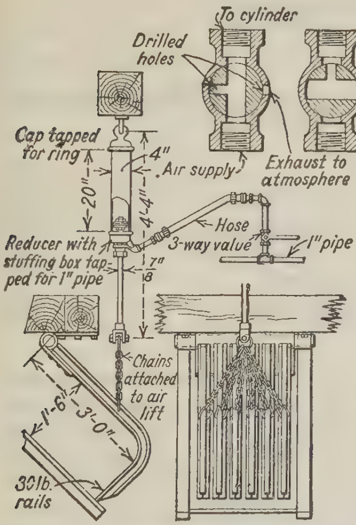


Fig. 35.—Air-operated finger gate.

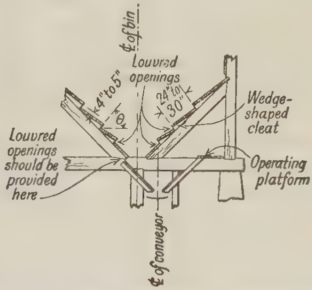


Fig. 36.—Louvres in hopper-bottom bin.

Moist material, especially when fine, may have an angle of repose of 90° or more and a sliding angle against wood or steel almost as great. The most difficulty is met in storage bins following intermediate crushing and those for mixed granular and flotation concentrate. Compressed air is most effective for starting such material. In bins a series of perforated compressed-air pipes set vertically and extending nearly to the bottom will, when blown, break up most hang-ups. Elsing (*89 J 203*) describes the use of a pointed 9-ft. length of 1¼-in. pipe supplied with air at 80 to 90 lb. per sq. in. pressure for unloading fine gravity concentrate from cars. Prior to its use it required 60 to 90 min. to bar material out of a 6-car train; with it, 15 to 20 min. was sufficient.

SECTION 20

TRANSPORT OF MATERIALS

BY

Henry A. Behre, Assistant Professor of Mining, Sheffield Scientific School, Yale University

Handling comprises transport over considerable distances, both vertical and horizontal; and movement through relatively short distances into and out of storage containers. The specific problems ordinarily encountered are: (*a*) handling run-of-mine ore from mine delivery point to the primary breaker; (*b*) handling coarsely-broken dry rock between primary and secondary crushers and into and away from the mill storage bins; (*c*) handling relatively coarse wet products, such as jig feed, jig products and the like in the mill; (*d*) handling wet sand and slime pulps; (*e*) handling the same material dry; (*f*) handling froth-flotation concentrates. The apparatus described in this section includes conveyors and elevators, chutes and launders, bins, tanks, feeders, pumps and air lifts. Transport by wagon, truck, railroad, rope haulage, etc., is treated of in Sec. 23.

Conveyors are used to transport material when the point of delivery is at the same level as or not too much above or below the point of departure (rarely, however, when the delivery point is below the starting point); and when the horizontal distance of transport is relatively short. There is no arbitrary limit of distance beyond which a conveyor system of transport cannot be used, but batch transport, as by railroad, is usually more economical when the distance exceeds a rather small number of hundred feet.

Types of conveyors. Belt conveyors are most commonly employed within the conveyor range. They consist of a continuous belt, fabricated of various materials, passing around head and tail pulleys, and supported at intervals along both upper and return runs by various kinds of idlers. Pan conveyors are similar to belt conveyors in method of drive and support, but differ in that the carrying surface consists of a series of articulated plates or shallow pans supported on rollers and tied together by pins. Bucket conveyors differ from pan conveyors in that buckets of rectangular horizontal cross-section are substituted for the shallow pans or plates of the latter. The buckets may be continuous, *i.e.*, overlapping, or spaced so that definite inter-

vals occur between successive buckets. FLIGHT CONVEYORS are essentially a trough through which a series of scrapers attached to a chain or rope is drawn. SCREW CONVEYORS push material along the bottom of a semi-cylindrical trough by means of a spiral screw revolving therein.

1. Belt conveyor

The carrying element is a continuous belt passing around a head pulley and a tail pulley, supported on its carrying run by troughing idlers (Fig. 1) properly spaced, and on the return run by return idlers (Fig. 1). The simple horizontal conveyor is loaded near the tail pulley through a chute and delivers over the head pulley. It is normally driven through the head pulley. This type of conveyor may be run on an incline up to somewhat more than 20° from the horizontal, the allowable slope depending upon the kind of material being transported, the speed of the belt and the character of the belt surface, but for ordinary service it is well to limit the inclination to 18°. Fig. 2 represents various types of belt-conveyor installations. In the fourth arrangement the radius of the curve should be at least 125 ft., if the belt is to remain on the troughing idlers when running empty. If this cannot be allowed, the third arrangement is frequently employed, but it is better to use two conveyors and do away with the necessity for running the belt around a snub pulley, as this grinds grit into the carrying face.

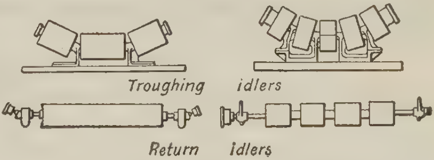


FIG. 1.—Belt-conveyor idlers.

Conveyor belting is usually so-called RUBBER BELT made of several layers or PLIES of heavy cotton duck bound together with rubber "friction" and covered completely with a layer of best-grade vulcanized rubber. The strength of the belt depends upon the weight and strength of the duck and the number of plies; also upon the way in which the joints in the plies are spaced. Belt duck is made with the lengthwise or WARP threads heavier, stronger and closer together than the transverse or FILLER threads. The strength of the duck depends upon the weight, which in turn depends on the number of yarns per thread and the degree of twist. Weights are stated as the number of ounces per piece 36-in. long in the direction of the warp threads and 42 in. wide. The ordinary weights are 28-oz. to 42-oz. The number of plies ranges from 3 for short, narrow belts to 9 or more for long, wide belts. The ultimate tensile strength of duck depends on the width of the test specimen, the method of holding in the test machine, the percentage of moisture and the rate of pulling. Under fair test conditions with not more than 6 per cent. moisture, the breaking strength of 28-oz. duck should be not less than 300 lb. per in. of width; of 30- or 32-oz., 325 lb.; 36-oz., 360 lb.

The life of belt under a given carrying service depends upon the character of the bond between plies, *i.e.*, "FRICTION," and the thickness and quality of the rubber cover. The friction should be of such strength that plies will not separate under the tension of the power pull or in passing around the pulleys. The ordinary specification, best directed toward ensuring this service, is to cut a 1-in. strip lengthwise of the belt, separate the plies at one end and suspend the strip by means of one of the outer plies, attaching to the lower end a 10-lb. weight to keep the strip vertical; then peel away the other plies, one at a time, by attaching thereto, by means of a coiled spring, a weight of from 13 to 18 lb. or more, specifying that the rate of separation of the plies shall not exceed 2 in. per min. The character of the rubber cover depends upon service demanded. The quality should be of best-grade pure vulcanized rubber in order to resist abrasion and keep out moisture. The minimum economical thickness is about 1/32 in., and this is too thin for any but the lightest service. On heavy service the pulley side of the belt has a cover 1/16 in. thick and the carrying side up to 1/4 in. The resistance of rubber covers to impact and abrasion depends on the tensile strength and stretching quality of the rubber compound. The ultimate tensile strengths as determined by the U. S. Bureau of Standards range from about 300 to upwards of 2500 lb. per sq. in. Results vary according to method of taking the test pieces and the manner of

testing, and hence should be definitely set forth in the specifications. The stretching quality should be such that, for instance, a piece 1 in. wide between marks 2 in. apart will stretch to, say, 6 or 12 in. before breaking and, if immediately released, will return so that after a few minutes' rest it will show not more than, say, 10 per cent. elongation between the marks. A so-called FLOATING PLY, or single layer of heavy duck, is sometimes imbedded in thick covers to prevent pieces of the cover from being gouged out. If belts in service wear out first in the middle of the carrying side, the cover may be stepped by rolling on an extra strip of cover rubber down the center, or the ordinary cover may be made thicker at the middle of the belt. The friction between cover and belt should be the same as between plies or greater. The edges of conveyor belts may be subjected to hard wear by rubbing against guide rollers,

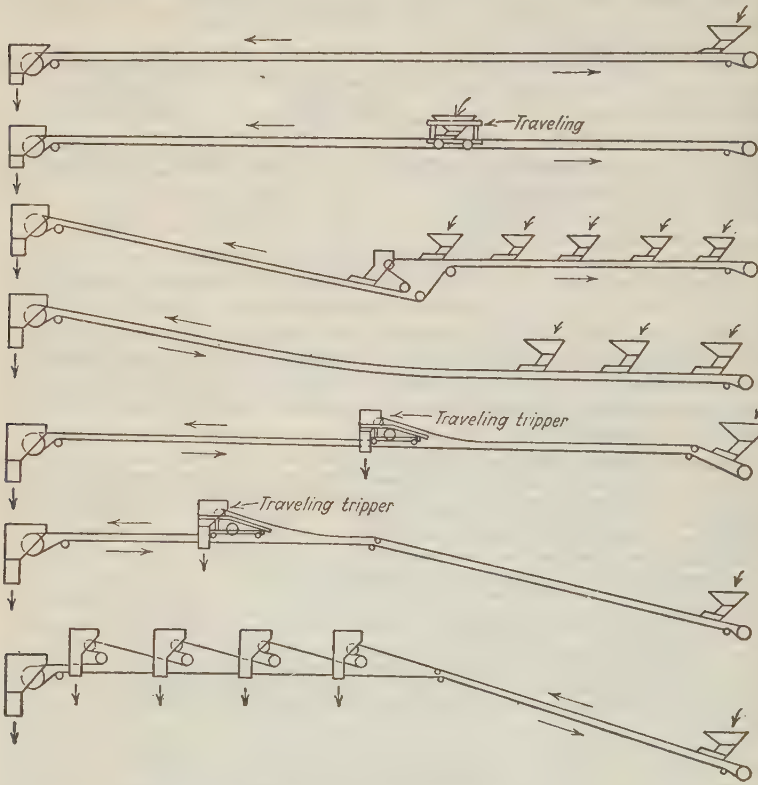


FIG. 2.—Arrangements of belt conveyors and trippers.

or the inside of discharge boxes, trippers and the like. They should, therefore, be protected with extra thicknesses of rubber, firmly bonded to the rest of the belt to prevent tearing away.

Age, light and heat are all destructive to rubber belts. Age causes hair-line cracking of the cover and this deterioration goes forward most rapidly in sunlight. Aging lessens both the tensile strength and stretch of rubber compounds, deterioration in both directions being most rapid in the first year and amounting to as much as 50 per cent. of the tensile strength and 63 per cent. of the stretch (ultimate elongation) in the case of samples tested by the U. S. Bureau of Standards. As a result of the consequent breakdown of the cover in service, dirt and moisture get into the belt fabric, the friction is destroyed, plies pull apart, and the belt is ruined. Heat causes deterioration of the rubber cover similar to that caused by age and also causes breakdown of the friction rubber with consequent loosening of plies and cover.

Special belts with under-vulcanized rubber for friction and cover are made for handling hot materials, but even such belts deteriorate rapidly.

Life of belt (*Q*) varied from 110 days for 20-in. belt at 190 ft. per min., carrying 25 tons per hr. to 6 yrs. for 24-in. belt at 380 ft. per min. carrying 200 tons per hr. A 20-in. 6-ply belt at 200 ft. per min, transporting material 4-in. max. size, carried 857,225 long tons of a magnetic iron ore at the WITHERBEE-SHERMAN Co. and a 20-in, 6-ply belt at 220 ft. per min, carried 764,224 long tons of – ¾-in. ore. At CHINO a 34-in. 6-ply stitched canvas belt at 325 ft. per min. lasted 550 days and carried 400 tons per hr. of – 9-in. material.

Balata belt is multiple-ply canvas belt without covering. The duck is impregnated with some waterproofing compound and balata gum is used for the friction material. This belt is said to be somewhat stronger than rubber belt for the same number of plies because the strength of the duck has not been impaired by the heat of vulcanizing. It cannot be used in hot locations because the gum softens at 120° F. It has been used extensively in South Africa but not much in the United States. Robertson and Johnston (*102 J 10*) report lives ranging in days from 414 to 672 and in tons handled from 400,000 to 800,000 for balata belts carrying crusher product (– 3 in.) in RAND gold mills, as compared with 345 to 1700 days and 200,000 to 850,000 tons for rubber belt in similar or harder service.

Idlers are used on both carrying and return runs. Carrying idlers are usually TROUGHED, *i.e.*, set so as to bend the belt up at the sides, and thus increase carrying capacity. Return idlers are invariably straight-faced. The carrying capacity of a belt loaded so that the upper surface of the load comes within a given distance of the edge increases with steepness of troughing up to 45°. The disturbance of the load in passing over the troughing idlers increases, likewise, with increase of side slope, and there is a corresponding increase in belt wear and power consumption. As a result, standard troughing rarely exceeds 30° maximum side slope. At this angle it is easier to keep the belt straight and there is less failure of belts by longitudinal cracking than is the case with steeper slopes.

Troughing idlers (Fig. 1) are commonly made with 3 or 5 rollers. In the 3-roll type, used with narrow belts, the usual slopes of the inclined rollers are 22½° and 30°; in the 5-roll type, commonly used for wide belts, the inner rollers are sloped 15° and the outer, 30°. The cheap grades of rollers are made of cast iron with babbitted hubs running on cast-iron shafts bored for grease-cup lubrication. Ball-bearing and roller-bearing idlers of pressed steel or cast iron show savings in power losses due to idler friction alone of as high as 60 per cent. The important economies effected in the use of such idlers lie, however, in the saving in labor for lubrication and in reduced wear on belts.

Troughing idlers are spaced according to width and stiffness of the belt and the load carried. With light loading and ordinary belt a spacing of 5 ft. is allowable but this should be the limit of spacing under any conditions; with heavy loading and wide belts, spacing should be reduced to 3 ft. or 3 ft. 6 in.

Return idlers are made of the same materials as troughing idlers, but ordinarily with one pulley only. They are 1 to 2 inches wider than the belt and are commonly spaced 10 ft. apart.

Side or guide idlers are set to bear at right angles to the edges of the belt at places where the latter tends to run off the troughing idlers. They are hard on the edges, both wearing the belt and tending to fold it back, causing a crack 1 or 2 in. in. They should not, therefore, be used, unless it is unavoidable. Belts can sometimes be straightened and side idlers avoided by canting troughing idlers slightly toward head pulley, thus giving the horizontal idler pulley more bearing and more effect on the belt, or they may be skewed. An automatic straightening idler (Fig. 3) has been successfully used at several of the large copper mills. It consists of an ordinary troughing idler (*a*) mounted on a pivoted plate (*b*) with guide idlers (*c*) carried on projecting arms. If the belt rides, *e.g.*, to the left, the troughing idler is swung forward on the right, which causes the belt to travel back toward that side.

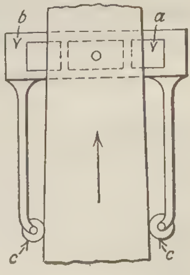


FIG. 3.—Automatic guide idler.

Pulleys should be crown-faced, one or two inches wider than the belt, mounted on sufficiently heavy shafts to insure against bending, and the driving pulley (usually the head pulley) should be securely keyed and clamped to the drive shaft. The diameter in inches should be at least five

times the number of plies and 8 to 10 times is better. Robertson and Johnston (*102 J 15*) tell of an 8-ply belt on a 20-in. pulley that separated along the plies in four months. The pulley at the other end of the conveyor from the driving pulley is fitted with a TAKE-UP MECHANISM (Fig. 4) to compensate for small changes in belt length and hold belt tension constant. The usual allowance is 12 to 18 in. for belts under 100-ft. centers; 18 to 24 in. from 100 to 200 ft. and 30 to 36 in. for longer belts.

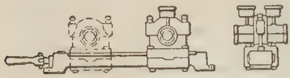


FIG. 4.—Take-up for conveyor tail pulley.

Capacity depends upon width of belt, degree of troughing, speed, slope, size and specific gravity of the material carried and its angle of repose.

Width of belt is determined primarily by the size of material to be carried, after that by the capacity desired. The minimum width is preferably fixed at 14 in. as a belt narrower than this will not conform to the troughing idlers when empty unless of very light-weight. The loading chute should not exceed two-thirds the projected width of the troughed belt, if material is to be prevented from falling off, and the chute should be more than three times the size of the particles, if they are of uniform size and more than twice as large as the largest where mixed sizes are handled. Troughing allows the load to be carried up nearer the edge of the belt than is safe on a flat belt.

Capacity formula. Manufacturers give formulas for capacity in terms of belt width in the form $V = KW^2$, in which V = cu. ft. per hr. at 100 ft. per min. belt speed, W = width of belt in in. and K is a constant ranging, according to the conservatism of the manufacturer from 2.8 (for narrow belts only) to 3.5. The smaller figure contemplates about 9 per cent. of the belt width unloaded at each edge; the larger, 4 per cent.

Speed depends upon the size of particles carried, character of material, width of belt and slope. The limiting speed in every case is that at which material is blown off the belt by air resistance. With small material that flows easily and steadily in a loading spout so that the belt is evenly loaded, belts may be run fast, but with coarse material the feed is necessarily irregular and bare spots will be left on a belt run too fast. With large lumps there is danger of throwing material off the belt in passing idlers when the belt is run too fast. If material is friable and breakage undesirable, as, for instance with coke and coal conveyors, belts must be run slowly if excessive breakage at the discharging end is to be avoided. Ordinary idlers are not carefully turned and balanced as is necessary for high speeds and will rattle and vibrate at speeds much over 400 ft. per min. This limitation does not apply with well-made ball- or roller-bearing idlers. Minimum speed is limited to the necessity for throwing material clear of the head pulley, if head room is an important consideration. The minimum speed to effect clean discharge is about 150 ft. per min. Lower speed is necessary on inclined runs than on horizontal due to the greater difficulty in bringing the loading material up to speed. Table 1 from *Hetzel* gives maximum safe speeds, under the considerations listed.

Table 1. Maximum advisable speeds for belt conveyors

Horizontal

Belt width, in.	12	14	16	18	20	24	30	36	42	48
Belt speed, feet per minute	300	300	300	350	350	400	450	500	550	600

Inclined

Angle of rise, degrees	5	10	13	16	19	22
Percentage of normal horizontal speed	91	83	78	73	67	61

Apart from the physical limitations stated, a good rule is to run the belt at as low a speed as it can be run to carry the load. In this way the percentage of the total load that comes into contact with the belt surface is least, the number of times that any given point on the

belt is subject to the abrasive action of the oncoming stream is a minimum and power consumption, and internal belt strain are least.

Slope. The maximum allowable slope varies with the size and shape of material, method of loading, speed of belt and moisture content. Coarse, rounded material requires a flatter slope than fine material or flat slabs; mixed sizes can be raised on a steeper slope than sized material and uniform feed permits steeper slope than intermittent feed. At normal speeds a slope of 18° is safe for −3 or −4-in. material; −¼-in. dry material can be run at 22° and at 25 or 26° if the conveyor is speeded to 350 or 400 ft. per min.; sand tailing with considerable moisture (15 to 20°) may run backward at 15°, but Delano (*102 J 27*) says that −2-mm. tailing containing 28 to 30 per cent. water was readily carried at a slope of 13° 10′ at the Bonne Terre mill (St. Jos. Lead Co.)

Power consumption depends upon the load carried, the inclination and speed of the belt, the spacing and kind of idlers and size of pulleys. The general formula is $Hp. = PS/33{,}000$, where P = pull on belt in pounds and S = speed in ft. per min. P is made up, in horizontal conveyors, of idler friction due to the empty belt plus that due to the load and may be written as $P = Lfd(X + Y + Z)/D$ where L = length in ft. center to center of tail and head pulleys; d and D are the diameters, respectively, of the idler bearings and the idler pulleys; f = coefficient of idler-bearing friction; X = weight of revolving part of idlers per ft. of length including troughing and return, Y = weight of 2 ft. of empty belt (carrying plus return) and Z = weight of material on 1 ft. of belt. By experiment, $f = 0.35$ for ordinary grease-lubricated idlers. $Z = 2000T/60S = 33.3T/S$, where T = tons per hour. Clearing and substituting, $Hp. = \frac{LSd}{100{,}000D}\left(X + Y + \frac{33.3T}{S}\right)$ (approximately). For an inclined conveyor the weight of the down-going belt balances that of the rising belt, so that the added power consumption is only that due to the load of material. This may be written $Hp. = 2000TH/60(33{,}000) = TH/990$, where H = lift in ft. In short conveyors the additional friction losses at the end pulleys require allowance. Jeffrey Manufacturing Co. recommends adding 20 per cent. to the above figures for conveyors less than 50 ft. in length, 10 per cent. for conveyors 50 to 100 ft. in length and 5 per cent. for those 100 to 150 ft. long. For belt trippers add according to the formula $Hp. = \frac{W^2}{432} + \frac{2}{3}$. In the drive add 5 per cent. for each speed reduction, except that with rough-cast gears this addition should be 10 per cent. An approximate rule, frequently used for horsepower of the conveyor alone is: $Hp.$ = 2 per cent. of the tons per hr. for every 100 ft. of length plus 1 per cent. of the tons per hr. for each 10 ft. vertical lift, or $Hp. = (0.02L/100 + 0.01H/10)T$.

Laboratory tests indicate that the coefficient of friction for oil-lubricated idlers is about 60 per cent., and for roller-bearing idlers 25 per cent. of that for grease-lubricated idlers. This does not mean, however, that 0.21 and 0.09 respectively can be substituted for 0.35 (= f above), on account of the fact the 0.35 takes into account many things beside actual idler friction. *Hetzel* gives the following empirical formula for long conveyors with ball-bearing idlers: $Hp. = (0.0087L/100 + 0.01H/10)T$.

Belt tension may be derived from horsepower as follows: If T_t = tension in the tight side of a belt as it goes onto a driving pulley and T_s the tension in the slack belt leaving the pulley, the driving pull = $T_t - T_s$. If f = coefficient of friction between belt and pulley and a the angle of contact between belt

and pulley in degrees, $T_t/T_s = 10^{0.00758fa}$. Experimental values of f are 0.25 for bare cast-iron pulleys and 0.35 for rubber-lagged pulleys. Table 2 gives values of ratio T_t/T_s for a wide range of conditions.

Table 2. Ratio of T_t/T_s. (*After Hetzel*)

Angle of belt wrap, degrees	T_t/T_s	
	Bare iron pulleys	Lagged pulleys
135	1.8	2.3
150	1.9	2.5
165	2.1	2.7
180	2.2	3.0
200	2.4	3.4
220	2.6	3.8
240	2.8	4.3
270	3.2	5.2
300	3.7	6.2
330	4.2	7.5
360	4.8	9.0
420	6.2	13.0
500	8.9	21.2
600	13.7	39.1
700	21.2	72.0

For a simple horizontal drive (a = 180°) on a bare iron pulley, T_t/T_s = 2.2. Effective pull = $T_e = T_t - T_s = T_t - 0.45\ T_t$, *i.e.*, the total pull = 1.8 times the effective pull. By using a rubber-lagged pulley, T_t/T_s = 3.0 and the total pull is reduced to 1.5 times the effective tension. By increasing the wrap on the driving pulley, by the use of a snub pulley, to say, 220°, the total pull is further reduced to 1.35 times the effective tension.

There is an additional tension on the belt due to its weight, when the belt is inclined at anything more than a very small angle. In Fig. 5, if B = weight of each run of the belt and i is the angle of inclination from the horizontal, the tension in the belt at the head pulley due to each run of the belt $= B \sin i$. The load on the idlers is $B \cos i$ and their resistance to turning under this load, *i.e.*, their resistance to a downhill run is $(fdB \cos i)/D$. The added tension on each run is, therefore, $T_i = B\left(\sin i - f\frac{d}{D}\cos i\right)$.

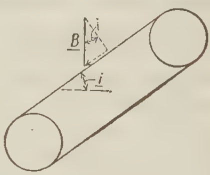
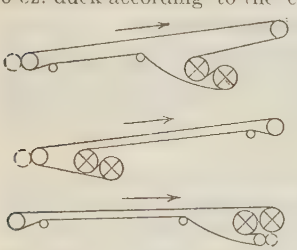


Fig. 5.

Calculation of belt weight. Determine Hp., then effective pull $T_e = 33{,}000Hp./S$ and from this and the ratio T_t/T_e, calculate T_t. Assign a safe value t for the working tension per inch, according to the weight and quality of duck used in making the belt. Figures used for t vary from 20 to 30 lb. for 28-oz. to 36-oz. duck according to the conservatism of the designer and the duty to which the belt is to be subjected. For long life, if the belt is well covered, use a low value for t; if the expected life is short, either by reason of hard service, light cover, or other reason, use the higher figures. The number of plies $= n = T_t/tW$.



The crossed pulleys are the driving pulleys and should be geared together. The third form should be used for the heaviest service. Driving pulleys should be lagged with rubber belting.

Fig. 6.—Drive arrangements for heavily-loaded conveyors.

Drive is ordinarily and best through the head pulley. Tail-pulley drive causes the loaded belt to run crooked. When the load is so heavy that insufficient wrap is obtained by simple head-pulley drive, some form of snubbing arrangement is necessary. (See Fig. 6.)

Feed to conveyors should be so delivered that at the point at which it reaches the belt it will be moving in the same direction as the belt and as nearly as

possible at the same velocity. This is accomplished by use of an inclined bottom in the feed chute at such an angle that the horizontal component of velocity of the feed passing from the chute will be about the same as the speed of belt travel. As the force due to the vertical component of motion of the feed must be withstood by the belt, the slope of the chute is generally a compromise to avoid this strain, or special arrangements are provided to relieve the belt of vertical shocks. Large pieces falling directly on the belt are liable to cut it. At a Rand crushing station one belt a few days old fed with lump material over a 40° grizzly had three holes 3 to 4 in. long (*122 P 313*).

Special feed chutes are sometimes used, which have a screen or grate in the bottom so that finer material falls on belt first and acts as a cushion for larger pieces which will not pass the grate. Clogging of the grate makes this device ineffective and it will not be satisfactory for all materials. A V-notch in the feed-chute bottom acts similarly and does not clog. The end of the chute may be made in a long curve ending at a tangent parallel to the belt and thus present the feed with little vertical force. This requires giving the feed sufficient velocity to pass along the curved part; considerable wear will take place on the chute. Specially-shaped chutes designed to keep the finer material near the bottom are also used.

The feed should be so delivered that it makes first contact with the belt just beyond an idler pulley; the belt then yields sufficiently to take up impact gradually and avoid cutting. To bring the material up to speed, especially if the belt is traveling fast, results in considerable slipping and tumbling with corresponding wear; this is more pronounced with inclined belts.

Skirt boards should always be provided to keep material from running off the belt before it has become settled on it. They are made of wood or steel and have a strip of belting nailed or bolted onto their lower edges, to close the space between the boards and the belt. On level conveyors skirt boards need be only 3 or 4 ft. long but on rapid or inclined conveyors they should be longer; on steep inclined conveyors carrying coarse material they are sometimes provided the entire length to prevent spill of any pieces that slide or roll back.

Discharge of conveyors may be by fall at the head pulley or by some device which removes the load before the head pulley is reached. The most satisfactory device of this sort is a tripper (Fig. 7). The tripper is usually mounted on wheels running on tracks so that the load can be delivered at several points, as over a long bin or into several bins; the discharge chute delivers to one or both sides of the belt. Trippers may be moved by hand or mechanically or may be automatic, traveling backward and forward and thus spreading the load uniformly. Ploughs or scrapers to remove the load cause extra wear on the belt and when scraping to one side only cause the belt to run out of line.

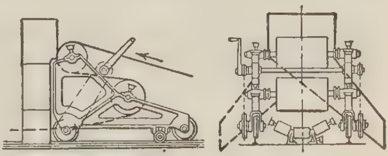


Fig. 7.—Tripper for belt conveyor.

Cleaning the belt of material that sticks is done by scrapers or revolving brushes placed under the head pulley; water sprays are used in some instances.

Support for a conveyor usually consists of wood or steel bents with longitudinal stringers across the caps on which the idlers are mounted; short conveyors are readily supported on the stringers alone. A walkway of light flooring (⅞-in. boards) is generally placed across the stringers between the upper and lower run of the belt to prevent material from falling onto the return side and possibly cutting the belt as it passes around the tail pulley.

Performance of a very large installation at H. C. Frick Coke Co. Colonial mine, East Roscoe, Pa. (*71 A 1123*) is shown in Table 3. All of the conveyors were 48-in. except No. 20,

Table 3. Conveyor installation at H. C. Frick Coke Co.

Conveyor data

Belt number	1 Length, feet	2 Net rise or drop	3 Estimated weight of running parts empty, tons	4 Estimated weight of coal at 1220 tons per hour, tons	5 Estimated weight of running parts loaded, tons	6 Tension load carried by pulley bearings, tons	7 Horsepower of empty conveyor	8 Horsepower to raise live load	9 Running horsepower without lift	10 Total running horsepower	11 Horsepower of electric motor
1	786	43.42	37	31	68	17	11	+60	51	111	150
2	417	8.30	20	17	37	9	17	+12	28	40	50
3	321	4.85	17	13	30	9	5	+ 7	23	30	50
4	1029	19.88	45	41	86	19	15	+27	65	92	125
5	1101	21.07	47	44	91	20	16	+29	69	98	125
6	1496	4.20	60	60	120	24	21	+ 6	91	97	150
7	1402	−12.23	57	56	113	24	20	−17	114	97	150
8	1500	1.47	60	60	120	25	21	+ 2	91	93	150
9	938	11.09	42	37	79	18	14	+15	59	74	100
10	1410	12.33	57	57	114	23	20	+17	86	103	150
11	1514	3.26	61	60	121	25	22	+ 5	92	97	150
12	1320	29.64	54	53	107	23	19	+40	81	121	175
13	1326	22.89	55	53	108	22	19	+31	81	112	175
14	1342	24.42	55	54	109	22	19	+33	82	115	175
15	1296	25.36	54	52	106	22	18	+35	80	115	175
16	1263	27.92	53	50	103	21	18	+38	77	115	175
17	1366	19.12	56	55	111	23	19	+26	84	110	175
18	1301	34.04	54	52	106	22	18	+46	80	126	175
19	1244	36.19	52	50	102	21	18	+49	77	126	175
20	558	20.20	31	34	65	10	11	+27	34	61	100
21 and 22	60	......	......	......	......	......	......	......	......	....	1–15 & 1–5 to each
Total	22,930	357.42	.....	......	......	......	......	......	......	1933	.....

Belt number	12 Start horsepower empty 15 sec.	13 Start horsepower loaded 15 sec.	14 Initial tension, lb.	15 Belt tension running, pounds	16 Belt tension start 15 sec. loaded, pounds	17 Estimated travel of loaded belt to stop: From feet	17 Estimated travel of loaded belt to stop: To feet	18 Lineal feet of belt required, net	19 Number of carriers required	20 Number of return rollers
1	70	200	1500	8050	13,350	23	35	1636-6	225	78
2	39	87	2000	4320	7,210	35	82	867–6	120	41
3	32	69	1500	3240	5,580	38	88	675–6	92	32
4	87	204	1500	6950	13,660	35	73	2113–6	294	102
5	91	217	1500	7290	14,390	35	73	2258–0	315	110
6	117	234	2000	7740	17,100	46	156	3047–6	428	149
7	111	245	2000	7800	16,610	43	349	2858–0	401	140
8	117	250	2000	7540	16,900	48	179	3055–6	429	150
9	81	176	1500	5840	12,000	40	102	1931–0	268	93
10	111	254	2000	8220	17,110	40	105	2875–0	403	140
11	120	255	2000	7710	17,150	47	164	3082–0	433	150
12	105	262	2000	9230	17,580	32	65	2695–0	378	131
13	107	254	2000	8690	17,110	35	77	2707–0	380	132
14	107	258	2000	8870	17,370	35	74	2740–0	384	134
15	104	253	2000	8810	17,080	34	72	2648–0	371	129
16	103	250	2000	8850	16,880	33	66	2582–0	361	126
17	109	255	2000	8540	17,200	37	87	2788–0	391	136
18	104	265	2000	9510	17,780	31	59	2659–0	372	130
19	101	260	2000	9500	17,460	30	55	2542–0	356	124
20	35	103	3000	8250	11,820	14	23	1151–0	197	54
21 and 22	......	....	....	....	....	......	......	134–0 each	......	......
Total	....	....	....	....	....	......	......	47,080–0	6598	......

Table 3. Conveyor installation at H. C. Frick Coke Co.—*Continued*

Actual test data after the installation

	1	2	3	4	5	6	7	8
Belt number	Length, feet	Lift, feet	Date of test	Tons carried for day	Total kilowatt-hours for day	Time that belt ran, minutes	Time that belt carried, minutes	Average load in tons per hour
1	786	43.4	8– 4–24	6346	460	368	352	1080
2	417	8.3	8– 5–24	6418	174	360	327	1180
3	321	4.8	8– 7–24	6460	144	350	326	1192
4	1028	19.9	8–11–24	6500	410	395	350	1115
5	1101	21.1	8–12–24	6854	420	390	333	1235
6	1496	4.2	8–14–24	7122	410	370	350	1222
7	1402	−12.2	8–15–24	6884	330	395	389	1060
8	1499	1.5	8–16–24	6750	400	415	361	1120
9	939	11.1	8–19–24	7032	358	376	361	1142
10	1410	12.3	8–21–24	7883	440	389	354	1336
11	1513	3.2	8–23–24	7250	400	388	359	1212
12	1321	29.6	8–25–24	7271	530	375	340	1282
13	1325	22.9	8–26–24	7252	480	374	342	1272
14	1342	24.4	8–28–24	7085	490	377	345	1232
15	1296	25.4	8–29–24	7341	500	367	334	1320
16	1263	27.9	8–30–24	7939	430	400	350	1362
17	1366	19.1	9– 1–24	6225	440	355	340	1100
18	1301	34.0	9– 2–24	7235	520	370	363	1196
19	1243	36.0	9– 4–24	7847	650	375	353	1334
20	558	20.7	9– 5–24	7421	307	440	283	1575

	9	10	11	12	13	14	15	16
Belt number	Kilowatt demand of empty belt	Additional kilowatts above empty demand to carry average load	Kilowatts to lift load average	Level belt demand measured as additional power above empty demand, Column 10, less 11		Constant kilowatt per 100 tons per 100 ft. based on Column 12		Kilowatts per 100 ft. of conveyor, empty belt
				Kilowatt	Horsepower	Kilowatt per 100 tons per 100 ft.	Horsepower per 100 tons per 100 ft.	
1	19.2	51.6	39.8	11.8	14.2	0.139	0.167	2.44
2	12.8	14.1	8.3	5.8	7.0	0.118	0.142	3.35
3	12.8	9.3	4.9	4.4	5.3	0.115	0.138	4.00
4	24.0	35.0	18.9	16.1	19.3	0.140	0.168	2.33
5	27.0	36.7	22.2	14.5	17.5	0.105	0.128	2.45
6	31.1	29.0	4.4	24.6	29.5	0.134	0.178	2.09
7	28.8	17.5	−11.0	28.5	35.2	0.192	0.230	2.05
8	28.8	27.9	1.4	26.5	31.8	0.158	0.189	1.92
9	25.6	26.0	10.8	15.2	18.2	0.142	0.170	2.73
10	24.0	37.0	14.0	23.0	27.6	0.122	0.147	1.70
11	28.8	30.2	3.3	26.9	32.2	0.146	0.176	1.90
12	32.0	52.0	32.3	19.7	23.6	0.116	0.139	2.42
13	26.5	47.2	24.8	22.4	26.9	0.132	0.159	2.00
14	26.0	49.0	25.6	23.4	28.0	0.141	0.169	1.94
15	24.0	49.9	28.5	21.4	25.7	0.125	0.150	1.85
16	26.4	50.1	32.3	17.8	21.4	0.103	0.124	2.09
17	26.0	42.0	17.9	24.1	28.9	0.160	0.192	1.90
18	24.0	53.3	34.6	18.7	22.5	0.120	0.144	1.84
19	28.8	71.7	40.8	30.9	37.1	0.186	0.223	2.32
20	12.8	48.7	27.8	20.9	25.1	0.238	0.285	2.30

Table 4. Cost of construction of belt conveyors at Arizona Copper Co. (*After Jones*)

Conveyor number	1	2	7^1, 7^2	8^1, 8^2, 8^3	9^1, 9^2, 9^3, 10^1, 10^2	12	13^1 and 13^2	15	3, 4, 5, 6, 11 and 14
Belt	686.80	432.38	1433.79	2124.73	3300.76	209.43	837.72	664.19	5351.03
Feeder belt	56.75	56.75	....	....	56.76	....	....	....	....
Conveyor material	1497.12	1367.50	1451.06	1793.55	3643.25	370.91	913.60	1231.91	4290.86
Control switch	34.00	34.00	72.40	108.60	181.05	34.00	68.00	36.20	136.00
Broken pulley	44.00	....	....	....	....	....	....	....	38.00
Miscellaneous material	307.73	317.14	44.44	63.66	166.55	10.20	24.11	92.55	468.62
Motors	87.05	87.04	295.55	569.82	435.20	87.04	174.08	138.75	1309.68
Trippers	....	....	2178.38	2850.00	....	....	1000.00	....	....
Lumber	....	....	253.74	237.13	755.61	30.97	123.90	67.05	....
Drive belts	....	....	54.80	72.41	106.28	....	30.79	20.82	....
Electrical supplies	....	....	97.26	145.89	243.13	....	....	....	....
Painting	....	....	20.88	31.32	52.19	....	....	....	....
Freight	233.74	203.22	471.37	721.87	815.41	62.50	300.13	....	911.43
Labor	310.92	355.19	564.34	1211.01	1912.20	164.15	476.29	422.77	1210.87
Total	3258.11	2853.22	6938.01	9929.99	11,668.39	969.20	3948.62	2674.24	13,716.49
Cost per foot	33.49	24.33	18.24	17.67	12.88	18.93	18.17	16.21	10.68

which was 60-in. The duty was transportation of run-of-mine bituminous coal from underground loading chutes to barges 4.3 miles distant. For conveyors at Reploglе Steel Co., see Sec. 2, Table 61; at New Jersey Zinc Co., Sec. 2, Table 84.

Cost of construction. At Arizona Copper Co. smelter, Clifton, Ariz. (*49 A 3*) the conveyors described in Table 4*a* were built according to the cost schedule in Table 4 (1912–1914). The costs given include all labor and material for installation, feeders, etc., but not the steel frames to which the idlers are attached. The cost of wooden supports and walkways is included in case of conveyors 9^1, 9^2, 9^3, 10^1 and 10^2.

2. Pan conveyor

Pan conveyors are used for lump material that would cut a belt, or for finer material when loading pressures would be excessive for belts, or when, on account of low speed imposed by other conditions, belt tension is excessive. They consist of articulated steel pans carried on chains; the chains run over head and tail sprockets and are supported on the run by wheels or rollers running on tracks on the supporting frame. Different types of pans are shown in Fig. 8.

The shallow V-shaped pan (*a*) has the advantage that it discharges higher at the head sprocket than the other types, but the stiffness is not so great on wide conveyors as that of the deeper pans and it cannot be used on as steep slopes (15 to 20° max.). Pan (*b*) can be used up to 25° slope; it is stiff and well suited for hardwood lining, as shown. With such lining the conveyor becomes substantially of the apron type and should not be used on slopes greater than 15°. Pan (*c*) can be used on slopes up to 30°.

Pans are made of $\frac{1}{4}$- to $\frac{3}{8}$-in. steel; they are replaceable on the chain links, but if wear is great they should be lined with metal or hardwood.

Chains are subject to great tension on account of the slow speeds at which the conveyors are usually run; working loads may run as high as 25,000 lb., but it is better to keep down to 8000 to 10,000 lb. per sq. in. pin pressures, if possible. For light service, malleable roller chain is used (Fig. 14, *D*); for heavy service, steel-bushed roller chain; and for heavy feeder service, heavy all-steel roller chain.

Apron conveyors (Fig. 9) are essentially very shallow pan conveyors;

Table 4a. Conveyors referred to in Table 4.

Conveyor	Length, feet and inches	Rise, feet and inches	Width of belt, inches	Speed, feet per minute	Capacity, tons per hour	Size of material, inches	Motor, horsepower
1	97–4	3–0	30	150	100	12	5
2	117–4	3–0	20	200	150	⅜ (conc.)	5
3	182–5½	46–0	20	250	150	⅜ (conc.)	20
4	220–9	64–0	20	250	100	. . . .	20
5	127–0	26–4	20	250	150	*a*	15
6	113–8	25–6	20	250	100	. . . .	10
7^1	180–3	8–9	20	300	150	⅜	7½
7^2	200–0	9–2	20	400	100	2½	10
8^1–8^2–8^3	1 & 2 186–9 3 191–9	1 & 2 3–0 3 6–0	20	400	150	⅜	15
9^1–9^2–9^3	198–4	Flat	20	300	100	. . . .	5
10^1	145–3	Flat	. . . .	300	100	. . . .	5
10^2	165–0	Flat	20	300	100	. . . .	5
11	369–8½	87	20	300	100	. . . .	20
12	51–3	8	20	300	100	. . . .	5
13^1–13^2	109–0	Flat	20	300	100	. . . .	5
14	271–5	71	20	300	100	. . . .	20
15	165–0	Flat	20	300	100	. . . .	7½

a Fines from sample mill.

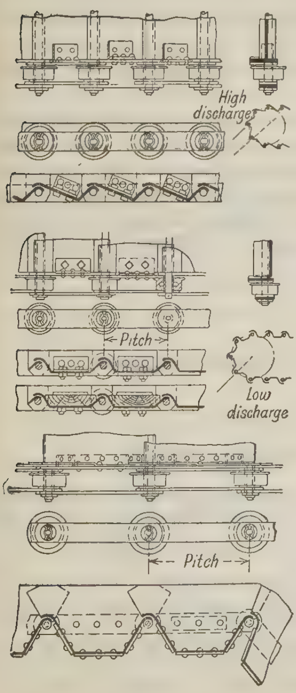


FIG. 8.—Pan conveyors.

in some cases the pans are without ends and spill is prevented by skirt boards. They are used for horizontal transport or inclinations up to 10 or 12°. Their principal application is in heavy feeder service. The lighter type (*a*) has the rollers at the side of the pans; in the heavy type (*b*) the rollers are underneath in order to cut down the unsupported span.

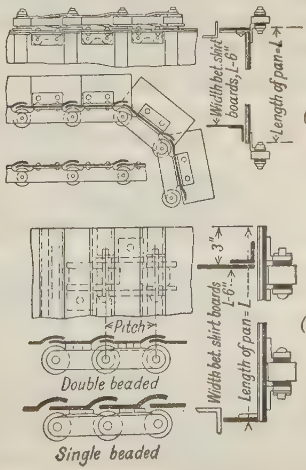


FIG. 9.—Apron conveyors.

Speed of pan and apron conveyors in metal-concentration plants rarely exceeds 50 ft. per min. and when they are used as feeders, speed usually ranges between 2.5 and 10 ft. per min. In the latter service they should be driven by some variable-speed device capable of ready speed change and convenient stopping and starting. In coal handling, the usual speeds are from 50 to 100 ft. per min.

Capacity may be estimated from Table 5. Ordinarily actual capacities will be below the tabular figures on account of irregularities in feeding; these irregularities are greater the greater the speed of the conveyor.

Table 5. Capacity of pan and apron conveyors at 20 ft. per minute.(*a*)
(*After Stephens-Adamson*)

Effective width, inches	Depth of material on conveyor, inches														
	2	3	4	5	6	7	8	10	12	14	16	18	20	24	30
12	10	15	20	25	30	35	40	50	60	70	80	90	100	120	150
18	15	22	30	37	45	52	60	75	90	105	120	135	150	180	225
24	20	30	40	50	60	70	80	100	120	140	160	180	200	240	300
30	25	37	50	62	75	87	100	125	150	175	200	225	250	300	375
36	30	45	60	75	90	105	120	150	180	210	240	270	300	360	450
42	35	52	70	87	105	122	140	175	210	245	280	315	350	420	525
48	40	60	80	100	120	140	160	200	240	280	320	360	400	480	600
54	45	67	90	112	135	157	180	225	270	315	360	405	450	540	675
60	50	75	100	125	150	175	200	250	300	350	400	450	500	600	750
72	60	90	120	150	180	210	240	300	360	420	480	540	600	720	900

a Based on material weighing 100 lb. per cubic foot. For other specific weights and for other speeds capacities are in direct proportion.

Power consumption. Stephens-Adamson Co. (*Catalog* 25) states that the chain pull at the head sprocket with machined bushed-roller chain, well lubricated, is 12 per cent. of the combined weight of load and conveyor for level conveyors; with rough, malleable roller chain, the corresponding figure is 24 per cent.; inclination lessens the frictional resistance in proportion to the cosine of the angle with the horizontal. They give the following formulas for chain pull P and horsepower $Hp.$:

For steel-bushed roller chain:

$$P = (4L + 33.3H)T/S + 0.24LW,$$
$$Hp. = (TL/8250 + SLW/137{,}500 + HT/990)(1 + 0.1N).$$

For rough malleable chain:

$$P = (8L + 33.3H)T/S + 0.48LW,$$
$$Hp. = (TL/4125 + SLW/68{,}750 + HT/990)(1 + 0.1N),$$

where L = horizontal projection of conveyor in ft.; H = total vertical lift in ft.; W = weight per ft. of conveyor in lb.; T = tons of material carried per hr.; S = speed in ft. per min.; N = number of gear-speed reductions.

Skirt boards for apron conveyors should be placed about 3 in. in from the edge of the pans (see Fig. 9) and should clear the pans just enough to insure against rubbing; this excludes entry of any material underneath the boards that cannot be broken without stalling the conveyor. Heavy angles lined with plate or heavy timbers lined with longitudinal steel straps are the usual skirt-board material; they must be stiff enough, as above indicated, to crush small particles between them and the pans, if necessary, without any noticeable deformation.

3. Bucket conveyor

Bucket conveyor (Fig. 10) consists of a continuous line of buckets attached by pivots to two endless roller chains running on tracks and driven by sprockets. The buckets are so pivoted that they always remain in an upright position. The carrier can transport either horizontally or vertically and thus is both a conveyor and an elevator. The path need not be rectangular but may be made almost any desired polygonal shape. Buckets are dumped by means of a cam placed to engage a shoe on the bucket, thus turning it into dumping position; the device can be placed anywhere on a horizontal or slightly-inclined run. The buckets have overlapping edges to avoid spilling when fed from a continuous stream; a special tripper reverses the overlap on the return trip so that it will not interfere as the buckets start rising. The carrier has been used most in coal handling but it is a most satisfactory device for use at custom mills and smelters where several varieties of material must be moved in and out of separate bins; thus mixed charges can be made up with one carrier fed simultaneously from a number of chutes.

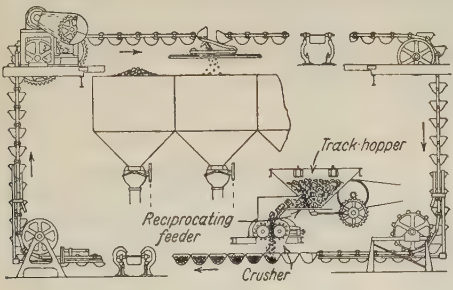


Fig. 10.—Bucket conveyors.

Table 6. Size, capacity and speed of Peck carriers. (*Link-Belt Co.*)

Bucket dimensions, inches: Pitch × Width	Pitch of chain, inches	Carrying capacity of bucket, cubic feet	Speed, feet per minute
18 × 15	18	0.68	30–40
18 × 18	18	0.81	30–40
18 × 21	18	0.94	30–40
24 × 18	24	1.68	40–50
24 × 24	24	2.24	40–50
24 × 30	24	2.80	40–50
24 × 36	24	3.36	40–50
30 × 24	30	3.50	45–60
30 × 30	30	4.37	45–60
30 × 36	30	5.25	45–60

Sizes and usual speeds of Peck carriers (Link Belt Co.) are shown in Table 6.

4. Flight conveyor

Flight conveyors (Fig. 11) consist of chain-drawn scrapers or flights running in a trough through which they drag the material to be transported. The trough may be placed on either the upper or lower or on both runs of the chain. Flights are usually made of malleable iron or steel and the troughs of steel, lined with steel or cast-iron wearing plates. Troughs may be placed at any angle from the

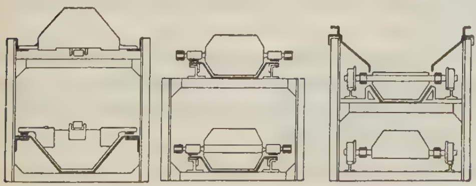


Fig. 11.—Flight conveyors.

horizontal to about 30°, but capacity is very much lowered as inclination increases.

Speed. The usual range is from 50 to 100 ft. per min.

Capacity may be estimated from Table 7.

Table 7. Capacity of flight conveyors. Tons of coal per hour at 100 ft. per minute
(*After Stephens-Adamson*)

Size of flight, inches	Horizontal				Inclined		
	Spaced			Pounds carried per flight	Spaced 24 in.		
	16 in.	18 in.	24 in.		10 deg.	20 deg.	30 deg.
4×10	34	30	22	15	18	14	10
4×12	43	38	28	19	24	18	13
5×12	52	46	34	23	28	22	16
5×15	70	62	46	31	40	31	22
6×18	....	80	60	40	49	40	31
8×18	....	120	90	60	72	57	48
8×20	....	....	105	70	84	66	56
8×24	....	....	135	90	120	96	72
10×24	....	....	172	115	150	120	90

Power consumption. L. D. Moss (*Peele*) states that for level conveyors anthracite requires about 3 hp.-hr. per ton-mile; bituminous coal, 3.5 to 4; and ashes, 4 to 6. Additional horse-power required for elevation may be obtained from the formula: hp. = tons per hr. × ft. lifted ÷ 990. Add to the total for horizontal transport plus elevation 10 per cent. for each gear speed reduction at the head sprocket.

Applicability. Flight conveyors are rarely used in metal-concentrating mills on account of excessive wear but are frequently used in coal washeries, where the material handled is less abrasive than ores. They are sometimes very useful for transporting short distances in confined spaces where no elevation may be lost or even slight elevation must be gained.

5. Screw conveyor

Screw conveyor consists of a spiral blade attached to a shaft which revolves in a horizontal or inclined trough. Material fed into one end of the trough is pushed toward the other end by the rotating spiral. The shaft is mounted on bearings at each end and also in the middle, if the conveyor is long. Spirals used for ores must be extra-heavy steel or cast iron.

Drive is through a pulley or sprocket mounted on one end of the shaft.

Power consumption is high. Horse-power required may be approximated by the formula: Hp. = $WLC/33{,}000$ (*S.-A. catalog*) where W = weight of material in pounds per min., L = length in ft., and C = 2.5 for cement, fine coal, etc.; 4.0 for ashes, sand, etc. For sand, gravel, ashes or similar materials screw conveyors may be obtained with diameters of 9, 12, 16 or 18 in.; running at speeds of 55, 50, 45, 40 r.p.m., respectively; with capacities of 230, 510, 1100, 1360 cu. ft. per hr. respectively. Higher speeds with greater capacities are used for lighter and less abrasive materials.

Screw conveyors are useful to transport dry or moist sandy material short distances where space for other devices is lacking, *e.g.*, to deliver return sands from a classifier to a ball mill or as a distributing feeder for vibrating screens but wear is great both on the blade and the trough.

6. Bucket elevator

A bucket elevator (Fig. 12) consists of a number of buckets (d) fastened to an endless chain or belt (a) running respectively on two sprockets or pulleys (b, c) at different elevations. Material is fed at (c) directly into the buckets or

is scooped up from the BOOT (*f*) and carried up and discharged into a receiving hopper (*g*) as the buckets pass over the upper (HEAD) wheel. The line joining the centers of the pulleys or sprockets may be inclined at any angle between 65 or 70° and the vertical. Bucket elevators are called CONTINUOUS if the buckets are spaced practically touching and CENTRIFUGAL-DISCHARGE if the buckets are spaced say one or more bucket-depths apart. The height of lift in concentrating mills is seldom over 75 ft. but there is no definite limit in the ordinary range of requirements.

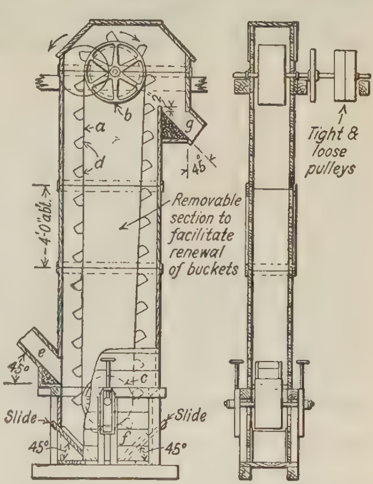


FIG. 12.—Belt-bucket elevator.

Drive is usually by a spur gear on the head shaft and pinion on a jack shaft belt-driven from a motor or line shaft. Direct belt drive from a line shaft to the head shaft is sometimes used; also direct connection of a motor to the pinion shaft. Gear drive is better than belt drive because it permits higher drive-belt speed and belt drive is better than direct connection because the belt will slip in case of a sudden jam and possibly save breakage of the bucket line.

Head shaft should be extra heavy and as short as possible. The greatest stress is due to the weight of the loaded bucket line and to sudden shocks arising from obstruction to the free motion of the line; a shaft strong enough to support this loading is more than large enough to transmit the necessary power. A light shaft that bends under load causes uneven and excessive wear on bearings.

Size of head shaft. Let w = total load in lb. of shaft, pulley, bucket line and ore; l = length in inches of the shaft between bearings; d = diameter of shaft in inches; z = section modulus = $\pi d^3/32$; s = permissible working stress in lb. per sq. in. = say 5000 lb. Then $wl/4 = sz = 5000\ \pi d^3/32$ and $d^3 = wl/1963.5$.

Bearings may be of standard pattern but preferably ball-and-socket, grease lubricated. Special COLLARS with an interlocking rim to cover the end of the bearing are sometimes used to exclude grit; closed ends aid exclusion. Shafting is frequently turned down on the ends to permit the use of smaller-sized bearings.

Rubber belt is the usual medium for carrying buckets in American concentrating-mill practice; BALATA BELTS have been widely used in South Africa. Rubber elevator belts are usually made with 32-oz. duck; for heavy work, 36- or 42-oz.; with a $\frac{1}{32}$- to $\frac{1}{8}$-in rubber cover on the pulley side for protection against pulley slip and some cover on the bucket side also, to provide for the wear of entering feed; for wet materials the cover on the pulley side is usually twice as thick as on the bucket side and ranges up to $\frac{1}{8}$-in. The edges usually have extra-heavy covering. The belt should be two to four in. wider than the bucket to prevent the buckets from catching on the housing or any other projection. Elevator belts are subject to heavy loads; surface wear is severe both on account of loading conditions and slip and creep at the head pulley; the perforations for bolts allow access of grit and water, and this in conjunction with acute bending around small boot pulleys, and frequent bending due to short length disintegrates the internal bond (FRICTION). (CREEP is change in belt length, due to difference in tension on the two sides of the head pulley, so that the belt shrinks in passing from the up side to the down side and this causes relative movement between the surface of the pulley and the belt).

Belt replacement is the most important item of upkeep in elevator operation so that precautions to extend belt life pay for themselves in short periods. Table 8 gives life of belts under various conditions. On elevators with buckets spaced some distance apart and run at low speeds, triangular strips of wood are sometimes fastened to the belt between buckets to prevent material from running along the belt and getting caught behind the buckets; this practice is reported to have greatly increased the life of belts (*59 A 225*).

Table 8. Performances of

Material	Maximum size material	Per cent. water	Lift, feed	Diameter, pulley, inches	Belt speed, feet per minute	Belt	
						Material	Width
Iron ore (magnetite)	4-in.	Dry	67	42	275	Rubber	26
Iron ore (magnetite)	¾-in.	Dry	47	36	220	Rubber	18
Iron ore (magnetite)	4-in.	Dry	88	42	220	Rubber	26
Iron ore (magnetite)	2-in.	Dry	88	50	338	Rubber	32
Copper ore	2-in.	2	64	36	357	Rubber	18
Porphyry copper ore	1-in.	60	54.5	60	450	Balata	26
Porphyry copper ore	0.09-in.	75	55	60	450	Balata	30
Porphyry copper ore	3-in.	9–12	44.5	60	450	Balata	36
Missouri lead ore	9-mm.	50	64	36	500	Rubber	15
Missouri lead ore	9-mm.	50	54.5	36	354	Rubber	18
Missouri lead ore	3-in.	1.5	32	36	232	Rubber	18
Missouri lead ore	150-mesh	90	37.5	24	450	Rubber	13
Missouri lead ore	⅝-in.	40	68.5	42	385	Rubber	20
Missouri lead ore	1-in.	4	64	48	385	Rubber	28
Missouri lead ore	12-mm.	40	61	42	385	Rubber	16
Missouri lead ore	12-mm.	4	60	42	385	Rubber	16
Lead-zinc ore	0.5-in.	85.7	57.5	43	517	Rubber	16
Lead-zinc ore	20-mesh	87.5	38	43	472	Rubber	16
Native copper ore	⅝-in.	50	28	36	300	Rubber	10
Low-grade gold ore	0.07-in.	72	74	60	393	Rubber	12
Lead-zinc ore	1-in.	25	38	38	450	Rubber	14
Reclaimed tailing	¼-in.	4	70	36	212	Rubber	18
Zinc ore	¾-in.	......	61	60	409	Rubber	36
Complex sulphide ore	¼-in.	21	83	60	658	Rubber	32
Complex sulphide ore	0.05-in.	33	55	48	503	Rubber	28
Tungsten ore	½-in.	80–85	38	36	368	Rubber	18
Low-grade copper ore	1-in.	60	60	42	372	Rubber	22

Splicing belts. Various methods are shown in Fig. 13. The JACKSON FASTENER (*d*) consists of stamped steel plates each with two counter-sunk bolts, two oval cup washers with prongs and two sleeve nuts. The bolts with cup washers are inserted into holes in the belt from the pulley side, the steel plate put on and the sleeve nut tightened. The cup washers cause the belt to be drawn up into the concave parts of the steel plate and the sleeve nuts wedge the warp threads of the duck together and a tight joint is made. BUTT-STRAP JOINT (*c*) is usually twice as long as the width of the belt or, with continuous buckets, the length of two buckets on each side of the joint. The lap-joint (*c*) is simple and easily made. With butt-strap and lap-joints on heavy belts (over 6-ply) the extra thickness caused by the double layer of belt at the joint causes movement between the belts when passing over the pulleys and tends to loosen the bolts and allow sand to enter between the layers. When a lap joint is used the direction of motion of the belt should be as indicated by the arrow to prevent turning over the end when slipping occurs at the head pulley.

Head pulley should be large enough to prevent undue internal strain between the plies of the belt; a diameter in inches at least four, better five, times the number of plies in the belt is satisfactory. The diameter is also limited by the belt speed and discharge requirements.

Width of face should be 2 to 4 in. greater than the width of the belt. The pulley face is usually crowned but high crowns must be avoided unless the buckets are placed in two rows. A solid cast pulley with split hub and one or two keys and set screws is best; as

belt-bucket elevators

Belt			Buckets				Tons solid per 24 hr.	Horse-power con-sumed
Plies	Cover, inches	Life	Material	Length× width× depth, inches	Spac-ing, inches	Life		
8	1/16	360,000 tons	Steel	24×13½×10	13½	600 da.	......	......
8	1/16	1,000,000 tons	Steel	16×6×6	18	1000 da.	......	......
8	1/16	185,315 tons	Steel	24×13½×10	13½	600 da.	2600	15
8	1/16	900,000 tons	Steel	30×15×12	15	800 da.	8400	20
6	1/8	....	Mall. C. I.	14×6×6	21	400 da.	840	4.3
12	....	105 da.	Mall. C. I.	12×8×7½ (2 rows)	12	120 da.	2400–3600	50
12	....	358 da.	Mall. C. I.	15×9×8 (2 rows)	16½	180 da.	3200	46
12	....	433 da.	Mall. C. I.	17½×10×8 (2 rows)	18	150 da.	5000	40
5–6–7	1/16	10 mo. to 2 yr.	Mall. C. I.	15×7×7	18	3-4 mo.	1500	13
8	3/32	400 da.	Mall. C. I.	16×8×7	18	250 da.	900	11
8	3/32	2 yr.	Mall. C. I.	16×8×7	18	90 da.	800	8
6	3/32	575 da.	Steel	12×5×4	12	....	45	......
10	1/16	1 yr.	Mall. steel	18×8×6½	22	7-8 mo.	3000	15
10	1/16	1 yr.	Mall. steel	14×7×5½ (2 rows)	24	4-5 mo.	2400	25
8	1/16	1 yr.	Mall. steel	14×7×5½	22	7-8 mo.	840	10
8	1/16	3 yr.	Mall. steel	14×7×5½	22	7-8 mo.	840	10
8	1/16 and 1/8	600 da.	Mall. I.	15×7¾×8½	16	6 mo.	450	8
8	1/16 and 1/8	1100 da.	Steel	16×7½×7¼	16	5 mo.	150	6
8	(Friction)	350 da.	Mall. I.	7½×5×4½	18	300 da.	125	......
8	1/16	730 da.	Mall. I.	12×9×6½	18	515 da.	88	15
10	1/8	150 da.	Mn. steel	12×7×7	16	120 da.	1200	12
10	1/8	2 yr.	Steel	16×8×12	Con-tinu-ous	3½ yr.	1860	15
14	1/8	453 da.	Mall. I.	18×8×8	16	540 da.	3600	36
12	1/8 and 1/16	98–401 da.	Steel	16×8×8 (2 rows)	8	9 mo.	3531	......
12	1/8 and 1/16	1510 da.	Steel	14×7×7	8	9 mo.	175	......
5	1/8	240 da.	Steel	16×8×5	19	270 da.	500	18
12	1/8	300 da.	Mall. I.	20×9×8	22	200 da.	2000	15

a further precaution against loosening a heavy band may be shrunk around the split hub or a keyed, solid-hub pulley pressed onto the shaft. Head pulleys are generally made extra heavy with two rows of arms on wide-face pulleys and long hubs extending almost the entire width of face.

Lagging increases traction between the pulley and belt and decreases wear on both. The most satisfactory lagging is 3- or 4-ply rubber belt or 2- or 3-ply belt with a 1/16- or 1/8-in. rubber cover, fastened by 1/4-in. flat-head bolts; the bolt holes in the rim should be countersunk outside so that the heads of the bolts are drawn down below the outer edge of the lagging, and thus prevented from wearing the belt.

Chains or link-belts running on sprocket wheels are commonly used for elevating broken stone. They have the great advantage of positive drive,

and are good for hot materials that might damage rubber belts. Their chief disadvantage is the great wear at the articulations, where lubrication is difficult or impossible; notwithstanding the use of special wear-resisting alloy steels, chain is unsatisfactory for wet pulps or dusty abrasive material.

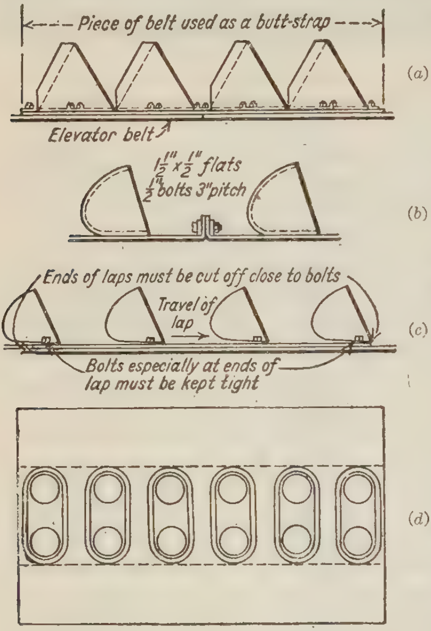


Fig. 13.—Methods of splicing elevator belts (*after Hetzel*).

The standard detachable chain Fig. 14, *A* is cheapest and is widely used; the links are easily replaced but no provision is made for lubrication or exclusion of grit; it can be obtained in manganese steel. Fig. 14, *B* shows interlocking riveted pintle chain, usually made of malleable iron; most of the strain comes on the links rather than the pins and grit is excluded to some extent. Ley bushed chains (Fig. 14, *C*) have malleable-iron links with case-hardened steel pins and bushings which are easily replaced. Special links (Fig. 15) carry the buckets.

An elevator may have one or two chains. Single chains are attached to the back of the buckets; double chains may be attached either to the back or sides. With elevators inclined at any considerable angles from the vertical, the chains are usually placed on the sides of the buckets and provided with rollers which run on a track, thus supporting the loaded side. Fig. 14, *D* shows link for malleable iron roller chain. For details of the numerous types of chains and link belt see catalogs of Stephen-Adamson Co., Link Belt Co., Jeffrey Mfg. Co., C. O. Bartlett and Snow, etc.

Housings. No housing is needed with coarse and nearly dry material except to guard against possible spill of material on men or machines near the

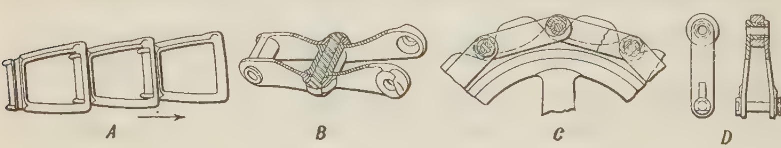


Fig. 14.—Chain-links for elevators.

elevator. If wet pulps or fine dusty materials are being elevated a housing should be provided to confine splash or dust. Wood is commonly used, steel

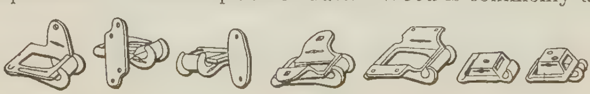


Fig. 15.—Links with attachments for holding buckets.

and concrete less frequently. The front of the housing (upcoming side) is made in removable sections and doors are provided to allow access to the boot.

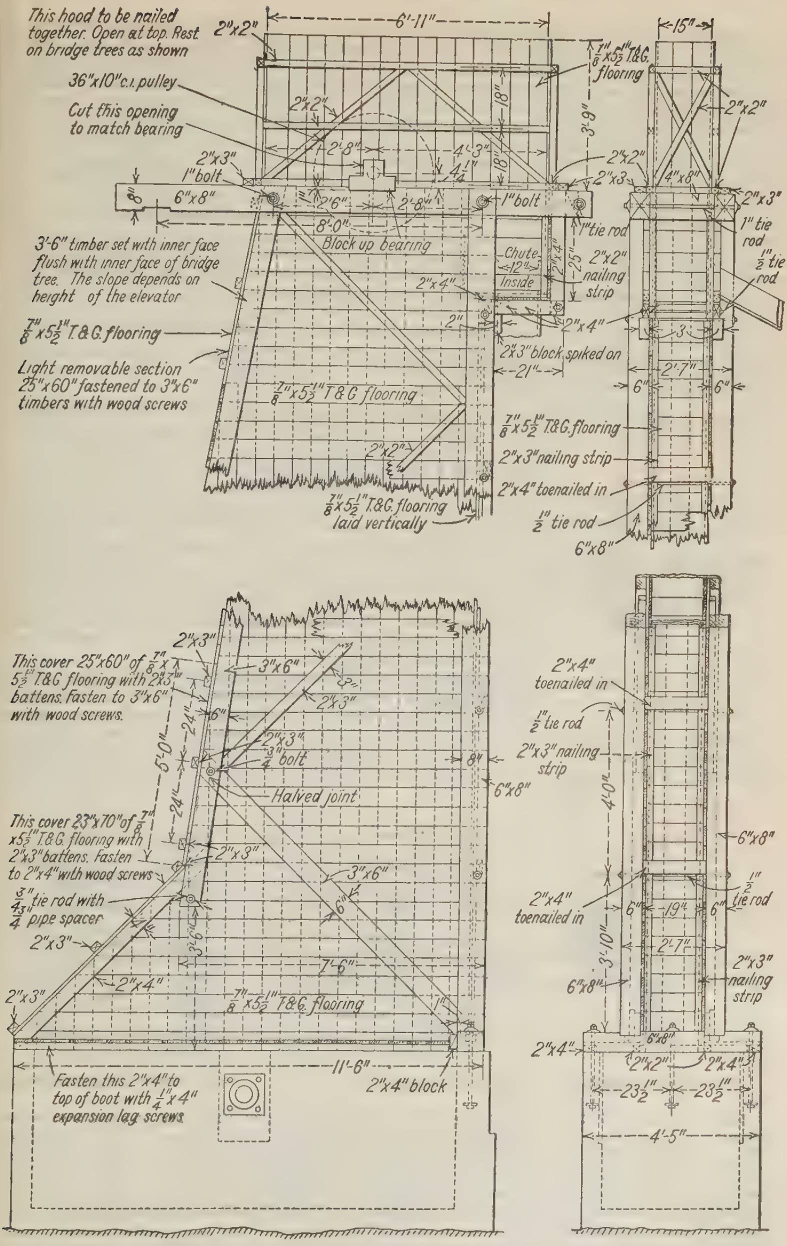


Fig. 16.—Wooden elevator housing.

The simplest WOOD HOUSING consists of 1- or 1½-in. matched boards nailed vertically to the inside of an outer framework. Fig. 16 shows details of a wooden elevator housing made up of two layers of matched boards; the outside layer is placed horizontally and nailed to the framework and the inside layer is nailed vertically to the outside layer. Building paper painted both sides with asphalt paint laid in wet between the two layers of boards prevents leakage. CLEARANCE between the belt and buckets and the housing should be not less than 4½ in. on the sides, better 6 to 8 in., and from 8 to 12 in. or more front and back. Boards running parallel to the belt are sometimes nailed on the inside of the housing to take any wear if the belt runs out of line. The joints on such boards should run in the direction of travel of the belt. The housing is usually light and is supported when possible on the floors of the mill building. SUPPORT FOR HEAD-PULLEY shaft and bearings is provided by special girders in the building frame. STEEL HOUSINGS are made in sections of light sheet steel with light angles riveted to the edges; sections are bolted together through the angles. REMOVABLE SECTIONS should be conveniently placed to allow access for observation and repair. A crane rail should be provided above the head pulley.

Boots for elevators may be small or large. Small boots are used for elevators lifting coarse material so that, if there is any spill, the material will be readily moved by the buckets.

Boots of cast iron or steel can be obtained from makers (Fig. 17). The bottoms are smooth and rounded to permit easy sliding of accumulated material in front of the buckets; removable liners are usually provided. Bearings for the boot-pulley shaft are sometimes supported on the boot with a take-up as shown in Fig. 17. CLEARANCE between the bucket tips and the bottom of the boot should be sufficient at the lowest position of the boot pulley to prevent the buckets from jamming against accumulated material; at least three times the diameter of the largest pieces handled should be allowed. For fine wet pulps the boots should be large to reduce wear on the sides and bottom. Such material is frequently fed directly to the boot and is scooped up by the buckets. The boot then may be either a concrete sump or a large wooden box 2 or 3 times the width of the housing and of greater length (see Fig. 16).

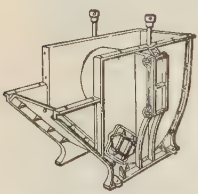


FIG. 17.—Cast-iron elevator boot.

Boot pulley is usually smaller than the head pulley, ranging from 6 in. less to two-thirds the diameter of the head pulley, but never less than 24 in. diameter. BOOT PULLEY SHAFT is usually lighter than the head shaft. BEARINGS are supported on the boot or better, independently outside. Proper lubrication of boot bearings is difficult and is usually neglected. Fig. 18 shows a bearing with flanges for bolting to the boot walls. Grease from cup at end moves in opposite direction from any entering grit and tends to keep the bearing clean. TAKE-UPS are necessary on vertical elevators to compensate for belt stretching. The usual form is supported on the boot and consists of a screw moving the bearing in guides. Special weighted automatic take-ups can be procured. A simple device in which the boot-pulley shaft is supported on wooden slides operated by levers is shown in Fig. 19. The amount of take-up is necessarily small and when the limit is reached the belt must be shortened. Take-ups can be omitted from inclined elevators; if the boot pulley shaft is placed behind the head pulley a horizontal distance equal to the diameter of the head pulley, the sag of the loaded belt is sufficient to allow as much stretch without loss of "wrap" on the

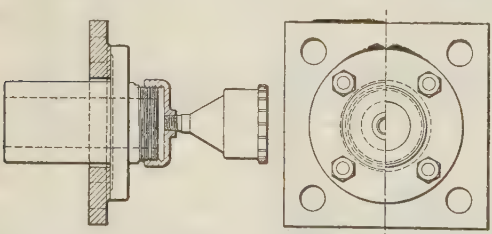


FIG. 18.—Special bearing for boot-pulley shaft.

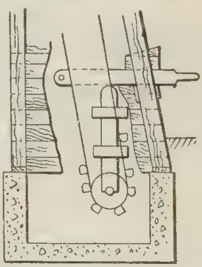


FIG. 19.—Lever take-up.

boot pulley as would be provided by a take-up. With inclined elevators extra tension is put in the belt, if the hang of the loose side is within the line of the catenary between the head and boot pulleys.

Head housing. A removable housing should be provided around the head pulley, extending a distance above the head shaft equal at least to the diameter of the head pulley. It should make a tight joint with the receiving hopper and be provided with a light, easily removable cover.

Receiving hopper is provided with a receiving iron (*a*) Fig. 16, to catch material as it discharges from the buckets. The receiving iron is placed as close to the buckets as possible (rarely more than 1 in. away from the edges of the buckets) and at a point low enough so that all material will be discharged from any bucket before it passes the lip of the iron. The best position to avoid too great a loss of lift varies from a few inches below the center of the head shaft for high-speed centrifugal-discharge elevators carrying a freely-flowing material to a point below a 45° line drawn tangent to the circumference of the head pulley for slow-speed elevators or those carrying sluggish material. The receiving hopper is usually made with rectangular bottom and material is allowed to bank up to its own angle of repose thus avoiding the use of liners. If discharging material strikes the sides of the hopper, liners are provided. The receiving iron in inclined elevators may be placed closer under the head pulley than in vertical, but the housing will be somewhat more complicated, greater floor space required, and if the inclination is great, rollers must be provided to support the loaded belt. Material discharges from the hopper through a chute or launder let into the side or end a few inches above the bottom.

7. Continuous-bucket elevator

This type is used for elevating coarse and comparatively dry material. They are usually run at low speeds so that the discharge takes place by dumping of the buckets as they pass over the head pulley, with little aid from centrifugal force. As the buckets dump, their load slides over the bottom of the preceding bucket and is caught in the discharge chute. Speed should not be so great as to cause excessive spilling at the feed chute and with material containing a large percentage of freely running fine material it should be fast enough so that enough throw will be given as the buckets pass over the head pulley to prevent the fine sand from running out the sides of the buckets and falling back down into the boot. With continuous chain-bucket elevators the speed is seldom greater than 100 ft. per minute; continuous belt-bucket elevators run at higher speeds; Table 9 gives proper speeds for various head-pulley diameters.

Table 9. Speeds for continuous-bucket elevators. (*After Hetzel*)

Head pulley		Belt speed, feet per minute
Diameter, inches	Revolutions per minute	
12	35	110
15	31	124
18	28	132
21	27	147
24	25	157
27	24	175
30	23	180
33	22	190
36	21	198

Buckets for continuous bucket elevators are shown in Fig. 20, and their dimensions, capacities and weights in Table 10. They are usually made of

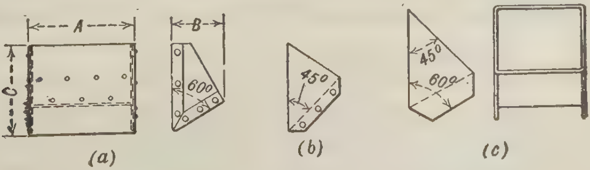


Fig. 20.—Buckets for continuous-bucket elevators.

Table 10. Dimensions and weights of standard steel ore-buckets for continuous-belt elevators (Fig. 20). (*Stephens-Adamson M'f'g Co.*)

Dimensions				Weight per 100, pounds				
Length, A inches	Width, B inches	Depth, C inches	Capacity, cubic inches	No. 16 steel	No. 14 steel	No. 12 steel	No. 10 steel	No. 8 steel
5	3	4	20	125	150	220	....	....
6	3½	5	35	160	200	280	....	....
7	4	6	56	170	220	320	....	....
8	4½	7	84	290	360	520	650	....
9	5	8	120	320	400	690	730	....
10	5½	9	165	390	490	720	890	....
11	5½	9½	190	530	660	960	1190	1460
12	6	10	240	550	690	1020	1250	1530
13	6½	11	314	650	810	1190	1470	1830
14	7	12	392	....	950	1380	1700	2090
15	7½	12	450	....	1040	1500	1870	2280
16	8	13	555	....	1180	1700	2100	2600
17	8	13	589	....	1220	1780	2130	2700
18	9	14	756	....	1430	2080	2580	3160
20	9	14	840	....	1550	2240	2780	3400
22	10	14	1016	....	1725	2500	3100	3790
24	10	15	1200	....	2000	2930	3540	4450

steel plates riveted together (a, b), with or without ears for lapping adjacent buckets. The transverse section depends upon the material carried and the slope of the elevator.

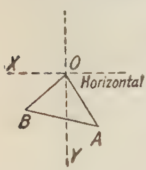


FIG. 21.

In Fig. 21, AOB represents a bucket in dumping position. Angle XOB should be 40 to 50° so that material dumped from the following bucket onto OB will slide freely; angle BOA should be 50 to 60° to prevent lumps of rock from wedging and lessen sticking of damp fine material. $BOA = 90 - XOB + YOA$. Setting $XOB = 50°$ and BOA at 60°. YOA, the inclination of the down-going belt, is 20°. The carrying capacity for any given inclination of elevator and length of bucket front depends upon angle BOA; for elevator slopes of 10° to 30° from the vertical, buckets with angle BOA between 60° and 70° have maximum carrying capacity.

Fastening buckets to belts is done by means of bolts. Other methods have been proposed but have not worked out satisfactorily. One or two rows of bolts may be used, the latter for all but very small buckets; two rows are spaced 1 to 2 in., and pulleys should be at least 30-in. diameter, if two rows are used. Buckets work loose readily and an elevator should be inspected at short intervals to tighten the buckets before they fall into the boot and tear off others. Double rows of narrow buckets set staggered on a wide belt are better than wide buckets because they allow the belt to conform to the crowned head pulley without undue strain on belts and bucket bolts. The gap between bucket and belt in passing over a pulley depends upon the length of the back of the bucket, the place of attachment and the diameter of the pulley. If the buckets are bolted along the upper edge of the back the gap at the bottom is large and in passing over the head pulley material will be spilled into the gap unless extraordinary precautions are taken; if bolted near the bottom, the large gap picks up material in passing through the boot and the buckets will probably be torn away. Center bolting is the compromise adopted. The gap $= g = \dfrac{(\sqrt{d^2 + l^2}) - d}{2}$, where d = diameter of head pulley and l = length of back of bucket; g should never exceed $l/8$.

Feed. A continuous-bucket elevator should be fed through a chute delivering to the rising side at a point such that the bottom of the feed chute is two bucket spaces above the foot shaft. This gives a chance to catch spill in a later bucket. CLEARANCE between the feed chute and the outer edge of the

bucket should be great enough to prevent a large piece of material from jamming.

Spill that falls into the boot must be dug out by the buckets, with consequent increase in wear and power consumption. Small accumulations in large boots may be removed manually through doors; Fig. 22 shows an ingenious method of removing large amounts mechanically. (*H. M. Roche*, Beach Glen Mine, Dover, N. J.)

Inclination is usually 10° to 30° from the vertical; this increases the chance of catching spill in succeeding buckets and also aids sliding discharge.

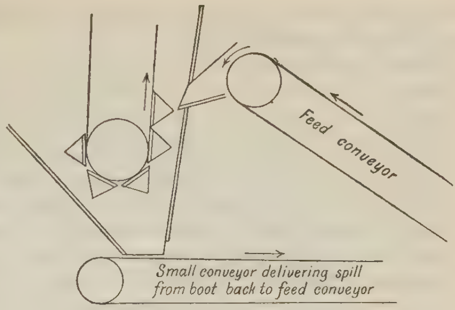


Fig. 22.—Arrangement for removing spill from elevator boot.

8. Centrifugal-discharge elevator

These elevators employ centrifugal force developed in passage over the head pulley to throw material clear of the buckets into the receiving hopper. As rising buckets reach the head pulley centrifugal force becomes effective and tends to push the bucket contents toward the lip. The force acting on the bucket loads is the resultant of gravity and this centrifugal force. Centrifugal force, $C = 2Wv^2/gD$ where v = peripheral velocity in ft. per sec., D = diameter in ft. of the circle described by the center of gravity of the load, W = weight of the bucket load in lbs., and g = acceleration due to gravity (32.2 ft. per sec. per sec.). Material should start to discharge from the bucket when the direction of the resultant of the forces becomes perpendicular to the front of the bucket; actually discharge begins somewhat later, the amount of lag varying with the fluidity of the load. Free-flowing material discharges close to the theoretical point; sluggish and sticky materials lag considerably.

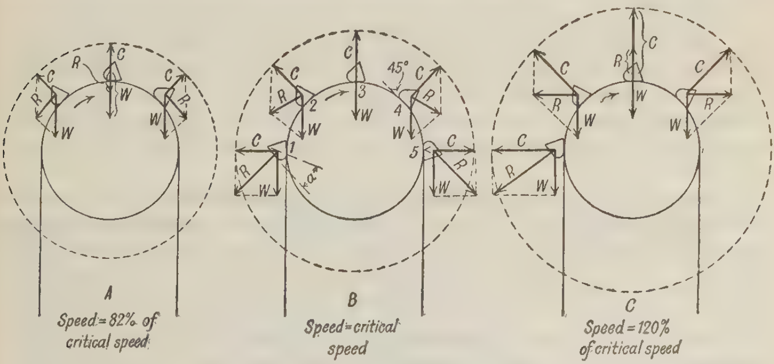


Fig. 23.—Forces at the head-pulley of centrifugal-discharge elevators.

Critical speed is that for which the centrifugal force is equal in magnitude and opposite in direction to gravity. At this speed the load is in equilibrium with respect to the bucket (see Fig. 23) and $C = W = 2Wv^2/gD$; $v^2 = gD/2$; $v = 4\sqrt{D}$ (approx.). To find the r.p.m. (N) for this condition: $v = \pi DN/60$; $\pi^2D^2N^2/3600 = .gD/2$; $N^2 = 1800g/\pi^2D$, and $N = 76.6/\sqrt{D}$. d (diam. of

head pulley) $= D - 2t - p$, where t = thickness of belt in feet and p = projection of the bucket in ft.

As a bucket starts onto the head pulley the sudden application of centrifugal force may, if the bucket is full of freely-flowing pulp, cause some spill as the load shifts to bring its surface perpendicular to the resultant force. As the bucket passes around the head pulley the resultant force decreases but it always has a component pointing towards the bottom of the bucket until the bucket is vertically above the pulley center. In Fig. 23, B, the buckets have a 45° bottom angle, and the resultant force at position (1) is perpendicular to the front of the bucket. The amount of material that the bucket will hold under this condition depends on the internal friction or fluidity of the mass; the load tends to take a position so that the angle (a) between the side of the bucket and the top surface of the load equals the angle of repose (Sec. 19, Table 2); with mobile pulps the carrying capacity of the bucket is distinctly limited, but with sticky or coarse dry material and buckets not too heavily loaded, practically no spill will occur; because the initial condition is instantaneous and is followed by more favorable conditions.

Free discharge occurs on the discharge side of the pulley when the direction of the resultant is within the parallels to the front and back drawn through the center of gravity of the load; in such cases there is no friction between the load and walls of the bucket. For the 45° buckets shown in Fig. 23, free discharge at critical speed extends over the full quadrant from the top to the point at which the belt leaves the pulley. For a bucket with bottom angle of 30°, free discharge extends only to a point 60° from the vertical; a bucket with 15° bottom angle will discharge freely only to a point 30° beyond the vertical. The bucket may still discharge beyond the point at which free discharge ends but discharge will be retarded by friction due to the component of the resultant perpendicular to the front of the bucket. Thus while a bucket with a small bottom angle has the greater carrying capacity its period of free discharge is so limited that net capacity is ordinarily less than that of buckets with more flare.

For belt speeds below the critical speed (Fig. 23, A), the bottom angle is unimportant except that material packs and holds back more with small angles. Under the conditions pictured in (A), while the bucket is moving from the top position to a point about 45° beyond, the resultant lies inside the tangent to the circle of rotation of the center of gravity of the load and the load tends to run out onto the belt; free discharge occurs between 45° and 90° beyond the vertical.

Overspeeds. Fig. 23, C, shows the effect of overspeed; 20 per cent. is probably the limit for efficient work. With 45° bottom angle, the resultant on the rising quadrant is always in a direction tending to push the load out; it reaches a minimum at a point 45° up from the horizontal. With a bottom angle between 35° and 40° the tendency to discharge is delayed until the bucket reaches 15° to 25° from the vertical. On the discharge side the resultant favors discharge over the whole quadrant; with a 45° bottom angle free discharge takes place after the bucket passes the 45° point; with smaller bottom angles free discharge starts later and *vice versa.*

The usual bottom angle for buckets handling ores or mill pulps is about 40°; this gives good capacity with a good range of free or nearly free discharge. Overspeeding usually causes incipient discharge in the rising quadrant but if overspeed is not excessive, most of the load is thrown well over into the receiving hopper. Excessive overspeed causes excessive discharge on the rising

side and much of the load falls back into the boot. Slow speeds cause delayed discharge, and the load may not be thrown clear, and there is considerable spill back into the boot, but the wear on the receiving hopper is not so great and friable material is not broken up as much as with the more violent ejection of higher speeds.

Loading conditions are affected by belt speed, if the elevator is boot fed. High speeds, especially with boot pulleys of small diameter, may produce such great centrifugal forces that material is prevented from entering the buckets in the lower portion of the run and it is then necessary to carry a deep bed of material in the boot to get proper loading; with heavy pulps and coarse material this causes excessive power consumption and damage to buckets. Table 11 gives speeds for various head-pulley diameters; critical speeds should

Table 11. Belt speeds for centrifugal-discharge elevators. (*After Hetzel*)

Diameter of head pulley, inches	Critical speed		82 per cent. of critical speed	
	Revolutions per minute	Belt speed, feet per minute	Revolutions per minute	Belt speed, feet per minute
12	69	217	56	176
15	62	247	51	200
18	56	264	46	217
21	53	292	43	237
24	50	314	41	257
27	47	333	39	276
30	45	353	37	290
33	43	372	36	311
36	41	386	34	320
39	40	408	33	337
42	39	429	32	352
48	37	465	30	377
54	35	495	28	396
60	33	518	27	424
66	32	553	26	449
72	31	584	25	471
84	29	637	23	506
96	27	679	22	554
108	25	707	....	....
120	24	754	....	....

be used for freely flowing materials, the lower speeds for coarse or dusty materials.

Material coarser than ¼-in. should not be boot fed.

Buckets for centrifugal-discharge elevators are usually of malleable iron; various styles are shown in Fig. 24. Corresponding dimensions, capacities and weights are shown in Table 12. Styles *A* and *AA* are most used; *AA* is heavily reinforced on the front edge and corners. Style *B* is used for elevators inclined over 30° to the vertical; the high back prevents spill. Wear is greatest at the corners, especially when the buckets dig material from the

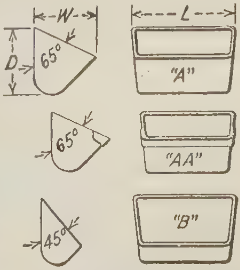


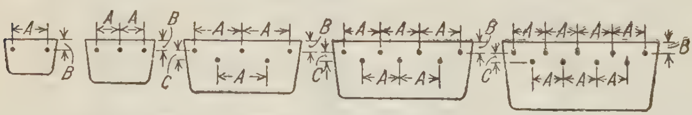
Fig. 24.—Standard malleable-iron buckets.

Table 12. Standard malleable-iron buckets (see Fig. 24)

Dimensions in inches			Style *A*		Style *AA*	
Length (*L*)	Projection (*W*)	Depth (*D*)	Contents, cubic inches	Weight, pounds	Contents, cubic inches	Weight, pounds
4	2¾	3	16	0.80	....	....
5	3½	3¾	36	1.50	....	....
6	4	4¼	55	2.50	55	2.75
7	4½	5	85	3.30	....	....
8	5	5½	115	4.25	115	4.75
10	6	6¼	204	6.75	204	7.50
11	6	6¼	223	6.90	223	7.75
12	6	6¼	246	7.25	246	8.40
12	7	7¼	332	8.20	332	9.00
14	7	7¼	391	9.50	391	10.70
14	8	8½	509	11.50	509	16.30
15	7	7¼	425	10.25	425	11.60
16	7	7¼	467	11.00	467	12.50
16	8	8½	593	12.75	....	....
18	8	8½	668	14.80	668	20.24
18	10	10½	1053	19.50	....	....
23	7	7¼	732	19.00	....	....
24	8	8½	887	23.00	928	26.00

boot. Replaceable wearing edges are sometimes used. LIFE of buckets under varying conditions is shown in Table 8. Sheet-steel buckets are rarely used. Buckets are fastened to the belt by special flat-head bolts in one or two rows near upper edge of the back of the buckets; Table 13 gives standard spacing.

Table 13. Standard spacing of bolt holes for malleable-iron buckets. (*After Heizel*)



Number of holes......	2		3				5			7		9	
Width of bucket, in...	4	5	6	7	8	9	10	11	12	14	16	18	20
A..................	3	3	2	2½	3	3½	3½	3¾	4⅞	3¼	4	3½	4
B..................	1	1	1	1	1	1	1	1	1	1	1	1	1
C..................	....	....	....	....	....	....	¾	¾	¾	¾	¾	¾	¾

Bolt holes 9/32-in. diameter for ¼-in. bolts.

Soft-rubber washers on narrow strips of belting placed between the buckets and the belt protect the belt from cutting by the upper edge of the buckets or by gritty material caught behind buckets.

Spacing of buckets should be such that discharging material from one bucket will clear the preceding bucket. *Hetzel's* recommendations for closest spacing are given in Table 14. Increase in speed causes discharge in a more nearly radial direction and therefore permits closer spacing. Sluggish material requires greater spacing to allow free pick-up in the boot. A rough rule

is to space the buckets a distance equal to twice their projection; usual spacings are from 12 to 24 in.

Load in buckets is rarely more than half the theoretical capacity. Buckets full of free-flowing pulps will spill on reaching the head pulley, causing waste of power. In design, buckets are calculated one-third full in determining capacity and full in estimating power requirements.

Table 14. Bucket spacing, centrifugal-discharge elevators. (*After Hetzel*)

Projection of bucket, inches	Closest spacing, inches: Freely-flowing materials; critical speeds (Table 11)	Closest spacing, inches: Coarse or granular material; underspeeds (82 per cent.) (Table 11)
2	5	6
2½	6	7
3	7	8
3½	8	9
4	9	10
4½	10	11
5	11	12
5½	12	13
6	12	14
6½	13	15
7	14	17
7½	15	....
8	16	20
9	....	22
10	....	24

Discharge of packed or sticky material is aided by punching holes or slots in the bottom of the buckets; by attaching short lengths of chain loosely in the bottom; or by a U-shaped wire with the ends bolted to the belt outside the bucket and the bottom of the U projecting to the bottom of the bucket so that as the belt bends around the head pulley, the wire moves and breaks up the packed mass in the bucket. Suspended boards or rods arranged to tap each bucket sharply as it comes over the head pulley are also used.

Power for elevators

Driving power for elevator belts is transmitted by friction between the face of the head pulley and the belt. The friction must be sufficient to overcome a force equal to the difference in tension between the tight and loose sides of the belt; if it is less, the belt will slip. Slip may be prevented by lagging the head pulley, or by decreasing the difference in tension between the two sides of the belt by light loading or tightening the belt. Excessive tightening overstrains the belt and brings an excessive load on the head-shaft bearings. The angle of wrap on the head pulley is about 180° for vertical elevators and slightly less for inclined elevators where the down side of the belt hangs loosely. The proper relation between the tensions on the tight and slack sides of the belt can be found from the formula $T_1/T_2 = 10^{0.00758fa}$ in which T_1 = tension on the tight side in lb., T_2 = tension on the slack side, f = coefficient of friction, a = angle of wrap in degrees. Table 15 gives safe

Table 15. Coefficients of friction and values of T_1/T_2 for rubber belts on elevators with wrap angle = 180°. (*After Hetzel*)

	Coefficient of friction	T_1/T_2
Clean iron pulley.................	0.25	2.19
Rubber-lagged pulley..............	0.35	3.00
Dusty work, iron pulleys...........	0.20	1.87
Dusty work, rubber-lagged pulleys..	0.27	2.33
Wet work.........................	0.20	1.87

values to use under various conditions. Dust or water between belt and pulley decreases the coefficient of friction.

Tension in the rising belt (T_1) is the sum of the weight of material in the buckets, the weight of the empty buckets and belt, the pull due to digging the load or filling the buckets and friction losses in the boot shaft. WEIGHTS OF MALLEABLE-IRON BUCKETS are given in Table 12. WEIGHT OF BELTING may be approximated as the product of the width in inches $\times$ number of plies $\times 0.03$. The PULL DUE TO LOADING AND PICK-UP from the boot can be taken as equivalent to the pull of the load of material in the buckets for a length of belt in feet of from $2D$ to $12D$, where D is the diameter of the boot pulley in feet. The most favorable condition arises in a slow-speed continuous-bucket elevator, which is front fed with practically no spill; for this a value of $2D$ may be used. For a continuous-bucket elevator for coarse material, picking up a large part of its load from the boot, $12D$ should be used; for front feeding of damp sand with little spill, $3D$; for thick pulps, 40 to 50 per cent water, $3D$ to $4D$; ordinary dilute mill pulps, boot fed, which do not settle readily, $4D$ to $6D$; and for boot-fed pulps which pack, $10D$. PULL DUE TO FRICTION LOSS at the boot is covered by adding 1 or 2 per cent. of the total calculated pull from other causes.

Tension on the down side of the belt (T_2) is equal to the weight of the empty buckets and belt.

If the ratio T_1/T_2 thus found is greater than $10^{0.00758fa}$ the tension must be increased by take-up and allowance must be made for the added tension in determining the size of belt needed. WORKING STRESSES in elevator belts should be kept below 35 lb. per in. per ply for 32-oz. duck and 40 lb. for 36-oz. duck; in most cases the stress can be kept below 25 lb. Extra plies are usually allowed when external wear is great.

Horsepower to drive the belt $= (T_1 - T_2) \times$ *belt speed in ft. per min.*/33,000.

To determine the horse-power to be delivered by the motor or line-shaft add 5 per cent. for each speed reduction through belts, chains or cut gears and 10 per cent. for each reduction through cast gears.

Design of a continuous-bucket elevator

To raise 50 tons per hr. of 2-in. material weighing 100 lb. per cu. ft. 50 ft. vertically, the elevator to slope 10° from the vertical.

As the slope angle is small it may be disregarded in calculation. The volume to be delivered per min. $= \dfrac{50 \times 2000}{100 \times 60} = 16.7$ cu. ft. Assume the buckets one-third full, then bucket capacity should be $16.7 \times 3 = 50$ cu. ft. per min. $= 86{,}400$ cu. in. per min. From Table 10 a $15 \times 7\frac{1}{2} \times 12$-in. bucket has 450 cu. in. capacity. Number of buckets per min. $= 86{,}400/450 = 192$. As the length of the bucket along the belt is 12 in. a belt speed of 192 ft. per min. will be necessary, and from Table 9, a 36-in. pulley will be needed. Gap $= \dfrac{\sqrt{d^2 + l^2} - d}{2} = \dfrac{\sqrt{36^2 + 12^2} - 36}{2} = 1$ in., which is less than $l/8$ and therefore satisfactory. If slower speed is desired, a $16 \times 8 \times 13$-in. bucket with capacity $= 555$ cu. in. gives $86{,}400/555 = 155$ buckets per min. or a belt speed of $\dfrac{13 \times 155}{12} = 168$ ft. per min. and the gap will be 1.13 for a 36-in. pulley or within the limit $l/8$. Using the latter size of buckets, the number of buckets on the upcoming side $= 50 \times 12/13 = 48$ (approx.). The load due to empty No. 10 steel buckets $= 48 \times 21 = 1008$ lb. The load in these buckets, assuming them full, $= 48 \times 555 \times 100/1728 = 1541$ lb. The belt width will be 18 in. Assuming 9-ply belt, the weight per ft. of length is approximately $18 \times 9 \times 0.03 = 4.86$ lb. and the weight on the loaded side $= 4.86 \times 50 = 243$ lb. Using a 30-in. boot pulley and assuming fair loading conditions, the pull due to digging in boot = the load carried on $6D$ ft. of belt $= \dfrac{6 \times 2.5}{50} \times 1541 = 462$ lb. The total tension on the loaded side $= 1008 + 1541 + 243$

+ 462 = 3254 lb. Add for friction between 1 and 2 per cent. say 46 lb. and T_1 = 3300 lb.

The tension per in. of width = 3300/18 = 183 lb. and with a working tension of 25 lb. per in. per ply, 183/25 = 7.32 or 8-ply belt will be needed. The head-pulley diameter in inches is 4.5 times the number of plies, which is satisfactory. The exact weight of 18-in. 8-ply belt with 36-oz. duck is within the value assumed above.

Tension on the down side of the belt will be 1008 + 243 = 1251 lb., hence T_1/T_2 = 3300/1251 = 2.64 and the coefficient of friction should equal 0.309 (from the formula $T_1/T_2 = 10^{0.00758fa}$). A rubber-lagged head pulley should be used (see Table 15). Since a wide allowance was made in taking the tension on the rising side that due to full buckets, there should be no slip under normal working conditions.

Horsepower to drive the belt = (3400 − 1251) × 168/33,000 = 11 hp. Add 10 per cent. (= 1.1 hp.) for reduction by cut gears and 5 per cent. (= about 0.6 hp.) for belt drive from motor to pinion shaft; the motor power required will be 12.7 hp.; a 15-hp. motor that will operate at high efficiency over a wide range should be chosen.

Size of head shaft. The total weight to be supported = the load due to belt, buckets and load on the loaded side, 3400 lb., + belt and buckets on the empty side, 1251 lb., + a 36 × 20-in. pulley, 660 lb., + the shaft (assumed), 200 lb., = 5511 lb. Assuming 4-in. clearance on each side of the pulley the thickness of the housing as 2 in., and placing 16-in. bearings just outside the housing, l = 20 + 8 + 4 + 16 = 48 in. d^3 = 5511 + 48/1963.5 = 134.8; d = about 5.12 in. The nearest standard size of shafting is 5$\frac{3}{16}$ in.

Design of centrifugal-discharge elevators. Similar procedure should be followed except that in addition belt speed and bucket spacing must be such as to give satisfactory discharge.

Performances. For elevators at Replogle Steel Co., see Sec. 2, Table 62; at New Jersey Zinc Co., Sec. 2, Table 85. See also elevators in the various flow-sheets in Sec. 2.

Cost

The principal item of cost other than power is belt and bucket replacement.

Table 16 (*52 A 116*) gives details of the cost for elevating fine thick pulp 55 ft.

Table 16. Cost of elevating pulp at Tonopah, Belmont

149 ft. of 20-in. Balata belt @ 2.80	$392.70
36 buckets @ 2.90	104.40
220 elevator bolts	20.24
7½ lb. washers	0.83
4$\frac{15}{16}$-in. shaft	22.22
2$\frac{15}{16}$-in. shaft	4.36
Babbitt	12.30
Labor to install new belt	193.41
Repair labor	107.12
Power at $8.82 per hp.-mo. (7.3 hp.)	772.56
Chain drive (replacement)	104.85
Total	$1,734.99
Wet tons elevated	1,045,162
Cost per ton	$0.00166

At Chino Copper Co. (*112 J 806*) the cost of elevating tailing during the sixteen months prior to Dec., 1920, was $0.01954 per dry ton. The lift was about 40 ft.; average tonnage, 5000 dry tons per 24 hr. in a pulp containing about 15 per cent. solids; dry tons elevated in 16 months, 2,459,300.

At Miami Copper Co., in 1915, two 24-in. bucket elevators in the coarse-crushing plant handled 1,348,122 tons of −¾-in roll product. The cost of maintenance per ton was as follows: Belt, $0.00104; buckets, 0.00025; miscellaneous supplies, 0.00044; labor, 0.00031; total, $0.00204.

Automatic skip hoists, which are used to a considerable extent in feeding iron blast-furnaces and in raising water from shallow mines, have been used at one plant, viz.: Alaska Gastineau (Sec. 2, Fig. 75) for elevation in the roll-screen circuit. For performance see Sec. 23, Art. 6.

Bibliography. F. V. Hetzel, *Belt conveyors and bucket elevators*, John Wiley & Sons, New York, 1922, contains a very complete discussion of belt elevators. See also A. O. Gates, *102 J 4*; *96 J 725*; and E. S. Wiard, *The theory and practice of ore dressing*, McGraw-Hill Book Co., New York, 1915.

9. Chutes

Chutes are steeply-inclined rectangular troughs used for transportation of dry or nearly-dry ore by gravity; they may or may not be covered. Vertical chutes are usually equi-dimensional and are closed on all four sides. Round-bottom chutes are sometimes used for transportation of coal to lessen breakage. Chutes are usually made of wood, lined with some wear-resisting material. Steel is frequently used for vertical or nearly-vertical chutes.

Chutes must be well supported and rugged enough to stand the shock of material passing through; they must be large enough to pass the desired tonnage, and slope enough to keep the material moving. Covered and vertical chutes should be made with removable covers to allow access for replacing lining or relieving stoppages.

Liners are always placed on the bottom and frequently on the sides of inclined chutes; vertical chutes are lined on all four sides. Materials used for lining are sheet steel, cast iron, chilled cast iron, manganese steel, steel rails, old iron or steel, wood, rubber or concrete. In some cases chutes are built in steps and material is allowed to build up; no lining is then needed.

Chute linings in anthracite breakers have commonly been sheets of blue annealed iron, (*66 A 422*) but these are rapidly corroded by acid water. Galvanized sheet resists corrosion and permits lower sliding angles. Vitrified clay pipe makes excellent lining and wears indefinitely. Glass has been found too brittle. HUDSON COAL Co. has used Corrosiron (cast iron with about 12 per cent. Si). It is made up in flanged U-shaped sections, 18 in. wide and 2 to 3 ft. long, both straight and spiral. AT the LOREE breaker this material had, in 1920, already outlasted ten sheet-steel linings and bade fair to last indefinitely. When new the surface is rough, hence the minimum angle is greater than after the chute has worn smooth. This difficulty is overcome by setting the chutes on the final slopes, lining with sheet iron, and removing the lining a sheet at a time, beginning at the lower end.

Size. The WIDTH of a chute should never be less than three times the size of the largest piece which it is to carry, if free running is desired. No definite rule can be set for the cross-section of a chute to carry a given tonnage. The width is usually fixed by the size of opening from which the chute takes or to which it delivers its load; DEPTH is usually made much greater than the depth of the stream both to prevent bounding out and for convenience in construction.

The minimum slope at which material will flow freely in a chute depends on the coefficient of friction between the chute lining and the material.

Holbrook & Fraser (*Bul. 234 USBM 42*) found that sliding angles on smooth steel surfaces varied not only with hardness and specific-gravity but also with the shape of the fragments. See Sec. 5, Tables 18, 19, 21 and 22. Cubical or rounded pieces slide at smaller angles than flat or wedge-shaped pieces. The distribution of pieces of different sizes in the materials is important, especially if moisture is present; very fine material tends to cake and retard motion; in general, sized material will flow on a flatter slope than unsized. SAFE SLOPES are greater than the angle of repose of the material; if smooth liners are used, such slopes are greater than those necessary for sliding on the lining, but the chutes will not clog if the lining becomes rough and uneven. General practice is to make slopes for dry material at least 40 to 45°. Too great slope causes excessive bounding with resultant breakage of material and excessive wear on liners. Convergence of the sides of chutes is to be avoided; if unavoidable, the chute must be designed so that the inclination to the horizontal of the line of intersection of the side and bottom is equal to or greater than the angle of repose of the material or the sliding angle, whichever is adopted as the minimum slope. Table 17 gives performances of various chutes.

Chutes for coal breakers are designed to reduce breakage of coal. This breakage may amount to as much as 10 or 12 per cent. of the total material reduced from domestic to steam sizes in the passage from jigs and screens to and through the loading pockets. For vertical drops, spiral chutes (Fig. 25) are commonly used. An automatically controlled box chute (Fig. 26) is also frequently used. So long as coal is traveling freely in the upper inclined chute, the counterweight keeps the hinged chute in the position shown and the bottom gate closed. When the vertical shaft is filled and coal begins to back up in the upper inclined chute, the weight of the coal depresses the hinged chute and opens the bottom gate

Table 17. Chutes in various mills

Ore	Size of pieces	Dimensions of chute		Slope, degrees and minutes
		Width, inches	Depth, inches	
Magnetic iron ore	−4-in. +2-in.	24	24	45–0
Magnetic iron ore	−2-in. +1-in.	10	8	45–0
Magnetic iron ore	−2-in. + ¾-in.	12	10	45–0
Magnetic iron ore	− ¾-in. + ⅜-in.	10	8	45–0
Magnetic iron ore	− ¼-in.	10	8	45–0
Porphyry copper ore	Coarse crushing plant			45–0
Low-grade copper ore	− ¾-in.	12	10	Almost vertical
Tungsten ore	Run-of-mine	24	24	30–15*a*
Tungsten ore	−2½-in.	12	24	39–47*a*
Missouri lead ore	Run-of-mine, −24-in. +7-in.	29	166	Vertical
Missouri lead ore	−2-in. +9-mm.	18	12	26–34
Low-grade gold ore	−2½-in.	22	20	43–0
Low-grade gold ore	−10-mesh	11½	10	47–0*b*

Ore	Tons handled per 24 hr.	Lining		
		Material	Thickness, inches	Life, days
Magnetic iron ore	900	Steel	¾	50
Magnetic iron ore	960	Chilled C. I.	1½	60
Magnetic iron ore	800	*c*	*c*	......
Magnetic iron ore	800	Chilled C. I.	1½	100
Magnetic iron ore	800	Chilled C. I.	1½	100
Porphyry copper ore	......	Steel plate	5/16	30–45
Low-grade copper ore	350	C. I.	......	120
Tungsten ore	300	Steel	......	......
Tungsten ore	300	Steel	......	365
Missouri lead ore	800	56-lb. steel rails	......	Sides 310, Bottom 30
Missouri lead ore	1000	Manganese steel	......	......
Low-grade gold ore	5000	Boiler plate and roll steel	1	365
Low-grade gold ore	833	C. I.	⅞	183

a Occasional clogging. *b* Clogs when moisture is over 6 per cent. *c* Bottom made in steps; material piles up between the steps and protects the bottom.

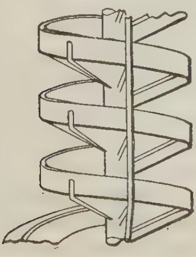


FIG. 25.—Spiral chute.

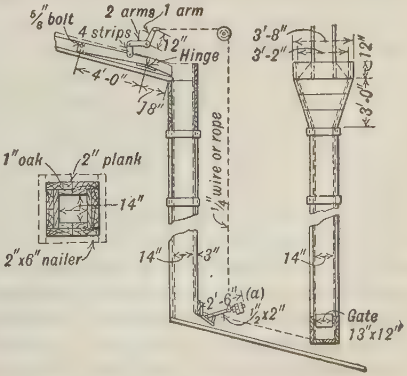
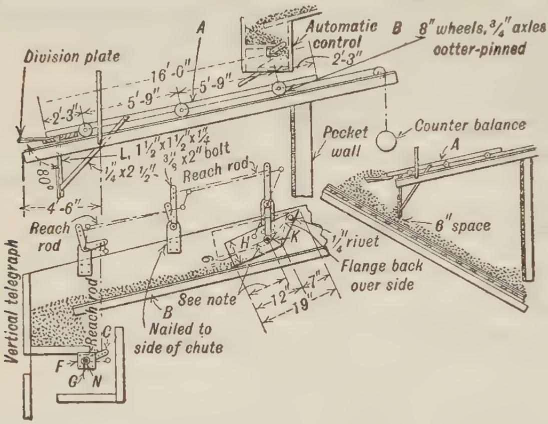


FIG. 26.—Telegraph chute (*after Ashmead*).

and flow starts again. If the vertical shaft is round, breakage is least. WHITE CHUTE (Fig. 27) is an automatic device for regulating the flow and fall into loading pockets. The counterweighted traveling carriage *A* is placed in position over the chute as shown. It is fed by an automatic control chute *B*, the control mechanism consisting of a balanced float *H* which connects by means of link rods with a tilting lip *C*. When coal is flowing down chute



Rod *N* is adjusted by moving *F* to desired position before nailing. Position of gate *G* is adjusted to suit conditions by moving lever *C* to any desired position and holding with set screw. Hole for rod *K* is located by trial so that float *H* balances with front on chute and back $1\frac{1}{2}$ to 3 in. off bottom, according to size of coal.

FIG. 27.—White automatic chute control and bin loader.

B the float lifts and thereby depresses the lip *C* and coal flows into the loading pocket. In the pocket the pressure of coal against the board *L* backs the movable chute *A* up slope and thereby permits complete automatic filling of the pocket with but little breakage. According to Ashmead (*66 A 422*) tests have shown 5 per cent. reduction in pocket-loading breakage due to this type of chute. It is to be noted that the control mechanism is adaptable to any chute.

10. Launders

Launders are narrow, inclined troughs used for transporting mixtures of pulverized solid and water by gravity flow. The solid is carried either by suspension, by sliding or rolling, or by a series of leaps or jumps, termed SALTATION. To obtain free motion of material in a launder, the slope must permit sufficient water velocity to overcome the inertia of the solid material and frictional resistance. The inertia of the solid particles varies with their size and density. Frictional resistance varies with velocity, the kind of lining, shape of launder, amount of material, proportion of solids in the flowing stream and the size, shape and density of the solid particles. Where particles are carried either by sliding or rolling the roughness of the lining may have important effect; if very smooth, angular pieces will probably slide and friction increase, while, if the lining is somewhat rough, the same pieces may roll or progress by leaps, with less friction loss. Very fine material is carried in suspension even at low velocities and with increasing velocity coarser material is held in suspension due to the more violent eddy currents. When velocity is low and the solid particles coarse, most of the movement is by rolling or sliding; as velocity increases, the movement is more by leaping with the finer particles going into suspension. Increase in the amount of very fine material in sus-

pension causes increase in density of the mass and the larger particles transported by rolling or jumping are more easily moved because of the buoyant effect of the denser medium. The velocity of the water is least at the bottom of the launder and increases, in a given cross-section, to a maximum at the surface of the stream.

Where solid material is carried only by rolling, the large pieces (extending higher in the stream) benefit by the higher velocity of the upper levels and tend to be carried faster than the finer particles. However, the path of a single small mass of liquid is not a straight line parallel to the axis of the launder but really sinuous, curving up and down and in and out from the sides. In this respect the flow may be considered different from that of a comparatively thin stream on a gradual slope, not narrowly confined. Roughness of the lining increases sinuosity of the stream by causing eddy currents.

When material fed to a launder starts practically from rest, the velocity at which it travels increases due to the component of gravity acceleration parallel to the bottom of the launder, reduced by frictional resistance both within the mass of flowing material (viscosity) and between the flowing material and the sides and bottom of the launder. Velocity will increase to the point where frictional resistance equals the gravitational force, thence onward the velocity will remain constant provided slope, shape, lining, etc., remain the same. Experimental evidence indicates that maximum velocity is quickly reached in launders on moderate slopes, and it is probable that in most cases launders are so fed that the initial velocity is close to or even greater than the velocity that would be reached starting from rest. If a launder of even slope is capable of holding solid material in suspension over the period of acceleration, it is capable of carrying a larger load after constant velocity is reached. Practice recognizes this by increasing the slope for a short distance at the head end; velocity is thus quickly increased and full advantage may be taken of the carrying capacity of the stream at constant velocity.

Calculation of minimum slope and proper size for a launder to serve under stated conditions is difficult because of the number of empirical constants needed. Considerable experimental work has been done, but the materials handled have been different in all cases and the variety limited in each instance, consequently widely divergent results are presented. Reports on working launders are not easy to analyze, since the slopes used are generally on the safe side and there is no clue as to the minimum slope. Tabulation and study of information available on about 150 launders carrying almost every variety of ores, concentrates and tailings failed to reveal more than broad generalities as to the influence of the various factors.

Velocity of the stream furnishes the force to move the solid which is to be transported. Velocity is either initial or that imparted by gravity. Retardation is due to frictional resistance or obstructions. Frictional resistance is decreased by making the cross-section of the stream such that the wetted perimeter is least per unit area of cross-section; by using a smooth lining; by varying the proportion of solid and the character of the solid. In general coarse material requires greater stream velocity than fine; heavy material greater than light; thick pulp greater than thin; angular material greater than rounded. Very rough lining and breaks at joints in the lining obstruct free flow by causing irregular movement in the body of the stream; their effect decreases with increase in depth of stream.

Slope. Figs. 28 and 29 show the recommendations of various experimenters.

Fig. 28 curves (*A*) and (*B*) are the minimum slopes determined by Blue (*8 J 536*); curve (*A*) is for beach sand and curve (*B*) for angular mill tailing with some slime present. Curve (*C*) gives slopes found suitable by Caldecott (*14 JCM 486*) for launders carrying sand containing about 4 per cent. pyrite, 10 per cent. of the sand being retained on a screen with 0.01 in. aperture; these values were calculated from the empirical formulas: $W = 12/(G - 1)$; $P = 100W/(W + 1)$ and $G = (W + 12)/W$, in which W = ratio by weight of water to solids, P = per cent. by weight of water, G = slope of launder, per cent. Curves (*D*), (*J*), (*K*) and (*L*) were plotted from results obtained by Browning (*29 M & M 300*); (*D*) is for $-10 + 30$-mesh limestone tailing

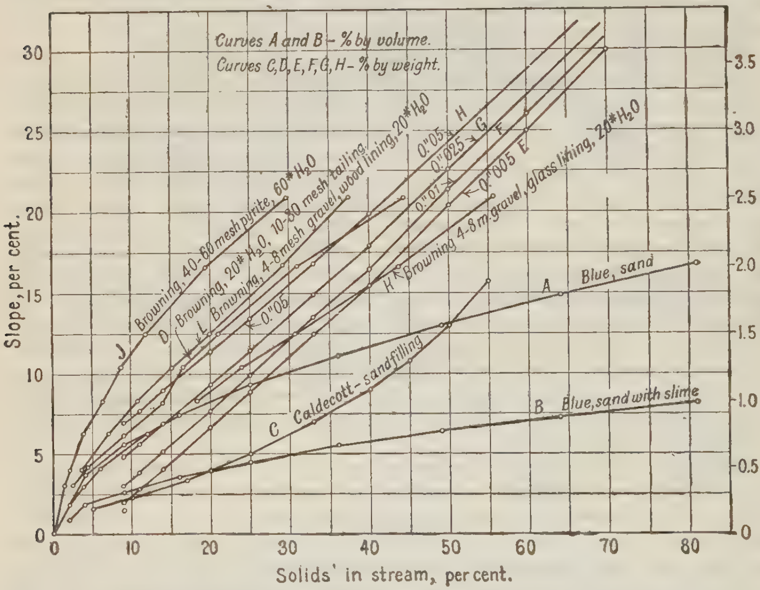


FIG. 28.—Relation between percentage of solids and slope of launders.

(40 per cent. +20-mesh and 60 per cent. $-20 + 30$-mesh) in a rectangular wooden launder 2½-in. wide; (*J*) is for $-40 + 60$-mesh pyrite table concentrate in a wooden launder 2½-in. wide.; (*K*) and (*L*) are for $-4 + 8$-mesh ordinary rounded pebbles, (*K*) with glass lining and (*L*) with wood. Curves (*E*), (*F*), (*G*) and (*H*) were plotted from a table by Julian, Smart & Allen (*Cyaniding gold and silver ores*), for sand of predominating size of 0.005-, 0.01-, 0.025- and 0.05-in. respectively. Caldecott and Powell (*14 JCM 121*) state that sand pulp will flow slowly with 30 per cent. water on a 30 per cent. slope, with 40 per cent. water, on 20 per cent. slope and with 60 per cent. water on a 10 per cent. slope.

The wide divergence of the curves is due to the variety of materials and conditions. The largest size investigated was 4-mesh rounded gravel. Fig. 29 (*2 RMP*) shows that the slope varies approximately as the square root of the particle size. The values from the curve are said to give velocities of 7 to 10 ft. per sec. The points off the curve are plotted from actual operations; most of them fall on the safe side above the curve; most of those below were dilute pulps (under 15 per cent. solids) but three ranged from 25 to 33 per cent.

Use of Curves. Bear in mind the characteristic of the particular material to be carried and err on the safe side; if insufficient slope is provided water must be added, which may be wasteful or undesirable in subsequent treatment.

GILBERT (*PP 86, USGS*) summarizing experiments on the carrying capacity of water in flumes, stated that much of the movement is by rolling or sliding, especially when the current is slow and the solid coarse; that with a swift current or fine solid the particles travel by saltation and the finest material is suspended. When the principal movement is by rolling or sliding, the capacity of the current increases with coarseness of the solid material, up to the point at which the current is barely able to start the particles. When the principal movement is by saltation, the capacity increases with fineness of solid up to and probably beyond the critical fineness at which the solid goes into suspension.

BLUE's experiments (*84 J 536*) were carried on in a U-shaped sheet-iron launder, 50 ft. long, 5 in. deep, with semi-circular bottom of 2-in. radius. Two classes of solid material were used, (*a*) beach sand (77 per cent. −40- +80-mesh; 1 per cent. −100-mesh; average size about 60-mesh), (*b*) sharp quartz sand containing varying amounts of clayey slime, in the proportions of about 10 per cent. of slime and 90 per cent. of sand; the sand averaged about 80-mesh, 90 per cent. −40 and 10 per cent. −200-mesh. Various mixtures of

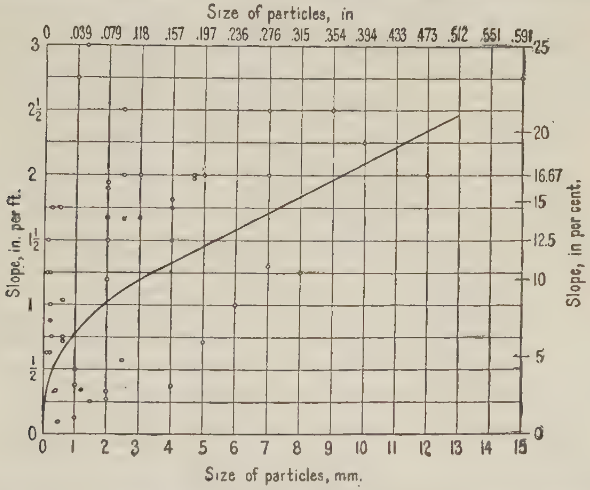


FIG. 29.—Relation between size of particles and slope of launder (*after Schmitt, 2 RMP*).

each of these solids with water were fed and observations made at the slope when the "least perceptible quantity of sand just began to fall to the bottom." Depth was measured to 1/16 in.; rate of flow and the proportion of solids by volume were determined by time samples taken at the discharge end and average velocities were calculated from these data. The results with beach sand indicated that the proportion by volume of wet solids in suspension, q, varied as the square of the slope S or, $S = k\sqrt{q}$. The sand-slime results were similar but less concordant; they indicated that with slime present more solid could be carried on a given slope with a given amount of water.

The value of k in above equation was determined to be 0.186 for the beach sand and 0.091 for the sand-slime mixture. Blue also used his results to calculate values of the COEFFICIENT OF ROUGHNESS, n, in Kutter's formula by solving for values of the constant C in the Chezy formula (Sec. 27, Art. 28) and substituting in Kutter's formula (Sec. 27, Art. 30). The values of n obtained indicated the relation, $n = 0.0125q^{0.055}$ for beach sand and $n = 0.0105q^{0.02}$ for the sand-slime mixture. These values of n can be used only when the roughness of the launder is about the same as that of the sheet-steel channel used. In the case of beach sand, an ill-defined relation exists between v and q, *e.g.*, $v = 8.148\sqrt[6]{q}$. The experiments covered slopes varying from 0.107 to 0.008, proportions of solids by volume from 0.0022 to 0.350; and depths from ⅝ in. up to 3¾ in. Calculated values of C varied from 72.2 to 135 and of n from 0.0080 to 0.0123. For the sand-slime mixture the slopes varied from 0.052 to 0.009, proportion of solids from 0.277 to 0.013, depths from ⅝ in. to 3¾ in., C from 70.0 to 128.0 and n from 0.0090 to 0.0118.

Julian and Smart (*Cyaniding gold and silver ores, J. B. Lippincott Co., 1921*) calculate the size of launders by use of the Chezy formula ($v = C\sqrt{rs}$) assuming a relation between width and depth to obtain the minimum wetted perimeter (for rectangular launders width = twice the depth) and making $C = 80$, which is an average value obtained from actual measurements of flow in launders carrying slime overflow and average stamp-battery pulp. Then if w = depth of the stream in ft., $2w$ = width, $2w^2$ = area of cross-section, $4w$ = wetted perimeter, $r = w/2$, and $v = C\sqrt{ws/2}$. If Q = cubic ft. per sec., $Q = 2vw^2 = 2Cw^2\sqrt{ws/2}$ or $Q^2 = 2C^2w^5s$, whence

$$w = \sqrt[5]{\frac{Q^2}{2C^2s}} = 0.151\sqrt[5]{\frac{Q^2}{s}}.$$

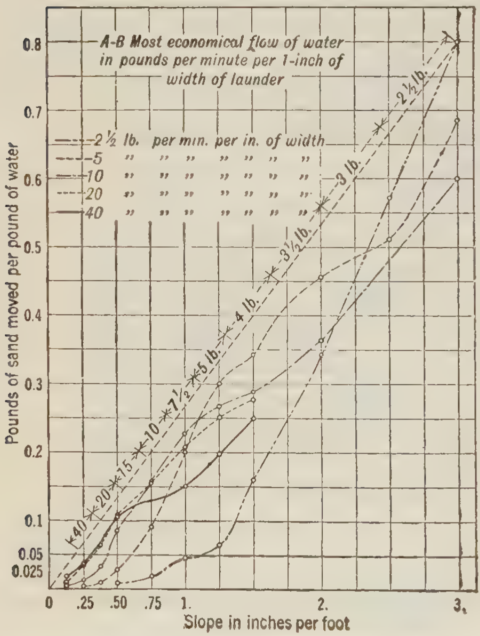


Fig. 30.—Launder flow (*after Overstrom and Richards*).

Any relation between the width and depth of the stream may be assumed; various assumptions should be tested until the size determined is best for the conditions at hand. In the construction of small wooden launders the width is influenced by the widths of good lumber available without ripping or making a longitudinal seam down the center.

Table 18 (*Gilbert, PP 86 USGS*) is of value in determining dimensions, using Julian and Smart's method for calculating launder size.

Richards (*3 OD 1592*) gives Fig. 30 from Overstrom's data. The most economical water quantity is determined by following the ordinate for a given slope to its intersection with the inclined straight line on which weight of water is indicated. The weight of sand that 1 lb. of water will transport is then found by following the same ordinate to its intersection with the curve for the economical water quantity and reading on the left-hand scale. To find the width of launder in inches divide the total weight of sand to be moved by the product of lb. of water per in. of width and lb. of sand moved per lb. of water.

Table 18. Depth of water in a rectangular wooden trough (planed and painted) 12 in. wide

Discharge, cubic feet per second	Discharge, cubic feet per second per inch of width	Slope, feet per foot	Depth, feet	Depth, inches
0.363	0.032	0.01	0.12	1.44
0.363	0.032	0.02	0.096	1.152
0.363	0.032	0.03	0.082	0.984
0.734	0.061	0.01	0.194	2.328
0.734	0.061	0.02	0.154	1.848
0.734	0.061	0.03	0.136	1.632

The diagram was plotted from experiments with quartz from 40- to 150-mesh flowing in a wooden launder 2½ in. wide. It indicates that the amount of solid carried is independent of the wetted perimeter. The curves indicate that there is a certain quantity of water for each slope that will carry a maximum amount of solids per lb.

Launder performances are given in Tables 19 to 22. Table 19 shows limiting values of slope for each class of material. CANANEA ore (Table 20) consists of copper sulphides, native copper, garnet, sphalerite, galena and oxides and carbonates of copper in gangue of limestone, quartzite, quartz porphyry and diorite country rock. ANACONDA ore (Table 21), see p. 82. The slopes given are not necessarily minimum but were considered safe; extra headroom was utilized when available. LUCKY TIGER ore is a complex sulphide (Table 22).

Shape of launder. Frictional resistance can be reduced in flumes or conduits for water by making the shape of the channel such that the wetted area is the smallest possible for the volume flowing. This is accomplished by making the bottom semicircular which gives the smallest wetted perimeter for a given cross-section. The same thing is true in transporting pulp in launders, when the solid material is practically all carried in suspension, but when the solid material is carried wholly or in great part by rolling, a semi-circular bottom restricts the area available for free motion of the particles, and a rectangular cross-section is better. Gilbert found that rectangular or box flumes can carry more coarse solid than semi-cylindrical flumes of similar width and the same flow of water. Reduction of wetted perimeter to a minimum leaves minimum area exposed to the abrasive action of solid material in the stream and liner consumption is thus decreased. But the condition of least wetted perimeter may not always be most desirable. Where comparatively coarse material is transported, chiefly by rolling or sliding, it may be advisable to use a shallow stream, only slightly deeper than the size of the largest pieces of solid; increase in depth beyond this point increases the amount of water used but only slightly increases the effective velocity in the lower zones of the stream. Gilbert, dealing with relatively coarse material in flumes, found that up to a certain limit, varying with conditions, the carrying capacity of the stream increased with increasing relative width of channel and concluded that the ratio of depth to width for highest efficiency under such conditions would rarely be greater than 1 : 10 and often as small as 1 : 30, but that on large operations the width would usually represent a compromise between efficiency and cost of construction and maintenance.

The use of wood for launders leads naturally to rectangular cross-section. Triangular-shaped cleats nailed into the corners of wooden launders make the shape of the bottom approach semi-circular. V-SHAPED LAUNDERS of wood are used to a limited extent; the advantage lies in the simplicity of construction. The shape may be easily made whatever is desired when sheet steel or cement is used.

Reinforced-concrete launder (Fig. 31) (*96 J 22*), was designed to carry overflow water 400 ft. It was cast in 16-ft. sections. The concrete mixture was 3 parts of ⅝-in. jig tailing, 3 parts ordinary mill tailing from ¼-in. to 10-mesh, one part fine sand and one part cement; the bottom was 2½ in. thick; the sides 3 in. at the top, increasing to 5½ in. at the bottom; a 4 × 1-in. ridge was carried along the bottom on each side; strands of worn hoisting rope were used for reinforcement. Single ½-in. strands were run along the top and two double ½-in. strands were placed along the bottom on each side. These were wrapped with wire about 5 ft. from the ends and two single strands, one from each pair, were taken up to the top of the side (C to A) while other two strands continued to the end of the section. Hooks (A) of ⅜-in. rod threaded at one end were used to draw the strands tight. Pieces of 1 × ⅛-in. strap, (B) were put in at 18-in. intervals; at every other strap a

Table 19. Minimum slopes of launders

Ore	Size of particles	Per cent. of solids by weight	Size of launder, width × depth, inches
Primary mill to classifier	− ⅜-in.	50	11½×10
Secondary mill to classifier	−20-mesh	50	11½×10
Tailing	−65-mesh	40	11¾×9½
Ball mill to classifier	44%+60	25	12″ wide
Cleaner-flotation concentrate	98%−100	20	10″ wide
Heavy hematite in taconite and quartz: Trommel to conveyor	+2⅞-in.	100	....
Heavy hematite in taconite and quartz: Apron under trommel	−1⅛-in.	100	....
Heavy hematite in taconite and quartz: Table feed	−3/16-in.	20	....
Sp. gr. 2.65	All −40-mesh 73%−200-mesh	50	8″ wide
Porphyry copper ore—sp. gr. 2.7	3.6%+ 40-m. 55.4%−200-m.	28.6	3′-4′ wide
	4.4%+8-mesh 19.2%−200-m.	10	....
	−0.6-mm. with slimes	37	7¾×9
Table concentrate	−0.6-mm.	....	....
Primary table concentrate	−4.7-mm.	....	7¾×9
General tailing	−0.6-mm.	30	13×7
Galena in limestone:	−9-mm. +2-mm.	40	12×12
Galena in limestone: Table tailing	−2-mm. +150-mesh	50	8×6
Galena in limestone: Table concentrate	−2-mm. +150-mesh	85	4×4
Gold ore. Gangue, quartz	1-mm.	15	44×22
Mill feed	−1½-in. +7-mm.	30	16×12
Concentrate	−7-mm.	20	12×9
	−100-mesh	16	16×9
Mill pulp	....	18	7×7
Concentrate	....	....	....
Tailing	− ¼-in.	0.4	36×12
Mill feed	−1-in.	40	12×8
Jig concentrate	7-mm.	10	5×7
Galena in limestone gangue: Jig tailing	−12-mm. +2-mm.	25	30×10
Galena in limestone gangue: Jig and table concentrates	−4-mesh	10	22×10
Galena in limestone gangue: Slime	−80-mesh	5	30×10
	− ¾-in.	50–70	15½×13½
	−6-mesh	40	15½×13½
	− ¾-in.	80–85	13½×11¾
Concentrate	−0.087-in.	....	....
Sands	−0.087-in.	....	....
Run-of-mine ore	−10-in.	Dry	24×24
	2½-in.	Dry	12×24
	−3/16-in.	12–16	12×11

for various ores at different plants

Slope of launder, inches per foot	Depth of stream, inches	Kind of lining	Tons solid per 24 hr.	Remarks	Mill
1¼ 1⅛ ½	....	....	1000 1500 1050	Too flat....	Utah Consolidated
1¹⁄₁₆ ⅝	....	No lining... No lining...	1200 15	Granular ore	Silver Dyke, Neihart, Mont.
8½ 7 2¼	......	....	....	....	Iron Mountain, Iron Mountain, Mo.
⅝″	......	Unlined flat bottom....	100	....	Mogul Mining Co.
⅜″ ¾″	....	.... Sheet iron...	15,000–18,000	....	Inspiration Consolidated Copper Co.
0.72 1.03 2.0 0.75	....	.... Wood-block. Cast iron... Wood-block.	....	.... 0.56-in. per ft. requires much water	Shattuck Arizona Copper Co.
2½ 1½ 4	1.0 1.0 ¼	Cast iron... Cast iron... Cast iron...	150 50 5	....	St. Joseph Lead Co., Bonne Terre, Mo.
0.38	4	2″ fir.......	5000	....	Alaska Gastineau
4 2 ³⁄₁₆	....	Cast iron... Cast iron... No lining...	1000 180 750	....	Moctezuma Copper Co., Nacozari, Sonora, Mex.
1.2 2.1	......	....	....	Minimum slope used..	Sunnyside Mining & Milling Co.
¼	7	Tank steel..	1800	....	Copper Range Co.
6 2½	1 1.5	Cast iron... Cast iron...	625 40	....	Bunker Hill & Sullivan Mining Co.
2 2 ⁵⁄₁₆	3 1 4½	Cast iron... Sheet steel.. Sheet steel..	1800 100 500	....	Federal Lead Co., Flat River, Mo.
2¾ 2 3¾ 1½ 1	3 2 4	White iron.. Concrete.... Concrete....	2400–3600 2400 2000–2500	.... Round bottom Round bottom	Chino Copper Co., Hurley, N. M.
7 10 0.72	 2	Steel....... Steel....... Wood......	300 300	Occasional clogging..	Tungsten Mines Co.

Table 20. Launders at Cananea Consolidated Copper Co. *(95 J 375)*

Material handled	Size of launder, depth × width, inches	Grade, inches per foot	Lining	Ratio solids, liquids	Largest particle, mm.	Per cent. weight on 60-mesh
Original feed to Section "C"	10×12	3½	C.I.—11¼″	1 : 1	30.0	82.0
Bull-jig tailing, to coarse rolls	10×9½	3	C.I.—9¼″	1 : 1.1	28.0	98.0
Roll-jig tailing, to fine rolls	10×7½	2¼	C.I.—7¼″	1 : 3	10.0	98.0
Coarse jig concentrate	10×7½	1⅔	C.I.—7¼″	1 : 2.9	35.0	95.0*a*
Undersize 2-mm. trommel to classifier	10×5¾	1½	C.I.—5¼″	1 : 4.4	4.0	52.0*a*
First spigot of classifier to sand jig	8×5¾	3⅓	C.I.—5¼″	1 : 1.5	4.0	90.0
Shaking launder (*b*) for sand-jig concentrate	6½×12	⅜	None—corners	1 : 19.0	4.0	84.0
Fine jig concentrate from shaking launder	10×7½	1¹³⁄₁₆	C.I.—7¼″	1 : 6.0	4.0	80.0
Bryan (Chile) mill discharge to drag belt	8¾×11½	⁹⁄₁₆	None—corners	1 : 1.7	2.5	26.0
Drag-belt sands to classifier distributor	10×7½	1⅔	C.I.—7¼″	1 : 0.9	2.5	46.0
No. 1 spigot of classifier to mud jigs	8×5½	1⅔	C.I.—5¼″	1 : 17.0	3.0	60.0
Table feed	7¼×5½	½	None—corners	1 : 8.4	1.0	0.3
Table concentrate, drag-belt launder (*c*)	9½×9½	⅛	1½″ boards	1 : 26.0	1.0	13.6
Table concentrate, shaking launders (*d*)	4×7½	³⁄₃₂	None—corners	1 : 7.4	0.5	35.2
Slimes, Section "C," settling tanks to Section "B" vanners	6½×9½	⁹⁄₁₆	None	1 : 96.0	0.17	0.0
Slime feed to vanners, Section "C"	9½×11½*e*	⅓	None	1 : 4.7	0.17	0.5
Vanner concentrates, drag-belt launder (*f*)	10×25×16	Level	None	1 : 31.0	0.2	1.0
Vanner and table concentrates	6½×6	1¹⁵⁄₁₆	None	1 : 20.0	2.0	27.2
Vanner tailing, Section "C"	9½×11½	⅓	None—corners	1 : 7.0	0.4	0.4
Coarse tailing, table and jig	9½×22	⅓	½″ boards and corners *g*	1 : 27.0	2.0	41.6*a*
Coarse sand tailings to dam No. 2	12¼×11½	1¹⁄₃₂	½″ boards and corners *g*	1 : 21.0	2.0	19.5*a*
Coarse sand tailings to dam No. 1	12×11	⁹⁄₃₂	½″ boards and corners *g*	1 : 21.0	2.0	19.6*a*
Slimes to mill No. 4	9½×12½	¼	None—corners	1 : 4.4	0.1	0.0
Sands and slimes to mill No. 3	10×11½	¼	None—corners	1 : 5.0	1.5	2.6
Feed to vanners, mill No. 4	6×7½	⅓	None	1 : 4.4	0.1	0.0
Slime concentrates, mill No. 4, drag-belt launder	12×24×18	Level	None *c*	....	0.1	0.2
Slime concentrates, mill No. 4, elevator to bins	7×7½	⅔	Glass—corners *h*	1 : 20.0	0.1	0.4
Slime concentrates, mill No. 3, elevator to bins	10×12	¹⁵⁄₁₆	Concrete	1 : 8.7	0.2	2.8
Slime tailing, mill No. 4, to mill No. 3 settling tanks	10×11	⁷⁄₃₂	None—corners	1 : 6.3	0.1	0.0

All cast-iron liners have 2-in. effective depth and 24-in. length, with corners rounded. *a* Liable to choke. *b* Speed 160 r.p.m., actuated by heavy head motion. "Corners" are strips of wood with cross-section of 45° triangle nailed in corners of launder. *c* Speed 75 ft. per min.; old-4-in. drive belts are used, no scrapers. *d* Speed 180 r.p.m. *e* Effective width 6 in. *f* Speed 75 ft. per minute. *g* Boards laid with grain across direction of flow. *h* Glass liners, ¾ × 4 × 14 in.

Table 21. Laundoers at the Washoe Concentrator, Anaconda Copper Mining Co. (*96 J 501*)

Material handled	Size of launder, inches		Grade, inches per foot	Lining	Ratio solids to water	Diameter largest particle, mm.	Depth of stream on launder, inches	Per cent. of solids on 60-mesh	Remarks
	Depth	Width							
Undersize ⅞-in. round-hole trommels	15	10	2.0	Cast iron	1 : 3.5	21.0	1.25	82.0	Contains slime
Undersize 5-mm. round-hole trommels	7	8½	3.0	Cast iron	1 : 5.2	5.0	1.25	75.0	Contains slime
Undersize 4-mm. round-hole trommels	9½	10½	1.75	Cast iron	1 : 5.0	4.0	1.50	50.0	Contains slime
Undersize 2½-mm. round-hole trommels	7	8½	2.5	Cast iron	1 : 8.2	2.5	0.50	62.0	Contains slime
Hutch product of Harz jigs	7	6½	1.0	Cast iron	1 : 22.7	6.0	1.00	95.0	No slime
Feed to Huntington mill	7	6	1.9	Cast iron	1 : 3.6	2.0	1.25	81.0	No slime
⅜-in. concentrate	10	9	1.3	Cast iron	1 : 12.8	7.0	1.00	100.0	No slime
Evans-jig concentrate	9½	9½	1.2	Cast iron	1 : 27.8	2.0	1.50	72.5	No slime
Coarse table feed	7½	7	1.2	No lining	1 : 14.2	0.3	0.50	7.0	Contains slime
Fine table feed	5½	4½	1.2	No lining	1 : 13.0	0.2	0.75	0.0	Contains slime
Table concentrate	11	8¼	0.75	No lining	1 : 15.6	0.2	0.75	0.0	No slime
Table middling	4½	5	0.75	No lining	1 : 15.6	0.2	0.75	0.0	No slime
Table tailing	7	7½	0.75	No lining	1 : 10.0	0.3	0.50	10.0	A little slime
Secondary table feed, remodeled section	7	7½	1.20	No lining	1 : 7.4	0.9	0.50	44.0	No slime
Fine primary table feed, remodeled section	7	7½	1.20	No lining	1 : 14.0	0.35	0.50	2.0	No slime
Coarse primary table feed, remodeled section	7	7½	1.20	No lining	1 : 4.4	0.9	0.50	46.5	No slime

Table 22. Launders at No. 2 mill, Lucky Tiger Mine, Sonora, Mex.

Material handled	Size of launder, inches		Slope, inches per foot	Ratio, solids to water	Size of material
	Depth	Width			
Stamps to classifier	8	8	⅛	1 : 6	47.8% −200 mesh
Classified coarse sand to Wilfleys	6	6	⅞	1 : 3	20-mesh
Classified fine sand to Wilfleys	6	6	¾	1 : 3	60-mesh to +200-mesh
Slime to dewatering tanks	8	8	⅛	1 : 20	−200-mesh
Thickened slime to Deister tables	6	6	⅜	1 : 4	−200-mesh
Sand tailing from Wilfley tables	8	8	1	1 : 5	20-mesh
Middling from Wilfleys to pan	4	4	1	1 : 2	8.4% +40-mesh
Concentrate from Wilfleys to bin	6	4	1¾	......	40-mesh to 200-mesh
Slime tailing from Deister tables	6	6	⅜	1 : 7	−200-mesh
Middling from Deister tables	4	4	⅝	1 : 4	−200-mesh
Concentrate from Deister tables	6	4	1⅛	......	−200-mesh

piece (*C*) was run across the bottom with a slight turn-up at each end. The bottom was reinforced longitudinally by three ½-in. strands above the straps. Forms of 1-in. board were built up as shown; the bottom form (*D*) was supported on wedges (*F*), made by ripping

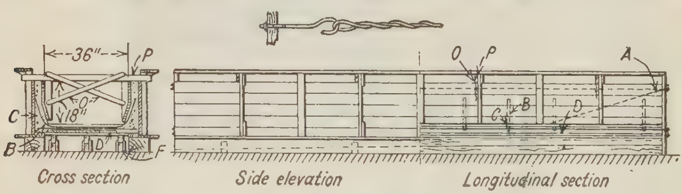


Fig. 31.—Reinforced-concrete launder.

2 × 10-in. hardwood planks, 16 ft. long, diagonally; it was easily removed by knocking out these wedges. The inside forms were held in place by ¼ × 1-in. iron braces (*O*), so arranged that the forms could be swung in and held in place away from concrete when removing by use of extra holes in the braces. The cost was $0.75 per ft., compared with $2.40 per ft. for iron pipe of equivalent capacity.

Turns and bends should be avoided, if possible; sharp turns cause clogging on flat slopes and bends cause excessive wear of linings. The slope at turns is generally increased to avoid clogging. Schmitt (2 *RMP 88*) recommends bend radii not less than 10 times the width of the launder and increase of from 1 to 2 per cent. in slope. Junctions should be made so that the direction of flow in each launder is approximately the same. Boxes or small sumps are frequently used at junction points and sharp turns; the exit should be raised a few inches above the bottom so that the bottom may fill with settled solid and decrease wear and splashing.

Depth of launder should be sufficiently greater than the depth of the stream to take care of splashing and of any sudden surges. *Julian, Smart and Allen* recommend making the launder 50 per cent. deeper than the stream for steady flow and 100 per cent. deeper where fluctuations are expected. Launders in practice show a much greater allowance in most cases. Of 53 launders, 45 had depths more than three times the wetted depth, the range was 1.3 times to 20 times and the average about 7.6 times.

Liners are important not only from the standpoint of frictional resistance but also because of the wearing quality and cost of different materials. Wood is usually the most easily available material but its life is short, except where only fine material in dilute suspension is flowing. Small wooden launders carrying material less than 1 mm. maximum size are generally left unlined; larger sizes are sometimes wood-lined. The life of wood in such service is long, it is easily replaced and its lightness saves in the amount of support necessary. The life can be increased indefinitely by placing cleats of wood or angle iron across the bottom at short intervals; solid material banks behind the cleats and makes a natural lining; frictional resistance is increased, however, and slope must be increased.

At the YMIR MINE, West Kootenay, B. C. (*34 A 602*) a 6 × 8-in. box-launder with 1-in. transverse riffles spaced 5 in. had a satisfactory life even when using soft hemlock or cedar lumber; the material was vanner tailing; the minimum slope was slightly under 5 per cent. (0.6 in. per ft.). At ALASKA GASTINEAU a 2-in. fir lining in a 22 × 44-in. launder carrying 5000 tons per 24 hr. of − 1-mm. material in a pulp containing 15 per cent. solids on a slope of 0.38 in. per ft. lasted 187 days.

RUBBER BELTING is a light and long-wearing lining, flexible and easily applied to wooden launders, but unless unfit for other service the cost is high.

At U.S.S.R. & M. Co. (Midvale) old ⅝-in. rubber belt used in 10 × 8-in. launder, sloped 3¾ in. per ft., carrying 25 tons per day of heavy − 1.0-mm. concentrate in a 1 : 1 pulp lasted two years. Similar belt in a 10 × 10-in. launder sloped 2¾ in. per ft. and carrying 85 tons per day of sand-table tailing of the same size in a pulp of 18 per cent. solids lasted over three years.

CANVAS BELTING is also sometimes used but the rougher surface increases frictional resistance. SLAG LINING was used at PHELPS DODGE CO., Morenci, Ariz. (*102 J 644*).

The slag castings were channel-shaped sections 12 × 12 × 2-in. weighing about 40 lb. each, reinforced with 1- or 1½-in. hexagonal 26- or 28-gage poultry wire. The slag was poured into metal molds standing on end with two pieces of reinforcing wire placed in each side; when the slag had chilled slightly, the molds were removed and the hot castings buried in a sand bed to allow slow solidification of the center and thus avoid shrinkage strains. The cost was not over 12 cents per ft. as compared to $0.90 to $3.00 per ft. for white-iron castings. Their use in the 1500-ton concentrator showed a saving in liner costs of $6000 per year over a period of 2½ years. In a two-mile tailing flume the cost in 1912 for cast-iron lining was $7015 or $0.01397 per ton. In 1914 cast iron and slag were both used; the combined cost was $1873.50 or $0.00407 per ton. From Jan. to July, 1915, slag lining cost $498.13 or $0.00197 per ton. In a launder taking roll product to an elevator, cast-iron liners lasted 90 days compared with 180 days for slag; cast-iron liners in launders carrying Wilfley-table feed lasted 60 days while slag liners were still in use after 240 days. In the tailing flume the slag lining had an estimated life of 6 years compared with yearly re-lining with 2-in. plank. Slag lining in chutes running dry was satisfactory for material up to ⅝-in. Such utilization of slag is a useful outlet for a product ordinarily waste; its use, however, is limited to plants where slag is readily available.

SHEET IRON OR STEEL, old boiler plate and discarded crusher steel and similar material are frequently used for launder linings. Table 23 shows the life of sheet-steel liners under various conditions.

CAST-IRON LINERS are usually made in short channel-shaped sections. The useful life of such lining is shown in Table 24.

The life of ⅝-in. cast-iron liners at FEDERAL LEAD CO., Flat River, Mo., in launders varying from 6 in. to 13 in. wide and carrying different materials, was from 8 to 10 months.

CONCRETE lining is readily made in convenient lengths and sizes and makes a very satisfactory and cheap lining.

At CONSOLIDATED MAIN REEF mill (*19 JCM 77*) U-shaped sections of concrete lining were cast in 18-in. lengths, 9 in. deep and 11½ or 13½-in. outside widths. The thickness

Table 23. Life of sheet-steel liners

Maximum size of material, mm.	Per cent. solids	Thickness of liners, inches	Life in days	Life in tons solid transported per inch of width	Grade, inches per foot
13	Dry	¼	120	3,840	4¼
7.0	33	¼	700	26,600	2½
7.0	40	⅜	350	33,250	5
6.0	0.4	⅜	450	22,500	¼
5.0	....	¼	180	5,220	2
5.0	10	12 gage (about 1/10 in.)	700	31,892	2
1.3	33	¼	400	6,000	3
0.8	33	¼	700	1,400	3¾
0.18	5	12 gage (about 1/10 in.)	1050	17,850	5/16

Table 24. Life of cast-iron liners

Maximum size of material, mm.	Per cent. solids	Thickness of liner, inches	Life in days	Life in tons of solid transported per inch of width	Grade of launder, inches per foot
15	50	1¾	90	5,130	2¾
12	25	⅝	180	10,800	2
7	20	½	360	16,000	4
2	42	1	365	9,125	1.67

Table 25. Round-bottom concrete lining at Chino Copper Co.

Maximum size of material, mm.	Size of launder, width × depth, inches	Solids, per cent.	Thickness of liner, inches	Life, days	Life in tons of solid transported per inch of width	Grade, inches per foot
19	13½ × 11¾	80	1½	270–365	51,770	3¾
3	15½ × 13½	40	1	180–300	37,200	2

Table 26. Life of concrete launder lining (*19 JCM 77*)

Description	Slope, per cent.	Screenings, grade of pulp			Life
		+60	+90	−90	
Mill launders, U-blocks.................	7½	53.55	11.38	35.07	3 yr. +
Wide launders, pulp from pump discharge to tube-mill cones, slabs..............	9½	53.55	11.38	35.07	3 yr. +
Wide launders, pulp from pump discharge to tube-mill cones, U-blocks..........	9½	....	....	....	....
Overflow launders from tube-mill diaphragm cones, U-blocks..............	18	9.6	22.4	68.0	2½ yr.
Overflow launders from tube-mill diaphragm cones, bends.................	18	9.6	22.4	68.0	4 mo.
Leaving amalgam tables, U-blocks.......	6	14.2	34.4	51.4	4 yr. +
Return-classifier cones to mill-pulp elevating pumps, U-blocks.................	8½	56.8	26.8	16.4	2½ yr. +
Launder under return-classifier cone discharge, U-blocks....................	10	56.8	26.8	16.4	5 mo.

at the bottom was 2 to 2¼ in., tapering to 1 in. on the sides at the top. The average weight of an 18-in. length was 75 lb. Bend blocks for the turns were made of the same cross-section. Concrete slabs, 2½ in. thick and of various lengths, were also used. The mixture was one part quartzite dump waste crushed to ½-in., one part of the − ¼-in. fine material from the above crushing, 1 part of washed drift sand and 1 part of Portland cement. After concrete was poured into the molds it was allowed to set for two days, then submerged in water for 6 or 7 days, when it was ready for use. The joints between lengths were sealed with quick-setting magnesia cement or with mill blanketing. Cost for U-shaped blocks, including labor, material and installation, was about $0.40 per running foot, which was little more than the cost of 1-in. pitch pine with 3 × 3-in. fillets and much less than the cost of lining with belting. Except at bends the concrete had a very long life (see Tables 25 and 26).

Steel bars set in concrete (*19 JCM 77*). Discarded Osborn-liner bars in 7-ft. lengths were set on edge and laid in cement on the bottom and sides of a 14 × 12-in. launder having triangular wooden fillets nailed in the corners. The lining was from ⅝ in. to 1 in. thick and could be placed and dried for use within 9 hr. Quick drying was accomplished by slowly burning old bags soaked in kerosene on top of the cement lining. Slope was 8.5 per cent; pulp was 74 to 61 per cent. +60-mesh, 14 to 20 per cent. +90-mesh and 12 to 19 per cent. −90-mesh. Life was 4½ years, except at points of division and at the feed intake where replacement was made at the end of 2 years. A similarly lined launder carrying stamp-mill discharge showed little wear after 2 years. The cost of lining 100 ft. of a 12 × 14-in. launder and using 3600 lb. of discarded steel was estimated at $19.50 for labor and material; this was about one-seventh of the cost of new belt lining which was estimated to last only 6 months.

Glass is sometimes used in launders carrying fine material. The smooth surface reduces frictional resistance so that a flatter slope than otherwise can be used. Wear is long but the glass tends to crack.

Resistance of different lining materials to flow. Schmitt (2 *RMP*) says that cement, wood, and rubber belt give practically the same resistance and that canvas belt decreases velocity.

Browning (*29 M & M, 300*) found that a given flow of water carried more angular tailing and pyrite on flat slopes in linoleum-lined and wood launders than in glass-lined ones. The capacity for rounded gravel was greater in every case in the glass-lined launder, probably due to absence of eddy currents and ease of rolling the rounded particles on the glass surface. In general, high velocities cause increased wear on lining.

Splitting the stream cannot be done satisfactorily by taking the laterals off directly; the best method is to use vanes to split into parallel streams and subsequently lead the streams apart.

Pipe launders. Pipes running partially full are sometimes used as launders. Ordinary iron pipe is most common, but wood-stave and vitrified clay pipe are also used. Large-sized iron pipes are sometimes lined with wood. When worn, a new surface is presented by turning the pipe.

At Guanajuato (*95 P 78*) 8-in. pipe was used for transporting −30-mesh, sharp, granular quartz-calcite tailing 5440 ft. The pipe was ⅝-in. thick; bell-and-spigot joints were calked with tarred hemp rope loosely driven; the outside was asphalted for protection against rust; curves were mostly 14-ft. radius; slope was 3½ per cent. for the first 800 ft. and 2¼ per cent. for the remainder. The pulp was reduced to 25 per cent. solids before entering the pipe; occasional increase in thickness caused some deposition of sand; delays due to clogging totaled four hours in 13 months. At full mill capacity, 250 tons per day, the pulp ran 1¾ in. deep, and traversed the pipe in 12 min., equivalent to 7.5 ft. per sec. The pipe could carry 1000 tons per day. No appreciable wear was noticed after 13 months (100,000 tons transported) and the life was estimated at 50 years. At about 500 tons per 24 hr. with 5 : 1 dilution the pipe ran a little less than half full (*95 P 457*). A 12-in. pipe running partially full carried sandy tailing at Homestake (*22 IMM 87*). Joints were made with a loose sleeve coupling held in place by wedges, which permitted turning without shutting down; pipes were turned 120°, so that three different wears were obtained from the same length of pipe by turning twice.

11. Centrifugal pumps

Single-stage centrifugal pumps are limited, in general, to lifts under 50 ft. but in several instances these pumps work against heads of from 60 to 80 ft.

Multi-stage pumps are used for larger lifts. Lifts greater than 50 ft. are rarely encountered in handling mill pulps so that very few multi-stage centrifugal pumps are found in mill work. The open-runner single-suction volute pump (Fig. 32) is most generally used. Material enters through a suction pipe and is delivered to the center of the revolving impeller, whence it is thrown by centrifugal force imparted by the blades to the periphery of the pump chamber and is discharged through the vertical pipe. Pressure to lift the fluid is obtained by the conversion of a large part of the velocity head produced by the rotation of the impeller to pressure head. The direction of motion of the fluid at the instant it leaves the impeller is the resultant of motions tangential to the periphery of the impeller and to the surface of the impeller blade. The volute chamber is designed so that the change from velocity head to pressure head shall take place with the smallest possible loss; thus in Fig. 32 the size of the chamber increases gradually towards the discharge pipe. The head which the pump must develop equals the difference in head between the intake and discharge levels, plus the velocity head required to give motion to the fluid, plus losses in the pump and losses due to friction in the pipe lines. The shape and curvature of the impeller blades are important considerations in design for any given service. Fig. 33, *A*, shows charac-

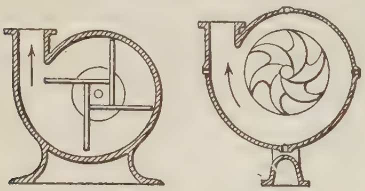


Fig. 32.—Centrifugal pump.

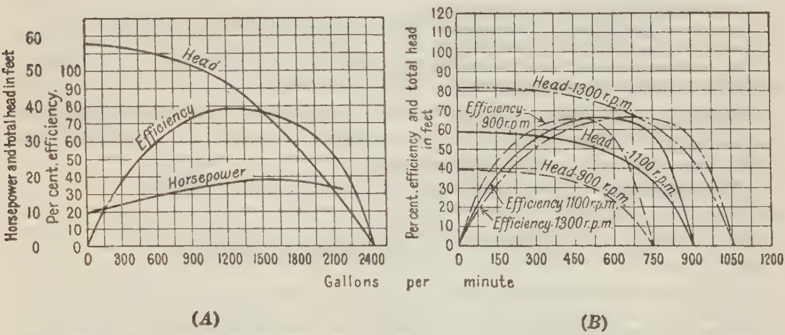


Fig. 33.—Characteristic curves of centrifugal pumps (*Morris Machine Works*).

teristic curves of a single-stage centrifugal pump operating at constant speed. Fig. 33, *B* shows the effect of change in speed. The general shape of the curves is the same for all speeds; it is determined by the design of the pump and especially the inclination and position of the vanes of the impeller. The curves for any given pump explain the changes which occur with changes in the conditions under which the pump operates. The size of a pump is usually designated by the size of the discharge pipe. Table 27 gives sizes and dimensions of one make; Table 28 gives the speed at which a pump should be run to deliver its most economical capacity against any required head.

Impellers may have 4, 6 or more blades; the shells may enclose liners or the shell itself may be designed to take the wear. Impellers and liners are generally made of cast-iron or

Table 27. Single-stage centrifugal pumps. (*Morris Machine Works*)

Number of pump (diam. discharge opening, in.)	Size of pipe flange on suction, inches	Economical capacity in gallons per minute	Power required for each foot of elevation, horse-power	Diameter and face of pulley, inches	Floor space required in inches; without primer	Shipping weight without primer, pounds
1	1¼	30	.025	4× 3¼	12× 17	85
1½	2	70	.058	6× 6	17× 31	175
1¾	2	90	.075	7× 8	21× 32	260
2	3	120	.10	8× 8	23× 37	350
2½	3	180	.15	8× 8	24× 38	360
3	4	260	.22	8× 8	25× 39	415
4	5	470	.30	10×10	29× 41	615
5	6	735	.45	12×12	34× 54	940
6	8	1,050	.59	15×12	37× 55	1,180
8	10	2,000	1.00	20×12	45× 64	2,065
10	12	3,000	1.52	24×12	51× 69	2,610
12	15	4,200	2.00	30×14	63× 71	3,615
12*a*	12	4,200	2.00	20×12	51× 59	2,800
15	18	7,000	3.50	40×15	77× 80	8,250
15*a*	18	7,000	3.50	30×15	60× 68	3,350
18	20	10,000	4.50	40×16	93×103	9,000
18*a*	20	10,000	4.50	30×16	66× 72	5,800
20	22	12,000	5.00	36×20	73× 83	7,000
24*a*	24	15,000	5.50	48×20	90× 98	10,800
24	24	15,000	5.50	48×36	94×137	....

a Refers to low-lift pumps, which are recommended for heads up to 40 ft.

Table 28. Revolution table for centrifugal pumps. (*Morris Machine Works*)

Number	Head in feet = lift + friction head													
	5	10	15	20	25	30	35	40	50	60	70	80	90	100
1	620	840	1020	1150	1260	1370	1470	1570	1760	1900	2060	2180	2320	2450
1½	428	604	739	854	955	1045	1131	1208	1351	1481	1599	1714	1813	1911
1¾	348	491	601	695	777	850	920	982	1099	1205	1301	1394	1475	1554
2	272	384	472	545	607	665	720	773	858	940	1030	1085	1150	1215
2½	272	384	472	545	607	665	720	773	858	940	1030	1085	1150	1215
3	272	384	472	545	607	665	720	773	858	940	1030	1085	1150	1215
4	230	364	447	515	574	630	680	727	812	890	960	1025	1085	1145
5	206	289	354	410	457	500	541	579	647	712	765	817	867	914
6	172	242	295	341	381	418	452	483	540	591	638	683	722	763
8	148	207	250	293	327	359	387	415	464	510	548	586	620	655
10	135	190	233	270	300	329	355	380	425	467	502	538	568	600
12	107	151	185	213	238	261	282	302	337	371	400	436	452	476
12*a*	206	289	354	410	457	500	541	579	647	712	765	817	867	914
15	83	116	142	164	183	201	217	232	259	286	307	328	348	368
15*a*	121	171	209	241	270	295	319	342	382	420	451	483	511	539
18	83	116	142	164	183	201	217	232	259	286	307	328	348	368
18*a*	121	171	209	241	270	295	319	342	382	420	451	483	511	539
20	115	162	197	228	255	280	302	323	361	395	426	456	483	510
24*a*	104	146	178	206	230	253	272	291	326	358	384	412	436	459
24	77	108	132	152	170	187	201	215	241	263	284	304	322	340

a Refers to low-lift pumps.

manganese steel. For special work with hot, acid or corrosive liquids, brass, rubber or other resistant materials are used. The greatest wear is on the impellers and liners and varies with the material being pumped; it varies with the size of particles in the pulp, hardness of the ore, shape of pieces, and, with the dilution; coarse material causes greater wear than fine; very dilute and very thick pulps wear less than those of intermediate density.

Table 29 summarizes data on life of impellers and liners of various sizes of centrifugal pumps at 16 different plants; the short life of the smaller-sized pumps raises a doubt as to their economy. LOST TIME varies from 0.5 per cent. to 5 per cent. of the possible running time; the chief cause is replacement of worn impellers and liners; in larger pumps several hours may be consumed in this work. Where pumps are to run continuously duplicate units should be installed to prevent shut-downs for repairs.

Table 29. Life of impellers and shells or liners in tons solid handled. (Based on data covering 35 pumps at 16 plants) (*a*)

Size of pump, inches	Fine material		Medium material		Coarse material	
	Impellers	Liners or shells	Impellers	Liners or shells	Impellers	Liners or shells
2	....	....	240	480	....	....
2½	....	....	900	1080	....	....
3	2,500–33,000	3,080–33,000	1000–8000	2,000–58,000	....	....
4	7,000–78,000	12,000–250,000	22,000	165,000	....	....
5	30,000	30,000	18,000	36,000	....	....
6	22,000–45,000	90,000–150,000	....	....	....	....
10	....	....	54,000	108,000	....	....
12	....	....	....	....	126,000	1,000,000
16	450,000	3,000,000	....	....	....	....

a Dilution of pulp varied from 60 per cent. water to 95 per cent. water. Lifts varied from 18 ft. to 80 ft. Speeds varied from 350 r.p.m. to 1200 r.p.m.

Drive may be either by belt or direct connection to a motor or steam engine. A direct-connected motor makes a compact, independent unit but is not advisable if the pump is located in a wet place or where there is danger of flooding. Belt drive is preferable in wet stations or where a line-shaft is convenient. If the drive belt is too tight, excessive wear of bearings results; if the pump turns hard, the cause should be sought in the pump itself.

Feed may be either suction or gravity; the latter is better. If suction feed is used, the pump must be primed when starting. Priming is best done by introducing water into the discharge pipe just above the level of the top of the pump; the water should be supplied through a separate water line with a valve. A tee in the discharge line with nipple, elbow and removable plug furnishes manual means of priming. Steam ejectors are sometimes furnished for priming large pumps. Priming is complete and the pump ready to work when the suction line and the pump itself are completely filled with water. Provision of a pet-cock let into the top of the pump allows the escape of any air which may be pocketed there during priming or accumulate during running. The SUCTION PIPE is usually one or two sizes larger than the discharge pipe and should be as straight and free from bends as possible. Bends should be of long radius to decrease friction losses. FOOT VALVES and STRAINERS are usually provided on the suction pipe; the free area through these should be about twice the inside cross-sectional area of the pipe. All joints in the suction pipe should be made carefully to prevent leakage of air, which would reduce or destroy suction.

Stuffing box. Leakage of air into the pump or leakage of fluid from the pump where the shaft passes through the shell is prevented by a stuffing box and gland. Water at a pressure slightly higher than that developed by the pump (about 1 lb. for every ft. of lift) is forced in to the bearing through a small hole tapped in the top and connected to the water line; this keeps gritty or acid material from getting into the bearing and also aids in keeping the packing in the stuffing box from hardening and cutting the shaft. Square flax PACKING treated liberally with oil and graphite and cut in rings to go just once around the shaft is used in the stuffing box. The GLAND should be made just tight enough so that 5 or 10 drops of water per min. leak out; excessive tightening causes heating and wear on the shaft with loss of power.

Suction lift. Table 30 gives the approximate theoretical maximum suction lifts in ft. at 60° F. for various altitudes and fluid densities; actual lifts ordinarily will not exceed one-half the theoretical. Increase in temperature reduces the maximum suction lift, due to increased liberation of vapor at higher temperatures. Table 31 shows the effect on suction lift with increase in temperature; very hot fluids should be fed by gravity.

Table 30. Approximate theoretical suction lifts at 60° F. (*17 CME 629*)

Altitude above sea level	Specific gravity of fluid, thousandths							
	1000	1100	1200	1300	1400	1500	1600	1700
	Maximum suction lift, feet							
Sea level	34	31	28	26	24	23	21	20
1,000	33	30	27	25	23	22	21	19
2,000	32	29	26	24	22	21	20	18
3,000	31	28	25	23	22	20	19	18
4,000	29	27	24	23	21	20	18	17
5,000	28	26	23	22	20	19	18	17
6,000	27	25	22	21	19	18	17	16
7,000	26	24	21	20	19	17	17	16
8,000	25	23	21	19	18	16	16	15
9,000	24	22	20	18	17	16	16	14
10,000	23	21	19	18	17	16	15	14

Table 31. Effect of temperature on suction lift. (*17 CME 629*)

Degrees F.	Approximate theoretical maximum suction lifts at sea level, feet
60	34
80	33
100	32
120	30
140	27
160	23
180	16
200	8

Discharge pipe layout should be carefully made; a level run may cause clogging; bends should be as few as practicable and of long radius; valves should not be placed in pulp lines, since closing may cause solid material to pack so that the line will not clear again when the valve is reopened; stopping the pump may cause similar trouble. Provision of a by-pass to be opened just before stopping the pump will clear the pipe; shutting off the pulp supply and substituting clear water before shutting down is also effective. The by-pass is the best arrangement, as the pipes can then be drained immediately, if the pump stops accidentally.

Velocities of 350 ft. per min. in discharge pipes and 200 ft. per min. in suction pipes will, in general, prove satisfactory.

Size of feed. Centrifugal pumps successfully handle pulps containing coarse particles. They are frequently used to elevate pulps containing up to ¼-in. particles; large pumps used in hydraulicking operations successfully handle gravels containing fair sized rocks.

A 12 × 24-in. pump in one mill was reported as handling 600 tons per 24 hr. of − 1¼-in. + 20-mesh material together with 4500 gal. of water per min.

Efficiency of well-designed pumps doing good work will vary from about 50 per cent. for the smaller sizes to about 75 per cent. for large sizes. These efficiencies are not always attained in practice as the pumps may not be working under the most favorable conditions. In handling pulps, the high velocity attained keeps the solid material in suspension. Possible dilution of certain pulps from the water admitted to the gland may make it desirable to choose some other means of elevation.

Advantages. The centrifugal pump is simple, requires little space and the attendance while running amounts to only a small portion of a man's time, to determine whether it is functioning properly.

Costs. Jones (*52 A 116*) [1915] gives the following figures covering the cost to operate a 4-in. solid-lined Krogh centrifugal pump elevating filter feed 55 ft.

	Cost, dollars	Per cent of total
Liners	482.57	37.97
Runners	126.72	9.97
Housings	81.00	6.37
Bushings	36.00	2.83
Shafts	22.84	1.80
Chain and sprocket	47.65	3.75
Gland	1.89	0.15
Strand	11.25	0.89
Labor	53.12	4.18
Power @ \$8.82 per hp.-month (39 hp.)	408.00	32.09
Total	1271.04	100.00
Wet tons pumped	755,255	
Cost per ton	\$0.00168	

Wilfley centrifugal pump (Fig. 34) has a centrifugal seal where the shaft passes out of the pump casing and thus avoids the need of a stuffing box. The success of the centrifugal seal in protecting the bearings from gritty and hot solutions permits the use of ball bearings on the shaft with consequent saving in wear and power. The pump has no suction; feed enters intake (*a*) thence through a passage to the enclosed-type impeller (*c*). The discharge pipe is screwed into the keeper (*l*), which is raised or lowered by bolt (*m*), when removing the case; the joint between the keeper and the case is made with a ring of square packing. The centrifugal seal consists of a revolving expeller (*d*) having radiating wings similar to an open runner and a stationary member having a projecting groove (*e*). Material leaking by the edge of the expeller encounters the groove (*e*) and is directed towards the wings of the expeller which eject it again. When the pump stops, an automatic check valve closes around the shaft, preventing any leakage. Wearing parts are easily and quickly removable by loosening two nuts which hold the shell (*f*), when the shell can be swung aside on arm (*g*), giving access to the impeller, the follower plate (*h*) and expeller. The joint between the follower plate and the frame is made with a ring of square packing (*i*), set in a groove in the frame; the joint

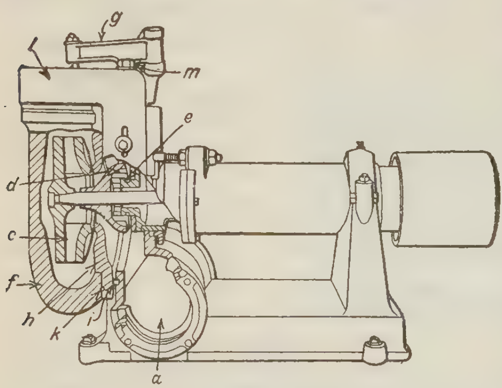


FIG. 34.—Wilfley centrifugal pump.

between the follower plate and the case is made with flat faces and a round rubber gasket (*k*), cemented into a groove in the follower plate.

This pump has been very successful in handling gritty or sandy materials. Tables 32 and 33 give details of capacity and speed of the various sizes.

Table 32. Wilfley centrifugal sand pump

Size of pump, inches	Pipe connections, inches		Normal capacity, gallons per minute	Intake head above shaft for normal capacity, feet	Approximate shipping weight, pounds
	Discharge	Intake			
2	2	4	200	3	660
3	3	5	300	3	900
4	4	6	500	3	1590
6	6	8	950	3	2550

Table 33. Speeds of Wilfley centrifugal sand pumps

Size of pump, inches	Standard pulley, inches		Static head plus friction head, in feet								
	Diameter	Face	20	30	40	50	60	70	80	90	100
2	6	6	990	1130	1255	1380	1500	1605	1720	......	......
3	8	6	820	935	1035	1135	1230	1320	1415	1500	1580
4	10	10	......	745	825	910	985	1060	1130	1195	1260
6	12	12	......	......	725	800	860	905	985	1035	1080

12. Spiral pump

Frenier pump (Fig. 35) consists of a spiral wheel, (*a*) revolving in box (*b*) which contains the fluid to be elevated. Material entering the spiral through the opening (*c*) advances toward the center and is discharged through hollow shaft (*d*) and the stuffing box and gland connection (*e*) to discharge pipe (*f*). The weighted lever (*g*) takes up the thrust of the delivery pipe. With each revolution of the wheel, fluid is scooped into the spiral and air enters during the remainder of the revolution so that a portion of each succeeding turn of the spiral is filled with the fluid to be lifted. The lifting force results from the difference between the sum of the hydrostatic heads resulting from the fluid in the spiral and the pressure induced as the fluid enters the spiral, and the head of aerated liquid in the discharge pipe. The lifting force is independent of the speed of revolution; it can be increased only by an increase in the number of turns of the spiral or diameter of the wheel. The makers recommend 20 r.p.m. as the most satisfactory speed. The radial distance between suc-

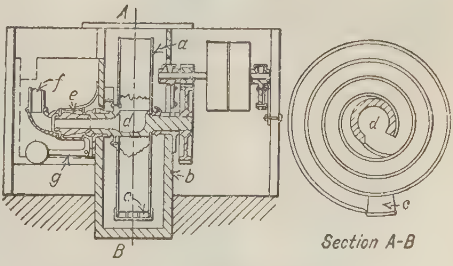


FIG. 35.—Frenier pump.

cessive turns of the spiral is generally 2½ in. Table 34 shows maximum capacities and lifts for the various standard sizes of pumps.

Table 34. Sizes and capacities of Frenier pumps. (*Power and Mining Mach. Co.*)

Size of wheel, inches		Capacity, gallons per hour, maximum	Maximum lift in feet	Shipping weight, lb.
Thickness	Diameter			
6	44	3000	12	1100
6	48	3200	16	1200
6	54	3500	22	1400
8	44	4000	12	1200
8	48	4200	16	1300
8	54	4500	22	1500
10	44	5000	12	1400
10	48	5200	16	1500
10	54	5500	22	1670

The Frenier pump is suitable for elevating solutions, sand or slime pulps to heights ordinarily not greater than 10 or 12 ft. but in several instances lifts as great as 16- and 20-ft. have been made. The pump will not operate satisfactorily under changing conditions. The level of fluid in the FEED box should be 7 in. below the shaft. If the pump is overloaded, it will stop working; it does not work well when under-fed; frequently it is fed at a rate slightly in excess of capacity and an overflow is provided on the feed box; the material overflowing is then handled by an auxiliary air-lift with float-control valve. The greatest WEAR takes place between the rotating tube at the end of the hollow shaft and at the bend of the discharge pipe. For most efficient operation the discharge pipe should be vertical and have a vertical discharge opening with free fall to allow the escape of air. The construction is simple and little attendance is required.

13. Diaphragm pump

DIAPHRAGM PUMP (Fig. 36) (*18 JCM 95*) consists of a hollow chamber with a valve in the bottom and a movable diaphragm, also valved, which closes the top of the chamber. The diaphragm is given an up and down motion by a connecting rod from an eccentric on a pulley- or gear-driven shaft. To start, the pump is primed by completely filling the chamber and the space above the diaphragm with water. On the up stroke of the diaphragm the lower valve opens and admits fluid to the chamber from the suction pipe; displacement of the fluid in the space above the diaphragm causes some of it to overflow the lip shown. At end of the up stroke the lower valve closes and on the down stroke the upper valve opens and allows fluid to flow from the underside of the diaphragm to the space above it. At end of the down stroke the lower valve again closes and the cycle is repeated. The DIAPHRAGM is made of a rubber sheet about ⅜ in. thick with a single ply of loosely-woven duck imbedded in it. The working surfaces of valves and seats are best made of high-grade rubber; this wears well and gives tight closure even when small

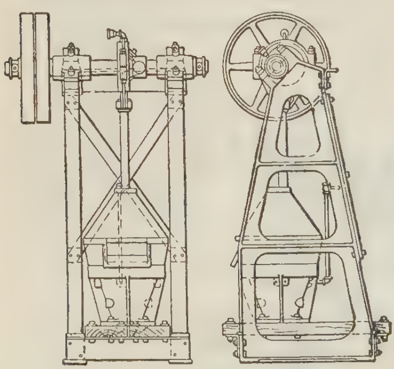


FIG. 36.—Diaphragm pump.

chips get between the valve and seat. The floating type of valve allows complete washing of the seat at each opening.

Height of lift of diaphragm pumps is limited by atmospheric pressure and also by the strength of the diaphragm; lifts should never exceed half the theoretical lifts of Table 30; ordinarily lifts are less than 8 ft. SPEED may be from 15 to 100 strokes per minute but the most satisfactory operation is with less than 40 strokes per min.; higher speeds cause splashing. STROKE may be lengthened as much as the diaphragm will stand. CAPACITY may be varied by changing the speed or length of stroke or it may be controlled by admission of small quantities of air into the chamber through a pipe with a needle valve.

The diaphragm pump has found its greatest application in elevating and controlling the underflow from thickeners. It is simple to operate and requires little attendance and repair. Pulps containing as much as 50 per cent. solids are easily handled.

At HOLLINGER (*57 A 150*) a No. 4 Goulds diaphragm pump with 3-in. stroke performed as follows:

No. strokes per min.	Vol. per stroke, cu. ft.	Sp. gr. of pulp	Per cent. solids	Tons solids pumped per day	Per cent. increase in speed	Per cent. increase in volume per stroke	Per cent. increase in tonnage
14.5	0.139	1.54	54.5	76.3			
23.0	0.148	1.48	50.5	114.5	58.5	6.5	50.0

The life of a diaphragm was more than one year when operating at from 14 to 18 strokes per min. while rubber valves and seats showed no perceptible wear in six months.

14. Tailing wheel

Tailing wheel (Fig. 37) consists of large rotating wheel with projecting buckets on one or both sides of the rim. When at the bottom the buckets scoop up pulp from a pit and at the top deliver it into a receiving trough or launder. The old type wheels were constructed of wood but newer ones are of steel, either rigid or of bicycle-spoke construction. Wood is best, even on steel wheels, for lining the rim flanges and the vanes forming the buckets. Tailing wheels have been most used on the Witwatersrand to elevate stamp-mill tailing. Wheels up to 60-ft. diameter have been made. The net lift, *i.e.*, the distance between the level of the feed launder and that of the receiving launder, is less than the diameter of the wheel to an extent determined by the inclination of the bucket vanes to the wheel radius. Julian and Smart give the RATIO OF DIAMETER TO NET LIFT is about 1.3 : 1 and recommend an angle of 65° between the radius and the bucket vanes. Usual SPEED is about one-third the critical speed at which centrifugal force would prevent bucket discharge. Table 35 gives critical and practical speeds for wheels of various diameter. The time required for buckets to discharge is given by Wood and Laschinger (*77 J 482*) as not less than 3 sec. on 40-ft. wheels or 5 sec. on 60-ft. wheels. DRIVE may be by belt and pulley mounted on the shaft or by pinions and gear, or by a sheave built into or around the outside of the wheel itself. Manila-rope drive on sheaves has been most favored.

The first COST of tailing wheels is high; large wheels require heavy construction and massive supports for the shaft bearings. Maintenance and repair costs are low. Reliability and durability are high. POWER EFFICIENCY is usually from 40 to 50 per cent. Centrifugal pumps and bucket elevators cost less, require less space, and are generally more desirable than tailing wheels.

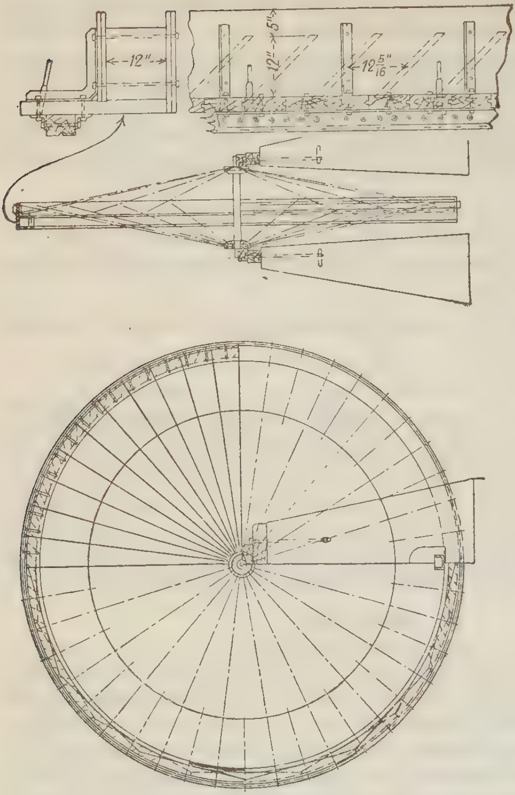


FIG. 37.—Tailing wheel.

At CALUMET AND HECLA plants (*90 J 218, 12 MH 308*) five tailing wheels, one 40-ft. diam., three 50-ft. and one 60-ft. elevate coarse mill tailing to launders running out over Torch Lake. The 60-ft. wheel at HECLA weighs 500 tons with base plates and columns; it is bicycle-type with a hollow shaft, 32 in. outside and 16-in. inside diameter by 27 ft. long; 4-in. spokes radiate from two 10-ton hubs to the outer rim upon which are mounted two spur gears with 12-in. face and 4.7-in. pitch. The buckets are 3 ft. wide × 4 ft. 6 in. long and are mounted on each side of the rim. A 37-in. pinion on the counter-shaft is driven by ropes from a 700-hp. motor. The speed is about 4 r.p.m., equivalent to about 12 ft. per sec. at the inside edge of the buckets. The buckets dip into a trough under the wheel and deliver to two troughs above, making a total lift of about 50 ft. CAPACITY is about 5500 gal. per revolution. The KENNEDY MINING AND MILLING Co., California, used four 56-ft. tailing wheels in tandem to elevate stamp-mill tailing to flumes (*97 J 517*). A wooden flume delivered to the first wheel about 1500 ft. from the mill; the second wheel was 100 ft. distant, the third, 820 ft., and the fourth, 200 ft. The LIFT was about 35 ft. in each

Table 35. Critical and actual speeds of tailing wheels

Diameter of wheel, feet	Critical speed		Actual speeds (⅓ critical speed)	
	Revolutions per minute	Peripheral velocity, feet per minute	Revolutions per minute	Peripheral velocity, feet per minute
10	24.218	761	8.07	254
20	17.125	1076	5.71	359
30	13.983	1318	4.66	439
40	12.110	1522	4.04	507
50	10.830	1701	3.61	567
60	9.887	1863	3.29	621
70	9.154	2013	3.05	671

wheel; the flumes were set on 2½ per cent. grade. The mill capacity was about 400 tons per day. At the old Belmont Mill, Millers, Nev. (*106 P 282*), stamp-mill pulp was lifted 48 ft. to classifiers by a tailing wheel. Operation was reliable and upkeep low; 87,952 tons were elevated in one year at a cost, including classifying, of 3.6¢ per ton. The Tonopah Mill, Millers, Nev. (*106 P 282*), had two wheels. No. 1 was 30-ft. diameter and elevated 500 tons of dry ore and 3500 tons of solution per day with a 7½-hp. motor. First cost was high but the wheels were efficient and maintenance was low compared to bucket-elevators. The cost of elevating and separating was 8¢ per ton. At the City and Suburban Mine, Witwatersrand (*77 J 481*) a 25-ft. wheel lifted 5549 lb. of tailing pulp per min. 19 ft. 1 in. The theoretical power requirement was 3.205 hp.; the actual power, including two intermediate countershafts was 6.635 hp.; total efficiency, 48.35 per cent. At Henry Nourse mine, Witwatersrand (Julian & Smart) a 60-ft. wheel was operated at 45 per cent. efficiency and consumed 15 hp. The drive sheave was 55-ft. diameter, driven at 3 r.p.m. by 1¾-in. manila rope from an 11-ft. sheave on the countershaft. The wheel axle was 20 ft. × 14 in. with bearings 2 ft. 6 in. × 12-in. diameter. The life of the rope was 3.5 years (*77 J 481*). At the Bullfinch Proprietary mill, W. A. (*13 CME 331*) a tailing wheel elevates the product of stamp mills to hydraulic classifiers. Diameter of wheel, 40 ft.; 112 @ 720-cu. in. buckets; peripheral speed, 400 ft. per min.; driven by manila rope, 6 in. in circumference, which lasts 60 days; power consumption, 7.3 hp.; about 2500 tons of ore and solution lifted per day.

15. Air-lift

The air-lift (Fig. 38) is used to elevate solutions, slime and, in certain cases, sand pulps. It consists of a delivery pipe (*a*) submerged for a part of its length in well or sump (*b*); compressed air is introduced into the delivery pipe at the foot-piece (*c*) through an air pipe (*d*). The air causes a decrease in the density of the fluid in the delivery pipe and consequent elevation by hydrostatic pressure and discharge at (*e*). Efficient operation depends on proper design of the lift and regulation of the air pressure. The known factors in any particular case are the amount of fluid to be elevated and the height to which it must be raised; the important items to be determined are the volume of air required, the pressure at which it must be delivered, the size of delivery and air pipes and the depth to which the delivery pipe must be submerged below the normal level of the liquid in the well or sump.

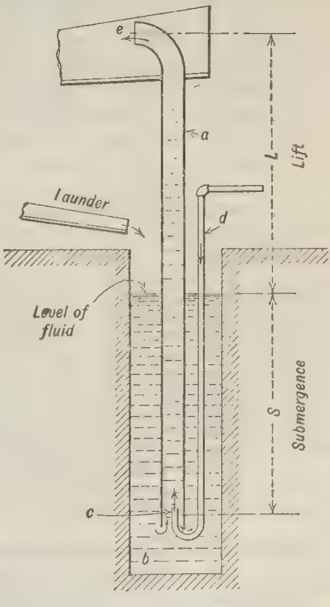


Fig. 38.—General arrangement of air-lift.

The depth of submergence (S) (Fig. 38) for most efficient operation at any lift (L) cannot be exactly determined in advance; if the air-lift is to handle large quantities of material so that a slight improvement in efficiency will cause considerable saving, provision should be made to vary submergence after operation is started. The figures in Table 36, based on average practice, may be used for calculation in design.

Air pressure. The pressure at which air must be delivered to the foot-piece is governed solely by the depth of submergence (S) and should be just sufficient to force air into the delivery pipe; $P_1 = 0.434gS + P$, where P is the atmospheric pressure in lb. per sq. in., P_1 is absolute pressure of air entering the foot-

piece and g the sp. gr. of the fluid to be elevated. A slightly higher pressure will be required in starting up. The pressures at which air is supplied at the foot-piece in operating lifts conforms closely to the theoretical pressures thus calculated. Excessive pressure causes air to escape from the bottom of the delivery pipe into the well; this reduces the density of the fluid therein and may stop the lift; at any rate, to keep the lift in operation under such circumstances will require a large excess in volume of air and so reduce efficiency.

Table 36. Submergences for air-lifts. (*Sullivan Machinery Co.*)

Lift in feet (L)	Submergence, per cent. $\left(\frac{100S}{S+L}\right)$
Up to 50	70–66
50–100	66–55
100–200	55–50
200–300	50–43
300–400	43–40
400–500	40–33

Volume of free air required may be calculated by the following formula: $V_a = 0.8\ L/C \log [(S + 34)/34]$ (**1**) in which V_a = cu. ft. of free air per gallon of fluid (at normal pressure, P_a, 14.7 lb. per sq. in. and 60° F.) and C = a constant varying with height of lift, L. Values of C are given in Table 37. Results obtained closely approximate actual values.

The size of delivery pipe required can be calculated from the amount of fluid to be delivered in a given time and the volume of air used during that time. Certain values for velocity have been found satisfactory in practice and by taking pipe of a size to keep within these limits losses due to friction will be kept within reasonable amounts. The VELOCITY OF THE FLUID in the discharge pipe increases as the top is neared due to expansion of the contained air. The velocity at the foot-piece may be taken at from 4 to 8 ft. per sec. and the velocity at discharge should be kept under 20 to 25 ft. per sec. Velocities should never be so low that solid material will settle out of the rising liquid or less than the velocity at which the largest air bubbles will rise in the fluid.

Table 37. Values of C in equation (1)

Lift in feet	C
10–60	243
61–200	233
201–500	216
501–650	185
651–750	156

Let V_1 = cu. ft. of air per min. at pressure P_1; P_1 = absolute pressure of air at the foot-piece, lb. per sq. in.; Q = cu. ft. of liquid per min.; v_1 = velocity in ft. per min. at foot-piece (assumed between 250 to 450); A = internal area of the delivery pipe in sq. ft.; q = gallons of liquid per min. Then $Q = q/7.48$ and $V_1 = qP_aV_a/P_1$. The total volume of air and liquid at the foot-piece equals $Q + V_1$ cu. ft. per min. and the required area of the delivery pipe = $A = (Q + V_1)/v_1$. Standard pipe nearest this size may be used.

The velocity of the air-liquid mixture at the discharge should be calculated to insure that it is not greater than 20 to 25 ft. per sec. Expansion of the air on rising may be considered isothermal since the air is in small bubbles surrounded by water.

Some authorities allow capacities of 10, 12 or 15 gallons of fluid per min. per sq. in. of cross-section of the delivery pipe and calculate the size of pipe on this allowance, but it is wise to check by calculating actual velocities at the foot-piece and discharge.

Size of air pipe can be found by reckoning air velocity at 20 to 30 ft. per sec.; velocities up to 70 ft. per sec. are sometimes used but friction loss is higher.

Foot-pieces are of various designs; the simplest is merely the projection of the air pipe into the lower end of the delivery pipe (Fig. 38). Other forms are shown in Figs. 39 and 40. The air should be dispersed as completely as practicable; large air bubbles rise faster than small with consequent greater slippage of the liquid past the bubbles. To reduce SLIPPAGE to a minimum, the more complicated foot-pieces introduce the air through many small holes, thus obtaining smaller bubbles and better dispersion.

Tests have shown that the efficiency of the system is increased appreciably by use of properly designed foot-pieces. At the East Rand Proprietary Mines, Ltd. (*105 EL 26*) efficiencies were 17.7 per cent. and 27.5 per cent. with a foot-piece with air admitted through a single 4-in. opening compared with efficiencies of 37.15 per cent. and 30.55 per cent. with the improved style shown in Fig. 40. These tests were made on lifts elevating slime mill pulps weighing 63.3 lb. per cu. ft. about 30 ft. A slight flare at the lower end of the foot-piece is advantageous in furnishing a gradual increase in velocity of entering fluid.

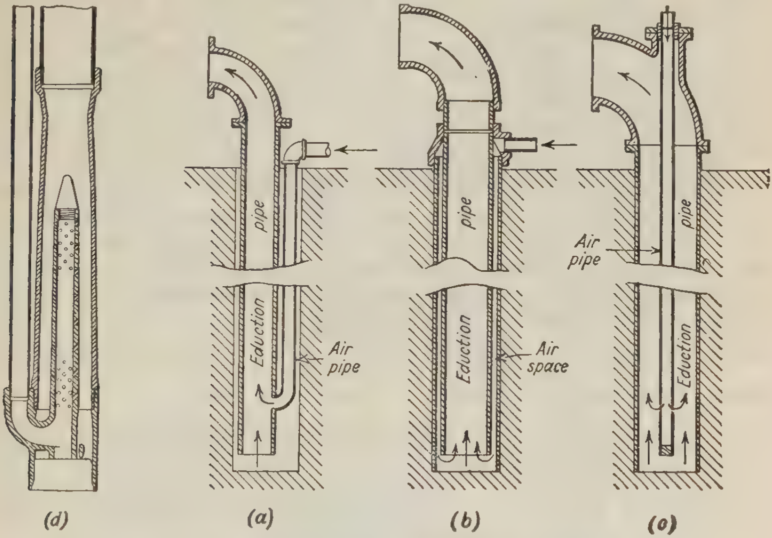
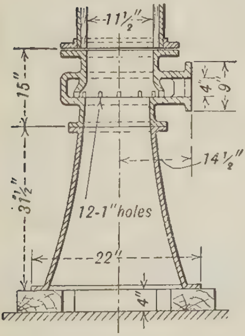


Fig. 39.—Foot-pieces for air-lifts (*Sullivan Machinery Co.*).



Diameter	Inches	
	No. 1	No. 2
Outside top flange	21	23
Inside pipe	11½	13½
Outside flange below air inlet	19	20
Inside bottom of bell	22	30

Fig. 40.—Foot-piece for sand and slime pulps (*105 EL 26*).

Discharge pipe is sometimes increased in size upward on long lifts to keep down the velocity. The advantages of changing to a larger size with ordinary reducers is questionable, as a sudden change of velocity with consequent eddy losses takes place; with a long gradual reducer eddy losses are cut down but gradual increase in pipe diameter is impractical except possibly for low lifts in agitating and aerating machines such as Pachuca or Parral tanks. The

best arrangement for discharge end is a vertical opening with a cusp-shaped umbrella (Fig. 41) to deflect the stream downward. The discharge may be turned horizontally by use of a long-radius elbow but it will not be as efficient; if the discharge pipe is turned and run any considerable distance horizontally allowance must be made for further losses.

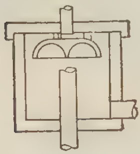


Fig. 41.—Umbrella top for air-lift.

Losses in an air-lift are friction in the pipe lines, slippage of fluid past rising air bubbles and velocity head lost at the discharge. Friction losses can be kept within reasonable limits and slippage can be reduced by increased dispersion of air. Ordinarily the velocity head lost is unavoidable on account of the necessity to keep the solid in suspension; some saving could be made by the use of increasing-diameter pipe, but this is generally impracticable.

Efficiency of the air-lift may be expressed as the ratio (multiplied by 100) of the theoretical horse-power needed to raise the liquid to the air horse-power required for adiabatic compression, or the indicated horsepower of the air-compressor cylinders, or the input horse-power to the compressor as determined from steam-indicator cards, or the motor power consumption. Water horse-power $= 62.5QgL/33{,}000$. Air horse-power with adiabatic compression $= \dfrac{144PVn}{33{,}000(n-1)}\left[\left(\dfrac{P_1}{P}\right)^{\frac{n-1}{n}} - 1\right]$, where $n = 1.406$ for single-stage adiabatic compression.

In making comparisons of air-lift efficiency with the efficiency of other systems, the work of the air-lift should not be condemned because of an inefficient compressor plant nor should the lost work occurring in the compression of the air be disregarded.

Efficiencies for good work in practice vary between 35 and 40 per cent.; instances of efficiencies up to 45 per cent. are claimed.

Advantages. The air-lift is easily constructed; lack of moving parts reduces wear, expensive repairs and shut-downs; attendance for the lift itself is insignificant but a proportionate part of compressor charges must be borne by it. Where aeration and agitation of the fluid to be elevated are desired it is especially suitable. Flexibility can be obtained by provision of a valve on the air line operated by a float on the surface of the liquid in the sump so that the air supply can be regulated to the amount to be lifted. When large variations occur for considerable lengths of time lift columns of various sizes may be provided so as to be easily interchanged. An air-lift can generally be easily cleaned out by shutting off the discharge and turning on the air at as high a pressure as possible thus forcing out and mixing any accumulations in the pipe lines; these will be removed readily on again opening the discharge. The first cost ordinarily will be less than for other devices. The greater power as compared to other systems may be compensated by savings in attendance and repairs. Air-lifts have been successfully applied to the elevation of sand and slime pulps. Where acid or other corrosive fluids are to be elevated the air-lift has a distinct advantage over pumps.

Disadvantages. Proper design is essential for operating efficiency. It is necessary to provide a sump or well of considerable depth; great depths can be avoided by compounding the lift, making it a series of short lifts, but this complicates and increases the piping. When compressed air is not already available, air-lifts would be of doubtful advantage unless large volumes were to be handled.

Application of air-lift to mill work is generally limited to lifts under 20 ft., due to the depth of submergence necessary. In such cases, especially where comparatively small tonnages are handled, little attention has been given to the refinements of design necessary for efficient operation, as the total power consumed is small. However, close attention to design and efficient operation have resulted in the installation of several plants for handling large tonnages of mill tailing at considerable saving over elevators, centrifugal pumps or tailing wheels. At Chino Copper Co. (*112 J 806*) an air-lift to elevate 12,000 dry tons per 24 hr. of mill tailing in a pulp containing about 15 per cent. of solids was installed in preference to a bucket-elevator system because of lower first cost and an estimated

operating cost of 72 per cent. of that with bucket elevators. Actual operation showed that operating costs were about 50 per cent. of those for bucket elevators. The general arrangement is shown in Fig. 42. Six well pipes were provided in a single shaft, with gates arranged to deflect the incoming pulp-stream to whichever one was being used. Three sizes of lift columns of standard, wrought-iron, flange-connected pipe were made as shown in Fig. 43 to be used with varying capacities and to determine which gave the highest efficiency. Details of the foot-piece are shown in Fig. 44, A. A traveling crane on a steel framework was installed to facilitate changing the lift columns; concrete umbrella deflectors on a mov-

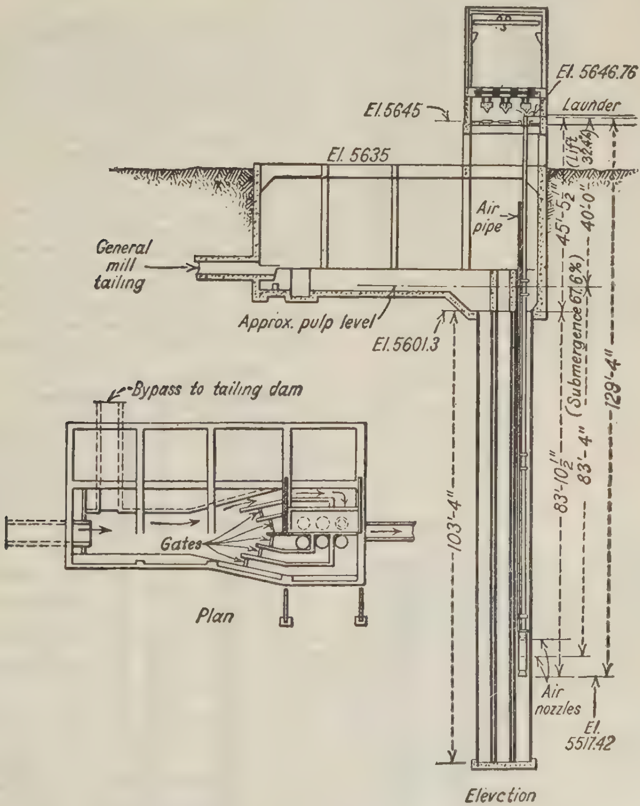


FIG. 42.—Arrangement of tailing air-lift at Chino Copper Co.

able carriage over the discharge ends of the columns caused the rising pulp to drop into a concrete box from which it flowed away in a launder. Air was furnished from the main power house, 530 ft. distant, by two Ingersoll-Rand, Imperial-type XPV-4 steam-driven compressors with compound steam cylinders 13- and 29-in. diameters and duplex single-stage air cylinders 22-in. diam. by 20-in. stroke. Special oil-pressure governors controlled the speed of the compressors so that with decrease of pressure or lowering of pulp in the well the governor caused the compressor to slow down until the wells filled again with pulp and the reverse action took place. Table 38 presents operating details for sixteen months of 1919 and 1920. For cost data see Sec. 23, Art. 3. The air-lift proved very successful in automatically taking care of fluctuations in feed rate, without change of lift-columns. Although ample provision was made for reserve wells and columns in case of clogging it was found that the lifts could free themselves and start up even after 32 ft. of sand had settled in the pit.

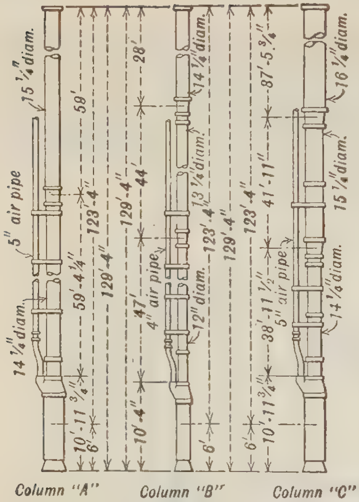


FIG. 43.—Lift columns for air-lift at Chino Copper Co.

Fig. 44, *B*, shows a foot-piece designed by Ingersoll-Rand to improve the efficiency of the CHINO lift. This foot-piece was attached to 43 ft. of standard 14-in. O. D. pipe followed by 76 ft. to the discharge of special welded pipe increasing uniformly from 14 to 16 in. outside diameter. With this column it was expected that power consumption would be cut 5 to 10 per cent. with a corresponding increase in efficiency of from 2 to 4 units per cent.

At another large copper mine in the southwest (*105 J 1177*), three air-lifts replaced three 10-in. centrifugal pumps direct-connected to 150-hp. motors to lift 7000 gal. per min. of tailing having 4 or 5 parts of water to 1 of solids. The air-lifts consisted of 10-in. discharge pipes in 20-in. wood stave pipes; a 40-hp. motor drove the compressor for each. Operating costs and repairs on the air lifts were a small fraction of those when pumps were used since excessive wear of sand in the pumps reduced the life of runners and liners to about 4 days, continuous running. At the Angelo and Cason mills of EAST RAND PROPRIETARY MINES, LTD., tests were made to determine best operating conditions. (*105 EL 26*.) The foot-piece used is shown in Fig. 40. The lower 35.4 ft. of discharge pipe

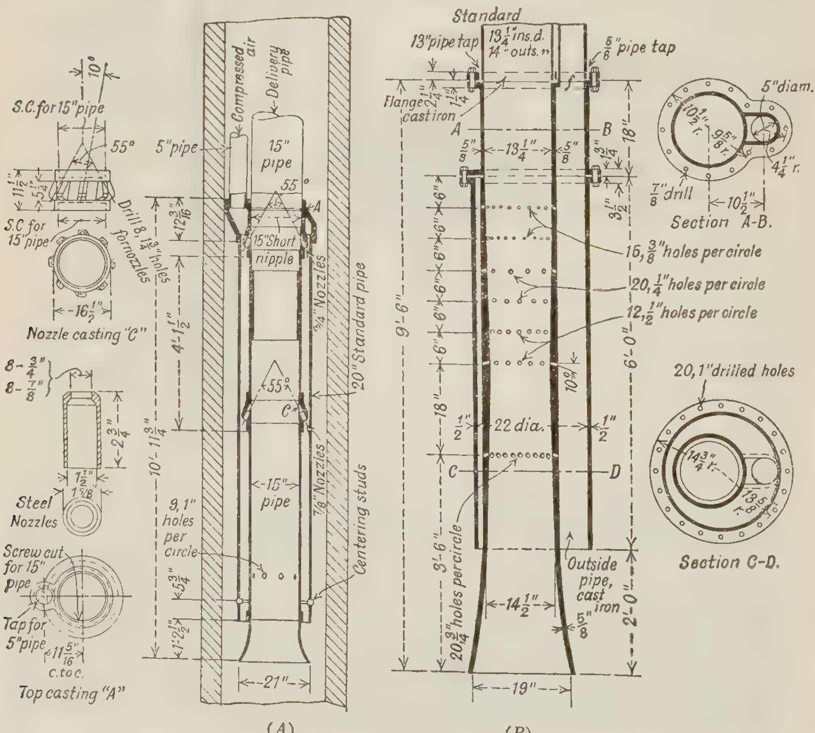


FIG. 44.—Foot-pieces for air-lift at Chino Copper Co.

was lined with wood, thus reducing the internal diameter of pipe on No. 1 from 14- to 11½-in. and on No. 2 from 16- to 13 ½-in. Table 39 gives results of some of the tests on slimes and sands. The slime pulp weighed 63.3 lb. per cu. ft. and the sand, 64.56 lb. per cu. ft. Columns 1, 2 and 3 show the effect of submergence on efficiency. Plotting the results of these and other tests gave a curve indicating a submergence of 1.83 to 1 for a maximum efficiency of 36.2 per cent. Column 5 on Cason slimes shows higher efficiency than this, but was attained with an increased amount of pulp.

Table 38. Operations of air-lift at Chino Copper Co.

Elevation above sea-level about	5600 ft.		
Height of lift	40 ft.		
Submergence	83 ft. 4 in.		
Best submergence (from 80 tests)	67.6 per cent.		
Averages	Month of highest efficiency	Month of lowest efficiency	Average of 16 mo.
Wet tons per day	36,445	18,700	31,723
Dry tons per day	5473	2805	4758
Revolutions per minute of compressors	86	66	85.7
Wet tons per minute	25.3	13.0	22.0
Dry tons per minute	3.8	1.94	3.30
Cubic feet of piston displacement per minute	1504.1	1154.3	1498.9
Actual air, cubic feet per minute	1240	952	1242
Gallons of pulp per minute	5514	2830	4793
Cubic feet of actual air per gallon	0.225	0.336	0.2595
Indicated horsepower in steam cylinders	150	115	150.2
Auxiliary horsepower (10 per cent. of above)	15.0	11.5	15.0
Total horsepower	165	126.5	165.2
Water horsepower	61.3	31.4	53.3
Efficiency based on indicated horsepower in steam cylinders	40.8	27.3	35.5
Efficiency based on total horsepower	37.2	24.8	32.2

Table 39. Tests on air-lifts at East Rand Proprietary

	1	2	3	4	5
Material elevated	Angelo slimes	Angelo slimes	Angelo slimes	Cason sands	Cason slimes
Foot-piece	No. 2	No. 2	No. 2	No. 1	No. 2
Submergence, ft.	52.528	45.696	39.895	78.17	37.54
Lift, ft.	26.8	33.6	39.5	43.0	17.33
Submergence : lift	1.96 : 1	1.36 : 1	1.01 : 1	1.817 : 1	2.166 : 1
Gage pressure	24	21.5	18	34.5	14.785
Free air per minute, cu. ft.	822.5	1410	2820	892.5	854.57
Free air per cubic foot of pulp	2.28	3.91	7.83	2.418	1.871
Cubic feet of pulp per minute	360	360	360	369	456.7
Throat velocity, feet per second	6.03	6.03	6.03	8.526	7.612
Theoretical horsepower	18.49	23.18	27.25	31.05	15.182
Horsepower per cubic foot of free air per minute	0.068	0.063	0.055	0.088	0.04775
Air horsepower (adiabatic)	55.93	88.83	155.6	78.54	40.706
Efficiency	33.1	26.1	17.5	39.5	37.206

16. Feeders

Feeders are necessary whenever it is desired to send a uniform stream of dry or moist ore to any kind of machine, since dry or moist ore, whether

coarse or fine, will not flow uniformly from a reservoir of any kind through a gate except when regulated by some type of mechanical feeding mechanism. The requirements of a satisfactory feeder are: (*a*) It must be positive (*b*) once set for a given rate it must deliver at that rate irrespective of the amount of ore ahead of it. (*c*) It must be readily subject to adjustment to vary its delivery rate. (*d*) It must start under load, and stop without spill. (*e*) It should be adapted to the size of material to be handled. The commonest types of feeders are: (1) traveling bands of articulated pans, called, APRON FEEDERS, or of belting, called BELT FEEDERS; (2) revolving pulleys or rollers with smooth or irregular surfaces, called ROLL OR PULLEY OR ROTARY FEEDERS; (3) shaking or reciprocating plates; (4) plungers; (5) screws; (6) revolving disks. Movable grizzlies (Sec. 5, Art. 4) are used for coarse-ore feeding and special forms of feeders are used with cylinder mills (Sec. 4).

Apron feeder (Fig. 45) is used for coarse ore. It consists of a short pan conveyor (Art. 2) set underneath the bin or hopper to be discharged in such a way that a part of the weight of the filling rests on the carrying surface.

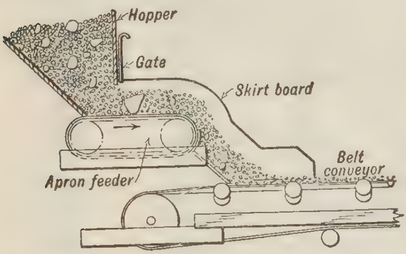


FIG. 45.—Apron feeder delivering to belt conveyor.

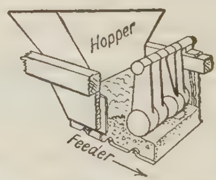


FIG. 46.—Swing-hammer feed regulator.

This surface should be sufficiently uneven to make the feeder positive. If the lumps are large and there is considerable pressure of material above the feeder tending to cause the material to pack, the pans should be deeply corrugated, but if the material is fine or does not tend to bridge at the hopper mouth, a smooth surface may be used. If desired, the bin gate can be eliminated and replaced by a swing-hammer regulator (Fig. 46). This device, as shown in the sketch, automatically adjusts itself to the passage of large lumps, while acting in general to maintain a layer of feed of even thickness on the apron. Heavy chains, steel rail and the like are used as substitutes for the swing hammer. A disadvantage of this device is the fact that if the swinging part breaks for any reason the broken steel is apt to go into the crusher. A device to prevent this contingency (*112 J 660*) consists of, say, three steel rods bolted at several places to the swinging part and fastened to the support; these will hold the parts of a broken hammer until it is discovered and removed.

Speed of apron FEEDERS ranges between 5 and 20 ft. per min. The usual widths are 24, 30, 36, 42, 48 and 60 in. and the distance between sprocket centers 36 to 42 in.

Capacity depends upon rate of travel, width, and size of material. The thickness of the layer must be at least as great as the size of the maximum lump, if a fixed slide gate is used on the bin; but if a swing-hammer regulator or its equivalent is used, the layer may be thinner. Tons per hour are approximately equal $3wts$, where w, t and s are width of the feeder in ft., thickness of the layer in ft. and speed in ft. per min. respectively. This assumes 20 cu. ft. of broken material as piled on the conveyor to weigh one ton, which is equivalent to an allowance of 56 per cent. voids.

Pan conveyors are used in place of apron feeders when it is desired to both transport and feed the material (see Fig. 47). At United Comstock a 48-in. pan conveyor with wood-cushioned pans was used to feed from the coarse-ore bin. It was driven at 9 to 15 ft.

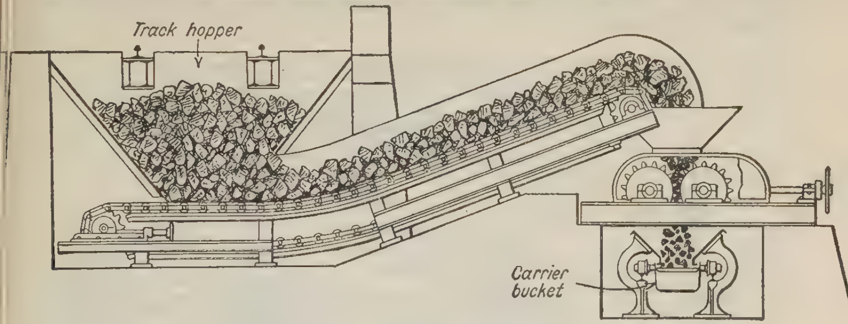


Fig. 47.—Pan-conveyor feeder from bin to crusher.

per min. from the head shaft by means of a direct-connected motor with double-gear speed reducer (*114 J 848*). This conveyor both fed and elevated to a grizzly preceding the primary crusher. At Ohio Copper Co. (*99 J 749*) stock apron feeders used for coarse ore worked well but the plates bent in time. At Inspiration (*121 J 726*) special feeders with manganese-steel plates (platens) of the form shown in Fig. 48 were used to feed run-of-mine ore from bins to the primary breakers. Instead of having rollers on the link pins to run on a track, the rollers were stationary and supported the links on the loaded run and the platen flange on the return. The platen width was 4 ft.; width between skirt plates, 42 in.; length between sprocket centers, 42 ft. 6 in.; maximum thickness of ore stream, 2 ft.; maximum load, 7 cu. ft. per ft. of length (= 0.35 ton); average maximum load delivered, 300 tons per hr., including many stops for picking waste and to prevent overloading the crusher. Motor input at 20 ft. per min., 5.36 hp. empty and 7.50 hp. loaded; at 10 ft. per min., 3.43 and 4.29 hp. respectively. Short feeders (7 ft. 6 in.), the same width drew 3.21 hp. empty and 3.75 hp. loaded when running 40 ft. per min. and fed about 12 tons per min.

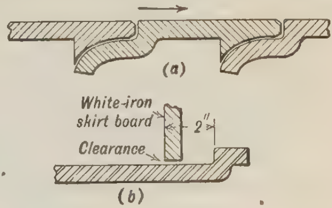


Fig. 48.—Transverse (*a*) and longitudinal (*b*) sections of manganese-steel platens on apron feeders at Inspiration.

Belt feeders are essentially short belt conveyors so placed under an inclined chute that they control the discharge of material therefrom. The method of placing relieves them from any part of the static load of the material in the bin. The head pulley must be placed so that the surface of the material at rest in the chute falls behind the vertical plane through the pulley shaft. Belt feeders frequently and properly replace apron conveyors for fine ore but are unsatisfactory for coarse ore since uneven loading by heavy lumps deforms them so that material spills, and because wear is excessive. Occasionally the belt surface is armored as shown in Fig. 49 (*111 J 63*). The material list for the feeder pictured follows:

Six common flat boxes, 1$^{15}/_{16}$-in. 12 bolts ½ × 7½-in. for same. 12 ½-in. mal. washers, for same. 1 shaft 1$^{15}/_{16}$ in. × 3 ft. 2 in.; keyseated for 10 × 15 pulley and ratchet wheel. 1 shaft 1$^{15}/_{16}$ in. × 2 ft. 6 in.; keyseated for 10 × 15 pulley. 1 shaft 1$^{15}/_{16}$ in. × 3 ft. 4 in.; keyseated for 14 × 4½-in. pulley and crank disk. 6 split safety collars, 1$^{15}/_{16}$-in. 9 ft. 3 in. of 4-ply rubber belt, armored as shown. 2 c. i. solid pulleys 10 × 15 in., 1$^{15}/_{16}$-in. bore. 1 c.i. solid pulley 14 × 4½ in., 1$^{15}/_{16}$-in. bore. 1 ratchet wheel (15½ in. outside diam.,

face 2½ in. ±), with arm; $1\frac{15}{16}$-in.-bore. 1 crank disk, 15 in. diameter, face 2 in. ±; $1\frac{15}{16}$-in. bore as shown. 1 crank pin $\frac{15}{16}$ in. × 4 in. ±. 1 connecting rod with ends, as shown. 1 pin, $\frac{15}{16}$ in. × 6 in. ±, with washers and keys. 1 pawl, 2 in. × 6 in. × 2½ in. with pin, washers, and keys. 1 pawl, 2 in. × 6 in. × 2½ in. with pin, washers, keys, and box. Armor plates and rivets for belt.

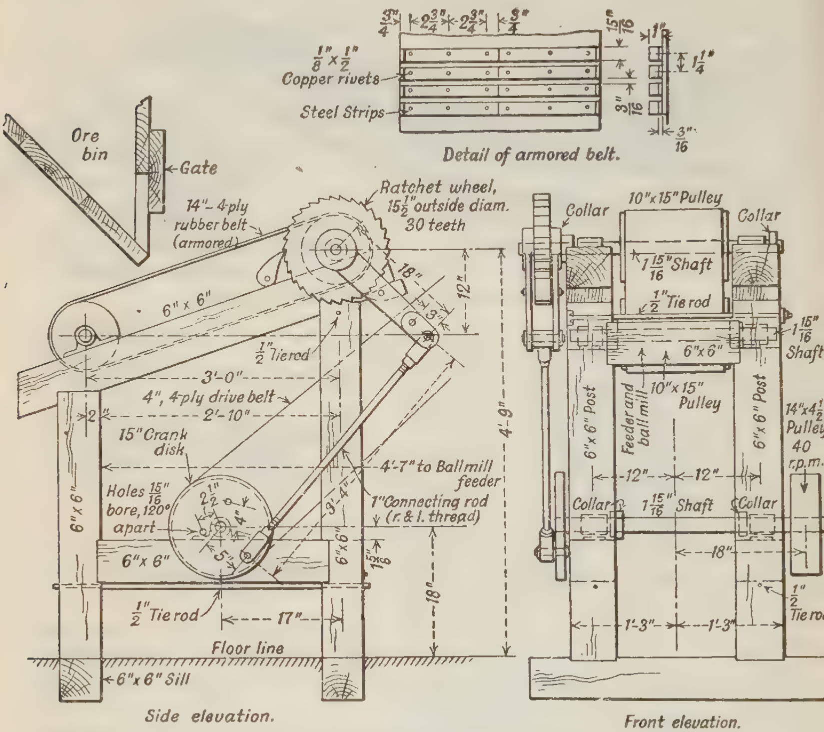


FIG. 49.—Belt-apron feeder.

The price in 1921 was $157 against $285 to $350 for plunger feeders and $300 for roll feeders of equal capacity.

Capacity may be calculated by the same formula as for apron feeders.

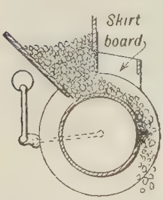


FIG. 50.—Roller feeder.

Roller feeder. (Fig. 50.) The diameter and setting of a feeder of this type should be such that a tangent to the front surface of the roll, drawn from the lower edge of the hopper gate, makes an angle with the horizontal less than that of the angle of repose (ϕ) of the material and less than the sliding angle (θ) of the material against the roll face, and at the same time the angle of the tangent to the roll face at the rearmost point of contact of the ore therewith should be less than θ. Skirt-boards must be used to confine material on the roller. Rolls from 12 × 12-in. to 48 × 48-in. are commonly used, the larger diameters for coarser feeds and the greater widths for larger capacities.

Advantages are simplicity of desigr, large capacity, low speed and low power consumption. Disadvantages are that delivery is almost directly below the feed point and there is considerable loss of head room.

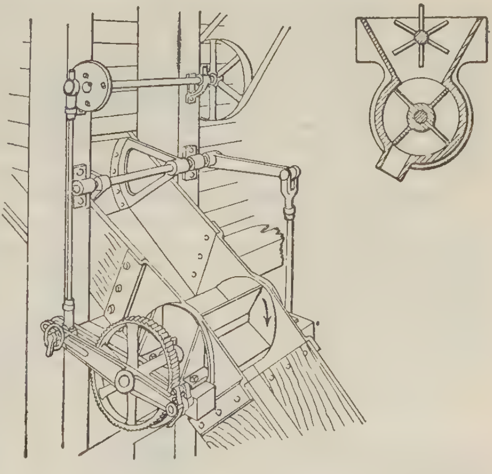
At Miami Copper Co., a 48 × 48-in. roller feeder had two spiders; face, 1½ in. thick, protected by ½-in. manganese steel plates bolted on. It was driven by ratchet and pawl with six possible speed changes.

Wear on drum surfaces can be materially lessened by grooving the surface transversely or by bolting on transverse angles, thus preventing sliding. Such provision is useful, also when material tends to pack and bridge at the nopper mouth. Rubber bands (see Sec. 13, Art. 5) should be useful in this service.

Peripheral speed of 5 to 20 f.p.m. is usual.

Capacity in tons per hr. is given approximately by the formula $T = 3\pi dwtn$, where d, w and t are diameter of roll, width of roll, and thickness of layer of material thereon respectively in feet and n = r.p.m. of roll.

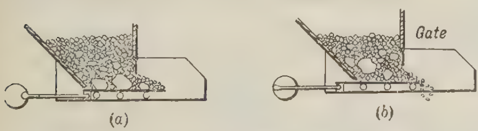
Rotary feeder (Fig. 51) is used where close regulation of fine feed is desired. It is usually placed in a hopper-discharge chute (*b*) and lacks the positive features of the apron and roller feeders but with a freely-flowing fine feed, or one that can be stirred by revolving prongs as indicated in (*b*) a remarkably close estimation of the quantity passed may be made by attaching a revolution counter to the feeder shaft and carefully calibrating the quantity delivered against r.p.m. The chute form (*a*) is less common. The figure illustrates the use of ratchet-and-pawl drive, which is probably the best for slow-moving feeder.



(*a*) (*b*)

Fig. 51.—Rotary feeder.

Reciprocating-plate feeder (Fig. 52). In moving forward into the position shown in (*a*) a layer of material is carried forward on the plate and more material settles from the hopper onto the rear portion of the plate. On the reverse stroke the plate slides under the material and that material at the forward end of the plate has its support withdrawn and falls. Widths range from 12 to 48 in. The plate may be supported on wheels, as indicated in the figure, or suspended by rods.



(*a*) (*b*)

Fig. 52.—Reciprocating-plate feeder.

The speed is usually below 30 r.p.m. Stroke length depends upon the size of material; it should be great enough to prevent bridging above the plate at the end of the forward stroke; usual lengths are between 4 and 12 in. Capacity may be estimated from the equation $T = 3ltwn$, where w, l and t are effective width of feeder, length of stroke and average thickness of layer on plate in front of hopper gate respectively, in feet, and n = r.p.m. This

feeder is suitable for both coarse and fine ore, provided there is no tendency to pack and bridge in hopper mouth. Wear on the plate is considerable and the hopper must be emptied in order to make renewals. The drive should be through a gear, and a flywheel to overcome the inertia at reversing is advisable.

When this type of feeder can be mounted so as to remove the static weight of the column of ore, *e.g.*, under the end of a chute, it may be run at higher speeds and becomes a shaking feeder. The operation is then the same as that of shaking screens (see Sec. 5, Art. 6) and, in fact, screen or grizzly bottoms are often provided. Such an arrangement is particularly desirable in loading a mixture of medium-coarse and fine material onto a belt conveyor, as it permits the fine material to load onto the belt first (see Fig. 53) and thus decreases wear and lessens slip and tumbling of the coarse lumps; material thus loaded is also excellently arranged for hand picking.

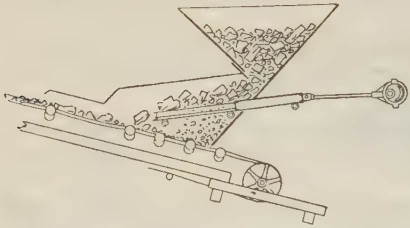


FIG. 53.—Shaking grizzly loading conveyor.

Vibrating-tray feeder is a form of shaking feeder; the feed trough, inclined slightly (5 to 10°) is hinged at the upper end and supported on an adjustable stirrup at the lower end so that it is just lifted by a projection on a revolving shaft, *e.g.*, a square corner or cam ear, and then dropped. The vibration thus set up is sufficient to move dry, pulverized material readily and if the number of vibrations is between 240 and 600 per min. will give substantially continuous, uniform movement. The feeder is not suitable for heavy loads nor coarse material.

Plunger feeder (Fig. 54) consists of an eccentric-driven plunger working in an open-top trough placed below the mass of ore to be fed, or replacing a side-draw chute; unless the hopper is shallow, the latter is the better position.

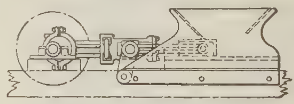


FIG. 54.—Plunger feeder.

SPEED is usually less than 30 strokes per min. and stroke length from 3 to 6 in. CAPACITY is estimated similarly to that of the preceding feeders.

Screw feeder (Fig. 55) is sometimes used for closely regulated positive feeding of fine, free-running soft material; it is useless for sticky material and wears rapidly with gritty substances. Screws range from 6 to 12 in. diameter, speed up to 80 r.p.m., capacity of a 6-in. screw at 80 r.p.m. is roughly 90 cu. ft. per hr. and of a 12-in. screw, 700 cu. ft.

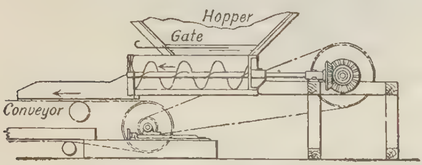


FIG. 55.—Screw feeder.

Challenge feeder (Fig. 56) is of the revolving-disk type; it consists essentially of an inclined circular plate or disk (J) forming the bottom of a chute or hopper (L) which is open at (M). Disk (J) is revolved slowly by bevel gears on shaft (I), which is driven by the friction pawl (D) attached to lever (B). When the feeder is used with gravity stamps, (B) is actuated by connecting rod (F) and lever (G) from a feeder tappet (H) on the center stem of the battery. The feeder may also be pulley driven, in which case a cam on the pulley shaft lifts (B). Speeds of (J), and hence feed rate, are varied by varying the down-

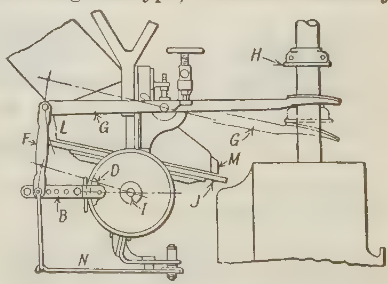


FIG. 56.—Challenge feeder.

ward return of (B) by means of the tension of spring (N). The pulley-driven form requires relatively little headroom, compared to other forms of feeders.

Stirrup feeder is essentially an undercut concave gate (Sec. 19, Fig. 29) eccentric-driven, with a transverse baffle placed centrally in the box above the swinging gate and clearing the latter sufficiently to prevent wedging. This baffle holds material that falls onto the gate in place so that the gate moves from under and drops it at each oscillation. It has been used frequently for fine-roll feeding.

17. Distributors

It is almost invariably necessary to split the stream of ore passing through a mill at some point or points in its flow in order to distribute the parts to parallel treatment processes; thus the product of a primary breaker must, frequently, be sent to two or more secondary breakers; the product of intermediate crushing to a battery of grinding machines and the product of one grinding machine to a battery of concentrators. The method employed for distribution of moving streams depends primarily upon accuracy required. CRUDE SPLITTING may be done by spreading the stream into a shallow and roughly-rectangular section and then inserting diverting vanes into the stream, at the central point, if halving is desired, etc. ACCURATE SPLITTING may be effected by the method employed in many mechanical samplers (Sec. 21) *i.e.*, by contracting the stream into a substantially equidimensional section and delivering it to a compartmented vertical drum, either the drum or the delivering mechanism being made to revolve on a vertical axis in such relation to the other that when the compartment-dividing walls cut the stream the stream is flowing substantially in the plane of the wall.

Coarse material. Except in sampling, distribution of streams of coarse material rarely requires accurate splitting; for crude work the stream from a chute, conveyor, or the like is allowed to fall freely onto the upper edge of vertical partition walls between fixed chutes. Only a two-way split makes any approach to accuracy owing to the fact that the center of the stream flows most rapidly and is coarsest. Bins with multiple gates are probably best for distribution of coarse dry material to a number of parallel machines.

Launder splitting is frequently employed for fine fluid pulps. The simplest arrangement is to fork the launder and place a hinged metal plate extending the partition wall upstream; this may be moved until the desired proportion passes down each fork before fastening into place. Multiple splitting is best effected by repetition of two-way splits.

Revolving distributors. The commonest form for fine pulp is typified by Fig. 57. The feed is led into the central compartment of the revolving tub (a), overflows at notch (b) (4 in. wide in the figure) into the annular launder (c) and thence through spout (d) successively into compartments (e) in the outer tub. Each compartment delivers through (f) to its proper pipe or launder. SPEED is 20 to 30 r.p.m.; pulsation of the resulting streams is lessened by making hole (f) small enough to prevent immediate emptying of its compartment.

Distributors with revolving compartmented drums are rare. They discharge through pipes at different radial distances into corresponding fixed annular troughs and thence to launders.

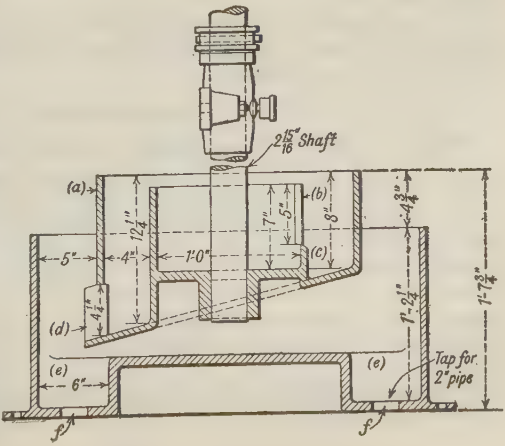


FIG. 57.—Revolving distributor.

SECTION 21

SAMPLING

BY

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Definition. Sampling is the operation of taking from a given lot of material a portion that is as nearly representative of the whole as is economically possible and is of convenient bulk for testing or analysis. It usually involves two operations, (*a*) crushing to reduce the size of particles, and (*b*) cutting down or dividing the lot into two parts, one of which is usually of much smaller bulk than the other and constitutes the sample. The scope of the present article is sampling of broken ore and various mill products. For sampling mines see mining texts and handbooks.

Factors affecting ore sampling. The principal elements in any sampling problem are (*a*) the weight of sample to be taken, and (*b*) the method of taking the sample.

The first question involves consideration of one or both of the following: the weight necessary to fulfill the purpose of the sample, and the minimum weight that will represent the lot.

1. Weight of sample

Weight necessary for assay sample will vary from a minimum of, say, 100 gm. to a maximum of 1 kg., the latter amount being sufficient to allow any necessary number of repeat or umpire assays. The amount actually weighed out for assay will vary from 0.5 gm. on high-grade base-metal concentrate to several assay-tons (29.166 gm.) on spotty gold ores. For SCREEN ANALYSIS the necessary weights range from 50 gm. for a single analysis of 100-mesh material or finer to 20 kg. for 1-in. material, and proportionately larger amounts for coarser sizes. For LABORATORY AND SMALL-SCALE MILL RUNS the necessary quantity will vary from a few pounds to many tons, depending upon the extent investigation.

Minimum weight of sample allowable in order that the sample may represent the lot depends upon (*a*) allowable error, (*b*) worth of valuable mineral,

(c) size of largest pieces in lot, (d) size and number of pieces of valuable mineral. Note that (b) and (d) are the elements of the grade of the ore.

Allowable error is determined by the purpose for which the sample is taken. The least error is allowed in taking samples for assay. Great error is allowable in the case of samples taken for preliminary treatment tests, and the like. The allowable error in sampling for assay is least when the assay is to form the basis for settlement between buyer and seller. The limit is practically fixed by the accuracy of the methods of assay used, it being, of course, unnecessary to carry refinement in sampling beyond the point reached by ordinary commercial methods of analysis. Accuracy of analytical methods on different products is presented in the following paragraphs.

Gold. Duplicate assays by the same assayer should check within 0.02 oz. on low-grade ores; the discrepancy should not exceed 0.05 oz. on 5-oz. ore, while a 0.1-oz. discrepancy is permissible on 5- to 20-oz. ore. (*Lodge, R. W., notes on assaying, John Wiley and Sons, 1919; Smith, E. A., The sampling and assay of the precious metals, Chas. Griffin & Co., Ltd., London, 1913.*) Two assayers working on the same pulp or on samples cut from a small, finely-pulverized lot should check within similar limits, according to most writers. Fulton (*TP 83, USBM*) states that differences of 0.02 to 0.05 oz. are usually split on 1- to 2-oz. ores and of 0.04-oz. on ores running above 2 oz. Barbour (*92 J 314*), gives 0.05 oz. as the splitting limit and Whitaker (*105 J 538*), 0.02 oz. Seamon (*A manual for assayers and chemists, John Wiley and Sons, 1910*) states that 0.01 and 0.2 oz. covers the usual splitting range, but recommends that umpiring be resorted to when the difference between buyer's and seller's assays on a 50-ton lot exceeds 0.1 oz. Table 1, arranged from Brunton (*40 A 567*), presents figures of actual mill results on a variety of ores. By comparison of the last four columns, the degree to which sampling checks assaying may be observed. Table 2 was prepared from data given by Woodbridge (*TP 86, USBM*) based on results obtained from shipments of one mine to various sampling plants. It shows the degree to which buyer and seller may be expected to check on the same samples of various grades of ore. The detail (not presented here) shows that buyer's assays are rather consistently lower than seller's which leads to the belief that deviations shown in this table are not due entirely to physical difficulties. Table 3, arranged from the same source as Table 2, is a comparison to indicate the degree to which duplicate samples may be expected to check. Note that, in general, greater difference exists between duplicate samples than between assays by buyer and seller.

The above figures indicate that sampling should be accurate to within at least 1 per cent. on a 5-oz. ore while an error of as much as 5 per cent. would be allowable on 0.5-oz. ore. A hard and fast rule as to allowable difference cannot be set for all cases. Many ores present physical difficulties in sampling and assaying which must be taken into account when setting a standard for accuracy. In any case, however, although individual assays may not check closely, an average of several or many assays on a sample should closely approximate the true value of the sample.

Silver. Duplicate assays by the same assayer, doing the most careful work, should check within 0.2 to 0.5 oz. for ores up to 50 oz. In ordinary work differences up to 1.0 or 2.0 oz. maximum may be expected (*Fulton, Seamon*). On high-grade ores duplicates should check within 1 per cent. of the total silver content. Assays on duplicate samples by different assayers should check within 0.5 to 1.0 oz. for average (10- to 20-oz.) ores (*Fulton*). On high-grade ores differences of 1 per cent. are allowable. The usual splitting limit between buyer and seller is 0.5 oz.

Tin. Ordinary cyanide fire assays by the same assayer should check within 0.2 per cent. metal for low-grade ores to 0.5 per cent. for high-grade ores. (*W. A. MacLeod and C. Walker, Metallurgical analysis and assaying, C. Griffin and Co., Ltd., London, 1903.*)

Other metals. Table 4 presents limits for commercial work using volumetric methods. Duplicate electrolytic assays for copper should check within 0.03 per cent. metal on 20 per cent. to 60 per cent. ores and 0.02 per cent. on ores less than 20 per cent. (*MacLeod and Walker*).

Worth of valuable mineral. The preceding text and tables show that on gold ores and concentrate the allowable error is, in general, under 0.05 oz.; on silver ores, under 0.5 oz.; on copper ores, from 1 to as high as 10 lb. per ton; and on zinc and lead, as high as 20 lb. per ton; increasing as the worth of the valuable mineral decreases.

Table 1. Sampling results on gold ores, Taylor and Brunton Sampling Co., Cripple Creek, Colo.

Lot	Sample 1, ounces of gold per ton		Sample 2, ounces of gold per ton		Average of 4 results, ounces per ton	Deviation of average				Difference between assayers on same sample		Difference between two samples	
						Below high assay		Above low assay					
	Assayer A	Assayer B	Assayer A	Assayer B		Ounce	Per cent.	Ounce	Per cent.	Sample 1	Sample 2	Assayer A	Assayer B
1	0.53	0.55	0.55	0.56	0.5475	0.0125	2.23	0.0175	3.30	0.02	0.01	0.02	0.01
2	1.11	1.10	1.07	1.09	1.0925	0.0175	1.58	0.0225	2.10	0.01	0.02	0.04	0.01
3	1.27	1.30	1.30	1.35	1.3050	0.0450	3.33	0.0350	2.76	0.03	0.05	0.03	0.05
4	1.27	1.24	1.27	1.28	1.2650	0.0150	1.17	0.0250	2.02	0.03	0.01	0.00	0.04
5	1.36	1.35	1.29	1.30	1.3250	0.0350	2.57	0.0350	2.71	0.01	0.01	0.07	0.05
6	1.77	1.72	1.75	1.71	1.7450	0.0250	1.41	0.0250	1.45	0.05	0.01	0.02	0.02
7	2.22	2.24	2.22	2.23	2.2275	0.0125	0.55	0.0075	0.34	0.02	0.01	0.00	0.01
8	2.33	2.34	2.34	2.36	2.3425	0.0175	0.74	0.0125	0.54	0.01	0.02	0.01	0.02
9	12.62	12.58	12.69	12.68	12.6425	0.0375	0.30	0.0625	0.59	0.04	0.01	0.07	0.10
10	115.05	115.25	114.90	115.20	115.1000	0.1500	0.13	0.2000	0.17	0.20	0.30	0.15	0.05
Average of 1–8	....	....	....	....	....	0.0225	1.70	0.0225	1.90	0.0238	0.0175	0.0238	0.0262

Table 2. Comparison of gold assays on same samples by buyer and seller

Grade of ore, ounces per ton	Number of samples	Difference, ounce per ton				Number of cases with difference of . . oz.														Number of cases where difference is less than probable difference
		Maximum	Minimum	Average	Probable (a)	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11 to 0.20	0.21 to 0.30	+0.30	
0.30-0.99	96	0.11	0.00	0.0212	0.0183	11	29	25	16	8	2	4	0	0	0	0	1	0	0	65
0.94-1.96	172	0.21	0.00	0.0492	0.0420	16	15	18	21	15	27	15	10	9	6	7	12	1	0	85
1.97-3.34	110	0.18	0.00	0.0578	0.0470	4	6	14	10	12	12	16	5	9	6	1	15	0	0	58
2.80-4.08	56	0.37	0.00	0.0964	0.0794	3	2	2	3	3	7	4	3	6	3	0	17	1	2	33
4.08-4.69	12	0.20	0.01	0.0867	0.0756	0	2	2	0	0	0	1	0	2	0	1	4	0	0	7
5.26-5.90	8	0.19	0.00	0.0850	0.0766	2	0	0	0	0	0	1	0	2	0	0	3	0	0	5
Totals...	454	0.37	0.00	....	....	36	54	61	50	38	48	41	18	28	15	9	52	2	2	253

a Probable difference $= 0.6745\sqrt{\frac{\Sigma d^2}{n-1}}$

Table 3. Comparison of gold assays on original and duplicate samples

Grade of ore, ounces per ton	Number of samples	Difference, ounce per ton				Number of cases with difference of .. oz.														Number of cases where difference is less than the probable difference
		Maximum	Minimum	Average	Probable (a)	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11 to 0.20	0.21 to 0.30	+0.30	
0.30-0.99	96	0.16	0.00	0.0194	0.0212	21	36	18	7	5	1	3	2	1	0	0	2	0	0	75
0.94-1.96	172	0.40	0.00	0.0580	0.0648	26	24	29	12	23	3	11	2	7	1	7	20	2	5	128
1.97-3.34	110	0.62	0.00	0.0958	0.1174	15	13	16	5	12	3	7	3	3	1	6	13	3	10	84
2.80-4.08	56	0.46	0.00	0.1210	0.1142	3	1	9	1	5	2	3	1	7	0	2	11	7	4	36
4.08-4.69	12	0.19	0.00	0.0500	0.0504	2	0	3	1	2	0	1	0	1	0	1	1	0	0	8
5.26-5.90	8	0.26	0.02	0.0850	0.0816	0	0	1	1	1	1	1	0	1	0	0	1	0	0	6
Totals...	454	0.62	0.00	....	....	67	74	76	27	48	10	26	8	20	2	16	48	13	19	337

a Probable difference $= \sqrt{\dfrac{\Sigma d^2}{n-1}}$

Table 4. Limits of accuracy in commercial volumetric assay

Metal, etc.	Duplicates, same assayer	Same pulp, different assayers	Between buyer and seller	
			Different assayers	Ordinary splitting limits
Copper	0.05 to 0.20% *b*	0.2%	0.25 to 0.5% *b*	0.2-0.5% *e*
Lead	0.05 to 0.2% *b*	0.2 to 0.8% *g*	0.5 to 1.0% *b*	0.5% to 0.6% *a*
Zinc	0.05 to 0.2% *b*	0.2% *f*	1.0% *b*	0.6%- to 1.0% *e*
Insoluble	0.3% *c*	....	1.0% to 2.0% *b*	0.5% to 1.0% *e*
Manganese	0.1% *d*	....	....	1.5% *f*
Lime	0.3% *c*	....	1.0% *b*	0.5%-1.0-1.5% *f*
Iron	0.3% *c*	....	1.0% to 2.0% *b*	0.5% to 1.0% *e*
Sulphur	0.3% *c*	....	1.0% *c*	0.5% to 1.0% *e*

Note—Percentages are percentages of metal, not of total metal content. *a* Set by committee for standardization of sampling, assays and settlements in Missouri, Kansas and Oklahoma, districts. *b TP 83, USBM.* *c 108 J 806.* *d MacLeod and Walker.* *e 105 J 538.* *f Seamon.* *g 121 P 866.*

Size of largest pieces in lot and size and number of pieces of valuable mineral present are interdependent in their effect on the minimum weight of sample allowable and affect this weight because of the fact that accuracy requires the inclusion in the sample of a certain minimum number of individual pieces rather than of any given weight of material. The significance of this statement will be seen from a consideration of the following possible and usual cases.

(a) **The lot is a mixture of individual particles of pure valuable mineral and pure gangue, all of the same size.** The valuable mineral may all be in one piece. In such a case nothing less than the whole lot is a correct sample. If, however, the lot contains a known large number of pieces of valuable mineral and after careful mixing is divided into parts. the probable number of pieces in excess or deficit in any part may be determined by experiment or mathematically. The part taken may then be said to represent the whole if the effect on its value of pieces in excess or deficit is within the allowable error for sampling operations. If the whole lot is now taken and crushed so that the diameter of each particle is reduced one-half, the number of particles then present will be eight times as many as originally. A sample portion of this crushed material, in order to contain the same number of pieces of valuable mineral as above and, therefore, to have the same probable error, need be only one-eighth of the weight taken above, *i.e.*, the sample weight varies as the cube of diameter of the particles. If w_1, w_2, w_3, etc., = the weights to be taken at sizes D_1, D_2, D_3, etc., respectively, then $w_1/w_2 = D_1^3/D_2^3$ or $w_1/D_1^3 = w_2/D_2^3 = w_3/D_3^3$, etc., or $w/D^3 = k$ (ore constant) and $w = kD^3$. Application of this formula for weight of sample cannot be made to usual cases met with in ore sampling, as the conditions obtaining in crushed ore are different from those assumed in deriving the formula.

(b) **The lot is a mixture of particles of different sizes, all of which contain both valuable mineral and gangue in varying proportions.** This is substantially the condition in all coarse ores. For any given ore there is a particle size above which no valuable mineral can occur as a separate piece and likewise a size above which no gangue can so occur. In this case the pieces containing the greatest amount of valuable mineral have the greatest effect on the value of the part taken for the sample. When in excess or deficit, the effect of such pieces is determined by the difference in grade between them and the pieces of average grade that they replace, or their effect is equivalent to that of smaller pieces of pure valuable mineral. The allowable number of pieces of highest grade in excess or deficit is greater than if these pieces were of pure valuable mineral, hence a smaller sample of material containing such pieces can be taken. With coarser and coarser pieces making up the ore mass, the difference in value between a piece of highest grade and pieces of average grade becomes less and less and the value of each piece comes nearer to that of the whole.

(c) **The lot is a mixture of pieces of pure valuable mineral, pure gangue and particles containing both valuable mineral and gangue.** This is the case most frequently met. If all the pieces are of the same size, the maximum error is introduced when a piece of pure valuable mineral replaces a piece of pure gangue or *vice versa*. However, the number of pieces of pure mineral is less than the total number of pieces containing valuable mineral and the probable number of such pieces in excess or deficit in the sample is, therefore, less, so that their effect on the value of the whole is less than if all pieces were of pure valuable mineral and pure gangue. Further, the probability that such pieces will replace pieces containing both valuable mineral and gangue is a factor that decreases their effect. If the material is composed of pieces of varying size, the coarser pieces are more likely to be those containing both valuable mineral and gangue while pieces of pure mineral and pieces of pure gangue are more likely to occur in the finer sizes and to have, therefore, small effect on the value of the sample. Consideration of the conditions discussed in (*b*) and (*c*) leads to the conclusion that if, at any given size, a sample is taken, the weight necessary to give the same degree of accuracy at finer sizes will be greater than that indicated by the rule ($w = kD^3$). Likewise at coarser sizes the weight of sample necessary will be less than indicated by the rule. Successive crushing operations on a lot of ore produce changes in the conditions in which valuable mineral and gangue occur. In the coarsest sizes practically all of the valuable mineral occurs together with gangue in all pieces. With decreasing size mineral is gradually freed from the gangue and in the finest sizes it is practically entirely free. Differences in friability of gangue and valuable mineral cause increase in the amount of one or the other in finer sizes at each stage as crushing progresses.

Relative specific gravities of the valuable and gangue minerals will determine the effect that one particle of a given size in excess or deficit will have on the value of the sample.

Table of sample weights. The difficulty of deriving mathematical formulas to take all of above factors into account is apparent. The usual way to determine the sample weight required at any size is to reckon on experience with the particular ore to be sampled or experience with ores in general. The tendency is to allow a sufficient factor of safety by taking a sample large enough to cover possible unforeseen sources of error. Variations in the weights taken in practice, on ores of same class are not always due to disregard or misunderstanding of the principles involved, but to local conditions and the facilities available.

Brunton (*25 A 826*) presents diagrams based on mathematical calculations to determine the weight of sample at different sizes when the specific gravity of the richest mineral and the ratio of grade of the richest mineral to the average grade of the ore is known. The diagrams are based on the reasoning that the allowable error is equal to the difference between the value of the probable number of pieces of valuable mineral and the same number of pieces of average value which they would replace if in excess or deficit. Results show that if such conditions remain the same throughout all sizes the weight of sample should vary as the cubes of the diameters. Full account is also taken of the fact that larger pieces than the theoretical cube are found in the undersize of a screen, due either to irregularity of the screen or the presence of elongated pieces. Use of these diagrams is limited to cases where conditions are known and where they remain the same throughout all sizes. This is not the case with ordinary ores.

Richards (*2 OD, Chap. 19*), reasoning from considerations similar to those discussed under (*b*) and (*c*) above, and noting that in several installations on a variety of different ores the weights at different sizes were taken proportional to the squares of the diameters, calculated a table for different grades of ore, using one figure from practice in each case and calculating the rest.

Columns 1–6 in Table 5 were prepared from Richards' table. Column 7 shows weights obtained by applying the rule that the sample weight should vary as the cube of the diameter. The calculation is based on the assumption that 0.1 assay ton (0.00643 lb.) is a correct sample of 100-mesh material. Column 8 gives figures presented by Philip Argall for sampling in successive cuts 200,000-lb. lots of 10- to 15-oz. gold ores from Cripple Creek. The weights in this column vary approximately as the squares of the diameters and lie somewhere between the figures in Columns 5 and 6 from Richards. The larger figures in parentheses are recommended for an added factor of safety for certain ores. Column 11 summarizes results obtained from examination of 55 flow-sheets of sampling plants. The weights are based on 100,000-lb. lots, the figures being averages for different sizes. In averaging, some excessively high or low weights for any one size were omitted. The results lie between columns 3 and 4 for medium ores as given by Richards. This is as might be expected and indicates that practice generally has followed the rule that sample weights should vary as the squares of particle diameters. Columns 9 and 10 give maximum and minimum figures from the flow-sheets used in averaging; figures in parentheses are highest or lowest figures.

Columns 12 to 15 inclusive, present sampling standards suggested by Demond and Halferdahl for base and precious metals. These data were set down in an attempt to obtain logical figures based on a combination of experiment and mathematics rather than on arbitrary application of the rule of squares. Their method is based on the assumption that the sample weight w should vary as some power a of the diameter D or $w = kD^a$ where k is a constant for any given ore. One gram for base-metal ores and one assay ton for precious-metal ores, the amounts usually taken for assays, were assumed to be correct sample weights for 150-mesh material. A portion of the ore at some larger size was divided into a number of equal parts and each part was crushed down and assayed. The probable error for any portion was then calculated and if larger than the allowable error the correct sample weight was calculated so as to obtain a probable error less than the allowable error in 90 to 95 per cent. of all cases. Substituting values of weights and sizes in the two instances in the formula $w = kD^a$, they solved for k and a, and then calculated the correct weights for other sizes. They demonstrated that values of a less than 1.5 should not be used, as the samples would be too small. The method is applicable to any given class of ores. With low-grade or very uniform ores a will have a low value and with high-grade or spotty ores a high value (see Table 5); for ores of intermediate grade or uniform value, a will lie between these extremes.

While the method of Demond and Halferdahl has much to commend it, it does not take into account the changes in relation between gangue and valuable mineral with change of

Table 5. Sample weights at different particle sizes (pounds)

Diameter of largest piece			Arranged from R. H. Richards						$W \alpha D^3$	Argall (*a*)
			1	2	3	4	5	6	7	8
Inches	Mm.	Mesh	Very low-grade or very uniform ores	Low-grade or uniform ores	Medium ores		Rich or spotty ores	Very rich or exclusively spotty ores	Starting with 0.1 assay ton of − 100-mesh material	Colorado practice gold ores
8			19,200	64,000						
6			10,800	36,000	80,000					
5			7,500	25,000	55,550					
4			4,800	16,000	35,556	80,000				
3			2,700	9,000	20,000	45,000			900,000	
2.5			1,875	6,250	13,888	31,250	80,000		521,600	
2			1,200	4,000	8,889	20,000	51,200		267,264	
1.5			675	2,250	5,000	11,250	28,800		112,500	
1.25			469	1,563	3,472	7,813	20,000		65,200	
1.0			300	1,000	2,222	5,000	12,800		33,408	40,000
0.75			169	563	1,250	2,813	7,200		14,063	
0.625			117	391	868	1,953	5,000		8,150	
0.500			75	250	556	1,250	3,200		4,176	
0.375			42	141	313	704	1,800		1,758	
0.3125			29	98	217	488	1,250		1,019	
0.250			19	63	139	313	800	27,550	522	2500 (4000)
0.1875			10.5	35	78	176	450	15,500	220	
0.131	3.327	6	5.15	17.2	38.1	86	220	7,560	75	
0.093	2.362	8	2.6	8.65	19.2	43	111	3,810	26.32	
0.065	1.651	10	1.29	4.3	9.5	21.5	55	1,890	9.32	157 (250)
0.046	1.168	14	0.65	2.16	4.8	10.75	28	953	3.29	
0.0328	0.833	20	0.322	1.075	2.37	5.38	13.76	473	1.16	
0.0232	0.589	28	0.162	0.539	1.20	2.69	6.90	238	0.41	10 (15)
0.0164	0.417	35	0.081	0.269	0.59	1.345	3.44	118	0.146	
0.0116	0.295	48	0.041	0.135	0.30	0.673	1.73	60	0.051	
0.0082	0.208	65	0.020	0.067	0.15	0.336	0.86	30	0.018	
0.0058	0.147	100	0.010	0.034	0.075	0.168	0.43	15	0.0064	
0.0041	0.104	150	0.005	0.017	0.038	0.084	0.215	8		
0.0029	0.074	200	0.0025	0.009	0.019	0.042	0.107			

a 10 *IMM* 234.

Table 5. Sample weights at different particle sizes (pounds)—*Continued*

Diameter of largest piece			Woodbridge (*b*)			Demond and Halferdahl (*c*) $w = kD^a$				Richards	*a* varies from 1.5 to 3.0	
			9	10	11	12	13	14	15	16	17	18
Inches	Mm.	Mesh	Maximum	Minimum	Averages from practice	Base-metal ores, $a = 1.5$	Base-metal ores, $a = 2.7$	Precious metal, ores, $a = 1.5$	Precious metal, $a = 2.7$	Gold ores, minimum	Gold ores	Base-metal ores
8												
6												
5												
4						71	290,000	2080	8,460,000		228,700	7833
3			50,000	10,000	25,000	46						
2.5			20,000	20,000	20,000	35.1						
2			25,000	4,000	16,000	25.1	43,900	735	1,302,000	10,000	80,860	2770
1.5			30,000 (100,000)	400	7,400	16.3				5,000		
1.25						12.7						
1.0			10,000 (25,000)	400	3,600	8.92	6860	260	200,000	2,000	24,890	852
0.75			4000	100	1,600	5.8				1,000		
0.625						4.4						
0.500			2000 (5000)	200 (4–6)	800	3.2	1040	92	30,800	400	6668	228
0.375			800	200	700	2.05				300		
0.3125						1.56						
0.250			1000 (2000)	8	300	1.10	160	33	4700	200	1667	57
0.1875										100		
0.131	3.327	6	250 (1200)	8	80					75	429.4	14.7
0.093	2.362	8								50		
0.065	1.651	10	20	20	20	0.14	3.8	4.1	110		93.4	3.2
0.046	1.168	14	12	10	12					25		
0.0328	0.833	20	10	3	8					10	19.0	0.65
0.0232	0.589	28		1.25								
0.0164	0.417	35		1.00						4	3.36	0.115
0.0116	0.295	48		1.25						1		
0.0082	0.208	65		1.25		0.0062	0.0143	0.183	0.419		0.515	0.0176
0.0058	0.147	100		0.25		0.0040	0.0066	0.119	0.192			
0.0041	0.104	150				0.002205	0.002205	0.064	0.064		0.064	0.002205
0.0029	0.074	200										

a 10 *IMM* 234. *b* Tech. Paper 86, USBM. *c* 114 *J* 280, 948.

particle size, but uses the same power of the diameter over the whole range of sizes. Columns 17 and 18 use one assay ton for precious-metal ores and 1 gm. for base-metal ores as correct weights at 150-mesh and take the exponent a as ranging from 3 for material between 150-mesh and 65-mesh to 1.5 for material between 2-in. and 4-in. This method tends to give larger samples at the smaller sizes and smaller samples in the larger sizes. If the assay charges need to be larger than one assay ton or one gm., the weight of any size may be found by multiplying the tabular figures by the number of assay tons or grams needed to give correct assays.

Column 16 gives the minimum permissible sample weights suggested by Richards for GOLD ORES. For SILVER ORES take one-tenth of these values. Inspection of figures in this column shows that there is no constant or regularly changing ratio between weights and diameters. These sample weights are evidently based on no rule except experience and personal judgment as to what weights can be taken at various sizes.

Examination of sample-mill flow-sheets indicates wide variation in sample size. The most obvious relation noticed is the taking of successive 20-per cent. cuts after each reduction in size by one-half or the use of a value for a of about 2.13.

Weight of coal samples used and recommended by U. S Bureau of Mines (*Bul. 116, USBM*) based on an original gross sample of 1000 lb. is given below.

Largest size of coal and impurities in sample before division, in	Weight of sample to be divided, lb.
1	1000 (or more)
¾	500
½	250
⅜	125
¼	60
3⁄16	30

Two 5-lb. samples are taken from the 15-lb. sample obtained by division of the 30-lb. at 3⁄16-in., (p. 1139) so that 5 lb. is the sample weight really taken at this size. Further reduction of the sample yields 6 oz. after crushing to 20-mesh (0.0328-in.) and 3 oz. after grinding to 60-mesh. From the 60-mesh sample 1-gm. amounts are used in making the various analyses. The weights in the coarser sizes vary as the squares of the diameters of the largest pieces. Assuming this relation necessary and sufficient, a 5-lb. sample at 3⁄16-in. is too small; if the 5-lb. sample is sufficient 6 oz. at 20-mesh is larger than necessary and 1 gm. (0.002205 lb.) at 60-mesh is far too small. If, however, 1-gm. is enough, an undue allowance is made in the larger sizes.

Weight of samples for screen test should be great enough to give an accurate sample at the sizes that are important. The allowable error is influenced by the care with which the screen test is made. The weights ordinarily taken for screen-test samples vary from say 100-gm. for −65-mesh material to 5 or 10 kg. of 1-in. material. Larger weights may be necessary in special cases depending on the accuracy desired and the end in view.

2. Segregation

It is most desirable that the mass to be sampled be thoroughly mixed. Practical homogeneity is attained with solutions, and the maximum of uniformity, lacking homogeneity, with fine suspensions. In such cases, any part can be taken at random as an accurate sample. But in dealing with broken ore, uniform mixing is difficult. Ore as mined varies in composition from place to place and in breaking, loading and crushing segregation increases. In crushing, some of the minerals present, due to differences in hardness and friability, break into smaller pieces than others. Vibration of a car transporting ore tends to sift the finer and the heavier pieces down through the coarser, so that the material in the car is not uniform in value from top to bottom. If ore is transported by means of inclined chutes or launders, segregation takes place therein. Coarser pieces bound or roll down the chute while finer pieces slide slowly along the bottom. Heavier pieces tend to settle near the bottom

in the sliding mass. If a chute ends in a free fall, the larger and heavier pieces, due to their greater momenta, fall further away from the end than the finer and lighter pieces. If the ore is piled in a heap, the coarser pieces roll down while the finer remain near the top. Thus, in order to take an accurate sample, either the material must be thoroughly mixed or the method must be so applied as to take segregation fully into account.

HAND SAMPLING

Sampling is done either by hand or by machine. Hand sampling is usually expensive, especially on large lots; it is slow, and the personal element enters so largely that it is difficult to obtain accurate results. Machine sampling should be used where possible and hand sampling applied only where material is not suitable for treatment by machine, as with sticky ore, or where machinery is not available, or the expense of its installation not justified.

Hand-sampling methods. The more common methods of hand sampling are: (*a*) grab sampling, (*b*) coning and quartering, (*c*) shovel sampling, (*d*) pipe sampling.

3. Grab sampling

This is the simplest form of hand sampling. It consists in taking small, equal portions by scoop or shovel at random or at regular intervals from the mass of material to be sampled. The following variations illustrate practice.

(a) **Grab samples of large heaps of ore** are obtained by taking a shovel- or scoopful at different points on the surface of the heap. The points may be located by dividing the surface into squares or rectangles, or the location may be left entirely to the judgment of the sampler. If large samples are desired, pits are dug at various points and the whole or part of the material excavated is taken for the sample.

(b) **Ore in railroad cars or boats** may be sampled as in (a). In this case a net is frequently placed over the surface of the ore and the sample is taken at the knots in the net, or the sampler measures off sample intervals from the sides and ends of the cars with a shovel or measuring stick.

(c) **As ore is being loaded or unloaded,** grab samples may be taken at intervals along the working faces as excavation progresses.

(d) **Where ore is being transported in small lots** by wheelbarrows, tram buckets, wagons or small cars, a GRAB may be taken from each load and thrown into a box. The box is emptied at the end of each day or shift and taken as a sample of the ore transported for such period of time.

Methods (*a*) and (*b*), taking samples from the surface only, assume that the character of the ore does not change with depth in the heap. Correct results can hardly be expected on such an assumption on account of segregation. Method (*c*) is more likely to give results that are accurate, since samples are taken from all parts of the mass. Method (*d*) is also more commendable than (*a*) and (*b*) and should, over a long period, give a fair average with certain classes of ore. However, it is probable that the samples would run regularly high or regularly low.

E. H. Dickenson (*35 SMQ 55*) gives the following figures from a small ALASKA mine: Ore, medium-grade copper containing chalcopyrite and pyrite in greywacke and slate. A 2-lb. sample was taken by an experienced man from each one-ton car dumped into a boat for shipment. The ore was again sampled at the smelter in a mechanical sampling mill. Figures based on two years of work with shipments varying from 200 to 2000 tons each showed that the average difference in assay between grab and mechanical samples was 7.1 per cent. of the copper content and the maximum difference, 28.5 per cent. The percentage of cases in which the grab sample was high was 55 and results were more than 10 per cent. wrong in 30 per cent. of the cases. Applying these figures to three ores running 2 per cent.,

10 per cent. and 20 per cent. copper, respectively, errors and the expectation of errors would be as follows:

	2-per cent. Cu ore.	10-per cent. Cu ore.	20-per cent. Cu ore.
	Per cent. Cu	Per cent. Cu	Per cent. Cu
Average error of grab sample	0.14	0.71	1.42
Maximum error of grab sample	0.57	2.86	5.72
Minimum error to be expected in 30 per cent. of the results	0.20	1.00	2.00

In this ore values did not tend to segregate in the fines to any great extent as shown by the following sizing-assay test:

Screen, mesh	Per cent. of total weight	Per cent. of total Cu
60	30.6	24.8
80	9.5	8.7
100	6.6	7.0
100	53.3	59.5
	100.0	100.0

Usually more than 55 per cent. of the results would be high. It is general experience that grab sampling gives high results, due probably to the fact that the sampler avoids large lumps.

Advantages of grab sampling are that it is cheap and quick. **Disadvantages:** It is difficult, when taking a small portion such as a shovel- or scoopful, to get full representation of all sizes of particles, especially when lumps are present. Just what part of a lump or how much of the coarser and finer pieces should be taken at each grab is left to the judgment of the sampler, and either by over-zealousness or by carelessness on his part, more of some size than is proper is almost certain to be taken. It is also practically impossible to take such an amount of material at each grab that every part of the lot shall have proportionate representation in the final sample.

Applicability. The ore should be crushed to 0.5-in. or smaller before sampling. Grab sampling should be applied only to low-grade or very uniform ores, or in cases where approximate results only are desired, or in order to detect salting.

Examples of grab sampling

Mogollon district, New Mexico (*72 A 535*). The ore contains gold and silver, both native and in argentite, and pyrite in a gangue of quartz, calcite and andesite. Yearly averages of composite daily grab samples taken from cars at the scale seldom show a variation of more than 10 per cent., generally 3 or 4 per cent., from the value determined from addition of the metal produced to that lost in tailing. (See *25 A 82C*.) Comparison follows:

	Case 1	Case 2
Weight of ore, tons	51,862	43,993
Value, mill average	\$11.084	\$10.254
Value, grab sample	\$11.218	\$11.193
Percentage excess of grab sample above mill average	1.20	9.17

Iron ores in cars or boats are usually sampled by grab-sampling. The methods described below are those used by U. S. Steel Corporation (*1 IEC 107*).

Car sampling. Samples of about 2 oz. to 3 oz. are taken with a garden trowel or small scoop (3½ × 2¼ × 1¼-in.) at 12 places in a 25-ton car and 15 places in 50-ton car. Either the PARALLEL or ZIGZAG SYSTEM is used (Fig. 1, *a*, *b*, *c*, *d*). Lumps met at sampling points are sampled by breaking off a small piece about the size of the first joint of the thumb. Samples from ten cars are combined, making about 15 lb. for ten 25-ton cars and 20-lb. for ten 50-ton cars. With very lumpy ore the ROPE-NET SYSTEM is used. Knots are about 18 in. apart and the car is sampled in about 32 places. MOISTURE SAMPLE is taken at three places along the center of each car after removing several inches of surface.

Boat sampling. The size of boats varies from 3000 to 12,000 tons capacity, one or two decks, 6 to 36 hatches with widths from 12 to 24 ft. Ores vary from very fine to all lump. The sample may be taken by small scoop (¼-, ½-, or 1-lb. capacity) from grab buckets, while unloading. This method is expensive as one man is needed for each grab bucket.

Sampling of cones is the term applied to the following method: The sampler starts at point "A" (Fig. 1, *e*), directly under the edge of the hatch and midway between the center and side of the boat and takes samples at points 1 ft. apart (measured by the trowel, which is 12 in. long), up the side of the cone, over the apex and down on the opposite side. Then similarly along the other diagonal. The amount thus taken is equivalent to about one-tenth of the total sample for the hatch. After the grab bucket has removed all ore within reach, samples are taken in parallel vertical lines up the face of the ore at 1-ft. intervals. The first line is 2 ft. from the side of the boat and succeeding lines are spaced at 4-ft. intervals (Fig. 1, *f*). This is repeated on the opposite face.

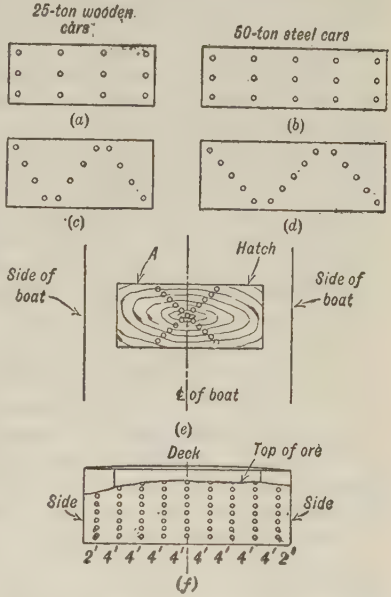


FIG. 1.—Sampling iron ores in cars and boats.

Round sampling is used when operation of the grab bucket makes sampling of cones impracticable or with boats with 24-ft. hatch centers and decks furnishing protection to samplers. When a 5- or 6-ft. face is exposed by the grab-bucket, the sampler takes samples of the face by the same method as in Fig. 1, *f*. The amount taken on this round is about one-third of the total. The second round is taken similarly after the grab bucket has removed all ore within reach; this comprises the remaining two-thirds of the sample.

4. Coning and quartering

This is one of the oldest forms of hand sampling. It was for many years the standard method of sampling throughout the western United States, especially for batches of ore whose value was to be determined between buyer and seller. It can be used on lots up to 50 tons. Where larger lots are to be sampled, the first cut is made by some other method and the sample thus obtained is further reduced by coning and quartering.

The method consists in piling the ore into a conical heap, spreading this out into a circular cake, dividing the cake radially into quarters, taking opposite quarters as sample and rejecting the other two. The ore should be crushed through 2-in. or smaller ring. The operation should be carried on in a room of sufficient size to allow convenient handling of material. The floor should be smooth and free from cracks, preferably made of concrete or steel sheets; it should first be swept thoroughly clean to avoid salting from

a previous lot. The ore is dumped on the floor in two or four piles, or in a circular ring. Shovelers then pile this ore into a right-conical heap conveniently placed, taking care to drop each shovelful directly onto the apex. The object of coning is to form a heap in which segregation shall be symmetrical with respect to the vertical axis. To insure this, successive shovel loads should be so taken as to contain similar sizes similarly segregated on the shovel and should be so dropped onto the cone that the segregation therein is symmetrical. This end is attained by having the shovelers take successive shovel loads from adjacent places around the periphery of the heaps from which they are shoveling and drop these loads from successively adjacent points around the cone that they are building. Each man should take a shovel load of the same size as each other shoveler, and shovel loads should be made smaller as the size of the heap being shoveled becomes less. When the material is placed in a ring the men move around the ring as they shovel onto the cone. When the ore is all heaped up into a cone, the floor is carefully swept and the fines collected are placed on the apex, not swept up against the bottom of the pile. The men then start at points near the bottom of the cone and, with their shovels held tangentially, drag the material down radially so as to form a truncated cone or flat circular cake. In this operation they should work around the cone. The cake is then marked off into quarters with a stick or board along diameters at right angles. Opposite quarters are shoveled out as reject. The remaining quarters, called the sample, are then shoveled into one or more piles or into a ring depending on the amount of material, and the coning and quartering operations are repeated until the sample is so small that further crushing is necessary before further reduction in bulk can be made. A cross made of sheet iron or wood with sharpened edges is often used to mark the truncated cone into quarters. This is placed on top of the cake with the center of the cross directly over the center of the cone and is then pressed into the ore until it touches the floor. Such an arrangement is preferable to marking off with a single board. Lines of division will be more exact and the cross can be left in place to hold the sides of the sample quarters vertical while the reject quarters are being shoveled away. Or the cross may be laid on the floor before starting the pile and each shovelful dropped over the intersection. The cone is then spread out to the thickness of the cross and the reject shoveled away as above.

Coning does not mix the ore uniformly. As material is shoveled onto the cone the coarser pieces roll down the sides and come to rest on the floor while the finest particles remain near the apex. Pieces of intermediate size arrange themselves on the slopes of the pile according to their size. The ultimate result desired is that the segregation be symmetrical with respect to the axis of the cone. If this condition is attained, any sector taken should correctly represent the whole.

Bench or cobbing system, sometimes used to get better distribution of the material in the cone, consists in first making a small cone of some of the ore and spreading it out into a cake, then making another cone on the center of the cake and spreading it out and repeating until all the ore is thus disposed of. This method tends to reduce the effects of accidental errors in flattening the cone.

Advantages of coning and quartering are few. Expensive equipment is not required, the only tools necessary being wheelbarrows, shovels and brooms. It can, therefore, be used in remote places where more elaborate sampling machinery is not available. It is applicable to practically all classes of ore and, in the case of small lots of high-grade material, there is no danger of such loss as might occur through leaks and hold-backs in a mechanical sampling mill. The fact that all ore is in plain sight throughout the period of sampling is probably the chief reason why this method has survived as long as it has, this being especially desirable

where ore is being sampled for sale. DISADVANTAGES. The method is expensive, requiring frequent handling of the ore by crude means, and much sweeping. The danger of salting is considerable. From the standpoint of accuracy the method is susceptible, either through accident or intention, to such manipulation during the operation that a true sample will not be obtained. As the men shovel from the piles of ore or ring onto the cone, they should all push their shovels either radially or at a tangent into the piles or ring. Otherwise the coarse material will be in a different position on each man's shovel and when the ore is thrown onto the cone the coarse material will segregate unsymmetrically. Further, it is difficult to keep the axis vertical. If the axis is inclined, more fine material will accumulate on the side toward which it leans. This is called "drawing the center" and it has frequently been brought about in order to make the sample of higher or lower grade, as the operator desired. To avoid "drawing the center" a vertical rod is sometimes placed to mark the axis of the cone. When spreading the cone into a flat cake, there is danger that more fine material from the apex will be spread over one part than another. During dragging out, coarser pieces are pushed along to the outside by the shovels while the finer particles sift through and tend to remain nearer the center. As spreading progresses the finer material from the center is spread out and the cake obtained tends to have a separation of the coarser particles on the bottom and the finer on the top. There is also difficulty in spreading the last small portion of fines evenly. For these reasons chances of error are great and the effect of carelessness or willful abuse is difficult to detect.

Time and labor required. The following data are given by Johnson (*53 J 111, 132*): EL PASO SMELTING WORKS. Mexican labor, coning and quartering a 2-ton sample taken by the tenth-shovel method:

4000-lb. lot.		
4 men, 1 on wheelbarrow from platform to crusher, 1 man at crusher, 2 men with wheelbarrows carrying ore from crusher and forming circular ring	45 min.	
4 men shoveling from ring to cone 6-ft. diam., 3.5 ft. high	20 min.	
4 men spreading to truncated cone 9 in. high at center, 5-in. at circumference	5 min.	
4 men cutting quarters and removing reject	10 min.	
Total, 4 men		80 min.
2000-lb. sample; 4 men.		
Shoveling to a ring	6 min.	
Making cone	9 min.	
Spreading	3 min.	
Halving	5 min.	
Total, 4 men		23 min.
1000-lb. sample; 4 men.		
Making ring and cone	12 min.	
Spreading	2 min.	
Halving	3 min.	
Total, 4 men		17 min.
500-lb. sample; 4 men.		
Re-crushed in rolls, 10 × 20-in.		35 min.
Total, 4 men		155 min.
500-lb. sample; 2 men.		
Piled in a heap, spread to a circle, pushed out to ring	15 min.	
Making cone (3 ft. diam., 1.75 ft. high)	5 min.	
Spreading (6.5 in. thick at center and 3 in. at circumference)	3 min.	
Halving	2 min.	
Total, 2 men		25 min.
250-lb. sample; 2 men.		
Ground in rolls to pass 10-mesh		35 min.
Making ring	2 min.	
Making cone	4 min.	
Spreading (5 ft. × 2 in.)	2 min.	
Halving	3 min.	
Total, 2 men		11 min.

125-lb. sample; 2 men.	
Making two heaps, coning, spreading, halving	8 min.
63-lb. sample; 2 men.	
Making two heaps, coning, spreading, halving	6 min.
32-lb. sample; 2 men.	
Coning and halving	5 min.
About 16 lb. sent to bucking room.	
Total, 2 men	90 min.

Total time, 4 men, 2 hr. 35 min. = 1.29 man shifts
Total time, 2 men, 1 hr. 30 min. = 0.38 man shifts

Total .... 1.67 man shifts

Total time to reduce 2 tons to a 15-lb. sample for delivery to assay office . . 4 hr. 5 min.

5. Shovel sampling

Shovel sampling, also called FRACTIONAL SHOVELING, or FRACTIONAL SELECTION, can be applied when ore is being loaded or unloaded, or moved from one place to another by shoveling, or it may replace coning and quartering for cutting down a given lot, with intermediate coning of the sample as previously described. Every alternate, or every third, fourth, fifth or tenth shovelful is taken for the sample, depending upon the size of sample permissible or desired. Common practice is to take the fifth or tenth shovels in unloading a car and to finish sampling by alternate shovels.

Advantages. Shovel sampling is applicable to larger lots than coning and quartering and when alternate shovel loads are taken, is more reliable and accurate; it is cheaper, quicker, and requires much less space. Accuracy is attained by making a large number of sample cuts, the number depending on the size of shovel used and the weight of the lot sampled.

DISADVANTAGES: Shovel sampling should not be used if large lumps, say greater than 2-in., are present. It is subject to manipulation by the sampler in choosing the shovelful to be taken for the sample, both as regards the amount taken and the kind of pieces. The cheapest kind of labor is commonly employed and when other than alternate shovels are taken, it is difficult for the shoveler to keep count of shovelfuls. When ore is piled in a heap in which segregation has taken place and is sampled by this method there is considerable danger of the sample shovelfuls being taken more from either the coarser or finer parts of the heap, so that every size of material is not represented in proper proportion in the sample.

Time required by four Mexican laborers unloading a 20-ton car and taking at the same time a tenth-shovel sample is given by Johnson as 2.5 hr. At EL PASO smelter four men take 4 hr. unloading a 50-ton car of Chino concentrate (15 per cent. moisture) and taking every tenth shovel for sample. (*117 J 13.*) In this instance, the sample is shoveled onto boards placed across the car above the load that is being shoveled; the sample remains in the car and is moved and dumped, samples from five cars being united.

Cost of shovel sampling varies with the character of material, moisture contained and facilities available for disposal of sample and reject.

The labor cost of the old method of sampling wet, sticky flotation concentrate at GARFIELD smelter, consisting in taking every 15th or 20th shovel for sample and cutting down further by alternate shovels, was from 6¢ to 8¢ per ton of concentrate before the war to 20¢ later. (*122 P 17.*)

A variation of shovel sampling used on high-grade silver ore from Cobalt follows. (*17 CMI 239.*)

Ore coming from a trommel with 0.25-in. openings is discharged into wheelbarrows and distributed in a long narrow ridge. The ridge is turned over once by shoveling and a new ridge formed, then divided by alternate shoveling into two ridges, the shovelfuls being carefully distributed over the tops of the ridges. These two ridges are each divided into two other ridges by the same method, four ridges resulting. Further reduction of each ridge

is then made by successive crushing and riffling. Table 6 gives the assays of samples from each of the four ridges with five different ores and indicates that this method gives good results even on ores as difficult to sample as these.

Table 6. Results of shovel sampling Cobalt silver ores

Ridge	Silver, ounces per ton				
	A	B	C	D	E
No. 1	2269.7	2082.8	1527.7	535.8	353.3
No. 2	2263.6	2079.2	1533.0	538.5	355.1
No. 3	2260.7	2072.6	1528.6	540.1	358.6
No. 4	2267.2	2080.4	1526.2	554.5	355.1
Average	2265.3	2078.8	1528.9	539.7	355.5
Maximum deviation from average, oz.	+4.4 −4.6	+4.0 −6.2	+4.1 −2.7	+4.8 −3.9	+3.1 −2.2
Maximum deviation, per cent. of total	−0.2%	−0.3%	−0.3%	+0.85%	+0.9%

Combination of hand-sampling methods is used by the U. S. Bureau of Mines in sampling coal. (*Bul. 116, USBM.*) The original or gross sample of 1000 lb. for ordinary lots (or 1500 lb. when large pieces of impurities are present, or 500 lb. with slack or smaller sizes of anthracite) is taken by grab sampling or shovel sampling during unloading. The amount taken at each grab or shovelful varies from 5 to 10 lb. for slack and smaller sizes to 10 to 30 lb. for run-of-mine coal. The sampling interval depends on the size of lot. When taking the sample from bottom-dump cars, while unloading, a special " ladle " is used having a handle 5 ft. long and bowl 1 ft. in diameter at top, 9 in. at bottom, and a depth of 9½ in. The " ladle " holds from 25 to 30 lb. of coal. Generally two ladlefuls are taken from each car by holding the ladle underneath the car when dumping. The gross sample is cut down by alternate shoveling to 250 lb. and coning and quartering to 15 lb. as follows:

1000 lb. gross sample. Crushed to 1-in.; mixed by shoveling into a conical pile, then shoveled into a pile 5 to 10 ft. long and alternate shovels taken giving a **500-lb. sample,** which is crushed to ¾-in. and treated as above, giving a **250-lb. sample.** This is crushed to ½-in., mixed, by shoveling, into a conical pile and then shoveled into another cone, flattened and quartered, giving a **125-lb. sample,** which is crushed to ⅜-in., placed on a blanket or canvas (6 ft. × 8 ft.), mixed by rolling, coned by drawing up the ends of the cloth, then flattened and quartered, giving a **60-lb. sample.** This is crushed to ¼-in., placed on the blanket and the preceding operation repeated, giving a **30-lb.** sample which is crushed to 3/16-in. and again subjected to rolling, coning, flattening and quartering, giving **15-lb. sample.** Two **5-lb.** samples are taken from the two 7½-lb. sample quarters, and are placed in sample containers.

Every precaution as described in Art. 4 is taken in forming the cones and shoveling from them. The first shovelful for forming the long piles is spread out in straight line, having a width equal to the width of the shovel and length of 5 to 10 ft. The next shovelful is spread directly over the top of the first but in opposite direction and so on, back and forth. The pile is occasionally flattened during this procedure. Alternate shoveling from the long pile begins at either end on one side and successive shovelfuls are taken the width of the shovel apart as the shoveler advances along one side and around the pile, always in the same direction. Crushing may be done mechanically or by hand with an iron tamping bar or sledge on a sheet-iron plate or solid floor. Flattening of the cone is accomplished by pressing the apex vertically with the flat of the shovel or a board.

Trench sampling (*Smith, loc. cit.*) consists in spreading the ore out into a flattened square or rectangle, about 2 or 3 ft. thick with 100-ton lots and from 1 to 2 ft. thick for smaller lots, and cutting trenches through the cake about 1 ft. wide, crossing at right angles at the center. All material thus excavated is taken for the sample or alternate shovelfuls may be taken. Sometimes all of the material from the trenches is rejected and samples are taken with a shovel from the sides of the four piles left. The accuracy of this method depends on the degree to which material is mixed before making the flattened rectangle. If the rectangle is made by spreading out from a cone, the sample will probably contain too great an amount of fine material, as the same width of cut is taken from the center as from the edges. (See Art. 4.) As the sides of the trenches take the form determined by the angle of repose of the material, it is difficult to be sure that the same proportion of ore is taken from the top as from the bottom, and if there is a difference between the top and bottom layers, the sample will not be accurate.

Quartering shovel (Fig. 2) is a shovel about 10 in. wide, with flanged edges and with two partitions on the blade dividing it into three spaces. The two outside spaces cover three-fourths of the area of the shovel blade and are open at the back. The center space, about 2.5 in. wide (one-fourth of the area) is closed at the back. The sampler pushes the shovel into an ore pile and then, with a quick rotary motion, swings the loaded shovel. The ore in the two outside spaces slides off the back of the shovel and goes to the reject, while the portion in the center space remains on the shovel and is thrown into a wheelbarrow or other receptacle as the sample.

This device does not give accurate results. There is a tendency for fines to be more in evidence in the central space and coarser pieces in the two outside spaces. The method is seldom used, as it offers no advantages over fractional shoveling and is less accurate.

Fig. 2.—Quartering shovel.

Fig. 3.—Split shovel.

Split shovel or U-shovel (Fig. 3) is made up of several troughs, generally about 12 to 18 in. long, 2 in. wide and 2.5 to 4 in. deep, with intervening spaces of the same width. Material to be sampled should be crushed to 0.5-in. or less. The shovel is laid on the ground or held by an operator and another man shovels the ore onto the split shovel with a square-end shovel. When the troughs are filled, the first man throws the split-shovel load to the sample heap. The troughs must not be allowed to overflow.

This method is slow, requires two conscientious and careful men, and is not as accurate as the alternate-shovel method.

6. Pipe sampling

Several varieties of pipe or gun samplers are used. The simplest form consists of a piece of pipe (0.5-, 1-, or 1.5-in. diam.) one end with sharpened edge, the other fitted with a tee and two short nipples to form a handle. The pipe should be long enough to reach the bottom of the heap to be sampled; it is thrust vertically to the bottom of the heap at regular intervals, and then withdrawn and pounded with a hammer, to release the sample.

Several forms are shown in Fig. 4. (*A*) consists of pipe and fittings made up as above, with a longitudinal slot cut out. It is pushed into the mass to be sampled, twisted around until filled, and then withdrawn. (*B*) is made up of two such slotted pipes, one smaller than and within the other. It is pushed into the ore with the pipes in position (*a*) until it reaches the bottom. The handle is then turned until the pipes take position (*b*) and is twisted back and forth until the inner pipe is filled with material. The pipes are then again turned to position (*a*) and withdrawn and emptied into the sample box or sack.

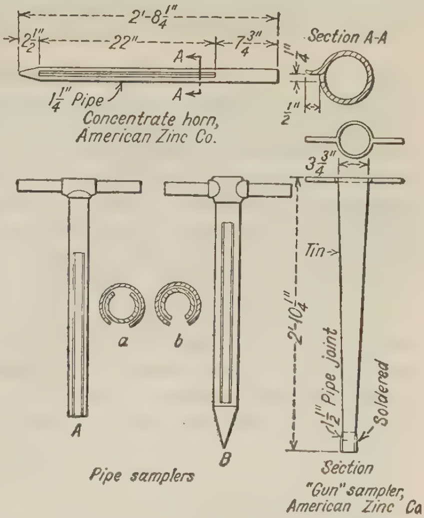


FIG. 4.—Pipe samplers.

Applicability. Pipe samplers are satisfactory for sampling fine concentrate in railroad cars or bins, provided the material has not packed down too hard. They are also used for sampling sand in vats before or after treating with solution in percolation extraction processes. They can be used on tailing heaps or on any finely-crushed ore. Small pipe samplers are used for sampling material in sacks or cans. Pipe sampling gives quick results, is cheap, and, for concentrate or vat charges, where the material is well mixed, results are as accurate as necessary. Pipe sampling of concentrate or other fine material in cars is usually done by taking samples at regularly spaced intervals.

In MONTANA (*121 P 866*) the usual method is to make insertions of pipe in parallel rows spaced at 2-ft. intervals. The pipes are 4 to 5 ft. long, 3 in. diam. at top and 2 in. at the cutting edge. About 250 lb. is usually obtained from 40 to 75 insertions of pipe. The expectable error is within 1 part in 100. Two men should sample the material in a 50-ton car in about 30 min.

Auger samplers, resembling ordinary earth augers, are also used in the same manner and on the same classes of material. They have the advantage that they can be used on material that has packed so hard that it is difficult to force a pipe or "gun" sampler into it.

MACHINE SAMPLING

Automatic machine samplers are devices designed to substitute mechanical processes for the undesirable human element in hand-sampling. By so doing they lessen or eliminate the accidental and intentional errors introduced by the personal factor, shorten the time required for cutting a sample and reduce the amount of operating labor. Against the reduction in labor and time are to be charged the consumption of a small amount of power, a capital investment with attendant interest and amortization, repair and maintenance charges. Further, in a custom sampling plant, cleaning out between different lots of ore is a tedious and difficult job. Machine samplers have difficulty in handling sticky ore.

The fact that ore being machine-sampled is not in sight throughout the operation in a custom mill, delayed the substitution of machine for hand methods, but at present practically all large custom mills use machine sampling and find marked advantages in the ease and uniform accuracy of operations.

Sampling machines require the ore in motion in order to present it to the cutters as a thin ribbon or stream. They are of two general types: (*a*) Those that take part of the stream all the time (stationary devices); (*b*) those that take all of the stream part of the time (moving devices).

7. Stationary machine samplers

Stationary cutters are designed and used on the assumption that there is no variation in the character of material in a transverse section of an ore stream. This is practically never the truth. Generally the material in a moving stream varies more irregularly and less gradually in a transverse section than along its direction of flow. In broken ore sliding down an inclined chute, the coarse pieces slide on top and toward the center, while the fines are on the bottom and against the sides. When such a stream changes direction, the coarser and heavier pieces are carried to the far side of the chute, due to their greater momentum, while the finer particles are not carried so far. The result is a stream made up of coarse on one side and fine on the other, which does not subsequently mix and become uniform. The following conditions may cause similar results (*A. W. Warwick, Notes on sampling, Industrial Print. and Pub. Co., Denver, 1903*): (*a*) Two elevators throwing ore into the same chute, one elevator raising low-grade ore and the other richer ore; if the streams do not cross, little mixing takes place. (*b*) Feeding an elevator from the side instead of from the front. (*c*) Irregular feed of coarse and fine material to a crusher, or coarse material on one side and fine on the other; (*d*) anything that produces an eddy causes segregation between different sizes in an ore stream. Stationary cutters can not be expected to deliver an accurate sample unless ore is presented thoroughly mixed.

Whistle-pipe sampler in its simplest form consists of a vertical iron or steel pipe with notched openings cut halfway through the pipe, each opening spaced 90° horizontally from the preceding. Rectangular steel plates are placed in the notches so that top edges coincide with a diameter of the pipe. Above each notch is placed a hopper-shaped cast-iron liner which gathers the ore into a compact stream before presenting it to the dividing edge. Ore is poured into the pipe at the top. When the stream reaches the first splitter, half is deflected outside the pipe and falls into the enclosing bin or housing as reject. The remainder continues down the pipe and is halved by each succeeding edge. The portion passing the last cutter constitutes the sample. With five dividing notches, *e.g.*, the sample is $\frac{1}{32}$ of the weight of the total material fed to the pipe.

Fig. 5 (*Warwick*) shows details of an improved type of whistle-pipe sampler, in which material is fed at each cut to a splitter (*m*) that divides the stream into quarters. Opposite quarters are rejected, the other two continue and are again split in the same manner. In this design deflectors are spaced at 45° horizontally.

Advantages of whistle-pipe samplers are cheapness of installation and operation, simplicity, and quick reduction of bulk. No power is needed. Disadvantages are such as to preclude their use when an accurate sample is desired. In the simple type first described, wear on the cutting edges causes displacement thereof so that a changing proportion of the stream is taken as wear progresses. The improved design with vertical cutting edges dividing the stream in quarters avoids this disadvantage. Fuse, waste, rags, wood and similar materials catch on cutting edges and cause improper splitting and may clog the openings. Clogging cannot be detected during operation. The general objections to machines that cut part of the stream all the time also apply. Uneven wear of cast-iron liners may cause segregation. No opportunity is afforded for re-crushing between cuts hence the whole lot must be reduced to a size commensurate with the bulk of the final sample. Expense of such crushing would be out of the question in a custom sampling mill, and would not be justified in any mill where the ore could be treated in a coarser state than sampling demanded.

Single-split samplers consist of a splitter or divider placed in such a way in a stream of material issuing from a launder or chute as to continuously deflect a portion of the stream into a sample hopper or container. They are more inaccurate than the whistle-pipe samplers and have little to commend them.

Bank or combination riffles are usually made of five sets of riffles arranged as shown in Fig. 6 (*TP 86, USBM*), each riffle made up of a number of troughs of equal width, separated by cutting edges, adjacent troughs discharging in opposite directions. For detailed description of single riffles see Art. 11. Material is fed in a uniform sheet over the first riffle and split into many streams. Half of these discharge one side and half

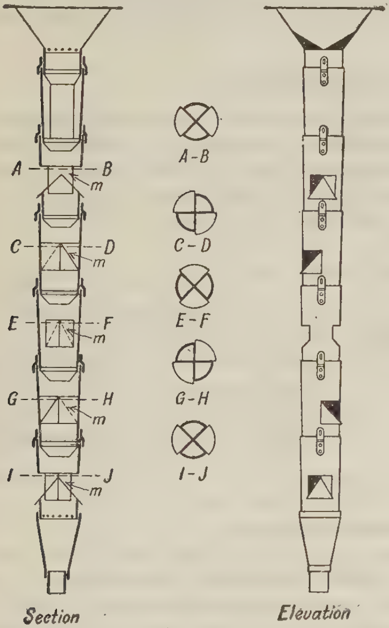


Fig. 5.—Whistle-pipe sampler.

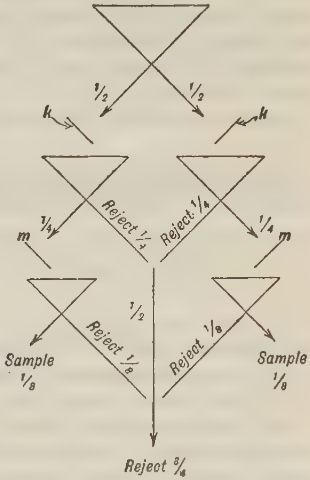


Fig. 6.—Riffle bank.

the other. Discharges from the first riffle impinge on inclined plates (k) and fall to a second set of riffles, smaller than the first. These riffles produce four main streams, two of which unite and pass in to the reject, while each of the others falls to deflecting plates (m) and then to a third set of riffles, smaller than the second, which again halve the streams. One-half from each of the last riffles is rejected, while duplicate samples, each representing one-eighth of the whole, are furnished by the others. The riffles are supported by a rigid frame or may be so hung that one or more sections can be swung across a falling stream of ore.

This machine makes many cuts of the ore stream and, therefore, has ADVANTAGES over methods that make only two or four cuts. It is simple of operation and, as it is generally used on fine ore, wear on the dividing edges is not great. The DISADVANTAGES are that the device is one of the class that cuts part of the stream all the time; construction is not simple; great care is required to insure that each compartment is of same width as others; there is a

possibility that during operation dividing edges will become bent or knocked out of shape so that they do not make a proper division; damp ore and foreign waste clog the riffles, and clogging may not be discovered for some time on account of inaccessibility. Clogging causes one part of the riffle to take no sample or deflects too much into the sample. Tapping with a hammer will obviate clogging, but may deform the riffle edges. If the feed is not carefully delivered to the first riffle, segregation of the coarse and fine material is probable, producing an improper sample. Damp, fine material sticks to the deflecting plates and may cause deflection of coarse pieces with resulting segregation. Riffles are sometimes swung or shaken to avert clogging and segregation.

8. Moving machine samplers

Principle. These take all of the stream part of the time and thus eliminate errors due to changes in character of the stream across its section. They remove equal sections across the stream at frequent regular intervals and since change in character of the material in a stream along its direction of flow is gradual the average of all cuts represents the average grade of the whole mass of material.

Suppose an ore worth $10 per ton passing through a mill. On top of this ore in the feed bin is a lot of ore running, say, $30 per ton. As ore is drawn and the $10 ore becomes nearly exhausted, the grade of ore in the stream gradually changes. If Fig. 7 represents the changes in value along the stream, and the diagonal lines represent sample cuts, it is apparent that the average of assays of the sections cut is the average of assays of all sections from the $10 section to the $30 section. The result is the same if even-numbered sections are taken. Abrupt changes require frequent cuts, the more frequent the cuts the more accurate the sample.

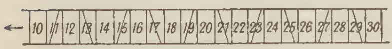


FIG. 7.—Sampling cuts in stream of changing value.

Requirements for accurate machine sampling are:

(1) The sampler should take the whole of the stream part of the time.

(2) Equal percentages should be taken from all parts of the stream. This necessitates that the cutter travel at a uniform rate while passing through the stream, and the scoop move completely across and out of the stream at each cut. If the rate of motion of the cutter is variable, the largest percentage will be taken from those parts of the stream where the cutter moves most slowly, and unless the cutter moves out of the stream on both sides, one side will be unduly weighted in the sample.

(3) The interval between cuts should be constant, otherwise all sections along the length of the stream will not be equally represented.

(4) Cuts should be frequent enough to insure representation in the sample of the most abrupt change in the character of the stream.

(5) The distance between cutting edges should be great enough to allow the largest pieces of material free passage between them; this distance should be at least three times the diameter of the largest piece, better four times or more.

(6) The depth of the scoop should be sufficient to prevent material from bounding or splashing out.

(7) Scoops with closed bottoms should be large enough to hold all of the material taken in one cut without danger of overflowing. The opening to discharge the sample should be large enough to empty the scoop quickly and prevent sedimentation.

(8) Feed should be constant; if the stream is intermittent, the cutter may receive no sample during some of its passages.

(9) Speed should not be great enough to knock away pieces that should go into the sample. In a revolving sampler speed should be so low that centrifugal force will neither throw material out nor prevent discharge.

(10) The stream delivered to the cutting edge should be as narrow as possible in order to lessen the time of passage of the cutter through the stream. The limit of this contraction is determined by the size of the largest particle.

(11) The ore should be thoroughly mixed and all devices that tend to cause segregation eliminated.

(12) The drive should produce uniform motion of the cutter. Relatively high speeds prevent jerky motion. Loose belts should be avoided. It may be advantageous to reduce from a high-speed belt-driven countershaft by gears.

General principles of sampler construction. The following, while not essential for accurate sampling, are desirable characteristics for any sampler. (*a*) The construction should be rugged enough for the service. (*b*) All parts should be easily accessible, both for observation and repairs. (*c*) The feed should be delivered to strike the cutting edge at right angles to the direction of motion of the edge. (*d*) Clearance between the feed spout and the cutting edge should be at least 1.5 times the diameter of the largest particle, to prevent clogging or jamming.

Many different samplers are used; some capable of accurate work, others not. A few machines have become well known and their design is more or less standardized. On the other hand many plants design and build their own samplers, intended to meet local requirements. Moving sampling devices may take the form of a deflecting surface diverting the sample out of the stream; or there may be a deflecting surface which diverts the whole stream, with an opening to allow the sample to pass through; or a scoop may cut through the stream and catch the sample, discharging either through an opening in the scoop or by dumping at some convenient point. The greatest variation in samplers of local design is in the method of driving.

9. Moving samplers for dry ore

Brunton vibrating sampler is shown in Fig. 8. Ore fed through chute (*c*) passes through spout (*b*) in a narrow stream or ribbon to cutting edge (*a*), consisting of a vane oscillating on shaft (*p*). The vane deflects the stream first into chute (*e*), then into chute (*d*). The ore delivered by one of these chutes is taken as a sample. The vibrating cutting edge is actuated by connecting rod (*j*), connected to sliding bar (*i*); motion is imparted to (*i*) through bent rods (*n*) attached thereto and pins (*l*) in wheel (*h*), which is mounted on a shaft with pulley (*g*) and driven by belt from a convenient source of power. Wheel (*h*) has two rows of 20 holes each in the rim to receive pins (*l*). Rows of holes are spaced the same distance apart as the sliding bar is required to travel. Frequency of cuts and the time that the cutting edge deflects the stream into sample is regulated by arrangement of pins. Each hole represents 5 per cent. of the time required for one revolution of the wheel.

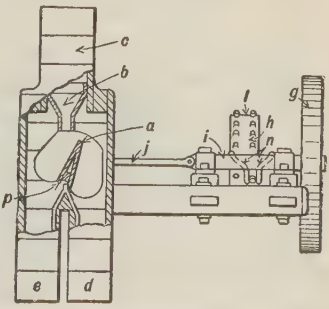


Fig. 8.—Brunton vibrating sampler.

The fundamental error in this machine is that it takes more from one side of the stream than from the other as shown in Fig. 9, *A*; if one side of the stream is the richer, an accurate sample is not secured.

Brunton oscillating sampler (Fig. 10) consists of a hopper (*a*) which receives ore and discharges through vertical chute (*b*) to an oscillating deflector consisting of two reject-deflecting surfaces (*m*) and a sample-deflecting surface (*n*), placed between and separated from the surfaces (*m*) by cutting edges. The machine is so arranged that sample and reject are discharged in opposite directions. The oscillator is made of stiff, strong sheet metal and is attached to shaft (*p*), which imparts an oscillating motion in a vertical plane to the cutter, by means of a counterbalanced crank and connecting rod. A housing of sheet iron surrounds both chute and oscillator. The sampler is usually arranged to take a 20-per cent. cut, but by the use of elliptical gears on the driving mechanism a 5 per cent. cut can be obtained. The cut may also be varied by changing the size of the sample-deflecting spout.

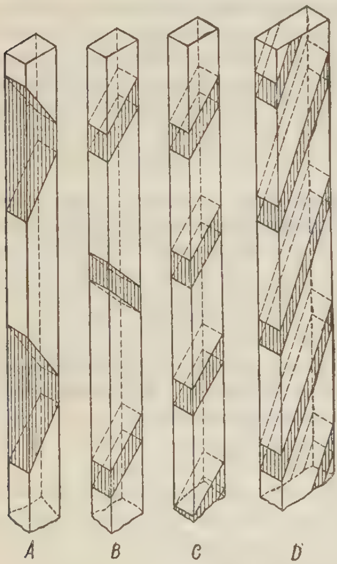


Fig. 9.—Cuts taken by samplers.

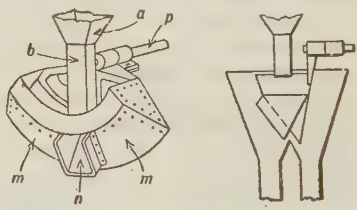


Fig. 10.—Brunton oscillating sampler.

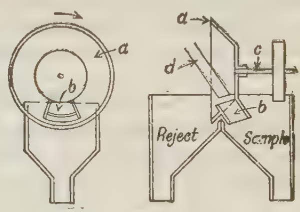


Fig. 11.—Snyder sampler.

At each oscillation the sample spout passes completely through and out of the stream. The cutting edges are short and made of special steel to prevent warping and excessive wear. The side walls of the sample spout should be in planes that pass through the center of the oscillating shaft; otherwise the angular sampling distance will change as the cutting edges wear. The speed may be as great as 72 oscillations per minute. Rapid motion aids materially in discharging, especially if the ore is wet or sticky, and tends to keep the cutting edges free from accumulations of foreign waste.

Advantages. The machine takes a cut of the stream as represented graphically in Fig. 9, *B*. This is correct sampling; the great number of small cuts taken insures an accurate sample. The sampler requires but little head room, is easily accessible for examination and repair, and is quickly cleaned. The fact that sticky ores might clog the spouts is the only serious disadvantage; this danger is common to practically all mechanical samplers.

Snyder sampler (Fig. 11) consists of a pan-shaped plate (*a*) with one or more sample spouts (*b*), supported on and rotated by a horizontal pulley-driven shaft (*c*). The plate and spouts are usually a single strong casting but may be fabricated of good sheet steel. Feed enters through chute (*d*), the

reject is deflected by the conical plate and the sample passes through the spout into the sample hopper. To prevent change in sample weight as the cutting edges wear, the side walls of the sample spout should lie in planes that pass through the axis of rotation. The amount taken for sample is varied by changing the size or number of sample openings; frequency of cuts is varied by changing the speed of rotation or number of openings. The shape of the cut is shown in Fig. 9, *C*.

ADVANTAGES.—The machine is simple in construction, easily cleaned and repaired and can be readily observed during operation. Foreign waste that catches on the cutting edges is readily freed as the disk revolves. Little head room is necessary. A serious DISADVANTAGE is that with wet or sticky ores, fine material adheres to the reject-deflecting surface and as the disk revolves falls through the sample opening; thus salting the sample.

Three sizes are made:

Diameter, inches	Maximum size of feed, inches
60	4
42	2½
27	1

Vezin sampler (Fig. 12) consists of two hollow truncated cones, (*a*) and (*b*), joined base-to-base, with one or more scoops (*c*) attached to the upper cone, all mounted on a vertical shaft. Ore fed through chute (*d*), which may be vertical or inclined, normally passes into hopper (*e*) and is rejected through spout (*f*). As the sampler revolves, chutes (*c*) cut through the stream and divert parts into the conical chamber and through spout (*g*) as sample. The inventor recommends an inclined chute delivering ore so that the horizontal component of velocity is the same as that of the cutting edges. This gives the effect of a piece falling vertically on these edges. Warwick (*loc. cit.*) states that the mode of delivery, whether vertical or inclined, has no effect on the accuracy of the sample.

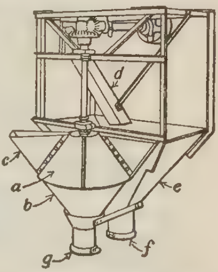


FIG. 12.—Vezin sampler, single.

The cutting edges should be radial to the axis of rotation and the sides of the scoops vertical; otherwise the scoop will take more from one part of the stream than from another. In Fig. 13, *A* the edges are radially placed, the angular velocity is the same at all points, and all parts of the stream, therefore, flow into the sample for the same time. This gives a correct cut. The shape is shown in Fig. 9, *D*. In 13, *B*, the lines of the cutting edges intersect on the scoop side of the center, the arcs subtended increase from inside to outside of the described annulus and a smaller proportion of the stream is cut from the inner edge than from the outer. The opposite case is illustrated in 13, *C*.

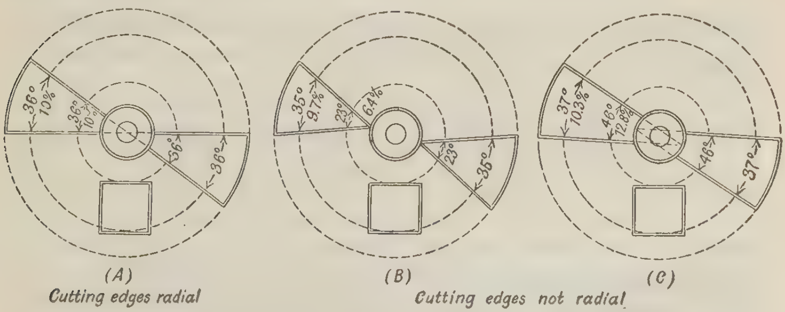


FIG. 13.—Arrangements of cutting edges in Vezin-type samplers.

The proportion of the stream length that is taken for the sample is determined by the ratio of the number of degrees of arc subtended by the sample spouts to 360°. For a 10-per cent. cut a single sample spout should subtend 36° of arc, or with two spouts each should subtend 18°. The speed has no effect on the amount taken for the sample but affects only the FREQUENCY of the cuts; frequency can be varied also by changing the number of scoops. Usually one or two scoops are used, sometimes four. Two samplers are sometimes mounted on opposite sides of the same reject hopper in order to take duplicate samples. SPEED should not exceed 3 ft. per sec. at a point on the scoop at the center of the feed stream, in order to prevent knocking pieces out of the stream, throwing them out of the scoops by centrifugal force, or hindering discharge. Width of scoop should be at least four times the diameter of the largest piece of material at the center of the stream and the opening into the cone should be at least 2.5 times this maximum diameter. Cutting edges should be made of hard, brittle steel to avoid distortion and wear; chips lost will probably be small and not materially affect operation; but bending changes the whole aperture.

Fig. 14 shows a simple and efficient design for cutting edges used in sampling COBALT silver ores. The scoop is made of tempered steel, tapered for a short distance at the top to make a knife edge. Small wedge-shaped pieces of steel are placed between the scoop walls to hold the edges rigidly in place. For ordinary use such careful sharpening of edges is not necessary or common but Cobalt ores are difficult to sample and it is necessary to take every precaution to attain accuracy.

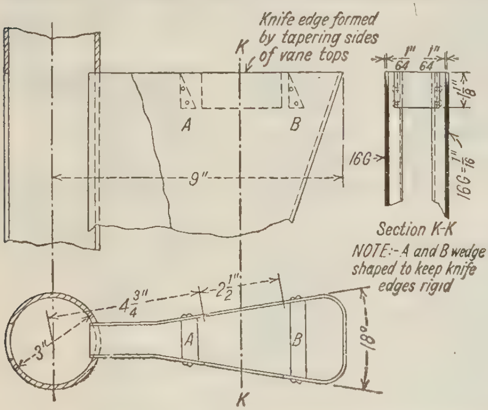


FIG. 14.—Cutting edges on Vezin samplers at Cobalt.

ADVANTAGES of Vezin samplers are the ability to take an accurate sample; simplicity of construction; and easy accessibility for observation and repair. DISADVANTAGES are the head room required, danger of clogging, lack of ruggedness, possible improper construction of scoops, and delivery of feed in a manner to cause segregation and inaccurate sampling. The machine should be constantly watched to detect lodgment of foreign material on the cutting edges and to be sure that the scoops are not clogged. Hammers or mallets should be used only with the greatest care, if at all, to free clogged chutes on account of danger to cutting edges.

Size. Vezin sampler is made in the following sizes:

Number	Max. size feed, inches	Weight, single, lb.	Weight, duplicate, lb.
1	1	500	850
2	2	850	1300
3	4	900	1600

Chas. Snyder sampler (Fig. 15) is similar to the Vezin sampler, except that it has four sample scoops and the feed chute, instead of being square or rectangular, is annular and extends over an arc of 90°. As a result there is a

sample scoop under the feed stream at all times. Feed is scattered by a number of short iron rods extending across the feed chute; these tend to reduce segregation.

Fig. 9, *D* shows the kind of cut made by this machine. The ADVANTAGES and DISADVANTAGES are the same as those of the Vezin.

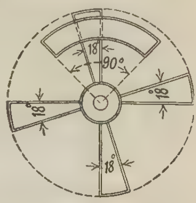


FIG. 15.—Chas. Snyder sampler for 20-per cent. cut.

Simplex sampler is fully shown by Fig. 16. It can be arranged to take cuts from 5 per cent. to 30 per cent. As in other rotating samplers, the sides of the cutter should be in planes passing through the axis of rotation.

The machine is made in the following sizes:

Size, in.	Sample, per cent.	Pulley, in.	Speed, r.p.m.	Weight, lb.
28	10	4½ × 24	24	600
44	10	4½ × 36	15	700
64	10	4½ × 48	10.5	850

ADVANTAGES are simplicity in construction; small head room required; easy inspection and repair. DISADVANTAGE. Wet ores stick in the deflecting scoops, are carried up and fall into the reject, causing, of course, inaccurate results.

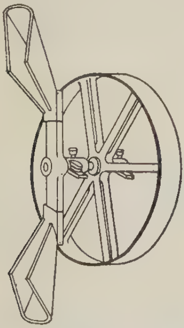


FIG. 16.—Simplex sampler.

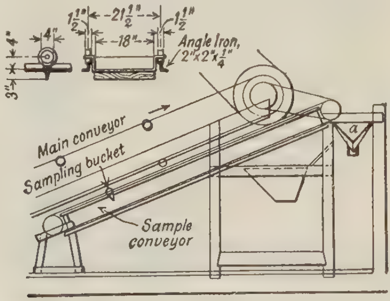


FIG. 17.—Van Mater sampler.

Van Mater sampler (Fig. 17) consists of a sprocket-chain sampling conveyor set under and carrying a sampling bucket through the discharge of the main conveyer. The sampling conveyor is driven by pulley and belt from the head end of the main conveyor. Any number of sampling buckets may be used according to the frequency and size of sample desired. The buckets must be at least as wide as the stream delivered by the main conveyor. The sample cut by the buckets is dumped into a sample hopper (*a*) as the bucket passes over the head sprockets. A wooden wearing block is fastened to the bottom of the buckets to take wear when the bucket passes through the stream of ore on the return trip. The proportion taken for a sample is the percentage of the total length of sprocket chain that is represented by the distance between the cutting edges of a bucket, multiplied by number of buckets. The bucket must be deep enough to prevent material from bounding out. *(88 J 1282.)*

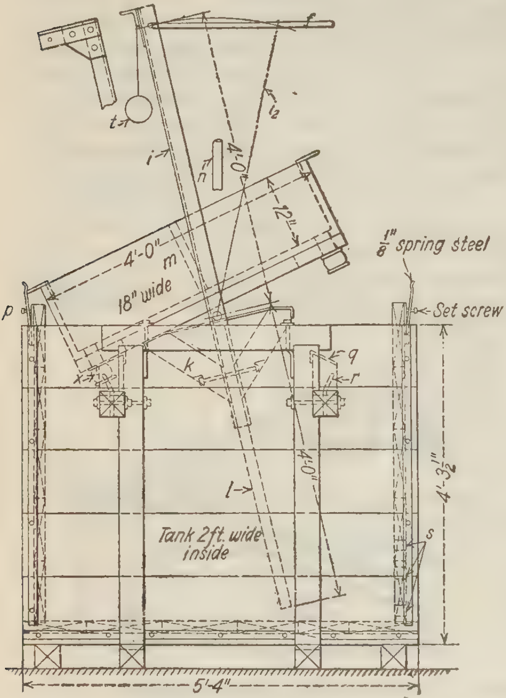
Damp and sticky ores. The machines thus far described are especially adapted for ores that are dry or nearly so. With damp or sticky ore that tends to ball up or stick to machine surfaces, they are unsatisfactory. Such ore is generally dried prior to sampling or some method of hand sampling is used.

Martin sampler was used to sample flotation concentrate at GARFIELD smelter. The machine was mounted on a traveling gantry. A 1¼-yd. clam-shell bucket unloaded concentrate from 75-ton flat-bottom railroad cars and dumped through a grizzly into a 20-ton hopper. The bottom of the hopper, 9 ft. square, consisted of a belt conveyor which carried material through a 6-in. slot 9 ft. wide in one side and discharged on to a "chopper" consisting of eleven 15-in. revolving disks spaced 12 in. on a shaft with No. 12 spring steel wires threaded through holes near the circumference parallel to the shaft. This cut the stream into strips which fell onto a 30-in. revolving hollow cylinder with a 16-in. section of the surface cut out along the entire length, making a 1/10 cut. A 12-in. conveyer inside the cylinder carried the sample to a smaller "chopper," thence to a second cylinder cutting out 1/15th and thence by another axial conveyer to a sample car. Rejects passed to a large conveyer which delivered to the bedding bins. (*114 P 17.*)

10. Sampling wet pulp

Sampling freely-flowing pulps is relatively simple; small cuts may be taken, the products are well mixed, of even grade, and, if flowing, the variations along the stream are gradual.

Cutters vary in shape according to the direction of travel of the stream as cut.



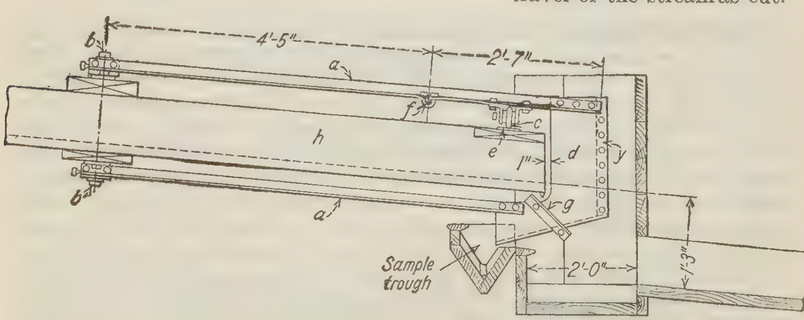


FIG. 18.—Water-tilting box and cutter.

The cutting edges should travel in a plane perpendicular to the stream when cutting. The scoop should be large enough to prevent loss by splashing and splash boards should exclude from the sample any part of the stream that does not enter the cutter directly. The cutting edges should be straight, sharp, easily accessible for observation and repair, and far enough apart to prevent clogging; the aperture is usually made as narrow as permissible to lessen the bulk of each cut and permit a large number of cuts in a small total sample.

Actuating devices are of many varieties. The simplest is a lever automatically operated by a tilting box; cranks, eccentric gears, various simple quick-return mechanisms; pistons, air-, steam-, or liquid-driven; and electrical devices are also used. The essential requirement is uniform, rapid movement to yield a small cut but not to exclude material on account of rapidity. There should be adequate insurance against sticking in mid-stream.

Timing devices are used to insure equal time intervals between cuts. They may be an integral part of the actuating mechanism, as the water feed to a tilting box; or entirely external, as a clockwork mechanism with electrical contacts to control a reversing motor, or tilting boxes to control piston valves. When external timers are used, one such frequently controls all samplers in a section or entire mill.

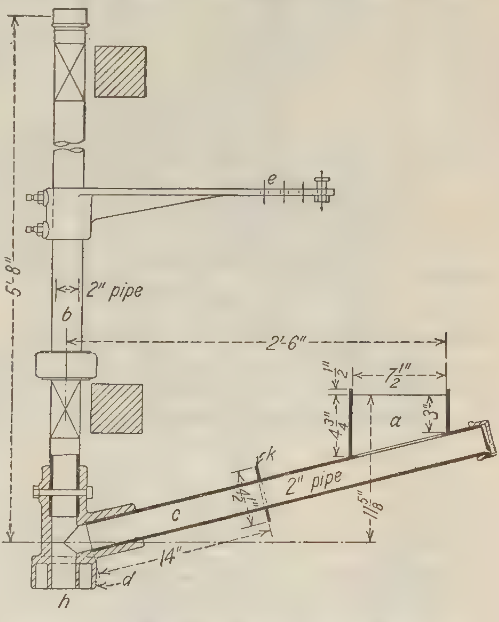


FIG. 19.—Pulley-driven sampler, St. Joseph Lead Co.

Vertical-edged cutter traveling on horizontal circle, actuated and timed by a tilting box is shown in Fig. 18. The scoop (d) is made of a piece of sheet steel cut and bent to proper slope for discharge of the sample and riveted against a ¾-in. wedge-shaped block to close the back; angles (g) prevent drip from sides into the sample. It is carried on a stiff frame (a), made of two pairs of 1½ × 1½-in. angles, back-to-back, carried on pivot bolts (b), roller (c) and plate (e). The carriage is dragged back and forth across the stream by rod (f) attached to upright (i) of a tilting box. The upright is pivoted on a shaft; it carries below the pivot a bumping frame (k) and, at the lower end, paddle (l). Teeter box (m) is also pivoted on the shaft. This box is divided by a central transverse partition and is provided with two check valves. Operation is as follows: With the box empty and in the position shown, water enters the upper compartment through pipe (n). When this compartment is full enough to cause release of the spring catch (p) the box tilts freely until it strikes the upper end of bumping frame (k), which then moves downward and swings the upright (i) across into position (i_2), and thus, through rod (f), drags cutter (d) across the stream. At the end of the swing the tilting box strikes bumper (q) and pin (r) opens check valve (x), allowing the water to run from the tilting box into the tank below. The other end of box (m) now

fills and the cycle is repeated. The level of water in the lower tank is regulated by plugs (*s*) at such a height that the paddle takes up the shock of the striking teeter box and causes uniform motion of the cutter. The sensitivity of the mechanism is increased by the suspended weight (*t*) which raises the center of gravity of the tilting frame; this weight also aids in holding upright (*i*) in off position against the weight of the paddles, and, by reason of its inertia during the swing, prevents stoppage at dead center. Timing is regulated by a valve on water line (*n*) which should be on an open-pressure tank with float valve or constant overflow.

Horizontal-edged cutter traveling on horizontal circle, actuated and timed by pulleys and gearing, is shown in Fig. 19. The cutter is a slotted pipe (*c*) with vertical cutting edges (*a*) in planes passing through the axis of shaft (*b*). This shaft is actuated by a reciprocating mechanism similar to the Scobey timer (see p. 1154). The sample receptacle is at (*h*). Ring (*d*) excludes drip.

Horizontal-edged cutter traveling on vertical circle, piston actuated is shown in Fig. 20. The cutter consists of steel sheets (*a*) mounted on a slot in inclined pipe (*b*) supported by stirrups (*c*) and swung through a stream issuing from the bottom of box (*d*) by movement of the water-actuated piston (*e*) controlled by a Scobey timer (see p. 1154). Reject discharges from the side of box (*f*), sample passes into (*g*) and through pipe (*h*) into sample receptacles (*k*).

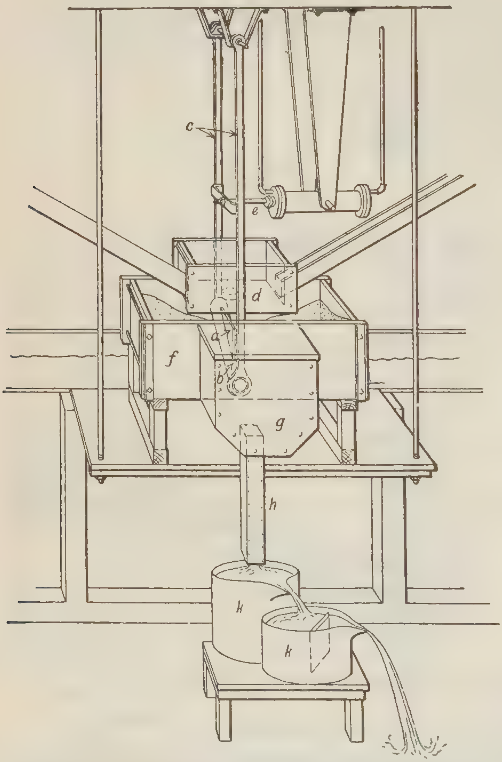


Fig. 20.—Piston-actuated sampler, Alaska Gold Mining Co.

Inclined-edged cutter with straight-line travel, electrically driven and timed is shown in Fig. 21. The cutter (*a*) is a special casting with replaceable cutting edges, suspended by arms (*b*) from a carriage (*c*) running on rollers on track (*d*). Motion is attained through rack (*e*), fastened to the carriage, and reducing gears (*f*) from reversing motor (*g*) controlled by a timing drum and electrical connections as shown in the figure. With the timing drum in position shown and contact blocks, operated by lug (*k*) and rod (*m*), in the right-hand position, the motor connections (*h*), read clockwise, are (*A*), (*C*), (*B*) and the motor runs counter-clockwise. With the timing drum at 180° from this position and the contact blocks in the left-hand position, the motor connections read (*A*), (*B*), (*C*) clockwise and the motor runs clockwise. This timer provides for one cut every 8 min.

Vertical-edged cutter with straight-line travel, actuated and timed by tilting box is shown in Fig. 21*a*. Here three scoops, sampling as many pulp streams, are mounted on one carriage driven by the tilting box.

Borcherdt sampler (*101 J 524*) (Fig. 22) uses the pulp stream for motive power. It consists of two wheels (*h*) and (*i*) mounted on ball bearings on a vertical shaft. Each wheel is divided

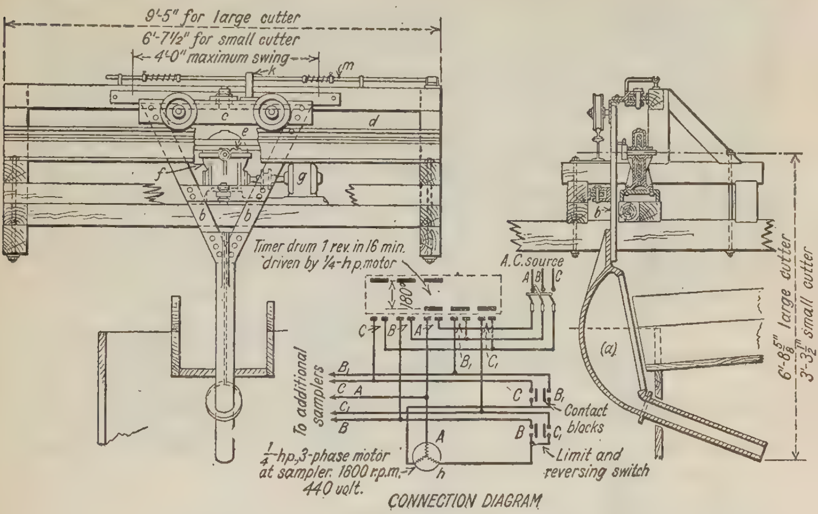


FIG. 21.—Electric sampler, Chino-Copper Co.

into a large number of equal pockets by means of thin pieces of sheet steel, the plane of each sheet passing through the axis of rotation. One pocket of each wheel is fitted with a false bottom (*f*) so arranged that any material entering the pocket is discharged toward the center of the wheel; the remaining pockets are open. The proportion of the stream cut as a sample depends on the number of pockets on the wheel. Pulp is delivered to the upper wheel from a covered box (*a*) through a number of short pipe nipples (*b*) slightly inclined so that pulp passing from them causes the upper wheel to rotate. Pulp caught by the closed pocket of the large wheel is delivered to a distributor (*c*) which spreads it over about one-third the area of the smaller wheel. The latter is rotated by the action of the falling pulp on eight inclined paddles (*d*) fastened to the periphery. Pulp caught in the closed pocket of the small wheel is delivered to the center and discharges through pipe (*e*), passing out of the bottom of box (*g*), which encloses the apparatus. The main pulp stream falls through both wheels and discharges through a launder let into the side of box (*g*). The two wheels rotate in opposite directions at about 7 r.p.m. for the larger

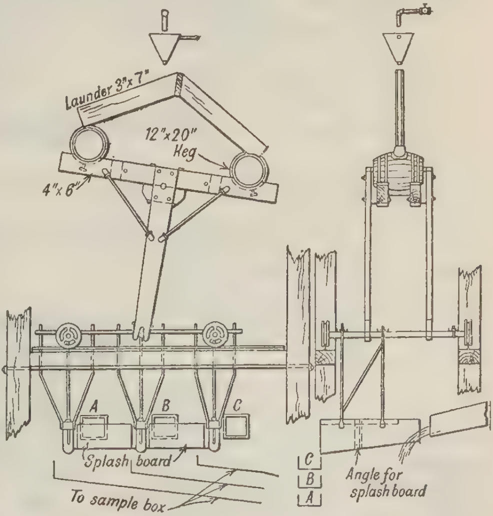


FIG. 21a.—Tilting-box sampler with three scoops.

and 13 for the smaller. The sampler works without splash on tailing from a 45-stamp mill carrying 180 tons of solids and 1440 tons of water per 24 hr.

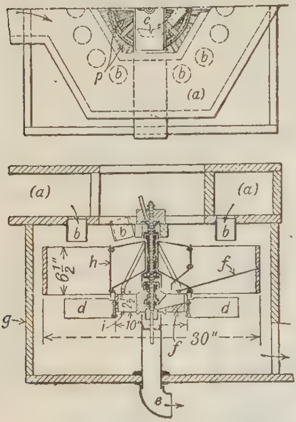


FIG. 22.—Borcherdt sampler.

Flood sampler (Fig. 23) has two vertical solenoids (*a*), energized through a timing clock. When energized, the solenoids draw up soft-iron cores (*b*) carrying a yoke (*c*) which is bored out in the center and has a lug that engages a helical groove in spindle (*d*) causing it to revolve through about 45° and swing the cutter (*f*) through a stream falling from spout (*e*). Shaft (*d*) is slightly tilted so that the cutter swings back by gravity into the position shown when the current is cut off. The master clock keeps the circuit closed a predetermined time, usually one to five seconds and then open for some other, usually much longer, interval. Sampler is made with 4-, 5-, or 6-in. cutter and weighs 35, 45 and 50 lb. respectively.

Tilting box as an external timer is illustrated in Fig. 24 where the actuating device on the cutter is a water-operated piston.

Scobey timing device (Fig. 25) is used to operate samplers directly or to control the power for the actuating mechanism. It is driven by belt from a convenient source of power at about 20 r.p.m. Motion is transmitted through bevel gears (*a*) to a chain of intermittent gears which move a sliding frame (*b*) from side to side. A lug (*c*) on the sliding frame causes arm (*d*), pivoted at (*e*) to swing

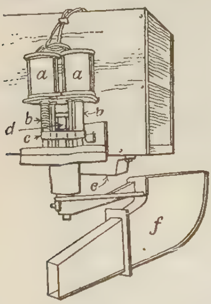


FIG. 23.—Flood sampler attached to bottom of launder.

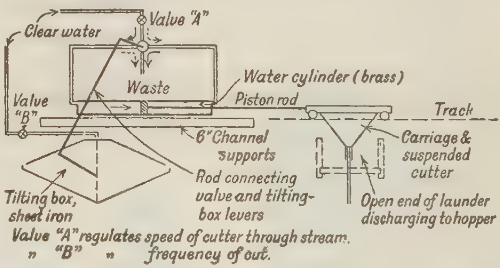


FIG. 24.—Tilting box as external timer.

back and forth. The cutter (*f*) is attached directly to the swinging arm or the arm is connected to a switch or valve controlling the power operating the samplers. The position of the pivot of the swinging arm may be varied by turning hand wheel (*g*), thus changing the length of this arm of the lever and consequently the speed at which the cutter moves. The arc which the arm describes may thus be varied from 10 to 22 in. in length. By using different combinations of intermittent gears, the intervals at which a stroke is made can be varied from 1 min. 20 sec. to 5 min. 20 sec., when the driving pulley is making 20 r.p.m. With a 2-in. cutter attached to the swinging arm and by varying adjustments as above, samples from 1/000th to 1/1000th part can be obtained. When arranged as shown and taking the discharge from a launder, only 10 in. fall is required.

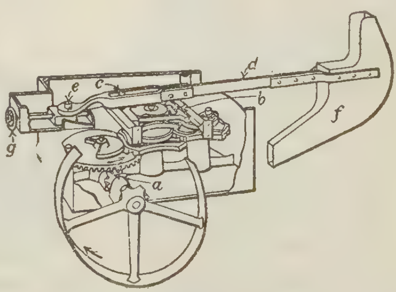


FIG. 25.—Scobey timer.

Galigher mechanism (Fig. 26) for actuating or timing samplers, can be used in the same manner as a tilting box. It may be belt-driven or direct-connected to a 1/6-hp. motor as shown. The motor drives shaft (*B*) through worm and worm gear. Cam (*A*) contacts with roller (*C*) on ratchet lever (*D*), raising the latter, and thus imparting slow motion to the shaft of ratchet wheel (*G*) through friction ratchet (*F*). Bevel pinion (*I*) on this shaft rotates bevel gears (*J*) on stationary shaft (*M*) in opposite directions. Lugs (*K*) on the bevel gears engage lugs (*L*) attached to loosely hanging pendent weights (*N*) and raise them in opposite directions. When either weight reaches an upright position and the center of gravity passes the vertical center-line, it falls. In falling, arm (*O*), attached to the weight, contacts with the cam surface on a trip arm (*P*) and forces it through an arc to the opposite position. The trip arms (*P*) are rigidly connected on opposite ends of shaft (*Q*) and thus cause it to rotate through an arc first in one direction, then in the opposite. Cutter (*S*), attached to shaft (*Q*) by a counterweighted bar, is thus made to move back and forth through the pulp stream discharging from the launder. Weights (*N*) are so placed that one

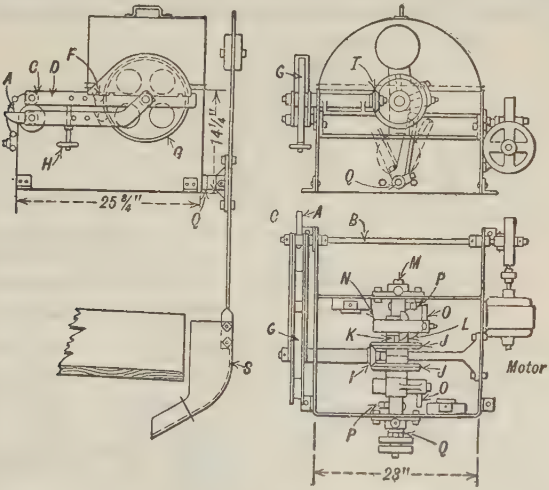


FIG. 26.—Galigher timing and actuating mechanism.

falls while the other is just starting its upward path. Variation of sampling intervals can be made between 10 and 60 minutes by adjusting the length of stroke of ratchet arm (*D*) through adjusting screw (*H*). The counterweight on the sampler bar balances the weight of the cutter and holds it in off position during the interval between falls of the weights.

The machine has advantages over the water tilting box; it is steadier in operation because danger of change of flow or clogging of water supply is eliminated; small space is required and it is easy to connect it up.

Clocks may be purchased which are arranged to open and close an electric circuit through a knife switch at predetermined intervals. They are useful in controlling movements of electric sampling devices such as the Flood.

11. Recording devices

It is essential to accurate sampling to install a recording device on automatic samplers in order to have a check on performance. The simplest recorder is a revolution counter, set to count each cut; the counter is read at beginning and end of the sample period giving the number of cuts and therefrom the average sampling interval. This device does no more, however, than point a lack of cuts in case a sampler is stuck for a part of the sampling period; it tells nothing as to when the hang-up occurred, nor as to whether

it occurred in midstream. Too few cuts indicates unreliability of the sample; too many may point to unreliability of the counter.

Graphical recorders of the Bristol or similar type, arranged to indicate the length of travel of the sample scoop at each cut and the time of the cut, give all information necessary as to performance, in the shape of a chart presenting graphically the relation between time and scoop travel. If the chart indicates that the scoop has stuck in mid-stream, the sample should, of course, be discarded.

12. Hand sample cutters

These are used for wet pulp in practically all plants; in some plants all wet sampling is done by hand; others use them only for taking special and occasional samples. Two forms are shown in Fig. 27. Hand sample cutters

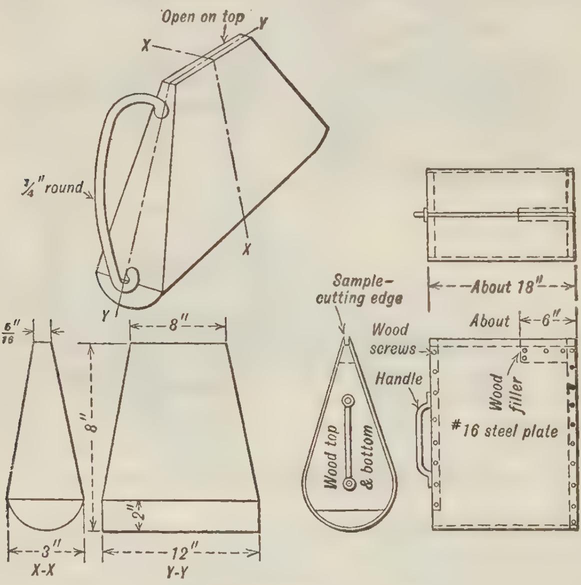


FIG. 27.—Hand sample cutters.

should be held with the edges perpendicular to the direction of motion of the cutter and passed completely through and out of the stream in one direction with a movement as nearly uniform as possible. The best results are obtained by taking cuts at regular intervals of time and by so using the cutter that its operation approaches that of a properly-run mechanical sampler. The cutter should be large enough so that there is no danger of overflow while making one passage through the stream of pulp. The aperture should be at least four times the diameter of the largest grain in the pulp; the cutter should be smooth inside, watertight, and designed to clean itself readily.

13. Preparing wet samples

Splitting wet pulp samples is frequently done before drying, to reduce the bulk. The riffle sampler (Fig. 28) or a device consisting of a revolving

distributor delivering to a compartmented tub may be used. The cutter that takes the sample can be arranged to deliver it over a series of riffles which automatically reduce the bulk.

When the solids are very fine a dip sample may be taken from thoroughly agitated pulp. Air is frequently used for agitation.

Dewatering samples is done by decanting, siphoning or filtering, final drying in any case being by evaporation. Electrolytes such as alum, lime, or sodium bicarbonate may be used to settle solids before decanting or siphoning; 1 to 5 gm. of electrolyte to a bucketful or tubful of slimes is sufficient. (See Sec. 16, Art. 4.) Pressure, vacuum or gravity filters are used. Where the amount of sample for a definite period is large, a series of small settling tanks may be used, arranged so that each succeeding one takes overflow from the preceding, overflow from the last being clear. Final dewatering of material remaining in the tanks is done by evaporation, preceded, if necessary, by filtration. Dewatering before evaporation must not be practiced, if metals are in solution, unless separate solution assays and moisture determinations are made. Great care must also be required of operators in decanting and siphoning to prevent slime losses.

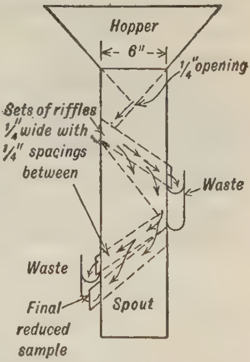


FIG. 28.—Wet riffle.

14. Tonnage determination

Determinations of weight of ore delivered to or passing through different sections of a mill are necessary in order to keep close check on operations. Where ore is being accepted at a mill from a seller, the weight must be determined as accurately as possible; in plants that are milling company ore the weight need be determined only approximately, as a rough check on working. Many schemes are used for tonnage determination, some are accurate and some mere approximations that depend on averages over long periods of time for dependable results.

Track scales are used for carload lots. They ordinarily require the services of an operator who balances the load on the scale beam and notes the weight on a suitable form. This method gives correct results within a small error (0.5 per cent.) but has the DISADVANTAGE of relying on the care and accuracy with which the operator balances the load and notes results. With recording scales the operator merely balances the load, then turns a screw which automatically punches a card and records the weight. Some track scales weigh a train of ore automatically as it passes slowly over the platform. This does away with the personal factor entirely except for occasional standardization. The car must stand free and completely on the scale; snow and ice should be removed before weighing.

Platform scales are used like track scales when wagons or trucks deliver the ore. The tare must be accurately determined. Unless the scales are in perfect adjustment, the load should be centered as nearly as possible at both loaded and empty weighings, or should stand at the same place for both weighings. Naturally, helpers or guests of the driver who weigh in on the full load should accompany the empty wagon over the scales. The wagon

should be at rest when weighed and, in case of animal-drawn vehicles, traces should hang loose.

Automatic dump scales consist of a hopper which receives ore until full, when the weight is recorded and the material run out automatically. This is accurate. Considerable space is required, however, and operation is intermittent, making it necessary to use two units where a continuous stream of material is wanted, so that one will be discharging while the other is filling. Care should be taken to insure that all material runs out before new material is run in.

Conveying weighers are attached to conveyors and automatically record the weight of material being carried. They occupy but little space and need no attendance except for occasional standardizing and adjustment. They are accurate to within 0.5 per cent. when kept in proper adjustment. Several different types may be obtained.

Blake-Denison weigher (Fig. 29) suspends a short section of conveyor from the short arm of steelyard (*a*), which is balanced to the weight of the unloaded conveyor. The load on the conveyor is balanced by a plunger (*b*) suspended in a mercury dashpot (*c*). Shaft (*d*), driven through bevel gears from a pulley revolved by the conveyor, is fitted with cams so that every time the conveyor travels through the suspended distance these cams operate a device (*e*), which grips the steelyard, and a measuring quadrant (*f*) that rotates the ratchet registering wheel (*g*) a distance depending on the position of the steelyard. This amount is recorded on indicator (*h*), calibrated for the units desired. The machine needs no attendance except occasional inspection and calibration with known weights.

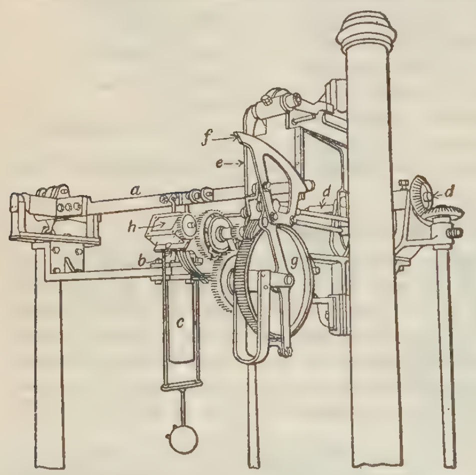


FIG. 29.—Weighing and recording mechanism of Blake-Denison automatic weigher.

Checking operation of the weigher is usually done by feeding known weights of ore to the conveyor at a regular rate for a given period of time. At one plant (*100 J 520*), if the automatic weigher checks within 50 lb. in 8000 lb. in comparison with hand-weighed feed, it is considered accurate enough; if a larger discrepancy is recorded, the weigher is adjusted. To insure proper operation, checking should be done at frequent and regular intervals.

Merrick weightometer (Fig. 30) is so arranged that, instead of weighing the load on the conveyor by a succession of weights of short sections, the weight is taken continuously by means of a specially designed integrator. A portion of the conveyor is suspended by means of rods (*a*) from weighing levers (*l*) that operate beam (*b*). The weight of the load at any instant is automatically counter-balanced by an iron float (*c*) attached to the beam. This float is partially immersed in a bath of mercury and thus as it rises or falls its gain or loss in buoyancy compensates for variations in load. The extreme end of the beam is connected by rod (*d*) to a totalizing mechanical integrator. The integrator (Fig. 31) consists of an aluminum disk (*X*) which has rollers (*Y*) around the periphery, the axes tangential to the edge of the disk and free to revolve. The disk is attached to shaft (*e*) which revolves in bearings on frame (*Z*). The frame is mounted on bearings at both ends which permits it to rotate on an axis that lies in the plane of the disk and passes through the center thereof. A link at one end of the disk frame is attached to rod (*h*) (*d*, Fig. 30) so that any movement of the beam causes the frame to tilt through an angle whose sine is proportional to the vertical

movement of the float which, in turn, is proportional to the load on the suspended portion of the conveyor. Four pulleys (U, U, Q, Q) drive an endless belt (W), which touches rollers (Y) at two points diametrically opposite on the axis of the frame (Z). Contact between the disk rollers and the belt is maintained by pressure rollers behind the belt. The take-up pulley (T) is weighted and insures an even tension of the belt and takes care of any stretch. The two pulleys (U) are geared together and driven by means of gears from a bend pulley under the return conveyor belt, or from a sprocket, if link belt is used. When the belt is running unloaded, the machine is adjusted by means of an adjusting weight on a screw attached to the beam so that the disk remains vertical. In this position motion of (W) causes rotation of rollers (Y) but no rotation of the disk. When a load comes on the conveyor, motion of the beam causes the disk frame to tilt, the axes of rollers (Y) become inclined to belt (W) and the belt pushes the rollers sidewise, causing the disk to rotate at a speed proportional to its inclination, *i.e.*, to the load on the conveyor. Revolution of the disk is recorded on a counter calibrated to read weight in the units for which the machine is designed. Thus the two factors, speed of belt and weight carried per unit of length are accounted for.

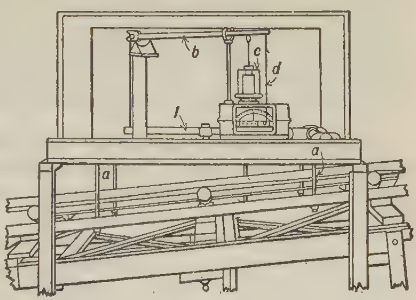


FIG. 30.—Merrick weightometer.

This device can be used for weighing material transported on belt and bucket conveyors, cable railways and overhead transporters. A special attachment can be procured that counterbalances a varying empty weight of conveyor, should the material handled adhere to the belt on the return trip.

Electric weigher operates on the principle that the amount of current flowing through an electric circuit is proportional to the product of the voltage and conductivity of the circuit. Voltage proportional to the speed of the conveyor is produced by driving a constant-field dynamo by gears or a chain and sprocket attached to the driving shaft of the conveyor. Conductivity is varied with change of load by a rheostat operated by a plunger in a mercury dashpot. An ampere-hour meter in circuit is graduated to read in units of weight. A section of the conveyor is suspended in a manner similar to that employed in the Merrick weightometer. A counterweight on the scale beam balances the empty weight of the conveyor while the weight on the beam due to the load carried is counterbalanced by motion of the plunger in the dashpot. To provide for varying weight of the empty belt, a corresponding length of the unloaded part of the conveyor is suspended so that it operates on the opposite side of the scale beam from the loaded portion. The machine is calibrated by moving riders astride two wires of a loop of the rheostat when standard weights are suspended from the scale and the dynamo is run at a known speed. Instruments may be attached to the system to record the rate of handling, time of starting and stopping, and continuity of operation; these instruments may be placed at any desired distance from the weigher. By connection of the scale with a device controlling the feeder, delivery of material at a certain rate or delivery of a certain predetermined amount may be effected.

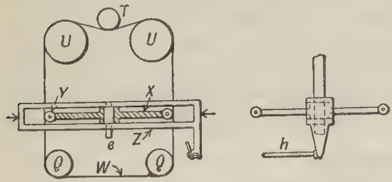


FIG. 31.—Merrick integrator.

Approximate methods of weighing are used when great accuracy is unnecessary or investment for a scale is not warranted. Carloads or trainloads are counted and the weight calculated from the volume of the cars and the weight of a unit volume of broken ore. In some cases carloads or trainloads are weighed occasionally and this weight used as a basis for calculating total tonnage from the number of cars or trainloads counted. Bin measurements of volume are frequently used as a rough check on other methods, or in company mills for inventory at the end of statement periods. Tonnage may be esti-

mated by counting the number of strokes or revolutions of feeding devices, the weight of ore fed at each stroke or revolution being determined experimentally.

At the Portland mill (*63 A 514*) the weight of ore passing was determined as follows: Once each day one Chilean mill was stopped and the feed discharged into a box for 20 strokes of the feeder plunger. This sample was weighed and the average of the five latest weights, applied as factor to the number of strokes of the feeder plunger as indicated by a counter, gave the tonnage per 24 hr. A similar method was used at the Rosebery concentrator. (*114 J 677*.) Ore at 1-in. size was sampled every 15 min. by catching the full discharge of the feeder for a period of 30 sec.; alternate-interval samples were weighed.

Wet-pulp tonnage is ordinarily computed from volume measurements and moisture determinations. The usual problem is that of finely-ground material suspended in water and flowing in pipes or launders. The whole stream is deflected into a suitable container during a period of time measured by a stop watch. Volume is determined by graduations on the container or by measurement. The proportion of solids is determined from a small dip sample taken while the container is filling. If the stream is small, the whole amount diverted may be used to determine the amount of solids in the pulp. Percentage of solids may be determined either (*a*) by weighing, dewatering, drying and weighing the dry solids; or (*b*) by calculation from the specific gravity of wet pulp and a value for the specific gravity of the dry material. The first method is the more accurate but takes longer; the specific gravity of the ore varies from time to time with corresponding effect on computation. Sometimes a cutter of known width is used to catch a time sample and the weight is multiplied by the ratio between width of stream and width of cutter to determine the amount carried by the whole stream. This method is not so accurate as the first because the stream varies in depth and density across its width. Results by this method will be most satisfactory when the material is very finely ground. Tonnage of ore in an agitating tank whose volume is known may be calculated from a dipper sample; the accuracy depends on the degree of pulp uniformity caused by the agitation. The volume of pulp passing through pipes is sometimes measured by solution meters and tonnage calculated from small specific-gravity samples.

Calculations and pulp formulas, see Sec. 22.

15. Moisture sampling

Moisture samples are necessary to determine net dry weight from gross weights obtained by any method. The same care should be used in taking moisture samples as is used in other sampling. Moisture samples should be weighed immediately or placed in tight containers that prevent evaporation until the samples can be weighed. Calculation is simplified by taking moisture samples weighing 100 gm. or 1000 gm. or small multiples of these weights. The samples are weighed wet, dried at a suitable temperature until all hygroscopic moisture is driven off, then weighed again; the difference represents moisture and is usually expressed as percentage of the wet weight.

Moisture samples should be taken at the time the material is weighed, if possible, to avoid errors due to evaporation or subsequent wetting, hence moisture samples are generally taken at different times than assay samples. Some form of grab sampling is frequently used so that the sample can be quickly collected and placed in tight containers. The assumption is that error due to crudeness of method is less than the error introduced by longer exposure of material during more elaborate sampling. Grab samples for

moisture are frequently taken from the end of a conveyor belt after material has passed over a conveying weigher.

It is difficult to obtain moisture samples that check within close limits. When ore is shipped in cars, the outer or top layers contain more or less moisture than the bulk, depending on climatic conditions; if shipped in bags, the material will probably be drier near the outside of the bag than in the middle. Thorough mixing must precede an accurate sample. If the regular sample mill is used for a moisture sample, drying in passage through the sampling mill is considerable and must be compensated in the calculation. This is usually done by an arbitrary percentage added to the percentage determined. Brunton (*40 A 567*) says that, in ordinary practice, this loss would not exceed 10 per cent. of the percentage determined in summer, nor 7 per cent. in winter. In buying and selling a factor is determined by agreement between the parties; in one instance 10 per cent. of the percentage determined was added with a maximum addition of 1 per cent. of water based on the wet weight (*TP 86*, *USBM*). Duplicate determinations on relatively dry material (say under 10 per cent. moisture) should check within 10 per cent. of the percentage present.

Tests by the U. S. Bur. of Mines of 254 pairs of duplicate moisture samples of coal showed an average difference in moisture content of 0.256 per cent. with a maximum difference of of 3.6 per cent. (*Bul. 116*, *USBM*.) The method employed was to use the same 5-lb. sample, obtained as described on p. 1139, for determination of moisture and for analysis; it was kept in sealed air-tight containers until the moisture determination.

16. Sampling mills

The usual procedure is to crush and remove a sample which is alternately re-crushed and re-sampled until reduced to the desired weight.

Lay-out of sampling mills. In many sampling mills, especially the older ones, all ore is elevated to the top of a tower-like building and then falls through successive crushers and sampling devices, the sample and reject being finally delivered at the lowest part of the building. This arrangement necessitates a series of superimposed floors and requires a tall building with sufficient strength to withstand the load and vibration of heavy machinery; the largest pieces of machinery are on the upper floors since the bulk of the sample is reduced as it progresses downward; much of the fall from floor to floor is wasted. The present tendency is to place the machinery as far as possible on one level and to use inclined belt conveyors to elevate to successive machines; this eliminates elevators which require much attendance, are difficult to clean and to observe. A level-site mill is cheaper to build and simpler to operate; one crane can serve all machines; power distribution, attendance, supervision and repairs are all facilitated. Rejects are collected on a conveyor running the length of the building.

Pulsifer (*121 P 866*) points out that by using straight-line single-bay mills the original cost of construction should decrease from 25 to 40 per cent. and that, with the resulting lower operating charges, the cost of sampling should be much lower than at present. The Washoe sampling plant (Fig. 34) illustrates the vertical type of mill; the new sampler at the Tacoma smelter (Fig. 25) the one-level type.

Size at which sampling begins depends on local conditions; in general it is best not to cut a sample from material containing pieces larger than 2-in., but in some cases a sample may be cut when pieces up to 4- or 5-in. or larger are present. In any case, the weight taken at any size should be sufficient to correctly represent the lot, as explained in Art. 1. The size interval between successive sample cuts depends on the amount of reduction in size of pieces

obtained in the crushing machinery used. Taking full advantage of the reduction that a given crusher can make may not always be feasible, as the power consumption may rise so high as to prove more expensive than an extra crushing and sampling operation.

Crushers of jaw or gyratory type (see Sec. 3) are used on coarse sizes down to 1- or 1½-in.; rolls (see Sec. 3) are almost universally used from this size down to ⅛-in..

Sampling machines. See Art. 7, 8 and 9.

Flow of material should be continuous and uniform, otherwise the scoops of a given sampler may take no material during one or more revolutions and this failure may be multiplied in succeeding machines by synchronism, or it may happen that two samplers will so synchronize for several revolutions that the scoop of the second receives all of the sample cut by first. Any case between these extremes may occur, and in any such case the correct proportion of material will not be taken and the sample will be unrepresentative. The means for avoiding such contingencies are storage hoppers, elevators, mixing drums, or feeders with some storage capacity, placed between successive sample cutters.

Feeders are usually placed ahead of the rolls preceding a sampler; they not only spread material and retard its motion sufficiently to give a continous stream, but they feed the rolls in a thin even stream, which aids in preventing uneven wear on roll shells. The feeders are usually of the shaking type. They should be of sufficient length to accomplish the necessary retardation of material and their slope should be low enough so that the larger pieces will not move faster than the main stream; a length of 6 ft. and slope not greater than 2 in. per ft. satisfies these requirements.

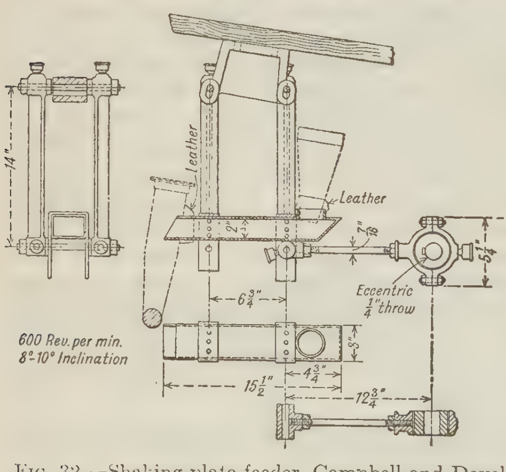


FIG. 32.—Shaking-plate feeder, Campbell and Deyell mill.

A small SHAKING FEEDER used at COBALT (*17 CMI 199*) is shown in Fig. 32. For other types of feeders see Sec. 20.

Drum mixers retard the stream sufficiently to make it continuous, but their ability to mix thoroughly is questionable. Fine material rides up the sides and slides back, coarse material rolls down on top of the fine and segregation rather than mixing is accomplished. Longitudinal ribs on the inside of the shell counteract this tendency; without ribs coarse material discharges mostly at one side and this causes uneven wear on the roll shells.

Drum mixers may be either cylindrical or conical; a ribbed cylindrical mixer is illustrated in Fig. 33. Drum mixers require more head room than shaking-plate feeders and are more expensive to construct. Either device requires but a small amount of power for operation.

Flexibility of plant is desirable when the size or character of ore shipments varies widely. Ordinarily a plant is designed to handle shipments of a certain size; if smaller lots arrive it may be necessary to by-pass the first sampler in order not to cut too small a sample. This can sometimes be done by stopping the sampler so that all material passes through into the sample chute; otherwise, suitable chutes should be provided to accomplish the same end. Allowance for sample variation may be made by varying the speed or cutter widths of the sample machines.

Accessibility. All parts of a sampling mill should be easily accessible for cleaning and observation. Housings on crushing and sampling machines should be detachable and easily-removable doors should be provided in all housings for elevators, chutes, launders, etc. Chutes and launders should be designed with minimum turns. Hoppers and bins should be constructed so that no

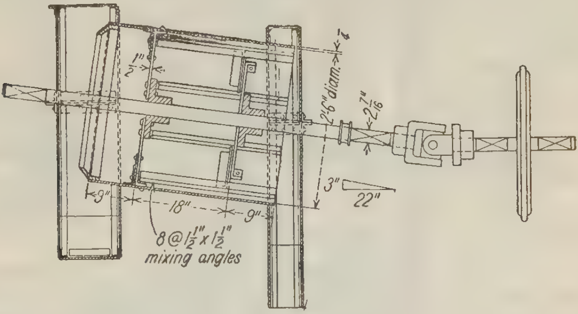


Fig. 33.—Revolving mixer.

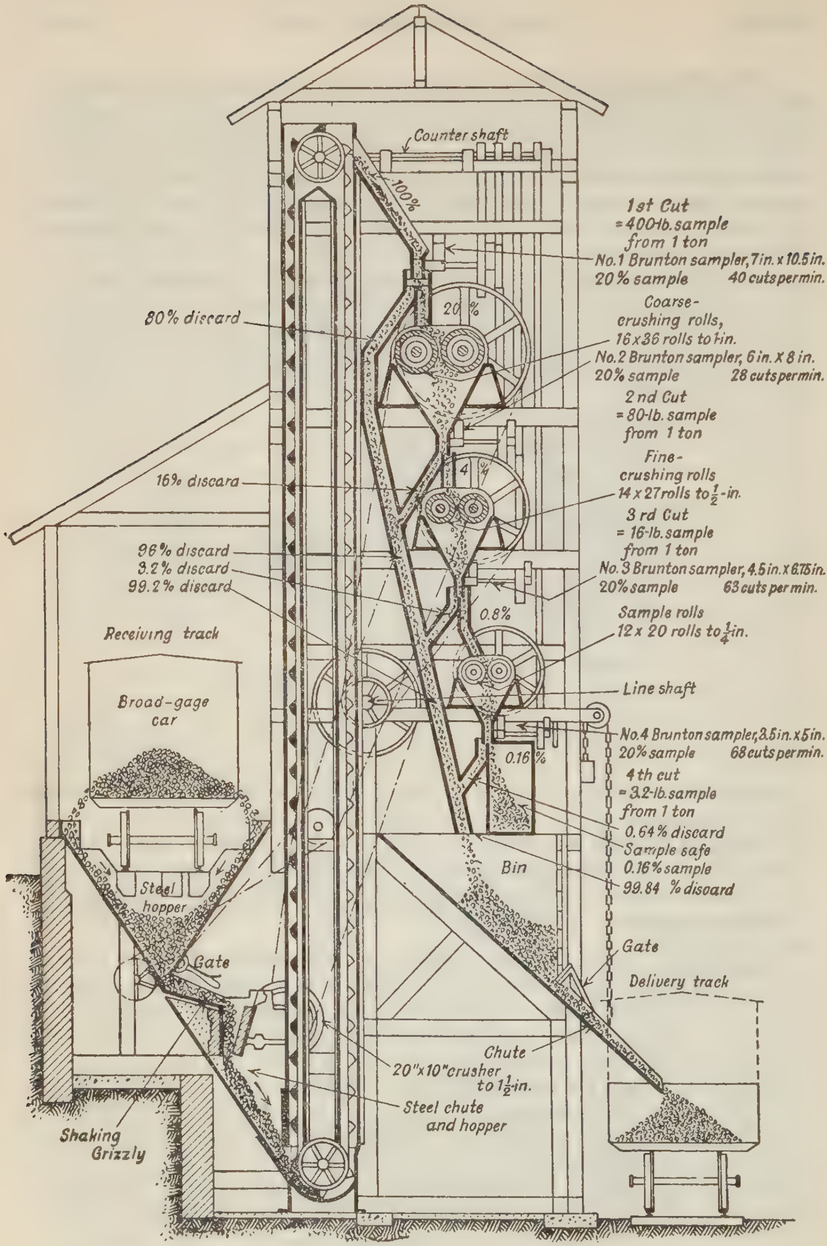
material hangs up when they are emptied. Every precaution should be taken to avoid the possibility of clogging or leakage of material.

Cleaning between successive lots of ore is essential in order to avoid salting. Ordinarily compressed air and brushes are used for cleaning fine material from machines and spouts; floors should be swept and all material collected put with the lot just run through, or so disposed of as to preclude contamination of succeeding lots.

Dust losses. When a sampling mill has many open windows and a strong wind is blowing, the dust loss may be 5 per cent. of the total lot. As fine material is generally of higher grade than coarse, serious losses may thus occur. Further, it is hard to keep good labor in a dusty plant. Where hand sampling is employed, the ore is frequently dampened to keep down dust; this causes fine material to coat the larger pieces and thus aids in mixing. Wetting causes material to stick in automatic samplers and is, therefore, not permissible, hence in sampling mills all sampling machinery and crushers should be enclosed in dust-proof housings and all chutes covered and free from leaks. If frequent observation of moving parts is necessary, small doors that can be readily opened or coverings with removable sections should be provided.

17. Custom sampling mills

These mills sample lots of ore to determine their value for sale. The charge for this service depends usually on the grade of ore and size of lot. In order to keep down the expense as much as is commensurate with accurate



Ore: various. *Capacity:* 500 tons in 8 hr.; 50 tons in about 25 min. On 100,000-lb. lot, *a sample* of 160 lb. is delivered to the sample safe. This is cut down to 25 to 40 lb. in the finishing room before further crushing.

FIG. 34.—Washoe sampling plant.

work, the sample cuts taken at the coarser sizes are made as small in bulk as possible, but a sufficient factor of safety in bulk of sample is allowed to obviate any chance of serious error. A portion of the reject from some point early in

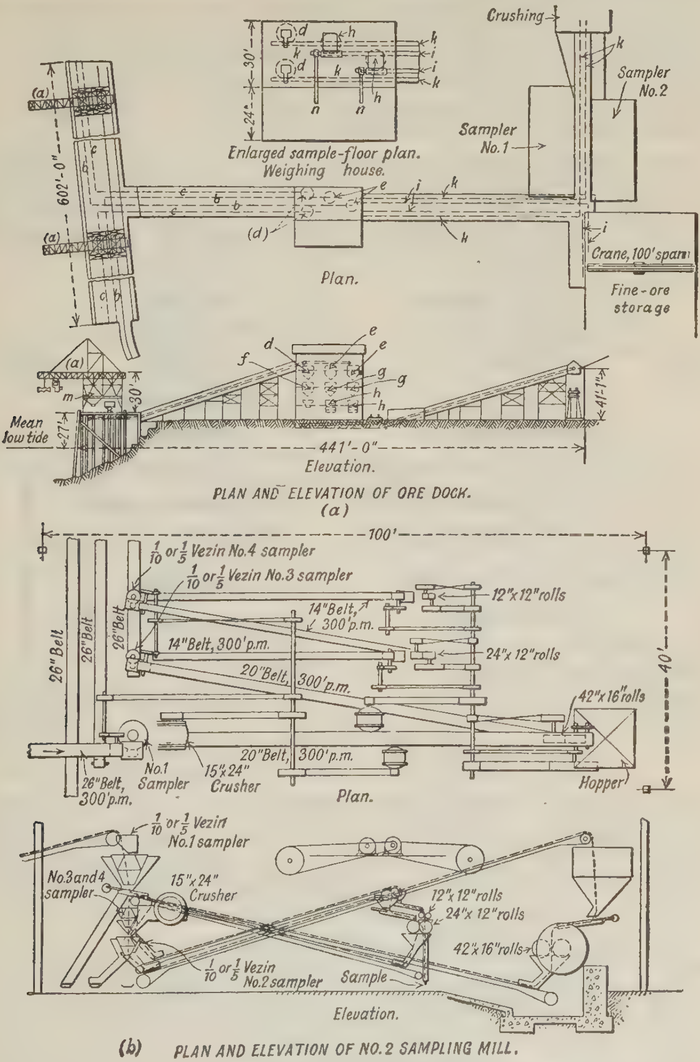


Fig. 35.—Tacoma sampling plant.

the sampling operation is usually kept until settlement is made, to use for re-sampling if necessary.

Washoe sampling plant (Fig. 34) at Butte, Mont., illustrates general practice in western United States.

Sampling plants at mills or smelters that purchase ores have the same characteristics as custom plants except that in cases where the ore is to be finely crushed for treatment in the plant, it may be broken to small size before samples are cut. Unless, however, the purchasing contract makes settlement compulsory on an umpire assay without re-sampling, such fine crushing of the lot must be done in the small crushers in the sampling plant with resulting loss in efficiency. If re-sampling is eliminated by contract, the lot can best be sampled in the mill at a point just prior to any division of the ore stream, all crushing thus far having been done in large mill crushers.

Tacoma smelter (Fig. 35) shows the scheme of unloading from boats and sampling at Point Defiance, Washington. Ore is lifted from boats by 42-cu. ft. Brown-hoist bucket (*a*) and dropped into hoppers (*m*) feeding onto 24-in. belt conveyors (*b*, *c*). Conveyors (*b*) deliver concentrate to 550-cu. ft. hoppers (*e*) and conveyors (*c*) deliver coarse ore to 550-cu. ft. hoppers (*d*) in the weighing house. Directly below hoppers (*d*) and (*e*) are weighing hoppers (*f*) and (*g*) equipped with 60-ton Fairbanks-Morse recording scales. Concentrate

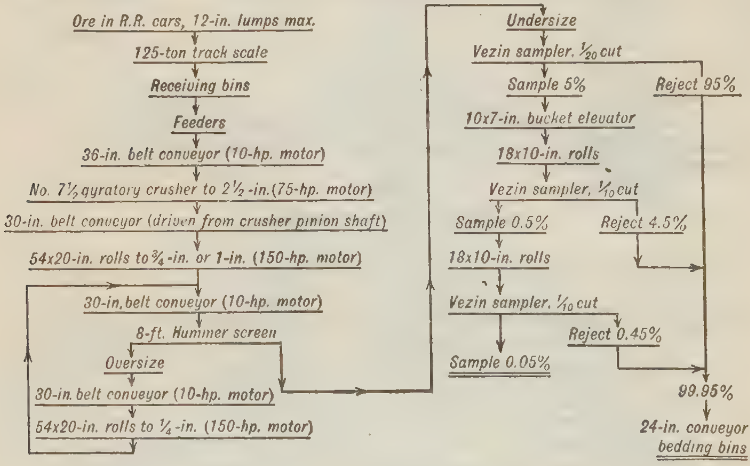


FIG. 36.—Sampling plant, Magma Copper Co. smelter.

passes from the weighing hoppers to Martin samplers (*h*) (see p. 1150) which cut a 1/100 sample in two cuts of 1/10 each. The samples are delivered by belt conveyors (*n*) to the finishing room for preparation of moisture and assay samples. Rejects from the Martin samplers are delivered to storage bins by conveyors (*i*). Ore after weighing is delivered to the sampling-plant bin by belt conveyors (*k*). The method of obtaining coarse-ore samples is shown in Fig. 35, *b* which, by variation in the percentage taken by each sampler, can deliver a final sample of 1/600 to 1/10,000 of the original lot. A large variety of gold-, silver- and copper ores and concentrates is handled (*119 J 557*).

Magma Copper Co. smelter. (Fig. 36.) Ore to be sampled: various copper ores; capacity, 50 to 60 tons per hr. The use of individual motors is noteworthy; a 30-hp. motor drives the two small rolls, elevator and samplers. (*118 J 685*.)

Cobalt sampling mill (*17 CMI 199*) was designed by Campbell and Deyell, Ltd., to sample high-grade silver ores of the Cobalt district. The fact that most of the values were in the form of nuggets or flakes of silver made sampling especially difficult. The flow-sheet is shown in Fig. 37. A specially-designed Krupp-type ball mill crushes all material to pass 8-mesh before any cut is made. A great part of the metallic particles remain in the mill until the whole lot is run through, then the mill is cleaned out and these particles are melted and credited to the lot. All sampling machines are of the Vezin type with special cutting edges. Accuracy is assured by dividing the whole lot into quarters and sampling each quarter separately. Close concordance of samples is accepted as conclusive evidence of accuracy. The QUARTERING MACHINE is a cylinder divided into quadrants, which rotates

on its vertical axis beneath a feed spout whose center is 5.5 in. from the center of revolution. Each quadrant has a hopper bottom with a spout that discharges into a separate annular chute. The machine runs at 30 r.p.m. and thus makes 120 cuts per minute. Each quarter is sampled separately by the same method. The final 100-mesh pulp is mixed by sifting through a 40-mesh screen onto a glass table, and divided as described under (*e*) on page 1177. Metallics on the 100-mesh screen are similarly divided on a glass plate. Metallic iron is removed with a magnet. If the metallics vary in size the different sizes are separated, divided into aliquot parts and combined to make a composite. The moisture sample is

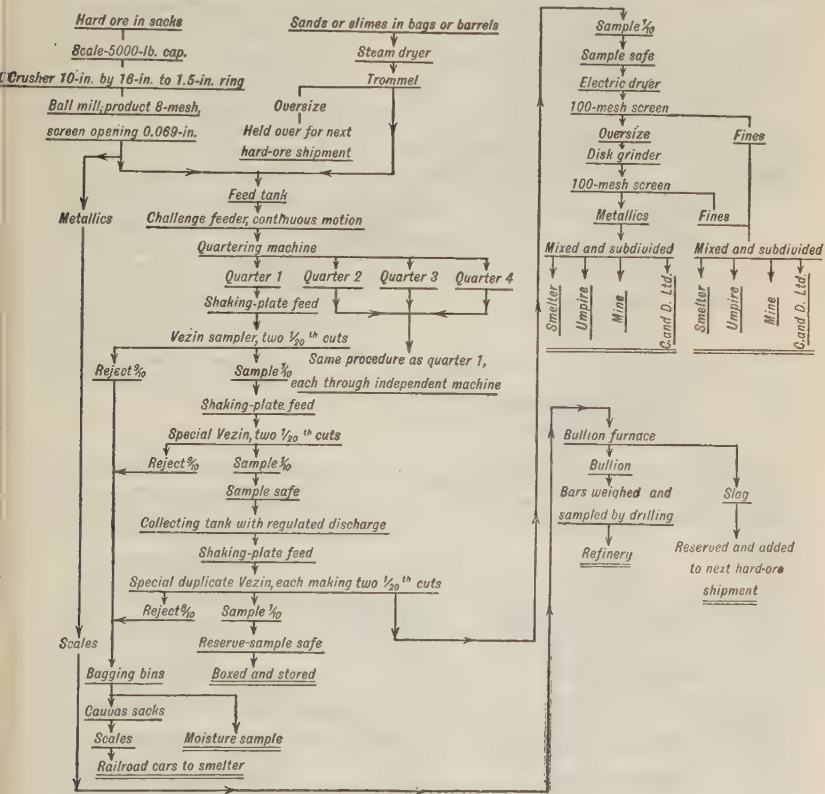


Fig. 37.—Campbell and Deyell sampling plant.

taken with a small scoop from the mouth of every fifth bag while sacking and is placed alternately into two tins. When ore is shipped in barrels a pipe sample is taken when the barrel is half-full and another from the top half, alternating into two tins as before.

18. Head sampling

Most large mills have separate plants to sample the mill feed. The usual practice is to place the head-sampling plant following the coarse crushing plant and deliver reject to the fine-ore bins. Practice is illustrated by the flow-sheets, Figs. 38 to 41, incl.

In porphyry-copper-plant practice, as typified by Chino Copper Co., (Fig. 38) the first cut rejects over 99 per cent. of the lot and the final machine-cut sample is less than one part

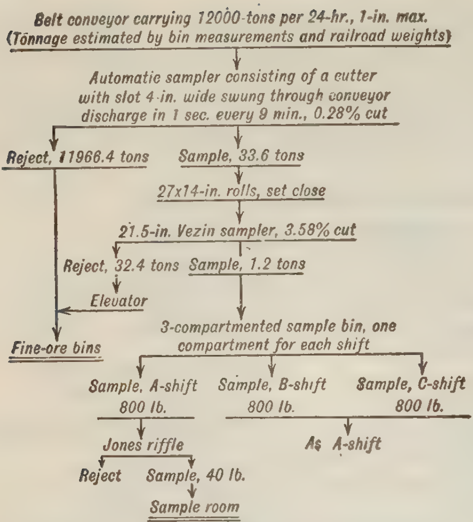


FIG. 38.—Sample plant at Chino Copper Co.

in 200,000. As the metal content of the ore or the value of the metal increases, or both, the proportion of initial reject decreases and the proportion of the final machine-cut sample to the initial lot increases.

In small mills and in mills treating low-grade ore or uniform ore, head sampling is usually accomplished in a simpler manner. In some cases an automatic sampler is used, cutting a small percentage at long intervals, this sample being reduced further by hand.

At ALASKA GASTINEAU (low-grade gold ore) the heads are sampled at 10-mesh size, just before passing to concentrating tables, by automatic cutters ½ in. wide, operated by air cylinders and timed by Scobey timers; the sampling interval is 12 min. The combined sample

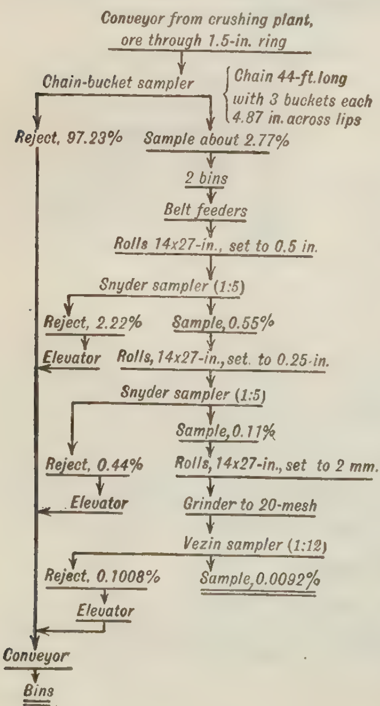


FIG. 39.—Sample plant at Cananea

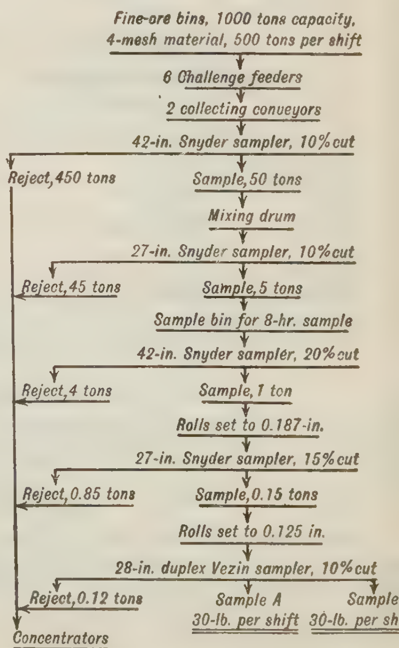
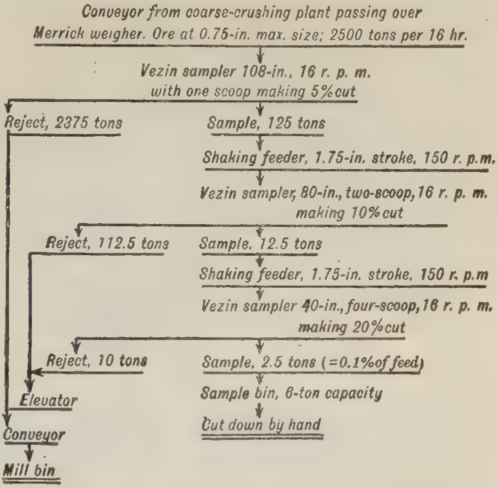


FIG. 40.—Sample plant, Phelps-Dodg

from 6000 tons averages about 3000 lb. (*63 A 502.*) At Belmont Milling Co. (*52 A 100*) (gold (0.32 oz.)-silver (3.2 oz.) ore with some copper and lead present) the ore is weighed by electric conveying weigher; a sample is cut at 1¼-in. size at the discharge of the conveyor every 68 sec. by a scoop mounted on a vertical intermittent-gear wheel. The ¹⁄₅₀₀th sample thus obtained is further reduced to 50 lb. by riffling after crushing to pass ¼-in. ring in a small laboratory gyratory crusher. On a 3500-ton lot this method checked a sampling mill (Western Ore Purchasing Co.) within 8¢ per ton.

At the Sullivan concentrator (*116 J 453*) (zinc-lead sulphide ore with pyrite, calcite and quartz gangue) the ore is weighed on a 150-ton track scale. A 1-per cent. cut is made by a bucket sampler after crushing through ¾-in. square holes. The sample passes over a circular mixing table and a second 1-per cent. cut is made by another bucket sampler at the end of a conveyor belt. The sample ratio is 1 : 10,000.



25-hp. motor runs entire plant. 2 men per shift.

Fig. 41.—Sample plant at Federal Lead Co., Mill No. 4.

At the Silver King Coalition mill (*116 J 369*), (silver-lead carbonate and sulphide ore) 300 tons per day is crushed through a 1½-in. grizzly, and a sample is cut as ore falls from the end of the main conveyor onto a 24-in. apron conveyor, one 6-in. section of which is removed. The 5-per cent. sample that falls through is crushed and again cut to 1 per cent. of the original lot.

Hand sampling of heads is practiced at the following plants:

At Witherbee-Sherman Co., (magnetite ore averaging 25 to 45 per cent. Fe). The head sample is taken from the discharge end of a conveyor carrying −4-in. material. A steel bucket, 12 × 12 × 6 in. deep, swung on an arm from a vertical shaft, is passed quickly through the stream every 30 min. The sample is crushed in a small crusher to 0.5-in. and riffled down. The results obtained check with results calculated from samples of concentrate and tailing taken in similar manner. At St. Joseph Lead Co., the ore is galena in limestone gangue. Hourly samples of the feed to six sections is taken with a hand cutter. The largest piece is 0.35-in. and the cutter aperture 1.5-in. The cuts are combined, mixed, coned and quartered at the end of each 8-hr. shift, then ground to 80-mesh and cut down to assay-sample size. Results check over a period.

At Utah Leasing Co., the ore is reclaimed tailing, −4-mesh and averages 0.7 per cent. copper as chalcopyrite. The head sample consists of a shovelful from each 5-ton car. This is quartered down to 5 lb.

When all of the ore is crushed finely before being divided or subjected to any concentration, the head sample is most conveniently taken from the discharge of the fine-grinding department.

At Sunnyside Mining and Milling Co. a sulphide ore containing Au, 0.06. oz; Ag, 4.5 oz.; Pb, 4.8 per cent.; Cu, 0.40 per cent.; Zn, 5.5 per cent.; Fe, 3.0 per cent. is sampled at the overflow of the Dorr classifier following the tube mills. The sample contains 20 per cent. solids, 1.5 per cent. +65-mesh (0.0082-in.) and 68 per cent. −200-mesh. An automatic cutter with 0.625-in. aperture is used, making a cut every 20 min. The cutter remains in the stream 5 sec. and cuts a sample 0.025 per cent. of the total stream. Reasonably close checks are reported. At United Eastern Mining Co., the ore is quartz and calcite with about 1 oz. Au and 0.57 oz. Ag per ton. A head sample of −8-mesh material is

Table 7. Practice in tailing sampling with automatic cutters

Name of company	Sampling interval, minutes	Maximum size of particle	Width of cutter aperture, inches	Weight of shift sample, pounds	Per cent. of whole	Kind of sampler	Ore
Timber Butte	12	0.3-mm.	0.25	5	0.002	....	Zinc-lead
St. Joseph Lead Co., Bonne Terre, Mo.	16	0.147-mm.	0.375	20	0.0067	....	Lead flotation tailing
Belmont-Surf Inlet	5	80-mesh	0.5	20	0.01	Timed by tilting box	Gold and silver ore with some copper
Baltic and Champion	60	0.25-in	0.75	4	0.00033	....	Native copper
Utah Leasing Co.	15	−35-mesh 2.5%	0.25	20	0.005	....	See page 99
Sunnyside Mining and Milling Co.	20	+65-mesh	0.5	....	0.025	Fig. 24	0.6% lead-0.8% zinc
Melones Mining Co., Melones, Cal.	10	20-mesh	0.5	75	0.0075	Cutter operated by tilting box	Gold, native and with pyrite
American Zinc, Lead and Smelting Co.	4.5	0.5-in.	1.25	70	0.01	....	See page 158
Alaska Gastineau	15	0.04-in.	0.25	18–25	0.0004	Automatic cutter controlled by Scoby timer	Low-grade gold ore
A porphyry-copper mill	5	0.04-in.	0.375	60	0.001	Electrically-timed cutter	....
A zinc-ore concentrator	8.5	80-mesh	0.25	40	0.007	Electrically operated	....
Chino Copper Co.	9	0.0232-in.	0.75	10	0.00016		Porphyry-copper, 1 to 2% Cu
Phelps-Dodge Corp., Morenci Branch	13	0.065-in.	0.25	20	0.0023	....	0.53% Cu
Moctezuma Copper Co.	20	8-mesh	0.375	10	0.002	....	Copper
Burro Mountain	8–10	....	0.375	60–70	0.004	....	Copper
Consolidated Arizona Smelting Co.	9	....	0.5	10	....	....	Copper
Federal Lead Co.	10	−12-mm. 0.6%	0.375	75	0.01	Tilting-box sampler	0.5% lead
Shattuck-Arizona Copper Co., Bisbee, Ariz.	1.5–2	+28-mesh	0.375	75–100	....	Cutter suspended from trolley on rail; operated by tilting box	Silver-lead carbonate ores: tailing 0.43% lead; 2.52 oz. silver
Cananea Consolidated Copper Co.	15	35-mesh	0.25	....	....	....	Copper
Engels Copper Co.	21	0.0116-in.	0.375	3	0.000635	....	Copper
Old Dominion Copper Co.	20	48-mesh	2	....	0.0056	....	Copper
Liberty Bell Gold Mining Co.	35	0.0116-in.	0.1875	2	0.0006	Specially-designed mechanically-operated cutter	Gold and silver. Tailing: 0.2 oz. Au, 0.89 oz. Ag
Morning Mill, Wallace, Idaho	6.25	80-mesh	0.625	20	0.012	Swinging cutter, electrically-operated	Lead-zinc-silver

taken by hand every 30 min. from the Marcy-mill discharge. Crushing is performed in cyanide solution that contains dissolved values. The sample is dried with all solution and a deduction is made for values in solution, as determined by a separate drip sample. The method checks within 3 per cent. of the calculated value from assays of tailing and bullion. At Elko Prince Mining Co. (silicious ore averaging $25 in gold per ton) a head sample is taken every 30 min. by a pan grab from the Marcy-mill discharge. The largest piece is 0.2-in. Results do not check well.

Accurate head samples are not, in general, obtained unless automatic cuts are taken at frequent intervals. Information obtained by crude methods is used as a rough check on operations. Such results, averaged over long period, may check with results obtained by calculation from accurately-taken samples of tailing and concentrate or bullion. The degree to which head-sample results will check a calculated head assay obtained from concentrate or bullion and tailing assays depends not only on the accuracy of the head sample but also on the accuracy of the head-tonnage determination. Probably the best place to sample heads in any mill is just before the first division of the ore stream; the ore is then generally rather finely crushed and well mixed and simple automatic sampling machines can be used.

19. Tailing sampling

Automatic wet-pulp samplers are best. Ordinarily the final tailing from a mill is low-grade, finely crushed and well mixed, and there are no sudden changes in value, hence small cuts and long sample intervals are permissible. Hand-sampling is more likely to be accurate than in sampling heads or concentrate. Practice in several plants is summarized in Tables 7 and 8.

Table 8. Practice in tailing sampling with hand cutters

Company	Sample interval	Maximum size of material	Width of cutter aperture, inch	Weight of shift sample, pounds	Per cent. of total
St. Joseph Lead Co., Rivermines	1 hr.	9-mm.	0.75	5–6	0.0001
St. Joseph Lead Co., Rivermines	....	150-mesh	0.5	....	....
St. Joseph Lead Co., Bonne Terre, Mo.	1 hr.	9-mm.	0.75	30	0.0025
Hedley G. M. Co.	1 hr.	−100-mesh	4	50	0.03
Calumet and Hecla	30–90 min.	0.25-in.	0.375	5	Less than 0.001
Tonapah Belmont	....	100-mesh	....	15	....
United Eastern, Oatman, Ariz.	30 min.	100-mesh	....	....	0.00093
Pittsburg Dolores Mining Co.	30 min.	100-mesh	....	10	0.01
Morning Mill, Wallace, Idaho.	60 min.	0.25-in.	0.75	10	0.005

20. Concentrate sampling

Automatic samplers are used to a great extent. The same forms of cutters as used for sampling tailing are suitable. Table 9 summarizes practice in several plants where automatic sampling of concentrate is used. Hand cutters, as described on p. 1156, are also used extensively. Pipe or gun samples and augers are satisfactory devices for sampling concentrate in bins or cars. Table 10 presents data on hand-sampling of concentrate in various plants.

In general automatic samplers are preferable for sampling fine concentrate or when the material is flowing in a stream with water. This is the case with most flotation concentrate. Some plants report that automatic sampling of flotation concentrate must be used to get

Table 9. Practice in concentrate sampling with automatic cutters

Company	Sample interval, minutes	Maximum size of material	Weight of shift sample, pounds	Per cent. of whole
Sunnyside Mining and Milling Co....	20	11% + 200-mesh	....	0.03
Alaska-Gastineau..................	4	0.07-in.	10.5–30	0.165
A porphyry-copper mill............	6	⅛-in.	200	0.02
A zinc-ore concentrator............	8.5	20-mesh	60	0.03
Chino Copper Co..................	9	0.075-in.	30	0.0056
Phelps-Dodge Corp., Morenci Branch	12	0.25-in.	19	0.04
Moctezuma Copper Co.............	20	4-mesh	20	0.0143
Burro Mountain..................	15	6% + 14-mesh	6	0.003
Shattuck-Arizona Copper Co........	3	1.5% + 150-mesh	....	....
Engels Copper Co.	3⅔	0.0116-in.	2.1	0.006
Liberty Bell Gold Mining Co........	30	0.07-in.	20	0.05

Company	Sampler	Tonnage determination	Moisture sample
Sunnyside Mining and Milling Co....	Fig. 24	Estimated in bins and R. R. weights	From car and filter cake sample. 1000 gm. taken
Alaska-Gastineau..................	Cutters timed by Scobey timer	Sacked and weighed	Pipe samples taken from sacks
A porphyry-copper mill............	....	Scale weights	50 oz. from cars
A zinc-ore concentrator............	Electrically-timed cutters	Bin measurements and R. R. weights	....
Chino Copper Co..................	....	Bin measurements	....
Phelps-Dodge Corp., Morenci Branch	....	R. R. weight	Taken by hand while loading
Moctezuma Copper Co.............	....	Weighed	Sample from cars
Burro Mountain..................	....	....	....
Shattuck-Arizona Copper Co........	Cutter operated by tilting box	Estimated and checked by car weights	Pipe sample from cars
Engels Copper Co.................	....	R. R. car weights	Sample taken from conveyor belt
Liberty Bell Gold Mining Co........	....	....	Grab sample

accurate results. However, if the concentrate discharged by a machine is thick and flows sluggishly, when water is added to wash it down launders it surges so much that at one instant concentrate and the next almost clear water is flowing. In such cases automatic samplers are not satisfactory and a hand cutter must be used at the discharge of the machine or some method of sampling concentrate in bins or cars after draining must be employed.

21. Miscellaneous mill sampling

Miscellaneous mill samples are taken to give information concerning the operation of a particular machine or set of machines. When accurate results are desired and the sample must be taken every shift, automatic samplers are best. This is the case when a mill is operated in sections and the feed and products of each section are sampled to check its work. Ordinarily intermediate samples are taken by hand by various operators to serve as guides

to operations of particular machines. Great accuracy is unnecessary and the use of hand cutters and long sample intervals are justified.

22. Preparing samples for assay

Marking samples. In custom-sampling mills, where many different lots are run in succession through the mill, ample provision must be made to avoid possibility of confusion of samples. A satisfactory method is to keep a complete record of the treatment of each lot on a suitable form convenient for filing. This form should accompany each sample until the work is finished and the sacked sample delivered to the assayer. It should then be filed for permanent record. In mill work, when samples are of small bulk and can be held in pails or pans, small numbered tags of copper or brass are convenient for identification. Complete identity of sample is kept in a note-book record on a form numbered the same as the pan or pail.

Samples in sacks may be marked with linen or paper tags. If there is danger of the tags becoming wet, a thin coating of hot paraffin or shellac will prevent the marking on the tag from being obliterated or changed.

DRYING AND CUTTING DOWN

Samples delivered to the bucking room or assay office may vary in weight from a few pounds to 100 or 200 lb. Before these samples can be assayed they must be dried and further reduced in bulk. If coarse, they must be crushed and then reduced in bulk to a few ounces or a pound of fine pulp.

Drying is done in ovens or on stoves or hot plates. Steam, electricity, or burning coal, wood, gas, oil or other material may be used to furnish heat. Exhaust-steam tables are particularly desirable. Steam is usually circulated through coiled pipes arranged in shelves on which the sample pans are placed, or through false-bottom steam tables on which samples are dumped. (See Sec. 22.)

Table 10. Practice in concentrate sampling with hand cutters

Company	Concentrate from	Method	Weight of shift sample	Per cent. of whole
St. Joseph Lead Co., Rivermines, Mo.	Jigs and tables	2-in. auger, 24 holes per car	50 lb. per car	0.055
Tungsten Mines Co.	Tables	1/10 shoveled when sacking; maximum size of particle, 0.1875-in.	120–300 lb. per 24 hr.	10.0
Copper Range Co.	Jigs and tables	Slotted pipe	....	....
Hedley Gold Mining Co., Hedley, B. C.	Tables	Try rod in concentrate boxes. Also each wagon of 8000 lb. by try rod in 8 different places	27–30 lb.	0.1
American Zinc, Lead and Smelting Co.	Jigs	Gun sampler	....	....
American Zinc, Lead and Smelting Co.	Tables	Hand cutter	....	....
Federal Lead Co.	Flotation, tables and jigs	Pipe sampler, 24 holes per car	....	....
Old Dominion Copper Co.	Flotation	Pipe sampler in cars	....	....
Morning Mill, Wallace, Idaho	Jigs and tables	Hand cutter	....	0.0001

Out of 31 plants reporting, 18 used steam for sample drying and most of these passed the steam through coiled pipes. Electric hot plates were reported by eight plants. Others reported coal, wood or kerosene stoves. One used heat from a Diesel engine exhaust by passing it through a box on which sample pans were placed.

Maximum temperature at which drying is done varies in different plants reporting from 132° F. to 450° F. Temperatures around 200° to 220° F. are most generally used. Ordinarily it is best not to use temperatures above 212° F. on account of danger of loss. This is especially true with easily oxidized sulphide ores which may lose sulphur on heating, or with minerals containing water of crystallization. In some cases temperatures even lower than 212° F. may be necessary, as in the case of certain clays in which water of crystallization is driven off before all hygroscopic moisture has evaporated.

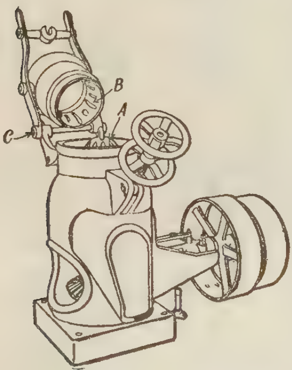


FIG. 42.—Engelbach grinder.

For air-drying samples for moisture determination the U. S. Bureau of Mines uses a temperature of 35° C. (95° F.).

Test for completion of drying may be made by holding a cool, dry watch glass or plate over the sample. Absence of condensed vapor on the glass indicates that the sample is dry. Stirring a hot sample with a spatula gives an indication of the presence of moisture by the vapor which rises. The most accurate test is to weigh until no further loss in weight takes place on further heating.

Crushing samples. Small jaw crushers are best for crushing material from 1- or 2-in. maximum down to about 0.5-in. Either the Blake or Dodge type is satisfactory. Small gyratory crushers may be used. Intermediate grinding is generally done in a COFFEE MILL (Engelbach) (Fig. 42) in which grinding is done between a rotating cone (*A*) mounted on a vertical, gear-driven spindle and stationary circular concaves (*B*) attached to the machine. The grinder is arranged so that the top part, containing the concaves, may be lifted on hinges (*C*) allowing access for cleaning and repairs. The following sizes may be obtained:

Capacity lb. per hr.	From	To	R.p.m.	Power required, hp.	Approx. weight, lb.
200	0.5-in.	10-mesh	225	4	850
100	0.37-5in.	10-mesh	250	3	550

Fine grinding of samples is best performed in DISK GRINDERS of either the Braun (Fig. 43) or McCool type.

Braun pulverizer. The ore is ground between two disks, one revolving (*a*) and other stationary (*b*); fineness of product is determined by adjustment of screw (*c*). The revolving disk is mounted on a horizontal shaft equipped with tight and loose pulleys (*d*). Material is fed through hopper (*e*) and enters the space between the disks at the center whence it is fed by gravity and centrifugal force to the periphery and there ground. Ground material passes between the disks and is discharged into a removable tray (*f*) beneath. Removable cover (*g*) and the hinged stationary disk permit easy access to the grinding parts for cleaning. Feed may be ¼-in. size. The principal dimensions are: length, 23 in.; width, 14 in.; weight, 235 lb., 850 r.p.m.; 1 hp. One set of grinding disks should handle from 2500 to 7500 ordinary ore samples. They are readily replaced.

McCool pulverizer is a similar machine except that slow relative motion of the disk centers is superimposed on rotation, which is supposed to accent the rubbing motion

between the disks. The machine is made in two sizes, the larger rated at 1 lb., the smaller at 0.5 lb. to 100-mesh in 30 sec. Dimensions follow:

Size	Length, in.	Width, in.	Height, in.	Pulleys, in.	Pulley speed,	Power, hp.	Shipping weight, lb.
Large........	40	18	17	12 × 4½	275	2	570
Small........	30	11	14½	11 × 4½	250	1	330

Bucking board consists of a flat cast-iron plate with finished surface on which ore is pulverized by means of a cast-iron muller on a wooden handle. The operation is usually manual, but mechanical drive may be used, if desired. Bucking boards are not as efficient or quick as disk grinders but furnish suitable means for grinding in places where power and more elaborate equipment are not available or for grinding very small quantities of material that might be lost in a mechanical grinder.

Grinding surfaces should be of material that is not so hard that it becomes polished, with consequent reduction in efficiency, nor so soft that the value of the sample is affected by admixture of fragments from the surfaces. If the surfaces are very soft, pieces of valuable mineral may be taken up, thus not only reducing the metal content of the sample being ground, but possibly salting succeeding samples. Grinding surfaces developing small holes or cavities should be avoided and discarded. Filling such holes with soft metal should not be practiced. Iron introduced into a sample by grinding can be removed with a magnet, if objectionable.

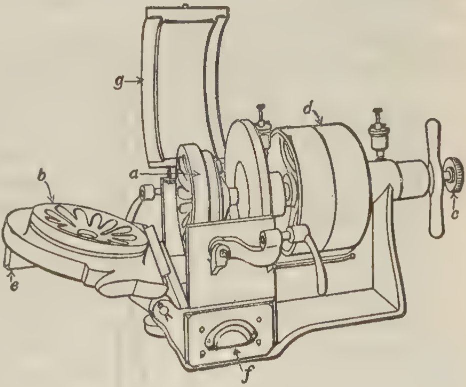


FIG. 43.—Braun disk pulverizer.

Fineness of sample required for assay varies with the character of ore. Since the actual material assayed must be cut out of the assay sample, the maximum size of grain must bear the same relation to the weight of the sample cut as at any other step in the sampling operation. Further, where a wet method of assay is employed, grinding must expose all of the valuable mineral to the action of the solvent. Usually a large factor of safety is allowed in the final sample and no sample for assay is coarser than will pass 80-mesh; many plants pass the final sample through 100-, 120-, 150-, or 200-mesh before delivering to the assayer.

Usual procedure in final grinding is to pass the sample through the grinder and separate undersize through a screen of the desired aperture. Oversize is returned to the grinder and the operation repeated until all passes the screen. Iron washers or stiff brushes are frequently used to aid in pushing material through the screen. Time is thus saved but washers cause wear on screen wires and in case of ores containing fine elongated pieces of valuable metal or mineral a piece may be forced through the screen that will have a serious effect on the value of the sample. With ores containing metallics it is often found impossible to pass the last flat metallic pieces through the screen, even on repeated grinding. In such cases the final oversize of metallic pieces is collected, weighed and assayed separately, and its value is allotted in proportion

to its weight and the weight of the sample. The screens used should be perfect.

Jones riffle (Fig. 44) is the most satisfactory device with which to make final reductions in bulk of sample prior to delivery to the assayer. It consists of an even number of equally sized chutes, adjacent chutes discharging at opposite sides. Ore is fed to the riffle from a scoop. Jones riffles may be procured in the following sizes:

Dimensions, in.	4 × 4	6 × 6	10 × 10	10 × 18
Width of troughs, in.	½	½	¾	¾
Number of troughs	6	8	10	18

Flat riffle (Fig. 45) is used for the same purpose as the Jones riffle, but is less convenient. It has alternate slots closed at the bottom so that half of the material fed is retained in alternate troughs while the other half falls through. In using this riffle care should be taken that the closed troughs do not fill up and overflow into the open troughs.

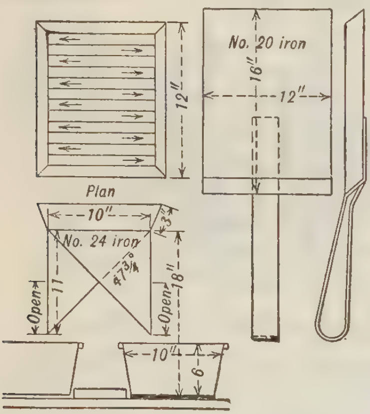


FIG. 44.—Jones riffle and scoop.

Use of riffles. Riffles should be of strong construction and carefully handled. Rough treatment causes distortion of the dividing edges between chutes with consequent inaccuracies in samples. Chutes should be wide enough to prevent bridging of coarse particles; at least three times the diameter of the largest particle in the feed and better more. Feed scoops should be of the same width as the total combined width of the riffle chutes; if greater, end chutes receive an undue proportion; if less, chutes receiving the edge of feed stream are under-loaded. The same number of riffles should discharge each side. Some manufacturers furnish riffles with an uneven number of chutes, so that both end chutes discharge the same side. This will cause material discharged on the favored side to contain an undue proportion of coarse material as well as to weigh more than the other portion. Such riffles should be rejected. Material should be spread evenly across the full width of the feed scoop and delivered to the riffles well toward the center. A good way to feed is to place the loaded scoop with ore at rest thereon so that the edge rests on top of the riffle-chute dividers along a line perpendicular to these and near the edge farthest from the operator; then draw the scoop smartly from under the charge, allowing it to fall evenly on the riffle. It is desirable to keep the edge of the scoop straight and to avoid a scoop that, through long usage, has a turned or uneven edge.

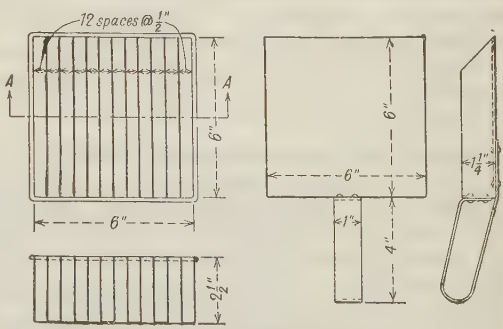


FIG. 45.—Flat riffle and scoop.

Mechanical devices are sometimes used to make the final reduction in bulk of sample. They have the advantage of eliminating the personal equation but it is questionable if their substitution for a Jones riffle gives any greater accuracy.

Umpire sampler (Fig. 46) has two buckets revolved in opposite directions by means of bevel gears. Each bucket is divided into four compartments by plates at right angles, two opposite compartments being open at the bottom while the other two are closed. Material is fed from a scoop (*a*) that is shaken by a cam (*b*) striking a strap (*c*) attached to the scoop and to the coiled spring (*d*). The sample passing through the second rotating bucket is caught in a bucket (*e*) and represents one-quarter of the material fed to the machine.

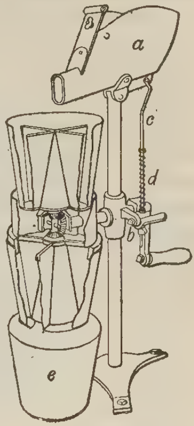


Fig. 46.—Umpire sampler.

Mixing is essential before dividing a sample or weighing out for assay. Simple methods are usually employed: (*a*) coning; (*b*) turning over and over with a spatula; (*c*) rolling on a piece of glazed paper, rubber or oil-cloth. Rolling is accomplished by drawing the corners of the paper or cloth horizontally toward diagonally opposite corners, causing the sample to roll over and over on itself. If the corner is lifted instead of drawn horizontally, the sample merely slides along the surface of the cloth and no mixing occurs. (*d*) Anaconda mixer, consisting of a cubical box rotating on a horizontal axis forming a diagonal of the box, makes an efficient mechanical mixer for small samples. (*e*) Jones riffle may be used for mixing finely-ground material, the two portions obtained being united and passed through again and again.

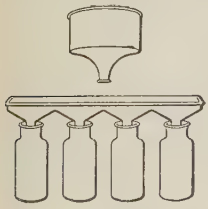


Fig. 47.—Bottle-filling device.

Final division of ground samples into small parcels for assay may be effected by riffles or machines as described above, or by one of the following methods: (*a*) Coning and quartering. (*b*) Spreading and taking small portions with a spatula from points scattered at random over the surface of material, taking care that the spatula tip goes down to the bottom of the pulp layer each time. (*c*) Pouring into bottles from a rolling cloth or scoop after mixing, using the bottle-filling device shown in Fig. 47. (*d*) Mechanical dividers of various types, usually diminutives of apparatus already described. (*e*) A method including mixing and dividing, successfully used on cobalt silver ores (*17 CMI 199*) is as follows:

The finely-ground sample (−100-mesh) is sifted several times through a 40-mesh screen onto a glass table, the screen being held about two inches above the material on the table. The pile is kept spread with a spatula. The glass table is marked off radially from a center into 16 equal sectors, and the material sifted is centered. After the final sifting the material is divided along radial lines. Material from alternate sectors is placed in separate packets, thus giving eight samples for such disposition as is desired. The balance of the material is rejected.

23. Cost of sampling

Information is limited and of such character that averages have little value. Costs at custom plants using machine-sampling methods vary from about 20 to 65¢ per ton (*Peele*), according to the character of the ore, location of plant, cost of labor, tonnage sampled and whether the ores are purchased or

not. The above figures include assaying and overhead but not the cost of buying and selling. In smelters cutting the first sample by hand methods, costs vary greatly according to whether the first handling is charged to sampling or other operations; they may be as high as $1.00 to $1.50 per ton. In mills having a mechanical head-sampling plant, the amount of labor required varies from 0.125 to 3 or more man-shifts per 24 hr. This does not include preparation of the sample for assay. For estimating the cost of general mill samples taken by hand, from 15 to 50 samples per man-shift, depending on local conditions, may be taken as a basis. From 12 to 45 samples can be prepared for assay per man shift, depending on the size of samples to be prepared character of material, degree of fine grinding necessary, equipment and general organization of the working force.

At HOLLINGER the mill-sampling cost per ton milled in 1923 was $0.0052 while assaying cost $0.0236 per ton milled. At UNITED EASTERN (*63 A 566*) the cost of general mill sampling in dollars per ton milled was as follows:

Year	Operating Labor	Supplies	Power	Total
1917	0.0011	.0034	.0005	.0050
1918	.0004	.0018	.0009	.0031
Average	.0007	.0026	.0007	.0040

The mill treats gold ore by cyaniding and has about 250 tons per day capacity.

SECTION 22

TESTING

Testing has two purposes, *viz.*: (1) determination of the best method for treating a given ore, and (2) determination of the best method of operating a given process. The purposes and methods of procedure overlap to a considerable extent, but in general, testing for a process demands the application of several different processes to the same ore while testing processes involves investigation of the effect of changes in the operating characteristics of a given process in treating the same or different ores. In either case the work requires painstaking attention to detail; close, careful and accurate observation and record of performance, and incessant speculation as to causes. Correct interpretation of results requires wide experience both in testing and mill operation and a healthy balance between pessimism and optimism. The best laboratory results can frequently be bettered in mill operation; on the other hand it is frequently impractical to expend in the mill the care lavished on laboratory operation and it is rarely that mill operation is economically subject to as close control in the matter of quantities treated and purity and uniformity of substances used as is the laboratory. This is particularly true in hydrometallurgical processes and flotation.

Time is ordinarily an element in all testing work. The experimenter is continually ground between the upper millstone of demand for quick results and the nether necessity for exhaustive investigation before announcing conclusions. Clear recognition of the situation should be sufficient guide.

The principal tools for testing are the assay, the testing sieve and the microscope. Others are the specific-gravity flask, heavy solutions, and small-scale replicas of or substitutes for mill machines. Among the latter are hammer and anvil for estimating crushing resistance; hand jig, pan, batea and plaque for gravity concentration; portable magnets, bottles for agitation-froth flotation tests, and the like.

1. Testing for a process

Outline of procedure

1. Obtain a proper sample.
2. Determine the qualitative mineralogical composition.
3. Determine the content of valuable mineral. (Assay.)
4. Determine the distribution of valuable mineral. (Sizing-assay test.)
5. Determine the aggregation of valuable mineral. (Microscopic examination and sizing-sorting-assay test.)
6. Study existing flow-sheets for the treatment of similar ores.
7. Devise a tentative flow-sheet for laboratory procedure.
8. Procure, by following this flow-sheet, material representing the feed to each machine in the tentative flow-sheet, and treat these materials in batches on the indicated machines, in order to determine the best possible conditions of treatment and the corresponding results.
9. Construct a metallurgical balance sheet of the results of the batch testing. (Four-column table.)
10. Make a continuous run to confirm the results shown in (9).

At this point the lines of procedure diverge according to the size of mill contemplated and the elaborateness of the laboratory equipment. It is rarely that a testing laboratory has sufficient equipment, properly balanced as to size, to make a continuous run on a scale that will yield a reliable indication of mill performance, except to an experienced interpreter, and the more experienced the interpreter the more doubt he is likely to feel of his interpretation. Consequently, if the size of the mill installation will justify the expense, the continuous run should be made in a pilot mill, built at the mine, fed with freshly-mined ore that is as nearly as possible representative of what the final mill will get, and run for a sufficient time to answer definitely, with as little interpretation necessary as possible, what the performance of each machine will be and what the performance of the whole mill. Practically all of the big mills in this country, when they have not been the result of progressive growth from small beginnings—which, of course, is the equivalent of a pilot mill—have been preceded by pilot mills of more or less elaborateness, in which, usually, a variety of different machines have been thoroughly tested for each step of the process. The cost of the pilot mill is usually only a small fraction of that of the big plant and the expense is readily justified as insurance and quickly saved if, as a result, the big mill falls more rapidly into regular operating stride.

When the contemplated mill is too small to justify the expense of a pilot mill, the continuous run or runs must be made in the laboratory. In the laboratory run it is usually impossible to duplicate mill conditions as to middling circulation and water reclamation. Except from the mechanical standpoint, however, *i.e.*, sedimentation and incrustation in the water channels and effects, reclaimed water makes no difference in the performance of gravity concentration. Its probable effect in flotation or in hydrometallurgical processes can be investigated by a series of small-scale tests supplementing the continuous run. On the other hand, middling must be circulated in the continuous run and eliminated from the final products. If the middling circuit differs, as it almost inevitably will, from that to be expected in the operating mill, its effect on performance must be closely studied and suitable allowance made in the final design. This is by far the most difficult part of testing and interpretation. No rules can be set down for guidance. Success

in handling the problem is in proportion to the ability of the experimenter to observe closely and to his experience in mill operation and in similar testing work.

Sample. The first essential is a correct sample, of such weight as to furnish all the material necessary for the initial small-scale laboratory tests. This weight will usually lie between 50 and 1000 lb. The smaller amount is ordinarily sufficient when enough is already known concerning the ore to indicate fine grinding and a simple method of treatment such as cyaniding or flotation. The larger amount is necessary when nothing is known or when what is known indicates a complicated flow-sheet.

Mineralogical composition. The first step in the testing work is to determine the approximate mineralogical composition of the ore, at least in so far as the principal minerals are concerned. With simple base-metal ores this can usually be done from hand specimens. (See Rogers, Dana, Davy and Farnham, Murdoch, Johannsen.) Precious-metal ores ordinarily require an assay to determine the presence of gold and silver. Complex base-metal ores demand microscopic examination (see p. 1192, also books on petrology).

Assaying is not within the scope of this book. The following books are reliable and complete: A. H. Low, *Technical methods of ore analysis*, John Wiley and Sons, Inc., 1919. E. A. Smith, *The sampling and assay of the precious metals*, J. B. Lippincott Co., Philadelphia, 1913. E. E. Bugbee, *A text book of fire assaying*, Wiley, 1922. C. H. Fulton, *A manual fire assaying*, McGraw-Hill Book Co., N. Y., 1911.

2. Sizing tests

Sizing analysis consists in quantitative separation of a mass of material of various sizes into a number of grades, each characterized by a relatively small size interval between largest and smallest particles. The means of grading are (*a*) screening and (*b*) settling in a fluid, usually water. Screen grading, used on granular material, is called SCREEN ANALYSIS; grading by settling in fluids, used for the finest sizes, is called ELUTRIATION. Frequently the various grades are assayed, in which case the operation is called a SIZING-ASSAY TEST.

Testing sieves (Fig. 1), used for screen analysis, are circular pans, 6 to 12 in. diameter and 1 to 2 in. deep with screen bottom. The frame is ordinarily made of brass with upper edge beaded on a stout wire and lower edge extended below the screen, crimped inward, and rolled to an easy fit with the upper edge of other screens in the set, in order that the screens may be nested.

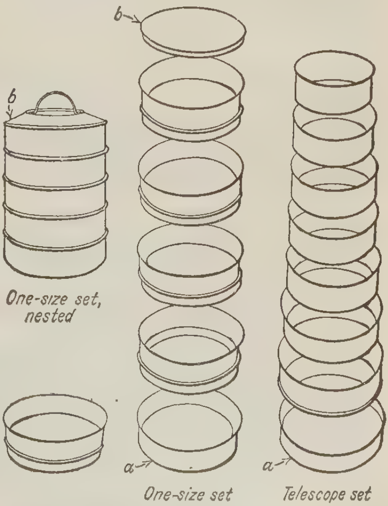


FIG. 1.—Testing sieves.

A pan (*a*) to collect undersize and cover (*b*) to confine oversize may also be obtained; their use lessens dust loss. TELESCOPE NESTS, usually made with

each finer screen of sufficiently smaller diameter than the preceding to permit telescopic packing, are made for field use. The pan of a telescope set is made of sufficient diameter to receive the nested set.

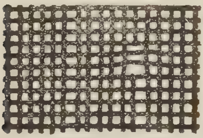


FIG. 2.—Microphotograph of 200-mesh screen cloth, magnified 12.5 diameters (warp wires vertical).

Standard testing sieves. The U. S. Bureau of Standards has done considerable work in the study of fine testing sieves and their performance, in connection with the determination of fineness of cement. The work early showed the irregularities in fine cloth (Fig. 2) and the first specification of a standard was designed to overcome the irregularity by setting up tolerances in diameter of wires and their spacing. (*USBS, Circ. 39.*) Further investigation (*USBS, TP 29 and 42*) led to a statement of larger tolerance as to mechanical construction, placing the principal weight on performance. The present specification follows:

"*Bureau of Standards Specifications for No. 200 Cement Sieves.*

"Wire cloth for standard sieves for cement shall be woven (not twilled) from brass, bronze, or other suitable wire and mounted on frames without distortion. The sieve frames shall be circular, about 20 cm. (7.87 inches) in diameter, 6 cm. (2.36 inches) high, and provided with a pan about 5 cm. (1.97 inches) deep and a cover.

"*No. 200 Cement sieve, 0.0029-inch opening.*

"The No. 200 sieve should have 200 wires per inch and the number of wires in any whole inch shall not be outside the limits 192 to 208. No opening between adjacent parallel wires shall be more than 0.0050 inch in width.

"The diameter of the wire should be 0.0021 inch, the average diameter shall not be outside the limits 0.0019 to 0.0023 inch.

"The sieving value of the sieve, as determined by sieving tests made in conformity with the standard specifications for these tests on a standardized cement which has a fineness of 75 to 80 per cent. passing the No. 200 sieve, or on other similarly graded material, shall not show a variation of more than 1.5 per cent. from the standards maintained at the Bureau of Standards.

"The Bureau also reserves the right to reject sieves for obvious imperfections in the sieve cloth or its mounting, as, for example, punctured, loose, or wavy cloth, imperfections in soldering, etc."

The 100-mesh and 20-mesh sieves are standardized on a mechanical basis only. For a nominal fee, the Bureau of Standards will test any sieve and certify its approach to the standard.

The Bureau investigation developed that single fineness determinations, made by experienced operators, rarely vary from the mean of several determinations by more than 0.5 on material testing between 75.00 to 85.00 fine and that the constant variations of invidual operators ("personal equation") were of the order of 0.1 to 0.2; that tests made when the humidity is high are unreliable; that the effect of humidity is different with different materials, least with cement, ground quartz sand and ground marble, of 11 materials tested, and greatest with trap rock and alumina.

Sieve scales. If a piece of substantially homogeneous rock is broken by any of the usual methods of crushing and the product is divided into a number of grades according to size, it will be found that the weights of the grades change gradually from size to size, that they pass through a maximum in one of the coarser sizes, and, sometimes, through a secondary maximum near the fine end, while the weight of the undersize of the finest screen is usually greater than that of the last preceding grades. This behavior is apparently a natural characteristic of crushed homogeneous material. The location of the maximum point in the curve varies according to the extent of the size reduction, the method of crushing, and perhaps also to other factors. If the rock

is non-homogeneous, the secondary maximum will usually be more pronounced and its position distinctly related to the rock structure, while the principal maximum will be located differently than with a homogeneous rock similarly treated. Graphical representation of these facts is given in Fig. 3. As the number of grades is decreased the smoothness of the curves decreases and a small amount of experimental work makes it apparent that irregular intervals between successive grade sizes increases the irregularity of the results. While this is an *ex post facto* explanation, it is the principal physical justification for a testing-sieve scale with regular size intervals. Testing-sieve scales have been proposed by Rittinger (*Aufbereitungskunde, 1867*), Richards (*OD*), DeKalb (*80 J 151*), Institution of Mining and Metallurgy (I.M.M.) (*19 IMM 486*), Hoover (*19 IMM 486*), W. S. Tyler Co. (*13 ASTM 1053*), and U. S. Bureau of Standards, the chronological order of the proposals being the same as the order of listing. Of these proposed scales only the I. M. M. and the Tyler series have taken form in actual testing sieves.

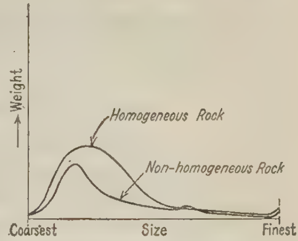


Fig. 3.—Direct plots of sizing tests of homogeneous- and non-homogeneous rock powders.

Rittinger series. The unit aperture is 1.0-mm. Apertures of other screens form a geometrical series both ways from the unit with a ratio of $\sqrt{2}(=1.414)$. Areas of individual apertures in successive screens vary, therefore, in the ratio of 2.

Richards series (also known as DOUBLE RITTINGER) starts from 1-mm. aperture and progresses both ways with the constant ratio $\sqrt[4]{2}(=1.189)$.

DeKalb series is pseudo-arithmetic. Starting with 0.003-in. as the smallest aperture, the succeeding apertures in the proposed series are found by adding $0.001n$ in. to the aperture of the preceding screen, n being the number of the screen in the series.

I. M. M. series (Institution of Mining and Metallurgy) has no regular basis, but contains screens whose mesh designation is in common use in the mills, *e.g.*, 20-, 30-, 40-mesh, etc. (see Table 1). In the hope to insure perfect locking of the wires on crimping, the wires in any given screen are chosen equal in diameter to the apertures. This specification makes the coarse screens too heavy for weaving and the fine too light for durability, while the percentage of opening in the intermediate screens (25 per cent.) is smaller than necessary and there is consequent reduction in rate of sifting. The aperture of any screen in the series, in inches, is the fraction $1/(2 \times \text{mesh-designation})$, (*e.g.*), the aperture of the 20-mesh screen is $\frac{1}{40}$ in.

Hoover series is a constant-ratio series (geometrical progression) with the ratio $\sqrt[3]{2}(=1.2599)$, starting with 1-in. as the unit aperture. It happens that the fifteenth screen, 0.0394-in., is 1-mm. aperture, so that this may be looked upon as the unit aperture for metric designation.

Tyler series is a geometrical progression with the multiplier $\sqrt{2}(=1.414)$, starting from the standard 200-mesh testing sieve (0.0029-in.). This is substantially the Rittinger series, except that instead of choosing 1-mm. aperture as the starting point, the series is based on a screen (200-mesh) already long established in testing-sieve practice and standardized by the U. S. Bureau of Standards. The 20-mesh (0.833-mm.) and 100-mesh (0.147-mm.) screens have also been standardized by the Bureau. The W. S. Tyler Co. made the series effective by manufacturing the screens in durable and attractive form and advertising them widely. As a result they have been adopted by a large number of mills, and for the first time, published screen tests of results at one mill can be interpreted in terms of screens familiar to readers at other mills in practically every part of the world. The Tyler Co. also manufactures sieves to the $\sqrt[4]{2}$ ratio, in order to give closer sizing, if desired. These start at the standard 200-mesh and hence fill in the gaps between the screens in the regular series.

U. S. B. S. series (United States Bureau of Standards) is the old Rittinger series, revivified by the Bureau several years after the introduction and adoption of the Tyler series. It includes the $\sqrt[4]{2}$ screens (double-Rittinger) below 1-mm. and thus brings in the standard 200-mesh sieve, but this sieve would not form a regular member of the series.

Table 1. Testing-sieve scales

Tyler series (a) Constant ratio, $\sqrt{2}$. Base, 0.074 mm.			I. M. M. series No constant ratio. No base			De Kalb series (c) Arithmetic series, constantly increasing increment. Base, 0.003 in.			Hoover series (c) Constant ratio, $\sqrt[3]{2}$. Bases, 1 in and 1 mm.			U. S. Bur. of Standards (c) Constant ratios, $\sqrt{2}$ and $\sqrt[4]{2}$. Base, 1 mm.		
Mesh	Inches	Mm.	Mesh	Inch	Mm.	Mesh (b)	Inches	Mm.	Mesh	Inches b	Mm.	Mesh	Inch (b)	Mm.
	1.050	26.7	5	0.100	2.54		1.038	26.4		1.000	25.4	2½	0.315	8.00
	(0.883)	(22.4)	8	0.062	1.57		0.993	25.2		0.794	20.2	3	0.265	6.73
	0.742	18.8	10	0.050	1.27		0.949	24.1		0.630	16.0	3½	0.223	5.66
	(0.624)	(15.8)	12	0.042	1.06		0.906	23.0		0.500	12.7	4	0.187	4.76
	0.525	13.3	16	0.031	0.79		0.864	22.0		0.397	10.1	5	0.157	4.00
	(0.441)	(11.2)	20	0.025	0.64		0.823	20.9	2½	0.315	8.00	6	0.132	3.36
	0.371	9.42	30	0.017	0.42		0.783	19.9	3	0.250	6.35	7	0.111	2.83
(2½)	(0.312)	(7.92)	40	0.012	0.32		0.744	18.9	3½	0.198	5.04	8	0.094	2.38
3	0.263	6.68	50	0.010	0.25		0.706	17.9	5	0.158	4.00	10	0.079	2.00
(3½)	(0.221)	(5.61)	60	0.0083	0.21		0.669	17.0	6½	0.125	3.17	12	0.066	1.68
4	0.185	4.70	70	0.0071	0.18		0.633	16.1	7½	0.099	2.52	14	0.056	1.41
(5)	(0.156)	(3.96)	80	0.0062	0.16		0.598	15.2	9	0.079	2.00	16	0.047	1.19
6	0.131	3.33	90	0.0055	0.14		0.564	14.3	9½	0.062	1.59	13	0.039	1.00
(7)	(0.110)	(2.79)	100	0.0050	0.13		0.531	13.5	10	0.049	1.26	20	0.033	0.84
8	0.093	2.36	120	0.0042	0.107		0.499	12.7	16	0.039	1.00	25	0.028	0.71
(9)	(0.078)	(1.98)	150	0.0033	0.084		0.468	11.9	18	0.031	0.794	30	0.023	0.59
10	0.065	1.65	200	0.0025	0.063		0.438	11.1	26	0.025	0.630	35	0.020	0.50
(12)	(0.055)	(1.40)					0.409	10.4	32	0.020	0.500	40	0.0165	0.42
14	0.046	1.17					0.381	9.68	40	0.0156	0.397	45	0.0138	0.35
(16)	(0.039)	(0.99)					0.334	8.99	45	0.0124	0.315	50	0.0117	0.30
20	0.033	0.83					0.328	8.33	60	0.0098	0.250	60	0.0098	0.25
(24)	(0.028)	(0.70)					0.303	7.70	70	0.0078	0.198	70	0.0083	0.21
28	0.023	0.59					0.279	7.09	80	0.0062	0.158	80	0.0070	0.177
(32)	(0.020)	(0.50)				3	0.256	6.50	115	0.0049	0.125	100	0.0059	0.149
35	0.016	0.42				3⅓	0.234	5.94	150	0.0039	0.099	120	0.0049	0.125
(42)	(0.014)	(0.35)				3½	0.213	5.41	200	0.0031	0.079	140	0.0041	0.105
48	0.0116	0.30				4	0.193	4.90	220	0.0025	0.062	170	0.0035	0.088
(60)	(0.0097)	(0.25)				4½	0.174	4.42	250	0.0020	0.050	200	0.0029	0.074
65	0.0082	0.21				5	0.156	3.96				230	0.0024	0.062
(80)	(0.0069)	(0.18)				6	0.139	3.53				270	0.0021	0.053
100	0.0058	0.15				6½	0.123	3.12				325	0.0017	0.044
(115)	(0.0049)	(0.12)				7	0.108	2.74						
150	0.0041	0.10				8	0.094	2.39						
(170)	(0.0035)	(0.088)				9	0.081	2.06						
200	0.0029	0.074				10	0.069	1.75						
						12	0.058	1.47						
						14	0.048	1.22						
						16	0.039	0.99						
						20	0.031	0.79						
						25	0.024	0.61						
						30	0.018	0.46						
						45	0.013	0.33						
						60	0.009	0.23						
						100	0.006	0.15						
						150	0.004	0.10						
						200	0.003	0.076						

a Alternate sieves in parenthesis, taken with the regular sieves, form a series with constant ratio = $\sqrt[4]{2}$. Sieves with 3-, 2-, and 1.5-in. openings which, for all practical purposes extend the $\sqrt{2}$ series upward, are obtainable from the Tyler Co. *b* Approximate. *c* Series not manufactured.

In view of the satisfactory quality and wide adoption of the Tyler series, the proposal of the Bureau is deplorable and it is to be hoped that it will not meet with acceptance by mill laboratories with resulting confusion in screen-test literature. Table 1 gives the mesh and apertures of the Tyler and I. M. M. testing sieves and, in parallel columns the apertures proposed in the other series.

Screens for coal testing. Holbrook and Fraser (*Bull. 234, USBM*) call attention to the fact that none of these scales is applicable for ordinary commercial coal testing for several reasons, *viz.*: (1) The proposed series all include such fine screens that woven-wire screen surface must be used, while most commercial coal preparation is done on round-hole screens and the testing sieves should correspond. (2) The series are based on the metric system while coal is sold in sizes commonly rated in common (English) units. (3) The majority of screens in the proposed scales are below 1-mm. while the usual coal investigations are interested in coarser sizes. (4) The usual testing sieves have 8-in. frames; coal testing, involving as it does much coarse material, requires larger samples than ore testing and consequently larger sieves. They propose a series with 1-in. as the base and a sieve ratio of two, viz.: 8-in. 4, 2, 1, ½, ¼, ⅛, 1/16, 1/32 and 1/64-in., the sieves to have round holes punched in steel or brass plate, mounted in frames 18 in. square. They suggest 6-in., 3-in. and 1½-in. sieves, if closer grading is desired in the coarser sizes. For pulverized coal the problem is different and the usual square-mesh wire-cloth sieves are suitable.

Methods of screen analysis. Crude analyses, suitable for all ordinary work, are made by placing a weighed dry sample of the material to be tested on the top or coarse screen of a nest, shaking the nest until most of the undersize has passed the coarse screens (one to two min.), then removing the screens one at a time, beginning at the top, shaking each separately over a pan until the amount passing through in a minute is less than 1 per cent. of that remaining on the screen. Undersize is added to the top screen of the remaining nest. On coarse screens (0.75-in. or larger apertures) pieces near the screen size may justifiably be tested and put through, if possible, by hand. Oversizes and final undersize should be weighed and kept separate until all have been weighed and the weight checked against the original weight. Weighings should be accurate to within 1 per cent. and the total weight of the grades should check the original weight of the sample within 1 per cent. Screens should be shaken in such a way that the material is caused to travel slowly in a thin sheet over the whole surface of the sieve and at the same time the sieve should be jarred in a way that will cause the cloth to vibrate gently in a direction perpendicular to its plane. In specifying a standard method for sieve testing of cement, the U. S. Bureau of Standards states that the sieve "shall be held in one hand in a slightly inclined position . . . , at the same time gently striking the side about 150 times per min. against the palm of the other hand on the up stroke. The sieve shall be turned every 25 strokes about one-sixth of a revolution in the same direction." The reason for the final specification appears in some work by Griesenauer (*70 EN 1296*) showing a variation of 1.4 per cent. in the mean of tests across warp and shoot wires. When the fine material is caked, it may be broken up by rubbing on the sieve with a bristle brush or a rubber cork. Cut-metal washers are sometimes placed on the finer sieves during shaking, but this practice wears and distorts the fine cloth and, with soft material, produces an improperly large amount in the finest size.

When coarse material is tested the sample must be large because of the impossibility of cutting down accurately. On the other hand, except for the most accurate work, a carefully riffled sample of −0.12-in. material weighing 250 to 300 gm. is large enough. Hence considerable time can be saved by sifting the large sample roughly on the 3.33-mm. screen, cutting down the undersize to 250 to 300 gm. and screening this on the finer screens while the oversize is re-screened beginning with the coarsest screen. The undersize from the 3.33-mm. screen on the re-screening may usually be safely considered

oversize on the next finer screen and so treated in calculating the re-distribution.

Wet samples are best handled by jigging on the finest screen at the surface of water in a pail and washing the material on the screen with a fine jet until substantially all of the slime has passed. Oversize is then dried and re-screened on a nest including the fine screen, and the undersize of the fine screen on the dry sifting is added to the dried undersize of the wet screening. This procedure is quicker and more accurate than preliminary drying and dry sifting, on account of the difficulty in breaking up slime cake formed in drying. Sampling wet pulp for the screen sample is, however, a difficult matter, and this consideration may justify drying, riffling and subsequent wetting to break slime cake.

Mechanical testing-sieve shakers. It requires from one to three hours to sift a 200-gm. sample containing 30 to 50 gm. of -200-mesh material by manual shaking. Mechanical shaking will reduce the time to from 30 to 45 min. and during the time the sieves are being shaken the operator is free for other laboratory duties.

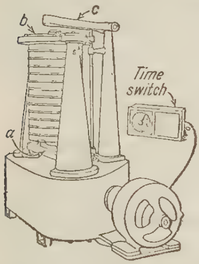


Fig. 4.—Ro-tap testing-sieve shaker.

The Ro-tap testing-sieve shaker (Fig. 4) made by W. S. Tyler Co., for 8-in. testing sieves, consists of a movable cage with base (*a*) and top plate (*b*) between which a nest of 13 half-height sieves or 7 full-height with pan and cover can be mounted and subjected to a rotary sifting motion while at the same time the lever (*c*) strikes the top plate once per revolution and produces vibration of the screen cloth. A time switch on the motor is useful. Duplicate samples sifted for equal periods of time on the same or different machines check well within the limits of sampling error. If the total amount of dust in the sample is important, the sieves after removal from the shaker, should be brushed around the inside of the rims with a soft brush and shaken individually for a short time as in hand sifting, as a small amount of dust collects around the edges of the coarser screens during the mechanical shaking and does not pass through.

Standard sizing test. No standard procedure for sizing tests is accepted or generally used, but Hersam and Gross (*Reports to Milling Comm. A. I. M. & M. E., Jan., 1924 and Jan., 1925*) have proposed the following method, after long study. The time required for performance is excessive except in cases where an apparent refinement is demanded that is greater than is actually possible by any method known at the present day. However, while adoption of the proposed method *in toto* is certainly neither justified nor desirable, many of the individual suggestions are valuable. A summary follows:

Table 2. Tolerances in testing sieves as recommended by U. S. Bureau of Standards

Screen opening, mm.	Tolerances in percentage		
	Wire diameter	Average opening	Maximum opening
2.83 +	10	1.5	10
1.00 +	10	3.0	10
0.35 +	10	4.5	25
0.125+	20	6.0	40
0.044+	20	7.5	60

Screen cloth should have square holes and be made of double-crimped wire of sufficient weight to resist deformation, but not so heavy as to permit enough wear to change the aperture materially. Sieve frames should be 8-in. diameter, strong and well-made, and free from crevices that can hold material. The actual average dimensions of apertures should be known and recorded and variation in aperture should not exceed U. S. Bur. of Standard tolerances. (See Table 2.) The sieve-scale ratio should not exceed $\sqrt{2}$. The sample should be dried at 110° C., riffled down to within 10 per cent. of the weight corresponding to maximum size of particle, according to Table 3. Sift wet on finest sieve,

using distilled water, until all slime is removed. This may be done either by placing the charge on the sieve and jigging it through the surface of a body of water in a container, or by decantation through the sieve. In either case the quantity of water used should be kept as small as possible. DRY oversize at 110° C., cool and weigh. SIFT OVERSIZE dry, beginning with the finest sieve, SHAKING as recommended by the Bureau of Standards (p. 1185). Sifting should be done with pan and cover, the sieve held over glazed paper to indicate and save SPILLS. When the undersize for one minute is less than 0.1 per cent. of the original weight of sample, remove the oversize, brush dust from both sides of the sieve, remove particles loosely held in the screen, then return all oversize and shake again until less than 0.05 per cent. of the weight of the original charge passes in one minute. Repeat the brushing operation, return the oversize and sift for one minute. If the undersize is less than 0.05 per cent. of the original charge weight, the END POINT has been reached, otherwise repeat the foregoing procedure until this condition is reached. Return the last-minute undersize to the oversize. Weigh final undersize and add it to the wet undersize. CLEAN SIEVE by tapping, brushing and rubbing with fingers, add material thus obtained to the oversize and weigh. Repeat sifting of oversize on successively coarser screens, weighing the final oversize in each sieve and obtain the WEIGHT OF UNDERSIZE by difference. This method credits the LOSS on each screen except the finest to the undersize of that screen, which, in lieu of other information, is logical. WEIGHT OF FINEST UNDERSIZE is obtained by drying at 110° C. and weighing. This material may be further sized, if desired, by ELUTRIATION. The sample for elutriation should be riffled down to about 20 gm. and separated by free settling in water into several grades. The proposed grades are shown in Table 4. SIZES OF ELUTRIATED PRODUCTS may be estimated from Fig. 5 or determined on wet samples by microscopic measurement (Art. 4). Return microscope samples, dry all products at 110° C., cool and weigh. When riffled samples have been used for elutriation, the weights of products must be re-calculated on the basis of the original fine undersize.

Table 3. Sample weights for sizing analysis, proposed by Hersam and Gross

Range of size of coarsest particles in sample, mm.	Sample weight, gm.
16.00–11.32	40,000
11.32– 8.00	12,500
8.00– 5.66	5,000
5.66– 4.00	2,000
4.00– 2.00	1,000
2.00– 1.00	500
1.00– 0.50	250
0.50– 0.25	100
0.25– 0.00	50

Table 4. Settling rates for elutriation, proposed by Hersam and Gross

Product	Material that settles one meter in	Material that does not settle one meter in
Slime...............	....	3 hr.
Finest sand........	3 hr.	2 hr. 40 min.
Next coarser sand...	2 hr. 40 min.	2 hr. 20 min.
Next coarser sand...	2 hr. 20 min.	1 hr. 35 min.
Next coarser sand...	1 hr. 35 min.	50 min.
Next coarser sand...	50 min.	24 min.(*a*)
Next coarser sand...	24 min.	12 min.(*b*)

a When aperture of finest screen is about 0.05 mm.
b When aperture of finest screen is about 0.07 mm.

Apart from the excessive amount of time required for this procedure the principal faults are the large sample weights recommended and the sifting of the dry material on progressively coarser screens. Duplicate screen tests of 10-kg. shovel samples of −16-mm. material will check well within the limits of the differences in sifting by two different operators and the same is true of samples of −0.12-in. material weighing between 250 and 300 gm. Placing coarse oversize on fine screens is equivalent to the use of metal washers, which is universally and rightly condemned.

Elutriation, as applied to ore dressing, is the process of grading finely-divided solid matter according to size by washing with water. There are two general methods, *viz.*: (1) by allowing the solids to settle freely in still water for varying periods of time, and (2) by subjecting the solids to rising water currents of different velocities.

Decantation is the simplest method of elutriation.

Take a tall beaker of 300 to 500 cc. capacity, place therein the sample to be sized, which

should weigh between 10 and 20 gm., fill with water to a predetermined depth, say 10 to 20 cm., stir thoroughly, then allow the beaker to stand for such a time that this time in seconds divided by the depth of water in millimeters will equal a predetermined settling rate corresponding to the largest-sized particle desired in the grade. The required time may be read from Fig. 5, or determined by application of Stokes' equation (Sec. 6, Art. 1). Now pour off the supernatant liquid with the non-settled solid in suspension, re-fill to the mark and again allow to settle for the same time. Repeat this until the appearance of the supernatant liquid indicates that all of the finest slime has been removed. Combine the decanted portions, take a sample, while wet, for microscopic determination of size (Art. 4), then dry at 110° C., cool and weigh. Re-fill the beaker to the mark with fresh water, and repeat the above operations, allowing a shorter time for settling corresponding to the next coarser grade desired, and repeating with this settling time until the supernatant liquid, at the end of the period, shows no solid in suspension. Repeat as above until the desired number of grades is obtained. If there is obvious chemical action during the operation, using tap water, *e.g.*, formation of gelatinous precipitates, distilled water should be used. If this does not remedy the trouble, weak solutions of acid or alkali may be used, but in such cases the presence of the added chemicals or their salts in the dried solid must be recognized. If microscopic sizing of the different grades is dispensed with, it must be recognized that there is considerable overlapping of sizes, due to the fact that not all of the solids start settling from the surface of the liquid in the beaker and that some fine particles that start settling from a point below the surface will reach bottom while coarser particles starting above them are still in suspension. Repetition, as directed, lessens this inaccuracy but does not eliminate it.

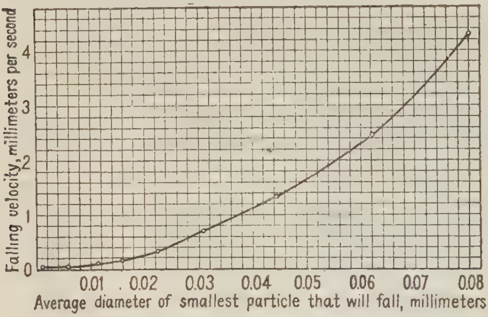


FIG. 5.—Free-falling velocities of quartz (*after Richards*).

Still-water settling tube is an improvement on decantation.

The apparatus (Fig. 6) consists of a glass tube of about 1.5-in. uniform diameter and 8 ft. long, converging in a 60° cone at the bottom to a stopcock with 1.5- to 2-mm. hole and expanded at the top into a funnel of about 200 cc. capacity, with 60° apex angle. Calibrate the tube from stopcock to base of funnel sufficiently closely that the length at any point in the tube corresponding to a given volume withdrawn at the stopcock will be known. From this calibration make a table of volumes for the required number of successive drafts such that the distance that the slowest settling particles are dropped, *i.e.*, that the surface originally at the base of the funnel drops, at each draft, is the same. Thus in Fig. 6 if the tube is calibrated for 10 draws, the successive volumes drawn should be such that the distance *ab*, *bc*, . . . *jk*, representing the successive drops of the surface originally at *a*, are equal. The charge of material to be sized, weighing from 20 to 50 gm. should be mixed with water to form a volume somewhat less than 200 cc. Place a closely-fitting disk of metal or fine screen cloth at the base of the funnel with a handle extending above the rim. Charge the feed carefully and wash in the last with a fine jet. Then, first observing and noting the time, remove the disk carefully to prevent the solid from plunging and allow settlement for a pre-determined time, or for a time dependent upon the appearance or amount of material at the stopcock. The tube should be jarred slightly at intervals to prevent material from clinging to the sides of the apparatus. Draw off now the volume corresponding to the first equal drop in surface of settling column and set aside. Make successive draws at inter-

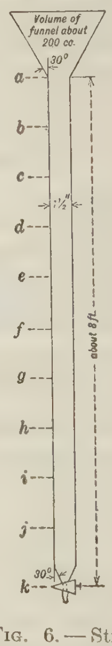


FIG. 6.—Still-water settling tube.

vals determined by the requirements of the analysis, *e.g.*, at equal intervals, or at intervals determined by Fig. 5 or by Stokes' equation to yield predetermined sizes, or at intervals which the settling behavior indicates will yield equal weights of solid. When the total volume of withdrawals is equal to the original volume of water in the settling column, there will remain a volume equal to that of the feed pulp and containing the finest slime. Remove this at one draw and class it as of slower settling rate than that of the slowest-settling particle in the preceding grade. Sizes of the particles in the different grades may be determined by microscopic measurement or may be estimated from the known settling rates. Decant clear water from the different grades, dry at 110° C., cool and weigh.

If l is the total length of settling column, measured from neck of funnel to top of stopcock, N = the total number of draws, and n = the number of any given draw, then the average distance D_n that the particles taken in the *nth* draw have settled is given by the equation $D_. = l(2N - 2n + 1)/2N$. If t_n = the total time elapsed from the beginning of settlement until the end of the *nth* interval, the average settling velocity V_n of the solids collected during the *nth* interval is given by the equation $V_n = l(2N - 2n + 1)/2Nt_n$. If equal time intervals t are taken, this equation become $V_n = l(2N - 2n + 1)/2Nnt$. The slowest-settling particle in the last grade settles through the distance $D_{SS} = l - l/N$ in the time t_n. Its settling velocity is, therefore, $V_{SS} = l(N - 1)/Nt_n$ and the most rapidly-settling particle in the final draft has a slower velocity.

Elutriation by rising currents is performed by subjecting the material to be graded to rising currents of different velocities and collecting separately the material lifted by each current.

The apparatus shown in Fig. 7 (*32 M & M 125*) is one of the simplest of these devices. As described, it was used merely to separate slime from a sample that was to be subsequently screen sized. The procedure was to set the dial cock (f) so that the current at the overflow level was sufficient to carry over all slime, then to close the rubber tube (g) with a pinch cock, charge the weighed sample into (a) and then release the pinch cock. When overflow was clear, the material in (e) and that remaining in the tube were collected and sized. The same apparatus may, however, be used to separate a number of different grades. There are two alternative methods of procedure, viz.: (1) To set the current at the lower part of constriction (c) to just prevent slime from settling, feed the sample into (a) and collect the overflow in a pail or tub. Then raise the current slightly, feed back the settled material collected in (e) and again collect the overflow, and repeat this procedure until the desired number of grades has been made. (2) Set the current originally so that the velocity at the lower part of section (c) will permit only the coarsest material to settle. Collect overflow and settled product. Separate the slime from the sand in the overflow by decantation. Slack off the current slightly, feed back the sandy portion of the first overflow and again collect the settled material. Repeat with gradually slower currents until as many grades as desired have been made. Final products will consist of the various settlings and the decanted portion of the first overflow. This apparatus is adapted to relatively coarse grading only.

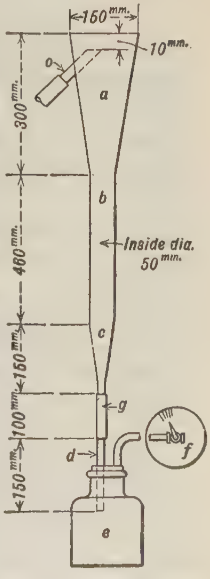


FIG. 7.—Rising-current sizing tube.

Schoene apparatus (Fig. 8) is distinguished by the fact that overflow is made through a piezometer, which permits ready setting of current velocities, once the piezometer is calibrated to the tube with which it is to be used. In the figure, (a) and (b) are sorting tubes of smaller and greater diameter respectively for coarse and fine sizing. The procedure consists in first determining the average cross-section of the cylindrical portions of the tubes by weighing the water drawn through the stopcocks corresponding to the measured length between two marks delimiting the cylindrical section. Knowing the average cross-section from this determination, average rising-current velocities can be determined by weighing overflows collected for known times. This is done and at the same time the corresponding piezometer readings are taken and a curve showing average rising velocities in terms of piezometer readings plotted. To make a sizing analysis, fill the feed-water tank and maintain sufficient inlet so that the tank will overflow throughout the test. Close the stopcock

at the lower end of the sorting tube and admit sufficient water from the tank so that the level in the sorting tube stands a couple of inches above the stopcock. Make up the weighed feed sample into a dilute pulp sufficiently small in volume to less than fill the tube and introduce this through the funnel. Close the funnel stopcock. Open the water cock slowly until the piezometer reading indicates the desired minimum current and continue this current until the discharge carries substantially no solids. Set aside the overflow, then increase the current and collect another grade and continue until all but the coarsest material is overflowed. Collect this last through the stopcock at the bottom of the sorting tube. Determine the size of material in different grades by microscopic methods or estimate from Fig. 5 or from Stokes' equation. Stadler (*22 IMM 686*) points out that, if the specific gravity of the material being sized is known, solid weights may be determined by weighing the material in a specific-gravity flask, or its equivalent (see Art. 23), thereby saving much time in drying.

According to Schoene a calibration curve is unnecessary and the velocity V corresponding to any piezometer reading h may be obtained from the equation $V = V_1\sqrt{\frac{h-c}{h_1-c}}$ when V_1 and h_1 are corresponding values from one observation and c is a factor, determined once for all, from the relation $c = (Q^2h_1 - Q_1{}^2h)/(Q^2 - Q_1{}^2)$ in which Q and Q_1 are quantities in cc. per sec. corresponding to piezometer readings h and h_1 respectively in cm.

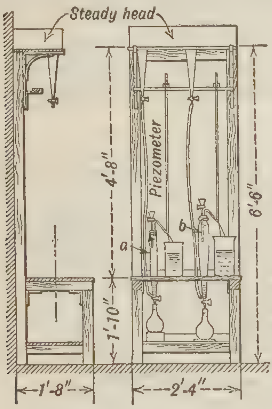


Fig. 8.—Schoene elutriation apparatus (modified).

Multi-tube elutriation. If a series of Schoene tubes of the type shown in Fig. 8, of different diameters, is set up without piezometers, the overflow tube of the smallest entering the apex of the next larger, and so on, and a charge of solid is placed in the first and a current started of sufficient velocity to lift all but the coarsest material out of the first tube, then if the last tube is of sufficient diameter to overflow only the finest slime, there will be collected in the successive tubes successively finer grades and the entire separation can be done at one operation. This is the best type of apparatus for routine tests, but lacks flexibility.

Interpretation of elutriation tests. The settling rate of solids in water is dependent upon the specific gravity and shape of the particles, upon the degree of packing in the sorting column and upon the uniformity of the sorting current (see Sec. 6, Art. 1). It is inevitable that, if the sample sorted contains grains of different specific gravities, the grades will contain grains of highly divergent sizes, and that, even if all of the material is of the same specific gravity, the coarser grades will contain finer grains carried down mechanically or by eddy currents, and the finer grades will contain flat scaly particles that settle much more slowly than their average "diameter" would indicate. Hence the only sure way to determine the average size of grains in a given grade is by microscopic measurement. If desired, this measurement may be made separately on the heavy and light particles and on the flat and rounded particles, and the analysis recast on the basis of these measurements.

Air elutriation. Pearson and Sligh (*TP 48 USBS*) review eight forms of air analyzers and describe in detail the construction and operation of a form developed by the Bureau.

This consists (Fig. 9) of a vertical cylindrical polished-brass sorting tube (a), 2.7 in. inside diameter by 60 in. long with a bulb (b) at the bottom for holding the sample, three nozzles (c), 0.04-, 0.09- and 0.13-in. diameter, for varying the velocity of the entering air and hence the intensity of agitation in the bulb and the velocity of the rising air current in the sorting tube, and a collector (d), about 10 in. diameter by 20 in. high, with canton-flannel covering, for receiving the various solid fractions while passing the air. Air is supplied at constant pressure (ordinarily about 1 lb. per sq. in.) from the reservoir, which is supplied by a motor-driven blower and controlled by a blow-off valve and an oil regulator. The latter

is a vertical pipe about 4 in. diameter by 5 ft. long, closed at the bottom and nearly filled with kerosene. A long glass tube, open at the lower end and connected at the top with the reservoir, extends to within a short distance of the bottom of the oil-filled cylinder and the back pressure is adjusted by changing the depth of submersion by raising or lowering the tube. In operation the blower is run at a speed sufficient to maintain slightly more than 1 lb. per sq. in. pressure with a small amount of blow-off and the regulator is then set by trial. The sample taken is −200-mesh material, weighs 30 to 50 gm., and is divided into four fractions. The flow of air in the sorting stack is reasonably constant in velocity and uniform across the section; air delivery is not retarded by the sample, if the nozzle tip is kept 1 cm. or more above the surface, nor by the back pressure of the collector sack. The sample must be kept stirred up in the bulb, if operation is to be satisfactory. The dependent factors are air pressure in reservoir, diameters of nozzle, bulb and stack, shape of bulb and quantity of sample. Lodgment of material in the separating stack, which would hinder separation, is guarded against by polishing all interior surfaces, beveling the upper rim of the stack to a knife edge, tapping the stack continually during operation with an electric tapper, and so designing the collector that material once therein cannot again enter the sorting tube.

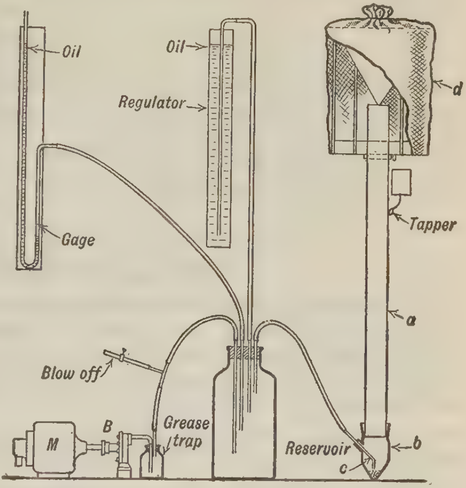


Fig. 9.—Air analyzer, Bureau of Standards.

Operation is as follows: (1) With the nozzle that is to be used for the finest separation connected to the reservoir, but not in place in the bulb, start the blower and adjust the speed, blow-off and oil regulator for a constant pressure of very slightly more than 1 lb. per sq. in. with the regulator blowing off a slight amount of excess air. Disconnect the nozzle, insert it in the bulb and weigh the two to the nearest 0.01 gm. Add the sample to the bulb (33.33 gm. if the 0.04-in. nozzle is to be used for the finest separation, otherwise 50 gm.), attach to the stack, start the tapper and connect the air tube. Continue elutriation until the loss per unit of time is at the rate of some pre-determined amount (Pearson and Sligh have adopted 0.02 gm. per min.). The end point is determined by stopping blowing some 15 or 20 min. before the estimated time of completion, allowing 30 sec. to 1 min. for material to settle from the stack, then weighing the bulb and contents. Continue blowing 10 min. and again weigh. Repeat until the 10-min. loss is less than 0.2 gm. Interpolate for the weight at the minute when the loss was at the rate of 0.02 gm. per min. Repeat with 0.09-in. and 0.13-in. nozzles.

Actual sizes of the various grades for a given apparatus and material must be determined by microscopic measurement, but once determined the sizes are reasonably constant for other samples of the same material in the same apparatus. With cement, experiment shows that the average diameter reckoned as $D = \sqrt[3]{blt}$, where b, l and t are the arithmetical means of a number of determinations of breadth, length, and thickness, respectively, of grains, checks very closely with the mean of b for both the air-elutriated grades and the material on 100- and 200-mesh sieves. Results of one determination are shown in Table 5.

Shape of bulb is not important except in so far as it determines the amount of material that can be treated with the minimum air stream. Diameter of nozzles need be neither accurately specified nor determined. Diameter of stack should be the least that is consistent with the finest separation, *i.e.*, the least that will give sufficiently low air velocity in the stack when the material in the bulb is properly permeated. At this, the agitation in the bulb when sufficient air is rising in the stack for the coarsest separation will be great. Length of stack has been tested over the range from 5 to 8 ft. and results with the short stack shown to be reasonably concordant with those of the long stack. Abrasion is negligible under conditions of minimum stack diameter. Atmospheric conditions have inconsiderable effects,

Small variations in pressure (less than 10 per cent. range) may be neglected. Size of sample may range between 25 and 50 gm. without noticeable effect on results.

Table 5. Comparison of average diameter with mean breadth of air-elutriated and screened particles. (*After Pearson and Sligh*)

Grade	Mean breadth, b, inch	Average diameter, $\sqrt[3]{lbt}$, inch
Rising with 0.04-in. nozzle.........	0.00062	0.00066
Rising with 0.09-in. nozzle.........	0.00130	0.00129
Rising with 0.13-in. nozzle.........	0.00181	0.00178
Not rising with 0.13-in. nozzle but passing 200-mesh sieve..........	0.00368	0.00371
Passing 100-mesh, retained on 200-mesh.........................	0.00703	0.00690

3. Testing with the microscope

The microscope is used in ore testing (1) to aid in mineral and rock identification and classification; (2) to aid in the study of mineral occurrence and the ore characteristics upon which the treatment scheme depends; (3) in quantitative mineralogical analysis; (4) in sizing analysis. Until the present time adoption of this tool in testing laboratories has been very slow and much time and money have been and still are wasted in work that could have been more quickly done with the microscope or which microscopic examination would have shown to be useless.

Microscopic mineralogy and petrography are familiar tools to the geologist and the literature is excellent and large. The principal apparatus required is a petrographic microscope. Samples for examination are prepared either in the form of thin sections or pulverized fragments. Detailed description of methods of study and interpretation of thin sections are given by *Johannsen*; L. M. Luquer, *Minerals in rock sections, Van Nostrand, 1913;* Winchell, N. H. and A. N., *Elements of optical mineralogy, Van Nostrand, 1909;* the study of pulverized fragments is best described by *Rogers*.

Mineragraphy is the study of opaque minerals, including native metals, most sulphides, certain base metals, oxides and the like, by means of the metallographic microscope. The methods include study of polished sections by reflected light, etching and various microchemical tests, hardness tests, etc. The methods are much more effective in identification of opaque minerals than either of the ordinary petrographic methods above mentioned.

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Of even greater importance to the ore tester than identification and the information concerning ore genesis afforded by mineragraphical work is the information that the polished section gives as to particle size and method of occurrence of the valuable mineral. The first fact can be established with coarse-grained ores by means of a sizing-sorting-assay test (Art. 7) and with such ores the second is not of much importance, but with finely disseminated

and complex ores the polished section quickly gives essential information that can be obtained in no other way. Determination of grain size will tell the size to which the ore must be crushed to free the valuable mineral and upon this fact and the kind of mineral and gangue, the principal elements of a tentative flow-sheet can be immediately founded. The appearance of the edges of the sulphide-mineral grains and of cracks traversing them will tell whether any alteration that would be likely to affect flotation has occurred. Inclusions of worthless or deleterious substances that would lower the grade of concentrate, such as silicates between the laminæ of graphite grains, blende in galena and the like, are immediately apparent in a polished section. The degree of admixture of sulphides in complex ores can be studied readily and in many cases one or two properly chosen sections are sufficient to tell the hopelessness of any attempt at separation by mechanical methods. Thus much time and money can frequently be saved and no test of any ore should proceed beyond the preliminary assay stage without such microscopic examination.

Quantitative mineralogical analysis is useful primarily in study of the products of a concentrating operation. The first step is identification of the important minerals and mineral groups. The analytical work is done with low magnification since identification of the valuable mineral, the accompanying heavy mineral or minerals, and the gangue minerals as a group is easy. Failing this, however, analysis must be preceded by identification, and for this work *Rogers'* crushed-fragment method with the petrographic microscope is best.

Apparatus: Low-power microscope, preferably binocular type, so mounted as to permit ready relative movement of stage and objectives. The stage should be ruled in squares about 10 mm. on a side and the microscope should have an eye-piece net micrometer with one of the squares subdivided. Zeiss and Bausch and Lomb both make binocular microscopes for ore-dressing work having magnification up to 20 diameters, fitted with racking mechanism in two directions at right-angles, and provided with net-ruled glass stage and eye-piece micrometer. For greater magnification a metallurgical binocular with 25- and 55-mm. objectives and 6 × and 10 × oculars is a suitable instrument. This instrument, which is obtainable from any microscope house, is not usually fitted with racking stage, and the sample, mounted on a suitably ruled slide, must be moved manually to change the field. This can, however, be done readily with a little practice.

Procedure consists in close and accurate sizing of the sample and a mineral count of a sufficient number of fields of each grade to establish a reliable number percentage. Sizing should be done by first washing out the slime through the finest screen, then drying and screening the oversize and, if necessary, grading the undersize by elutriation. Washing through the finest screen may be done with a soap or saponin solution, which prevents skin flotation and the formation of floccules that will not pass the screen and also lessens the surface tension of the liquid films that form across the screen apertures and thus aids the passage of undersize.

Coarser grades can be transferred to and distributed on the slide by means of a small spatula. The layer should be only one grain deep and if the grains are spaced at least one diameter, which may usually be accomplished by tapping the slide, counting will be made much easier. Finer grades are best transferred to the slide by shaking them on through a suitable fine screen. If, as is rarely the case, the analysis must be carried down into very fine water-separated grades, the methods of mounting employed in microscopic sizing analysis should be used.

Sampling of the grades to obtain a representative lot for counting is one of the most difficult and unsatisfactory parts of the whole procedure. The most accurate cutting down can be effected on the unsized sample, but if this is made too small, inaccuracies and losses in sizing become proportionately great. It is probably best to set 25 gm. as the lower limit of screen-sample size and to approach this for samples containing 0.5-mm. maximum grains and finer and run up to 50 to 75 gm. for samples up to 1.5-mm. size. For material coarser than this the method becomes distinctly inaccurate.

Irrespective of the size of sample taken for screen analysis, the bulk of most of the grades will be too great to permit mounting of the entire grade on one or even on several slides, and it is necessary to sample the individual grades. Probably the best method of doing this is as follows: Pour the sample through a small-necked funnel on to a glass plate, so as to form

a cone-shaped heap; flatten out the heap with a small spatula to make a circular cake, taking care that so far as possible all particles that move are displaced radially outward; divide the heap into quarters by first making a diametral cut with the edge of the spatula and pushing one-half with the spatula face a distance of about one inch in a direction at right angles to the first cut, then split each half into two equal parts by cuts along the radii at right-angles to the initial cut; finally reject opposite quarters, combine the remainder and repeat, if necessary, until a sample is obtained that will give the required layer one grain deep on the slide.

Counting should proceed systematically. In the largest sizes, with a few particles only in each square, it is best to count all of the grains in the ruled field; in the intermediate sizes only the grains in selected squares need be counted, the squares selected being regularly spaced to cover the ruled field and the number counted being proportioned to the size of grain, so that each size shall be analyzed with substantially the same degree of accuracy. Application of this criterion requires experience and judgment on the part of the analyst. The factors involved are: degree of uniformity of distribution of the different kinds of grains in the field, degree of uniformity in size of particles, amount of composite material (middling grains), relation of the weight of one particle to the weight of material in one square and in the whole field, relation of the weight of material in the field to the weight of the grade, and relation of the weight of the grade to the weight of original unsized sample. In the finest size the squares counted are the smallest in the micrometer net and one or several of the squares in this net should be counted from each selected square on the stage net, according to the above criteria. Some analysts recommend counting all or selected squares along one or both diagonals of the net, thus substantially cutting sections through the center of the field; others choose squares equally spaced in both directions covering the entire area. In no case should random selection or selection involving judgment on the part of the operator be permitted.

In counting the grains in any square it is best to carry through and record the count on each kind of grain separately. If one kind of grain breaks in such a fashion that its volume is consistently different from that of the grains of the other minerals in the grade, this fact should be noted and the relative volume established or approximated. In counting grains that are composites of the varieties sought to be separately reported, the relative volumes of the constituents must be estimated and if, as is usually the case, the relative proportions vary in different grains, the count should be classified on the basis of this proportion. Four classes are enough in any case, viz.: less than 25 per cent., 25 to 50 per cent., 50 to 75 per cent., and more than 75 per cent. Grains containing traces of, say, sulphide should be classed as tailing and those containing traces of gangue as concentrate. For most purposes three or even two classes are sufficient.

It should be borne in mind in microscopic estimation that the aspect presented to the eye is a surface aspect while relative volume is the figure sought, and the visual impression must, therefore, be suitably weighted in recording the percentage estimate. Thus Coghill and Bonardi (*see below*), analyzing a molybdenite flotation concentrate containing pyrite and silica, weighted FeS_2 and the MoS_2 locked in middling at two, compared to one for silica, basing the rating on the relative specific gravities, but the free MoS_2 was rated at one on account of its flaky character and the fact that the free flakes lay flat. Their microscopic analysis, thus weighted, showed 41.4 per cent. MoS_2 against 38.2 per cent. by chemical assay.

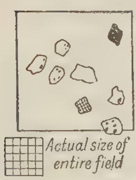

FIG. 10.—Sketch of molybdenite flotation concentrate (*after Coghill and Bonardi*).

Fig. 10 shows one of the fields counted, sketched by means of a camera lucida. This sketch, showing that most of the silica in this particular concentrate was in the form of middling, pointed the obvious conclusion that the proper method to raise the grade of concentrate was to grind finer.

Time for a test will range from 3 to 8 hr. according to the size of sample and the number of minerals counted.

Bibliography: Thomas and Apgar, *Approximate determination of minerals in concentrates by means of the microscope, 18 CME 514.* Clayton, C. Y., *The microscope in ore dressing, 19 CME 61.* Hynes, D. P., *Degree of crushing to free economic minerals, 110 P* 994. Coghill and Bonardi, *Approximate quantitative microscopy of pulverized ores, TP 211, USBM. Examination of ores and ore-dressing products, 105 CME 723.*

4. Microscopic sizing analysis

Sizing by screens below 200-mesh (0.074-mm.) is highly inaccurate, both on account of the inaccuracy in screen manufacture and because of the great difficulty in getting material through the screen. Elutriation and air classification alone are similarly undependable because of difference in settling rates induced by differences in shape and specific gravity of particles. Microscopic sizing has been highly developed in recent years to supplement or supplant the settling methods. With proper equipment and good technique microscopic sizing can be carried down to less than one micron (1 MICRON = 0.001 mm.) and the average size of excessively fine products stated with a precision of one- or two-tenths micron.

Green (*Jour. Frank. Inst., Nov., 1921*) predicts that precision will shortly be increased to hundredths or even thousandths of a micron. Such fine sizing is not necessary in concentration but is of importance in many commercial grinding operations.

Apparatus. The principal piece of equipment required is a monocular microscope with 16-, 8-, 4- and 1.9- or 2-mm. objectives. A petrographic microscope has the advantage that by the use of crossed nicols floccules in the field may be identified. Necessary accessories are indicated in the following descriptions of methods.

Direct measurement. Weigel (*TP 296, USBM*) describes two methods. In one, the −200-mesh material was elutriated into five fractions and each one was measured separately; in the other, a representative sample of the whole −200-mesh material was measured. Microscopic work was the same in both cases.

Preparation of slides. Slides should be about 50 × 75 mm. Dilute the sample in a test-tube with distilled water until it appears cloudy or very slightly milky on shaking. A drop or two of ammonia aids dispersion with clays. Pour off until the tube is about one-third full, agitate by blowing through a pipette then quickly transfer a few drops to the slide, covering an area about 20 mm. diameter. Dry in an air bath at 105° C. Particles should be spaced with little or no overlapping and no flocculation. New slides should be prepared until this condition obtains. The slide may be used uncovered for work with 4-mm. and longer objectives and these should, therefore, be objectives corrected for use without a cover glass. For the finest material (1 micron or less) an oil-immersion lens must be used and the slide must be mounted with a cover glass. Glycerine (index ref. = 1.47) is satisfactory for most non-metallic minerals. Methylene iodide (ref. index = 1.74) is used for clays and tales, if necessary, but it produces diffraction rings. To mount, place a drop of the mounting liquid on the center of the dried sample, drop the cover (35 × 50 mm.) into place and work it down with slight pressure and a rotary motion. Remove excess liquid with a filter or blotting paper, then run a ring of melted paraffin around the edge, using a fine camel's-hair brush. An excess of mounting liquid may result in washing fine material from under the edges of the cover and ruining the slide.

Measuring. Use a micrometer disk in the ocular, ruled with 6 to 8 squares on a side of the inscribed square of the field and with one of the larger squares subdivided. The size of most particles can be estimated from the large open squares, but the subdivided square can be used, if necessary, to measure the smallest particles. Particles in outer squares are distorted at high magnifications, hence use only the inner squares. Determine the undistorted area for each objective by calibration with a stage micrometer. Set the sample slide so that the field is in one corner. Have the microscope fitted with lowest-power objective. Search the field for the largest particle, estimate the mean of the two principal dimensions exposed to the nearest 5 microns, then, beginning at one corner of the inner (undistorted) square work over all squares in order, counting and recording the number of particles within this range. Repeat this process with successive fields until the whole slide is worked over, making as many counts as seem necessary to obtain a fair average of the grains per field. Repeat with the higher-powered objectives, narrowing the range of sizes each time, and counting fewer squares per field but the same number of fields as with the first objective. The range of the 16-mm. objective with 10 × ocular (100 diam.) is, conveniently, down to 30 microns, the 8-mm. objective (200 diam.) from 30 to 10 microns, the 4-mm. (430 diam.) from 10 to 1 micron. and the 1.9-mm. oil-immersion lens (950 diam.) for finer than 1 micron. Check on the relation of counts with different objectives was obtained by Weigel by re-counting the smaller size with one objective with the next higher power and reducing the results to equivalent areas. Eighty fields were counted with the 16-mm.

objective, 40 with the 8-mm. and 20 each with the 4-mm. and 1.9-mm. The more uniform in size the material counted the smaller the number of fields necessary.

Procedure in count of total sample is the same, except that more fields and more squares per field must be counted on each slide, yet the total microscopic work is less than when elutriation is practiced. The time for a total-sample measurement, including calculation was 5 to 6 hours. An elutriation test requires about 3 days and the count and calculation one day more. Less skill and experience are needed for the elutriation test.

Indirect measurement is described by Green (*loc. cit.*) and is adapted to very fine materials of greater uniformity than the non-metallic natural fillers measured by Weigel. The method involves making a slide, photographing one or more fields, projecting the negative on a screen and measuring and counting the grains thereon.

Mounting. Place about one mg. of the material on the center of a slide and cover with a drop of re-distilled turpentine. Rub out with a straight, smooth glass rod, stroking lengthwise of the slide. Continue rubbing until the mixture is thick enough to prevent particles from floating and flocculating but thin enough not to streak, then stop with a slight lift on the last stroke to leave a wedge-shaped deposit on the slide and yield fields of different intensities from which to choose the one for photographing. Evaporate the turpentine completely on a hot plate at a temperature that completes the drying in 40 to 50 sec. Mount with glycerine as described by Weigel (p. 1195).

Photograph with transmitted light that is absolutely axial. Fine-grained contrast plates and hydroquinone developer are easiest to handle but panchromatic plates and pyro developer give better detail. The important point in the photography is sharp definition of particle edges. The negative should show 200 to 250 particles.

Measurement. Project on a screen so that the total magnification is 20,000 to 25,000 diam., measure with a millimeter rule to the nearest whole millimeter and record. Fig. 11 shows a form of record calculation.

Millimeters, mm.	Zinc Oxide # 3B 1st 12–17–20	Frequency, f.	f×mm.	v	v^2	$f \times v^2$ (Σv^2)
6						
7						
8						
9						
10						
11	𝍸 |||	8	48	4.44	19.71	157.68
12	𝍸 |	6	42	3.44	11.83	70.98
13	𝍸 𝍸 𝍸 ||||	19	152	2.44	5.95	113.05
14	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 |||	53	477	1.44	2.07	109.71
15	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 ||	82	820	.44	.19	15.58
16	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 |	46	506	.56	.31	14.26
17	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 ||||	34	408	1.56	2.43	82.62
18	𝍸 𝍸 ||	12	156	2.56	6.55	78.60
19	𝍸	5	70	3.56	12.67	63.35
20	||	2	30	4.56	20.79	41.58
21	||	2	32	5.56	30.91	30.90
22	|	1	17	6.56	43.03	43.03
23	|	1	18	7.56	57.15	57.15
19	|	1	19	8.56	73.27	73.27
20	|	1	20	9.56	91.39	91.39
21	|	1	21	10.56	112.40	112.40
22	|	1	22	11.56	134.60	134.60
23	|	1	23	12.56	158.80	158.80
		276	2881			1449

FIG. 11.—Record of count of microphotograph (*after Green*).

5. Average size of particles

Mineral particles produced by crushing and grinding show an almost infinite variety of shape and size. No simple and accurate numerical expression of the dimensions of a single particle nor of the average dimensions of a group is possible; the best that can be done in any case is an approximation which is ordinarily expressed as a single number, as though the particles were spheres or cubes. This number is called the DIAMETER or SIZE of an individual particle or the AVERAGE DIAMETER or AVERAGE SIZE of a group of particles.

Diameter of a particle. The fundamental assumption of particle measurement is that it has three principal axes at right angles and that its dimensions are completely stated when the distances between the intercepts of the surface on the respective axes are given. Starting with this assumption, which is, on its face, only a crude approximation except in the case of materials with cubical cleavage, averaging of the three principal dimensions into a single figure is attempted by one of several different methods, as follows:

$$d = b, \qquad (1) \qquad\qquad d = \frac{l+b}{2}, \qquad (2)$$

$$d = \frac{l+b+t}{3}, \qquad (3) \qquad\qquad d = \sqrt{lb}, \qquad (4)$$

$$d = \sqrt[3]{lbt}, \qquad (5) \qquad\qquad d = \frac{\sqrt{2lb + 2bt + 2lt}}{6}, \qquad (6)$$

$$d = \frac{3lbt}{lb + lt + bt}, \qquad (7)$$

where d = "diameter" and l, b and t are respectively the distances between the intercepts of the surface on the long, intermediate and short axes, or, in common parlance, length, breadth and thickness of the particle. The significance of the first five approximations is immediately apparent; the sixth gives the edge of a cube whose total surface is equal to the total surface of a rectangular parallelopiped, the dimensions of which are equal to the principal dimensions of the particle. The seventh is the harmonic mean of the dimensions. Green (*loc. cit.*) develops the fact that this harmonic mean is a factor in the expression for the SPECIFIC SURFACE of a mass of particles, *i.e.*, the total surface per unit weight of the material. Thus, for a mass of spherical or cubical particles of the same size, $S = 6/\rho\mu$, where S = specific surface, ρ = specific gravity and μ = the diameter. But $S = Ns$, where N = the number of particles per unit weight and s = the surface of each particle, whence, if the particles are taken as rectangular parallelopipeds, $S = (1/\rho lbt)(2(lb + lt + bt)) = 2(lb + lt + bt)/\rho lbt$, and, by substitution in the first equation, $\mu = 3lbt/(lb + lt + bt)$.

Which one of equations (1) to (6) is to be used in any given determination of particle size depends upon the method of measurement and the predilection of the experimenter. When screens are used the first assumption is adopted perforce. When elutriation is employed without subsequent microscopic measurement of the fractions the fifth assumption is necessarily involved. When individual particles are actually measured, any one of the seven assumptions may be employed, but the second and fourth are the ones most commonly used. The sixth is no more accurate than the fifth and involves much more computation, while the seventh requires so much measurement and calcula-

tion, if a large number of particles is to be measured, as to be substantially impracticable.

Average diameter is calculated by some method of averaging the mean or equivalent diameters of a number of particles. Perrott and Kinney (*The meaning and microscopic measurement of average particle size*; 6 *Jour. Am. Ceramic Soc.* 417) give the following summary of suggested methods:

1. Arithmetical mean $D = \frac{d_1 + d_2}{2}$.
2. Geometrical mean $D = \sqrt{d_1 d_2}$.
3. Laschinger's mean $D = \frac{d_1 - d_2}{\log_e d_1 - \log_e d_2}$.
4. Mellor's mean $D = \sqrt[3]{\frac{(d_1 + d_2)(d_1^2 + d_2^2)}{4}}$.
5. Mean of form $D = \frac{4}{5}\left(\frac{d_1^5 - d_2^5}{d_1^4 - d_2^4}\right)$.
6. Von Reytt's mean $D = 0.435(d_1 + d_2)$
7. Number mean $D = \frac{\Sigma nd}{\Sigma n}$.
8. Length mean $D = \frac{\Sigma nd^2}{\Sigma nd}$.
9. Surface mean $D = \frac{\Sigma nd^3}{\Sigma nd^2}$.
10. Volume mean $D = \frac{\Sigma nd^4}{\Sigma nd^3}$.

To these should be added

11. $D = \sqrt{\frac{\Sigma nd^2}{\Sigma n}}$.
12. $D = \sqrt[3]{\frac{\Sigma nd^3}{\Sigma n}}$,

where D is the mean diameter, d_1 and d_2 are the maximum and minimum mean particle diameters, respectively; d represents the successive mean particle diameters in a sizing operation, and n the numerical frequency of the corresponding d.

Formulas 1 to 5 inclusive are based on the assumption of an even gradation in size from maximum to minimum and necessarily also on an equal number of particles in each size group, or else they disregard the effect of frequency of occurrence of particles of the different sizes. Formula 6 is clearly an approximation to formula 1. Formulas 7 to 10 consider and weight the intermediate sizes on the bases, respectively, of (*a*) number of particles in the successive grades; (*b*) total length of mean diameters in these grades, *i.e.*, number of particles in a grade × mean particle diameter; (*c*) total surface, *e.g.*, number of particles in the grades × mean particle diameter squared; (*d*) and total volume, *i.e.*, number of particles × mean particle volume.

Equation 7 has the physical significance that if the particles under investigation were laid side-by-side in a line and the length of the line were divided by the number of particles the quotient would be the number D. If the area covered by these particles were divided by the length of the line, the quotient would be the value D given in equation 8. On this base, the average height of a parallelopipedon equal in volume to the total volume of Σn particles is the D of equation 9. Formula 10 is the equivalent of $D = \Sigma wd/\Sigma w$ where w = percentage weights of the different grades and the other letters have the significance already assigned. This is the formula commonly used when sizing is done by screens, sedimentation or elutriation, and when the amounts of the various grades are determined by weight. Formula 11 gives the edge of a cube whose surface multiplied by the total number of particles in a given mass of particles is equal to the total surface of the mass, if the particles are taken as cubes. Formula 12 gives the edge of a cube whose volume multiplied by the total number of particles is equal to the total volume of the particles sized, considered as cubes.

Comparison of methods. Perrott and Kinney (*loc. cit.*) give the microscopic sizing analysis shown in Table 6. Comparison of the average diameters calculated by the different formulas points clearly the uncertainty of meaning of this term and the necessity for stating the method of calculation when giving a numerical result. The result by formulas 1 to 6 incl. is not affected in any way by the amounts in any of the grades. These formulas will, therefore, each give the same result for any mass of grains of mixed sizes, irrespective of the size composition of the mass, if, only, the largest and smallest particles are in every case of the same sizes. This fact condemns these for-

Table 6. Comparison of methods of calculating average diameter

Microscopic analysis

Diameter, microns (d) ...	60	50	40	30	20	10	5	2
Number of particles (n) ..	87	100	156	660	1750	6200	25,600	155,000

Percentages

$\frac{n}{\Sigma n}$	0.05	0.05	0.1	0.3	0.9	3.3	13.5	81.8
$\frac{nd}{\Sigma nd}$	0.9	0.9	1.1	3.4	6.1	10.7	23.7	53.2
$\frac{nd^2}{\Sigma nd^2}$	7.8	6.3	6.3	14.9	17.5	15.5	16.1	15.6
$\frac{nd^3}{\Sigma nd^3}$	22.4	14.9	11.9	21.2	16.7	7.4	3.8	1.5

Average Diameter

Formula	Microns
1. $D = (d_1 + d_2)/2 = (60 + 2)/2$	= 31
2. $D = \sqrt{d_1d_2} = \sqrt{60 \times 2}$	= 11
3. $D = (d_1 - d_2)/(\log_e d_1 - \log_e d_2) = (60 - 2)/(2.303[1.7782 - 0.3010])$	= 17.0
4. $D = \sqrt[3]{(d_1 + d_2)(d_1^2 + d_2^2)/4} = \sqrt[3]{(60 + 2)(3600 + 4)/4}$	= 38.2
5. $D = 4(d_1^5 - d_2^5)/5(d_1^4 - d_2^4) = 4(60^5 - 2^5)/5(60^4 - 2^4)$	= 48
6. $D = 0.435(d_1 + d_2) = 0.435(60 + 2)$	= 27
7. $D = \Sigma nd/\Sigma n = (0.05 \times 60 + \ldots 81.8 \times 2)/100$	= 3.0
8. $D = \Sigma nd^2/\Sigma nd = (0.9 \times 60 + \ldots 53.2 \times 2)/100$	= 7.0
9. $D = \Sigma nd^3/\Sigma nd^2 = (7.8 \times 60 + \ldots 15.6 \times 2)/100$	= 21.0
10. $D = \Sigma nd^4/\Sigma nd^3 = (22.4 \times 60 + \ldots 1.5 \times 2)/100$	= 36.4
11. $D = \sqrt{\Sigma nd^2/\Sigma n} = \sqrt{(87 \times 60^2 + \ldots 155{,}000 \times 2^2)/(87 + \ldots 155{,}000)}$	= 4.6
12. $D = \sqrt[3]{\Sigma nd^3/\Sigma n} = \sqrt{(87 \times 60^3 + \ldots 155{,}000 \times 2^3)/(87 + \ldots 155{,}000)}$	= 7.6

mulas for anything but the crudest kind of work. Formulas 9 and 10 place too much weight on the coarser sizes and both they and No. 8 give results that have no real meaning in terms of the diameter of ideal particles which could be substituted for the actual particles. On the other hand, No. 9 is useful when specific surface is important. Results by formulas 7, 11 and 12 have the meanings already developed. Which of the three should be used in any given case is a matter of individual preference, since it is impossible to choose scientifically. No. 7 gives the easiest calculations and the physical significance is most readily visualized. It weights the finest particles most heavily and, therefore, gives an average that leans toward the fine end. No. 12 weights the coarsest particles most heavily and consequently the result leans toward the coarse end. No. 11 is intermediate between the other two and would, on that score alone, seem to be preferable. It is distinctly to be preferred when surface is the valuable property of the material.

Uniformity. The complete information regarding the texture of a mass of broken material is not expressed by the average size. Thus in Fig. 12 curves

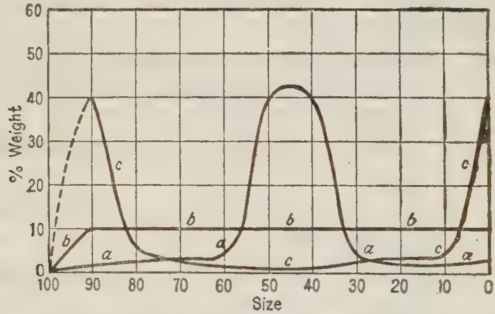

FIG. 12.—Plots of three products showing the same average size $D = \Sigma wd/\Sigma w$.

(*a*), (*b*) and (*c*) represent three possible sizing tests which indicate the same average size of material, but (*c*), on account of the coarse material present, will appear coarser than either of the other two while (*a*) on account of the lack of fines, may appear coarser than *b*. Green (*loc. cit.*) gives the following equation from statistical mathematics for expressing the degree of uniformity as a coefficient, $U = \Delta x\sqrt{\frac{n}{2\Sigma v^2}}$ where Δx is the difference between two successive values on the X-axis (size-axis), made equal to unity for convenience in calculation; n is the total number of particles measured, and v is the difference in units of length between the measured diameter of a given particle or group of particles of substantially the same size and the average diameter of the whole number of particles n.

Thus in Table 7, columns 1 and 2 represent the results of a particle count on a slide at a magnification of 20,000 diameters. Average particle size as measured at 20,000 magnification is

$$D_{(\times 20,000)} = \frac{\Sigma nd}{\Sigma n} = \frac{2881}{276} = 10.44 \text{ mm.}$$

and $D = 0.522$ micron. Column 4 gives deviations, in millimeters, at 20,000 magnification, between the diameters of the particles in the corresponding size groups and the mean diameter (*i.e.*, 10.44 less the number in Column 1). Column 5 gives the squares of the

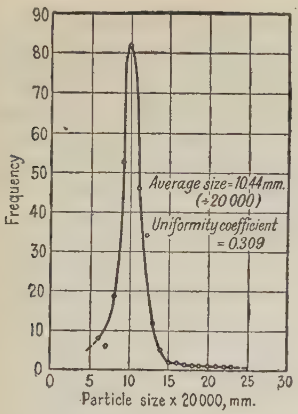

FIG. 13.—Frequency plot of data in Table 7.

numbers in Column 4 and Column 6 the product of columns 2 and 5. The total of Column 6 is Σv^2 of the formula. The total of column 2 is n.

Hence $$U = 1\sqrt{\frac{276}{2(1449)}} = 0.309.$$

The uniformity coefficient ranges between 0, representing complete lack of uniformity, or no two particles of the same size, and ∞, which indicates all particles of the same size. The significance of the number 0.309, given above, is indicated by Fig. 13, which is a frequency plot of the data given in Table 7.

Table 7. Microscopic sizing analysis

Col. 1	Col. 2	Col. 3	Col. 4	Col. 5	Col. 6
Diameter, mm., d	Number of particles, n. Total = Σn(a)	nd Total = Σnd	$v = \frac{\Sigma nd}{\Sigma n} - d$	v^2	nv_2 Total = Σv^2
6	8	48	4.44	19.71	157.68
7	6	42	3.44	11.83	.70.98
8	19	152	2.44	5.95	113.05
9	53	477	1.44	2.07	109.71
10	82	820	0.44	0.19	15.58
11	46	506	0.56	0.31	14.26
12	34	408	1.56	2.43	82.62
13	12	156	2.56	6.55	78.60
14	5	70	3.56	12.67	63.35
15	2	30	4.56	20.79	41.58
16	2	32	5.56	30.91	30.90
17	1	17	6.56	43.03	43.03
18	1	18	7.56	57.15	57.15
19	1	19	8.56	73.27	73.27
20	1	20	9.56	91.39	91.39
21	1	21	10.56	112.40	112.40
22	1	22	11.56	134.60	134.60
23	1	23	12.56	158.80	158.80
Totals	276	2881	....	....	1449

a Frequency.

6. Plotting sizing tests

Sizing tests are most easily compared and their significance, frequently, most readily understood from graphical representation. A number of different methods of plotting are shown in Fig. 14.

Direct plot is shown by curve 1. It has the advantage of familiarity but the disadvantage that the fine sizes are so crowded as to lose substantially all of their significance.

Cumulative direct plot is shown by curve 2. This curve reads directly at any point the

total oversize and undersize of a screen of given aperture, but has the same disadvantage of crowding in the fine sizes as the first curve.

Direct-logarithmic plot is illustrated by curve 3. Weights are plotted directly as ordinates, while the abscissas are the logarithms of the screen apertures. With screens whose apertures increase according to a constant ratio the logarithms of the apertures increase by a constant, hence the points corresponding to successive apertures are equally spaced along

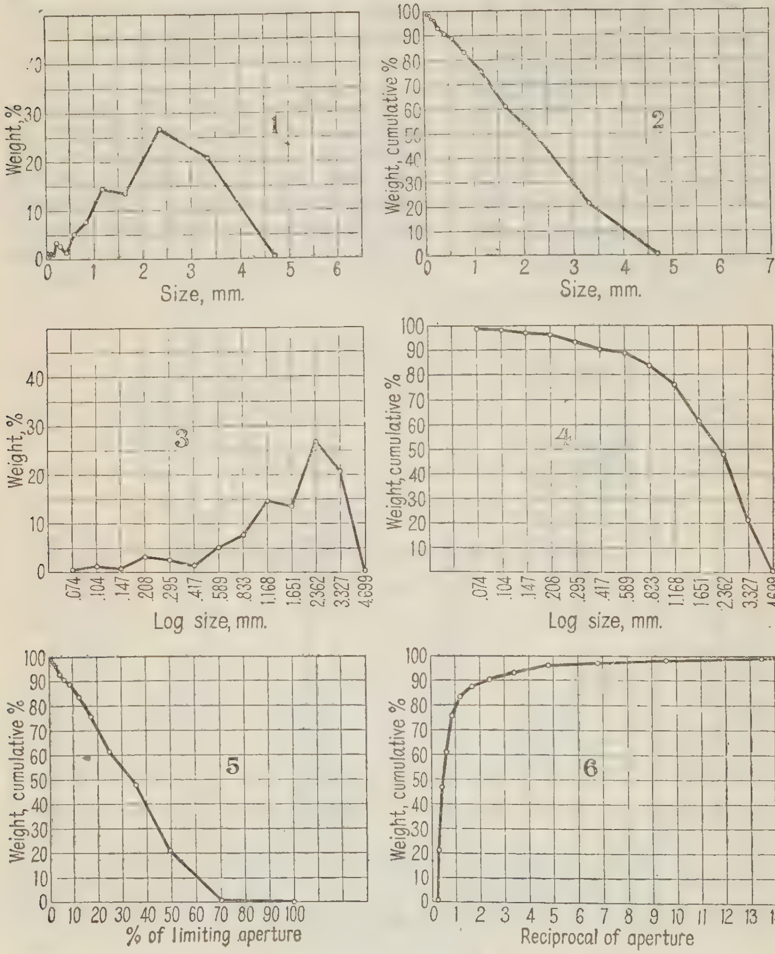

FIG. 14.—Methods of plotting sizing tests.

the horizontal axis of the plot. The curve necessarily terminates with the oversize of the finest screen because of lack of knowledge of the size of the finer material and the fact that the logarithm of zero is infinity. Comparing curves 1 and 3, it is to be seen that the latter spreads out the fine part of the curve and compresses the coarse and that it is much easier to read.

Cumulative-logarithmic plot is shown in curve 4. This has the advantages of both logarithmic and cumulative plots and is probably the most convenient for all ordinary presentations.

Percentage-aperture plot, curve 5, is particularly useful in comparing screen tests of products in different size ranges, as, for instance, the discharges of a jaw crusher and a ball mill. It reduces both ordinates and abscissas to the same scale and all curves have common end points.

Reciprocal plot, curve 6, reverses the crowding of the direct plot, and is useful to spread out the points when the amounts of fine material are small and of large material great, as in the case plotted. In the reverse condition, the curve becomes very crowded in the coarse end. This curve (and any of the other cumulative curves) may be turned over by making the ordinates percentages through rather than percentages on the particular screens. Gates adopts the reciprocal plot in estimating crushing efficiency. (See Sec. 4, Art. 27.)

Frequency curves are plots in which, ordinarily, the number of particles of a given size, or some function thereof, is plotted against the size itself or a function. The simplest form is that of number against size (Fig. 13). Martin, Blyth and Tongue (*Researches on the theory of fine grinding, Part I, British Portland Cement Research Association, Pamphlet No. 4, 1924*), plot increase in number of particles per unit increase in diameter ($\delta N/\delta X$) against diameter (x). In this curve any ordinate represents the rate of increase in number of particles at the corresponding value of particle diameter and the area between the curve, the X-axis and any two adjacent ordinates is numerically equal to the number of particles between the corresponding diameters.

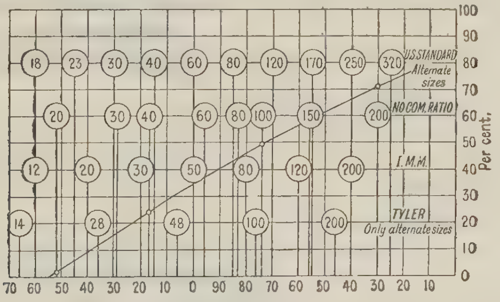

FIG. 15.—Multiple-scale plot.

Multiple-scale plot (Fig. 15) was suggested by Randall (57 *A* 481) to enable ready interpretation of screen tests by users of different testing sieves. Otherwise it is the usual cumulative-logarithmic plot.

Callow method for graphical representation of screen analyses and sizing-assay tests is shown in Figs. 16 and 17. (*92 J 884.*) Fig. 16 shows in block (1) a screen analysis of the product of a Dodge crusher; in block (2) the product of two sets of rolls taking the material plotted in (1); and (3) shows the product of a Chilean mill fed with the roll discharge. The dotted rectangles in block (2) represent the coarse material that was contained in the roll feed which has been broken, and the dot-hatched rectangles in block (2) represent additions to the line-hatched rectangles of block (1). Similarly the open rectangles in block (3) represent coarse sizes in block (2) that have disappeared, and the cross-hatching represent fines produced from this material. This is an excellent picture, simple to construct. Fig. 17 pictures the products and performance of an hydraulic classifier showing both distribution of sizes and of valuable content. This likewise presents clearly data hard to comprehend from an array of figures. Presentation of results of sizing-assay tests is shown in Tables 8 and 9 and Fig. 17.

Table 8. Results of sizing-assay test.

Screen mesh	Weight, gm.	Per cent. of total weight	Assays per ton of 2000 lb. Gold, dollars	Silver, ounces	Total, dollars	Per cent. of total values Gold	Silver	Total	Values in each product per ton of original ore Gold	Silver	Total
+10	250	50	11.54	0.18	11.63	40.7	16.0	40.2	5.77	0.09	5.81
20	113.5	22.7	14.83	0.18	14.92	23.8	7.3	23.4	3.37	0.04	3.38
60	53	10.6	15.06	0.28	15.20	11.3	5.3	11.2	1.60	0.03	1.62
120	25	5.0	15.66	1.13	16.18	5.5	10.0	5.6	0.78	0.05	0.81
200	14	2.8	16.27	2.76	17.65	3.2	13.7	3.4	0.45	0.08	0.49
−200	44.5	8.9	24.72	3.02	26.63	15.5	47.7	16.2	2.20	0.27	2.34
Total......	500	100	......	......	......	......	......	......	14.17	0.56	14.45

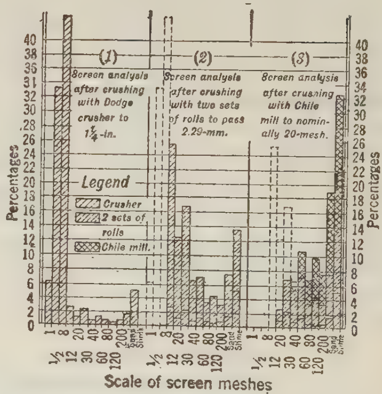

FIG. 16.—Diagram of crushing performance.

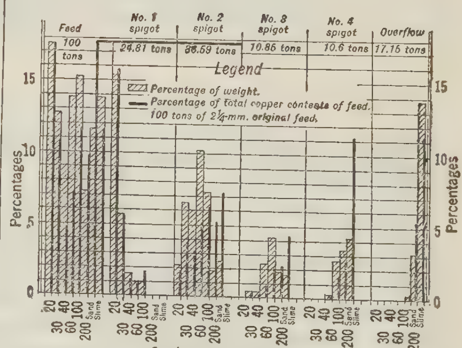

FIG. 17.—Diagram of performance of hydraulic classifier.

Table 9. Sizing-sorting-assay test of −10-mm. lead ore.

Line number	Screen, mm.	Weight, gm.	Tons per 100 tons	Assays, per cent. Pb	Tons Pb per 100 tons of feed	Per cent. of total lead content	Concentrate: Weight, gm.	Concentrate: Tons per 100 tons of feed	Concentrate: Assay, per cent. Pb	Concentrate: Tons Pb per 100 tons of feed
1	6.68	55.6	2.02	3.14	0.0634	1.20	....	....	....	....
2	4.70	213.0	7.73	3.41	0.2635	5.00	0.44	0.012	86.8	0.0104
3	3.33	315.5	11.44	3.66	0.4190	7.96	1.09	0.030	86.7	0.0260
4	2.36	465.5	16.90	3.85	0.6510	12.36	4.80	0.132	80.3	0.1059
5	1.65	442.0	16.09	4.73	0.7615	14.46	5.93	0.163	82.0	0.1336
6	1.17	248.5	9.04	4.75	0.4295	8.15	4.33	0.119	81.6	0.0972
7	0.83	85.0	3.08	4.97	0.1531	2.90	2.11	0.058	82.4	0.0476
8	0.59	102.4	3.72	6.10	0.2268	4.30	5.02	0.138	81.9	0.1127
9	0.42	84.6	3.07	7.08	0.2175	4.12	6.55	0.180	81.4	0.1469
10	0.30	80.7	2.93	8.02	0.2352	4.46	8.55	0.235	79.7	0.1874
11	0.21	61.2	2.22	9.06	0.2012	3.81	8.47	0.233	76.2	0.1775
12	0.15	80.0	2.90	10.35	0.3003	5.68	13.73	0.378	74.5	0.2815
13	0.10	73.9	2.68	9.89	0.2648	5.02				
14	0.07	55.4	2.01	8.00	0.1608	3.04	57.80	2.100	63.2	1.3254
15	−0.07	390.0	14.17	6.53	0.9250	17.54				
16	Total	2753.3	100.00	5.27	5.2726	100.00	118.82	3.778	70.1	2.6521
		Distributed middling.....		....	....	....	....	3.710	65.0a	2.4095
		Calculated totals........		....	....	....	....	7.488	67.6	5.0616

Line number	Screen, mm.	Middling: Weight, gm.	Middling: Tons per 100 tons of feed	Middling: Assay, per cent. Pb	Middling: Tons Pb per 100 tons of feed	Tailing: Weight, gm.	Tailing: Tons per 100 tons of feed	Tailing: Assay, per cent. Pb	Tailing: Tons Pb per 100 tons of feed
1	6.68	19.3	0.702	8.62	0.0605	36.3	1.318	0.22	0.0029
2	4.70	59.4	2.164	11.25	0.2434	153.2	5.554	0.18	0.0100
3	3.33	81.3	2.959	12.60	0.3726	233.1	8.451	0.24	0.0203
4	2.36	104.8	3.814	13.70	0.5227	355.9	12.954	0.17	0.0220
5	1.65	91.7	3.329	18.10	0.6025	344.4	12.598	0.20	0.0252
6	1.17	46.8	1.701	18.59	0.3163	197.4	7.220	0.22	0.0159
7	0.83	14.1	0.513	18.86	0.0967	68.8	2.509	0.35	0.0088
8	0.59	15.7	0.571	18.47	0.1055	81.7	3.011	0.29	0.0087
9	0.42	11.8	0.430	15.02	0.0646	66.2	2.460	0.24	0.0059
10	0.30	10.2	0.371	11.36	0.0421	61.9	2.324	0.23	0.0054
11	0.21	6.9	0.252	8.19	0.0206	45.8	1.635	0.19	0.0031
12	0.15	8.0	0.290	5.34	0.0155	58.3	2.332	0.14	0.0033
13	0.10								
14	0.07	....	....	....	....	461.5	16.760	0.15	0.0252
15	−0.07								
16	Total	470.0	17.096	14.42	2.4630	2164.5	79.126	0.20	0.1567
		Distributed middling........		....	....	....	13.386	0.40a	0.0535
		Calculated totals........		....	....	....	92.512	0.227	0.2102

a Assumed.

Recovery, from weights $= \frac{5.0616}{5.2718} = 96.0$

Ratio of concentration $= \frac{100}{7.488} = 13.3$

Recovery, from formula $= \frac{67.6(5.27-0.227)}{5.27(67.6-0.227)} = 96.0$

Ratio of concentration $= \frac{67.6-0.227}{5.27-0.227} = 13.4$

7. Sizing-sorting-assay test

A sizing-sorting-assay test affords an excellent basis for estimate of the recovery that it is possible to make on an ore and of the flow-sheet needed for treatment.

Procedure consists in making a sizing test of a sample ground to approximately the size at which clean mineral or clean tailing can be made, separating each of the grades by appropriate means into concentrate, middling and tailing, and weighing and assaying the several products. Hand picking can be used for separation down to the oversize on a 1-mm. screen and panning on the finer sizes, or, if the bulk of the finer sizes is large enough, they may be combined into a sample for a flotation test. Table 9 shows the results of a test thus run and the method of calculation.

Interpretation. Neglecting the middling, or, what amounts to the equivalent, assuming that if it were re-ground and put back into circuit, the grades of concentrate and tailing would not be affected, the recovery indicated in this test is 96 per cent. It is improbable, however, that middling assaying 14.42 per cent. Pb would yield as low a tailing as that yielded by the original ore, assaying 5.27 per cent. Pb, especially since in concentrating the original ore a relatively high-grade middling was made. If the assumption is made that the concentrate obtained by re-treating middling separately would average 65 per cent. Pb and the tailing 0.40 per cent., the recovery on middling would be 97.8 per cent. Such an assumption was justified in this test on the grounds that microscopic examination of the middling showed that substantially all mineral would be free at 100-mesh, that the concentrate obtained by floating 14-per cent. feed would be of somewhat higher grade than that from 5-per cent. feed, and that the assays of flotation tailings from the two feeds would be roughly in proportion to the feed assays. Such middling re-treatment would add 3.710 tons of concentrate containing 2.4095 tons of lead and 13.386 tons of tailing containing 0.0535 ton of lead. If any doubt exists as to the behavior of the middling on re-treatment, it should be actually ground and treated.

The test presented would indicate a flow-sheet in which the material should be crushed through 10-mm., screened on 2-mm., the oversize sent to jigs making finished concentrate and tailing, and middling for re-crushing; undersize to a classifier for de-sliming at 100-mesh; classifier sand to tables making finished concentrate and tailing, and a middling for re-crushing; classifier overflow to flotation, making finished concentrate and tailing. On account of the high grade of the middling, it would appear best to grind the combined middling directly to flotation size and treat it separately, but, if this proved unsatisfactory on test, the re-ground material could be returned into the de-sliming classifier. The table indicates the tonnages that would go to the different machines on first pass. The amount that the middling would add would depend on how finely it was ground and whether it was treated separately or thrown back into circuit.

8. Testing of machines

Crushing. The behavior of ore in both coarse and intermediate crushing and in grinding should be investigated, for the reason that the behavior of a rock when the work consists in breaking apart mineral aggregates, as in coarse and intermediate crushing, may be entirely different from its behavior when individual mineral grains are being broken down, as in grinding. The safest method of testing for capacity, barring actual trial on full mill scale, is to make parallel tests on a rock of known crushing characteristics and the unknown rock. These tests should be made in machines of the types used in mill treatment of the known rock and to be used in treatment of the unknown. Laboratory crushing tests that are to be used as a basis of mill installation are about 5 per cent. manipulation and 95 per cent. interpretation based on experience, in any case, and an experienced investigator with a few lumps of the unknown and some known rocks and a hammer and anvil can tell more than an inexperienced man with a most complete laboratory crushing equipment.

Classifier tests give information, not yielded by screen tests, as to the distribution of material to be expected in a gravity-concentration plant; and, in conjunction with screen tests, form a basis for prediction of the results to be expected from gravity-concentration treatment. Testing classifiers are also used to prepare material for concentration tests on shaking tables and the like. Elutriation tests (Art. 2) are classifier tests applied to sizing fine material, but these same tests, subjected to different interpretation, give useful information regarding gravity-concentration.

Beaker classification involves the same manipulation, in so far as separation into different grades is concerned, as the decantation method of sizing by elutriation. When applied to coarse material (0.5 mm. to say 2.5-mm.) the time required for settlement is so short that there is much overlapping of the grades and results are very different from those attainable in a rising-current classifier.

Sorting tubes are made in glass for both free-settling and hindered-settling classification.

A free-settling tube (designed by H. S. Munroe and manufactured by Eimer and Amend and by Emil Greiner, N. Y.) is shown in Fig. 18. The essential parts are sorting column (*C*) with feed inlets (*F*), vortex fitting (*D*), overflow tube (*B*), and feed and spigot flasks (*A*) and (*E*) respectively. Joints between flasks and apparatus are made with rubber tubing. The ends entering the flasks are expanded by means of brass thimbles of suitable diameter slipped inside the tubing. Joints between (*B–C*) and (*C–D*) are made tight by rubber tubing slipped over the lower ends of (*B*) and (*C*) respectively. The annular space between the lower end of (*B*) and constricted portion of (*C*) should be about ⅛ in. wide. The lower end of (*C*) should project about ½ in. below the bottom of the water inlet in (*D*). When set up, the tubes (*B*) and (*C*) should be clamped rigidly in vertical position in a ringstand which carries also two large rings near the top in which flasks (*A*) may hang. Flask (*E*) should rest on the base of the stand. There should be no kinks in the rubber tubes connecting the flasks. Tubes are made in several sizes, ranging from about 1- to 3-in. diameter of sorting column (*C*). The actual average internal diameter of (*C*) is obtained by measuring the water drawn from between spaced marks and applying equation $q = av$. (see Sec. 27, Art. 28). The quantity of water necessary to overflow in a given time in order to produce any required average rising velocity in (*C*) may then be calculated from the same equation and the desired current set by bringing the overflow to this figure.

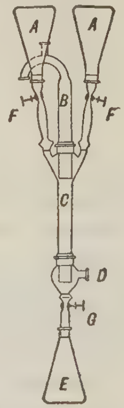

Fig. 18.—Munroe tube classifier.

Procedure. A weighed sample of ore is divided about equally between the two flasks (*A*). Enough water is added to moisten the ore thoroughly, with shaking; the bottles are then filled with water, and put into position for feeding, the cocks (*F*) being closed. The rubber tubes must be full of water both above and below (*F*), because an air bubble prevents proper discharge of ore. Having adjusted the flow of water up the column to the desired velocity, cocks (*F*) are slightly opened to allow ore to drop slowly. Light material will be carried over, while the heavier will fall through the rising current and into flask (*E*). When all ore is out of the upper flasks, a few minutes are allowed for the sorting column to clear partially, the current is then shut off and a little more time allowed for matter still in suspension to settle. Cock (*G*) is then closed, flask (*E*) removed, and its contents transferred to feed flasks (*A*). This last operation may be simplified by having at hand an extra flask (*E*) filled with water; when about half the ore has fallen into the first flask, it may be replaced by the second flask, after closing (*G*) temporarily. Two flasks (*E*) may then be put into the position first occupied by flasks (*A*). Current is now adjusted for the next faster velocity desired, and the operations repeated. While working with velocities up to, say, 20 mm. per second, it is well to catch the entire overflow, water and solids, in pails which may be set aside for a sufficient length of time to permit solids to settle perfectly; with the higher velocities, overflow may be led to pans in which solids will settle while water overflows. Solid matter carried over at each velocity is caught separately, dried, and weighed.

Hindered-settling tube (Fig. 19) consists of a glass tube (*A*) about 1-in. diameter and 30 in. long, drawn down at the lower end to smaller diameter, and provided with a side

tube (B) for water inlet. The ratio of the diameter of section (C) to that of section (A) for all-around work on feeds ranging from, say, 2-mm. maximum to 0.5-mm. maximum should be 1 : 2, but a ratio of 1 : 1.5 is better for the coarse separations and one of 1 : 3 or 4 for fine sizes, and a series of tubes is best for close work. The length of section (C) should be 3 to 4 in. Inlet (B) and the spigot should be about ¼-in. internal diameter. A galvanized-iron or sheet-copper funnel with side tube about ¾-in. diameter should be provided for feeding and collecting overflow, and a number of small-necked flasks for collecting spigot products. The joint between tube and flask is a rubber tube controlled by a stopcock.

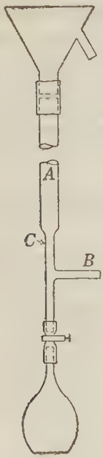

FIG. 19.—Hindered-settling tube.

Final settlement of any particle is determined by its ability to pass the constricted section (C), free-settling against a given current. Hence this is the important diameter and currents are set for section (C) by the methods described under "Free-settling tube" above.

Operation starts by filling the entire system with water, then setting the minimum current (about 5 mm. per sec.) and feeding material slowly into the funnel. The feed should be wet to prevent skin flotation and the best results will be obtained if the slimes are slowly decanted into the funnel several times before any of the sands are poured in, taking care that the rate of feeding slimy water is insufficient to make the rising current at the overflow level equal to or greater than that in section (C). Collect the slime in a pail and set aside. The water in the flask should be slime free; if not, re-feed the spigot product. Collect the spigot product. Set the current at the maximum rate, re-feed the first spigot product, run until only an occasional grain falls through section (C), then close the stopcock, collect the spigot product and replace the flask, full of water; slack off the current gradually until the next lower current is reached, open the stopcock, feed back any sand that overflowed in the preceding operation, and run as before until settling substantially ceases. Repeat with decreasing current velocities until the 5-mm. velocity is again reached. It is better to run at this current a second time, adding the overflow and material remaining in the teeter-chamber (A) to the first overflow. Dry and weigh the products.

Laboratory classifiers. Miniatures of mill-type classifiers (Sec. 6), are used. The most satisfactory miniatures are those of the tank type, either hindered-settling or free-settling. Fig. 20 shows a useful size, which has a capacity of about 1 kg. per min. Laboratory classifiers are much more con-

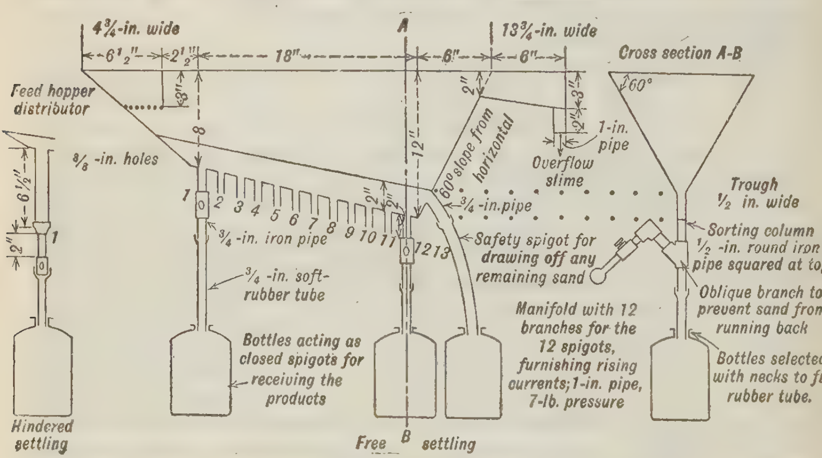

FIG. 20.—Laboratory classifier (*after Richards, 91 J 415*).

veniently operated with closed spigot, when possible, as it is then unnecessary to supply and dispose of large quantities of spigot water.

Interpretation of classifier tests. The weights of the products tell the tonnages of different grades for which concentrating machines must be provided. Assays of the combined products do not yield much information concerning probable mill performances, on account of the middling present; actual concentration tests should be made. Sizing the products on a series of screens will approximate the work of shaking tables, the finest sizes representing concentrate, the coarsest the tailing, and the intermediate middling. If the sized products are laid out on a ruled board in such a way that the abscissa of any grade is the number of the classifier spigot from which it came and its ordinate the screen size, an excellent picture of the work of the classifier is obtained. The best classification is represented by maximum spread on this board between the heaps of pure heavy mineral and pure gangue. The smaller the middling heaps, the heavier the loads that can be put on the following shaking tables and, of course, the size of the middling heaps indicates the tonnage that must be re-ground.

Settling ratio is the ratio of the average diameter of light-mineral grains to that of the heavy-mineral grains in a classified product. It may be calculated from the layout described in the preceding paragraph by sorting out the middling grains in the various heaps, assaying the residues for heavy mineral, and then calculating average sizes of the two kinds of grains in each spigot product. The larger the settling ratio the easier the work of the concentrator treating the product and the better the work of the classifier. *Richards* has determined the average free-settling ratio of quartz and galena as about 4 and the hindered-settling ratio slightly less than 7. These figures were obtained by careful laboratory work with artificial mixtures of the two minerals and making a large number of successive spigot products with small current differences. Excellent mill work will show not more than 2.5 and 4 respectively for these same minerals, on account of the smaller number of spigots, the interference by middling grains and overloading of the classifier.

Mechanical-classifier tests can be made properly only in machines of the type concerning which information is sought, and, since so many other factors than simple settlement in water are involved, *e.g.*, agitation of the rakes, time of draining sand, slope of draining surface, depth of pool, and the like, it is almost essential that the testing machine be a replica of the full-size machine in longitudinal section, at least. An experienced man can tell something by beaker tests on the settling rate of the sands in pulps of the same density as those that are to be classified, but at the best his conclusions from such work are little more than guesses.

Thickening. For methods of testing, see Sec. 16, Art. 10.

Testing for vacuum filtration. The usual apparatus is a small filtering surface reproducing essentially the type of surface to be investigated.

The Oliver Filter Co. uses a surface 0.5 sq. ft. in area made as follows: Make a piece of 1-in. board, square or round, exactly 0.5 sq. ft. in area. On one side tack parallel cleats, ¼ in. wide by ½ in. deep, spaced ¾ in. face to face with ends staggered so that there is a passage from every point on the riffled surface to the center point. Bore a hole for a ½-in. nipple at the center and set a short nipple in place from the back side, not projecting through the front surface. Paint the edges and bottom of the board thoroughly with No. 2 P.-and-B. paint or its equivalent. Cut a piece of stiff screen cloth the size of the board. Make a band of ¼-in. × 3½-in. or 4-in. strap iron with inside dimensions about 1/32 in. larger than the board. Cut a piece of canvas large enough to cover the cleated side and turn down over the edges of the board. Place the screen on the cleats with the canvas over it, then force down the strap-iron band so that the canvas makes a tight fit

at the edges. Connect the nipple to a receiver, which is best a wide-mouthed bottle with 2-hole cork, and connect the receiver with a vacuum pump.

A test leaf may also be made on a ½-in. pipe frame having the inner element of the bottom and side pipes perforated on 1-in. centers with ⅟₁₆-in. holes. The corners should be made with tees. Set ¼ × ½-in. wooden spacers in place by notching the ends and wedging them between top and bottom pipes. Sew on a canvas cover, making sure that the seam is tight. A good corner joint may be made by placing capped nipples in the outer ends of the tees and wrapping the canvas tightly with twine around the nipples, then painting the joints.

Procedure consists in immersing the leaf, canvas side down, in agitated pulp for different times until the time required to produce cake of maximum thickness is determined. Next dry the cake for a definite time, holding it with canvas side up. The time allowed for drying should be about the same as the time of submergence. Blow off the cake with water or air. Determine the percentage of moisture and dry weight of solids. These data give the capacity per half sq. ft. (with the wooden leaf) per revolution of a continuous filter. The time per revolution will depend on the periods required for forming cake and drying and the usual percentages of submergence. If the test showed 4 min. for forming cake and 4 min. for drying, and 40 per cent. submergence were used, the rate of revolution would be once in 10 min. If the leaf made 1 lb. of dry cake in 4 min., this is equivalent to 2 lb. per sq. ft. per revolution or 288 lb. per sq. ft. per 24 hr. If 10,000 lb. of solid is to be filtered per 24 hr. it will require 10,000 ÷ 288 = 34.7 sq. ft. The corresponding size of commercial filter can be chosen from this figure.

The Oliver Filter Co. makes a laboratory-size filter, with drum substantially 3 ft. diam. by 6 in. long. A single-leaf American filter, suitable for laboratory purposes can also be purchased. Miniature Moore and Butters plants are not difficult to construct. None of these, however, gives any more information than is obtainable from the test leaf.

9. Concentration tests

Hand-picking often affords valuable information, even when not considered a practicable method of treating a given ore under existing conditions. It may be applied to ore as fine as pea sizes; and, in the investigation of screened products, may be carried down to 1.0- or even 0.5-mm. with the aid of a hand glass or a low-power binocular microscope. For satisfactory hand-picking, ore should be sized between rather close limits, and should be clean; washing brings out the distinctive color or luster of some minerals.

Practicability of hand-picking on a commercial scale may be tested by attempting to make the following products, or so many of them as practicable: (*a*) Rich minerals, fit for market or for metallurgical treatment. These will include not only such minerals as galena, blende, chalcopyrite, etc., nearly or quite pure, but also rich mineral which possibly can be treated better by some metallurgical process than by mechanical means, for example copper carbonates, silver chloride, finely disseminated silver ore, etc. (*b*) Rich ore, with coarse disseminated mineral, for coarse crushing and jigging. (*c*) Fine-disseminated ore, usually poor, in which the useful mineral occurs in such small particles that exceedingly fine crushing will be necessary to liberate it. (*d*) Barren waste, or material that does not seem to be mineralized. Several classes may sometimes be made of this, according to gangue minerals or rocks in the ore, or when a difference of color, texture, or other characteristic indicates a possible difference in richness. Assays of these different classes of barren or seemingly barren material will show which may be thrown away, and which should be included with the ore for milling. Finally, inspection of the different products may show that some classes are present in small quantity only, and that perhaps several sorts may be combined without disadvantage.

Economy of hand-picking may be investigated by means of the formulas in Art. 22.

Hand jigging, for testing ore ranging in size from 10- to 2-mm., requires a tub of water and a few small screens of differing mesh. Before jigging, the ore should be sized fairly closely.

Put about 2 in. depth of sized ore into a sieve having a mesh fine enough to retain it, and jig for several minutes under the surface of the water, with a long, slow stroke for the coarser sizes, and a shorter and quicker stroke for finer sizes. A quick down stroke combined with a slower up stroke is best. Care should be taken to keep the sieve level and to avoid any horizontal or overturning movement of the mass of ore. When the tailing appears to be clean, scrape off the upper layer and replace it with an equal amount of ore, and resume jigging. Middling and concentrate should be allowed to accumulate unless the layers become too thick. The main object of this preliminary jigging is to produce tailing as poor as possible. When the whole sample has been treated, the accumulated concentrate is cleaned up by careful jigging, aided by handpicking if necessary. Skimmings from the cleaning operation should be added to the middling and the whole re-jigged and reduced to the smallest possible bulk by combining part with the concentrate and part with the tailing. Assays and microscopic examination of the products will indicate the maximum size at which jigging can profitably be begun.

A rather more elaborate hand jig, described by *Richards*, consists of a square frame about 12 × 12 × 6 in., made of heavy galvanized sheet (10 to 14 gage) turned over ⅛ × ¼-in. strap top and bottom for stiffness. Slats of the same strap to support the screen should be placed 2 or 3 in. above the bottom of the frame and the screen wired thereto. It is desirable to solder the joint between screen and sides, but this makes it difficult to change screens, if desired. Otherwise some leak at the edges must be tolerated. In operation the screen is hung from a helical spring at the desired height above the tub in which jigging is to be done and is then plunged and lifted in the usual manner.

Elmore (*18 CA 1132*) recommends a large hand jig for coal testing. For description and performance of such a jig on bituminous coal see Sec. 9, Art. 9.

Machine jigging. Experimental jigs for testing purposes, operated by hand or power, are made by manufacturers of commercial jigs; they are useful equipment for testing laboratories, but their results are no more illuminating than those obtained by hand jigging. All small-scale continuous jigs are subject to the disadvantage that the ratio of dead space along the sides to total sieve area is relatively large and that an undue amount of heavy grains travels along these dead sides into subsequent compartments or even into the tailing.

Pan. Gold pan. Miner's pan. These are different names for the same device. (See Sec. 8, Art. 8.) The pan has three principal uses, *viz.*: (1) to assay gold-bearing gravels in prospecting, and less frequently, to make a rough assay of crushed vein material for gold; (2) to work gold-bearing gravel on a small scale; (3) to make gravity-concentration tests on heavy-metal ores. The procedure differs according to the material being panned.

Method of gold panning is to take a pan load, submerge it, then work the material over carefully with the hands, rejecting large boulders that are free of adhering fines and clayey material, until all of the coarse material is removed and all clayey or cemented material is disintegrated. Next with the pan submerged or with the material in the pan submerged, holding the bottom substantially horizontal, subject the pan to a rotary motion of sufficient intensity to produce suspension of the solids. The heavy particles, having the faster settling rate in water, settle toward the bottom in the loose suspension and leave the surface metal-free. Now, with the pan above the water surface, tilt it slightly away from the operator and at the same time rock it from side to side. The resulting action of the water will tend to wash solid matter to the lower edge of the pan, and the amount of tilt and speed of rocking should be such that only the surface layers of particles are washed down into a "toe" at the lower rim. The toe is next washed off by alternately dipping and raising it through the water surface. These three operations are repeated in order until the amount of material remaining in the pan is small and con-

sists of heavy minerals and some fine light sand. Further separation can sometimes be made by moving the pan in such a way that a small amount of water therein will course around the trough formed by the intersection of side and bottom. Such treatment strings out the material with the lightest sand ahead and the heavy material bringing up the rear, and it is often possible, by careful manipulation to thus work out a further quantity of light sand. This treatment will usually, also, bring any gold to light at the tail of the fan of material. Final separation of gold from the heavy minerals is made by amalgamation or by drying, separating magnetite with a magnet and the balance of the waste by blowing.

Testing an ore differs from the above procedure in that (1) the ore must first be ground to a size that will free a goodly part of the valuable mineral, (2) the weight of the sample taken can be and should be determined, (3) stratification requires more careful and prolonged swirling and (4) the thickness of the surface layer removed in each cycle must be thinner. The operation should be conducted over a tub in which the first tailing is collected for re-panning. Concentrate will always contain considerable gangue and the tailing some valuable mineral. In the hands of an experienced operator the recovery will be about the same as is possible in mill work, but mill concentrate, especially the coarser sizes, will assay higher.

An experienced panner, working steadily, can treat about 100 pans of uncemented gold gravel per 10-hr. day and proportionately less according to the degree of cementation. The same man cannot run down more than one-third as many samples of galena-quartz ore and even less of ores in which the difference in specific gravities of heavy and light minerals is smaller.

Vanning is somewhat similar to panning but not applicable to as coarse material (0.5-mm max.) and limited to much smaller quantities, *e.g.*, about 50 gm. Vanning plaque is made of enameled iron in the shape of a spherical segment, about 12-in. diameter and ¾ in. deep at the center. The Cornish vanning shovel is essentially a plaque mounted on a handle about 24 in. long. A batea or a large watch glass may be used as a substitute for a plaque.

Procedure. The sample, if dry, is first wet down carefully, taking pains to prevent skin flotation. The wet pulp is then swirled vigorously to get the slime in suspension, next more slowly to let all granular material settle, after which slime is decanted. This operation is repeated until all slime is removed. Thereafter the elements of a vanning operation are three-fold, *viz.*: (1) stratification as in panning, (2) throwing the lower stratum (heavy concentrate) to one edge of the plaque while the upper stratum remains nearer the center, and (3) washing the upper stratum to and over the side of the plaque away from the head of concentrate. Stratification is effected by simple swirling of the pulp by a horizontal rotary motion of the plaque. In order to throw up the head of concentrate the plaque, held on opposite edges in the two hands, is moved as in swirling with one hand while the other describes, at each revolution, a small vertical circle, say 1 in. in diameter, in a clockwise direction, moving downward more rapidly than upward. The swirling motion keeps the upper stratum in suspension while the lower stratum hugs the surface, hence as the plaque moves away from the operator the heavy material moves with it, but it moves from under the gangue in suspension. The rapid down stroke drops the plaque from under the heavy mineral and when the latter again reaches the plaque surface it rests at a point further away from the operator than before. The head of concentrate is, in this wise, made to travel up onto the edge of the plaque away from the operator while the lighter sands remain nearer the center. Horizontal separation is continued by imparting a gentle swirl that moves the water only, and giving the plaque a smart shake at the time that the swirling water is traveling away from the head of concentrate. In this way the sand is washed down slope toward the operator by film-sizing action (Sec. 8, Art. 14). Some operators manipulate the second phase of the separation so as to draw the head toward them and then wash tailing off the far side. Others throw up the head by simple swirling with one hand and jarring the side of the plaque at each revolution against the heel of the other hand. In the latter case the mechanics of the horizontal separation is different, but the result is the same. This method is easier to learn than the other, but is very tiring, if much vanning is to be done. With a vanning shovel the head is thrown up by a slight side fillip on the backward stroke of the swirl.

Principal uses of the vanning plaque are in examination of finely-ground mill products, and in assaying, particularly in CORNWALL to assay tin ore. Vanning has the advantage over a chemical assay that it gives some idea of the amount of middling grains and of the size of the free-mineral grains. It is now being superseded in Cornwall tin assaying by chemical methods, but for

many years it was the principal, if not the only method of assaying used in that district. It had, of course, the apparent advantage that it indicated only recoverable tin, *i.e.*, free cassiterite of a size that could be won by gravity concentration, and this was all that most millmen were interested in, but careful experiment showed that skilful operators reported discrepant results on the same sample and that none, of course, checked the chemical assay.

Shaking tables. Mine and Smelter Supply Co. manufactures a Wilfley table with 12 × 30-in. deck; Deister Machine Co. makes the Plat-O-table in quarter size, 3 × 7-ft. deck, and Deister Concentrator Co., the Deister-Overstrom table also quarter size, with 3½ × 7½-ft. deck. Interchangeable decks for coarse and fine feeds are obtainable with all three tables. The little Wilfley table is the most convenient for small-scale tests, of course, as it can be made to give an excellent result on a sample as small as 1 lb., but larger tables are better when a considerable amount of material is to be run. The difference in performance between the different makes of table on most ores is so small that results on any one can safely be used as indicative of results to be expected on any of the others. The work of the small tables is substantially the same as that of full-size machines. Tables should be set up with variable-speed drive and with provision for making more than the usual number of splits of the discharge. Provision should also be made for collecting all of the various products separately. Much information can be gained by microscopic study of products in addition to the assays. Relative capacities of small and full-scale tables is best established by a series of tests on the two machines with the same feed, but a fair approximation can be made by multiplying the capacity of the small table by the ratio of deck areas. The capacity of the large table will usually exceed somewhat the figure thus obtained.

Vanners and buddles. Performances can be approximated by use of a shallow trough, 3 or 4 ft. long and 8 to 12 in. wide, with a bottom of planed board, ground glass, linoleum, or the like, resting on adjustable supports.

A more elaborate form (designed by H. S. Munroe and made by Eimer & Amend, N. Y., Fig. 21) consists of a strong metal frame about 5 ft. long in which flat plates of pine, maple,

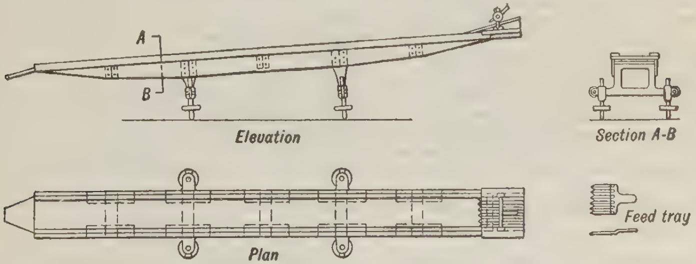

Fig. 21.—Laboratory film-sizing table.

slate, glass or other material may be secured. These plates are held by springs, and the joint between frame and plate is made tight by inserting a strand of cotton wicking. Inclination of the table is adjusted by leveling screws, and amount of water by a dial cock. These adjustments are controlled by testing a small sample of the material to be treated, and giving to the table such inclination and amount of water as will effect the desired separation. A portion of wet ore is gradually pushed under water jets at the head of table, care being taken not to feed too fast. After 2 or 3 min. the feed is interrupted and material on the table is washed 2 or 3 min. until most of the tailing has run off. The tailing pan is then removed, and middling and concentrate are successively washed off with a jet of water, into separate pans. The operation is then repeated with

another portion of ore, first replacing the tailing pan. By making at first a large proportion of middling, clean concentrate and low-grade tailing may be secured. Next wash the middling by itself, varying the amount of water and inclination of table to suit. Collect concentrate, middling, if any, and tailing; dry, weigh, and assay.

The performance of the laboratory table gives no direct information as to the capacity of mill-size machines, except in so far as it shows separation to be difficult or the reverse. See also *27 A 76*.

10. Heavy solutions

This method of separation is much more useful in the laboratory than in the mill for the reason that the cost of supplies is a minor item. A complete series of solutions ranging in specific gravity from well under 1.0 to about 3.5 may be made or bought and intermediate densities of any desired value may be obtained by diluting or mixing the proper solutions in the proper proportions. *Johannsen* describes in detail the preparation and use of these liquids. The following abridged descriptions cover the more common.

Sonstadt (Thoulet) solution is an aqueous solution of potassium mercuric iodide. It is made by dissolving 270 gm. of mercuric iodide (HgI_2) and 230 gm. potassium iodide in 80 cc. of cold distilled water, then evaporating on a water bath until crystallization just begins. On cooling, the specific gravity is 3.196. The solution may be diluted to any desired density with water and re-concentrated on a water bath. In warm, humid weather the density drops materially. Dense solution filters readily. Metals and organic matter decompose it; it is highly poisonous and corrodes the skin. If iodine separates, the solution may be restored by re-concentrating with the addition of a small amount of metallic mercury.

Klein solution is an aqueous solution of cadmium borotungstate

$$(2\ Cd(OH)_2 \cdot B_2O_3 \cdot 9\ WO_3 \cdot 16\ H_2O).$$

The concentrated solution, sp. gr. 3.36, is obtained by evaporation of a more dilute solution on a water bath, and any lesser specific gravities are obtained by dilution. This solution is not very poisonous, it filters readily and is not decomposed by filter paper nor organic dust and does not affect the skin. It may be used for years. It is decomposed by iron, lead and zinc and reacts with carbonates. When the latter are suspected the pulp should be washed with dilute acetic acid.

Rohrbach solution, sp. gr. > 3.55, is an aqueous solution of barium-mercuric iodide made by heating 100 parts barium iodide and 130 parts mercuric iodide and 20 parts distilled water on an oil bath at 150 to 200° C. The solution is non-filterable, highly poisonous, very hygroscopic, and difficult to dilute. This is best done by slowly adding a dilute solution of the same substance. It is not decomposed by carbonates, but the material to be separated must be absolutely dry. Separated solids should be washed with a very dilute solution of potassium iodide. Separations should be made in closed vessels.

Methylene iodide (CH_2I_2) is a very mobile, easily handled separating fluid whose specific gravity varies from 3.3485 at 5° C. (freezing point) to 3.2890 at 33° C. It is diluted with benzol, carbon bisulphide or chloroform, but, while it is unchanged in the atmosphere, if concentrated, the specific gravity rises rather rapidly by evaporation of the dilutant. It filters readily and is unaffected by the paper. It does not attack the skin and is not decomposed by metals or carbonates but is by sulphur. It is readily washed away from solids by benzol.

Retger's solutions are a variety of mixtures of higher specific gravities than those already described. They are uncertain and difficult to make and handle.

Procedure is simple. It consists in placing the powdered mixture in a suitable solution and diluting until the desired constituent or constituents sink, then separating the floating from the sunken portion, separating the solution from each portion, and washing the solid free of solution. Johannsen describes a large number of separating vessels, chiefly applicable to close separations of a number of constituents. Tomlinson (*52 A 852*) describes a method of mineralogical analysis of sand that is adaptable to most ore-dressing work.

Preparation. The material to be investigated should first be carefully sized and the weights of the grades recorded. Apart from the information thus obtained, sized products are more accurately separated in heavy solutions than unsized.

Solution. Sonstadt solution is the easiest to make and handle. The concentrated solution will drop all sulphide minerals and rich middling while holding up most gangue minerals and lean middling. If separation of the gangue minerals is desirable, a density of 2.9 will drop practically all of the dark silicates, 2.7 will drop calcite and dolomite and float quartz, and 2.59 will drop quartz and float feldspar, although this separation will not be as close as the others.

Separation is most readily made in a Harada tube, which is a special separatory funnel (see *Johannsen*), but it may be made satisfactorily in beakers. Use a beaker whose capacity is about six times the volume of the lot of sand to be separated, dry it thoroughly, add the solution (two volumes of solution of proper density to one of sand), stir in the sand slowly, allow it to stratify, skim the bulk of the float with a glass or platinum spoon, then pour off some of the liquid with the balance of the floating material. Some floating sand will usually stick to the beaker in pouring off. If so, push it away on the sides of the beaker with a glass rod until sufficient clear space is available to pour out the settled solid with the balance of the liquid without contamination. Wash the separates repeatedly with water to remove and save solution. If several grades are to be made, it may be well to separate first at an intermediate density and re-work these rough grades at higher and lower densities respectively, making sure that they are thoroughly dry before re-treatment.

Coal analysis is the most important application of heavy-solution testing. Any of the solutions and the method described may be used, but the volume of sample is usually so large and the coal so light that special methods and cheaper solutions are usually employed.

Sink-and-float test for coal

The heavy-solution method is particularly suitable for coal analysis for the reason that with any given coal the ash content has a direct and substantially constant relation to the specific gravity of the coal, so that once an allowable ash content for such a coal has been set and the specific gravity of the highest-ash particle therein has been determined, a solution of that specific gravity will float good coal and everything that sinks will be of higher ash content than the permissible limit. Table 10 shows a sink-and-float test on a low-grade bituminous coal. The specific gravities of the usual impurities associated with coal compared with those of coal are shown in Table 11. It is apparent that a heavy solution of specific gravity between 1.50 and 1.65 would put all of the distinct impurities and some of the bone coal into the "sink" fraction. With different coals the ash content and the specific gravity have distinctly different relations, as is shown in Table 12. Hence, as above intimated, it is necessary to establish the ash-specific-gravity relation and the critical specific gravity for each coal.

Table 10. Results of a sink-and-float test on a bituminous coal. (*After McMillan and Bird*)

Specific gravity	Weight, per cent.	Ash, per cent.	Cumulative per cent.	
			Weight	Ash
Under 1.30	15.8	4.1	15.8	4.1
1.30–1.35	27.2	9.0	43.0	7.2
1.35–1.40	15.3	16.5	58.3	9.6
1.40–1.45	6.9	23.1	65.2	11.1
1.45–1.50	6.3	29.3	71.5	12.7
1.50–1.55	5.4	35.0	76.9	14.2
1.55–1.70	7.5	43.9	84.4	16.9
Over 1.70	15.6	71.5	15.6	....
Total	100.0	24.8	100.0	25.4

Table 11. Specific gravities of common ingredients of raw bituminous coal. (*After Drakeley*)

Material	Specific gravity	
	Range	Average
Coal	1.17–1.35	1.30
Bone	1.35–1.65	1.50
Carboniferous shale	1.65–2.15	1.85
Shale	2.15–2.55	2.40
Coal or shale with pyrite	2.55–4.80	3.65
Pyrite	4.80–5.20	5.00
Quartz	....	2.60
Calcite	....	2.71+
Feldspar	....	2.40

Table 12. Relation between specific gravity and ash content of various Vancouver coals. (*After Garman*)

Order of increasing ash(*a*)	Clean coal		Refuse	
	Specific gravity	Ash, per cent.	Specific gravity	Ash, per cent.
1	1.30	3.86	1.76	42.22
2	1.25	3.90	1.57	44.37
3	1.32	7.12	1.91	57.13
4	1.28	7.50	1.75	62.25
5	1.24	7.63	2.00	68.50
6	1.60	8.22	1.77	75.34
7	1.30	9.00	2.25	84.55
8	1.31	15.57	2.60	88.38
9	1.38	17.55	....	....
10	1.43	26.25	....	....

a There is no necessary relation between the clean coal and the refuse in any given horizontal line; they may not be from the same coal.

Solutions. The usual heavy solution is a water solution of zinc chloride. This gives a maximum density of about 1.80 at ordinary temperature. Calcium chloride is sometimes used. Sinnat and Mitton (*67 IME 494*) used mixtures of carbon tetrachloride (sp. gr. 1.582 at 21° C.) and toluene (sp. gr. 0.8708). For sizes finer than 20-mesh McMillen and Bird (*Bul. 28, UW Ser. 61*) used a mixture of carbon tetrachloride with benzene or bromoform, according to whether densities below or above 1.58 were desired.

In changing solution strengths, the volume, x, of old liquid to be withdrawn and new solution to be added may be calculated by the equation, $x = (O - N)V/(O - A)$, where V is the volume of testing solution before and after the change in density, and O, N and A are the specific gravities of the old, new, and added solutions respectively. If the testing tank is of constant cross-section, x and V may be expressed in uuits of depth of liquid in the tank, if there is no appreciable amount of solid material in the tank.

Size of sample depends on the size of the particles. McMillan and Bird (*loc. cit.*) recommend 500 lb. for egg-size coal, $-3 + 1\frac{1}{2}$-in.; 250 lb. for nut-size, $-1\frac{1}{2} + \frac{3}{4}$-in.; 125 lb. for pea, $-\frac{3}{4} + \frac{3}{8}$-in.; 50 lb. for buckwheat, $-\frac{3}{8} + \frac{3}{16}$-in.; 25 lb. for birdseye, $-\frac{3}{16}$-in. + 20-mesh; and 0.5 lb. for -20-mesh. (See also Sec. 2, Art. 12.)

Apparatus. The essential elements of the apparatus for making tests conveniently are a large-enough container and means for removing the sink and float fractions separately.

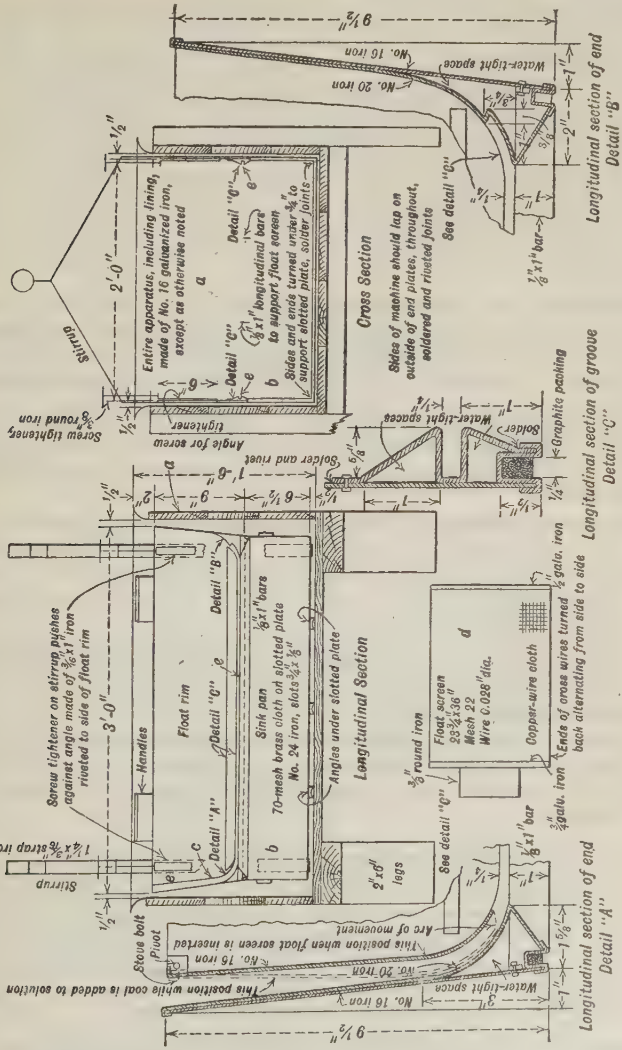

FIG. 22.—Apparatus for sink-and-float testing of coal.

Fig. 22 represents an apparatus for coarse sizes developed by the U. S. Bur. of Mines. It consists essentially of a wood or zinc-lined tank (*a*) and two perforated-bottom containers for removing the separated fractions. The sink pan (*b*) rests on the bottom of the tank and has a permanent perforated bottom and handles at the end. The float rim (*c*) rests on the upper edge of the sink pan. As set into place for a test it is merely a 4-sided rim

extending the sides of the sink pan above the surface of the liquid in the tank. After the charge has separated into sink and float fractions, a flexible screen (*d*) is slid down through and along the groove (*e, e, e*) in the float rim until it forms a perforated divider between the fractions and a support on which to lift out the float fraction when the float rim is removed. Details of construction are shown in the figure.

Procedure, especially when the feed contains impurities that disintegrate in water, is to start with the heaviest solution (1.70 sp. gr. for bituminous coals and 1.80 for anthracite will float all material of any value). The sample is shoveled into the assembled apparatus, float being skimmed as necessary to maintain the layer of float at not more than three particles deep. It is necessary to make certain, before any material is skimmed, that the specific gravity of the solution is exactly that or less than that at which separation is intended. Float should be stirred before skimming to prevent removal of entangled sink material. When all of the sample is in, the specific gravity of the solution should be brought to exactly the desired figure (using an accurate hydrometer for testing), the sink should be stirred to free any entangled float, and then the screen inserted into the float rim (Fig. 22) and the latter lifted out and placed on a suitable draining board arranged to collect drippings. The sink should then be removed, drained, washed free of heavy liquid and dried. Next reassemble the apparatus, dilute the solution to the next lower separating density, and re-treat the preceding float. Repeat until the desired number of cuts has been made.

Fine material. For material finer than 20-mesh a two-liter bottle makes a good separating container and carbon tetrachloride with benzene or bromoform good heavy solutions. Float material may be drawn into a two-liter vacuum flask by means of a glass tube. Separating solutions must be maintained at constant temperature in order to maintain constant specific gravity. The sample should be 200 gm. of air-dried (105° C.) coal which has been thoroughly wetted before separation. Twenty-four hours may be required for complete separation. The specific gravity of the solution must not be adjusted during a given separation.

Testing coal to determine a method of treatment involves the same principles as ore testing for the same purpose. The testing campaign should include sink-and-float tests, with ash and sulphur determinations; a sizing test with sink-and-float tests of the sizes will yield valuable information; and the microscope should be used freely.

The sink-and-float test on the whole feed will tell what yield and ash content of washed coal are to be expected; it will give information concerning the composition of the washed product and as to the possibility of taking an intermediate product for mine-plant fuel.

The sizing test with sink-and-float tests on the grades tells the size to which crushing must be carried to free the economic maximum of waste. The same information may be obtained by crushing different samples to different sizes and testing these.

Tables 13 and 14 present the results of two such tests on a coking coal. Table 13 indicates that further crushing from 1-in. to ⅜- freed some slate and distributed it principally in the coarser sizes of the − ⅜-in. product. The effect is more clearly shown in Table 14. There is more low-ash material and more free slate in the finer product. On the other hand, for a yield of, say, 90 per cent. (see Table 14), the 1-in. crushing gives a clean product containing about 7.3 per cent. ash against 6.5 per cent. in the − ⅜-in. product and the probability is that in actual washing the difficulties due to treating the finer product would prevent realization of this difference in the washery product and that the improvement in result, if any, would not pay for the increased treatment cost.

In general, to estimate from float-and-sink tests the amount of sulphur to be expected in washed coal, multiply the percentage of sulphur in the float by 0.9.

Microscopic examination will give an idea of the mode of occurrence of the ash and the possibilities of concentration (washing).

Table 13. Sizing-assay tests of a raw coal crushed to different sizes. (*After Yancey, Bul. 16, CIT*)

Screen size, inch	1-in. crushing, per cent.			⅜-in. crushing, per cent.	
	Weight	Ash	S	Weight	Ash
+1	15.4	14.6	0.66	....	....
½	41.9	10.3	0.77	....	....
¼	18.5	10.3	0.90	22.6	12.5
⅛	10.4	10.4	1.06	39.5	10.8
1/16	6.1	10.2	1.06	17.0	10.2
1/32	4.3	10.5	1.15	9.4	9.3
1/64	1.8	12.4	1.23	2.5	9.6
−1/64	1.6	12.9	1.32	9.0	11.1
Totals and averages..........	100.0	11.1	0.86	100.0	10.9

See also Table 14.

Table 14. Float-and-sink tests on the same coal (Table 13) crushed to different sizes. (*After Yancey*)

Specific gravity	1-in. size, per cent.				− ⅜-in. size, per cent.			
	Weight		Ash		Weight		Ash	
	Direct	Cumulative	Direct	Cumulative	Direct	Cumulative	Direct	Cumulative
−1.30	58.2	58.2	3.8	3.8	66.3	66.3	3.5	3.5
+1.30–1.40	24.8	83.0	10.9	5.9	17.6	83.9	12.1	5.3
+1.40–1.45	6.3	89.3	22.5	7.1	4.0	87.9	21.3	6.0
+1.45–1.50	2.1	91.4	28.4	7.6	2.6	90.5	26.6	6.6
+1.50–1.60	2.1	93.5	33.4	8.0	2.8	93.3	35.2	7.5
+1.60–1.70	2.4	95.9	41.8	9.0	1.7	95.0	42.8	8.1
+1.70	4.1	4.1	62.9	11.2	5.0	5.0	63.1	10.9
Original	100.0	100.0	11.0	....	100.0	100.0	11.0	....

For methods of ash and sulphur determination see: *Methods of analyzing coal and coke*, F. M. Stanton and A. C. Fieldner, *TP 8, USBM*; *Standard methods of laboratory sampling and analysis of coal, A.S.T.M. Standards, 1921, p. 760.*

Following the small-scale tests, washing tests should next be made on a large hand jig (see Sec. 9, Art. 9) or better yet on a full-sized coal jig. These tests should be made at the mine, where there is sufficient material readily available for an exhaustive testing campaign and on a scale sufficiently large, in the case of a coking coal, to yield tonnage for full-scale coking tests. From these data a flow-sheet can be laid out intelligently.

It will generally be found in these tests that practically everything of less than 1.50 sp. gr. will go into the washed coal and everything heavier than 1.75 into the reject. The distribution of the material intermediate in specific gravity will depend on the type of washer (concentrator) and the way in which it is run. Tank, trough and tubular washers, unless run with the greatest care and not overloaded, will put much of this material into the cleaned coal. Jigs, unless badly overloaded or operating on too large a range of sizes can be made to deliver the bulk of this bony material either into the clean coal or

the refuse, especially if the jig is of more than one compartment or two jigs are run in series. Shaking tables will make a good separation if a middling product can be shaken out, otherwise the bone will distribute between both products. Except in the case of machines treating fine feeds the refuse should not contain more than 5 per cent. of clean coal and this should be well less than 1 per cent. of the feed to the machine.

Delameter (*22 CA 751*) gives, in considerable detail, the results of an elaborate series of tests to determine whether it was necessary or best to wash all of the coal and, if not, at what size it was best to split, jigging oversize and mixing the washed product back with the unwashed screenings. The results are summarized in Fig. 23. Plot *A* shows sizing-

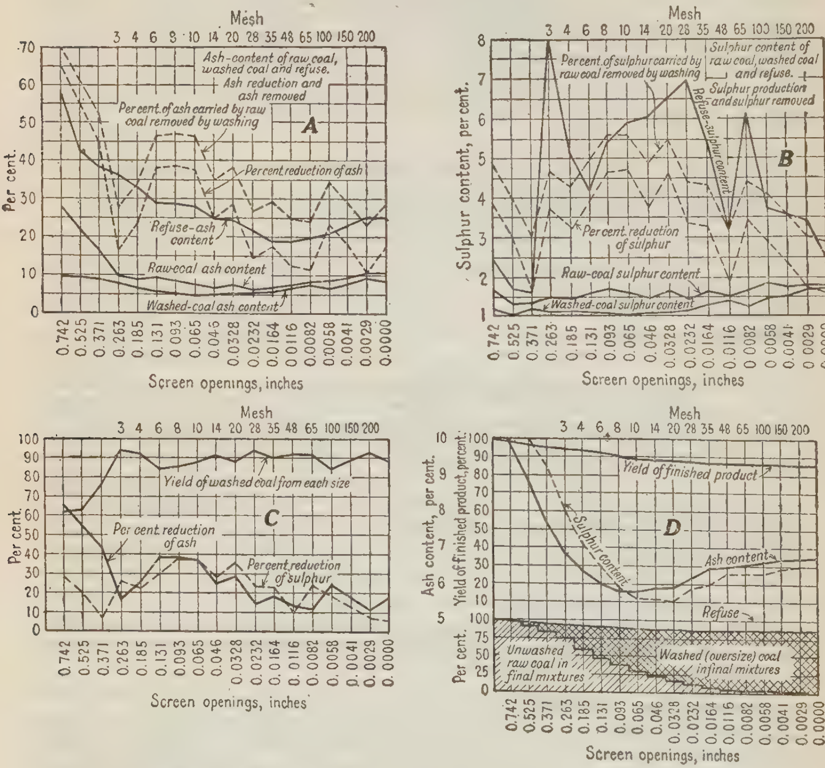

FIG. 23.—Results of washing tests on bituminous screenings (*after Delameter*).

assay tests giving the ash content of raw coal, washed coal and refuse in a representative run. *B* gives the same data for sulphur. *C* shows the relation between yield and ash and sulphur reduction. *D* shows the yield and the ash and sulphur content of the cleaned coal, assuming the raw coal (− 1.05-in.) sized on any given screen (abscissa), the oversize jigged and the unwashed undersize added to the washed oversize to make the final cleaned coal. According to this curve, if all of the − 1.05-in. coal is washed, the yield will be about 87 per cent., the ash content of the cleaned coal 6.7 per cent. and the sulphur content 1.15 per cent. On the other hand, by sizing on an 8-mesh screen and washing only the oversize, re-mixing unwashed undersize with the washed oversize, the yield would be 90 per cent., the ash content of the washed coal 5.8 per cent. and the sulphur 1.09 per cent. At this mine 1.2 per cent. of sulphur was allowable, which permitted the

separation to be made at 4-mesh, at which size the yield was between 93 and 94 per cent. and the ash content of the cleaned coal about 6.4 per cent. This is a rather surprising result in view of the fact that the fine sizes of the raw coal contained the most sulphur (see *B*) and not the least ash (see *A*).

Washery control. Sink-and-float tests form an excellent rule for judging the efficiency of the operation of any machine or operation. The feed and products of the machine or operation should be tested in a solution of PERMISSIBLE DENSITY, *i.e.*, the density pre-determined to yield a washed coal of allowable ash content. From the results, efficiency may be calculated by any of the methods of Art. 28. For this kind of work it is unnecessary to dry the sink-and-float fractions before weighing, since the results will be sufficiently accurate using moist weights, and the time saved is considerable.

11. Flotation

Testing flotation processes is more difficult than testing other methods of concentration for the reason that the controlling conditions are less well recognized and the phenomena, which are physico-chemical rather than strictly physical, are sometimes vastly affected by apparently slight changes in these controlling conditions. Minute amounts of foreign substances may exert enormous effect, temperature is of considerable importance, the character of the water and, in some cases, even, the material of which the machine is made may affect performance. It is particularly necessary, therefore, to make a careful, detailed record, *during the course of the test*, of every feature connected with the test, making special note of all conditions surrounding any unusual performance.

The conditions affecting froth flotation are: (1) the mineralogical character of the feed, the size and method of grinding; (2) the character and quantity of flotation agents; (3) the pulp thickness and the amount and character of suspended and dissolved organic and inorganic material in the water; (4) the kind of process; (5) the kind of apparatus; (6) the degree of agitation; (7) the duration of treatment; (8) the temperature.

The indicia of effect are copiousness and consistency of froth; size of bubbles; amount and kind of solid load; the degree of dispersion of the pulp; rate of flotation, and finally the recovery and grade of concentrate.

Record form. The record of tests is best kept on a printed or mimeographed form that will act as a check list for the information to be obtained. This form should include the following items: TEST; number, date, purpose. FEED; origin, history, approximate mineral composition, sizing test, assay, record of microscopic examination, weight. MACHINE; type, size. WATER; source, litmus reaction. REAGENTS; description, quantity, order of adding, method of mixing, duration of mixing period, pulp density and temperature and litmus reaction during mixing. ROUGHING (or concentrate-making) period; duration, degree of agitation or aeration; pulp density, temperature and litmus reaction; character of froth, including texture, size of bubbles at water line and overflow level, elastic or effervescent nature of bubbles, viscosity or tenderness of froth, persistence, mineralization, litmus reaction, percentage of solids. CLEANER (or middling-making) period: Collect the same data as in roughing period. ROUGHER TAILING; litmus reaction, degree of dispersion; rate of settling, temperature, density. CLEANER TAILING; the same. ASSAYS. METALLURGICAL RESULTS. NAME OF OPERATOR. It is well to define the meaning of certain of the descriptive terms. The following definitions are useful. TEXTURE: Examine the "grain" of the froth. Mental reference to the grain of a rock or other non-homogeneous mixture will help in choosing the proper descriptive term. The texture should be described as "even" when all, or a great majority of the bubbles in a given horizontal line against the glass walls of the testing machine are of approximately the same size; "uneven" describes the reverse condition. Texture should be described as "fine" when the average diameter of bubbles at ½ in. above the water line is ⅛ in. or less. CHARACTER OF BUBBLES: *Elastic* indicates that the surface bubbles in the machine are relatively

persistent, that they may be deformed considerably without bursting, and that they elongate markedly in overflowing. *Effervescent* bubbles burst with considerable violence soon after reaching the surface layer. QUALITY is *viscous* when patches of the froth act almost as solids and the body of froth is sluggish in the cell; *tender* describes a homogeneous, fluffy froth that is active on the surface of the cell. MINERALIZATION is best determined by examining a film under a low-power (15 to 20 ×) microscope. *Heavy* describes the mineralization of a film that is crowded with solid, opaque, and that draws back slowly when punctured; *light* describes the complete reverse of this condition; *medium* describes a wide intermediate range. *Gangue* and *sulphide* indicate the predominating mineral in the solid load. PERSISTENCE. *High* describes the fact that overflowed froth shows little or no tendency to break down after standing 10 min.; *medium* describes a froth that breaks down spontaneously to about one-half its volume in 10 min. after removal; *low* describes substantially complete breaking down in 10 min. SETTLING RATE of tailing gives a pseudo-quantitative measure of degree of dispersion. It is best measured by allowing one minute to elapse after flotation ceases, then measuring the distance from a fixed point on the side of the machine to the top of the subsiding solids and again measuring after the lapse of one minute. An experienced operator can keep this record with very little more time than is required for the test itself; it serves as a guide to methodical and careful observation; and makes it possible for persons who have not seen the test to visualize the performance and interpret the results.

Machines

For crude, small-scale tests by the agitation-froth process, agitation may be effected by hand shaking in a test-tube or a wide-mouth bottle, or in a milk-shake machine, and froth separated with a spoon or by overflow by introducing sufficient water through a tube below the pulp level. A separatory funnel may be used for shaking (*111 P 122*) in which case the tailing can be drawn off from the bottom without dilution. A motor-driven bar mixer with a small square glass jar is an excellent device for preliminary tests.

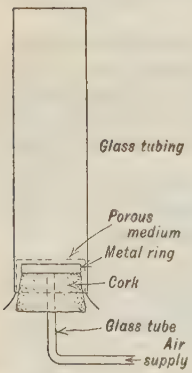

FIG. 24.—Tube for pneumatic flotation tests.

Preliminary pneumatic-process tests may be made with a small machine made of a short length (8 to 12 in.) of 1.5- or 2-in. tubing, a 1-hole rubber cork and a piece of finely porous cloth, arranged as shown in Fig. 24. Air may be supplied by a bicycle or automobile pump or at the worst an ordinary atomizer or syringe bulb. It is well to have a receiver made, say, of a 2-liter acid bottle, between the pump and cell, arranged so that the air supply to the cell can be regulated by means of a pinchcock.

These preliminary tests will be valuable principally to determine amenability and, perhaps, to give some idea of the grade of concentrate that can be made. They will not yield good recoveries.

The most convenient machines for laboratory work are single-cell models of mill machines taking charges of 500 to 1500 gm. of solid with provision for froth overflow and arranged for pulp circulation. Such machines yield results that are, in most cases, directly comparable to mill operations.

Parsons (*Bul 617, Canada Dept. of Mines, Mines Branch, 1923*) recommends that for differential-flotation tests a continuous-flow testing unit, consisting of a ball mill in series with the flotation machines, be used. He says that batch work gives better results than can be obtained in mill operation.

Janney mechanical machine (Fig. 25) is a miniature copy of one cell of the mill machine, differing therefrom in that the volume and area of the spitzkasten (*c*) are somewhat less in proportion to the size of the agitating compart-

ment (a) than in the larger machine and that the bottom of the spitzkasten of the laboratory machine is higher than the bottom of the agitating compartment, while the reverse is true of the mill cell. Pulp circulation is, of course,

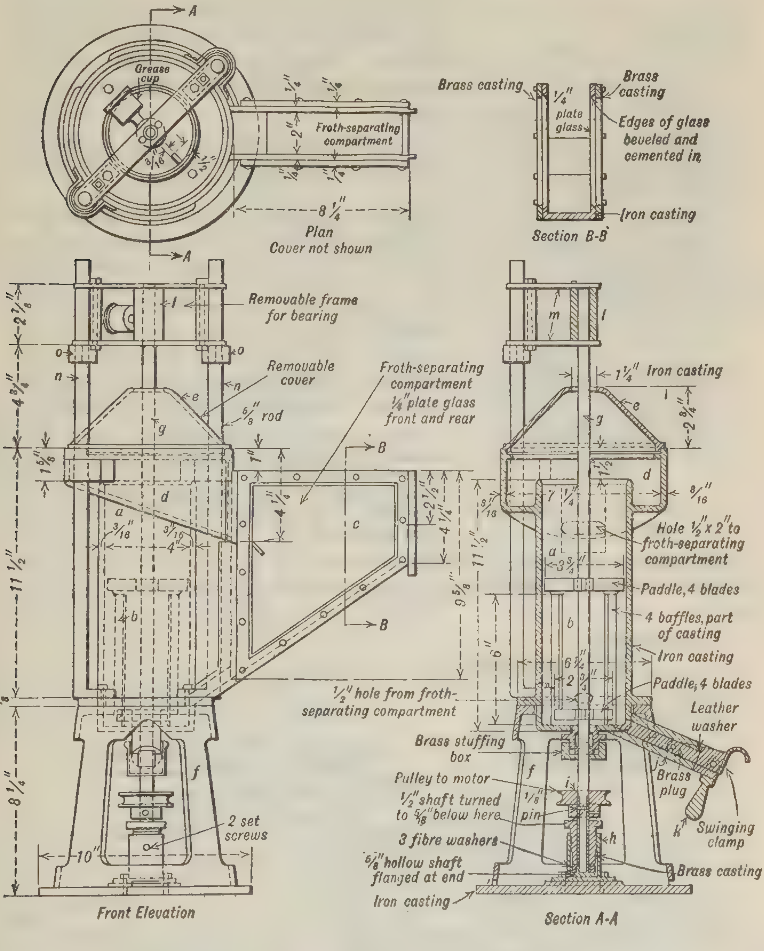

FIG. 25.—Janney laboratory flotation machine.

restricted to the one agitating compartment and one spitzkasten. The machine is made of cast iron or aluminum. A suspended impeller shaft, ¾-in. diameter, carried on roller bearings, is better than the under-driven type pictured.

A ¼-hp. motor capable of sufficient speed variation to give a range of agitator speeds

between 500 and 2400 r.p.m. should be provided. The ore charge for a pulp containing 20 to 25 per cent. solids is 500 to 600 gm.

Minerals Separation machine. A one-compartment model of the mill cell arranged for continuous pulp circulation is built by the Denver Fire-Clay Co., Denver, Col., under the name of Case laboratory machine. The form shown in Fig. 26 is just as satisfactory and is easily built in any mine shop.

A ¼-hp. motor giving the agitator a speed range from 800 to 2000 r.p.m. is desirable. The machine shown takes an ore charge of 750 gm.

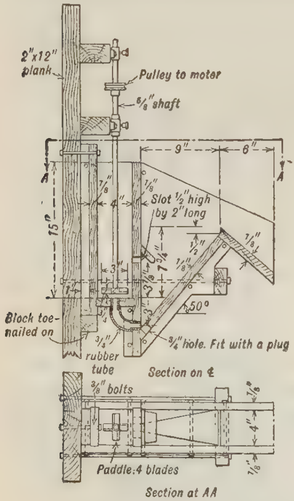

Fig. 26.—M. S. laboratory flotation machine.

Callow machine. General Engineering Co., Salt Lake City, Utah, makes a two-cell laboratory unit consisting of a rougher, 2 × 15 in., a cleaner, 2 × 7 in., a Pachuca tank for mixing, and an air-lift for circulating cleaner tailing back to the head of the rougher. The apparatus requires provision for 11 cu. ft. of free air per min. at about 5 lb. pressure. The ore charge is 1000 to 2000 gm.

The pneumatic machine shown in Fig. 27 can be built in any good mine shop. It can be arranged either for continuous or batch treatment, according to the position of the tailing-discharge slide. The ore charge for a pulp containing 20 to 25 per cent. solids is 1500 gm.

Sub-aeration machine (Fig. 28) is probably the most flexible of the laboratory types. It is not manufactured for sale but is easily made in any machine shop.

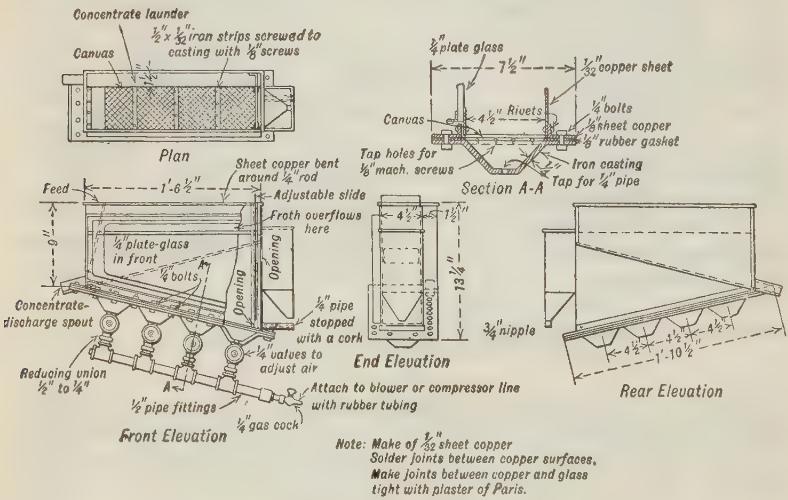

Fig. 27.—Laboratory-size Callow cell.

It requires a $\frac{1}{4}$-hp. motor which should give an impeller-speed range from 500 to 2000 r.p.m. The machine shown requires a charge of 750 gm. to give a pulp containing 20 to 25 per cent. solids.

Other testing machines. See A. F. Taggart, *Manual of flotation processes*, John Wiley & Sons, Inc., 1921.

Motors. An agitation-froth machine with 3- or 4-in. impeller requires a $\frac{1}{4}$-hp. motor. For direct current the General Electric Co. type DSD, constant-speed, shunt-wound motor, with a speed of 1700 r.p.m.; maximum voltage 250, drawing 1.25 to 0.63 amp., wired with a field rheostat in the armature circuit, is eminently satisfactory. Type SD is a similar motor for 110 volts. For alternating current Ralston (*112 P 8*) vouches for the satisfactory behavior of the General Electric Co. repulsion-induction motor, single-phase, 60-cycle, 1780 r.p.m. full speed, drawing 4.2 amp. at 110 volts or 2.1 amp. at 220 volts, either voltage being acceptable. Speed varies with the load and voltage and is controlled by a field rheostat in series with the motor.

Testing procedure

General. All apparatus should be scrupulously clean. Cleaning is best done by scrubbing with strong solution of sodium carbonate or sodium hydroxide, followed by a neutralizing wash with sulphuric acid solution and then by a blank run with finely-ground waste rock. If there is no frothing in the blank run the machine is clean and should then be washed free of solid in preparation for the ore charge.

It is most convenient to measure all quantities in metric units. Small quantities of mobile liquid reagents are readily measured with a Mohr pipette and the quantities converted into weights, taking into account the specific gravity; viscous liquids may be measured by counting drops, after the necessary calibration; solid reagents are best added in solution, if possible, otherwise they are weighed in.

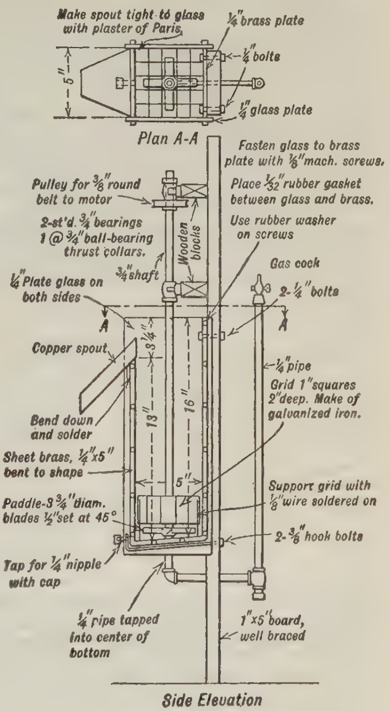

Fig. 28.—Laboratory sub-aeration machine.

The method of preparing the ore charge depends upon the ore, the nature of the test and the nature of the reagents. Dry grinding in a disk sample grinder or a small ball or pebble mill is most convenient, as it permits preparation of a large number of samples at one time, but the nature of such dry-ground material is not necessarily the same as that of the same ore wet-ground. Dry-ground particles may be of different shape from wet-ground, the relative average sizes of gangue and mineral will probably be different, and the heat developed in dry grinding together with the exposure to air may oxidize the surface of some of the floatable particles. Wright (*120 P 459*) says that wet-crushed samples have, in his experience, generally yielded recoveries from 5 to 10 per cent. higher than those from dry-crushed samples. The dispersion of viscous reagents is best effected in a thick pulp in a cylinder grinding mill. Hence wet grinding, simulating mill conditions, should be practiced unless it is established that dry grinding produces no essential difference in results.

In wet grinding the ore is first reduced to pass 10- or 20-mesh dry. The desired charge is then weighed into a small batch cylinder mill, water in an amount equivalent to 30 to 50 per cent. of the combined weight is added, flotation agent is put in, if desired, and the charge ground for the pre-determined time required to produce the desired size. A convenient form of batch-grinding apparatus may be made of a piece of 8-in. pipe, 10 in. long, with special caps with lugs for ease in unscrewing, to form the grinding cylinder; and two 4-in. pipe rollers with gudgeon ends in bearings, one roller driven and the other loose, in the same horizontal plane, spaced 9 in., on which the grinding cylinder is supported and rotated. The speed of the cylinder should be 40 to 50 r.p.m.

When the sample for test is a mill pulp, it should not be dried prior to testing.

In preliminary testing, as *e.g.*, in determining a suitable reagent, rough tests of metal content of the tailing may, with many ores, be made by vanning, and assays need not be made unless the vanning test indicates good recovery.

Methods of testing with the laboratory machines described are given in succeeding paragraphs. For methods of testing with other machines and by other processes see *Manual of flotation processes*.

Testing in agitation-froth machine. Start the impeller at slow speed. Add the charge of oiled pulp. (If feed has been dry ground, make a pulp in the machine to contain 50 per cent solids, add the collecting agent and agitate at full speed for one minute.) Bring the impeller up to full speed, add water until the level of the pulp stands one or two inches below the overflow lip, then add the frothing agent into the agitation compartment. Allow froth to build up until it stands ½ to 1 in. above the overflow lip, then scrape off as fast as it builds to this height. Add water as necessary to keep the pulp level at the proper height, which will increase as the test proceeds. Additional frothing agent may be added, if necessary to obtain froth toward the end of the test. Procedure varies somewhat according to the type of flotation flow-sheet that is being investigated. If this is of the rougher-cleaner variety, froth may be scraped deeply and removed rapidly during the roughing period in the endeavor to make clean tailing. Tailing is then removed from the machine and all of the overflow from one or more roughing operations, depending upon the amount of rougher froth produced, is charged back into the machine and frothed slowly to produce clean concentrate. The percentage of solids in the cleaning operation should ordinarily be between 6 and 12 at the beginning of the run. In a concentrate-middling test the first froth is removed slowly in an endeavor to make clean concentrate. Thereafter froth is removed as rapidly as possible and even a small amount of pulp may be overflowed in the endeavor to produce low-grade final tailing.

The rate of flow through mill machines is such that pulp remains in a given compartment of the primary machine for about two minutes. Hence each two-minute period in the test machine corresponds to treatment in one compartment of a mill-size machine and the probable mill installation can be estimated from the duration of the test run.

Collect concentrate, middling, tailing, and, if desired, the material that collects in the bottom of the agitating compartment ("untreated material") separately, weigh wet, dry and weigh, sample and assay.

Testing in pneumatic machine. Usually the feed is ground wet in a thick pulp with the collecting and dispersion agents. If dry-ground feed is used, 10-min. mixing with the agents in a 1 : 1 pulp in the batch-grinding mill with a light ball or pebble load is best. Thin the charge of pulp with water, add with frothing agent to the machine with air turned on slightly and tailing exit closed. Regulate the air in the different compartments so that there is no marked horizontal travel of air or bubble column, then add more water, if necessary, until froth overflows freely. Continue frothing until the residue in the cell shows marked impoverishment or until 8 or 10 min. have elapsed. This time corresponds to that required for the passage of pulp through a standard Callow cell under ordinary mill conditions. If no marked impoverishment occurs, it may be concluded that the particular flotation agents used are unsuited to the ore and others should be tried. In the case of a good roughing test, draw the tailing and return the rougher froth to the machine for cleaning. This operation requires only one-half to one-quarter the quantity of air used in roughing, the pulp should be quite dilute and the rate of overflow slow. Collect cleaner concentrate, cleaner tailing (middling) and rougher tailing separately, weigh wet, dry and weigh, sample and assay.

Testing in the sub-aeration machine is akin to testing in the pneumatic machine except that mixing is readily performed in the cell itself, as in the agitation-froth machine. Otherwise, with apparent modifications, the preceding instructions will serve for this machine.

Continuous tests in machines of the size described will not give satisfactory indications of mill results. Machines for such testing should have a capacity of several hundred pounds per hour and should be preceded by a ball mill and classifier. A flow-sheet consisting of a 3-ft. ball mill, in closed circuit with a simplex Dorr classifier sending overflow to a 1 × 5-ft. rougher Callow cell and a 6-in. × 3-ft. cleaner, with suitable accessory pumps and sampling apparatus will, however, treat ore at the rate of 5 tons per 24 hr., if already crushed to ½-in. size, and will give results that are comparable to mill performances. Several hundred to 1000 lb. of ore are required to charge such a plant and one or two hours running to allow it to settle down, so that at least one ton of sample is required for a test and two to five tons is safer. Flotation feed, unlike the feed in gravity concentration, cannot be made up of re-combined products from a previous run.

Purpose of flotation testing may be (*a*), to investigate the amenability of an ore to flotation, (*b*) to determine the best method of treatment, (*c*) to investigate a flotation agent, (*d*) to investigate a process.

Amenability. Actual flotation should be preceded by microscopic examination to determine the approximate qualitative mineralogical composition and state of the ore as regards oxidation. Preliminary examination with a binocular microscope followed by study of a polished section, supplemented, if necessary, by study of a thin section or some fragments with a petrographic microscope, will usually save time and permit much more intelligent prosecution of the actual testing work. If microscopic study shows unaltered sulphide associated with the ordinary rock-forming gangue minerals, it is a safe conclusion that the ore is amenable to flotation and actual tests in the machine may be directed rather toward determination of the best method of treatment than toward confirmation of amenability. If microscopic examination shows mixed sulphides, that fact will indicate low-grade concentrate and point the way toward differential flotation. If there are signs of oxidation, trouble is to be anticipated and the question of amenability cannot be answered without actual flotation testing.

The following classification of commonly-used flotation agents will serve as a guide in the choice of agents. It is not, however, to be looked upon in any way as rigid or complete, but merely as an aid to the beginner in flotation testing.

Frothing agents. (See p. 842.)

Agents that stiffen froths. Wood tar and wood-tar oils. Petroleum and petroleum derivatives.

Collecting agents. (See p. 842.)

Dispersion agents. (See p. 844.)

Agents for copper ores. Tars and their derivatives. Thiocarbanilid and xanthates. (See also p. 847.)

Agents for lead ores. Pine oil. Wood creosotes. Thiocarbanilid and xanthates. (See also 851.)

Agents for zinc ores. Pine oil. Coal-tar creosote. Thiocarbanilid and xanthates. Copper sulphate. (See also p. 851.)

Agents for differential flotation. See Sec. 12, Arts. 16 and 19–24.

Agents for oxide flotation. See Sec. 12, Art. 25.

Grade of concentrate can ordinarily be improved, if poor, by decreasing the rate of froth overflow. This may be accomplished by decreasing aeration, diluting the pulp, using less or a less-powerful frothing agent, by eliminating an agent that stiffens the froth, by choosing an agent that is more highly selective, or by the use of dispersing agents such as sulphuric acid, lime, and certain inorganic salts.

Recovery will usually be increased by increasing the rate of froth overflow. This may be done by increasing aeration, increasing the percentage of solids, using more frothing agent or a more powerful frothing agent, or adding a froth stiffener. Recovery may also be increased by increasing selection, either by the use of a different collecting agent or by adding a dispersion agent.

Tests for a method of flotation are in no way fundamentally different from tests for amenability; they differ only in that they are more thorough and exhaustive and should, ordinarily, be carried out on a larger scale. They should start with knowledge of amenability and of one method of treatment, and should comprise investigation of the effect of changes in flotation process, in grinding, dilution, reagents, temperature, water supply, etc., on recovery and grade of concentrate. The following facts should be kept in mind in this testing campaign:

(*a*) In the laboratory the agitation-froth process is easier to control and tests are more quickly run than by the pneumatic process.

(*b*) An ore that can be successfully concentrated by flotation in an agita-

tion-froth machine can be successfully concentrated, with certain changes in the accessory details of operation, in a pneumatic machine, and *vice versa*.

(*c*) Before a mill is built the process worked out in the laboratory should be tried out on something approximating a mill scale in a test plant.

(*d*) The flotation process operates most easily and with greatest leeway on a pulp containing from 15 to 20 per cent. solids. On the other hand, power consumption, mill equipment and reagent consumption are lessened as the percentage of solids in the pulp is increased.

(*e*) A change of reagents in an operating mill may be a serious matter, involving considerable laboratory experimental work and costly interference with mill operation. Hence the reagent chosen should be one of which a supply at a fair price is reasonably assured, and the reagents tried in the testing work should be of this class. The important members of the class are: pine oil, coal tar, coal-tar creosote, wood tar, wood-tar creosote, petroleum residuum, the low-grade kerosene commonly known as stove oil, thiocarbanilid, sodium and potassium xanthates, sodium sulphide, alkaline cyanides, sodium carbonate and bicarbonate, sulphuric acid, zinc and copper sulphates, xylidin, orthotoluidin. All of these substances are available at fair prices and in good supply. Certain other substances may be locally abundant and their use, temporarily at least, may be justifiable on that score, but a suitable flotation agent consisting of one or more of the above-mentioned substances should be determined and the best available supply investigated against the time when the supply of the local substance is exhausted.

Testing flotation agents. This is probably the most common type of testing at an operating plant. The first step is, of course, to find by actual test, first in the laboratory and then in the mill, whether the new agent will produce satisfactory recovery and grade of concentrate with the mill feed. In this investigation it is essential that the conditions of contemplated use be duplicated. This is not always an easy matter. For example, if the mill-flotation feed is re-ground residue from gravity concentration and reclaimed water is used in the mill and the new reagent is to replace one or more of the reagents in regular use, it is substantially impossible to obtain a sample for laboratory testing that has the size and mineralogical composition of mill-flotation feed and is uncontaminated with existing reagent. Under such circumstances the best approximation to mill conditions will be attained by taking some of the original ore, concentrating it in the laboratory by gravity treatment patterned on the mill scheme, then grinding the residue in the laboratory to mill size. It is not safe to take a sample in the mill and dry it to drive off mill reagents. Such dried pulp will almost invariable float more poorly than properly prepared fresh pulp.

Assuming favorable flotation results from the laboratory tests, the questions of quantity required, supply and price should next be investigated. Unless the potential supply is large, the price is sure to advance with adoption of the agent by other companies, and deliveries may be difficult, with resulting interruption to mill operation. The probable effect of storage at the mill should be looked into; the behavior in the water-recovery system; and the effect on flotation of such part of the reagent as comes back with reclaimed water. Some reagents that have been suggested have unpleasant or dangerous physiological effects, *e.g.*, amyl acetate, valerianic acid, hydrogen sulphide, sulphur dioxide, and alkaline cyanides. This fact must be given due weight.

When a new reagent is proposed to be used in combination with old reagents, the laboratory investigation should include parallel runs in which every con-

dition is the same except that in one run the proposed new reagent is present and in the other it is absent. A considerable number of reagents have been patented that will not stand this test.

Testing flotation processes involves no different principles, in so far as determining suitability for mill operation is concerned, from those already outlined. If the process is a new one, it is well to bear in mind that the inventor may know little about it himself and that, unfortunately, he does not always put all of that little into his patent. As a result the early trials may be a succession of failures. The history of the agitation-froth process typifies the first contingency. The discovery was made in treating high-grade zinc-bearing tailing in Australia. Great difficulty was encountered five years later in applying the process to Butte zinc ores, and its subsequent application to low-grade copper ores was established as the result of literally thousands of trials by mill operators in the face of statements by representatives of the patentees that their process was not suited to the treatment of such ores.

When testing is directed toward establishment of the physical and chemical phenomena underlying a flotation process, actual operation of the process is of little avail. The essential phenomena involved lie in the field of molecular physics and chemistry, and, in the operation of the complete process, are so complex and masked that it is impossible to segregate and observe them. Such testing requires resources both of equipment and personnel different from those available at a plant laboratory. The experiments usually bear but little apparent resemblance to the process. The danger to guard against is that of overlooking, in a simple experiment that involves only one of the elements, the effect of the simultaneous action of the other elements.

Oil testing. The usual purpose of oil testing in flotation work is to determine that the oil in question is similar in physical properties and, therefore, probably in its behavior in the flotation cell to a given prior shipment, or in order to set specifications for flotation oil purchase. For details see *Manual of flotation processes*. For results of tests on a number of oils see Sec. 12, Tables 15 and 16.

Interpretation of flotation-test results. Translation of laboratory results into terms of mill-scale operations is, in the usual case, less difficult in flotation than in gravity concentration, and in all cases more certain than where chemical reactions such as occur in leaching and precipitation operations are concerned. Any flotation result that can be obtained in a laboratory machine can be obtained in mill operation, if the essential laboratory conditions are duplicated. The converse of this statement is also true, except that the mill-sized machine is capable of handling a somewhat coarser feed than can be handled in the laboratory machine. Considering the essential elements of pulp treatment in detail, the translation from laboratory results to mill results will be as follows:

Average size of feed may be slightly coarser in the mill than in the laboratory or, if the grinding in the mill is carried to the same extent as in the laboratory, a somewhat better result, other conditions being equal, may be expected in the mill than in the laboratory.

Water may make a considerable difference between laboratory results and mill results and this difference may be either in favor of or to the detriment of the mill. The former will ordinarily be the case if a portion of the mill water is reclaimed and reused. Under these circumstances it will ordinarily be found that the flotation agent brought back by the mill water will lessen, to a considerable extent, the amount of new flotation agent that it is necessary to add,

and that froth will be more easily obtained with this reclaimed water mixed in. If, however, there is any considerable amount of soluble salts in the ore, or if the settling ponds are of considerable area and in an arid region and there is any considerable amount of dissolved solids in the new water, then the salts in the water may have a harmful effect on flotation.

Flotation agents in the mill will be the same as in the laboratory except that it will generally be possible in the mill to lessen, to some extent, the proportion of so-called frothing oil in the mixture.

The peripheral speed of the agitators in agitation-type machines may, in general, be somewhat less in the mill than in the laboratory.

The air consumption per cubic foot of pulp treated in pneumatic machines will usually be less in the mill than in the laboratory. The pressure on the under side of the blanket will be necessarily higher in the mill machine than in the laboratory machines described, on account of the greater head on the pulp side of the blanket.

Time of treatment necessary in the mill will be very closely the same for a given recovery and grade of concentrate as in the laboratory. The grade of final concentrate obtained in the mill will be close to that obtained in the laboratory. The recovery will come close to the indicated extraction calculated by the formula (10), p. 1236, from laboratory results, if, in the calculation, the figure for grade of concentrate is that obtained from the cleaner operation, the figure for rougher tailing is that obtained from the rougher operation, and the middling or cleaner tailing obtained in the laboratory is disregarded, provided that the grade of this middling product is not more than twice the grade of the original heads, and that the mineralogical character of the middling is not markedly different from that of the original feed.

Table 15 (*56 A 676*) shows comparison between laboratory tests and mill results. The first three columns compare mill results and two different laboratory results in miniature

Table 15. Comparison of laboratory tests with large-scale operations, Consolidated Nevada-Utah Corporation

	Mill results August 14, 1916	Miniature machine on feed, August 14, 1916, Salt Lake City water	Miniature machine results with head sample and mine water, August 14, 1916	5-ton laboratory test of April 15, 1916	Miniature results prior to 5-ton laboratory tests
Assay of feed, Zn per cent	17.20	17.56	17.56	17.54	17.53
Assay of total zinc concentrates, Zn per cent	41.80	40.48	40.17	42.41	41.91
Assay of total tailing, Zn per cent	5.60	4.89	4.89	3.64	4.71
Ratio of concentration	....	2.91	2.81	2.87	2.94
Recovery of Zn in zinc concentrate	77.80	78.58	79.07	84.12	81.48
Assay of all table zinc concentrate, Zn per cent	42.60	40.63	40.63	41.32	41.77
Assay of flotation zinc concentrate, Zn per cent	37.00	39.80	38.52	46.77	43.19
Assay of zinc in total iron concentrate, Zn per cent	10.10	14.30	....	12.88	10.06

machines on the same material. The last two columns compare miniature-machine (1000-gm.) results with 5-ton laboratory-machine results on the same sample. The discrepancy between the April and August tests is no more than is to be expected from different lots of ore, and the difference in miniature results on the two dates compared with difference between miniature results and larger-scale results on the same date, indicates that the discrepancy on the different dates is rather due to the ore than to the difference in scale of operation.

Mill tests. It cannot be too strongly urged that before a mill is erected, some testing work be done on mill-sized flotation machinery. This work should be done in a test mill at the mine, on ore whose prior handling corresponds as closely as possible with the scheme to be followed in the finished mill, and the water used should be as near as possible of the character of the water that is to be used in the operating plant. If such a test does no more than confirm the laboratory results, it will pay for itself in the information that it gives concerning mill operation on the ore and it may be that the test will bring up conditions which were overlooked in the laboratory testing work. Some of the equipment used in such a test can ordinarily be utilized in the final plant so that it need not all be charged against the testing work.

At Utah Copper Co. (Nov. 29, 1916) with Janney mechanical-air cells in the mill, a sample of mill pulp was taken and tested in the laboratory Janney mechanical machine. The mill machines averaged 0.120 per cent. Cu for the day, the laboratory-machine rougher tailing was 0.130 per cent. Cleaner concentrate was about 26 per cent. Cu in both places.

12. Magnetic and electrostatic concentration

For small-scale tests for permeability, to remove iron introduced in grinding small samples, or to remove magnetite from pan concentrate and the like, a small electromagnet such as that shown in Fig. 29 is satisfactory.

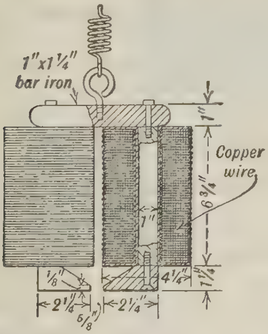

Fig. 29.—Electromagnet for laboratory testing (*after Richards*).

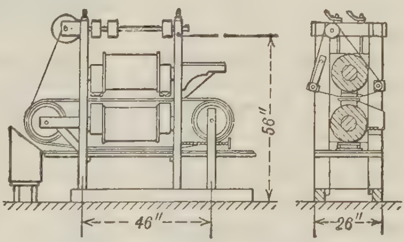

Fig. 30.—Laboratory-size Wetherill separator.

Cores and pole-pieces should be made of soft iron. According to *Richards*, the magnet shown, when wound with 5000 ft. of No. 21 cotton-covered copper wire on each pole, making a total of 6760 turns, carries a maximum of 0.8 amp. at 50 volts without undue heating. For continuous tests Richards recommends the small Wetherill-type machine shown in Fig. 30. Each magnet carries 100,000 ampere turns and with proper rheostat control can be used to treat minerals of both low and high permeability.

Electrostatic machine for laboratory testing is made by Huff Electrostatic Separator Co. to sell at about $1500. This apparatus is not justified in the ordinary testing laboratory.

Hydrometallurgy. See Sec. 15.

Sample dryers. A useful form for large samples is shown in Fig. 31. It consists of a large 3-walled rectangular pan with a steam chamber for a bottom, all mounted on a suitable framework at a convenient distance from the floor. The steam drying rack (Fig. 32) and gas dryer (Fig. 33) are suitable for smaller samples. An enclosing cupboard with good ventilation is essential to the best performance of both of these apparatus.

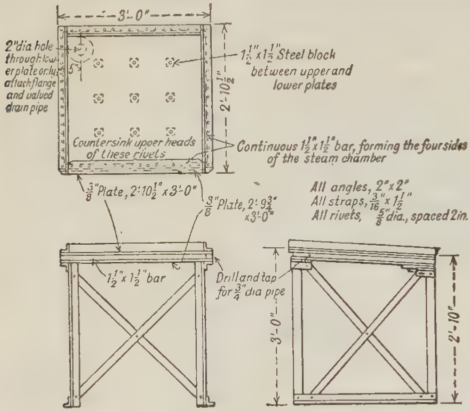

FIG. 31.—Steam drying plate.

13. Ore-dressing laboratory

General. The most desirable arrangement is a separate building with railroad spur for delivery of heavy machinery and ore samples and river, storm-sewer connection or the like for disposal of finely ground waste. The spur track should preferably approach the building at a sufficient elevation to permit gravity unloading of bulk-ore samples into a receiving bin and the unloading platform should be served by a crane that serves the milling room also. The building proper should consist of a large milling room and a number of small rooms for microscopic work, assaying, flotation testing, screen analysis, shop, small-sample storage, junk pile, etc. The best arrangement for the milling room is a large light room with high roof, preferably having, at one end or side, a strong skeleton framework with three or four floors at 8- to 10-ft. intervals, stepped back sufficiently to permit ready crane delivery, and a gallery around the rest of the room. The whole room should be crane-served. All but the heaviest apparatus should be set up as a self-contained portable unit, capable of being picked up by the crane and set down in the desired position on the floor or on the skeleton frame for use, and thereafter returned

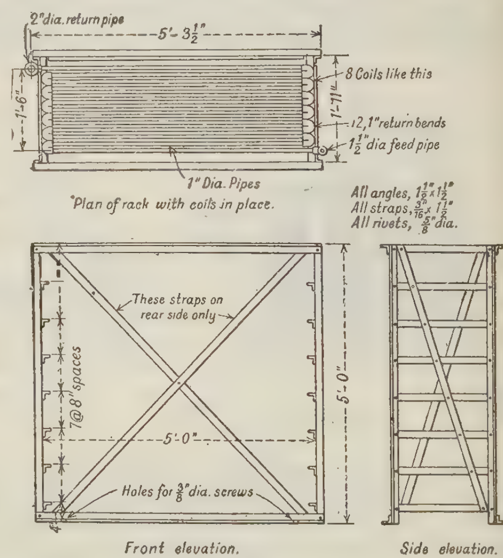

FIG. 32.—Steam drying rack.

to a place on the gallery for storage. The feed bins should discharge by feeders to a screen, oversize to a jaw crusher, undersize and crusher product to a conveyor and thence to a screen, oversize to a gyratory, undersize and crusher product to a conveyor to a screen, oversize to rolls, undersize and roll product to a bucket elevator to another screen returning oversize to the rolls and undersize to a compartmented fine-ore storage bin. The ideal condition would be to have each of the preceding crushers and screens so placed that it may be picked up and set down in the main milling room for individual test, but if this is not possible the arrangement must be such that each crusher is capable of individual test for capacity, power consumption and size of feed and product. There should be a bucket elevator in the main milling room delivering to the top of the skeleton frame and a battery of centrifugal pumps piped to deliver to any desired level of this frame. The fine-ore storage bins should be arranged for delivery to the elevator boot or to a rod or ball mill and the latter should, of course, be arranged for closed-circuit grinding, if in permanent locations, with finished product going to one of the centrifugal pumps. Set-ups for substantially any concentration or hydrometallurgical process with gravity flow of pulp can be made on the skeleton framework. This framework should be floored with removable slatted wooden sections about 2 ft. wide by 5 ft. long built up of 2 × 4-in. yellow pine. A dust-collecting system with suitable inlets at each permanent dry-crushing or screening-location and a service line into which temporary inlets may be connected is desirable although not essential. There should be a large sump in the floor of the main room, toward which the floor drains from all directions, with sufficient capacity to hold all excess water from a one- or two-hour concentrating run involving wet screening, hydraulic classification, jigging and tabling. The sump should discharge to waste by gravity either from bottom outlets or by decantation. A centrifugal pump should be provided to return water from the sump to the water-feed tank. This should be a tank about 6-ft. diameter by 6 ft. deep, located at the highest point of the laboratory, fed by the new-water system and by the sump pump, discharged into the laboratory-supply line by an outlet placed about 6 in. above the bottom, and having an overflow pipe of generous dimensions returning to the sump, a 1-in. tell-tale line returning with the overflow, and a small drain line from the center of a slightly-dished bottom. The water line from this supply tank should be of 3- or 4-in. diameter running around the walls of the laboratory, with permanent branch lines at strategic points and numerous plugged tees and valved outlets, a part of which should be fitted with ¾-in. and others with 1½-in. male hose bibbs. There should be a half dozen or more portable cylindrical tanks, 6- and 8-ft. diameter by 2 ft. high, of heavy galvanized-iron sheet, fitted with several 1½-in. outlets through floor flanges at one place on the rim and one such opening on the wall at the bottom. These are useful for collecting and dewatering sand products, or for holding dewatered slimes, and to keep solid matter out of the sump.

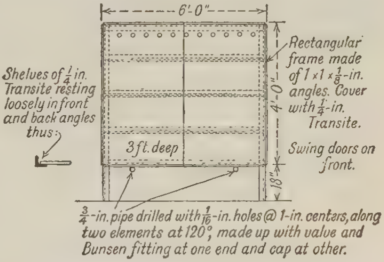

Fig. 33.—Gas dryer.

Cleaning up. All apparatus should be capable of complete and ready clean-up. Detail of the necessary arrangement to secure this end will vary with different apparatus, but in general all tanks and hoppers should have sloping bottoms where possible, all machines should be so placed that the material left-in-process can be washed or brushed into receptacles set under them, and all apparatus should be so designed that the amount of material left therein at the end of the run is a minimum.

Sampling and weighing equipment. Mechanical sampler for dry ore. Platform scales weighing in metric and common units to about 1000 lb., accurate to 0.5 lb. Druggist's trip scale weighing in metric and common units to 10 lb. with slide scale graduated to 10 gm. and ¼ oz., sensitive to 0.5 gm. Pulp balance weighing to 250 gm. and sensitive to 0.1 gm. Analytical balance. Suitable weights for the preceding. Square-edge D-handle shovels. Cross for cone-and-quarter sampling. Jones riffle with about 16 @ 1-in. chutes and another with about 16 @ 0.5-in. chutes. (The riffles should have an even number of chutes.) Galvanized ash cans, coal hods and pails. A cement sampling floor free from cracks or two @ ¼-in. steel sheets about 6 × 6 ft. Floor brush. Coffee mill. Disc pulverizer.

Assay laboratory with full equipment of apparatus and reagents for both wet and dry assaying. See a good book on assaying (p. 1181) for a check list. There should be gas-fired pot and muffle furnaces that will serve also as roasting furnaces for testing in magnetic concentration and hydrometallurgical work.

Microscope equipment. Hand lens, 10 × to 15 ×. Binocular microscope with interchangeable open-foot and stage mounting, 25-, 40- and 55-mm. objectives and 6 × and 10 × oculars. Petrographic microscope. Metallurgical microscope. Grinding and polishing laps. Mounting equipment. Apparatus and chemicals for microchemical work. See Chamot, Murdock, Davy and Farnham, Johannsen, p. 1192.

Equipment for sizing tests. Set of 8-in. Tyler standard sieve-scale testing screens to $\sqrt[4]{2}$ ratio up to 1.050 in. (see Table 1) and square-mesh wire screens in wooden frames about 18-in. square on approximately $\sqrt{2}$ scale from 1.05-in. to 4-in. Testing-sieve shaker. Sample pans, 2, 4, 6, 8 and 12 in. diameter (patty tins and pudding tins will serve). Scoops. Counter brushes. Round assay-button brushes. 1-in. flat camel's-hair brushes. Glazed paper. Paper bags and sample envelopes. Eye-piece and stage micrometers for the microscopes (see Microscopic sizing, Art. 4). Elutriation apparatus (see Art. 2).

Crushing equipment. Jaw crusher, Blake type; about 10 × 7 in. No. O gyratory crusher (see Sec. 3, Table 12). Crushing rolls, 12 to 16 × 10-in. Ball mill, not less than 4-ft. diameter, if feed is to be more than ½-in. maximum size; in no case less than 3-ft. diameter. Rod mill, preferably 24-in. diameter by 5 or 6 ft. long. This can also be used as a tube mill. Batch ball or pebble mill. The large crushing and grinding machines should be driven in such a way that reasonably accurate power readings can be obtained.

Grading equipment. Vibrating screen with good assortment of screen cloth. Glass classifiers, both free- and hindered-settling. Tank-type laboratory-size hindered-settling classifier. Mechanical classifiers of suitable size for the ball and rod mills. Diaphragm cones for feeding concentrating tables. 6 × 6-ft. Dorr thickener with diaphragm-pump discharge. Filter leaf. Drying rack.

Concentrators. Hand-jig sieves. Gold pan. Vanning plaque. Galvanized wash tubs. Laboratory-size Harz jig. Small shaking table (12 × 30-in. deck) with interchangeable decks and Wilfley, Garfield and Butchart riffling. Quarter-size shaking tables for sands. Slime table. Experimental film sizer. Miniature agitation-froth and pneumatic flotation machines. Larger flotation machines of about ¼ ton per hr. capacity. Blower. Electro-magnet. Dry-separating table.

Handling equipment. Trucks, conveyors, elevators for dry material and centrifugal pumps for fine wet material.

Feeders. Bins should be provided with automatic feeders with variable-speed control and movement-recording apparatus. Belt feeders with ratchet-and-pawl drive and revolution counters on the head shaft are most satisfactory. Self-contained portable vibrating or shaking-tray feeders mounted with small feed hoppers are best for feeding individual machines with dry or moist sands. An inclined trough along which a weighed quantity of dry or moist sand or slime is uniformly spread and from which a given length (weight) of charge is washed out per minute with a constant stream of water is the most satisfactory wet feeder for the laboratory. Travel of the feed-water stream may be made automatic by mounting the nozzle on a mechanically-driven screw.

Miscellaneous. Good equipment of tools for simple carpentry, plumbing, tin-smithing and machine work. Racks for storage of material and small samples. Room for barrel or bin storage of larger samples. Room for a junk pile.

14. Metallurgical calculations

Computations of performance in milling are complicated by the fact that it is a continuous rather than a batch operation, that the quantities of solid materials handled are large and usually mixed with water, hence difficult or impossible to weigh. Fortunately it is possible to determine many facts concerning performance without knowing weights, if the value of various constituents is known in some common unit, *e.g.*, the content of some particular metal or mineral, or of water, or of particles of a particular size or falling within some particular size range, or the like. Metal content may be expressed in per cent. by weight or volume, or in units of weight as oz. per ton, or even in units of value, as in dollars per ton, provided, in the latter case, that the value is in direct proportion to the metal content and not an artificial value dependent both upon metal content and lack of other content, as is frequently the case in valuation of concentrate.

Definitions and notation. The following notation and definitions obtain throughout all of the calculations.

C = weight of concentrate, expressed in any units, but necessarily in the same units as F. CONCENTRATE may be defined, for the purpose of these calculations, as any product of the treatment of a given feed that is richer in content of a given ingredient than the feed.

c = Assay of concentrate. See notes to f.

c_a, c_b, c_c. See notes to f_a, substituting the word "concentrate" for "feed."

E = EFFICIENCY. This term is usually applied to a two-product classifier operation. So applied, it indicates the ratio of the weight of classified material in the overflow to the weight of classifiable material in the feed. See development of formula (38) for further discussion.

F = weight of feed. This may be expressed in any units. The word FEED applies to the material entering treatment in any machine or operation, *e.g.*, the ore entering a treatment plant, or the material entering an individual machine.

f = assay of feed. As stated in the introductory paragraph, this assay may be given in any units, *e.g.*, per cent. Pb, oz. gold per ton, lb. Cu per ton; dollars per ton, when the value is directly proportional to the weight of a given constituent, say gold or silver, but not when the value is the combined value of the gold and silver, unless the ratio of weight of gold to silver is the same in all products as in the feed; per cent. — 1-mm. material, per cent. moisture, per cent. ash (in coal), etc.

f_a, f_b, f_c, etc. When the feed contains more than one ingredient of value, assays are made for each, and these are expressed separately in the formulas, distinguished by the subscripts. Thus, in an ore containing zinc, lead, and silver, f_a might be chosen to represent the per cent. zinc in the feed, f_b the per cent. lead, and f_c the oz. silver per ton of feed.

K = RATIO OF CONCENTRATION. This term is defined as the ratio of the weight of the feed in a given operation to the weight of concentrate obtained from it; or, stated another way, as the number of tons of feed required to produce one ton of concentrate.

M = weight of middling, expressed in any unit, but necessarily in the same unit as F. MIDDLING is defined, for the purpose of these calculations, as that product of the treatment of a given feed whose content of a given ingredient lies between that of the tailing and that of the concentrate.

m = assay of middling. See notes to f.

m_a, m_b, m_c. See notes to f_a, substituting the word "middling" for "feed."

R. RECOVERY is the ratio, expressed as percentage, of the weight of the sought-for ingredient, in the finished product of a given operation, to the weight of the same ingredient in the material entering the operation. Thus if there are 20 lb. of copper per ton in the material entering a given mill and 19 lb. are obtained in the form of concentrate from each ton entering, the recovery, $R = 19/20 = 95$ per cent.

The terms EXTRACTION, INDICATED EXTRACTION, ESTIMATED EXTRACTION and ACTUAL EXTRACTION are sometimes used to indicate recovery. The terminology is so confused that the meaning of none of the terms can be safely assumed without indication of the method of determination. Some writers have attempted to distinguish between extraction and recovery, making the former term mean the value of R as calculated from assays of feed and products (see Formulas 10, 24, 34 and 35) while recovery indicates the ratio of the weight of metal in concentrate actually recovered (= actual weight of concentrate × assay of concentrate) to the actual weight of metal in the feed (= actual weight of feed

× assay of feed). Other writers use the term indicated extraction or estimated extraction to signify R from assays alone and actual extraction to signify the percentage of the total metal fed that is actually recovered in concentrate. In this book the word recovery is used generally to describe the result by either method of calculation and the term ACTUAL RECOVERY to distinguish, when necessary, the case in which calculation is based on actual weights.

T = weight of tailing, expressed in any unit, but necessarily in the same unit as F. TAILING is defined, for the purpose of these calculations, as that product of the treatment of a given feed which is distinctly impoverished in content of a given ingredient as compared to the feed.

t = assay of tailing. See notes to f.

t_a, t_b, t_c. See notes to f_a, substituting the word "tailing" for "feed."

15. Two-product formulas

The simplest case is that in which two products only, *viz.:* concentrate and tailing, are made from the treatment of a given feed. Under such circumstances:

$$F = C + T, \quad (1)$$

$$Ff = Cc + Tt. \quad (2)$$

Multiply equation (1) by t, eliminate T, and

$$C = F(f - t)/(c - t), \quad (3)$$

$$F = C(c - t)/(f - t), \quad (4)$$

By similar manipulation,

$$T = F(c - f)/(c - t), \quad (5)$$

$$F = T(c - t)/(c - f), \quad (6)$$

$$T = C(c - f)/(f - t), \quad (7)$$

$$C = T(f - t)/(c - f). \quad (8)$$

By definition, $K = F/C$, whence, from equation (3),

$$K = (c - t)/(f - t). \quad (9)$$

By definition, $R = 100Cc/Ff$, whence from equation (3),

$$R = 100c(f - t)/f(c - t). \quad (10)$$

16. Three products, one metal

When three products are made and assays are reported in terms of one ingredient only, if no weights are known, determinations of recovery and ratio of concentration are strictly indeterminate problems. If enough is known with respect to the performance of the ore to justify an assumption as to the effect of middling re-treatment on the assays of final concentrate and tailing, correction of concentrate and tailing assays may be made on the basis of the assumption, and the preceding formulas may then be applied. For an example, see Art. 7. If the assumption be made that re-treatment of middling will result in distribution of the same into concentrate and tailing of the same assays as the corresponding products made in the operation that pro-

duced the middling, then if X and Y represent the final weights of concentrate and tailing respectively from the original and re-treatment operations,

$$C + M + T = X + Y, \qquad (11)$$

and

$$Cc + Mm + Tt = Xc + Yt. \qquad (12)$$

Eliminating Y,

$$X = [C(c - t) + M(m - t)]/(c - t) \qquad (13)$$

and

$$Y = 1 - X = [(c - t)(1 - C) - M(m - t)]/(c - t) \qquad (14)$$

Equations (3) and (5) will, however, give the same result.

If two of the weights F, C, M, T are known in addition to the assays, the other weights and the recovery and ratio of concentration may be calculated by manipulation of the two following equations:

$$F = C + M + T \quad (15) \qquad \text{and} \qquad Ff = Cc + Mm + Tt \quad (16)$$

The resulting equations are:

$$C = \frac{F(f-t) - M(m-t)}{(c - t)} = \frac{T(m-t) - F(m-f)}{(c - m)} = \frac{T(f-t) - M(m-f)}{(c - f)}; \qquad (17)$$

$$M = \frac{F(f-t) - C(c-t)}{(m - t)} = \frac{F(c-f) - T(c-t)}{(c - m)} = \frac{T(f-t) - C(c-f)}{(m - f)}; \qquad (18)$$

$$F = \frac{C(c-t) + M(m-t)}{(f - t)} = \frac{T(m-t) - C(c-m)}{(m - f)} = \frac{M(c-m) + T(c-t)}{(c - f)}; \qquad (19)$$

$$T = \frac{C(c-m) + F(m-f)}{(m - t)} = \frac{F(c-f) - M(c-m)}{(c - t)} = \frac{C(c-f) + M(m-f)}{(f - t)}. \qquad (20)$$

$$c = \frac{f - Mm - Tt}{1 - M - T}, \qquad (21)$$

$$m = \frac{f - Cc - Tt}{1 - C - T}, \qquad (22)$$

$$t = \frac{f - Cc - Mm}{1 - C - M}. \qquad (23)$$

$$R = 100\left[\frac{c(f - t)}{f(c - t)} - \frac{Mc(m - t)}{Ff(c - t)}\right] = 100\left[\frac{c(m - f)}{f(c - m)} - \frac{Tc(m - t)}{Ff(c - m)}\right]$$

$$= 100\left(1 - \frac{Tt}{f}\right) = 100\left[\frac{Cc(f - t) + Mm(f - t)}{Cf(c - t) + Mf(m - t)}\right]. \qquad (24)$$

$$K = \frac{F(c - t)}{F(f - t) - M(m - t)} = \frac{F(c - m)}{T(m - t) - F(m - f)}$$

$$= \frac{C(c - t) + M(m - t)}{C(f - t) + M(f - t)} \qquad (25)$$

Other equations for K and R may be written by manipulation of equations 15 to 20, but the simpler procedure is to solve for weights of feed and concentrate, after which solutions of K and R are obtainable from the definition equations.

17. Three-product formulas

(For 3-product formulas involving one ingredient only, see Art. 16.) When a feed containing, say, metal "a" and metal "b" is so treated as to make three products, *e.g.*, concentrate rich in metal "a," another concentrate (or middling) rich in metal "b," and a tailing impoverished in both "a" and "b," equations may be written that express the recoveries and ratios of concentration in terms of assays alone, and the weights of the different products in terms of assays and the weight of the feed.

Thus:

$$F = C + M + T, \qquad (26)$$

$$Ff_a = Cc_a + Mm_a + Tt_a, \qquad (27)$$

$$Ff_b = Cc_b + Mm_b + Tt_b. \qquad (28)$$

Then, from the solution of these simultaneous equations (most readily made by the method of determinants),

$$C = F\left[\frac{(f_a - m_a)(m_b - t_b) - (f_b - m_b)(m_a - t_a)}{(c_a - m_a)(m_b - t_b) - (c_b - m_b)(m_a - t_a)}\right]; \qquad (29)$$

$$M = F\left[\frac{(c_a - f_a)(f_b - t_b) - (c_b - f_b)(f_a - t_a)}{(c_a - m_a)(m_b - t_b) - (c_b - m_b)(m_a - t_a)}\right]; \qquad (30)$$

$$T = F\left[\frac{(c_a - m_a)(m_b - f_b) - (c_b - m_b)(m_a - f_a)}{(c_a - m_a)(m_b - t_b) - (c_b - m_b)(m_a - t_a)}\right]; \qquad (31)$$

where C = weight of concentrate rich in metal "a" and M = weight of concentrate rich in metal "b."

Ratio of concentration with respect to metal "a" is, by definition, $K_a = F/C$. Substituting in this equation the value of C from equation (29):

$$K_a = \frac{(c_a - m_a)(m_b - t_b) - (c_b - m_b)(m_a - t_a)}{(f_a - m_a)(m_b - t_b) - (f_b - m_b)(m_a - t_a)}. \qquad (32)$$

Similarly, $K_b = F/M$, and from equation (30):

$$K_b = \frac{(c_a - m_a)(m_b - t_b) - (c_b - m_b)(m_a - t_a)}{(c_a - f_a)(f_b - t_b) - (c_b - f_b)(f_a - t_a)}. \qquad (33)$$

Recovery of metal "a" in C is given by the equation $R_a = 100\,Cc_a/Ff_a$. Substituting in this equation the value of C from equation (29)

$$R_a = \frac{100\,c_a[(f_a - m_a)(m_b - t_b) - (f_b - m_b)(m_a - t_a)]}{f_a[(c_a - m_a)(m_b - t_b) - (c_b - m_b)(m_a - t_a)]} \qquad (34)$$

Similarly, $R_b = 100\,Mm_b/Ff_b$ and from equation (30)

$$R_b = \frac{100\,m_b[(c_a - f_a)(f_b - t_b) - (c_b - f_b)(f_a - t_a)]}{f_b[(c_a - m_a)(m_b - t_b) - (c_b - m_b)(m_a - t_a)]} \qquad (35)$$

Instead of working from the formulas, the weights of the products may be solved for by writing equations (26), (27) and (28) with the assay values written in and solving directly by determinants.

Example. Given assays as follows: Feed, 7.7 per cent. Pb and 11.9 per cent. Zn; lead concentrate, 50 per cent. Pb and 5 per cent. Zn; zinc concentrate, 50 per cent. Zn and 10 per cent. Pb; tailing, 1 per cent. Pb and 2 per cent. Zn. Write three simultaneous equations as follows:

Weights: $F = C + M + T$ (C = lead conc., M = zinc conc.)
Lead assays: $7.7\ F = 50\ C + 10\ M + T$
Zinc assays: $11.9\ F = 5\ C + 50\ M + 2T$.

Write and solve the determinant for C (see Sec. 24, Art. 14):

$$C = \frac{\begin{vmatrix} 1 & 1 & 1 \\ 7.7 & 10 & 1 \\ 11.9 & 50 & 2 \end{vmatrix}}{\begin{vmatrix} 1 & 1 & 1 \\ 50 & 10 & 1 \\ 5 & 50 & 2 \end{vmatrix}} = \frac{\begin{vmatrix} 2.3 & -9 \\ 38.1 & -48 \end{vmatrix}}{\begin{vmatrix} -40 & -9 \\ 45 & -48 \end{vmatrix}} = \frac{232.5}{2325} = 0.10.$$

Similarly write and solve the determinant for M:

$$M = \frac{\begin{vmatrix} 1 & 1 & 1 \\ 7.7 & 50 & 1 \\ 11.9 & 5 & 2 \end{vmatrix}}{\begin{vmatrix} 1 & 1 & 1 \\ 10 & 50 & 1 \\ 50 & 5 & 2 \end{vmatrix}} = \frac{\begin{vmatrix} 42.3 & -49 \\ -6.9 & -3 \end{vmatrix}}{\begin{vmatrix} 40 & -49 \\ -45 & -3 \end{vmatrix}} = \frac{-465}{-2325} = 0.20.$$

Since $F = 1$, $T = 1 - (0.10 + 0.20) = 0.7$

Recovery of lead in lead concentrate $= \dfrac{0.1 \times 50}{1 \times 7.7} = 64.9$ per cent.

Recovery of zinc in zinc concentrate $= \dfrac{0.2 \times 50}{1 \times 11.9} = 84.1$ per cent.

Ratio of concentration: lead $= F/C = 1/0.1 = 10$; zinc $= F/M = 1/0.2 = 5$.

18. N-product formulas

When any number of products, say n, are made, formulas giving the weights of each in terms of the weight of feed, the recovery, and the ratio of concentration, may be written by the method illustrated in the preceding article, and these formulas may be solved provided accurate assays of the feed and all of the products are given in $n - 1$ independent ways. The formulas, however, become so long and involved that it is usually simpler to write the simultaneous equations with the numerical values inserted and solve for the percentage weights by determinants, as illustrated in the preceding paragraph. Thus for four products C, S, M, T whose corresponding assays in three metals a, b and c are c_a, c_b, c_c; s_a, s_b, s_c; m_a, m_b, m_c; t_a, t_b, t_c; made from a feed F whose assay is f_a, f_b, f_c, the formula for C is

$$C = F\left[\frac{\left\{\begin{aligned}&(s_c-t_c)(m_a-f_a)(t_b-m_b)-(t_a-m_a)(s_c-t_c)(m_b-f_b)\\ &\quad+(m_b-f_b)(t_c-m_c)(s_a-t_a)-(m_a-f_a)(t_c-m_c)(s_b-t_b)\\ &\quad+(t_a-m_a)(s_b-t_b)(m_c-f_c)-(t_b-m_b)(s_a-t_a)(m_c-f_c)\end{aligned}\right\}}{\left\{\begin{aligned}&(s_c-t_c)(m_a-c_a)(t_b-m_b)-(t_a-m_a)(s_c-t_c)(m_b-c_b)\\ &\quad+(m_b-c_b)(t_c-m_c)(s_a-t_a)-(m_a-c_a)(t_c-m_c)(s_b-t_b)\\ &\quad+(t_a-m_a)(s_b-t_b)(m_c-c_c)-(t_b-m_b)(s_a-t_a)(m_c-c_c)\end{aligned}\right\}}\right]. \qquad (35)$$

The formulas for S, M and T may be written from equation (36) by symmetry, *e.g.*, $S = F[X/Y]$ in which X is obtained from the numerator of (36) by substituting f for s, wherever the latter appears and substituting c in place of f. Thus the first term of the numerator becomes $(f_c - t_c)(m_a - c_a)\ (t_b - m_b)$.

By similar substitution, the first term in the formula for M, is $(s_c - t_c)$ $(f_a - c_a)\ (t_b - f_b)$, and for T it is $(s_c - f_c)\ (m_a - c_a)\ (f_b - m_b)$. Y is the same as the denominator of equation (36) in all cases.

Example of the use of determinants for a 4-product problem follows: Given assays as in Table 16, write the equations in the following form:

Table 16. Specimen assays for 4-product determinant solution

	Per cent. Pb	Per cent. Zn	Per cent. Cu
Feed	7.1	5.7	2.26
Lead concentrate	60	1	1
Zinc concentrate	1	40	1
Copper concentrate	2	2	10
Tailing	1	2	0.1

	Feed		Lead conc.		Zinc conc.		Copper conc.		Tailing
Weight equation:	F	=	C	+	S	+	M	+	T
Lead equation:	7.1 F	=	60 C	+	S	+	2 M	+	T
Zinc equation	5.7 F	=	C	+	40 S	+	2 M	+	2 T
Copper equation:	2.26 F	=	C	+	S	+	10 M	+	0.1 T

Then

$$C = \frac{\begin{vmatrix} 1 & 1 & 1 & 1 \\ 7.1 & 1 & 2 & 1 \\ 5.7 & 40 & 2 & 2 \\ 2.26 & 1 & 10 & 0.1 \end{vmatrix}}{\begin{vmatrix} 1 & 1 & 1 & 1 \\ 60 & 1 & 2 & 1 \\ 1 & 40 & 2 & 2 \\ 1 & 1 & 10 & 0.1 \end{vmatrix}} = \frac{\begin{vmatrix} -6.1 & 1 & -1 \\ 34.3 & -38 & 0 \\ -1.26 & 9 & -9.9 \end{vmatrix}}{\begin{vmatrix} -59 & 1 & -1 \\ 39 & -38 & 0 \\ 0 & 9 & -9.9 \end{vmatrix}} = \frac{\begin{vmatrix} 197.5 & -38 \\ -53.64 & -0.9 \end{vmatrix}}{\begin{vmatrix} 2203 & -38 \\ -531 & -0.9 \end{vmatrix}} = \frac{2216.07}{22160.7} = 0.10$$

Similarly $S = 0.10$, $M = 0.20$ and $T = 0.60$. From these values the recovery of each metal in its respective concentrate and the respective ratios of concentration may be written.

Limitations of multi-product formulas. The formulas above given are theoretically correct, but the accuracy of the answers that they give is, of course, wholly dependent upon the accuracy of the sampling and assaying. The formulas for two-product treatment are not particularly sensitive to small errors in data or calculation, hence recovery and weight of concentrate should check smelter returns (or their equivalent). If they do not check, mill operation should be examined for spills, losses, hold-backs as in tanks, etc., and shipping and smelter sampling should be carefully analyzed.

The formulas for three or more products are more sensitive to small errors both in data and calculation, especially when one of the products is of relatively small weight and low assay.

Fry (*114 J 493*) cites an example of the effect of such errors. His actual assays and assumed erroneous assays together with weights of products calculated from both by determinants, using the four possible equations three at a time, are shown in Table 17. The

Table 17. Possible effect of small errors in assaying and sampling on calculated weights in 3-product concentration. (Assays from *Fry, 114 J 493*)

	Pb, per cent.	Zn, per cent.	Ag, ounces per ton	Calculated weights, per cent., Data used(a) WLZ	WLS	WZS	LZS
Actual assays:							
Feed	11.34	31.85	13.2	100.69	100.4	100.0	102.84
Lead concentrate	21.4	9.1	25.1	39.93	40.4	40.0	39.74
Zinc concentrate	5.4	56.2	6.2	50.9	50.0	50.0	49.68
Tailing	0.8	1.1	0.6	9.86	10.0	10.0	13.42
Possible erroneous assays:							
Feed	11.3	31.8	13.3	99.93	100.07	....	....
Lead concentrate	21.3	9.0	25.0	39.93	12.17	....	....
Zinc concentrate	5.4	56.0	6.3	50.2	172.2	....	....
Tailing	0.7	1.0	0.7	9.8	−84.3	....	....

a W weight equation. L Lead equation. Z Zinc equation. S Silver equation.

calculated weight of feed in the columns whose heading contains the letter *W* (which indicates that one of the simultaneous equations was $F = L + Z + T$) should, of course, be 100.0. The difference is due to slide-rule discrepancies, which, in some cases may become very important. The difference between the calculated weight and 100 in the last column measures the effect of both assays and slide rule. It will be noted that the errors that Fry assumed made but little difference in calculated weights in the case where the silver assay was not used, but that impossible results were obtained when the erroneous silver assays were included in the calculation.

Graphical solution. Fig. 34 shows a method, proposed by Fry, for a graphical solution designed to point out inaccuracies in sampling and assaying and at the same time average them, if they are not too large to be averaged. The chart is constructed as follows: Assume two values for the weight of tailing (T), one less than the probable, the other more. Using these assumed values, *e.g.*, in the case of the erroneous assays given, 5 and 15 per cent., write equations in L (weight of lead concentrate) as follows:

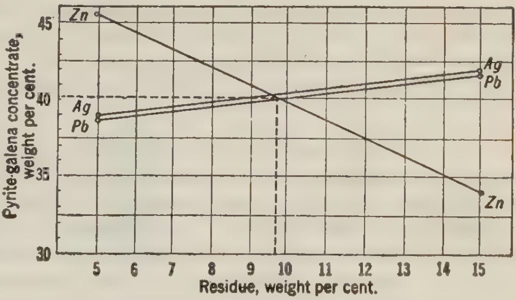

FIG. 34.—Graphical solution of 3-product problem.

Using the zinc assays,

$$31.8\ (100) = 9L + 56\ (95 - L) + \ (5)\ (1),\text{ from which } L = 45.6.$$
$$31.8\ (100) = 9L + 56\ (85 - L) + (15)\ (1),\text{ from which } L = 33.9.$$

Using the lead assays,

$$11.3\ (100) = 21.3L + 5.4\ (95 - L) + \ 5\ (0.7)\text{ from which } L = 38.6.$$
$$11.3\ (100) = 21.3L + 5.4\ (85 - L) + 15\ (0.7)\text{ from which } L = 41.6.$$

Using the silver assays,

$$13.3\ (100) = 25L + 6.3\ (95 - L) + \ 5\ (0.7)\text{ from which } L = 38.9.$$
$$13.3\ (100) = 25L + 6.3\ (85 - L) + 15\ (0.7)\text{ from which } L = 41.9.$$

Plot the assumed values of T as abscissas and the derived weights of L as ordinates and connect the corresponding values with straight lines. Read the mean of the two intersections as mean values of L and T. In this case L was read off as 40.1 and T as 9.65 which gave Z (weight of zinc concentrate) 50.25 by difference.

19. Applications of formulas

The most usual applications of the formulas are to control the operations of individual machines and of the whole mill and to check smelter returns. If feed is weighed in automatically and accurate moisture determinations are made, the calculated weight of concentrate over a period such as a month should check dry weight of concentrate shipped within reasonably close limits, and, over a period of a year should check very closely. Discrepancy in one direction only should be closely investigated; differences should be both positive and negative over a period of time.

In mills where the feed is not weighed, tailing should be time-sampled. Weights of concentrate and tailing may then be calculated and the former weight checked against the shipping weight.

As indicated in the introductory paragraph, the two-product formulas may be applied to investigations of the performance of screens and two-product classifiers. (See also Sec. 5, Art. 2.) In these cases the assays are usually screen analyses. The size split for statement of assays should be a screen that gives 5 per cent., or more if possible, as the minimum assay used. Ordinarily cumulative percentages on or through the key screen are chosen as the assays rather than individual percentages. For another formula for investigating classifier performance, see equation (38).

In laboratory testing three products are usually made, *viz.*: concentrate, middling and tailing. As discussed in Art. 16, some assumption must be made with respect to middling distribution, if the two-product formulas are to apply. In flotation testing it is ordinarily justifiable to assume that retreatment of middling would not affect primary concentrate or tailing assays, but the same assumption cannot ordinarily be made in gravity concentration.

Efficiency of concentration. It is apparent from inspection of the two-product formulas for recovery and ratio of concentration that if all of the feed were merely shoveled to one side and called concentrate, the recovery would be 100 per cent. The ratio of concentration would be 1, which would, of course, tell what had been done. This shows that recovery and ratio of concentration, or, at least, assay of concentrate, must be stated together in order that the figures may give a measure of the efficiency of the concentrating operation. Several proposals (see Art. 28) have been made to combine the numerical values of recovery and ratio of concentration, or ratio of assay of concentrate to assay of feed or tailing into one number called an efficiency index. Hancock (*19 MM 144*; *109 J 842*) proposes a number derived like that for classifier efficiency (E) (Art. 20). No proposal, however, shows a logical method of combination or one that has a physical significance that can be visualized. Hence it is better to give the two numbers, as is usual.

20. Formulas for classifier efficiency

Equation (10) may be, and ordinarily is used, together with equation (9) to give an idea of the relative amounts of sand discharge and overflow. Dean proposed defining CLASSIFIER EFFICIENCY as the ratio, expressed as percentage, of the weight of classified material in the overflow to the weight of classifiable

material in the feed. Overflow having the same sizing test as the feed is not said to be "classified material." Thus, if the separating size were 35-mesh and 100 tons of classifier feed contained 50 per cent. of −35-mesh material while the 50 tons of classifier overflow contained 90 per cent. −35-mesh, the 5 tons of +35-mesh in the overflow plus 5 tons of −35- mesh would comprise 10 tons of overflow of the same size composition as the feed. The net classified material in the overflow would, then, be 40 tons and efficiency, $E = 100 \times 40/50 = 80$ per cent. This can be expressed in terms of assays alone as follows: Let $100 - f$ = per cent. of oversize in feed, and $100 - c$ = per cent. of oversize in overflow. Then actual weight of oversize in overflow $= (100 - c)C/100$, and

$$E = \frac{C - \dfrac{(100 - c)C}{100 - f}}{\dfrac{fF}{100}} = 100\frac{C}{F} \cdot \frac{(c - f)}{f(100 - f)}. \quad . \quad . \quad . \quad (37)$$

But $C/F = (f - t)/(c - t)$. Substituting this value for C/F in equation (37),

$$E = \frac{100(c - f)(f - t)}{f(100 - f)(c - t)}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (38)$$

In the example in the preceding text $c = 90$, $t = 10$ and $f = 50$, whence $E = 80$ per cent. If the finished product contains only desired material, *i.e.*, $c = 100$, this equation becomes the same as the two-product recovery formula (10).

If this formula is used for concentrating operations, assays should be expressed as percentages of valuable mineral rather than as percentages of metal, otherwise the operation is penalized (*i.e.*, efficiency is low) because it does not separate metal from the chemically-combined elements.

Screen efficiency. Usual method of determination is to apply formula (10). Formula (38) may also be used.

21. Tonnages in milling circuits

Tonnages may frequently be determined by application of the two-product formulas (9) and (10). Fig. 35 shows four typical closed crushing circuits.

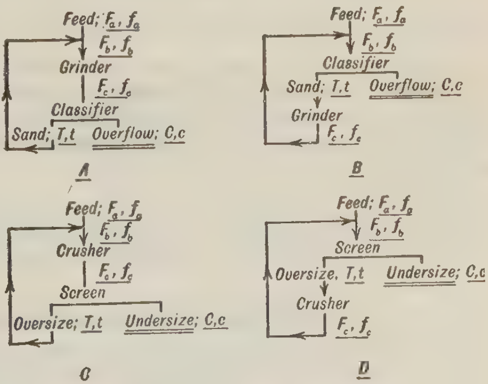

Fig. 35.—Typical closed crushing circuits.

In Fig. 35, *A*: $K = F_c/C = (c - t)/(f_c - t)$ from Eq. 9. But $C = F_a$. Hence, $F_c = F_a(c - t)/(f_c - t) = F_b$. And,

$$T = F_c - C = \frac{F_a(c - t)}{(f_c - t)} - F_a = \frac{F_a(c - f_c)}{(f_c - t)}.$$

Also $f_b = (F_a f_a + Tt)/(F_a + T)$.

In Fig. 35, B: $K = F_b/C = (c - t)/(f_b - t)$ from Eq. 9. But $C = F_a$, hence $F_b = F_a(c - t)/(f_b - t)$; and

$$T = F_c = F_b - C = F_b - \frac{F_b(f_b - t)}{c - t} = \frac{F_b(c - f_b)}{c - t}.$$

$$f_b = \frac{F_a f_a + F_c f_c}{F_a + F_c}. \qquad F_c = \frac{F_a(f_a - f_b)}{f_b - f_c}.$$

From inspection of the figure, $F_a f_a + F_c f_c = T_t + C_c$. But $F_c = T$ and $F_a = C$, hence $T(f_c - t) = F_a(c - f_a)$, and $T = F_a(c - f_a)/(f_c - t)$.

Adding C to both sides of the preceding equation, $T + C = F_a(c - f_a)/(f_c - t) + C$. But $T + C = F_b$ and $C = F_a$,

hence $F_b = F_a \left(\frac{c - f_a}{f_c - t} + 1\right)$.

By inspection of Fig. 35, B: $F_b f_b = F_a f_a + F_c f_c$.

$$f_b = \frac{F_a f_a + F_c f_c}{F_b} = \frac{F_a f_a + \frac{F_a(c - f_a) f_c}{(f_c - t)}}{F_a\left(\frac{c - f_a}{f_c - t} + 1\right)} = \frac{c f_c - t f_a}{c - f_a + f_c - t}.$$

$$f_c = \frac{F_a(c - f_a - t) + F_b t}{F_b - F_a}.$$

$$f_a = \frac{f_c(f_b - c) + f_b(c - t)}{f_b - t} = c + (f_c - t)[1 - F_b/F_a].$$

By equation (10) the value of R in Fig. 35, B, is $R = c(f_b - t)/f_b(c - t)$. Substituting in this equation the value of f_b, $R = \dfrac{c(f_c - t)}{c f_c - t f_a}$.

Tonnages in closed circuits may be estimated from sizing tests of the feed and product of a crusher on single pass by reckoning the percentage (r) of the original feed weight (a) requiring crushing (*i.e.*, with fines eliminated) that failed to be reduced on the first pass, by means of the formula for the sum of a geometric series, *viz.*: $S = a/(1 - r)$.

Thus if 100 tons per day feed to a set of rolls contained 30 tons undersize of the limiting screen, $a = 70$, and if the product of a single pass showed 65 tons undersize, $r = 35/70 = 0.5$, and S (total feed to the rolls per 24 hr.) $= 30 + 70/(1 - 0.5) = 170$. If the screen returns some undersize with the oversize, r must be increased accordingly. For another method of calculation, see *112 J 1050*.

22. Formulas involving sorting, roughing, etc.

The following formulas are useful to determine the monetary saving to be expected from sorting, roughing or other treatment in which a finished product, either concentrate or tailing, is to be removed in advance of the place that it is removed in the treatment scheme taken as standard.

Removal of concentrate. When the material to be removed is finished concentrate, let H and h = weight in tons and assay respectively of original feed; P and p = weight in tons and assay respectively of the concentrate to be produced by the proposed new operation; M and m = weight in tons and assay respectively of the residue from the proposed new operation, T = tons of final mill tailing without the new operation, T' = tons of final

mill tailing to be produced from M tons of residue from the new operation, C = tons of mill concentrate under normal operation and C' = tons of mill concentrate to be produced from M tons of residue from the new operation; c and t = assay respectively of C, C' and T, T'; V = value in dollars per unit of metal in concentrate to be produced by the new operation; V' = value in dollars per unit of metal in mill concentrate produced by either operation, R = cost of new operation in dollars per ton of concentrate produced thereby (P); S = cost in dollars per ton of milling H tons of original ore; and S' = cost in dollars per ton of milling M tons of residue from the new operation.

The assumption that the assays of tailing and concentrate made from original and sorted ore would be the same is justified by experience to the effect that relatively small changes in the tonnage or assay of mill feed have little effect on the assays of mill products. The values assigned to V and V' should be net and should take full account of penalties, freight, smelting charges, etc.

If the proposed operation is employed, the net return from P tons of concentrate so produced is its value less its cost of production $= PpV - PR$. Return from milling M tons of residue $= C'cV' - MS'$, and total return $= P(pV - R) + C'cV' - MS'$.

If the operation is omitted and the total feed is subjected to the same treatment that M is subjected to above, the return $= CcV' - HS$.

The saving in dollars (or loss, if the sign is negative) to be expected from adoption of the proposed operation

$$= [P(pV - R) + C'cV' - MS'] - (CcV' - HS).$$

The SAVING PER TON OF ORIGINAL FEED is

$$= \frac{P}{H}(pV - R) - cV'\left(\frac{C}{H} - \frac{C'}{H}\right) - \frac{M}{H}S' + S. \quad . \quad . \quad . \quad (39)$$

But $P/H = (h - m)/(p - m)$, from (9); $C/H = (h - t)/(c - t)$, from (9); $M/H = (p - h)/(p - m)$ (from the relation $M = H - P$); $C' = M(m - t)/(c - t)$, from (9). Then

$$\frac{C'}{H} = \frac{M}{H}\left(\frac{m - t}{c - t}\right) = \left(\frac{p - h}{p - m}\right)\left(\frac{m - t}{c - t}\right),$$

whence, by substitution of the above values in equation (39) the SAVING PER TON OF ORIGINAL MATERIAL to be expected from installation of the proposed process

$$= \frac{(h - m)(pV - R) - (p - h)S'}{p - m} - cV'\left[\frac{h - t}{c - t} - \frac{(p - h)(m - t)}{(p - m)(c - t)}\right] + S. \quad (40)$$

Removal of waste. If, instead of concentrate, W tons of waste assaying w is discarded at a cost of R dollars per ton discarded, then when the new operation is employed, the loss due to discarding waste is $= WwV' + WR$, and the return from milling the remainder $= C'cV' - MS$. Net return $= C'cV' - MS - W(wV' + R)$.

If the proposed operation is omitted, the net return $= CcV' - HS$, whence saving (or loss, if of negative sign) to be expected from installation of the new operation $= C'cV' - MS' - W(wV' + R) - CcV' + HS$.

SAVING PER TON OF ORIGINAL MATERIAL

$$= S - cV'\left(\frac{C}{H} - \frac{C'}{H}\right) - \frac{M}{H}S' - \frac{W}{H}(wV' + R).$$

Substituting assay values, as in the preceding development, the saving per ton of original material

$$= S - cV'\left[\frac{h-t}{c-t} - \frac{(m-t)(h-w)}{(c-t)(m-w)}\right] - \frac{(h-w)S' + (m-h)(wV'+R)}{m-w}. \quad (41)$$

Saving (or loss) to be expected from cleaning a salable product instead of shipping directly may be obtained by a similar method of analysis.

Let H' = tons of material, h' = assay of H', C' = tons of cleaned product, c' = assay of C', T' = tons of material that will be reject of the cleaning operation, t' = assay of T', R = cost of treatment in dollars per ton of H', V = value, in dollars per unit of metal, of C'; V' = value, in dollars per unit of metal, of H'.

If the material is shipped directly, the return $= H'h'V'$. If cleaning is practiced, the return $= C'c'V - H'R$.

Saving (or loss, if negative) to be expected from the proposed cleaning operation $= (C'c'V - H'R) - H'h'V' = C'c'V - H'(R + h'V')$.

Saving per ton of original material

$$= \frac{C'c'V}{H'} - (R + h'V') = \frac{c'V(h'-t')}{c'-t'} - (R + h'V'). \quad . \quad . \quad (42)$$

Saving to be expected by further treatment of a tailing product is the excess value of the concentrate produced by the treatment over the cost of treatment.

If H'' = tons of original tailing, C'' = tons of additional concentrate produced by further treatment of H'', T'' = tons of cleaned tailing, h'', c'', t'' = assays of H'', C'' and T'', respectively, R = cost of treatment per ton of H'', V = value in dollars per unit of metal in the concentrate C''.

Then profit (or loss, if of negative sign) to be expected from the proposed additional treatment $= C''c''V - H''R$

Profit per ton of original material (H'')

$$= \frac{C''}{H''}c''V - R = \frac{c''(h''-t'')}{c''-t''}V - R. \quad . \quad . \quad . \quad . \quad (43)$$

23. Specific-gravity assay

When an ore is a mixture of two minerals only or of one valuable mineral and a mixture of gangue minerals whose relative proportions are substantially constant, it is possible to make rapid approximate assays of a mixture of valuable mineral and gangue by determining the specific gravity of the mixture, provided the individual specific gravities of valuable mineral and gangue are already known. The usual method is by use of a specific-gravity flask.

Weigh the flask empty and when full of water. Dry the flask, introduce the ore sample, weigh; fill with water, taking care to remove all air bubbles, and again weigh. In centimeter-gram units, if S_o = specific gravity of ore, S_m of "mineral" and S_g of gangue; F = weight of dry flask, W = weight of water required to fill the flask, O = weight of dry ore, and T = total weight of flask + ore + water, then the weight of water required to fill the flask, with ore in it $= T - (O + F)$ = volume of water in flask, and volume of ore in flask $= W - [T - (O + F)]$.

$$S_o = \frac{O}{W - [T - (O+F)]} = \frac{O}{F + W + O + T} \quad . \quad . \quad . \quad (44)$$

If m = per cent. of "mineral" in ore = weight of mineral per unit weight of ore; $1 - m$ = per cent. of gangue; m/S_m = volume of "mineral" per unit weight of ore; $(1 - m)/S_g$ = volume of gangue per unit weight of ore; $1/S_o$ = volume of unit weight of ore.

Then $\dfrac{m}{S_m} + \dfrac{1 - m}{S_g} - \dfrac{1}{S_o} = 0$, and

$$m = \frac{S_m(S_o - S_g)}{S_o(S_m - S_g)}. \qquad (45)$$

24. Cyanidation

Let F weight of feed = 1; C = weight of concentrate (if made), T = weight of tailing from concentrating operation (= F, if no concentrate is made), A = weight of sand feed, B = weight of slime feed, X = weight of sand tailing and Y = weight of slime tailing, all expressed as decimal parts of F; and f, c, t, a, b, x and y = respective assays of the above products.

Then $F = C + T$; $T = A + B$; $f = cC + tT$; $Tt = aA + bB$; $Tt = f - cC$. But $T = F - C = 1 - C$. Hence $(1 - C)t = f - cC$ and

$$C = (f - t)/(c - t). \qquad (46)$$

Similarly

$$T = (c - f)/(c - t). \qquad (47)$$

$Aa = Tt - Bb$. But $B = T - A$, hence $Aa = Tt - (T - A)b$, and $A/T = (t - b)/(a - b)$. Substitute value of T from (Eq. 47), then

$$A = \frac{(c - f)(t - b)}{(c - t)(a - b)}, \qquad (48)$$

$$B = T - A = \frac{c - f}{c - t} - \frac{(c - f)(t - b)}{(c - t)(a - b)} = \frac{(c - f)(t - a)}{(c - t)(b - a)}. \qquad (49)$$

$R = [Cc + (Aa - Xx) + (Bb - Yy)]/Ff$. But $F = 1$, and for all practical purposes, $A = X$ and $B = Y$. Hence, substituting A for X and B for Y and then substituting for C, A and B their equivalents in terms of assays above given, and clearing

$$R = \frac{c(f - t) + \dfrac{c - f}{a - b}[(t - b)(a - x) - (t - a)(b - y)]}{f(c - t)}. \qquad (50)$$

25. Voids

Let w = weight of solid per unit volume. If material is moist, let w include also the weight of moisture. Let S = sp. gr. of dry solid, p = per cent. solids, V = PERCENTAGE OF VOIDS = percentage of unit volume unoccupied when that volume contains the weight w of solid or solid + water; B = per cent. of unit volume occupied by air, H = per cent. occupied by water and T = per cent. occupied by air + water. Quantities p, V, B, H and T are expressed as decimal parts of the unit, and gram-centimeter units are most readily used.

For dry material,

$$V = 1 - w/S. \qquad (51)$$

For wet material pw = weight of dry solid in unit volume; pw/S = volume of solid; and

$$T = 1 - wp/S. \qquad (52)$$

The weight of water in a weight w of wet material $= w - pw$. Since the numbers representing weight and volumes of water are interchangeable in gram-centimeter measure,

$$H = w(1 - p). \qquad (53)$$

$$B = T - H = \frac{wp(S - 1) - S(w - 1)}{S} \qquad (54)$$

Voids in broken rock smaller than 3-in. range between 40 and 45 per cent. when fines are present and 45 to 50 per cent. with fines removed. (*Bul. 5 UI No. 23.*)

Specific volume is defined as the weight of dry solid per unit volume of broken solid and is commonly stated in lb. per cu. ft. If Q = specific volume in gram-centimeter units, m = percentage of moisture in the mass of broken solid by weight, and v = percentage moisture by volume, then the volume of actual solid + water per unit volume of broken material $= 1 - V$ and the actual solid in this weight is $Q = S(1 - V)(1 - v)$. But $v = \dfrac{mS}{mS + 1 - m}$ (from Eq. 66), hence

$$Q = S(1 - V)\left[\frac{1 - m}{m(s - 1) + 1}\right] \qquad (55)$$

In common units this becomes

$$Q = 62.5S(1 - V)\left[\frac{1 - m}{m(S - 1) + 1}\right] \text{ lb. per cu. ft.} \qquad (56)$$

26. Pulp consistency

Let p = PER CENT. SOLIDS = weight of solids in unit weight of pulp; D = DILUTION, = water-solid ratio = parts water, by weight, per part of solid, usually written, *e.g.*, 6 : 1, 3.2 : 1, etc.; S = specific gravity of dry ore; d = specific gravity of pulp.

Then p/S = volume of solids in unit weight of pulp, $1 - p$ = volume of water in unit weight of pulp, $1/d$ = volume of unit weight of pulp, and $p/S + 1 - p = 1/d$, from which

$$d = \frac{S}{p + S(1 - p)} = \frac{S}{S - p(S - 1)} = \frac{D + 1}{D + \frac{1}{S}}, \qquad (57)$$

$$p = \frac{S(d - 1)}{d(S - 1)} = \frac{1}{D + 1}, \qquad (58)$$

$$S = \frac{dp}{1 - d(1 - p)} = \frac{d}{1 - D(d - 1)}, \qquad (59)$$

$$D = \frac{1 - p}{p} = \frac{S - d}{S(d - 1)} = \frac{1}{p} - 1. \qquad (60)$$

The relations between p, S and d are shown graphically in Fig. 36.

If Z = SOLID FACTOR = tons solid per FLUID TON (= 32 cu. ft.) of pulp; G = fluid tons of pulp per ton of dry solids; and q = per cent. solids by volume, then, since the weight of 1 cu. ft. of pulp = 62.5 d lb., the weight of one fluid ton = 32 × 62.5 d and

$$Z = \frac{32 \times 62.5d \times p}{2000} = pd. \qquad (61)$$

By substitution in equation (61) of values of p and d from equations (57) to (60),

$$Z = \frac{d}{D+1} = \frac{pS}{S - p(S-1)} = \frac{S}{DS+1} = \frac{S(d-1)}{S-1} \quad . \quad . \quad (62)$$

By definition $G = 1/Z$. Hence, by equations (61) and (62)

$$G = 1/pd = (D+1)/d. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (63)$$

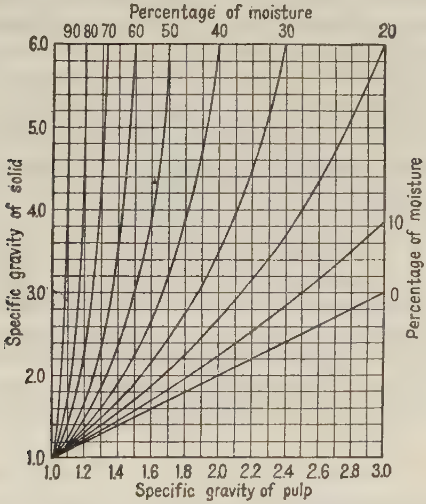

FIG. 36.—Relation between percentage of solids and specific gravity of ore pulps.

By definition $q = \dfrac{Z(2000)}{S(62.5 \times 32)} = Z/S$, hence, by equations (57) and (62),

$$q = \frac{d-1}{S-1} = \frac{p}{S - p(S-1)}. \quad . \quad . \quad . \quad . \quad (64)$$

27. Counting assay

If all particles are of substantially the same shape (that they are of substantially the same intermediate dimension may be assured by sizing), the volume percentages are equal to the number percentages. If there is a distinct difference in average shape of particles, the number percentages must be adjusted by factors expressing relative volumes of the mean shapes, in order to get volume percentages. If v = per cent. by volume of "mineral" in sample = volume per unit volume and S_m and S_g = specific gravities of "mineral" and gangue respectively, then weight of "mineral" in a unit volume of sample = vS_m and weight of gangue = $(1 - v)S_g$. The percentage of mineral by weight is

$$m = \frac{vS_m}{vS_m + (1-v)S_g} = \frac{vS_m}{v(S_m - S_g) + S_g}, \quad . \quad . \quad . \quad . \quad (65)$$

$$v = \frac{mS_g}{mS_g + S_m(1-m)} \quad . \quad . \quad . \quad . \quad . \quad (66)$$

These formulas are applicable to the case of mixtures of solid and water, in which case, if m and v are taken as percentages of solid, S_m is specific gravity of solid and $S_g = 1$.

28. Efficiency in coal washing

In coal washing, efficiency is judged on somewhat different grounds from those used in metal concentration and hydrometallurgy. In these latter processes the valuable material is a definite substance (metal) that can be accurately measured by assay, and, so measured, has definite significance. But the combustible material in coal is not so definitely measurable and it is never, practically, completely separable from the non-combustible, so that clean coal is not clean combustible but rather a mixture of "middlings" of varying fuel value from the cleanest that can be obtained to the dirtiest that can be sold.

Recovery. The assays made are usually for impurities, *i.e.*, ash and sulphur. If sulphur is reckoned as combustible,

$$R = \frac{(1 - c_1)(t_1 - h)}{(1 - h_1)(t_1 - c_1)}, \quad \ldots \ldots \quad (67)$$

where h_1, c_1, and t_1 are assays for ash in raw coal, washed coal and refuse respectively. If sulphur is reckoned as non-combustible, h_1, c_1 and t_1 must be taken as the sum of the percentages of sulphur and ash in the respective products.

Tonnage calculations may be made by the formula

$$X = \frac{t_1 - h_1}{t_1 - c_1}, \quad \ldots \ldots \quad (68)$$

where X = percentage weight of washed coal. $100/X$ = the ratio of concentration (page 1235).

Dodds (*23 CA 635*) points out that if sulphur and ash concentrate in different proportions in the washing process the tonnages calculated by equation (68) will differ according to whether ash or sulphur or the sum of the assays is used and that the latter gives the correct result.

Percentages of reduction of ash and of sulphur effected are used as measures of efficiency. These percentages are expressed as the difference between the respective percentages in raw and clean coal divided by the percentage in raw coal. It is not possible to effect 100 per cent. reduction of either substance by mechanical means.

Sink-and-float tests. On the basis of sink-and-float tests (Art. 10), the percentage of the total raw coal that will float in a solution, the specific gravity of which is that of the maximum specific gravity of salable coal (PERMISSIBLE DENSITY) represents the highest possible percentage recovery of salable coal, and the percentage of total combustible therein contained represents maximum recovery of combustible. No commercial process of extraction will reach this maximum and, therefore, if it is set equal to 100 per cent., the actual performances may be stated as percentages thereof.

Drakeley efficiency formulas. If X = weight of washed coal expressed as percentage of raw coal; a = weight of CLEAN COAL (*i.e.*, float in the solution whose specific gravity is such that it floats a coal of maximum salable ash content) expressed as a percentage of the weight of raw coal treated; and

b = percentage weight of clean coal in the washed coal (determined by a sink-and-float test), recovery

$$R' = 100Xb/a. \qquad (69)$$

This number according to Drakeley (*54 IME 428*) is the QUANTITATIVE EFFICIENCY. Drakeley suggests the term QUALITATIVE EFFICIENCY (Q) to express the relation

$$Q = 100(b - a)/(100 - a), \qquad (70)$$

and the term GENERAL EFFICIENCY to express the relation

$$G = R'Q/100. \qquad (71)$$

This attempt to combine recovery and a number akin to the ratio of concentration in metal concentration, is similar to attempts that have been made in the latter art, and no more successful. Such a number has no physical significance but merely relative value, and its magnitude, depending although it does on the product of two significant numbers, may be the same with a low value of R' and high value of Q or the reverse or with intermediate values of both, and is, therefore, not even numerically significant.

Delameter efficiency formulas. If E = percentage efficiency, a = percentage weight of raw coal floated on the PERMISSIBLE BATH (heavy solution that floats coal of permitted ash content), b = ratio of weight of washed coal to raw coal, expressed as percentage, d = percentage of ash in raw coal, e = percentage of ash in permissible float from raw coal, and f = percentage of ash in washed coal, Delameter proposes (*5 CA 723*) the following formulas for efficiency.

(1) If $f > e$ and $b = a$; $$E = (d - f)/(d - e). \qquad (72)$$

(2) If $f > e$ and $b > a$; $$E = \frac{1}{2}\left(\frac{b - a}{100 - a} + \frac{d - f}{d - e}\right). \qquad (73)$$

(3) If $f \ll e$ and $b \ll a$; $$E = \frac{1}{2}\left(\frac{f}{e} + \frac{b}{a}\right). \qquad (74)$$

(4) If $f > e$ and $b < a$; $$E = \frac{1}{2}\left(\frac{d - f}{d - e} + \frac{b}{a}\right). \qquad (75)$$

Formula (72) gives a true efficiency number based on percentage ash reduction. The first term in the numerator of formula (73) is roughly the percentage weight of the total sink fraction of the raw coal that appears in the washed coal. For perfect work the first term of the numerator should be zero and the second term one, in which case $E = 1/2$. This is clearly not a significant efficiency number since the same value for E would be obtained if both terms in the numerator equaled 1/2, which would, of course represent very poor work. Equation (74) gives $E = 1$ corresponding to perfect work and is, to that extent, a significant efficiency number. It also measures, within the limits set, the approach to two important goals in washing operations. Equation (75) likewise gives $E = 1$ for perfect work.

The great weakness of the equations is the fact that it is necessary to specify which equation has been used in determining the efficiency number stated for any given operation and also to give the values for a, b, d, e and f, in order that their relative effects may be judged; hence the efficiency number alone is in no case significant.

Fraser and Yancey. (*69 A 456*) propose the efficiency formula

$$E = \frac{YR}{Y_1R_1} = \frac{(c - a)(a - b)}{B(c - b)(a - f)}. \qquad (76)$$

where Y = the actual yield of washed coal = (*weight of washed coal*)/(*weight of raw coal*); Y_1 = standard yield = (*weight of raw-coal float*)/(*weight of raw coal*); R = actual ash reduction, per cent.; R_1 = standard ash reduction, based on sink-and-float test on raw coal, per cent.; a = per cent. of ash in raw coal; b = per cent. of ash in washed coal; c = per cent. of ash in refuse; f = per cent. of ash in raw-coal float; and B = weight of raw-coal float.

This number suffers from the disadvantage that, although perfect work gives $E = 1$, the same value may be obtained when $R/R_1 < 1$, if, at the same time $Y/Y_1 > 1$, both of which suppositions will commonly occur.

Hamilton (*69 A 475*) proposes

$$E = \frac{a(100 - c)}{b},$$

where a = per cent. of ash in raw-coal float, b = per cent. of ash in washed coal, and c = per cent. of clean coal lost = per cent. of refuse $\times$ per cent. of float in refuse. When $a = b$ and $c = o$, which represents perfect work, $E = 100$.

The weakness of this equation lies in the fact that it does not measure the recovery of combustible and that loss of clean coal does not penalize the efficiency number to anything like the same extent that it is penalized by failure to reduce ash.

Hancock (*69 A 476*) proposes the formula

$$E = \frac{I_r - YI_w}{I_r}, \quad \ldots\ldots\ldots \quad (77)$$

where I_r = per cent. of a given impurity in the raw coal, I_w = per cent. of the same impurity in the washed coal and Y = yield = (*weight of washed coal*)/(*weight of raw coal*).

This formula merely measures the efficiency of the operation in reduction of impurity with no penalty for loss of coal, in fact, if $Y = 0$ in the equation, $E = 1$.

Lincoln (*11 Bul. UI, No. 9*) suggests assaying the washed products by sink-and-float methods and calling half of the sum of the percentage float in the coal and sink in the ash the efficiency.

When the figure thus obtained is high, it has definite and useful significance, but otherwise it does not tell whether the performance fails to be good by reason of low-grade coal or high-grade refuse.

29. Statistical calculation

Numerical data covering mill operations rarely involve two variables only and for that reason the relation of effect to cause cannot be found by the ordinary methods of two-variable investigation such as plotting to rectangular co-ordinates, etc. If the ordinary methods are employed to determine the relation between any two of a number of variables, the investigation involves such doubtful assumptions with respect to the other variables (the usual assumption, *e.g.*, being that their effect is negligible) that the relation evolved is probably wrong and certainly untrustworthy. Thus in any ordinary mill operation of the flotation process, in which the quantity of oil per ton of feed varies from day to day and recovery also varies, direct comparison of recovery with oil quantity involves neglect of other elements of the operation, likewise variable, such as percentage of solids in feed pulp, fineness of grinding, mineral content of feed, grade of concentrate, etc., each of which has an effect on the recovery that cannot, justifiably, be neglected. The method of STATISTICS attempts to solve a situation such as that presented by supplying a

number called the COEFFICIENT OF CORRELATION, that is a measure of the extent of the directness or linearity of the relation between, say, oil quantity and recovery, if all of the other above-mentioned variables were held constant; another number, called the PROBABLE ERROR OF THE PARTIAL COEFFICIENT, that is a measure of the reliability and significance of the first; a third number, called the COEFFICIENT OF MULTIPLE CORRELATION, which measures the approach to directness or linearity of the relation between, say, oil quantity, and all of the other variables considered together; and a fourth number, called the PROBABLE ERROR OF THE MULTIPLE CORRELATION, that measures the reliability and significance of the third.

Notation. Coefficients of partial correlation are indicated by r with suitable subscripts indicating the variables considered. The arrangement of the subscripts is indicative of the nature of the coefficient, thus r_{12} is a SIMPLE OR ZERO-ORDER COEFFICIENT of partial correlation between variables *1* and *2*, in finding which the effect of all other variables has been neglected. When the calculation has been extended, as is explained later, to eliminate the effect of one of the neglected variables, the notation for the FIRST-ORDER COEFFICIENT thus obtained is $r_{12.3}$, etc.

The number $r_{12.3\ldots n}$ is called the coefficient of partial correlation between the variables *1* and *2*, say oil quantity and recovery; $e_{r_{12.3\ldots n}}$ is the probable error in $r_{12.3\ldots n}$; $R_{1(2,3\ldots n)}$ is called the coefficient of multiple correlation between the variable *1* (*e.g.*, oil quantity) and all of the other variables; and $e_{R_{1(2,3\ldots n)}}$ is the probable error in $R_{1(2,3\ldots n)}$.

Coefficient of partial correlation. With data available, as *e.g.*, in Table 18, which represent daily performances of a copper-flotation plant covering the period from Aug. 7, 1916 to March 12, 1917, incl., the first step is to copy these data on cards, one card for each set of figures, *i.e.*, one day in the present case. This is done for ease in compiling the subsequent calculations. Next arrange the cards in order of increase of one of the variables to be investigated.

For example, in the present calculation, the first relation investigated was that between "Lb. of oil per ton" (O) and "Dilution" (D), and the cards were arranged in order of increasing amounts of oil. With the cards thus arranged, Table 19 was built as follows:

The difference between maximum and minimum oil quantities was divided by a number between 15 and 30, chosen primarily to give a quotient easy to add and subtract. The maximum and minimum oil quantities were 3.8 and 0.6 respectively. The difference, 3.2, was divided by 16, which gives a common difference of 0.2. Seventeen classes with this difference embrace the entire range of oil quantity. Table 19 proper was, therefore, divided into 17 vertical columns headed 0.65, 0.85 . . . 3.85 lb.oil per ton as shown. The column headed 0.65 embraces all quantities of oil between 0.55 and 0.75, that headed 0.85 embraces all quantities from 0.75 to 0.95, etc. The cards were next arranged in order of increasing dilution, the range was found to be from 4.8 to 15.8, and an interval difference of 0.4 gave 28 classes or horizontal columns. A blank sheet was next ruled as in Table 19, providing for five columns or lines to the left of and above the double rulings respectively and 17 vertical columns and 28 horizontal lines in the space below and to the right of the double ruling. Vertical columns were headed with the determined oil-quantity means and horizontal lines by dilution means as shown in line O and column D respectively. All cards with values of O between 0.55 and 0.75 were next grouped into values of D corresponding to the means in column D and the number of cards in each group entered in the rectangle corresponding to the corresponding values of O and D. Thus on Aug. 17, Jan. 7 and Jan. 27 the amounts of oil used were 0.7, 0.7 and 0.6 lb. per ton respectively, which group under $O = 0.65$, and the corresponding dilutions were 6.0, 5.4 and 5.2 respectively. These group as one occurrence under $D = 6.1$ and two under $D = 5.3$. The numbers 1 and 2 were, therefore, entered in the corresponding rectangles as described and shown in the table. By similar procedure the other FREQUENCIES were entered in the proper places. The numbers in column "f_D" are the sums of all of the frequencies in the same horizontal line and the numbers in line "f_O," the sums of all the frequencies in the corresponding vertical columns. From inspection of columns "D" and "f_D" the mean value of D is apparently in the range $D = 6.1$. Similarly, the mean value of O appears to be in the range $O = 1.45$.

Table 18. Record of daily quantities in copper-flotation plant

Date	Assays, per cent. Cu		Recovery, per cent.	Oil (*a*), pounds per ton	Acid, pounds per ton	Dilution (*b*)
	Feed	Concentrate				
1916		(C)	(R)	(O)	(A)	(D)
August						
7	1.01	25.6	92	1.0	6.5	5.4
8	.95	29.4	92	1.1	7.0	5.8
9	.93	28.1	93	1.0	7.5	5.6
10	1.00	28.4	94	1.0	7.0	5 0
11	1.01	25.7	92	1.3	9.0	4.8
12	.99	27.8	92	1.0	7.5	5.0
13	.99	29.1	90	1.1	6.5	4.8
14	.95	28.4	91	1.1	6.5	5.2
15	1.03	25.7	91	1.3	6.5	4.8
16	1.03	29.9	89	1.2	6.0	6.0
17	.91	29.0	90	0.7	7.5	6.0
18	.88	29.5	90	1.1	7.5	5.4
19	.85	30.2	90	1.2	6.5	6.8
20	.91	26.4	93	0.8	6.5	6.2
21	.97	30.2	91	0.9	7.0	6.8
22	1.04	30.5	92	1.4	7.5	6.8
23	.93	28.4	89	1.2	6.5	6.2
24	.99	26.8	90	1.2	6.5	6.0
25	1.07	29.2	91	1.3	7.0	5.6
26	.91	26.0	91	1.3	7.0	6.6
27	.92	25.5	92	0.9	6.5	6.2
28	.98	25.3	88	1.3	6.5	6.4
29	1.05	28.3	88	1.2	7.0	6.0
30	.96	27.7	90	.9	5.0	6.2
31	.95	25.8	91	1.2	6.5	6.2
September						
1	.94	26.5	91	1.3	5.5	5.8
2	.96	26.9	91	1.2	5.5	5.6
3	1.11	27.7	90	1.3	5.5	5.6
4	1.16	33.6	90	1.0	5.0	5.0
5	1.10	27.4	90	1.3	5.5	5.8
6	1.07	27.0	89	0.8	5.0	5.2
7	1.19	28.1	91	1.2	5.0	5.0
8	1.07	26.7	90	1.2	5.5	6.2
9	1.05	25.1	92	1.4	5.5	6.0
10	1.12	25.8	93	1.2	5.5	5.6
11	1.08	28.2	92	0.9	5.0	5.4
12	1.05	26.9	92	1.2	6.0	5.8
13	1.07	27.8	92	1.2	6.0	4.8
14	1.04	24.4	93	1.5	6.0	5.6
15	1.02	22.5	92	1.3	5.5	6.0
16	1.06	25.3	92	1.2	5.0	5.8
17	.87	21.7	91	1.4	5.5	5.6
18	.87	21.6	91	1.3	5.5	5.6
19	.90	21.9	87	1.4	6.0	5.8
20	1.01	25.4	89	1.0	5.5	5.4
21	1.00	25.9	87	1.3	5.5	5.8
22	1.05	25.7	86	1.6	7.5	5.2
23	1.05	27.4	88	1.4	6.5	5.8
24	.94	26.0	88	1.5	6.5	5.8
25	.97	29.8	88	1.8	7.5	5.8
26	1.01	27.6	87	1.7	6.5	6.2
27	1.12	28.0	82	1.2	5.0	5.2
28	.99	23.8	80	2.8	10.5	7.8
29	.95	24.5	87	1.0	5.5	5.2
30	.95	24.3	83	1.7	5.5	5.4

a Same oil throughout. *b* 100 ÷ per cent. solids.

Table 18. Record of daily quantities in copper-flotation plant—*Continued*

Date	Assays, per cent. Cu		Recovery, per cent.	Oil (*a*), pounds per ton	Acid, pounds per ton	Dilution (*b*)
	Feed	Concentrate				
1916		(C)	(R)	(O)	(A)	(D)
October						
1	1.00	24.7	87	1.1	5.0	5.4
2	1.09	28.0	86	1.3	6.0	6.4
3	1.07	25.4	85	1.3	6.0	6.0
4	1.09	24.3	88	1.5	6.5	5.2
5	1.15	27.6	83	1.6	7.0	5.4
6	1.04	24.0	89	1.6	6.5	6.0
7	1.12	26.9	91	1.4	6.0	6.2
13	.99	27.3	87	1.5	6.0	5.4
14	1.10	25.3	92	2.2	11.0	8.2
15	.97	27.8	92	2.4	10.5	8.2
16	.99	29.6	93	1.7	8.0	7.8
17	1.01	23.7	92	2.0	9.0	6.2
18	1.20	28.3	90	1.5	6.0	6.6
19	1.06	25.7	89	2.1	8.5	7.6
20	1.07	20.1	84	1.5	6.5	7.0
21	1.12	25.8	88	1.2	6.0	6.2
22	1.03	25.0	89	.9	5.5	6.0
23	1.06	28.4	88	1.1	5.0	5.6
24	1.07	25.1	88	1.4	5.0	6.6
25	1.14	28.0	89	1.4	6.5	6.6
26	1.36	28.8	95	1.6	6.5	6.0
27	1.32	25.5	94	1.8	7.5	7.2
28	1.06	23.5	92	2.3	9.0	6.2
29	1.15	23.7	93	1.9	7.5	6.8
30	1.19	27.0	91	1.5	5.5	5.6
31	1.31	28.3	81	1.4	6.0	5.4
November						
1	1.24	32.4	90	1.4	5.5	5.8
2	1.22	29.5	88	1.6	7.5	6.2
3	1.14	30.7	91	1.3	5.0	6.0
4	1.14	28.5	91	1.6	6.0	6.2
5	1.03	28.5	90	1.4	6.0	6.0
6	1.05	28.4	89	1.5	5.5	5.8
7	1.00	25.2	90	1.3	5.5	6.0
8	1.04	26.8	88	1.4	5.5	6.2
9	1.04	26.2	89	1.1	5.5	6.6
10	1.00	24.7	90	1.5	6.5	6.6
11	.94	23.4	90	1.4	5.5	6.2
12	.95	23.8	89	1.4	5.5	6.0
13	.97	23.8	89	1.2	5.5	5.6
14	.98	19.4	91	2.3	9.0	7.0
15	.93	22.5	91	1.3	5.0	6.0
16	.85	22.3	90	1.4	5.5	7.4
17	1.13	29.3	90	1.2	5.0	6.0
18	1.04	27.3	90	1.5	5.0	6.0
19	1.10	25.2	89	1.2	5.0	6.6
20	1.12	25.2	82	1.1	5.5	6.8
22	1.11	26.6	86	1.3	5.5	6.4
23	1.14	27.3	89	1.2	5.0	5.8
24	1.10	24.7	91	1.6	6.0	8.0
25	1.07	26.4	92	1.6	6.0	9.2
26	1.12	23.9	92	1.6	6.0	7.0
27	1.24	28.3	91	1.2	6.0	6.6
28	1.09	25.6	88	1.3	5.0	6.8
29	1.13	27.0	91	1.1	6.0	6.6
30	1.15	28.0	89	1.1	5.0	6.2

a Same oil throughout. *b* 100 ÷ per cent. solids.

Table 18. Record of daily quantities in copper-flotation plant—*Continued*

Date	Assays, per cent. Cu		Recovery, per cent.	Oil (*a*), pounds per ton	Acid, pounds per ton	Dilution (*b*)
	Feed	Concentrate				
		(C)	(R)	(O)	(A)	(D)
1916 December						
1	1.04	24.3	90	1.3	5.5	6.8
2	1.04	25.7	89	0.9	5.5	5.4
3	1.03	22.4	89	1.9	7.0	9.2
4	1.08	22.5	91	1.9	9.0	8.0
5	1.02	23.6	93	1.9	7.5	7.2
6	1.04	26.1	90	1.1	5.5	7.4
7	1.03	28.8	89	1.1	5.0	7.4
8	1.01	24.9	92	1.0	5.0	6.4
9	.97	26.6	91	1.0	4.5	5.6
10	.94	24.8	90	1.2	6.0	6.6
11	.97	24.6	89	1.0	5.0	6.2
12	1.02	24.6	90	1.1	6.0	6.8
13	1.05	29.6	90	0.9	4.5	6.4
14	1.11	27.1	87	1.0	4.5	6.6
15	1.01	25.3	89	0.8	4.5	6.6
16	1.26	26.6	90	1.3	5.0	7.2
17	1.33	28.7	92	1.4	4.5	7.2
18	1.13	30.8	92	0.8	4.0	6.2
19	1.18	25.6	92	1.1	4.5	7.4
20	1.07	26.6	90	1.0	3.5	6.8
21	1.13	28.5	89	1.0	4.5	6.8
22	1.30	22.7	88	1.8	5.0	7.8
23	.95	21.7	89	1.5	5.5	7.4
24	1.32	31.0	87	1.1	4.0	6.6
25	1.14	25.2	90	2.0	7.5	6.2
27	1.75	24.3	79	3.2	9.5	6.0
28	1.18	23.3	73	3.4	15.0	10.8
29	.92	20.6	89	3.5	14.5	15.8
30	.95	20.0	90	3.0	12.5	12.0
31	1.10	24.0	90	2.0	7.0	9.4
1917 January						
1	1.00	25.3	86	1.4	8.5	6.0
2	.88	17.0	89	2.0	13.0	6.4
3	.92	23.6	88	1.3	9.0	5.6
4	.88	22.1	91	1.6	11.0	5.6
5	.88	15.7	91	1.4	8.0	5.8
6	1.04	19.5	92	1.0	6.0	5.0
7	1.15	21.7	88	0.7	6.0	5.4
11	1.05	24.4	89	0.8	4.5	6.0
12	1.06	24.4	83	0.8	4.0	6.0
13	1.13	23.8	89	1.1	5.5	5.6
14	.99	23.6	89	1.0	4.5	6.2
15	1.15	26.4	90	0.8	4.5	5.4
16	1.13	26.8	91	1.8	7.5	5.8
17	1.11	21.5	91	1.5	7.5	6.6
18	1.05	20.0	92	1.2	6.5	5.2
19	1.03	25.8	93	1.1	5.5	5.6
20	1.10	22.4	93	1.3	5.5	5.4
21	.95	22.0	92	1.4	6.5	4.8
22	.97	22.0	91	1.8	7.0	6.0
23	.94	19.7	92	1.2	7.5	6.6
24	1.11	22.8	91	1.2	7.5	5.4
25	.95	23.1	87	1.3	4.5	5.6

a Same oil throughout. *b* 100 ÷ per cent. solids.

Table 18. Record of daily quantities in copper-flotation plant—*Continued*

Date	Assays, per cent. Cu		Recovery, per cent.	Oil (a), pounds per ton,	Acid, pounds per ton	Dilution (b)
	Feed	Concentrate				
1917		(C)	(R)	(O)	(A)	(D)
January						
26	.87	21.2	78	1.2	6.0	6.0
27	1.00	21.5	87	0.6	4.0	5.2
28	1.05	22.0	81	1.1	5.5	6.8
29	.98	27.5	90	1.1	6.5	5.6
30	1.10	21.9	90	1.2	6.5	5.6
31	1.05	23.1	85	1.4	6.0	5.4
February						
8	.73	19.8	83	1.2	6.0	5.8
9	1.05	22.0	87	1.5	7.0	5.8
10	1.10	23.1	89	1.3	5.5	5.2
11	1.10	23.8	90	1.5	6.0	5.6
12	1.12	23.0	93	1.5	7.0	5.6
13	1.06	25.7	91	1.2	5.5	5.4
14	1.04	20.1	93	1.7	6.5	5.0
15	.94	18.6	93	1.4	6.0	5.2
16	.98	20.5	89	1.3	5.5	6.0
17	1.14	22.0	89	1.5	6.0	5.8
18	1.02	18.5	92	2.0	6.5	5.4
19	1.12	24.4	92	1.8	6.0	5.2
20	1.04	19.1	93	1.7	6.0	6.0
21	1.11	20.8	85	2.0	6.0	5.2
22	1.17	23.4	80	1.4	6.5	5.0
23	1.19	17.6	85	3.8	11.5	9.6
24	1.05	20.0	75	1.8	7.0	6.4
25	1.10	18.2	71	2.2	9.0	6.8
26	1.14	18.9	82	1.9	9.0	5.8
27	1.12	17.4	85	2.9	10.5	5.8
28	1.16	18.2	76	2.1	8.0	5.4
March						
2	1.29	20.6	93	2.5	9.5	5.4
3	1.13	22.1	85	2.1	8.5	5.2
6	1.06	19.7	83	1.5	6.5	5.8
7	1.17	22.7	91	2.5	8.5	7.8
8	1.16	22.8	85	2.0	7.0	5.4
9	1.08	22.6	85	1.8	8.5	7.4
10	.95	20.9	90	1.9	9.5	4.8
11	1.10	19.3	88	3.2	11.0	9.2
12	.99	19.6	82	2.8	10.0	5.4

a Same oil throughout. *b* 100 ÷ per cent. solids.

Assuming, for the present, that these are the correct means, the individual deviations of the other values of D and O, by number of groups, are set down in corresponding files in the vertical column and the horizontal line marked "d_D" and "d_O" respectively. For example, the first number in column d_D is -3, indicating a deviation by three groups, of 0.4 value each, of $D = 4.9$ from $D = 6.1$, the assumed mean. The files marked "fd" show the products of the corresponding values of f and d and the numbers in the files marked "fd^2" are derived from those in files "fd" by multiplying by the corresponding values of d. The DEVIATION (c_O) OF THE TRUE MEAN of the values of O from the assumed mean may now be found by dividing the algebraic sum of file "f_Od_O" by the total number of observations $N(=\Sigma f)$ or $c_O = \Sigma f_Od_O/N = \Sigma f_Od_O/\Sigma f = +8/198 = +0.04$. Similarly $c_D = \Sigma f_Dd_D/N = \Sigma f_Dd_D/\Sigma f = +65/198 = 0.328$. The STANDARD DEVIATIONS, σ_O and σ_D respectively are obtained from the general equation

$$\sigma = \sqrt{\frac{\Sigma fd^2}{\Sigma f} - c^2}. \qquad (78)$$

Table 19. Arrangement of data for calculation of partial coefficient of correlation.

					Lb. oil per ton																		
D (= dilution)	f_D	d_D	$f_D d_D$	$f_D d_D^2$	0.65	0.85	1.05	1.25	1.45	1.65	1.85	2.05	2.25	2.45	2.65	2.85	3.05	3.25	3.45	3.65	3.85	Totals	0 (= oil)
					3	14	34	53	39	15	14	10	4	3		3	1	2	2		1	198	f_o
					−4	−3	−2	−1		1	2	3	4	5		7	8	9	10		12		d_o
					−12	−42	−68	−53		15	28	30	16	15		21	8	18	20		12	+8	$f_o d_o$
					48	126	136	53		15	56	90	64	75		147	64	162	200		144	1,380	$f_o d_o^2$
4.9	13	−3	−39	117			5^{30}	4^{12}	2	1^{-3}	1^{-6}												+33
5.3	34	−2	−68	136	2^{16}	4^{24}	6^{24}	6^{12}	5	3^{-6}	1^{-4}	5^{-30}		1^{-10}		1^{-14}							+12
5.7	42	−1	−42	42			7^{14}	16^{16}	14	1^{-1}	3^{-6}					1^{-7}							+16
6.1	45				1	7	3	15	8	6	1	2	1					1					
6.5	22	+1	22	22		2^{-6}	5^{-10}	8^{-8}	5		1^{2}	1^{3}											−19
6.9	15	+2	30	60		1^{-6}	5^{-20}	3^{-6}	2	1^{2}	1^{4}		2^{16}										10
7.3	10	+3	30	90			3^{-18}	1^{-3}	3		3^{18}												−3
7.7	5	+4	20	80						1^{4}	1^{8}	1^{12}		1^{20}		1^{28}							+72
8.1	4	+5	20	100						1^{5}	1^{10}		1^{20}	1^{25}									+60
8.5																							
8.9																							
9.3	4	+8	32	256						1^{8}	1^{-16}	1^{24}						1^{72}					+120
9.7	1	+9	9	81																	1^{108}		+108
10.1																							
10.5																							
10.9	1	+12	12	144															1^{120}				+120
11.3																							
11.7																							
12.1	1	+15	15	225													1^{120}						+120
12.5																							
12.9																							
13.3																							
13.7																							
14.1																							
14.5																							
14.9																							
15.3																							
15.7	1	+24	24	576															1^{240}				+240
Totals	198		+65	1929																			901 32 32

Hence $\sigma_O = \sqrt{\frac{1380}{198} - 0.0016} = 2.64$ and $\sigma_D = \sqrt{\frac{1929}{198} - 0.1074} = 3.10$. The coefficient of partial correlation (r) is derived from the equation

$$r = \frac{p}{\sigma_O \sigma_D}, \quad \ldots \quad (79)$$

in which

$$p = \frac{\Sigma f d_O d_D}{\Sigma f} - c_O c_D. \quad \ldots \quad (80)$$

The individual values of fd_Od_D, with appropriate signs, are shown in small figures in the small co-ordinate rectangles with the corresponding frequencies. These figures are summed horizontally and the totals shown in the last vertical file, and these semi-totals are then summed vertically, yielding the total $\Sigma fd_Od_D = 869$, for the present set of observations. Then $p = 869/198 - 0.04 \times 0.328 = 4.37$, and $r_{OD} = 4.37/2.64 \times 3.10 = 0.535 \pm e_{r_{OD}}$.

Probable error in partial coefficient, $e_{r_{12.3\ldots n}}$ is derived from the formula

$$e_{r_{12.3\ldots n}} = \frac{0.6745(1 - r^2_{12.3\ldots n})}{\sqrt{N}}. \quad \ldots \quad (81)$$

In the present case $e_{r_{OD}} = 0.6745(1 - \overline{0.535}^2)/\sqrt{198} = 0.034$; hence $r_{OD} = 0.535 \pm 0.034$.

Significance of $r_{12.3\ldots n}$ **and** $e_{r_{12.3\ldots n}}$. The numerical value of $r_{12.3\ldots n}$ ranges between -1 and $+1$ as limits, a negative value indicating an inverse relation between the variables 1 and 2 and a positive value a direct relation. The absolute value of r is, however, the significant matter. If $r = 0$, entire lack of correlation is indicated. The significance of other values is not capable of mathematical demonstration, but is based on experience in the use of the method. Rugg (*Statistical methods applied to education, Houghton-Mifflin Co.*) states that in educational statistics a value or r between 0 and $|0.15|$ or $|0.20|$ indicates negligible correlation, a value from $|0.15|$ or $|0.20|$ to $|0.35|$ or $|0.40|$ indicates that correlation is present but of low degree, a value between $|0.35|$ or $|0.40|$ and $|0.50|$ or $|0.60|$ indicates marked correlation, while a value greater than 0.60 corresponds to a high degree of correlation. When the data are numerous, when they may be selected so as to exclude one or more variables that would enter into a consideration of unselected data, and when rejection of obviously erratic values is possible, as is frequently the case in metallurgical data, a more exacting standard of significance of values of r obtains. For such data the following scale is more suitable: 0 to $|0.25|$, negligible correlation; $|0.25|$ to $|0.5|$, low; from $|0.5|$ to $|0.7|$, medium; $|0.7|$ to $|0.9|$, high; $|0.9|$ to $|0.95|$, very high; and $|0.95|$ to $|1.00|$, practically perfect.

The significance of r is also affected by the relative magnitude of r and e. From the definition of e it follows mathematically that 50 per cent. of all values of r will fall between $r + e$ and $r - e$ and that 99+ per cent. will fall between $r + 4e$ and $r - 4e$. If, therefore, e is large with respect to r (r being positive), $r - 4e$ may well be a negative number, from which it follows that the value of r may be zero or thereabouts, indicating negligible or no correlation. In the present problem $r + 4e = 0.669$ and $r - 4e = 0.401$ and medium correlation is indicated.

MULTIPLE CORRELATION

Zero-order coefficients. The coefficient of partial correlation r_{12}, as above developed gives, by definition, an indication of the linearity of the rela-

tion between the variables 1 and 2, on the assumption that the variables 3 . . . n are constants. Since 3 . . . n are, however, variables, this partial coefficient is insufficient basis for judgment as to the actual approach to linearity of the relation between 1 and 2, and it is necessary to go through a further series of operations in which the effect of each variable on the relation between each of the other pairs is eliminated. The first step in this operation is to determine the other coefficients of partial correlation of the zero order, *e.g.*, r_{13}, r_{14}, etc., or, in the present case r_{OA}, r_{AD}, etc., as in Table 20. This is done in the same way

Table 20. Coefficients of partial correlation of zero order (r_{12}, etc.)

$r_{OD} = 0.535 \pm 0.034$	$r_{OR} = -0.288 \pm 0.044$	$r_{CR} = 0.265 \pm 0.045$
$r_{DA} = 0.794 \pm 0.018$	$r_{AR} = -0.196 \pm 0.046$	$r_{CA} = -0.371 \pm 0.041$
$r_{AD} = 0.434 \pm 0.039$	$r_{DR} = -0.070 \pm 0.048$	$r_{OC} = -0.444 \pm 0.038$
	$r_{CD} = -0.124 \pm 0.047$	

A Pounds of acid per ton of ore. *C* Grade of concentrate, per cent. copper. *D* Dilution (ratio of water to ore, by weight). *O* Pounds of oil per ton of ore. *R* Recovery of copper, per cent.

as r_{OD} was determined. The next step is the elimination of the effect of each variable, in order, and the establishment of second-degree coefficients $r_{13.2}$, $r_{12.3}$, $r_{23.1}$, etc., or, taking the data of the present problem $r_{OD \cdot A}$, $r_{OA \cdot D}$, etc. These are called FIRST-ORDER COEFFICIENTS. The first-order coefficient $r_{OD \cdot A}$ indicates the degree of linearity of the relation between lb. of oil per ton and dilution with the effect of acid quantity eliminated; $r_{OA \cdot D}$ similarly measures the relation between the quantities of oil and acid added when the effect of dilution is eliminated; etc., the assumption being still made, as above that the other variables are constant.

Derivation of partial coefficients of higher order is effected by suitable application of the general formula

$$r_{12.3 \ldots n} = \frac{r_{12.3 \ldots (n-1)} - r_{1n.3 \ldots (n-1)}\, r_{2n.3 \ldots (n-1)}}{\sqrt{1 - r^2_{1n.3 \ldots (n-1)}}\ \sqrt{1 - r^2_{2n.3 \ldots (n-1)}}}. \quad . \quad . \quad (82)$$

where n is the total number of variables considered at that step.

First-order coefficients. In the problem in hand the first-order coefficients are derived from those given in Table 20 by application of the general formula as follows, e.g.:

$$r_{OD \cdot A} = \frac{r_{OD} - r_{OA}\, r_{AD}}{\sqrt{(1 - r^2_{OA})(1 - r^2_{AD})}} = \frac{0.535 - (0.794)(0.434)}{(1 - 0.794^2)(1 - 0.434^2)} = 0.345.$$

Similarly the other first-order coefficient shown in Table 21 are calculated.

Table 21. Coefficient of partial correlation of the first order

$r_{OD \cdot A} = +0.345$	$r_{DR \cdot O} = +0.064$	$r_{DC \cdot R} = -0.109$
$r_{OA \cdot D} = +0.738$	$r_{DR \cdot A} = +0.017$	$r_{RC \cdot O} = +0.160$
$r_{DA \cdot O} = +0.015$	$r_{OD \cdot C} = +0.540$	$r_{RC \cdot D} = +0.259$
$r_{OD \cdot R} = +0.538$	$r_{OA \cdot C} = +0.756$	$r_{RC \cdot A} = +0.211$
$r_{OA \cdot R} = +0.786$	$r_{DA \cdot C} = +0.421$	$r_{AC \cdot O} = -0.035$
$r_{DA \cdot R} = +0.429$	$r_{OR \cdot C} = -0.197$	$r_{AC \cdot D} = -0.354$
$r_{OR \cdot D} = -0.298$	$r_{AR \cdot C} = -0.110$	$r_{AC \cdot R} = -0.338$
$r_{OR \cdot A} = -0.224$	$r_{DR \cdot C} = -0.039$	$r_{OC \cdot D} = -0.451$
$r_{AR \cdot O} = +0.055$	$r_{DC \cdot O} = +0.151$	$r_{OC \cdot A} = -0.266$
$r_{AR \cdot D} = -0.184$	$r_{DC \cdot A} = +0.044$	$r_{OC \cdot R} = -0.398$

Second-order coefficients ($r_{12.34}$, $r_{13.24}$, etc.) are similarly calculated from the first-order coefficients, a typical example of the application of the general formula being

$$r_{OD \cdot AR} = \frac{r_{OD \cdot A} - r_{OR \cdot A}\, r_{DR \cdot A}}{\sqrt{(1 - r^2_{OR \cdot A})(1 - r^2_{DR \cdot A})}} = \frac{0.345 - (-0.224)(0.017)}{\sqrt{(1 - (-.224)^2)(1 - (0.017)^2)}} = +0.358.$$

The values of the second-order coefficients for the present problem are given in Table 22.

Table 22. Coefficients of partial correlation of the second order

$r_{OD \cdot AR} = +0.358$	$r_{OC \cdot DR} = -0.406$	$r_{DC \cdot AR} = +0.042$
$r_{OD \cdot AC} = +0.371$	$r_{OC \cdot AR} = -0.230$	$r_{AR \cdot OD} = +0.054$
$r_{OD \cdot RC} = +0.544$	$r_{DA \cdot OR} = +0.011$	$r_{AR \cdot OC} = +0.061$
$r_{OA \cdot DR} = +0.728$	$r_{DA \cdot OC} = +0.020$	$r_{AR \cdot DC} = -0.102$
$r_{OA \cdot DC} = +0.693$	$r_{DA \cdot RC} = +0.419$	$r_{AC \cdot OD} = -0.033$
$r_{OA \cdot RC} = +0.756$	$r_{DR \cdot OA} = +0.063$	$r_{AC \cdot OR} = -0.045$
$r_{OR \cdot DA} = -0.247$	$r_{DR \cdot OC} = +0.041$	$r_{AC \cdot DR} = -0.322$
$r_{OR \cdot DC} = -0.210$	$r_{DR \cdot AC} = +0.008$	$r_{RC \cdot OD} = +0.152$
$r_{OR \cdot AC} = -0.179$	$r_{DC \cdot OA} = +0.152$	$r_{RC \cdot OA} = +0.158$
$r_{OC \cdot DA} = -0.305$	$r_{DC \cdot OR} = +0.143$	$r_{RC \cdot DA} = +0.211$

Third-order coefficients ($r_{12.345}$, $r_{13.245}$, etc.) are similarly calculated from the coefficients of the second order. A typical example applicable to the present problem is

$$r_{OD \cdot ARC} = \frac{r_{OD \cdot AR} - r_{OC \cdot AR}\, r_{DC \cdot AR}}{\sqrt{(1 - r^2_{OC \cdot AR})(1 - r^2_{DC \cdot AR})}} = \frac{0.358 - (-0.230)(0.042)}{\sqrt{(1 - (-0.230)^2)(1 - (0.042)^2)}} = 0.379.$$

The values of the third-order coefficients in the present problem are given in Table 23.

Table 23. Coefficients of partial correlation of the third order

Symbol	Coefficient	Probable error
$r_{OD \cdot ARC}$	+0.379	±0.041
$r_{OA \cdot DRC}$	+0.692	±0.025
$r_{OR \cdot DAC}$	−0.197	±0.046
$r_{OC \cdot DAR}$	−0.267	±0.044
$r_{DA \cdot ORC}$	+0.017	±0.048
$r_{DR \cdot OAC}$	+0.040	±0.048
$r_{DC \cdot OAR}$	+0.144	±0.047
$r_{AR \cdot ODC}$	+0.060	±0.048
$r_{AC \cdot ODR}$	−0.042	±0.048
$r_{RC \cdot ODA}$	+0.154	±0.047

Significance of $r_{OD \cdot DRC}$ and $e_{r_{OD \cdot ARC}}$, etc., in Table 23 is, according to the standard set up on p. 1259, as follows:

1. The correlation between oil and acid is medium-high. This was, of course, to be expected from the facts of plant operation, where the usual tendency of the operators was to increase both oil and acid when froth overflow became sparse and to decrease the two together when froth overflow was too rapid.

2. The correlation between oil and dilution is low but distinct. Physically this low correlation is in part explained by the fact that changes in oil lagged behind the cause that required changes. Increase in dilution, if the quantity of oil fed remains constant, causes decrease in frothing and in carrying power of the froth, or what the flotation operator describes as an under-oiled condition, and increase causes the reverse condition, which he describes as over-oiling. But these changes in machine performance lag considerably behind the changes in dilution causing them, hence for certain periods high dilution and low oil quantity go together and for certain other periods low dilution and high oil, notwithstanding that the operator endeavors to maintain high oil with high dilution and *vice versa*. Mathematically, also, the correlation suffers in that grade of feed, which is another variable affecting oil quantity, is not considered in the correlation. If it were considered and properly eliminated, the effect should be to raise $r_{OD \cdot A} \cdots$

3. Correlation between quantity of oil and recovery and that between quantity of oil and grade of concentrate is in the zone of low to negligible, according to the data analyzed. The interpretation of this result is that under the conditions of the mill operations studied, changes of one- or two-tenths of a pound of oil made little difference in recovery or grade of concentrate. The negative sign of these two coefficients indicates, in so far as the low value of the coefficient permits any distinct indication, that increase in oil in the particular operations resulted in decrease in both grade and recovery.

4. Negligible correlation is indicated in all of the other cases.

Coefficient of multiple correlation is a number (R) that measures the degree of completeness with which the variations of any one variable are accounted for by consideration of the other variables entering into the calculation. The value of R for any variable ranges between 0 and +1. In any problem its value must be greater than the absolute value of the greatest of the partial coefficients involving the same variable. It is obtained from the formula

$$R = \sqrt{1 - (1 - r^2_{12})(1 - r^2_{13.2})(1 - r^2_{14.23}) \ldots (1 - r^2_{1n.234) \ldots (n-1)}} \quad (83)$$

In the present example

$$\begin{aligned} R_O &= \sqrt{1 - (1 - r^2_{OD})(1 - r^2_{OA\cdot D})(1 - r^2_{OR\cdot DA})(1 - r^2_{OC\cdot DAR})} \\ &= \sqrt{1 - (1 - \overline{0.535}^2)(1 - \overline{0.738}^2)(1 - [-0.210]^2)(1 - [-0.267]^2)} \\ &= 0.844. \end{aligned}$$

Other values of R are:

$$R_D = 0.553; \quad R_C = 0.487; \quad R_R = 0.333; \quad R_A = 0.795.$$

Multiple correlation is very high if R is larger than 0.90, high from 0.75 to 0.90, medium from 0.60 to 0.75, low from 0.30 to 0.60, and negligible below 0.30.

Reliability of coefficient of multiple correlation is to be judged from the probable error, obtained by the formula

$$E_R = \frac{k(1 - R^2)}{\sqrt{N}}, \quad (84)$$

where k is a coefficient given by Table 24 (Pearson, *6 Biometrika 59*).

Table 24. Value of coefficient k in equation for E_R

$n' - 1$	k
1	0.674
2	1.177
3	1.538
4	1.832
5	2.086
6	2.313
7	2.519
8	2.710

n', Number of values of R investigated.

For E_{R_O}, $K = 1.832$, $R_O = 0.844$, $N = 198$ and

$$E_{R_O} = \frac{1.832(1 - \overline{0.844}^2)}{\sqrt{198}} = \pm 0.037.$$

$$E_{R_D} = \pm 0.090; \; E_{R_C} = \pm 0.099; \; E_{R_R} = \pm 0.116 \text{ and } E_{R_A} = \pm 0.048.$$

Hence the final values for R are $R_O = 0.844 \pm 0.037$; $R_D = 0.553 \pm 0.090$; $R_C = 0.487 \pm 0.099$; $R_R = 0.333 \pm 0.116$; $R_A = 0.795 \pm 0.048$. These figures indicate that the variations in oil and acid are accounted for with a high degree of completeness and that the values of R are reliable; variations in dilution and grade of concentrate are not particularly well accounted for, and the data are highly incomplete in so far as the investigation of recovery is concerned.

SECTION 23

DESIGN AND CONSTRUCTION OF ORE-TREATMENT PLANTS

BY

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1. Situation of mill with respect to mine

General Principles. Since one of the main purposes of concentration is to reduce the expense of delivering mine product to the smelter, a concentrating mill should ordinarily be situated as close as practicable to the source of its ore supply. In general those mines at which the ratio of concentration is high are most benefited by placing the mill near the mine, but this conclusion may be modified by one or more of the following facts: (*a*) freight rates of public carriers usually advance in direct relation to the unit value of the material carried (see Table 10); (*b*) mill concentrate produced by wet methods may carry from 4 to 15 per cent. water, even after draining or filtering; (*c*) the site near the mine may lack adequate supply of mill water with low expense for elevation; (*d*) suitable space for tailing disposal, and (*e*) advantageous topography for a mill site. Examples of a variety of practices are given in Table 1.

In the case of a large-scale operation, the ability to move great tonnages by the most efficient available motive power over privately owned track to a mill adjacent to or in the direction of the smelter, may offset some of the advantages of proximity of mill to mine. Some notable examples of long hauls are given in Table 2.

Of the mills cited, those of ANACONDA, NEVADA CONSOLIDATED, and RAY CONSOLIDATED are closely adjacent to smelters; water supply was an important consideration in selecting sites for the CHINO, COPPER RANGE, and NEVADA CONSOLIDATED mills; while the disposal of tailing contributed largely to the selection of sites for NEVADA CONSOLIDATED and the

Table 1. Concentration ratios and mill location

Mine and mill	Assay of ore	Distance hauled to mill, miles	Ratio of concentration
Calumet and Hecla	1.75% Cu	5	35 : 1
Copper Range	1.88% Cu	14	40 : 1
Chino	1.60% Cu	10	14–22 : 1
Inspiration	1.14% Cu	1.6	27.5 : 1
Miami	2.00% Cu	0	27 : 1
Ray	1.66% Cu	25	13–15 : 1
Federal No. 3	3.00% Pb	1–2	27 : 1
Federal No. 4	4.75% Pb	0	18.5 : 1
Engels	2.25% Cu	0 and 2.5	15 : 1
St. Joseph Lead Co., Bonne Terre	4.00% Pb	0	20 : 1
Joplin District	0.5–1.25% Zn	0–0.5	35–55 : 1
Alaska-Gastineau	....	3.5	1157 : 1
Mascot	3.77% Zn	0 and 0.3	19.7 : 1
Cananea, Mex	1.76% Cu	1–3	3.5 : 1
Hedley, B. C.	$10.50 gold (*a*)	3.5	16–20 : 1
Moctezuma, Mex	3.3% Cu	5	4.5 : 1
Phelps-Dodge, Morenci	1.8% Cu	0–0.75	9 : 1
Timber Butte	15% Zn	....	3.5 : 1
Utah Copper	1.2% Cu	18–20	17.5 : 1
Britannia Mining and Smelting	1.6% Cu	5	10 : 1

Mine and mill	Assay of concentrate	Water in concentrate, per cent.	Distance concentrate hauled, miles
Calumet and Hecla	62 % Cu	12	0.5
Copper Range	65 % Cu	7.5	19
Chino	15.5% Cu	14	130
Inspiration	30.2% Cu	15	1
Miami	42 % Cu	10–12	1
Ray	19 % Cu	12	1
Federal No. 3	Table, 70% Pb	4	100
	Flot., 49% Pb	6	100
Federal No. 4	Table, 73% Pb	4	100
	Flot., 55% Pb	6	100
Engels	30.0% Cu	12	600
St. Joseph Lead Co., Bonne Terre	70.0% Pb	5	30
Joplin District	60.0% Zn	....	50–1000
Alaska-Gastineau	33% Pb; Au,16oz.; Ag,32oz.	10	2000 *b*
Mascot	60% Zn	8	550
Cananea, Mex	5.6% Cu	10	1
Hedley, B. C.	As,35%; Au,2.5oz.	10	400
Moctezuma, Mex	12.8% Cu	5	78
Phelps-Dodge, Morenci	12.0% Cu	Table, 9.5; Flot., 13.0	0.75
Timber Butte	54% Zn	8.5	1200
Utah Copper	16.5% Cu	13–14	3
Britannia Mining and Smelting	12.0% Cu	10–12	130*b*

a In arsenopyrite. *b* Steamer.

Utah Copper Co.'s mills. In no case, however, was the decision based upon one favorable condition alone.

Placing a mill near the mine it serves, tends to minimize irregularities in receipt of ore. The ill effect of mill stoppage for lack of ore can be guarded against to a limited extent by providing storage bins, but it is seldom practicable to store more than two or three days' supply, and in the case of some

Table 2. Haulage of ore over privately controlled standard-gage railways

Mine or carrier	Year	Miles hauled	Tonnage	Cost		
				Total	Per ton, cents	Per ton-mile, cents
Butte, A. & P. (*d*)	1922	32	90,348,000*a*	$991,732*b*	......	1.098*c*
Chino (via A. T. & S. F.)	1922	10	1,416,869	154,198	10.8	1.08
Copper Range	1922	14	33,396,000*a*	884,483*b*	......	2.648*c*
Duluth and Iron Range	1922	227	507,545,000*a*	6,037,000*b*	......	1.190*c*
Duluth, Missabe and Nor.	1922	335	1,194,982,000*a*	13,301,000*b*	......	1.113*c*
Nevada Consolidated	1922	26	518,523	195,899	37.7	1.45
Ray Consolidated	1922	25*e*	1,178,350	258,437	22.0	0.88
United Verde	1918	6.7	861,250	198,687	23.0	3.4
United Verde Ext.	1922	6.1	136,980	22,148	16.2	2.6
Utah Copper	1922	18	4,364,251	651,096	15.0	0.83

a Revenue-ton-miles. *b* Freight earnings. *c* Freight earnings per revenue-ton-mile. *d* Electrically operated; see page 1267. *e* Of which 10 miles over private road; remainder by track license over Arizona Eastern R. R.

large mills, operations would cease if the ore supply were interrupted for a single day (see Table 12).

METHODS OF TRANSPORTING ORE TO MILL

The following means are available: 1. Mine-car or other small-car tramming: (*a*) hand; (*b*) animal; (*c*) cable; (*d*) storage-battery locomotive; (*e*) trolley locomotive; (*f*) gasoline locomotive; (*g*) narrow-gage steam locomotive; (*h*) mono-rail system. 2. Wagon or truck haulage: (*a*) animal; (*b*) motor trucks; (*c*) tractors and trailers. 3. Belt conveyors. 4. Aerial tramway: (*a*) reversible; (*b*) continuous. 5. Standard-gage railroad.

Hand tramming is frequently adopted at small mines for distances up to 300 ft., especially when the mine cars themselves can be conveniently trammed to the mill. For such surface tramming, the track can usually be maintained in better condition than underground, and can be laid to the most advantageous grade, say 0.6 per cent. in favor of the loaded run. On well-graded track, and with the usual metal-mine car in good condition, one man can push a net load of 2000 lb. at speed of 1½ miles per hour (130 ft. per min.); maintaining the same speed on the return trip would give an ore-carrying capacity of 0.75 ton-mile per hour *in motion*. Dumping time, which is approximately ½ min., will range from 40 per cent. of the total working time (excluding loading) on a 50-ft. tramming distance, to 10 per cent. on a 300-ft. tram. Loading time will vary between the extremes of 2 min. (from chutes) to 30 min. per ton (shoveling from rough bottom). (For further data on loading and tramming by hand, see *Peele*.)

Animal tramming is applied to trains of mine cars, and also to one or several cars of large size receiving ore from a dumping or storage pocket. The method is not more efficient than hand tramming over short distances and for traffic of less than two ore-ton-miles per hour, but competes favorably with mechanical systems up to 25 ore-ton-miles per hour, at distances up to ½ mile and with grades below 1.25 per cent. On good track, with moderate grades in favor of the load, one mule can pull a gross load of 7 to 12 tons, and haul

from 2 to 4 ore-ton-miles per hour of total working time at distances of 700 to 3000 ft. The tractive force of an average horse, at 2.5 miles per hour for 8 hours, is 125 lb.

Cable haulage may be applied to gravity plane, engine plane, or tail-rope system.

Gravity plane. Loaded cars, of modern design and well lubricated, will run freely down a grade of 2 per cent., but if empty cars are to be hauled up by the action of the descending loads, transmitted through a cable, 5 to 5½ per cent. is about the minimum grade that will operate with a plane 500 ft. long; longer planes require steeper grades, up to 10 or 15 per cent. Cars or skips discharging automatically into a receiving bin at the bottom of the incline are frequently used.

Engine plane may be operated on any grade steep enough to allow the empty trip to descend by gravity, while overcoming the added friction of cable and engine drum. In practice, 3½ per cent. has been found to be the minimum satisfactory grade.

The NIPISSING low-grade mill, at Cobalt, receives part of its ore by an engine plane 2200 ft. long, hauling a trip of 4 cars, or 3.85 tons of ore, up a steady grade of 10 per cent., with a 30-hp. electric hoist and a ½-in. cable. Operating at 90 per cent. capacity, the plane makes one trip in 25 min. The total cost of operation (1922) was 34.2¢ per ton hauled, or 82¢ per ton-mile. The cost of installation, in 1913, was $6500. At the TROJAN mine, S. D., one electric hoist is used both to haul trips of eight 1.5-ton cars up a 1500-ft., 6 per cent. slope out of the mine, and to lower them 2000 ft. down a 15 per cent. incline to a gathering station from which they are hauled to the mill by gasoline locomotive.

Tail-rope system is best adapted to nearly level or irregular grades, and can be applied to curved track; it is commonly used at coal mines.

A well-known application to surface tramming in the United States is in the TRI-STATE DISTRICT, hauling ore from one or more outlying shafts to a central mill, an average distance of 800 ft., with ½ mile as a maximum (10-ton steam and 5-ton gasoline locomotives are used for the same service). Track is 36-in. gage with 40-lb. rail, and cost (1923) an average of $1.56 per ft., including grading, laying, and all track materials. Cars are of steel, with roller bearings, and of 2.5- and 3.5-ton capacity (44 and 62 cu. ft.). At GOLDEN ROD mill (*H. H. Wallower, PC*) a 14-car train with gross weight of 60 tons is hauled by a tail-rope system a distance of 1800 ft. over level track containing frequent curves. The driver is a band-friction, tandem, double-drum hoist, belt-driven by 75-hp. motor; after starting, little power is needed. The cost of this method is compared with that of steam-locomotive haulage on the same property during 1923 in Table 3.

Table 3. Haulage costs at Golden Rod Mill, 1923

	Cable system	Steam locomotive
Distance, ft.	1800	2600
Grades	Level	900 ft. @ 1% (Remainder level)
Tons trammed	144,019	114,179
Cost per ton, cents:		
Labor	4.09	6.06
Power	0.27	0.93
Supplies	1.82	3.54
Total	6.18	10.53
Total cost per ton-mile, cents	18.13	21.40

Limitations of the tail-rope system as to grades and distances have not been reached in the Tri-State district; curves present slight difficulty.

Storage-battery locomotives might be advantageous to haul trains of ore cars directly to the mill from a mine in which battery locomotives for any reason are preferred underground. Gages range from 18 to 44 in. In general, battery locomotives are equipped with motors aggregating 4 hp. per ton of total weight; the range is from 2.5 to 8 tons. On clean rails they exert a drawbar pull of one-fourth their weight when moving at 3½ miles per hour; they have been operated effectively on grades up to 10 per cent., but in general they give best results on moderate grades.

In the BUTTE & SUPERIOR mine, in narrow, tortuous drifts with irregular grades (*123 P 751*), a 3½-ton locomotive, exerting a draw-bar pull of 1500 lb., at 4 miles per hour, hauls a train of 10 roller-bearing cars, each of 2250 lb. gross weight. During 1919, nine locomotives hauled 737,787 cars of ore and waste an average distance of 790 ft., at cost per car-trip of: power, 0.41¢; locomotive repairs, 3.21¢; motorman and helper, 6.90¢; total, 10.52¢.

Exhaustive tests at COPPER QUEEN showed that battery locomotives required 1.6 kw.-hr. at the power house per ore-ton-mile. In the UNITED VERDE mine (*66 A 177*) Baldwin-Westinghouse 3-ton locomotives gave a drawbar pull of 800 lb. at 3½ miles per hr. Batteries (80 Edison A–4 cells) were re-charged after each shift, with 250-volt current. Hauling 215 tons ore per shift, a distance of 200 ft. up 0.4 per cent. grade, the cost in cents per ton was (1918): Labor (1 motorman and 2 loaders), 8.4; power (25 kw.-hr. per shift), 0.4; depreciation of motor, 0.6; repairs and inspection, 0.2. Total, 9.6.

Trolley locomotives, in addition to hauling trains of ore cars direct from mine workings to the mill, where this is possible, are often installed independently for surface haulage from a loading pocket at the mine, as at ALASKA GASTINEAU, BRITANNIA, HEDLEY, MELONES, TONOPAH EXTENSION, BUNKER HILL & SULLIVAN, and many other mines. For this purpose, more powerful and wider-gage locomotives are applicable than for mine haulage and higher voltages are permissible. Trolley locomotives are usually equipped with motors aggregating 10 hp. per ton of total weight (which ranges from 3 to 30 tons), and give a draw-bar pull, with steel-tired wheels, estimated at 25 per cent. of the total weight of the locomotive.

At UNITED VERDE mine, Jerome, Ariz. (*66 A 177*) in the Hopewell main-haulage tunnel, 25-ton Baldwin-Westinghouse locomotives are used, equipped with two 75-hp. 250-volt, direct-current motors and exerting a draw-bar pull of 12,500 lb. at 7.1 miles per hr.; total haul, 8900 ft. Bottom-dump, standard-gage cars of 220-cu. ft. nominal capacity are loaded from chutes with 280 cu. ft. or 20 tons of heavy sulphide ore. The trains are 14 cars, making 425 tons gross per locomotive. The cost in cents per ton for 1918, on 861,250 dry tons, was: Labor, including loading and dumping, 5.4; supplies, 0.1; power, 0.5; repairs, 3.5. Total, 9.5, or 5.6¢ per ore-ton-mile. At ALASKA-JUNEAU (*120 P 251*) the mine output was hauled two miles down a 0.5-per cent. grade, 30-in. gage, double-track, 32-car trains; each car weighed 3 tons empty and carried 10 tons of ore; total weight of train, 96 tons empty, 416 tons loaded. The locomotive was in two sections, of 9 tons each, having two direct-current Westinghouse No. 905 motors in each unit and was small enough to pass through a rotary car dump discharging four cars at a time. The front section of the locomotive could be quickly detached and used for switching. Power for operation was obtained from two 300-kw. rotary converters, each with an individual 6-phase transformer.

For other data on performance and operating costs of trolley locomotives, see *Peele*.

The BUTTE, ANACONDA & PACIFIC RY. transports ANACONDA ore (over 5,000,000 tons per year) from mines on Butte Hill to the Washoe concentrator at Anaconda; total haul, 32 miles (*Elec. Ry. Jour.*, Mar. 14, 1914). The General Electric locomotives used weigh 80 tons, have 1050-hp. continuous rating, and exert a tractive force of 25,000 lb. at 15 miles per hr. Trolley wire is No. 0000 and carries 2400 volts direct-current. The cars weigh 18 tons and carry 50–53 tons of ore; two locomotives per train. Operating data are given in Table 4.

This road, formerly steam-operated, was electrified during 1912–13, at a cost of $1,-201,000; the first year's electric operation indicated a saving of 36 per cent. in direct costs, as shown in Table 5. In the item of power alone, the saving was 48 per cent. compared with steam operation. The saving in trainmen's wages was due principally to the higher speed and greater regularity in train movement.

Table 4. Haulage of Anaconda ore by Butte, Anaconda and Pacific Ry.

	On Butte Hill, Mines to Rocker	Main Line, Rocker to E. Anaconda	On Smelter Hill, E. Anaconda to Concentrator
Distance, miles	4.6	20.1	7.2
Average grade, loaded	−2.5%	−0.3%	+1.1%
Cars in train	30	65	20
Gross train load, tons	2000	4620	1400
Speed, miles per hour	12	16–21	16

Table 5. Comparative costs of steam and electric haulage at Anaconda

	1913, steam	1914, electric
Locomotive operation only (fuel or power, oil, crew, maintenance, etc.)	$594,921	$357,339
Trainmen's wages	147,632	116,486
Total direct operation	$742,553	$473,825
Ton-miles hauled, net	158,917,720	172,855,856
Cents per ore-ton-mile	0.467	0.274

Gasoline locomotives are built up to 15 tons in weight, with draw-bar pulls of 20 per cent. their weight and geared to speeds of 6 to 12 miles per hour. The makers estimate gasoline consumption at 0.1 gal. (0.6 lb.) per hp.-hr.; tests have indicated 0.73 to 1.2 lb. of gasoline per brake hp.-hr. at full load and full speed, 1.2 to 2.2 lb. at half load and half speed, and higher consumptions for reduced loads.

At numerous bituminous-coal mines and a few metal mines of the United States (*Peele*) with hauls of 1800 to 3400 ft., the operating cost varies from 2.5 to 5¢ per mineral-ton-mile.

At the New York Zinc Co., Edwards, N. Y. (*W. R. Wade, PC*), a Fate-Root-Heath 25-hp. gasoline locomotive hauls a train of 2 or 3 cars, each weighing 3000 lb. empty and carrying 3 tons of ore, an average distance of 800 ft. down a 0.5-per cent. grade at a speed of 15 miles per hour. The track is 3-ft. gage with 25-lb. rails. The locomotive also hauls waste (about one-fourth of the ore tonnage) an average distance of 400 ft. In a 9-hr. shift, it averages 250 tons ore and waste. Gasoline consumed, 1 gal. per hour; estimated operating expense (excluding wages), $2 per shift; repairs, not over 0.1¢ per ton. Allowing wages of $4 a day for one engineer, the estimated total direct cost for haulage only is 17.7¢ per ton-mile. The first cost of the locomotive was $1815. At the Trojan mine, S. D., a Milwaukee gasoline locomotive hauls trains of eight 1.5-ton cars a distance of 2000 ft. to the mill at cost of 6¢ per ton (1922) or 15.8¢ per ton-mile.

Narrow-gage steam locomotives are found in many districts, hauling for long distances, where rough topography makes construction of standard-gage roadbed unjustifiably expensive. A simple steam locomotive consumes 4.5 to 8 lb. of coal or 2 to 5 lb. of fuel oil, and 27 to 32 lb. water per hp.-hr. while in motion; the total fuel consumption, covering delays, may be 50 to 100 per cent. greater.

At Bawdwin mines, Burma, the total length of narrow-gage line is 46 miles with grades of 4 to 5.5 per cent. in places and numerous sharp curves. Gage is 2 ft. The principal traffic is down-grade, 33 miles, at a cost of 7¢ per ton-mile.

Mono-rail system, well developed in Europe, has only one noteworthy installation in the United States, *viz.*: near Searles Lake, Cal. (*116 J 100.*)

The total length is 28 miles, of which 8 miles is level but the remainder traverses extremely rugged country, requiring sharp curves and grades of 8 to 10 per cent. The wooden

framework supporting the rail is seldom more than 3 ft. high. Estimated cost was $8000 per mile. A specially fitted Fordson tractor makes 10–15 miles per hour on the level, and 8 miles per hr. up grade. Cost of transportation was estimated at less than $1 per ton.

Horse-drawn wagon haulage. For general data on loads, speeds, grades, tractive resistance, road surfaces, etc., see *Peele.*

At Philipsburg, Mont., the ALGONQUIN mine formerly shipped manganese ore 2.7 miles to the mill by 6- or 8-horse wagon and trailer carrying together 5 to 6 tons. The team made two round trips per day. In the DOLORES district, Colo., picked vanadium ore is hauled on contract by 4- or 6-horse wagon and trailer 50 to 90 miles to Placerville. The average load is 1000 to 4000 lb. per horse, depending on grades and road condition. Speed on rough roads, 16 miles per day; on good roads, 22 to 25 miles per day. During the early development of the MAGMA mine (1914) the rate for wagon haulage of supplies, 32 miles up-grade from Webster, was $10 per ton; when the wagons hauled both ways the rate was $8 up, $5 down.

Motor trucks of 1- to 6-ton capacity are rapidly supplanting horse-drawn wagons where the roads are suitable.

In the vanadium district of SOUTH-WEST COLORADO, 5-ton trucks, generally overloaded, make from 50 to 80 or 90 miles per day over roads in fair condition.

At the SILVER DYKE mine, Neihart, Mont. (*W. E. Wampler, PC*), teaming contractors haul concentrate 3.5 miles down a mountain road having an average grade of 4 per cent. for $1.50 per ton, returning with freight, against a maximum adverse grade of 7 per cent., at $2 per ton; they use wagons, sleds, or motor trucks, according to the condition of the road. The motor trucks are 3-ton International, but are usually loaded with 5 tons; they make three round trips in an 8-hr. shift, loaded both ways. Driver's wages, $5.50 per shift; gasoline consumption, 8 gal. per shift; oil, 1 quart. The original cost of the truck (1923) was $3600.

Contractors hauling supplies to the MOGOLLON mines from Silver City, N. M., 75 miles over good roads, use White trucks of 2- and 5-ton capacity (*72 A 533*). The total freight hauled in 1922 was 1484 tons at a cost of 26.5¢ per ton-mile. The contractors haul concentrate by motor trucks from the BETTY O'NEAL mill, Lewis, Nev., to Battle Mountain, over 12 miles of dirt road in good condition, except during rains, for $5 per ton, or $0.42 per ton-mile (*117 J 449*). ARMSTEAD mill, Talache, Id. (*A. H. Burroughs, Jr., PC.*), ships concentrate to Eagle, 6.5 miles over good gravel road down an average grade of 2.5 per cent. with maximum of 6 per cent., in 2½-ton Standard motor trucks usually loaded with 3 tons. During 1923, 4160 tons of concentrate was hauled down and 2000 tons freight up. Contract price, $1.80 per ton in summer, $2 in winter (aver. 30¢ per ton-mile). More recently, a 5-ton GMC truck has proved cheaper to operate.

Tractors and trailers move at relatively slow speed, but can haul heavy loads over roads rough enough to be damaging to motor trucks.

At Philipsburg, Mont., the ALGONQUIN mine shipped manganese ore 2.7 miles to the mill in trains of five 6-ton trailers drawn by a 75-hp. Holt caterpillar tractor, making 3 trips or 90 tons a day, at a cost of 72¢ per ton, or 27¢ per ton-mile. TREADWELL-YUKON Co., Mayo district, Yukon Terr., employs a 10-ton Holt caterpillar tractor to haul 4 sleds loaded with 33 to 54 tons of sacked ore a distance of 42 miles down very rough mountain road, with frequent grades that require splitting the load. During 5 months (November, 1922, to Mar., 1923) the train made 54 round trips (4636 miles) delivering 2501 tons ore (46 tons per trip). The temperature ranged from $+10°$ to $-50°$ F. and snow was 4 ft. deep on the level but there was no interruption to tractor service. The loaded trip required 20 to 24 hr., the return trip with a light load of supplies, 16 to 20 hr. Gasoline consumption was 2 gal. per mile loaded; 1 gal. per mile returning. Total cost (fuel, oil and wages, including return trips) was $2.60 per ton, or 6.2¢ per ore-ton-mile.

Belt conveyors are frequently used for bringing ore into a mill from a hoist pocket, train-dump pocket, or coarse-crushing plant. They may also serve for hand-picking of the ore in transit, and to elevate to the top of the mill. For details of construction, see Sec. 20, Art. 1. With a 36-in. belt, and material of 100 lb. per cu. ft., 1 ton-mile of transport on level ground requires 1.25 hp.-hr. of work; in general, for lighter materials and narrower belts, 1 hp.-hr. moves 1 ton 1 mile, on a level haul.

A down-hill belt conveyor was installed by the MOUNTAIN COPPER Co. to convey the pyrite output of its Hornet mine, crushed to 4-in. maximum size, a vertical distance of 189 ft. from mine level to railroad track in a horizontal distance of 300 ft. The two terminal bins and an intermediate tower bin supplied all but 60 ft. of the vertical drop, which was divided between two conveyors. The upper, 24 in. wide, sloped 9° 40′ and traveled 125 ft. per min., governed by a small motor and two sets of spur gears; the lower, 20 in. wide, sloped 14° 40′, traveled 150 ft. per min., and was connected to the same shaft that drove a trommel (*119 P 638*). In a similar manner, at the ENGELS mill (*123 P 183*) flotation concentrate (70 tons, dry, per day) carrying 10 to 15 per cent. water is lowered a vertical distance of 100 ft. on a 12-in. belt sloping 34°.

Reversible aerial tramways are designed for distances up to 2000 ft., and bucket loads of 400 to 2500 lb. Intermediate supports may be used, but the speed is then limited to 1000 ft. per min.; on an unsupported span, 1500 ft. per min. is safe. There may be one or two carrier cables; in the latter case the economical limit is approximately 20,000 ft.-tons per hour; for a single cable the capacity is less than 10,000 ft.-tons per hour. (*Peele*). Data supplied by the American Steel and Wire Co. are given in Table 6.

Table 6. Capacities and weights of reversible tramways

Load per bucket, lb.		400	800	1500	2000	2500
Diameter of track cable, in.		⅞	1⅛	1⅜	1½	1⅝
Diameter of tractor rope, in.		⅜	½	½	½	9/16
Length, feet	Minutes per trip (*a*)					
Single-cable tram		Capacity, tons per hour				
500	2.4	5	10	18.7	25	31.2
1000	4.0	3	6	11.2	15	18.7
2000	7.5	1.6	3.2	6.0	8	10.0
		Weight of entire equipment, tons (*b*)				
500	....	4.62	5.22	6.20	6.76	7.24
1000	....	5.64	6.64	8.20	9.23	10.03
2000	....	7.67	9.48	12.20	14.15	15.60
Double-cable tram		Capacity, tons per hour				
500	1.5	8	16	30	40	50
1000	2.4	5	10	18.8	25	31.2
2000	4.0	3	6	11.2	15	18.8
		Weight of entire equipment, tons (*b*)				
500	....	7.26	9.71	11.04	11.89	12.79
1000	....	8.87	12.02	14.32	15.92	17.42
2000	....	12.10	16.65	20.90	24.00	26.70

a Based on travel of 600 ft. per minute, with allowance of 40 to 50 sec. for loading and dumping. *b* Weight may be used for estimating cost of equipment at factory, since unit price per pound is practically constant throughout the stated ranges of length and capacity; in 1917 it was quoted as 19¢ per pound.

At the OTTAWA mill, near Slocan, B. C., a two-bucket reversible gravity tram 200 ft. long, with rated capacity of 10 tons per hour, cost $8000 in 1921, including the erection of terminals and clearing of way. Rugged country and local scarcity of timber added to the expense. Cost of tramming was 10¢ per ton.

Continuous aerial tramways often afford the only practicable means of conveying ore from the mine or concentrates from the mill in steep or rough

districts. The economical minimum limit of capacity is 10 tons per hour; the maximum limit varies from 100 tons per hour in general to 200 tons under exceptionally favorable conditions. They are not usually warranted for distances of less than 1000 ft. and the maximum length of a unit is about 4 miles; two or more units can be erected in tandem, with transfer of buckets for greater distances. Anchor and tension stations must be situated at intervals of 3000 to 5000 ft. Loads per bucket range between 500 and 2000 lb., and three buckets per minute is about as rapidly as they can be handled at terminals; two buckets per minute is nearer the average. (For additional data see *Peele*.) Structural and general operating data supplied by the American Steel and Wire Co. are given in Tables 7 and 8. Data on specific operations are given in Table 9.

Table 7. Trenton-Bleichert continuous tramways

Capacity, tons per hour	Cable diameter		Carrier buckets			Weight of line and buckets, pounds per 100 ft. (*b*)	Weight of Terminal equipment, tons (*c*)	Weight of anchor and tension stations, tons (*d*)
	Tracks	Traction	Load, pounds (*e*)	Spaced, ft.	Number per hour (*a*)			
10– 25	1 and ⅞	½	500	750–300	40–100	832– 970	9.9	3.55
25– 50	1⅛ and ⅞	½	600	360–180	84–168	1035–1205	9.9	3.55
50–100	1⅜ and ⅞	⅝	1200	360–180	84–168	1360–1627	14.0	4.45
100–150	1½ and ⅞	⅝	1500	225–150	133–200	1451–1867	14.4	4.55
150–200	1½ and 1	¾	2000	200–150	150–200	1964–2164	17.6	4.55
Approximate cost.......	....	......	....	....	....	*f*	*g*	*h*

a At speed of 500 ft. per minute. *b* Locked-coil track cables; lang-lay, crucible-steel traction rope; buckets and carriers with Webber grips; tower saddles; rollers and bolts, telephone line and instruments; that is, all equipment proportioned to length of tramway, but not including towers. *c* Complete equipment for both terminals, non-automatic type, but not including bins nor power connections. *d* Complete equipment for anchor and tension station; one such station required for each mile of length or major portion thereof beyond the first mile. *e* Material of 100 lb. per cubic foot. *f* Cost per pound for line equipment quoted at 19 to 19.8¢ in 1917. *g* Cost per pound for terminal equipment quoted at 18.7¢ in 1917. *h* Cost per pound for anchor and tension stations quoted at 12 to 12.7¢ in 1917.

Table 8. Labor and other charges for operating Trenton-Bleichert tramways

Capacity, tons per hour	Men required per shift (*a*)				Man-hour per ton	Additional charges
	Loading terminal	Discharge terminal	Line rider or other help	Total		
10	1	1	....	2	0.20	If direct labor cost be
25	1	1	1	3	0.12	taken as 100%, add for:
40	2	1	1	4	0.10	Supplies and renewals. 18%
60	2	2	1	5	0.08	Repairs............. 33%
75	3	2	1	6	0.08	Miscellaneous....... 13%
100	3	3	1	7	0.07	
150	4	3	1	8	0.053	Total additional... 64%
200	5	3	1	9	0.045	of direct labor cost.

a At least one man should be a skilled mechanic, at wages about double that of the others.

Table 9. Performance of continuous double-rope tramways

	Tomboy, Telluride, Colo.	Utah Cons., Toole, Utah	Walker, Plumas Co., Cal.	Magma, Superior, Ariz.	Nipissing, Cobalt, Ont.
Type	Bleichert	Bleichert	Leschen	Bleichert	Bleichert
Length, miles	1	4	8.6	0.5	0.72
Elevation of discharging with respect to loading terminal, ft.	−2500	−1260	−2130*a*	−250	+100
Driven by	35-hp. motor	Motor	Motor	10-hp. motor as governor	Motor
Power required, hp.	None; generates	30	12–18, after starting	None; develops 2.5 hp.	15
Materials transported	*c*	Ore	Flot.con.(*d*)	Ore	Ore
Speed, feet per minute	280	600	475	440	690
Rated capacity per hour, tons	20	100	9–10	37	30
Actual delivery, tons	75 in 7 hr.	1100 in 12 hr.	90–105 in 10 hr.	600 in 16 hr.	180 in 9 hr.
Cost of installation	$43,600*e*	$190,000*f*	$175,000*f*	$8132*f*	$18,300
Operating cost, per ton	$0.56	$0.28	$1.00*b*	$0.125	$0.093
Operating cost, per ton-mile	0.56	0.07	0.116	0.25	0.133
Built	1912	1910	1919	1914	1912

a Goes over a high point 1020 ft. above loading terminal. *b* During winter, with heavy snow, men and all supplies are carried by this tram, necessitating extra guards. *c* Concentrate down; coal and all other supplies up. *d* 10% water. *e* Including bins. *f* Excluding bins.

Standard-gage steam locomotives are employed by the largest mines, not only for long hauls of ore but also for disposal of waste. In many cases, the tracks form part of the mine equipment; in others, the roads serve as public carriers operated by organizations subsidiary to the mining companies; while in others, the ore traffic runs partly or entirely over independent roads, sometimes under a trackage license. A few notable examples of relatively long hauls over standard-gage railroads were given in Table 2. Examples of rates charged by common carrier railways in the West are given in Table 10.

Copper Queen, since 1921, has hauled waste rock from Sacramento Hill down grade to a dump two miles away (*68 A 251*). Of 15 standard-gage Porter locomotives used (4 driving wheels and saddle tanks), three weigh 53 tons each and exert a tractive force of 20,436 lb.; the other 12, with superheaters, weigh 54.5 tons each and exert 23,063 lb. pull; gage pressure, 175 lb. in both cases. Cars are all-steel with compressed-air dump, have capacities of 20 and 25 cu. yd., and weigh, respectively, 55,400 and 60,700 lb. The average performance per locomotive-shift is 450 cu. yd. (solid; estimated at 1035 tons) at a cost of 17.7¢ per cu. yd., or 3.8¢ per ton-mile.

Shay geared locomotive affords the only practicable motive power for numerous railroads in mountainous districts, and could be used advantageously elsewhere as a pusher for slow-moving traffic on grades not too steep for ordinary locomotives. In addition to negotiating sharp curves, it has the advantage of small weight in proportion to its tractive force, since all its weight rests on driving wheels, and its force is well maintained even on steep grades; in the example noted below, on a 6 per cent. grade the Mikado locomotive loses more than one-half of its level draw-bar pull while the Shay loses only 32 per cent.

Greenbriar, Cheat & Elk R.R., with 115 miles of track in W. Va., having 7 per cent grades and 32° curves, uses 150-ton, 3-truck, 3-cylinder (17 × 18-in.) Shay locomotives,

Table 10. Freight rates on ores and concentrates charged by certain common-carrier railways, September, 1923. Cents per ton-mile. (*116 J 480*)

From	To	Miles	Road	Value of material, per ton				
				$20	$30	$50	$70–75	$100
				Cents per ton-mile				
Creede	Durango	270	D. & R. G.*a*	1.30	1.50	1.76	2.13	2.63
Ouray	Durango	173	D. & R. G.; R. G. So.	2.78	2.93	3.42	3.65	4.46
Silverton	Durango	46	D. & R. G.	3.10	....	.....	5.64	6.20
Park City	Murray	37	D. & R. G.	3.44	4.13	5.51	......	6.88
Eureka	Midvale	83	D. & R. G.	1.51	1.81	2.41	3.01	3.92
Ray Junction	Hayden	15	Arizona Eastern	2.74	3.42	6.16	6.84	7.53
Tyrone	Douglas	204	E. P. & S. W.	0.93	1.08	1.37	1.67	1.86
Tyrone	El Paso	173	E. P. & S. W.	1.10	1.27	1.62	1.96	2.20
Burke-Wallace	E. Helena	260	Northern Pacific	....	1.15	1.35	1.54	1.83
Tonopah	Midvale	740	Via Ogden	0.69	0.78	0.97	1.16	1.45
Rossland	Tadanac	12	C. P. R.	8.33	12.5	13.33	14.16	14.16

a Narrow gage.

geared to a ratio of 1 : 2.45, made by the Lima Locomotive Works (*71 Railway Age 1209*); comparison of this locomotive with one of the heavy Mikado type (8 drivers) having equal tractive effort is shown in Table 11.

Table 11. Comparison of Mikado-type and Shay locomotives

	Mikado	Shay
Tractive effort, lb.	60,000	59,740
Total weight, engine and tender, lb.	497,000	308,000
Weight on drivers, lb.	240,000	308,000
Wheel diameter, in.	63	48
Wheel base, total driving, ft.	16.75	49
Wheel base, rigid, ft.	16.75	5.7
Minimum radius of track, ft.	300	179
Total evaporating surface, sq. ft.	4297	1882
Draw-bar pull	Pounds	Pounds
Level	56,000	58,500
1 per cent	51,000	55,400
2 per cent	46,000	52,350
4 per cent	36,100	46,200
6 per cent	26,150	40,000
8 per cent	16,200	33,850

At Mountain Copper Co., a narrow-gage Shay locomotive with seven 20-ton cars delivers ore from mines on Iron Mt. to the mill at Minnesota, a descent of 800 ft. in 5 miles, or 3 per cent. average grade.

Storage bins are almost invariably provided at some point between the mine workings and the mill, or that part of the mill where the major crushing operations are conducted. This storage not only obviates the necessity for stopping the mill when delays occur at the mine or on the transport system, but also makes it possible to operate mine or traffic on part time while the mill runs continuously. Conversely, delays in the mill need not affect the continuity of mining or transportation. At many mines, the ore receives its first reduc-

tion at an independent crushing plant, in some cases remote from the mill, which is equipped with its own receiving and discharging pockets, and is thereby enabled likewise to work intermittently, if desired. In general, the storage capacity should suffice to maintain the mill in operation during any mine or traffic interruption that is at all probable; it will therefore be roughly proportioned to the milling capacity, and to the difficulties inherent in the method of conveying the ore into the mill. In the case of mills served by standard-gage railroads, the loaded cars in transit afford a notable volume of storage. At mills of large capacity, especially those at which the crushing is performed in a centralized machine, as in the LAKE SUPERIOR copper mills, structural difficulties may prevent the erection of storage bins to hold more than a few hours' supply of ore. Table 12 gives data on a few modern mills.

2. Situation of mill with respect to water supply

General considerations. All wet concentrating mills require so large an amount of water that securing an adequate supply often becomes an important engineering problem; in many cases, it has been the deciding factor in the selection of a mill site. Where railroad connections permit cheap haulage, it has often proved economical to bring the crude ore many miles to a convenient source of water instead of pumping the required amount of water to a mill in the neighborhood of the mine. Even with an abundant supply of water in proximity to the mill, if this must be pumped to storage sufficiently elevated above the mill to afford rapid flow, the pumping of 4 to 40 tons of water, compared with elevating 1 ton of ore, becomes an important factor in the design and location of a mill. For mills using flotation, the chemical composition and uniform character of the water supply are matters of prime importance.

Water requirements in the milling circuit depend primarily on the nature of the process and equipment; jigs and tables require the most; flotation a less amount; cyaniding and other lixiviation processes the least volume of water. The supply of new water depends upon the facilities employed, if any, for recovering clarified water from tailing, concentrate, or intermediate products in the mill. The practice of recovering water is naturally most common in districts where water is scarce; also at those flotation mills in which soluble reagents have a value; but the desirability of clarifying and recovering water at any convenient stage in the milling system, at an elevation above that of final discharge, always deserves investigation as to its possible economy in pumping. Data on the water requirements of representative mills are given in Table 13.

Gravity supply is an important advantage when obtainable. The water may be conducted by ditch, flume, or pipe-line; the latter often permits the selection of a shorter route, and may be constructed of wood in manufactured sections, wood-stave pipe assembled on the ground, riveted steel pipe, or heavier forms of metal if great pressures must be carried, as at the bottom of an inverted siphon.

Ditches. See Sec. 27, Art. 30.

Flumes are commonly of rectangular section, for ease of construction and repair with local materials; the width is usually twice the wetted depth. Sills and posts should be spaced not more than 4 ft., and 3 ft. is better. No lumber less than 1½ in. thick should be used. According to *Wilson* a 3 × 3-ft. flume with 3 × 4-in. sills, posts and caps, requires 139,600 bd. ft. of lumber per mile, including battens and walk-way, but not the longitudinal stringers and trestle work. Semi-circular flumes of wood or steel, having the advantage of conforming more closely to the best principles of hydraulic flow, are in the market; their principal drawback is the inconvenience of repairs.

Table 12. Storage-bin capacities at representative mills

Mill	Daily capacity, tons	Storage capacity				Ore received via
		As mined	Partly crushed	Total, tons	Total, days	
Alaska-Gastineau	7,000	6,600	5,100	11,700	1.7	Standard-gage railroad, 3.5 miles
Anaconda, copper department	15,300	9,600	....	9,600	15 hr.	Standard-gage electric railroad, 32 miles
Anaconda, zinc department	2,000	800	....	800	10 hr.	Standard-gage electric railroad, 32 miles
Belmont-Surf Inlet	400	....	630	630	1.6	Conveyor; near shaft
Britannia *a*	3,000	2,500	3,600	6,100	2.0	Electric mine tramming
Bunker Hill and Sullivan, West No. 2	1,500	1,400	1,700	3,100	2.0	Electric mine tramming
Butte and Superior	1,750	1,800	2,750	4,550	2.7	Mine strip
Calumet and Hecla, "Conglomerate"	11,000	450	....	450	1 hr.	Standard-gage railroad, 5 miles
Cananea Consolidated	1,000	2,000	2,400	4,400	4.4	Narrow-gage railroad, maximum 3 miles
Chino	14,000	18,000	14,000	32,000	2.4	Standard-gage railroad, 10 miles
Copper Queen	4,000	500	10,000	10,500	2.6	Standard-gage railroad, 3-4 miles
Copper Range	2,100	....	500	500	6 hr.	Standard-gage railroad, 14 miles
Engels	1,200	850	3,400	4,250	3.5	Half from skip; half by aerial tram
Federal No. 4, Flat River, Mo.	3,000	1,500	2,500	4,000	1.3	Conveyor; near shaft
Homestake, South	1,800	*a*	7,200	7,200	4.0	Mine electric tram
Inspiration	18,000	2,500	28,500 *b*	31,000	1.7	Standard-gage railroad, 1.5 mile
Kimberley	3,000	*a*	3,500	3,500	1.2	Standard-gage railroad, 4 miles
Liberty Bell	500	....	....	1,500	3.0	Aerial tram, 7000 ft.
McIntyre-Porcupine	525	....	....	1,100	2.0	Aerial tram
Melones	600	....	....	2,350	4.0	Electric tram, 1 mile
Moctezuma	2,500	800	4,000	4,800	1.9	Narrow-gage railroad, 5 miles
Mountain Copper	550	*a*	800	800	1.5	Narrow-gage railroad, 5 miles
New Cornelia	5,000	5,000	10,000	15,000	3.0	Standard-gage railroad, 1 mile
Old Dominion	1,000	....	2,400	2,400	2.4	Standard-gage railroad
Ray Consolidated	10,000	....	....	26,000	2.6	Standard-gage railroad, 25 miles
St. Joseph Lead, Bonne Terre	2,600	....	115	115	1 hr.	Mine skip
Sta. Barbara	500	500	500	1,000	2.0	Mine tram
Silver King Coalition	300	2,700	2,200	4,900	16.0	Electric mine train, short distance
Sunnyside	550	....	....	4,200	8.4	Aerial tram, 3 miles
Tennessee Copper Co.	600	300	400	700	1.2	Standard-gage railroad, 2-3 miles
Timber Butte	700	600	4,500	5,100	7.0	Standard-gage railroad
Tonopah-Belmont	500	....	....	1,500	3.0	Automatic tram, 300 ft.
Tonopah Extension	350	....	....	500	1.5	Electric tram, 4000 ft.
United Comstock	2,000	1,600	3,200	4,800	2.4	Electric mine train, short distance
Utah Copper (two mills)	40,000	*c*	27,840	27,840	0.7	Standard-gage railroad, 18 miles

a Large reserve storage in mine. *b* "Available" storage only, including 2500 tons in 3 ore trains; actual cubic capacity of storage bins for crushed ore is 50,000 tons. *c* Utah Copper Co. has no storage bins for uncrushed ore, but loaded cars standing or in transit contain 26,000 tons.

Table 13. Water requirements and water recovery at representative mills

Mill	Nature of process	Daily ore tonnage	Tons of water in circuit per ton of ore	Recovery, amount and method
Alaska Gastineau	Tables and vanners	7,000	5.2	None
Belmont-Surf Inlet	Tables and flotation	400	4.0	None
Calumet & Hecla "Conglomerate"	Jigs and tables	11,000	28.0	None
Cananea	Jigs, tables and flotation	1,000	8.0	None
Chino	Tables, vanners and flotation	14,000	7.1	*a*
Copper Queen	Tables and flotation	4,000	6.3	75% *b*
Copper Range	Jigs and tables	2,100	30.0	None
Engels	Flotation only	1,200	3.5	None
Federal Lead, Nos. 3 and 4	Jigs, tables and flotation	7,800	7.0	85% *h*
Hedley G. M. Co.	Vanners and cyanidation	210	10.0	None
Inspiration	Flotation and tables	14,700	3.8	*c*
Iron Cap	Tables and flotation	300	6.5	36%
Liberty Bell	Amalgamation, tables, cyaniding	500	10–20	None
Mascot	Jigs, tables, flotation	2,400	16.0	40%
McIntyre-Porcupine	Cyanidation only	525	1.0	None
Miami	Flotation only	7,000	3.5	2.1 tons *d*
Moctezuma	Tables, vanners, flotation	2,500	10.6	75% *e*
Neveda Packard	Cyanidation only	100	0.75	None
Phelps-Dodge, Morenci	Tables and flotation	4,500	4.4	70% *b*
Ray Consolidated	Jigs, tables, vanners, flotation	10,000	6.0	*f*
St. Joseph Lead Co., Bonne Terre	Jigs, tables, flotation	2,600	8.0	90% *e*
Sta. Barbara	Sand and slime tables	500	13.8	91.3% *g*
Sunnyside	Flotation only	500	4.9	None
Timber Butte	Tables and flotation	700	4.0	None
Tonopah Extension	Cyanidation only	350	1.3	None
United Eastern	Cyanidation only	300	0.6	33%

a 34% from tailing pond, 48% from settlers in mill. ***b*** From settlers and filters See page 1280. ***c*** 1.56 tons from thickeners, 0.32 ton from tailing pond. ***d*** From tailing dam. ***e*** From slime pond. ***f*** None from tailing. 2.5 tons recovered inside the mill. ***g*** In three 50-ft. Dorr tanks. ***h*** From thickeners and settling ponds.

Manufactured wood pipe is available in inside diameters of 2 to 48 in., piece lengths of 6 to 25 ft., and to withstand heads up to 400 ft. of water. ADVANTAGES: light weight, ease and rapidity of laying, resistance to corrosion even in acid water, and small frictional resistance to flow. Comparing 16-in. pipes under heads up to 6 ft., a riveted-steel and a new cast-iron pipe will deliver respectively 80 and 85 per cent. as much as a wooden pipe; the frictional resistance in the latter, furthermore, does not tend to increase with age. The woods commonly employed are cypress, white pine, Douglas fir, and California redwood. Bondings are of wire or steel ribbon; the wood is commonly treated with creosote, and the outside protected with asphalt.

Wood-stave pipe, assembled on the job with specially formed staves and rod bands, is suitable for diameters of 16 in. to 20 ft., and hydraulic heads of 150 ft. for the larger up to 400 ft. for the smaller diameters. ADVANTAGES are: ease and rapidity of construction, applicability to rough country, ability to form curves of radius 60 times the pipe diameter, resistance to corrosion, low frictional resistance. A pipe buried in clean soil has longer life than one lying on the surface; decay from the outside seems to be promoted by dents and abrasions given by rough workmanship during construction. Initial leakage can be stopped quickly by introducing bran at the inlet of the pipe.

Nevada Cons. mill, at McGill, Nev., receives most of its water by gravity from a reservoir on Duck Creek, 9.5 miles away and 105 ft. higher than the receiving tank at the mill, through a 32-in. pipe of which 40,000 ft. was laid with Douglas fir staves and ½-in. round bands; sills were spaced at 3 ft. (*109 J 857*). The total first cost (1907), including 10,000 ft. of 32-in. riveted-steel pipe in the line, was $180,000, or $4 per ft. Flow capacity, 20 to 21 cu. ft. per sec. In 1920, the ends of the staves were found to be rotted; repairs were made by enclosing the affected portions of the pipe in concrete jackets, about 6 in. thick and reinforced with small rods; the total length thus repaired was 1500 ft. This line was still in use early in 1924 and was expected to last 2 years longer (*C. B. Lakenan, PC*).

Sheet-metal riveted pipe is of two general types: longitudinal lap or butt jointed, and spiral jointed; the former is generally adopted for large lines and high heads, the thickness of metal and the manner of riveting being selected to suit the pressure, while the latter has a wider application to smaller flows and pressures. The spiral-riveted lap seams give the pipe added strength as a beam, thereby reducing the amount of support to be provided when laying a line over the surface; it is lighter than cast-iron pipe of corresponding size and strength, and is practically equal in flow capacity to cast-iron or smooth steel pipe when the latter has become tubercular. Riveted pipe sections are connected by slip joints, for light pressures, or by bolted flanges or other special forms of bolted joints which permit a slight deflection.

Spiral-riveted pipe is available in diameters of 3 to 42 in., sectional lengths up to 20 ft. for galvanized and 40 ft. for asphalted, steel thicknesses from 16 gage to ¼ in., and for withstanding bursting pressures of 400 to 900 lb. per sq. in. for the larger diameters up to 1800 lb. for smaller pipes. A safety factor of 5 is commonly employed. Table 14 gives data

Table 14. Spiral riveted pipe

Inside diameter, inches	Thickness, U. S. gage	Weight per foot, asphalted, pounds	Approximate bursting strength, pounds per square inch
6	14	6.6	1560
7	14	7.7	1340
8	14	8.8	1170
9	14	9.9	1045
10	14	11.0	935
11	14	12.0	850
12	14	13.0	780
13	12	19.7	1010
14	12	22.2	940
15	12	23.7	875
16	12	25.2	820
18	12	27.6	730
20	12	30.6	660
22	10	42.2	765
24	10	45.7	705

(The data relate to sizes of intermediate, or "Extra Heavy" weight; each size listed is also made in lighter and heavier sheet metal.)

selected from publications of the American Spiral Pipe Works. Prices per pound for plain-ended pipe were quoted in 1924 at 12.2 to 13.2¢ for asphalted, and 17.1 to 18.5¢ for galvanized, the higher price applying to the lighter-gage metal.

Couplings most commonly used, other than slip joints for small pressures, are flanges, preferably of forged steel, and bolted sleeve joints with rubber packing. On a line of asphalted pipe, in 40-ft. sections, flanges attached at the factory will cost an additional 30 per cent. over the price of the plain pipe in sizes from 6- to 30-in.; bolted sleeve joints cost 13 per cent. the price of plain pipe. For a line of galvanized pipe, in 20-ft. sections, galvanized flanges attached at the factory will cost 50 to 60 per cent. the price of the plain

pipe, and bolted sleeve couplings (not galvanized) will amount to 15 to 20 per cent. the cost of plain pipe in the line.

The Nevada Consolidated Copper Co., in connection with its central reservoir on Duck Creek (*109 J 857*), laid 54,800 ft. of feeder lines to convey water from four tributary streams and thereby maintain their flow during the winter. Of this total, 45,000 ft. (at three tributaries) consisted of 8- and 12-in. riveted, slip-jointed, light-weight pipe in 20-ft. sections, made in the company's shop at a cost of 7¢ per lb., and asphalted. The lines were laid so as to develop little pressure. Joints were made by warming the larger end to expand it and soften the asphalt; the small end of the next section was then driven in by a ram supported on a light tripod. Three men laid 1500 to 2000 ft. of 8-in. pipe in this manner per day. The total cost of these three lines was 62¢ per ft. for the 8-in. and 75¢ per ft. for the 12-in. line. The fourth line, 9800 ft. of 6-in. pipe designed to carry 150 lb. per sq. in., cost $1.53 per ft.

Flow in pipes and open channels. See Sec. 27, Arts. 29 and 30.

Other sources of water. Mine water is often available for milling purposes; its composition may require correction, especially for flotation processes. Springs, wells, tunnels in water-bearing strata, streams, ponds, lakes, and the ocean have all been drawn upon for mill water.

Nevada Consolidated Copper Co., just below its main gravity storage dam on Duck Creek, drove a tunnel 1150 ft. long across the valley at the bottom of gravelly soil, 40 ft. deep, resting on hardpan. This intercepted 2 cu. ft. per sec. of the seepage from the reservoir, which was raised by a hydraulic ejector operated by a high-pressure gravity line. McGill springs, at the Nevada Consolidated smelter, deliver 9 to 12 cu. ft. per sec. This was formerly pumped to mill-supply tanks by two motor-driven centrifugal pumps, each with capacity of 4.9 cu. ft. per sec., at cost of 0.77 to 1.04¢ per ton of water. One triple-expansion steam pump, with maximum capacity of 12 cu. ft. per sec. against 480-ft. head did the work at about half the cost of the electric installation (*109 J 857*). New Cornelia's water comes from a 2-compartment shaft 600 ft. deep, sunk for the purpose at a point eight miles northeast of Ajo. A duplex, double-acting pump, direct-connected to a synchronous motor, delivers 800,000 to 1,000,000 gal. per day through 6.7 miles of 10-in. iron pipe against a total head (including friction) of 1375 ft. (*60 A 22*). The 2000-ton Allenby mill of the Granby Consolidated, requiring 4.6 tons of water per ton of ore, is supplied from the Similkameen River, one mile away, by three centrifugal pumps (one in reserve) which have a capacity of 800 gal. per min. each, discharging through 5000 ft. of 14-in. pipe against 600-ft. static head (*115 J 989*). At Alaska-Juneau salt water was pumped from Gastineau channel, with two centrifugal pumps, 3000 gal. per min. each; each pump direct-connected to a 400-hp. squirrel-cage induction motor (*120 P 251*). At Alaska-Gastineau, treating 10,000 tons a day with 5.2 tons water per ton ore, there is sufficient gravity supply for five summer months; the remaining months salt water is pumped with three 2-stage Byron Jackson turbines of 1000, 2000, and 3000-gal. per min. capacity respectively against a head of 275 ft., with the expenditure of 700 hp. (*63 A 488*). Water for the Chino mill, Hurley, N. M., is obtained from wells and streams, respectively five miles south, one mile northeast, four miles and five miles west, the last two using the same steel pipe line. The pump stations are equipped with Aldrich electric-driven pumps (*104 P 464*). Storage at Hurley is 3,000,000 gal. New water amounts to 18 per cent. of the 7.1 tons in circulation per ton of ore.

At the 4000-ton Panda concentrator at Katanga water is pumped from the river, two miles distant, by two 3-stage centrifugal pumps, each of 3300 gal. per min. capacity and each direct-connected to a 525-hp. synchronous motor. The pumps deliver through 24-, 26-, and 28-in. spiral-riveted pipe against 300 ft. head. After passing through the powerhouse condensers, the water is elevated another 200 ft. by three 2-stage pumps, each driven by 350-hp. synchronous motors. The capacity of the two mill reservoirs is 3,820,000 gal. (*29 MM 137*).

Inspiration, treating 14,700 tons ore per day (1924) obtains its daily supply of 6,700,000 gal. (1.89 tons per ton of ore) of new water from six wells, 300 ft. to 1400 ft. deep in gravel, at a point 14,500 ft. from the mill (*G. H. Ruggles, PC*). Well pumps deliver through 1000 ft. of 10-in. pipe against a static head of 80 ft. to the station pumps operating against a static head of 520 ft. and delivering to a 3,000,000-gal. mill reservoir through a 20-in. pipe line. The line is partly of steel with leaded bell-and-spigot joints, partly of flanged pipe. Power requirements are 3.49 kw.-hr. per 1000 gal. The approximate total cost is 5¢ per 1000 gal. (= 2.2¢ per ton of ore). For recovery of old water see Art. 3.

At the Ray Consolidated mill, Hayden, Ariz., treating 10,000 tons of ore per day, 35,000 tons of water per day is pumped from wells near the Gila River through 9000 ft. of 26-in.

wood-stave pipe against head of 310 ft. (*W. S. Boyd, PC*). The initial cost of the water plant was $400,000. Power for pumping is 1.3 kw.-hr. per 1000 gal. (1.09 kw.-hr. per ton of ore) showing an over-all efficiency of 75 per cent.; the total cost of water is 3.4¢ per 1000 gal. (2.85¢ per ton of ore). No provision for recovery of water has been found necessary. The water circulation in the gravity section of the mill (jigs, tables, vanners) was 6 tons per ton ore.

At Miami, treating 7000 tons of ore a day with an average water circulation of 3.5 tons per ton of ore, 1600 gal. of new water per min. (1.4 tons per ton of ore) is pumped from wells and mine shafts four miles away, against a total head of 862 ft. with an expenditure of 3.5 kw.-hr. per 1000 gal. at a cost of 7¢ per 1000 gal. (2.3¢ per ton of ore). Old water to the amount of 2500 gal. per min. (2.1 tons per ton of ore) is reclaimed from the tailing dam, against head of 160 ft., with an expenditure of 235 hp. and at cost of 1.5¢ per ton of ore. (For methods of reclaiming, see Art. 3.) In 1922 Miami spent 4.6¢ per ton of ore for mill water, or 7.1 per cent. of the total cost of milling.

Chemical composition of the mill water may be important, especially when mine water is to be used. Mechanical troubles may be caused by the corrosion of metal by acid water, or by lime accretions deposited by hard water. Flotation is highly susceptible to variation in character of the water, with respect both to its soluble and its suspended matter, and certain special flotation methods require careful control of the character of water entering the mill.

In South-east Missouri it is common practice to use mine water as far as it goes, supplemented by pumping stations on Big River and Flat River. The water in circulation averages 7 to 11 tons per ton of ore, and a 3000- to 4000-ton mill can usually operate with a supply of new water amounting to 700 gal. per min. (4200 tons per day), the principal loss occurring with flotation tailing. The mine water often carries as much as 30 grains of calcium and magnesium salts per gal., and the piping, launders and tables of the mills require frequent cleaning (*57 A 322*). At Tul Mi Chung mill, Korea, concentrating a highly complex sulphide ore exclusively by flotation (*33 IMM 2*), water is obtained by pumping from a reservoir fed by a stream which, during nine months of the year, carries a large proportion of suspended clay in addition to dissolved salts corresponding to 44 parts total CaO and 12 parts MgO per million. Coagulation is effected by adding 300 lb. of lime to the 1700 tons of water consumed per day (0.18 lb. lime per ton) followed by settling before pumping. Undissolved lime is injurious to flotation of this ore, but free alkalinity of 0.0004 per cent. CaO is helpful. Lime is not added during the 3 months that the water is clear.

At Consolidated Coppermines milk of lime was added to the discharged tailing just before it enters the final Dorr thickener, in order to overcome interference with flotation caused by sulphuric acid and iron salts. All mine water used in the mill was first sent to the same thickener, which was thereby made to act as a water-treatment plant, the overflow being returned to the main supply tank, while precipitated calcium sulphate and iron hydroxides were discharged with the thickened tailings. By a similar arrangement of flow at the Utah Consolidated mill, constant character of mill water was maintained, although chemical treatment was not required. At the Utah Apex mill, acid salts in the mine-water supply were neutralized, and at the same time a desirable flotation reagent was introduced, by adding sodium sulphide at the fine-crushing mills and combining the flotation tailing and new mine water in the same Dorr tank, the overflow of which returned to the storage supply, while the thickened tailing, carrying also the precipitated impurities from the mine water, passed over concentrating tables and thence to waste.

Water conservation may become necessary or advisable: (*a*) to economize in the supply of new water, where the latter is scarce or expensive to procure; (*b*) to save cost in pumping new water, where the escaping water can be collected at an elevation above that of its original source; (*c*) to avoid loss of useful material in solution; (*d*) to escape prosecution for polluting streams.

The most notable examples of systematic conservation in the United States are found among the lead-ore concentrating mills of south-east Missouri, which recover 80 to 90 per cent. of the mill water; the zinc mills of the Joplin district, and the copper-flotation mills of the South-west, whose average return of water ranges from 70 to 85 per cent. The Sta. Barbara lead-carbonate concentrator in Mexico returns over 90 per cent. of its circulating mill water. (See Table 13.)

A tailing pond is the most common means for recovery, sometimes in combination with mechanical devices, but frequently alone. For methods of forming retaining dams, see Art. 3. Water is clarified by natural sedimentation, or by discharging the slime portion of the tailing on top of a growing pile of sand tailing, which acts as a filter (*57 A 332, 420*). This latter method was formerly the usual practice in South-east Missouri, but is now largely abandoned in favor of settling ponds.

Mechanical settlers, designed primarily for dewatering concentrate, sand tailing, etc., but incidentally yielding an overflow sufficiently clear to be returned to the mill circuit, can often be so placed as to afford considerable economy in returning water to the central supply tank. Devices of this type include the Allen, Boylan, Callow, Caldecott and similar settling cones, the Akins, Dorr, Esperanza, and other varieties of screw or scraper dewaterers, and Dorr settling tanks of the smaller sizes. Filter water is also suitable for returning. If not wanted where it can be immediately conducted by gravity, the clear water can be returned to the main supply system by pumps discharging directly into the main water pipes in the mill, thereby avoiding a separate column pipe from each pump to the main pressure tank.

Clarifying tanks, for the primary purpose of reclaiming water and discharging thickened slime, are now commonly of the Dorr type, having the advantage of large capacity with small loss of mill height. Tanks of the larger sizes, 50 to 200-ft. diam., are frequently placed out-doors (except in cold climates) set with the bottom below the general level of the ground.

Performance. See Table 13. At Phelps Dodge, Morenci branch, reclaimed water (2310 gal. per. min.) is returned from a common sump against a head of 235 ft. by pumps requiring 200 hp. At the Chino mill (see Table 13) the total power for water circulation is 1850 hp., including pumping of the original supply (see p. 1278). At Copper Queen (see Table 13) the water content of discharged tailing is reduced from 75 per cent. to 35 per cent. before discharging to the sand retaining dam, where an additional recovery of water is secured. Water recovery at Inspiration is described in Art. 3. South-east Missouri practice is, in general, to dewater sand tailing by drag scrapers or shovel wheels, thus obtaining clear water, and to impound slime tailing.

3. Situation of mill with respect to tailing disposal

General. The necessity of providing for the economical and satisfactory disposal of tailing, usually amounting to 75 per cent. and frequently approaching 100 per cent. of the tonnage delivered to a mill, has often been the deciding factor in selection of a mill site, especially for a large plant treating ore from a mine with an assured long life. Some of the factors that enter the problem are: (*a*) maximum economy requires gravity flow of the tailing to its destination; (*b*) gravity disposal involves the use of water as a carrier and not all of this can be recovered; (*c*) mechanical elevation of tailing and the accompanying water may be a necessary preliminary to gravity disposal; mechanical stacking of dewatered sand tailing is an alternative; (*d*) areas of land may have to be acquired solely for tailing disposal; (*e*) special precautions may need to be taken against pollution of streams by soluble or suspended matter; (*f*) special haulage facilities may be needed; (*g*) tailing from which it is hoped to extract valuable ingredients by subsequent operations will usually require additional expense for suitable storage; (*h*) in a few cases, the tailing has enough present market value to become a source of revenue, or at least to pay for disposal.

Direct loading into cars is a common method of disposing of coarse waste, especially that produced by hand sorting or by dry methods of concentration, as in Lake Superior "rock houses," Wisconsin zinc mills, coal washers,

magnetic iron concentrators, etc. At wet concentrating mills, the advantage of this method, even where gravity disposal is utilized for finer tailing, is that it avoids waste of the large volume of water that would be required to flush coarse material down a flume.

Former practice in SOUTH-EASTERN MISSOURI was to dewater sand tailing in an elevated tank from which the settled product passed through spigots into cars drawn to a dump by locomotive. This plan is now obsolete, but in modified form (tailing mechanically dewatered before transfer to a bin) is still practiced at ST. JOSEPH LEAD CO., Bonne Terre mill (*57 A 420*). At BUNKER HILL & SULLIVAN, sand tailing is occasionally diverted from an elevated gravity flume into railroad cars and hauled away for ballast. Tailing from the leaching vats at Ajo is loaded by truss excavator into the same cars that bring the crushed, raw ore to the plant. Unusual circumstances at the MOCTEZUMA mill formerly required conveyance of de-slimed sand tailing across a gulch by aerial tram; the tailing now is all slime and flows by gravity to settling ponds.

Gravity flushing of wet tailing of medium to fine size is always adopted if conditions permit; the method may require assistance of mechanical elevators, and is frequently employed in such a manner as to permit recovery of water. Design of a tailing launder requires special attention because: (*a*) it is usually longer than any launder inside a mill and, if given a grade steeper than is actually necessary, will sacrifice an undue amount of available head; (*b*) it carries a larger volume than individual mill launders and should be designed to do this without the addition of valuable water for flushing; (*c*) abrasion, while no more severe than in mill launders conveying the same material, demands more extensive renewals; linings should therefore be specially designed for durability and ultimate economy. (For general data on dimensions and slopes of launders, see Art. 6 and Sec. 20, Art. 10.)

At BUNKER HILL & SULLIVAN the sides of a wooden tailing launder, 24 in. wide, are lined with concrete slabs, 1 ft. square and 1.5 in. thick, or white-iron plates of the same size, 1 in. thick, or with discarded elevator belt with the rubber-protected side against the walls. The bottom is riffled transversely, at 24-in. intervals, with ¾-in. white-iron strips 6 in. wide, standing on edge; sand resting between the cleats prevents wear on the bottom (*S. A. Easton, PC*; *120 P 525*). The slope is ⅜ in. per ft. At UNITED EASTERN fine-crushed cyanide tailing (46 per cent. water) is distributed by gravity through 6-in. slip-jointed, iron pipe laid with 3° (⅝ in. per ft.) minimum slope.

Impounding of tailing has three purposes: (*a*) to save the solid for future treatment, as at Anaconda, Panda, and Chino; (*b*) to give opportunity for settling and recovering water, as in Missouri, the South-west, Mexico, and elsewhere; (*c*) to avoid pollution of streams, as at Engelmine, Cal., certain anthracite washeries, and at numerous cyanide mills. Tailing ponds may be formed by excavation in earth, as at Anaconda; by concrete dams, as at Engelmine; or, more commonly, by dams formed from the tailing itself; the latter is common practice in the South-west. Impounding dams are frequently used to catch the water remaining in the thickened product of mechanical dewaterers.

Tailing dams are built by discharging a stream of pulp from a launder elevated on a trestle of light construction, from which, as the pile accumulates, some of the horizontal timber can be recovered. To build a stable dam with maximum slope of face requires that slime be removed from the sand before the latter is placed. The slime, deposited separately above the sand dam, effectively fills the pores in the upper layers of sand, but does not increase the tendency of the dam to slide. De-sliming may be done in the mill, discharging the two tailings through separate launders, or in improvised or regularly-constructed classifiers, situated on the tailing-dam trestle, and delivering slime overflow at a distance from the sand. In some cases the topography permits the tailing sluice or pipe line to be laid on the surface.

At Old Dominion 700 tons per day of flotation tailing (practically all −48-mesh, and 56 per cent. −200-mesh) is carried by gravity in an unthickened pulp of 20 to 25 per cent. solids through 8-in. steel pipe having a minimum length of 1600 ft. and maximum of 3600 ft. (See Fig. 1.) The total head is 50 ft., and the pipe is laid at any convenient grade (*C. E. Chaffin, PC*). The pipe delivers to a Dorr duplex classifier at the face of the dam, mounted with its motor on a truck, periodically moved ahead on a 20-ft. section of track, when a corresponding section of pipe is introduced. (See Fig. 2.) Sands, constituting about one-third of the solids, are deposited ahead of the classifier, forming a long, narrow dam on a hillside, while slimes discharge sideways into the pond. The dam section in process of building is 12 ft. high and 12 ft. wide at the top, with side slopes of 45°. When a 12-ft. layer has been completed to the limits of its width (200 ft.), another layer is started on top. When the maximum height of the present dam, *viz.* 30 ft. above ground, has been reached, another will be started at a lower point. One man attends the classifier and shapes the dam. The cost of disposal, 2½¢ per ton, is relatively high because of the long, narrow shape of the dam. Water is not reclaimed, but overflows by a wooden standpipe and flume through the dam.

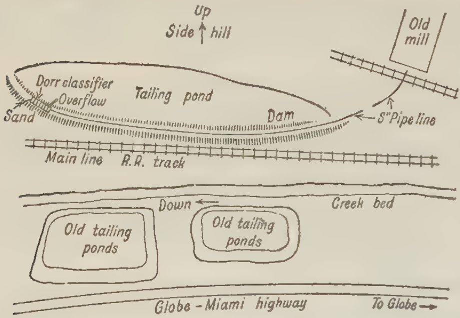


Fig. 1.—Plan of tailing dam, Old Dominion.

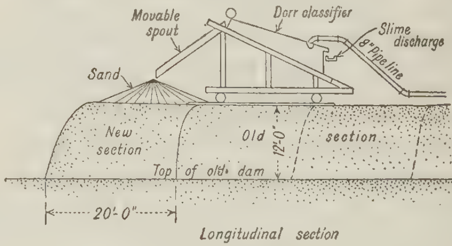


Fig. 2.—Old Dominion method of building tailing dam.

At Inspiration 14,200 tons per day of table and flotation tailing (94 per cent. −48 and 52 per cent. −200-mesh) is disposed of entirely by gravity (*C. E. Chaffin, PC*). All table tailing and part of the flotation tailing are thickened just outside the mill to recover water at as high a level as possible. The feed to the roughing tanks carries 22.(per cent. solids; the overflow going to the Dorr thickeners carries 22 per cent. solids. The total water recovered is 4600 gal per min., or nearly 50 per cent of the whole circulation. The final discharge to the tailing dam containing all thickened products and the unthickened portion o flotation tailing, averaged 31. per cent. solids during 1922 This flows in an open launde 3000 ft. to the nearest corner o the tailing dam, roughly squar in plan, and then in either on of two elevated launders, abou 5000 ft. long, each extendin half-way around the perimeter The launders are 4 ft. wid and 2 ft. deep, made of 2-in plank with battened joints and have a uniform grade of 14 in. per 100 ft. Collars a

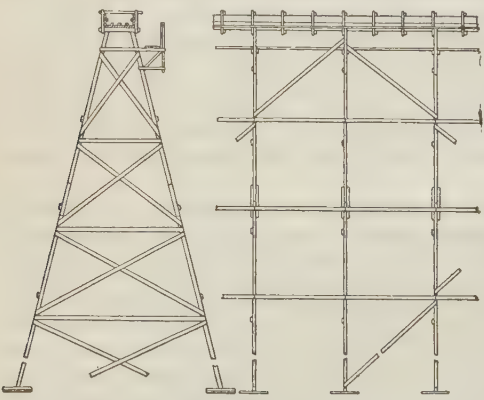


Fig. 3.—Tailing launder and trestle, Inspiration.

3-ft. intervals are made of 4 × 6-in. sill and 2 × 4-in. posts and cap. The trestle (Fig. 3) ranges from 30 to 70 ft. high. Bents are spaced 12 ft. For bents up to 50 ft. the legs and caps are 4 × 6-in., battered 3 in 12, transversely braced in 12-ft. panels with 2 × 6-in. cross braces. For higher bents, the posts are increased 50 to 100 per cent. in section. A walkway 2 ft. wide with hand-rail is carried at one side of the flume. Longitudinal bracing is effected by means of purlins spaced 12 ft. and slant bracing as shown in Fig. 3. The total load is 750 lb. per linear ft. Total lumber for a 30-ft. trestle is 900 bd. ft. per 12-ft. section.

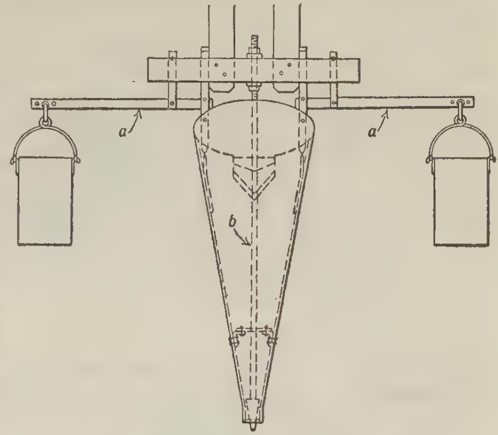


Fig. 4.—De-sliming cones, Inspiration tailing dam.

Sand is de-slimed in automatic cones (Fig. 4) 15 ½-in. inside diameter at the top, 2-in. diameter at the bottom, and about 3 ft. 2 in. deep, made of No. 20 galvanized sheet. The cone is suspended by straps from counterweighted levers (*a*). The valve rod (*b*) is fixed in position so that when the cone is filled with pulp at low density the valve is closed. As sand collects and the weight of the cone increases, it drops somewhat and the valve opens, discharging thickened sand.

The cones are spaced at 12-ft. centers along the trestle. They are fed through ¾-in. pipes inserted through the side of the flume. Sand and slime are discharged through short spouts, the former to a marginal pile and the latter to the enclosed pond. (See Fig. 5.) Of 400 cones installed, 250 are in use at a time; one man per shift watches 80 cones and 1000 ft. of flume. Other labor includes a boss and three men per shift. Wet sand settles to a slope of 1½ to 1. The marginal pile is raised to height of 40 ft., with a top width of 12 ft. When the enclosed area is filled with slime, a new trestle is erected on top and about 80 ft. back from the edge of the preceding pile; the resulting terraces are necessary to prevent leakage. In Feb., 1923, the fourth marginal pile had reached 126 ft. above the lowest point of ground, and was expected to impound two years' tailing; lower ponds will then be started, as no higher level can be reached without pumping. Table 15 gives costs. Prior to 1920, about the same annual tonnage was distributed to four smaller ponds at costs ranging from 2.65¢ to 3.71¢ per ton.

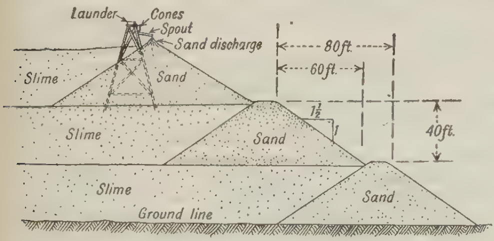


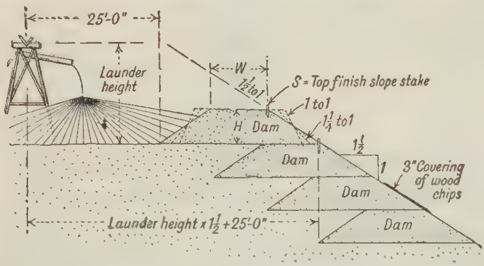
Fig. 5.—Section of tailing dam, Inspiration.

Table 15. Cost of tailing disposal at Inspiration mill (cents per dry ton)

Year	Tons solid	Launders and trestles	Drain	Operating labor	Miscel.	Total
1920	4,943,000	0.488	0.168	0.490	0.224	1.370
1921	884,950	1.420	0.463	0.539	0.168	2.690
1922	3,534,000	1.120	0.090	0.550	0.460	2.220
1923	4,957,730	1.620	0.100	0.570	0.120	2.410

See Table 13 for water recovery. Triplex pumps deliver the overflow of the thickene against a head of 125 ft. with power consumption of 0.801 kw.-hr. per 1000 gal. at a tot cost of 1.25¢ per 1000 gal. Water from the pond is returned by centrifugal pumps again a head of 280 ft., with consumption of 3.23 kw.-hr. per 1000 gal. at a total cost of 5¢ p 1000 gal.

MIAMI, since Dec., 1923, has disposed by gravity of a daily average of 7000 tons flotation tailing (98 per cent. −48- and 55 per cent. −200-mesh) in an unthickened pu carrying about 27 per cent. solids, or at the rate of approximately 3300 gal. per min. (*C. Chaffin, PC*). This flows through 4000 ft. of 18-in. redwood stave pipe under a head 26 ft. The pipe is large enough to allow sand to settle on its lower side and protect it fro wear. It lies on the ground, at no special grade (at one point passing over a low hill except where it crosses the slime pond on a trestle. When it is necessary to raise this trestl sand is allowed to flow through holes in the bottom of the pipe, forming a ridge across t middle of the pond. The pipe delivers to a distribution box from which two launders e tend, one for 2400 ft. and one for 1600 ft., in opposite directions along the face of the dar The launders are V-shaped, with 24-in. sides, made of 2-in. plank, set on a grade of to 3/16 in. per ft., and have V-notches to the full depth on the dam side at 20-ft. interval They are supported on a trestle (Fig. 6), 30 to 60 ft. high, 25 ft. back from the outer edge of t dam. The bents, 10 ft. apa are made of 4 × 4-in. pos and 2 × 4-in. braces. T entire flow in each launder discharged through its lowe notch, and directed towa the face of the dam by apron 9 ft. long, 4 ft. wide a 1 ft. deep, made of 1-in. boar The sands settle on the ban while slime flows inwardly t ward the pond. When t sands have accumulated to depth of 6 ft., the next high notch is opened, and the pro ess is repeated until a 6- layer has been deposited ba to the distribution box. T part of the operation requi 1 man per shift, with anoth man and a foreman on d shift, or 5 men per day. dam 6 ft. high is next formed along the outer edge of the pile by an American railw ditcher operated by three men and requiring 45 days (one shift) to complete a dam alo the whole length of the tailing pile. The outer face of this dam, sloping 1½ to 1, is co tinuous with the general outer slope of the whole pile; no terracing is necessary. O 6-ft. dam provides retaining space for 6 months, or 1,250,000 tons. The highest point the dam (early 1924) was 200 ft. above ground; additional height will require pumpi of tailing, as was formerly practiced when the mill discharged from a lower level. Wo chips, collected on screens attached to Dorr classifiers in the mill and amounting to 3 4 tons (dry) per day, are spread loosely in a 3-in. layer over the outer slope of the pile alleviate nuisance from blowing dust. The cost of gravity disposal (1923) including tres and launder building, ditcher operation, labor and maintenance was 1.61¢ per ton; t additional cost of collecting, transporting and spreading wood chips was 0.75¢ per t of tailing.



W = finished width of top when dry. *H* = height of dam for stacking dry sand. For stacking wet sand for finished top width *W*, place stake at *S*; for semi-wet sand move slope stakes in *H*/4 from *S*.

FIG. 6.—Section of tailing dam, Miami Copper Co.

Clarification of tailing water to avoid pollution of streams is required the laws of certain states.

At ENGELS, a concrete tailing dam, preceded by three Esperanza drag classifiers in ser and one 80-ft. Dorr thickener, is used. Lime is added in the proportion of 1.76 lb. C per ton of ore, to assist settling. The cost of clarification (1920) was 17¢ per ton of milled; water is not recovered (*123 P 183*). Settling ponds are frequently inadequate the desired degree of clarification at PENNSYLVANIA ANTHRACITE WASHERIES and electroly methods of coagulating fine suspended matter are under development.

Elevation of tailing is adopted, either to stack comparatively dry taili in a pile or to extend the radius of gravity disposal. The most commo

employed means of elevation are: (*a*) inclined trams, as at anthracite breakers and some So. African gold mills; (*b*) sand wheels, notably at some Lake Superior copper mills; (*c*) belt-bucket elevators, as in the Joplin district, and formerly at Chino; (*d*) inclined belt conveyors, as in south-east Missouri and more recently at Joplin; (*e*) centrifugal sand pumps, as at Bunker Hill and Sullivan, and formerly at Miami; (*f*) air-lifts, of which there is a noteworthy installation at the Chino mill.

Sand wheels up to 60-ft. diameter have been used in So. Africa and are still retained at Calumet and Hecla copper mills, for transferring tailing from the main concentrator to re-treatment works, on account of their simplicity, durability and efficiency. For details see Sec. 20, Art. 14.

Belt-bucket elevators are frequently used for tailing disposal in the Tri-State district but are gradually being replaced by inclined conveyors handling dewatered tailing (*H. H. Wallower, PC*). When the disposal space within reach of the mill elevator and its distributing launder is exhausted, a second elevator of the same height is erected on top of the first pile, receiving its pulp through launder from the top of the first elevator, and distributes its discharge by a second launder. The system is extended in this manner until as many as three or four intermediate " dummy " elevators are installed in addition to the mill elevator and the final stacker.

Owing to the grade (2 in. per ft.) required of the connecting launders, and an unavoidable drop of 6 ft. at the upper end of each elevator, the combined lift of the elevators will be about twice the height of the final pile, and all water required for flushing through the successive launders (1.7 tons per ton of tailing) must be elevated the same distance. An average mill of the district discharges 25 tons and a large mill 45 tons of tailing per hour. At an average mill, elevator belts are 8-ply, 24 in. wide, with buckets 24 by 7 in., made of 10-gage steel, spaced on 18-in. centers; this permits 3 shifts of buckets during the life of one belt, which is usually about 18 months. Average speed is 275 ft. per min.; average power for a system containing one or two dummies, 0.44 hp. per ton of pulp, equivalent to 1.18 hp. per ton of dry tailing deposited.

A new 60-ft. dummy elevator costs about $2000 complete, itemized as follows; 136 ft. of belt @ $3.85 = $524; 90 buckets @ $1.31 = $118; 1 spout, $40; 15-hp. motor and material and erection of elevator structure, $1318. Cost of operation of a system containing five elevators will average, per year: power (300 days of 20 hr., 75 hp. @ 1.5¢ per hp.-hr.), $6750; labor for maintenance and attendance, $250; total, $7000, equivalent to 4.7¢ per ton for a mill of average size, or 2.6¢ per ton for a large mill.

Inclined belt conveyors represent standard practice in the lead belt of south-east Missouri, where a single mill may dispose of 2000 to 4000 tons of sand tailing per day (*57 A 322*).

The sands are mechanically dewatered (shovel wheels are preferred) to 8 to 20 per cent. moisture and delivered to a rubber-covered belt conveyor, usually 24 in. wide, inclined 16°, and moving 300 to 350 ft. per min.; in one exceptional case, the speed was 475 ft. per min. up 23° slope; in general, 18 per cent. moisture will cause tailing to slide on a 16° slope. The belt is usually lengthened by units of 75 ft., as the pile gains height. Idlers for this wet work must stand hard usage. The standard 5-pulley grease-lubricated type is in general use, experiments with ball-bearing idlers having proved disappointing. Discharging from the belt may require assistance by air or water jets. Tailing is distributed from the end of the belt through semi-circular launders (9-in. radius) of No. 10 sheet iron in 4-ft. sections. They are laid on the pile with a slope of 2.5 to 3 in. per ft. Flow is assisted by water (formerly also by flotation tailing) pumped through a pipe from the mill to the upper end of the conveyor; in some cases part of this flushing liquid is dropped on the flank of the sand pile, washing out a cavity to be concurrently filled with sand from the conveyor. Slime tailing was sometimes filtered through sand piles, yielding clear water to the reclaiming pond, but impounding dams for slime tailing are now practically always used.

At certain mills of the Tri-State district (*H. H. Wallower, PC*) the tailing, previously dewatered by cones, drag scrapers, shovel wheels, screens, etc., is now stacked by inclined belt conveyors instead of by elevators. Fig. 7 illustrates one method of arrangement. One 250-ft. conveyor inclined at 20° will replace a mill elevator, final stacking elevator

and two dummies, with their connecting launders; the power consumption will be about one-third that for the four elevators, and the life of conveyor belt is 2½ to 3 times that of elevator belts; attendance is also reduced. At the GOLDEN ROD mill, one 450-ft. inclined belt replaced five elevators and avoided erection of a sixth and was expected to save $21,100 in a four years in addition to refunding the cost of the conveyor ($6300). The usual belt in the Tri-State district is 18 in. wide, inclined at 20°. Speeds are: 175 ft. per min. for 3[illegible] tons per hour; 220 ft. per min. for 45 tons; 230 ft. per min. and a 20-in. belt for 55 tons per hour. At one plant, stacking by a conveyor 260 ft. long rising 85 ft., requires 0.5[illegible] hp.-hr. per ton, including dewatering wheel, conveyor, and slime pump.

Tailing is distributed from the head of the conveyor with or without the addition of water. A common means for distribution without additional water is an auxiliary horizontal reversible conveyor, at right-angles to the elevating conveyor, and extending in both directions. The GOLDEN ROD mill will install a 40-ft. distributing conveyor pivoted at its feed end and supported by cable from an A-frame at the head of the inclined conveyor;

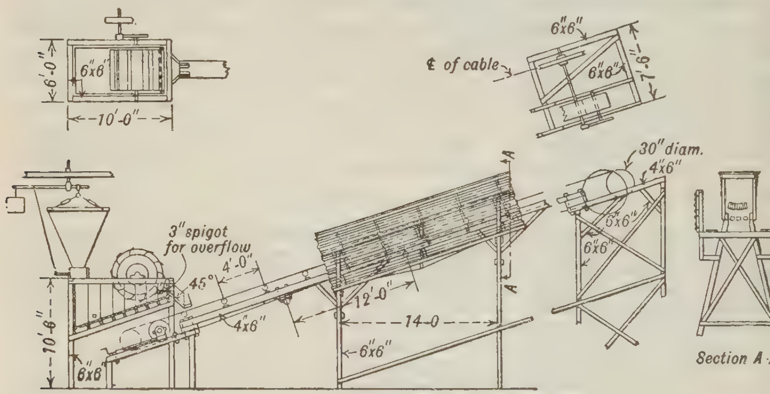


FIG. 7.—Tailing conveyor.

this can then swing 180° and may be inclined to afford additional elevation. In the we[illegible] method, the inclined conveyor discharges into a box which also receives flotation tailin[illegible] pumped up from the mill; the combined pulp then flows through a steel launder slopin[illegible] 2 in. per ft. The average mill, discharging 45 tons sand tailing per hour, will pump 25[illegible] gal. per min of flotation-tailing pulp the solids in which average 98 per cent. −100-mesh

Centrifugal sand pumps are now available in durable types and adequate sizes to stand the severe service of pumping large volumes of tailing. (For pumps in general mill service, see Art. 6; also Sec. 20, Art. 11.) Sands up to ⅛-in. size can be pumped safely, if carried by at least three parts of water; coarser sands are being pumped, but at the cost of excessive wear. Pulps below 48-mesh can be pumped at a dilution of 1 : 1; and, in general, trouble due to sedimentation is less likely to occur with thick than with thin pulps. Pipes through which coarse pulps are being pumped should be inclined at least 20° from the horizontal, to permit complete drainage when the pump stops.

At UTAH CONSOLIDATED 950 tons per day of tailing (80 per cent. −200-mesh) in pu[illegible] carrying 40 per cent. solids is pumped against a head of 35 ft., using a 6-in. Wilfley pum[illegible] BUNKER HILL & SULLIVAN lifts 600 tons of general tailing with 24,000 tons of water per d[illegible] against a 25-ft. head (to a gravity flume) with a 12-in. Byron Jackson pump direct-co[illegible] nected to a 75-hp. motor at 900 r.p.m. (*120 P 525*). MIAMI formerly used three 6-in. W[illegible] fley pumps (with three others always in reserve) to lift 3300 gal. per min. of tailing pu[illegible] carrying 22 to 27 per cent. solids (averaging 6000 tons dry solid per day crushed to 18 mes[illegible] against a total head of 46 ft. Each pump was driven at 1020 r.p.m. by belt from a 50-h[illegible] motor; the power requirement was 0.45 kw.-hr. per dry ton. The delivery pipe was 18 i[illegible]

diameter and was steel for a short distance from the pumps, the remainder redwood; total length, 1280 ft. The average pulp velocity was 4.2 ft. per sec. The expense per dry ton was: power, 0.594¢; labor and supplies for operation, 0.224¢; maintenance of pumping plant, 0.212¢; total, 1.032¢, not including labor on the dump. Wood pipe showed no wear after 18 months (*C. E. Chaffin, PC*). This system is now displaced, rearrangement of the milling process having made it possible to discard tailing by gravity from a higher level (see p. 1284).

Air-lifts were installed at the Chino Copper Co. in 1919 to raise 12,000 (dry) tons of tailing per 24 hr. to a height of 40 ft., delivering into a gravity launder (*J. M. Sully, PC; 112 J 806*). For details of construction and operation, see Sec. 20, Art. 15. The costs for the initial period of 16 months (Sept., 1919 to Dec., 1920), for 9 months of 1922 (resumed Apr. 1 after a year's suspension), and for the year 1923, are compared in Table 16.

Table 16. Operating cost, Chino tailing air-lift

	16 months, 1919–1920	9 months, 1922	Year, 1923
Ore milled, tons	....	1,416,869	2,785,564
Tailing lifted, tons	2,207,922	1,348,619	2,645,182
	Cents per dry ton	Cents per dry ton	Cents per dry ton
Steam power	0.687	0.675	0.721
Electric power (condenser auxiliaries)	.102	.091	.078
Compressor labor (operation and repairs)	.120	.063	.064
Plant labor (mostly repairs)	.022	.194	.111
Compressor supplies	.040	.059	.021
Plant supplies	.017	.261	.087
Total per dry ton lifted	0.988	1.343	1.082
Total per ton milled	....	1.278	1.028
Total per ton of pulp	0.148	....	....

During the first 16 months, a belt-bucket elevator in the mill was lifting a slightly greater volume of the same pulp to a corresponding height at total cost of 1.954¢ per dry ton (0.293¢ per ton of pulp). In addition to the indicated saving of nearly 50 per cent. in operating cost, the air-lift had the additional advantage of causing no delay for replacements, and of being more elastic in its power requirements. The air-lift was able to start and clear itself after the foot-piece had been buried under 32 ft. of settled slime. The estimated cost of a bucket-elevator installation for the same service as the air-lift plant was 33 per cent. greater than the estimated cost of the latter. During 1921, experiments demonstrated the possibility of air-lift elevation for tailing from the coarse-concentrating department, containing everything up to 1-in. size in a pulp with 50 per cent. solids in suspension.

Possibility of utilizing tailing may have a bearing on the selection of the method for temporary disposal. Common uses for mill tailing are: (*a*) mine filling, as at certain mines in the Lake Superior copper district, Butte, So. Africa, and West Australia; (*b*) for re-treatment, as at the Calumet & Hecla, Utah Copper, Anaconda, Chino, and Panda mills; (*c*) for road building, railroad ballast, and structural purposes; (*d*) for recovery of soluble constituents by natural efflorescence, as at certain cyanide mills; (*e*) for agricultural purposes, as at Mascot, Tenn.

Mine filling. At the Champion mine deficiency of coarse waste filling is made up by hauling mill tailing up to ¼-in. size in railroad cars and dumping into a raise connecting with the working levels. The sand is distributed in the stopes by blowing through pipes up to 250 ft. long from a tank holding a charge of 1.5 tons, using air at 75 lb. pressure. The cost is 2¢ per ton of sand moved. Burning stopes in the Leonard, Tramway and West Colusa mines, Butte, were filled with flotation tailing, both old and currently produced, of which 50 per cent. was −280-mesh. The material was sluiced through a 3650-ft. flume, 23 in. wide, on 2-per cent. grade, in a pulp carrying 20 to 50 per cent. solids and 250 to 600 gal. water per min. Pulp was conducted down the shaft in a 6-in. cast-iron pipe and dis-

tributed 800 to 1000 ft. laterally through 4-in. pipe (cast and wrought) with bends of 4-f radius. A head of 100 ft. was usually satisfactory for distribution (*68 A 61*).

Re-treatment of tailing by tabling, ammonia leaching, and flotation, forms an importar part of the present milling operations at CALUMET & HECLA (*117 J 277*), the material havin been formerly deposited under water and along the shore of Torch Lake, from which it reclaimed by suction dredges. (See pp. 78, 967.) At ANACONDA, when starting the mi in 1901, slime tailing was saved for future treatment by collecting it in six earth pond each 300 by 600 ft. in area and averaging 15.5 ft. deep, the flow, when possible, being cor tinuous through three ponds in series. Settled material was excavated by drag-line scrape and piled alongside the ponds (*46 A 249*). In subsequent years, this accumulation afforde notable amounts of cheaply recovered copper. At CHINO a similar plan was followe during early operations, and later a 1000-ton mill was erected to treat the accumulate tailing. At PANDA, 4000 tons per day of a copper-carbonate ore is concentrated by jig and tables. The tailing averages 5 per cent. Cu, easily recoverable by leaching, for whic treatment it is being impounded (*29 MM 137*, *5 MMt 55*). At the SWEENEY mill, Ke logg, Id., 1,200,000 tons of jig, table, and flotation tailing, −20-mm. in size and averagin 1.9 per cent. Pb, was re-treated. The pile had been formed by the Joplin method of eleva tors and launders (see p. 1285); it covered 20 acres, had maximum height of 70 ft., max mum slope of 20° 07′, a minimum natural slope of 4° 30′, and weighed 110 lb. per cu. f It was reclaimed by steam shovel, standard-gage cars and locomotive, and inclined be conveyor. (*119 P 289*.) At UTAH LEASING Co. (p. 99) old tailing (0.7 per cent. Cu a chalcopyrite) from the Cactus mill was reclaimed at the rate of 620 tons per day by stea shovel and 5-ton cars hauled 1000 ft. by a steam locomotive.

In general, when disposing of tailing in expectation of subsequent re-treatment, th following precautions should be observed, where practicable, even at some additional co (*58 A 178*): (*a*) when starting a mill, the earliest tailing should not be placed so that will be buried under later accumulation, which will probably be of lower grade; (*b*) mid dling should not be mixed with tailing; (*c*) tailing should be placed in a deep and narro rather than a broad and shallow deposit, to facilitate reclamation; (*d*) sand and slime tai ing should be separated; the latter is not only lower in grade but also needs no re-crushin before further treatment.

Road building and railroad ballasting consume important amounts of coarse mill tailin in the tri-state, south-eastern Missouri, Lake Superior, northern New Jersey and nume ous other districts situated near populous communities. At the RICHARD mine, Dove N. J., a large part of the hard, light-colored tailing from dry concentration of iron ore sold for structural purposes within a radius of 100 miles (*115 J 973*). Two coarse grade −2 + ¾-in. and − ¾ + ⅛-in., sell for $1 per ton, and sand − ⅛-in. for 80¢ per to The income from this source is a set-off against fine crushing for recovery of addition iron. The jig tailing from the AMERICAN ZINC, LEAD AND SMELTING Co. mill at MASCO Tenn. (⅝-in. maximum size), is re-screened and classified into sized products suitable fo various structural purposes; the largest consumption is for railroad ballast, but some shipped as far south as Tampa, Fla. A finer material (80 per cent. −200-mesh) is shippe as far as New Orleans, Memphis, and Washington, for use in street and road constructio (*H. I. Young, PC*).

Brick of excellent quality and high acid resistance is made from ANACONDA flotatio tailing, which averages 20 per cent. alumina; the same practice is applied to mill an cyanide tailing at the OPP mine, Oregon (*115 J 492*).

Agricultural uses. The fine limestone flotation tailing (85 per cent. −100-mesh at MASCOT finds a good local market as soil dressing because of its lime content.

4. Topography of mill-site

General. The slope of the ground selected for the mill-site is not only a de termining factor in the cost of construction, but also has an important bearin on the cost of operation. Classified with respect to the slope of the site, mi buildings are of four general types: (*a*) Terraced, on steep hillside. EXAMPLES Alaska-Gastineau, Britannia, Copper Queen, Engels, Kimberley (B. C. Le Roi No. 2, Liberty Bell, Nevada Consolidated, Panda, Silver Dyke, Silve King, Sunnyside, Utah Consolidated, Utah Copper. (*b*) Low, broad buildin on practically level ground. EXAMPLES: Armstead, St. Joseph Lead Co (Bonne Terre), Federal Lead Co. No. 3 (Flat River), American Z. L. & S. Co (Mascot), and the typical mills of the Joplin district. (*c*) Tall and relativel narrow structure on level or slightly sloping ground. EXAMPLES: man anthracite breakers, Bunker Hill & Sullivan, Calumet & Hecla, Northern O

Co. (Edwards, N. Y.), Richard. (*d*) Composite type, represented by United Comstock, Ray, Miami, and numerous other cyanide and flotation mills; in these, the coarse-crushers and screens are housed in a tall structure (sometimes separated by some distance from the concentrator) while agitators, settling tanks, tables or flotation machines are covered by a low broad building with little or no slope.

The CONSTRUCTION FEATURES most affected by slope of site and type of mill are: excavation, retaining and foundation walls, design and erection of frame, with special reference to support of heavy or vibrating machinery and snow loads; natural lighting, and ease of enlarging. The OPERATING FACTORS most strongly influenced are: elevation and re-elevation of ore and its products, pumping and re-pumping of water, labor required for supervision, facility in making repairs and replacements.

Excavations required for a terraced site are not only larger in volume than for a level or gently sloping site, but are also more expensive per cubic yard, owing to the difficulty of employing horse or mechanical traction in the constricted working spaces. For methods and cost of excavation, see *Peele, Merriman, Gillette.* See also Table 43.

Retaining walls are an essential feature only in a terraced mill, although occasionally required elsewhere. On a steep hillside in loose material, retaining walls must have exceptional solidity to withstand the combined settling effect of drainage and the jarring of heavy machinery. When benches are in rock, retaining walls become more or less ornamental only, and are sometimes omitted. Retaining walls generally serve also as foundations for building and equipment. See Sec. 27, Art. 14.

Foundations for the mill building (aside from retaining walls) need have only moderate bearing strength for mills of low average height; in such mills, both terraced and level, the massive equipment rests on the ground, leaving little but the walls, roof, cranes and elevators to be supported by the vertical members of the structure. In tall mills, while it is possible likewise to place the heaviest crushers on the ground, provision must usually be made for carrying heavy and vibrating loads on fairly long vertical columns, and the foundations for these must be substantial. Safe bearing pressures for earth and rock are given in Table 17.

Table 17. Safe bearing pressures, tons per square foot. (*After Baker*)

	Minimum	Maximum
Rock equal to the best ashlar masonry...	25	30
Rock equal to the best brick masonry...	15	20
Rock equal to poor brick masonry......	5	10
Clay in thick beds, always dry..........	6	8
Clay in thick beds, moderately dry......	4	6
Clay in soft beds....................	1	2
Gravel and coarse sand, well cemented...	8	10
Sand, dry, compact, well cemented......	4	6
Sand, clean, dry......................	2	4
Quicksand, alluvial soils...............	0.5	1

Design and erection. In a terraced mill considerable diversity will occur in the dimensions of the several panels and their individual members, calling for a large amount of detailed designing. Erection is retarded by the necessary distribution of structural elements over a large area and at numerous levels. A tall mill also requires skillful and detailed designing; its erection is usually more rapid than that of a terraced mill because the materials can be assembled at a few convenient points, and hoisted into place by one or two derricks. A low, flat building has the advantage of simplicity in design; standardized, stock types of trusses can be selected, and the height of vertical members can be reduced to a few dimensions by bringing foundation walls or piers up to a common level at small expense. Erection of such a building can be done by men working mainly from the ground, with the assistance of a few gin poles. Cost data relating to the erection of typical mills are given in Art. 11.

Enlargements are made most easily at a terraced mill, since the direction of flow is usually directly down the slope, with only subordinate amounts of lateral transfer of prod-

ucts, and additional and similar sections at either end can be readily supplied with or by an extension of the receiving bins, or by a distributing conveyor. A flat mill can usually be extended in the same manner, provided it has been designed with a flow mainly i one direction. A tall building offers the most difficulty, since its ore must be received a a central point and the flow is mainly vertical. BUNKER HILL & SULLIVAN doubled th original capacity of its No. 2 West mill by erecting a duplicate 5-story addition on th opposite side of its receiving bin.

Gravity flow of feed and products, with the accompanying large volume of water, an as nearly complete elimination of elevators and pumps as is possible, are the main object sought by the terraced and tall types of mill. In districts such as south-west Colorad and the coasts of Alaska and British Columbia, choice is practically limited to the terrace site, and in other localities, where ore or water or both can come by gravity to a mill havin the necessary space for tailing disposal, also by gravity, the advantages of a terraced sit probably outweigh its drawbacks. In other situations, requiring initial elevation of ore c water or mechanical elevation of tailing, the choice of a site and the decision as to type c mill demand careful investigation.

Type of mill. As between a tall or terraced and a low mill structure the decision is based on comparison of the costs of the following items: (a Initial elevation of ore. (b) Initial elevation of water. (c) Final elevation c tailing. (d) Final elevation of concentrate. (e) Intermediate elevation of or and products. (f) Re-elevation of water. (g) Interest and amortization o excess cost of erecting and installing a terraced or tall mill. (h) Increase labor charges, due to inconvenience of supervising operations on numerou floors; efficiency of extraction may be adversely affected for the same reason

Initial elevation of ore to the top of the mill by means of the mine-shaf hoist can be performed at little or no expense above that required to delive the ore at ground level; this is general practice in the Tri-State district and often done elsewhere, as at Mascot, Butte & Superior, Engelmine, an Bonne Terre. Other common methods of initial elevation are by chain-buck elevator, as at Utah Apex; inclined pan conveyor, as at United Comstock an at Federal Lead Co., No. 4 mill; inclined belt conveyor as at Bunker Hill Sullivan, Chino, Copper Queen, Santa Barbara, and many others; and incline trams, as at those mills of the Joplin district that receive ore from other source than adjacent shafts.

At BUNKER HILL & SULLIVAN (*S. A. Easton, PC*) the entire feed for No. 2 West m (50 tons per hr. of −1.25-in. material weighing 150 lb. per cu. ft.) is elevated by a be conveyor 227 ft. long, inclined 22°, and rising 52 ft.; 5-ply belt is 20 in. wide, speed 187 f per min. Head pulley 42 in., tail pulley 30 in., 5-pulley troughing idlers. Operati power, including five feeders, one weigher, one pump, one distributer, and one sampler 18 hp. Operating cost, per ton: equipment (troughing and idler pulleys, etc.), 0.029 belting, total length, 465 ft., 0.068¢; power, 0.009¢; labor (cleaning, oiling, etc.), 0.217 total, 0.323¢ per ton.

At UTAH APEX (*J. H. Manwaring, PC*) a chain-bucket elevator 76 ft. long, incline 71° and moving at 63 ft. per min., is used to lift the entire mill feed of 100 tons per h The drive is a 25-hp. motor which draws 11.2 kw. at full load, 7.9 kw. empty. Run-of-mi ore is fed from a track hopper by a steel-apron feeder driven from the tail pulley of t elevator. Keystone rivetless manganese 9-in. chain lasted 5 years; 9-in. Simplex rivetle manganese chain, was still in use at the end of five years. Buckets, 108 spaced 18 in., a 24 × 14 × 17½-in., No. 8 steel; cost $10.61 each. The original cost of the elevator 1913, f.o.b. plant was $3093. The total cost of repairs during 10 years, including o entire replacement of chains and sprockets, was 0.715¢ per ton; power (@ 1¢ per kw.-hr. 0.112¢ per ton.

Initial elevation of water. See Art. 2. In those cases where wat must be pumped, the cost of elevating water to the storage reservoir may more costly than elevation of ore. In general, pumps are more efficien mechanically, than ore elevators, but the fact that they may be required lift 4 to 40 tons of water for every ton of ore, and also to a higher level, makes

essential to keep down mill height, unless abundant water under natural head is available.

Final elevation of tailing is ordinarily unavoidable at any mill on level ground. If treating an ore from which it is possible to extract clean tailing at a fairly coarse stage, a tall or a terraced mill may have the advantage of being able to dispose of this portion of its tailing without further elevation; but at Bunker Hill & Sullivan, a tall mill on level ground at which some coarse-sand tailing is discharged, it is necessary to use a large tailing elevator with a lift of 25 ft. (*120 P 525*). Modern practice discourages coarse tailing; hence the disposal problem has less bearing upon the type of mill than upon the selection of a site. (See Art. 3.)

Elevation of concentrate. The importance of this feature is often exaggerated. Concentrate rarely exceeds 20 per cent. the weight of original ore, and at most of the largest mills it ranges from 3 per cent. to 7 per cent.; hence, any design that provides an additional story or terrace solely to permit gravity disposal of concentrate will be unnecessarily expensive to construct, and is likely to be uneconomical in operation, if it throws additional work upon pumps or elevators at earlier stages of the process.

Intermediate elevation of ore in process can rarely be wholly avoided, even by the most steeply terraced mill.

Britannia, a flotation mill with seven terraces and a drop of 250 ft. in a horizontal length of 209 ft., requires only one elevator. Alaska-Gastineau, a table mill on a steeply-terraced hillside, requires two sets of automatic skips (with 100-ft. lifts) and two bucket elevators. Silver Dyke mill (tables and flotation) with 11 terraces and a total drop of 150 ft. in a horizontal length of 277 ft., requires one elevator of 27-ft. lift and an inclined conveyor with 19-ft. rise, in addition to a number of sand pumps.

In general, the necessity for intermediate elevation is most pronounced in those mills treating an ore in which the valuable minerals occur as particles of graduated size, recoverable by a stage process of reduction and giving rise to important quantities of middling products which must be returned for re-crushing; in such mills, terracing shows the least advantage over the flat system because no practicable amount of terracing (except in rare instances) can provide an exclusively gravity flow for all unfinished product. On the other hand, ores containing only finely-disseminated minerals can often be reduced at once to the final size by consecutive crushing operations, most of the unavoidable re-elevation being performed by scraper classifiers, and separation is made on tables or flotation machines requiring but little mill height. For a milling operation of this character, a gravity flow through the whole concentrating division of the plant can be obtained by a moderate amount of terracing; the re-elevation saved thereby, as compared with a flat mill, while of relatively small lift is large in total volume, the pulp usually containing 75 per cent. or more of water. Methods of re-elevating ore, dry or in pulp, are described in Art. 6 and Sec. 20.

Re-elevation of water. Assuming water reclamation necessary, the advantage is with a low mill. A tall or terraced mill can sometimes utilize the tailing water from jigs treating sized or washed ore as wash water on tables at a lower elevation, but in general clarified water in large amounts is not obtainable until near the end of the concentrating scheme, and most frequently is not recovered until after it has left the mill at the lowest point. If ore is re-elevated by pumps, it must be accompanied by at least an equal weight of water, and may require three times that amount, or more, depending upon its coarseness.

Interest and amortization. The excess cost of a terraced or tall mill as

compared with a low mill of corresponding capacity should be taken into account in comparison of operating costs. The amortization charge per ton will obviously be small for a mill of long life. An elaborate and expensive mill would be injudicious for a mine of dubious life. This explains the character of mill structures commonly erected in the Wisconsin and Joplin zinc districts.

Supervision of machines is greatly impeded when the millmen are compelled to walk up and down stairs; hence in a large mill, justifying the employment of specialized millmen, it is advisable to place as many as possible of the machines of one kind on the same level; in a small mill, as much as possible of the whole equipment should be on one floor, to insure equality of supervision. Facility of supervision not only fixes the number of millmen to be employed at a small mill, but may influence the tonnage and recovery attained. A mill using several dissimilar crushers, or operating a process requiring a variety of concentrating apparatus or involving extended use of hydraulic classifiers, will need more supervision than a mill of the same capacity using only ball mills and flotation. Any mill, however small, needs a certain minimum of supervision, while in another mill of the same type a much larger tonnage may often be treated with the same labor force. A large mill is justified in the installation of mechanical equipment for performing certain operations, such as collection of concentrate, which can more profitably be done in a small mill by hand labor.

Repairs and renewals are much simplified in a level-site mill, where workshops and the operating floor can be joined by a level traveling way or overhead crane.

Lack of working space on the ball-mill terrace was one defect of the ALASKA-JUNEAU mill; the ALASKA-GASTINEAU provides a shop on every terrace; at the UNITED COMSTOCK commodious shops serve every part of the ball-mill floor by track and crane (*114 J 846*; *117 J 516*).

5. Materials for mill construction

The durability of a mill building should bear an approximate relation to its expected life; any expense incurred for making the structure outlast the mine is wasted. The useful life of a mill cannot always be predicted, but during the development of any mine or group of mines up to the stage at which erection of a mill becomes justified, enough information should have been obtained to permit a reasonable estimate of the minimum tonnage likely to be available. The largest and most expensive mills are not undertaken until the factor of ultimate tonnage has been most carefully investigated.

Foundations. Concrete is almost universally employed, except at certain small mills on ground not likely to be heaved by frost, where wooden blocking is sometimes sufficient. Materials for aggregate, already crushed, are usually readily available around any mine, particularly if a mill has previously been operated on the premises.

At ALASKA-GASTINEAU all of the concrete was made with rock and sand provided by excavation of an underground storage pocket adjacent to the mill.

The customary batter of the outside face of terrace retaining walls is 1 to 1¼ in. per ft. but at ANACONDA and SANTA BARBARA retaining walls are vertical. Concrete foundations for vibrating equipment, notably stamp batteries, require special solidity, and the upper surfaces should be protected by a semi-elastic mat. See Sec. 3, Art. 17.

At ALASKA-JUNEAU, oak blocks were necessary under the ends of the vertical columns carrying jaw crushers, to avoid disintegration of the concrete foundations.

Framework. Timber, steel, and reinforced concrete are the customary materials. Wood is the only material found in older mills and is still occasionally employed for framing large mills and nearly always for small ones. Wood has the advantages of: (*a*) lower cost per unit of weight, and also, in most places, per unit of strength; (*b*) reduced amount of detailed designing; (*c*) quicker delivery; (*d*) cheaper erection, by less highly skilled and more easily obtained labor; (*e*) elasticity, as compared with reinforced concrete. Its principal drawbacks are inflammability and liability to decay, especially in floors and mud-sills, which are unavoidably damp.

Structural steel is the prevailing material for large permanent mills, such as Britannia (1426 tons of steel in a terraced mill covering a horizontal area of 56,400 sq. ft. or 51 lb. per sq. ft.), Copper Queen, Chino, Kimberley, Panda, Silver King, Homestake (South), Utah Copper, and the cyanide division of the United Comstock mill (441 tons of steel in a building of 75 ft. maximum height covering 2¼ acres, or 9.2 lb. per sq. ft. of area). In addition to being fireproof and durable (if painted), steel has the advantage of permitting longer trusses and wider spacing, thus reducing the necessary number of columns which occupy or obstruct useful floor space (66-ft. spacing both ways at United Comstock); the columns themselves can also be longer without requiring such extensive cross bracing as would be needed for a wooden structure of same height. Many of the heavy loads (up to 60 tons) commonly carried by cranes serving the crushing departments of modern mills could not be supported safely on any practicable timber structure. While steel structures call for more detailed designing than wooden frames, much of this work can be delegated to the steel-work contractors; many steel fabricators carry in stock or can quickly supply columns, trusses, or whole buildings to cover specified floor areas with a roof of given height. Bearings or hangers for line shafts can be easily attached to steel beams by clamps; they are readily shifted and do not weaken the members by bolt holes, as is the case with wooden beams.

Reinforced concrete has not yet been widely adopted for structural framework, although it was used in the coarse-crushing, fine-crushing, and concentrating departments of the United Comstock mill (*114 J 846, 117 J 516*); in this instance, owing to the easily available aggregate for concrete, and the relatively high cost of structural steel, reinforced concrete was estimated to be nearly 20 per cent. cheaper than steel. Estimates of wooden construction were 30 per cent. lower than for concrete, but the saving on insurance within three years was enough to offset this difference. The cost of the structural concrete work (excluding 15,000 cu. yd. of foundation and retaining walls) was: Concrete, 132 cu. yd., \$1980; reinforcing steel, 21,600 lb., \$1050; forms, 9500 sq. ft., \$1700; total, \$4730, or \$35.83 per cu. yd. For all-steel construction, the corresponding cost for 106,020 lb. of steel, at 8¢ erected, was estimated at \$8481. In a mill of reinforced concrete, special attention must be given to support of oscillating machinery, on account of the severity of the stresses caused by vibration continuously in one direction.

Walls. Corrugated iron, galvanized or asbestos coated, is the commonest material for walls. A wooden frame is usually sheathed with boards, to which corrugated iron is nailed over a layer of paper. On a steel frame the corrugated sheet is attached by bolts, clamps, or wire, with no wood sheathing. Where this construction would not afford the necessary protection against cold, the interior can be coated with gunite, applied against woven-wire or expanded-metal reinforcement, with as much insulating material as desired.

The walls of the United Comstock cyanide section are composed of an outer layer of No. 24 corrugated and an inner layer of No. 28 plain galvanized steel, separated by two thicknesses of tar paper (see below). At the N. Y. Zinc Co. 5-story mill at Edwards, N. Y., the walls are hollow tile, and the equipment is entirely supported by a steel framework having no connection with the walls (*116 J 95*). The tall, steel-framed Richard mill has

hollow-tile walls for the lower half of its height, and asbestos-coated corrugated iron for the upper half (*115 J 973*).

Tile walls are much improved in appearance by an outer coating of mortar or GUNITE; when this finish is intended, the tile should be placed rough side out. The best mixture for applying with a cement gun is four parts clean sharp sand to one part cement; in a dry climate, the sand may be increased to six parts. The outer face of the tile must be thoroughly moistened before applying the mortar, but no reinforcement should be necessary, as would be required with a wooden wall. When applied with a cement gun (*Bul. 114, Cement Gun Co.*) one bag of cement and three cu. ft. of sand will cover 22 sq. ft. with a layer 1 in. thick. A force of eight men at $41 per day (including the operator of a portable compressor, which may not be necessary at an established mine plant) can average 1300 sq. ft. of gunite per day. The cost of a cement gun, not including a portable compressor, was (1923) from $1325 to $1565 corresponding to free-air capacities of 100 and 225 cu. ft. per min. respectively.

A comparison of estimates of costs and heat-insulating characteristics of different types of walls for the UNITED COMSTOCK cyanide building is shown in Table 18.

Table 18. Comparison of estimates of costs and heat-insulating characteristics of different types of wall

	Cost per square foot, cents	Boiler horse-power to heat plant
Plain galvanized corrugated sheet	14.0	220
Gunite, 1 ½ in. on metal lath	19.6	116
Hollow clay tile, 4-in.	35.0	93
Galvanized corrugated steel outside, plain galvanized inside, 2 layers of tar paper between	19.5	125
Wood sheathing, tar paper, galvanized corrugated steel	24.5	93

Roofs. Corrugated galvanized iron is the commonest roofing material. In a cool climate, a board sheathing is almost indispensable to prevent condensation and dripping of moisture; the corrugated sheet is fastened down to this by lead-washered nails.

BRITANNIA mill required 96 tons of corrugated sheet for a total roof area of 58,100 sq. ft. SMUGGLER UNION mill in Colorado (cold winters with heavy snow) has a roof of No. 22 corrugated iron laid on one layer of 1⁄16-in. asbestos sheet and two layers of tar paper; the inside is gunite on woven-wire reinforcement. The roof over the cyanide department of UNITED COMSTOCK (96,400 sq. ft.) is made of 1 ¾-in. Oregon pine, tongued and grooved, covered with No. 20 corrugated sheet. The roof of the KIMBERLEY steel-framed mill (77,000 sq. ft.) is composed of 2 × 4-in. timber on edge laid face to face and covered with felt, tar and gravel (*115 J 244, 116 J 453*). The "Malthoid" roof originally put on the ALASKA-JUNEAU mill proved unsatisfactory, and was replaced by corrugated iron.

Asbestos materials are durable without painting and resist fire. They are also good heat insulators.

In ONTARIO, where winters are long and severe and forest fires have been frequent, the usual roof construction is board sheathing, an intermediate layer of quilted hair felt with paper covers, and an outer layer of Flexstone asbestos-asphalt sheeting, or Transite asbestos-cement boards. Keystone hair felt (*J. W. Morrison, PC; Johns-Mansville, Inc.*) is sold in folded strips 3 ft. wide containing 500 sq. ft.; the edges are beveled and bound to give smooth lapping. Flexstone 3-ply roofing comes in rolls of 100 sq. ft., 32-in. wide; the 4-ply comes only in flat sheets 32 × 80 in. Transite boards, corrugated or plain, are made in thicknesses of ¼ and ½ in. and cut into sheets 42 in. wide by 4, 5, 6, 7 and 8 ft. long; they are largely used for walls as well as for roofs having a pitch of 2 in. or more per ft. The ¼-in. boards can be laid directly on purlins spaced up to 45 in.; the ½-in. boards on supports spaced up to 60 in.

Snow, in addition to its dead weight of 12 lb. per cu. ft. (*Trautwine*), interferes with roof lighting and may cause damage by sliding in heavy masses. This may be prevented by: (*a*) snow guards; (*b*) giving the roof sufficient slope (30° or more) to cause the snow to slide soon after it falls (ALASKA-JUNEAU installed salt water sprays to flush the snow off the

roofs as fast as it fell); (c) making roof so nearly level that snow cannot slide (as at HOMESTAKE SOUTH mill). According to *Kidder*, metal roofs in the Rocky Mountain and northwestern states should be designed for the following snow loads per sq. ft. of roof surface: ½-pitch, none; ⅓-pitch, 10 to 12 lb.; ¼-pitch, 20 to 25 lb.; ⅕-pitch, 27 to 37 lb.; ⅙-pitch or less, 35 to 45 lb. UNITED COMSTOCK roof, with an unbroken slope of 20° (⅓-pitch) was designed for a snow load of 40 lb. and a wind load of 20 lb. per sq. ft.

Windows and skylights. Daylight is of the utmost importance in practically every department of a concentrating mill. It is easily provided in a tall structure, the walls of which can be composed of glass to a large extent, as in anthracite breakers. A terraced mill can be amply lighted from its sides, and also by vertical windows in the breaks of the roof.

The steeply-terraced BRITANNIA mill has 22,300 sq. ft. of windows for lighting a horizontal area of 56,400 sq. ft. (*29 MM 204*).

A low, flat mill can be lighted by skylights in the roof, subject to interference by snow and probability of leakage unless constructed with special care; a turreted or saw-tooth roof with windows in all available vertical spaces is frequently used for industrial plants and might be equally useful for ore dressing plants.

Fenestra steel sashes have been installed in large numbers at UNITED COMSTOCK, HOMESTAKE, and numerous other mills with both steel and wood construction. At COPPER QUEEN the windows are glazed with ribbed glass, for diffusion of sunlight, and the skylights are of RUBBER GLASS. This material is used for skylights in many other mills in the southwest, where snow is unknown, rain not abundant, and the sunlight so intense as to make the yellow tint imparted agreeable.

Floors. Wood and concrete are used indiscriminately in mills of either wood or steel construction. A wooden floor in the wet-concentrating section of a mill should be tight, to prevent loss of concentrate, and should preferably be laid with green timber, to avoid warping. Concrete floors are usually spread in place.

UNITED COMSTOCK used pre-cast concrete slabs supported by steel frames. At the steeply-terraced mills of the SILVER DYKE, NATIONAL COPPERMINES, and UTAH CONSOLIDATED mills, reinforcement was obtained by discarded hoisting rope in lengths running unbroken from the highest to the lowest floor, passing horizontally through every floor and downward through the intervening retaining walls, thus tying the latter securely together.

The SLOPE of floors in the wet parts of a mill should be sufficient to permit rapid drainage; dry floors are usually cleaned by sweeping and the floors require no slope. Slopes of ¼ and ½ in. per ft. are most common, the latter preferred. Slopes up to 2 in. per ft. are sometimes adopted to permit launders to be laid on the floor, but this is unnecessary and makes walking unsafe. The discomfort of walking on a concrete floor, especially if wet, should be ameliorated by providing walk-ways of wooden slats.

Painting. Black corrugated iron should be protected by at least two coats of paint, renewed at frequent intervals. Galvanized iron does not usually require paint protection, although it is subject to rapid deterioration anywhere in the vicinity of a smelter. A durable paint for new galvanized iron has not yet been found, but after the metal has been exposed for two or three years any good paint will adhere firmly. The inside of a mill should be painted white or a light tint. An air brush is a useful tool for applying oil paint. Whitewash does not adhere strongly enough to resist loosening under continuous vibration.

6. Arrangement of mill equipment

The grouping of mill equipment should aim to satisfy the following requirements: (*a*) Mechanical transfer of feed and products by the shortest and most direct routes, utilizing gravity flow so far as economically practicable (see Art. 4). (*b*) Convenience of superintendence (see Art. 4). (*c*) Economical connection with driving power (see Art. 7). (*d*) Facility of repairs (see Art. 9). If possible, provision should be made for by-passing any piece of machinery undergoing repair, to avoid complete suspension of operations; in a small mill, without duplicate equipment, this may not be possible. (*e*) Facility of enlargement or remodeling (see Art. 4). (*f*) Isolation of dust-making operations, with or without arrangements for collecting the dust (see Art. 8).

Grouping in plan. Dimension sheets giving over-all dimensions of the individual pieces of ore-dressing equipment are usually obtainable from the makers. The additional necessary allowance of floor space between and around the machines composing a single group should provide for: (*a*) Walkways for inspection and adjustment of machinery; also, in some cases, for manual disposal of concentrate by wheelbarrow or tram car. (*b*) Working space for making repairs with minimum amount of carriage. (*c*) Launders, chutes, or conveyors for the several products of the operation. (*d*) Motors, in case of individual drives, with speed reducers when required. (*e*) Elevators or pumps, possibly serving other groups of mill equipment.

Jigs, tables, pneumatic flotation cells, and other equipment to which a sufficient amount of attention can be given from one side, are often consolidated into pairs, or blocks of four, mainly to facilitate feeding and to combine discharges, but also to economize floor area. Ball, pebble, or rod mills are usually set as close as possible to a sand-slime separator. Screens and hydraulic classifiers seldom require special allowance of floor space, since they can usually be supported overhead. Spacing of coarse crushers is frequently determined by the dimensions of the bins from which they receive their feed or to which they deliver, rather than by the amount of floor space they actually require. Table 19 gives total floor area of representative mills.

Grouping in vertical relation. The difference in elevation between the points at which a given piece of apparatus receives its feed and discharges its products is most important from the standpoint of mill design, since it is this dimension that fixes the position of the machine with respect to the "stream line" of the mill. Over-all vertical dimensions are needed for placing foundations and making allowance for head room; the latter should make due provision for hoisting out parts of machines requiring replacement.

Chutes and launders. (See Sec. 20, Art. 9, 10.) In general, a chute for nominally dry material should have a slope of not less than 45°, to avoid choking in case of accidental or temporary wetting. A launder that proves to have insufficient grade can be corrected, if it is impossible to increase its slope or if inadvisable to increase the proportion of water, by diminishing its width or by introducing liners of smooth material with low coefficient of friction. Rubber, both of crude crepe and vulcanized varieties, is receiving wide attention as lining material for launders subjected to hard service; in spite of its higher cost per pound, it is usually cheaper than steel or chilled iron per sq. ft. of surface, and compares favorably with those materials in withstanding wear. Locally made concrete slabs are usually much cheaper per sq. ft. than iron. In places where sufficient gravity flow is not easily obtainable, as in collection of concentrate or tailing from a long line of jigs or tables, a shaking launder on a grade as low as 3/8 in. per ft., suspended by flexible wood strips and oscillated

Table 19. Total floor areas of typical mill buildings

Mill and general nature of process	Daily tonnage	Department	Sectional area, square feet per ton-day	Total area, square feet per ton-day
Allenby, B. C., fine crushing and flotation.	2,000	....	....	33.12
Anaconda (one of 8 sections), jigs, tables, flotation (as of 1919).	2,000	Bin, Blake, 2 rolls, elevators......	2.18	
		36 @ 3-cell Evans jigs...........	1.56	
		2 rolls, 18 Wilfleys, 5 elevators....	2.60	
		6 Hardinge, 10-ft.; and trackway..	2.50	
		4 M. S., 16-cell; 1 elevator.......	2.50	11.34
Annapolis, Mo., jigs, tables and flotation	1,000	1 gyratory, 2 rolls, 2 screens.....	3.7	
		2 Hancock jigs, 2 rod mills.......	3.2	
		16 Wilfleys, cones and elevators...	4.0	
		2 Callow, 11-cell; classifier and filter	3.2	
		2 Dorr settlers, 20-ft., Lowden dryer	4.0	18.1
Armstead, tables and flotation.	150	Bins, jaw crusher, elevators, screens	16.8	
		Hardinge, 8-ft.; Dorr classifier, 6 ft.	4.8	
		5 Wilfleys, 2 Callow, 10-cell.......	22.0	
		Thickeners, filter, concentrate bin..	15.4	59.0
Betty O'Neal (one of 2 sections), flotation only.	150	Receiving bin, ore partly crushed..	2.8	
		2 ball mills, classifier, etc.........	8.2	
		M. S., 12-cell, 18-in.............	5.3	
		Table, 10-ft. thickener, pumps....	7.0	23.3
Britannia, B. C., flotation only.	2,700	....	....	21.0
Chino, tables and flotation.	14,000	(No coarse crushing in this building)......................	....	8.6
Consolidated Coppermines, flotation and tables.	1,000	Bins and coarse crushing.........	4.88	
		Fine crushing, Hardinge	3.25	
		Flotation, 30 @ 7-ft. Callow cells.	3.46	
		Tables, 22 Wilfleys..............	4.32	
		Thickening and filtering.........	7.55	
		Boilers, engines, etc.............	9.28	32.74
Homestake South, amalgamation (coarse crushing in separate building).	1,800	7200-ton bin, 120 stamps, 6 rod mills and classifiers, 24 amalgamating plates, 30 motors (no cyanidation in this building)....	....	13.3
Joplin, typical mill built by United Iron Works, jigs and tables.	200	2 bins.........................	4.9	
		4 rolls, 6 elevators, 4 screens, 4 jigs	15.7	
		6 Wilfley tables................	7.6	
		Engines and boilers.............	12.2	40.4

Table 19. Total floor areas of typical mill buildings—*Continued*

Mill and general nature of process	Daily tonnage	Department	Sectional area, square feet per ton-day	Total area, square feet per ton-day
Kimberley, B. C., flotation only.	2,500	....	....	31.0
Panda, Belgian Congo, jigs and tables.	4,000	Coarse crushing, hand picking....	6.0	
		Jigs, tables, fine crushers, etc.....	11.5	17.5
Santa Barbara, tables only. (Coarse crushing in separate building.)	500	2 rod mills, 5 elevators, 8 screens..	8.0	
		40 Deister tables................	15.7	
		Dorr thickener, 5 elevators, 14 dewatering tables and screens.....	8.9	32.6
Silver King Coalition, jigs, tables, flotation	300	2 bins, total 4900 tons...........	12.0	
		Gyratory and grizzly............	5.3	
		12 Harz 2-comp. jigs............	7.7	
		Roll, elevator and screen.........	4.7	
		1 Hardinge, 1 Marcy, 26 tables...	18.3	
		Flotation, thickening, filtering, etc.	15.2	63.2
United Comstock, tables and cyaniding (operations in 4 separate buildings).	2,000	Coarse crushing, gyratory and rolls	4.4	
		Fine crushing and tables.........	10.7	
		Cyanide treatment..............	48.2	
		Precipitation and refining........	1.8	65.1
United Eastern, fine crushing and cyanidation	400	Gyratory, 2 Marcy, 3 tube-mills, leaching and settling tanks, etc.	....	70.0
Utah Consolidated, flotation only.	1,000	Coarse crushing, gyratory and rolls	1.86	
		Fine bin, screens and elevator....	1.75	
		Fine crushing, 6 ball mills, 6 classifiers, 6 motors................	7.30	
		Flotation, Callow, 120 cells......	4.50	
		Settling and filtering............	4.74	20.05
Utah Copper Co. (Arthur), tables, vanners, flotation (as of 1918).	16,000	Coarse breaking................	0.36	
		Secondary crushing.............	1.57	
		Fine crushing, tables, vanners....	13.30	
		Flotation, including blowers......	4.47	
		Filtering......................	0.65	20.35
		22 Dorr tanks, 75-ft. (out doors)...	(12)	
Silver Dyke, flotation and tables.	450	Bin, coarse crushing, conveyor....	5.36	
		Fine crushing, Marcy, etc........	5.08	
		Flotation, Callow, 48 cells........	4.54	
		Tables, 12 Deister..............	6.40	
		Thickeners, concentrate and tailing	12.00	33.38

by an ordinary table head-motion at one end, has been found useful at New Jersey Zinc Co. and Santa Barbara (*112 J 1050*). Another method applicable to jig tailing is to dewater on a screen and deliver to a horizontal belt conveyor.

Launder slopes. The slopes given in Table 20 are suggested as working limits for rectangular wooden launders conveying average ores of pyrite, zinc, and copper, with concentrating ratios between 4 and 20 to 1, and dilutions of not less than 2 water to 1 ore. See also Sec. 20, Art. 10.

Table 20. Average slopes for launders

Jig and table mills	Inches per foot	Fine-grinding and flotation mills	Inches per foot
Trommel product:		Sands, 20 per cent. moisture, classifier to tube-mill:	
+20-mm	5–6	30-mesh	4–6
−20+10-mm	4–5	80-mesh	3–4
−10+5-mm	3–4	Tube-mill discharge to classifier:	
−5+2½-mm	2–3	30-mesh	2–2½
−2½-mm	1½–2	80-mesh	1½–2
Table feed, 20-mesh	1¼–1½	Classifier overflow to flotation:	
Table tailing, 20-mesh	1½	48-mesh	½–¾
Table middling, 20-mesh	1½	80-mesh	¼–½
Table concentrate, 20-mesh	2½	Flotation concentrate	2–3
Tail race, mixed sizes	⅜–¾	Flotation tailing	¼–½
		Tail race	3⁄16–¼

For zinc-lead and iron-oxide ores of low concentrating ratio, the launder slopes would need to be probably 25 per cent. steeper than those stated above.

Cost of launders. From the figures in Table 21, relating to reconstruction of the Phelps-Dodge Morenci mill in 1924, it is seen that the labor cost to install $1 worth of material ranges from $1 to $1.25. In this mill, only 14.4 per cent. of the whole launder system was over 16 in. wide.

Elevation of ore in transit through a mill is practically unavoidable, even on steeply terraced sites; on flat sites, elevators or sand pumps constitute a large and expensive part of the mill equipment, beside requiring close attention to maintenance. Structural details of inclined belt or pan conveyors, belt-bucket or chain-bucket elevators, sand pumps, centrifugal pumps, and suction pumps for ore pulps are given in Sec. 20.

Inclined belt conveyors (see Sec. 20, Art. 1) for initial delivery of ore to the mill are discussed in Art. 1, and those for disposal of tailing in Art. 3. Inside a mill, they are largely employed in the coarse-crushing department, being especially useful in delivery of ore into a long bin or receipt of ore from a row of chutes. Data on a few noteworthy installations are given in Table 22. Distribution of ore from a belt conveyor can be accomplished by (*a*) movable tripper, shifted by hand from place to place; (*b*) automatic traveling tripper, actuated by the belt itself; (*c*) shuttle conveyor mounted on a truck running on longitudinal rails and discharging only from its end; this system is particularly useful for materials which have to be handled only at long or irregular intervals.

Inclined pan conveyors (see Sec. 20, Art. 2) are most used for elevating crude ore in lumps of such size (say 4-in. and over) that they would be injurious to rubber belts; incidentally, they may serve as feed regulators for coarse crushers.

At United Comstock (*114 J 846, 117 J 516*) a 48-in. Stephens-Adamson pan conveyor 91 ft. long and rising 30 ft. (20°), driven at 9 to 15 ft. per min. by a 20-hp. variable-

speed, slip-ring motor, carries 125 tons of mine-run ore per hour. At the COPPER QUEEN concentrator are two 54-in. pan conveyors in parallel, each capable of carrying 1800 tons −8-in. ore per hour (250 tons normal rate); length, 61 ft.; rise, 21 ft.; slope, 20° 10′.

Table 21. Cost of launder construction, Morenci mill, Phelps-Dodge Corporation

Department	Length, feet	Width, inches	Cost per linear foot	Lumber, board feet	Cost per M.B.M.
Tables and de-slimers.	1866 204 100 18 2188	12 16 18 24	Labor.....$1.46 Lumber.... .64 Cast-iron lining.... .65 $2.75	11,403	Labor....$34.00 Materials. 18.65 Total, including lining...$52.65
Primary grinding, classification and flotation.	363 500 863	12 16	Labor.....$2.01 Lumber.... 2.92 Wrought-iron lining...... 0.10 $5.03	26,308	Labor... $68.23 Materials 97.47 Total, including lining..$165.70
Secondary grinding, classification and flotation.	801 505 72 133 60 1571	12 16 18 22 36	Labor.....$1.84 Lumber.... 1.17 $3.01	19,519	Labor...$147.78 Lumber.. 94.44 $242.22
Concentrate dewatering.	1195 186 134 101 1616	6 12 16 30	Labor.....$0.66 Lumber.... .63 $1.29	10,230	Labor......$104 Lumber..... 100 $204
Tailing dewatering.	90 909 588 70 1650	12 16 24 36	Labor.....$2.01 Lumber.... 1.51 $3.52	33,496	Labor... $99.00 Lumber.. 74.40 $173.40
Total, all launders.	7888		Labor.....$1.64 Materials.. 1.27 $2.91	100,956	Labor...$128.23 Materials 99.50 $227.73

Belt-bucket elevators (see also Sec. 20, Art. 6) are by far the most common appliance for elevating ore, whether wet or dry. For wet work they have the advantage over chain-bucket elevators of requiring no difficult and expensive lubrication and of having fewer wearing and friction-producing surfaces. The capacity of a given belt, however, is limited by its adhesion to the head pulley; this can be augmented by a wrapping of less slippery material around the pulley. As compared with inclined conveyors, they occupy much less floor space, and can also elevate thin pulps. It is important to provide means for emptying the boot of an elevator when repairs are needed; so placed that the discharged material can be sluiced or easily transferred by other means to an adjoining elevator.

Chain-bucket elevators (Sec. 20, Art. 7), in spite of their multiplicity of wearing and breaking elements, can be used satisfactorily for dry work but must then be lubricated with heavy grease to minimize the effect of grit; their capacity is limited only by the size of buckets that can be supported. Certain types have a pronounced advantage in being able to receive or discharge at a number of points in their travel, thus combining the functions of elevator and conveyor.

Table 22. Examples of inclined belt conveyors in mills

Mill	Length, feet	Width, inches	Slope, degrees	Speed, feet per minute	Size of limiting screen, inches	Capacity, tons per hour	Power installed, horsepower
Copper Queen	335	42	....	200	1	600	100
Panda	320	36	20	....	1	500	100
Richard	135	24	19	300	2	62	....
	90	24	20	300	2	62	....
United Comstock	122	30	20.6	....	3	125	20
United Eastern	106	18	20	....	3	36	10
Various examples from Stephens Adamson Co.	330	42	19	400	½	700*a*	250*a*
	380	42	17	450	½	700*a*	250*a*
	140	42	0	450	7	700	10
	30	36	18	100	7	180	....
	175	24	16	300	3½	180	15
	65	30	22	330	2	400	25
	80	24	20	330	1	220	15
	310	48	15	450	7	700	100

a In tandem.

In one 250-ton mill, a single Peck conveyor-elevator was devised to perform all the following operations: (*a*) receive crushed ore via belt conveyor from a sampler; (*b*) elevate and deliver this ore, at will, into smelter bin, mill bin, storage bin for any class of ore (with the aid of a distributing belt conveyor), or bins for doubtful ore (pending assay); (*c*) withdraw doubtful ore from bins and deliver to smelter, storage, or mill bins; (*d*) withdraw ore from any storage bin (with the aid of a belt conveyor) and deliver to a smelter bin or mill bin.

Sand pumps. (See Sec. 20, Arts. 11, 12, 13.) Centrifugal pumps for elevating gritty pulp must be specially lined, and even this affords little protection to the shaft, which is a source of weakness in all but the latest form of Wilfley pump. For pulp carrying solids coarser than ⅛-in., a bucket elevator will usually require less repair than any centrifugal pump; for fine pulps (−30-mesh), centrifugal pumps are generally better than bucket elevators.

At the Clifton mill of Arizona Copper Co. an Aldridge plunger pump, specially designed as to valves and plunger clearances, was used to dispose of table and vanner tailing. At Magma Copper Co. all mill concentrate (5 per cent. +20-, 63 per cent. −200-mesh), derived about equally from tables and flotation, is pumped in the form of a thickened pulp (70 per cent. solids) to filters at the smelter, which discharge cake directly into the smelter bins, thereby dispensing with at least one handling (*J. W. Thompson, PC*). The pipe line is 4 in. diameter, 2760 ft. long, and rises to height of 92 ft. near its middle point with slopes of 7°. No trouble has been experienced from sedimentation so long as the pulp consistency is held at 70 per cent. solids. A 4-in. Wilfley pump, driven at 1600 r.p.m. by a 75-hp. motor, delivers 165 gal. of this pulp per min., equivalent to 60 tons solids per hour. The operating costs per ton of concentrate (dry) in 1925 were: repair parts, etc., 3.00¢; labor, operating and repairs, 1.94¢; power, 0.60¢; total, 5.54¢.

Air-lifts (see Sec. 20, Art. 15) for elevating ore pulps in process are satisfactorily employed in the Nevada Consolidated, Chino, Ray, New Cornelia, and Copper Queen mills, and in the older types of cyanide plants.

Skips are used for elevating roll products to screens at two points in the intermediate-crushing department of the Alaska-Gastineau mill (*63 A 488*). The product from the

primary rolls, set at 1-in., is hoisted in four skips of 5-ton capacity; that from the secondary rolls, at 10-mesh, in ten skips of the same size. The lift is 100 ft. in both cases. The skips are operated automatically and in counterbalance by a 75- to 135-hp. hoist motor for each pair; plow-steel rope is flat, $\frac{3}{8} \times$ 5-in. The loading gate is operated by a compressed-air cylinder controlled through a 3-way valve actuated by the descending skip; high-pressure water can also be used for operating the gate. Loading time is governed by an oil dash-pot which also throws the motor switch. Loading takes 11 sec., and the complete cycle for two skip loads is 1 min. 50 sec. The cost of operation compares favorably with that of dry elevators; maintenance is 0.9¢ per ton milled.

7. Driving power for mills

Individual power requirements for commonly employed machines, operating under stated conditions, are given in connection with the descriptions of the various machines. Table 23 gives the total motive power (installed or used) at a few mills of representative types.

Line-shaft drives are used at small mills operated from a central power unit, whether water, steam, oil, or electric; large and small mills operated by steam power; and large motor-driven mills operating long blocks of similar machines having relatively small power requirements. At mills of the first two types line shafts are unavoidable; in the third type they reduce the number of motors necessary and the large motors cost less per unit of power and require no more frequent attention than small ones. Some large, modern mills, however, as Copper Queen, have introduced individual 1-hp. motors for driving concentrating tables. The use of line shafts for heavy-duty equipment has the disadvantage of requiring numerous friction clutches (frequently a source of trouble) while, if electrically driven, the stopping of an individual machine produces a bad effect on the power factor of the motor, the capacity of which is fixed by the total connected load; this irregularity can be more efficiently compensated by steam or oil-driven engines.

Installation of line-shaft drives. For the elements of design, refer to mechanical-engineering handbooks (Kent, Marks, etc.; see also *107 J 1132*). The following general suggestions refer particularly to ore-dressing works. (*a*) Shafts should have a wide margin of strength above that required for transmission alone; this is to provide against abnormal loads on pulleys due to shrinkage of damp belts, excessive shortening of belts to take up slack, etc. (*b*) Pulleys, couplings, and clutches should be closely adjacent to bearings; the end of an oiled bearing, however, should lie 1 to 2 in. outside the nearer edge of an adjoining pulley, so that oil drippings will not fall on belt or pulley. (*c*) Hangers with ball-socket journal boxes are as satisfactory as pillow blocks for shafts of moderate sizes, up to $2\frac{7}{16}$-in. Larger shafts should be carried in pillow blocks, and for extra heavy service, a shaft should be mounted near the ground, on concrete piers if possible. (*d*) Ball-socket, or other self-aligning pillow blocks and journal boxes should always be installed for long shafts, and preferably also for short ones. (*e*) Ring-oiling journal boxes are satisfactory for general service; in dusty places, heavy-grease lubrication affords better protection. (*f*) Leather belts are admissible only in permanently dry places. Canvas belts deteriorate with moisture, and lack pliability in large sizes. Rubber-covered belts give the best service in all parts of a wet mill; the most satisfactory weights are: 4-ply for 4- and 6-in.; 6-ply for 8- to 12-in.; 8-ply for 18-in. (*g*) Vertical belts should be avoided. Quarter-turn belts are admissible in sizes up to 8-in., and are unavoidable for driving a group of vanners placed side by side and operated from a single line shaft. The minimum length between centers for belts without tightening idlers should be five times the diameter of the larger pulley. (*h*) Tightening idlers give good service, when needed, but require

Table 23. Total motive power for representative mills

Mill	Daily tonnage	Nature of operation	Total power, horse-power	Horse-power per ton-day
Alaska-Gastineau.	7,000	Crushing mine-run to 57 per cent. +48-mesh	1909*a*	0.2727*a*
		Concentrating (tables only), elevating, etc.	1532	.2188
		Pumping, lighting, etc.	448	.0641
			3889*a*	0.5556*a*
Armstead.	150	Blake, rolls, Hardinge, Callow flotation, tables	318*b*	2.1*b*
Consolidated Coppermines.	1,000	Blake, Hardinge, flotation, tables. (Steam)	520*b*	0.52*b*
Honduras Rosario.	428	Coarse crushing, 2 gyratories	15*a*	0.0342*a*
		Stamps, 20 @ 1850-lb.	117	.2722
		Tube mills, 3 @ 5×22-ft.	233	.5443
		Pumps, 10 triplex, various sizes	142	.3326
		Agitation tanks and compressors	140	.3282
			647*a*	1.5115*a*
Inspiration (1922).	11,437	Coarse crushing	340*a*	0.0297*a*
		Fine crushing and tables, 94 per cent. −48-mesh, 52 per cent. −200-mesh	6686	.5846
		Flotation blowers	1498	.1310
		Filtering and reclaiming water	487	.0426
		Lighting	77	.0067
			9088*a*	0.7946*a*
Mountain Copper.	600	Crushing to 2.5-in., 2 gyratories in series, 8 hr.	100*b*	0.167*b*
		Fine crushing and M. S. flotation	592	.970
			692*b*	1.137*b*
Ottawa, Slocan, B. C.	50	Jaw, tube mill, M. S. flotation. Water power	110*b*	2.2*b*
Panda.	4,000	Crushing: 10 gyratories, 6 rolls, to 1-in., 8 hr.	1500*b*	0.375*b*
		Conveying and distributing, 8 hr.	285	.071
		Jigs, tables, re-crushing to 1-mm.	1800	.450
			3585*b*	0.896*b*
Richard.	1,500	Gyratory, rolls, magnetic separation beginning at 2-in.; finest crushing, ⅛-in.	200*b*	0.13 *b*
St. Joseph Lead Co., Bonne Terre (1917).	2,700	Jigs, tables, flotation. Concentration begins at 9-mm.	925*a*	0.343 *a*
St. Joseph Lead Co., Rivermines (1917).	4,000	Jigs, tables, flotation. Concentration begins at 9-mm.	1840*a*	0.460 *a*

a Power consumed. *b* Power installed.

Table 23. Total motive power for representative mills—*Continued*

Mill	Daily tonnage	Nature of operation	Total power, horse-power	Horse-power per ton-day
Santa Barbara.	500	Crushing all to 2-mm., gyratory, disk, rod mill	300*b*	0.600 *b*
		40 Deister tables	33	.066
		Conveyors, elevators, screens, classifiers	207	.414
		Pumps, tram, settlers	51	.102
			591*b*	1.182 *b*
Silver King.	300	Jigs, tables, flotation; crushing to 16-mesh.	425*b*	1.420*b*
S. E. Missouri, average practice.	2,000 to 4,000	Crushing: gyratory, disk, rolls, ball and rod-mills		0.157 *a*
		Concentration: Hancock jigs, tables		.095
		Flotation: Federal, Janney, K. & K., Callow		.062
		Water service		.081
		(General tailing averages 55 per cent. on 8-mesh and 75 per cent. on 20-mesh.)		0.395 *a*
Sunnyside	550	Crushing to 70 per cent. —200-mesh, flotation and tables	900*b*	1.64 *b*
Tul Mi Chung, Korea.	470	3 Jaw crushers (to 1-in.), trommel, conveyors	47 *a*	0.1009 *a*
		6 Hardinge (ball and pebble), 6 Dorr classifiers	393	.8369
		M. S. flotation (8 @ 29½-in.; 8 @ 16-in. square)	74	.1570
		5 Deister tables	6	.0130
		Re-grinding table tailing: 5 × 5-ft. ball granulator	64	.1357
		Tailing disposal (3600 tons 1 : 7 pulp), centrifugal pump	27	.0569
		Water supply (1700 tons pumped 1½ miles)	53	.1129
			664*a*	1.4133*a*
United Comstock.	2,000	Gyratory, rolls, ball-mills, agitation, cyaniding	2800*b*	1.400 *b*
United Eastern.	300	Gyratory, ball and tube-mills, cyanidation (85 per cent. —200-mesh)	330*a*	1.100*a*
United Verde crushing plant.	6,000 (24 hr.)	4 jaw crushers (36 × 24) to 3-in., 4000 tons per 8 hr.	500*b*	0.0833*b*
		3 vertical disks (48-in.) to ¾-in., 4000 tons per 16 hr.	225	.0375
		3 rolls (56 × 24) to ¼-in., 4000 tons per 16 hr.	450	.0750
		2 exhaust fans (45,000 cu. ft. per minute each)	250	.0417
		Conveyors, feeders, elevators	1200	.2000
			2625*b*	0.4375*b*

a Power consumed. *b* Power installed.

endless belts, since no form of lacing or coupling has proved satisfactory in this service; endless belts of leather can be spliced by glueing on the premises (*107 J 1132*) but endless rubber belt can be made only at the factory. (*i*) Chain drives are useful for transmissions too short-centered for belts or where the desired speed ratio would require too small a pulley at one end; also for transmitting small power at low speed in dusty places. Their efficiency is high, and as they require no tension on the slack side, the journal friction at both ends is less than that caused by a short, tight belt. They are particularly applicable to the driving of tubular mills, blowers, etc.; for a good example, see *119 P 7*.

Stopping and starting from line shafts. Tight-and-loose pulleys with a belt shifter are suitable for machines of relatively small power requirements, up to 25 or 30 hp., with belts up to 8 in. wide; for larger powers, belt shifting not only throws excessive stresses on all moving parts, but the price of a loose pulley of large size and the extra cost for a large double-width flat-faced pulley will usually be greater than the cost of a suitable clutch. Dental or jaw clutches for throwing the load on a moving shaft can be used safely for speeds under 30 r.p.m. Clutch pulleys are unsatisfactory where their power is likely to be unutilized for considerable periods, owing to rapid wear of the bushings, which cannot be constructed or lubricated like a standard journal box; this is particularly objectionable in case of a stationary pulley riding on a moving shaft, since the wear will then be eccentric and likely to cause breakage of the clutch pulley. Friction-clutch couplings, of band, jaw, or disk types, connected with a short jack-shaft, or with a quill, and supported by adequate journal boxes, are much better than clutch pulleys. The quill system is well adapted to driving a row of machines from a long main shaft: It requires two large journal boxes for each quill, in addition to the smaller boxes for the line shaft, and the whole installation must be rigidly supported, preferably on concrete piers, to avoid internal friction from distorted alignment.

Slack belt, with a swinging tension idler, was formerly the standard method of driving individual stamp batteries operated by a single line shaft; this practice is now almost obsolete, modern stamp mills using an individual motor for every five or ten stamps.

Individual motor drives are almost universally installed at modern mills, for all classes of equipment. For tables and other small-power apparatus which can be closely grouped into rows or blocks and easily driven from line shafts, the economy of individual drives, whether as to cost of installation or expense for maintenance, has not yet been fully established; manufacturers expect soon to market a 1-hp. motor and speed reducer for $100, which is no greater than the cost per table for installing equivalent line-shaft drive. For tables, the present advantage of individual drives is mainly their ease of installation; they make accurate alignment unnecessary, avoid overhead supports for line-shafts, and are easy to maintain. For driving heavy equipment, individual motors offer the same advantages, with the added economies introduced by operating at higher efficiencies and power factors.

Motor characteristics. Large ore-dressing plants in North America operate, almost without exception, with 3-phase alternating current, usually at 60 cycles but occasionally at 25 (50 cycles is standard practice in Europe). The advantages over direct current lie in the simplicity of apparatus, and, usually, lower cost of equipment. Standard rugged designs are cheapest both in first cost and maintenance expense. Motors having the same speed and size of shaft extension should be installed so far as is consistent with good efficiency, to reduce the number of spares carried.

Standard voltages are given in Table 24. Large a-c. motors (more than 50-hp.) are commonly 2200- or 2300-volt; 440-volt current is used for large

Table 24. Standard voltages

Direct current		Alternating current		
Generator	Motor	Generator	Transformer	Motor
125	115	120	115	110
250	230	240	230	220
575	550	480	460	440
....	....	600	575	550
....	....	2,300	2,300	2,200[a]
....	....	6,600	6,600	6,600
....	....	11,000	11,000	11,000
....	....	13,200	13,200	13,200

a With synchronous motors it is customary to design the motor to operate at 2300 volts.

motors in exposed conditions and for motors from 5- to 50-hp.; 220- or 110-volt for 5-hp. and smaller.

At Honduras Rosario mill (*115 J 1147*) all but two of the motors of 15-hp. and larger are on 2200 volts, whereas at the United Comstock (*114 J 846*, *117 J 516*) all of the hundred motors (some of 150- and 200-hp.), aggregating 2800 hp., are on 440-volt current.

Low-voltage operation causes higher operating losses and higher cost of conductor, but the equipment cost is less than in high-voltage work. In general, the economic voltage depends upon the load factor, the power consumed, and the distance from the generator to the point of consumption. (See Table 25.) Distribution and interruption of current in a large mill are difficult at low voltage, and expansion is expensive, nevertheless the usual tendency is to adopt too low rather than too high voltage.

Motor speed. High-speed motors are lighter, cheaper and usually more efficient than low-speed, provided they are equipped with efficient speed-reducing drives. Speeds recommended (*109 J 849*) for 60-cycle synchronous motors in general mill service are shown in Table 27. E. Bachman (*PC*) recommends the speeds given in Table 28 for motors for general service.

Outboard bearings should be specified for geared motors of 75 hp. and larger.

Power rating. Highest efficiency and best power factor are obtained by operating a motor continuously at as near its rated capacity as possible. Hence motors for driving stamp batteries, ball, rod or pebble mills, compressors or blowers, pumps, and other steady-load apparatus, should have capacities corresponding as closely as practicable with the power requirements. For driving coarse crushers, rolls, Chilean mills, tables, agitators or settling tanks, elevators, conveyors, and other equipment subject to irregularity in feed or to excessive and variable friction, a motor should have a capacity two or three times greater than the normal requirement. The bad effect of so many oversized motors on the power factor of a whole mill is not unduly serious, because in most mills, particularly those compelled to crush fine, a large part (50 to 75 per cent.) of the total mill power is applied to the driving of steady-load equipment by efficient motors.

Table 25. Distances of transmission

Distances to which 100-kw. three-phase current can be transmitted over different sizes of wires at different potentials, assuming an energy loss of 10 per cent. and a power factor of 85 per cent.

Number B. & S.	Area in circular mils	Voltages 2,000	3,000	4,000	5,000	6,000	8,000
		Distance of transmission in miles for various potentials at receiving end					
6	26,250	1.32	2.98	5.28	8.27	11.9	21.1
5	33,100	1.66	3.75	6.64	10.4	15.0	26.6
4	41,740	2.10	4.74	8.40	13.2	19.0	33.6
3	52,630	2.54	5.96	10.2	16.6	23.8	40.6
2	66,370	3.33	7.51	13.3	20.9	30.0	53.3
1	83,690	4.21	9.48	16.8	26.3	37.9	67.4
0	105,500	5.29	11.9	21.2	33.1	47.7	84.6
00	133,100	6.71	15.1	26.8	42.0	60.4	107
000	167,800	8.45	19.0	33.8	52.9	76.2	135
0000	211,600	10.6	23.9	42.5	66.4	95.7	170
	250,000	12.6	28.3	50.3	78.7	113	201
	500,000	25.2	56.7	101	157	227	403

Number B. & S.	Area in circular mils	Voltages 10,000	12,000	15,000	20,000	25,000	30,000
		Distance of transmission in miles for various potentials at receiving end					
6	26,250	33.1	47.7	74.5	132	207	289
5	33,100	41.6	60.0	93.7	166	260	375
4	41,740	52.6	75.8	119	210	329	474
3	52,630	66.2	95.4	149	255	414	596
2	66,370	83.4	120	188	334	521	751
1	83,690	105	152	212	421	658	948
0	105,500	132	192	298	530	828	1190
00	133,100	168	242	378	672	1050	1510
000	167,800	211	305	476	846	1320	1900
0000	211,600	266	283	598	1060	1660	2390
	250,000	315	453	708	1260	1970	2830
	500,000	629	907	1420	2520	3930	5660

Courtesy General Electric Co.

Table 26. Current in three-phase circuits with varying loads and voltages, 100 per cent. power factor

Volts	Kilowatts, amperes per phase				
	50	100	200	300	400
110	262.431	524.863	1049.727	1574.591	2099.455
220	131.215	262.431	524.863	787.295	1049.727
330	87.477	174.954	349.909	524.863	699.818
440	65.607	131.215	262.431	393.647	524.863
600	48.112	96.225	192.450	288.675	384.900
1,150	25.102	50.204	100.408	150.613	200.817
2,300	12.551	25.102	50.204	75.306	100.408
3,300	8.747	17.495	34.990	52.486	69.981
4,400	6.560	13.121	26.243	29.364	52.485
6,600	4.373	8.747	17.495	26.243	34.990
11,000	2.624	5.248	10.497	15.745	20.994
13,200	2.186	4.373	8.747	13.121	17.495
22,000	1.312	2.624	5.248	7.872	10.497
30,000	.962	1.924	3.849	5.773	7.698
33,000	.874	1.749	3.499	5.248	6.998
45,000	.641	1.283	2.566	3.849	5.132
60,000	.481	.962	1.924	2.886	3.849
100,000	.288	.577	1.154	1.732	2.309

Volts	Kilowatts, amperes per phase				
	500	600	700	800	900
110	2624.319	3149.183	3674.047	4198.911	4723.774
220	1312.159	1575.591	1837.023	2099.455	2361.887
330	874.773	1049.727	1224.682	1399.637	1574.591
440	656.079	787.295	918.511	1049.727	1180.943
600	481.125	577.350	673.575	769.800	866.025
1,150	251.021	301.226	351.430	401.634	451.839
2,300	125.510	150.613	175.715	200.817	225.919
3,300	87.477	104.972	122.468	139.963	157.459
4,400	65.607	78.729	91.851	104.972	118.094
6,600	43.738	52.486	61.234	69.981	78.729
11,000	26.243	31.491	36.740	41.989	47.237
13,200	21.869	26.243	30.617	34.990	39.364
22,000	18.121	15.745	18.370	20.994	23.618
30,000	9.622	11.547	13.471	15.396	17.320
33,000	8.747	10.497	12.240	13.996	15.745
45,000	6.415	7.698	8.981	10.264	11.547
60,000	4.811	5.773	6.735	7.698	8.660
100,000	2.886	3.464	4.041	4.618	5.196

Courtesy General Electric Co.

Table 27. Speed of synchronous motors

Horsepower rating	Speeds, revolutions per minute	
	Belts	Gears(*a*)
1–2	1800	900
5	1200	900
7½–10	1200	720
15	1200	600
20–50	900	600
75–100	720	514
150–200	600	450

a Plain spur gears; with herring bone gear, chain drive, or a flexible coupling on motor-shaft extension, the same speed may be used as recommended for belts.

Table 28. Speed of general industrial motors

Speeds, revolutions per minute	Maximum horsepower of motor	
	Belted	Geared, steel pinions
1700 or 1800	40	5
1440 or 1500	50	10
1150 or 1200	75	25
850 or 900	125	50
720 or 750	....	75

Applicability of motor types (*115 J 1147, 109 J 849, 118 P 819, Peele; E Bachman, PC*). The special fields for the various types of motors are given below; direct-current motors are included in view of their occasional desirability:

DIRECT-CURRENT MOTORS

Shunt-wound

When the work is of a fairly steady nature.
When a considerable range of speed adjustment is desired.
When fairly close speed regulation is required.

Compound-wound

When there are sudden heavy loads of short duration.
When heavy starting duty is required.
Where, for these duties, series motors could not be used on account of excessive light-load speeds.

Series

When excessive starting torques are required, but only when speed regulation is not important and when light-load speeds do not become dangerous.

ALTERNATING-CURRENT MOTORS

Squirrel-cage

For constant speed with normal starting duty.

Squirrel-cage with high-resistance rotors

For constant speed and heavy starting duty or when heavy loads of short duration are expected.

Double squirrel-cage

For excessive starting duty when the running duty would require a standard constant-speed squirrel-cage motor. Simplifies control.

Slip-ring

For heavy duty when some speed adjustment is necessary. These motors have the characteristics, not of d-c. adjustable-speed motors, but of d-c. motors with armature control, the current consumed in the resistance to reduce speed being wasted.

Multi-speed

When direct current is not available and three or four definite speeds, in conjunction with gear changes, will give the desired speeds.

Synchronous

When load can be started by clutch or when the starting load is light. Applicable to centrifugal pumps starting under no load, and to air compressors or blowers where the load is increased after full speed is reached.

Super-synchronous or "Built-in Clutch" type synchronous

When the load requires high starting duty but low and constant speed; ball and tube mills are typical examples.

"Squirrel-cage" synchronous

When the normal starting duty is too great for a standard synchronous motor, *e.g.*, automatically-started compressors, pumps, etc.

The distinguishing features of the three simple types of alternating-current motors compare as follows:

SYNCHRONOUS	SQUIRREL-CAGE INDUCTION	SLIP-RING INDUCTION
Constant speed.	Constant speed, fixed by frequency of the supply current.	Variable speed, usually by induction of resistance in the secondary circuit.
Adaptable to high voltages, thereby often obviating transformers.	Adaptable to high voltages.	Adaptable to high voltages.
Good efficiency and high power factor at all loads.	Good efficiency. Power factor lower than synchronous motor, especially at fractional loads.	Lower efficiency than squirrel-cage.
Power factor under control. Can be used to equalize poor power-factor caused by induction motors.	A low power-factor does not materially increase the power cost for driving the motor, but introduces greater voltage loss in the whole system, involving larger transformers, wiring, etc.	
Low starting torque (except with special design).	Good starting torque except with low frequency.	High starting torque.
Requires d-c. excitation.	Large current at start.	Small current at start.
Has collector rings.	No moving contacts.	Moving contacts may suffer from dust or moisture.
Skillful manipulation required at starting.	Easily started.	Easily started.
Costs more than induction motor.	Relatively cheap.	Relatively expensive.
	Principal uses	
Compressors or blowers.	Coarse crushers.	Intermittent hoist service.
Centrifugal pumps.	Rolls.	Variable-speed drives. Starting under heavy load, as elevators, plunger pumps, tube-mills without friction clutches, agitators and settlers.
Tube mills, if provided with friction clutches. Stamp batteries.	Groups of any equipment that can be started under small load.	
In general, for loads that can be added gradually.		

At United Comstock, operating at 440 volts exclusively (stepped from 60,000 and 2300 volts), the tube mills are driven by slip-ring motors with reversible controllers; more than ninety other motors in the plant (some of 200-hp.) are of squirrel-cage type, with automatic compensators and push-button starters (*114 J 846*, *117 J 516*). At Honduras Rosario, mainly on 2200-volts, 20 stamps are driven by a 200-hp. synchronous motor with belted exciter, started by a 35-hp. induction motor and automatic synchronizer; three tube-mills each have a 75-hp. slip-ring motor with flexible coupling and reversing starter (no clutch); all plunger pumps have slip-ring motors; the agitator compressor has a 75-hp. squirrel-cage motor. At Panda six synchronous motors (three of 350 and three of 525 hp., each at 6600 volts) are used to drive 2-stage centrifugal pumps for water supply of 10,000 gal. per min. against a head of 500 ft. (*29 MM 137*). The Alaska-Juneau mill contained several unusual features, made possible by an abundant supply of power from its own generators and those of the adjoining Alaska-Treadwell mine (*120 P 251*). Each of the two coarse-crushing sections contained one 36 × 48-in. jaw crusher and two No. 9 gyratories; these three crushers were driven by clutch pulleys on one shaft direct-connected to a 350-hp. synchronous motor of 2200 volts and 360 r.p.m., with direct-connected exciter. The starting torque was sufficient to start all connected equipment without releasing the clutches. The slow speed of this motor obviated countershafts and other reducers, space for which was wanting. Fine crushing, in two stages, was done in twelve ball mills (8 × 6-ft.) each driven by one 225-hp. motor, and twelve tube mills (6 × 12-ft.) each with one 150-hp. motor; all of these motors were of squirrel-cage type, 2200-volt, 435-r.p.m., specially designed for large starting torque. The mills had friction clutches, but it was

seldom necessary to use them; motors were thrown directly on the full line voltage by special circuit breakers and without compensators, but extra-heavy feeder lines were installed. The unbalancing caused by these motors was compensated by the synchronous motors at the coarse crushers. Two centrifugal pumps for mill water, 3000 gal. per min. each, were each driven by a 400-hp., direct-connected, squirrel-cage motor, not specially designed for high starting torque; a synchronous torque 33 per cent. of the full-load torque proved satisfactory. These motors were excited by independent d-c. circuit. At the SANTA BARBARA mill (*112 J 1050*), with nineteen motors aggregating 590 hp., three slip-ring motors of 100 hp. each are used for coarse crushing and rod-mill grinding, but all others are of the squirrel-cage type.

Mechanical transmission in some form is necessary between the motor and the driven machine.

Long belts are efficient, but may occupy space needed for other purposes. The belt span should not be less than five times the diameter of the larger pulley, and the more nearly horizontal the better; inclinations up to 60° are allowable.

Short belt, with a tightening idler permits the motor to be placed at either side or above or below the driven pulley, with a clear distance as small as 2 or 3 ft. between faces of pulleys; endless belt is practically unavoidable.

Chain drive is as efficient mechanically as a short belt with tightener and has the advantage of producing less journal friction in both sprocket shafts. The shaft must be accurately aligned and so mounted that the chain may be tightened. Wherever possible, vertical chain drives should be avoided and the machines should be mounted so that the chain may be conveniently taken off and replaced without undue loss of time. For this reason the sprockets should not be too close to walls or other machines.

Spur gearing is an accepted method of driving tube mills. Owing to the impossibility of preventing some longitudinal motion in the mill which will be transmitted to the pinion shaft, a motor cannot safely be rigidly connected to the shaft, but must drive through a flexible coupling; or by belt or chain, if further reduction of speed is desired. A further purpose of flexible couplings is to avoid minutely accurate alignment of the motor. A spur gear is often applied to an elevator drive, in which case the reduction should be made near the motor, the elevator being driven by belt from a jack-shaft. Geared machines must be accurately aligned and rigidly fastened to a common base. An error of a few thousandths of an inch may produce serious vibration and ultimately break the shaft or wreck the apparatus. When gears are properly meshed, it should be possible to pull a thin piece of paper from between the teeth without tearing.

Herringbone gear is more expensive than spur gears, but permits larger speed reduction and is sufficiently more efficient in power transmission to pay for itself easily in tube- and ball-mill drives where the motor is direct-connected to the pinion shaft.

Worm gear with high ratio of speed reduction is applied to such slow-moving apparatus as thickening tanks, and with a lower reduction ratio, in head motions of unit-driven tables and for conveyors. For driving inclined conveyors and all forms of bucket elevators, worm-gear drive avoids the necessity of safety devices to prevent back travel on stopping.

Bevel gears are found chiefly in gyratories, Huntington and Chilean mills, trommels, and M.S. flotation cells; they are usually a source of annoyance, and are to be avoided wherever possible.

Direct connection is desirable for high-speed and constant-speed apparatus, such as centrifugal pumps of moderate size, fans, blowers, etc., and even for slower-speed equipment, the advantages of direct drive may sometimes justify the additional expense of a slow-speed motor. Direct-connected machines must be mounted on and rigidly fastened to a common base in order to maintain accurate alignment and flexible couplings must be used to prevent trouble from end thrust. Couplings should be aligned with a straight edge placed against the outer diameter at top, bottom, and sides, with the rotating parts standing in different positions.

Electrical power transmission. (*E. Bachman, PC.*) The relation between kv-a., power, current, potential, and power factor in the different types of circuits is as follows:

Direct current. See **Fig.** *8a*. I = current per wire, in amperes; E = potential between wires, in volts; $P = EI/1000$ in kw.

Alternating current, single phase. See Fig. 8*b*. $\cos \phi$ = power-factor. Apparent power = Kv-a. = $EI/1000$ and $P = EI \cos \phi/1000$ in kw.

Alternating-current, 3-phase, 3-wire. See Fig. 8*c*. This is the most common form in which alternating-current energy for power purposes is produced. Potential to neutral, in volts = $e = E/\sqrt{3}$. Apparent power = Kv-a. = $\sqrt{3}\,EI/1000 = 3\,eI/1000$; and $P = \sqrt{3}\,EI \cos \phi/1000 = 3\,eI \cos \phi/1000$ in kw.

Alternating-current, 3-phase, 4-wire. A 3-phase, 4-wire system is commonly used in cases where it is desired to supply power at maximum potential to a number of single-phase independent circuits, which may and probably will be unbalanced; and at the same time have maximum assurance against insulation breakdown. For example, a 4000-volt Y-connected generator with the neutral grounded and brought out to form the fourth wire of a 3-phase, 4-wire system, may be used to supply power to single-phase transformers having 2300-volt primary windings and connected between one of the phase wires and the neutral. It is advisable to endeavor to balance this type of single-phase load so as not to overload any one phase of the generator while other phases remain underloaded. For a balanced load (equal load between each conductor and neutral) no current will flow through the neutral and the circuit may be considered simply as 3-phase, 3-wire. In case of an unbalanced load, the unbalanced current will flow through the neutral.

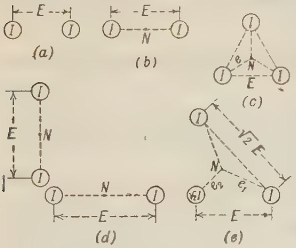


FIG. 8.—Power circuits.

Alternating-current, 2-phase, 4-wire. See Fig. 8*d*. Potential to neutral, in volts = $e = E/2$. Apparent power = Kv-a. = $2\,EI/1000$; $P = 2\,EI \cos \phi/1000$ in kw.

Alternating-current, 2-phase, 3-wire. I = current in outside wire, in amperes; $2I$ = current in common wire, in amperes; E = potential between outside and common wire, volts; $e_1 = 0.745\,E$ = potential between outside wire and neutral; $e_2 = 0.471\,E$ = potential between common wire and neutral. Apparent power = Kv-a. = $2\,EI/1000$; $P = 2\,EI \cos \phi/1000$ in kw.

Conductors are of the following classes: (*a*) Exposed bare conductors for ground connections. (*b*) Exposed insulated conductors mounted on insulators, for low and medium potentials. (*c*) Exposed bare conductors mounted on insulators, for high potentials. (*d*) Small insulated conductors in iron conduit. (*e*) Large insulated conductors in fiber or tile duct.

Short-circuit currents and their effects. An installation must be constructed proof against any reasonably conceivable abnormal condition. The most important, and possibly most frequent, cause of trouble is short-circuiting, despite most elaborate precautions. Hence all measures within reasonable expense should be employed to limit the destructive effects. The initial short circuit usually results from failure of insulation. Any circuit or piece of apparatus subject to short circuits should be removed from service as soon after the short circuit has occurred as is reasonably possible.

Four important provisions against short-circuiting must always be made: (*a*) Material of fireproof or fire-resisting nature should be used whenever possible; if not possible, apparatus and conductors should be segregated and enclosed by suitable barriers or cells, so that a fire resulting from a short circuit will be confined to a small part of the installation. (*b*) All apparatus and material should have ample strength to resist the mechanical stresses resulting from the abnormal flow of current during a short circuit. (*c*) All circuit breakers that may be called upon to open a short circuit should have ample capacity to rupture the maximum current that can flow at the instant they are called upon to operate. It is necessary not only that breakers be amply insulated to operate safely at the installation potential, and that the contacts have sufficient area to carry the maximum current during normal operation; but it is also necessary to know the maximum short-circuit current that they may be required to open and to be certain that each breaker selected has ample rupturing capacity to open that circuit safely. This is one of the most important points to be considered but unfortunately is the one most frequently neglected, often with disastrous results. (*d*) Current should not be allowed to flow through a short circuit any longer than can be avoided.

Bare conductors are usually employed when the voltage exceeds 15,000. These consist of solid wire, copper tubing, or iron pipe. The use of tubing makes it possible to reduce the number of expensive insulators used for sup-

port. It is expensive and quite unnecessary to insulate such high-voltage conductors because when properly installed they are widely spaced and kept well away from the ground.

Cable is used for inter-connection of apparatus and generally for low-voltage distribution. That for inter-connection should be of the best grade of material and installed in the best possible manner. The cost is but a small percentage of the total. Transmission cables may represent a considerable part of the total investment in electrical equipment when used for distribution of power to a number of isolated loads.

When large cables are multiplied to carry the current for low-voltage machines it is necessary to arrange and group the phases so that each conductor will have about the same reactance and thus the current will be properly apportioned between the different conductors in parallel. When dealing with large conductors, special care should be taken to support them rigidly so that they will not be torn from their supports should a severe short circuit occur.

Exposed cable runs. The best arrangement, provided the number of cables in close proximity does not make the group too congested or hazardous, is to use wires or cable insulated for full potential and rigidly supported on insulators good for full potential. This arrangement is safe, since each method of insulation affords full protection. Exposed runs are under constant observation. VARNISHED CAMBRIC is the most lasting insulation because it does not absorb moisture, like paper insulation, and does not deteriorate as rapidly as rubber. It should be thoroughly covered with a fireproof braid or tape to minimize communication between circuits in the event of a short circuit or ground on one circuit or conductor. A 3/32-in. asbestos tape with a flame-proof braid covering gives excellent fire protection.

Cables in conduits or ducts should be stranded. Iron conduit is ordinarily used up to 4-in. diameter; above this size fiber or tile is satisfactory and less expensive. Iron should not be used with alternating current unless all conductors of the circuit are in the same conduit. Rubber-covered standard conductors with double waterproofed braid (or tape and braid) are generally used for conductors up to 600 volts and as large as 0000 B. & S. gauge. Varnished-cambric insulation is by far the best for larger cables and higher voltages, provided it is covered with good weatherproofed braid for protection against abrasion. Cambric-covered cables may be run in fiber ducts, if they are carefully drained. Cables for main connections of considerable importance are frequently installed with insulation suitable for voltages 50 per cent. in excess of the rated voltage.

Lead-covered cable used on a-c. circuits should be of the multiple-conductor type. Eddy currents in the lead sheaths of single-conductor cables increase the energy losses. Single-conductor lead-covered cables should not, in general, be used for heavy a-c. circuits.

Armored cables and conduits. On d-c. circuits, where maximum copper area is desired, single conductors should be used. Single-conductor cable is preferable for low-tension, d-c. mains on account of the simplicity of service connections. Concentric cables give maximum ampere capacity per duct for single-phase alternating current, but flat twin cables are usually more convenient and cheaper for mains and lines where the area is less than 250,000 circular mils. They are liable to kink and are not recommended in sizes larger than 250,000 circular mils. For two-conductor cables larger than 250,000 circular mils, either round cable (two conductors twisted together) or concentric cable is best. Triple conductor is almost a necessity in three-phase, a-c. work in order to avoid disturbance to parallel circuits and reduce loss in the lead sheath to a minimum. (If single-conductor lead-covered cables are used in a-c. circuits the sheaths should not be in contact nor connected by any low-resistance path.) Multiple-conductor cable is best for a number of small conductors run in one duct.

The cheapest cable insulation for UNDERGROUND CIRCUITS is paper and it can be used safely when the lead cover is not subject to corrosion. The insulation on sizes smaller than No. 6 B. & S. is likely to be injured by sharp bending or handling, and for smaller conductors, rubber insulation is best. It is also best for cables placed in very wet ducts, where severe corrosion is to be expected.

Thickness of insulation is principally dependent on mechanical requirements with low-tension cables (less than 1000-volt); for higher voltages the thickness is governed by the dielectric strength required. For 3-phase alternating current ungrounded or delta-connected, one-half the thickness of insulation should be put around each conductor and the other half surrounding the three conductors. This gives the same total thickness of insulation between conductors and between each conductor and ground. For 3-phase circuits Y-connected, with the neutral grounded, the thickness of the outer jacket may be reduced to roughly one-half the thickness of the insulation on the individual conductors,

since the pressure to ground is approximately 60 per cent. of the pressure between conductors.

Finish on cables in clean, dry ducts or wooden boxes underground or in any situation where the cable is not subject to mechanical injury or liable to corrosion should be plain lead. For wet ducts, or other places where corrosion (but not mechanical injury) is to be feared, and for burying directly in clean earth, with a protecting plank or tile laid above the cable, use a jute and asphalt jacket over the lead. When mechanical injury is to be guarded against, use band-steel armor over the jute and asphalted lead. If the cable is not to be supported practically continuously use wire-armored instead of band-steel armored cables. Such cable may be suspended for any practical distance, practically the entire strain being taken by the wire armor.

Current-carrying capacity of an insulated conductor is determined within practical limits by the maximum temperature the insulation will withstand. The temperature should not be above 85° C. for saturated paper, 75° for cambric, and 60° for rubber. The limit is lower for high voltage. The following maximum safe temperatures at the surface of the conductor in the cable are recommended in the Standardization Rules of the A. I. E. E.: For impregnated-paper insulation, $85-E$; for varnished-cambric insulation, $75-E$; for rubber compound insulation $60-0.25E$, where E = the effective operating voltage in kilovolts between conductors.

The maximum safe continuous load for any given cable is higher with direct than with alternating current, but the difference is of slight practical importance for conductors less than 500,000 circular mils in area. The safe load is less for 60 cycles than for 25 cycles; less in the tropics than in temperate zones; it increases markedly in winter; it decreases with the number of loaded cables adjacent to each other; *e.g.*, the average safe load in a four-duct line would be about 60 per cent. greater than in a 16-duct line; and cables immersed in water carry at least 50 per cent. more current than those buried in earth.

Table 29 shows the carrying capacity of different sizes of cables. The figures in the table are based on concentric stranded conductors. If solid conductors are used, reduce

Table 29. Allowable current in copper wire and cable

Size B. & S. gage	Circular mils	Amperes		Circular mils	Amperes	
		Rubber insulation	Other insulation		Rubber insulation	Other insulation
18	1,624	3	5	250,000	237	350
16	2,583	6	10	300,000	275	400
14	4,107	15	20	350,000	300	450
12	6,530	20	25	400,000	325	500
10	10,380	25	30	450,000	362	550
8	16,510	35	50	500,000	400	600
6	26,250	50	70	600,000	450	680
4	41,740	70	90	700,000	500	760
2	66,370	90	125	800,000	550	840
1	83,690	100	150	1,000,000	650	1000
0	105,500	125	200	1,250,000	750	1180
00	133,100	150	225	1,500,000	850	1360
000	167,800	175	275	1,750,000	950	1520
0000	211,600	225	325	2,000,000	1050	1670

the current given (for 30° C rise) by 7 per cent. For two-conductor cables, either round or flat, decrease the current given for single conductor by 15 per cent. For two-conductor concentric cables, decrease the current given for single conductor by 25 per cent. For four-conductor cables, reduce the current given for three-conductor cables by 12.5 per cent. For higher-voltage cable reduce table values by 1 per cent. for each 2000 volts that the working pressure exceeds 3000 volts; for example, 25,000 volt cable 11 per cent. less than the tabulated values.

Conduit sizes. See Table 30.

Current in motor terminals. See Table 31.

Table 30. Conduit sizes for wire and cable
(As adopted by the National Fire Protection Association)

Conductors in conduit	1	2	3	4	Conductors in conduit	1	2	3	4
Wire, B. & S. gage	Size of conduit, inches				Wire, circular mils	Size of conduit, inches			
14	½	½	½	¾	200,000	1¼	2	2½	2½
12	½	¾	¾	¾	250,000	1¼	2½	2½	3
10	½	¾	¾	1	300,000	1¼	2½	2½	3
8	½	1	1	1	400,000	1¼	3	3	3½
6	½	1	1¼	1¼	500,000	1½	3	3	3½
5	¾	1¼	1¼	1¼	600,000	1½	3	3½	......
4	¾	1¼	1¼	1½	700,000	2	3½	3½	......
3	¾	1¼	1¼	1½	800,000	2	3½	4	......
2	¾	1¼	1½	1½	900,000	2	3½	4	......
1	¾	1½	1½	2	1,000,000	2	4	4	......
0	1	1½	2	2	1,250,000	2½	4½	4½	......
00	1	2	2	2½	1,500,000	2½	4½	5	......
000	1	2	2	2½	1,750,000	3	5	5	......
0000	1¼	2	2½	2½	2,000,000	3	5	6	......

Table 31. Current in motor terminals
(Approximate amperes per terminal for a-c. induction motors, for determining size of wires, capacity of fuses, and setting of circuit breakers)

Horsepower of motor	Voltage, three-phase				
	110	220	440	550	2200
1	6.5	3.2	1.6	....	....
2	12	6	3	2.5	....
3	17	9	4.5	3.5	....
5	30	15	7.5	6	....
7½	45	22	11	9	....
10	59	29	14	11	....
15	84	41	20	16	4.5
20	....	55	27	22	5.5
25	....	62	31	25	7
30	....	81	40	32	8
35	....	94	47	38	9.5
40	....	109	54	44	11
50	....	127	64	52	13
75	....	192	96	77	20
100	....	248	124	100	25
150	....	366	183	147	40
200	....	475	237	192	49
250	....	590	290	237	62
300	....	700	350	285	74

For single-phase motors, multiply the current for three-phase motors by 1.73. For two-phase motors, multiply the current for three-phase motors by 0.886.

Power generation. The great majority of American ore-dressing mills of medium and large size operate electrically; public-service water-generated power is transmitted for this purpose as far as 175 miles (to Butte), 150 miles (to Mascot, Tenn.), and 100 miles (to Tonopah), while transmission of 50 to 75 miles is common.

Hydro-electric generators are rarely warranted for an individual mill of short or unknown life owing to the heavy expense usually required for storage reservoirs, although the generating station itself may be no more expensive than a steam plant of corresponding size.

In the rugged and rainy coasts of British Columbia and southern Alaska exceptionally favorable hydraulic conditions justified several of the large mining and milling companies in erecting water-driven generators. The total cost of hydro-electric plants (before 1917) including dams, ditches, and all equipment, was estimated at $100 to $200 per kw. capacity (*Peele*). On the score of reliability of operation, a hydro-electric plant is equal to steam and superior to gas-engine, the most common source of delay being ice at the intake; a long transmission line, however, is always subject to interruption.

Steam generation in plants designed for the utmost efficiency is the usual practice at many large mills in the United States, and steam generation is usually preferred at small mills where first cost is considered more important than fuel economy. Coal or oil is used as fuel, depending upon availability; the convenience of the latter may offset higher unit cost. The plant should be designed to furnish power at a minimum cost per hp.-hr. delivered (not rated output), including interest and amortization of plant within the life of the mill. The necessity for uninterrupted service may justify a greater outlay than would be warranted on grounds of fuel or labor economy.

The essential elements of a steam plant are boiler, engine and feed-water pump; further equipment intended to economize fuel or labor, stated in the order in which it is usually adopted, is feed-water heater, compound engine, condenser, superheater, automatic stoker (*107 J 1121*). Fuel economy and reduced cost per unit of power generated are thus gained at the expense of simplicity (entailing more skilled supervision) and of original outlay. Turbines are cheaper and simpler than reciprocating engines, but require more skilled supervision. The cost of steam-driven generating plants (before 1917) including buildings and all equipment (*Peele*) varied from $90 to $140 per rated kw. for reciprocating engines, and from $40 to $90 per rated kw. for turbines.

Diesel engine has about the same weight and cost, per unit of installed power, as a steam-engine plant of the more elaborate types; it is particularly noteworthy for its high fuel efficiency up to the safe limit, which is fixed by the weight of air that can be admitted to its cylinders, but it is incapable of carrying sustained overloads. It requires skillful supervision, and repairs and renewals constitute a large item of expense, which, however, can be diminished by regular replacement of worn valves, etc., before failure. Its best field is at plants of moderate size, where coal would be uneconomical and fuel oil expensive; its consumption averages only 30 to 40 per cent. of the amount required at a high-class oil-fired steam plant of corresponding size.

Notable installations of Diesel-driven generators have been made by the Phelps Dodge Corp. at Tyrone, Morenci, Globe, and Nacozari. Those at Tyrone, at 5950-ft. elevation (*119 P 369*), are vertical, 2-cycle, with 5 cylinders 20.6-in. diameter and 26-in. stroke; speed, 180 r.p.m.; rated at 1250 brake hp. at sea level. The scavenger pump is large enough to maintain a pressure of 2 to 3 lb. per sq. in. in the cylinder at the beginning of the stroke, thereby simulating sea-level conditions; this plan increases fuel consumption per available hp., but makes the total cost lower by permitting investment in a smaller engine. A compressor for the oil injector, and to store compressed air for starting, is direct-connected to the engine. Each engine is direct-connected to a 60-cycle alternator of 815 to 850 kv-a.

capacity. Fuel is the same quality as that used under steam boilers, California 14° to 20° Bé. occasionally carrying 2 per cent. sulphur; some oil of 24° Bé. is fed just before stopping an engine, so that its first supply on starting will ignite more readily. The heavier oil is warmed by coils carrying cooling water from the exhaust jackets of the engines. Fuel consumption, per hour, varies directly with the load, from 175 lb. at 100 kw. to 420 lb. at 600 kw.; above that point, consumption increases somewhat more rapidly, being 570 lb. at 800 kw. A steam plant with the same output as the Diesel engines at Tyrone is estimated to require five times more water, 2¼ to 2½ times more fuel, but only one-third as much expense for repairs and replacements. Costs for labor and miscellaneous supplies are practically identical. A IRON MT., Mo., two 500-hp. Diesel engines drive two 436-kv-a. generators at 2300 volts. The cost of the whole plant (1923), including building and all accessories, erected and running, was $100,000. Operating at half capacity, with oil at $1.65 per bbl., the cost was 1.78¢ per kw.-hr. including interest and amortization (15 yr.). BETTY O'NEAL mill (and mine) Lewis, Nev., operates two Fairbanks-Morse 6-cylinder semi-Diesels of 300- hp., each direct-connected to a 200-kw., 460-volt alternator of 60 cycles. Fuel is 27° Bé. distillate, costing 7.1¢ per gal. at the mine; each engine consumes 17 gal. per hr. The total power cost is $6 per hp.-month (*117 J 449*). Cooling water from the mine is chemically softened before use.

8. Lighting

Artificial lighting (*E. Bachman, PC*) is necessary for safety and to enable workmen to inspect operations. The usual practice is to install a lighting circuit of 110 volts with separate transformer, or an individual dynamo so that lighting need not be subject to interruption. It is possible to secure lamps for 220-volt circuits but these are not so efficient, sturdy or generally useful as 110-volt lamps. Exposed wires are permissible in a steel structure; iron pipe or flexible metallic conduit is safer for wooden mills.

Incandescent lamps of high intensity on series circuits are installed at UTAH CONSOLIDATED, CHIEF CONSOLIDATED, SILVER DYKE, U. S. SMELTING R. & M. CO. and other mills. This system is more efficient, lower in first cost, and easier to maintain than multiple systems. The layout is the same as for multiple circuits, except that series sockets, lamps, and reflectors such as are used for street illumination are necessary. Lead-covered, single-conductor cable, No. 8-gage is most economical, since it can be run on girders, etc., without insulators or pipe conduits. For comparative efficiencies of the two systems, see Fig. 9. Series circuits are fully safeguarded from the effect of high voltage coming from

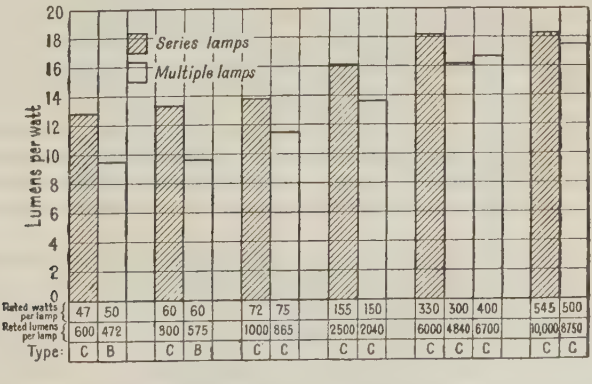


FIG. 9.—Relative efficiencies of multiple and series Edison Mazda lamps.

an open-circuited secondary, lamp burn-out, etc., by suitable protective devices. Lamp operation is protected by a film cut-out in each socket. In case a lamp burns out, the film cut-out is punctured in the socket, which permits current to flow through the circuit without interrupting the operation of other lamps.

Methods of illumination (*123 P 751, 61 EW 628, 70 EW 6, 201; 72 ER 907, 9 Trans. Illum. Eng. Soc., 814*). In a small mill, 50-watt lamps or larger, with reflectors and waterproof sockets, are suspended by cords, wherever needed; they should be within reach for easy cleaning and replacement, and the number should not be stinted. With power at 1¢ per kw.-hr., the total cost for lighting with Mazda lamps (including replacements) is 4¢ to 5¢ per 1000 candle-power-hours (c-p.-hr.) One c-p. = 0.75 watt in this type of lamp. Globes and reflectors should be kept clean. Theft-proof sockets are on the market. Drop lamps are necessary for close work, but produce glare, which reduces acuity of sight and tires the whole body. Breakage of lamps is liable to be excessive and the cost of wiring is high. General illumination with a relatively small number of large, efficient lamps is now in extensive use for all types of industrial lighting and is well suited to large ore-dressing plants. When the arrangement of a mill is such that the greater part of the area needs only relatively low intensity of general illumination, but some few operations require much higher intensity, the most economical plan is to supplement overhead lighting with local lamps judiciously placed, *e.g.*, at tables and jigs. In some of the most modern plants portable lamps are issued in the same way as special tools and conveniently located wall outlets are installed for attaching them.

Types of lamps. The inside-frosted Mazda C lamp is most generally applicable for general illumination. It is efficient and sturdy; the smooth white mineral coating on the lower portion of the bulb diffuses the light, eliminates glare, and makes the shadows soft and feathery rather than dense and harsh. Special features of construction, such as the short stem and flexible supports, give these lamps greater strength than the old Mazda B lamps.

Cooper Hewitt mercury tubes have proved satisfactory at MIAMI and other large mills. They are particularly useful where it is difficult to recognize the valuable minerals by incandescent light. Iron arcs are used at the Franklin plant of NEW JERSEY ZINC CO. to show apple-green willemite on the tables.

Reflectors. The RLM standard dome reflector is most used for general or localized general illumination. It gives a desirable distribution of light with adequate illumination on vertical surfaces, is efficient, easy to clean, and so designed that direct glare is reduced to a satisfactory minimum. The deep-bowl metal reflector gives a lower cut off. This tends to reduce the amount of light on vertical surfaces and although there may be excellent illumination on the working plane a room often appears dull. The deep-bowl reflector is especially serviceable in local lighting with low-hung lamps. Angle metal reflectors are used where especially high illumination is required on vertical surfaces and where lighting units must be located on the side walls. They are frequently placed below a crane track and should generally be supplemented by overhead units.

A number of types of metal reflectors specially designed for industrial lighting provide additional means of diffusing the light. Sometimes a polished-metal cap is placed over the lower half of the lamp bulb to cut off the direct light. Diffusion is sometimes accomplished by opal-glass diffusing caps, and some devices employ a shield of metal on the level with the filament. If the reflector itself is properly designed, these additional accessories tend to eliminate sharp shadows and annoying reflections from the work at a reduction in total output of light and a somewhat higher cost of equipment. Prismatic, mirrored, and dense opal, deep-bowl glass reflectors are also used. These may be very efficient and give suitable distribution of light. The translucent types produce a bright, cheerful room. While it is evident that breakage is likely to

be greater than with metal reflectors, the hazard is not so great as often assumed. Enclosing, or semi-enclosing units of opalescent or diffusing glass find quite an extensive application; they produce excellent diffusion with slight sacrifice of efficiency.

Lamp spacing. The standard dome reflector and bowl-enameled Mazda C lamp have become almost standard equipment for mill lighting. Assuming this unit gives out a certain definite amount of light the advisable spacings of different sizes are as shown in Table 32.

Table 32. Spacing of lights

Ceiling height, feet	Unit lights, watt	Maximum spacing, feet
Rough work, requiring no discernment of detail; general illumination of approximately 2.5 foot-candles:		
Less than 12...	75	12
	100	15
	150	18
12 to 16.......	100	15
	150	18
	200	22
More than 16..	150	18
	200	22
Work requiring observation of machine operation; general illumination of approximately 5 foot-candles:		
Less than 12...	100	10
	150	13
12 to 16.......	150	13
	200	16
More than 16..	200	16
	300	20
Work requiring discrimination of detail; general illumination of approximately 8 foot-candles:		
Less than 12...	100	8
	150	10
	200	13
12 to 16.......	150	10
	200	13
More than 16..	200	13
	300	16

9. Heating

Heating of mills in cold climates is necessary not only for comfort but to avoid delays due to freezing. Practically every important mill in the United States, except a few in the Southwest, makes special provision for heating during the winter; in some cases, stoves are sufficient, in others exhaust from steam engines is utilized, but the general practice is to install radiators connected to steam boilers set aside for this purpose.

In Ontario, heating during 5 to 6 months of the year becomes a heavy item of expense. At McIntyre-Porcupine, treating 241,000 tons, the annual heating cost is $15,000, or 5.6 per cent. of the total milling expense; at Nipissing the annual cost for heating the low-grade mill is $22,000, or 6.3 per cent. of the total milling cost for 82,000 tons.

Steam-heating system for mills in the western and north-western United States may be designed on the following principles: (*a*) An average radiator surface of 1 sq. ft. (with steam at 5-lb. pressure) suffices for 160 cu. ft. of mill interior; in a terraced mill, radiator area in the lower sections should be somewhat above average, and less than average in the upper sections. (*b*) Based on the area of radiators alone, the boiler rating should be 1 hp. per 50 sq. ft. (*c*) Including the radiating surface of supply- and return-steam lines, 1 boiler hp. per 80 sq. ft. of total radiating surface is usually ample. (*d*) Cost of radiators, pipe and installation, but not including boilers, is 0.4 to 0.5¢ per cu. ft. of mill interior. (*e*) A reducing valve set to deliver at, say, 5-lb. pressure, should be placed in the supply main ahead of the first radiator branch. (*f*) A drip pocket, connected to a steam trap, should be placed in the high-pressure line near the

boiler, or at any other low points in that line. (*g*) A steam trap should also be attached at the end of the return line, delivering its water to the sump for the boiler-feed pump; at a small mill or at one having no parts subject to a sudden drop in temperature, the return line may lead directly to the boiler.

Design of radiators. (*a*) A continuous-pipe radiator, with U-joints, requires no left-hand threading, and is cheap and easy to construct; where large heating effect is required, this design is objectionable in that the cumulative effect of condensation may overload the lower pipes in the series with water and retard steam circulation. (*b*) A grid of radiator pipes rigidly connecting two parallel headers requires right- and left-hand threading, is difficult to make tight at all joints, and is likely to develop leaks with unequal expansion; pipes can be welded into a wrought-iron header by a skillful operator, but the expense is likely to exceed that of threading and tapping. (*c*) The design shown in Fig. 10 has the advantages of using only stock materials (headers are cast-iron), being easily and quickly constructed, and having ample allowance for unequal expansion; the latter provision could also be made, with parallel headers, by a right-angled bend, setting the radiator in a corner. For stated lengths *A*, the amounts of 1-in. pipe and the total radiating areas are as given in Table 33.

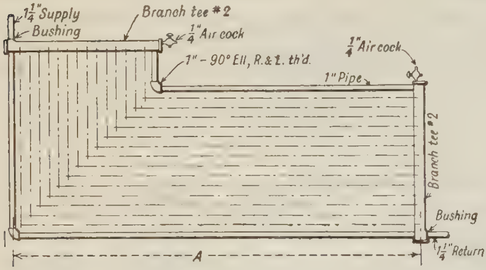


FIG. 10.—Outline sketch of mill radiator.

Table 33. Data for radiator shown in Fig. 10

A, feet	1-in. pipe, linear feet	Radiation, square feet	Cost (1924), complete
10	168	56	$63.25
12	200	67	67.05
14	232	78	70.85
16	264	89	74.65
18	296	100	78.45

Radiator fittings	Connecting fittings
2 Crane c.-i. branch tees No. 2, with 16 @ 1-in. branches, ¼-in. tap at inlet end, 1½-in. tap at outlet. 2 Bushings, 1½- to 1¼-in. 2 Air cocks, ¼-in. 16 @ 1-in. 90° ells, right- and left-hand threaded. 8 Hook plates for 1-in. pipe, 4 hooks each, spaced 2½-in.	2 Kewanee unions, 1¼-in. 2 Gate valves, 1¼-in. 3 Standard 90° ells, 1¼-in. 2 Street 90° ells, 1¼-in. (The above fittings allow for a swing joint in the supply line, and one ell in the return line.)

Table 34 gives the radiator areas that can be adequately supplied with steam at 5-lb. pressure through supply pipes of stated lengths and diameters; also the diameters of suitable return lines. Knowing the total radiator area and the distance from mill to boiler,

the necessary sizes of supply and return lines can be calculated. In similar manner, the diameters of branch lines serving different sections of a large mill can be estimated.

Table 34. Dimensions of pipe lines to supply stated radiator areas

Diameter of supply line, inches	Length of supply line, feet										Diameter of return line, inches
	100	200	300	400	500	600	700	800	900	1000	
	Radiator areas, square feet										
1¼	108	......	......	......	......	......	......	......	......	......	1
1½	216	154	125	108	......	......	......	......	......	......	1¼
2	432	307	251	216	195	177	164	151	143	138	1½
2½	792	563	460	396	357	325	301	277	261	254	2
3	1,350	959	783	675	608	554	513	473	446	432	2½
3½	1,980	1,406	1,149	990	891	812	753	693	654	634	2½
4	2,880	2,045	1,671	1440	1296	1181	1095	1008	951	922	3
4½	4,140	2,940	2,401	2070	1863	1697	1573	1449	1366	1325	3
5	5,580	3,962	3,237	2790	2511	2288	2120	1953	1842	1786	3½
6	9,000	6,390	5,220	4500	4050	3690	3420	3150	2970	2880	3½
7	13,500	9,585	7,830	6750	6075	5535	5130	4725	4455	4320	4
8	19,440	13,803	11,275	9720	8748	7971	7387	6804	6415	6221	4

10. Fire Protection

Fire protection in a wood-frame mill should include as many of the following precautions as necessary: (*a*) Portable extinguishers, desirable in all mills. (*b*) Automatic water sprinklers, advisable in all wooden mills of large extent. If sprinkler lines are liable to freeze, the dry system of control must be installed. (*c*) Fire mains under constant pressure and used for no other purpose, with hydrant valves at frequent and convenient intervals, to which hoses and nozzles or monitor nozzles are permanently connected; this system is considered adequate for small mills, and may render the installation of a sprinkler system unnecessary. (*d*) Well organized and regularly trained force of fire fighters, without whom the most elaborate system of equipment would be of little value in an emergency. (*e*) Cleanliness in the use of flotation and lubricating oils, with special precautions against accumulations of either. Advice on the general problem of fire protection, as well as detailed specifications for the most suitable installation in a given mill may be obtained gratis from: National Board of Fire Underwriters, New York; State boards of insurance inspectors; reputable fire insurance companies, all of whom maintain engineers for this express purpose.

Portable extinguishers have important advantages for protection of mills, since during two-thirds of the day the number of men who can be instantly summoned is reduced to the minimum required for the supervision of operations. In a wooden mill, the points that should receive best protection are: vicinity of flotation-oil feeders and storage, boiler rooms, store rooms, electric switchboards, and the neighborhood of all equipment requiring copious lubrication.

Soda-acid, 2½-gal. extinguisher is satisfactory for small fires in wood, rubbish, etc., not impregnated with oils or grease, but should not be used against fire in proximity to high-tension electric circuits; it must be protected against freezing. It is made also in 20- and 40-gal. sizes mounted on wheels.

Anti-freezing. 2½-gal. tank contains calcium chloride solution which is discharged by pressure generated by an automatically ignited cartridge; this has the same limitations as the soda-acid type, but does not require protection against cold.

Foamite extinguishers in 2½-gal. hand tanks and 40-gal. wheel-mounted tanks, contain solutions of aluminum sulphate, sodium bicarbonate, and a froth stabilizer (licorice extract). On mixing these solutions by inverting the tank, a frothy stream having about eight times the original bulk is discharged. The smothering effect is due to tenacious bubbles of carbon dioxide aided by films of aluminum hydroxide. It is suitable for ordinary fires, and particularly for those involving oils and grease, but is not safe for fires near high-voltage electric circuits although the stream of foam, after the momentary discharge of ungasified solution, is a poor conductor; foamite does not injure electric insulation. Foamite tanks must be protected from freezing.

Carbon tetrachloride is the only fire-extinguishing medium that can be used with safety around high-tension electric circuits. For other fires, including those in oil or grease, it is also serviceable, but the small size of the containers adopted by manufacturers (usually 1 or 1½ qt.) limits its effectiveness on a fire that has gained much headway, while the necessity for pumping the container is also a drawback to its efficiency. Larger containers are being developed. The tetrachloride supplied by extinguisher manufacturers contains ingredients to reduce its freezing point, and no protection against cold has to be given; no other than the special tetrachloride should be used. In the vicinity of electrical equipment or oil supplies, a unit of two tetrachloride extinguishers should be provided for 2500 sq. ft. of floor, and within 15 ft. of any specially hazardous point.

Automatic sprinklers, held closed by fusible links, are recognized by insurance companies as the most efficient form of protection against incipient fires. If sprinklers are installed, the standpipe and hose equipment may safely be reduced. The following recommendations apply particularly to the type of construction commonly found in wood-framed mills (*Regulations of the National Board of Fire Underwriters governing the installation of automatic and open sprinkler equipment; New York, 1922*):

(*a*) The entire mill should be equipped with sprinklers, not merely that portion where fire is most likely to begin; any unsprinklered areas must be cut off by fire-proof walls. (*b*) Sprinklers must be in upright position (deflector on top), must have 24 in. of wholly clear space below them, and should be 6 to 8 in. below the ceiling or the bottom of joists. (*c*) One sprinkler must be provided for 80 sq. ft. of area, spaced, say, 8 ft. on lines 10 ft. apart, the sprinklers being in staggered rows. No sprinklers should come within 12 in. of a post, hanger, or other vertical obstruction, nor within 2 ft. of wall or partition. (*d*) Under pitched roof, one line of sprinklers should be installed under the peak, or two lines each not more than 2½ ft. from the peak. (*e*) The maximum number of sprinklers to be served by pipes of stated sizes is given in Table 35. Preferably, no branch line should carry more than eight sprinklers. (*f*) Reducers, not bushings, should be used for connecting pipes of different size; couplings should not be used unless practically unavoidable. (*g*) DRY-PIPE SYSTEM should be adopted only in those parts of a mill where water pipes would be liable to freeze; the air capacity of a dry-pipe system controlled by a single valve should not exceed 115,500 cu. in. (*h*) The air pressure in a dry system (maintained by compressor) should not be more than 15 to 20 lb. in excess of the normal tripping pressure of the automatic water valve; the dry system should not leak more than 10 lb. pressure per week. (For detailed specifications, see the cited publication, obtainable on request.)

Table 35. Size of pipe and number of sprinklers

Pipe, inches	Maximum number of sprinklers	Pipe, inches	Maximum number of sprinklers
¾	1	3	36
1	2	3½	55
1¼	3	4	80
1½	5	5	140
2	10	6	200
2½	20		

Water mains exclusively for fire-fighting purposes should be connected with tanks of ample size (50,000 gal. at TIMBER BUTTE) maintained constantly at full capacity; this is conveniently arranged by setting the fire tank first

in series with the mill-water tanks and allowing it to overflow into them. Fire mains should also be connected with other sources of water supply available either instantly or by starting special fire pumps. If connection for this purpose must be made between the mill system and the domestic supply of the community, precautions must be taken to avoid contamination of the latter by installing a tight check valve which will not yield until the pressure on the mill side is considerably below that on the domestic side (this valve must not be forgotten during any readjustments which may affect the pressure on either side); a safer method is to discharge the domestic water into the fire tank from an open pipe.

A minimum pressure of 75 lb. per sq. in. should be provided for the highest fire-hose connection. Mains and principal hydrants should be situated at least 50 ft. outside the mill; near the most hazardous points inside the mill, other connections should be provided for lines of 1 ¼- or 1 ½-in. hose, which is as large as one man can handle at adequate protective pressures. Hose permanently connected to outside hydrants, not over 100 ft. in length (but with other 50-ft. lengths at hand), should be of standard 2- or 2½-in. size without rubber. The standard Nozzle for this hose has 1⅛-in. orifice (passing 250 gal. per min. at 45-lb. nozzle pressure), but with the higher pressures available at a well protected mill, orifices of ¾- and 1-in. are sufficient. For the smaller hose, the nozzle opening should be ½-in.

Fire-fighting forces are of little use unless well organized and thoroughly trained by frequent practice. Every member should be instructed in the manner of reporting a fire, use of portable extinguishers, the location of hydrants, the fire-piping system and the inter-relation of its valves, the water resources at immediate command and methods of adding to them. All of this information can be best communicated by printed instructions, as at Timber Butte mill (*116 J 811*), supplemented by regular practice drills with teams so organized that a sufficient number of trained men will be on duty every shift. Suggestions for organization and training are published by the National Board of Fire Underwriters.

Cleanliness, always desirable on general principles, is particularly necessary at those parts of a wooden mill subject to spillage of lubricating and flotation oils. Journals that require frequent oiling should have drip pans or similar devices. Feeders for flotation oils should be similarly equipped. In a small mill, where flotation oils are transferred directly from barrels to the feeders, the area of floor used for this purpose should be covered with sheet iron with raised edges; sand is then spread on the floor, to be shoveled out and replaced as often as necessary.

11. Dust collection

Dust is likely to be produced in troublesome amounts by almost any operation (crushing, screening, elevating, etc.) performed on material carrying less than 7 per cent. moisture. If allowed to migrate freely, it is chiefly objectionable because of disagreeable or unhealthful working conditions, increased abrasion of lubricated bearings, expense for cleaning floors and equipment and possible loss of valuable ore. Motors are the most seriously affected; they can be somewhat protected by grouping in an isolated room and driving the mill by line shaft, or by interposing an extra long shaft between a piece of equipment and its direct-connected motor, which can then be situated in an adjoining and dust-proof room, as in the Symons crushing department at Ajo (*114 J 184*). The most satisfactory method of control is to confine and collect the dust at its sources; the worst dust-producing

machines, as a rule, can be enclosed readily with sheet-metal housings connected with an exhaust system, with or without appliances for recovering the dust.

Where collectors have been installed, as at Kalgoorlie, and Waihi (*114 J 1070*), and at Morenci and Nacozari (*117 J 165*), the value of the dust, which usually assays higher than the original ore, has been more than enough to defray the whole expense of its collection.

Design of suction system. The following points were established at Morenci in 1917 (*22 CME 207*). The hood should be so designed that the sum of its air inlets does not exceed the area of its exhaust pipe. The exhaust pipe should be attached at that point of the hood towards which the dust is naturally impelled by centrifugal or other forces or by air currents. Connections between the hood and the suction pipe should be by bolted flanges, to permit quick dismantling; other joints in the suction line should be riveted and soldered, with laps in such direction as to cause minimum friction. A velocity of 4000 ft. per min. will carry any ordinary dust. A main header should increase progressively in area, always being slightly larger than the sum of its branches. The system as a whole should be designed to maintain the desired velocities in mains and branches without recourse to valves and dampers for regulation. For straight pipe, 16- or 20-gage galvanized steel is best; on curves, 16-gage. Radius of bends (measured from the center line) should not be less than 1 ½ times the diameter of the pipe, but there is no advantage in a radius greater than twice the diameter. Unless the headers are equipped with hopper pockets and gates at short intervals, they, as well as all other pipes, should have an inclination of 30° or more (preferably 45°) to allow dust to be withdrawn on stopping operations. The simplest mechanism for recovering dust is a dry cyclone separator; this will catch 75 to 80 per cent. of any ordinary dust. Of the remaining finest particles, 98 per cent. can be caught in a cloth-walled chamber, bag house, or wet centrifugal collector; the expense of a Cottrell precipitator will not usually be justified for this purpose.

At a dry-crushing mill at Kalgoorlie (*114 J 1070*) one 36-in. Sturtevant suction fan, at 1000 r.p.m., drawing 5000 cu. ft. of air per min., collects dust from six large crushers of Griffin, Krupp, or Symons type. Branch pipes are 9-in. diameter; main header, 18-in., equipped with V-traps and vertical risers for collection of the coarser particles. The dust assays higher than the original ore. It is caught in a cyclone collector followed, by a chamber having double walls of fine burlap, with rapping devices, and hoppered bottom with screw conveyor. The original installation at Arizona Copper Co., Morenci, in 1917 (*22 CME 207*), when crushing 250 tons per hr. to ¾-in., the ore averaging 3.85 per cent. moisture, required seven hoods to cover one gyratory and grizzly, two horizontal disk crushers, and two pairs of rolls. The suction pipes were 8-in. diameter, except that from the gyratory hood, which was 10-in. The wet collector was a cylindrical tank of 3⁄16-in. steel, 10.5 ft. diameter by 15.5 ft. tall, with tangential inlet near the top. The exhaust, through the top, came from inside a cylindrical shroud, 7 ft. diameter, extending down to within 6 ft. of the bottom of the tank. Water sprays impinged against this shroud, inside and out, and muddy water collected in a concrete basin on which the tank rested, flowing out through a water-sealed trap to flotation. The suction fan, following the wet collector, was a Sturtevant No. 70, exhausting 11,100 cu. ft. per min. at 577 r.p.m., and 3.5-oz. negative pressure; motor input, 23 hp. The dust collected was 0.0367 per cent. of the weight of ore crushed, assayed 2.34 per cent. Cu, and consumed 6552 gal. of water per ton, which was probably more than actually necessary. A later installation by Phelps Dodge Co. (*117 J 165*) has two similar units each comprising a motor-driven Sturtevant fan No. 80, dry collector and wet collector. On fairly dry freshly-mined ore, the system collected 8 to 10 tons of dust per shift (80 per cent. from the dry collector), of which 1 per cent. was coarser than 65-mesh and 93 per cent. finer than 200-mesh; its assay was about double that of the original ore. Each wet collector used 10 gal. of water per min. When working on oxidized ore from the stockpile, about twice as much dust, of the same fineness, was collected, its assay being the same as that of the ore. At Nacozari, a similar but smaller installation collects three tons of dust per shift, produced entirely by coarse crushing, and assaying twice as much copper as the original ore. At a rock crushing plant at Corona, Cal. (*116 J 889*), trommels are enclosed in jackets connected by suction pipes to two Sly type-A units, each having 3000 sq. ft. of filter cloth and passing 10,000 cu. ft. of air per min. They collect 1.5 tons of dust per hr., coarse particles having been settled previously in a baffled chamber. Suction is stopped for 10 minutes every 4 hours to rap the filters. At United Verde crushing plant, Clarkdale, Ariz. (*117 J 396*), dust is withdrawn from groups of four jaw crushers, four vibrating screens, four disk crushers, and four rolls, by a separate duct from each group. The ducts have cleaning pockets and gates every 15 ft. Fine dust is caught by plate-and-wire Cottrell precipitators, collected in hoppers and discharged into cars delivering to roasters. A negative pressure of 6-in. water-gage is produced

by two 45,000-cu. ft. fans, each direct-connected to a 125-hp. motor. At Ajo leaching plant (*114 J 184*), disk crushers stand above a tunnel containing a belt conveyor on which the crushers discharge; when operating, the ends of the tunnel are closed by doors, and air is exhausted by a suction fan delivering to a group of cyclone separators.

12. Shops, repairs, supplies, etc.

Character of shop work. At an operating mill, shop work is confined almost exclusively to repairs; while a mill is under erection, however, the shops are a most valuable adjunct, and may well be the first buildings set up. A long, simple structure can be partitioned off, the most convenient arrangement being, starting at one end: office, warehouse, machine shop, carpenter shop. Blacksmith work can usually be done most conveniently at the mine shop. The scope of work to be arranged for depends on the size and elaborateness of the mill, and its situation with respect to other established shops capable of doing at least some of the required work. The foundries and machine shops of an enterprise like Utah Copper Co., for example, providing for the operation of mines, railroad, and two large mills, compare favorably as to size, arrangement, and completeness with the best industrial machine works. A small mill in an outlying district would be justified in the installation of fairly complete shop machinery, even if not all continuously occupied, so as to save time in repairs and avoid some duplication of expensive mill equipment to guard against delays. Some means of transferring parts of heavy mill equipment into the shop without excessive effort must be provided by car tracks, crane, or both.

List of equipment for a machine shop for a small mill: Hand punch and shear; lathe, 18-in. swing, 8- to 10-ft. bed; pipe-threading machine, 1¼ to 4 in.; oxy-acetylene torch outfit complete; bolt- and pipe-threading machine for ½- to 1½-in. bolts and ⅜- to 1-in. pipe; electric- or air-driven hand drill; portable forge, anvil and blacksmith tools; radial drill press, 48-in.; power hack saw. The usual assortment of hand tools, vises, benches, etc. The carpenter shop, besides benches and hand tools, should have a saw bench with 16-in. circular saws and a buzz planer.

At one large mill in the Southwest, the following machine tools, all with individual motors, are installed: 27-in. × 28-ft. lathe, 14-in. × 12-ft. lathe, 6-ft. radial drill, 16-in. spindle drill, 28-in. swing drill, 100-in. boring mill, 42-in. × 12-ft. planer, 24-in. shaper, No. 2 and No. 6 pipe machines, friction saw, punch and shears, blacksmith forges, welding equipment. One five-ton crane serves these machine tools. A five-ton crane and a sixty-ton crane with a ten-ton auxiliary are installed over the secondary ball mills to carry loads from the mill to the machine shops which are nearby.

Repairs, in most case, must be made without moving the machine from its place; small repairs can be made by the mill crew, who should be provided with benches and hand tools at convenient places in the mill. Worn parts of crushing machines can usually be repaired more conveniently in the shop, where screw-jacks, hydraulic presses, etc., are available. With such machines, the best practice is to keep one or more spare parts on hand, assembled and ready to install; this applies particularly to gyratory spindle and crushing head, rolls, cylinder-mill scoops, and entire mills.

Cranes or crawls are essential in every part of a mill containing equipment too heavy to be lifted by rope or chain tackle and temporary staging; they also avoid the trouble and delay in erecting the latter, even where they might be strong enough. The usefulness of a crane begins while a mill is under construction, and it should be installed as soon as the columns and beams are in place. Care should be taken to insure that the crane is able to reach every unit of equipment it can advantageously serve, including motors, cylinder-mill scoops, classifiers, heads of elevators, pumps, etc., in addition to the heavier crushing equipment. A crane should also be able to travel either

into the repair shop or far enough to deliver its load to a car or another crane serving the shop; if possible, the cranes in those parts of a mill having the heaviest equipment should reach also the railroad or supply yard.

At the UNITED COMSTOCK (*114 J 846*, *117 J 516*) the coarse-crushing department, containing No. 7½ gyratories, 72-in. rolls, and 200-hp. motors, has a 20-ton crane reaching the railroad and the working space between the shops; the fine-crushing section, with 7 × 6-ft. ball mills and 150-hp. motors, also has a 20-ton crane traveling over the supply yard and into the shops; the cyanide department, with basket filters aggregating 74,400 sq. ft. of filtering surface, contains two 40-ton traveling cranes. The COPPER QUEEN mill at Bisbee has an unusually complete equipment of electric cranes, as shown in Table 36.

Table 36. Cranes in Copper Queen mill

Serving	Span of craneway, feet	Lifting capacity, tons
1 jaw crusher, 66×84-in.; 250-hp. motor	75	50
2 gyratories, Gates No. 9; 2 motors, 125-hp	39	20
4 Symon's disks, 48-in.; grizzlies, etc	38	20
8 Marcy rod mills, 6½×12-ft	30	60 (2 hoists)
40 Deister tables, Dorr classifier, etc	28.5	5
8 Marcy rod mills, 6½×12-ft., classifiers, etc	30	60 (2 hoists)
Callow flotation cells; 1256 sq. ft of bottom	37	5
80 Deister tables, etc	37	5

Hand-operated chain blocks are available with lifting capacities up to 20 tons and reaches up to 18 ft.; small, self-contained electric hoists, for suspending from girders, have capacities up to 10 tons with a lift of 10 ft. The simplest crawl consists of a 4-wheel trolley truck running on the lower flange of an I-beam; trolleys for loads up to 10 tons are on the market.

Warehouse and supply department is essential to efficient operation; under competent administration it reduces waste, facilitates cost accounting, and avoids loss of operating time resulting from lack of necessary materials. The warehouse should adjoin the railroad and be as close as practicable to the mill and shops. Materials regularly consumed, such as balls and pebbles, flotation oils and reagents, should be stored as near as practicable to the places where they are used, though still under the jurisdiction of the storekeeper; other supplies, used intermittently, should be kept in the storehouse and issued only on requisition. Paints, lubricating oils, and similar highly inflammable materials should have a separate fire-proof storage. Lumber should be protected from weather.

13. Methods of practical design

Preliminary layout. A FLOW-SHEET is the first requisite. This must have been exhaustively studied and thoroughly and finally decided upon in respect to every important detail, including: number, types, sizes and capacities of individual machines; their relative positions with respect to one another; quantities of ore and of water in transit at every point throughout the mill; power required by every machine; number, sizes and types of all motors and the speed reducers connected with each. The design of the flow-sheet will be based on the results of laboratory or pilot-mill experiments, interpreted in the light of experience, and will be guided by the available supply of water, sources of power, type of mill structure applicable to the site,

and other practical considerations. A TOPOGRAPHIC MAP of the site is required, not only to select the general type of structure, but to determine the amount and character of the required excavations and the placing of walls, foundations, etc., so as to reduce the amount of this work, after the manner of a railroad location. The map, which will usually have been made early in the investigation, should cover a sufficient area to allow a choice among equally accessible sites and permit the final and precise selection to be deferred until the more important structural features have been determined. PRELIMINARY SKETCHES in three projections, not necessarily to scale, are a valuable means of crystallizing ideas, gaining a sense of proportion, and transmitting suggestions for execution of the next step. PRELIMINARY SCALE DRAWINGS should be confined to outlines, preferably on a scale of $\frac{1}{20}$ in. to the foot; adoption of this small scale protects the draftsman from the constant temptation to go too far into detail, thereby obscuring general features. The three projections of these drawings should appear on a single sheet, so as to give better idea of space proportions. On these drawings, the draftsman should prepare and make constant use of templates representing the outlines of individual machines or groups of closely related machines, such as rolls, elevator and trommel; ball-mill, motor and classifier; these templates (used under transparent paper) not only save much time during the initial period when erasures are likely to be frequent, but are of great assistance in fixing the most advantageous position of the equipment. PRELIMINARY ESTIMATES of structural elements can be made from these small-scale drawings. These estimates, as well as those that follow, and all calculations on which they are based, should be preserved in bound notebooks (conveniently 8×10 in. or larger).

Detailed general drawings, in three projections on a scale of $\frac{1}{8}$ to $\frac{1}{4}$ in. to the foot, depending on overall dimensions, must be complete as to every item, and show as much detail as possible at that scale. Nothing having any bearing on the erection of the building and installation of machines should be omitted; the more complete these drawings, the quicker the construction can be finished and the less the chance for overlooking essential features. Among the items that must be plainly shown on the general drawings are: foundations and floors, frame, walls and roof, windows, cranes and crawls, all machinery, motors and drives, shafting, clutches and pulleys, launders and pipe lines. A small mill should be drawn in entirety, a separate sheet for plan, front elevation, and end elevation or sections; for a larger mill, a typical unit will suffice at this larger scale, if accompanied by a complete assembly at a smaller scale.

Construction details. The amount of large-scale drafting of structural details depends upon the size of mill and type of construction. The following detailed drawings should be supplied for all mills: (*a*) EXCAVATIONS: volume and character. (*b*) FOUNDATIONS: footings, dimensions and batters, copings, reinforcements, drains or "weep" holes, elevations of the top of each wall, pier or foundation above the datum plane, and the position of anchor bolts. (*c*) MECHANICAL EQUIPMENT: where machines of standard types and sizes are adopted, dimensions and foundation plans of which are supplied by manufacturers, the drawings to be made for construction purposes will relate mainly to their support and the facilities for conveying ore to and from them; for the installation of the large variety of other equipment, such as trommels, elevators, conveyors, feeders, distributors, samplers, and all other devices that must be adapted in size and arrangement to suit the particular mill, com-

pletely detailed drawings should be prepared, including their connecting elements. (*d*) Shafting and all its appurtenances, including hangers or pillow blocks, journal boxes, clutches, pulleys, collars, and couplings; descriptions of all these elements should appear as completely as possible on the drawings, so that the accompanying written specifications need be little more than a reference list. (*e*) Piping system as a whole can be shown most easily and clearly by an isometric projection (see Fig. 11), on which the dimensions of all main and branch lines, valves and fittings can be indicated; for minor details of particular connections, a sketch on larger scale should be appended as an insert on the main drawing. (*f*) Launder system must be worked out in all its details and subjected to careful scrutiny as to the ability of each member to carry the desired volume of its given pulp; this is a particularly important feature of design, and should not be left to the discretion of the millwright, since a launder once installed at too low a slope is difficult to reconstruct. (*g*) Wiring diagram, conveniently indicated by colored lines on a copy of the general drawings, should state the sizes of conductors, character of insulation, position of switches, fuses, lights, etc.

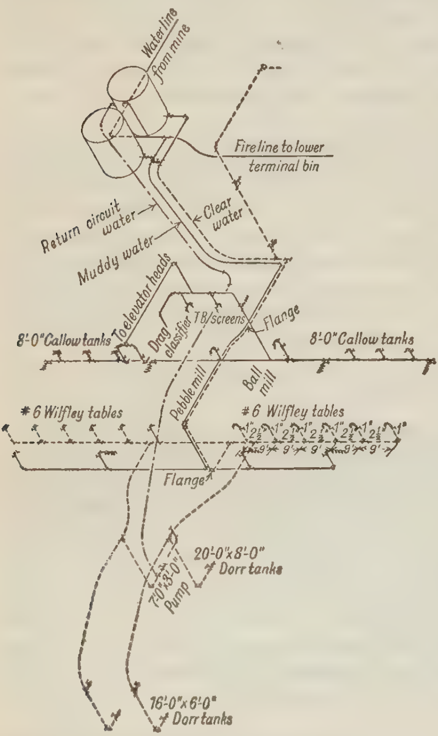


Fig. 11.—Typical arrangement for lay-out of mill water piping.

Steel structures. It is rarely necessary for the mill designer to prepare detailed structural drawings for steel work. If the plant can be protected by one or a group of simple rectangular buildings, these can be purchased ready-made from a number of fabricators. For a larger and more complicated structure, the usual procedure is to prepare several sets of general plans, accompanied by a statement of the roof load, floor load, crane capacities, character of wall material, etc., and submit these to steel-work fabricators, who thereupon assume responsibility for detailed working drawings. Bids from these fabricators are scrutinized and compared, and the low bids carefully checked before awarding contracts.

Wooden structures. If time permits, it is helpful to provide the millwright with a separate set of drawings of the structure dissociated from the mechanical equipment; this is easily done by tracing from the general detailed plans. If time is short, a set of the general plans can be amended by addition

Table 37. Approximate costs of erecting and equipping mills

Mill, location and year	Nature of process	Daily capacity, tons	Character of building	Total cost of building and equipment	Cost per ton-day, capacity
Tri-state District:					
Media, Webb City, 1915	Rolls, jigs, tables	1500	Wood	$50,000	$34
Typical jig mill, 1916	Rolls, jigs	250	Wood	7,200	30
Typical jig-table mill, 1923	Rolls, jigs, tables	720	Wood	75,000*b*	105
Coarse concentration and flotation:					
Annapolis, Mo., 1922	Jigs, tables, flotation	500	Steel(*a*)	194,760*b*	390
Silver King, Park City, Utah, 1921	Jigs, tables, flotation	300	Steel and concrete	300,000	1000
Tables and flotation:					
Armstead, Talache, Idaho, 1921	Tables and flotation	150	Wood frame	128,154	854
Magma, Superior, Ariz., 1914	Tables and flotation	150	Wood frame	86,420	576
National Copper, Mullan, Idaho, 1914	Tables and flotation	500	Wood frame	162,740	326
Fine crushing, flotation only:					
Betty O'Neal, Lewis, Nev., 1922–3	Ball mills, M. S. flotation	300	Wood frame	200,000	667
Allenby, Princeton, B. C., 1919		2000	Wood frame	1,300,000	650
Shattuck-Arizona, Bisbee, 1917–8	Ball mills, K. & K. flotation	400	Wood frame	294,900	737
Ottawa, Slocan, B. C., 1921	Ball mills, M. S. flotation	50	Wood	30,000	600
Carbon Mt., Ala., 1917	Fine crushing, Callow flotation	240	Wood	45,657*e*	190
Fine crushing, cyanidation:					
United Comstock, Nev., 1921–2		2000	Steel and concrete	1,739,300	870
United Eastern, Oatman, Ariz., 1916		300	Wood frame	200,000	667
Stargo, Morenci, Ariz., 1922		75	Wood frame	77,000*c*	1027
Winnemucca, Nev., 1916		50	Steel	41,850*d*	837
Wright-Hargreaves, Ont., 1919		200	Wood; asbestos covered	161,945	810
Stamps and amalgamation:					
Moore, Jackson, Cal., 1923	20 stamps	100		77,030*f*	770

a Building for 1000-ton capacity; equipped for 500 tons. *b* Excluding power generating plant. *c* Used large proportion of second-hand machinery. *d* Principal steel work second-hand. *e* For soft graphite rock. *f* Including shops and warehouse, but not tailing-disposal plant.

Table 38. Distributed costs of representative mills

Mill	Type	Machinery, F.O.B. factory	Building	Installation	Total	Building and installation per dollar of machinery
Annapolis	Jigs, tables, flotation	$92,365	$102,392*b*	*c*	$194,757	$1.11
Armstead	Tables, flotation	62,497	27,866	$16,707	128,154*a*	1.05
Burro Mountain	Tables, flotation	317,140	198,794	118,695	634,629	1.00
Magma(*d*)	Tables, flotation	50,663	35,756	*c*	86,419	0.71
Winnemucca	Cyanide	25,850	11,756*f*	3,985	41,591	0.61
United Comstock	Cyanide	779,829	732,595	226,883	1,739,307	1.23
United Eastern	Cyanide	110,952	62,079	24,726	197,757	0.78
Media	Jigs, tables	25,410	19,150	5,730	50,290	0.98
Richard	Magnetic separation	72,246	69,334	*c*	141,580	0.96
Carbon Mt.	Flotation	25,715	9,641	5,056	45,660*a*	0.78
Wright-Hargraves	Cyanide	108,618	53,327	*c*	161,945	0.50

a Total includes miscellaneous items, such as freight, engineering fees, supervision, etc., not distributable among the three principal headings.
b Building for twice the capacity of machinery installed. *c* Included under building. *d* Original mill, 1914. *f* Structural steel all second-hand.

of dimensions and such details as are not shown with sufficient clearness for the millwright's use, notably trusses and bins.

14. Methods of cost estimating

A preliminary, approximate idea of the ultimate cost of a complete mill of given type may be gained from the following factors representing in round numbers the range in total cost of erection and installation per ton-day (24-hr.) capacity:

Jig-table mill, Tri-State district (1923), buying electric power....	$105
Coarse concentration, no flotation........	$300– 450
Jigs and/or tables, and flotation...........	800–1200
Fine crushing, all flotation	600– 800
Fine crushing and cyanidation............	1000–1400
Stamps, amalgamation, and vanners........	475– 550

Tri-State mills do not contain fine-crushing machinery; tables (though common) are not universal; flotation has not been generally adopted, owing to the comparatively small proportion of the original feed that is reduced to suitable size; and the usual life of a mill demands cheapness and simplicity of construction.

Table 37 gives examples of mills representing the above types.

Erection and installation. When the prices of all essential machinery have been ascertained, the probable cost of the completed mill can be approximated by the empirical rule that for each $1 in machinery, f.o.b. factory, the cost of erecting the building and installing the equipment will range, under average conditions, from $0.75 to $1 per dollar of machinery cost, running up to $1.25 in exceptional circumstances, such as expensive building materials, inefficient labor, delays, etc.

Table 39. Volume costs of mill buildings

Mill	Type of construction	Cost of mill building only	Enclosed volume of building, cubic feet	Cost of building, per cubic foot, cents
Alamo, Sandon, B. C.	Wood frame	....	461,000	3.458*
Annapolis	Steel	$102,392*a*	716,604	14.3
Armstead	Wood frame	27,866	373,000	7.5
Burro Mountain, 1914–16:				
Crusher building	Steel	35,097	357,424	9.8
Mill & sampling plant	Steel	109,319	1,200,000	9.1
Machine shop	Steel	10,268	108,760	9.4
Carbon Mt.	Wood	9,641	190,476	5.0
Imlay, Imlay, Nev.	Wood frame	....	88,740	3.328*
Magma	Wood frame	35,756*a*	293,738	12.2*b*
National Copper Co.	Wood frame	....	738,458	2.917*
Utah Consolidated	Wood frame	....	959,880	5.87*
Watters, Sheridan, Mont.	Wood frame	....	273,831	1.809*
Winnemucca	Steel (mostly second-hand)	11,756	123,160	9.5

a Includes installation of equipment. *b* Includes expensive excavations and foundations, amounting to 40 per cent. of the total cost. * Labor only.

Table 40. Erection-cost factors

Mill, year	Type	Total lumber used, board feet	Labor costs: Frame only, per 1000 bd. ft.	Labor costs: Complete building, per 1000 bd. ft.
Magma, Superior, Ariz., 1914	Wood frame, iron roof and sides.	228,000	$14.93	$20.60
National, Mullan, Ida., 1915	Wood frame, boards and iron roof and sides.	443,000	....	23.79
Watters, Sheridan, Mont., 1912	Wood frame, roof and sides.	155,000	17.17	19.00
Imlay, Imlay, Nev., 1909	Wood frame, roof and sides.	55,000	....	25.12
Utah-Apex, Bingham, Utah, 1909	Wood frame	108,000	*a*	15.01
Alamo, Sandon, B. C., 1918.	Wood frame	354,000	....	33.30
Utah Consolidated, Tooele, Utah, 1921–2	Wood frame	763,000	....	37.51
Armstead, Talache, Idaho, 1921–2	Wood frame	271,000	....	29.65
		Steel, pounds	Per pound	
Annapolis, Mo., 1922	Steel	320,000	6.47¢ erected and with two coats of paint	

a Remodeling and enlarging old mill.

In one case of an elaborate and complicated ore-dressing and metallurgical plant, requiring unusual provisions for storage and widely extended steel structures, designed for a mine in Utah, the carefully estimated cost of erection and installation was $1.04 for $1 worth of machinery, at factories.

Table 38 gives the relation between the total cost of a few representative mills and the cost of their machinery.

Cost-volume ratios. Under average conditions in United States mining districts, the cost for labor and materials of buildings alone will approximate, per cubic foot of enclosed space: Wooden buildings, 3 to 6¢; steel buildings, 7 to 10¢. Table 39 gives examples.

Erection, labor-cost factors. Based on the QUANTITY OF PRINCIPAL STRUCTURAL MATERIAL required, the cost of erection can be estimated roughly as follows: Wooden structure, $30 to $40 per M bd. ft.; steel structure, 0.75 to 1.5¢ per lb. See Table 40.

Based on TONS OF EQUIPMENT INSTALLED, Table 41 gives the labor cost alone, covering the erection of building and installation of machinery, at representative mills.

Table 41. Labor costs per ton of machinery installed

Mill, year	Machinery installed, tons	Labor costs per ton of machinery			Wages	
		Complete mill building	Installing machinery	Total	Mechanics	Labor
Alamo, Sandon, B. C., 1918	161	$99.30	$62.10	$161.40	$6.75	$5.00
Armstead, Talache, Idaho, 1921	160	70.08	53.70	123.78	5.75	3.75
Magma, Superior, Ariz., 1914	140	85.60	49.00	134.60	4.50	3.00
National, Mullan, Idaho, 1914	265	86.03	44.27	130.30	4.75	3.50
New Haven, Bingham, Utah, 1909	66	25.53	53.47	79.00	5.00	3.00
Imlay, Imlay, Nev., 1909	48	62.38	35.70	98.08	5.00	3.50
Utah Apex, Bingham, Utah, 1909	74	24.60	50.65	75.25	5.00	3.00
Utah Consolidated, Toole, Utah, 1921–2	786	69.30	49.50	118.80	6.50	4.75
Watters, Sheridan, Mont., 1912	81	37.99	40.36	98.35	5.00	3.50

The labor cost for ERECTION OF INDIVIDUAL MACHINES of stated types and sizes is given in Table 42, together with weights and approximate factory prices (1924). Erection costs, of course, will vary with local conditions and wages; hence it should be understood that the figures in Table 42 are intended only for approximate, preliminary estimates.

Table 42. Weights, approximate (1924) prices, and labor cost for erecting the principal items of mill equipment

	Factory weight, pounds	Factory price	Labor cost of erection	Remarks
Crushers, Blake jaw:				
15× 9-in.	12,000	$1,100	$100	All chilled-iron fitted.
20× 10-in.	15,000	1,400	123	
24× 12-in.	25,000	2,200	206	
30× 18-in.	45,000	3,900	387	
36× 24-in.	86,000	8,300	680	
42× 40-in.	135,000	11,200	1172	
Crushers, gyratory (primary):				
5× 18-in. opening (*a*)	5,800	1,060	56	If manganese-steel fitted, add 10 per cent. to prices up to and including 14× 52-in.; on larger sizes, add 15 per cent. (*a*) Two-armed spider.
6× 21-in. opening	8,350	1,215	85	
7× 22-in. opening	14,500	1,750	135	
8× 30-in. opening	21,800	2,210	212	
10× 38-in. opening	32,300	3,040	288	
12× 44-in. opening	47,100	4,120	396	
14× 52-in. opening	68,100	5,550	603	
18× 68-in. opening	101,200	8,375	900	
21× 76-in. opening	156,000	11,600	1450	
Crushers, gyratory (Bull-dog type):				
14× 52-in. opening (*a*)	60,000	5,550	603	Same as preceding. (*a*) Two-armed spider.
18× 68-in. opening	84,500	8,200	850	
21× 76-in. opening	135,000	11,000	1400	
Rolls (belt-driven):				
24× 14-in.	14,000	$1,850	$60	If manganese-steel fitted, add 10 per cent. to prices up to and including 14× 52-in.; on larger sizes, add 15 per cent.
30× 14-in.	18,300	2,500	145	
36× 16-in.	24,500	3,100	280	
42× 16-in.	39,000	4,500	346	
54× 20-in.	102,700	9,600	927	
60× 24-in.	112,000	10,700	1350	
72× 24-in.	186,000	16,750	1990	
Grinders, grate ball-mills:				
3× 3-ft.	10,700	2,000	60	Fitted with Wuest gears, hard-iron liners, but no motor or ball charge. Weight of charge is approximately 25 to 30 per cent. of the weight of mill and liners.
4× 4-ft.	25,500	3,940	130	
5× 4-ft.	31,000	4,800	225	
6× 4-ft.	44,000	5,900	375	
7× 5-ft.	66,000	8,350	660	
8× 6-ft.	95,000	11,500	835	
Grinders, rod-mills:				
2× 4-ft.	8,000	1,600	55	
3× 6-ft.	23,000	4,550	185	
3× 8-ft.	25,000	4,775	220	
4× 8-ft.	44,000	6,600	375	
4× 10-in.	46,500	6,850	430	
5× 10-ft.	88,000	12,500	800	
6× 12-ft.	110,000	19,000	1120	
Grinders, ball-pebble mills:				
5× 10-ft.	34,800	$5,160	$305	Fitted with Wuest gears, hard-iron liners, but no motor or ball charge. Weight of charge is approximately 25 to 30 per cent. of the weight of mill and liners.
5× 12-ft.	37,800	5,280	330	
6× 12-ft.	43,200	6,540	380	
7× 14-ft.	110,000	9,300	970	

Table 42. Weights, approximate (1924) prices, and labor cost for erecting the principal items of mill equipment—*Continued*

		Factory weight, pounds	Factory price	Labor cost of erection	Remarks
Grinders, conical ball mills:					
4.5-ft.×16-in.		11,400	$3,200	$116	
6-ft. ×22-in.		22,100	6,200	208	
7-ft. ×22-in.		28,000	8,300	235	
8-ft. ×22-in.		38,000	10,500	300	
8-ft. ×48-in.		50,000	12,500	375	
Grinders, conical pebble mills:					
6-ft.×22-in.		17,500	4,500	200	
8-ft.×36-in.		32,000	6,000	350	
8-ft.×48-in.		37,000	6,200	400	
Screens, revolving stone:					
30-in.×10-ft.		5,900	850	45	All iron and wood work; no screen covering.
40-in.×10-ft.		8,500	995	70	
48-in.×12-ft.		13,800	1,370	150	
48-in.×16-ft.		15,800	1,550	190	
60-in.×16-ft.		31,900	2,585	390	
Screens, traveling-belt (Callow duplex):					
24-in.×4.5-ft.		3,600	600	65	With screen cloth.
Screens, vibrating (Hum-mer):					
3×5-ft., type 37 (open)		1,270	530	75	Without screen cloth.
Motor-driven generator		600	550		
Screens, impact (Colorado Iron Works Co.):					
3×4-ft.		1,300	350	60	Without screen cloth.
3×6-ft.		1,450	475	75	
Screens, trommels:					
30× 60-in.		2,000	378	75	All iron work; no screen covering or housing.
36× 72-in.		2.400	420	85	
42× 90-in.		3,300	565	115	
48×108-in.		4,000	620	145	
Jigs, Harz type, single:					
Compartments	Sizes, inches				
2	18×36	3,000	575	75	All wood and iron work complete.
2	24×36	3,450	650	90	
3	18×36	4,000	800	115	
3	24×36	4,800	900	140	
4	18×36	5,000	1,025	150	
4	24×36	6,150	1,150	180	
Jigs, Hancock:					
3.5×4.5×25-ft. tank		19,000	4,000	540	As above.
Jigs, Cooley:					
Compartments	Size, inches				
6	48×36	32,000	1,160	500	As above.
7	42×36	32,000	1,175	500	
5	42×36	28,000	905	400	

Table 42. Weights, approximate (1924) prices, and labor cost for erecting the principal items of mill equipment—*Continued*

	Factory weight, pounds	Factory price	Labor cost of erection	Remarks
Tables, Wilfley type:				
No. 6	3,100	$500	$45	Complete, without channel-iron base.
No. 11-D	1,850	450		
Thickeners, Callow-cone:				
8-ft.	650	125	20	
Thickeners, Dorr:				
30×10-ft.	25,000	2,500	300	Wood tank and steel superstructure.
50×12-ft.	52,000	5,800	500	
80×10-ft.	166,000	8,500	1300	As above.
120×12-ft.	185,000	19,000	1500	As above.
200×12-ft.	85,000	17,000	2500	Iron work only *
Classifiers, Dorr, Duplex, type D:				
4-ft. 6-in.×14-ft. 8-in.	4,500	1,440	56	With steel tanks.
5-ft. 6-in.×14-ft. 8-in.	5,800	1,610	70	
6-ft.×18-ft. 4-in.	12,500	2,930	160	
8-ft.×18-ft. 4-in.	13,900	3,710	175	
Classifiers, Akins, heavy-duty, submerged:				
45-in.	8,000	1,800	30	With steel tanks and lifting device.
54-in.	14,400	2,850	50	
60-in.	17,500	3,500	75	
78-in.	34,000	4,700	125	
96-in.	44,000	5,800	150	
Classifiers, Allen cone, type 40:				
3.5-ft.	675	400	15	
4.5-ft.	825	450	20	
6-ft.	1,050	550	25	
8-ft.	1,600	700	35	
Classifiers, Richards hindered-settling:				
3-spigot	700	428	15	Iron and woodwork complete.
Filters, vacuum-drum type:				
5-ft. 4-in.×6-ft. = 108 sq. ft. area	8,000	2,475	140	Not including vacuum pump, receiver, etc.
8-ft.×8-ft. = 200 sq. ft. area	12,500	3,590	250	
12-ft.×12-ft. = 432 sq. ft. area	26,000	5,600	520	
Filters, vacuum-disk type:				
2 @ 6-ft. disks = 100 sq. ft. area	6,300	2,100	65	
4 @ 6-ft. disks = 200 sq. ft. area	8,100	3,000	85	As above.
4 @ 8.5-ft. disks = 400 sq. ft. area	14,600	4,800	250	
6 @ 8.5-ft. disks = 600 sq. ft. area	19,000	6,000	325	

* Concrete tank, including excavations and all other items = $31,000.

Table 42. Weights, approximate (1924) prices, and labor cost for erecting the principal items of mill equipment—*Continued*

	Factory weight, pounds	Factory price	Labor cost of erection	Remarks
Filters, pressure, plate-and-frame:				
30 @ 18 × 18-in. sections = 117 sq. ft. area	3,900	$460	$35	
30 @ 24 × 24-in. sections = 212 sq. ft. area	5,800	595	52	
36 @ 30 × 30-in. sections = 400 sq. ft. area	12,300	915	105	
60 @ 36 × 36-in. sections = 977 sq. ft. area	26,415	1,725	130	
Flotation machines, Minerals Separation Co.:				
24-in. @ 16-cell, standard	45,500	10,000	600	Without motor.
24-in. @ 16-cell, sub-aeration	32,500	8,200	450	Without motor.
Flotation machines, Callow pneumatic:				
3 comp., double, 3 × 3-ft. unit	4,050	737	60	Without blower, motor or blower piping.
6 comp., double, 3 × 3-ft. unit	7,800	1,332	100	
9 comp., double, 3 × 3-ft. unit	10,700	1,947	135	
12 comp., double, 3 × 3-ft. unit	13,700	2,564	175	
Feeders, steel-apron type:				
18-in. × 3-ft. 2-in. long	2,000	360	15	Complete with driving mechanism.
30-in. × 4-ft. 2-in. long	2,550	480	30	
48-in. × 5-ft. 2-in. long	3,200	760	40	
Feeders, armored-belt:				
18-in. × 8-ft.	1,235	324	30	As above.
24-in. × 10-ft.	1,870	448	45	
30-in. × 12-ft.	2,460	590	60	
Pumps, centrifugal, water (Gould):				
2-in. = 120 gal. per min. (single suction)	190	95	7.50	
3-in. = 265 gal. per min. (single suction)	280	135	12.50	Single-stage.
4-in. = 470 gal. per min. (single suction)	400	160	20	
6-in. = 1000 gal. per min. (single suction)	600	270	35	
8-in. = 2000 gal. per min. (single suction)	1,000	375	60	
Pumps, triplex, plunger:				
6.5 × 8-in. = 241 gal. per min.	3,700	850	50	Good for 100-ft. lift.
7.5 × 8-in. = 321 gal. per min.	4,600	1,020	65	Good for 100-ft. lift.
9.5 × 8-in. = 501 gal. per min.	5,800	1,190	85	Good for 75-ft. lift.
13 × 8-in. = 975 gal. per min.	10,900	2,210	140	Good for 75-ft. lift.

Table 42. Weights, approximate (1924) prices, and labor cost for erecting the principal items of mill equipment—*Continued*

	Factory weight, pounds	Factory price	Labor cost of erection	Remarks
Pumps, centrifugal, sand (Wilfley):				
2-in. = 175 gal. per min. . .	660	$300	$10	
3-in. = 300 gal. per min. . .	900	400	15	
4-in. = 500 gal. per min. . .	1,650	550	20	
6-in. = 950 gal. per min. . .	2,550	750	30	
8-in. = 2000 gal. per min. . .	4,200	1,200	45	
Compressors and vacuum pumps:				
7.5 × 6-in. = 106 cu. ft. per min. . .	1,400	434	15	
11 × 12-in. = 350 cu. ft. per min. . .	5,400	957	55	Feather valve, belt driven.
15 × 14-in. = 695 cu. ft. per min. . .	8,500	1,520	90	
18 × 16-in. = 1030 cu. ft. per min. . .	12,500	1,891	135	
Blowers, Roots cycloidal:				
No. 2, 600–800 cu. ft. per min.	2,200	463	25	Good for 5 lb. per square inch.
No. 3, 800–1400 cu. ft. per min.	3,300	598	35	
No. 4, 1400–2200 cu. ft. per min.	4,800	764	50	
No. 5, 2200–3000 cu. ft. per min.	8,500	1,089	90	
No. 6, 3000–4600 cu. ft. per min.	14,000	1,596	150	
Elevators, continuous-bucket (Dry):				
9 × 6 × 9-in. buckets, 30-ft. centers.	3,000	405	120	Complete with woodwork.
9 × 6 × 9-in. buckets, 70-ft. centers.	5,200	715	280	
16 × 8 × 11-in. buckets, 30-ft. centers.	4,100	675	135	
16 × 8 × 11-in. buckets, 70-ft. centers.	7,250	1,175	315	
30 × 12 × 13-in. buckets, 30-ft. centers.	9,700	1,650	225	
30 × 12 × 13-in. buckets, 70-ft. centers.	18,000	2,750	525	

Elevators, belt-and-bucket (Wet):

Buckets	Spaced	Centers	Factory weight, pounds	Factory price	Labor cost of erection	Remarks
2 @ 14-in.	18-in.	60-ft.	11,800	2,364	600	All ironwork, belt and buckets, and wood housing.
2 @ 14-in.	18-in.	30-ft.	6,400	1,277	300	
1 @ 16-in.	18-in.	60-ft.	6,300	1,400	450	
1 @ 16-in.	18-in.	30-ft.	3,600	792	225	
1 @ 12-in.	16-in.	60-ft.	4,400	1,018	390	
1 @ 12-in.	16-in.	30-ft.	2,700	551	195	
1 @ 8-in.	12-in.	60-ft.	2,630	700	300	
1 @ 8-in.	12-in.	30-ft.	1,770	418	150	

Table 42. Weights, approximate (1924) prices, and labor cost for erecting the principal items of mill equipment—*Continued*

	Factory weight, pounds	Factory price	Labor cost of erection	Remarks
Tanks, wood, for water, etc.:				
8× 8-ft. = 2,625 gal. (2-in. staves)	1,332	$85	$20	Prices for Douglas fir; for redwood, add approximately 15 per cent.
10×10-ft. = 5,260 gal. (2-in. staves)	2,024	126	25	
12×12-ft. = 9,280 gal. (3-in. staves)	2,914	262	30	
16×12-ft. = 16,000 gal. (3-in. staves)	6,588	389	65	
20×16-ft. = 34,700 gal. (3-in. staves)	11,532	675	120	
30×20-ft. = 100,000 gal. (3-in. staves)	24,980	1,470	250	
40×20-ft. = 178,500 gal. (3-in. staves)	38,213	2,270	380	
Conveyors, belt:				
18-in. belt, 450-ft. centers	15,000	3,834	1,350	All ironwork, belt and rollers; also wood frames and stringers.
18-in. belt, 250-ft. centers	8,100	2,129	750	
18-in. belt, 50-ft. centers	2,420	545	150	
24-in. belt, 450-ft. centers	19,400	4,680	1800	
24-in. belt, 250-ft. centers	11,000	2,600	1000	
24-in. belt, 50-ft. centers	3,300	520	200	
30-in. belt, 450-ft. centers	24,200	6,007	2250	
30-in. belt, 250-ft. centers	14,550	3,337	1250	
30-in. belt, 50-ft. centers	3,850	667	250	
36-in. belt, 450-ft. centers	29,900	7,200	2700	
36-in. belt, 250-ft. centers	16,500	4,000	1500	
36-in. belt, 50-ft. centers	4,500	800	300	
42-in. belt, 450-ft. centers	33,600	9,787	3150	
42-in. belt, 250-ft. centers	19,800	5,437	1750	
42-in. belt, 50-ft. centers	5,750	1,087	350	
Boilers, tubular:				
60-hp., 125-lb. pressure	11,300	950	350	Full fronts, complete with stacks.
100-hp., 125-lb. pressure	22,700	1,400	670	
150-hp., 125-lb. pressure	29,200	1,800	875	
200-hp., 125-lb. pressure	35,700	2,320	1,075	
Boilers, Sterling:				
200-hp. (hand-fired)	45,000	4,750	2500	Including brick work, but not including stack or foundation.
300-hp. (hand-fired)	65,000	6,500	2750	
500-hp. (hand-fired)	90,000	9,000	3500	
750-hp. (chain-grate stoker)	160,000	16,000	6500	
1000-hp. (chain-grate stoker)	210,000	20,000	8500	
Dryers, Ruggles-Coles (class A type):				
48-in.×20-ft.	24,000	3,900	960	All steel and iron work.
60-in.×30-ft.	38,000	5,900	1520	
70-in.×35-ft.	50,000	7,900	2000	
80-in.×48-ft.	85,000	11,000	3400	
Roasters, McDougall:				
20-ft. dia. and 7 hearths	525,000	22,000	2600	Factory prices and labor erecting include both iron and brick work.
16½-ft. dia. and 7 hearths	300,000	13,000	1500	
10-ft. dia. and 7 hearths	200,000	8,000	1000	

Table 42. Weights, approximate (1924) prices, and labor cost for erecting the principal items of mill equipment—*Continued*

	Factory weight, pounds	Factory price	Labor cost of erection	Remarks
Electric motors, squirrel-cage type:				
At full load				
5-hp., 1750 r.p.m.	353	$129	$7.50	Includes auto-starter, overload relays, and no-voltage release. All below 2200 volts.
10-hp., 1160 r.p.m.	620	252	12.50	
15-hp., 1155 r.p.m.	815	292	17	
20-hp., 1160-r.p.m.	825	330	20	
30-hp., 1150 r.p.m.	1,550	399	30	
50-hp., 1155 r.p.m.	1,580	560	35	
75-hp., 865 r.p.m.	2,940	798	50	
100-hp., 860 r.p.m.	3,630	991	60	
Transformers, 44,000-volt primary current:				
3, 100-k.v.a.	21,000	6,700	700	Includes accessories.
3, 200-k.v.a.	30,000	8,100	850	
3, 300-k.v.a.	42,000	9,000	900	
3, 400-k.v.a.	49,000	10,000	1000	
3, 500-k.v.a.	52,000	10.700	1100	
Electric circuits and wiring:				
Interior lighting:				
$8 to $10 per drop	....	....	....	Labor and materials.
Interior power:				
$2.50 to $5 per hp.				
Ore-bin gates:				
18×24-in.	455	91	10	Double rack, heavy pattern.
24×32-in.	585	117	15	
30×36-in.	728	145	20	
Grizzlies, bar:				
2×6-ft. @ 1.5-in. spaces	684	67	10	Bars, ¾ and ⅜×2-in.
3×9-ft. @ 2-in. spaces	1,296	151	15	
4×12-ft. @ 2-in. spaces	2,310	268	20	
Chain blocks:				
Geared hoist				
2-ton, 9-ft. lift	184	87.50		
3-ton, 10-ft. lift	267	112.50		
5-ton, 12-ft. lift	443	175		
10-ton, 12-ft. lift	652	300		
Crawls, geared:				
2-ton	90	46	....	7, 8, or 9-in. I-Beam.
3-ton	118	54	....	8, 9, or 10-in. I-Beam.
5-ton	220	74.75	....	10, 12, or 15-in. I-Beam.
10-ton	480	132.50	....	18, 20, or 24-in. I-Beam.
Railroad track scales:				
125-ton, 50-ft.	18,000	2,500	600	
100-ton, 50-ft.	15,200	1,900	500	
Samplers:				
Brunton No. 3	1,500	250	20	Wood and ironwork.
Tipping box	540	110	25	
Geco automatic	275	200	7.50	

Table 42. Weights, approximate (1924) prices, and labor cost for erecting the principal items of mill equipment—*Continued*

	Factory weight, pounds	Factory price	Labor cost of erection	Remarks
Cranes:				
5-ton, 20-ft. span	3,600	$800	....	Hand operated.
10-ton, 20-ft. span	9,500	1,700	....	Hand operated.
25-ton, 25-ft. span	26,000	3,250	....	Hand operated.
50-ton, 30-ft. span	70,000	10,200	....	Motor operated.
75-ton, 40-ft. span	140,000	18,500	....	5-ton auxiliaries, 4 a-c. motors.
Log washers, double, steel:				
4×20-ft.	20,000	1,750	$200	Not including woodwork.
4×25-ft.	25,000	1,875	250	
Machine tools:				
Lathe 16-in.×8-ft.	2,150	869	25	Complete.
Lathe 30-in.×20-ft.	5,600	1423	50	Complete.
Lathe 13-in.×8-ft.	1,200	394	20	Complete.
Radial drill press, 48-in.	7,500	2935	80	
Upright drill press, 14-in.	1,900	440	20	
Upright drill press, 30-in.	2,200	700	25	
Planer, 24×24-in.×6-ft.	7,400	2025	80	
Shaper, 20-in.	3,150	925	35	
Shaper, 16-in.	2,200	780	25	
Milling machine, No. 2	3,600	1800	40	
Pipe-threading machine, 4-in. to 1-in.	1,368	520	20	
Pipe-threading machine, 8-in. to 2.5-in.	6,810	1317	75	
Bolt-threading machine, 2-in. to 0.5-in.	2,500	425	30	
Hacksaw, 18-in. blade	950	300	10	
Air hammer, 1200 lb.	34,000	4350	250	For belt or motor drive.
Blacksmith forge	275	61.50	5	
Air hammer and calker	12.5	50	....	
72-in. boring mill	31,000	7700	300	Motor drive.
Power punch and shear, up to 0.5-in. plate	5,100	1157	25	Motor drive.
Acetylene cutting and welding outfit	45	163	....	
Generator and truck for same	1170	555	....	
Electric drill	25	98	....	
Power rip saw, 14-in.	1350	325	20	
Jointer, 16-in.	1800	488	30	
Band saw, 36-in.	1700	360	30	
150-ton hydraulic press, 12-in. stroke	4000	514	....	
Weightometer, Merrick Scale Co.:				
16 or 18-in. belt	2300	2335	50	
24-in belt	2500	2400	50	
30-in. belt	2600	2500	50	

Shafting, diameter, inches	Pounds per foot	Per foot	Remarks
1 7/16–2 7/16	5.52–15.86	$0.39–$1.06	5 to 24-ft. lengths; cold-rolled and turned.
2 11/16–3 7/16	19.29–31.56	1.29– 2.17	
3 11/16–4 7/16	36.31–52.58	2.59– 3.89	
4 15/16–5 15/16	65.10–94.14	4.88– 7.63	

Table 42. Weights, approximate (1924) prices, and labor cost for erecting the principal items of mill equipment—*Continued*

	Per pound, cents	Remarks
Pulleys.	10–14	Solid, cast iron, key-seated.
Boxes.	22–31	

Belting, rubber	Pounds per foot, 1 in. wide	Per pound	First quality
4-ply	0.13	\$0.80–1.00	1 to 24 in. wide.
6-ply	0.18	0.90–1.05	2 to 24 in. wide.
8-ply	0.24	0.85–0.95	6 to 24 in. wide.

Couplings, diameter, inches	Pounds	Each	Remarks
$1\frac{7}{16}$–$2\frac{7}{16}$	20– 60	\$11.55–\$17.20	Flange fitted to shafts, and faced.
$2\frac{11}{16}$–$3\frac{7}{16}$	80–140	20.35– 32.70	
$3\frac{11}{16}$–$4\frac{7}{16}$	165–260	37.65– 63.80	
$4\frac{15}{16}$–$5\frac{15}{16}$	360–550	82.50–132.00	

Clutches, horsepower			
10– 25	200– 500	\$100–158	Clutch only (on pulleys). Horsepower figured at 100 r.p.m.
45– 90	650–1250	201–289	
125–275	1500–3200	345–722	

Collars, diameter, inches			
$1\frac{7}{16}$–$2\frac{7}{16}$	1.25–3.25	\$0.30–0.55	Solid
$2\frac{11}{16}$–$2\frac{7}{16}$	3.75–6.51	0.65–0.90	
$3\frac{11}{16}$–$4\frac{7}{16}$	9 –14	1.00–1.88	
$4\frac{15}{16}$–$5\frac{15}{16}$	15 –24	2.35–3.45	

	Per pound
Complete mill transmission equipment. . . .	16–30¢
Labor erecting.	3– 4¢

Lumber	Weight per M.B.M., pounds	Price per M.B.M.(*a*)	Erection cost per M.B.M.
Oregon fir, common.	3300	\$20	\$25–35
Oregon fir, mine grade.	3300	14	25–35
Redwood.	2750	85	25–35
Reclaimed lumber.	3000	7.50–10	25–35

a Lumber is usually bought at a price of f.o.b. railroad point nearest the place of consumption; freights, while variable, will average about equal to the saw-mill cost of common lumber.

It is intended that the foregoing list shall be used for estimating purposes only. To the factory price of machinery should be added enough to cover (1) freight charges from factory to the railroad point nearest the mill (approximately \$1.50 per 100 lb., or say 2¢ per ton mile, for carload, and twice as much for less than carloads), (2) the cost of hauling from railroad to millsite (say 40¢ per ton-mile for teams, or 20¢ per ton-mile for motor trucks), and (3) the cost of unloading and distributing machinery on the millsite (about \$2.50 per ton),

Excavation that was required at a few mills, and the cost per cubic yard, are given in Table 43.

Table 43. Amount and cost of excavation at typical mills

Mill	Type	Excavation, cubic yards	Cost per cubic yard	Labor wages	Character of material
Magma, Superior, Ariz.	Terraced	1800	$2.47	$2.25	Solid, hard rock.
Imlay, Imlay, Nev.	Terraced	915	0.55	3.50	Medium soft earth.
National, Mullan, Idaho	Terraced	5000	0.88	3.50	Loose rock and earth
Watters, Sheridan, Mont.	Terraced	....	1.50	......	Medium hard rock.
Winnemucca, Nev.	Flat	1155	0.75	......	
Utah Consolidated, Tooele, Utah	Terraced	6000	1.17	4.50	Medium soft earth.

Concrete work, its volume and cost, covering walls, foundations and floors at typical mills, is shown in Table 44.

Table 44. Volume and cost of concrete work at mills

Mill	Type	Total cubic yards	Cost per cubic yard						
			Sand	Rock	Cement	Lumber, etc.	Labor forms	Labor, mixing and placing	Total
Imlay, Imlay, Nev.	Terraced	166	$1.87	*a*	$2.43	.....	$1.40	$3.50	$9.20
Magma, Superior, Ariz.	Terraced	965	1.63	*a*	4.51	$1.80	1.27	2.06	11.27
National, Mullan, Idaho.	Terraced	1950	1.37	*a*	1.92	0.65	1.10	1.80	6.84
Utah Cons., Tooele, Utah	Terraced(*b*)	1469	1.33	$0.50	3.45	0.60	4.12	2.71	12.71
Utah Cons., Tooele, Utah	Terraced(*c*)	814	2.00	0.50	4.27	0.60	4.12	2.71	14.20
Armstead, Talache, Id.	Flat	528	.....	.....	.....	.....	......	......	10.40
Winnemucca, Nev.	Flat	312	.....	.....	.....	.....	......	......	9.50

a Gravel, included with sand. *b* Retaining walls and building foundations. *c* Floors and machinery foundations.

Final estimates are based on completed plans and specifications, previously described (Art. 13). The first step is to prepare an ITEMIZED LIST of all equipment and materials required; much depends on the completeness of this list; nothing should be omitted that will contribute in the least degree to the satisfactory operation of the mill. A few of the items most likely to be overlooked are given in the following list:

Windows. Screen covering. Steel balls. Ore bin, rods and washers. Building rods, bolts, drift spikes and washers. Blower air pipe. Master valves for cells. Main power cable. Main water pipe and cost of digging ditch. Main water tank. Main water pump. Lime bins and feeders (where lime is used). Cranes, hoists, trolleys, chain blocks and beams. Lighting transformers. Fire hydrants. Cable, conduits and connections. Paint and painting. Steam plant and all necessary piping and connections. Electric wiring, material and labor. Equalizing tank and float. Brick (where boiler is used). Fire and water hose. Coal for heating mill during construction. Electric-power poles. Agitator for stock tank. All gear housings. Steel chutes. Bin, launder, chute and spout linings. Steam-pipe covering. Tank for condensation water (where this will not return by gravity). Pump for same. Feed distributors. Belt fasteners. Elevator bolts and washers. Lighting cables, cords, shades and globes. Sampler equipment. Reagent feeders. Reagent storage. Walkways. Stairs. Signal systems. Cleaning up.

Underestimating the cost of a proposed job is invariably due to failure properly to survey and list all the items composing the installation and the work to be done to complete the plant to the point of actual operation. Care in this particular is really more important than precision as to each item, for in the very long list of items almost always needed, individual errors will tend to compensate.

The list should next be SEGREGATED according to the character and source of the materials: ore-dressing equipment (further subdivided as to specialties offered by particular manufacturers); motors and electric appliances; shafting, pulleys, clutches, speed reducers, etc.; belts; piping, valves and fittings; lumber; roofing and siding materials; windows; paints; structural steel-work (not usually necessary to list in detail—see Art. 13); boilers; engines. Bids are then invited from manufacturers of the respective classes, stating weights, prices, and time of delivery. Bids should be scrutinized, not only as to prices and quality, but also to insure that each one covers exactly the amount and kind of material wanted; freight charges to the mill site may affect comparison of factory prices. A convenient form for final presentation is suggested in Fig. 12.

		Coarse crusher & sampler	Concentrator	Tailing disposal	Water system	Stores, shops, etc.	Transportation facilities	Power plant	Totals	
									Cost	Weights
Mach'y & miscellaneous iron work. F. O. B. factory		$	$	$	$	$	$	$	$	lb.
Lumber and/or steel work. Other structural mat'ls.	1									
Labor erecting mach'y etc. Labor erecting bldgs. etc.	2									
Freight & unloading mach'y etc. Freight & unloading build'g mat'l	3									
Excavation Foundations etc.	4									
Electric wiring etc. Piping etc.	5									
Contingent fund @ 5% Engineering	6									
Total of items	7									

Fig. 12. **A form for final cost estimate.**

This generalized tabulation should be accompanied by individual lists relating to each separately-housed department of the plant, following the same itemizing as above, but in more detail; thus, each article of principal equipment should be listed as to weight and price; volumes of excavations and foundations should be stated, with their unit costs; lumber, quantity and unit cost; and similarly as to all other structural materials.

SECTION 24

MATHEMATICS

BY

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NOTE—Black-face numbers in parenthesis in the text refer to the bibliography.

ARITHMETIC

1. Approximate computation

Numerical engineering data are subject to errors of various kinds and should be written so that no ambiguity can exist as to the significance of the figures. In the number 0.002953 it would naturally be understood that four SIGNIFICANT FIGURES 2, 9, 5, 3 are intended. In 2,953,000, however, it should be made clear whether the three zeros are in doubt or not. Attention to significant figures is especially important in performing the four fundamental operations of arithmetic, viz., addition, subtraction, multiplication and division. (1)

Addition. A doubtful figure in any of the numbers makes the sum of the column in which it lies doubtful.

Example. In the sum retain figures for those columns only in which all figures are significant. Doubtful figures are indicated by sign x.

2.953xx
0.8942x
0.06483
3.912xx

Subtraction. Reject any column containing a doubtful figure. Ordinary method of subtraction may be replaced with advantage by "SHOP" OR "COMPLEMENTARY" METHOD based upon subtraction as the inverse of addition.

Example. Subtract the sum of 6439 and 954 from 8532. Answer is 1139.

Example illustrates the advantage of the method in working out, in one step, problems involving both addition and subtraction.

8532	$4 + 9 + \underline{9} = 22.$	Set down 9, carry 2.
6439	$7 + 3 + \underline{3} = 13.$	" " 3, " 1.
954	$10 + 4 + \underline{1} = 15.$	" " 1, " 1.
1139	$1 + 6 + \underline{1} = 8.$	" " 1.

Multiplication. It is convenient to arrange the work so that the figures of the multiplier are used from left to right, since doubtful figures are thus displayed prominently to the right of the vertical line. It is unnecessary to write the doubtful figures. The operation may be abbreviated by dropping the right-hand figures of the multiplicand one by one as multiplication is completed by successive figures of the multiplier, making proper allowance for so doing in the products. POSITION OF THE DECIMAL POINT may be determined in the usual way (as many places of decimals in product as in multiplicand and multiplier combined), or, better still, estimated from the given numbers as below.

2,953	
4,128	
11,812	
295	3
59	06
23	624
12,19x	xxx

2,953
4,128
11,812
295
59
24
12,19x

Thus to point off 29.53 × 412.8, think of result as about equal to 30 × 400 = 12,000. Product required contains five figures to left of decimal point. Hence 29.53 × 412.8 = 12,190. Again 0.00002953 × 4128 is best treated by thinking of the multiplier as about 4000, that is, 4 × 1000, the last factor shifting decimal point three places to the right. Hence 0.00002953 × 4128 = 0.1219.

Division. Work may be much abbreviated for numbers with limited significant figures by cutting off one figure of the divisor at each division, instead of adding a doubtful zero from the dividend to the remainder. This method gives rise to no loss in accuracy, and leads quickly to the desired result. Work may be further shortened by omitting the multiples of the divisor, writing down remainders only, using shop method of subtraction.

4128)	12190	(2
	8256	
413)	3934	(9
	3717	
41)	217	(5
	305	
4)	12	(3

4128	12,190	2	$2 \times 8 = 16, + 4 = 20.$	Write down 4, carry 2.
413	3,934	9	$2 \times 2 = 4, + 2 + 3 = 9.$	Write down 3.
41	217	5	$2 \times 1 = 2, + 9 = 11.$	Write down 9, carry 1.
4	12	3	$2 \times 4 = 8, + 1 + 3 = 12.$	Write down 3.

POSITION OF DECIMAL POINT is best determined by estimating the quotient in a manner analogous to that explained for multiplication. Thus in $0.001219 \div 0.4128$, think of the divisor as 4 and point off one less place in the quotient. Answer is 0.002953.

Notation using powers of 10. Engineering tables often employ powers of 10 as factors to bring significant figures into prominence and also to save space.

A power of 10 is indicated by an EXPONENT, which is an integer written at the right and above 10, as 10^2, 10^{-3}. A POSITIVE EXPONENT indicates a product in which 10 appears as a factor a number of times equal to the exponent. Thus $10^2 = 10 \times 10$. A NEGATIVE EXPONENT indicates the reciprocal of (unity divided by) the corresponding positive power. Thus $10^{-3} = 1/10^3 = 1/1000 = 0.001$. A negative power of 10 equals unity written in that place of decimals which corresponds to the numerical value of the exponent. Thus $10^{-1} = 0.1$, $10^{-4} = 0.0001$. The product, or quotient of two powers of 10 is also a power of 10. RULES: $10^a \times 10^b = 10^{a+b}$. $10^a/10^b = 10^{a-b}$. If 2, 9, 5, 3 are the significant figures in 2,953,000, write it 2953×10^3. Also $0.00002953 = 2.953 \times 10^{-5}$. Rule is to write down all significant figures and multiply by the correct power of 10 to take care of decimal point. Multiplication or division of very small or very large numbers is performed conveniently by first writing them with powers of 10 as factors.

Examples. Multiply 0.000002953 by 412,800. Work is, $2.953 \times 10^{-6} \times 4.128 \times 10^5 = 12.19 \times 10^{-1} = 1.219$. *Ans.* Divide 0.001219 by 0.4128. Work is, $12.19 \times 10^{-4} \div 4.128 \times 10^{-1} = 2.953 \times 10^{-3} = 0.002953$. *Ans.*

The above illustrations afford examples of the rule to be followed in DISCARDING SUPERFLUOUS FIGURES, *viz.*: if the first figure of a number discarded is five or more, add a unit to preceding figure.

For greater refinement, when the number rejected is known to be *exactly* five, add a unit to preceding figure when this is odd, leave it unchanged when even. Thus $7.5 \times 75 = 562.5 = 562$ to three figures. $15 \times 22.5 = 337.5 = 338$ to three figures.

Fractions. Product of two fractions is the product of the numerators divided by the product of the denominators. To divide one fraction by another, invert the divisor and multiply.

Ratio and Proportion. Ratio of one number to another is the quotient of the first by second.

Example. Ratio of 7 to 4 is $1\frac{3}{4}$.

Ratio of two numbers is often indicated by the sign : between them. Four numbers are said to be in proportion when the ratio of the first to the second equals the ratio of the third to the fourth.

Example. 2, 3, 8, 12 are in proportion. This is written $2 : 3 = 8 : 12$.

In a proportion, the first and last numbers are called EXTREMES, second and third numbers are called MEANS. A proportion remains true if any of following changes are made: (1) interchange extremes; (2) interchange means; (3) make extremes the means, and means the extremes. In any proportion, product of means equals product of extremes.

Example. Given $2 : 3 = 8 : 12$. (1) $12 : 3 = 8 : 2$; (2) $2 : 8 = 3 : 12$; (3) $3 : 2 = 12 : 8$.

Variation. One number is said to VARY DIRECTLY as another number when their ratio is constant, and to VARY INVERSELY as another number when their product is constant.

Charts to display tables of numerical data. A graphical representation of a table giving corresponding pairs of numbers is made by drawing two lines at right angles from a common point and laying off on one line a scale by which to plot one set of numbers, on the other line an appropriate scale for the second set of numbers. A point on the paper can then be marked for each pair of corresponding numbers, as in Fig. 1.

Example. Chart showing increase of population of United States.

Year	1850	1860	1870	1880	1890	1900	1910
Population in millions	23.2	31.4	38.6	50.2	62.6	76.0	92.0

Scale of years is laid off on horizontal line as marked, and scale of millions in population on vertical line, each division on the latter representing 5,000,000. A point on the vertical line 5 units upward represents population in 1850. For each decade a point is marked directly above the corresponding point for the year at a distance representing population for that year. For example, for 1890, point is 62.6 ÷ 5 = 12.5 divisions upward.

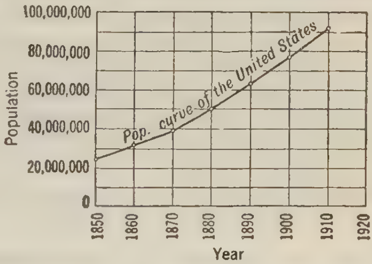

FIG. 1.

2. Errors

A definite number of significant figures is usually implied in a numerical table. The last right-hand digit may be in error, at most, half a unit.

For example, from Table 3, the value for the square root of 7 = 2.646, which means that the exact value lies between 2.6455 and 2.6465, that is, the error lies between − .0005 and + .0005.

In adding numbers taken from a table, errors may accumulate. The last right-hand digit may then be in error several units, and this fact should be noted carefully in any computation. Similar observations apply to other operations of arithmetic. In general the result obtained by computing with numerical tables of say four significant figures cannot be depended upon beyond three significant figures. Hence to reach a desired degree of accuracy it is essential to carry along one extra figure up to the last step.

The ABSOLUTE ERROR in any approximation is the actual difference between the approximate and the exact value. The limits between which the absolute error lies are usually all that is known of its magnitude.

Thus when we take $\sqrt{7} = 2.646$, absolute error lies between -0.0005 and $+0.0005$ and is numerically < 0.0005.

The RELATIVE ERROR is the ratio of absolute error to exact value. Usually in any given case an upper limit only of the relative error is known.

Thus in the preceding illustration, the relative error in $\sqrt{7} = 2.646$ is less than "one part in 2645."

Following are simple theorems on errors.

IN A SUM OR DIFFERENCE, ABSOLUTE ERROR OF THE RESULT IS NOT GREATER THAN THE SUM OF THE ABSOLUTE ERRORS OF THE GIVEN TERMS.

IN A PRODUCT OR QUOTIENT, RELATIVE ERROR OF THE RESULT IS NOT GREATER THAN THE SUM OF THE RELATIVE ERRORS OF THE TERMS (practically).

In calculating, from any formula, a value depending upon given approximate values, limits of the absolute and relative errors may often be found by aid of the Calculus. See Art. 59.

3. Interpolation in Tables

It is often convenient to consult tables for values not directly given, but which may be found by INTERPOLATION BY FIRST DIFFERENCES. This technical phrase is based upon a simple assumption and its application is merely the principle of proportion in arithmetic.

Suppose that in a Table of Square Roots, entries are for numbers of three significant figures, and the square root of a number with four significant figures is wanted to a fair degree of accuracy. Assume that the table gives $\sqrt{211} = 14.525$ and $\sqrt{212} = 14.560$ and that $\sqrt{211.4}$ is desired. Note that the square root increases .035 when the number increases 1 unit. Interpolation is accomplished on the assumption that a fractional increase in the number will cause the same fractional (proportional) increase in the square root. In this case, therefore, the increase of 0.4 (ratio 0.4 : 1) in the number should increase the square root by $.035 \times 0.4 = .014$. Hence $\sqrt{211.4} = 14.525 + .014 = 14.539$. If the tabular square roots have five significant figures, the interpolated result cannot be depended upon to the same degree, but the last figure is in doubt by one unit at most.

4. Logarithms

Explanation of Table 16. The tables are COMMON LOGARITHMS. The number 10 is called the BASE of the system, and the common logarithm of a given number is that power to which 10 must be raised to equal the number. In other words, common logarithms are powers (not necessarily integral) of 10. From the relations $10^{-3} = 0.001$, $10^{-2} = 0.01$, $10^{-1} = 0.1$, $10^0 = 1$, $10^1 = 10$, $10^2 = 100$, $10^3 = 1000$, etc., it is seen that the logarithm of $0.001 = -3$ (written $\log 0.001 = -3$), $\log 0.01 = -2$, $\log 0.1 = -1$, $\log 1 = 0$, $\log 10 = 1$, $\log 100 = 2$, $\log 1000 = 3$, etc.

To find the logarithm of a given number. When the number is greater than 1, the logarithm is positive. To find the decimal part, called MANTISSA, consider the first three significant figures of the number, paying no attention to the position of the decimal point, at present. Table 16 gives the decimal part for any three significant figures, correct to four decimal places.

Thus, the decimal part of log 3.21 = 0.5065; of log 14,000 = 0.1461, of log 98.6 = 0.9939.

If the number has four significant figures and the first figure is 1, the correct decimal part of the logarithm is given immediately in Table 16 (p. 1472), the fourth significant figure being found in the top row.

Thus for log 12.15, decimal part = 0.0846.

When the first figure is 2 or more, pp. 1474–1475 give a CORRECTION for a fourth significant figure at the right of the page. This correction is to be added to the number given in the body of the table.

Thus for log 2634, on the line for 26, the correction under 4 is 7. Hence add 7 to 4200, giving mantissa of log 2634 = 0.4207.

The part of the logarithm preceding the decimal point, called the CHARACTERISTIC, is the number which is one less than the number of digits in the given number preceding the decimal point.

The characteristic of log 26.45 is 1, of log 36,840 is 4, of log 3.862 is 0.

The complete logarithm of a number consists of the characteristic followed by the mantissa.

Thus log 13.15 = 1.1189; log 986.5 = 2.9941; log 3.216 = 0.5073.

If given number has more than four significant figures, cut off the excess figures. (Art. 1.)

When the number is less than 1, the logarithm is negative. To find the mantissa, which is taken positive, follow the same rule as before.

Thus, mantissa of log 0.2652 or log 0.0002652, is same as for 2652.

To the mantissa add the characteristic, which is now *negative* and equal to the number of zeros between the decimal point and the first significant figure of the number, increased by 1.

Thus the characteristic of log 0.2652 = − 1; of log 0.0002652 = − 4. Hence log 0.2652 = 0.4235 − 1(= − 0.5765); log 0.0002652 = 0.4235 − 4(= − 3.5765).

For convenience in calculation, it is customary not to perform the operation of subtracting the characteristic from the mantissa, but to write the characteristic preceding the mantissa, as when positive, and to place a minus sign above it.

For example, write 0.4235–4 as $\bar{4}.4235$. It must not be forgotten, however, that $\bar{4}.4235 = -4 + 0.4235$.

Some computers prefer to add and subtract 10 from a negative logarithm, so as to have a positive number in the units place. In this case − 4 + 0.4235 would be written 6.4235 − 10.

To find a number whose logarithm is given is the inverse of that just described.

Example. To find number whose logarithm = 2.6049. Mantissa is 0.6049. Find in the body of the table the next smaller mantissa. This is 0.6042, corresponding to significant figures 402. A fourth figure must give an added correction 6049 − 6042 = 7, and hence this figure may be either 6 or 7 from the right-hand column. Choosing 6, significant figures are 4026. Characteristic being 2, point off three places. Hence required number is 402.6. If logarithm is $\bar{1}.6049$, corresponding number is 0.4026.

Remark. Above process may lead to an error of at most one unit in last right-hand digit.

Fundamental properties of logarithms. The application of logarithms to computation depends upon the following relations:

(1) $\log (ab) = \log a + \log b$ (3) $\log (a)^n = n \log a$

(2) $\log (a \div b) = \log a - \log b$ (4) $\log \sqrt[n]{a} = (\log a) \div n$

Multiplication by using logarithms. Find the logarithm of each factor and add. The sum is the logarithm of the required product.

Example. Compute $x = 4128 \times 0.00002953$.

$$\begin{aligned} \log 4128 &= 3.6157 \\ \log 0.00002953 &= 0.4702 - 5 \\ \text{Adding, } \log x &= 4.0859 - 5 \text{ or } \bar{1}.0859. \quad \text{Hence } x = 0.1219. \end{aligned}$$

Products of more than two factors are worked out in same manner.

Division by using logarithms. Find the logarithm of the numerator and subtract from it the logarithm of the denominator. The remainder is the logarithm of the quotient. Shop method of subtraction is recommended.

Example. To compute $x = \dfrac{4.128}{0.02953}$

$$\begin{aligned} \log 4.128 &= 0.6157 \\ \log 0.02953 &= \bar{2}.4702 \\ \text{Subtracting, } \log x &= 2.1455 \\ x &= 139.8 \text{ } Ans. \end{aligned}$$

In computing by use of logarithms, negative mantissas must be avoided, since tables give only positive numbers as mantissas. Hence in subtracting a mantissa from a smaller one, first add and subtract 1 from the latter.

Example. Compute $x = \dfrac{0.02953}{0.4128}$

$$\begin{aligned} \log 0.02953 &= -3 + 1.4702 \\ \log 0.4128 &= -1 + 0.6157 \\ \log x &= \bar{2}.8545 \\ x &= 0.07153 \text{ } Ans. \end{aligned}$$

Instead of dividing by a number N, we may multiply by its reciprocal $1/N$. Log of reciprocal of a number is called the COLOGARITHM. Hence in using logarithms in division, we may add the cologarithm of N in place of subtracting log N. This device is useful especially where the divisor is itself a product. Some caution must be observed in using cologarithms. *Colog* N = *log* 1 − *log* N, and *log* 1 = 0. Illustrations following will show the method.

$$\begin{aligned} \log 1 &= 1.0000 - 1 \\ \log 0.2953 &= 0.4702 - 1 \\ \text{Colog } 0.2953 &= 0.5298 \end{aligned} \qquad \begin{aligned} \log 1 &= 2.0000 - 2 \\ \log 41.28 &= 1.6157 \\ \text{Colog } 41.28 &= \bar{2}.3843 \end{aligned}$$

In computation using cologarithms Tables of Cologarithms are useful.

To compute any power of a number using logarithms. Find the logarithm of the number and multiply it by the given power. The product is the logarithm of the desired result.

Example. Compute $x = (0.2953)^5$

$$\begin{aligned} \log 0.2953 &= -1 + 0.4702 \\ \text{Multiply by } & 5 \\ \log x &= -5 + 2.3510 = \bar{3}.3510 \\ x &= 0.002244 \text{ } Ans. \end{aligned}$$

To compute any root of a number using logarithms. Find the logarithm of the given number and divide it by the given index of the root. Quotient is logarithm of desired result. Care is necessary to avoid the introduction of fractional characteristics in the case of roots of numbers less than unity.

Examples. **1.** Compute $x = \sqrt[5]{0.4128}$

$$\begin{aligned} \log 0.4128 &= -1 + 0.6157 \\ \text{(Adding and subtracting 4)} &= -5 + 4.6157 \\ \log x &= \frac{-5 + 4.6157}{5} = \bar{1}.923 \\ x &= 0.8377 \text{ } Ans. \end{aligned}$$

2. Compute $x = 0.073^{0.062}$

$$\begin{aligned} \textit{Solution}, \log x &= 0.062 \log 0.073, \\ \log 0.073 &= -2 + 0.8663 = -1.1337, \\ \log x &= 0.062 \times -1.1337 \\ &= -0.0703 = -1 + 0.9297 \\ x &= 0.8506. \quad Ans. \end{aligned}$$

In computations such as are illustrated above, errors in final logarithms are introduced which may be appreciably greater than the error of the tables. The latter may be assumed to be not greater than half a unit in the fourth decimal place, although it may, by interpolation, be a whole unit at times. Hence, for example, in raising a number to fifth power, an error of five units may occur in the final logarithm and this may even lead to an error, in the number itself, of one unit in the third significant figure. Computation with

four-place tables does not ensure more than three-figure accuracy. Tables with mantissas carried out to five or more places are essential for accuracy to four or more significant figures. Such tables are available in great variety. (2)

5. Slide rule

A logarithmic scale is constructed as follows. On a line (Fig. 2) choose a convenient length OZ. Take OZ as unit of length. Mark at each point on the line the number whose logarithm is the distance of that point from O, *e.g.*, if $OA = \log a$, mark at point A the number a. If any other unit of length is more convenient, the ratio $OA : OZ$ is $\log a$. Point O is marked 1 ($\log 1 = 0$), point Z is marked 10 ($\log 10 = 1$). Integers 2, 3, etc., fall on the scale as marked. Graduations between integers will make it possible to read the number corresponding to any point.

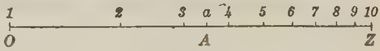

Fig. 2.

Let two such logarithmic scales of the same length be made on cardboard and placed as in Fig. 3 with O_1 on lower scale beneath A on upper scale. Let reading at A be a, at A_1 be a_1. Then reading at B is product aa_1. For $OA + O_1A_1 = OB$. But $OA = \log a$ and $O_1A_1 = \log a_1$. Hence $OB = \log a + \log a_1 = \log aa_1$ (Art. 4). Fig. 3 illustrates multiplication by adding two logarithmic scales. If A_1 comes beyond Z, place scales as in Fig. 4. Then reading at B is product aa_1 divided by 10. For $OA = OB + BA$

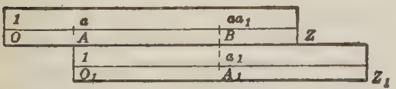

Fig. 3.

Fig. 4.

$= OB + O_1Z_1 - O_1A_1$. Hence $OB = OA + O_1A_1 - O_1Z_1 = \log a + \log a_1 - \log 10 = \log aa_1/10$. Reading at B is the product, as before, save for the position of the decimal point. By writing any number as a product of a number between 1 and 10 by a power of 10, multiplication of any two numbers can be performed as in Fig. 3 or Fig. 4, the position of the decimal point being determined by inspection. In fact, to locate the point on the scale corresponding to any number, consider its significant figures only, and disregard the position of the decimal point.

If readings a and x on the upper scale are directly over readings b and y, respectively, on the lower scale, (Fig. 5), $x/a = y/b$. (For $AX = BY$. But $AX = \log x - \log a = \log x/a$. And $BY = \log y/b$.) Write this equation in the form of a proportion $x : y = a : b$, then for any position of the scales, the readings at two points in a vertical line have a constant ratio.

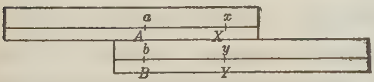

Fig. 5.

Fig. 6.

The logarithmic scale of Fig. 2 is extended to include numbers from 10 to 100 by reproducing the same scale to the right of Z, moving O over to fall on Z,

and replacing the marks 2, 3, 4, etc., by 20, 30, 40, etc. The reason for this lies in the fact that multiplication of a number by 10 increases its logarithm by 1. The scale is now a double logarithmic scale. An immediate advantage of such a scale is that two points on the scale can be marked for any number, according to whether that number is written as the product of a power of 10 by a number between 1 and 10, or between 10 and 100.

Example. 0.0265 can be marked at 2.65, or 26.5.

In the following discussion, assume a pair of double logarithmic scales as in Fig. 6.

Product of two numbers ab. Locate a on upper scale between 1 and 10 and b on lower scale between 1 and 10. Move the lower scale to the right until 1 is in line with a. Product ab is read on the upper scale over b (Fig. 7). Or, locate a on upper scale between 10 and 100, bring 100 on slide in line with a, and read product over b.

Quotient of two numbers a/b. Locate a on upper scale between 10 and 100 and b on lower scale between 1 and 10. Bring b under a. (Fig. 8.) Quotient $x = a/b$ is read on upper scale over 1 on lower scale ($x : 1 = a : b$).

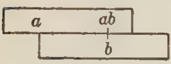

FIG. 7.

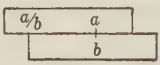

FIG. 8.

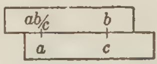

FIG. 9.

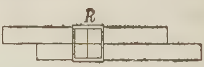

FIG. 10.

Combination of multiplication and division, ab/c. Locate c on lower scale and b on upper scale, and place b over c. (Fig. 9.) Read value $x = ab/c$ on upper scale over a. ($x : a = b : c$.) Or b and c can be marked on one scale and a on the other scale brought over c, when the value of x is read on line with b. ($x : b = a : c$).

To calculate x from

$$x = \frac{abcde\ldots}{pqrs\ldots} = \overset{(1)}{\frac{ab}{p}} \cdot \overset{(2)}{\frac{c}{q}} \cdot \overset{(3)}{\frac{d}{r}} \cdot \overset{(4)}{\frac{e}{s}} \cdot \ldots$$

(1) Place scales so that value of ab/p appears on the upper scale. A RUNNER (R, Fig. 10) is now necessary, consisting of a metal frame with upper and lower edges fitted to the slides so as to move freely back and forth, and carrying a transparent (glass) plate with a visible vertical hair line. Move the runner so that the hair line falls on the reading for ab/p on upper scale. (2) Keeping runner fixed on the upper scale, move the lower scale until q is under the hair line. Value of $\frac{ab}{p} \cdot \frac{c}{q}$ is then on upper scale over c. Move the runner until the hair line falls on this reading. (3) (4), etc.: proceed as in (2). Note that the number of factors in the numerator must be one more than in denominator. Adding the factor 1 a certain number of times may be necessary to accomplish this.

Example. Find value of x when $x = \dfrac{35 \times 144}{1.7 \times 60 \times 6.25}$.

Write $x = \dfrac{35 \times 144}{1.7} \times \dfrac{1}{60} \times \dfrac{1}{6.25}$.

Successive steps are shown in the figures (*1*), (*2*), (*3*), Fig. 11. The result 790 is read over 1 in the position given in the last figure. Decimal point is seen to be after 7. Hence $x = 7.90$ *Ans.*

The ordinary slide rule (Fig. 12) consists of a RULE $ABCD$ with a double logarithmic scale AB (called TOP SCALE) and a single logarithmic scale CD (BOTTOM SCALE) of equal length with AB, a SLIDE F having a double logarithmic scale on its upper edge identical with AB, a single logarithmic scale on the lower edge, congruent with CD, and a RUNNER. (3) The slide has also three scales on the reverse side, to which reference is made later. Multiplication and division are performed as already explained by using scale AB and the upper edge of the slide, or scale CD and the lower edge of the slide. The latter scales are best used, because graduated more finely.

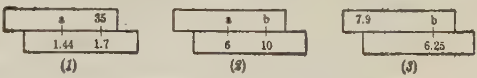

FIG. 11.

Scales are graduated (reference is to the standard 10-in. or 25-cm. rule) as follows: DOUBLE SCALE, each division between 1 and 2 is an increase of .02; between 2 and 5, an increase of .05; between 5 and 10, an increase of 0.1. SINGLE SCALE, each division between 1 and 2 is an increase of 0.01; between 2 and 4, increase of 0.02; between 4 and 10, increase of 0.05. On commercial rules 10, 20, etc., on the top scale are marked 1, 2, etc.

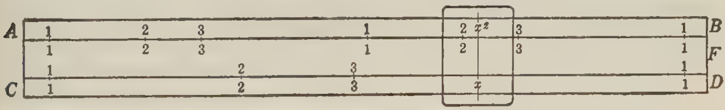

FIG. 12.

Squares and square roots. If the runner is placed in any position, the number under the hair line on AB is the square of the number under the hair line on CD (Fig. 12), proper attention being paid to the decimal point. To find the square root of any number, estimate the first figure of the answer and place the number on AB so that the number under it on CD will begin with the correct figure.

Example. For $\sqrt{18.5}$, first figure is 4. Hence place the runner on 185 on the right-hand half of AB. For $\sqrt{0.0185}$, first figure is 1, hence place the runner on 185 on the left-hand half of AB.

FIG. 13.

Cubes of numbers. To find x^3, set runner over x on CD, and multiply x^2 on AB by x (Fig. 13).

Cube root. Some slide rules are provided with a triple scale on the lower edge of CD of the same length as CD. The hair line on the runner projects over its lower edge, so that cubes of numbers on CD can be read on the triple scale, and cube roots of numbers on the latter can be read on CD.

If the slide is pulled out and turned over, its under side (Fig. 14) shows three scales of the same length as the upper scales. The top scale (marked S) is called S SCALE, the bottom scale (marked T) is called T SCALE, and the middle is called the CENTRAL SCALE. The last is simply the length of the rule subdivided into 500 equal parts, numbered from right to left.

Logarithms of numbers. If the slide is pulled out, inverted, turned end for end and run home, points on scale CD (Fig. 12) will be in line with numbers on the central scale (Fig. 14) thus giving the distances of these points

from C, measured with length $CD = 10$. Hence, numbers on the central scale give mantissas of logarithms of the numbers on the bottom scale.

S-scale (Fig. 14). This is a LOGARITHMIC SINE SCALE, constructed as follows: Length OZ equals length of scales on rule. Mark Z as 90°, and any point A as an angle x satisfying the equation $\log 100 \sin x = 2\ OA/OZ$. Using Table 18 of logarithmic sines (p. 1476) this scale is readily constructed. The point at the left end is 34′, since $\log \sin 34' = -2$. Let the slide be pulled out, inverted, and run back (Fig. 15), so that S-scale comes below scale AB on the rule. A point on the latter is marked a, satisfying $\log a = 2OA/OZ$. Hence, corresponding readings a on AB and x on S-scale satisfy $\sin x = a/100$. Hence, sines of angles between 34′ and 90° may be read off immediately. (Note that $\sin 34' = 0.01$.)

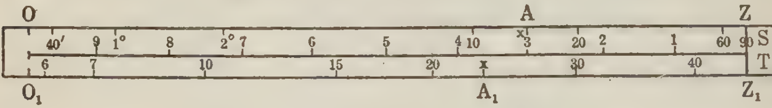

FIG. 14.

T-scale, at bottom of Fig. 14 is a LOGARITHMIC TANGENT SCALE. Point Z_1 is marked 45°, any point A_1 as the angle x for which $\log 10 \tan x = O_1A_1/O_1Z_1$. Let the slide be pulled out, inverted, and run back, as before, the T-scale coming above scale CD on the rule (Fig. 15). Readings x on T-scale and c on CD, in line, satisfy $10 \tan x = c$. Hence, tangents of angles from 5° 43′ to 45° can be read. ($\tan 5° 43' = 0.1$.) For angles greater than 45°, use the formula $\tan (90 - x) = 1/\tan x$. For angles less than 5° 43′, *sine* = *tangent* with sufficient accuracy.

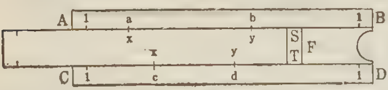

FIG. 15.

Multiplication or division using S-scale or T-scale presents no new principle. For position shown in Fig. 15, the following relations hold: $a : \sin x = b : \sin y$, and $c : \tan x = d : \tan y$. Comparison of these equations with the basic equation $x : y = a : b$ (p. 1351) shows how calculations involving sines or tangents can be made.

Assuming the slide in usual position with number scales in view, turn the rule over and pull out the slide. S- and T-scale will appear as in Fig. 16. A piece of transparent celluloid with hair line H is placed in the end of the rule over the slide as in the figure. It is easy to see that: (1) The reading on the central scale is the logarithm of the number on scale CD under 1 on slide. (2) The reading on the S-scale is the angle whose sine equals the number on the slide beneath 100 on scale AB, divided by 100. (3) The reading on the T-scale is the angle whose tangent equals the number on the slide above 10 on scale CD, divided by 10.

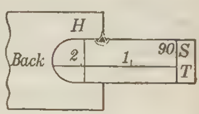

FIG. 16.

For use of the slide rule in trigonometry, see Art. 34.

ALGEBRA

6. Fundamental operations

Notation. The use of letters of the alphabet to represent numbers is the most striking difference between arithmetic and algebra. Different numbers may be represented by the same letter with accents (primes) as a', a'' (read "a-prime," "a-second"), or subscripts as a_1, a_2 (read "a-sub-one,"

"a-sub-two"). When letters occur in a product the multiplication sign ($\times$) is omitted. Thus $3a = 3 \times a$; $2xy = 2 \times x \times y$. The product of a number by itself one or more times is abbreviated by the use of an exponent. Thus $a \times a \times a = a^3$. An EXPONENT is an integer written at the right and above another number to show how many times the latter is to be taken as a factor. Exponent 1 is not expressed. Thus, write a, not a^1. (See also Art. 15.) Letters in algebra stand for negative numbers as well as positive. (4)

Addition and subtraction. In arithmetic, where numbers are always positive, a number may be subtracted from a greater number only. In algebra it is necessary to subtract numbers regardless of their relative magnitudes. To do this, arrange positive and negative numbers on a NUMBER SCALE (Fig. 17).

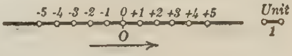

FIG. 17.

Beginning on a line at a point marked zero, lay off equal distances and mark successive points as indicated, positive numbers to the right, negative to the left. To add the positive number a to the number b begin at b on the number scale and count a spaces to the right. To subtract the positive number a from the number b, begin at b and count off a spaces to the left. Thus, $+2+3=5$; $-5+2=-3$; $-2-3=-5$.

To add a negative number is the same operation as subtracting that number taken with a positive sign. Hence, to add a negative number n to a given number, begin at the latter on the scale and count off n spaces to the left. Subtracting a negative number is the same, in effect, as adding the corresponding positive number. Rule for subtraction is usually stated: Change the sign of the subtrahend and add.

Absolute value of a number is its value without regard to sign. Absolute value of a is written $|a|$. Absolute values of 3, $-1\frac{1}{4}$, -0.51 are 3, $1\frac{1}{4}$, 0.51, respectively. ($|3| = 3$, $|-1\frac{1}{4}| = 1\frac{1}{4}$.)

Sign. To add two or more positive (or negative) numbers, find the sum of their absolute values and prefix to this sum the plus (or minus) sign.

To add a positive and a negative number, find the difference of their absolute values, and prefix to the result the sign of the one which has the greater absolute value.

The product of two numbers having like signs is a positive number and the product of two numbers having unlike signs is a negative number, *e.g.*, $+a \times +b = +ab$, $-a \times +b = -ab$, $+a \times -b = -ab$, $-a \times -b = +ab$. If one factor in a product is zero, the product itself is zero.

The quotient of two numbers having like signs is a positive number, and the quotient of two numbers having unlike signs is a negative number. Division by zero is always excluded.

A MONOMIAL or TERM is an indicated product of two or more numbers. Any factor in a product is called the COEFFICIENT of the product of the other factors, *e.g.*, in $3axy$, $3a$ is coefficient of xy. Terms alike in all respects except coefficients are called SIMILAR. Thus, a^2x, $-3a^2x$, $7a^2x$ are similar terms; also $\sqrt{2}$, $-2\sqrt{2}$, and $3\sqrt{2}$.

To add similar terms, find the algebraic sum of the numerical coefficients and prefix the result to the common part.

Example. $a^2x - 3a^2x + 7a^2x = (1 - 3 + 7)a^2x = 5a^2x$.

To subtract similar terms, change the sign of the subtrahend and add.

A POLYNOMIAL is an expression indicating algebraic addition of two or more dissimilar terms.

Addition of polynomials. Write similar terms in the same column. Find the sum of the terms in each column and add the results.

Example. Add $9ac - bc$, $7ab - 3ac$, and $-12ab + bc$.

$$\begin{array}{rrr} & 9ac & -\ bc \\ 7ab & -\ 3ac & \\ -\ 12ab & & +\ bc \\ \hline \textit{Ans.}\ -\ 5ab & +\ 6ac & \end{array}$$

Use of parentheses. In solving problems it is often necessary to inclose several terms in a parenthesis and such expressions as $(5x - 6y) - [-4x + (4z - y) - 2z]$ occur frequently. A parenthesis preceded by a plus sign may be removed without change in signs of the terms enclosed by that parenthesis. A parenthesis preceded by a minus sign may be removed provided the sign of each term enclosed by that parenthesis is changed. First rewrite the given expression removing the innermost parenthesis, and repeat until all parentheses are removed.

Example. $(5x - 6y) - [-4x + (4z - y) - 2z] = (5x - 6y) - [-4x + 4z - y - 2z] = 5x - 6y + 4x - 4z + y + 2z = 9x - 5y - 2z$. *Ans.*

One or more terms may be placed within a parenthesis preceded by a plus sign without changing the sign of any terms; or preceded by a minus sign provided the sign of each term enclosed is changed.

Example. $a + 2b - c = a + (2b - c) = a - (c - 2b)$.

Multiplication

The factors in a product may be written in any order. Product of a number by itself, a^2, a^4, etc., is called a POWER of that number. The exponent of the product of powers of the same number is the sum of exponents of factors, *e.g.*, $a^m \times a^n = a^{m+n}$.

To multiply two monomials. Obeying the rule of signs for multiplication (see above), write the product of the numerical coefficients followed by all the letters in multiplier and multiplicand, each letter having as its exponent the sum of its exponents in multiplier and multiplicand.

Example. $(3ax)(-2a^2x)(-7ax^3) = 42a^4x^5$.

To multiply a polynomial by a monomial, multiply each term of the polynomial by the monomial and write the algebraic sum of the resulting terms.

Example. Multiply $5x^2 - 2x - 4y$ by $-2xy$.

$$\begin{array}{r} 5x^2 - 2x - 4y \\ -\ 2xy \qquad\qquad \\ \hline -10x^3y + 4x^2y + 8xy^2. \end{array}\quad Ans.$$

To multiply polynomials. Multiply the multiplicand by each term of the multiplier in turn, and add the partial products.

Example. Multiply $x^2 - xy + y^2$ by $x + y$.

Before multiplication, a polynomial should be arranged according to descending powers of one of its letters, that is, so that exponents of that letter in successive terms decrease from left to right. Multiplier and multiplicand should both be arranged with respect to same letter, when possible. DEGREE OF A POLYNOMIAL in any letter equals the greatest exponent of that letter.

$$\begin{array}{lllll} x^2 & - xy & + y^2 & & \\ x & + y & & & \\ \hline x^3 & - x^2y & + xy^2 & & \\ & + x^2y & - xy^2 & + y^3 & \\ \hline x^3 & & & + y^3. & Ans. \end{array}$$

Division

Formula for quotient of powers of a number, $a^m \div a^n = a^{m-n}$ (m greater than n).

To divide one monomial by another divide the numerical coefficient of the dividend by the numerical coefficient of the divisor, following rule of signs in division (see above). Write after this quotient all letters of the dividend except those having the same exponent in divisor and dividend, giving each letter an exponent equal to the difference of exponents in dividend

and divisor. Write any letters in divisor not occurring in dividend under the preceding result as a denominator.

Examples. $\dfrac{-36a^4xb^8}{9a^3xb^7} = -4ab.$ $\dfrac{12ax^3}{-3bx^3} = -4\dfrac{a}{b}.$

To divide a polynomial by a monomial, write down the algebraic sum of the quotients of terms of the polynomial by the monomial.

Example. $\dfrac{15a^2b^2 + 9a^4b^3 - 30a^6b^4}{-3a^2b^2} = -5 - 3a^2b + 10a^4b^2.$ *Ans.*

To divide one polynomial by another. Arrange dividend and divisor according to descending powers of some common letter called the LETTER OF ARRANGEMENT. Divide the first term of the dividend by the first term of the divisor and write the result as the first term of the quotient. Multiply the entire divisor by the first term of the quotient, write the result under the dividend, and subtract. Treat the remainder as a new dividend, and proceed as before, obtaining second term of divisor. Continue until the remainder is zero, or of lower degree than the divisor in the letter of arrangement.

Example. Divide $16x + 12x^3 - 15 - 22x^2$ by $2x - 3$.

$$
\begin{array}{rrrr|ll}
12x^3 & -\,22x^2 & +\,16x & -\,15 & 2x - 3 & \text{Divisor} \\
\cline{5-5}
12x^3 & -\,18x^2 & & & 6x^2 - 2x + 5 & \text{Quotient} \\
\cline{1-2}\cline{5-5}
 & -\,4x^2 & +\,16x & -\,15 & & \\
 & -\,4x^2 & +\,6x & & & \\
\cline{2-4}
 & & 10x & -\,15 & & \\
 & & 10x & -\,15 & & \\
\cline{3-4}
\end{array}
$$

Examples in division should be checked by multiplication, using the formula: dividend = quotient *times* divisor + remainder.

Important special products. $(a + b)(x + y) = ax + ay + bx + by$; $(a + b)(a - b) = a^2 - b^2$; $(a + b)^2 = a^2 + 2ab + b^2$; $(a - b)^2 = a^2 - 2ab + b^2$; $(x + a)(x + b) = x^2 + (a + b)x + ab$; $(ax + b)(cx + d) = acx^2 + (ad + bc)x + bd$; $(a + b)(a^2 - ab + b^2) = a^3 + b^3$; $(a - b)(a^2 + ab + b^2) = a^3 - b^3$. These formulas aid in factoring a polynomial, that is, resolving it into other polynomials of which it is the product. Suggestions that aid in factoring are: (I) look for a monomial factor, and if there is one, separate polynomial into the product consisting of the greatest monomial factor and the quotient of the polynomial by it. (II) From the form of the polynomial factor determine under which of the above types it should be classed and, use the corresponding special-product formula to determine the factors.

Examples. $x^3 - 4x = x(x^2 - 4) = x(x + 2)(x - 2)$.
$9x^2 + 6x + 1 = (3x + 1)^2$. $x^2 - 5x - 6 = (x - 6)(x + 1)$.
$15a^2 + 16ab + 4b^2 = (5a + 2b)(3a + 2b)$.
$x^6 - 64y^{12} = (x^2 - 4y^4)(x^4 + 4x^2y^4 + 16y^8)$.

7. Fractions

The following formulas are important:

$$\frac{a}{b} = \frac{na}{nb} = \frac{a \div n}{b \div n}; \quad +\frac{a}{b} = -\frac{-a}{b} = -\frac{a}{-b}; \quad \frac{a}{c} + \frac{b}{c} = \frac{a + b}{c}.$$

The value of a fraction is unchanged by multiplying numerator and denominator by same number, or by changing the sign of the fraction simul-

taneously with the sign of either numerator or denominator. Sum of fractions with a common denominator equals a fraction with the same denominator and with a numerator equal to the sum of the given numerators. A fraction is in its LOWEST TERMS when no factor except 1 is common to both numerator and denominator.

To reduce a fraction to lowest terms, resolve numerator and denominator into their prime factors, and cancel the factors common to both. (Cancellation is equivalent to dividing numerator and denominator by the product of the common factors.)

Examples in reducing to lowest terms follow:

1. $\dfrac{m^2 - n^2}{m^3 - n^3} = \dfrac{\cancel{(m - n)}(m + n)}{\cancel{(m - n)}(m^2 + mn + n^2)} = \dfrac{m + n}{m^2 + mn + n^2}.$ *Ans.*

2. $\dfrac{a^4 - x^4}{a^4 + 3a^2x^2 + 2x^4} = \dfrac{\cancel{(a^2 + x^2)}(a + x)(a - x)}{\cancel{(a^2 + x^2)}(a^2 + 2x^2)} = \dfrac{a^2 - x^2}{a^2 + 2x^2}.$ *Ans.*

Addition and subtraction

Addition and subtraction of fractions with unlike denominators is accomplished by reduction to a common denominator, as in arithmetic. The LEAST COMMON DENOMINATOR (L.C.D.) of given fractions is the LEAST COMMON MULTIPLE (L.C.M.) of their denominators, namely, that expression having the least number of factors which is divisible by each denominator.

To find L.C.M., resolve each given expression into prime factors. L.C.M. is product of all different prime factors, using each the greatest number of times it occurs in any one expression.

Examples. 1. Find L.C.M. of $6a^2b$, $9ab^2c$, $3ab^3c^3$.

Solution. $6a^2b = 2 \cdot 3 \cdot a^2b$; $9ab^2c = 3^2ab^2c$; $30b^3c^3 = 2 \cdot 3 \cdot 5b^3c^3$.
L.C.M. is $2 \cdot 3^2 \cdot 5 \cdot a^2b^3c^3 = 90a^2b^3c^3$. *Ans.*

2. Find L.C.M. of $ax - ay$, $x^2 + xy$, $x^2 - y^2$.

Solution. $ax - ay = a(x - y)$; $x^2 + xy = x(x + y)$; $x^2 - y^2 = (x + y)(x - y)$.
L.C.M. is $ax(x - y)(x + y) = ax^3 - axy^2$. *Ans.*

To find the algebraic sum of two or more fractions (in their lowest terms). Reduce the given fractions to equivalent fractions having the L.C.D. Write in succession over L.C.D. the numerators of equivalent fractions, inclosing each numerator in a parenthesis preceded by the sign of the corresponding fraction. In this fraction, remove parentheses, combine like terms, and reduce the result to lowest terms, if necessary.

Example. 1. Reduce $\dfrac{4x^2 - 5}{3x^2} - \dfrac{2 - 3x}{2x} + \dfrac{3x - 7}{5x^3}.$

Solution. L.C.D. $= 30x^3$.
Hence,

$$\frac{(4x^2 - 5)10x}{3x^2 \cdot 10x} - \frac{(2 - 3x)15x^2}{2x \cdot 15x^2} + \frac{(3x - 7)6}{5x^3 \cdot 6} = \frac{10x(4x^2 - 5) - 15x^2(2 - 3x) + 6(3x - 7)}{30x^3}$$

$$= \frac{85x^3 - 30x^2 - 32x - 42}{30x^3}. \quad \textit{Ans.}$$

2. Reduce $\dfrac{2}{x^2 - 7x} - \dfrac{3}{x} + \dfrac{3}{x - 7}.$

Solution. L.C.D. $= x(x - 7)$.

Hence, $\dfrac{2}{x(x - 7)} - \dfrac{3(x - 7)}{x(x - 7)} + \dfrac{3x}{x(x - 7)} = \dfrac{2 - 3(x - 7) + 3x}{x(x - 7)} = \dfrac{23}{x^2 - 7x}.$ *Ans.*

Multiplication of fractions follows the formula $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$. Numerators and denominators must be resolved into prime factors and the common factors in the products of numerators and of denominators canceled.

Division of fractions. Invert divisor, and proceed as in multiplication.

Example. Reduce $\frac{x^2 + x - 30}{x^2 + 5x - 6} \div \frac{x^2 - 25}{x^2 - 6x + 5}$.

Solution. Factoring,

$$\frac{\cancel{(x+6)}(x-5)}{\cancel{(x+6)}(x-1)} \div \frac{\cancel{(x-5)}(x+5)}{\cancel{(x-5)}(x-1)} = \frac{(x-5)\cancel{(x-1)}}{\cancel{(x-1)}(x+5)} = \frac{x-5}{x+5}. \quad \textit{Ans.}$$

8. Equations

An equation is a statement of equality between number symbols. In algebra, equations contain certain letters, called UNKNOWNS, and it is desired to find numerical values to be assigned to these unknowns such that the equations are satisfied. This process is called SOLVING THE EQUATIONS. The following statements suggest methods for simple equations. If the same number is added to, or subtracted from, each member of an equation, the result is an equation. If each member of an equation is multiplied or divided by the same number, the result is an equation.

Example. Solve the equation $5x - 8 = 2x + 19$ for x.

Solution. Subtracting $2x$ from each member, $3x - 8 = 19$.
Adding 8 to each member, $3x = 27$.
Dividing each member by 3, $x = 9$. *Ans.*

Check. Substituting $x = 9$ in given equation, $5(9) - 8 = 2(9) + 19$, or $37 = 37$.

The first of the preceding statements is replaced by the RULE FOR TRANSPOSITION of terms from one member to the other, which is: A term may be omitted from one member of an equation, provided the same term with sign changed is written in the other member.

Equations often contain other letters beside the unknowns. In solving such equations, called LITERAL EQUATIONS, answers will usually involve the other letters. Unknowns are generally denoted by letters near the end of the alphabet, as x and y. Equations containing fractions are CLEARED OF FRACTIONS when both members are multiplied by the L.C.M. of the denominators of the fractions.

Example. Solve $ax + 2ab = 2a^2 + bx$.

Solution. Transposing

$$ax - bx = 2a^2 - 2ab$$

Factoring first member,

$$(a - b)x = 2a^2 - 2ab.$$

$$\therefore x = \frac{2a^2 - 2ab}{a - b} = 2a. \quad \textit{Ans.}$$

Check. $a \cdot 2a + 2ab = 2a^2 + b \cdot 2a,$
$2a^2 + 2ab = 2a^2 + 2ab$

A LINEAR EQUATION (OR EQUATION OF FIRST DEGREE) is one in which the unknown does not appear in any denominator, and its first power only is involved. A QUADRATIC EQUATION (OR EQUATION OF SECOND DEGREE) is one which, when cleared of fractions, involves the square of the unknown but no higher power.

Many quadratic equations can be solved immediately by factoring. Transpose all terms to the first member. Factor this, set each factor containing the unknown equal to zero, and solve the resulting equations.

Algebraic methods are widely used for solving problems expressible in form of equations.

Example. An automobile makes a run of 120 miles. The driver on the return route increases the speed by 5 mi. per hr., and reduces the time by 4 hr. Find speed going and returning.

Solution. Let x = speed (mi. per hr.) going, then $x + 5$ = speed (mi. per hr.) returning. Also $\frac{120}{x}$ = time (hr.) going, and $\frac{120}{x+5}$ = time (hr.) returning. $\therefore \frac{120}{x} = \frac{120}{x+5} + 4$.

L.C.M. of denominators is $x(x + 5)$. Multiply both members by this

$$120(x + 5) = 120x + 4x(x + 5).$$

Transposing and reducing, $x^2 + 5x - 150 = 0$. Factoring $(x + 15)(x - 10) = 0$. $\therefore x + 15 = 0$, or $x = -15$. $x - 10 = 0$, or $x = 10$.

Disregarding the negative answer, speed going is 10 mi. per hr.; speed returning, 15 mi. per hr. *Ans.*

9. Simultaneous linear equations

System of two equations with two unknowns. The typical form to which such a system can be reduced is $a_1x + b_1y = c_1$, $a_2x + b_2y = c_2$. The system may be solved by (a) ADDITION and SUBTRACTION, or by (b) SUBSTITUTION. (For determinants, see Art. 14.) In either case, the principle used is to obtain from the given equations one equation containing only one of the unknowns (ELIMINATION of the other unknown).

Equations of a system are INCOMPATIBLE when there is no common solution; DEPENDENT when there is an indeterminate number of solutions. Thus $x + 2y = 4$, and $x + 2y = 8$ are obviously incompatible, while $x + 2y = 4$, and $2x + 4y = 8$ are dependent, the second arising from the first by using the multiplier 2.

Example. Solve the simultaneous equations: (1) $5x - 3y = -1$, (2) $x + 2y = 5$.

(a) *Solution by addition and subtraction*: Eliminate x by multiplying members of (2) by 5:

$$\begin{array}{ll} (1) & 5x - 3y = -1 \\ (3) & \underline{5x + 10y = 25} \end{array}$$

Subtract: $-13y = -26$. $\therefore y = 2$.

Substitute in (2): $x + 4 = 5$. $\therefore x = 1$.

(b) *Solution by substitution*: Solve (2) for x, giving $x = 5 - 2y$. Substitute in (1): $5(5 - 2y) - 3y = 1$. $\therefore -13y = -26$. $y = 2$.

System of three equations with three unknowns.

Example. Solve the simultaneous system

(1) $4x - 2y + z = 9$, (2) $3x + y + 2z = 13$, (3) $2x + 3y - 3z = -2$.

Solution. Eliminate y from (1) and (2), and (2) and (3).

$$\begin{array}{llll} (1) & 4x - 2y + z = 9. & (3) & 2x + 3y - 3z = -2 \\ (2)\ \textit{times}\ 2 & \underline{6x + 2y + 4z = 26} & (2)\ \textit{times}\ 3 & \underline{9x + 3y + 6z = 39} \\ (4)\ \text{Add} & 10x + 5z = 35 & (5)\ \text{Subtract,} & -7x - 9z = -41 \end{array}$$

Solving (4) and (5) for x and z, $x = 2$, $z = 3$. Substituting in (1), $y = 1$.

Check. Substitute values found in (2) and (3).

A system of any number of linear equations in an equal number of unknowns may be solved in a similar manner by eliminating one variable from the system, thus reducing number of equations and number of variables by one, then another variable from the new system, etc. As in the case of two variables, incompatible or dependent equations may be contained in the given system or reduced system.

10. Quadratic equations

Typical form for quadratic equations is $ax^2 + bx + c = 0$. The equation has two solutions or ROOTS, x_1 and x_2.

Solution by formula.

$$x_1 = (-b + \sqrt{b^2 - 4ac})/2a, \; x_2 = (-b - \sqrt{b^2 - 4ac})/2a.$$

Sum of roots, $x_1 + x_2 = -b/a$. **Product of roots,** $x_1x_2 = c/a$.

Solution of a quadratic equation may lead to roots which are IMAGINARY NUMBERS, that is, numbers involving the square root of a negative number, *e.g.*, $3 + \sqrt{-11}$. Numbers not imaginary are called REAL NUMBERS. The equation has EQUAL ROOTS when $b^2 - 4ac = 0$; IMAGINARY ROOTS when $b^2 - 4ac < 0$. (See p. 1364.)

Solution by factoring. See Art. 6. A formula sometimes useful in factoring is

$$ax^2 + bx + c = a\left(x + \frac{b - \sqrt{b^2 - 4ac}}{2a}\right)\left(x + \frac{b + \sqrt{b^2 - 4ac}}{2a}\right).$$

Solution by completing the square. Transpose terms containing x to first member and other terms to second member. Divide both members by the coefficient of x^2. Add to both members the square of half the coefficient of x, thereby making the first member a perfect square. Rewrite the equation, expressing the first member as a square, and reduce the second member to its simplest form. Extract the square root of both members, writing the sign $\pm$ (plus and minus) before the square root of the second member, thus obtaining two linear equations.

Example. Solve $2x^2 - 9x + 4 = 0$. Successive steps give the equations, $2x^2 - 9x = -4$; $x^2 - \frac{9}{2}x = -2$; $x^2 - \frac{9}{2}x + (\frac{9}{4})^2 = -2 + (\frac{9}{4})^2$; $(x - \frac{9}{4})^2 = \frac{49}{16}$; $\therefore x - \frac{9}{4} = \pm \frac{7}{4}$.
Hence, $x = \frac{9}{4} + \frac{7}{4} = 4$, and $x = \frac{9}{4} - \frac{7}{4} = \frac{1}{2}$. *Ans.*
Check by substituting $x = 4$ and $x = \frac{1}{2}$ in the given equation.

System involving a linear and a quadratic equation, two unknowns. A quadratic equation in two unknowns contains one or more terms of the second degree (such as x^2, xy, y^2), but no term of higher degree in the unknowns. (The DEGREE OF A TERM IN x AND y is the sum of the exponents of these letters.)

To solve a system of the kind named, solve the linear equation for one unknown in terms of the other, substitute this value in the quadratic equation, and solve the resulting quadratic equation in one unknown. Substitute each root of this equation in the linear equation to find the corresponding value of the other unknown.

Example. Solve the system (1) $x^2 + 3xy = 25$, (2) $2x + y = 10$. From (2), $y = 10 - 2x$. Substituting in (1), $x^2 + 3x(10 - 2x) = 25$; reducing, $x^2 - 6x + 5 = 0$. Solving, $x = 1$ and 5. Substitute in (2): $y = 8$ and 0. Check. Substitute in (1) corresponding pairs of solutions $x = 1$, $y = 8$, and $x = 5$, $y = 0$.

11. Graphs of equations

Definitions. Draw two lines at right angles to each other, as $X'OX$, called X-AXIS, and $Y'OY$, called Y-AXIS, as in Fig. 18. Construct a NUMBER SCALE (Art. 6) on each line, point O being marked zero on both. Length of a division on the scales, called UNIT OF LENGTH, is (usually) the same on both axes. From any point (not on either axis) draw lines parallel to each axis, terminating in an axis. Distances parallel to X-axis are called ABSCISSAS, and are positive when to the right of Y-axis, negative on the left. Distances parallel to Y-axis and called ORDINATES, and are positive when above the X-axis, negative when below. For any point, the two distances are called CO-ORDINATES of the point, and their lengths are written in parenthesis, as (3, −4), the abscissa first. Axes divide the plane into four QUADRANTS, as marked in the figure. Signs of co-ordinates for points in different quadrants

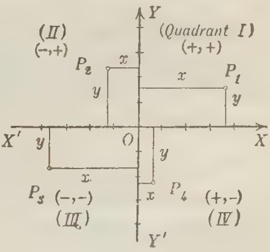

FIG. 18.

are indicated in parentheses. Every point (including those on axes) corresponds to a pair of real numbers, namely, its co-ordinates. Every pair of real numbers will determine a point, of which the given numbers are the co-ordinates. The point O, whose co-ordinates are (0, 0) is called the ORIGIN. Locating points with given co-ordinates is called PLOTTING POINTS. Axes are called AXES OF CO-ORDINATES, X-axis is the AXIS OF ABSCISSAS, and Y-axis the AXIS OF ORDINATES. (**5**)

Graph of linear equation in two unknowns. An indefinite number of corresponding pairs of values of x and y can be found to satisfy a linear equation in these unknowns. It can be shown that all such points lie on a straight line (See Art. 46). This line is called the GRAPH OF THE EQUATION. The graph of a linear equation in one unknown, as $x = 3$, is a straight line parallel to the Y-axis, 3 units to the right. Graph of $y = -1$ is straight line parallel to X-axis, 1 unit below it.

Example. Draw graph of $2x - 3y + 6 = 0$.
Solution. Solve for y, $y = \frac{2}{3}x + 2$. Assume values for x, calculate corresponding values of y, arranging in table. Plot points, and draw line through them (Fig. 19).

x	y	x	y
0	2	0	2
1	$2\frac{2}{3}$	-1	$1\frac{1}{3}$
2	$3\frac{1}{3}$	-2	$\frac{2}{3}$
3	4	-3	0
etc.	etc.	etc.	etc.

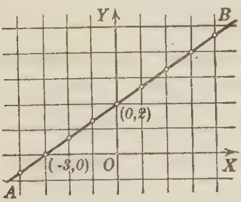

FIG. 19.

Graphical solution of system of two linear equations, two unknowns. Plot straight lines which are graphs of the given equations. Co-ordinates of the point of intersection are numbers satisfying both equations, hence the solution.

Graph of a quadratic equation. Given a quadratic equation in one unknown with the second member zero, as $ax^2 + bx + c = 0$, all points whose co-ordinates satisfy $y = ax^2 + bx + c$ lie on a curve (a parabola, see Art. 49), called the GRAPH OF QUADRATIC EQUATION. Real roots of a quadratic equation are abscissas of points at which the graph meets the X-axis. ($y = 0$ for these values of x.) Graph of $ax^2 + bx + c = 0$ will cross, touch, or not meet the X-axis according as the roots are real and unequal, equal, or imaginary.

Fig. 20 shows graph of $x^2 - 2x - 3 = 0$. Roots are 3, − 1.

Graphical solution of a quadratic equation. Write the equation in the form $x^2 = ax + b$. Plot the parabola $y = x^2$, and the straight line $y = ax + b$. Abscissas of points of intersection are the required roots.

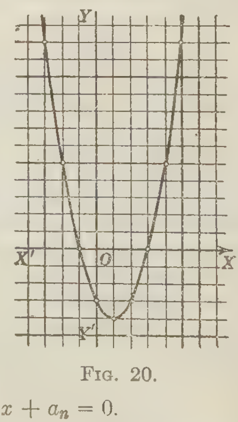

FIG. 20.

12. Graph of numerical equation of higher degree (than 2) in one unknown

Location of real roots. Typical form of equation of degree n is

$$(A) \qquad a_0x^n + a_1x^{n-1} + a_2x^{n-1} + \dots . a_{n-1}x + a_n = 0.$$

(Reference is made below to this equation as "equation (A)".) It is assumed hereafter that all terms in an equation have been transposed to the first

member. (5) The curve passing through all points whose co-ordinates satisfy the equation obtained by setting y equal to the first member is called the graph of the equation. The abscissa of any point of intersection of the graph and the X-axis is a real root of the equation ($y = 0$). Plotting the graph will often show the existence of one or more real roots, either integral, or lying between successive integers. For a short method of calculating ordinates, see Synthetic division, Art. 13.

Art. 13. Theorems on equations of degree n

Remainder theorem. When a polynomial in x is divided by $x - (r)$, the remainder obtained is the value of the polynomial resulting when r is substituted for x.

Example. When $3x^3 + 4x^2 - 6x + 5$ is actually divided by $x - 2$, the remainder is 33. For $x = 2$, $3.2^3 + 4.2^2 - 6.2 + 5 = 24 + 16 - 12 + 5 = 33 =$ remainder.

Linear factors and roots. A number r is a root of an equation with all terms transposed to the first member when and only when that member is exactly divisible by x minus the number r, *i.e.*, by $(x - r)$. Roots of numerical equations of degree greater than two may often be found by trial. The remainder theorem states, for example, that trial of 3 as a root of $3x^3 + 6x^2 - 8x + 4 = 0$ may be made by dividing the first member by $x - 3$, instead of substituting $x = 3$. Application of the theorem is made easy because of an abbreviated method of dividing a polynomial by a linear factor of the form $x - (r)$. The method is called SYNTHETIC DIVISION. To divide a polynomial in x by a linear factor of the form $x - (r)$, write down the successive coefficients a_0, a_1, a_2, etc., of descending powers of the polynomial in a horizontal line from left to right. Bring down the first coefficient a_0. Multiply a_0 by r and add the product to the second coefficient a_1. Multiply the sum $(a_0r + a_1)$ so obtained by r, and add to the third coefficient a_2. Continuing this process, the last sum will be the remainder, and the preceding sums from left to right the successive coefficients of descending powers of the quotient.

Explanation. The number $2(=r)$ is bracketed to the right of the line of coefficients. The first coefficient 3 is written down below the original ones. Multiply it by 2 and add the product $(3 \times 2 = 6)$ to the second coefficient 4. Multiply sum $(4 + 6 = 10)$ by 2, and add the product $(10 \times 2 = 20)$ to the third coefficient -6. Continuing, the last sum 33 is the remainder, and the preceding sums 3, 10, and 14, are successive coefficients of the quotient $3x^2 + 10x + 14$.

Example. Divide $3x^3 + 4x^2 - 6x + 5$ by $x - 2$.

Solution

$$\begin{array}{l} 3 + 4 - 6 + 5 \,\underline{|\,2} \\ + 6 + 20 + 28 \\ \hline \underbrace{3 + 10 + 14} + 33 = \text{Remainder} \end{array}$$

Coefficients of quotient, $3x^2 + 10x + 14$.

Note. If powers of x are missing in the given polynomial, their places must be supplied to zero coefficients. Thus to divide $x^3 + 8$ by $x + 2$, write $x^3 + 8 = x^3 + 0 \cdot x^2 + 0 \cdot x + 8$, and $x + 2 = x - (-2)$. Hence the work is

$$\begin{array}{l} 1 \quad\; 0 \quad\; 0 + 8 \,\underline{|\,-2} \\ - 2 + 4 - 8 \\ \hline 1 - 2 + 4 \quad 0 \end{array}$$

$\therefore x^3 + 8 = (x^2 - 2x + 4)(x + 2)$.

Synthetic division should be used to calculate the ordinates of a graph.

Example above shows that the graph of $3x^3 + 4x^2 - 6x + 5 = 0$ passes through (2, 33) the value 33 being found by synthetic division.

Number of linear factors and roots. An equation of degree n has precisely n linear factors of the form $x - r$. Since each factor corresponds to a root, an equation of degree n has n roots. A linear factor may be repeated (twice,

three times, etc.), in which case the equation has a MULTIPLE ROOT (DOUBLE ROOT, TRIPLE ROOT, etc.). If $r_1, r_2, \ldots, r_n$ are roots of the equation (A), (page 1362) it may be written

$$(B) \qquad a_0(x - r_1)(x - r_2) \ldots (x - r_n) = 0.$$

Relations between coefficients and roots. In an equation in which the coefficient of the highest power is unity, the coefficient of the second term with sign changed equals the sum of all the roots; the coefficient of the third term equals the sum of all products formed from two of the roots at a time; the coefficient of the fourth term with sign changed equals the sum of all the products formed from three of the roots at a time; etc. When the degree is even, the last coefficient equals the product of all roots; when odd, the last coefficient, with sign changed, equals the product of all roots.

Imaginary roots. If the imaginary number $a + b\sqrt{-1}$ (a and b being real numbers) is a root of an equation with real coefficients, $a - b\sqrt{-1}$ is a root also. Imaginary roots occur only in conjugate pairs in equations with real coefficients.

The imaginary numbers $a + b\sqrt{-1}$, $a - b\sqrt{-1}$ are called CONJUGATE IMAGINARIES, and differ only in sign of the imaginary part. The corresponding factors are $x - (a + b\sqrt{-1})$, and $x - (a - b\sqrt{-1})$. Their product is $x^2 - 2ax + a^2 + b^2$, and this product is a REAL QUADRATIC FACTOR of the equation. Number $a + b\sqrt{-1}$ ($a \neq 0$) is called, more exactly, a COMPLEX NUMBER (Art. 21).

Integral roots. Every integral root of an equation with integral coefficients is a factor of the last term. By this theorem all possible integral roots are found in advance by factoring the last term.

Solution of numerical equation of higher degree by trial. Clear the equation of fractional coefficients. If one integral root can be found, a linear factor is known, and the quotient is quickly found by synthetic division. The remaining roots are roots of the equation obtained by setting the quotient equal to zero, an equation of degree less by one than original. Factor last term. Since $+1$ and -1 are always factors, each should be tried for a root (by inspection). If either is a root, remove corresponding factor $(x - 1)$ or $(x + 1)$. Other factors of last term should be tried by Synthetic division. When enough factors have been removed to reduce the remaining quotient to a quadratic, this may be solved as usual.

Example. Solve $x^4 - x^3 - 9x^2 + 11x + 6 = 0$ by trial. Try $+1$ and -1. Substituting these values in turn, neither is a root. Other factors of 6 are ± 2, ± 3, ± 6. Trying $+2$, it turns out to be a root.

The equation now may be written

$$(x - 2)(x^3 + x^2 - 7x - 3) = 0.$$

The second factor shows that only $+3$ or -3 should be tried. Try -3. It is a root. Equation now is $(x-2)(x+3)(x^2-2x-1) = 0$. Solving for roots of quadratic factor, they are found to be $1 \pm \sqrt{2}$. Hence the roots of the given equation are $2, -3, 1+\sqrt{2}, 1-\sqrt{2}$.

$$\begin{array}{rrrrrl} 1 & -1 & -9 & +11 & +6 & \lfloor 2 \\ & +2 & +2 & -14 & -6 & \\ \hline 1 & +1 & -7 & -3 & 0 & \end{array}$$

$$\begin{array}{rrrrl} 1 & +1 & -7 & -3 & \lfloor -3 \\ & -3 & +6 & +3 & \\ \hline 1 & -2 & -1 & 0 & \end{array}$$

Fractional roots. Equations of degree higher than two with no integral roots cannot be solved by the method just explained. Such equations may, however, have fractional roots. When the coefficient of the highest power is not 1, divide by it. The remaining coefficients may then be reduced to integers as follows. After dividing the equation by the coefficient of the highest power, write in any missing powers of x with zero coefficients. Insert factors $m, m^2, m^3 \ldots$ in second, third, fourth, $\ldots$ terms. Choose for m

the smallest integer which will reduce all coefficients to integers. Roots of this transformed equation will be m-times the roots of the original equation. This transformation is called MULTIPLYING THE ROOTS BY m. Find integral roots of transformed equation by trial and divide each by the value of m. Quotients are roots of original equation.

Example. Solve $36x^4 - 55x^2 - 35x - 6 = 0$. Divide by 36, and write $x^4 + 0 \cdot x^3 - \frac{55x}{36} - \frac{35x}{36} - \frac{1}{6} = 0$. Putting in powers of m, $x^4 + 0 \cdot mx^3 - \frac{55}{36}m^2x^2 - \frac{35}{36}m^3x - \frac{1}{6}m^4 = 0$. Choose $m = 6$, giving $x^4 + 0 \cdot x^3 - 55x^2 - 210x - 216 = 0$. Roots of this equation are $-2, -3, -4, 9$.

Hence roots of original equation are

$$-\tfrac{1}{3}, \quad -\tfrac{1}{2}, \quad -\tfrac{2}{3}, \quad \tfrac{3}{2}. \quad Ans.$$

All possible fractional roots may be found in this way because of the theorem: An equation in which the coefficient of the highest power is unity, and other coefficients integral, cannot have a fractional root.

Changing signs of roots. If the signs of alternate terms ($m = -1$) are changed, the transformed and original equations have roots differing only in sign.

Incommensurable roots

Real roots that are neither integral nor fractional may be computed to any number of decimal places by various methods. The existence of real roots is indicated by the LOCATION THEOREM: Let two real numbers a and b be substituted for x in the first member of the equation; if the resulting numerical values differ in sign, an odd number of roots will lie between $x = a$ and $x = b$. The graph of the equation must join points on opposite sides of the x-axis, hence must cross this axis an odd number of times. (See Art. 11.) When a and b are successive integers, only one root will, as a rule, lie between them.

Example. In equation $x^3 - 6x^2 + 3x + 5 = 0$, putting y = the first member, corresponding values of x and y are as shown in table. (Values of y are found by synthetic division.) One root lies between -1 and 0, one between 1 and 2, and one between 5 and 6. These roots are incommensurable, since there are no fractional nor integral roots.

x	y
-1	-5
0	$+5$
$+1$	$+3$
$+2$	-5
$+5$	-5
$+6$	23

Horner's method. Assume a positive root located between $x = h$ and $x = h + 1$, where h is an integer not zero. Substitute $x = y + h$ in the equation. Roots of this new equation in y will be roots of the original equation diminished by h. (Since $y = x - h$.) Hence the equation in y must have a root between 0 and 1. The first significant figure of this root (in first place of decimals) will be the second figure of the root of the original equation. If this root of the equation in y is located between $y = k$ and $y = k + 0.1$ (k being an integer in the tenth's place), put $y = z + k$. The equation in z will have a root between 0 and 0.1, and its first significant figure in the hundredth's place will be

Example. Diminish the roots of $x^3 - 6x^2 + 3x + 5 = 0$ by 1.

$$\begin{array}{l}
1 - 6 + 3 + 5 \;\underline{|\,1} \\
\underline{\quad + 1 - 5 - 2} \\
1 - 5 - 2(+3) = \text{1st remainder} \\
\underline{\quad + 1 - 4} \\
1 - 4(-6) = \text{2nd remainder} \\
\underline{\quad + 1} \\
1(-3) = \text{3rd remainder}
\end{array}$$

The transformed equation is $x^3 - 3x^2 - 6x + 3 = 0$. (The example shows compact arrangement of the work by synthetic division.)

the third figure of the root sought of the equation in x. This outline of HORNER'S METHOD FOR POSITIVE ROOTS indicates that a suite of equations (A), (B), (C), . . . is obtained by transformations which diminish the roots by a given number.

To diminish the roots of an equation by any given number h. Divide the given equation by $x - h$. The remainder is the last coefficient in the transformed equation. Divide the quotient obtained by $x - h$. The remainder is the coefficient of the next to last term in the transformed equation. Continue this process, the last remainder being the coefficient of the second term in the new equation. The coefficients of the highest powers are the same in the original and transformed equations.

Negative roots. Write down an equation with roots differing only in sign from those of the given equation (p. 1365), and calculate the positive roots of this equation. In locating roots greater than 10 and less than 100, it suffices to use the intervals 10, 20, 30, etc. To avoid diminishing by a number greater than 10, multiply roots by 0.1 (p. 1364). In the transformed equation, the root desired will lie between successive integers. For a root greater than 100, multiply roots by 0.01. In reduction by a figure in the tenth's place, decimals can be avoided by first multiplying roots by 10. Similarly for a figure in any succeeding place. Diminishing by a figure in unit's place establishes the sign of the first remainder (constant term), which must remain unchanged in the succeeding equations. The correct figure in a decimal place (not tenth's place) may usually be found by rejecting all but the last two terms of the equation, the assumption being that the value of the terms rejected will not affect the result. Caution must be observed in doing this, however. At a certain stage more than one figure can be found by dividing the last by the preceding coefficient (with sign changed).

Example. Calculate the root of $x^3 - 6x^2 + 3x + 5 = 0$ lying between 1 and 2. *Solution.* Diminish the roots by 1. New equation is $x^3 - 3x^2 - 6x + 3 = 0$. This equation has a root between 0.4 and 0.5. Diminish the roots by 0.4.

$$\begin{array}{l}
1 - 3.0 - 6.00 + 3.000 \;\underline{|\,0.4}\\
\underline{\quad + 0.4 - 1.04 - 2.816}\\
1 - 2.6 - 7.04(+ 0.184)\\
\underline{\quad + 0.4 - 0.88}\\
1 - 2.2(- 7.92)\\
\underline{\quad + 0.4}\\
1(- 1.8)
\end{array}$$

The transformed equation is $x^3 - 1.8x^2 - 7.92x + 0.184 = 0$. It has a root between 0 and 0.1. Neglecting the two first terms, $- 7.92x + 0.184 = 0$, $x = 0.02$. Diminish by 0.02. Transformed equation is $x^3 - 1.74x^2 - 7.9908x + 0.024888 = 0$. The root of this equation between 0 and 0.01 is $x = \dfrac{0.024888}{7.9908} = 0.003$. Hence root is 1.423 to three places.

The last division may be carried farther. $x = 0.024888 \div 7.9908 = 0.00311$. Value of rejected terms $(x^3 - 1.74x^2)$ for x between 0.003 and 0.004 lies between $- 0.000015$ and $- 0.000028$, therefore three figures may be found by division. The root is 1.42311 to 5 places.

Number of real roots. When two consecutive coefficients in an equation have like signs a PERMANENCE OF SIGN is said to occur, if unlike signs, a VARIATION OF SIGN occurs. DESCARTES' RULE OF SIGNS states that the number of positive roots cannot exceed the number of variations of sign, nor can the number of negative roots exceed the number of permanences of sign. Existence of imaginary roots may often be established by this rule.

Example. In $x^3 + 5x + 7 = 0$, no variation of sign occurs, therefore there is no positive root. Writing in x^2 with coefficient $- 0$, signs are $+ - + +$, only one permanence,

hence not more than one negative root. Equation has one real root which is negative and a pair of conjugate imaginary roots.

Sturm's theorem gives a method of determining the number of real roots between two numbers $x = a$, $x = b$. (**5**)

Solution of cubic equation, $x^3 + 3Hx + G = 0$.

Graphical solution. Plot $y = x^3$, and $y = -3Hx - G$. Abscissas of points of intersection are roots.

Formula. Put $J = G^2 + 4H^3$.

Case I. J positive. One root only is real, $x = \sqrt[3]{-\frac{1}{2}G + \sqrt{J}} + \sqrt[3]{-\frac{1}{2}G - \sqrt{J}}$.

Case II. $J = 0$. All roots are real and two equal $x_1 = 2\sqrt[3]{-\frac{1}{2}G} = x_2$.

Case III. J negative. Roots all real and distinct. Determine the angle t between 0° and 180° for which $\cos t = -G/2H\sqrt{-H}$. Roots are $x_1 = 2\sqrt{-H}\cos \frac{1}{3}t$, $x_2 = 2\sqrt{-H}\cos(\frac{1}{3}t + 120°)$, $x_3 = 2\sqrt{-H}\cos(\frac{1}{3}t + 240°)$.

To eliminate x^2 from $ax^3 + bx^2 + cx + d = 0$, put $z = ax + \frac{1}{3}b$, or $x = (3z - b)/3a$.

Graphical solution of any equation. Transpose such terms to second member as may be desirable, plot y = first member, and y = second member. Abscissas of points of intersection of graphs are roots.

14. Determinants

Formulas for solving linear systems can be written down in compact form by use of determinants, which are arrangements of numbers in the form of squares. (**5**) Examples and definitions are as follows:

Second order. $$\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1b_2 - a_2b_1.$$

Third order. $$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - a_2 \begin{vmatrix} b_1 & c_1 \\ b_3 & c_3 \end{vmatrix} + a_3 \begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}.$$

$$= a_1b_2c_3 - a_1b_3c_2 - a_2b_1c_3 + a_2b_3c_1 + a_3b_1c_2 - a_3b_2c_1.$$

Fourth order. $$\begin{vmatrix} a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \\ a_3 & b_3 & c_3 & d_3 \\ a_4 & b_4 & c_4 & d_4 \end{vmatrix} = a_1 \begin{vmatrix} b_2 & c_2 & d_2 \\ b_3 & c_3 & d_3 \\ b_4 & c_4 & d_4 \end{vmatrix} - a_2 \begin{vmatrix} b_1 & c_1 & d_1 \\ b_3 & c_3 & d_3 \\ b_4 & c_4 & d_4 \end{vmatrix}$$

$$+ a_3 \begin{vmatrix} b_1 & c_1 & d_1 \\ b_2 & c_2 & d_2 \\ b_4 & c_4 & d_4 \end{vmatrix} - a_4 \begin{vmatrix} b_1 & c_1 & d_1 \\ b_2 & c_2 & d_2 \\ b_3 & c_3 & d_3 \end{vmatrix}.$$

The numbers are called ELEMENTS. If the row and column in which an element lies are erased, the determinant remaining is called the CORRESPONDING MINOR of that element. The value of a determinant is expressed as a sum of products of successive elements of a row (or column) by corresponding minors with alternating signs, as shown above for the elements of the first column.

Solution of linear systems

Two unknowns. $a_1x + b_1y = c_1$, $a_2x + b_2y = c_2$.

$$x = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix} \div \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}, \quad y = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix} \div \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}.$$

Three unknowns. $a_1x + b_1y + c_1z = d_1$, $a_2x + b_2y + c_2z = d_2$, $a_3x + b_3y + c_3z = d_3$.

$$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \neq 0. \quad x = \frac{\begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}}{D}, \quad y = \frac{\begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}}{D}, \quad z = \frac{\begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}}{D}.$$

Similar formulas hold for any number of unknowns.

Properties of determinants. (1) Interchanging the corresponding elements of two columns (or rows) changes the sign of the determinant. (2) If corresponding elements of two rows (or columns) are identical, the determinant is zero. (3) Columns may be changed to rows and rows to columns. (4) If all the elements of a column are multiplied by a number m, the value of the determinant is multiplied by m. (5) The value of a determinant is unchanged, if the elements of a column are multiplied by m and added to corresponding elements of any other column.

15. Logarithms

Exponents. For any numbers m, n, the following formulas hold:—$a^m \times a^n = a^{m+n}$; $a^m/a^n = a^{m-n}$; $(a^m)^n = a^{mn}$; $a^0 = 1$; $1/a^n = a^{-n}$; $a^m b^m = (ab)^m$; $a^{\frac{1}{r}} = \sqrt[r]{a}$; $\sqrt[r]{a} \times \sqrt[r]{b} = \sqrt[r]{ab}$ (r a positive integer).

Radicals. A radical is an indicated root of an algebraic or arithmetic expression. Operations with radicals are performed by changing to exponents. $\sqrt[r]{A} = A^{1/r}$.

Example. $\sqrt{2} \times \sqrt[3]{3} = 2^{1/2} \times 3^{1/3} = 2^{3/6} \times 3^{2/6} = (2^3)^{1/6} \times (3^2)^{1/6} = 8^{1/6} \times 9^{1/6} = (72)^{1/6} = \sqrt[6]{72}$. *Ans.*

Division by radicals may be avoided by RATIONALIZING.

Example. To compute $\dfrac{\sqrt{3} + \sqrt{2}}{\sqrt{5} - \sqrt{3}}$, multiply numerator and denominator by $\sqrt{5} + \sqrt{3}$. This rationalizes the denominator, for $(\sqrt{5} - \sqrt{3})(\sqrt{5} + \sqrt{3}) = 5 - 3 = 2$, hence the value $\frac{1}{2}(\sqrt{3} + \sqrt{2})(\sqrt{5} + \sqrt{3}) = \frac{1}{2}(3 + \sqrt{6} + \sqrt{10} + \sqrt{15})$.

Logarithms are exponents of a given number called the BASE. (5) The system commonly used, COMMON LOGARITHMS, has the base 10. (See also Art. 4.) The common logarithm of any number is the exponent or power to which 10 must be raised to equal the number. If $10^x = N$, then $x = \log N$.

From $10^2 = 100$, and $0.1 = 1/10 = 10^{-1}$, it appears that $\log 100 = 2$, and $\log 0.1 = -1$. Also $10^0 = 1$, $\therefore \log 1 = 0$

From $10^x = m$, $10^y = n$, multiplying, $10^{x+y} = mn$. Hence the logarithm of a product equals the sum of the logarithms of the factors. For other rules, see Art. 4.

Any number may be written as the product of a number between 1 and 10 multiplied by an integral power of 10, positive or negative. The logarithm of a number between 1 and 10 is a number between 0 and 1. Hence the logarithm of any number may be written as the sum of a decimal (less than 1) called the MANTISSA, and an integer (positive or negative) called the CHARACTERISTIC. Tables of common logarithms give logarithms of numbers from 0 to 10 ($\log 0 = \infty$) to three, four, five, or more places of decimals (THREE-PLACE, FOUR-PLACE, FIVE-PLACE TABLES). (See Table 16 and "Use of Tables," Art. 75.)

Natural, or Napierian, logarithms. The base of this system is 2.71828 . . . (to five places), denoted by e. $10^{0.43429 \cdots} = 2.71828$. Common logarithm equals the natural logarithm times 0.43429, which number is called the MODULUS of the common system. Natural log = common log times 2.3026. The abbreviation NAP LOG is used for natural or Napierian logarithm.

16. Permutations and combinations

A PERMUTATION of any number of things is a group of some or all of them, arranged in a definite order. (**5**) The number of permutations of m different things taken r at a time is the product of successive decreasing integers from m down to $m - r + 1$. The number of permutations of m different things taken all together is the product of all integers from 1 to m. The product $1 \cdot 2 \cdot 3 \cdot \ldots \cdot m$ is called FACTORIAL m, and written $m!$ The formula for the number of permutations of m things r at a time is $m!/(m - r)!$

A COMBINATION of any number of things is a group of some or all of them, without reference to order in the group. The number of combinations of m different things r at a time equals the number of permutations divided by factorial r, namely $m!/(m - r)!r!$. Given two groups of m and n things respectively, the number of selections (combinations) of $r + s$ things, r from first group and s from second group equals $m!\ n!/(m - r)!\ (n - s)!\ r!\ s!$

17. Chance

The PROBABILITY that an event will happen is the ratio of the number of favorable cases to the whole number of cases that can occur. (**6**)

Example. From an urn containing 5 black and 4 white balls, 3 balls are drawn at random. What is the probability that 2 will be black and 1 white?

From 9 balls 3 may be drawn in $9 \times 8 \times 7/3! = 84$ ways. (This is the number of combinations of 9 things 3 at a time.) The number of favorable cases (formula above) is $5!\ 4!/3!\ 3!\ 2! = 40$. Hence the required probability is $40/84 = 10/21$, *i.e.*, 10 chances in 21.

If the probabilities of two events are a and b, respectively, the probability of simultaneous occurrence is ab, and of occurrence of one or the other is $a + b$.

Example. The probability of drawing a knave from a full pack of cards is 1/13. The probability of drawing a spade is 1/4. Hence the probability of drawing the knave of spades is $1/13 \cdot 1/4 = 1/52$. Probability of drawing a knave or an ace is $1/13 + 1/13 = 2/13$.

18. Progressions

An arithmetic progression is a sequence of terms each of which differs from the preceding by the same number d, called the COMMON DIFFERENCE. If n = number of terms, a = first term, l = last term, s = sum of n terms, then $l = a + (n - 1)d$, and $s = \frac{n}{2}(a + l)$. ARITHMETIC MEAN of two numbers is half their sum.

A geometric progression is a sequence of terms each of which is obtained from the preceding by multiplying it by a fixed number r, called the RATIO. If n = number terms, a = first term, l = last term, s = sum of n terms, then $l = ar^{n-1}$, $s = (rl - a)/(r - l)$. GEOMETRIC MEAN of two numbers m, n is $\sqrt{mn}$.

19. Binomial theorem

The binomial theorem is a formula for expanding a power of the sum of two terms.

$$(a + b)^n = a^n + na^{n-1}b + \frac{n(n-1)}{1 \cdot 2}a^{n-2}b^2 + \frac{n(n-1)(n-2)}{1 \cdot 2 \cdot 3}a^{n-3}b^3 + \ldots$$

If n is a positive integer, the right hand member contains $n + 1$ terms and the last term is b^n. The product of the coefficient in any term and the exponent

of a in that term divided by the exponent of b increased by 1 gives the coefficient of the next term. If n is not a positive integer, the sequence of terms of the second member does not terminate but leads to an INFINITE SERIES. The formula holds then only when $|b| < |a|$. (See under Series, Art. 62.)

Coefficients $1, n, \frac{n(n-1)}{1\cdot 2}, \frac{n(n-1)(n-2)}{1\cdot 2\cdot 3}, \frac{n(n-1)(n-2)(n-3)}{4!}$, etc.

are called BINOMIAL COEFFICIENTS. A convenient notation is $n = \binom{n}{1}$, $\frac{n(n-1)}{1\cdot 2} = \binom{n}{2}, \frac{n(n-1)(n-2)}{1\cdot 2\cdot 3} = \binom{n}{3}$, etc. When n = a positive integer, the rth coefficient from the beginning and the rth from the last are equal.

Binomial coefficients

r	$\binom{r}{2}$	$\binom{r}{3}$	$\binom{r}{4}$	$\binom{r}{5}$
0.1	−0.0450	0.0285	−0.0207	0.0161
0.2	−0.0800	0.0480	−0.0336	0.0255
0.3	−0.1050	0.0595	−0.0402	0.0297
0.4	−0.1200	0.0640	−0.0416	0.0300
0.5	−0.1250	0.0625	−0.0391	0.0273
0.6	−0.1200	0.0560	−0.0336	0.0228
0.7	−0.1050	0.0455	−0.0262	0.0173
0.8	−0.0800	0.0320	−0.0176	0.0113
0.9	−0.0450	0.0165	−0.0087	0.0054

20. Interpolation

Series of differences. From a given sequence a_1, a_2, a_3, a_4, etc., form the sequence of FIRST DIFFERENCES $a_2 - a_1, a_3 - a_2$, etc., obtained by subtracting each term from the preceding term. From this sequence a third series of SECOND DIFFERENCES may be formed, and so on.

Let b_1, b_2, b_3, etc., be the series of first differences;
c_1, c_2, c_3, etc., be the series of second differences;
d_1, d_2, d_3, etc., be the series of third differences.

The formula for $(n + 1)$th term of original sequence is $a_{n+1} = a_1 + nb_1 + \frac{n(n-1)}{1\cdot 2}c_1 + \frac{n(n-1)(n-2)}{1\cdot 2\cdot 3}d_1 + \ldots$ Formula for sum $(=s)$ of n terms is,

$$s = na_1 + \frac{n(n-1)}{1\cdot 2}b_1 + \frac{n(n-1)(n-2)}{1\cdot 2\cdot 3}c_1 + \ldots$$

Application of the formula is to cases when differences of a certain order all vanish. In an arithmetic progression, for example, second-order differences are zero.

Example. Find the sum of 11 terms of the series 1, 5, 12, 24, 43, 71, etc.

Solution. $a_1 = 1, b_1 = 4, c_1 = 3, d_1 = 2, n = 11.$
$s = 11 + 220 + 495 + 660 = 1386.$ *Ans.*

```
1   5   12   24   43   71
  4   7   12   19   28
    3   5    7    9
      2    2    2
        0    0
```

Sums of powers of integers 1, 2, 3, . . . n.

Sum of first powers $= \frac{1}{2}n(n + 1)$.
Sum of second powers $= \frac{1}{6}n(n + 1)(2n + 1)$.
Sum of third powers $= (\frac{1}{2}n(n + 1))^2$.

Interpolation by differences of higher order. A numerical table usually gives values $y_1, y_2, y_3, \ldots$ of a function y corresponding to values $x_1, x_2, x_3, \ldots$ of x. Successive values of x differ by a constant difference d (form an arithmetic progression.) From the values of y, form a series of differences of the first order, second order, etc., and let D_1, D_2, etc., be the first terms, respectively, of these series. To interpolate a value of y, say y', corresponding to the value $x' = x_1 + rd(d = x_2 - x_1,\ r < 1)$ of x between x_1 and x_2, use INTERPOLATION FORMULA,

$$y' = y_1 + rD_1 + \frac{r(r-1)}{1 \cdot 2} D_2 + \frac{r(r-1)(r-2)}{1 \cdot 2 \cdot 3} D_3 + \ldots$$

Differences above a certain order are assumed very small.

If differences above the first order are negligible, the formula $(y' = y' + rD_1)$ becomes the usual proportion $(y' - y_1) : (x' - x_1) = (y_2 - y_1) : (x_2 - x_1)$, for INTERPOLATION BY FIRST DIFFERENCES.

In using the above formula attention should be paid to the remarks under "addition" in approximate computation (Art. 1).

21. Complex numbers

If a and b are real numbers, $a + b\sqrt{-1}$ is a COMPLEX NUMBER. Write $i = \sqrt{-1}$, $i^2 = -1$. Number i is called IMAGINARY UNIT. When $a = 0$, the complex number is a pure imaginary, or simply imaginary, number. Complex numbers differing only in the sign of the imaginary part are said to be CONJUGATE. Point with coordinates (a, b) represents $a + bi$ graphically. (Fig. 21.) Real numbers are represented by points on X-axis (number scale), imaginary numbers by points on Y-axis. Formal laws of operation in algebra apply to complex numbers, remembering always that $i^2 = -1$. This representation of complex numbers is a counterpart of the number scale for real numbers, Art. 6.

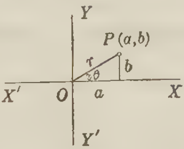

FIG. 21.

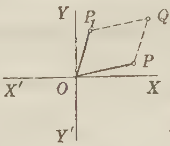

FIG. 22.

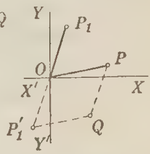

FIG. 23.

Addition. The sum of $a + bi$ and $a_1 + b_1 i$ is $(a + a_1) + (b + b_1)i$.

To construct graphically point Q corresponding to the sum of numbers represented by P and P_1, draw parallelogram $OPQP_1$. (Fig. 22.) Regarding OP and OP_1 as vectors, the operation of addition is VECTOR ADDITION.

Subtraction. Difference of $a + bi$ and $a_1 + b_1 i$ is $(a - a_1) + (b - b_1)i$.

To construct corresponding point Q, plot $P_1'(-a_1, -b_1)$, and draw parallelogram $OPQP_1'$. (Fig. 23.)

The absolute value and amplitude of $a + bi$ are respectively the polar coordinates r and θ (Art. 48) of $P(a, b)$. (Fig. 21.)

$$r = +\sqrt{a^2 + b^2} = |a + bi|. \quad a = r\cos\theta, \quad b = r\sin\theta.$$

Hence $a + bi = r(\cos\theta + i\sin\theta)$.

Absolute value r is also called MODULUS, and amplitude θ ANGLE or ARGUMENT. The amplitude given by $a = r\cos\theta$, $b = r\sin\theta$ is a unique angle less than 360°.

Multiplication of $a + bi$ and $a_1 + b_1i$. Write in polar form $a + bi = r(\cos\theta + i\sin\theta)$, $a_1 + b_1i = r_1(\cos\theta_1 + i\sin\theta_1)$. Then $(a + bi)(a_1 + b_1i) = rr_1[\cos(\theta + \theta_1) + i\sin(\theta + \theta_1)]$.

The absolute value of the product of two complex numbers is the product of the absolute values, and the amplitude is the sum of the amplitudes. Multiplication by i has the effect of rotating a point through 90°, by $i^2 = -1$ through 180°, by $i^3 = -i$, through 270°.

Division of two complex numbers $a + bi$ and $c + di$ is worked as below.

$$\frac{a + bi}{c + di} = \frac{a + bi}{c + di} \times \frac{c - di}{c - di} = \frac{ac + bd + (bc - ad)i}{c^2 + d^2}.$$

Absolute value of the quotient of two complex numbers is the quotient of the absolute values of numerator and denominator, and the amplitude of the quotient is the difference of the amplitudes of numerator and denominator.

Powers and roots. For a positive integer n, $(a + bi)^n = r^n(\cos\theta + i\sin\theta)^n = r^n(\cos n\theta + i\sin n\theta)$.

When $r = 1$, this is known as DeMoivre's theorem.

$$\sqrt[n]{a + bi} = \sqrt[n]{r}\,(\cos\theta + i\sin\theta)^{\frac{1}{n}} = \sqrt[n]{r}\,(\cos\theta/n + i\sin\theta/n).$$

In this formula, θ has n values differing by 360°, namely, θ_1 (amplitude of $a + bi < 360°$), $\theta_1 + 360°$, $\theta_1 + 720°$, . . . , $\theta_1 + (n - 1)$ 360°. The nth root of a real number has n distinct values, one real, the others complex.

Cube roots of unity are found by setting $n = 3$, $r = 1$, $\theta = 0$, 360°, 720°. They are 1, $\cos 120° + i\sin 120°$, $\cos 240° + i\sin 240°$, that is, 1, $-1/2 + \sqrt{-3/2}$, $-1/2 - \sqrt{-3/2}$. Points representing these roots are the vertices of an equilateral triangle inscribed in a circle drawn with unit radius about the origin, one vertex being (1, 0). Similarly, the vertices of a regular polygor of n sides inscribed in this circle, one vertex (1, 0), will represent the nth roots of unity.

Exponentials, trigonometric functions, and complex numbers. The relations are

$$e^{ix} = \cos x + i\sin x,$$
$$\sin x = (e^{ix} - e^{-ix})/2i;$$
$$\cos x = (e^{ix} + e^{-ix})/2.$$

From these relations, $e^{2\pi i} = 1$; $e^{\pi i} = \cos\pi + i\sin\pi = -1$; $e^{\pi i/2} = i$; $e^{\pi i/4} = +\sqrt{i} = \cos\pi/4 + i\sin\pi/4$.

Logarithm of a complex number $a + bi = re^{i\theta}$ is a complex number, $\log r + i\theta$. Since θ may be replaced by $\theta + 2n\pi$ (n any integer), $\log(a + bi) = \log r + i(\theta + 2\pi n)$. Points representing $\log(a + bi)$ lie on a line parallel to Y-axis, to the right a distance equal to $\log r$.

22. Precision of measurements. Least squares. Probable error.

If a number of measurements are made to determine directly the unknown magnitude of a certain object, all measurements being made with equal skill and care, the most probable value of the unknown is the arithmetical mean of all the measurements. That is, with n measurements $x_1, x_2, x_3 \ldots x_n$, the most probable value x_0 is $x_0 = (x_1 + x_2 + x_3 + \cdots + x_n)/n$. (6)

The basis for this assumption is that, on the average, errors in excess (positive errors), and errors in defect (negative errors) are evenly balanced, so that the sum of the errors is zero. If the unknown magnitude is x, it can be readily shown by Calculus (Art. 57) that

the most probable value is that for which the SUM OF THE SQUARES OF THE ERRORS, viz.: $(x_1 - x)^2 + (x_2 - x)^2 + \ldots + (x_n - x)^2$ is a minimum. Hence the name LEAST SQUARES.

Weighted measurements. If the method of making measurements shows that weights w_1, w_2, etc., should be assigned to measurements x_1, x_2, etc., then the most probable value of the unknown is the WEIGHTED MEAN, $x_0 = (w_1x_1 + w_2x_2 + \ldots + w_nx_n)/(w_1 + w_2 + \ldots + w_n)$.

Residuals are differences $x_1 - x_0$, $x_2 - x_0$, etc., between the observed values and the most probable value, and are denoted by v_1, v_2, etc. Evidently $v_1 + v_2 + \ldots + v_n = 0$.

Probable error of an observation is a number such that the actual error of that observation may with equal chances be greater or less than the probable error. For example, if a measurement is 30.726 with a probable error of $\pm$ 0.014, the meaning is that the correct value is just as likely to lie between 30.712 and 30.740 as outside these limits. The probable error gives a measure of the precision of a measurement. Weights to be attached to different determinations of the same quantity are inversely proportional to the squares of the probable errors. Formulas for computing probable error follow. Numerical values of residuals are $|v_1|$, $|v_2|$, etc., number of observations is n.

Probable error (r) **of single observation,**

$$r = \frac{0.6745}{\sqrt{n-1}}\sqrt{v_1^2 + v_2^2 + \ldots + v_n^2}. \qquad \text{(STANDARD FORMULA.)}$$

$$r = \frac{0.8453}{\sqrt{n(n-1)}}(|v_1| + |v_2| + \ldots |v_n|).$$

(PETER'S FORMULA, approximate.)

Probable error (r_0) **of arithmetical mean,**

$$r_0 = r/\sqrt{n}; \quad r_0 = \frac{0.6745}{\sqrt{n(n-1)}}\sqrt{v_1^2 + v_2^2 + \ldots + v_n^2}.$$

(STANDARD FORMULA.)

$$r_0 = \frac{0.8453}{n\sqrt{n-1}}(|v_1| + |v_2| + \ldots + |v_n|).$$

(PETER'S FORMULA, approximate.)

For tables of values of coefficient of parentheses, see (6).

Example. The following are ten measurements M of the length of a base line. Below are given the values of residuals, v, and the squares of residuals.

M: 455.35, 455.35, 455.20, 455.05, 455.75, 455.40, 455.10, 455.30, 455.50, 455.30.

M_0 = arithmetic mean = 455.330.

v: .02, .02, $-$.13, $-$.28, .42, .07, $-$.23, $-$.03, .17, $-$.03.

v^2: .0004, .0004, .0169, .0784, .1764, .0049, .0529, .0009 .0289, .0009.

Hence $v_1^2 + v_2^2 + \ldots + v_n^2 = .3610$. And $|v_1| + |v_2| + \ldots + |v_n| = 1.40$.

$\therefore$ by the standard formulas, $r = \frac{0.6745}{\sqrt{9}}\sqrt{.3610} = .13$, $r_0 = r/\sqrt{10} = .042$.

By the approximate formulas, $r = \frac{0.8453}{\sqrt{90}}(1.40) = .12$. $r_0 = .039$.

For the most probable length of the base line, the result is 455.330 with probable error $\pm$ .042 (using result given by standard formula), usually written 455.330 $\pm$ .042. Note also that five of the residuals are numerically less than the probable error of a single observation. In fact, in any considerable number of observations it should be the case that half of the residuals are less than the probable error.

Probable error of a function of a single measured quantity equals the product of the derivative of the function (Art. 56) by the probable error of the measured quantity.

To test a given set of measurements of an unknown quantity, make use of the normal distribution of residuals for the purpose of obtaining a comparison of the data of the experiment with results established by theory and confirmed by practice. By the table below, the normal number of residuals ($= y$) numerically less than an assumed value ($= a$) can be written down. In the table the number of measurements is n, and the probable error of a single observation is r

Table of values of ratio y/n

a/r	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
0	.000	.054	.107	.160	.213	.264	.314	.363	.4105	.456
1	.500	.542	.582	.619	.655	.688	.7195	.7485	.775	.800
2	.823	.843	.862	.879	.8945	.908	.9205	.931	.941	.9495
3	.957	.9635	.969	.974	.978	.982	.985	.987	.990	.9915
4	.993	.994	.995	.996	.997	.998	.998	.9985	.9988	.999
5	.999									
∞	1.000									

Example. In an experiment 40 measurements were made and the probable error of a single observation was $r = 0.136$. For $a = 0.05$, $a/r = 0.37$. The above table gives $y/n = 0.196$. Hence $y = 7.84$, that is, there should be normally 8 residuals not exceeding 0.05 in numerical value. Values of y are tabulated corresponding to assumed values of a, and under Y is written down the normal distribution of residuals for the experiment, namely, 8 residuals not exceeding 0.05, 7 residuals numerically between 0.05 and 0.10, inclusive, etc. Actual numbers of residuals are given in the last column, and comparison shows a satisfactory result.

a	a/r	y	Y	
0.00	0.00	0.00		
0.05	0.37	7.84	8	9
0.10	0.74	15.2	7	6
0.20	1.5	27.5	12	12
0.30	2.2	34.5	7	8
0.40	2.9	38	4	3
∞	∞	40	2	2

To test whether any measurement should be rejected. Calculate y/n by the formula $y/n = (2n - 1)/2n$, and from the above table find a/r, and then a. This value of a gives a maximum numerical value for all residuals, and any measurement in which the residual exceeds this value should be rejected.

Example. In the experiment above, $n = 40$, and $y/n = (80 - 1)/80 = 0.987$. Hence $a/r = 3.7$, and $a = 3.7 \times 0.136 = 0.50$. No residual should exceed 0.50 in numerical values. Maximum residual in the experiment was 0.45.

Constant errors due to some fixed cause such as conditions under which the experiment was made, defects in instruments, personal peculiarity of observer, etc., are supposed to have been detected and eliminated before the methods explained above are applied.

Solution of a system of linear equations by least squares. The problem of determining the empirical law satisfied by given data (Art. 71) involves the solution of a system of linear equations in which the number of equations exceeds the number of unknowns. For such a system the question is the most probable values of the unknowns. The method of solution of a system with three unknowns x, y, z illustrates the general method. **(6)**

Example. The data of the experiment establish a system of n OBSERVATION EQUATIONS with unknowns x, y, z. Form a system of NORMAL EQUATIONS equal in number to the number of unknowns as follows: (1) Multiply each observation equation by the coefficient of x in that equation, and add the resulting equations. This gives the first normal equation in which $a_1^2 + a_2^2 + \ldots + a_n^2 = [aa]$; $a_1b_1 + a_2b_2 + \ldots + a_n b_n = [ab]$, etc., adopting a convenient notation. (2) Multiply each observation equation by the coefficient of y in that equation, and add the resulting equations. This gives the second normal equation. (3) Multiply each observation equation by the coefficient of z in that equation, and add the resulting equations. This gives the third and, in this example, last normal equation. (4) Solve the normal equations for x, y, z. For these values, x', y', z', the sum of the squares of the left-hand members of the observation equations will be less than for any other values of x, y, z.

$$\begin{gathered} a_1x + b_1y + c_1z + d_1 = 0 \\ a_2x + b_2y + c_2z + d_2 = 0 \\ \cdot \quad \cdot \quad \cdot \quad \cdot \\ \cdot \quad \cdot \quad \cdot \quad \cdot \\ \cdot \quad \cdot \quad \cdot \quad \cdot \\ a_nx + b_ny + c_nz + d_n = 0. \end{gathered}$$

Normal equations

$$\begin{aligned} [aa]x + [ab]y + [ac]z + [ad] &= 0 \\ [ba]x + [bb]y + [bc]z + [bd] &= 0 \\ [ca]x + [cb]y + [cc]z + [cd] &= 0 \end{aligned}$$

The problem is to be regarded as one in which n observations are made upon the linear function $ax + by + cz + d$ of x, y, z, each observation establishing a set of values of a, b, c, d. The probable error r of a single observation is given by

$$r = \frac{0.6745}{\sqrt{n - q}}\sqrt{v_1^2 + v_2^2 + \ldots + v_n^2}, \text{ (standard formula)},$$

or

$$r = \frac{0.8453}{\sqrt{n(n - q)}}(|v_1| + |v_2| + \ldots + |v_n|), \text{ (approximate formula)},$$

where

$$q = \text{the number of unknowns}, \; v_1 = a_1x' + b_1y' + c_1z' + d_1,$$
$$v_2 = a_2'x' + b_2y' + c_2z' + d_2, \text{ etc.}$$

23. Interest and annuities

Compound interest. Let P = principal, i = interest on one dollar per year, n = number of years, A = amount after n years. Then $A = P(1 + i)^n$, when interest is compounded annually.

$A = P(1 + i/t)^{nt}$, when interest is compounded t times per year. From the first formula, $P = A/(1 + i)^n$, which gives the principal when the other quantities are known; $n = (\log A - \log P)/\log (1 + i)$, giving time.

Table 29, p. 1488, gives values of A when $P = 1$, $t = 1$, that is, gives values of A computed by the formula $A = (1 + i)^n$. This table can be used also when interest is compounded more often than once a year by taking i/t as percentage rate, and nt as number of years. For example, if interest is compounded semi-annually, halve the rate and double the time. To find amount, multiply the number in the table by the principal. Table 30, p. 1489, gives the principal which will amount to (present value of) one dollar in a given time at a given rate when interest is compounded annually, that is, is computed from $P = 1/(1 + i)^n$. For interest periods less than one year, proceed as above. To find the present value of any sum (discount any sum), multiply tabular number by this sum. (7)

An annuity is a series of equal payments made at equal periods of time, the first payment being made at the end of the first period. Let R = payment, i = interest on one dollar per year, n = number of years during which payments are continued, A = amount of annuity after n years, P = present value, then $A = R[(1 + i)^n - 1]/i$, when payments are made annually and interest is compounded annually.

Table 31, p. 1490, gives the amount of an annuity of one dollar for various rates and periods. For the amount of an annuity of any sum, multiply the tabular number by the sum. Formula for present value is $P = R(1 - (1 + i)^{-n})/i$. Table 32, p. 1491, gives the present value of an annuity of one dollar extending over a given period with money at a

given rate. For the present value of an annuity of any sum, multiply the tabular number by the sum. Table 33, p. 1492, gives the annuity which will amount to one dollar in a given time at a given rate. (Tabular numbers are reciprocals of those in Table 31.) To find the annual payment which will amount to a given sum in a given time at a given rate (SINKING FUND), multiply the tabular number by the sum.

Example. To provide a sinking fund of \$100,000 in 20 years if money accumulates at 4 per cent, the annual sum to be set aside equals 100,000 $\times$ 0.033582 (tabular number in Table 33 opposite 20 years under 4 per cent.), that is \$3358.20. *Ans.*

Annuity which a given sum P will purchase, when the annuity is to extend over a given period and money is worth a given rate, is found as follows: add the given rate in cents to the tabular number in Table 33, which corresponds to period and rate, and multiply the sum by P.

Example. For 20 years at rate 4 per cent., the tabular number in Table 33 is 0.033582. Adding .04 gives 0.073582. Hence \$100,000 will purchase an annuity of \$7358.20 terminating after 20 years, when money is worth 4 per cent.

24. Mine valuation (8)

The annual profit earned by a mine may be regarded as an annuity which must provide a dividend return on capital plus an annual sum to be set aside for a sinking fund that will amortize the capital. The value of the mine is the present value of the annual profit so regarded. If P = the annual profit assured for a period of years, value of mine is found as follows: Find the tabular number for period and rate earned by payments into sinking fund from Table 33, p. 1492. Add the dividend on one dollar to this number and divide the annual profit by the sum. (Table 5 is useful to replace division by multiplication.)

Example. Annual profit is estimated at \$200,000 for ten years. Payments into sinking fund earn 4 per cent. and dividend on capital invested is 7 per cent. Tabular number (Table 33) for 10 years and 4 per cent. is 0.083291. Adding dividend rate .07, gives 0.153291. Reciprocal of 0.1533 (Table 5) = 6.523. Present value of mine = 6.523 $\times$ 200,000 = \$1,304,600. *Ans.*

To determine the number of years that a given rate of income on capital must continue in order to amortize capital, pay a given rate of interest on capital, with annual payments into a sinking fund accumulating at a given rate per cent., proceed as follows: In Table 33, in the column under "Rate of accumulating sinking fund," find the number equal to the difference between the rate of income earned and the dividend rate. Then the time required is the corresponding value of time in the first column (interpolation may be used if justified by the problem.)

Example. Rate of income earned is 10 per cent., dividends paid 7 per cent., sinking fund accumulates at 4 per cent. Then from Table 33, under 4 per cent., number .03(= .10 − .07) occurs for a time between 21 and 22 years. (Interpolation gives 21.6 years.)

ELEMENTARY GEOMETRY

25. Plane figures

Triangles. The sum of the angles equals 180°. $\angle XAB$ is an exterior angle of ΔABC (Fig. 24). The exterior angle equals the sum of the opposite interior angles. ($\angle XAB = \angle B + \angle C$.) The MEDIAN of a triangle is the line joining a vertex to the midpoint of the opposite side. The medians

(Fig. 24) of a triangle meet in a point G, which is the center of gravity of the triangle, and G trisects each median. ($AG = \frac{2}{3}AD$.) BISECTORS OF ANGLES of a triangle meet in a point M equidistant from all sides. (Fig. 25.) M is the center of the circle tangent to the sides of the triangle, called INSCRIBED CIRCLE. M is INCENTER of triangle. The bisector of an angle of a triangle divides the opposite side into segments proportional to the other two sides. $AE : EC = AB : BC$. (Fig. 25.) ALTITUDES of a triangle meet in a point, the ORTHOCENTER. The perpendicular bisectors of the sides of a triangle meet in a point O (Fig. 26) equidistant from all vertices. O is the center of a circle passing through all vertices, called CIRCUMSCRIBED CIRCLE. O is CIRCUMCENTER of triangle. The longest side of a triangle is opposite the largest angle and *vice versa*. The line joining the mid-points of two sides of a triangle is parallel to the third side and half its length.

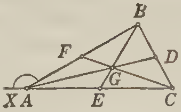

FIG. 24.

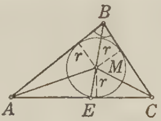

FIG. 25.

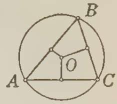

FIG. 26.

Orthogonal projection. In Figs. 27 and 28, AE is the orthogonal projection of AB on AC, BE being perpendicular to AC. The square of the side opposite an acute angle equals the sum of the squares of the other two sides diminished by twice the product of one of these sides by the orthogonal projection of the other side upon it. In Fig. 27, $a^2 = b^2 + c^2 - 2b \cdot AE$. The square of the side opposite an obtuse angle equals the sum of the squares of the other two sides increased by twice the product of one of these sides by the orthogonal projection of the other side upon it. In Fig. 28, $a^2 = b^2 + c^2 + 2b \cdot AE$.

Right triangle (Fig. 29). $\angle A + \angle B = 90°$. $c^2 = a^2 + b^2$. The ALTITUDE CE ($= h$) drawn from vertex of right angle C upon HYPOTENUSE (c), divides the hypotenuse into segments AE ($= m$) and BE ($= n$). Then $h^2 = mn$, $b^2 = cm$, $a^2 = cn$. The median drawn from C equals $\frac{1}{2}c$.

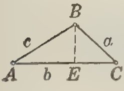

FIG. 27.

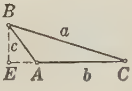

FIG. 28.

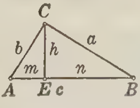

FIG. 29.

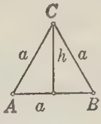

FIG. 30.

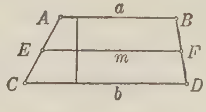

FIG. 31.

Equilateral triangle. (Fig. 30.) Side $= a$. Each angle $= 60°$. Altitude, $h = \frac{1}{2}a\sqrt{3}$. Radius of the circumscribed circle, $R = \frac{1}{3}a\sqrt{3}$. Radius of the inscribed circle, $r = \frac{1}{6}a\sqrt{3}$.

A trapezoid is a figure bounded by four lines, two of which are parallel (Fig. 31). ALTITUDE is the perpendicular distance between the parallel sides. MID-LINE (EF) joins the midpoints of the non-parallel sides and equals half the sum of the parallel sides. ($m = (a + b)/2$).

Sum of interior angles of a polygon equals 180° multiplied by the number of sides less two.

Circles. A straight line perpendicular to a radius of a circle at its extremity is tangent to the circle. Parallel lines intercept equal arcs on a circle.

When two circles intersect, the line of centers bisects the common chord at right angles. If two circles are tangent to each other, the line of centers passes through the point of contact.

Areas of two circles have the same ratio as the squares of their diameters or radii.

Angle measurement. The angle inscribed in a semi-circle is a right angle. The angle formed by two chords intersecting on a circle (INSCRIBED ANGLE) is measured by half the arc intercepted between its sides.

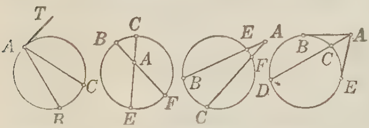

Fig. 32. Fig. 33. Fig. 34. Fig. 35.

In Fig. 32, $\angle BAC$ is measured by half arc BC.

The angle formed by a tangent and a chord drawn from the point of contact is measured by half the intercepted arc.

In Fig. 32, $\angle BAT$ is measured by $\frac{1}{2}$ arc BCA.

The angle formed by two chords intersecting within a circle is measured by half the sum of the intercepted arcs.

In Fig. 33, $\angle BAC$ (or EAF) is measured by $\frac{1}{2}$ (arc BC + arc EF).

The angle formed by two secants, or two tangents, or a tangent and a secant, intersecting without a circle, is measured by half the difference of the intercepted arcs.

In Fig. 34, $\angle BAC$ is measured by $\frac{1}{2}$ (arc BC − arc EF). In Fig. 35, $\angle BAE$ is measured by $\frac{1}{2}$ (arc BDE − arc BCE). In Fig. 35, $\angle BAC$ is measured by $\frac{1}{2}$ (arc BD − arc BC).

Chords, secants, and tangents. In Fig. 33, the product of segments AC and AE equals the product of segments AB and AF. In Fig. 34, the product of the whole secant AB and the external segment AE equals the product of the whole secant AC and its external segment AF. In Fig. 35, the product of the whole secant AD and the external segment AC equals the square of tangent AB (or AE).

Similar figures. Similar polygons have their angles respectively equal and the homologous sides proportional. Two triangles are similar if their angles are respectively equal, or if their sides are respectively proportional, or if they have an angle of one equal to an angle of the other and the including sides proportional. Areas of two similar polygons are to each other as the squares of homologous sides.

Regular figures. A regular polygon is one with equal sides and equal angles. A circle may be drawn to pass through all vertices of a regular polygon (CIRCUMSCRIBED CIRCLE), or to be tangent to every side (INSCRIBED CIRCLE). Two regular polygons of the same number of sides are similar.

26. Geometrical constructions

To bisect a given straight line AB (Fig. 36). With A and B as centers and with equal radii sufficiently great, describe arcs intersecting at C and D. Draw CD, cutting AB at E. Then E is the mid-point of AB.

To bisect a given angle CAB (Fig. 37). With A as center, and with any convenient radius AD describe an arc cutting AC in D and AB in E.

With D and E as centers and with equal radii sufficiently great, describe arcs intersecting in F. Draw AF. Then AF bisects $\angle A$.

To erect a perpendicular at a given point C, **in a line** AB. (1) Lay off $CD = CE$, (Fig. 38), and with D and E as centers and with equal radii sufficiently great, describe arcs intersecting at F. Draw FC, the perpendicular required. Or (2), (Fig. 39), take any convenient point D not on AB and with radius DC describe a circle cutting AB in E. Draw diameter EDF. Then FC is the required perpendicular.

To drop a perpendicular from a point C **to a line** AB. (1) With C as center and a radius sufficiently great (Fig. 40), describe an arc cutting AB in D and E. With D and E as centers, describe arcs intersecting at F. Draw CF, cutting AB in G. Then CF is the required perpendicular. (2) When C is nearly above one end of the line (Fig. 41), with C as center and radius 5 convenient units describe an arc cutting AB at D. Lay off DF equal to 4 units and draw CF, the required perpendicular.

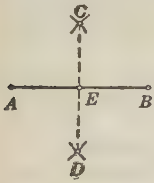

Fig. 36.

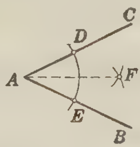

Fig. 37.

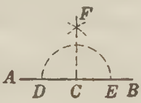

Fig. 38.

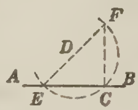

Fig. 39.

Fig. 40.

To draw a line through a given point C **parallel to given line** AB (Fig. 42). Draw CD perpendicular to AB, and erect a perpendicular CE to CD at C. Then CE is parallel to AB.

To draw a line parallel to a given line AB **at a given distance from it** (Fig. 43). With the given distance as radius and with any centers m and n on AB describe arcs xy and zw, respectively. Draw CD touching these arcs. Then CD is the required line.

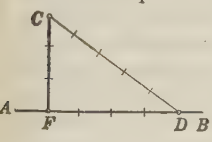

Fig. 41.

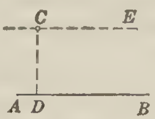

Fig. 42.

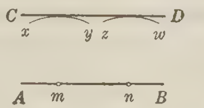

Fig. 43.

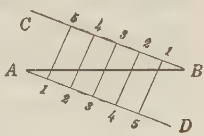

Fig. 44.

To divide a line AB **into a given number of equal parts** (Fig. 44). Draw AD making any angle with AB and draw BC parallel to AD. With dividers lay off equal lengths on AD and BC a number of times one less than the number of equal parts into which AB is to be divided. Number the points of division consecutively from A and from B and join as in the figure. The connecting lines will divide AB as required.

To construct an angle equal to a given angle ABC **when one side** FG **and the vertex** F **are given** (Fig. 45). With center B and a convenient radius BD describe arc DE. With the same radius and center F, draw arc KL. With radius equal to chord DE and with center K draw an arc cutting the arc KL at H. Draw FH. Then HFG is the required angle.

To lay out a circular arc of large radius. (Fig. 46.) Let AB be the chord of the desired arc and MC the height, MC being the perpendicular bisector of AB. Draw CB, and at B erect BF perpendicular to CB and BG perpendicular to AB. Divide CF, MB, BG into the same number of equal parts. Connect corresponding points of division on CF and MB, and draw lines from C to points of division on BG, as in the figure. In this manner as many points on the required arc can be determined as desired.

Let $CM = h$, $AB = 2b$. Then radius of arc $= (b^2 + h^2)/2h$.

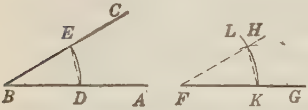

FIG. 45.

FIG. 46.

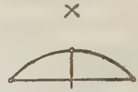

FIG. 47.

To bisect a given arc of a circle, draw the perpendicular bisector of the chord of the arc (Fig. 47). The point in which this bisector meets the arc is the required mid-point.

To circumscribe a circle about a given triangle (Fig. 48). Construct perpendicular bisectors of two sides. Their point of intersection is the center (circumcenter) of the required circle.

To inscribe a circle in a given triangle (Fig. 49). Draw bisectors of two angles intersecting in O (incenter). From O, draw OD perpendicular to BC. Then a circle with center O and radius OD is the required circle.

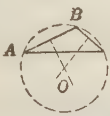

FIG. 48.

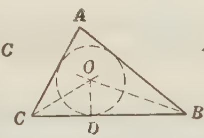

FIG. 49.

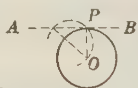

FIG. 50.

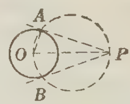

FIG. 51.

To draw a tangent to a given circle through a given point (P). (1) *When P is on circle,* (Fig. 50), draw radius OP, and construct AB perpendicular to OP at P. Then AB is the required tangent. (2) *When P is without the circle* draw a line joining P and O, the center of circle. (Fig. 51.) With OP as diameter describe a circle intersecting the given circle at A and B. Draw PA and PB. Each of these lines is tangent to the given circle.

To draw a common tangent to two given circles, with centers O and O' and unequal radii r and r', $r > r'$. When the given circles do not intersect (Fig. 52), to draw a COMMON INTERNAL TANGENT, construct a circle having same center O as larger circle and a radius equal to the sum of the radii of the given circles $(r+r')$. Construct a tangent $O'P'$ from center O' of the small circle to this circle. Construct $O'N$ perpendicular to this tangent. Draw OP'. The line MN joining the extremities of the radii OM and $O'N$ will be a common tangent. The figure shows two such common internal tangents. To draw a COMMON EXTERNAL TANGENT to two circles (Fig. 53), construct a circle having

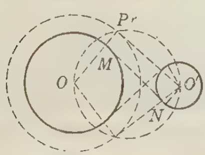

FIG. 52.

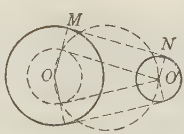

FIG. 53.

a common center with the larger circle and radius equal to the difference of radii $(r - r')$. Construct a tangent to this circle from the center of the smaller circle. The line joining the extremities M, N, of the radii of the given circles perpendicular to this tangent is the required common tangent. There are two such tangents.

In Fig. 52, $MN = \sqrt{c^2 - (r + r')^2}$, when $OO' = c$. In Fig. 53, $MN = \sqrt{c^2 - (r - r')^2}$.

To divide a given line AB into segments proportional to any number of given lines (Fig. 54). Draw AC, making any convenient angle with AB. Lay off AD, DE, and EF equal to the given lines m, n, p. Draw FB, and construct EH and DG parallel to FB. Then AG, GH, HB are the required segments.

To construct the mean proportional between two given lines (Fig. 55). Draw a line AC on which AB and BC equal the given lines, a, b. Construct semi-circle on AC as diameter. Erect a perpendicular to AC at B, intersecting the semi-circle at D. Then BD is the mean proportional between AB and BC.

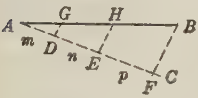

FIG. 54.

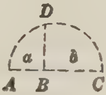

FIG. 55.

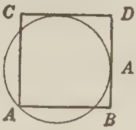

FIG. 56.

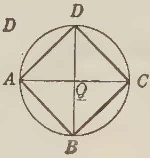

FIG. 57.

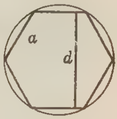

FIG. 58.

To construct a square with a given side AB (Fig. 56), erect a perpendicular at A, lay off $AC = AB$, and from C and B as centers with radius equal to AB describes arcs intersecting at the fourth vertex D.

To construct a square with a given diagonal AC (Fig. 57), draw a circle on AC as diameter, and erect the diameter BD perpendicular to AC. Then $ABCD$ is the required square.

To inscribe a square in a given circle, draw perpendicular diameters AC and BD (Fig. 57). Their extremities are the vertices of the inscribed square.

To inscribe a hexagon in a circle, step around the circle with dividers set to the radius (Fig. 58).

To construct a hexagon with a side of given length, draw a circle with radius equal to given side, and inscribe a regular hexagon in this circle.

If the side of a regular hexagon $= a$, and the distance between parallel sides $= d$, then $d = a\sqrt{3} = 1.732\,a$.

To draw a regular polygon of any desired number of sides, when one side AB is given, draw a semi-circle with radius AB (Fig. 59), and divide this semi-circle into as many equal parts BC, CD, DE, etc., as the required polygon has sides. Draw a line from A through each point of division except the last, and complete the construction as in the figure.

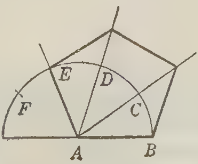

FIG. 59.

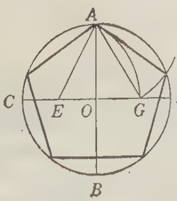

FIG. 60.

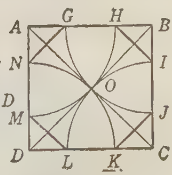

FIG. 61.

To inscribe a regular pentagon in a given circle, draw perpendicular diameters AB and CD (Fig. 60). Bisect radius OC at E. With E as center, strike an arc through A cutting OD at G. Then AG equals a side of the pentagon required.

To inscribe a regular octagon in a circle, draw perpendicular diameters, and bisect each of the four equal arcs determined by the extremities of these diameters.

To convert a square into an octagon, draw diagonals AC and BD, intersecting at O (Fig. 61). With vertices A, B, C, D as centers and radius AO draw arcs cutting the sides of the square in eight points. These points are vertices of a regular octagon.

27. Loci

All points equidistant from a given point lie on a circle whose center is the given point. All points equidistant from two given points lie on perpendicular bisector of the line joining the given points. All points equidistant from the sides of a given angle lie on the bisector of the angle. If a point moves. so that the ratio of its distances from two fixed points remains constant, it will lie upon a circle. In Fig. 62, PA/PB remains constant while P describes the circle. Diameter DE is determined by drawing bisectors of angles APB and BPE. (See Art. 25.) If a side AB and the opposite angle of a triangle (Fig. 63) are given, vertex E of angle will lie on a circle of which the given side is a chord. This circle is constructed as follows: Construct $\angle ABC$ equal to the given angle. Erect BM perpendicular to CB and draw the perpendicular bisector DP of AB. The point of intersection of BM and DP is the center of the required circle.

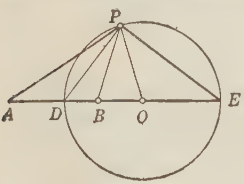

FIG. 62.

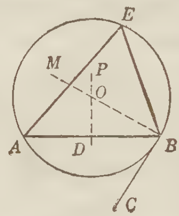

FIG. 63.

28. Solid geometry

Common solids. Figures 64–77 illustrate solids of common occurrence.

FIG. 64.

FIG. 65.

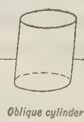

FIG. 66.

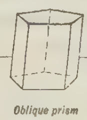

FIG. 67.

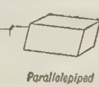

FIG. 68.

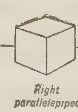

FIG. 69.

FIG. 70.

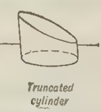

FIG. 71.

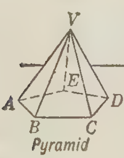

FIG. 72.

FIG. 73.

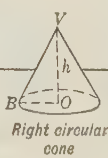

FIG. 74.

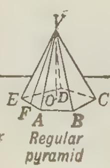

FIG. 75.

FIG. 76.

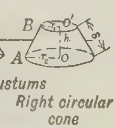

FIG. 77.

Dihedral angle (Fig. 78) is a figure formed by two planes meeting in a line, called an EDGE, and not extending beyond the line. The planes are called FACES. The angle formed by two lines ($\angle ABC$), one in each face drawn perpendicular to the edge at the same point, is called the PLANE ANGLE of the dihedral angle. Two dihedral angles are equal if their plane angles are equal. A dihedral angle is measured by its plane angle. If the plane angle is a right angle, the planes are perpendicular and form a RIGHT DIHEDRAL ANGLE. Through a given line oblique or parallel to a given plane one and only one plane can be passed perpendicular to the given plane. The line of intersection CD (Fig. 79) is the ORTHOGONAL PROJECTION of line AB upon plane P. The ANGLE BETWEEN A LINE AND A PLANE is the angle that the line (produced if necessary) makes with its orthogonal projection on the plane. This angle is the least angle which the line makes with any line in the plane.

A dihedral angle is designated by its edge and one point in each face; as dihedral angle P-BD-Q. Fig. 78.

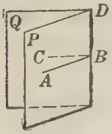

FIG. 78.

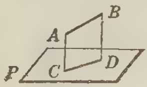

FIG. 79.

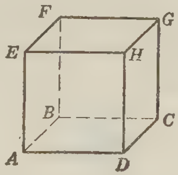

FIG. 80.

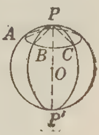

FIG. 81.

A polyhedron is a solid bounded by planes.

Figure 80 shows a common polyhedron, a CUBE, with twelve edges AB, BC, CD, etc.; six faces $ABCD$, $ABFE$, etc.; eight vertices A, B, C, D, etc., and twelve right dihedral angles E-AB-C, etc. A polyhedron is said to be CONVEX if it lies entirely on the same side of the plane of every one of its faces. A TETRAHEDRON is bounded by four triangles. The four perpendiculars erected at circumcenters of the four faces meet in a point equidistant from all vertices, which is the center of the circumscribed sphere. The four medians, joining each vertex with the center of gravity of the opposite face, meet in a point, which is the CENTER OF GRAVITY of the tetrahedron. This point is ¾ of the distance from each vertex along a median. The four altitudes meet in a point, called the ORTHOCENTER of the tetrahedron. The six planes bisecting the six dihedral angles meet in a point equidistant from all faces, the center of the inscribed sphere.

Sphere. Every plane section of a sphere is a circle. This circle is a GREAT CIRCLE when its plane passes through the center of the sphere. The POLES of a circle (P, P', Fig. 81) are extremities of the diameter of the sphere which is perpendicular to the plane of the circle. Through two points on a spherical surface not extremities of a diameter one great circle can be passed. The shortest line that can be drawn on the surface of a sphere between two points is an arc of great circle less than a semi-circumference joining those points. If two spherical surfaces intersect, their line of intersection is a circle whose plane is perpendicular to the line of centers, and whose center lies on this line.

On the surface of the earth the arcs of great circles are called GEODESICS. A SPHERICAL ANGLE is a figure formed by two arcs of great circles drawn from the same point (VERTEX), and is measured by the plane angle formed by tangents to its sides at its vertex. If planes of great circles are perpendicular, the angle is a RIGHT SPHERICAL ANGLE. A SPHERICAL POLYGON is a figure on a spherical surface bounded by three or more arcs of great circles. The sum of the angles of a spherical triangle is greater than two right angles and less than six right

angles. Two spherical polygons on the same sphere or on equal spheres are SYMMETRIC if the sides and angles of one are respectively equal to those of the other, but arranged in reverse order. Symmetric spherical polygons have equal areas.

A polyhedral angle (Fig. 82) is a figure formed by three or more RAYS, OA, OB, etc., drawn from a common origin or VERTEX O, no three of which lines lie in a plane. OA, OB, OC, etc., are called EDGES; AOB, BOC, COD, etc., FACE ANGLES; $A\text{-}OB\text{-}C$, $B\text{-}OC\text{-}D$, etc., DIHEDRAL ANGLES. A polyhedral angle is called TRIHEDRAL, TETRAHEDRAL, etc., according as it is formed by three rays, four rays, etc. The sum of two face angles of a trihedral angle is greater than the third face angle. The sum of all the face angles of any convex polyhedral angle is less than four right angles. When the edges of a polyhderal angle are produced through the vertex, a SYMMETRIC POLYHEDRAL ANGLE is formed. The face angles and dihedral angles of two symmetric polyhedral angles are equal but are arranged in reverse order.

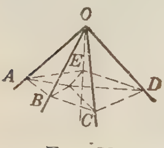

FIG. 82.

Regular solids. There are five and only five convex polyhedrons bounded by equal regular polygons. These are shown in Fig. 83, and are, from left to

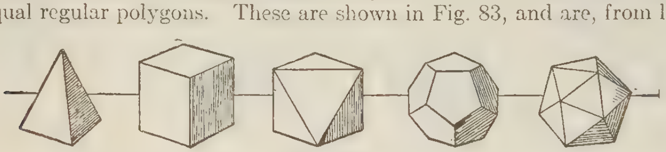

FIG. 83.

right, REGULAR TETRAHEDRON with four faces that are equilateral triangles; CUBE, with six square faces; REGULAR OCTAHEDRON, with eight faces that are equilateral triangles, REGULAR DODECAHEDRON, with twelve faces that are regular pentagons; and a REGULAR ICOSAHEDRON, with twenty faces that are equilateral triangles.

29. Areas and lengths in plane figures

Square (Fig. 84). Side $= a$, diagonal $= d$.

$d = a\sqrt{2} = 1.414a$. $a = \frac{1}{2}d\sqrt{2} = 0.707d$. $Area = a^2 = \frac{1}{2}d^2$.

Rectangle (Fig. 85). Sides $= a, b$; diagonal $= d$.

$d^2 = a^2 + b^2$. $Area = ab$.

Parallelogram (Fig. 86). Sides $= a, b$; altitude on side $a = h$; diagonals $= d_1, d_2$; acute angle at vertex $= C$.

$d_2^2 = a^2 + b^2 - 2ab \cos C\cdot$ $d_1^2 = a^2 + b^2 + 2ab \cos C$. $2(a^2 + b^2) = d_1^2 + d_2^2$. $Area = ah$.

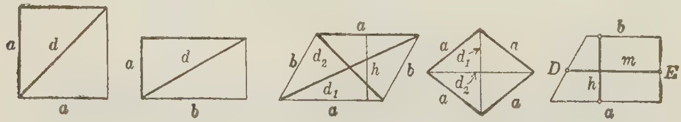

FIG. 84. FIG. 85. FIG. 86. FIG. 87. FIG. 88.

Rhombus (Fig. 87.) Each side $= a$; diagonals $= d_1, d_2$.

$d_1^2 + d_2^2 = 4a^2$. $Area = \frac{1}{2}d_1d_2$.

Trapezoid (Fig. 88). Parallel sides $= a$, b; altitude $= h$; mid-line $DE = m$.

Area $= \frac{1}{2}h(a + b) = hm$.

Triangle (Fig. 89). Sides $= a, b, c$; angles $= A, B, C$; altitude on side $a = h$; radius of inscribed circle $= r$, radius of circumscribed circle $= R$. Let $s = \frac{1}{2}(a + b + c)$.

$r = \sqrt{(s - a)(s - b)(s - c)/s}$; $R = \frac{1}{2}a/\sin A = \frac{1}{2}b/\sin B = \frac{1}{2}c/\sin C$.

Area $= \frac{1}{2}$ *base* $\times$ *altitude* $= \frac{1}{2}ah = \frac{1}{2}ab \sin C = \frac{1}{2}bc \sin A = \frac{1}{2}ca \sin B = rs = \frac{1}{4}abc/R = \sqrt{s(s - a)(s - b)(s - c)} = 2R^2 \sin A \sin B \sin C = r^2 \cot \frac{1}{2}A \cot \frac{1}{2}B \cot \frac{1}{2}C = \frac{1}{2}a^2 \sin B \sin C/\sin A$.

Length of median from vertex $A = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$. Length of bisector of angle $A = \sqrt{ab[(a + b)^2 - c^2]/(a + b)}$.

Equilateral triangle (Fig. 90). Side $= a$.

Altitude, $h = \frac{1}{2}a\sqrt{3} = 0.86603a$. *Area* $= \frac{1}{4}a^2\sqrt{3} = 0.43301a^2$.

Right triangle (Fig. 91). Sides $= a, b$; hypotenuse $= c$.

$c^2 = a^2 + b^2$. *Area* $= \frac{1}{2}ab = \frac{1}{2}a^2 \tan B = \frac{1}{2}a^2 \cot A = \frac{1}{4}c^2 \sin 2A$.

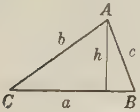

FIG. 89.

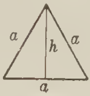

FIG. 90.

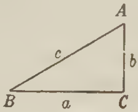

FIG. 91.

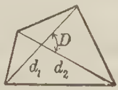

FIG. 92.

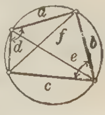

FIG. 93.

Any quadrilateral (Fig. 92). Sides $= a, b, c, d$; diagonals $= d_1, d_2$ forming angle D; line joining mid-points of diagonals $= m$.

$a^2 + b^2 + c^2 + d^2 = d_1^2 + d_2^2 + 4m^2$. *Area* $= \frac{1}{2}d_1d_2 \sin D$.

Quadrilateral inscribed in a circle (Fig. 93). Sum of opposite angles $= 180°$. Sides $= a, b, c, d$; angle formed by sides a and $b = X$; diagonals $= e, f$.

$ac + bd = ef$. Let $s = \frac{1}{2}(a + b + c + d)$.

Area $= \sqrt{(s - a)(s - b)(s - c)(s - d)} = (ab + cd) \sin X$.

Regular polygons (Fig. 94). Side $= a$, number of sides $= n$.

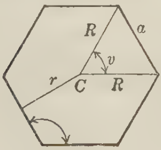

FIG. 94.

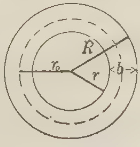

FIG. 95.

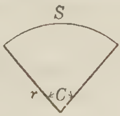

FIG. 96.

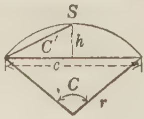

FIG. 97.

Vertex angle $= 360°(n - 2)/n$. Central angle (as v) $= 360°/n$. Radius of inscribed circle $= r = \frac{1}{2}a \cot \frac{1}{2}v = \frac{1}{2}a \tan \frac{1}{2}A$. Radius of circumscribed circle $= R = \frac{1}{2}a/\sin \frac{1}{2}v = \frac{1}{2}a/\cos \frac{1}{2}A$. *Area* $= \frac{1}{2}nar = a^2(\frac{1}{4}n \cot \frac{1}{2}v) = a^2k$. Values of k are given in following table:

n	3	4	5	6	7	8	9	10	11	12
k	0.433	1.000	1.720	2.598	3.634	4.828	6.182	7.694	9.366	11.196

Circle. Radius $= r$, diameter $= d$.

Ratio of circumference to diameter $= \pi = 3.1415927$. Circumference $= 2\pi r = \pi d$. *Area* $= \pi r^2 = \frac{1}{4}\pi d^2 = 0.7854d^2$. See Table 6, p. 1460.

Circular ring (or ANNULUS). (Fig. 95.) Outer radius $= R$, inner radius $= r$.

Mean radius $= r_0 = \frac{1}{2}(R + r)$; width $= b = R - r$. *Area* $= \pi(R^2 - r^2) = \pi(R + r)(R - r) = 2\pi r_0 b$.

Circular arc (Fig. 96). Radius $= r$, length of arc $= S$, central angle $= C$.

$S = rC$ (when C is expressed in radians) $= \pi rC/180 = 0.01745rC$ (when C is expressed in degrees). See Tables 10–13, p. 1466. If chord of arc (Fig. 97) $=$ S, and chord of half the arc $= c'$, then $S = 2c' + \frac{1}{3}(2c' - c)$, approximately (Huyghen's formula).

Circular sector (Fig. 96). *Area* $= \frac{1}{2}Sr = \frac{1}{2}r^2C$ (when C is expressed in radians) $= \pi r^2C/360$ (when C is expressed in degrees) $= 0.008727r^2C$.

Circular segment (Fig. 97). Height of segment $= h$, chord $= c$, chord of half arc $= C'$.

$C'^2 = 2hr$; $c = 2r \sin \frac{1}{2}C$; $h = r(1 - \cos \frac{1}{2}C)$, $r = (\frac{1}{4}c^2 + h^2)/2h$. *Area* $= \frac{1}{2}r^2(C - \sin C)$, (angle C in radians) $= \frac{1}{2}(rS + ch - cr)$. Approximate formula: *Area* $= \frac{4}{3}h^2\sqrt{2r/h - 0.608}$. See Table 9, p. 1464.

Parabolic segment (Fig. 98). Chord $= c$, height $= h$, arc $= S$.

$$Area = \tfrac{2}{3}ch. \quad S = \tfrac{1}{2}\sqrt{c^2 + 16h^2} + \frac{c^2}{8h} \text{ Nap log } (4h + \sqrt{c^2 + 16h^2})/c.$$

Area between two chords c, C, parallel and at a distance d apart $= \frac{2}{3}d\,\dfrac{C^3 - c^3}{C^2 - c^2}$.

Ellipse (Fig. 99). Semi-axes, major $= a$, minor $= b$.

Area between BB' *and* PP' $= xy + ab \sin^{-1} x/a$. *Area of sector* AOP $= \frac{1}{2}ab \cos^{-1} x/a$. *Area of ellipse* $= \pi ab$. *Perimeter of ellipse* $= 4aE$. Values of E for $t = b/a$ are as follows:

t	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
E	1.000	1.016	1.051	1.097	1.151	1.211	1.276	1.346	1.418	1.493	1.571

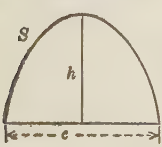

FIG. 98.

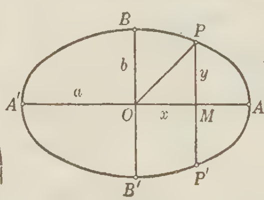

FIG. 99.

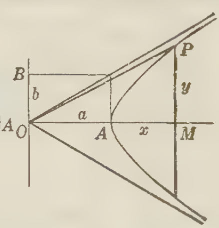

FIG. 100.

Hyperbola (Fig. 100). Semi-axes, transverse $= a$, conjugate $= b$.

Area $AMP = \frac{1}{2}xy - \frac{1}{2}ab$ Nap log $(x/a + y/b)$. *Area* $OAP = \frac{1}{2}ab$ Nap log $(x/a + y/b) = \frac{1}{2}abu$, where $x = a \cosh u$.

Cycloid. (See Fig. 177, p. 1411.) Radius of rolling circle $= a$.

Length of arc $OP = 4a(1 - \cos \frac{1}{2}\theta)$. Length of entire arc $OMN = 8a$.

Area between arc OMN and base $OX = 3\pi a^2 =$ three times area of rolling circle.

Epicycloid. (See Fig. 179, p. 1412.) Arc $AP = 4r\,(R + r)\,(1 - \cos (R\theta/2r))/R$. *Area* $AOP = [r(R + r)(R + 2r)](R\theta/r - \sin R\theta/r)/2R$.

Hypocycloid. (See Fig. 180, p. 1412.) Formulas for arc and area are the same as for epicycloid with sign of r changed.

Catenary. (See Fig. 184, p. 1413.) Arc $BP = a \sinh x/a = \sqrt{y^2 - a^2}$ where x, y, are co-ordinates of P. *Area* $OMPB = a^2 \sinh x/a = a\sqrt{y^2 - a^2}$.

Spiral of Archimedes. (See Fig. 185, p. 1413.) Arc $OP = \frac{1}{2}a[\theta\sqrt{1 + \theta^2} + \text{Nap log}\,(\theta + \sqrt{1 + \theta^2})]$, θ in radians.

Simpson's rule for approximate measure of an area (Fig. 101). Divide the area by equidistant parallel lines into an even number n of strips each of width b, and denote the lengths of the parallel sides of these strips successively by $y_0, y_1, y_2, y_3 \ldots, y_n$. Then an approximate value for the area is, *Area* $= \frac{1}{3}b(y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + \ldots + 4y_{n-1} + y_n)$, or

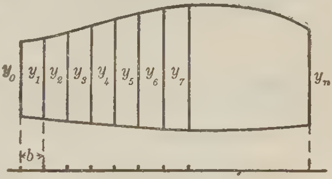

FIG. 101.

$$= \tfrac{1}{3}b[(y_0 + y_n) + 4(y_1 + y_3 + y_5 + \ldots y_{n-1}) + 2(y_2 + y_4 + y_6 + \ldots y_{n-2})].$$

Increasing the number of strips gives a closer approximation.

30. Surfaces and volumes of solids

Prism (Fig. 102). Perimeter of a plane section $FGHKL$ cutting all edges at right angles (right section) $= c$; length of lateral edge $AA' = l$; altitude (perpendicular distance between upper and lower bases) $= h$, area of base $ABCDE = B$.

Lateral surface $= cl$. *Volume* $= Bh$.

Right-circular cylinder (Fig. 64). Diameter of base $= d$, altitude $= h$. *Lateral surface* $= \pi dh$. *Total surface* $= \pi d(h + \frac{1}{2}d)$. *Volume* $= \pi d^2h/4$.

Truncated right-circular cylinder (Fig. 103). Diameter of circular base $= d$, arithmetic mean of longest and shortest elements $= h (= \frac{1}{2}(AB + CD))$.

Lateral surface $= \pi dh$. *Volume* $= \pi d^2h/4$.

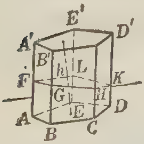

FIG. 102.

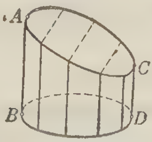

FIG. 103.

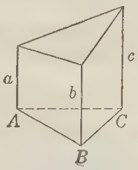

FIG. 104.

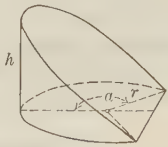

FIG. 105.

Truncated triangular prism (Fig. 104). Area of base $ABC = B$, lengths of edges $= a, b, c$.

Volume $= \frac{1}{3}B(a + b + c)$.

Ungula of right-circular cylinder (Fig. 105). Base is a circular segment with central angle $2a$ (radians) and area B. (See Art. 29.)

Lateral area $= hr(2 \sin a - a \cos a)/(1 - \cos a)$. *Volume* $= h(\frac{2}{3}r^2 \sin^3 a - B \cos a)/(1 - \cos a)$.

Hollow right-circular cylinder. Outer and inner diameters = D, d, altitude = h, thickness = t.

Volume $= \pi ht(D + d)/2$.

Regular pyramid (Fig. 75). Base $ABCDE$ is a regular polygon, lateral faces are equal isosceles triangles. Side of base = a, area of base = B (see Art. 29), altitude of a lateral face (slant height of pyramid) = s (= VF), altitude of pyramid (= VO) = h.

Lateral area $= \frac{1}{2}nas$. *Volume* $= \frac{1}{3}Bh$.

Right-circular cone (Fig. 74). Radius of base = r (= OB), slant height $VB = s$, altitude = $h = VO$.

$s = \sqrt{h^2 + r^2}$. *Lateral area* $= \pi rs$. *Total area* $= \pi r(s + r)$. *Volume* $= \frac{1}{3}\pi hr^2$.

Frustum of a regular pyramid (Fig. 76). Altitude of a lateral face (slant height of frustum FF') = s, perimeter of bases = p, P; areas of bases = b, B; altitude of frustum = $h = OO'$.

Lateral area $= \frac{1}{2}s(p + P)$. *Volume* $= \frac{1}{3}h(B + b + \sqrt{Bb})$.

Frustum of right-circular cone (Fig. 77). Slant height = s, radii of bases = r_1, r_2, altitude = $h = OO'$

$s = \sqrt{h^2 + (r_2 - r_1)^2}$. *Lateral area* $= \pi s(r_1 + r_2)$. *Total area* $= \pi(r_1^2 + r_2^2 + s(r_1 + r_2))$. *Volume* $= \frac{1}{3}\pi h(r_1^2 + r_2^2 + r_1r_2)$.

Any pyramid or cone. Altitude = h, area of base = B.

Volume $= \frac{1}{3}Bh$.

Frustum of any pyramid or cone. Altitude = h, areas of bases = b, B.

Volume $= \frac{1}{3}h(B + b + \sqrt{Bb})$.

Regular polyhedrons (see Art. 28). Edge = a.

	Tetrahedron	*Cube*	*Octahedron*	*Dodecahedron*	*Icosahedron*
Area.	$1.7321a^2$	$6a^2$	$3.4641a^2$	$20.646a^2$	$8.6603a^2$
Volume.	$0.1179a^3$	a^3	$0.4714a^3$	$7.6631a^3$	$2.1817a^3$

Sphere. Radius = r, diameter = d.

Area of surface $= 4\pi r^2 = \pi d^2 =$ *four great circles*. Volume $= \frac{4}{3}\pi r^3 = \frac{1}{6}\pi d^3$. Table 7.

Spherical polygon (see Art. 28.) Sum of angles in degrees = S, number of sides = n. Spherical excess $= E = S - (n - 2)180$ (in degrees).

Area $= \pi r^2E/180$.

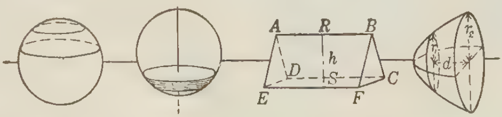

FIG. 106. FIG. 107. FIG. 108. FIG. 109.

Zone (Fig. 106) is a portion of a spherical surface included between two parallel plane sections. Altitude = h (perpendicular distance between parallel planes), radius of sphere = r.

Area of zone $= 2\pi rh$ (product of circumference of a great circle and altitude of zone).

Spherical segment (Fig. 107) is the portion of sphere included between two parallel plane sections called bases. Radii of bases = b, B; altitude of segment = h (perpendicular distance between bases).

Volume $= \frac{1}{2}\pi h(B^2 + b^2 + \frac{1}{3}h^2)$.

Wedge (Fig. 108). Base $DEFC$ is a rectangle, $CF = b$, $EF = L$, edge $AB = l$, at perpendicular distance $RS = h$ from base.

Volume $= \frac{1}{6}bh(2L + l)$.

Prismatoid. Bases are polygons in parallel planes, areas = b, B. Lateral faces are trapezoids or triangles whose vertices coincide with certain vertices of the bases. Altitude = h (perpendicular distance between bases), area of plane section midway between bases = m.

Volume $= \frac{1}{6}h(b + B + 4m)$.

Segment of paraboloid (Fig. 109). Radii of bases = r_1, r_2; altitude = d.

Volume $= \frac{1}{2}\pi d(r_1^2 + r_2^2)$.

Ellipsoid (Fig. 110). Semi-axes = a, b, c.

Volume $= \frac{4}{3}\pi abc$.

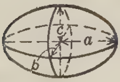

Fig. 110.

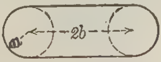

Fig. 111.

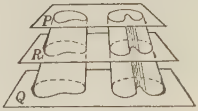

Fig. 112.

Theorems of Pappus. (1) If a closed plane figure is revolved about an exterior axis in its own plane, the volume of the solid generated equals the product of the area of the plane figure by the length of the circular path described by its center of gravity. (2) If a plane curve is revolved about an exterior axis in its plane, the area of the surface generated equals the product of the arc of the curve by the length of the circular path described by its center of gravity. These theorems are useful in determining centers of gravity when other quantities involved are known, and also in calculating areas and volumes when centers of gravity are given.

Torus (ANCHOR RING). (Fig. 111.) Application of preceding Theorems of Pappus gives the following results:

Area $= 2\pi a \times 2\pi b = 4\pi^2 ab$. *Volume* $= \pi a^2 \times 2\pi b = 2\pi^2 a^2 b$.

Cavalieri's theorem (Fig. 112). If two solids are included between a pair of parallel planes, and if the two sections cut from them by any plane parallel to the including planes are equal in area, then the volumes of the solids are equal.

TRIGONOMETRY

31. Trigonometric functions of an acute angle

In Fig. 113 take a point B on side AD of angle A, and draw BC perpendicular to side AE, forming the right triangle BAC. In this triangle, for

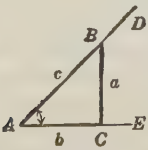

Fig. 113.

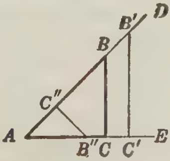

Fig. 114.

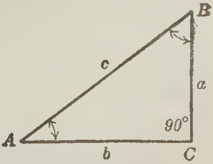

Fig. 115.

angle A, $BC(=a)$ is called OPPOSITE SIDE, and $AC\ (=b)$, ADJACENT SIDE. The trigonometric functions of A are sine of A, cosine of A, tangent of A, cotangent of A, secant of A, and cosecant of A, written, respectively sin A, cos A, tan A, cot A, sec A, csc A, and defined as equal to ratios as follows:

$$\sin A = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{a}{c}, \qquad \csc A = \frac{\text{hypotenuse}}{\text{opposite side}} = \frac{c}{a},$$

$$\tan A = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{a}{b}, \qquad \cot A = \frac{\text{adjacent side}}{\text{opposite side}} = \frac{b}{a},$$

$$\cos A = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{b}{c}, \qquad \sec A = \frac{\text{hypotenuse}}{\text{adjacent side}} = \frac{c}{b}.$$

Sine and cosine of A are called CO-FUNCTIONS of each other, as also are tangent, and cotangent, secant and cosecant.

Functions of A are independent of the position of B on AD. In fact, by plane geometry (Art. 25), in Fig. 114, when $B'C'$ is perpendicular to AE, and $B''C''$ perpendicular to AD, $BC/AB = B'C'/AB' = B''C''/AB''$. That is, sin A equals any one of these ratios. Any one of the triangles ABC, $AB'C'$, $AB''C''$ might be used to give trigonometric functions of A.

In the right triangle ABC (Fig. 115), angles A and B are COMPLEMENTARY angles (sum $= 90°$). Sin $A = \cos B$, $\tan A = \cot B$, $\sec A = \csc B$, $\cos A = \sin B$, $\cot A = \tan B$, $\csc A = \sec B$. Any function of an angle equals the corresponding co-function of the complementary angle.

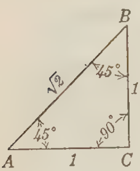

FIG. 116.

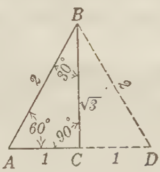

FIG. 117.

Functions of 30°, 45° and 60°. In Fig. 116, triangle ABC is an ISOSCELES RIGHT TRIANGLE, $AC = BC$, angle $A =$ angle $B = 45°$, sides AC and BC are of unit length and $AB = \sqrt{2}$. In Fig. 117, triangle ABD is equilateral with sides 2 units in length, each angle is 60°. In right triangle ABC, $A = 60°$, $B = 30°$, $BC = \sqrt{3}$.

Angle	Sin	Cos	Tan	Cot	Sec	Csc
30°	1/2	$\sqrt{3}/2$	$\sqrt{3}/3$	$\sqrt{3}$	$2\sqrt{3}/3$	2
45°	$\sqrt{2}/2$	$\sqrt{2}/2$	1	1	$\sqrt{2}$	$\sqrt{2}$
60°	$\sqrt{3}/2$	1/2	$\sqrt{3}$	$\sqrt{3}/3$	2	$2\sqrt{3}/3$

In addition to the six functions defined above, the following are also used. Versine of A = vers $A = 1 - \cos A$. Coversine of A = covers $A = 1 - \sin A$. Haversine of A = havers $A = \frac{1}{2}(1 - \cos A)$.

Tables for trigonometric functions. Natural values. Tables 14, 15, pp. 1468-1471, give values of sine, cosine, tangent and cotangent for all angles less than 90° at 10-minute intervals. (For 90°, see Art. 38.) For intermediate angles interpolation may be used, with the caution that sine and tangent increase as angle increases, while cosine and cotangent decrease as angle increases. Values of sine and tangent are under captions at the top of pages; of cosine and cotangent above captions at the bottom of pages.

Examples. Sine 25° 12′ = 0.4258. Cos 25° 12′ = 0.9048. Tan 36° 18′ = 0.7346 = tan 36° 20′ *minus* the correction for 2′ (= 9 units). Cot 26° 17′ = 2.024 = cot 26° 20′

plus the correction for 3′ (= 4 units). For values of secant and cosecant, use the relations secant = 1/cosine, cosecant = 1/sine and Table 5 (p. 1456).

Natural values by slide rule. See Art. 5.

Tables of logarithms of trigonometric functions. Tables 18, 19.

Annex − 10 to the given tabular values. Interpolation will be inaccurate for logarithmic sines and tangents of angles less than 6°, and for cosines and cotangents of angles greater than 83°. Five- or more-place tables should be consulted for such angles. In interpolation, follow the directions given above for natural values.

Example. Log sin 28° 32′ = 9.6792 − 10. Log cot 19° 38′ = 0.4477.

Relations between functions. $\sin^2 A + \cos^2 A = 1$. $\sec^2 A = 1 + \tan^2 A$. $\csc^2 A = 1 + \cot^2 A$. $\sin A = 1/\csc A$. $\tan A = 1/\cot A$. $\sec A = 1/\csc A$.

Expressions for sin, cos, and tan in terms of each of the other functions.

$$\sin A = \sqrt{1 - \cos^2 A} = \tan A/\sqrt{1 + \tan^2 A} = 1/\sqrt{1 + \cot^2 A}$$
$$= \sqrt{\sec^2 A - 1}/\sec A = 1/\csc A.$$

$$\tan A = \sin A/\sqrt{1 - \sin^2 A} = \sqrt{1 - \cos^2 A}/\cos A = 1/\cot A$$
$$= \sqrt{\sec^2 A - 1} = 1/\sqrt{\csc^2 A - 1}.$$

$$\cos A = \sqrt{1 - \sin^2 A} = 1/\sqrt{\tan^2 A + 1} = \cot A/\sqrt{\cot^2 A + 1}$$
$$= 1/\sec A = \sqrt{\csc^2 A - 1}/\csc A.$$

32. Functions of an obtuse angle

Line definitions. In Fig. 118, the ratio definitions of the functions of angle AOP (Art. 31) are replaced by line definitions as marked, the unit of length being the radius of the circle (unit circle). For an OBTUSE ANGLE (angle between 90° and 180°), the functions are defined for angle AOP as in Fig. 119, with the following conventions (in both figures) as to positive and negative: Horizontal lines OQ (= cosine) and BC (= cotangent) to the right of the vertical diameter (Fig. 118) are positive, to left (Fig. 119) are negative. Vertical line as PQ (= sine) above horizontal diameter is positive, AT (= tangent) below (Fig. 119) is negative. OC (= cosecant) extending *along* terminal side OP, is positive, while OT (= secant), measured along OP *produced* (Fig. 119) is negative. For an obtuse angle, the sine and cosecant are positive, and the other functions negative. To find the natural values of the functions of an obtuse angle, write the angle in one of the forms 90° + x or 180° − x, where x is less than 90°, and use the following formulas:

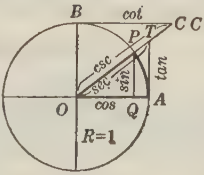

FIG. 118.

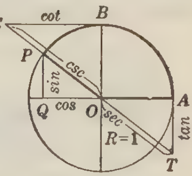

FIG. 119.

A	Sin A	Cos A	Tan A	Cot A	Sec A	Csc A
90° + x	cos x	− sin x	− cot x	− tan x	− csc x	sec x
180° − x	sin x	− cos x	− tan x	− cot x	− sec x	csc x

Example. Find sin 108°. Write 108° = 90° + 18°, or 180° − 72°. Hence sin 108° = cos 18° = sin 72° = 0.9511. Find tan 128°. Write 128° = 90° + 38°, or 180° − 52°. Then tan 128° = − cot 38° = − tan 52° = − 1.2799.

33. Oblique triangles. Formulas

Sides a, b, c, angles A, B, C, angle A opposite side a, etc.

Law of sines. $a/\sin A = b/\sin B = c/\sin C$. (See also Art. 29.)

Law of tangents. $(a + b)/(a - b) = \tan \frac{1}{2}(A + B)/\tan \frac{1}{2}(A - B)$.

Law of cosines. $a^2 = b^2 + c^2 - 2bc \cos A$. $b^2 = c^2 + a^2 - 2ca \cos B$. $c^2 = a^2 + b^2 - 2ab \cos C$. $\operatorname{Cos} A = (b^2 + c^2 - a^2)/2bc$. $\operatorname{Cos} B = (c^2 + a^2 - b^2)/2ac$. $\operatorname{Cos} C = (a^2 + b^2 - c^2)/2ab$.

If $s = \frac{1}{2}(a + b + c)$ and $r = \sqrt{(s - a)(s - b)(s - c)/s}$, $\sin \frac{1}{2} A = \sqrt{(s - b)(s - c)/bc}$. $\operatorname{Cos} \frac{1}{2}A = \sqrt{s(s - a)/bc}$, $\tan \frac{1}{2} A = r/(s - a)$.

Area $= \sqrt{s(s - a)(s - b)(s - c)} = \frac{1}{2}bc \sin A = \frac{1}{2}a^2 \sin B \sin C/\sin A$.

$\operatorname{Sin} A = 2\ \mathit{Area}/bc$. $\operatorname{Sin} B = 2\ \mathit{Area}/ca$. $\operatorname{Sin} C = 2\ \mathit{Area}/bc$.

Sides. $a = b \cos C + c \cos B$. $b = \cos A + a \cos C$. $c = a \cos B + b \cos A$.

Sum of sines. $\operatorname{Sin} A + \sin B + \sin C = 4 \cos \frac{1}{2}A \cos \frac{1}{2}B \cos \frac{1}{2}C$.

Sum of cosines. $\operatorname{Cos} A + \cos B + \cos C = 4 \sin \frac{1}{2}A \sin \frac{1}{2}B \sin \frac{1}{2}C + 1$.

Sum of tangents. $\operatorname{Tan} A + \tan B + \tan C = \tan A \tan B \tan C$.

Squares of sines. $\operatorname{Sin}^2 A + \sin^2 B + \sin^2 C = 2 \cos A \cos B \cos C + 2$.

Products of cotangents. $\operatorname{Cot} A \cot B + \cot B \cot C + \cot C \cot A = 1$.

Sines of twice the angles. $\operatorname{Sin} 2A + \sin 2B + \sin 2C = 4 \sin 2A \sin 2B \sin 2C$.

34. Solution of plane triangles

A triangle has six parts, the sides a, b, c, and angles A, B, C, angle A being opposite a, etc. Certain necessary relations exist between the parts, as that the sum of the angles $= 180°$, the sum of two sides is greater than the third side, and the greater side is opposite the greater angle. With these restrictions, a triangle can be determined when one side and two other parts are given. The determination of unknown parts is called SOLVING THE TRIANGLE. The formulas given below for the usual cases will give the unknown parts by substitution of numerical data, using tables of natural values and computing by slide rule, or using logarithms and tables.

Right triangles

(Fig. 113). $\angle A + \angle B = 90°$. $\angle C = 90°$.

Given an angle and opposite side, as A a. Unknowns, B, c, b. Formulas: $B = 90° - A$; $c = a/\sin A$, $b = a \cot A$.

Given an angle and adjacent side, as A, b. Unknowns, B, a, c. Formulas: $B = 90° - A$. $a = b \tan A$. $c = b/\cos A = b \sec A$

Given two sides a, b. Formulas: $\tan A = a/b$. $B = 90° - A$. $c = \sqrt{a^2 + b^2} = a/\sin A = a \csc A$.

Given a side, as a, **and hypotenuse** c. Formulas: $\sin A = a/c$. $B = 90° - A$. $b = \sqrt{(c + a)(c - a)}$.

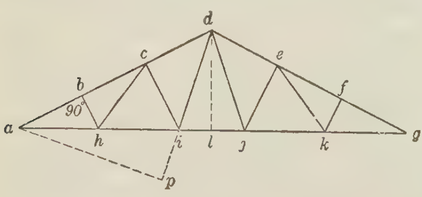

FIG. 120.

Example. In the roof truss (Fig. 120), with normal struts bh, ci, etc., the span $= ag = 60$ ft., rise $= dl = 15$ ft., find angle dil, and length of perpendicular ap let fall from a upon member di produced.

Solution. In the right triangle adl, $al = 30$ ft., $dl = 15$ ft. Hence $\tan \alpha = dl/al = 0.5$, and angle $\alpha = 26° 34'$ (Table 15). $ad = al \sec \alpha = 30 \times 1.118 = 33.54$. In the right

triangle *aci*, $ac = \frac{2}{3}\ ad = 22.36$. $ai = ac \sec a = 22.36 \times (1.118) = 25$. In right triangle *dil*, $il = al - ai = 5$, $dl = 15$. $\tan \angle dil = 3$. Angle $dil = 71° 34'$. In right triangle *aip*, $ai = 25$, angle $aip = 71° 34'$ (being equal to angle *dil*). $ap = ai \sin \angle aip = 25 \times 0.9487 = 23.72$. *Ans.*

Oblique triangles

Before solving, it is well to construct a triangle with the given parts to scale, using a protractor for laying off the given angle or angles. The unknown parts may then be measured and estimated.

Given one side and any two angles, as a, A, B. $C = 180° - (A + B)$. By the law of sines (Art. 33) $b = a \sin B/\sin A$ and $c = a \sin C/\sin A$.

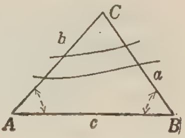

FIG. 121.

Example. A and B are stations 1800 ft. apart on one side of a river, C a station on the opposite side. To determine the distances from A and B to C, angles CAB and CBA are measured and found to be $48° 30'$, and $54° 24'$, respectively. Find AC and BC.

Solution. (Fig. 121.) A side c and two angles are given. $c = 1800$, angle $CAB = A = 48° 30'$, angle $CBA = B = 54° 24'$. Hence angle $C = 180° - (48° 30' + 54° 24') = 77° 6'$.

$a = 1800 \sin 48° 30'/\sin 77° 6'$. $\log a = \log c + \log \sin A - \log \sin c$.

$b = 1800 \sin 54° 24'/\sin 77° 6'$. $\log b = \log c + \log \sin B - \log \sin C$.

c	log 1800	= 3.2553	c	log 1800	= 3.2553
A	log sin 48° 30′	= 9.8745	B	log sin 54° 24′	= 9.9101
	Sum	= 3.1298		Sum	= 3.1654
C	log sin 77° 6′	= 9.9889	C	log sin 77° 6′	= 9.9889
a	log 1383	= 3.1409	b	log 1501	= 3.1765

Given two sides and included angle, as a, b, C. Let the greater of the given sides be a. Then $\frac{1}{2}(A + B) = 90° - \frac{1}{2}C$. By Art. 33 (law of tangents) $\tan \frac{1}{2}(A - B) = (a - b) \cot \frac{1}{2}C/(a + b)$, giving $\frac{1}{2}(A - B)$. Then $A = \frac{1}{2}(A + B) + \frac{1}{2}(A - B)$. $B = \frac{1}{2}(A + B) - \frac{1}{2}(A - B)$. $c = a \sin C/\sin A$. *Check* by the law of sines (Art. 33).

Alternative method. Use the law of cosines (Art. 33) to find the unknown side. $c^2 = a^2 + b^2 - 2ab \cos C$. $\sin B = b \sin C/c$. $(B < 90°)$. $A = 180° - (B + C)$. This method is useful when the third side only is required.

Formulas for sin B and sin A, using the law of sines do not determine these angles without ambiguity, since an angle (x) and its supplement $(180° - x)$ have the same sine (Art. 32). But it $b < a$, angle B must be acute. From $A = 180° - (B + C)$, angle A may be found and the result checked by the law of tangents.

Example. Two stations A and B on opposite sides of a mountain are both visible from a third station C. Distances AC and BC are 11.5 mi. and 9.4 mi., respectively; angle $ACB = 59° 31'$. Find the distance from A to B.

Solution. Two sides a, b, and the included angle C are given. Side c is required. $a = 9.4$, $b = 11.5$, $C = 59° 31'$.

$$c = \sqrt{a^2 + b^2 - 2ab \cos C} = \sqrt{88.36 + 132.25 - 109.68} = \sqrt{110.9} = 10.53 \text{ mi.}$$

Given three sides, a, b, c.

$$s = \tfrac{1}{2}(a + b + c). \quad r = \sqrt{(s - a)(s - b)(s - c)/s}.$$

$$\tan \tfrac{1}{2}A = \frac{r}{s - a}, \quad \tan \tfrac{1}{2}B = \frac{r}{s - b}, \quad \tan \tfrac{1}{2}C = \frac{r}{s - c}.$$

Check. $A + B + C = 180°$.

Alternative method. $\cos A = (b^2 + c^2 - a^2)/2bc$. If $b < a$, find B (one acute angle) from $\sin B = b \sin A/a$. $C = 180° - (A + B)$.

Given two sides and angle opposite one of them, as, a, b, A.

(1) *There are two solutions*, if A is acute and the value of a lies between b and $b \sin A$, as in Fig. 122. Here $b \sin A = CP$, and triangles ACB' and ACB are both solutions. To see if two solutions exist, examine the given data and calculate $b \sin A$. If $a < b$ and $> b \sin A$, there are two solutions. Solve triangle ABC by the formulas $\sin B = b \sin A/a$; angle $ACB = 180° - (A + B)$; $AB = a \sin \angle ACB/\sin A$. Then in triangle $AB'C$, angle $AB'C = 180° - B$; angle $ACB' = 180° - A - \angle AB'C$; $AB' = a \sin \angle ACB'/\sin A$.

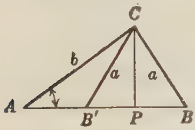

FIG. 122.

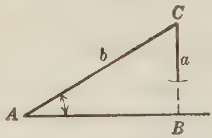

FIG. 123.

(2) *There is no solution* if A is acute and the value of a is less than $b \sin A$, because the side opposite A is not long enough to reach the opposite side, *i.e.*, $a < CB$. (Fig. 123.)

(3) *There is one solution* in all other cases. Solve as in triangle ABC above.

35. Angles of any magnitude

The notion of magnitude of an angle must be generalized for purposes of trigonometric analysis. The general concept may be stated thus: An angle is considered as generated by a line (GENERATING LINE) which first coincides with one side (INITIAL SIDE) of the angle, then revolves about the vertex and finally coincides with other side (TERMINAL SIDE). An angle generated by counterclockwise rotation is POSITIVE (Fig. 124a), by clockwise rotation, NEGATIVE (Fig. 124b). The generating line may be regarded as making any number of complete revolutions before coinciding finally with the terminal side, hence the magnitude of an angle may be as large as we choose.

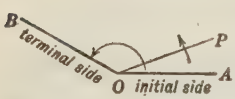

FIG. 124a.

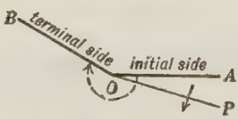

FIG. 124b.

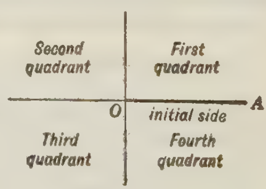

FIG. 125.

Four quadrants. It is customary to divide the plane about the vertex of an angle into four quadrants by drawing two mutually perpendicular lines through the vertex, one of these lines (OA) coinciding with the initial side of all angles (Fig. 125). An angle is said to lie in a certain quadrant when its terminal side falls in that quadrant. Angles between 0° and 90° lie in first quadrant, between 90° and 180° in second quadrant, between 180° and 270° in third quadrant, between 270° and 360° in fourth quadrant. For *negative* angles, those between 0° and $-90°$ lie in fourth quadrant, between $-90°$ and $-180°$ in third quadrant, between $-180°$ and $-270°$ in second quadrant, between $-270°$ and $-360°$ in first quadrant. Addition to or subtraction from an angle of multiples of 360° gives an angle of different magnitude but the same terminal side, hence in the same quadrant.

36. Angle measurement

Two systems of measuring angles are in use in engineering.

Degree measure. The unit angle is one DEGREE, viz., an angle at the center of a circle whose intercepted arc equals 1/360th of the circumference.

Circular measure. The unit angle is one RADIAN, viz.: an angle at the center of a circle intercepting an arc equal in length to the radius of the circle. (Fig. 126.) For any central angle in a circle, the number of radians in the angle = *length of intercepted arc/length of radius.*

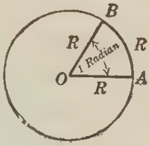

FIG. 126.

Relation between unit angles. 1 degree = 0.01745 radian = $\pi/180$ radian. 1 radian = 57.2957 degrees = $180/\pi$ degrees. 90° = $\pi/2$ radians. 180° = π radians. 270° = $3\pi/2$ radians. 360° = 2π radians (π = 3.14159265). See also Tables 10 to 13, inclusive.

37. Functions of an angle in any quadrant

In the first and second quadrants, functions are defined by the lengths of lines in the unit circle as in Figs. 118 and 119, in the third and fourth quadrants as in Figs. 127 and 128, respectively. For conventions as to positive and negative lines, see Art. 32. The rule for signs of functions of angles in different quadrants is shown in Fig 129. To write down the numerical

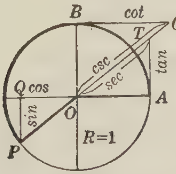

FIG. 127.

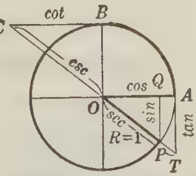

FIG. 128.

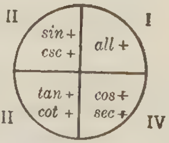

FIG. 129.

values of functions of an angle of any magnitude, express the angle in degrees, and, if necessary, add or subtract multiples of 360° until the magnitude lies between 0° and 360°. Then observe in what quadrant the angle lies. If in the first quadrant, the functions are given by Tables 14 and 15. If in the second quadrant, proceed as in Art. 32.

Third-quadrant angles. Write the magnitude of the angle, which is between 180° and 270°, in the form $180° + x$, or $270° - y$, where x and y are between 0° and 90°, and use the following formulas:

A	Sin A	Cos A	Tan A	Cot A	Sec A	Csc A
$180° + x$	$-\sin x$	$-\cos x$	$\tan x$	$\cot x$	$-\sec x$	$-\csc x$
$270° - y$	$-\cos y$	$-\sin y$	$\cot y$	$\tan y$	$-\csc y$	$-\sec y$

Fourth-quadrant angles. Express the angle, which now lies between 270° and 360°, in either of the forms $270° + y$ or $360 - x$, where x and y are between 0° and 90°, and use the following formulas:

A	Sin A	Cos A	Tan A	Cot A	Sec A	Csc A
$360° - x$	$-\sin x$	$\cos x$	$-\tan x$	$-\cot x$	$\sec x$	$-\csc x$
$270° + y$	$-\cos y$	$\sin y$	$-\cot y$	$-\tan y$	$\csc y$	$-\sec y$

Examples. To find tan 985° 15′. Reduce to an angle between 0° and 360° by subtracting $2 \times 360°$. $985° 15' - 720° = 265° 15'$, which is an angle in the third quadrant. $265° 15' = 180° + 85° 15' = 270° - (4° 45')$. Tan 265° 15′ = tan 85° 15′ = cot 4° 15′ = 13.46. Hence tan 985° 15′ = 13.46. *Ans.*

To find csc (− 385° 20′). Adding 720°, $720° - 385° 20' = 334° 40'$, which is an angle in the fourth quadrant. $334° 40' = 360° - 25° 20' = 270° + 64° 40'$. Csc 334° 40′ = − csc 25° 20′ = − sec 64° 40′ = − 2.337. Hence csc (− 385° 20′) = − 2.337.

Relations between functions of positive and negative angles which are numerically equal are: $\sin(-A) = -\sin A$; $\cos(-A) = \cos A$; $\tan(-A) = -\tan A$; $\cot(-A) = -\cot A$; $\sec(-A) = \sec A$; $\csc(-A) = -\csc A$.

38. Functions of 0°, 90°, 180°, 270°, 360°

Angle	Sin	Cos	Tan	Cot	Sec	Csc
0° and 360°	0	1	0	∞	1	∞
90°	1	0	∞	0	∞	1
180°	0	− 1	0	∞	− 1	∞
270°	− 1	0	∞	0	∞	− 1

39. Angles for which one function has a given value

All angles whose magnitude is expressible as $n \times 180° + (-1)^n \times A$ where n is any integer, positive or negative, and A is expressed in degrees, have the same sine and cosecant as A. (In radians, $\pi n + (-1)^n A$.) If expressible as $n \times 180° + A$, they have the same tangent and cotangent as A. (In radians, $\pi n + A$.) All angles expressible as $n \times 360° \pm A$ have the same secant and cosine as A. (In radians, $\pi n + A$.)

Example. Solve $2 \sin A - \cos A = 0$ for A. Transposing $\cos A$ to the right-hand member and dividing both members by $\cos A$ gives $2 \sin A/\cos A = 1$, or $\tan A = \frac{1}{2}$. The value of A between 0° and 90° satisfying this equation is $A = 26° 34'$. (Table 15.) Hence the solution is: $n \times 180° + 26° 34'$, where n is any integer, positive or negative. Angles between 0° and 360° satisfying the equation are 26° 34′, and 206° 34′ ($n = 1$).

40. Graphs of trigonometric functions

Graphs of trigonometric functions are drawn by plotting values of the angle as abscissas and values of the function as ordinates. See Figs. 130–133. Points on the X-axis corresponding to angles for which the function is zero or infinity, are marked in radians. Graphs of $\sin x$ and $\csc x$ are obtained from

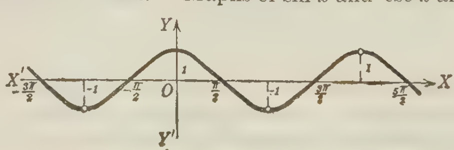

FIG. 130.—Graph of $y = \cos x$.

Figs. 130 and 133, respectively, by drawing the Y-axis through the point $x = -\pi/2$.

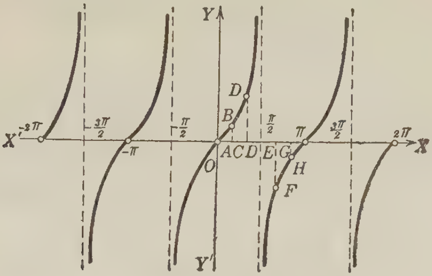

FIG. 131.—Graph of $y = \tan x$.

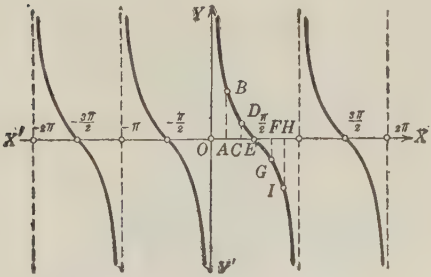

FIG. 132.—Graph of $y = \cot x$.

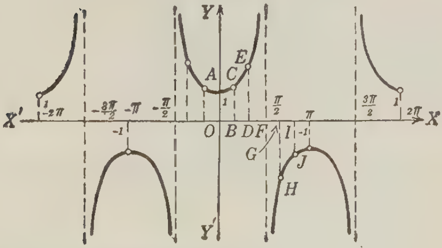

FIG. 133.—Graph of $y = \sec x$.

41. Formulas in plane trigonometry

Relations between functions of the same angle, any quadrant. See Art. 31.

Functions of multiple angles. Sin $2A = 2 \sin A \cos A$. Cos $2A = \cos^2 A - \sin^2 A = 1 - 2 \sin^2 A = 2 \cos^2 A - 1$. Tan $2A = 2 \tan A/(1 - \tan^2 A)$. Cot $2A = (\cot^2 A - 1)/2 \cot A$.

Sin $3A = 3 \sin A - 4 \sin^3 A$. Cos $3A = 4 \cos^3 A - 3 \cos A$. Tan $3A = (3 \tan A - \tan^3 A)/(1 - 3 \tan^2 A)$.

Sin $4A = 4 \sin A \cos A - 8 \sin^3 A \cos A$. Cos $4A = 8 \cos^4 A - 8 \cos^2 A + 1$.

Sin $nA = \sin (n-1) A \cos A + \cos (n-1) A \sin A$. Cos $nA = \cos (n-1) A \cos A - \sin (n-1) A \sin A$.

*Sin $nA = n \cos^{n-1} A \sin A - \binom{n}{3} \cos^{n-3} A \sin^3 A + \binom{n}{5} \cos^{n-5} A \sin^5 A -$, etc.

*Cos $nA = \cos^n A - \binom{n}{2} \cos^{n-2} A \sin^2 A + \binom{n}{4} \cos^{n-4} A \sin^4 A -$, etc.

* In these formulas $\binom{n}{2}$, $\binom{n}{3}$, etc., are binomial coefficients. See Art. 18.

Functions of half angles. Sin $\frac{1}{2}A = \sqrt{\frac{1}{2}(1 - \cos A)}$. Cos $\frac{1}{2}A = \sqrt{\frac{1}{2}(1 + \cos A)}$.

Tan $\frac{1}{2}A = \sqrt{1 - \cos A}/\sqrt{1 + \cos A} = \sin A/(1 + \cos A) = (1 - \cos A)/\sin A$.

Powers in terms of multiple angles. Sin$^2 A = \frac{1}{2}(1 - \cos 2A)$. Cos$^2 A = \frac{1}{2}(1 + \cos 2A)$.

Sin$^3 A = \frac{1}{4}(3 \sin A - \sin 3A)$. Cos$^3 A = \frac{1}{4}(\cos 3A + 3 \cos A)$.

Functions of sum or difference of two angles. Sin $(A + B) = \sin A \cos B + \cos A \sin B$. Sin $(A - B) = \sin A \cos B - \cos A \sin B$.

Cos $(A + B) = \cos A \cos B - \sin A \sin B$.

Cos $(A - B) = \cos A \cos B + \sin A \sin B$.

Tan $(A + B) = (\tan A + \tan B)/(1 - \tan A \tan B)$. Tan $(A - B) = (\tan A - \tan B)/(1 + \tan A \tan B)$.

Cot $(A + B) = (\cot A \cot B - 1)/(\cot A + \cot B)$.

Cot $(A - B) = (\cot A \cot B + 1)/(\cot B - \cot A)$.

Sums, differences, and products of functions.

Sin $A + \sin B = 2 \sin \frac{1}{2}(A + B) \cos \frac{1}{2}(A - B$.

Sin $A - \sin B = 2 \cos \frac{1}{2}(A + B) \sin \frac{1}{2}(A - B)$.

Cos $A + \cos B = 2 \cos \frac{1}{2}(A + B) \cos \frac{1}{2}(A - B)$.

Cos $A - \cos B = -2 \sin \frac{1}{2}(A + B) \sin \frac{1}{2}(A - B)$.

Tan $A + \tan B = \sin (A + B)/\cos A \cos B$.

Tan $A - \tan B = \sin (A - B)/\cos A \cos B$.

Sin$^2 A - \sin^2 B = \sin (A + B) \sin (A - B)$.

Cos$^2 A - \cos^2 B = \cos (A + B) \sin (A - B)$.

Cos$^2 A - \sin^2 B = \cos (A + B) \cos (A - B)$.

Sin $A \sin B = \frac{1}{2} \cos (A - B) - \frac{1}{2} \cos (A + B)$.

Sin $A \cos B = \frac{1}{2} \sin (A + B) + \frac{1}{2} \sin (A - B)$.

Cos $A \cos B = \frac{1}{2} \cos (A + B) + \frac{1}{2} \cos (A - B)$.

Series. (See also Art. 62.)

Sin $x = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \ldots$ for all values of x in radians.

Cos $x = 1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \ldots$ for all values of x in radians.

Tan $x = x + \frac{1}{3}x^3 + \frac{2}{15}x^5 + \frac{17}{315}x^7 + \cdot - \ldots$ for values of x in radians numerically less than $\frac{1}{2}\pi$.

Trigonometric functions and exponentials. (See Art. 21.)

Sin $x = \dfrac{e^{ix} - e^{-ix}}{2i}$. Cos $x = \dfrac{e^{ix} + e^{-ix}}{2}$. Tan $x = \dfrac{e^{ix} - e^{-ix}}{e^{ix} + e^{-ix}}$.

$$(\text{Cos } x + i \sin x)^n = \cos nx + i \sin nx.$$

42. Inverse functions

The equation $\sin 30° = \frac{1}{2}$ is also written $30° = \sin^{-1} \frac{1}{2}$. The expression $\sin^{-1} \frac{1}{2}$ is called INVERSE SINE of $\frac{1}{2}$, or "angle whose sine is $\frac{1}{2}$." Similarly $\cos^{-1} a$, $\tan^{-1} a$, $\sec^{-1} a$, $\csc^{-1} a$, $\cot^{-1} a$, are inverse functions, *e.g.*, inverse cosine of a, or angle whose cosine equals a, etc.

Relations between inverse functions. $\text{Sin}^{-1} a = \cos^{-1}\sqrt{1 - a^2} = \tan^{-1} a/\sqrt{1 - a^2} = \cot^{-1}\sqrt{1 - a^2}/a = \sec^{-1} 1/\sqrt{1 - a^2} = \csc^{-1} 1/a$.

$\text{Tan}^{-1} a = \sin^{-1} a/\sqrt{1 + a^2} = \cot^{-1} 1/a = \cos^{-1} 1/\sqrt{1+a^2} = \sec^{-1}\sqrt{1+a^2} = \csc^{-1}\sqrt{1 + a^2}/a$.

$\text{Cos}^{-1} a = \sin^{-1}\sqrt{1 - a^2} = \tan^{-1}\sqrt{1 - a^2}/a = \cot^{-1} a/\sqrt{1 - a^2} = \sec^{-1} 1/a = \csc^{-1} 1\sqrt{1 - a^2}$.

$\text{Sin}^{-1} a \pm \sin^{-1} b = \sin^{-1} (a\sqrt{1 - b^2} \pm b\sqrt{1 - a^2})$.

$\text{Cos}^{-1} a \pm \cos^{-1} b = \cos^{-1} (ab \pm \sqrt{(1 - a^2)(1 - b^2)})$.

$\text{Tan}^{-1} a \pm \tan^{-1} b = \tan^{-1} [(a \pm b)/(1 \mp ab)]$.

43. Spherical trigonometry

Right spherical triangles have six parts, namely, the sides a, b, c, and the angles A, B, C ($= 90°$). (Fig. 134.) Each part is less than 180°. When two of the parts a, b, c, A, B, are given, assuming certain conditions are met (see below), the triangle can be solved, *i.e.*, the unknown parts can be determined. The necessary conditions are: $A + B > 90°$ (since the sum of the three angles must be greater than 180° (Art. 28); an angle and opposite side are either both greater than 90°, both equal to 90°, or both less than 90°.

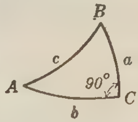

FIG. 134.

Formulas for solving right spherical triangles. $\text{Cos } c = \cos a \cos b$. $\text{Sin } b = \tan a \cot A$. $\text{Sin } a = \tan b \cot B$. $\text{Sin } a = \sin c \sin A$. $\text{Cos } B = \tan a \cot c$. $\text{Cos } A = \cos a \sin B$. $\text{Cos } c = \cot A \cot B$. $\text{Sin } b = \sin c \sin B$. $\text{Cos } B = \cos b \sin A$. $\text{Cos } A = \tan b \cot c$.

In applying the formulas careful attention must be paid to the algebraic signs of the given parts. When an angle and its opposite side are given, there are two solutions.

Example. Solve the right spherical triangle given $B = 33° 50'$, $a = 108°$.

Solution. The formulas in the above group containing both B and a are:

$$\sin a = \tan b \cot B, \qquad \cos B = \tan a \cot c, \qquad \cos A = \cos a \sin B.$$

Solving the first two of these for the unknown part, the results are (1) $\tan b = \sin a \tan B$, (2) $\cot c = \cot a \cos B$. From (1) since $\sin a$ and $\tan B$ are positive, $\tan b$ is positive and b lies between 0° and 90°. By logarithms, $b = 32° 31'$. In (2), $\cot a$ is negative (Art. 32), hence $\cot c$ is negative and c lies between 90° and 180°. In solving (2) by logarithms proceed without regard to algebraic sign of $\cot a$, and find the angle less than 90° whose cot equals $\cot a \cos B$ numerically. This angle is found to be $74° 54'$. Hence $c = 180° - 74° 54' = 105° 6'$. Finally in finding A from $\cos A = \cos a \sin B$, since $\cos a$ is negative, proceed as in determining c. Answer for A is found to be $99° 54'$.

ANALYTIC GEOMETRY

44. Formulas using co-ordinates

Definitions. The RECTANGULAR CO-ORDINATES of a point P in a plane are the distances of the point from two perpendicular lines XX', YY', called AXES OF CO-ORDINATES. (Fig. 135. See also Art. 11.) The distance from YY'

is the ABSCISSA of the point, and is POSITIVE when P is to right of YY', negative when to left. The distance from XX' is the ORDINATE of the point, POSITIVE when P is above XX', NEGATIVE when below. The axes divide the plane into four QUADRANTS. Signs of co-ordinates for points in various quadrants are indicated in Fig. 136. Co-ordinates a and b of point P are written $P(a, b)$, the first number being the abscissa. The intersection O of the axes is called the ORIGIN. The abscissa of any point on YY' (Y-axis) is zero and the ordinate of any point on XX' (X-axis) is zero. Co-ordinates of the origin are (0, 0) (**12**). OBLIQUE CO-ORDINATES are sometimes used, the axes forming an angle A not equal to 90°. The co-ordinates of a point are the lengths of lines drawn from the point to the axes parallel to respective axes. POLAR CO-ORDINATES, see Art. 48.

Length l of line joining two points (x_1, y_1) **and** (x_2, y_2) **is**

$$l = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}.$$

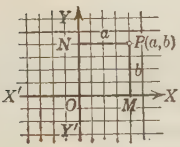

FIG. 135.

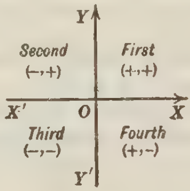

FIG. 136.

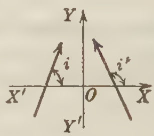

FIG. 137.

Slope of a line, produced if necessary to meet X-axis, is the tangent of the angle measured from the X-axis around to the line in a counterclockwise direction. The angle is called the INCLINATION. In Fig. 137, the inclination is marked, i, i' for the respective lines. Slope is denoted by letters m, m', etc. Thus $m = \tan i$, $m' = \tan i'$, etc. The slope may have any value, positive or negative. Parallel lines have equal slopes. For two perpendicular lines, the slope of one is the negative reciprocal of the slope of the other. If (x_1, y_1) and (x_2, y_2) are points on a line, the slope is

$$m = (y_1 - y_2)/(x_1 - x_2).$$

Lines parallel to OX have slope zero, those parallel to OY have slope equal to infinity. ($\tan 0° = 0$, $\tan 90° = \infty$, Art. 38.)

Point of division. If $P_1(x_1, y_1)$, $P_2(x_2, y_2)$, $P(x, y)$ are three points on a line, then $x = (x_1 + rx_2)/(1 + r)$ and $y = (y_1 + ry_2/(1 + r)$, where r is numerically equal to the ratio of lengths P_1P and PP_2; r is positive when P is on segment P_1P_2, negative when outside.

Mid-point. Co-ordinates (x, y) of mid-point ($r = 1$) are $= \frac{1}{2}(x_1 + x_2)$, $\frac{1}{2}(y_1 + y_2)$.

Area of triangle with vertices (x_1, y_1), (x_2, y_2), (x_3, y_3) is $\frac{1}{2}(x_1y_2 - x_2y_1 + x_2y_3 - x_3y_2 + x_3y_1 - x_1y_3)$.

This may be written by determinants (Art. 14). The formula gives an area with a negative sign when the order of vertices on the perimeter is clockwise.

$$\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

A convenient way to find the area in any given numerical case is the following: Write abscissas and ordinates in rows as indicated, repeating the first abscissa and ordinate. (1) Multiply each abscissa by the ordinate in the next column, and add the results. (2) Multiply each ordinate by the abscissa in next column and add results. (3) Subtract the last sum from the first sum and divide by 2.

$$\begin{matrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{matrix}$$

Area of a polygon with vertices given may be found in the same manner by writing down the co-ordinates of successive points on the perimeter as in the annexed scheme and following the rule as just given.

$$\begin{matrix} x_1x_2 \ . \ . \ . \ x_nx_1 \\ y_1y_2 \ . \ . \ . \ x_nx_1 \end{matrix}$$

45. Curve and equation

Locus of an equation in two variables (usually x and y) representing rectangular co-ordinates is a curve (or group of curves) passing through all points whose co-ordinates satisfy that equation, and through such points only. To PLOT the locus of a given equation, solve the equation for one variable in terms of the other variable, assume values for the latter, compute corresponding values of the first variable, plot the points with co-ordinates thus determined, and draw a smooth curve through these points.

Example. (Fig. 138.) Plot locus of

$$x^2 + y^2 + 6x - 16 = 0.$$

Solving for y,

$$y = \pm\sqrt{16 - 6x - x^2}.$$

Compute y for assumed values of x as shown in the adjacent table.

x	y	x	y
0	± 4	0	± 4
1	± 3	-1	± 4.6
2	0	-2	± 4.9
3	imag.	-3	± 5
4	"	-4	± 4.9
5	"	-5	± 4.6
6	"	-6	± 4
7	"	-7	± 3
		-8	0
		-9	imag.

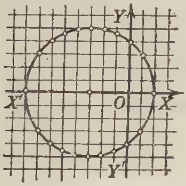

FIG. 138.

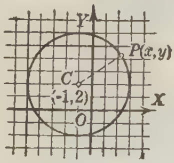

FIG. 139.

Symmetry of a curve with respect to the axes or origin should be noticed, and may be determined by the following tests: Curve is symmetric with respect to the X-axis, or to the Y-axis, or to the origin, when the given equality is unaffected by replacing y by $-y$, or x by $-x$, or x and y simultaneously by $-x$ and $-y$, respectively.

In the example given, the circle is symmetric with respect to the X-axis.

Equation of a curve (or locus of a point satisfying a given condition) is that equation in two variables (usually x and y) representing co-ordinates such that the co-ordinates of every point on the curve will satisfy the equation, and conversely, every point whose co-ordinates satisfy the equation will lie on the curve.

Example. Find the equation of the circle whose center is $C(-1, 2)$ and radius 4.
Let $P(x, y)$ be any point on the circle. (Fig. 139.) Then $PC = 4$ by definition of circle. But PC = length of line joining (x, y) and $(-1, 2) = \sqrt{(x + 1)^2 + (y - 2)^2}$ (Art. 44). Substitute this expression in $PC = 4$, square both sides, transpose and reduce. Result is $x^2 + y^2 + 2x - 4y - 11 = 0$, the required equation of the circle.

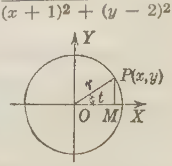

FIG. 140.

Points of intersection of two curves (loci) whose equations are given are those points whose co-ordinates satisfy both equation. These co-ordinates are found by solving the given equations. See Art. 9, 10.

Parametric equations of a curve arise when the co-ordinates x and y of a point on the curve are each expressed in terms of a third variable (called PARAMETER).

Example. In the circle of Fig. 140, if $OP = r$, angle $MOP = t$, then $x = OM = r \cos t$, and $y = MP = r \sin t$, are parametric equations of the circle. Squaring and adding gives the rectangular equation $x^2 + y^2 = r^2$.

Elimination of parameter gives the rectangular equation. Parametric equations for a curve may be found in an infinite variety of ways. When a point describes a curve in a given manner, the parametric equations may often be found directly, and rectangular equations from these, if desired. See Art. 54.

46. Straight lines

Notation (Fig. 141). OA = intercept on X-axis $= a$. OB = intercept on Y-axis $= b$. ON = perpendicular distance from origin $= p$. Slope $m = \tan i$ (Art. 44). The angle that ON forms with OX, measured counter-clockwise, $= n$.

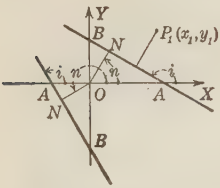

FIG. 141.

Equations of straight lines. Let $P(x, y)$ be any point on the line.

$y = mx + b$, given m and b.

$x/a + y/b = 1$, given a and b.

$y - y_1 = m(x - x_1)$, given one point (x_1, y_1) and m.

$(y - y_1)/(x - x_1) = (y_1 - y_2)/(x_1 - x_2)$, given two points (x_1, y_1), (x_2, y_2).

$x \cos n + y \sin n = p$, given p and n.

Line parallel to X-axis; $y = b$; parallel to Y-axis, $x = a$. Equation of X-axis, $y = 0$; of Y-axis, $x = 0$.

Polar equation. Art. 48.

General equation of first degree in x, y, is $Ax + By + C = 0$. The locus is a straight line. $m = \tan i = -A/B$; $a = -C/A$; $b = -C/B$; $p = \pm C/\sqrt{A^2 + B^2}$, $n = \sin^{-1} B/\sqrt{A^2 + B^2} = \cos^{-1} A/\sqrt{A^2 + B^2}$. Choose the sign of the radical opposite to the sign of C.

Perpendicular distance d **from line** $Ax + By + C = 0$ **to point** $P_1(x_1, y_1)$. $d = (Ax_1) + By_1 + C)/\sqrt{A^2 + B^2}$.

If $C \neq 0$, and the radical is given the sign opposite to C, d is positive when P_1 and O are on opposite sides of the line, negative when on the same side.

Relations between two lines $A_1x + B_1y + C_1 = 0$ and $A_2x + B_2y + C_2 = 0$. The lines are parallel if $A_1B_2 - A_2B_1 = 0$; they are perpendicular if $A_1A_2 + B_1B_2 = 0$. The angle between the lines $= \tan^{-1} (A_1B_2 - A_2B_1)/(A_1A_2 + B_1B_2)$.

Locus of an equation of higher degree in x **and** y is a group of straight lines when the equation can be written as the product of real factors of the first degree equal to zero. The locus consists of the lines obtained by setting the factors equal to zero.

Example. Equation $9x^2 - y^2 = 0$ may be written $(3x - y)(3x + y) = 0$. Locus is pair of intersecting lines $3x - y = 0$, $3x + y = 0$.

47. Circles

Notation. Center $C\ (a, b)$; radius $= r$; $P(x, y)$ = any point on the circumference.

Equations of circles.

$(x - a)^2 + (y - b)^2 = r^2$, given radius r and center (a, b).
$x^2 + y^2 - 2ax = 0$, center (a, o) on X-axis; passes through origin.
$x^2 + y^2 - 2by = o$, center (o, b) on Y-axis; passes through origin.
$x^2 + y^2 = r^2$, center at origin.

Polar equation. Art. 48.

Locus of $x^2 + y^2 + Dx + Ey + F = 0$ is a circle when $D^2 + E^2 - 4F$ is positive. Center is $(-\frac{1}{2}D, -\frac{1}{2}E)$, radius $= \frac{1}{2}\sqrt{D^2 + E^2 - 4F}$. The locus is a POINT CIRCLE when $r = o$. The equation has no locus when $D^2 + E^2 - 4F < 0$.

48. Polar co-ordinates

Polar co-ordinates of a point P in a plane (Fig. 142) are its distance OP from a fixed point O (the POLE) and the angle AOP between OP and a fixed line OA (POLAR AXIS). Distance OP is called the RADIUS VECTOR of P, angle AOP is called the VECTORIAL ANGLE. The latter is positive or negative as

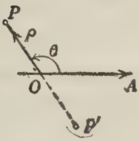

FIG. 142.

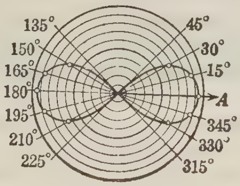

FIG. 143.

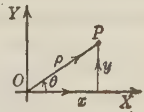

FIG. 144.

in Trigonometry (Art. 35). The radius vector is positive when P lies on the terminal side of the vectorial angle, negative when on the terminal side produced through the pole. Polar co-ordinates are denoted by Greek letters ρ (*rho*) and θ (*theta*) for polar distance and vectorial angle, respectively. Equations in polar co-ordinates ρ and θ (POLAR EQUATIONS) are plotted by calculating corresponding values of ρ and θ, plotting the points, and drawing a smooth curve through them.

Example. Figure 143 shows the locus of

$$\rho^2 = a^2 \cos 2\theta.$$

The adjoining table gives corresponding values of ρ and θ.

$\rho^2 = a^2 \cos 2\theta$			
θ	2θ	$\cos 2\theta$	ρ
0	0	1	$\pm a$
15°	30°	.866	$\pm .93a$
30°	60°	.500	$\pm .7a$
45°	90°	0	0

Length l of line joining (ρ_1, θ_1) and $\rho_2, \theta_2)$ is

$$l = \sqrt{\rho_1^2 + \rho_2^2 - 2\rho_1\rho_2 \cos (\theta_1 - \theta_2)}.$$

Relations between rectangular and polar co-ordinates of a point P. (Fig. 144.) $x = \rho \cos \theta$, $y = \rho \sin \theta$. $\rho = \pm\sqrt{x^2 + y^2}$. $\theta = \tan^{-1} y/x$.

Polar equations for a straight line and a circle. $P(\rho,\theta)$ any point on line, or circumference.

Straight line

$\rho = p \cos (\theta - n)$, given n and p. (Art. 46.)
Locus of $\rho(A \cos \theta + B \sin \theta) + C = 0$ is a straight line.
Equation of a line through pole: $\theta = a$ *constant.*

Circle

$\rho = 2r \cos \theta$; center on polar axis, radius r, circle passes through the pole.
$\rho = 2r \sin \theta$; center $(r, 90°)$, radius r, circle passes through pole.
$\rho = r$, center at pole.
Locus of $\rho^2 + \rho (D \cos \theta + E \sin \theta) + F = 0$ is a circle. (See Art. 47.)

49. Parabola

Definitions. A PARABOLA is a curve described by a point moving so that it remains always equidistant from a fixed point (FOCUS) and a fixed line (DIRECTRIX). NOTATION. (Figs. 145, 146.) Focus F, directrix DD', distance from focus to directrix $= p$. The line drawn through the focus perpendicular to the directrix is the AXIS of the curve. The curve is symmetric with respect to its axis. The point on the axis midway between focus and directrix is the VERTEX. The vertex lies on the parabola. The chord drawn through the focus parallel to the directrix is the LATUS RECTUM; length $= 2p$.

Equations of parabola

$y^2 = 2px$; vertex at origin, axis of curve along X-axis, focus $(\frac{1}{2}p, 0)$, equation of directrix $x = -\frac{1}{2}p$. $P(x, y)$ any point on curve. (Fig. 145.)

$x^2 = 2py$; vertex at origin, axis of curve along Y-axis, focus $(0, \frac{1}{2}p)$, equation of directrix, $y = -\frac{1}{2}p$. (Fig. 146.)

$(y - b)^2 = 2p(x - a)$; vertex (a, b), axis of curve parallel to X-axis.

$(x - a)^2 = 2p(y - b)$; vertex (a, b) axis of curve parallel to Y-axis.

Polar equation. $\rho = p/(1 - \cos \theta)$; pole at focus, polar axis along axis of curve.

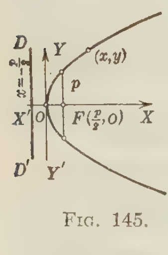

FIG. 145.

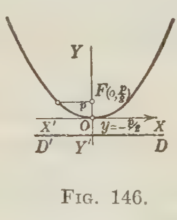

FIG. 146.

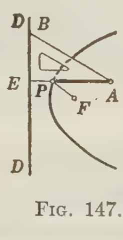

FIG. 147.

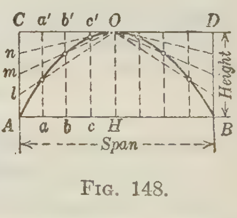

FIG. 148.

Locus of $x^2 + Dx + Ey + F = 0$, E **not zero, is a parabola with vertex**

$$(-\tfrac{1}{2}D, (D^2 - 4F)/4E),$$

axis parallel to Y-axis, latus rectum $= -E$.

Locus of $y^2 + Dx + Ey + F = 0$, D **not zero, is a parabola with vertex**

$$((E^2 - 4F)/4D, -\tfrac{1}{2}E),$$

axis parallel to X-axis, latus rectum $= -D$.

Parabola with axis not parallel to XX' or YY', see Art. 53.

Construction of parabola. Given focus F and directrix DD'. (Fig. 149.) Draw axis MX. Bisect FM, giving vertex V. Through any point A to right of V draw AB parallel to DD'. From F as center with a radius equal to MA strike arcs to intersect AB at P and Q. Then P and Q are points on the curve. Any number of points may be constructed in this manner. Figure 147 shows how to trace a parabola by sliding a triangle ABE along DD. Ends of string of length AE are fastened at A and F.

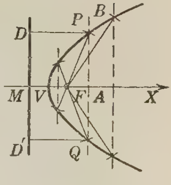

FIG. 149.

To construct a parabolic arch with given span ($2a$) and height (h). (Fig. 148.) Draw rectangle $ABCD$, divide AC, AH, OC into the same number of equal parts, draw lines aa', bb', cc', On, Om, and Ol. The intersections are points on the arch. With OH as Y-axis, CD as X-axis, the equation of the parabola in Fig. 148 is $x^2 = a^2y/h$.

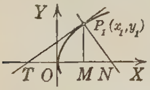

FIG. 150.

Tangent and normal (Fig. 150).

$y_1y = p(x + x_1)$ is the equation of the tangent to $y^2 = 2px$ at $P_1(x_1, y_1)$.

$y_1(x-x_1) + p(y-y_1) = 0$ is the equation of the normal P_1N. SUBTANGENT ($=MT$) is bisected at the vertex ($TO = OM$). SUBNORMAL ($=MN$) is of constant length $= p$.

To construct a tangent at a given point P_1 (Fig. 150), draw P_1M through P_1 perpendicular to the axis OX, lay off $OT = OM$, draw TP_1, which is the tangent required. Line $y = mx + c$ is a tangent to $y^2 = 2px$ when $c = p/2m$. Two perpendicular tangents intersect on the directrix and the line joining the points of contact passes through the focus. The foot of the perpendicular drawn from the focus upon a tangent lies on the tangent drawn at the vertex. The tangent and normal bisect the angles formed by lines drawn through the point of contact and the focus and through the point of contact parallel to the axis. (Fig. 151.) Two parabolas with common focus and axis (CONFOCAL PARABOLAS) and vertices upon opposite sides of the focus intersect at right angles. The equation $y^2 = 2px + p^2$ represents a system of confocal parabolas, p having any constant value. The equation of a parabola referred to the tangents drawn at the extremities of the latus rectum as axes is $\sqrt{x} + \sqrt{y} = \sqrt{a}$, where $a = p\sqrt{2}$. (Fig. 152).

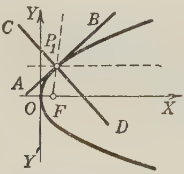

FIG. 151.

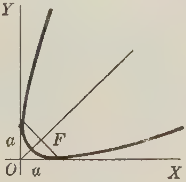

FIG. 152.

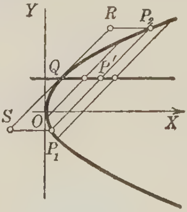

FIG. 153.

A diameter is a line drawn parallel to the axis of a parabola (Fig. 153). The diameter bisects all chords parallel to the tangent at the point where the diameter meets the parabola. The distance of a diameter from the axis $=p/m$ where m = the slope of the chords. The area of the parabolic segment P_1QP_2, cut off by any chord P_1P_2, = ⅔ area of parallelogram P_1P_2RS. (Fig. 153.)

50. Ellipse

Definitions. An ellipse is a curve described by a point moving so that the sum of its distances from two fixed points (FOCI) remains constant. NOTATION. (Figs. 154, 155.) Foci F, F'. $PF + PF' = 2a$. $FF' = 2c$. AA' = MAJOR AXIS $= 2a$. BB' = MINOR AXIS $= 2b$. $a > b$. VERTICES A, A' are the extremities of the major axis. CENTER, O. LATUS RECTUM is the chord drawn

through the focus perpendicular to the major axis, length $= 2b^2/a$. ECCENTRICITY, $e = c/a$. $e < 1$. Lines DD, $D'D'$, (Fig. 156) drawn parallel to the minor axis, at distances $\pm a^2/c$ from the center, are DIRECTRICES. $PF = e \times PE$, $PF' = e \times PE'$. An ellipse is also a curve described by a point moving so that the ratio of its distances from a fixed point (focus) and fixed line (directrix) remains constant (eccentricity) and less than unity.

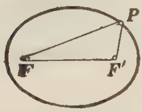

FIG. 154.

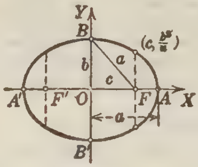

FIG. 155.

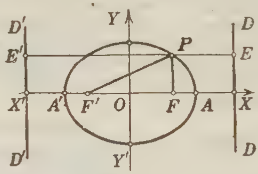

FIG. 156.

Formulas. $a^2 = b^2 + c^2$. $c = ae$. $b^2 = a^2(1 - e^2)$. $e = \sqrt{1 - b^2/a^2}$.

Equations of ellipse.

$x^2/a^2 + y^2/b^2 = 1$ or $y = \pm b\sqrt{a^2 - x^2}/a$; center (0, 0), foci on X-axis, $P(x, y)$ any point on curve.

$(x - h)^2/a^2 + (y - k)^2/b^2 = 1$, center (h, k), major axis parallel to X-axis.

Interchange a and b in the two preceding equations if the major axis is vertical.

Parametric equations. $x = a \cos \phi$, $y = b \sin \phi$, where $\phi =$ ECCENTRIC ANGLE of point $P(x, y)$, $(= \angle XOM$, Fig. 156).

Focal radii of P (x, y, Fig. 156). $F'P = a + ex$, $FP = a - ex$.

Polar equation. $\rho = a(1 - e^2)/(1 - e \cos \theta)$; pole at left-hand focus, polar axis drawn through other focus. (Fig. 156.) $F'P = \rho$, $\angle OF'P = \theta$.

Locus of $Ax^2 + By^2 + Dx + Ey + F = 0$, A and B positive numbers (not zero), is an ellipse when $R = \frac{1}{4}(D^2/A + E^2/B - 4F) > 0$. Center is $(-D/2A, -E/2B)$; semi-axes $(\sqrt{R/A}, \sqrt{R/B})$ major-axis is parallel to X-axis or Y-axis according as $A < B$, or $A > B$. For $A = B$, locus is a circle. For $R = 0$, locus reduces to point $(-D/2A, -E/2A)$, a POINT-ELLIPSE. The equation has no locus when $R < 0$.

For an ellipse with major axis oblique to the X-axis, see Art. 53.

Construction of ellipse. When the major and minor axes AA', BB' are given, draw circles upon those lines as diameters (Fig. 157) and from any point M on the larger circle draw radius OM, and draw SP and MR parallel to OA and OB, respectively, to intersect at P, on the ellipse. An ellipse is a flattened circle, in the sense that the ordinates of the large circle in Fig. 157 are reduced in the ratio $b : a$ to the given ordinates of the ellipse. Fig. 158 shows an instrument for drawing ellipses. A, B are adjustable nuts on the crossbar PAB. These slide in perpendicular grooves XX', YY'. When the crossbar is moved, P traces an ellipse with semi-axes AP, BP.

For other constructions, Art. 54.

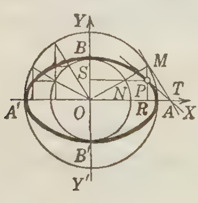

FIG. 157.

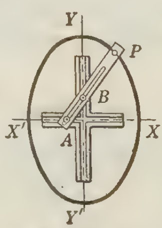

FIG. 158.

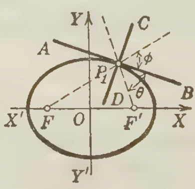

FIG. 159.

Tangent and normal to ellipse $b^2x^2 + a^2y^2 = a^2b^2$.
$b^2x_1x + a^2y_1y = a^2b^2$ is the equation of the tangent at (x_1, y_1).
$a^2y_1x - b^2x_1y = (a^2 - b^2)x_1y_1$ is the equation of the normal through (x_1y_1).
$y = mx + c$ is a tangent when $c = \pm\sqrt{a^2m^2 + b^2}$.

To construct a tangent at P, Fig. 157, extend ordinate RP to meet the outer circle at M and draw tangent MT; then TP is tangent to the ellipse. Two perpendicular tangents intersect on the circle $x^2 + y^2 = a^2 + b^2$ (DIRECTOR CIRCLE). The foot of the perpendicular from the focus to a tangent lies on the circle $x^2 + y^2 = a^2$. The tangent and normal bisect angles formed by focal radii through the point of contact (Fig. 159).

To draw tangents from an external point P (Fig. 160). Draw secants PQR, PST; chords QT, RS, intersecting at L, and secants QS, RT, intersecting at M. Then LM intersects the ellipse in points of contact A, B.

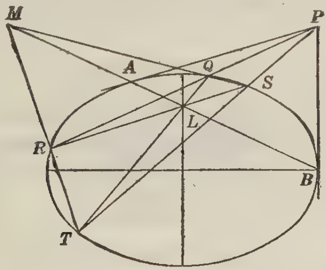

FIG. 160.

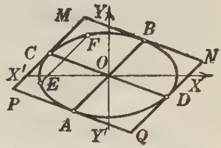

FIG. 161.

Diameter is a line drawn through the center. Tangents drawn at the extremities of a diameter are parallel. All chords parallel to these tangents are bisected by the diameter. In Fig. 161, AB and CD are CONJUGATE DIAMETERS, each being parallel to chords bisected by the other. If m, m' are slopes of the conjugate diameters, $mm' = -b^2/a^2$. The area of the circumscribed parallelogram $MNQP$ formed by tangents drawn at the extremities of conjugate diameters equals $4ab$. If $2a'$, $2b'$, are the lengths of AB, CD, then $a'^2 + b'^2 = a^2 + b^2$.

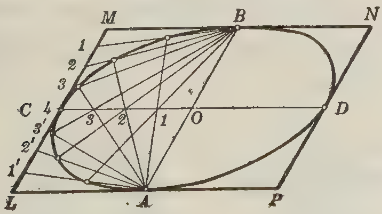

FIG. 162.

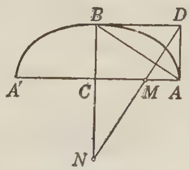

FIG. 163.

To construct an ellipse, given a pair of conjugate diameters AB, CD (Fig. 162). Draw the parallelogram $LMNP$, divide MC and OC into the same number of equal parts, draw lines from B and A, through like-numbered points. These lines intersect on the ellipse.

Radii of curvature (Art. 61) for points A, B, at extremities of the axes are determined (Fig. 163) by drawing a line from vertex D of the rectangle $ADBC$ perpendicular to the chord AB. Then M, N are centers of curvature for A, B, respectively, and AM, BN, are radii of curvature. $AM = b^2/a$, $BN = a^2/b$.

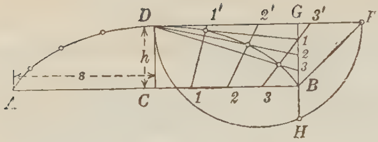

FIG. 164.

To construct an elliptic arch of given height (h) **and span** ($2s$) **with axes in a given ratio** (m). (Fig. 164) $GH = mh$, DF is diameter of the circle drawn through H. Lines BG, BC, DF are divided into the same number of equal parts and lines drawn to intersect on the ellipse as shown. Axes are $h + s^2/m^2h$ (along CD produced), and $mh + s^2/mh$.

Approximate construction of ellipse by circular arcs.

With four centers (Fig. 165)

Lay off $CF = OB - OC$, bisect BF at S, and draw SH perpendicular to BC. Set off $OG' = OG$, and $OH' = OH$. Then the four centers are G, H, G', H', and the radii are GB, HC, $G'A$, $H'D$.

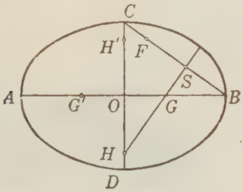

FIG. 165.

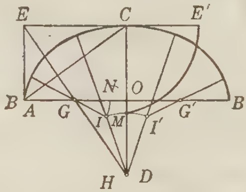

FIG. 166.

With eight centers (Fig. 166)

$OB = a$, $OC = b$. Draw EGH perpendicular to AC. Then G, H are centers with radii GA, HC (radii of curvature at A and C). Produce EC to E' making $CE' = OC$. Upon EE' as a diameter draw a circle cutting CD at M. Then $CM = \sqrt{ab}$, and is the third radius. (See Fig. 55.) Lay off $AN = CM$, and from G, H strike arcs with radii GN, HM, intersecting at I, the third center for the elliptic arc from A to C. Eight centers are necessary to complete the ellipse. Radius $AG = b^2/a$, $HC = a^2/b$, and the third radius CM is the mean proportional between AG and HC.

51. Hyperbola

Definitions. A hyperbola is a curve traced by a point moving so that the difference of its distances from two fixed points (FOCI) remains constant. NOTATION. Figs. 167, 168. Foci F, F'. $FF' = 2c$. $PF' - PF = 2a$. AA' = TRANSVERSE AXIS $= 2a$. BB' = CONJUGATE AXIS $= 2b$. VERTICES are A, A'.

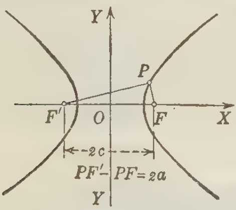

FIG. 167.

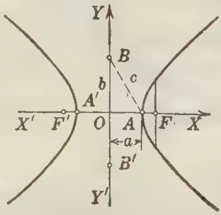

FIG. 168.

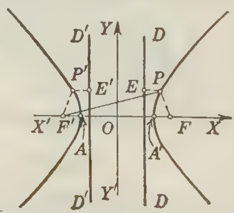

FIG. 169.

CENTER, is O. LATUS RECTUM is a chord drawn through the focus perpendicular to the transverse axis produced; length $= 2b^2/a$. ECCENTRICITY $e = c/a$ $e > 1$. Lines DD, $D'D'$, (Fig. 169) drawn parallel to the conjugate axis at distances $\pm a^2/c$ from the center are DIRECTRICES. $PF = e \times PE$, $P'F' = e \times P'E'$. The hyperbola is also a curve traced by a point so moving that the

ratio of its distances from a fixed point (focus) and fixed line (directrix) remain constant (eccentricity) and greater than unity.

Formulas. $c^2 = a^2 + b^2$. $c = ae$. $b^2 = a^2(e^2 - 1)$. $e = \sqrt{1 + b^2/a^2}$.

Equations of a hyperbola.

$x^2/a^2 - y^2/b^2 = 1$ or $y = \pm b\sqrt{x^2 - a^2}/a$; center (0, 0); foci on X-axis, $P(x, y)$ any point on curve.

$(x - h)^2/a^2 - (y - k)^2/b^2 = 1$, center (h, k), transverse axis parallel to X-axis.

If the transverse axis is vertical, interchange x and y, and h and k, in the two preceding equations.

Parametric equations. $x = a \cosh u$, $y = b \sinh u$. (For the meaning of u, see Art. 29.)

Focal radii of P are $F'P = ex + a$, $FP = ex - a$. (Fig. 167.)

Polar equation, $\rho = a(e^2 - 1)/(e \cos\theta - 1)$; pole at left-hand focus, polar axis drawn through other focus. (Fig. 167.) $F'P = \rho$, $\angle XF'P = \theta$.

Conjugate hyperbolas have the property that the transverse and conjugate axes of one are respectively the conjugate and transverse axes of the others. Fig. 170. The equations differ only in the sign of the constant term and are $x^2/a^2 - y^2/b^2 = \pm 1$.

Asymptotes of $x^2/a^2 - y^2/b^2 = 1$ are the lines $x/a - y/b = 0$, $x/a + y/b = 0$. Conjugate hyperbolas have the same asymptotes. The diagonals of the rectangle in Fig. 170 lie on the asymptotes. The branches of the hyperbola approach infinitely near its asymptotes as they recede to infinity.

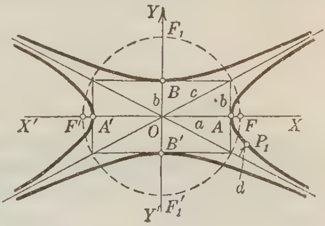

FIG. 170.

Rectangular hyperbola, has equal axes ($a = b$), and asymptotes perpendicular. $e = \sqrt{2}$. The equation is, $x^2 - y^2 = a^2$; foci $(\pm a\sqrt{2}, 0)$. The equation referred to the asymptotes as axes is, $2xy = \pm a^2$. A rectangular hyperbola is also called an EQUILATERAL HYPERBOLA. (See also below.)

Locus of $Ax^2 - By^2 + Dx + Ey + F = 0$, A and B positive numbers (not zero), is a hyperbola when $R = \frac{1}{4}(D^2/A - E^2/B - 4F)$ is not zero. Center is $(-D/2A, E/2B)$; semi-axes $\sqrt{|R/A|}$, $\sqrt{|R/B|}$ (absolute values, see Art. 6); transverse axis is parallel to X-axis or Y-axis according as $R > 0$, or < 0. When $A = B$, the locus is a rectangular hyperbola. If $R = 0$, the locus is a pair of lines intersecting at $(-D/2A, E/2B)$, with slopes $\pm\sqrt{A/B}$.

Locus of $2Cxy + Dx + Ey + F = 0$, C not zero, is a rectangular hyperbola when $R = \frac{1}{4}(DE - CF)/C^2$ is not zero. Center is $(-E/2C, -D/2C)$; $a^2 = |R|$; asymptotes are parallel to OX, OY. When $R = 0$, the locus is a pair of lines $x + E/2C = 0$, $y + D/2C = 0$. See also Art. 53.

Construction of hyperbola. When foci F, F_1, and transverse axis AB are given, points on the curve may be constructed as in Fig. 171. $FR = FJ =$ any radius $> AB$. $JE = AB$. $F_1R = FE = FJ - AB$.

In Fig. 172, focus and directrix are given. Triangle JUV is moved along the ruler edge (directrix), one end of a string of length JV is fastened at J, the other end at F (focus), and kept taut by a pencil point at P. $PF = PV$. Distance from P to the ruler $= PV \sin \angle UVJ = PF \sin \angle UVJ$. Hence eccentricity $= \csc \angle UVJ$.

When asymptotes OX, OY and one point A on the curve are given, construction is shown in Fig. 173 (which is drawn for a rectangular hyperbola): $AN_2 = M_2P_2$; $AN_1 = M_1P_1$, etc.

Tangent and normal to $b^2x^2 - a^2y^2 = a^2b^2$.

$b^2x_1x - a^2y_1y = a^2b^2$, is the equation of the tangent at (x_1, y_1).

$a^2y_1x - b^2x_1y = (a^2 - b^2)x_1y_1$, is the normal through (x_1, y_1).

$y = mx + c$ is a tangent when $c = \pm\sqrt{a^2m^2 - b^2}$.

To construct a tangent at Q (Fig. 171), draw FQ, F_1Q, and construct the bisector of the angle FQF_1. This is the desired tangent. Two perpendicular tangents intersect on the circle $x^2 + y^2 = a^2 - b^2$. (Impossible when $a < b$.) The foot of the perpendicular from the focus upon a tangent lies on circle $x^2 + y^2 = a^2$. Tangent and normal bisect angles formed by the focal radii through the point of contact. The portion of a tangent lying between its intersection with asymptotes is bisected at the point of contact, and forms with the asymptotes a triangle of constant area ab. Hence to draw a tangent at A (Fig. 173) when the asymptotes are given, draw AM, AR, each parallel to an asymptote. Lay off $MT = OM$, $RN = OR$, and draw NT. This is the tangent required.

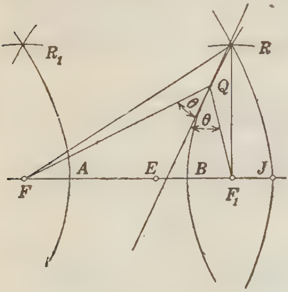

FIG. 171.

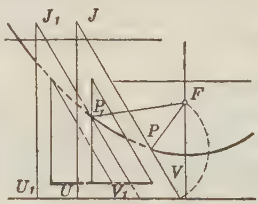

FIG. 172.

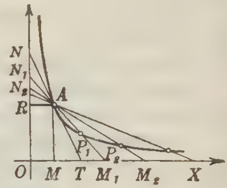

FIG. 173.

Diameter is a line drawn through the center, and will intersect the hyperbola or conjugate hyperbola (asymptotes excluded). The tangents drawn at the extremities of a diameter are parallel. All chords parallel to these tangents terminating in a conjugate hyperbola are bisected by the diameter. In Fig. 174, AB, CD are conjugate diameters, each bisects chords parallel to the other. Product of slopes $= b^2/a^2$. The area of the parallelogram formed by tangents at A, B, C, $D = 4ab$. The vertices are on the asymptotes. $\overline{OB}^2 + \overline{OC}^2 = a^2 + b^2$.

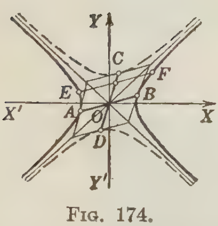

FIG. 174.

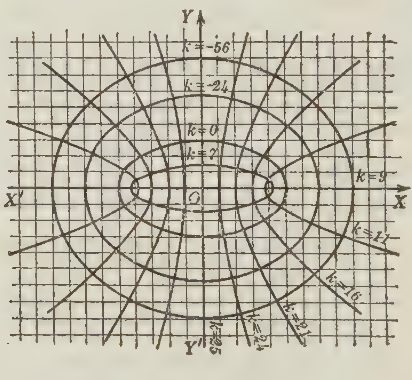

FIG. 175.

Confocal ellipses and hyperbolas. An ellipse and a hyperbola with common foci intersect at right angles. The equation $x^2/(a^2 + k) + y^2(b^2 + k) = 1$, for values of $k > -a^2$, represents a system of curves composed of confocal ellipses and hyperbolas. (Fig. 175. $a = 5$, $b = 3$.)

52. Transformation of rectangular co-ordinates

Notation. New axes of co-ordinates $O'X'$, $O'Y'$, are drawn through a new rigin $O'(h, k)$. Co-ordinates of any point P are (x, y) referred to old axes and $x', y')$ referred to the new axes.

Translation. If the new axes are parallel to the original axes, $x = x' + h$, $y = y' + k$.

Translation and rotation. If the new axis $O'X'$ makes an angle θ with the riginal X-axis, $x = x' \cos\theta - y' \sin\theta + h$, $y = x' \sin\theta + y' \cos\theta + k$.

By these formulas a given equation can be transformed so that the new axes f co-ordinates assume any desired position. By suitable choice of h, k, and θ, eductions of equations to simpler forms may often be effected. (See below.)

53. Locus of $Ax^2 + 2Bxy + Cy^2 + 2Dx + 2Ey + F = O$ (general equation of the second degree)

By rotation of the axes of co-ordinates through the angle $\theta = \frac{1}{2} \tan^{-1} 2B/(A - C)$, the equation will assume a form in which the xy-term is lacking. Hence the locus comes under the cases already discussed.

Tests are given in the following table, excluding the case of no locus.

Test	General case	Exceptional case
$B^2 - AC = 0$	Parabola	Two parallel lines; one line
$B^2 - AC < 0$	Ellipse	Point ellipse
$B^2 - AC > 0$	Hyperbola	Two intersecting lines

A conic section is a curve traced by a point moving so that the ratio (called ECCENTRICITY) of its distance from a fixed point (FOCUS) and a fixed line (DIRECTRIX) is constant. The polar equation, is $\rho = ep/(1 - e\cos\theta)$, pole at focus, polar axis perpendicular to directrix, distance of focus from directrix $= p$, eccentricity $= e$. A conic section is a parabola, ellipse, or hyperbola according as $e = 1$, $e < 1$, or $e > 1$. Plane sections of a right circular conical surface are conic sections, a parabola when the plane is parallel to an element, an ellipse when the plane cuts all elements, a hyperbola in other cases.

54. Locus problems and parametric equations.

It is often easier to express the rectangular co-ordinates x, y of a moving point P in terms of a parameter (parametric equations of the locus of P, Art. 45) than to derive the rectangular equation directly. Study of the problem will determine the choice of parameter.

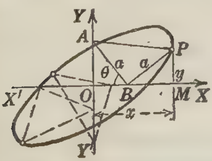

FIG. 176.

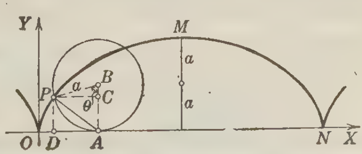

FIG. 177.

Example. A rigid isosceles right-triangular frame moves so that the extremities of one side move on perpendicular lines. Find the curve traced by the third vertex.

Solution. In Fig. 176, choose $\angle ABO = \theta$ for a parameter. The side AB of the triangle $= a$. Then $x = OB + BM = AB \cos\theta + BP \cos(90° - \theta)$, $y = MP = BP \sin(90° - \theta)$. Hence $x = a(\cos\theta + \sin\theta)$, $y = a\cos\theta$. The rectangular equation is $x^2 - 2xy + 2y^2 = a^2$, and the locus is an ellipse. (See Art. 53.)

Cycloid is a curve traced by a point P on the circumference of a circl which rolls without slipping on a straight line. The parametric equation (Fig. 177), are $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$, θ in radians. The rect angular equation is, $x = a\ \text{vers}^{-1}y/a - \sqrt{2ay - y^2}$.

Construction of cycloid, is shown in Fig. 178, in which $M_1D_1 = CC_1$, $M_2D_2 = CC_2$ $M_3D_3 = CC_3$, etc.

The NORMAL to the cycloid at P passes through A (Fig. 177). A point Q on the radiu BP (Fig. 177), or the radius produced, describes a PROLATE CYCLOID if Q is inside of th rolling circle, and a CURTATE CYCLOID if Q is outside. The equations of such a cycloid ar $x = a\theta - b\sin\theta$, $y = a - b\cos\theta$ when $b = BQ$.

Epicycloid (or **hypocycloid**) is a curve traced by a point on the circum ference of a circle which rolls without slipping on the outside (or inside) of a fixed circle. Let the radius of the fixed circle $= R$, radius of rolling circle $= r$ The parametric equations are:

Epicycloid (Fig. 179). $x = (R + r)\cos\theta - r\cos(\theta + R\theta/r)$, $y = (R + r)$ $\sin\theta - r\sin(\theta + R\theta/r)$. **Hypocycloid** (Fig. 180), the same, with the sign of r changed. Parameter θ = the angle described by the line of centers

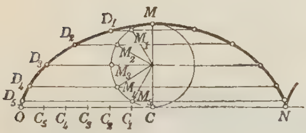

FIG. 178.

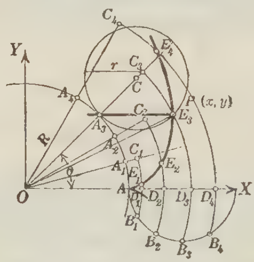

FIG. 179.

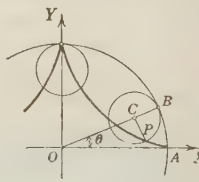

FIG. 180.

measured from a position when the tracing point is on the fixed circle. Th normal to the curve for any position of the tracing point passes through th point of contact of the circles. (E_3A_3, Fig. 179.)

Construction of epicycloid (Fig. 179). Arcs AA_1, AA_2, etc., on the fixed circle equa arcs AB_1, AB_2, etc. on the rolling circle. Arcs B_1C_1, B_2C_2, etc., are drawn with O as cente and $D_1B_1 = C_1E_1$, $D_2B_2 = C_2E_2$, etc. A similar construction holds for the hypocycloid.

Epitrochoid (or **Hypotrochoid**) is the curve traced by a point (Fig. 17 or 180), on the radius of the rolling circle at a fixed distance b from its center The parametric equations of the curve are,

$$x = (R + r)\cos\theta - b\cos(\theta + R\theta/r),$$

$$y = (R + r)\sin\theta - b\sin(\theta + R\theta/r).$$

(For a hypotrochoid change signs of r and b.) If $R = 2r$, the hypocycloi is a diameter of the fixed circle, and the hypotrochoid is an ellipse. If $R = 4r$ the hypocycloid has four cusps and is called the ASTROID (Fig. 181). Its rect angular equation is $x^{2/3} + y^{2/3} = R^{2/3}$. If $R = 2r$, the epicycloid is the CAR DIOID. (Fig. 182.) When each chord drawn from a fixed point of a circle i produced a distance equal to the diameter, the extremities will lie on a car dioid.

The polar equation of the cardioid is $\rho = a(1 + \cos\theta)$; (Fig. 182). The rectangular equation is $(x^2 + y^2 - ax)^2 = a^2(x^2 + y^2)$.

Involute of a circle is the curve traced by the end of a taut string wrapped around a fixed circle and unwinding. The parametric equations are (Fig. 183),

$$x = r\cos\theta + r\theta\sin\theta;\quad y = r\sin\theta - r\theta\cos\theta,\ (\theta \text{ in radians}).$$

At any instant the portion of string unwound (as BP) is normal to the involute and tangent to the circle, and $BP = arc\ AB$.

Catenary is the curve assumed by a flexible cord of uniform density when suspended at its extremities. Its equation (Fig. 184) is $y = a\cosh(x/a)$. (Table 21.) The constant a equals the ratio of the horizontal component of tension in the cord to the weight of cord per unit length. The length of the normal drawn from $P(x, y)$ on the curve and intercepted by $OX = y^2/a$.

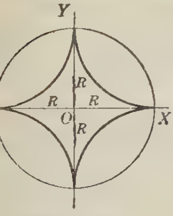

Fig. 181.

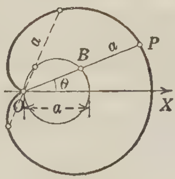

Fig. 182.

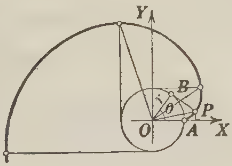

Fig. 183.

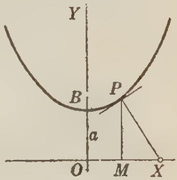

Fig. 184.

Spiral of Archimedes is the curve traced by a point moving with constant speed v on a line which turns about a fixed point on itself with uniform angular velocity w. (Fig. 185.) The polar equation is $\rho = a\theta$ (θ in radians), where $a = v/w$, w being expressed in radians per sec., or $\rho = b\theta/360°$ (θ in degrees, $b = 2\pi a$).

To construct the curve, draw radial lines OR_1, OR_2, OR_3, etc., forming angles α, 2α, 3α, etc., with OX; lay off $OP_1 = a\alpha$ (α in radians) $= b\alpha/360$ (α in degrees), $OP_2 = 2OP_1$, $OP_3 = 3OP_1$, etc. Consecutive coils of the spiral intercept equal lengths ($2\pi a$) on any radius. The normal at any point P cuts off a constant distance a from O on a line ON drawn perpendicular to OP.

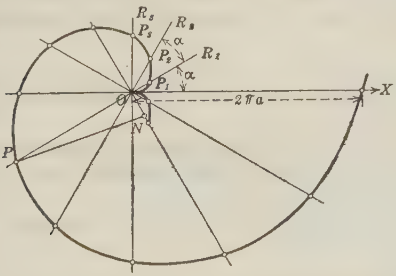

Fig. 185.

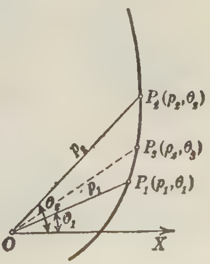

Fig. 186.

Logarithmic spiral is a curve cutting all radii drawn to it from a fixed point at the same angle. The polar equation is, $\rho = be^{a\theta}$ (θ in radians). (Fig. 186.) The cotangent of the angle between radius OP_1 and the tangent at P_1 equals a. The radius equals b when $\theta = 0$. If P_1 and P_2 are points on the

curve, and angle P_1OP_2 is bisected and OP_3 is laid off on this bisector equal to the mean proportional between OP_1, and OP_2 ($OP_3 = \sqrt{OP_1 \cdot OP_2}$), then P_3 is on the curve. For negative values of θ increasing numerically, the spiral winds about O with decreasing radius.

To construct a normal at P, lay off on OP from P a length equal to 1, at this point erect a perpendicular to OP of length a, then the normal passes through the end of this perpendicular.

A helix is the path of a point which moves on a right circular cylinder of radius r at constant speed v parallel to the axis and at the same time rotates about the axis with uniform angular velocity w (radians per sec.). The curve cuts all elements at an angle A whose tangent equals rw/v. Consecutive coils intercept equal lengths $h(= 2\pi r \cot A)$ on an element. The length of a helix for one complete turn equals $2\pi r/\sin A$. The curve formed by a screw thread is a helix, and h = the pitch of the thread.

CALCULUS

55. Functions

Definitions. When two variables x and y are so related that the value of y depends on the value of x, then y is said to be a FUNCTION of x. The functional relation is written $y = f(x)$ [read f of x]. If $x = x_0$ gives a corresponding value of y as y_0, then $y_0 = f(x_0)$. The GRAPH OF A FUNCTION is the curve obtained by plotting corresponding values of x and y as rectangular co-ordinates (Art. 11). A mathematical expression containing a variable is a FUNCTION of this variable. In $y = f(x)$, x is called the INDEPENDENT VARIABLE, y the DEPENDENT variable.

From $y = \log_e x$, it follows that $x = e^y$ (Art. 15). Functions $\log_e x$ and e^y are called INVERSE FUNCTIONS.

An INCREMENT of a variable is an increase (+) or a decrease (−) in its value. Increment of x is denoted by Δx (read "*delta* x"), of y by Δy; etc. When values of the independent variable (x) and its increment (Δx) are given, the increment of dependent variable $y = f(x)$ is $\Delta y = f(x + \Delta x) - f(x)$, that is Δy is the difference of the values of the function for the two values ($x + \Delta x$ and x) of the independent variable.

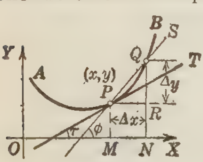

FIG. 187.

Example. If $y = 2 \sin x$, the increment of y when $x = 30°$ and $\Delta x = 2°$ is $\Delta y = 2 \sin 32° - 2 \sin 30° = 2(0.5299 - 0.5000) = 0.060$.

The RATIO OF CORRESPONDING INCREMENTS $\Delta y/\Delta x$ is the slope of the secant drawn through points on the graph of the function with co-ordinates (x, y) $(x + \Delta x, y + \Delta y)$. Thus, in Fig. 187, $\Delta y/\Delta x = RQ/PR = \tan \angle RPQ$ = the slope of the secant line PS through $P(x, y)$ and $Q(x + \Delta x, y + \Delta y)$.

56. Derivatives

The derivative of a function is obtained by calculating the ratio of corresponding increments of the function and independent variable and finding the LIMITING VALUE of this ratio when the increment of the independent variable decreases numerically and becomes zero. The fact that a variable as Δx

decreases numerically and becomes zero is written $\Delta x \doteq 0$ (read Δx approaches zero as a limit).

Notation for derivative of y with respect to x is dy/dx (or $D_x y$). From the above definition, $\frac{dy}{dx}$ = the limiting value of $\left(\frac{\Delta y}{\Delta x}\right)$ when $\Delta x \doteq 0$, or $\underset{\Delta x \doteq 0}{\text{Limit}} \left(\frac{\Delta y}{\Delta x}\right)$, as it is usually abbreviated.

Notation for derivative of $f(x)$ with respect to x is

$$\frac{d}{dx} f(x) = f'(x) = \underset{\Delta x \doteq 0}{\text{Limit}} \left(\frac{f(x + \Delta x) - f(x)}{\Delta x}\right).$$

This equation shows formally the steps necessary to calculate the derivative of a function. For actual work formulas for derivatives (or DIFFERENTIALS) are at hand, and those given below suffice for engineering problems.

Slope of tangent at a point on the graph of $y = f(x)$ is given immediately by the derivative of the function, that is, the value of the derivative for a given value of x equals the tangent of the angle formed with the X-axis by the line drawn tangent to the graph at the corresponding point.

In Fig. 187 when $\Delta x = 0$, the point Q takes successive positions on the arc PQ nearer P, the secant PS turns about P, and, eventually, when Δx becomes zero, secant PS becomes tangent to the graph at P, its slope becoming, in the limit, the slope of PT.

Formulas for derivatives, in which c, n, are constants, e the Napierian base (Art. 15), and u, v, w, are functions of x.

(1) $\frac{dc}{dx} = 0.$

(2) $\frac{dx}{dx} = 1.$

(3) $\frac{d}{dx}(u + v - w) = \frac{du}{dx} + \frac{dv}{dx} - \frac{dw}{dx}.$

(4) $\frac{d}{dx}(cv) = c\frac{dv}{dx}.$

(5) $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}.$

(6) $\frac{d}{dx}(v)^n = n(v)^{n-1}\frac{dv}{dx}.$

(7) $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{1}{v^2}\left(v\frac{du}{dx} - u\frac{dv}{dx}\right).$

(8) $\frac{d}{dx}\log_e v = \frac{1}{v}\frac{dv}{dx}.$

(9) $\frac{d}{dx}\log_{10} v = \frac{\log_{10} e}{v}\frac{dv}{dx}.$

(10) $\frac{d}{dx}e^v = e^v\frac{dv}{dx}.$

(11) $\frac{d}{dx}c^v = c^v \log_e c \frac{dv}{dx}.$

(12)* $\frac{d}{dx}\sin v = \cos v\frac{dv}{dx}.$

(13)* $\frac{d}{dx}\cos v = -\sin v\frac{dv}{dx}.$

(14)* $\frac{d}{dx}\tan v = \sec^2 v\frac{dv}{dx}.$

(15)* $\frac{d}{dx}\cot v = -\csc^2 v\frac{dv}{dx}.$

(16)* $\frac{d}{dx}\sec v = \sec v \tan v\frac{dv}{dx}.$

(17)* $\frac{d}{dx}\csc v = -\csc v \cot v\frac{dv}{dx}.$

(18) $\frac{d}{dx}\sin^{-1} v = \frac{1}{\sqrt{1 - v^2}}\frac{dv}{dx}.$

(19) $\frac{d}{dx}\cos^{-1} v = -\frac{1}{\sqrt{1 - v^2}}\frac{dv}{dx}.$

(20) $\frac{d}{dx}\tan^{-1} v = \frac{1}{1 + v^2}\frac{dv}{dx}.$

* In 12–17, v must be measured in radians.

(21) $\dfrac{d}{dx}\cot^{-1} v = -\dfrac{1}{1+v^2}\dfrac{dv}{dx}.$

(22) $\dfrac{d}{dx}\sec^{-1} v = \dfrac{1}{v\sqrt{v^2-1}}\dfrac{dv}{dx}.$

(23) $\dfrac{d}{dx}\csc^{-1} v = -\dfrac{1}{v\sqrt{v^2-1}}\dfrac{dv}{dx}.$

(24) $\dfrac{d}{dx}\sinh v = \cosh v\dfrac{dv}{dx}.$

(25) $\dfrac{d}{dx}\cosh v = \sinh v\dfrac{dv}{dx}.$

(26) $\dfrac{d}{dx}\tanh v = \text{sech}^2\, v\dfrac{dv}{dx}.$

(27) $\dfrac{d}{dx}\coth^2 v = -\text{csch}^2\, v\dfrac{dv}{dx}.$

(28) $\dfrac{d}{dx}\text{sech}\, v = -\text{sech}\, v\tanh v\dfrac{dv}{dx}.$

(29) $\dfrac{d}{dx}\text{csch}\, v = -\text{csch}\, v\coth v\dfrac{dv}{dx}.$

(30) $\dfrac{d}{dx}\sinh^{-1} v = \dfrac{1}{\sqrt{v^2+1}}\dfrac{dv}{dx}.$

(31) $\dfrac{d}{dx}\cosh^{-1} v = \dfrac{1}{\sqrt{v^2-1}}\dfrac{dv}{dx}.$

(32) $\dfrac{dv}{dx} = \dfrac{dv}{dt}\cdot\dfrac{dt}{dx}.$

(33) $\dfrac{dv}{dx} = 1\Big/\dfrac{dx}{dv}.$

Differentiation of a function is accomplished by applying in succession formulas from the above list.

Examples. (1) Find $\frac{d}{dx}3\sqrt{9+x^2}$. Apply (4) with $c = 3$, $v = \sqrt{9+x^2}$; the result is $3\frac{d}{dx}\sqrt{9+x^2}$. Change the square root to the power ½ and apply (6) with $v = 9 + x^2$, $n = \frac{1}{2}$. This gives $3 \times \frac{1}{2}(9+x^2)^{-\frac{1}{2}}\frac{d}{dx}(9+x^2)$. Now apply (3). $\frac{d}{dx}(9+x^2) = \frac{d}{dx}9 + \frac{d}{dx}x^2$. By (1) $\frac{d}{dx}9 = 0$, and by (6) and (2) $\frac{d}{dx}x^2 = 2x$. The final result is $3x(9+x^2)^{-\frac{1}{2}}$, or $3x/\sqrt{9+x^2}$. *Ans.*

(2) Find $\frac{d}{dx}(25\cos x - 12.8\cot x)$. Applying (3), $\frac{d}{dx}(25\cos x) - \frac{d}{dx}(12.8\cot x)$. Apply (5) in each term and then (13) and (15). The result is $-25\sin x + 12.8\csc^2 x$. *Ans.*

Logarithmic differentiation is a method under which the derivative of the Nap log of a function is worked out and this result multiplied by the function, the final product being the derivative of the latter.

Example. Find $\frac{d}{dx}\frac{x}{\sqrt{9-x^2}}$. $\log_e \frac{x}{\sqrt{9-x^2}} = \log_e x - \frac{1}{2}\log_e(9-x^2)$. (See Art. 4) Differentiating this by (3), (4), (8), successively, the result is $\frac{1}{x} + \frac{x}{9-x^2} = \frac{9}{x(9-x^2)}$. Multiplying this by $\frac{x}{\sqrt{9-x^2}}$, the product is $9/(9-x^2)^{\frac{3}{2}}$. *Ans.*

57. Applications of differential calculus

Maximum and minimum values of a function. In Fig. 188, which represents a graph of $f(x)$, the value of $f(x)$ for $x = OM$ is represented by MA, and this value of $f(x)$ is said to be a MAXIMUM, because it is greater than the values of the function immediately preceding or following it. Similarly the function has a MINIMUM value ($= NB$) when $x = ON$. The slope of the graph is zero at A and also at B (horizontal tangents). Passing along the graph through A from left to right, the slope changes sign from + to −

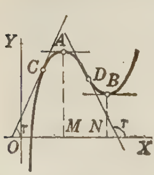

FIG. 188.

while at B, the slope changes sign from $-$ to $+$. Hence $f(x)$ is a maximum for $x = a$ when its derivative vanishes for $x = a$ and changes sign from $+$ to $-$ as x increases through a; $f(x)$ is a minimum for $x = a$ when its derivative vanishes for $x = a$ and changes sign from $-$ to $+$ as x increases through a.

To examine a function for maximum or minimum values, find its derivative, set it equal to zero, and solve this equation for values (CRITICAL VALUES) of the variable. Consider one critical value at a time and determine the sign of the derivative, first for a value of the variable a trifle less than the critical value, and second for a value a trifle greater. Change in sign from $+$ to $-$ shows a maximum value of the function for the particular critical value, of $-$ to $+$, a minimum value.

In many problems the function to be examined must be derived and it is often advisable to graph this function. The conditions of the problem usually determine without testing whether the function is a maximum or a minimum.

Examples. (1) An opening is to be dug from a point A to a point B, 300 ft. lower than A and 500 ft. east of A. On level A the opening is dug through earth, below A through solid rock. The cost through earth is \$1 per linear foot, through rock \$3 per linear foot. Find the distance tunneled on level A for least cost, and find the least cost.

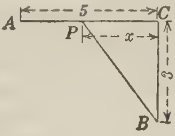

FIG. 189.

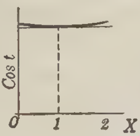

FIG. 190.

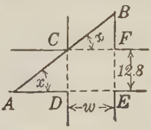

FIG. 191.

In Fig. 189 the distances are marked in hundred feet. The cost of a tunnel from A to P is $100(5 - x)$ dollars, and of the incline from P to B is $100 \times PB \times 3 = 300\sqrt{9 + x^2}$ dollars. Hence the total cost is $100[5 - x + 3\sqrt{9 + x^2}]$ dollars. Fig. 190 shows the graph of this function. For minimum cost the derivative must vanish, hence $\frac{d}{dx}[5 - x + \sqrt{9 + x^2}] = 0$. The result (see Ex. 1, Art. 56) is $-1 + 3x/\sqrt{9 + x^2} = 0$, or $\sqrt{9 + x^2} = 3x$. Squaring and solving for x, $x = \sqrt{9/8} = 3\sqrt{2}/4 = 1.06$. Hence the length of the tunnel section is $500 - 106 = 394$ ft. The length of the inclined part through rock is $\sqrt{9 + 9/8} = 3.18$ hundred feet. The minimum cost is $394 + 954 = \$1348$. *Ans.*

(2) A girder 25 ft. long is moved on rollers along a passageway 12.8 ft. wide and thence into a corridor at right angles to the passageway. Neglecting the width of the girder, how wide must the corridor be?

In Fig. 191, $w = AE - AD = AB \cos x - CD \cot x$, or $w = 25 \cos x - 12.8 \cot x$. When the girder just swings clear, w is a maximum, hence $dw/dx = 0$. This gives (by Ex. 2, Art. 56) $-25 \sin x + 12.8 \csc^2 x = 0$. From Art. 31, $\csc x = 1/\sin x$, hence $\sin^3 x = 12.8/25 = 64/125$. Then $\sin x = 4/5$, and $\cos x = 3/5$, $\cot x = 3/4$, $w = 25(3/5) - 12.8(3/4) = 15 - 9.6 = 5.4$ ft. *Ans.*

Derivatives of higher order arise when the first derivative is differentiated, this result differentiated, etc. The order of a derivative is indicated by the number of times the original function is differentiated. A second derivative is the derivative of the first derivative. The notation is $\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d^2y}{dx^2}$ (second derivative); $\frac{d^3y}{dx^3}, \frac{d^4y}{dx^4}, \ldots, \frac{d^ny}{dx^n}$, third, fourth, . . . , nth, derivative, respectively. For $f(x)$, successive derivatives are indicated by placing an accent on the f, as $f'(x), f''(x), f'''(x), f^{\text{IV}}(x) \ldots f^{(n)}(x)$.

Sign of second derivative for a given value of x determines whether the graph is below the tangent at the corresponding point on the graph, *i.e.*, whether the graph is "concave downward," as at A, Fig. 188, or above the tangent, *i.e.*, "concave upward," as at B, Fig. 188. The rule is, if the value of the second derivative is positive, the graph is concave upward; if negative, the graph is concave downward.

Second test for maximum and minimum values of a function. $f(x)$ is a maximum for $x = a$ if $f'(a) = 0$ and $f''(a)$ is negative, and a minimum if $f'(a) = 0$ and $f''(a)$ is positive. This second test is convenient when the second derivative can be readily calculated.

58. Differentials

Definitions. The DIFFERENTIAL OF A FUNCTION is the product of its derivative by the increment of the independent variable. $d[f(x)] = f'(x)\Delta x$. Then $d(x) = 1 \times \Delta x$ (written $dx = \Delta x$). Also $dy = dy/dx$ times dx. The differential and the increment of an independent variable are identical, but the differential and increment of a function are not. In Fig. 192, when passing from P to P', $\Delta x = MM' = PQ = dx$, increment of function $y = f(x)$ is QP', but the differential of the function is QT, that is the differential of the function is the increment of the ordinate of the tangent to the graph at the point under consideration. Differentials are INFINITESIMALS, that is, variables whose numerical values decrease and ultimately become zero. The preceding definitions give the derivative equal to the quotient of corresponding differentials dy and dx (a DIFFERENTIAL QUOTIENT). The differential of a function is the principal part of its increment. The ratio of the increment to the differential becomes unity when the increment of the variable becomes zero. In problems involving infinitesimals, the increment of a function is replaced by its differential. This principle of replacing the increment by its differential has wide application in calculus. More generally, one infinitesimal may be replaced by another if the limit of their ratio is unity.

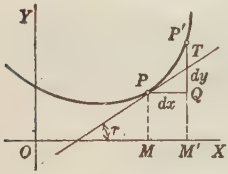

FIG. 192.

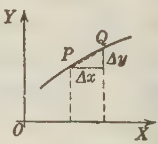

FIG. 193.

Example. Find the differential of the arc of a curve. Passing from P to Q along the curve (Fig. 193), the increment of arc is arc $PQ = \Delta s$. The chord $PQ = \sqrt{(\Delta x)^2 + (\Delta y)^2}$. When $\Delta x \doteq 0$, the ratio of arc PQ to chord PQ becomes unity. Hence chord PQ may be replaced by Δs, this by ds, and Δx and Δy by dx and dy, respectively. Hence $ds = \sqrt{dx^2 + dy^2}$. *Ans.*

Differential of arc s of a curve. In rectangular co-ordinates, $ds = \sqrt{dx^2 + dy^2} = \sqrt{1 + (dy/dx)^2}\ dx = \sqrt{1 + (dx/dy)^2}\ dy$. In polar coordinates $ds = \sqrt{d\rho^2 + \rho^2 d\theta^2} = \sqrt{\rho^2 + (d\rho/d\theta)^2}\, d\theta$.

59. Small errors

If an error h is made in measuring x, the corresponding error in calculating $f(x)$ is $f(x + h) - f(x)$ = increment of $f(x)$. When h is numerically small, the increment of the function may be replaced by its differential, that is, *error in $f(x) = f'(x)$ times error in x.* This formula determines the degree of approximation in value of a function when the degree of accuracy of the measurement of the variable is known.

Example. An angle x is measured as 45° with a possible error of $\pm$ 1′. What is the corresponding error in tan x? *Solution.* The error in tan $x = \frac{d}{dx}$ (tan x) times the error in x (in radians) = sec² x times *error in x*. By Table 11, 1′ = 0.000291 rad. Sec² 45° = 2. Hence the required error is $\pm$ 0.000582 unit, that is, tan x lies between 0.999 and 1.001, using three decimal places.

Probable error of $y = f(x)$ is $f'(x)$ *times the probable error in* x. (Art. 22.)

60. Rates

In Fig. 192 rate of change of y along the tangent PT per unit change in x is constant and equals numerically the slope of the tangent. But the rate of change of the function at P (instantaneous rate) per unit change in x is the same as the rate of change at y along the tangent at P. This gives an interpretation of dy/dx as a rate, namely, rate of change in y per unit change in x. This fact in connection with derivatives is of great importance in applied science.

Examples. (1) The velocity of a point moving along a line at a distance x from a fixed point on that line is dx/dt, (variable t representing time), since VELOCITY is the rate of change of distance per unit change in time (time rate of change of distance). ACCELERATION $= dv/dt$, the time-rate of change of velocity.

(2) Sand pouring from a chute at the rate of 50 cu. ft. per min. forms a conical pile whose base has a diameter always equal to its height. How rapidly is the height of the pile increasing when it is 10 ft. high?

In Fig. 194 the volume of the pile $= V = \frac{1}{3}\pi x \cdot x^2/4$ (Art. 30). Hence $V = \pi x^3/12$. Differentiating with respect to t, the result is $\frac{dV}{dt} = \pi/12 \times 3x^2 dx/dt$. Now dV/dt = time-rate of change of volume = 50 cu. ft. per min. (from the given data), and x = 10 ft. Also dx/dt is the rate at which the height of the pile is increasing per unit time. This is the unknown sought. Solving, $dx/dt = 4\frac{dV}{dt}/\pi x^2 = 200/100\pi = 0.64$ ft. per min. *Ans.*

61. Curvature

Definitions. The CURVATURE OF A CURVE relates to the rate at which a point describing the curve changes the direction of its motion. In mathematical terms, CURVATURE AT A POINT is the rate of change of the angle which the tangent forms with a fixed line per unit arc on the curve (arc-rate of change of direction). In Fig. 195 the tangent turns through angle Δi as P moves to P', a distance Δs along the curve. The curvature at $P = di/ds$. Angle i is measured in radians, and curvature is therefore expressed in radians per unit length of arc. The CURVATURE OF A CIRCLE is the same at all points and equals the reciprocal of the radius, numerically. CIRCLE OF CURVATURE (Fig. 196) at point P on a curve is the circle tangent at P whose curvature equals that of the curve at P. The center of this circle is the CENTER OF CURVATURE of the curve. The circle of curvature crosses the curve at the point of contact.

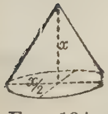

FIG. 194.

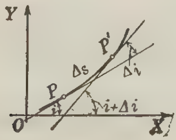

FIG. 195.

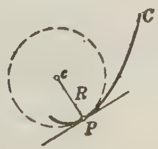

FIG. 196.

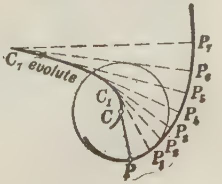

FIG. 197.

Formulas for curvature

Rectangular co-ordinates. K = curvature, R = radius of curvature, (a, b) = center of curvature, $p = dy/dx$, $q = d^2y/dx^2$.

$K = q/(1 + p^2)^{3/2}$. $R = 1/K$. $a = x - p(1 + p^2)/q$. $b = y + (1 + p^2)/q$.

Polar co-ordinates. $s = d\rho/d\theta$, $t = d^2\rho/d\theta^2$.

$K = (\rho^2 - t\rho + 2s^2)/(\rho^2 + s^2)^{3/2}$. $R = 1/K$.

The point of intersection of normals drawn at consecutive points $P(x, y)$ and $Q(x + \Delta x, y + \Delta y)$ approches the center of curvature at P when Q moves along the curve toward P.

Evolute of a curve (Fig. 197) is the locus of the centers of curvature of that curve. To find the equation of the evolute from the rectangular equation of the curve, work out expressions for a and b by the formulas given above, and eliminate x and y from these equations by means of the equation of the given curve.

Formulas for radii of curvature (R) for standard curves, and their evolutes

Parabola $(y^2 = 2px)$. $R = (p + 2x)^{3/2}/\sqrt{p} = (p^2 + y^2)^{3/2}/p^2$ at (x, y).
Evolute $27py^2 = 8(x - p)^3$.

Ellipse $(b^2x^2 + a^2y^2 = a^2b^2)$. $R = (b^4x^2 + a^4y^2)^{3/2}/a^4b^4$ at (x, y).
Evolute $(ax)^{2/3} + (by)^{2/3} = (a^2 - b^2)^{2/3}$.

Hyperbola $(b^2x^2 - a^2y^2 = a^2b^2)$. $R = (b^4x^2 + a^4y^2)^{3/2}/a^4b^4$ at (x, y).
Evolute $(ax)^{2/3} - (by)^{2/3} = (a^2 + b^2)^{2/3}$.

Rectangular hyperbola $(2xy = a^2)$. $R = (x^2 + y^2)^{3/2}/a^2$ at (x, y).
Evolute $(x + y)^{2/3} - (x - y)^{2/3} = 2a^{2/3}$.

Cycloid (equations, Art. 54). $R = 4a \sin \theta/2 = 2AP$ (Fig. 177).
Evolute $x = a(\theta' - \sin \theta')$, $y = a(1 - \cos \theta')$, $(\theta' = \theta + \pi)$, an equal cycloid.

Epicycloid (equations, Art. 54). $R = 4(R' + r)r \sin (R'\theta/2r)/(R' + 2r)$. ($R'$ = the radius of the fixed circle.)

Hypocycloid (equations, Art. 54). $R = 4\,(R' - r)r \sin (R'\theta/2r)/(R' - 2r)$. ($R'$ = the radius of the fixed circle.)

Catenary (Art. 54), $(y = a \cosh x/a)$. $R = y^2/a$, at any point (x, y).

Spiral of Archimedes (Art. 54), ($\rho = a\theta$ [θ in radians]) $R = (\rho^2 + a^2)^{3/2}/(\rho^2 + 2a^2)$.

Logarithmic spiral (Art. 54), $(\rho = be^{a\theta})$ $R = \rho\sqrt{1 + a^2}$.

Properties of an evolute. The normal to a given curve is tangent to the evolute at the corresponding center of curvature. The length of the arc of the evolute between two centers of curvature equals the difference of radii of the circles of curvature of the given curve having these centers.

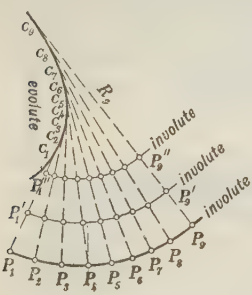

FIG. 198.

In Fig. 197, arc C_1C_7 on the evolute = radius C_7P_7 − radius C_1P_1.

Curves having a given curve as a common evolute are called INVOLUTES of that curve. (Also called PARALLEL CURVES.)

Fig. 198 shows three involutes with a common evolute. If a flexible string is made to take the form of the evolute, and the free end is unwound and kept taut, this end will move along an involute.

62. Series

Definitions. A POWER SERIES consists of an indicated sum of terms each of which is the product of a constant and a positive integral power of a variable, arranged according to ascending powers of the variable, the number of terms being unlimited (INFINITE SERIES).

Example. $1 + \frac{1}{2}x + \frac{1}{3}x^2 + \frac{1}{4}x^3 + \ldots$ *ad infinitum.*

For a given value of the variable, the sum S_n of n (any number of) terms is a function of n. If, as n increases indefinitely, S_n remains finite and approaches a definite value S, that is, if $\underset{n \doteq \infty}{\text{Limit}}\ S_n = S$, then the series is said to be CONVERGENT for this value of the variable and to have a sum equal to S. A power series which is convergent for values of the variable within a certain interval is called a CONVERGENT SERIES. A power series convergent for no value of the variable is called DIVERGENT.

Expansion of a function in a power series is accomplished by means of the following formulas.

Taylor's series

Powers of $(x - a)$, where a is a given value of x.

$$f(x) = f(a) + f'(a)(x - a) + f''(a)\frac{(x-a)^2}{1\cdot 2} + f'''(a)\frac{(x-a)^3}{1\cdot 2\cdot 3} + \ldots + f^n(a)\frac{(x-a)^n}{n!} + \ldots$$

Maclaurin's series

Powers of x.

$$f(x) = f(o) + f'(o)x + f''(o)\frac{x^2}{1\cdot 2} + f'''(o)\frac{x^3}{1\cdot 2\cdot 3} + \ldots + f^{(n)}(o)\frac{x^n}{n!} + \ldots.$$

Such an expansion is valid only for values of x for which the series is convergent. Determination of the interval of convergence of a given series is a necessary precaution before using it instead of the function in numerical calculations. (See below where the interval of convergence is given for many series.) Expansion of a function as a polynomial of nth degree with a remainder is given by Taylor's Series using first $(n + 1)$ terms and adding the REMAINDER TERM, $f^{(n+1)}(z)x^{n+1}/(n + 1)!$, where the value of z lies between a and x. This polynomial gives an approximate representation of the function for those values of x for which the remainder term is numerically less than the limit of error determined by the problem in hand.

Alternating series have the characteristic that successive terms have opposite signs. Such a series converges if successive terms decrease in numerical value and tend toward a limit zero. The sum is then less than the first term.

Convergent power series. The interval of convergence is written after each series.

1. **Binomial Series** (Theorem) — Interval of convergence

$$(1+x)^{-1/2} = 1 - \tfrac{1}{2}x + \frac{1\cdot 3}{2\cdot 4}x^2 - \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}x^3 + \ldots \qquad x^2 < 1.$$

$$(a+bx)^{-1} = \frac{1}{a}\left(1 - \frac{bx}{a} + \frac{b^2x^2}{a^2} - \frac{b^3x^3}{a^3} + \ldots\right) \qquad b^2x^2 < a^2.$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \ldots \qquad x^2 < 1$$

2. $$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots + \frac{x^n}{n!} + \ldots \qquad \text{all values of } x.$$

3. $$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots \qquad \text{all values of } x.$$

4. $$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots \qquad \text{all values of } x.$$

5. $$\tan x = x + \frac{x^3}{3} + \frac{2}{15}x^5 + \frac{17}{315}x^7 + \ldots \qquad x^2 < \tfrac{1}{4}\pi^2.$$

6. $$\log_e x = 2\left(\frac{x-1}{x+1} + \frac{1}{3}\left(\frac{x-1}{x+1}\right)^3 + \frac{1}{5}\left(\frac{x-1}{x+1}\right)^5 + \ldots\right) \qquad x > 0.$$

7. $$\sec x = 1 + \frac{x^2}{2!} + \frac{5x^4}{4!} + \frac{61x^6}{6!} + \ldots \qquad x^2 < \tfrac{1}{4}\pi^2$$

8. $$\sin^{-1} x = x + \frac{1}{6}x^3 + \frac{1\cdot 3}{2\cdot 4}\frac{x^5}{5} + \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\frac{x^7}{7} + \ldots \qquad x^2 < 1.$$

9. $$\tan^{-1} x = x - \tfrac{1}{3}x^3 + \tfrac{1}{5}x^5 - \tfrac{1}{7}x^7 + \ldots. \qquad x^2 < 1.$$

10. $$\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \ldots \qquad \text{all values of } x.$$

11. $$\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \ldots \qquad \text{all values of } x.$$

An approximate formula for a function results when the function is set equal to a finite number of terms of its expansion in a convergent series. Such a formula holds with a certain degree of exactness for values of the variable for which the series is convergent. If the series is alternating, the error is less than the term in the series following the last term in the formula used.

Example. $1/\sqrt{1+x} = 1 - \tfrac{1}{2}x$ for $x^2 < 1$. The error is numerically less than $3x^2/8$. This result follows directly from the first series given above.

Computation by series is often a useful means of calculation when Tables are not at hand, or when the degree of approximation desired exceeds that given in Tables, or obtainable by the logarithms which are accessible.

Example. Calculate $(34)^{2/5}$ to four decimal places. The nearest fifth power to 34 is $32 = 2^5$. Write the required quantity $(32 + 2)^{2/5} = (32(1 + 1/16))^{2/5} = (32)^{2/5}(1 + 1/16)^{2/5} = 4(1 + 1/16)^{2/5}$. Now $(1 + x)^{2/5} = 1 + 2x/5 - 3x^2/25 + 8x^3/125 - 26x^4/625 +$ etc. Put $x = 1/16$. Then $(34)^{2/5} = 4 + 0.1 - 0.001875 + 0.0000625 - 0.000003 + \ldots$ Taking 4 terms, the result is 4.0981875 and the error is less than the fifth term 0.000003. Hence 4.09818 is correct to five decimal places. (See Table 27.)

63. Partial derivatives

A function of several independent variables $x, y, z, \ldots$ is represented by $f(x, y, z, \ldots)$. Derivatives may be formed by varying one variable only, and are called PARTIAL DERIVATIVES. NOTATION: $\delta f/\delta x$, $\delta f/\delta y$, $\delta f/\delta z$, etc., or D_xf, D_yf, D_zf, etc. The TOTAL DIFFERENTIAL is $df = \frac{\delta f}{\delta x}dx + \frac{\delta f}{\delta y}dy + \frac{\delta f}{\delta z}dz + \ldots$

When the independent variables receive increments dx, dy, dz, etc., the PRINCIPAL PART of the increment of the function is df. When $x, y, z \ldots$ are functions of the same variable t, the TOTAL DERIVATIVE of f with respect to t is

$$\frac{df}{dt} = \frac{\delta f}{\delta x}\frac{dx}{dt} + \frac{\delta f}{\delta y}\frac{dy}{dt} + \frac{\delta f}{dz}\frac{dz}{dt} + \cdots .$$

This formula gives the rate of change of a function in terms of the rates of change of the independent variables.

Small errors. When the value of a function is determined by measurement of several variables $x, y, z, \ldots$ with small errors $dx, dy, dz, \ldots$, the corresponding error df in the function is given by the above formula for total differential df.

Example. Two sides x, y of a triangle are 81.25 in., 91.04 in., respectively, with possible errors $\pm$ .01 in. in each. The included angle C is 30° with a possible error of 1′. Find the maximum error possible in the area.

Solution. Area $= u = \frac{1}{2}xy \sin C$ (Art. 29). Form du by the above formula. Then $du = \frac{1}{2}y \sin C\,dx + \frac{1}{2}x \sin C dy + \frac{1}{2}xy \cos C dC$. Substitute $x = 81.25$, $y = 91.04$, $dx = dy = .01$, $\sin C = 0.5$, $\cos C = 0.8660$, $dC = 0.0002909$ (radians in 1′). Result is $du = 1.36$ sq. in. *Ans.* Note that the area computed by the given values is 1850 sq. in.

Percentage error is the relative error du/u(d Nap log u) times 100.

In the above example, $du/u = dx/x + dy/y + \cot CdC$, that is, the relative error in area equals the sum of the relative errors in measurements of the sides x and y increased by the relative error in sin C. (See Art. 2.) With the same absolute error in reading the angle C, the relative error in the value of sin C decreases as C increases. For the values given, $du/u = 0.000735$. Hence the percentage error is 7/100 of 1 per cent.

The probable error e of a function $f(x, y, z, \ldots)$ in terms of the probable errors e_1, e_2, e_3 . . . of measured quantities x, y, z, is given by formula

$$e = \sqrt{(\delta f/\delta x)^2 e_1^2 + (\delta f/\delta y)^2 e_2^2 + (\delta f/\delta z)^2 e_3^2 + \cdots}.$$

INTEGRAL CALCULUS

64. Integration

Definitions. INTEGRATION is the process of finding a function of which a given function is the derivative, hence it is the inverse of differentiation. The sign of integration is $\int$ read "*integral of.*" The process is indicated by writing the integral sign before the given function multiplied by the differential of the variable.

Example. $\int 6xdx = 3x^2$, since $\frac{d}{dx}(3x^2) = 6x$. Also $\int 6xdx = 3x^2 + C$, where C is any constant, called the CONSTANT OF INTEGRATION.

Integration is accomplished by reference to a Table of Integrals. The given integral is compared with one of similar form in the table, and, if necessary, made identical with this by simple substitutions. The examples given below illustrate the method.

65. Table of integrals*

General forms

(1) $\int d[f(x)](dx = f(x).$

(2) $\int cf(x)dx = c\int f(x)dx$, where c is a constant.

(3) $\int (u + v)dx = \int udx + \int vdx.$

(4) $\int udv = uv - \int vdu.$

Remark. Formula 4, "integration by parts" is very useful. It may be written

$$\int h(x)d[f(x)] = h(x)f(x) - \int f(x)d[h(x)].$$

The integral remaining in the right-hand member may often be found in tables when the original integral is not. In applying the formula, the given integrand is to be factored into the product of a function $h(x)$ by the differential of another function $f(x)$.

Integrals involving $a + bx$

(5) $\int (a + bx)^n dx = \dfrac{(a + bx)^{n+1}}{b(n + 1)}$, unless $n = -1$, then use (6).

(6) $\int \dfrac{dx}{a + bx} = \dfrac{1}{b}$ Nap log $(a + bx)$.

(7) $\int \dfrac{xdx}{a + bx} = \dfrac{1}{b^2}[a + bx - a$ Nap log $(a + bx)]$.

(8) $\int \dfrac{x^2dx}{a + bx} = \dfrac{1}{b^3}[\tfrac{1}{2}(a + bx)^2 - 2a(a + bx) + a^2$ Nap log $(a + bx)]$.

(9) $\int \dfrac{dx}{x(a + bx)} = -\dfrac{1}{a}$ Nap log $\dfrac{a + bx}{x}$.

(10) $\int \dfrac{xdx}{(a + bx)^2} = \dfrac{1}{b^2}\left[\text{Nap log } (a + bx) + \dfrac{a}{a + bx}\right]$.

* The constant of integration C should be added to the right-hand member in every formula. a, b, c, m, n, p, are constants.

(11) $$\int \frac{x^2 dx}{(a+bx)^2} = \frac{1}{b^3}\left[a + bx - 2a \text{ Nap log } (a+bx) - \frac{a^2}{a+bx}\right].$$

(12) $$\int x\sqrt{a+bx}\,dx = -\frac{2(2a-3bx)\sqrt{(a+bx)^3}}{15b^2}.$$

(13) $$\int \frac{dx}{x\sqrt{a+bx}} = \frac{1}{\sqrt{a}} \text{ Nap log } \frac{\sqrt{a+bx}-\sqrt{a}}{\sqrt{a+bx}+\sqrt{a}}, \text{ for } a > 0.$$

(14) $$\int \frac{dx}{x\sqrt{a+bx}} = \frac{2}{\sqrt{-a}} \tan^{-1}\sqrt{\frac{a+bx}{-a}}, \text{ for } a < 0.$$

(15) $$\int \frac{x\,dx}{\sqrt{a+bx}} = -\frac{2(2a-bx)}{3b^2}\sqrt{a+bx}.$$

(16) $$\int \frac{\sqrt{a+bx}\,dx}{x} = 2\sqrt{a+bx} + a\int \frac{dx}{x\sqrt{a+bx}}.$$

(17) $$\int \frac{x^n dx}{\sqrt{a+bx}} = \frac{2x^n\sqrt{a+bx}}{b(2n+1)} - \frac{2an}{b(2n+1)}\int \frac{x^{n-1}dx}{\sqrt{a+bx}}.$$

(18) $$\int \frac{dx}{x^n\sqrt{a+bx}} = -\frac{\sqrt{a+bx}}{(n-1)ax^{n-1}} - \frac{(2n-3)b}{2(n-1)a}\int \frac{dx}{x^{n-1}\sqrt{a+bx}},$$ unless $n = -1$, then see 13, 14.

Integrals involving $a^2 - x^2$, or $x^2 - a^2$, or $x^2 + a^2$

(19) $$\int \frac{dx}{a^2 - b^2x^2} = \frac{1}{2ab} \text{ Nap log } \frac{a+bx}{a-bx}.$$

(20) $$\int \frac{dx}{a^2 + b^2x^2} = \frac{1}{ab} \tan^{-1}\frac{bx}{a}.$$

(21) $$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\frac{x}{a}.$$

(22) $$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \text{Nap log } (x + \sqrt{x^2 \pm a^2}).$$

(23) $$\int \frac{dx}{\sqrt{x^2 + a^2}} = \sinh^{-1}\frac{x}{a}.$$

(24) $$\int \frac{dx}{\sqrt{x^2 - a^2}} = \cosh^{-1}\frac{x}{a}.$$

(25) $$\int x(a+bx^2)^p dx = \frac{(a+bx^2)^{p+1}}{2b(p+1)},$$ unless $p = -1$, then use (26).

(26) $$\int \frac{x\,dx}{a+bx^2} = \frac{1}{2b} \text{ Nap log } (a+bx^2).$$

(27) $\int \sqrt{a^2 - x^2}\, dx = \tfrac{1}{2}x\sqrt{a^2 - x^2} + \tfrac{1}{2}a^2 \sin^{-1}\frac{x}{a}.$

(28) $\int \sqrt{x^2 \pm a^2}\, dx = \tfrac{1}{2}x\sqrt{x^2 \pm a^2} \pm \tfrac{1}{2}a^2 \text{ Nap. log } (x + \sqrt{x^2 \pm a^2}).$

(29) $\int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a}\cos^{-1}\frac{a}{x}.$

(30) $\int \frac{dx}{x\sqrt{a^2 \pm x^2}} = -\frac{1}{a}\text{Nap log}\left(\frac{a + \sqrt{a^2 \pm x^2}}{x}\right).$

(31) $\int \frac{\sqrt{a^2 \pm x^2}\, dx}{x} = \sqrt{a^2 \pm x^2} - a \text{ Nap log}\left(\frac{a + \sqrt{a^2 \pm x^2}}{x}\right).$

(32) $\int \frac{\sqrt{x^2 - a^2}\, dx}{x} = \sqrt{x^2 - a^2} - a\cos^{-1}\frac{a}{x}.$

(33) $\int \frac{\sqrt{a^2 - x^2}}{x^2}dx = -\frac{\sqrt{a^2 - x^2}}{x} - \sin^{-1}\frac{x}{a}.$

(34) $\int \frac{\sqrt{x^2 \pm a^2}}{x^2}\, dx = -\frac{\sqrt{x^2 \pm a^2}}{x} + \text{Nap log } (x + \sqrt{x^2 \pm a^2}).$

(35) $\int \frac{x^m dx}{(a^2 - x^2)^{1/2}} = -\frac{x^{m-1}}{m}\sqrt{a^2 - x^2} + \frac{(m-1)a^2}{m}\int \frac{x^{m-2}}{(a^2 - x^2)^{1/2}}dx.$

(36) $\int \frac{x^m dx}{\sqrt{x^2 \pm a^2}} = \frac{x^{m-1}}{m}\sqrt{x^2 \pm a^2} \mp \frac{a^2(m-1)}{m}\int \frac{x^{m-2}dx}{\sqrt{x^2 \pm a^2}}.$

Integrals involving $a + bx + cx^2$

(37) $\int \frac{dx}{a + bx + cx^2} = \frac{2}{\sqrt{4ac - b^2}}\tan^{-1}\frac{2cx + b}{\sqrt{4ac - b^2}},$ when $b^2 < 4ac$.

(38) $\int \frac{dx}{a + bx + cx^2} = \frac{1}{\sqrt{b^2 - 4ac}}\text{ Nap log }\frac{2cx + b - \sqrt{b^2 - 4ac}}{2cx + b + \sqrt{b^2 - 4ac}},$

when $b^2 > 4ac$.

(39) $\int \frac{dx}{\sqrt{a + bx - cx^2}} = \frac{1}{\sqrt{c}}\sin^{-1}\frac{2cx - b}{\sqrt{b^2 + 4ac}}.$

(40) $\int \frac{dx}{\sqrt{a + bx + cx^2}} = \frac{1}{\sqrt{c}}\text{ Nap log } (2cx + b + 2\sqrt{c}\sqrt{a + bx + cx^2}).$

(41) $\int \frac{x\, dx}{a + bx + cx^2} = \frac{1}{2c}\text{ Nap log } (a + bx + cx^2) - \frac{b}{2c}\int \frac{dx}{a + bx + cx^2}.$

(42) $\int \frac{x\, dx}{\sqrt{a + bx + cx^2}} = \frac{1}{c}\sqrt{a + bx + cx^2} - \frac{b}{2c}\int \frac{dx}{\sqrt{a + bx + cx^2}}.$

Other algebraic integrals

(43) $\int x^{n-1}(a + bx^n)^p dx = \frac{(a + bx^n)^{p+1}}{bn(p + 1)}$, unless $p = -1$, then use 44.

(44) $\int \frac{x^{n-1}dx}{a + bx^n} = \frac{1}{nb}$ Nap log $(a + bx^n)$.

Integrals involving transcendental functions

(45) $\int e^{ax}dx = e^{ax}/a.$

(46) $\int a^{nx}dx = a^{nx}/n$ Nap log a.

(47) $\int$ Nap log $nx\, dx = x($Nap log $nx - 1)$.

(48) $\int \sin ax\, dx = -\cos ax/a.$

(49) $\int \cos ax\, dx = \sin ax/a.$

(50) $\int \tan ax\, dx =$ Nap log $\sec ax/a$.

(51) $\int \cot ax\, dx =$ Nap log $\sin ax/a$.

(52) $\int \sec ax\, dx =$ Nap log $(\sec ax + \tan ax)/a$.

(53) $\int \csc ax\, dx =$ Nap log $(\csc ax - \cot ax)/a$.

(54) $\int \sin^2 ax\, dx = \left(\frac{a}{2}x - \frac{1}{4}\sin 2ax\right)/a.$

(55) $\int \cos^2 ax\, dx = \left(\frac{ax}{2} + \frac{1}{4}\sin 2ax\right)/a.$

(56) $\int \tan^2 ax\, dx = \tan ax/a - x.$

(57) $\int \cot^2 ax\, dx = -\cot ax/a - x.$

(58) $\int \sec^2 ax\, dx = \tan ax/a.$

(59) $\int \csc^2 ax\, dx = -\cot ax/a.$

(60) $\int \sec ax \tan ax \, dx = \sec ax/a.$

(61) $\int \csc ax \cot ax \, dx = - \csc ax/a.$

(62) $\int (\sin x)^n dx = -\dfrac{(\sin x)^{n-1} \cos x}{n} + \dfrac{n-1}{n} \int (\sin x)^{n-2} dx.$

(63) $\int (\cos x)^n \, dx = \dfrac{(\cos x)^{\;-1} \sin x}{n} + \dfrac{n-1}{n} \int (\cos x)^{n-2} \, dx.$

(64) $\int (\sin x)^n \cos x \, dx = \dfrac{(\sin x)^{n-1}}{n+1}.$

(65) $\int (\cos x)^n \sin x \, dx = -\dfrac{(\cos x)^{n+1}}{n+1}.$

(66) $\int (\tan x)^n \, dx = \dfrac{(\tan x)^{n-1}}{n-1} - \int (\tan x)^{n-2} \, dx.$

(67) $\int (\cot x)^n \, dx = -\dfrac{(\cot x)^{n-1}}{n-1} - \int (\cot x)^{n-2} dx.$

(68) $\int x e^{ax} dx = \dfrac{e^{ax}}{a^2}(ax - 1).$

(69) $\int x^n e^{ax} dx = \dfrac{x^n e^{ax}}{a} - \dfrac{n}{a} \int x^{n-1} \, e^{ax} dx.$

(70) $\int x^n \text{ Nap} \log x \, dx = x^{n+1}\left[\dfrac{\text{Nap} \log x}{n+1} - \dfrac{1}{(n+1)^2}\right].$

(71) $\int x^m \sin x \, dx = - x^m \cos x + m \int x^{m-1} \cos x \, dx.$

(72) $\int x^m \cos x \, dx = x^m \sin x - m \int x^{m-1} \sin x \, dx.$

(73) $\int e^{ax} \sin nx \, dx = \dfrac{e^{ax} (a \sin nx - n \cos nx)}{a^2 + n^2}.$

(74) $\int e^{ax} \cos nx \, dx = \dfrac{e^{ax} (n \sin nx + a \cos nx)}{a^2 + n^2}.$

(75) $\int \sin^{-1} x \, dx = x \sin^{-1} x + \sqrt{1 - x^2}.$

(76) $\int \cos^{-1} x \, dx = x \cos^{-1} x - \sqrt{1 - x^2}.$

(77) $\int \tan^{-1} x \, dx = x \tan^{-1} x - \tfrac{1}{2} \text{ Nap} \log (1 + x^2).$

(78) $\int \sec^{-1} x\,dx = x \sec^{-1} x - \text{Nap}\log(x + \sqrt{x^2 - 1})$.

(79) $\int \sinh x\,dx = \cosh x$.

(80) $\int \cosh x\,dx = \sinh x$.

(81) $\int \tanh x\,dx = \log\cosh x$.

(82) $\int \coth x\,dx = \log\sinh x$.

For a more extended table, see B. O. Peirce, *A short table of integrals*, by Ginn and Company, Boston, Mass.

Examples in integration

(1) Work out $\int \sqrt{9 - 4x^2}\,dx$.

Since $\sqrt{9 - 4x^2} = 2\sqrt{(3/2)^2 - x^2}$, then $\int \sqrt{9 - 4x^2}\,dx = \int 2\sqrt{(3/2)^2 - x^2}\,dx = 2\int \sqrt{(3/2)^2 - x^2}\,dx$, by (2). The integral remaining is now in the form (27), with $a = 3/2$.
$\therefore \int \sqrt{9 - 4x^2}\,dx = x\sqrt{(3/2)^2 - x^2} + (3/2)^2 \sin^{-1} 2x/3 + C$. *Ans.*

(2) Work out $\int x \sin 2x\,dx$.

This resembles formula 71. Put $v = 2x$, or $x = \frac{1}{2}v$. Hence $dx = \frac{1}{2}dv$. Then $\int x \sin 2x\,dx = \int \frac{1}{2}v \sin v\,\frac{1}{2}dv = \frac{1}{4}\int v \sin v\,dv$, by (2). The remaining integral is now (71), with $m = 1$. Using this and then putting $v = 2x$, the result is $\frac{1}{4}\sin 2x - \frac{1}{2}x\cos 2x + C$. *Ans.*

(3) Work out $\int \frac{x^2dx}{4 - x^2}$.

This example illustrates cases when the division indicated in the integral can be performed. For $x^2/(4 - x^2) = -1 + 4/(4 - x^2)$.
$\therefore \int \frac{x^2dx}{4 - x^2} = -\int dx + 4\int \frac{dx}{4 - x^2} = -x + \text{Nap}\log\frac{2 + x}{2 - x} + C$. *Ans.*

As a rule, in a quotient of two polynomials, divide numerator by denominator when the latter is of lower degree than the former or of the same degree.

66. Constant of integration

Integration by tables gives INDEFINITE INTEGRALS. Two indefinite integrals of a function differ by a constant. The CONSTANT OF INTEGRATION appearing in an indefinite integral takes on a definite value when the data of the problem are properly given.

Example. The slope of a certain curve at any point (x, y) is $-3x/2y$, and the curve passes through (4, 6). Find the equation of curve.

By Art. 56, the slope $= dy/dx$. Hence $dy/dx = -3x/2y$, from which the differential equation $2y\,dy = -3x\,dx$ results. Integrating both members, $y^2 = -3x^2/2 + C$, when C is to be determined so that the curve passes through (4, 6). Putting $x = 4$, $y = 6$, then $36 = -24 + C$, hence $C = 60$. The required equation is $y^2 = -3x^2/2 + 60$, or $3x^2 + 2y^2 - 120 = 0$, an ellipse. *Ans.*

67. Definite integral

Integrating between limits (DEFINITE INTEGRAL) consists in evaluating an indefinite integral for two values of variable x, say $x = b$, and $x = a$, and subtracting the results. The constant of integration then disappears. The notation follows:

If $\int f(x)dx = F(x) + C$, then $\int_a^b f(x)dx = F(b) - F(a)$. b is the UPPER LIMIT; a the LOWER LIMIT. Read the second equation, "integral from a to b of $f(x)\ dx$ equals," etc. Interchanging the limits changes the sign of the definite integral.

68. Applications of integral calculus

Area between the curve $y = f(x)$, the X-axis, and the ordinates at $x = a$, $x = b$ (Fig. 199), such as area $CDPFE$), is given by the definite integral

$$\text{Area} = \int_a^b y\,dx, \text{ when } y = f(x).$$

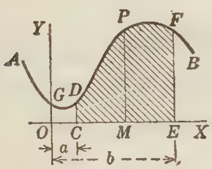

FIG. 199.

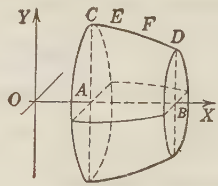

FIG. 200.

Area swept over by radius vector ρ of a curve $\rho = f(\theta)$ (polar equation, Art. 48) turning from $\theta = \alpha$ to $\theta = \beta$ is

$$\text{Area} = \tfrac{1}{2}\int_\alpha^\beta \rho^2 d\theta, \text{ when } \rho = f(\theta).$$

Length of arcs. End-points (x_1, y_1), (x_2, y_2), in rectangular co-ordinates; or (ρ_1, θ_1), (ρ_2, θ_2), in polar co-ordinates.

$$s = \int \sqrt{dx^2 + dy^2} = \int_{x_1}^{x_2} \sqrt{1 + (dy/dx)^2}\,dx = \int_{y_1}^{y_2} \sqrt{(dx/dy)^2 + 1}\,dy.$$

$$s = \int \sqrt{d\rho^2 + \rho^2 d\theta^2} = \int_{\theta_1}^{\theta_2} \sqrt{(d\rho/d\theta)^2 + \rho^2}\,d\theta = \int_{\rho_1}^{\rho_2} \sqrt{1 + (\rho d\theta/d\rho)^2}\,d\rho.$$

The derivative dy/dx, or $d\rho/d\theta$, must be found from the equation (rectangular or polar) of the curve, and the radical reduced to a function of the variable involved in the differential.

Volume of a solid of revolution. Axis of revolution OX, meridian section $y = f(x)$, $x = a$, $x = b$, at extremities of meridian section. (Fig. 200.) $OA = a$, $OB = b$, equation of $CEFD$ is $y = f(x)$,

$$V = \pi \int_a^b y^2 dx.$$

Axis of revolution OY, meridian section $x = h(y)$, $y = c$, $y = d$, at extremities of meridian section.

$$V = \pi \int_c^d x^2 dy.$$

Area of surface of revolution. (x, y) is a point on the meridian section.

$$S = 2\pi \int y \, ds, \text{ or } S = 2\pi \int x \, ds,$$

according to whether the axis of revolution is OX, or OY.

The value of ds is worked out by Art. 58, and the limits of the integral are determined by the co-ordinates of the extremities of the meridian section.

Examples. (1) Find the length of arc of one arch of a cycloid (Art. 54) with the equations $x = a(\theta - \sin \theta)$, $y = a(1 - \cos \theta)$.

Differentiating, $dx = a(1 - \cos \theta)d\theta$, $dy = a \sin \theta \, d\theta$.

Hence $dx^2 + dy^2 = a^2[(1 - \cos \theta)^2 + \sin^2 \theta]d\theta^2 = 2a^2(1 - \cos \theta)d\theta = 4a^2 \sin^2 (\frac{1}{2}\theta) \, d\theta$ (Art. 41).

Then $s = \int \sqrt{dx^2 + dy^2} = \int 2a \sin \frac{1}{2}\theta \, d\theta$. Limits are 0° and 360°.

$$s = -4a \cos \tfrac{1}{2}\theta \Big]_0^{360°} = 4a + 4a = 8a. \quad Ans.$$

(2) Find the volume and surface of the paraboloid, Fig. 201, if $OA = h$, $AB = AD = a$.

The equation of the meridian section OPB is $y^2 = a^2x/h$. Then $V = \pi \int y^2 dx = \pi \int a^2x/h \cdot dx = \pi a^2x^2/2h$, with limits $x = h$, $x = 0$. Hence $V = \pi a^2 h/2$, that is, the area of the base $BDCE$ times half of the height OA.

To find the area of the surface, calculate $ds = \sqrt{1 + (dy/dx)^2} \, dx$. Differentiating $y^2 = a^2x/h$, the result is $2y \, dy/dx = a^2/h$, from which $dy/dx = a^2/2hy$. Hence $ds = \sqrt{1 + a^4/4h^2y^2} \, dx = \sqrt{4h^2y^2 + a^4} \, dx/2hy$.

Then $S = 2\pi \int y ds = \pi/h \int \sqrt{4h^2y^2 + a^4} \, dx = a\pi/h \int \sqrt{4hx + a^2} \, dx = a\pi(4hx + a^2)^{3/2}/6h^2$ (using formula (43), Art. 65, $n = 1$). Limits are $x = h$, $x = 0$.

Hence $S = \pi a[(4h^2 + a^2)^{3/2} - a^3]/6h^2$. *Ans.*

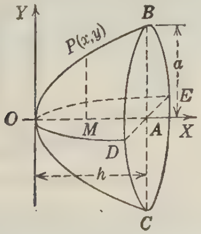

Fig. 201.

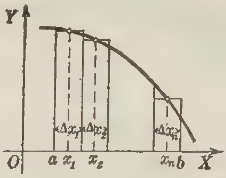

Fig. 202.

69. Integration as a summation

A definite integral is the limit of a sum of differential (infinitesimal) elements. Many useful applications of the integral calculus depend upon the fundamental theorem of integral calculus, which, stated abstractly, is to the following effect: Given a function $f(x)$ continuous in interval from $x = a$ to $x = b$. Let this interval (Fig. 202) be divided into any number (n) of parts of respective lengths Δx_1, Δx_2 . . ., Δx_n. On each of these segments (Fig. 202)

take a point, and let the values of x at these points be $x_1, x_2, \ldots, x_n$, respectively. Then the corresponding values of $f(x)$, are $f(x_1), f(x_2), \ldots, f(x_n)$. Form the sum of the products of each of these values of $f(x)$ by the length of the corresponding segment, namely the sum, $f(x_1)\Delta x_1 + f(x_2)\Delta x_2 + \ldots + f(x_n)\Delta x_n$. Let the number of segments be increased indefinitely so that the length of each approaches zero, then the limiting value of the above sum is the value of the integral of $f(x)dx$ evaluated for limits $x = a$, $x = b$. In the usual notation, $\int_a^b f(x)dx = \underset{n = \infty}{\text{Limit}} \ (f(x_1)\Delta x_1 + f(x_2)\Delta x_2 + \ldots + f(x_n)\Delta x_n)$.

Each term of the sum in the right-hand member is a DIFFERENTIAL ELEMENT. The value of the theorem lies in affording means of calculating the limit of a sum of differential elements of a certain form. The theorem has wide application.

In Fig. 202, the curve is $y = f(x)$; the products $f(x_1)\Delta x_1$, etc., are the areas of rectangles; the sum of the products is approximately the area under the curve, and the limit of the sum is the exact area. But this also equals the definite integral in the left-hand member in the above equation (Art. 68). Hence the result as above.

Examples. (1) The area swept over by the radius vector in Fig. 203 is the limit of the sum of the circular sectors with radii ρ_1, ρ_2, etc., and central angles $\Delta\theta_1$, $\Delta\theta_2$, etc., hence of area $\frac{1}{2}\rho_1{}^2\Delta\theta_1$, $\frac{1}{2}\rho_2{}^2\Delta\theta_2$, etc. (See Art. 29.) The fundamental theorem leads at once to the formula (Art. 68) $A = \frac{1}{2}\int \rho^2 d\theta$.

(2) The volume of the solid of revolution about OX (Fig. 204) may be considered the limit of the sum of circular cylinders with altitudes Δx_1, Δx_2, etc., and base radii y_1, y_2, etc., hence of volumes $\pi y_1{}^2\Delta x_1$, $\pi y_2{}^2\Delta x_2$, etc. The fundamental theorem gives immediately $V = \pi\int y^2 dx$.

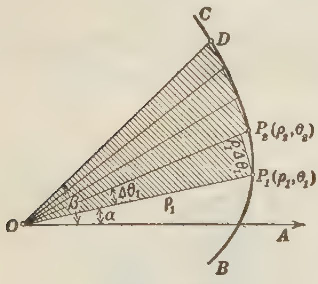

FIG. 203.

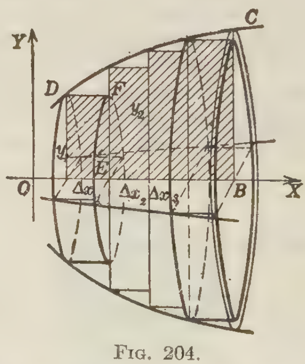

FIG. 204.

(3) A certain solid answers the following description: the base is an ellipse with semi-axes a, b (Art. 50); plane sections standing on the base and perpendicular to the major axis are squares. Find the volume.

With axes OX, OY in the base (Fig. 205), the equation of the ellipse is $b^2x^2 + a^2y^2 = a^2b^2$, or $y^2 = b^2(a^2 - x^2)/a^2$. Divide the solid into slices of equal infinitesimal thickness Δx by sections perpendicular to OX, and consider these slices trimmed into rectangular blocks such as $PQMN - P'Q'M'N'$. The co-ordinates of M are (x, y). The volume of this block $= 4y^2\Delta x$. Then the required volume is the limit of the sum of all such blocks when the thickness (Δx) approaches zero. Hence

$$V = \int 4y^2 dx = 4\int b^2(a^2 - x^2)/a^2 \cdot dx = 4b^2(a^2x - \tfrac{1}{3}x^3)/a^2,$$

substituting the value of y^2 from equation of base. The limits are $x = -a$, $x = a$, giving $V = 16ab^2/3$. *Ans.*

(4) **Work expended in friction in thrust bearings.** A vertical shaft M turns in a bearing (Fig. 206). At a distance x in. from the axis of the shaft let the normal pressure on the bearing due to load P lb. in the direction of the shaft axis be p lb. per sq. in. (unit normal pressure). Then the normal pressure on an infinitesimal ring of radius x and width dx is $p \times 2\pi x\, dx$ (since $2\pi x\, dx$ = the area of the ring). Then the load P must equal the sum of the normal pressures on all such infinitesimal rings, that is, $P = 2\pi \int px\, dx$. Let f = the coefficient of sliding friction. Then the work expended in one revolution in friction on an infinitesimal ring $= fp \times 2\pi x\, dx$ times the circumference of the ring $(2\pi x)$. Hence the total work expended, W, is the sum of the above elements of work for all such rings, that is $W = 4\pi^2 f \int px^2 dx$ in in.-lb. The relation between W and P is $W = 2\pi fP \int px^2 dx \div \int px\, dx$. For a collar bearing (Fig. 206) the limits are $x = \frac{1}{2}d$, $x = \frac{1}{2}D$. If p is everywhere constant (the usual assumption),

$$W = fP \times 4\pi(D^3 - d^3)/3(D^2 - d^2).$$

(5) **Center of gravity. Plane line.** Divide the line into infinitesimal parts of length ds, multiply each ds by its perpendicular distance r from a given axis in the plane of the line,

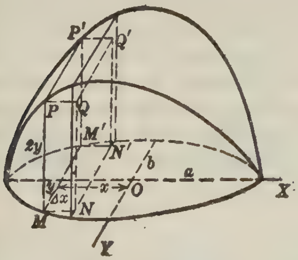

FIG. 205.

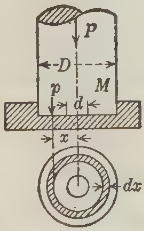

FIG. 206.

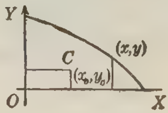

FIG. 207.

add all such products, which gives the STATIC MOMENT M of the line with respect to the axis, or $M = \int r\, ds$. Divide M by the length of the line and the result is the perpendicular distance of the center of gravity C from the given axis. A similar argument holds for plane areas, solids of revolution, etc. The formulas follow. Center of gravity is C.

Plane line. Let $C = (x_0, y_0)$; (x, y) any point on the line; L = length of line; $x_0 = \int x\, ds/L_1$, $y_0 = \int y\, ds/L_1$.

Plane area, bounded by a curve and the rectangular axes OX, OY (Fig. 207). $C = (x_0, y_0)$. A = area. (x, y) = any point on the boundary curve. $x_0 = \int xy\, dx/A$, $y_0 = \int xy\, dy/A$.

Solid of revolution. Let OX be the axis of revolution, V = volume, x_0 = distance of C from origin. The equation of the meridian section is $y = f(x)$. $x_0 = \pi \int xy^2 dx/V$.

(6) **Moment of inertia. Plane line.** The argument is the same as in Ex. 5 above, except that each infinitesimal length is multiplied by r^2. Hence the moment of inertia $I = \int r^2 ds$. In the following formulas, the subscript indicates the AXIS OF REFERENCE.

Plane line $I_x = \int y^2 ds$, $I_y = \int x^2 ds$; (x, y) = any point on line.

Plane areas as in Fig. 207. $I_x = \int y^2 x\, dy$, $I_y = \int x^2 y\, dx$; (x, y) = any point on boundary curve.

70. Approximate integration

Evaluation of $\int_a^b f(x)dx$. When a definite integral cannot be worked out by the tables available, an approximate value may be found by one of the following rules: (1) Expand $f(x)$ into a power series (Art. 62), integrate term by term, and evaluate the new series for the given limits.

(2) Plot the curve $y = f(x)$. The numerical measure of the area between the curve, the X-axis, and the ordinates at $x = a$ and $x = b$ is the value of the integral.

(*a*) Find this area by counting squares of cross-section paper.
(*b*) Use Simpson's Rule, Art. 29.
(*c*) Use a planimeter or integraph (MECHANICAL QUADRATURE). **(16)**

71. Derivation of formulas for given experimental data

To determine a formula (EMPIRICAL EQUATION) satisfied by given experimental data, when the law obtaining in the experiment is unknown, the data should be plotted on cross-section paper and the equation of a curve to "fit the points" derived. (Fig. 208.) The curve need not pass through all plotted points; derivation of an equation satisfied exactly by the given data is not the object, since such data are subject to errors of determination; a simple formula fulfilling the conditions with a degree of accuracy warranted by the data is the end desired.

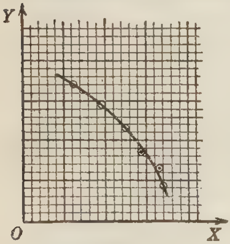

FIG. 208.

Acquaintance with several standard curves (Figs. 209–212) will assist in choosing the formula to be tested. Comparison of the curve suggested by the graph of the given data with these figures will suggest the law to be tried. Type laws are:

1. STRAIGHT-LINE LAW: $y = mx + b$.
2. POWER LAW: $y = ax^n$. (Figs. 209, 210.)
3. EXPONENTIAL LAW: $y = be^{ax}$. (Fig. 211.)
4. HYPERBOLIC LAWS: $y = (ax + b)/x$. (Fig. 216, I);
$y = x/(ax + b)$. (Fig. 216, II).

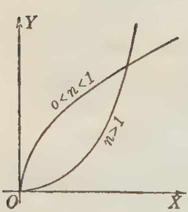

FIG. 209.

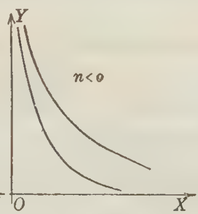

FIG. 210.

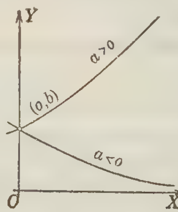

FIG. 211.

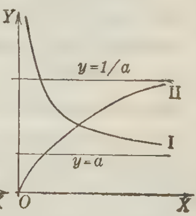

FIG. 212.

Test for a straight-line law may be made with a straight-edge; the test is affirmative when deviations from the straight line are small, some positive and some negative, and curve has no systematic curvature.

Constants in a straight-line formula may be determined by:

1. *Selected points.* Two points on the plot may be selected such that a straight line drawn through them gives satisfactory distribution of all points.

If the corresponding values selected are (x_1, y_1) and (x_2, y_2), solve the simultaneous equations $y_1 = mx_1 + b$ and $y_2 = mx_2 + b$, for the constants m and b, and place these values in the formula $y = mx + b$.

2. *Method of averages.* Substitute each pair of given values of x and y in the formula $y = mx + b$, thus forming the OBSERVATION EQUATIONS (equal in number to the number of determinations of x, y in the experiment); divide these observation equations into two groups as nearly equal in number as possible; add together all equations of each group to obtain one final equation for each group, and solve these final equations for m and b. Place the resulting values of m and b in the formula $y = mx + b$.

W	E	W	E
10	3¼	60	10½
20	4⅞	70	12¼
30	6¼	80	13¾
40	7½	90	15
50	9	100	16½
(150)	(30⅞)	(400)	(68)
	(Totals)		

Example. In an experiment with a pulley, the effort E lb. required to raise a load W lb. was found to be as in the accompanying table.

Solution. (Fig. 213.) **(1)** Straight line drawn through (30,6¼) and (100,16½) fits points well. Substituting these values in $y = mx + b$, equations are $6.25 = 30m + b$, $16.5 = 100m + b$. Solving, $m = 0.146$, $b = 1.86$. Hence the law is $E = 0.146W + 1.86$. *Ans.*

(2) Take first 5 observation equations, $3\frac{1}{4} = 10m + b, \ldots, 9 = 50m + b$, and add them; result is $30\frac{7}{8} = 150m + 5b$. Taking last 5, namely $10\frac{1}{2} = 60m + b, \ldots 16\frac{1}{2} = 100m + b$, and adding, $68 = 400m + 5b$. Solving, $m = 0.148$, $b = 1.72$. Hence result, $E = 0.148W + 1.72$. *Ans.*

3. *Method of least squares.* If the degree of accuracy of the experiment warrants, which is not often the case with engineering data, the method explained in Art. 22 may be used.

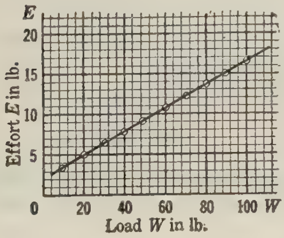

FIG. 213.

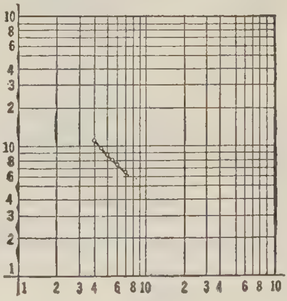

FIG. 214.

Test for power law $y = ax^n$. If the graph suggests a curve of the type in Fig. 209 or 210, the test to apply is to plot the points $(\log x, \log y)$. If a power law holds, a straight line will fit these points, because from $y = ax^n$, it follows that $\log y = \log a + n \log x$; or, putting $\log y = y'$, $\log a = a'$ and $\log x = x'$, the result is $y' = a' + nx'$, which is the equation of a straight line. The constants n and a' are found as described above; and a comes from $\log a = a'$. Values of a and n placed in $y = ax^n$ will give the desired equation.

Logarithmic cross-section paper is constructed by laying off logarithmic scales (Art. 5) on each of two perpendicular axes (Fig. 214) with the intersection marked 1 on both, and drawing lines through the points of division parallel to the axes. (Such paper, as well as semi-logarithmic paper (see below) can be purchased.) The experimental values, plotted to the scales shown in Fig. 214, are actually plotted as their logarithms with respect to ordinary

rectangular co-ordinate scales, and permit testing for a power law without looking up logarithms.

Example. Pressure (p = pounds per square inch) and volume (v = cubic feet) of 1 lb. of saturated steam were measured as in the adjoined table. Find a law for the data.

v	p
4	110
4.5	97.1
5	86.8
5.5	78.4
6	71.5
7	60.7

v'	p'	v'	p'
0.6021	2.0414	0.7404	1.8943
0.6532	1.9872	0.7782	1.8543
0.6990	1.9385	0.8451	1.7832
(1.9543)	(5.9671)	(2.3637)	(5.5318)

Solution. The curve suggested in Fig. 215 is of type in Fig. 210. A test on logarithmic cross-section paper, Fig. 214, shows that a power law, $p = av^n$ applies. To find the equation, tabulate values of $\log v = v'$ and $\log p = p'$, the relation being $p' = a' + nv'$ ($\log a = a'$), and apply the method of averages. Add the first three observation equations, and the last three (see table)

The results are, $5.9671 = 3a' + 1.9543n$ and $5.5318 = 3a' + 2.3637n$. Solving, $n = -1.063$, $a' = 0.6925$. $a = 480$. $p = 480v^{-1.063}$, or $pv^{1.063} = 480$. *Ans.*

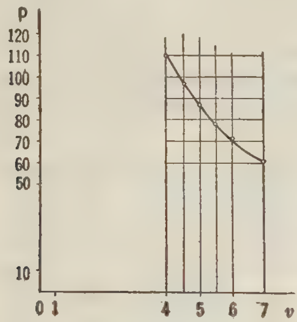

FIG. 215.

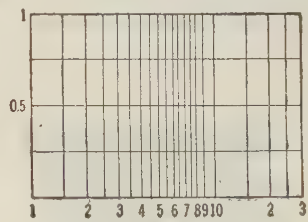

FIG. 216.

Test for exponential law $y = be^{ax}$. Type of curve, Fig. 211. Plot the experimental points as (x, $\log y$). If a straight line fits these points, an exponential law holds.

To verify this statement, take logarithms of both members of the equation, which gives $\log y = \log b + ax \log e$. Putting $\log y = y'$, $\log b = b'$, $a \log e = a'$, the result is $y' = b' + a'x$, which is the equation of a straight line.

Semi-logarithmic cross-section paper is constructed by laying off a logarithmic scale (Art. 5) from the origin on one axis of co-ordinates, and drawing lines through points of division parallel to the other axis. Draw equidistant lines parallel to the first axis. (Fig. 216.) Points (x, $\log y$) can be plotted quickly on such paper without determination of the actual logarithms and testing the data for an exponential law is thus expedited.

Test for hyperbolic laws. (See Fig. 212.) Equation $y = (ax + b)/x$ may be written $xy = ax + b$. Hence points (x, xy) lie on a straight line. Equation $y = x/(ax + b)$ is the same as $x/y = ax + b$, and therefore the points (x, x/y) lie on a straight line.

Other laws. The preceding five equations are simple "TWO-CONSTANT LAWS" and suffice for solution of many problems. Laws with three constants are:

(5) $y = a + bx + cx^2$ (PARABOLIC LAW),
(6) $\log y = a + bx + cx^2$,
(7) $y = ax^n + c$,
(8) $y = be^{ax} + c$.

When given values of x differ by a constant increment Δx (Art. 55), points $(x, \Delta y)$ will follow a straight-line law if (5) applies, and points $(x, \Delta \log y)$, if (6) holds. Curves for (7) and (8) differ from Figs. 209–210, and Fig. 211, respectively, only in the position of the X-axis. (Line $y - c = 0$ has the same relation to the curves as the X-axis in the figures named.) Value of c can often be found by trial and y replaced by $y - c$ in the methods explained above. (**16, 17, 18**.)

72. Charts for engineering formulas

The following material deals with methods of so treating formulas involving several variables that corresponding values of these variables can be read immediately from a chart. The methods possess wide application.

Notation. The variables are denoted by z, z_1, z_2, z_3, etc., and functions $f(z)$, $f(z_1)$, $F(z_1)$, $g(z_2)$, etc. of them by f, f_1, F_1, g_2, etc.

Function scales. The logarithmic scale (Art. 5) is a simple example of a function scale. In general, to construct a function scale for $f(z)$, lay off on a line a uniform scale of lengths with decimal subdivisions, calculate $f(z)$ for a number of values of z occurring in the given problem, lay off these values on the scale of lengths, and inscribe at each point thus located the corresponding value of z. If values of $f(z)$ are denoted by x, then $x = f(z)$ is called the equation of the function scale. In practice a SCALE FACTOR m (a constant) is introduced, *i.e.*, the equation of the scale is written $x = m\,f(z)$. The value of the scale factor in any given case is found as follows:

Example. Required a scale 10 in. long for $\log z$ when z varies from 0.01 to 100. *Solution.* Values of $\log z$ run from -2 to 2, a range of 4 units. Since $10/4 = 2.5$, the equation of the scale is $x = 2.5 \log z$. Generally, if the length of the scale is L in. and the range of values of the function is Z units, $m = L/Z$. Only a limited number of values of z appear on the scale; division marks indicate intermediate values. The subdivisions of the uniform scale of lengths may be retained when readings of values of $f(z)$ are desired. The function scales for $m\,f(z)$ and $m\,f(z) + c$ (c = a constant), are identical; the addition of a constant simply shifts the scale constructed for $m\,f(z)$ along the scale of lengths. If values of $f(z)$ are large, either a scale for $m\,f(z)$, where m is small, may be constructed; or a scale for $f(z) - c$, where c is large. The function scales thus far discussed, being laid off on a straight line, are called STRAIGHT SCALES.

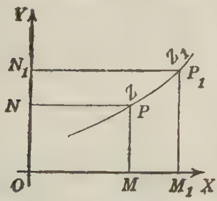

FIG. 217.

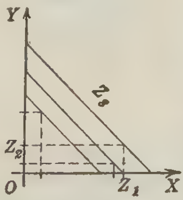

FIG. 218.

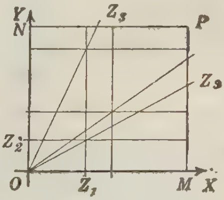

FIG. 219.

Curved scales. Let the straight scales $x = f(z)$ and $y = g(z)$ be constructed on perpendicular axes. Let M and N (Fig. 217) be points with the same value of z. Plot $x = OM$, $y = ON$. Again, let z have the same value z_1 at M_1, N_1. Plot P_1. Draw a smooth curve through all such points, and mark at each point corresponding values of z. The result is a CURVED SCALE. Equations $x = f(z)$, $y = g(z)$ are parametric equations of the curve. (Art. 45.) An indefinite number of curved scales may be constructed on any curve, corresponding to different parametric equations for that curve.

73. Charts with a network of lines

Three variables. From the explanation below it will appear that the object is to construct three systems of curves such that corresponding values of the variables are attached to three curves through a common point. In the examples given such a system of curves is composed of straight lines and easily constructed. In other cases the labor involved is so great that another type of chart is to be preferred. (See Art. 74.)

Type 1. $f_2 = f_1 + f_3$. On perpendicular axes OX, OY (Fig. 218) construct the scales $x = m_1 f_1$, $y = m_2 f_2$, and draw a net work of parallels to the axes through the points of division. Construct the system of parallel lines $y = m_2\, x/m_1 + m_2 f_3$ which result by assigning to z_3 the values used in the problem, and mark on each line the corresponding value of z_3, called the PARAMETER of the system. The values of z_1, z_2, z_3 attached to any three lines in the diagram through a point will satisfy the equation. From the given data it may be desirable to use the scales $x = m_1 f_1 + c_1$, $y = m_2 f_2 + c_2$. Parallel lines are now $y = m_2(x - c_1)/m_1 + m_2 f_3 + c_2$. In practice such constants are usually introduced. See Ex. 1 below.

Type 2. $f_2 = f_1 f_3$. Construct scales $x = m_1 f_1$, $y = m_2 f_2$ as in Type 1, and the system of lines $y = m_2 f_3 x/m_1$ intersecting at $O(x = 0,\ y = 0)$. (Fig. 219.) If MP and NP are drawn through the extremities of the scales on OX and OY, the values of z_3 may be conveniently written along these lines. If the scales $x = m_1 f_1 + c_1$, $y = m_2 f_2 + c_2$, are used, the system of lines is $y - c_2 = m_2 f_3\,(x - c_1)/m_1$, passing through (c_1, c_2). Transforming $f_2 = f_1 f_3$ by taking logarithms gives $\log f_2 = \log f_1 + \log f_3$, which is in the form of Type 1.

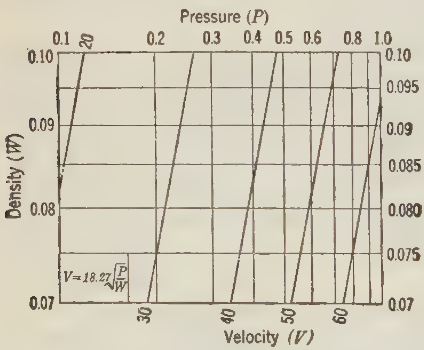

FIG. 220.

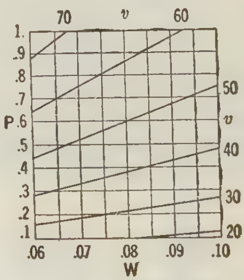

FIG. 221.

Example 1. Fig. 220 is a chart for $V = 18.27\sqrt{P/W}$, the formula for measurement of velocity of air by a Pitot tube. Taking logarithms and transposing, $\log W = \log P - 2 \log (V/18.27)$ which is Type 1. The scales used are $y = 25 \log 10W$, $x = 5 \log 10P$. The system of parallel lines is $y = 5x - 50 \log (V/18.27)$. Values of V are marked on the ends of the lines. This example illustrates logarithmic transformation of a given formula to bring it under Type 1.

Example 2. The preceding equation may be written $P = 0.003\ Wv^2$, and then comes under Type 2. Fig. 221 shows a chart, with scales $y = 5P$, $x = 100\ W$, and lines $y/x = 0.00015\ v^2$. Equations of the bounding vertical lines are $x = 6$, $x = 10$; of the bounding horizontal lines, $y = 0.5$, $y = 5$. Uniform scales are laid off on these lines. The system of lines for v may be plotted by calculating the points of intersection with the bounding lines, or also, by drawing lines through $x = 0$, $y = 0$, which is in this case a point off of the chart.

Four variables. Extensions of the preceding methods are shown in Figs. 222–224.

Type 3. $f_1 + f_2 = f_3 + f_4$. (Fig. 222.) Construct the function scales $x = m_1 f_1 + c_1$ on OM, $x' = m_3 f_3 + c_3$ on PQ, and two systems of parallel lines, (1) $y = m(x - c_1)/m_1 + mf_2$, with parameter z_2 (AB, etc.); (2) $y = m(x' - c_3)/m_3 + mf_4$, with parameter z_4 (CD, etc.). Corresponding values of z_1, z_2, z_3, z_4 are determined as indicated in the figure.

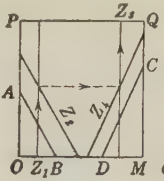

Fig. 222.

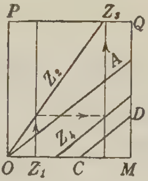

Fig. 223.

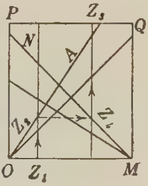

Fig. 224.

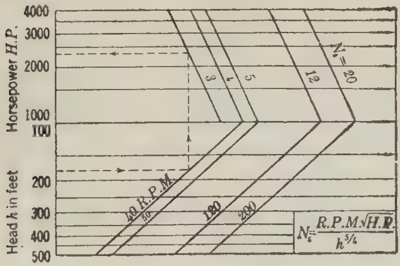

Fig. 225.

Type 4. $f_1 f_2 = f_3 + f_4$. (Fig. 223.) Construct function scales $x = m_1 f_1 + c_1$ on OM, $x' = m_3 f_3 + c_3$ on PQ, and two systems of lines, (1) $y = mf_2\,(x - c_1)/m_1$, with parameter z_2 (lines through $O(c_1, 0)$, OA, etc.), (2) $y = m(x' - c_3)/m_3 + mf_4$, with parameter z_4 (parallel lines CD, etc.). Corresponding values of z_1, z_2, z_3, z_4 are indicated in the figure.

Type 5. $f_1 f_2 = f_3 f_4$. (Fig. 224.) Construct function scales $x = m_1 f_1 + c_1$ on OM, $x' = m_3 f_3 + a$ on PQ, ($OM = a$), and two systems of lines; (1) $y = mf_2\,(x - c_1)/m_1$, with parameter z_2 (lines through $O(c_1,\ 0)$), and (2) $y = mf_4(x' - a)/m_3$, with parameter z_4 (lines through M). Corresponding values of z_1, z_2, z_3, z_4 appear in the figure.

Example 1. Fig. 225 shows a portion of a chart for $N_S = \text{R.P.M.}\sqrt{\text{H.P.}}/h^{5/4}$, which is the formula for the specific speed (N_S) for impulse and reaction water wheels. Taking logarithms, $\log N_S - \frac{1}{2}\log \text{H.P.} = \log \text{R.P.M.} - 5/4 \log h$. This comes under Type 3. The scales are $y = 3.3 \log 0.001$ H.P., $y' = -3.3 \log 0.01\, h$, both of which are laid off on the line $x = 3 \log 3$. The systems of parallel lines are $4.4\,x - 5\,y' = 13.2 \log 0.3$ R.P.M., $2.2x + y = 6.6 \log 3\,N_S$. The Y-axis is off the chart. For lines of the two systems intersecting on the X-axis (horizontal line in figure through $h = 100$), R.P.M. $= 10\,N_S$.

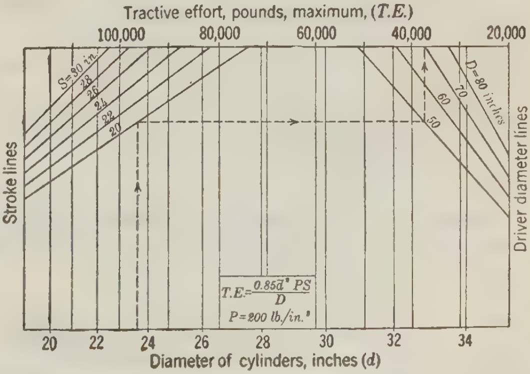

Fig. 226.

Example 2. Fig. 226 shows the lower part of a chart for T.E. $= 0.85\, d^2\, PS/D$, which is the formula for the tractive effort of a simple two-cylinder locomotive, with boiler pressure $P = 200$ lb./in.² The formula may be written $170\, d^2\, S = \text{T.E.} \times D$, and comes

under Type 5. The scales are $x = 0.0085\ d^2$, $x' = 12 - 0.75\ T/10{,}000$, both laid off on horizontal lines. The two systems of lines are (1) $y = Sx/30$, (2) $y = D(12 - x')/45$.

74. Alignment charts

Principle. For many formulas in engineering it is possible to construct function scales in such a way that readings on them at points on one or more lines will satisfy the formula. Many of these charts are simple and easily constructed. Alignment charts for the types in Art. 73 are given below.

Type 6. $f_1 + f_2 = f$. (Fig. 227.) Scales for m_1f_1, m_2f_2, mf are constructed on parallel axes with the conditions $m_1/m_2 = a/b$, $m = m_1m_2/(m_1 + m_2)$, and adjusted so that readings at M_1, M, M_2, will satisfy the equation. Then readings at any three points P_1, P, P_2, in line, will also satisfy the formula.

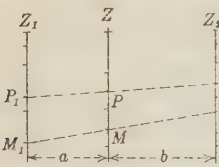

FIG. 227.

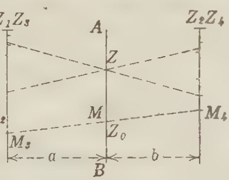

FIG. 228.

Type 7. $f_1 + f_2 = f_3 + f_4$. (Fig. 228.) Scales for m_1f_1, m_3f_3 are constructed on one axis and, scales for m_2f_2, m_4f_4, on a parallel axis, with $m_1 = m_3$, $m_2 = m_4$. A third axis AB is then drawn so that $m_1/m_2 = a/b$. Set $f_1 + f_2 = z$, and $f_3 + f_4 = z$. Chart the first of these equations as in Type 6, and compute z $(= z_0)$ at some point as M. No scale for z on AB need be constructed. Chart the second equation, adjusting the scales for m_3f_3, m_4f_4 so that readings at M_3, $M(z_0)$, M_4 on a line $M_3M\ M_4$ will correspond. If any two lines intersect on AB, readings for z_1, z_2, on one line, and z_3, z_4, on the other line, will satisfy the equation.

Example. Fig. 229 is a chart for $N_c = 1{,}550{,}000\ d^2/W^{1/2}l^{3/2}$, which is a formula for the determination of the critical speed of a shaft when the load is concentrated midway between the bearings. Taking logarithms, $\log N_c + \frac{1}{2}\log W - \log 1{,}550{,}000 = 2\log d - \frac{3}{2}\log l$, which is the form of Type 7. Scales are laid off for $10 \log d$, and $3.75 \log l$ with $m_3/m_4 = -2$, hence with axis AB to the right. The line through $d = 1$, $l = 1$ cuts AB in $z = 0$. For N_c, the scale is $5 \log N_c$. The line joining $N_c = 15{,}500$ and $z = 0$ must cut the W-scale at $W = 10{,}000$. The scale for W is $-1.25 \log W$.

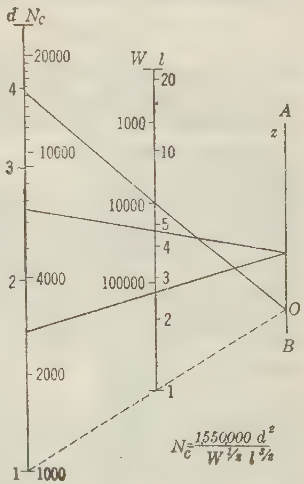

FIG. 229.

Type 8. $f = f_1/f_2$. (Fig. 230.) Construct scales for m_1f_1, $-m_2f_2$ on parallel axes, and adjust these so that line AB drawn from $A(f_1 = 0)$ to $B(f_2 = 0)$ makes a convenient angle with AM (usually 45°). On AM construct a scale for $x = am_1f/(m_2 + m_1f)$. Project these graduations on to AB by lines parallel to the axes, and mark at these points on AB the same values for z as on AM. Values of z_1, z, z_2 on any transversal will satisfy the equation. This type is called a Z-chart.

Type 9. $f_1/f_2 = f_3/f_4$. (Fig. 231.) Construct scales for m_1f_1, $-m_2f_2$, m_3f_3, $-m_4f_4$, with $m_1/m_2 = m_3/m_4$, on axes as indicated, with $f_1 = f_3 = 0$ at

A, $f_2 = f_4 = 0$ at B. Transversals intersecting on diagonal AB will cut the scales in corresponding values of z_1, z_2, z_3, z_4.

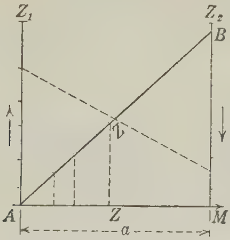

FIG. 230.

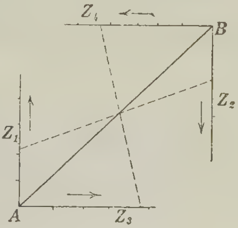

FIG. 231.

Type 10. $f_1 + f_2 = f_3/f_4$. (Fig. 232.) Lay off scales for m_1f_1, m_3f_3, on AM starting at A ($f_1 = f_3 = 0$), scale for m_4f_4 on AB from A ($f_4 = 0$), scale downward for m_2f_2 from B ($f_2 = 0$) on BQ, with $m_1 = m_2$, and m_3, $m_4 = m_1/AB$. Parallel index lines will cut the scales in corresponding values of the variables, z_3, z_4, on one line, and z_1, z_2, on the other.

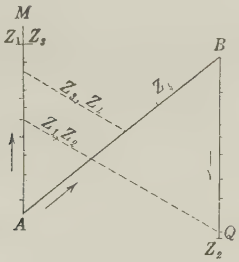

FIG. 232.

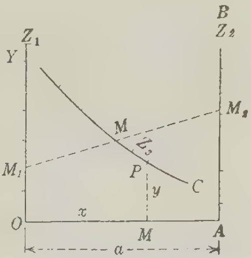

FIG. 233.

Type 11. $f_1 + f_2f_3 = F_3$. (Fig. 233.) This type brings in a curved scale. (Art. 72.) Construct the scales $y_1 = OM_1 = m_1f_1 + c_1$, $y_2 = AM_2 = m_2f_2 + c_2$, and the curved scale $x = OM = am_1f_3/(m_2 + m_1f_3)$, $y = MP = (m_1m_2F_3 + m_2c_1 + m_1c_2f_3)/(m_2 + m_1f_3)$, on the curve C. Readings on any transversal $M_1M\ M_2$ will satisfy the equation.

For other types, some of which are extensions of the preceding, see (**16, 17, 18, 19**).

MATHEMATICAL TABLES

75. Explanation of tables

Table 1. Squares of numbers with three significant figures from 1.00 to 9.99 are tabulated. Corrections to be *added* for a fourth figure 1, 2, 3, 4, 5 are given in right-hand columns. For a fourth figure 6, 7, 8, 9, take the

tabular entry for the next larger number of three figures and *subtract* the correction given for 4, 3, 2, 1, respectively.

Examples. $(1.285)^2 = 1.638 + .013 = 1.651$. $(1.286)^2 = 1.664 - .010 = 1.654$.

If the decimal point is moved *one* place in N, move it *two* places in the tabular number.

$$(0.128)^2 = 0.01638. \quad (12.85)^2 = 165.1.$$

Table 2. Cubes of numbers. Method of correction for a fourth significant figure is the same as above. Interpolate for a fourth significant figure when corrections in the right-hand column are missing. If the decimal point is moved *one* place in N, move it *three* places in the tabular number.

$$(9.86)^3 = 958.6. \quad (0.986)^3 = 0.9586. \quad (98.6)^3 = 958600.$$

Table 3. Square roots. If the decimal point is moved two places in N, move it one place in the tabular number.

$$\sqrt{3.142} = 1.773. \quad \sqrt{314.2} = 17.72. \quad \sqrt{0.03142} = 0.1773.$$

Table 4. Cube roots. If the decimal point is moved *three* places in N, move it one place in the tabular number.

$$\sqrt[3]{34.98} = 3.270. \quad \sqrt[3]{34980} = 32.70. \quad \sqrt[3]{0.03498} = 0.3270.$$

Table 5. Reciprocals. The tabular values *decrease* with *increasing* N, hence the correction given in the right-hand column for a fourth figure 1, 2, 3, 4, 5 must be *subtracted* from the tabular value given for the first three figures of N. For a fourth figure 6, 7, 8, 9, *add* to the tabular value given for the next larger number the correction for 4, 3, 2, 1, respectively.

$$1 \div 3.654 = 0.2736. \quad 1 \div 3.658 = 0.2734.$$

If the decimal place in N is moved to *right* (or *left*), move it the same number of places to *left* (or *right*) in the tabular value.

$$1 \div 0.3654 = 2.736. \quad 1 \div 0.003654 = 273.6.$$

Tables 6–8. Circumferences and areas of circles; volumes of spheres. Formulas for calculation are $Circumference = \pi D$, $Area = \pi D^2/4$, $Volume = \pi D^3/6$, $D = diameter$, $\pi = 3.141593$. In Tables 6–8, move the decimal point one place in **circumference,** two places in **area,** three places in **volume,** if the decimal point is moved one place in D. If the circumference C, area A, or volume V is given, diameter D is found by

$$D = 0.31831C = 1.12838\sqrt{A} = 1.24070\sqrt[3]{V}.$$

Table 9. Circular segments. (Art. 29, Fig. 97.) To find the height or chord for any radius, multiply the corresponding tabular number by the radius. To find the area, multiply the tabular number by the square of the radius.

Given a central angle 36° and radius 2, then height $= 0.0489 \times 2 = 0.0978$, chord $= 0.6180 \times 2 = 1.2360$, area $= 0.02027 \times 4 = 0.0811$.

Tables 10–19 are self-explanatory.

Table 20. Exponentials. Values of e^u and e^{-u}. For definition of e, see Art. 15. Interpolation will be inaccurate for values of u greater than 1. The last column gives the common log of e^u or multiples of $\log_{10} e = 0.434294$. Interpolation should not be used in this column.

Table 21. Hyperbolic functions are expressions of frequent occurrence involving e^u. Those tabulated are the hyperbolic sine ($\sinh u$), hyperbolic cosine ($\cosh u$), hyperbolic tanh ($\tanh u$), defined in terms of e^u as follows: $\sinh u = \frac{1}{2}(e^u - e^{-u})$, $\cosh u = \frac{1}{2}(e^u + e^{-u})$, $\tanh u = \sinh u/\cosh u$. Other hyperbolic functions are $\operatorname{sech} u = 1/\cosh u$, $\operatorname{cosech} u = 1/\sinh u$, $\coth u = 1/\tanh u$. Other relations between hyperbolic functions are $\cosh^2 u - \sinh^2 u = 1$, $\operatorname{sech}^2 u = 1 - \tanh^2 u$. Avoid interpolation for $u > 1$

Trigonometric and hyperbolic functions satisfy the equations $\sin ui = i \sinh u$, $\cos ui = \cosh u$, $(i = \sqrt{-1})$, see Art 21.)

Bibliography. (1) A good presentation of computation, errors, etc., is given by Langley, *A treatise on computation* (Longmans). (2) A short list follows. Wentworth-Smith, *Trigonometric and logarithmic tables, five places* (Ginn and Co.); *Macmillan's Logarithmic and trigonometric tables, five places* (Macmillan Co.); Wells, *New six-place logarithmic tables* (Heath and Co.); Hutton, *Mathmetical tables, seven places* (London); Schrön, *Seven-place tables* (German) (Brunswick). (3) A good explanation of the slide rule with numerous problems s given by Dunlop and Jackson, *Slide-rule notes* (Longmans). See also pamphlets published by the manufacturers, for example, Cox, *The Mannheim slide rule* (Keuffel and Esser Co.). Many other types of slide rules are on the market, some of which are designed for the solution of special problems. For illustrations and descriptions of these as well as a great variety of calculating machines see *Methods of calculation, a handbook of the Exhibition at the Napier Tercentenary Celebration* (G Bell and Sons, London). (4) For an excellent textbook on elementary algebra see Hawkes, Luby, and Touton, *Complete school algebra* (Ginn and Co.). (5) Hawkes, *Advanced algebra* (Ginn and Co.); Hall and Knight, *Algebra for colleges and schools* (Macmillan Co.); Wentworth, *College algebra* (Ginn and Co.) (6) Merriman, *A textbook on the method of least squares* (Wiley and Sons). (7) A good presentation of financial arithmetic is given in Skinner, *The mathematical theory of investment* (Ginn and Co.). (8) Hoover, *Principles of mining* (Hill Publishing Co.). (9) Inwood's *Tables of interest, etc.* (London). (10) Cranville, *Plane and spherical trigonometry and tables* (Ginn and Co.). (11) Moritz, *Elements of plane trigonometry* (Wiley and Sons). (12) Riggs, *Analytic geometry* (Macmillan); (13) Smith and Gale, *New analytic geometry* (Ginn and Co.). (14) Granville, *Differential and integral calculus (revised)* (Ginn and Co.). (15) Osgood, *Differential and integral calculus* (Macmillan Co.). (16) For explanation of the underlying theory of mechanical quadrature and description of various types of planimeters and the integraph see Lipka, *Graphical and mechanical computation* (John Wiley and Sons), p. 246. (17) Saxelby, *Practical mathematics* (Longmans); (18) Running, *Empirical formulas* (John Wiley and Sons). (17) Peddle, *The construction of graphical charts* (McGraw-Hill); (18) d'Ocagne, *Traité de nomographie* (Paris); (19) Strachan, *Nomographic solutions for formulas of various types* (*Trans. Am. Soc. Civil Engineers, vol.* 78, 1915). (20) For squares, cubes, square roots, cube roots, and reciprocals to 10 significant figures see Barlow, *New mathematical tables* (London). (21) More extensive tables are *Smithsonian tables, Hyperbolic functions* (Washington, 1909).

Table 1. Squares of numbers (see Art. 75)

N	0	1	2	3	4	5	6	7	8	9	Prop. parts 1	2	3	4	5
1.0	1.000	1.020	1.040	1.061	1.082	1.102	1.124	1.145	1.166	1.188	2	4	6	8	11
1.1	1.210	1.232	1.254	1.277	1.300	1.322	1.346	1.369	1.392	1.416	2	5	7	9	12
1.2	1.440	1.464	1.488	1.513	1.538	1.562	1.588	1.613	1.638	1.664	3	5	8	10	13
1.3	1.690	1.716	1.742	1.769	1.796	1.822	1.850	1.877	1.904	1.932	3	5	8	11	14
1.4	1.960	1.988	2.016	2.045	2.074	2.102	2.132	2.161	2.190	2.220	3	6	9	12	15
1.5	2.250	2.280	2.310	2.341	2.372	2.402	2.434	2.465	2.496	2.528	3	6	9	12	16
1.6	2.560	2.592	2.624	2.657	2.690	2.722	2.756	2.789	2.822	2.856	3	7	10	13	17
1.7	2.890	2.924	2.958	2.993	3.028	3.062	3.098	3.133	3.168	3.204	4	7	11	14	18
1.8	3.240	3.276	3.312	3.349	3.386	3.422	3.460	3.497	3.534	3.572	4	7	11	15	19
1.9	3.610	3.648	3.686	3.725	3.764	3.802	3.842	3.881	3.920	3.960	4	8	12	16	20
2.0	4.000	4.040	4.080	4.121	4.162	4.202	4.244	4.285	4.326	4.368	4	8	12	16	21
2.1	4.410	4.452	4.494	4.537	4.580	4.622	4.666	4.709	4.752	4.796	4	9	13	17	22
2.2	4.840	4.884	4.928	4.973	5.018	5.062	5.108	5.153	5.198	5.244	5	9	14	18	23
2.3	5.290	5.336	5.382	5.429	5.476	5.522	5.570	5.617	5.664	5.712	5	9	14	19	24
2.4	5.760	5.808	5.856	5.905	5.954	6.002	6.052	6.101	6.150	6.200	5	10	15	20	25
2.5	6.250	6.300	6.350	6.401	6.452	6.502	6.554	6.605	6.656	6.708	5	10	15	20	26
2.6	6.760	6.812	6.864	6.917	6.970	7.022	7.076	7.129	7.182	7.236	5	11	16	21	27
2.7	7.290	7.344	7.398	7.453	7.508	7.562	7.618	7.673	7.728	7.784	6	11	17	22	28
2.8	7.840	7.896	7.952	8.009	8.066	8.122	8.180	8.237	8.294	8.352	6	11	17	23	29
2.9	8.410	8.468	8.526	8.585	8.644	8.702	8.762	8.821	8.880	8.940	6	12	18	24	30
3.0	9.000	9.060	9.120	9.181	9.242	9.302	9.364	9.425	9.486	9.548	6	12	18	24	31
3.1	9.610	9.672	9.734	9.797	9.860	9.922	9.986	10.049	......	......	6	13	19	25	32
3.1	......	......	......	......	......	......	......	10.05	10.11	10.18	1	1	2	3	
3.2	10.24	10.30	10.37	10.43	10.50	10.56	10.63	10.69	10.76	10.82	1	1	2	3	
3.3	10.89	10.96	11.02	11.09	11.16	11.22	11.29	11.36	11.42	11.49	1	1	2	3	
3.4	11.56	11.63	11.70	11.76	11.83	11.90	11.97	12.04	12.11	12.18	1	1	2	3	
3.5	12.25	12.32	12.39	12.46	12.53	12.60	12.67	12.74	12.82	12.89	1	1	2	3	
3.6	12.96	13.03	13.10	13.18	13.25	13.32	13.40	13.47	13.54	13.62	1	1	2	3	
3.7	13.69	13.76	13.84	13.91	13.99	14.06	14.14	14.21	14.29	14.36	1	2	2	3	
3.8	14.44	14.52	14.59	14.67	14.75	14.82	14.90	14.98	15.05	15.13	1	2	2	3	
3.9	15.21	15.29	15.37	15.44	15.52	15.60	15.68	15.76	15.84	15.92	1	2	2	3	
4.0	16.00	16.08	16.16	16.24	16.32	16.40	16.48	16.56	16.65	16.73	1	2	2	3	
4.1	16.81	16.89	16.97	17.06	17.14	17.22	17.31	17.39	17.47	17.56	1	2	2	3	
4.2	17.64	17.72	17.81	17.89	17.98	18.06	18.15	18.23	18.32	18.40	1	2	3	3	
4.3	18.49	18.58	18.66	18.75	18.84	18.92	19.01	19.10	19.18	19.27	1	2	3	3	
4.4	19.36	19.45	19.54	19.62	19.71	19.80	19.89	19.98	20.07	20.16	1	2	3	4	
4.5	20.25	20.34	20.43	20.52	20.61	20.70	20.79	20.88	20.98	21.07	1	2	3	4	
4.6	21.16	21.25	21.34	21.44	21.53	21.62	21.72	21.81	21.90	22.00	1	2	3	4	
4.7	22.09	22.18	22.28	22.37	22.47	22.56	22.66	22.75	22.85	22.94	1	2	3	4	
4.8	23.04	23.14	23.23	23.33	23.43	23.52	23.62	23.72	23.81	23.91	1	2	3	4	
4.9	24.01	24.11	24.21	24.30	24.40	24.50	24.60	24.70	24.80	24.90	1	2	3	4	
5.0	25.00	25.10	25.20	25.30	25.40	25.50	25.60	25.70	25.81	25.91	1	2	3	4	
5.1	26.01	26.11	26.21	26.32	26.42	26.52	26.63	26.73	26.83	26.94	1	2	3	4	
5.2	27.04	27.14	27.25	27.35	27.46	27.56	27.67	27.77	27.88	27.98	1	2	3	4	
5.3	28.09	28.20	28.30	28.41	28.52	28.62	28.73	28.84	28.94	29.05	1	2	3	4	
5.4	29.16	29.27	29.38	29.48	29.59	29.70	29.81	29.92	30.03	30.14	1	2	3	4	

Table 1. Squares of numbers—*Continued* (see Art. 75)

N	0	1	2	3	4	5	6	7	8	9	Prop. parts 1	2	3	4	5
5.5	30.25	30.36	30.47	30.58	30.69	30.80	30.91	31.02	31.14	31.25	1	2	3	4	6
5.6	31.36	31.47	31.58	31.70	31.81	31.92	32.04	32.15	32.26	32.38	1	2	3	5	6
5.7	32.49	32.60	32.72	32.83	32.95	33.06	33.18	33.29	33.41	33.52	1	2	3	5	6
5.8	33.64	33.76	33.87	33.99	34.11	34.22	34.34	34.46	34.57	34.69	1	2	4	5	6
5.9	34.81	34.93	35.05	35.16	35.28	35.40	35.52	35.64	35.76	35.88	1	2	4	5	6
6.0	36.00	36.12	36.24	36.36	36.48	36.60	36.72	36.84	36.97	37.09	1	2	4	5	6
6.1	37.21	37.33	37.45	37.58	37.70	37.82	37.95	38.07	38.19	38.32	1	2	4	5	6
6.2	38.44	38.56	38.69	38.81	38.94	39.06	39.19	39.31	39.44	39.56	1	3	4	5	6
6.3	39.69	39.82	39.94	40.07	40.20	40.32	40.45	40.58	40.70	40.83	1	3	4	5	6
6.4	40.96	41.09	41.22	41.34	41.47	41.60	41.73	41.86	41.99	42.12	1	3	4	5	6
6.5	42.25	42.38	42.51	42.64	42.77	42.90	43.03	43.16	43.30	43.43	1	3	4	5	7
6.6	43.56	43.69	43.82	43.96	44.09	44.22	44.36	44.49	44.62	44.76	1	3	4	5	7
6.7	44.89	45.02	45.16	45.29	45.43	45.56	45.70	45.83	45.97	46.10	1	3	4	5	7
6.8	46.24	46.38	46.51	46.65	46.79	46.92	47.06	47.20	47.33	47.47	1	3	4	5	7
6.9	47.61	47.75	47.89	48.02	48.16	48.30	48.44	48.58	48.72	48.86	1	3	4	5	7
7.0	49.00	49.14	49.28	49.42	49.56	49.70	49.84	49.98	50.13	50.27	1	3	4	6	7
7.1	50.41	50.55	50.69	50.84	50.98	51.12	51.27	51.41	51.55	51.70	1	3	4	6	7
7.2	51.84	51.98	52.13	52.27	52.42	52.56	52.71	52.85	53.00	53.14	1	3	4	6	7
7.3	53.29	53.44	53.58	53.73	53.88	54.02	54.17	54.32	54.46	54.61	1	3	4	6	7
7.4	54.76	54.91	55.06	55.20	55.35	55.50	55.65	55.80	55.95	56.10	1	3	4	6	7
7.5	56.25	56.40	56.55	56.70	56.85	57.00	57.15	57.30	57.46	57.61	2	3	5	6	8
7.6	57.76	57.91	58.06	58.22	58.37	58.52	58.68	58.83	58.98	59.14	2	3	5	6	8
7.7	59.29	59.44	59.60	59.75	59.91	60.06	60.22	60.37	60.53	60.68	2	3	5	6	8
7.8	60.84	61.00	61.15	61.31	61.47	61.62	61.78	61.94	62.09	62.25	2	3	5	6	8
7.9	62.41	62.57	62.73	62.88	63.04	63.20	63.36	63.52	63.68	63.84	2	3	5	6	8
8.0	64.00	64.16	64.32	64.48	64.64	64.80	64.96	65.12	65.29	65.45	2	3	5	6	8
8.1	65.61	65.77	65.93	66.10	66.26	66.42	66.59	66.75	66.91	67.08	2	3	5	7	8
8.2	67.24	67.40	67.57	67.73	67.90	68.06	68.23	68.39	68.56	68.72	2	3	5	7	8
8.3	68.89	69.06	69.22	69.39	69.56	69.72	69.89	70.06	70.22	70.39	2	3	5	7	8
8.4	70.56	70.73	70.90	71.06	71.23	71,40	71.57	71.74	71.91	72.08	2	3	5	7	8
8.5	72.25	72.42	72.59	72.76	72.93	73.10	73.27	73.44	73.62	73.79	2	3	5	7	9
8.6	73.96	74.13	74.30	74.48	74.65	74.82	75.00	75.17	75.34	75.52	2	3	5	7	9
8.7	75.69	75.86	76.04	76.21	76.39	76.56	76.74	76.91	77.09	77.26	2	4	5	7	9
8.8	77.44	77.62	77.79	77.97	78.15	78.32	78.50	78.68	78.85	79.03	2	4	5	7	9
8.9	79.21	79.39	79.57	79.74	79.92	80.10	80.28	80.46	80.64	80.82	2	4	5	7	9
9.0	81.00	81.18	81.36	81.54	81.72	81.90	82.08	82.26	82.45	82.63	2	4	5	7	9
9.1	82.81	82.99	83.17	83.36	83.54	83.72	83.91	84.09	84.27	84.46	2	4	5	7	9
9.2	84.64	84.82	85.01	85.19	85.38	85.56	85.75	85.93	86.12	86.30	2	4	6	7	9
9.3	86.49	86.68	86.86	87.05	87.24	87.42	87.61	87.80	87.98	88.17	2	4	6	7	9
9.4	88.36	88.55	88.74	88.92	89.11	89.30	89.49	89.68	89.87	90.06	2	4	6	8	9
9.5	90.25	90.44	90.63	90.82	91.01	91.20	91.39	91.58	91.78	91.97	2	4	6	8	10
9.6	92.16	92.35	92.54	92.74	92.93	93.12	93.32	93.51	93.70	93.90	2	4	6	8	10
9.7	94.09	94.28	94.48	94.67	94.87	95.06	95.26	95.45	95.65	95.84	2	4	6	8	10
9.8	96.04	96.24	96.43	96.63	96.83	97.02	97.22	97.42	97.61	97.81	2	4	6	8	10
9.9	98.01	98.21	98.41	98.60	98.80	99.00	99.20	99.40	99.60	99.80	2	4	6	8	10

Table 2. Cubes of numbers (see Art. 75)

N	0	1	2	3	4	5	6	7	8	9	Prop. parts 1	2	3	4	5
1.0	1.000	1.030	1.061	1.093	1.125	1.158	1.191	1.225	1.260	1.295	Necessary to interpolate here			See Art. 75	
1.1	1.331	1.368	1.405	1.443	1.482	1.521	1.561	1.602	1.643	1.685					
1.2	1.728	1.772	1.816	1.861	1.907	1.953	2.000	2.048	2.097	2.147					
1.3	2.197	2.248	2.300	2.353	2.406	2.460	2.515	2.571	2.628	2.686					
1.4	2.744	2.803	2.863	2.924	2.986	3.049	3.112	3.177	3.242	3.308					
1.5	3.375	3.443	3.512	3.582	3.652	3.724	3.796	3.870	3.944	4.020					
1.6	4.096	4.173	4.252	4.331	4.411	4.492	4.574	4.657	4.742	4.827					
1.7	4.913	5.000	5.088	5.178	5.268	5.359	5.452	5.545	5.640	5.735					
1.8	5.832	5.930	6.029	6.128	6.230	6.332	6.435	6.539	6.645	6.751					
1.9	6.859	6.968	7.078	7.189	7.301	7.415	7.530	7.645	7.762	7.881					
2.0	8.000	8.121	8.242	8.365	8.490	8.615	8.742	8.870	8.999	9.129					
2.1	9.261	9.394	9.528	9.664	9.800	9.938	10.078	......	......	......					
2.1	......	......	......	......	......	......	10.08	10.22	10.36	10.50	1	3	4	6	7
2.2	10.65	10.79	10.94	11.09	11.24	11.39	11.54	11.70	11.85	12.01	2	3	5	6	8
2.3	12.17	12.33	12.49	12.65	12.81	12.98	13.14	13.31	13.48	13.65	2	3	5	7	8
2.4	13.82	14.00	14.17	14.35	14.53	14.71	14.89	15.07	15.25	15.44	2	4	5	7	9
2.5	15.63	15.81	16.00	16.19	16.39	16.58	16.78	16.97	17.17	17.37	2	4	6	8	10
2.6	17.58	17.78	17.98	18.19	18.40	18.61	18.82	19.03	19.25	19.47	2	4	5	8	11
2.7	19.68	19.90	20.12	20.35	20.57	20.80	21.02	21.25	21.48	21.72	2	5	7	9	11
2.8	21.95	22.19	22.43	22.67	22.91	23.15	23.39	23.64	23.89	24.14	2	5	7	10	12
2.9	24.39	24.64	24.90	25.15	25.41	25.67	25.93	26.20	26.46	26.73	3	5	8	10	13
3.0	27.00	27.27	27.54	27.82	28.09	28.37	28.65	28.93	29.22	29.50	3	6	8	11	14
3.1	29.79	30.08	30.37	30.66	30.96	31.26	31.55	31.86	32.16	32.46	3	6	9	12	15
3.2	32.77	33.08	33.39	33.70	34.01	34.33	34.65	34.97	35.29	35.61	3	6	10	13	16
3.3	35.94	36.26	36.59	36.93	37.26	37.60	37.93	38.27	38.61	38.96	3	7	10	13	17
3.4	39.30	39.65	40.00	40.35	40.71	41.06	41.42	41.78	42.14	42.51	4	7	11	14	18
3.5	42.88	43.24	43.61	43.99	44.36	44.74	45.12	45.50	45.88	46.27	4	7	11	15	19
3.6	46.66	47.05	47.44	47.83	48.23	48.63	49.03	49.43	49.84	50.24	4	8	12	16	20
3.7	50.65	51.06	51.48	51.90	52.31	52.73	53.16	53.58	54.01	54.44	4	8	13	17	21
3.8	54.87	55.31	55.74	56.18	56.62	57.07	57.51	57.96	58.41	58.86	4	9	13	18	22
3.9	59.32	59.78	60.24	60.70	61.16	61.63	62.10	62.57	63.04	63.52	5	9	14	19	
4.0	64.00	64.48	64.96	65.45	65.94	66.43	66.92	67.42	67.92	68.42	5	10	15	20	Interpolate here. See Art. 75
4.1	68.92	69.43	69.93	70.44	70.96	71.47	71.99	72.51	73.03	73.56	5	10	16		
4.2	74.09	74.62	75.15	75.69	76.23	76.77	77.31	77.85	78.40	78.95	5	11	16		
4.3	79.51	80.06	80.62	81.18	81.75	82.31	82.88	83.45	84.03	84.60	6	11	17		
4.4	85.18	85.77	86.35	86.94	87.53	88.12	88.72	89.31	89.92	90.52	6	12	18		
4.5	91.13	91.73	92.35	92.96	93.58	94.20	94.82	95.44	96.07	96.70	6	13	19		
4.6	97.34	97.97	98.61	99.25	99.90	100.54	......	......	......	......	6	13	19		
4.6	......	......	......	......	......	100.5	101.2	101.8	102.5	103.2	1	1	2	3	
4.7	103.8	104.5	105.2	105.8	106.5	107.2	107.9	108.5	109.2	109.9	1	1	2	3	
4.8	110.6	111.3	112.0	112.7	113.4	114.1	114.8	115.5	116.2	116.9	1	1	2	3	
4.9	117.6	118.4	119.1	119.8	120.6	121.3	122.0	122.8	123.5	124.3	1	1	2	3	
5.0	125.0	125.8	126.5	127.3	128.0	128.8	129.6	130.3	131.1	131.9	1	2	2	3	
5.1	132.7	133.4	134.2	135.0	135.8	136.6	137.4	138.2	139.0	139.8	1	2	2	3	
5.2	140.6	141.4	142.2	143.1	143.9	144.7	145.5	146.4	147.2	148.0	1	2	2	3	
5.3	148.9	149.7	150.6	151.4	152.3	153.1	154.0	154.9	155.7	156.6	1	2	3	3	
5.4	157.5	158.3	159.2	160.1	161.0	161.9	162.8	163.7	164.6	165.5	1	2	3	4	
N	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5

Table 2. Cubes of numbers—*Continued* (see Art. 75)

N	0	1	2	3	4	5	6	7	8	9	Prop. parts				
											1	2	3	4	5
5.5	166.4	167.3	168.2	169.1	170.0	171.0	171.9	172.8	173.7	174.7	1	2	3	4	5
5.6	175.6	176.6	177.5	178.5	179.4	180.4	181.3	182.3	183.3	184.2	1	2	3	4	5
5.7	185.2	186.2	187.1	188.1	189.1	190.1	191.1	192.1	193.1	194.1	1	2	3	4	5
5.8	195.1	196.1	197.1	198.2	199.2	200.2	201.2	202.3	203.3	204.3	1	2	3	4	5
5.9	205.4	206.4	207.5	208.5	209.6	210.6	211.7	212.8	213.8	214.9	1	2	3	4	5
6.0	216.0	217.1	218.2	219.3	220.3	221.4	222.5	223.6	224.8	225.9	1	2	3	4	6
6.1	227.0	228.1	229.2	230.3	231.5	232.6	233.7	234.9	236.0	237.2	1	2	3	5	6
6.2	238.3	239.5	240.6	241.8	243.0	244.1	245.3	246.5	247.7	248.9	1	2	4	5	6
6.3	250.0	251.2	252.4	253.6	254.8	256.0	257.3	258.5	259.7	260.9	1	2	4	5	6
6.4	262.1	263.4	264.6	265.8	267.1	268.3	269.6	270.8	272.1	273.4	1	3	4	5	6
6.5	274.6	275.9	277.2	278.4	279.7	281.0	282.3	283.6	284.9	286.2	1	3	4	5	7
6.6	287.5	288.8	290.1	291.4	292.8	294.1	295.4	296.7	298.1	299.4	1	3	4	5	7
6.7	300.8	302.1	303.5	304.8	306.2	307.5	308.9	310.3	311.7	313.0	1	3	4	6	7
6.8	314.4	315.8	317.2	318.6	320.0	321.4	322.8	324.2	325.7	327.1	1	3	4	6	7
6.9	328.5	329.9	331.4	332.8	334.3	335.7	337.2	338.6	340.1	341.5	1	3	4	6	7
7.0	343.0	344.5	345.9	347.4	348.9	350.4	351.9	353.4	354.9	356.4	1	3	4	6	7
7.1	357.9	359.4	360.9	362.5	364.0	365.5	367.1	368.6	370.1	371.7	2	3	5	6	8
7.2	373.2	374.8	376.4	377.9	379.5	381.1	382.7	384.2	385.8	387.4	2	3	5	6	8
7.3	389.0	390.6	392.2	393.8	395.4	397.1	398.7	400.3	401.9	403.6	2	3	5	7	8
7.4	405.2	406.9	408.5	410.2	411.8	413.5	415.2	416.8	418.5	420.2	2	3	5	7	8
7.5	421.9	423.6	425.3	427.0	428.7	430.4	432.1	433.8	435.5	437.2	2	3	5	7	9
7.6	439.0	440.7	442.5	444.2	445.9	447.7	449.5	451.2	453.0	454.8	2	3	5	7	9
7.7	456.5	458.3	460.1	461.9	463.7	465.5	467.3	469.1	470.9	472.7	2	4	5	7	9
7.8	474.6	476.4	478.2	480.0	481.9	483.7	485.6	487.4	489.3	491.2	2	4	6	7	9
7.9	493.0	494.9	496.8	498.7	500.6	502.5	504.4	506.3	508.2	510.1	2	4	6	8	9
8.0	512.0	513.9	515.8	517.8	519.7	521.7	523.6	525.6	527.5	529.5	2	4	6	8	10
8.1	531.4	533.4	535.4	537.4	539.4	541.3	543.3	545.3	547.3	549.4	2	4	6	8	10
8.2	551.4	553.4	555.4	557.4	559.5	561.5	563.6	565.6	567.7	569.7	2	4	6	8	10
8.3	571.8	573.9	575.9	578.0	580.1	582.2	584.3	586.4	588.5	590.6	2	4	6	8	10
8.4	592.7	594.8	596.9	599.1	601.2	603.4	605.5	607.6	609.8	612.0	2	4	6	9	11
8.5	614.1	616.3	618.5	620.7	622.8	625.0	627.2	629.4	631.6	633.8	2	4	7	9	11
8.6	636.1	638.3	640.5	642.7	645.0	647.2	649.5	651.7	654.0	656.2	2	4	7	9	11
8.7	658.5	660.8	663.1	665.3	667.6	669.9	672.2	674.5	676.8	679.2	2	5	7	9	11
8.8	681.5	683.8	686.1	688.5	690.8	693.2	695.5	697.9	700.2	702.6	2	5	7	9	12
8.9	705.0	707.3	709.7	712.1	714.5	716.9	719.3	721.7	724.2	726.6	2	5	7	10	12
9.0	729.0	731.4	733.9	736.3	738.8	741.2	743.7	746.1	748.6	751.1	2	5	7	10	12
9.1	753.6	756.1	758.6	761.0	763.6	766.1	768.6	771.1	773.6	776.2	3	5	7	10	13
9.2	778.7	781.2	783.8	786.3	788.9	791.5	794.0	796.6	799.2	801.8	3	5	8	10	13
9.3	804.4	807.0	809.6	812.2	814.8	817.4	820.0	822.7	825.3	827.9	3	5	8	10	13
9.4	830.6	833.2	835.9	838.6	841.2	843.9	846.6	849.3	852.0	854.7	3	5	8	11	13
9.5	857.4	860.1	862.8	865.5	868.3	871.0	873.7	876.5	879.2	882.0	3	5	8	11	14
9.6	884.7	887.5	890.3	893.1	895.8	898.6	901.4	904.2	907.0	909.9	3	6	8	11	14
9.7	912.7	915.5	918.3	921.2	924.0	926.9	929.7	932.6	935.4	938.3	3	6	9	11	14
9.8	941.2	944.1	947.0	949.9	952.8	955.7	958.6	961.5	964.4	967.4	3	6	9	12	15
9.9	970.3	973.2	976.2	979.1	982.1	985.1	988.0	991.0	994.0	997.0	3	6	9	12	15
N	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5

Table 3. Square roots of numbers 1.0–5.49 (see Art. 75)

N	0	1	2	3	4	5	6	7	8	9	Prop. parts 1	2	3	4	5
1.0	1.000	1.005	1.010	1.015	1.020	1.025	1.030	1.034	1.039	1.044	0	1	1	2	2
1.1	1.049	1.054	1.058	1.063	1.068	1.072	1.077	1.082	1.086	1.091	0	1	1	2	2
1.2	1.095	1.100	1.105	1.109	1.114	1.118	1.122	1.127	1.131	1.136	0	1	1	2	2
1.3	1.140	1.145	1.149	1.153	1.158	1.162	1.166	1.170	1.175	1.179	0	1	1	2	2
1.4	1.183	1.187	1.192	1.196	1.200	1.204	1.208	1.212	1.217	1.221	0	1	1	2	2
1.5	1.225	1.229	1.233	1.237	1.241	1.245	1.249	1.253	1.257	1.261	0	1	1	2	2
1.6	1.265	1.269	1.273	1.277	1.281	1.285	1.288	1.292	1.296	1.300	0	1	1	2	2
1.7	1.304	1.308	1.311	1.315	1.319	1.323	1.327	1.330	1.334	1.338	0	1	1	2	2
1.8	1.342	1.345	1.349	1.353	1.356	1.360	1.364	1.367	1.371	1.375	0	1	1	1	2
1.9	1.378	1.382	1.386	1.389	1.393	1.396	1.400	1.404	1.407	1.411	0	1	1	1	2
2.0	1.414	1.418	1.421	1.425	1.428	1.432	1.435	1.439	1.442	1.446	0	1	1	1	2
2.1	1.449	1.453	1.456	1.459	1.463	1.466	1.470	1.473	1.476	1.480	0	1	1	1	2
2.2	1.483	1.487	1.490	1.493	1.497	1.500	1.503	1.507	1.510	1.513	0	1	1	1	2
2.3	1.517	1.520	1.523	1.526	1.530	1.533	1.536	1.539	1.543	1.546	0	1	1	1	2
2.4	1.549	1.552	1.556	1.559	1.562	1.565	1.568	1.572	1.575	1.578	0	1	1	1	2
2.5	1.581	1.584	1.587	1.591	1.594	1.597	1.600	1.603	1.606	1.609	0	1	1	1	2
2.6	1.612	1.616	1.619	1.622	1.625	1.628	1.631	1.634	1.637	1.640	0	1	1	1	2
2.7	1.643	1.646	1.649	1.652	1.655	1.658	1.661	1.664	1.667	1.670	0	1	1	1	2
2.8	1.673	1.676	1.679	1.682	1.685	1.688	1.691	1.694	1.697	1.700	0	1	1	1	1
2.9	1.703	1.706	1.709	1.712	1.715	1.718	1.720	1.723	1.726	1.729	0	1	1	1	1
3.0	1.732	1.735	1.738	1.741	1.744	1.746	1.749	1.752	1.755	1.758	0	1	1	1	1
3.1	1.761	1.764	1.766	1.769	1.772	1.775	1.778	1.780	1.783	1.786	0	1	1	1	1
3.2	1.789	1.792	1.794	1.797	1.800	1.803	1.806	1.808	1.811	1.814	0	1	1	1	1
3.3	1.817	1.819	1.822	1.825	1.828	1.830	1.833	1.836	1.838	1.841	0	1	1	1	1
3.4	1.844	1.847	1.849	1.852	1.855	1.857	1.860	1.863	1.865	1.868	0	1	1	1	1
3.5	1.871	1.873	1.876	1.879	1.881	1.884	1.887	1.889	1.892	1.895	0	1	1	1	1
3.6	1.897	1.900	1.903	1.905	1.908	1.910	1.913	1.916	1.918	1.921	0	1	1	1	1
3.7	1.924	1.926	1.929	1.931	1.934	1.936	1.939	1.942	1.944	1.947	0	1	1	1	1
3.8	1.949	1.952	1.954	1.957	1.960	1.962	1.965	1.967	1.970	1.972	0	1	1	1	1
3.9	1.975	1.977	1.980	1.982	1.985	1.987	1.990	1.992	1.995	1.997	0	1	1	1	1
4.0	2.000	2.002	2.005	2.007	2.010	2.012	2.015	2.017	2.020	2.022	0	0	1	1	1
4.1	2.025	2.027	2.030	2.032	2.035	2.037	2.040	2.042	2.045	2.047	0	0	1	1	1
4.2	2.049	2.052	2.054	2.057	2.059	2.062	2.064	2.066	2.069	2.071	0	0	1	1	1
4.3	2.074	2.076	2.078	2.081	2.083	2.086	2.088	2.090	2.093	2.095	0	0	1	1	1
4.4	2.098	2.100	2.102	2.105	2.107	2.110	2.112	2.114	2.117	2.119	0	0	1	1	1
4.5	2.121	2.124	2.126	2.128	2.131	2.133	2.135	2.138	2.140	2.142	0	0	1	1	1
4.6	2.145	2.147	2.149	2.152	2.154	2.156	2.159	2.161	2.163	2.166	0	0	1	1	1
4.7	2.168	2.170	2.173	2.175	2.177	2.179	2.182	2.184	2.186	2.189	0	0	1	1	1
4.8	2.191	2.193	2.195	2.198	2.200	2.202	2.205	2.207	2.209	2.211	0	0	1	1	1
4.9	2.214	2.216	2.218	2.220	2.223	2.225	2.227	2.229	2.232	2.234	0	0	1	1	1
5.0	2.236	2.238	2.241	2.243	2.245	2.247	2.249	2.252	2.254	2.256	0	0	1	1	1
5.1	2.258	2.261	2.263	2.265	2.267	2.269	2.272	2.274	2.276	2.278	0	0	1	1	1
5.2	2.280	2.283	2.285	2.287	2.289	2.291	2.293	2.296	2.298	2.300	0	0	1	1	1
5.3	2.302	2.304	2.307	2.309	2.311	2.313	2.315	2.317	2.319	2.322	0	0	1	1	1
5.4	2.324	2.326	2.328	2.330	2.332	2.335	2.337	2.339	2.341	2.343	0	0	1	1	1

Table 3. Square roots of numbers 5.50–9.99—*Continued* (see Art. 75)

N	0	1	2	3	4	5	6	7	8	9	Prop. parts				
											1	2	3	4	5
5.5	2.345	2.347	2.349	2.352	2.354	2.356	2.358	2.360	2.362	2.364	0	0	1	1	1
5.6	2.366	2.369	2.371	2.373	2.375	2.377	2.379	2.381	2.383	2.385	0	0	1	1	1
5.7	2.387	2.390	2.392	2.394	2.396	2.398	2.400	2.402	2.404	2.406	0	0	1	1	1
5.8	2.408	2.410	2.412	2.415	2.417	2.419	2.421	2.423	2.425	2.427	0	0	1	1	1
5.9	2.429	2.431	2.433	2.435	2.437	2.439	2.441	2.443	2.445	2.447	0	0	1	1	1
6.0	2.449	2.452	2.454	2.456	2.458	2.460	2.462	2.464	2.466	2.468	0	0	1	1	1
6.1	2.470	2.472	2.474	2.476	2.478	2.480	2.482	2.484	2.486	2.488	0	0	1	1	1
6.2	2.490	2.492	2.494	2.496	2.498	2.500	2.502	2.504	2.506	2.508	0	0	1	1	1
6.3	2.510	2.512	2.514	2.516	2.518	2.520	2.522	2.524	2.526	2.528	0	0	1	1	1
6.4	2.530	2.532	2.534	2.536	2.538	2.540	2.542	2.544	2.546	2.548	0	0	1	1	1
6.5	2.550	2.551	2.553	2.555	2.557	2.559	2.561	2.563	2.565	2.567	0	0	1	1	1
6.6	2.569	2.571	2.573	2.575	2.577	2.579	2.581	2.583	2.585	2.587	0	0	1	1	1
6.7	2.588	2.590	2.592	2.594	2.596	2.598	2.600	2.602	2.604	2.606	0	0	1	1	1
6.8	2.608	2.610	2.612	2.613	2.615	2.617	2.619	2.621	2.623	2.625	0	0	1	1	1
6.9	2.627	2.629	2.631	2.632	2.634	2.636	2.638	2.640	2.642	2.644	0	0	1	1	1
7.0	2.646	2.648	2.650	2.651	2.653	2.655	2.657	2.659	2.661	2.663	0	0	1	1	1
7.1	2.665	2.666	2.668	2.670	2.672	2.674	2.676	2.678	2.680	2.681	0	0	1	1	1
7.2	2.683	2.685	2.687	2.689	2.691	2.693	2.694	2.696	2.698	2.700	0	0	1	1	1
7.3	2.702	2.704	2.706	2.707	2.709	2.711	2.713	2.715	2.717	2.718	0	0	1	1	1
7.4	2.720	2.722	2.724	2.726	2.728	2.729	2.731	2.733	2.735	2.737	0	0	1	1	1
7.5	2.739	2.740	2.742	2.744	2.746	2.748	2.750	2.751	2.753	2.755	0	0	1	1	1
7.6	2.757	2.759	2.760	2.762	2.764	2.766	2.768	2.769	2.771	2.773	0	0	1	1	1
7.7	2.775	2.777	2.778	2.780	2.782	2.784	2.786	2.787	2.789	2.791	0	0	1	1	1
7.8	2.793	2.795	2.796	2.798	2.800	2.802	2.804	2.805	2.807	2.809	0	0	1	1	1
7.9	2.811	2.812	2.814	2.816	2.818	2.820	2.821	2.823	2.825	2.827	0	0	1	1	1
8.0	2.828	2.830	2.832	2.834	2.835	2.837	2.839	2.841	2.843	2.844	0	0	1	1	1
8.1	2.846	2.848	2.850	2.851	2.853	2.855	2.857	2.858	2.860	2.862	0	0	1	1	1
8.2	2.864	2.865	2.867	2.869	2.871	2.872	2.874	2.876	2.877	2.879	0	0	1	1	1
8.3	2.881	2.883	2.884	2.886	2.888	2.890	2.891	2.893	2.895	2.897	0	0	1	1	1
8.4	2.898	2.900	2.902	2.903	2.905	2.907	2.909	2.910	2.912	2.914	0	0	1	1	1
8.5	2.915	2.917	2.919	2.921	2.922	2.924	2.926	2.927	2.929	2.931	0	0	1	1	1
8.6	2.933	2.934	2.936	2.938	2.939	2.941	2.943	2.944	2.946	2.948	0	0	1	1	1
8.7	2.950	2.951	2.953	2.955	2.956	2.958	2.960	2.961	2.963	2.965	0	0	1	1	1
8.8	2.966	2.968	2.970	2.972	2.973	2.975	2.977	2.978	2.980	2.982	0	0	1	1	1
8.9	2.983	2.985	2.987	2.988	2.990	2.992	2.993	2.995	2.997	2.998	0	0	1	1	1
9.0	3.000	3.002	3.003	3.005	3.007	3.008	3.010	3.012	3.013	3.015	0	0	0	1	1
9.1	3.017	3.018	3.020	3.022	3.023	3.025	3.027	3.028	3.030	3.032	0	0	0	1	1
9.2	3.033	3.035	3.036	3.038	3.040	3.041	3.043	3.045	3.046	3.048	0	0	0	1	1
9.3	3.050	3.051	3.053	3.055	3.056	3.058	3.059	3.061	3.063	3.064	0	0	0	1	1
9.4	3.066	3.068	3.069	3.071	3.072	2.074	3.076	3.077	3.079	3.081	0	0	0	1	1
9.5	3.082	3.084	3.085	3.087	3.089	3.090	3.092	3.094	3.095	3.097	0	0	0	1	1
9.6	3.098	3.100	3.102	3.103	3.105	3.106	3.108	3.110	3.111	3.113	0	0	0	1	1
9.7	3.114	3.116	3.118	3.119	3.121	3.122	3.124	3.126	3.127	3.129	0	0	0	1	1
9.8	3.130	3.132	3.134	3.135	3.137	3.138	3.140	3.142	3.143	3.145	0	0	0	1	1
9.9	3.146	3.148	3.150	3.151	3.153	3.154	3.156	3.158	3.159	3.161	0	0	0	1	1

Table 3. Square roots of numbers 10–54.9—*Continued* (see Art. 75)

N	0.	1	2	3	4	5	6	7	8	9	Prop. parts 1	2	3	4	5
10.	3.162	3.178	3.194	3.209	3.225	3.240	3.256	3.271	3.286	3.302	2	3	5	6	8
11.	3.317	3.332	3.347	3.362	3.276	3.291	3.406	3.421	3.435	3.450	1	3	4	6	7
12.	3.464	3.479	3.493	3.507	3.521	3.536	3.550	3.564	3.578	3.592	1	3	4	6	7
13.	3.606	3.619	3.633	3.647	3.661	3.674	3.688	3.701	3.715	3.728	1	3	4	5	7
14.	3.742	3.755	3.768	3.782	3.795	3.808	3.821	3.834	3.847	3.860	1	3	4	5	7
15.	3.873	3.886	3.899	3.912	3.924	3.937	3.950	3.962	3.975	3.987	1	3	4	5	6
16.	4.000	4.012	4.025	4.037	4.050	4.062	4.074	4.087	4.099	4.111	1	2	4	5	6
17.	4.123	4.135	4.147	4.159	4.171	4.183	4.195	4.207	4.219	4.231	1	2	4	5	6
18.	4.243	4.254	4.266	4.278	4.290	4.301	4.313	4.324	4.336	4.347	1	2	3	5	6
19.	4.359	4.370	4.382	4.393	4.405	4.416	4.427	4.438	4.450	4.461	1	2	3	5	6
20.	4.472	4.483	4.494	4.506	4.517	4.528	4.539	4.550	4.561	4.572	1	2	3	4	6
21.	4.583	4.593	4.604	4.615	4.626	4.637	4.648	4.658	4.669	4.680	1	2	3	4	5
22.	4.690	4.701	4.712	4.722	4.733	4.743	4.754	4.764	4.775	4.785	1	2	3	4	5
23.	4.796	4.806	4.817	4.827	4.837	4.848	4.858	4.868	4.879	4.889	1	2	3	4	5
24.	4.899	4.909	4.919	4.930	4.940	4.950	4.960	4.970	4.980	4.990	1	2	3	4	5
25.	5.000	5.010	5.020	5.030	5.040	5.050	5.060	5.070	5.079	5.089	1	2	3	4	5
26.	5.099	5.109	5.119	5.128	5.138	5.148	5.158	5.167	5.177	5.187	1	2	3	4	5
27.	5.196	5.206	5.215	5.225	5.235	5.244	5.254	5.263	5.273	5.282	1	2	3	4	5
28.	5.292	5.301	5.310	5.320	5.329	5.339	5.348	5.357	5.367	5.376	1	2	3	4	5
29.	5.385	5.394	5.404	5.413	5.422	5.431	5.441	5.450	5.459	5.468	1	2	3	4	5
30.	5.477	5.486	5.495	5.505	5.514	5.523	5.532	5.541	5.550	5.559	1	2	3	4	5
31.	5.568	5.577	5.586	5.595	5.604	5.612	5.621	5.630	5.639	5.648	1	2	3	4	4
32.	5.657	5.666	5.675	5.683	5.692	5.701	5.710	5.718	5.727	5.736	1	2	3	4	4
33.	5.745	5.753	5.762	5.771	5.779	5.788	5.797	5.805	5.814	5.822	1	2	3	3	4
34.	5.831	5.840	5.848	5.857	5.865	5.874	5.882	5.891	5.899	5.908	1	2	3	3	4
35.	5.916	5.925	5.933	5.941	5.950	5.958	5.967	5.975	5.983	5.992	1	2	3	3	4
36.	6.000	6.008	6.017	6.025	6.033	6.042	6.050	6.058	6.066	6.075	1	2	2	3	4
37.	6.083	6.091	6.099	6.107	6.116	6.124	6.132	6.140	6.148	6.156	1	2	2	3	4
38.	6.164	6.173	6.181	6.189	6.197	6.205	6.213	6.221	6.229	6.237	1	2	2	3	4
39.	6.245	6.253	6.261	6.269	6.277	6.285	6.293	6.301	6.309	6.317	1	2	2	3	4
40.	6.325	6.332	6.340	6.348	6.356	6.364	6.372	6.380	6.387	6.395	1	2	2	3	4
41.	6.403	6.411	6.419	6.427	6.434	6.442	6.450	6.458	6.465	6.473	1	2	2	3	4
42.	6.481	6.488	6.496	6.504	6.512	6.519	6.527	6.535	6.542	6.550	1	2	2	3	4
43.	6.557	6.565	6.573	6.580	6.588	6.595	6.603	6.611	6.618	6.626	1	2	2	3	4
44.	6.633	6.641	6.648	6.656	6.663	6.671	6.678	6.686	6.693	6.701	1	2	2	3	4
45.	6.708	6.716	6.723	6.731	6.738	6.745	6.753	6.760	6.768	6.775	1	1	2	3	4
46.	6.782	6.790	6.797	6.804	6.812	6.819	6.826	6.834	6.841	6.848	1	1	2	3	4
47.	6.856	6.863	6.870	6.877	6.885	6.892	6.899	6.907	6.914	6.921	1	1	2	3	4
48.	6.928	6.935	6.943	6.950	6.957	6.964	6.971	6.979	6.986	6.993	1	1	2	3	4
49.	7.000	7.007	7.014	7.021	7.029	7.036	7.043	7.050	7.057	7.064	1	1	2	3	4
50.	7.071	7.078	7.085	7.092	7.099	7.106	7.113	7.120	7.127	7.134	1	1	2	3	4
51.	7.141	7.148	7.155	7.162	7.169	7.176	7.183	7.190	7.197	7.204	1	1	2	3	3
52.	7.211	7.218	7.225	7.232	7.239	7.246	7.253	7.259	7.266	7.273	1	1	2	3	3
53.	7.280	7.287	7.294	7.301	7.308	7.314	7.321	7.328	7.335	7.342	1	1	2	3	3
54.	7.348	7.355	7.362	7.369	7.376	7.382	7.389	7.396	7.403	7.409	1	1	2	3	3

Table 3. Square roots of numbers 55–99.9—*Continued* (see Art. 75)

N	0	1	2	3	4	5	6	7	8	9	Prop. parts				
											1	2	3	4	5
55.	7.416	7.423	7.430	7.436	7.443	7.450	7.457	7.463	7.470	7.477	1	1	2	3	3
56.	7.483	7.490	7.497	7.503	7.510	7.517	7.523	7.530	7.537	7.543	1	1	2	3	3
57.	7.550	7.556	7.563	7.570	7.576	7.583	7.589	7.596	7.603	7.609	1	1	2	3	3
58.	7.616	7.622	7.629	7.635	7.642	7.649	7.655	7.662	7.668	7.675	1	1	2	3	3
59.	7.681	7.688	7.694	7.701	7.707	7.714	7.720	7.727	7.733	7.740	1	1	2	3	3
60.	7.746	7.752	7.759	7.765	7.772	7.778	7.785	7.791	7.797	7.804	1	1	2	3	3
61.	7.810	7.817	7.823	7.829	7.836	7.842	7.849	7.855	7.861	7.868	1	1	2	3	3
62.	7.874	7.880	7.887	7.893	7.899	7.906	7.912	7.918	7.925	7.931	1	1	2	3	3
63.	7.937	7.944	7.950	7.956	7.962	7.969	7.975	7.981	7.987	7.994	1	1	2	3	3
64.	8.000	8.006	8.012	8.019	8.025	8.031	8.037	8.044	8.050	8.056	1	1	2	2	3
65.	8.062	8.068	8.075	8.081	8.087	8.093	8.099	8.106	8.112	8.118	1	1	2	2	3
66.	8.124	8.130	8.136	8.142	8.149	8.155	8.161	8.167	8.173	8.179	1	1	2	2	3
67.	8.185	8.191	8.198	8.204	8.210	8.216	8.222	8.228	8.234	8.240	1	1	2	2	3
68.	8.246	8.252	8.258	8.264	8.270	8.276	8.283	8.289	8.295	8.301	1	1	2	2	3
69.	8.307	8.313	8.319	8.325	8.331	8.337	8.343	8.349	8.355	8.361	1	1	2	2	3
70.	8.367	8.373	8.379	8.385	8.390	8.396	8.402	8.408	8.414	8.420	1	1	2	2	3
71.	8.426	8.432	8.438	8.444	8.450	8.456	8.462	8.468	8.473	8.479	1	1	2	2	3
72.	8.485	8.491	8.497	8.503	8.509	8.515	8.521	8.526	8.532	8.538	1	1	2	2	3
73.	8.544	8.550	8.556	8.562	8.567	8.573	8.579	8.585	8.591	8.597	1	1	2	2	3
74.	8.602	8.608	8.614	8.620	8.626	8.631	8.637	8.643	8.649	8.654	1	1	2	2	3
75.	8.660	8.666	8.672	8.678	8.683	8.689	8.695	8.701	8.706	8.712	1	1	2	2	3
76.	8.718	8.724	8.729	8.735	8.741	8.746	8.752	8.758	8.764	8.769	1	1	2	2	3
77.	8.775	8.781	8.786	8.792	8.798	8.803	8.809	8.815	8.820	8.826	1	1	2	2	3
78.	8.832	8.837	8.843	8.849	8.854	8.860	8.866	8.871	8.877	8.883	1	1	2	2	3
79.	8.888	8.894	8.899	8.905	8.911	8.916	8.922	8.927	8.933	8.939	1	1	2	2	3
80.	8.944	8.950	8.955	8.961	8.967	8.972	8.978	8.983	8.989	8.994	1	1	2	2	3
81.	9.000	9.006	9.011	9.017	9.022	9.028	9.033	9.039	9.044	9.050	1	1	2	2	3
82.	9.055	9.061	9.066	9.072	9.077	9.083	9.088	9.094	9.099	9.105	1	1	2	2	3
83.	9.110	9.116	9.121	9.127	9.132	9.138	9.143	9.149	9.154	9.160	1	1	2	2	3
84.	9.165	9.171	9.176	9.182	9.187	9.192	9.198	9.203	9.209	9.214	1	1	2	2	3
85.	9.220	9.225	9.230	9.236	9.241	9.247	9.252	9.257	9.263	9.268	1	1	2	2	3
86.	9.274	9.279	9.284	9.290	9.295	9.301	9.306	9.311	9.317	9.322	1	1	2	2	3
87.	9.327	9.333	9.338	9.343	9.349	9.354	9.359	9.365	9.370	9.375	1	1	2	2	3
88.	9.381	9.386	9.391	9.397	9.402	9.407	9.413	9.418	9.423	9.429	1	1	2	2	3
89.	9.434	9.439	9.445	9.450	9.455	9.460	9.466	9.471	9.476	9.482	1	1	2	2	3
90.	9.487	9.492	9.497	9.503	9.508	9.513	9.518	9.524	9.529	9.534	1	1	2	2	3
91.	9.539	9.545	9.550	9.555	9.560	9.566	9.571	9.576	9.581	9.586	1	1	2	2	3
92.	9.592	9.597	9.602	9.607	9.612	9.618	9.623	9.628	9.633	9.638	1	1	2	2	3
93.	9.644	9.649	9.654	9.659	9.664	9.670	9.675	9.680	9.685	9.690	1	1	2	2	3
94.	9.695	9.701	9.706	9.711	9.716	9.721	9.726	9.731	9.737	9.742	1	1	2	2	3
95.	9.747	9.752	9.757	9.762	9.767	9.772	9.778	9.783	9.788	9.793	1	1	2	2	3
96.	9.798	9.803	9.808	9.813	9.818	9.823	9.829	9.834	9.839	9.844	1	1	2	2	3
97.	9.849	9.854	9.859	9.864	9.869	9.874	9.879	9.884	9.889	9.894	1	1	2	2	3
98.	9.899	9.905	9.910	9.915	9.920	9.925	9.930	9.935	9.940	9.945	1	1	2	2	3
99.	9.950	9.955	9.960	9.965	9.970	9.975	9.980	9.985	9.990	9.995	1	1	2	2	3

Table 4. Cube roots of numbers 1.0–49.9 (see Art. 75)

N	0	1	2	3	4	5	6	7	8	9	Prop. parts 1	2	3	4	5
1.	1.000	1.032	1.063	1.091	1.119	1.145	1.170	1.193	1.216	1.239	..	..	..	..	..
2.	1.260	1.281	1.301	1.320	1.339	1.357	1.375	1.392	1.409	1.426	..	..	..	..	..
3.	1.442	1.458	1.474	1.489	1.504	1.518	1.533	1.547	1.560	1.574	1	3	4	6	7
4.	1.587	1.601	1.613	1.626	1.639	1.651	1.663	1.675	1.687	1.698	1	2	4	5	6
5.	1.710	1.721	1.732	1.744	1.754	1.765	1.776	1.786	1.797	1.807	1	2	3	4	5
6.	1.817	1.827	1.837	1.847	1.857	1.866	1.876	1.885	1.895	1.904	1	2	3	4	5
7.	1.913	1.922	1.931	1.940	1.949	1.957	1.966	1.975	1.983	1.992	1	2	3	3	4
8.	2.000	2.008	2.017	2.025	2.033	2.041	2.049	2.057	2.065	2.072	1	2	2	3	4
9.	2.080	2.088	2.095	2.103	2.110	2.118	2.125	2.133	2.140	2.147	1	2	2	3	4
10.	2.154	2.162	2.169	2.176	2.183	2.190	2.197	2.204	2.210	2.217	1	2	2	3	3
11.	2.224	2.231	2.237	2.244	2.251	2.257	2.264	2.270	2.277	2.283	1	1	2	3	3
12.	2.289	2.296	2.302	2.308	2.315	2.321	2.327	2.333	2.339	2.345	1	1	2	2	3
13.	2.351	2.357	2.363	2.369	2.375	2.381	2.387	2.393	2.399	2.404	1	1	2	2	3
14.	2.410	2.416	2.422	2.427	2.433	2.438	2.444	2.450	2.455	2.461	1	1	2	2	3
15.	2.466	2.472	2.477	2.483	2.488	2.493	2.499	2.504	2.509	2.515	1	1	2	2	3
16.	2.520	2.525	2.530	2.535	2.541	2.546	2.551	2.556	2.561	2.566	1	1	2	2	3
17.	2.571	2.576	2.581	2.586	2.591	2.596	2.601	2.606	2.611	2.616	0	1	1	2	2
18.	2.621	2.626	2.630	2.635	2.640	2.645	2.650	2.654	2.659	2.664	0	1	1	2	2
19.	2.668	2.673	2.678	2.682	2.687	2.692	2.696	2.701	2.705	2.710	0	1	1	2	2
20.	2.714	2.719	2.723	2.728	2.732	2.737	2.741	2.746	2.750	2.755	0	1	1	2	2
21.	2.759	2.763	2.768	2.772	2.776	2.781	2.785	2.789	2.794	2.798	0	1	1	2	2
22.	2.802	2.806	2.811	2.815	2.819	2.823	2.827	2.831	2.836	2.840	0	1	1	2	2
23.	2.844	2.848	2.852	2.856	2.860	2.864	2.868	2.872	2.876	2.880	0	1	1	2	2
24.	2.884	2.888	2.892	2.896	2.900	2.904	2.908	2.912	2.916	2.920	0	1	1	2	2
25.	2.924	2.928	2.932	2.936	2.940	2.943	2.947	2.951	2.955	2.959	0	1	1	2	2
26.	2.962	2.966	2.970	2.974	2.978	2.981	2.985	2.989	2.993	2.996	0	1	1	2	2
27.	3.000	3.004	3.007	3.011	3.015	3.018	3.022	3.026	3.029	3.033	0	1	1	1	2
28.	3.037	3.040	3.044	3.047	3.051	3.055	3.058	3.062	3.065	3.069	0	1	1	1	2
29.	3.072	3.076	3.079	3.083	3.086	3.090	3.093	3.097	3.100	3.104	0	1	1	1	2
30.	3.107	3.111	3.114	3.118	3.121	3.124	3.128	3.131	3.135	3.138	0	1	1	1	2
31.	3.141	3.145	3.148	3.151	3.155	3.158	3.162	3.165	3.168	3.171	0	1	1	1	2
32.	3.175	3.178	3.181	3.185	3.188	3.191	3.195	3.198	3.201	3.204	0	1	1	1	2
33.	3.208	3.211	3.214	3.217	3.220	3.224	3.227	3.230	3.233	3.236	0	1	1	1	2
34.	3.240	3.243	3.246	3.249	3.252	3.255	3.259	3.262	3.265	3.268	0	1	1	1	2
35.	3.271	3.274	3.277	3.280	3.283	3.287	3.290	3.293	3.296	3.299	0	1	1	1	2
36.	3.302	3.305	3.308	3.311	3.314	3.317	3.320	3.323	3.326	3.329	0	1	1	1	2
37.	3.332	3.335	3.338	3.341	3.344	3.347	3.350	3.353	3.356	3.359	0	1	1	1	1
38.	3.362	3.365	3.368	3.371	3.374	3.377	3.380	3.382	3.385	3.388	0	1	1	1	1
39.	3.391	3.394	3.397	3.400	3.403	3.406	3.409	3.411	3.414	3.417	0	1	1	1	1
40.	3.420	3.423	3.426	3.428	3.431	3.434	3.437	3.440	3.443	3.445	0	1	1	1	1
41.	3.448	3.451	3.454	3.457	3.459	3.462	3.465	3.468	3.471	3.473	0	1	1	1	1
42.	3.476	3.479	3.482	3.484	3.487	3.490	3.493	3.495	3.498	3.501	0	1	1	1	1
43.	3.503	3.506	3.509	3.512	3.514	3.517	3.520	3.522	3.525	3.528	0	1	1	1	1
44.	3.530	3.533	3.536	3.538	3.541	3.544	3.546	3.549	3.552	3.554	0	1	1	1	1
45.	3.557	3.560	3.562	3.565	3.567	3.570	3.573	3.575	3.578	3.580	0	1	1	1	1
46.	3.583	3.586	3.588	3.591	3.593	3.596	3.599	3.601	3.604	3.606	0	1	1	1	1
47.	3.609	3.611	3.614	3.616	3.619	3.622	3.624	3.627	3.629	3.632	0	1	1	1	1
48.	3.634	3.637	3.639	3.642	3.644	3.647	3.649	3.652	3.654	3.657	0	1	1	1	1
49.	3.659	3.662	3.664	3.667	3.669	3.672	3.674	3.677	3.679	3.682	0	0	1	1	1
N	0	1	2	3	4	5	6	7	8	9	1 Prop. parts	2	3	4	5

Table 4. Cube roots of numbers 50.0–99.9

N	0	1	2	3	4	5	6	7	8	9	Prop. parts 1	2	3	4	5
50.	3.684	3.686	3.689	3.691	3.694	3.696	3.699	3.701	3.704	3.706	0	0	1	1	1
51.	3.708	3.711	3.713	3.716	3.718	3.721	3.723	3.725	3.728	3.730	0	0	1	1	1
52.	3.733	3.735	3.737	3.740	3.742	3.744	3.747	3.749	3.752	3.754	0	0	1	1	1
53.	3.756	3.759	3.761	3.763	3.766	3.768	3.770	3.773	3.775	3.777	0	0	1	1	1
54.	3.780	3.782	3.784	3.787	3.789	3.791	3.794	3.796	3.798	3.801	0	0	1	1	1
55.	3.803	3.805	3.808	3.810	3.812	3.814	3.817	3.819	3.821	3.824	0	0	1	1	1
56.	3.826	3.828	3.830	3.833	3.835	3.837	3.839	3.842	3.844	3.846	0	0	1	1	1
57.	3.849	3.851	3.853	3.855	3.857	3.860	3.862	3.864	3.866	3.869	0	0	1	1	1
58.	3.871	3.873	3.875	3.878	3.880	3.882	3.884	3.886	3.889	3.891	0	0	1	1	1
59.	3.893	3.895	3.897	3.900	3.902	3.904	3.906	3.908	3.911	3.913	0	0	1	1	1
60.	3.915	3.917	3.919	3.921	3.924	3.926	3.928	3.930	3.932	3.934	0	0	1	1	1
61.	3.936	3.939	3.941	3.943	3.945	3.947	3.949	3.951	3.954	3.956	0	0	1	1	1
62.	3.958	3.960	3.962	3.964	3.966	3.968	3.971	3.973	3.975	3.977	0	0	1	1	1
63.	3.979	3.981	3.983	3.985	3.987	3.990	3.992	3.994	3.996	3.998	0	0	1	1	1
64.	4.000	4.002	4.004	4.006	4.008	4.010	4.012	4.015	4.017	4.019	0	0	1	1	1
65.	4.021	4.023	4.025	4.027	4.029	4.031	4.033	4.035	4.037	4.039	0	0	1	1	1
66.	4.041	4.043	4.045	4.047	4.049	4.051	4.053	4.055	4.058	4.060	0	0	1	1	1
67.	4.062	4.064	4.066	4.068	4.070	4.072	4.074	4.076	4.078	4.080	0	0	1	1	1
68.	4.082	4.084	4.086	4.088	4.090	4.092	4.094	4.096	4.098	4.100	0	0	1	1	1
69.	4.102	4.104	4.106	4.108	4.109	4.111	4.113	4.115	4.117	4.119	0	0	1	1	1
70.	4.121	4.123	4.125	4.127	4.129	4.131	4.133	4.135	4.137	4.139	0	0	1	1	1
71.	4.141	4.143	4.145	4.147	4.149	4.151	4.152	4.154	4.156	4.158	0	0	1	1	1
72.	4.160	4.162	4.164	4.166	4.168	4.170	4.172	4.174	4.176	4.177	0	0	1	1	1
73.	4.179	4.181	4.183	4.185	4.187	4.189	4.191	4.193	4.195	4.196	0	0	1	1	1
74.	4.198	4.200	4.202	4.204	4.206	4.208	4.210	4.212	4.213	4.215	0	0	1	1	1
75.	4.217	4.219	4.221	4.223	4.225	4.227	4.228	4.230	4.232	4.234	0	0	1	1	1
76.	4.236	4.238	4.240	4.241	4.243	4.245	4.247	4.249	4.251	4.252	0	0	1	1	1
77.	4.254	4.256	4.258	4.260	4.262	4.264	4.265	4.267	4.269	4.271	0	0	1	1	1
78.	4.273	4.274	4.276	4.278	4.280	4.282	4.284	4.285	4.287	4.289	0	0	1	1	1
79.	4.291	4.293	4.294	4.296	4.298	4.300	4.302	4.303	4.305	4.307	0	0	1	1	1
80.	4.309	4.311	4.312	4.314	4.316	4.318	4.320	4.321	4.323	4.325	0	0	1	1	1
81.	4.327	4.329	4.330	4.332	4.334	4.336	4.337	4.339	4.341	4.343	0	0	1	1	1
82.	4.344	4.346	4.348	4.350	4.352	4.353	4.355	4.357	4.359	4.360	0	0	1	1	1
83.	4.362	4.364	4.366	4.367	4.369	4.371	4.373	4.374	4.376	4.378	0	0	1	1	1
84.	4.380	4.381	4.383	4.385	4.386	4.388	4.390	4.392	4.393	4.395	0	0	1	1	1
85.	4.397	4.399	4.400	4.402	4.404	4.405	4.407	4.409	4.411	4.412	0	0	1	1	1
86.	4.414	4.416	4.417	4.419	4.421	4.423	4.424	4.426	4.428	4.429	0	0	1	1	1
87.	4.431	4.433	4.434	4.436	4.438	4.440	4.441	4.443	4.445	4.446	0	0	1	1	1
88.	4.448	4.450	4.451	4.453	4.455	4.456	4.458	4.460	4.461	4.463	0	0	1	1	1
89.	4.465	4.466	4.468	4.470	4.471	4.473	4.475	4.476	4.478	4.480	0	0	0	1	1
90.	4.481	4.483	4.485	4.486	4.488	4.490	4.491	4.493	4.495	4.496	0	0	0	1	1
91.	4.498	4.500	4.501	4.503	4.505	4.506	4.508	4.509	4.511	4.513	0	0	0	1	1
92.	4.514	4.516	4.518	4.519	4.521	4.523	4.524	4.526	4.527	4.529	0	0	0	1	1
93.	4.531	4.532	4.534	4.536	4.537	4.539	4.540	4.542	4.544	4.545	0	0	0	1	1
94.	4.547	4.548	4.550	4.552	4.553	4.555	4.556	4.558	4.560	4.561	0	0	0	1	1
95.	4.563	4.565	4.566	4.568	4.569	4.571	4.572	4.574	4.576	4.577	0	0	0	1	1
96.	4.579	4.580	4.582	4.584	4.585	4.587	4.588	4.590	4.592	4.593	0	0	0	1	1
97.	4.595	4.596	4.598	4.599	4.601	4.603	4.604	4.606	4.607	4.609	0	0	0	1	1
98.	4.610	4.612	4.614	4.615	4.617	4.618	4.620	4.621	4.623	4.625	0	0	0	1	1
99.	4.626	4.628	4.629	4.631	4.632	4.634	4.635	4.637	4.638	4.640	0	0	0	1	1
N	0	1	2	3	4	5	6	7	8	9	Prop. parts 1	2	3	4	5

Table 4. Cube roots of numbers 100–549

N	0	1	2	3	4	5	6	7	8	9	Prop. parts 1	2	3	4	5
10.	4.642	4.657	4.672	4.688	4.703	4.718	4.733	4.747	4.762	4.777	1	3	4	6	7
11.	4.791	4.806	4.820	4.835	4.849	4.863	4.877	4.891	4.905	4.919	1	3	4	6	7
12.	4.932	4.946	4.960	4.973	4.987	5.000	5.013	5.027	5.040	5.053	1	3	4	5	7
13.	5.066	5.079	5.092	5.104	5.117	5.130	5.143	5.155	5.168	5.180	1	3	4	5	6
14.	5.192	5.205	5.217	5.229	5.241	5.254	5.266	5.278	5.290	5.301	1	2	4	5	6
15.	5.313	5.325	5.337	5.348	5.360	5.372	5.383	5.395	5.406	5.418	1	2	3	5	6
16.	5.429	5.440	5.451	5.463	5.474	5.485	5.496	5.507	5.518	5.529	1	2	3	4	6
17.	5.540	5.550	5.561	5.572	5.583	5.593	5.604	5.615	5.625	5.636	1	2	3	4	5
18.	5.646	5.657	5.667	5.677	5.688	5.698	5.708	5.718	5.729	5.739	1	2	3	4	5
19.	5.749	5.759	5.769	5.779	5.789	5.799	5.809	5.819	5.828	5.838	1	2	3	4	5
20.	5.848	5.858	5.867	5.877	5.887	5.896	5.906	5.915	5.925	5.934	1	2	3	4	5
21.	5.944	5.953	5.963	5.972	5.981	5.991	6.000	6.009	6.018	6.028	1	2	3	4	5
22.	6.037	6.046	6.055	6.064	6.073	6.082	6.091	6.100	6.109	6.118	1	2	3	4	5
23.	6.127	6.136	6.145	6.153	6.162	6.171	6.180	6.188	6.197	6.206	1	2	3	4	4
24.	6.214	6.223	6.232	6.240	6.249	6.257	6.266	6.274	6.283	6.291	1	2	3	3	4
25.	6.300	6.308	6.316	6.325	6.333	6.341	6.350	6.358	6.366	6.374	1	2	2	3	4
26.	6.383	6.391	6.399	6.407	6.415	6.423	6.431	6.439	6.447	6.455	1	2	2	3	4
27.	6.463	6.471	6.479	6.487	6.495	6.503	6.511	6.519	6.527	6.534	1	2	2	3	4
28.	6.542	6.550	6.558	6.565	6.573	6.581	6.589	6.596	6.604	6.611	1	2	2	3	4
29.	6.619	6.627	6.634	6.642	6.649	6.657	6.664	6.672	6.679	6.687	1	2	2	3	4
30.	6.694	6.702	6.709	6.717	6.724	6.731	6.739	6.746	6.753	6.761	1	1	2	3	4
31.	6.768	6.775	6.782	6.790	6.797	6.804	6.811	6.818	6.826	6.833	1	1	2	3	4
32.	6.840	6.847	6.854	6.861	6.868	6.875	6.882	6.889	6.896	6.903	1	1	2	3	4
33.	6.910	6.917	6.924	6.931	6.938	6.945	6.952	6.959	6.966	6.973	1	1	2	3	3
34.	6.980	6.986	6.993	7.000	7.007	7.014	7.020	7.027	7.034	7.041	1	1	2	3	3
35.	7.047	7.054	7.061	7.067	7.074	7.081	7.087	7.094	7.101	7.107	1	1	2	3	3
36.	7.114	7.120	7.127	7.133	7.140	7.147	7.153	7.160	7.166	7.173	1	1	2	3	3
37.	7.179	7.186	7.192	7.198	7.205	7.211	7.218	7.224	7.230	7.237	1	1	2	3	3
38.	7.243	7.250	7.256	7.262	7.268	7.275	7.281	7.287	7.294	7.300	1	1	2	3	3
39.	7.306	7.312	7.319	7.325	7.331	7.337	7.343	7.350	7.356	7.362	1	1	2	2	3
40.	7.368	7.374	7.380	7.386	7.393	7.399	7.405	7.411	7.417	7.423	1	1	2	2	3
41.	7.429	7.435	7.441	7.447	7.453	7.459	7.465	7.471	7.477	7.483	1	1	2	2	3
42.	7.489	7.495	7.501	7.507	7.513	7.518	7.524	7.530	7.536	7.542	1	1	2	2	3
43.	7.548	7.554	7.560	7.565	7.571	7.577	7.583	7.589	7.594	7.600	1	1	2	2	3
44.	7.606	7.612	7.617	7.623	7.629	7.635	7.640	7.646	7.652	7.657	1	1	2	2	3
45.	7.663	7.669	7.674	7.680	7.686	7.691	7.697	7.703	7.708	7.714	1	1	2	2	3
46.	7.719	7.725	7.731	7.736	7.742	7.747	7.753	7.758	7.764	7.769	1	1	2	2	3
47.	7.775	7.780	7.786	7.791	7.797	7.802	7.808	7.813	7.819	7.824	1	1	2	2	3
48.	7.830	7.835	7.841	7.846	7.851	7.857	7.862	7.868	7.873	7.878	1	1	2	2	3
49.	7.884	7.889	7.894	7.900	7.905	7.910	7.916	7.921	7.926	7.932	1	1	2	2	3
50.	7.937	7.942	7.948	7.953	7.958	7.963	7.969	7.974	7.979	7.984	1	1	2	2	3
51.	7.990	7.995	8.000	8.005	8.010	8.016	8.021	8.026	8.031	8.036	1	1	2	2	3
52.	8.041	8.047	8.052	8.057	8.062	8.067	8.072	8.077	8.082	8.088	1	1	2	2	3
53.	8.093	8.098	8.103	8.108	8.113	8.118	8.123	8.128	8.133	8.138	1	1	2	2	3
54.	8.143	8.148	8.153	8.158	8.163	8.168	8.173	8.178	8.183	8.188	0	1	1	2	2
N	0	1	2	3	4	5	6	7	8	9	1 Prop. parts	2	3	4	5

Table 4. Cube roots of numbers 550–999

N	0	1	2	3	4	5	6	7	8	9	Prop. parts 1	2	3	4	5
55.	8.193	8.198	8.203	8.208	8.213	8.218	8.223	8.228	8.233	8.238	0	1	1	2	2
56.	8.243	8.247	8.252	8.257	8.262	8.267	8.272	8.277	8.282	8.286	0	1	1	2	2
57.	8.291	8.296	8.301	8.306	8.311	8.316	8.320	8.325	8.330	8.335	0	1	1	2	2
58.	8.340	8.344	8.349	8.354	8.359	8.363	8.368	8.373	8.378	8.382	0	1	1	2	2
59.	8.387	8.392	8.397	8.401	8.406	8.411	8.416	8.420	8.425	8.430	0	1	1	2	2
60.	8.434	8.439	8.444	8.448	8.453	8.458	8.462	8.467	8.472	8.476	0	1	1	2	2
61.	8.481	8.486	8.490	8.495	8.499	8.504	8.509	8.513	8.518	8.522	0	1	1	2	2
62.	8.527	8.532	8.536	8.541	8.545	8.550	8.554	8.559	8.564	8.568	0	1	1	2	2
63.	8.573	8.577	8.582	8.586	8.591	8.595	8.600	8.604	8.609	8.613	0	1	1	2	2
64.	8.618	8.622	8.627	8.631	8.636	8.640	8.645	8.649	8.653	8.658	0	1	1	2	2
65.	8.662	8.667	8.671	8.676	8.680	8.685	8.689	8.693	8.698	8.702	0	1	1	2	2
66.	8.707	8.711	8.715	8.720	8.724	8.729	8.733	8.737	8.742	8.746	0	1	1	2	2
67.	8.750	8.755	8.759	8.763	8.768	8.772	8.776	8.781	8.785	8.789	0	1	1	2	2
68.	8.794	8.798	8.802	8.807	8.811	8.815	8.819	8.824	8.828	8.832	0	1	1	2	2
69.	8.837	8.841	8.845	8.849	8.854	8.858	8.862	8.866	8.871	8.875	0	1	1	2	2
70.	8.879	8.883	8.887	8.892	8.896	8.900	8.904	8.909	8.913	8.917	0	1	1	2	2
71.	8.921	8.925	8.929	8.934	8.938	8.942	8.946	8.950	8.955	8.959	0	1	1	2	2
72.	8.963	8.967	8.971	8.975	8.979	8.984	8.988	8.992	8.996	9.000	0	1	1	2	2
73.	9.004	9.008	9.012	9.016	9.021	9.025	9.029	9.033	9.037	9.041	0	1	1	2	2
74.	9.045	9.049	9.053	9.057	9.061	9.065	9.069	9.073	9.078	9.082	0	1	1	2	2
75.	9.086	9.090	9.094	9.098	9.102	9.106	9.110	9.114	9.118	9.122	0	1	1	2	2
76.	9.126	9.130	9.134	9.138	9.142	9.146	9.150	9.154	9.158	9.162	0	1	1	2	2
77.	9.166	9.170	9.174	9.178	9.182	9.185	9.189	9.193	9.197	9.201	0	1	1	2	2
78.	9.205	9.209	9.213	9.217	9.221	9.225	9.229	9.233	9.237	9.240	0	1	1	2	2
79.	9.244	9.248	9.252	9.256	9.260	9.264	9.268	9.272	9.275	9.279	0	1	1	2	2
80.	9.283	9.287	9.291	9.295	9.299	9.302	9.306	9.310	9.314	9.318	0	1	1	2	2
81.	9.322	9.326	9.329	9.333	9.337	9.341	9.345	9.348	9.352	9.356	0	1	1	2	2
82.	9.360	9.364	9.368	9.371	9.375	9.379	9.383	9.386	9.390	9.394	0	1	1	2	2
83.	9.398	9.402	9.405	9.409	9.413	9.417	9.420	9.424	9.428	9.432	0	1	1	2	2
84.	9.435	9.439	9.443	9.447	9.450	9.454	9.458	9.462	9.465	9.469	0	1	1	1	2
85.	9.473	9.476	9.480	9.484	9.488	9.491	9.495	9.499	9.502	9.506	0	1	1	1	2
86	9.510	9.513	9.517	9.521	9.524	9.528	9.532	9.535	9.539	9.543	0	1	1	1	2
87.	9.546	9.550	9.554	9.557	9.561	9.565	9.568	9.572	9.576	9.579	0	1	1	1	2
88.	9.583	9.586	9.590	9.594	9.597	9.601	9.605	9.608	9.612	9.615	0	1	1	1	2
89.	9.619	9.623	9.626	9.630	9.633	9.637	9.641	9.644	9.648	9.651	0	1	1	1	2
90.	9.655	9.658	9.662	9.666	9.669	9.673	9.676	9.680	9.683	9.687	0	1	1	1	2
91.	9.691	9.694	9.698	9.701	9.705	9.708	9.712	9.715	9.719	9.722	0	1	1	1	2
92.	9.726	9.729	9.733	9.736	9.740	9.743	9.747	9.750	9.754	9.758	0	1	1	1	2
93.	9.761	9.764	9.768	9.771	9.775	9.778	9.782	9.785	9.789	9.792	0	1	1	1	2
94.	9.796	9.799	9.803	9.806	9.810	9.813	9.817	9.820	9.824	9.827	0	1	1	1	2
95.	9.830	9.834	9.837	9.841	9.844	9.848	9.851	9.855	9.858	9.861	0	1	1	1	2
96.	9.865	9.868	9.872	9.875	9.879	9.882	9.885	9.889	9.892	9.896	0	1	1	1	2
97.	9.899	9.902	9.906	9.909	9.913	9.916	9.919	9.923	9.926	9.930	0	1	1	1	2
98.	9.933	9.936	9.940	9.943	9.946	9.950	9.953	9.956	9.960	9.963	0	1	1	1	2
99.	9.967	9.970	9.973	9.977	9.980	9.983	9.987	9.990	9.993	9.997	0	1	1	1	2
											1	2	3	4	5
N	0	1	2	3	4	5	6	7	8	9	Prop. parts				

Table 5. Reciprocals of numbers (see Art. 75)

N	0	1	2	3	4	5	6	7	8	9	Prop. parts 1	2	3	4	5
1.0	1.000	.9901	.9804	.9709	.9615	.9524	.9434	.9346	.9259	.9174	Interpolate			(See Art. 75)	
1.1	.9091	.9009	.8929	.8850	.8772	.8696	.8621	.8547	.8475	.8403					
1.2	.8333	.8264	.8197	.8130	.8064	.8000	.7937	.7874	.7813	.7752					
1.3	.7692	.7634	.7576	.7519	.7463	.7407	.7353	.7299	.7246	.7194					
1.4	.7143	.7092	.7042	.6993	.6944	.6897	.6849	.6803	.6757	.6711					
1.5	.6667	.6623	.6579	.6536	.6494	.6452	.6410	.6369	.6329	.6289					
1.6	.6250	.6211	.6173	.6135	.6098	.6061	.6024	.5988	.5952	.5917	4	7	11		
1.7	.5882	.5848	.5814	.5780	.5747	.5714	.5682	.5650	.5618	.5587	3	7	10		
1.8	.5556	.5525	.5495	.5464	.5435	.5405	.5376	.5348	.5319	.5291	3	6	9		
1.9	.5263	.5236	.5208	.5181	.5155	.5128	.5102	.5076	.5051	.5025	3	5	8	11	13
2.0	.5000	.4975	.4950	.4926	.4902	.4878	.4854	.4831	.4808	.4785	2	5	7	10	12
2.1	.4762	.4739	.4717	.4695	.4673	.4651	.4630	.4608	.4587	.4566	2	4	6	9	11
2.2	.4545	.4525	.4505	.4484	.4464	.4444	.4425	.4405	.4386	.4367	2	4	6	8	10
2.3	.4348	.4329	.4310	.4292	.4274	.4255	.4237	.4219	.4202	.4184	2	4	5	7	9
2.4	.4167	.4149	.4132	.4115	.4098	.4082	.4065	.4049	.4032	.4016	2	3	5	7	8
2.5	.4000	.3984	.3968	.3953	.3937	.3922	.3906	.3891	.3876	.3861	2	3	5	6	8
2.6	.3846	.3831	.3817	.3802	.3788	.3774	.3759	.3745	.3731	.3717	1	3	4	6	7
2.7	.3704	.3690	.3676	.3663	.3650	.3636	.3623	.3610	.3597	.3584	1	3	4	5	7
2.8	.3571	.3559	.3546	.3534	.3521	.3509	.3497	.3484	.3472	.3460	1	2	4	5	6
2.9	.3448	.3436	.3425	.3413	.3401	.3390	.3378	.3367	.3356	.3344	1	2	3	5	6
3.0	.3333	.3322	.3311	.3300	.3289	.3279	.3268	.3257	.3247	.3236	1	2	3	4	5
3.1	.3226	.3215	.3205	.3195	.3185	.3175	.3165	.3155	.3145	.3135	1	2	3	4	5
3.2	.3125	.3115	.3106	.3096	.3086	.3077	.3067	.3058	.3049	.3040	1	2	3	4	5
3.3	.3030	.3021	.3012	.3003	.2994	.2985	.2976	.2967	.2959	.2950	1	2	3	4	4
3.4	.2941	.2933	.2924	.2915	.2907	.2899	.2890	.2882	.2874	.2865	1	2	3	3	4
3.5	.2857	.2849	.2841	.2833	.2825	.2817	.2809	.2801	.2793	.2786	1	2	2	3	4
3.6	.2778	.2770	.2762	.2555	.2747	.2740	.2732	.2725	.2717	.2710	1	2	2	3	4
3.7	.2703	.2695	.2688	.2681	.2674	.2667	.2660	.2653	.2646	.2639	1	1	2	3	4
3.8	.2632	.2625	.2618	.2611	.2604	.2597	.2591	.2584	.2577	.2571	1	1	2	3	3
3.9	.2564	.2558	.2551	.2545	.2538	.2532	.2525	.2519	.2513	.2506	1	1	2	3	3
4.0	.2500	.2494	.2488	.2481	.2475	.2469	.2463	.2457	.2451	.2445	1	1	2	2	3
4.1	.2439	.2433	.2427	.2421	.2415	.2410	.2404	.2398	.2392	.2387	1	1	2	2	3
4.2	.2381	.2375	.2370	.2364	.2358	.2353	.2347	.2342	.2336	.2331	1	1	2	2	3
4.3	.2326	.2320	.2315	.2309	.2304	.2299	.2294	2288	.2283	.2278	1	1	2	2	3
4.4	.2273	.2268	.2262	.2257	.2252	.2247	.2242	.2237	.2232	.2227	1	1	2	2	3
4.5	.2222	.2217	.2212	.2208	.2203	.2198	.2193	.2188	.2183	.2179	0	1	1	2	2
4.6	.2174	.2169	.2165	.2160	.2155	.2151	.2146	.2141	.2137	.2132	0	1	1	2	2
4.7	.2128	.2123	.2119	.2114	.2110	.2105	.2100	.2096	.2092	.2088	0	1	1	2	2
4.8	.2083	.2079	.2075	.2070	.2066	.2062	.2058	.2053	.2049	.2045	0	1	1	2	2
4.9	.2041	.2037	.2033	.2028	.2024	.2020	.2016	.2012	.2008	.2004	0	1	1	2	2
5.0	.2000	.1996	.1992	.1988	.1984	.1980	.1976	.1972	.1969	.1965	0	1	1	2	2
5.1	.1961	.1957	.1953	.1949	.1946	.1942	.1938	.1934	.1931	.1927	0	1	1	2	2
5.2	.1923	.1919	.1916	.1912	.1908	.1905	.1901	.1898	.1894	.1890	0	1	1	1	2
5.3	.1887	.1883	.1880	.1876	.1873	.1869	.1866	.1862	.1859	.1855	0	1	1	1	2
5.4	.1852	.1848	.1845	.1842	.1838	.1835	.1832	.1828	.1825	.1821	0	1	1	1	2
N	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5

Table 5. Reciprocals of numbers—*Continued*

N	0	1	2	3	4	5	6	7	8	9	Prop. parts				
											1	2	3	4	5
5.5	.1818	.1815	.1812	.1808	.1805	.1802	.1799	.1795	.1792	.1789	0	1	1	1	2
5.6	.1786	.1783	.1779	.1776	.1773	.1770	.1767	.1764	.1761	.1757	0	1	1	1	2
5.7	.1754	.1751	.1748	.1745	.1742	.1739	.1736	.1733	.1730	.1727	0	1	1	1	2
5.8	.1724	.1721	.1718	.1715	.1712	.1709	.1706	.1704	.1701	.1698	0	1	1	1	1
5.9	.1695	.1692	.1689	.1686	.1684	.1681	.1678	.1675	.1672	.1669	0	1	1	1	1
6.0	.1667	.1664	.1661	.1658	.1656	.1653	.1650	.1647	.1645	.1642	0	1	1	1	1
6.1	.1639	.1637	.1634	.1631	.1629	.1626	.1623	.1621	.1618	.1616	0	1	1	1	1
6.2	.1613	.1610	.1608	.1605	.1603	.1600	.1597	.1595	.1592	.1590	0	1	1	1	1
6.3	.1587	.1585	.1582	.1580	.1577	.1575	.1572	.1570	.1567	.1565	0	0	1	1	1
6.4	.1563	.1560	.1558	.1555	.1553	.1550	.1548	.1546	.1543	.1541	0	0	1	1	1
6.5	.1538	.1536	.1534	.1531	.1529	.1527	.1524	.1522	.1520	.1517	0	0	1	1	1
6.6	.1515	.1513	.1511	.1508	.1506	.1504	.1502	.1499	.1497	.1495	0	0	1	1	1
6.7	.1493	.1490	.1488	.1486	.1484	.1481	.1479	.1477	.1475	.1473	0	0	1	1	1
6.8	.1471	.1468	.1466	.1464	.1462	.1460	.1458	.1456	.1453	.1451	0	0	1	1	1
6.9	.1449	.1447	.1445	.1443	.1441	.1439	.1437	.1435	.1433	.1431	0	0	1	1	1
7.0	.1429	.1427	.1425	.1422	.1420	.1418	.1416	.1414	.1412	.1410	0	0	1	1	1
7.1	.1408	.1406	.1404	.1403	.1401	.1399	.1397	.1395	.1393	.1391	0	0	1	1	1
7.2	.1389	.1387	.1385	.1383	.1381	.1379	.1377	.1376	.1374	.1372	0	0	1	1	1
7.3	.1370	.1368	.1366	.1364	.1362	.1361	.1359	.1357	.1355	.1353	0	0	1	1	1
7.4	.1351	.1350	.1348	.1346	.1344	.1342	.1340	.1339	.1337	.1335	0	0	1	1	1
7.5	.1333	.1332	.1330	.1328	.1326	.1325	.1323	.1321	.1319	.1318	0	0	1	1	1
7.6	.1316	.1314	.1312	.1311	.1309	.1307	.1305	.1304	.1302	.1300	0	0	1	1	1
7.7	.1299	.1297	.1295	.1294	.1292	.1290	.1289	.1287	.1285	.1284	0	0	1	1	1
7.8	.1282	.1280	.1279	.1277	.1276	.1274	.1272	.1271	.1269	.1267	0	0	0	1	1
7.9	.1266	.1264	.1263	.1261	.1259	.1258	.1256	.1255	.1253	.1252	0	0	0	1	1
8.0	.1250	.1248	.1247	.1245	.1244	.1242	.1241	.1239	.1238	.1236	0	0	0	1	1
8.1	.1235	.1233	.1232	.1230	.1229	.1227	.1225	.1224	.1222	.1221	0	0	0	1	1
8.2	.1220	.1218	.1217	.1215	.1214	.1212	.1211	.1209	.1208	.1206	0	0	0	1	1
8.3	.1205	.1203	.1202	.1200	.1199	.1198	.1196	.1195	.1193	.1192	0	0	0	1	1
8.4	.1190	.1189	.1188	.1186	.1185	.1183	.1182	.1181	.1179	.1178	0	0	0	1	1
8.5	.1176	.1175	.1174	.1172	.1171	.1170	.1168	.1167	.1166	.1164	0	0	0	1	1
8.6	.1163	.1161	.1160	.1159	.1157	.1156	.1155	.1153	.1152	.1151	0	0	0	1	1
8.7	.1149	.1148	.1147	.1145	.1144	.1143	.1142	.1140	.1139	.1138	0	0	0	1	1
8.8	.1136	.1135	.1134	.1133	.1131	.1130	.1129	.1127	.1126	.1125	0	0	0	1	1
8.9	.1124	.1122	.1121	.1120	.1119	.1117	.1116	.1115	.1114	.1112	0	0	0	0	1
9.0	.1111	.1110	.1109	.1107	.1106	.1105	.1104	.1103	.1101	.1100	0	0	0	0	1
9.1	.1099	.1098	.1096	.1095	.1094	.1093	.1092	.1091	.1089	.1088	0	0	0	0	1
9.2	.1087	.1086	.1085	.1083	.1082	.1081	.1080	.1079	.1078	.1076	0	0	0	0	1
9.3	.1075	.1074	.1073	.1072	.1071	.1070	.1068	.1067	.1066	.1065	0	0	0	0	1
9.4	.1064	.1063	.1062	.1060	.1059	.1058	.1057	.1056	.1055	.1054	0	0	0	0	1
9.5	.1053	.1052	.1050	.1049	.1048	.1047	.1046	.1045	.1044	.1043	0	0	0	0	1
9.6	.1042	.1041	.1040	.1038	.1037	.1036	.1035	.1034	.1033	.1032	0	0	0	0	1
9.7	.1031	.1030	.1029	.1028	.1027	.1026	.1025	.1024	.1022	.1021	0	0	0	0	1
9.8	.1020	.1019	.1018	.1017	.1016	.1015	.1014	.1013	.1012	.1011	0	0	0	0	1
9.9	.1010	.1009	.1008	.1007	.1006	.1005	.1004	.1003	.1002	.1001	0	0	0	0	1
N	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5

Table 6. Circles, circumferences and areas (diameters in hundredths) (see Art. 75)

Diameter	Circumference	Area	Diameter	Circumference	Area	Diameter	Circumference	Area	Diameter	Circumference	Area
1.00	3.142	.7854	1.50	4.712	1.767	2.00	6.283	3.142	2.50	7.854	4.909
1.01	3.173	.8012	1.51	4.744	1.791	2.01	6.315	3.173	2.51	7.885	4.948
1.02	3.204	.8171	1.52	4.775	1.815	2.02	6.346	3.205	2.52	7.917	4.988
1.03	3.236	.8332	1.53	4.807	1.839	2.03	6.377	3.237	2.53	7.948	5.027
1.04	3.267	.8495	1.54	4.838	1.863	2.04	6.409	3.269	2.54	7.980	5.067
1.05	3.299	.8659	1.55	4.869	1.887	2.05	6.440	3.301	2.55	8.011	5.107
1.06	3.330	.8825	1.56	4.901	1.911	2.06	6.472	3.333	2.56	8.043	5.147
1.07	3.362	.8992	1.57	4.932	1.936	2.07	6.503	3.365	2.57	8.074	5.187
1.08	3.393	.9161	1.58	4.964	1.961	2.08	6.535	3.398	2.58	8.105	5.228
1.09	3.424	.9331	1.59	4.995	1.986	2.09	6.566	3.431	2.59	8.137	5.269
1.10	3.456	.9503	1.60	5.027	2.011	2.10	6.597	3.464	2.60	8.168	5.309
1.11	3.487	.9677	1.61	5.058	2.036	2.11	6.629	3.497	2.61	8.200	5.350
1.12	3.519	.9852	1.62	5.089	2.061	2.12	6.660	3.530	2.62	8.231	5.391
1.13	3.550	1.003	1.63	5.121	2.087	2.13	6.692	3.563	2.63	8.262	5.433
1.14	3.581	1.021	1.64	5.152	2.112	2.14	6.723	3.597	2.64	8.294	5.474
1.15	3.613	1.039	1.65	5.184	2.138	2.15	6.754	3.631	2.65	8.325	5.515
1.16	3.644	1.057	1.66	5.215	2.164	2.16	6.786	3.664	2.66	8.357	5.557
1.17	3.676	1.075	1.67	5.246	2.190	2.17	6.817	3.698	2.67	8.388	5.599
1.18	3.707	1.094	1.68	5.278	2.217	2.18	6.849	3.733	2.68	8.419	5.641
1.19	3.738	1.112	1.69	5.309	2.243	2.19	6.880	3.767	2.69	8.451	5.683
1.20	3.770	1.131	1.70	5.341	2.270	2.20	6.912	3.801	2.70	8.482	5.726
1.21	3.801	1.150	1.71	5.372	2.297	2.21	6.943	3.836	2.71	8.514	5.768
1.22	3.833	1.169	1.72	5.404	2.324	2.22	6.974	3.871	2.72	8.545	5.811
1.23	3.864	1.188	1.73	5.435	2.351	2.23	7.006	3.906	2.73	8.577	5.853
1.24	3.896	1.208	1.74	5.466	2.378	2.24	7.037	3.941	2.74	8.608	5.896
1.25	3.927	1.227	1.75	5.498	2.405	2.25	7.069	3.976	2.75	8.639	5.940
1.26	3.958	1.247	1.76	5.529	2.433	2.26	7.100	4.012	2.76	8.671	5.983
1.27	3.990	1.267	1.77	5.561	2.461	2.27	7.131	4.047	2.77	8.702	6.026
1.28	4.021	1.287	1.78	5.592	2.488	2.28	7.163	4.083	2.78	8.734	6.070
1.29	4.053	1.307	1.79	5.623	2.516	2.29	7.194	4.119	2.79	8.765	6.114
1.30	4.084	1.327	1.80	5.655	2.545	2.30	7.226	4.155	2.80	8.796	6.158
1.31	4.115	1.348	1.81	5.686	2.573	2.31	7.257	4.191	2.81	8.828	6.202
1.32	4.147	1.368	1.82	5.718	2.602	2.32	7.288	4.227	2.82	8.859	6.246
1.33	4.178	1.389	1.83	5.749	2.630	2.33	7.320	4.264	2.83	8.891	6.290
1.34	4.210	1.410	1.84	5.781	2.659	2.34	7.351	4.301	2.84	8.922	6.335
1.35	4.241	1.431	1.85	5.812	2.688	2.35	7.383	4.337	2.85	8.954	6.379
1.36	4.273	1.453	1.86	5.843	2.717	2.36	7.414	4.374	2.86	8.985	6.424
1.37	4.304	1.474	1.87	5.875	2.746	2.37	7.446	4.412	2.87	9.016	6.469
1.38	4.335	1.496	1.88	5.906	2.776	2.38	7.477	4.449	2.88	9.048	6.514
1.39	4.367	1.517	1.89	5.938	2.806	2.39	7.508	4.486	2.89	9.079	6.560
1.40	4.398	1.539	1.90	5.969	2.835	2.40	7.540	4.524	2.90	9.111	6.605
1.41	4.430	1.561	1.91	6.000	2.865	2.41	7.571	4.562	2.91	9.142	6.651
1.42	4.461	1.584	1.92	6.032	2.895	2.42	7.603	4.600	2.92	9.173	6.697
1.43	4.492	1.606	1.93	6.063	2.926	2.43	7.634	4.638	2.93	9.205	6.743
1.44	4.524	1.629	1.94	6.095	2.956	2.44	7.665	4.676	2.94	9.236	6.789
1.45	4.555	1.651	1.95	6.126	2.986	2.45	7.697	4.714	2.95	9.268	6.835
1.46	4.587	1.674	1.96	6.158	3.017	2.46	7.728	4.753	2.96	9.299	6.881
1.47	4.618	1.697	1.97	6.189	3.048	2.47	7.760	4.792	2.97	9.331	6.928
1.48	4.650	1.720	1.98	6.220	3.079	2.48	7.791	4.831	2.98	9.362	6.975
1.49	4.681	1.744	1.99	6.252	3.110	2.49	7.823	4.870	2.99	9.393	7.022

Table 6. Circles, circumferences and areas (diameters in hundredths)—*Continued*

Diameter	Circumference	Area	Diameter	Circumference	Area	Diameter	Circumference	Area	Diameter	Circumference	Area
3.00	9.425	7.069	3.50	11.00	9.621	4.00	12.57	12.57	4.50	14.14	15.90
3.01	9.456	7.116	3.51	11.03	9.676	4.01	12.60	12.63	4.51	14.17	15.98
3.02	9.488	7.163	3.52	11.06	9.731	4.02	12.63	12.69	4.52	14.20	16.05
3.03	9.519	7.211	3.53	11.09	9.787	4.03	12.66	12.76	4.53	14.23	16.12
3.04	9.550	7.258	3.54	11.12	9.842	4.04	12.69	12.82	4.54	14.26	16.19
3.05	9.582	7.306	3.55	11.15	9.898	4.05	12.72	12.88	4.55	14.29	16.26
3.06	9.613	7.354	3.56	11.18	9.954	4.06	12.75	12.95	4.56	14.33	16.33
3.07	9.645	7.402	3.57	11.22	10.01	4.07	12.79	13.01	4.57	14.36	16.40
3.08	9.676	7.451	3.58	11.25	10.07	4.08	12.82	13.07	4.58	14.39	16.47
3.09	9.708	7.499	3.59	11.28	10.12	4.09	12.85	13.14	4.59	14.42	16.55
3.10	9.739	7.548	3.60	11.31	10.18	4.10	12.88	13.20	4.60	14.45	16.62
3.11	9.770	7.596	3.61	11.34	10.24	4.11	12.91	13.27	4.61	14.48	16.69
3.12	9.802	7.645	3.62	11.37	10.29	4.12	12.94	13.33	4.62	14.51	16.76
3.13	9.833	7.694	3.63	11.40	10.35	4.13	12.97	13.40	4.63	14.55	16.84
3.14	9.865	7.744	3.64	11.44	10.41	4.14	13.01	13.46	4.64	14.58	16.91
3.15	9.896	7.793	3.65	11.47	10.46	4.15	13.04	13.53	4.65	14.61	16.98
3.16	9.927	7.843	3.66	11.50	10.52	4.16	13.07	13.59	4.66	14.64	17.06
3.17	9.959	7.892	3.67	11.53	10.58	4.17	13.10	13.66	4.67	14.67	17.13
3.18	9.990	7.942	3.68	11.56	10.64	4.18	13.13	13.72	4.68	14.70	17.20
3.19	10.02	7.992	3.69	11.59	10.69	4.19	13.16	13.79	4.69	14.73	17.28
3.20	10.05	8.042	3.70	11.62	10.75	4.20	13.19	13.85	4.70	14.77	17.35
3.21	10.08	8.093	3.71	11.66	10.81	4.21	13.23	13.92	4.71	14.80	17.42
3.22	10.12	8.143	3.72	11.69	10.87	4.22	13.26	13.99	4.72	14.83	17.50
3.23	10.15	8.194	3.73	11.72	10.93	4.23	13.29	14.05	4.73	14.86	17.57
3.24	10.18	8.245	3.74	11.75	10.99	4.24	13.32	14.12	4.74	14.89	17.65
3.25	10.21	8.296	3.75	11.78	11.04	4.25	13.35	14.19	4.75	14.92	17.72
3.26	10.24	8.347	3.76	11.81	11.10	4.26	13.38	14.25	4.76	14.95	17.80
3.27	10.27	8.398	3.77	11.84	11.16	4.27	13.41	14.32	4.77	14.99	17.87
3.28	10.30	8.450	3.78	11.88	11.22	4.28	13.45	14.39	4.78	15.02	17.95
3.29	10.34	8.501	3.79	11.91	11.28	4.29	13.48	14.45	4.79	15.05	18.02
3.30	10.37	8.553	3.80	11.94	11.34	4.30	13.51	14.52	4.80	15.08	18.10
3.31	10.40	8.605	3.81	11.97	11.40	4.31	13.54	14.59	4.81	15.11	18.17
3.32	10.43	8.657	3.82	12.00	11.46	4.32	13.57	14.66	4.82	15.14	18.25
3.33	10.46	8.709	3.83	12.03	11.52	4.33	13.60	14.73	4.83	15.17	18.32
3.34	10.49	8.762	3.84	12.06	11.58	4.34	13.63	14.79	4.84	15.21	18.40
3.35	10.52	8.814	3.85	12.10	11.64	4.35	13.67	14.86	4.85	15.24	18.47
3.36	10.56	8.867	3.86	12.13	11.70	4.36	13.70	14.93	4.86	15.27	18.55
3.37	10.59	8.920	3.87	12.16	11.76	4.37	13.73	15.00	4.87	15.30	18.63
3.38	10.62	8.973	3.88	12.19	11.82	4.38	13.76	15.07	4.88	15.33	18.70
3.39	10.65	9.026	3.89	12.22	11.88	4.39	13.79	15.14	4.89	15.36	18.78
3.40	10.68	9.079	3.90	12.25	11.95	4.40	13.82	15.21	4.90	15.39	18.86
3.41	10.71	9.133	3.91	12.28	12.01	4.41	13.85	15.27	4.91	15.43	18.93
3.42	10.74	9.186	3.92	12.32	12.07	4.42	13.89	15.34	4.92	15.46	19.01
3.43	10.78	9.240	3.93	12.35	12.13	4.43	13.92	15.41	4.93	15.49	19.09
3.44	10.81	9.294	3.94	12.38	12.19	4.44	13.95	15.48	4.94	15.52	19.17
3.45	10.84	9.348	3.95	12.41	12.25	4.45	13.98	15.55	4.95	15.55	19.24
3.46	10.87	9.402	3.96	12.44	12.32	4.46	14.01	15.62	4.96	15.58	19.32
3.47	10.90	9.457	3.97	12.47	12.38	4.47	14.04	15.69	4.97	15.61	19.40
3.48	10.93	9.511	3.98	12.50	12.44	4.48	14.07	15.76	4.98	15.65	19.48
3.49	10.96	9.566	3.99	12.53	12.50	4.49	14.11	15.83	4.99	15.68	19.56

Table 6. Circles, circumferences and areas (diameters in hundredths)—*Continued*

Diameter	Circumference	Area	Diameter	Circumference	Area	Diameter	Circumference	Area	Diameter	Circumference	Area
5.00	15.71	19.63	5.50	17.28	23.76	6.00	18.85	28.27	6.50	20.42	33.18
5.01	15.74	19.71	5.51	17.31	23.84	6.01	18.88	28.37	6.51	20.45	33.29
5.02	15.77	19.79	5.52	17.34	23.93	6.02	18.91	28.46	6.52	20.48	33.39
5.03	15.80	19.87	5.53	17.37	24.02	6.03	18.94	28.56	6.53	20.51	33.49
5.04	15.83	19.95	5.54	17.40	24.11	6.04	18.98	28.65	6.54	20.55	33.59
5.05	15.87	20.03	5.55	17.44	24.19	6.05	19.01	28.75	6.55	20.58	33.70
5.06	15.90	20.11	5.56	17.47	24.28	6.06	19.04	28.84	6.56	20.61	33.80
5.07	15.93	20.19	5.57	17.50	24.37	6.07	19.07	28.94	6.57	20.64	33.90
5.08	15.96	20.27	5.58	17.53	24.45	6.08	19.10	29.03	6.58	20.67	34.00
5.09	15.99	20.35	5.59	17.56	24.54	6.09	19.13	29.13	6.59	20.70	34.11
5.10	16.02	20.43	5.60	17.59	24.63	6.10	19.16	29.22	6.60	20.73	34.21
5.11	16.05	20.51	5.61	17.62	24.72	6.11	19.20	29.32	6.61	20.77	34.32
5.12	16.08	20.59	5.62	17.66	24.81	6.12	19.23	29.42	6.62	20.80	34.42
5.13	16.12	20.67	5.63	17.69	24.89	6.13	19.26	29.51	6.63	20.83	34.52
5.14	16.15	20.75	5.64	17.72	24.98	6.14	19.29	29.61	6.64	20.86	34.63
5.15	16.18	20.83	5.65	17.75	25.07	6.15	19.32	29.71	6.65	20.89	34.73
5.16	16.21	20.91	5.66	17.78	25.16	6.16	19.35	29.80	6.66	20.92	34.84
5.17	16.24	20.99	5.67	17.81	25.25	6.17	19.38	29.90	6.67	20.95	34.94
5.18	16.27	21.07	5.68	17.84	25.34	6.18	19.42	30.00	6.68	20.99	35.05
5.19	16.30	21.16	5.69	17.88	25.43	6.19	19.45	30.09	6.69	21.02	35.15
5.20	16.34	21.24	5.70	17.91	25.52	6.20	19.48	30.19	6.70	21.05	35.26
5.21	16.37	21.32	5.71	17.94	25.61	6.21	19.51	30.29	6.71	21.08	35.36
5.22	16.40	21.40	5.72	17.97	25.70	6.22	19.54	30.39	6.72	21.11	35.47
5.23	16.43	21.48	5.73	18.00	25.79	6.23	19.57	30.48	6.73	21.14	35.57
5.24	16.46	21.56	5.74	18.03	25.88	6.24	19.60	30.58	6.74	21.17	35.68
5.25	16.49	21.65	5.75	18.06	25.97	6.25	19.63	30.68	6.75	21.21	35.78
5.26	16.52	21.73	5.76	18.10	26.06	6.26	19.67	30.78	6.76	21.24	35.89
5.27	16.56	21.81	5.77	18.13	26.15	6.27	19.70	30.88	6.77	21.27	36.00
5.28	16.59	21.90	5.78	18.16	26.24	6.28	19.73	30.97	6.78	21.30	36.10
5.29	16.62	21.98	5.79	18.19	26.33	6.29	19.76	31.07	6.79	21.33	36.21
5.30	16.65	22.06	5.80	18.22	26.42	6.30	19.79	31.17	6.80	21.36	36.32
5.31	16.68	22.15	5.81	18.25	26.51	6.31	19.82	31.27	6.81	21.39	36.42
5.32	16.71	22.23	5.82	18.28	26.60	6.32	19.85	31.37	6.82	21.43	36.53
5.33	16.74	22.31	5.83	18.32	26.69	6.33	19.89	31.47	6.83	21.46	36.64
5.34	16.78	22.40	5.84	18.35	26.79	6.34	19.92	31.57	6.84	21.49	36.75
5.35	16.81	22.48	5.85	18.38	26.88	6.35	19.95	31.67	6.85	21.52	36.85
5.36	16.84	22.56	5.86	18.41	26.97	6.36	19.98	31.77	6.86	21.55	36.96
5.37	16.87	22.65	5.87	18.44	27.06	6.37	20.01	31.87	6.87	21.58	37.07
5.38	16.90	22.73	5.88	18.47	27.15	6.38	20.04	31.97	6.88	21.61	37.18
5.39	16.93	22.82	5.89	18.50	27.25	6.39	20.07	32.07	6.89	21.65	37.28
5.40	16.96	22.90	5.90	18.54	27.34	6.40	20.11	32.17	6.90	21.68	37.39
5.41	17.00	22.99	5.91	18.57	27.43	6.41	20.14	32.27	6.91	21.71	37.50
5.42	17.03	23.07	5.92	18.60	27.53	6.42	20.17	32.37	6.92	21.74	37.61
5.43	17.06	23.16	5.93	18.63	27.62	6.43	20.20	32.47	6.93	21.77	37.72
5.44	17.09	23.24	5.94	18.66	27.71	6.44	20.23	32.57	6.94	21.80	37.83
5.45	17.12	23.33	5.95	18.69	27.81	6.45	20.26	32.67	6.95	21.83	37.94
5.46	17.15	23.41	5.96	18.72	27.90	6.46	20.29	32.78	6.96	21.87	38.05
5.47	17.18	23.50	5.97	18.76	27.99	6.47	20.33	32.88	6.97	21.90	38.16
5.48	17.22	23.59	5.98	18.79	28.09	6.48	20.36	32.98	6.98	21.93	38.26
5.49	17.25	23.67	5.99	18.82	28.18	6.49	20.39	33.08	6.99	21.96	38.37

Table 6. Circles, circumferences and areas (diameters in hundredths)—*Continued*

Diameter	Circumference	Area	Diameter	Circumference	Area	Diameter	Circumference	Area	Diameter	Circumference	Area
7.00	21.99	38.48	7.50	23.56	44.18	8.00	25.13	50.27	8.50	26.70	56.75
7.01	22.02	38.59	7.51	23.59	44.30	8.01	25.16	50.39	8.51	26.73	56.88
7.02	22.05	38.70	7.52	23.62	44.41	8.02	25.20	50.52	8.52	26.77	57.01
7.03	22.09	38.82	7.53	23.66	44.53	8.03	25.23	50.64	8.53	26.80	57.15
7.04	22.12	38.93	7.54	23.69	44.65	8.04	25.26	50.77	8.54	26.83	57.28
7.05	22.15	39.04	7.55	23.72	44.77	8.05	25.29	50.90	8.55	26.86	57.41
7.06	22.18	39.15	7.56	23.75	44.89	8.06	25.32	51.02	8.56	26.89	57.55
7.07	22.21	39.26	7.57	23.78	45.01	8.07	25.35	51.15	8.57	26.92	57.68
7.08	22.24	39.37	7.58	23.81	45.13	8.08	25.38	51.28	8.58	26.95	57.82
7.09	22.27	39.48	7.59	23.84	45.25	8.09	25.42	51.40	8.59	26.99	57.95
7.10	22.31	39.59	7.60	23.88	45.36	8.10	25.45	51.53	8.60	27.02	58.09
7.11	22.34	39.70	7.61	23.91	45.48	8.11	25.48	51.66	8.61	27.05	58.22
7.12	22.37	39.82	7.62	23.94	45.60	8.12	25.51	51.78	8.62	27.08	58.36
7.13	22.40	39.93	7.63	23.97	45.72	8.13	25.54	51.91	8.63	27.11	58.49
7.14	22.43	40.04	7.64	24.00	45.84	8.14	25.57	52.04	8.64	27.14	58.63
7.15	22.46	40.15	7.65	24.03	45.96	8.15	25.60	52.17	8.65	27.17	58.77
7.16	22.49	40.26	7.66	24.06	46.08	8.16	25.64	52.30	8.66	27.21	58.90
7.17	22.53	40.38	7.67	24.10	46.20	8.17	25.67	52.42	8.67	27.24	59.04
7.18	22.56	40.49	7.68	24.13	46.32	8.18	25.70	52.55	8.68	27.27	59.17
7.19	22.59	40.60	7.69	24.16	46.45	8.19	25.73	52.68	8.69	27.30	59.31
7.20	22.62	40.72	7.70	24.19	46.57	8.20	25.76	52.81	8.70	27.33	59.45
8.21	22.65	40.83	7.71	24.22	46.69	8.21	25.79	52.94	8.71	27.36	59.58
7.22	22.68	40.94	7.72	24.25	46.81	8.22	25.82	53.07	8.72	27.39	59.72
7.23	22.71	41.06	7.73	24.28	46.93	8.23	25.86	53.20	8.73	27.43	59.86
7.24	22.75	41.17	7.74	24.32	47.05	8.24	25.89	53.33	8.74	27.46	59.99
7.25	22.78	41.28	7.75	24.35	47.17	8.25	25.92	53.46	8.75	27.49	60.13
7.26	22.81	41.40	7.76	24.38	47.29	8.26	25.95	53.59	8.76	27.52	60.27
7.27	22.84	41.51	7.77	24.41	47.42	8.27	25.98	53.72	8.77	27.55	60.41
7.28	22.87	41.62	7.78	24.44	47.54	8.28	26.01	53.85	8.78	27.58	60.55
7.29	22.90	41.74	7.79	24.47	47.66	8.29	26.04	53.98	8.79	27.61	60.68
7.30	22.93	41.85	7.80	24.50	47.78	8.30	26.08	54.11	8.80	27.65	60.82
7.31	22.97	41.97	7.81	24.54	47.91	8.31	26.11	54.24	8.81	27.68	60.96
7.32	23.00	42.08	7.82	24.57	48.03	8.32	26.14	54.37	8.82	27.71	61.10
7.33	23.03	42.20	7.83	24.60	48.15	8.33	26.17	54.50	8.83	27.74	61.24
7.34	23.06	42.31	7.84	24.63	48.28	8.34	26.20	54.63	8.84	27.77	61.38
7.35	23.09	42.43	7.85	24.66	48.40	8.35	26.23	54.76	8.85	27.80	61.51
7.36	23.12	42.54	7.86	24.69	48.52	8.36	26.26	54.89	8.86	27.83	61.65
7.37	23.15	42.66	7.87	24.72	48.65	8.37	26.30	55.02	8.87	27.87	61.79
7.38	23.18	42.78	7.88	24.76	48.77	8.38	26.33	55.15	8.88	27.90	61.93
7.39	23.22	42.89	7.89	24.79	48.89	8.39	26.36	55.29	8.89	27.93	62.07
7.40	23.25	43.01	7.90	24.82	49.02	8.40	26.39	55.42	8.90	27.96	62.21
7.41	23.28	43.12	7.91	24.85	49.14	8.41	26.42	55.55	8.91	27.99	62.35
7.42	23.31	43.24	7.92	24.88	49.27	8.42	26.45	55.68	8.92	28.02	62.49
7.43	23.34	43.36	7.93	24.91	49.39	8.43	26.48	55.81	8.93	28.05	62.63
7.44	23.37	43.47	7.94	24.94	49.51	8.44	26.52	55.95	8.94	28.09	62.77
7.45	23.40	43.59	7.95	24.98	49.64	8.45	26.55	56.08	8.95	28.12	62.91
7.46	23.44	43.71	7.96	25.01	49.76	8.46	26.58	56.21	8.96	28.15	63.05
7.47	23.47	43.83	7.97	25.04	49.89	8.47	26.61	56.35	8.97	28.18	63.19
7.48	23.50	43.94	7.98	25.07	50.01	8.48	26.64	56.48	8.98	28.21	63.33
7.49	23.53	44.06	7.99	25.10	50.14	8.49	26.67	56.61	8.99	28.24	63.48

Table 6. Circles, circumferences and areas—*Continued*

Diameter	Circumference	Area	Diameter	Circumference	Area
9.00	28.27	63.62	9.50	29.85	70.88
9.01	28.31	63.76	9.51	29.88	71.03
9.02	28.34	63.90	9.52	29.91	71.18
9.03	28.37	64.04	9.53	29.94	71.33
9.04	28.40	64.18	9.54	29.97	71.48
9.05	28.43	64.33	9.55	30.00	71.63
9.06	28.46	64.47	9.56	30.03	71.78
9.07	28.49	64.61	9.57	30.07	71.93
9.08	28.53	64.75	9.58	30.10	72.08
9.09	28.56	64.90	9.59	30.13	72.23
9.10	28.59	65.04	9.60	30.16	72.38
9.11	28.62	65.18	9.61	30.19	72.53
9.12	28.65	65.33	9.62	30.22	72.68
9.13	28.68	65.47	9.63	30.25	72.84
9.14	28.71	65.61	9.64	30.28	72.99
9.15	28.75	65.76	9.65	30.32	73.14
9.16	28.78	65.90	9.66	30.35	73.29
9.17	28.81	66.04	9.67	30.38	73.44
9.18	28.84	66.19	9.68	30.41	73.59
9.19	28.87	66.33	9.69	30.44	73.75
9.20	28.90	66.48	9.70	30.47	73.90
9.21	28.93	66.62	9.71	30.50	74.05
9.22	28.97	66.77	9.72	30.54	74.20
9.23	29.00	66.91	9.73	30.57	74.36
9.24	29.03	67.06	9.74	30.60	74.51
9.25	29.06	67.20	9.75	30.63	74.66
9.26	29.09	67.35	9.76	30.66	74.82
9.27	29.12	67.49	9.77	30.69	74.97
9.28	29.15	67.64	9.78	30.72	75.12
9.29	29 19	67.78	9.79	30.76	75.28
9.30	29.22	67.93	9.80	30.79	75.43
9.31	29.25	68.08	9.81	30.82	75.58
9.32	29.28	68.22	9.82	30.85	75.74
9.33	29.31	68.37	9.83	30.88	75.89
9.34	29.34	68.51	9.84	30.91	76.05
9.35	29.37	68.66	9.85	30.94	76.20
9.36	29.41	68.81	9.86	30.98	76.36
9.37	29.44	68.96	9.87	31.01	76.51
9.38	29.47	69.10	9.88	31.04	76.67
9.39	29.50	69.25	9.89	31.07	76.82
9.40	29.53	69.40	9.90	31.10	76.98
9.41	29.56	69.55	9.91	31.13	77.13
9.42	29.59	69.69	9.92	31.16	77.29
9.43	29.63	69.84	9.93	31.20	77.44
9.44	29.66	69.99	9.94	31.23	77.60
9.45	29.69	70.14	9.95	31.26	77.76
9.46	29.72	70.29	9.96	31.29	77.91
9.47	29.75	70.44	9.97	31.32	78.07
9.48	29.78	70.58	9.98	31.35	78.23
9.49	29.81	70.73	9.99	31.38	78.38

Table 7. Volumes of spheres

Diameter	Volume	Diameter	Volume	Diameter	Volume
1.0	.5236	6.0	113.1	11.0	696.9
1.1	.6969	6.1	118.8	11.1	716.1
1.2	.9048	6.2	124.8	11.2	735.6
1.3	1.150	6.3	130.9	11.3	755.5
1.4	1.437	6.4	137.3	11.4	775.7
1.5	1.767	6.5	143.8	11.5	796.3
1.6	2.145	6.6	150.5	11.6	817.3
1.7	2.572	6.7	157.5	11.7	838.6
1.8	3.054	6.8	164.6	11.8	860.3
1.9	3.591	6.9	172.0	11.9	882.3
2.0	4.189	7.0	179.6	12.0	904.8
2.1	4.849	7.1	187.4	12.1	927.6
2.2	5.575	7.2	195.4	12.2	950.8
2.3	6.371	7.3	203.7	12.3	974.3
2.4	7.238	7.4	212.2	12.4	998.3
2.5	8.181	7.5	220.9	12.5	1023
2.6	9.203	7.6	229.8	12.6	1047
2.7	10.31	7.7	239.0	12.7	1073
2.8	11.49	7.8	248.5	12.8	1098
2.9	12.77	7.9	258.2	12.9	1124
3.0	14.14	8.0	268.1	13.0	1150
3.1	15.60	8.1	278.3	13.1	1177
3.2	17.16	8.2	288.7	13.2	1204
3.3	18.82	8.3	299.4	13.3	1232
3.4	20.58	8.4	310.3	13.4	1260
3.5	22.45	8.5	321.6	13.5	1288
3.6	24.43	8.6	333.0	13.6	1317
3.7	26.52	8.7	344.8	13.7	1346
3.8	28.73	8.8	356.8	13.8	1376
3.9	31.06	8.9	369.1	13.9	1406
4.0	33.51	9.0	381.7	14.0	1437
4.1	36.09	9.1	394.6	14.1	1468
4.2	38.79	9.2	407.7	14.2	1499
4.3	41.63	9.3	421.2	14.3	1531
4.4	44.60	9.4	434.9	14.4	1563
4.5	47.71	9.5	448.9	14.5	1596
4.6	50.97	9.6	463.2	14.6	1630
4.7	54.36	9.7	477.9	14.7	1663
4.8	57.91	9.8	492.8	14.8	1697
4.9	61.60	9.9	508.0	14.9	1732
5.0	65.45	10.0	523.6	15.0	1767
5.1	69.46	10.1	539.5	15.1	1803
5.2	73.62	10.2	555.6	15.2	1839
5.3	77.95	10.3	572.2	15.3	1875
5.4	82.45	10.4	589.0	15.4	1912
5.5	87.11	10.5	606.1	15.5	1950
5.6	91.95	10.6	623.6	15.6	1988
5.7	96.97	10.7	641.4	15.7	2026
5.8	102.2	10.8	659.6	15.8	2065
5.9	107.5	10.9	678.1	15.9	2105

Table 8. Circles, circumferences and areas (diameters in eighths)

Diameter	Circum-ference	Area	Diameter	Circum-ference	Area	Diameter	Circum-ference	Area	Diameter	Circum-ference	Area
0	....	....	6	18.85	28.27	12	37.70	113.1	18	56.55	254.5
⅛	.3927	.01227	⅛	19.24	29.46	⅛	38.09	115.5	⅛	56.94	258.0
¼	.7854	.04909	¼	19.63	30.68	¼	38.48	117.9	¼	57.33	261.6
⅜	1.178	.1104	⅜	20.03	31.92	⅜	38.88	120.3	⅜	57.73	265.2
½	1.571	.1963	½	20.42	33.18	½	39.27	122.7	½	58.12	268.8
⅝	1.963	.3068	⅝	20.81	34.47	⅝	39.66	125.2	⅝	58.51	272.4
¾	2.356	.4418	¾	21.21	35.78	¾	40.06	127.7	¾	58.90	276.1
⅞	2.749	.6013	⅞	21.60	37.12	⅞	40.45	130.2	⅞	59.30	279.8
1	3.142	.7854	7	21.99	38.48	13	40.84	132.7	19	59.69	283.5
⅛	3.534	.9940	⅛	22.38	39.87	⅛	41.23	135.3	⅛	60.08	287.3
¼	3.927	1.227	¼	22.78	41.28	¼	41.63	137.9	¼	60.48	291.0
⅜	4.320	1.485	⅜	23.17	42.72	⅜	42.02	140.5	⅜	60.87	294.8
½	4.712	1.767	½	23.56	44.18	½	42.41	143.1	½	61.26	298.6
⅝	5.105	2.074	⅝	23.95	45.66	⅝	42.80	145.8	⅝	61.65	302.5
¾	5.498	2.405	¾	24.35	47.17	¾	43.20	148.5	¾	62.05	306.4
⅞	5.891	2.761	⅞	24.74	48.71	⅞	43.59	151.2	⅞	62.44	310.2
2	6.283	3.142	8	25.13	50.27	14	43.98	153.9	20	62.83	314.2
⅛	6.676	3.547	⅛	25.52	51.85	⅛	44.37	156.7	⅛	63.22	318.1
¼	7.069	3.976	¼	25.92	53.46	¼	44.77	159.5	¼	63.62	322.1
⅜	7.461	4.430	⅜	26.31	55.09	⅜	45.16	162.3	⅜	64.01	326.1
½	7.854	4.909	½	26.70	56.75	½	45.55	165.1	½	64.40	330.1
⅝	8.247	5.412	⅝	27.10	58.43	⅝	45.95	168.0	⅝	64.80	334.1
¾	8.639	5.940	¾	27.49	60.13	¾	46.34	170.9	¾	65.19	338.2
⅞	9.032	6.492	⅞	27.88	61.86	⅞	46.73	173.8	⅞	65.58	342.3
3	9.425	7.069	9	28.27	63.62	15	47.12	176.7	21	65.97	346.4
⅛	9.817	7.670	⅛	28.67	65.40	⅛	47.52	179.7	⅛	66.37	350.5
¼	10.21	8.296	¼	29.06	67.20	¼	47.91	182.7	¼	66.76	354.7
⅜	10.60	8.946	⅜	29.45	69.03	⅜	48.30	185.7	⅜	67.15	358.8
½	11.00	9.621	½	29.85	70.88	½	48.69	188.7	½	67.54	363.1
⅝	11.39	10.32	⅝	30.24	72.76	⅝	49.09	191.7	⅝	67.94	367.3
¾	11.78	11.04	¾	30.63	74.66	¾	49.48	194.8	¾	68.33	371.5
⅞	12.17	11.79	⅞	31.02	76.59	⅞	49.87	197.9	⅞	68.72	375.8
4	12.57	12.57	10	31.42	78.54	16	50.27	201.1	22	69.12	380.1
⅛	12.96	13.36	⅛	31.81	80.52	⅛	50.66	204.2	⅛	69.51	384.5
¼	13.35	14.19	¼	32.20	82.52	¼	51.05	207.4	¼	69.90	388.8
⅜	13.74	15.03	⅜	32.59	84.54	⅜	51.44	210.6	⅜	70.29	393.2
½	14.14	15.90	½	32.99	86.59	½	51.84	213.8	½	70.69	397.6
⅝	14.53	16.80	⅝	33.38	88.66	⅝	52.23	217.1	⅝	71.08	402.0
¾	14.92	17.72	¾	33.77	90.76	¾	52.62	220.4	¾	71.47	406.5
⅞	15.32	18.67	⅞	34.16	92.89	⅞	53.01	223.7	⅞	71.86	411.0
5	15.71	19.63	11	34.56	95.03	17	53.41	227.0	23	72.26	415.5
⅛	16.10	20.63	⅛	34.95	97.21	⅛	53.80	230.3	⅛	72.65	420.0
¼	16.49	21.65	¼	35.34	99.40	¼	54.19	233.7	¼	73.04	424.6
⅜	16.89	22.69	⅜	35.74	101.6	⅜	54.59	237.1	⅜	73.43	429.1
½	17.28	23.76	½	36.13	103.9	½	54.98	240.5	½	73.83	433.7
⅝	17.67	24.85	⅝	36.52	106.1	⅝	55.37	244.0	⅝	74.22	438.4
¾	18.06	25.97	¾	36.91	108.4	¾	55.76	247.4	¾	74.61	443.0
⅞	18.46	27.11	⅞	37.31	110.8	⅞	56.16	250.9	⅞	75.01	447.7

Table 9. Circular segments (see Art. 75)

Central angle in degrees	Height / R	Chord / R	Height / Chord	Area / R^2	Central angle in degrees	Height / R	Chord / R	Height / Chord	Area / R^2
1	.0000	.0175	.0022	.00000	46	.0795	.7815	.1017	.04176
2	.0002	.0349	.0044	.00000	47	.0829	.7975	.1040	.04448
3	.0003	.0524	.0066	.00001	48	.0865	.8135	.1063	.04731
4	.0006	.0698	.0087	.00003	49	.0900	.8294	.1086	.05025
5	.0010	.0872	.0109	.00006	50	.0937	.8452	.1108	.05331
6	.0014	.1047	.0131	.00010	51	.0974	.8610	.1131	.05649
7	.0019	.1221	.0153	.00015	52	.1012	.8767	.1154	.05978
8	.0024	.1395	.0175	.00023	53	.1051	.8924	.1177	.06319
9	.0031	.1569	.0196	.00032	54	.1090	.9080	.1200	.06673
10	.0038	.1743	.0218	.00044	55	.1130	.9235	.1223	.07039
11	.0046	.1917	.0240	.00059	56	.1171	.9389	.1247	.07417
12	.0055	.2091	.0262	.00076	57	.1212	.9543	.1270	.07808
13	.0064	.2264	.0284	.00097	58	.1254	.9696	.1293	.08212
14	.0075	.2437	.0306	.00121	59	.1296	.9848	.1316	.08629
15	.0086	.2611	.0328	.00149	60	.1340	1.0000	.1340	.09059
16	.0097	.2783	.0350	.00181	61	.1384	1.015	.1363	.09502
17	.0110	.2956	.0372	.00217	62	.1428	1.030	.1387	.09958
18	.0123	.3129	.0394	.00257	63	.1474	1.045	.1410	.10428
19	.0137	.3301	.0415	.00302	64	.1520	1.060	.1434	.10911
20	.0152	.3473	.0437	.00352	65	.1566	1.075	.1457	.11408
21	.0167	.3645	.0459	.00408	66	.1613	1.089	.1481	.11919
22	.0184	.3816	.0481	.00468	67	.1661	1.104	.1505	.12443
23	.0201	.3987	.0503	.00535	68	.1710	1.118	.1529	.12982
24	.0219	.4158	.0526	.00607	69	.1759	1.133	.1553	.13535
25	.0237	.4329	.0548	.00686	70	.1808	1.147	.1576	.14102
26	.0256	.4499	.0570	.00771	71	.1859	1.161	.1601	.14683
27	.0276	.4669	.0592	.00862	72	.1910	1.176	.1625	.15279
28	.0297	.4838	.0614	.00961	73	.1961	1.190	.1649	.15889
29	.0319	.5008	.0636	.01067	74	.2014	1.204	.1673	.16514
30	.0341	.5176	.0658	.01180	75	.2066	1.218	.1697	.17154
31	.0364	.5345	.0680	.01301	76	.2120	1.231	.1722	.17808
32	.0387	.5513	.0703	.01429	77	.2174	1.245	.1746	.18477
33	.0412	.5680	.0725	.01566	78	.2229	1.259	.1771	.19160
34	.0437	.5847	.0747	.01711	79	.2284	1.272	.1795	.19859
35	.0463	.6014	.0770	.01864	80	.2340	1.286	.1820	.20573
36	.0489	.6180	.0792	.02027	81	.2396	1.299	.1845	.21301
37	.0517	.6346	.0814	.02198	82	.2453	1.312	.1869	.22045
38	.0545	.6511	.0837	.02378	83	.2510	1.325	.1894	.22804
39	.0574	.6676	.0859	.02568	84	.2569	1.338	.1919	.23578
40	.0603	.6840	.0882	.02767	85	.2627	1.351	.1944	.24367
41	.0633	.7004	.0904	.02976	86	.2686	1.364	.1970	.25171
42	.0664	.7167	.0927	.03195	87	.2746	1.377	.1995	.25990
43	.0696	.7330	.0949	.03425	88	.2807	1.389	.2020	.26825
44	.0728	.7492	.0972	.03664	89	.2867	1.402	.2046	.27675
45	.0761	.7654	.0995	.03915	90	.2929	1.414	.2071	.28540

Table 9. Circular segments—*Continued*

Central angle in degrees	Height / R	Chord / R	Height / Chord	Area / R^2	Central angle in degrees	Height / R	Chord / R	Height / Chord	Area / R^2
91	.2991	1.427	.2097	.29420	136	.6254	1.854	.3373	.83949
92	.3053	1.439	.2122	.30316	137	.6335	1.861	.3404	.85455
93	.3116	1.451	.2148	.31226	138	.6416	1.867	.3436	.86971
94	.3180	1.463	.2174	.32152	139	.6498	1.873	.3469	.88497
95	.3244	1.475	.2200	.33093	140	.6580	1.879	.3501	.90034
96	.3309	1.486	.2226	.34050	141	.6662	1.885	.3534	.91580
97	.3374	1.498	.2252	.35021	142	.6744	1.891	.3566	.93135
98	.3439	1.509	.2279	.36008	143	.6827	1.897	.3599	.94700
99	.3506	1.521	.2305	.37009	144	.6910	1.902	.3633	.96274
100	.3572	1.532	.2332	.38026	145	.6993	1.907	.3666	.97858
101	.3639	1.543	.2358	.39058	146	.7076	1.913	.3700	.99449
102	.3707	1.554	.2385	.40104	147	.7160	1.918	.3734	1.0105
103	.3775	1.565	.2412	.41166	148	.7244	1.923	.3768	1.0266
104	.3843	1.576	.2439	.42242	149	.7328	1.927	.3802	1.0428
105	.3912	1.587	.2466	.43333	150	.7412	1.932	.3837	1.0590
106	.3982	1.597	.2493	.44439	151	.7496	1.936	.3871	1.0753
107	.4052	1.608	.2520	.45560	152	.7581	1.941	.3906	1.0917
108	.4122	1.618	.2548	.46695	153	.7666	1.945	.3942	1.1082
109	.4193	1.628	.2575	.47844	154	.7750	1.949	.3977	1.1247
110	.4264	1.638	.2603	.49008	155	.7836	1.953	.4013	1.1413
111	.4336	1.648	.2631	.50187	156	.7921	1.956	.4049	1.1580
112	.4408	1.658	.2659	.51379	157	.8006	1.960	.4085	1.1747
113	.4481	1.668	.2687	.52586	158	.8092	1.963	.4122	1.1915
114	.4554	1.677	.2715	.53807	159	.8178	1.967	.4158	1.2084
115	.4627	1.687	.2743	.55041	160	.8264	1.970	.4195	1.2253
116	.4701	1.696	.2772	.56289	161	.8350	1.973	.4233	1.2422
117	.4775	1.705	.2800	.57551	162	.8436	1.975	.4270	1.2592
118	.4850	1.714	.2829	.58827	163	.8522	1.978	.4308	1.2763
119	.4925	1.723	.2858	.60116	164	.8608	1.981	.4346	1.2934
120	.5000	1.732	.2887	.61418	165	.8695	1.983	.4385	1.3105
121	.5076	1.741	.2916	.62734	166	.8781	1.985	.4424	1.3277
122	.5152	1.749	.2945	.64063	167	.8868	1.987	.4463	1.3449
123	.5228	1.758	.2975	.65404	168	.8955	1.989	.4502	1.3621
124	.5305	1.766	.3004	.66759	169	.9042	1.991	.4542	1.3794
125	.5383	1.774	.3034	.68125	170	.9128	1.992	.4582	1.3967
126	.5460	1.782	.3064	.69505	171	.9215	1.994	.4622	1.4140
127	.5538	1.790	.3094	.70897	172	.9302	1.995	.4663	1.4314
128	.5616	1.798	.3124	.72301	173	.9390	1.996	.4704	1.4488
129	.5695	1.805	.3155	.73716	174	.9477	1.997	.4745	1.4662
130	.5774	1.813	.3185	.75114	175	.9564	1.998	.4786	1.4836
131	.5853	1.820	.3216	.76584	176	.9651	1.999	.4828	1.5010
132	.5933	1.827	.3247	.78034	177	.9738	1.999	.4871	1.5184
133	.6013	1.834	.3278	.79497	178	.9825	2.000	.4914	1.5359
134	.6093	1.841	.3309	.80970	179	.9913	2.000	.4957	1.5533
135	.6173	1.848	.3341	.82454	180	1.000	2.000	.5000	1.5708

Table 10. Degrees to radians

Deg.	Rad.	Deg.	Rad.	Deg.	Rad.	Deg.	Rad.	Deg.	Rad.	Deg.	Rad.
1	.0175	31	.5411	61	1.0647	91	1.5882	121	2.1118	151	2.6354
2	.0349	32	.5585	62	1.0821	92	1.6057	122	2.1293	152	2.6529
3	.0524	33	.5760	63	1.0996	93	1.6232	123	2.1468	153	2.6704
4	.0698	34	.5934	64	1.1170	94	1.6406	124	2.1642	154	2.6878
5	.0873	35	.6109	65	1.1345	95	1.6581	125	2.1817	155	2.7053
6	.1047	36	.6283	66	1.1519	96	1.6755	126	2.1991	156	2.7227
7	.1222	37	.6458	67	1.1694	97	1.6930	127	2.2166	157	2.7402
8	.1396	38	.6632	68	1.1868	98	1.7104	128	2.2340	158	2.7576
9	.1571	39	.6807	69	1.2043	99	1.7279	129	2.2515	159	2.7751
10	.1745	40	.6981	70	1.2217	100	1.7453	130	2.2689	160	2.7925
11	.1920	41	.7156	71	1.2392	101	1.7628	131	2.2864	161	2.8100
12	.2094	42	.7330	72	1.2566	102	1.7802	132	2.3038	162	2.8274
13	.2269	43	.7505	73	1.2741	103	1.7977	133	2.3213	163	2.8449
14	.2443	44	.7679	74	1.2915	104	1.8151	134	2.3387	164	2.8623
15	.2618	45	.7854	75	1.3090	105	1.8326	135	2.3562	165	2.8798
16	.2793	46	.8029	76	1.3265	106	1.8500	136	2.3736	166	2.8972
17	.2967	47	.8203	77	1.3439	107	1.8675	137	2.3911	167	2.9147
18	.3142	48	.8378	78	1.3614	108	1.8850	138	2.4086	168	2.9322
19	.3316	49	.8552	79	1.3788	109	1.9024	139	2.4260	169	2.9496
20	.3491	50	.8727	80	1.3963	110	1.9199	140	2.4435	170	2.9671
21	.3665	51	.8901	81	1.4137	111	1.9373	141	2.4609	171	2.9845
22	.3840	52	.9076	82	1.4312	112	1.9548	142	2.4784	172	3.0020
23	.4014	53	.9250	83	1.4486	113	1.9722	143	2.4958	173	3.0194
24	.4189	54	.9425	84	1.4661	114	1.9897	144	2.5133	174	3.0369
25	.4363	55	.9599	85	1.4835	115	2.0071	145	2.5307	175	3.0543
26	.4538	56	.9774	86	1.5010	116	2.0246	146	2.5482	176	3.0718
27	.4712	57	.9948	87	1.5184	117	2.0420	147	2.5656	177	3.0892
28	.4887	58	1.0123	88	1.5359	118	2.0595	148	2.5831	178	3.1067
29	.5061	59	1.0297	89	1.5533	110	2.0769	149	2.6005	179	3.1241
30	.5236	60	1.0472	90	1.5708	120	2.0944	150	2.6180	180	3.1416

Table 11. Minutes to radians

Min.	Rad.	Min.	Rad.	Min.	Rad.	Min.	Rad.	Min.	Rad.	Min.	Rad.
1	.0003	11	.0032	21	.0061	31	.0090	41	.0119	51	.0148
2	.0006	12	.0035	22	.0064	32	.0093	42	.0122	52	.0151
3	.0009	13	.0038	23	.0067	33	.0096	43	.0125	53	.0154
4	.0012	14	.0041	24	.0070	34	.0099	44	.0128	54	.0157
5	.0015	15	.0044	25	.0073	35	.0102	45	.0131	55	.0160
6	.0017	16	.0047	26	.0076	36	.0105	46	.0134	56	.0163
7	.0020	17	.0049	27	.0079	37	.0108	47	.0137	57	.0166
8	.0023	18	.0052	28	.0081	38	.0111	48	.0140	58	.0169
9	.0026	19	.0055	29	.0084	39	.0113	49	.0143	59	.0172
10	.0029	20	.0058	30	.0087	40	.0116	50	.0145	60	.0175

Table 12. Decimal parts of a degree to minutes

D.	M.	D.	M.	D.	M.	D.	M.	D.	M.	D.	M.	D.	M.	D.	M.	D.	M.	D.	M.
.01	0.6	.11	6.6	.21	12.6	.31	18.6	.41	24.6	.51	30.6	.61	36.6	.71	42.6	.81	48.6	.91	54.6
.02	1.2	.12	7.2	.22	13.2	.32	19.2	.42	25.2	.52	31.2	.62	37.2	.72	43.2	.82	49.2	.92	55.2
.03	1.8	.13	7.8	.23	13.8	.33	19.8	.43	25.8	.53	31.8	.63	37.8	.73	43.8	.83	49.8	.93	55.8
.04	2.4	.14	8.4	.24	14.4	.34	20.4	.44	26.4	.54	32.4	.64	38.4	.74	44.4	.84	50.4	.94	56.4
.05	3.0	.15	9.0	.25	15.0	.35	21.0	.45	27.0	.55	33.0	.65	39.0	.75	45.0	.85	51.0	.95	57.0
.06	3.6	.16	9.6	.26	15.6	.36	21.6	.46	27.6	.56	33.6	.66	39.6	.76	45.6	.86	51.6	.96	57.6
.07	4.2	.17	10.2	.27	16.2	.37	22.2	.47	28.2	.57	34.2	.67	40.2	.77	46.2	.87	52.2	.97	58.2
.08	4.8	.18	10.8	.28	16.8	.38	22.8	.48	28.8	.58	34.8	.68	40.8	.78	46.8	.88	52.8	.98	58.8
.09	5.4	.19	11.4	.29	17.4	.39	23.4	.49	29.4	.59	35.4	.69	41.4	.79	47.4	.89	53.4	.99	59.4
.10	6.0	.20	12.0	.30	18.0	.40	24.0	.50	30.0	.60	36.0	.70	42.0	.80	48.0	.90	54.0	1.00	60.0

Table 13. Radians to degrees and minutes

Radians	Deg. °	min. ′	Radians	Deg. °	min. ′	Radians	Deg. °	min. ′	Radians	Deg. °	min. ′	Radians	Deg. °	min. ′	Radians	Deg. °	min. ′
.001	0	3	0.47	26	56	1.02	58	27	1.57	89	57	2.12	121	28	2.67	152	59
.002	0	7	.48	27	30	.03	59	1	.58	90	32	.13	122	2	.68	153	33
.003	0	10	.49	28	4	.04	59	35	.59	91	6	.14	122	37	.69	154	8
.004	0	14	0.50	28	39	1.05	60	10	1.60	91	40	2.15	123	11	2.70	154	42
.005	0	17	.51	29	13	.06	60	44	.61	92	15	.16	123	46	.71	155	16
.006	0	21	.52	29	48	.07	61	18	.62	92	49	.17	124	20	.72	155	51
.007	0	24	.53	30	22	.08	61	53	.63	93	24	.18	124	54	.73	156	25
.008	0	28	.54	30	56	.09	62	27	.64	93	58	.19	125	29	.74	156	59
.009	0	31	0.55	31	31	1.10	63	2	1.65	94	32	2.20	126	3	2.75	157	34
0.01	0	34	.56	32	5	.11	63	36	.66	95	6	.21	126	37	.76	158	8
.02	1	9	.57	32	40	.12	64	10	.67	95	41	.22	127	12	.77	158	43
.03	1	43	.58	33	14	.13	64	45	.68	96	15	.23	127	46	.78	159	17
.04	2	18	.59	33	48	.14	65	19	.69	96	50	.24	128	21	.79	159	51
0.05	2	52	0.60	34	23	1.15	65	53	1.70	97	24	2.25	128	55	2.80	160	26
.06	3	26	.61	34	57	.16	66	28	.71	97	59	.26	129	29	.81	161	0
.07	4	1	.62	35	31	.17	67	2	.72	98	33	.27	130	4	.82	161	34
.08	4	35	.63	36	6	.18	67	37	.73	99	7	.28	130	38	.83	162	9
.09	5	9	.64	36	40	.19	68	11	.74	99	42	.29	131	12	.84	162	43
0.10	5	44	0.65	37	15	1.20	68	45	1.75	100	16	2.30	131	47	2.85	163	18
.11	6	18	.66	37	49	.21	69	20	.76	100	50	.31	132	21	.86	163	52
.12	6	53	.67	38	23	.22	69	54	.77	101	25	.32	132	56	.87	164	26
.13	7	27	.68	38	58	.23	70	28	.78	101	59	.33	133	30	.88	165	1
.14	8	1	.69	39	32	.24	71	3	.79	102	34	.34	134	4	.89	165	35
0.15	8	36	0.70	40	6	1.25	71	37	1.80	103	8	2.35	134	39	2.90	166	9
.16	9	10	.71	40	41	.26	72	12	.81	103	42	.36	135	13	.91	166	44
.17	9	44	.72	41	15	.27	72	46	.82	104	17	.37	135	47	.92	167	18
.18	10	19	.73	41	50	.28	73	20	.83	104	51	.38	136	22	.93	167	53
.19	10	53	.74	42	24	.29	73	55	.84	105	25	.39	136	56	.94	168	27
0.20	11	28	0.75	42	58	1.30	74	29	1.85	106	0	2.40	137	31	2.95	169	1
.21	12	2	.76	43	33	.31	75	3	.86	106	34	.41	138	5	.96	169	36
.22	12	36	.77	44	7	.32	75	38	.87	107	9	.42	138	39	.97	170	10
.23	13	11	.78	44	41	.33	76	12	.88	107	43	.43	139	14	.98	170	44
.24	13	45	.79	45	16	.34	76	47	.89	108	17	.44	139	48	.99	171	19
0.25	14	19	0.80	45	50	1.35	77	21	1.90	108	52	2.45	140	22	3.00	171	53
.26	14	54	.81	46	25	.36	77	55	.91	109	26	.46	140	57	.01	172	28
.27	15	28	.82	46	59	.37	78	30	.92	110	0	.47	141	31	.02	173	2
.28	16	3	.83	47	33	.38	79	4	.93	110	35	.48	142	6	.03	173	36
.29	16	37	.84	48	8	.39	79	38	.94	111	9	.49	142	40	.04	174	11
0.30	17	11	0.85	48	42	1.40	80	13	1.95	111	44	2.50	143	14	3.05	174	45
.31	17	46	.86	49	16	.41	80	47	.96	112	18	.51	143	49	.06	175	20
.32	18	20	.87	49	51	.42	81	22	.97	112	52	.52	144	23	.07	175	54
.33	18	54	.88	50	25	.43	81	56	.98	113	27	.53	144	57	.08	176	28
.34	19	29	.89	51	0	.44	82	30	.99	114	1	.54	145	32	.09	177	3
0.35	20	31	0.90	51	34	1.45	83	5	2.00	114	35	2.55	146	6	3.10	177	37
.36	20	38	.91	52	8	.46	83	39	.01	115	10	.56	146	41	.11	178	11
.37	21	12	.92	52	43	.47	84	13	.02	115	44	.57	147	15	.12	178	46
.38	21	46	.93	53	17	.48	84	48	.03	116	19	.58	147	49	.13	179	20
.39	22	21	.94	53	51	.49	85	22	.04	116	53	.59	148	24	.14	179	55
0.40	22	55	0.95	54	26	1.50	85	57	2.05	117	27	2.60	148	58	3.15	180	29
.41	23	29	.96	55	0	.51	86	31	.06	118	2	.61	149	32	3.1416	180	π
.42	24	4	.97	55	35	.52	87	5	.07	118	36	.62	150	7	6.2832	360	2π
.43	24	38	.98	56	9	.53	87	40	.08	119	11	.63	150	41	9.4248	540	3π
.44	25	13	.99	56	43	.54	88	14	.09	119	45	.64	151	16	12.5664	720	4π
0.45	25	47	1.00	57	18	1.55	88	49	2.10	120	19	2.65	151	50	15.7080	900	5π
.46	26	21	.01	57	52	.56	89	23	.11	120	54	.66	152	24	18.8496	1080	6π

Table 14. Natural sines and cosines (see Art. 31)

Angle, degrees	Sine								Prop. parts				
	0′	10′	20′	30′	40′	50′	60′		1′	2′	3′	4′	5′
0	0.0000	0.0029	0.0058	0.0087	0.0116	0.0145	0.0175	89	3	6	9	12	15
1	0.0175	0.0204	0.0233	0.0262	0.0291	0.0320	0.0349	88	3	6	9	12	15
2	0.0349	0.0378	0.0407	0.0436	0.0465	0.0494	0.0523	87	3	6	9	12	15
3	0.0523	0.0552	0.0581	0.0610	0.0640	0.0669	0.0698	86	3	6	9	12	15
4	0.0698	0.0727	0.0756	0.0785	0.0814	0.0843	0.0872	85	3	6	9	12	14
5	0.0872	0.0901	0.0929	0.0958	0.0987	0.1016	0.1045	84	3	6	9	12	14
6	0.1045	0.1074	0.1103	0.1132	0.1161	0.1190	0.1219	83	3	6	9	12	14
7	0.1219	0.1248	0.1276	0.1305	0.1334	0.1363	0.1392	82	3	6	9	12	14
8	0.1392	0.1421	0.1449	0.1478	0.1507	0.1536	0.1564	81	3	6	9	12	14
9	0.1564	0.1593	0.1622	0.1650	0.1679	0.1708	0.1736	80	3	6	9	11	14
10	0.1736	0.1765	0.1794	0.1822	0.1851	0.1880	0.1908	79	3	6	9	11	14
11	0.1908	0.1937	0.1965	0.1994	0.2022	0.2051	0.2079	78	3	6	9	11	14
12	0.2079	0.2108	0.2136	0.2164	0.2193	0.2221	0.2250	77	3	6	9	11	14
13	0.2250	0.2278	0.2306	0.2334	0.2363	0.2391	0.2419	76	3	6	8	11	14
14	0.2419	0.2447	0.2476	0.2504	0.2532	0.2560	0.2588	75	3	6	8	11	14
15	0.2588	0.2616	0.2644	0.2672	0.2700	0.2728	0.2756	74	3	6	8	11	14
16	0.2756	0.2784	0.2812	0.2840	0.2868	0.2896	0.2924	73	3	6	8	11	14
17	0.2924	0.2952	0.2979	0.3007	0.3035	0.3062	0.3090	72	3	6	8	11	14
18	0.3090	0.3118	0.3145	0.3173	0.3201	0.3228	0.3256	71	3	6	8	11	14
19	0.3256	0.3283	0.3311	0.3338	0.3365	0.3393	0.3420	70	3	5	8	11	14
20	0.3420	0.3448	0.3475	0.3502	0.3529	0.3557	0.3584	69	3	5	8	11	14
21	0.3584	0.3611	0.3638	0.3665	0.3692	0.3719	0.3746	68	3	5	8	11	14
22	0.3746	0.3773	0.3800	0.3827	0.3854	0.3881	0.3907	67	3	5	8	11	13
23	0.3907	0.3934	0.3961	0.3987	0.4014	0.4041	0.4067	66	3	5	8	11	13
24	0.4067	0.4094	0.4120	0.4147	0.4173	0.4200	0.4226	65	3	5	8	11	13
25	0.4226	0.4253	0.4279	0.4305	0.4331	0.4358	0.4384	64	3	5	8	11	13
26	0.4384	0.4410	0.4436	0.4462	0.4488	0.4514	0.4540	63	3	5	8	10	13
27	0.4540	0.4566	0.4592	0.4617	0.4643	0.4669	0.4695	62	3	5	8	10	13
28	0.4695	0.4720	0.4746	0.4772	0.4797	0.4823	0.4848	61	3	5	8	10	13
29	0.4848	0.4874	0.4899	0.4924	0.4950	0.4975	0.5000	60	3	5	8	10	13
30	0.5000	0.5025	0.5050	0.5075	0.5100	0.5125	0.5150	59	3	5	8	10	13
31	0.5150	0.5175	0.5200	0.5225	0.5250	0.5275	0.5299	58	2	5	7	10	12
32	0.5299	0.5324	0.5348	0.5373	0.5398	0.5422	0.5446	57	2	5	7	10	12
33	0.5446	0.5471	0.5495	0.5519	0.5544	0.5568	0.5592	56	2	5	7	10	12
34	0.5592	0.5616	0.5640	0.5664	0.5688	0.5712	0.5736	55	2	5	7	10	12
35	0.5736	0.5760	0.5783	0.5807	0.5831	0.5854	0.5878	54	2	5	7	9	12
36	0.5878	0.5901	0.5925	0.5948	0.5972	0.5995	0.6018	53	2	5	7	9	12
37	0.6018	0.6041	0.6065	0.6088	0.6111	0.6134	0.6157	52	2	5	7	9	12
38	0.6157	0.6180	0.6202	0.6225	0.6248	0.6271	0.6293	51	2	5	7	9	11
39	0.6293	0.6316	0.6338	0.6361	0.6383	0.6406	0.6428	50	2	4	7	9	11
40	0.6428	0.6450	0.6472	0.6494	0.6517	0.6539	0.6561	49	2	4	7	9	11
41	0.6561	0.6583	0.6604	0.6626	0.6648	0.6670	0.6691	48	2	4	7	9	11
42	0.6691	0.6713	0.6734	0.6756	0.6777	0.6799	0.6820	47	2	4	6	9	11
43	0.6820	0.6841	0.6862	0.6884	0.6905	0.6926	0.6947	46	2	4	6	8	11
44	0.6947	0.6967	0.6988	0.7009	0.7030	0.7050	0.7071	45	2	4	6	8	10
	60′	50′	40′	30′	20′	10′	0′	Angle, degrees	1′	2′	3′	4′	5′
	Cosine												

Table 14. Natural sines and cosines—*Continued*

Angle, degrees	Sine								Prop. parts				
	0′	10′	20′	30′	40′	50′	60′		1′	2′	3′	4′	5′
45	0.7071	0.7092	0.7112	0.7133	0.7153	0.7173	0.7193	44	2	4	6	8	10
46	0.7193	0.7214	0.7234	0.7254	0.7274	0.7294	0.7314	43	2	4	6	8	10
47	0.7314	0.7333	0.7353	0.7373	0.7392	0.7412	0.7431	42	2	4	6	8	10
48	0.7431	0.7451	0.7470	0.7490	0.7509	0.7528	0.7547	41	2	4	6	8	10
49	0.7547	0.7566	0.7585	0.7604	0.7623	0.7642	0.7660	40	2	4	6	8	9
50	0.7660	0.7679	0.7698	0.7716	0.7735	0.7753	0.7771	39	2	4	6	7	9
51	0.7771	0.7790	0.7808	0.7826	0.7844	0.7862	0.7880	38	2	4	5	7	9
52	0.7880	0.7898	0.7916	0.7934	0.7951	0.7969	0.7986	37	2	4	5	7	9
53	0.7986	0.8004	0.8021	0.8039	0.8056	0.8073	0.8090	36	2	3	5	7	9
54	0.8090	0.8107	0.8124	0.8141	0.8158	0.8175	0.8192	35	2	3	5	7	8
55	0.8192	0.8208	0.8225	0.8241	0.8258	0.8274	0.8290	34	2	3	5	7	8
56	0.8290	0.8307	0.8323	0.8339	0.8355	0.8371	0.8387	33	2	3	5	6	8
57	0.8387	0.8403	0.8418	0.8434	0.8450	0.8465	0.8480	32	2	3	5	6	8
58	0.8480	0.8496	0.8511	0.8526	0.8542	0.8557	0.8572	31	2	3	5	6	8
59	0.8572	0.8587	0.8601	0.8616	0.8631	0.8646	0.8660	30	1	3	4	6	7
60	0.8660	0.8675	0.8689	0.8704	0.8718	0.8732	0.8746	29	1	3	4	6	7
61	0.8746	0.8760	0.8774	0.8788	0.8802	0.8816	0.8829	28	1	3	4	6	7
62	0.8829	0.8843	0.8857	0.8870	0.8884	0.8897	0.8910	27	1	3	4	5	7
63	0.8910	0.8923	0.8936	0.8949	0.8962	0.8975	0.8988	26	1	3	4	5	6
64	0.8988	0.9001	0.9013	0.9026	0.9038	0.9051	0.9063	25	1	3	4	5	6
65	0.9063	0.9075	0.9088	0.9100	0.9112	0.9124	0.9135	24	1	2	4	5	6
66	0.9135	0.9147	0.9159	0.9171	0.9182	0.9194	0.9205	23	1	2	3	5	6
67	0.9205	0.9216	0.9228	0.9239	0.9250	0.9261	0.9272	22	1	2	3	4	6
68	0.9272	0.9283	0.9293	0.9304	0.9315	0.9325	0.9336	21	1	2	3	4	5
69	0.9336	0.9346	0.9356	0.9367	0.9377	0.9387	0.9397	20	1	2	3	4	5
70	0.9397	0.9407	0.9417	0.9426	0.9436	0.9446	0.9455	19	1	2	3	4	5
71	0.9455	0.9465	0.9474	0.9483	0.9492	0.9502	0.9511	18	1	2	3	4	5
72	0.9511	0.9520	0.9528	0.9537	0.9546	0.9555	0.9563	17	1	2	3	3	4
73	0.9563	0.9572	0.9580	0.9588	0.9596	0.9605	0.9613	16	1	2	2	3	4
74	0.9613	0.9621	0.9628	0.9636	0.9644	0.9652	0.9659	15	1	2	2	3	4
75	0.9659	0.9667	0.9674	0.9681	0.9689	0.9696	0.9703	14	1	1	2	3	4
76	0.9703	0.9710	0.9717	0.9724	0.9730	0.9737	0.9744	13	1	1	2	3	3
77	0.9744	0.9750	0.9757	0.9763	0.9769	0.9775	0.9781	12	1	1	2	3	3
78	0.9781	0.9787	0.9793	0.9799	0.9805	0.9811	0.9816	11	1	1	2	2	3
79	0.9816	0.9822	0.9827	0.9833	0.9838	0.9843	0.9848	10	1	1	2	2	3
80	0.9848	0.9853	0.9858	0.9863	0.9868	0.9872	0.9877	9	0	1	1	2	2
81	0.9877	0.9881	0.9886	0.9890	0.9894	0.9899	0.9903	8	0	1	1	2	2
82	0.9903	0.9907	0.9911	0.9914	0.9918	0.9922	0.9925	7	0	1	1	2	2
83	0.9925	0.9929	0.9932	0.9936	0.9939	0.9942	0.9945	6	0	1	1	1	2
84	0.9945	0.9948	0.9951	0.9954	0.9957	0.9959	0.9962	5	0	1	1	1	1
85	0.9962	0.9964	0.9967	0.9969	0.9971	0.9974	0.9976	4	0	0	1	1	1
86	0.9976	0.9978	0.9980	0.9981	0.9983	0.9985	0.9986	3	0	0	1	1	1
87	0.9986	0.9988	0.9989	0.9990	0.9992	0.9993	0.9994	2	0	0	0	1	1
88	0.9994	0.9995	0.9996	0.9997	0.9997	0.9998	0.9998	1	0	0	0	0	0
89	0.9998	0.9999	0.9999	1.0000	1.0000	1.0000	1.0000	0	0	0	0	0	0
	60′	50′	40′	30′	20′	10′	0′	Angle, degrees	1′	2′	3′	4′	5′
	Cosine												

Table 15. Natural tangents and cotangents (see Art. 31)

Angle, degrees	Tangent 0′	10′	20′	30′	40′	50′	60′		Prop. parts 1′	2′	3′	4′	5′
0	0.0000	0.0029	0.0058	0.0087	0.0116	0.0145	0.0175	89	3	6	9	12	15
1	0.0175	0.0204	0.0233	0.0262	0.0291	0.0320	0.0349	88	3	6	9	12	15
2	0.0349	0.0378	0.0407	0.0437	0.0466	0.0495	0.0524	87	3	6	9	12	15
3	0.0524	0.0553	0.0582	0.0612	0.0641	0.0670	0.0699	86	3	6	9	12	15
4	0.0699	0.0729	0.0758	0.0787	0.0816	0.0846	0.0875	85	3	6	9	12	15
5	0.0875	0.0904	0.0934	0.0963	0.0992	0.1022	0.1051	84	3	6	9	12	15
6	0.1051	0.1080	0.1110	0.1139	0.1169	0.1198	0.1228	83	3	6	9	12	15
7	0.1228	0.1257	0.1287	0.1317	0.1346	0.1376	0.1405	82	3	6	9	12	15
8	0.1405	0.1435	0.1465	0.1495	0.1524	0.1554	0.1584	81	3	6	9	12	15
9	0.1584	0.1614	0.1644	0.1673	0.1703	0.1733	0.1763	80	3	6	9	12	15
10	0.1763	0.1793	0.1823	0.1853	0.1883	0.1914	0.1944	79	3	6	9	12	15
11	0.1944	0.1974	0.2004	0.2035	0.2065	0.2095	0.2126	78	3	6	9	12	15
12	0.2126	0.2156	0.2186	0.2217	0.2247	0.2278	0.2309	77	3	6	9	12	15
13	0.2309	0.2339	0.2370	0.2401	0.2432	0.2462	0.2493	76	3	6	9	12	15
14	0.2493	0.2524	0.2555	0.2586	0.2617	0.2648	0.2679	75	3	6	9	12	16
15	0.2679	0.2711	0.2742	0.2773	0.2805	0.2836	0.2867	74	3	6	9	13	16
16	0.2867	0.2899	0.2931	0.2962	0.2994	0.3026	0.3057	73	3	6	9	13	16
17	0.3057	0.3089	0.3121	0.3153	0.3185	0.3217	0.3249	72	3	6	10	13	16
18	0.3249	0.3281	0.3314	0.3346	0.3378	0.3411	0.3443	71	3	6	10	13	16
19	0.3443	0.3476	0.3508	0.3541	0.3574	0.3607	0.3640	70	3	7	10	13	16
20	0.3640	0.3673	0.3706	0.3739	0.3772	0.3805	0.3839	69	3	7	10	13	17
21	0.3839	0.3872	0.3906	0.3939	0.3973	0.4006	0.4040	68	3	7	10	13	17
22	0.4040	0.4074	0.4108	0.4142	0.4176	0.4210	0.4245	67	3	7	10	14	17
23	0.4245	0.4279	0.4314	0.4348	0.4383	0.4417	0.4452	66	3	7	10	14	17
24	0.4452	0.4487	0.4522	0.4557	0.4592	0.4628	0.4663	65	4	7	11	14	18
25	0.4663	0.4699	0.4734	0.4770	0.4806	0.4841	0.4877	64	4	7	11	14	18
26	0.4877	0.4913	0.4950	0.4986	0.5022	0.5059	0.5095	63	4	7	11	15	18
27	0.5095	0.5132	0.5169	0.5206	0.5243	0.5280	0.5317	62	4	7	11	15	18
28	0.5317	0.5354	0.5392	0.5430	0.5467	0.5505	0.5543	61	4	8	11	15	19
29	0.5543	0.5581	0.5619	0.5658	0.5696	0.5735	0.5774	60	4	8	12	15	19
30	0.5774	0.5812	0.5851	0.5890	0.5930	0.5969	0.6009	59	4	8	12	16	19
31	0.6009	0.6048	0.6088	0.6128	0.6168	0.6208	0.6249	58	4	8	12	16	20
32	0.6249	0.6289	0.6330	0.6371	0.6412	0.6453	0.6494	57	4	8	12	16	20
33	0.6494	0.6536	0.6577	0.6619	0.6661	0.6703	0.6745	56	4	8	13	17	21
34	0.6745	0.6787	0.6830	0.6873	0.6916	0.6959	0.7002	55	4	9	13	17	21
35	0.7002	0.7046	0.7089	0.7133	0.7177	0.7221	0.7265	54	4	9	13	18	22
36	0.7265	0.7310	0.7355	0.7400	0.7445	0.7490	0.7536	53	5	9	14	18	23
37	0.7536	0.7581	0.7627	0.7673	0.7720	0.7766	0.7813	52	5	9	14	18	23
38	0.7813	0.7860	0.7907	0.7954	0.8002	0.8050	0.8098	51	5	10	14	19	24
39	0.8098	0.8146	0.8195	0.8243	0.8292	0.8342	0.8391	50	5	10	15	20	24
40	0.8391	0.8441	0.8491	0.8541	0.8591	0.8642	0.8693	49	5	10	15	20	25
41	0.8693	0.8744	0.8796	0.8847	0.8899	0.8952	0.9004	48	5	10	16	21	26
42	0.9004	0.9057	0.9110	0.9163	0.9217	0.9271	0.9325	47	5	11	16	21	27
43	0.9325	0.9380	0.9435	0.9490	0.9545	0.9601	0.9657	46	6	11	17	22	28
44	0.9657	0.9713	0.9770	0.9827	0.9884	0.9942	1.0000	45	6	11	17	23	29
	60′	50′	40′	30′	20′	10′	0′	Angle, degrees	1′	2′	3′	4′	5′
	Cotangent								Prop. parts				

Table 15. Natural tangents and cotangents—*Continued*

Angle, degrees	Tangent 0′	10′	20′	30′	40′	50′	60′		Prop. parts 1′	2′	3′	4′	5′
45	1.000	1.006	1.012	1.018	1.024	1.030	1.036	44	1	1	2	2	3
46	1.036	1.042	1.048	1.054	1.060	1.066	1.072	43	1	1	2	2	3
47	1.072	1.079	1.085	1.091	1.098	1.104	1.111	42	1	1	2	3	3
48	1.111	1.117	1.124	1.130	1.137	1.144	1.150	41	1	1	2	3	3
49	1.150	1.157	1.164	1.171	1.178	1.185	1.192	40	1	1	2	3	3
50	1.192	1.199	1.206	1.213	1.220	1.228	1.235	39	1	1	2	3	4
51	1.235	1.242	1.250	1.257	1.265	1.272	1.280	38	1	1	2	3	4
52	1.280	1.288	1.295	1.303	1.311	1.319	1.327	37	1	2	2	3	4
53	1.327	1.335	1.343	1.351	1.360	1.368	1.376	36	1	2	2	3	4
54	1.376	1.385	1.393	1.402	1.411	1.419	1.428	35	1	2	3	3	4
55	1.428	1.437	1.446	1.455	1.464	1.473	1.483	34	1	2	3	4	5
56	1.483	1.492	1.501	1.511	1.520	1.530	1.540	33	1	2	3	4	5
57	1.540	1.550	1.560	1.570	1.580	1.590	1.600	32	1	2	3	4	5
58	1.600	1.611	1.621	1.632	1.643	1.653	1.664	31	1	2	3	4	5
59	1.664	1.675	1.686	1.698	1.709	1.720	1.732	30	1	2	3	5	6
60	1.732	1.744	1.756	1.767	1.780	1.792	1.804	29	1	2	4	5	6
61	1.804	1.816	1.829	1.842	1.855	1.868	1.881	28	1	3	4	5	6
62	1.881	1.894	1.907	1.921	1.935	1.949	1.963	27	1	3	4	5	7
63	1.963	1.977	1.991	2.006	2.020	2.035	2.050	26	1	3	4	6	7
64	2.050	2.066	2.081	2.097	2.112	2.128	2.145	25	2	3	5	6	8
65	2.145	2.161	2.177	2.194	2.211	2.229	2.246	24	2	3	5	7	8
66	2.246	2.264	2.282	2.300	2.318	2.337	2.356	23	2	4	5	7	9
67	2.356	2.375	2.394	2.414	2.434	2.455	2.475	22	2	4	6	8	10
68	2.475	2.496	2.517	2.539	2.560	2.583	2.605	21	2	4	6	9	11
69	2.605	2.628	2.651	2.675	2.699	2.723	2.747	20	2	5	7	10	12
70	2.747	2.773	2.798	2.824	2.850	2.877	2.904	19	3	5	8	11	13
71	2.904	2.932	2.960	2.989	3.018	3.047	3.078	18	3	6	9	12	14
72	3.078	3.108	3.140	3.172	3.204	3.237	3.271	17	3	6	10	13	16
73	3.271	3.305	3.340	3.376	3.412	3.450	3.487	16	4	7	11	14	18
74	3.487	3.526	3.566	3.606	3.647	3.689	3.732	15					
75	3.732	3.776	3.821	3.867	3.914	3.962	4.011	14	Interpolate.				
76	4.011	4.061	4.113	4.165	4.219	4.275	4.331	13					
77	4.331	4.390	4.449	4.511	4.574	4.638	4.705	12					
78	4.705	4.773	4.843	4.915	4.989	5.066	5.145	11					
79	5.145	5.226	5.309	5.396	5.485	5.576	5.671	10					
80	5.671	5.769	5.871	5.976	6.084	6.197	6.314	9					
81	6.314	6.435	6.561	6.691	6.827	6.968	7.115	8					
82	7.115	7.269	7.429	7.596	7.770	7.953	8.144	7					
83	8.144	8.345	8.556	8.777	9.010	9.255	9.514	6					
84	9.514	9.788	10.08	10.39	10.71	11.06	11.43	5					
85	11.43	11.83	12.25	12.71	13.20	13.73	14.30	4	Do not interpolate here.				
86	14.30	14.92	15.60	16.35	17.17	18.07	19.08	3					
87	19.08	20.21	21.47	22.90	24.54	26.43	28.64	2					
88	28.64	31.24	34.37	38.19	42.96	49.10	57.29	1					
89	57.29	68.75	85.94	114.6	171.9	343.8	∞	0					
	60′	50′	40′	30′	20′	10′	0′	Angle, degrees	1′	2′	3′	4′	5′
	Cotangent								Prop. parts				

Table 16. Mantissas of common logarithms, base 10 (see Art. 4)

N	0	1	2	3	4	5	6	7	8	9
100	.0000	.0004	.0009	.0013	.0017	.0022	.0026	.0030	.0035	.0039
101	.0043	.0048	.0052	.0056	.0060	.0065	.0069	.0073	.0077	.0082
102	.0086	.0090	.0095	.0099	.0103	.0107	.0111	.0116	.0120	.0124
103	.0128	.0133	.0137	.0141	.0145	.0149	.0154	.0158	.0162	.0166
104	.0170	.0175	.0179	.0183	.0187	.0191	.0195	.0199	.0204	.0208
105	.0212	.0216	.0220	.0224	.0228	.0233	.0237	.0241	.0245	.0249
106	.0253	.0257	.0261	.0265	.0269	.0273	.0278	.0282	.0286	.0290
107	.0294	.0298	.0302	.0306	.0310	.0314	.0318	.0322	.0326	.0330
108	.0334	.0338	.0342	.0346	.0350	.0354	.0358	.0362	.0366	.0370
109	.0374	.0378	.0382	.0386	.0390	.0394	.0398	.0402	.0406	.0410
110	.0414	.0418	.0422	.0426	.0430	.0434	.0438	.0441	.0445	.0449
111	.0453	.0457	.0461	.0465	.0469	.0473	.0477	.0481	.0484	.0488
112	.0492	.0496	.0500	.0504	.0508	.0512	.0515	.0519	.0523	.0527
113	.0531	.0535	.0538	.0542	.0546	.0550	.0554	.0558	.0561	.0565
114	.0569	.0573	.0577	.0580	.0584	.0588	.0592	.0596	.0599	.0603
115	.0607	.0611	.0615	.0618	.0622	.0626	.0630	.0633	.0637	.0641
116	.0645	.0648	.0652	.0656	.0660	.0663	.0667	.0671	.0674	.0678
117	.0682	.0686	.0689	.0693	.0697	.0700	.0704	.0708	.0711	.0715
118	.0719	.0722	.0726	.0730	.0734	.0737	.0741	.0745	.0748	.0752
119	.0755	.0759	.0763	.0766	.0770	.0774	.0777	.0781	.0785	.0788
120	.0792	.0795	.0799	.0803	.0806	.0810	.0813	.0817	.0821	.0824
121	.0828	.0831	.0835	.0839	.0842	.0846	.0849	.0853	.0856	.0860
122	.0864	.0867	.0871	.0874	.0878	.0881	.0885	.0888	.0892	.0896
123	.0899	.0903	.0906	.0910	.0913	.0917	.0920	.0924	.0927	.0931
124	.0934	.0938	.0941	.0945	.0948	.0952	.0955	.0959	.0962	.0966
125	.0969	.0973	.0976	.0980	.0983	.0986	.0990	.0993	.0997	.1000
126	.1004	.1007	.1011	.1014	.1017	.1021	.1024	.1028	.1031	.1035
127	.1038	.1041	.1045	.1048	.1052	.1055	.1059	.1062	.1065	.1069
128	.1072	.1075	.1079	.1082	.1086	.1089	.1092	.1096	.1099	.1103
129	.1106	.1109	.1113	.1116	.1119	.1123	.1126	.1129	.1133	.1136
130	.1139	.1143	.1146	.1149	.1153	.1156	.1159	.1163	.1166	.1169
131	.1173	.1176	.1179	.1183	.1186	.1189	.1193	.1196	.1199	.1202
132	.1206	.1209	.1212	.1216	.1219	.1222	.1225	.1229	.1232	.1235
133	.1239	.1242	.1245	.1248	.1252	.1255	.1258	.1261	.1265	.1268
134	.1271	.1274	.1278	.1281	.1284	.1287	.1290	.1294	.1297	.1300
135	.1303	.1307	.1310	.1313	.1316	.1319	.1323	.1326	.1329	.1332
136	.1335	.1339	.1342	.1345	.1348	.1351	.1355	.1358	.1361	.1364
137	.1367	.1370	.1374	.1377	.1380	.1383	.1386	.1389	.1392	.1396
138	.1399	.1402	.1405	.1408	.1411	.1414	.1418	.1421	.1424	.1427
139	.1430	.1433	.1436	.1440	.1443	.1446	.1449	.1452	.1455	.1458
140	.1461	.1464	.1467	.1471	.1474	.1477	.1480	.1483	.1486	.1489
141	.1492	.1495	.1498	.1501	.1504	.1508	.1511	.1514	.1517	.1520
142	.1523	.1526	.1529	.1532	.1535	.1538	.1541	.1544	.1547	.1550
143	.1553	.1556	.1559	.1562	.1565	.1569	.1572	.1575	.1578	.1581
144	.1584	.1587	.1590	.1593	.1596	.1599	.1602	.1605	.1608	.1611
145	.1614	.1617	.1620	.1623	.1626	.1629	.1632	.1635	.1638	.1641
146	.1644	.1647	.1649	.1652	.1655	.1658	.1661	.1664	.1667	.1670
147	.1673	.1676	.1679	.1682	.1685	.1688	.1691	.1694	.1697	.1700
148	.1703	.1706	.1708	.1711	.1714	.1717	.1720	.1723	.1726	.1729
149	.1732	.1735	.1738	.1741	.1744	.1746	.1749	.1752	.1755	.1758
150	.1761	.1764	.1767	.1770	.1772	.1775	.1778	.1781	.1784	.1787
N	0	1	2	3	4	5	6	7	8	9

Table 16. Mantissas of common logarithms, base 10—*Continued*

N	0	1	2	3	4	5	6	7	8	9
150	.1761	.1764	.1767	.1770	.1772	.1775	.1778	.1781	.1784	.1787
151	.1790	.1793	.1796	.1798	.1801	.1804	.1807	.1810	.1813	.1816
152	.1818	.1821	.1824	.1827	.1830	.1833	.1836	.1838	.1841	.1844
153	.1847	.1850	.1853	.1855	.1858	.1861	.1864	.1867	.1870	.1872
154	.1875	.1878	.1881	.1884	.1886	.1889	.1892	.1895	.1898	.1901
155	.1903	.1906	.1909	.1912	.1915	.1917	.1920	.1923	.1926	.1928
156	.1931	.1934	.1937	.1940	.1942	.1945	.1948	.1951	.1953	.1956
157	.1959	.1962	.1965	.1967	.1970	.1973	.1976	.1978	.1981	.1984
158	.1987	.1989	.1992	.1995	.1998	.2000	.2003	.2006	.2009	.2011
159	.2014	.2017	.2019	.2022	.2025	.2028	.2030	.2033	.2036	.2038
160	.2041	.2044	.2047	.2049	.2052	.2055	.2057	.2060	.2063	.2066
161	.2068	.2071	.2074	.2076	.2079	.2082	.2084	.2087	.2090	.2092
162	.2095	.2098	.2101	.2103	.2106	.2109	.2111	.2114	.2117	.2119
163	.2122	.2125	.2127	.2130	.2133	.2135	.2138	.2140	.2143	.2146
164	.2148	.2151	.2154	.2156	.2159	.2162	.2164	.2167	.2170	.2172
165	.2175	.2177	.2180	.2183	.2185	.2188	.2191	.2193	.2196	.2198
166	.2201	.2204	.2206	.2209	.2212	.2214	.2217	.2219	.2222	.2225
167	.2227	.2230	.2232	.2235	.2238	.2240	.2243	.2245	.2248	.2251
168	.2253	.2256	.2258	.2261	.2263	.2266	.2269	.2271	.2274	.2276
169	.2279	.2281	.2284	.2287	.2289	.2292	.2294	.2297	.2299	.2302
170	.2304	.2307	.2310	.2312	.2315	.2317	.2320	.2322	.2325	.2327
171	.2330	.2333	.2335	.2338	.2340	.2343	.2345	.2348	.2350	.2353
172	.2355	.2358	.2360	.2363	.2365	.2368	.2370	.2373	.2375	.2378
173	.2380	.2383	.2385	.2388	.2390	.2393	.2395	.2398	.2400	.2403
174	.2405	.2408	.2410	.2413	.2415	.2418	.2420	.2423	.2425	.2428
175	.2430	.2433	.2435	.2438	.2440	.2443	.2445	.2448	.2450	.2453
176	.2455	.2458	.2460	.2463	.2465	.2467	.2470	.2472	.2475	.2477
177	.2480	.2482	.2485	.2487	.2490	.2492	.2494	.2497	.2499	.2502
178	.2504	.2507	.2509	.2512	.2514	.2516	.2519	.2521	.2524	.2526
179	.2529	.2531	.2533	.2536	.2538	.2541	.2543	.2545	.2548	.2550
180	.2553	.2555	.2558	.2560	.2562	.2565	.2567	.2570	.2572	.2574
181	.2577	.2579	.2582	.2584	.2586	.2589	.2591	.2594	.2596	.2598
182	.2601	.2603	.2605	.2608	.2610	.2613	.2615	.2617	.2620	.2622
183	.2625	.2627	.2629	.2632	.2634	.2636	.2639	.2641	.2643	.2646
184	.2648	.2651	.2653	.2655	.2658	.2660	.2662	.2665	.2667	.2669
185	.2672	.2674	.2676	.2679	.2681	.2683	.2686	.2688	.2690	.2693
186	.2695	.2697	.2700	.2702	.2704	.2707	.2709	.2711	.2714	.2716
187	.2718	.2721	.2723	.2725	.2728	.2730	.2732	.2735	.2737	.2739
188	.2742	.2744	.2746	.2749	.2751	.2753	.2755	.2758	.2760	.2762
189	.2765	.2767	.2769	.2772	.2774	.2776	.2778	.2781	.2783	.2785
190	.2788	.2790	.2792	.2794	.2797	.2799	.2801	.2804	.2806	.2808
191	.2810	.2813	.2815	.2817	.2819	.2822	.2824	.2826	.2828	.2831
192	.2833	.2835	.2838	.2840	.2842	.2844	.2847	.2849	.2851	.2853
193	.2856	.2858	.2860	.2862	.2865	.2867	.2869	.2871	.2874	.2876
194	.2878	.2880	.2882	.2885	.2887	.2889	.2891	.2894	.2896	.2898
195	.2900	.2903	.2905	.2907	.2909	.2911	.2914	.2916	.2918	.2920
196	.2923	.2925	.2927	.2929	.2931	.2934	.2936	.2938	.2940	.2942
197	.2945	.2947	.2949	.2951	.2953	.2956	.2958	.2960	.2962	.2964
198	.2967	.2969	.2971	.2973	.2975	.2978	.2980	.2982	.2984	.2986
199	.2989	.2991	.2993	.2995	.2997	.2999	.3002	.3004	.3006	.3008
200	.3010	.3012	.3015	.3017	.3019	.3021	.3023	.3025	.3028	.3030
N	0	1	2	3	4	5	6	7	8	9

Table 16. Mantissas of common logarithms, base 10—*Continued*

No.	0	1	2	3	4	5	6	7	8	9	Prop. parts 1	2	3	4	5	6	7	8	9
20	.3010	.3032	.3054	.3075	.3096	.3118	.3139	.3160	.3181	.3201	2	4	6	8	11	13	15	17	19
21	.3222	.3243	.3263	.3284	.3304	.3324	.3345	.3365	.3385	.3404	2	4	6	8	10	12	14	16	18
22	.3424	.3444	.3464	.3483	.3502	.3522	.3541	.3560	.3579	.3598	2	4	6	8	10	12	14	15	17
23	.3617	.3636	.3655	.3674	.3692	.3711	.3729	.3747	.3766	.3784	2	4	6	7	9	11	13	15	17
24	.3802	.3820	.3838	.3856	.3874	.3892	.3909	.3927	.3945	.3962	2	4	5	7	9	11	12	14	16
25	.3979	.3997	.4014	.4031	.4048	.4065	.4082	.4099	.4116	.4133	2	3	5	7	9	10	12	14	15
26	.4150	.4166	.4183	.4200	.4216	.4232	.4249	.4265	.4281	.4298	2	3	5	7	8	10	11	13	15
27	.4314	.4330	.4346	.4362	.4378	.4393	.4409	.4425	.4440	.4456	2	3	5	6	8	9	11	13	14
28	.4472	.4487	.4502	.4518	.4533	.4548	.4564	.4579	.4594	.4609	2	3	5	6	8	9	11	12	14
29	.4624	.4639	.4654	.4669	.4683	.4698	.4713	.4728	.4742	.4757	1	3	4	6	7	9	10	12	13
30	.4771	.4786	.4800	.4814	.4829	.4843	.4857	.4871	.4886	.4900	1	3	4	6	7	9	10	11	13
31	.4914	.4928	.4942	.4955	.4969	.4983	.4997	.5011	.5024	.5038	1	3	4	6	7	8	10	11	12
32	.5051	.5065	.5079	.5092	.5105	.5119	.5132	.5145	.5159	.5172	1	3	4	5	7	8	9	11	12
33	.5185	.5198	.5211	.5224	.5237	.5250	.5263	.5276	.5289	.5302	1	3	4	5	6	8	9	10	12
34	.5315	.5328	.5340	.5353	.5366	.5378	.5391	.5403	.5416	.5428	1	3	4	5	6	8	9	10	11
35	.5441	.5453	.5465	.5478	.5490	.5502	.5514	.5527	.5539	.5551	1	2	4	5	6	7	9	10	11
36	.5563	.5575	.5587	.5599	.5611	.5623	.5635	.5647	.5658	.5670	1	2	4	5	6	7	8	10	11
37	.5682	.5694	.5705	.5717	.5729	.5740	.5752	.5763	.5775	.5786	1	2	3	5	6	7	8	9	10
38	.5798	.5809	.5821	.5832	.5843	.5855	.5866	.5877	.5888	.5899	1	2	3	5	6	7	8	9	10
39	.5911	.5922	.5933	.5944	.5955	.5966	.5977	.5988	.5999	.6010	1	2	3	4	5	7	8	9	10
40	.6021	.6031	.6042	.6053	.6064	.6075	.6085	.6096	.6107	.6117	1	2	3	4	5	6	8	9	10
41	.6128	.6138	.6149	.6160	.6170	.6180	.6191	.6201	.6212	.6222	1	2	3	4	5	6	7	8	9
42	.6232	.6243	.6253	.6263	.6274	.6284	.6294	.6304	.6314	.6325	1	2	3	4	5	6	7	8	9
43	.6335	.6345	.6355	.6365	.6375	.6385	.6395	.6405	.6415	.6425	1	2	3	4	5	6	7	8	9
44	.6435	.6444	.6454	.6464	.6474	.6484	.6493	.6503	.6513	.6522	1	2	3	4	5	6	7	8	9
45	.6532	.6542	.6551	.6561	.6571	.6580	.6590	.6599	.6609	.6618	1	2	3	4	5	6	7	8	9
46	.6628	.6637	.6646	.6656	.6665	.6675	.6684	.6693	.6702	.6712	1	2	3	4	5	6	7	7	8
47	.6721	.6730	.6739	.6749	.6758	.6767	.6776	.6785	.6794	.6803	1	2	3	4	5	5	6	7	8
48	.6812	.6821	.6830	.6839	.6848	.6857	.6866	.6875	.6884	.6893	1	2	3	4	4	5	6	7	8
49	.6902	.6911	.6920	.6928	.6937	.6946	.6955	.6964	.6972	.6981	1	2	3	4	4	5	6	7	8
50	.6990	.6998	.7007	.7016	.7024	.7033	.7042	.7050	.7059	.7067	1	2	3	3	4	5	6	7	8
51	.7076	.7084	.7093	.7101	.7110	.7118	.7126	.7135	.7143	.7152	1	2	3	3	4	5	6	7	8
52	.7160	.7168	.7177	.7185	.7193	.7202	.7210	.7218	.7226	.7235	1	2	2	3	4	5	6	7	7
53	.7243	.7251	.7259	.7267	.7275	.7284	.7292	.7300	.7308	.7316	1	2	2	3	4	5	6	6	7
54	.7324	.7332	.7340	.7348	.7356	.7364	.7372	.7380	.7388	.7396	1	2	2	3	4	5	6	6	7
55	.7404	.7412	.7419	.7427	.7435	.7443	.7451	.7459	.7466	.7474	1	2	2	3	4	5	5	6	7
56	.7482	.7490	.7497	.7505	.7513	.7520	.7528	.7536	.7543	.7551	1	2	2	3	4	5	5	6	7
57	.7559	.7566	.7574	.7582	.7589	.7597	.7604	.7612	.7619	.7627	1	2	2	3	4	5	5	6	7
58	.7634	.7642	.7649	.7657	.7664	.7672	.7679	.7686	.7694	.7701	1	1	2	3	4	4	5	6	7
59	.7709	.7716	.7723	.7731	.7738	.7745	.7752	.7760	.7767	.7774	1	1	2	3	4	4	5	6	7
60	.7782	.7789	.7796	.7803	.7810	.7818	.7825	.7832	.7839	.7846	1	1	2	3	4	4	5	6	6
61	.7853	.7860	.7868	.7875	.7882	.7889	.7896	.7903	.7910	.7917	1	1	2	3	4	4	5	6	6
62	.7924	.7931	.7938	.7945	.7952	.7959	.7966	.7973	.7980	.7987	1	1	2	3	3	4	5	6	6
63	.7993	.8000	.8007	.8014	.8021	.8028	.8035	.8041	.8048	.8055	1	1	2	3	3	4	5	5	6
64	.8062	.8069	.8075	.8082	.8089	.8096	.8102	.8109	.8116	.8122	1	1	2	3	3	4	5	5	6
65	.8129	.8136	.8142	.8149	.8156	.8162	.8169	.8176	.8182	.8189	1	1	2	3	3	4	5	5	6
66	.8195	.8202	.8209	.8215	.8222	.8228	.8235	.8241	.8248	.8254	1	1	2	3	3	4	5	5	6
67	.8261	.8267	.8274	.8280	.8287	.8293	.8299	.8306	.8312	.8319	1	1	2	3	3	4	5	5	6
68	.8325	.8331	.8338	.8344	.8351	.8357	.8363	.8370	.8376	.8382	1	1	2	3	3	4	4	5	6
69	.8388	.8395	.8401	.8407	.8414	.8420	.8426	.8432	.8439	.8445	1	1	2	3	3	4	4	5	6
70	.8451	.8457	.8463	.8470	.8476	.8482	.8488	.8494	.8500	.8506	1	1	2	2	3	4	4	5	6
No.	0	1	2	3	4	5	6	7	8	9									

Table 16. Mantissas of common logarithms, base 10—*Continued*

No.	0	1	2	3	4	5	6	7	8	9	Prop. parts 1	2	3	4	5	6	7	8	9
70	.8451	.8457	.8463	.8470	.8476	.8482	.8488	.8494	.8500	.8506	1	1	2	2	3	4	4	5	6
71	.8513	.8519	.8525	.8531	.8537	.8543	.8549	.8555	.8561	.8567	1	1	2	2	3	4	4	5	5
72	.8573	.8579	.8585	.8591	.8597	.8603	.8609	.8615	.8621	.8627	1	1	2	2	3	4	4	5	5
73	.8633	.8639	.8645	.8651	.8657	.8663	.8669	.8675	.8681	.8686	1	1	2	2	3	4	4	5	5
74	.8692	.8698	.8704	.8710	.8716	.8722	.8727	.8733	.8739	.8745	1	1	2	2	3	3	4	5	5
75	.8751	.8756	.8762	.8768	.8774	.8779	.8785	.8791	.8797	.8802	1	1	2	2	3	3	4	5	5
76	.8808	.8814	.8820	.8825	.8831	.8837	.8842	.8848	.8854	.8859	1	1	2	2	3	3	4	5	5
77	.8865	.8871	.8876	.8882	.8887	.8893	.8899	.8904	.8910	.8915	1	1	2	2	3	3	4	4	5
78	.8921	.8927	.8932	.8938	.8943	.8949	.8954	.8960	.8965	.8971	1	1	2	2	3	3	4	4	5
79	.8976	.8982	.8987	.8993	.8998	.9004	.9009	.9015	.9020	.9025	1	1	2	2	3	3	4	4	5
80	.9031	.9036	.9042	.9047	.9053	.9058	.9063	.9069	.9074	.9079	1	1	2	2	3	3	4	4	5
81	.9085	.9090	.9096	.9101	.9106	.9112	.9117	.9122	.9128	.9133	1	1	2	2	3	3	4	4	5
82	.9138	.9143	.9149	.9154	.9159	.9165	.9170	.9175	.9180	.9186	1	1	2	2	3	3	4	4	5
83	.9191	.9196	.9201	.9206	.9212	.9217	.9222	.9227	.9232	.9238	1	1	2	2	3	3	4	4	5
84	.9243	.9248	.9253	.9258	.9263	.9269	.9274	.9279	.9284	.9289	1	1	2	2	3	3	4	4	5
85	.9294	.9299	.9304	.9309	.9315	.9320	.9325	.9330	.9335	.9340	1	1	2	2	3	3	4	4	5
86	.9345	.9350	.9355	.9360	.9365	.9370	.9375	.9380	.9385	.9390	1	1	2	2	3	3	4	4	5
87	.9395	.9400	.9405	.9410	.9415	.9420	.9425	.9430	.9435	.9440	0	1	1	2	2	3	3	4	4
88	.9445	.9450	.9455	.9460	.9465	.9469	.9474	.9479	.9484	.9489	0	1	1	2	2	3	3	4	4
89	.9494	.9499	.9504	.9509	.9513	.9518	.9523	.9528	.9533	.9538	0	1	1	2	2	3	3	4	4
90	.9542	.9547	.9552	.9557	.9562	.9566	.9571	.9576	.9581	.9586	0	1	1	2	2	3	3	4	4
91	.9590	.9595	.9600	.9605	.9609	.9614	.9619	.9624	.9628	.9633	0	1	1	2	2	3	3	4	4
92	.9638	.9643	.9647	.9652	.9657	.9661	.9666	.9671	.9675	.9680	0	1	1	2	2	3	3	4	4
93	.9685	.9689	.9694	.9699	.9703	.9708	.9713	.9717	.9722	.9727	0	1	1	2	2	3	3	4	4
94	.9731	.9736	.9741	.9745	.9750	.9754	.9759	.9763	.9768	.9773	0	1	1	2	2	3	3	4	4
95	.9777	.9782	.9786	.9791	.9795	.9800	.9805	.9809	.9814	.9818	0	1	1	2	2	3	3	4	4
96	.9823	.9827	.9832	.9836	.9841	.9845	.9850	.9854	.9859	.9863	0	1	1	2	2	3	3	4	4
97	.9868	.9872	.9877	.9881	.9886	.9890	.9894	.9899	.9903	.9908	0	1	1	2	2	3	3	4	4
98	.9912	.9917	.9921	.9926	.9930	.9934	.9939	.9943	.9948	.9952	0	1	1	2	2	3	3	4	4
99	.9956	.9961	.9965	.9969	.9974	.9978	.9983	.9987	.9991	.9196	0	1	1	2	2	3	3	3	4
100	.0000	.0004	.0009	.0013	.0017	.0022	.0026	.0030	.0035	.0039									
No.	0	1	2	3	4	5	6	7	8	9									

Table 17. Numerical constants

Constant	Value	Constant	Value	Constant	Value
e	2.718282	π	3.141593	$\sqrt{2}$	1.414214
$1/e$	0.367879	$1/\pi$	0.318310	$\sqrt{3}$	1.732051
e^2	7.389056	π^2	9.869604	$\sqrt{5}$	2.236068
$1/e^2$	0.135335	$1/\pi^2$	0.101321	$\sqrt[3]{2}$	1.259921
$\sqrt{e}$	1.648721	$\sqrt{\pi}$	1.772454	$\sqrt[3]{3}$	1.442250
$\sqrt[3]{e}$	1.395612	$1/\sqrt{\pi}$	0.564190	1 radian	57.295780 degrees
$\log_{10} e$	0.434294	π^3	31.00628	1 radian	3437.7468 minutes
$1/\log_{10} e$	2.302585	$1/\pi^3$	0.032252	1 radian	206264.81 seconds
$\log_{10} \pi$	0.497150	$\sqrt[3]{\pi}$	1.464592	1 degree	0.017453 radian
$\log_e \pi$	1.144730	$1/\sqrt[3]{\pi}$	0.682784	1 minute	0.0002909 radian
$\log_e 10$	2.302585	$\pi/4$	0.785398	1 second	0.00000485 radian

Table 18. Logarithmic sines (see Art. 31)

Degrees	0′	10′	20′	30′	40′	50′	60′		Prop. parts				
									1′	2′	3′	4′	5′
0	Inf. Neg.	7.4637	7.7648	7.9408	8.0658	8.1627	8.2419	89	*	See Art. 31			
1	8.2419	8.3088	8.3668	8.4179	8.4637	8.5050	8.5428	88	*				
2	8.5428	8.5776	8.6097	8.6397	8.6677	8.6940	8.7188	87	*				
3	8.7188	8.7423	8.7645	8.7857	8.8059	8.8251	8.8436	86	*				
4	8.8436	8.8613	8.8783	8.8946	8.9104	8.9256	8.9403	85	*				
5	8.9403	8.9545	8.9682	8.9816	8.9945	9.0070	9.0192	84	*				
6	9.0192	9.0311	9.0426	9.0539	9.0648	9.0755	9.0859	83		Interpolate			
7	9.0859	9.0961	9.1060	9.1157	9.1252	9.1345	9.1436	82					
8	9.1436	9.1525	9.1612	9.1697	9.1781	9.1863	9.1943	81					
9	9.1943	9.2022	9.2100	9.2176	9.2251	9.2324	9.2397	80					
10	9.2397	9.2468	9.2538	9.2606	9.2674	9.2740	9.2806	79					
11	9.2806	9.2870	9.2934	9.2997	9.3058	9.3119	9.3179	78					
12	9.3179	9.3238	9.3296	9.3353	9.3410	9.3466	9.3521	77					
13	9.3521	9.3575	9.3629	9.3682	9.3734	9.3786	9.3837	76					
14	9.3837	9.3887	9.3937	9.3986	9.4035	9.4083	9.4130	75					
15	9.4130	9.4177	9.4223	9.4269	9.4314	9.4359	9.4403	74					
16	9.4403	9.4447	9.4491	9.4533	9.4576	9.4618	9.4659	73					
17	9.4659	9.4700	9.4741	9.4781	9.4821	9.4861	9.4900	72	4	8	12	16	20
18	9.4900	9.4939	9.4977	9.5015	9.5052	9.5090	9.5126	71	4	8	11	15	19
19	9.5126	9.5163	9.5199	9.5235	9.5270	9.5306	9.5341	70	4	7	11	14	18
20	9.5341	9.5375	9.5409	9.5443	9.5477	9.5510	9.5543	69	3	7	10	14	17
21	9.5543	9.5576	9.5609	9.5641	9.5673	9.5704	9.5736	68	3	6	10	13	16
22	9.5736	9.5767	9.5798	9.5828	9.5859	9.5889	9.5919	67	3	6	9	12	15
23	9.5919	9.5948	9.5978	9.6007	9.6036	9.6065	9.6093	66	3	6	9	12	15
24	9.6093	9.6121	9.6149	9.6177	9.6205	9.6232	9.6259	65	3	6	8	11	14
25	9.6259	9.6286	9.6313	9.6340	9.6366	9.6392	9.6418	64	3	5	8	11	13
26	9.6418	9.6444	9.6470	9.6495	9.6521	9.6546	9.6570	63	3	5	8	10	13
27	9.6570	9.6595	9.6620	9.6644	9.6668	9.6692	9.6716	62	2	5	7	10	12
28	9.6716	9.6740	9.6763	9.6787	9.6810	9.6833	9.6856	61	2	5	7	9	12
29	9.6856	9.6878	9.6901	9.6923	9.6946	9.6968	9.6990	60	2	4	7	9	11
30	9.6990	9.7012	9.7033	9.7055	9.7076	9.7097	9.7118	59	2	4	6	9	11
31	9.7118	9.7139	9.7160	9.7181	9.7201	9.7222	9.7242	58	2	4	6	8	10
32	9.7242	9.7262	9.7282	9.7302	9.7322	9.7342	9.7361	57	2	4	6	8	10
33	9.7361	9.7380	9.7400	9.7419	9.7438	9.7457	9.7476	56	2	4	6	8	10
34	9.7476	9.7494	9.7513	9.7531	9.7550	9.7568	9.7586	55	2	4	6	7	9
35	9.7586	9.7604	9.7622	9.7640	9.7657	9.7675	9.7692	54	2	4	5	7	9
36	9.7692	9.7710	9.7727	9.7744	9.7761	9.7778	9.7795	53	2	3	5	7	9
37	9.7795	9.7811	9.7828	9.7844	9.7861	9.7877	9.7893	52	2	3	5	7	8
38	9.7893	9.7910	9.7926	9.7941	9.7957	9.7973	9.7989	51	2	3	5	6	8
39	9.7989	9.8004	9.8020	9.8035	9.8050	9.8066	9.8081	50	2	3	5	6	8
40	9.8081	9.8096	9.8111	9.8125	9.8140	9.8155	9.8169	49	1	3	4	6	7
41	9.8169	9.8184	9.8198	9.8213	9.8227	9.8241	9.8255	48	1	3	4	6	7
42	9.8255	9.8269	9.8283	9.8297	9.8311	9.8324	9.8338	47	1	3	4	6	7
43	9.8338	9.8351	9.8365	9.8378	9.8391	9.8405	9.8418	46	1	3	4	5	7
44	9.8418	9.8431	9.8444	9.8457	9.8469	9.8482	9.8495	45	1	3	4	5	6
									1′	2′	3′	4′	5′
	60′	50′	40′	30′	20′	10′	0′	Degrees	Prop. parts				

Logarithmic cosines

Table 18. Logarithmic sines—*Continued*

Degrees	0′	10′	20′	30′	40′	50′	60′		Prop. parts 1′	2′	3′	4′	5′
45	9.8495	9.8507	9.8520	9.8532	9.8545	9.8557	9.8569	44	1	2	4	5	6
46	9.8569	9.8582	9.8594	9.8606	9.8618	9.8629	9.8641	43	1	2	4	5	6
47	9.8641	9.8653	9.8665	9.8676	9.8688	9.8699	9.8711	42	1	2	3	5	6
48	9.8711	9.8722	9.8733	9.8745	9.8756	9.8767	9.8778	41	1	2	3	4	6
49	9.8778	9.8789	9.8800	9.8810	9.8821	9.8832	9.8843	40	1	2	3	4	5
50	9.8843	9.8853	9.8864	9.8874	9.8884	9.8895	9.8905	39	1	2	3	4	5
51	9.8905	9.8915	9.8925	9.8935	9.8945	9.8955	9.8965	38	1	2	3	4	5
52	9.8965	9.8975	9.8985	9.8995	9.9004	9.9014	9.9023	37	1	2	3	4	5
53	9.9023	9.9033	9.9042	9.9052	9.9061	9.9070	9.9080	36	1	2	3	4	5
54	9.9080	9.9089	9.9098	9.9107	9.9116	9.9125	9.9134	35	1	2	3	4	5
55	9.9134	9.9142	9.9151	9.9160	9.9169	9.9177	9.9186	34	1	2	3	3	4
56	9.9186	9.9194	9.9203	9.9211	9.9219	9.9228	9.9236	33	1	2	3	3	4
57	9.9236	9.9244	9.9252	9.9260	9.9268	9.9276	9.9284	32	1	2	2	3	4
58	9.9284	9.9292	9.9300	9.9308	9.9315	9.9323	9.9331	31	1	2	2	3	4
59	9.9331	9.9338	9.9346	9.9353	9.9361	9.9368	9.9375	30	1	1	2	3	4
60	9.9375	9.9383	9.9390	9.9397	9.9404	9.9411	9.9418	29	1	1	2	3	4
61	9.9418	9.9425	9.9432	9.9439	9.9446	9.9453	9.9459	28	1	1	2	3	3
62	9.9459	9.9466	9.9473	9.9479	9.9486	9.9492	9.9499	27	1	1	2	3	3
63	9.9499	9.9505	9.9512	9.9518	9.9524	9.9530	9.9537	26	1	1	2	3	3
64	9.9537	9.9543	9.9549	9.9555	9.9561	9.9567	9.9573	25	1	1	2	2	3
65	9.9573	9.9579	9.9584	9.9590	9.9596	9.9602	9.9607	24	1	1	2	2	3
66	9.9607	9.9613	9.9618	9.9624	9.9629	9.9635	9.9640	23	1	1	2	2	3
67	9.9640	9.9646	9.9651	9.9656	9.9661	9.9667	9.9672	22	1	1	2	2	3
68	9.9672	9.9677	9.9682	9.9687	9.9692	9.9697	9.9702	21	0	1	1	2	2
69	9.9702	9.9706	9.9711	9.9716	9.9721	9.9725	9.9730	20	0	1	1	2	2
70	9.9730	9.9734	9.9739	9.9743	9.9748	9.9752	9.9757	19	0	1	1	2	2
71	9.9757	9.9761	9.9765	9.9770	9.9774	9.9778	9.9782	18	0	1	1	2	2
72	9.9782	9.9786	9.9790	9.9794	9.9798	9.9802	9.9806	17	0	1	1	2	2
73	9.9806	9.9810	9.9814	9.9817	9.9821	9.9825	9.9828	16	0	1	1	1	2
74	9.9828	9.9832	9.9836	9.9839	9.9843	9.9846	9.9849	15	0	1	1	1	2
75	9.9849	9.9853	9.9856	9.9859	9.9863	9.9866	9.9869	14	0	1	1	1	2
76	9.9869	9.9872	9.9875	9.9878	9.9881	9.9884	9.9887	13	0	1	1	1	2
77	9.9887	9.9890	9.9893	9.9896	9.9899	9.9901	9.9904	12	0	1	1	1	1
78	9.9904	9.9907	9.9909	9.9912	9.9914	9.9917	9.9919	11	0	1	1	1	1
79	9.9919	9.9922	9.9924	9.9927	9.9929	9.9931	9.9934	10	0	0	1	1	1
80	9.9934	9.9936	9.9938	9.9940	9.9942	9.9944	9.9946	9	0	0	1	1	1
81	9.9946	9.9948	9.9950	9.9952	9.9954	9.9956	9.9958	8	0	0	1	1	1
82	9.9958	9.9959	9.9961	9.9963	9.9964	9.9966	9.9968	7	0	0	0	1	1
83	9.9968	9.9969	9.9971	9.9972	9.9973	9.9975	9.9976	6	0	0	0	1	1
84	9.9976	9.9977	9.9979	9.9980	9.9981	9.9982	9.9983	5	0	0	0	0	1
85	9.9983	9.9985	9.9986	9.9987	9.9988	9.9989	9.9989	4	0	0	0	0	0
86	9.9989	9.9990	9.9991	9.9992	9.9993	9.9993	9.9994	3	0	0	0	0	0
87	9.9994	9.9995	9.9995	9.9996	9.9996	9.9997	9.9997	2	0	0	0	0	0
88	9.9997	9.9998	9.9998	9.9999	9.9999	9.9999	9.9999	1	0	0	0	0	0
89	9.9999	10.000	10.000	10.000	10.000	10.000	10.0000	0	0	0	0	0	0
	60′	50′	40′	30′	20′	10′	0′	Degrees	Prop. parts 1′	2′	3′	4′	5′

Logarithmic cosines

Table 19. Logarithmic tangents (see Art. 31)

Degrees	0′	10′	20′	30′	40′	50′	60′		Prop. parts 1′	2′	3′	4′	5′
0	Inf. Neg.	7.4637	7.7648	7.9409	8.0658	8.1627	8.2419	89	*	See Art. 31			
1	8.2419	8.3089	8.3669	8.4181	8.4638	8.5053	8.5431	88	*				
2	8.5431	8.5779	8.6101	8.6401	8.6682	8.6945	8.7194	87	*				
3	8.7194	8.7429	8.7652	8.7865	8.8067	8.8261	8.8446	86	*				
4	8.8446	8.8624	8.8795	8.8960	8.9118	8.9272	8.9420	85	*				
5	8.9420	8.9563	8.9701	8.9836	8.9966	9.0093	9.0216	84	*				
6	9.0216	9.0336	9.0453	9.0567	9.0678	9.0786	9.0891	83		Interpolate			
7	9.0891	9.0995	9.1096	9.1194	9.1291	9.1385	9.1478	82					
8	9.1478	9.1569	9.1658	9.1745	9.1831	9.1915	9.1997	81					
9	9.1997	9.2078	9.2158	9.2236	9.2313	9.2389	9.2463	80					
10	9.2463	9.2536	9.2609	9.2680	9.2750	9.2819	9.2887	79					
11	9.2887	9.2953	9.3020	9.3085	9.3149	9.3212	9.3275	78					
12	9.3275	9.3336	9.3397	9.3458	9.3517	9.3576	9.3634	77					
13	9.3634	9.3691	9.3748	9.3804	9.3859	9.3914	9.3968	76					
14	9.3968	9.4021	9.4074	9.4127	9.4178	9.4230	9.4281	75					
15	9.4281	9.4331	9.4381	9.4430	9.4479	9.4527	9.4575	74					
16	9.4575	9.4622	9.4669	9.4716	9.4762	9.4808	9.4853	73	5	9	14	19	23
17	9.4853	9.4898	9.4943	9.4987	9.5031	9.5075	9.5118	72	4	9	13	18	22
18	9.5118	9.5161	9.5203	9.5245	9.5287	9.5329	9.5370	71	4	8	13	17	21
19	9.5370	9.5411	9.5451	9.5491	9.5531	9.5571	9.5611	70	4	8	12	16	20
20	9.5611	9.5650	9.5689	9.5727	9.5766	9.5804	9.5842	69	4	8	12	15	19
21	9.5842	9.5879	9.5917	9.5954	9.5991	9.6028	9.6064	68	4	7	11	15	19
22	9.6064	9.6100	9.6136	9.6172	9.6208	9.6243	9.6279	67	4	7	11	14	18
23	9.6279	9.6314	9.6348	9.6383	9.6417	9.6452	9.6486	66	3	7	10	14	17
24	9.6486	9.6520	9.6553	9.6587	9.6620	9.6654	9.6687	65	3	7	10	13	17
25	9.6687	9.6720	9.6752	9.6785	9.6817	9.6850	9.6882	64	3	7	10	13	16
26	9.6882	9.6914	9.6946	9.6977	9.7009	9.7040	9.7072	63	3	6	9	13	16
27	9.7072	9.7103	9.7134	9.7165	9.7196	9.7226	9.7257	62	3	6	9	12	15
28	9.7257	9.7287	9.7317	9.7348	9.7378	9.7408	9.7438	61	3	6	9	12	15
29	9.7438	9.7467	9.7497	9.7526	9.7556	9.7585	9.7614	60	3	6	9	12	15
30	9.7614	9.7644	9.7673	9.7701	9.7730	9.7759	9.7788	59	3	6	9	12	14
31	9.7788	9.7816	9.7845	9.7873	9.7902	9.7930	9.7958	58	3	6	9	11	14
32	9.7958	9.7986	9.8014	9.8042	9.8070	9.8097	9.8125	57	3	6	8	11	14
33	9.8125	9.8153	9.8180	9.8208	9.8235	9.8263	9.8290	56	3	5	8	11	14
34	9.8290	9.8317	9.8344	9.8371	9.8398	9.8425	9.8452	55	3	5	8	11	14
35	9.8452	9.8479	9.8506	9.8533	9.8559	9.8586	9.8613	54	3	5	8	11	13
36	9.8613	9.8639	9.8666	9.8692	9.8718	9.8745	9.8771	53	3	5	8	11	13
37	9.8771	9.8797	9.8824	9.8850	9.8876	9.8902	9.8928	52	3	5	8	10	13
38	9.8928	9.8954	9.8980	9.9006	9.9032	9.9058	9.9084	51	3	5	8	10	13
39	9.9084	9.9110	9.9135	9.9161	9.9187	9.9212	9.9238	50	3	5	8	10	13
40	9.9238	9.9264	9.9289	9.9315	9.9341	9.9366	9.9392	49	3	5	8	10	13
41	9.9392	9.9417	9.9443	9.9468	9.9494	9.9519	9.9544	48	3	5	8	10	13
42	9.9544	9.9570	9.9595	9.9621	9.9646	9.9671	9.9697	47	3	5	8	10	13
43	9.9697	9.9722	9.9747	9.9772	9.9798	9.9823	9.9848	46	3	5	8	10	13
44	9.9848	9.9874	9.9899	9.9924	9.9949	9.9975	10.0000	45	3	5	8	10	13
	60′	50′	40′	30′	20′	10′	0′	Degrees	1′	2′	3′	4′	5′
									Prop. parts				

Logarithmic cotangents

Table 19. Logarithmic tangents—*Continued*

Degrees	0′	10′	20′	30′	40′	50′	60′		Prop. parts 1′	2′	3′	4′	5′
45	0.0000	0.0025	0.0051	0.0076	0.0101	0.0126	0.0152	44	3	5	8	10	13
46	0.0152	0.0177	0.0202	0.0228	0.0253	0.0278	0.0303	43	3	5	8	10	13
47	0.0303	0.0329	0.0354	0.0379	0.0405	0.0430	0.0456	42	3	5	8	10	13
48	0.0456	0.0481	0.0506	0.0532	0.0557	0.0583	0.0608	41	3	5	8	10	13
49	0.0608	0.0634	0.0659	0.0685	0.0711	0.0736	0.0762	40	3	5	8	10	13
50	0.0762	0.0788	0.0813	0.0839	0.0865	0.0890	0.0916	39	3	5	8	10	13
51	0.0916	0.0942	0.0968	0.0994	0.1020	0.1046	0.1072	38	3	5	8	10	13
52	0.1072	0.1098	0.1124	0.1150	0.1176	0.1203	0.1229	37	3	5	8	10	13
53	0.1229	0.1255	0.1282	0.1308	0.1334	0.1361	0.1387	36	3	5	8	11	13
54	0.1387	0.1414	0.1441	0.1467	0.1494	0.1521	0.1548	35	3	5	8	11	13
55	0.1548	0.1575	0.1602	0.1629	0.1656	0.1683	0.1710	34	3	5	8	11	14
56	0.1710	0.1737	0.1765	0.1792	0.1820	0.1847	0.1875	33	3	5	8	11	14
57	0.1875	0.1903	0.1930	0.1958	0.1986	0.2014	0.2042	32	3	6	8	11	14
58	0.2042	0.2070	0.2098	0.2127	0.2155	0.2184	0.2212	31	3	6	9	11	14
59	0.2212	0.2241	0.2270	0.2299	0.2327	0.2356	0.2386	30	3	6	9	12	14
60	0.2386	0.2415	0.2444	0.2474	0.2503	0.2533	0.2562	29	3	6	9	12	15
61	0.2562	0.2592	0.2622	0.2652	0.2683	0.2713	0.2743	28	3	6	9	12	15
62	0.2743	0.2774	0.2804	0.2835	0.2866	0.2897	0.2928	27	3	6	9	12	15
63	0.2928	0.2960	0.2991	0.3023	0.3054	0.3086	0.3118	26	3	6	9	13	16
64	0.3118	0.3150	0.3183	0.3215	0.3248	0.3280	0.3313	25	3	7	10	13	16
65	0.3313	0.3346	0.3380	0.3413	0.3447	0.3480	0.3514	24	3	7	10	13	17
66	0.3514	0.3548	0.3583	0.3617	0.3652	0.3686	0.3721	23	3	7	10	14	17
67	0.3721	0.3757	0.3792	0.3828	0.3864	0.3900	0.3936	22	4	7	11	14	18
68	0.3936	0.3972	0.4009	0.4046	0.4083	0.4121	0.4158	21	4	7	11	15	19
69	0.4158	0.4196	0.4234	0.4273	0.4311	0.4350	0.4389	20	4	8	12	15	19
70	0.4389	0.4429	0.4469	0.4509	0.4549	0.4589	0.4630	19	4	8	12	16	20
71	0.4630	0.4671	0.4713	0.4755	0.4797	0.4839	0.4882	18	4	8	13	17	21
72	0.4882	0.4925	0.4969	0.5013	0.5057	0.5102	0.5147	17	4	9	13	18	22
73	0.5147	0.5192	0.5238	0.5284	0.5331	0.5378	0.5425	16	5	9	14	19	23
74	0.5425	0.5473	0.5521	0.5570	0.5619	0.5669	0.5719	15					
75	0.5719	0.5770	0.5822	0.5873	0.5926	0.5979	0.6032	14					
76	0.6032	0.6086	0.6141	0.6196	0.6252	0.6309	0.6366	13					
77	0.6366	0.6424	0.6483	0.6542	0.6603	0.6664	0.6725	12					
78	0.6725	0.6788	0.6851	0.6915	0.6980	0.7047	0.7113	11		Interpolate			
79	0.7113	0.7181	0.7250	0.7320	0.7391	0.7464	0.7537	10					
80	0.7537	0.7611	0.7687	0.7764	0.7842	0.7922	0.8003	9					
81	0.8003	0.8085	0.8169	0.8255	0.8342	0.8431	0.8522	8					
82	0.8522	0.8615	0.8709	0.8806	0.8904	0.9005	0.9109	7					
83	0.9109	0.9214	0.9322	0.9433	0.9547	0.9664	0.9784	6					
84	0.9784	0.9907	1.0034	1.0164	1.0299	1.0437	1.0580	5	*				
85	1.0580	1.0728	1.0882	1.1040	1.1205	1.1376	1.1554	4	*				
86	1.1554	1.1739	1.1933	1.2135	1.2348	1.2571	1.2806	3	*				
87	1.2806	1.3055	1.3318	1.3599	1.3899	1.4221	1.4569	2	*	See Art. 31			
88	1.4569	1.4947	1.5362	1.5819	1.6331	1.6911	1.7581	1	*				
89	1.7581	1.8373	1.9342	2.0591	2.2352	2.5363	Infinite	0	*				
	60′	50′	40′	30′	20′	10′	0′	Degrees	1′	2′	3′	4′	5′ Prop. parts

Logarithmic cotangents

Table 20. Exponentials e^u and e^{-u} (see Art. 75)

u	e^u	e^{-u}	u	e^u	e^{-u}	u	e^u	e^{-u}	u	$\log e^u = u \log e$
.00	1.000	1.000	.50	1.649	.6065	1.0	2.718	.3679	.01	0.00434
.01	1.010	.9900	.51	1.665	.6005	1.1	3.004	.3329	.02	0.00869
.02	1.020	.9802	.52	1.682	.5945	1.2	3.320	.3012	.03	0.01303
.03	1.030	.9704	.53	1.699	.5886	1.3	3.669	.2725	.04	0.01737
.04	1.041	.9608	.54	1.716	.5827	1.4	4.055	.2466	.05	0.02171
.05	1.051	.9512	.55	1.733	.5769	1.5	4.482	.2231	.06	0.02606
.06	1.062	.9418	.56	1.751	.5712	1.6	4.953	.2019	.07	0.03040
.07	1.073	.9324	.57	1.768	.5655	1.7	5.474	.1827	.08	0.03474
.08	1.083	.9231	.58	1.786	.5599	1.8	6.050	.1653	.09	0.03909
.09	1.094	.9139	.59	1.804	.5543	1.9	6.686	.1496	.1	0.04343
.10	1.105	.9048	.60	1.822	.5488	2.0	7.389	.1353	.2	0.08686
.11	1.116	.8958	.61	1.840	.5434	2.1	8.166	.1225	.3	0.13029
.12	1.127	.8869	.62	1.859	.5379	2.2	9.025	.1108	.4	0.17372
.13	1.139	.8781	.63	1.878	.5326	2.3	9.974	.1003	.5	0.21715
.14	1.150	.8694	.64	1.896	.5273	2.4	11.02	.09072	.6	0.26058
.15	1.162	.8607	.65	1.916	.5220	2.5	12.18	.08209	.7	0.30401
.16	1.174	.8521	.66	1.935	.5169	2.6	13.46	.07427	.8	0.34744
.17	1.185	.8437	.67	1.954	.5117	2.7	14.88	.06721	.9	0.39087
.18	1.197	.8353	.68	1.974	.5066	2.8	16.44	.06081	1	0.43429
.19	1.209	.8270	.69	1.994	.5016	2.9	18.17	.05502	2	0.86859
.20	1.221	.8187	.70	2.014	.4966	3.0	20.09	.04979	3	1.30288
.21	1.234	.8106	.71	2.034	.4916	3.1	22.20	.04505	4	1.73718
.22	1.246	.8025	.72	2.054	.4868	3.2	24.53	.04076	5	2.17147
.23	1.259	.7945	.73	2.075	.4819	3.3	27.11	.03688	6	2.60577
.24	1.271	.7866	.74	2.096	.4771	3.4	29.96	.03337	7	3.04006
.25	1.284	.7788	.75	2.117	.4724	3.5	33.12	.03020	8	3.47436
.26	1.297	.7711	.76	2.138	.4677	3.6	36.60	.02732	9	3.90865
.27	1.310	.7634	.77	2.160	.4630	3.7	40.45	.02472	10	4.34294
.28	1.323	.7558	.78	2.181	.4584	3.8	44.70	.02237	20	8.68589
.29	1.336	.7483	.79	2.203	.4538	3.9	49.40	.02024	30	13.02883
.30	1.350	.7408	.80	2.226	.4493	4.0	54.60	.01832	40	17.37178
.31	1.363	.7334	.81	2.248	.4449	4.1	60.34	.01657	50	21.71472
.32	1.377	.7261	.82	2.271	.4404	4.2	66.69	.01500	60	26.05767
.33	1.391	.7189	.83	2.293	.4361	4.3	73.70	.01357	70	30.40061
.34	1.405	.7118	.84	2.316	.4317	4.4	81.45	.01228	80	34.74356
.35	1.419	.7047	.85	2.340	.4274	4.5	90.02	.01111	90	39.08650
.36	1.433	.6977	.86	2.363	.4232	4.6	99.48	.01005	$\pi/4$	0.34109
.37	1.448	.6907	.87	2.387	.4190	4.7	109.9	.00910	$\pi/2$	0.68219
.38	1.462	.6839	.88	2.411	.4148	4.8	121.5	.00823	$3\pi/4$	1.02328
.39	1.477	.6771	.89	2.435	.4107	4.9	134.3	.00745	π	1.36438
.40	1.492	.6703	.90	2.460	.4066	5.0	148.4	.00674	$5\pi/4$	1.70547
.41	1.507	.6637	.91	2.484	.4025	5.1	164.0	.00610	$3\pi/2$	2.04656
.42	1.522	.6570	.92	2.509	.3985	5.2	181.3	.00552	$7\pi/4$	2.38766
.43	1.537	.6505	.93	2.535	.3946	5.3	200.3	.00499	2π	2.72875
.44	1.553	.6440	.94	2.560	.3906	5.4	221.4	.00452	$5\pi/2$	3.41094
.45	1.568	.6376	.95	2.586	.3867	5.5	244.7	.00409	3π	4.09313
.46	1.584	.6313	.96	2.612	.3829	5.6	270.4	.00370	$7\pi/2$	4.77532
.47	1.600	.6250	.97	2.638	.3791	5.7	298.9	.00335	4π	5.45751
.48	1.616	.6188	.98	2.664	.3753	5.8	330.3	.00303	$9\pi/2$	6.13969
.49	1.632	.6126	.99	2.691	.3716	5.9	365.0	.00274	5π	6.82188

Table 21. Hyperbolic functions (see Art. 75)

u	Sinh u	Cosh u	Tanh u	u	Sinh u	Cosh u	Tanh u	u	Sinh u	Cosh u	Tanh u
.00	.0000	1.000	.0000	.50	.5211	1.128	.4621	1.0	1.175	1.543	.7616
.01	.0100	1.000	.0100	.51	.5324	1.133	.4700	1.1	1.336	1.669	.8005
.02	.0200	1.000	.0200	.52	.5438	1.138	.4777	1.2	1.509	1.811	.8337
.03	.0300	1.000	.0300	.53	.5552	1.144	.4854	1.3	1.698	1.971	.8617
.04	.0400	1.001	.0400	.54	.5666	1.149	.4930	1.4	1.904	2.151	.8854
.05	.0500	1.001	.0500	.55	.5782	1.155	.5005	1.5	2.129	2.352	.9052
.06	.0600	1.002	.0599	.56	.5897	1.161	.5080	1.6	2.376	2.577	.9217
.07	.0701	1.002	.0699	.57	.6014	1.167	.5154	1.7	2.646	2.828	.9354
.08	.0801	1.003	.0798	.58	.6131	1.173	.5227	1.8	2.942	3.107	.9468
.09	.0901	1.004	.0898	.59	.6248	1.179	.5299	1.9	3.268	3.418	.9562
.10	.1002	1.005	.0997	.60	.6367	1.185	.5370	2.0	3.627	3.762	.9640
.11	.1102	1.006	.1096	.61	.6485	1.192	.5441	2.1	4.022	4.144	.9705
.12	.1203	1.007	.1194	.62	.6605	1.198	.5511	2.2	4.457	4.568	.9757
.13	.1304	1.008	.1293	.63	.6725	1.205	.5581	2.3	4.937	5.037	.9801
.14	.1405	1.010	.1391	.64	.6846	1.212	.5649	2.4	5.466	5.557	.9837
.15	.1506	1.011	.1489	.65	.6967	1.219	.5717	2.5	6.050	6.132	.9866
.16	.1607	1.013	.1587	.66	.7090	1.226	.5784	2.6	6.695	6.769	.9890
.17	.1708	1.014	.1684	.67	.7213	1.233	.5850	2.7	7.406	7.473	.9910
.18	.1810	1.016	.1781	.68	.7336	1.240	.5915	2.8	8.192	8.253	.9926
.19	.1911	1.018	.1878	.69	.7461	1.248	.5980	2.9	9.060	9.115	.9940
.20	.2013	1.020	.1974	.70	.7586	1.255	.6044	3.0	10.02	10.07	.9951
.21	.2115	1.022	.2070	.71	.7712	1.263	.6107	3.1	11.08	11.12	.9960
.22	.2218	1.024	.2165	.72	.7838	1.271	.6169	3.2	12.25	12.29	.9967
.23	.2320	1.027	.2260	.73	.7966	1.278	.6231	3.3	13.54	13.57	.9973
.24	.2423	1.029	.2355	.74	.8094	1.287	.6291	3.4	14.97	15.00	.9978
.25	.2526	1.031	.2449	.75	.8223	1.295	.6352	3.5	16.54	16.57	.9982
.26	.2629	1.034	.2543	.76	.8353	1.303	.6411	3.6	18.29	18.31	.9985
.27	.2733	1.037	.2636	.77	.8484	1.311	.6469	3.7	20.21	20.24	.9988
.28	.2837	1.039	.2729	.78	.8615	1.320	.6527	3.8	22.34	22.36	.9990
.29	.2941	1.042	.2821	.79	.8748	1.329	.6584	3.9	24.69	24.71	.9992
.30	.3045	1.045	.2913	.80	.8881	1.337	.6640	4.0	27.29	27.31	.9993
.31	.3150	1.048	.3004	.81	.9015	1.346	.6696	4.1	30.16	30.18	.9995
.32	.3255	1.052	.3095	.82	.9150	1.355	.6751	4.2	33.34	33.35	.9996
.33	.3360	1.055	.3185	.83	.9286	1.365	.6805	4.3	36.84	36.86	.9996
.34	.3466	1.058	.3275	.84	.9423	1.374	.6858	4.4	40.72	40.73	.9997
.35	.3572	1.062	.3364	.85	.9561	1.384	.6911	4.5	45.00	45.01	.9998
.36	.3678	1.066	.3452	.86	.9700	1.393	.6963	4.6	49.74	49.75	.9998
.37	.3785	1.069	.3540	.87	.9840	1.403	.7014	4.7	54.97	54.98	.9998
.38	.3892	1.073	.3627	.88	.9981	1.413	.7064	4.8	60.75	60.76	.9999
.39	.4000	1.077	.3714	.89	1.012	1.423	.7114	4.9	67.14	67.15	.9999
.40	.4108	1.081	.3800	.90	1.027	1.433	.7163	5.0	74.20	74.21	.9999
.41	.4216	1.085	.3885	.91	1.041	1.443	.7211	5.1	82.01	82.01	.9999
.42	.4325	1.090	.3969	.92	1.055	1.454	.7259	5.2	90.63	90.64	.9999
.43	.4434	1.094	.4053	.93	1.070	1.465	.7306	5.3	100.17	100.17	1.0000
.44	.4543	1.098	.4136	.94	1.085	1.475	.7352	5.4	110.70	110.71	1.0000
.45	.4653	1.103	.4219	.95	1.099	1.486	.7398	5.5	122.34	122.35	1.0000
.46	.4764	1.108	.4301	.96	1.114	1.497	.7443	5.6	135.21	135.22	1.0000
.47	.4875	1.112	.4382	.97	1.129	1.509	.7487	5.7	149.43	149.44	1.0000
.48	.4986	1.117	.4462	.98	1.145	1.520	.7531	5.8	165.15	165.15	1.0000
.49	.5098	1.122	.4542	.99	1.160	1.531	.7574	5.9	182.52	182.52	1.0000

Table 22. Napierian (natural) logarithms. Base e = 2.71828

N	0	1	2	3	4	5	6	7	8	9	Prop. parts 1	2	3	4	5
1.0	0.0000	0.0100	0.0198	0.0296	0.0392	0.0488	0.0583	0.0677	0.0770	0.0862	Interpolate				
1.1	0.0953	0.1044	0.1133	0.1222	0.1310	0.1398	0.1484	0.1570	0.1655	0.1740					
1.2	0.1823	0.1906	0.1989	0.2070	0.2151	0.2231	0.2311	0.2390	0.2469	0.2546					
1.3	0.2624	0.2700	0.2776	0.2852	0.2927	0.3001	0.3075	0.3148	0.3221	0.3293					
1.4	0.3365	0.3436	0.3507	0.3577	0.3646	0.3716	0.3784	0.3853	0.3920	0.3988					
1.5	0.4055	0.4121	0.4187	0.4253	0.4318	0.4383	0.4447	0.4511	0.4574	0.4637					
1.6	0.4700	0.4762	0.4824	0.4886	0.4947	0.5008	0.5068	0.5128	0.5188	0.5247					
1.7	0.5306	0.5365	0.5423	0.5481	0.5539	0.5596	0.5653	0.5710	0.5766	0.5822					
1.8	0.5878	0.5933	0.5988	0.6043	0.6098	0.6152	0.6206	0.6259	0.6313	0.6366					
1.9	0.6419	0.6471	0.6523	0.6575	0.6627	0.6678	0.6729	0.6780	0.6831	0.6881					
2.0	0.6931	0.6981	0.7031	0.7080	0.7129	0.7178	0.7227	0.7275	0.7324	0.7372					
2.1	0.7419	0.7467	0.7514	0.7561	0.7608	0.7655	0.7701	0.7747	0.7793	0.7839	5	9	14	19	23
2.2	0.7885	0.7930	0.7975	0.8020	0.8065	0.8109	0.8154	0.8198	0.8242	0.8285	4	9	13	18	22
2.3	0.8329	0.8372	0.8416	0.8459	0.8502	0.8544	0.8587	0.8629	0.8671	0.8713	4	9	13	17	21
2.4	0.8755	0.8796	0.8838	0.8879	0.8920	0.8961	0.9002	0.9042	0.9083	0.9123	4	8	12	16	20
2.5	0.9163	0.9203	0.9243	0.9282	0.9322	0.9361	0.9400	0.9439	0.9478	0.9517	4	8	12	16	20
2.6	0.9555	0.9594	0.9632	0.9670	0.9708	0.9746	0.9783	0.9821	0.9858	0.9895	4	8	11	15	19
2.7	0.9933	0.9969	1.0006	1.0043	1.0080	1.0116	1.0152	1.0188	1.0225	1.0260	4	7	11	15	18
2.8	1.0296	1.0332	1.0367	1.0403	1.0438	1.0473	1.0508	1.0543	1.0578	1.0613	4	7	11	14	18
2.9	1.0647	1.0682	1.0716	1.0750	1.0784	1.0818	1.0852	1.0886	1.0919	1.0953	3	7	10	14	17
3.0	1.0986	1.1019	1.1053	1.1086	1.1119	1.1151	1.1184	1.1217	1.1249	1.1282	3	7	10	13	16
3.1	1.1314	1.1346	1.1378	1.1410	1.1442	1.1474	1.1506	1.1537	1.1569	1.1600	3	6	10	13	16
3.2	1.1632	1.1663	1.1694	1.1725	1.1756	1.1787	1.1817	1.1848	1.1878	1.1909	3	6	9	12	15
3.3	1.1939	1.1969	1.2000	1.2030	1.2060	1.2090	1.2119	1.2149	1.2179	1.2208	3	6	9	12	15
3.4	1.2238	1.2267	1.2296	1.2326	1.2355	1.2384	1.2413	1.2442	1.2470	1.2499	3	6	9	12	14
3.5	1.2528	1.2556	1.2585	1.2613	1.2641	1.2669	1.2698	1.2726	1.2754	1.2782	3	6	8	11	14
3.6	1.2809	1.2837	1.2865	1.2892	1.2920	1.2947	1.2975	1.3002	1.3029	1.3056	3	5	8	11	14
3.7	1.3083	1.3110	1.3137	1.3164	1.3191	1.3218	1.3244	1.3271	1.3297	1.3324	3	5	8	11	13
3.8	1.3350	1.3376	1.3403	1.3429	1.3455	1.3481	1.3507	1.3533	1.3558	1.3584	3	5	8	10	13
3.9	1.3610	1.3635	1.3661	1.3686	1.3712	1.3737	1.3762	1.3788	1.3813	1.3838	3	5	8	10	13
4.0	1.3863	1.3888	1.3913	1.3938	1.3962	1.3987	1.4012	1.4036	1.4061	1.4085	2	5	7	10	12
4.1	1.4110	1.4134	1.4159	1.4183	1.4207	1.4231	1.4255	1.4279	1.4303	1.4327	2	5	7	10	12
4.2	1.4351	1.4375	1.4398	1.4422	1.4446	1.4469	1.4493	1.4516	1.4540	1.4563	2	5	7	9	12
4.3	1.4586	1.4609	1.4633	1.4656	1.4679	1.4702	1.4725	1.4748	1.4770	1.4793	2	5	7	9	11
4.4	1.4816	1.4839	1.4861	1.4884	1.4907	1.4929	1.4951	1.4974	1.4996	1.5019	2	4	7	9	11
4.5	1.5041	1.5063	1.5085	1.5107	1.5129	1.5151	1.5173	1.5195	1.5217	1.5239	2	4	7	9	11
4.6	1.5261	1.5282	1.5304	1.5326	1.5347	1.5369	1.5390	1.5412	1.5433	1.5454	2	4	6	9	11
4.7	1.5476	1.5497	1.5518	1.5539	1.5560	1.5581	1.5602	1.5623	1.5644	1.5665	2	4	6	8	11
4.8	1.5686	1.5707	1.5728	1.5748	1.5769	1.5790	1.5810	1.5831	1.5851	1.5872	2	4	6	8	10
4.9	1.5892	1.5913	1.5933	1.5953	1.5974	1.5994	1.6014	1.6034	1.6054	1.6074	2	4	6	8	10
5.0	1.6094	1.6114	1.6134	1.6154	1.6174	1.6194	1.6214	1.6233	1.6253	1.6273	2	4	6	8	10
5.1	1.6292	1.6312	1.6332	1.6351	1.6371	1.6390	1.6409	1.6429	1.6448	1.6467	2	4	6	8	10
5.2	1.6487	1.6506	1.6525	1.6544	1.6563	1.6582	1.6601	1.6620	1.6639	1.6658	2	4	6	8	10
5.3	1.6677	1.6696	1.6715	1.6734	1.6752	1.6771	1.6790	1.6808	1.6827	1.6845	2	4	6	7	9
5.4	1.6864	1.6882	1.6901	1.6919	1.6938	1.6956	1.6974	1.6993	1.7011	1.7029	2	4	6	7	9
5.5	1.7047	1.7066	1.7084	1.7102	1.7120	1.7138	1.7156	1.7174	1.7192	1.7210	2	4	5	7	9
5.6	1.7228	1.7246	1.7263	1.7281	1.7299	1.7317	1.7334	1.7352	1.7370	1.7387	2	4	5	7	9
5.7	1.7405	1.7422	1.7440	1.7457	1.7475	1.7492	1.7509	1.7527	1.7544	1.7561	2	3	5	7	9
5.8	1.7579	1.7596	1.7613	1.7630	1.7647	1.7664	1.7681	1.7699	1.7716	1.7733	2	3	5	7	9
5.9	1.7750	1.7766	1.7783	1.7800	1.7817	1.7834	1.7851	1.7867	1.7884	1.7901	2	3	5	7	8
N	0	1	2	3	4	5	6	7	8	9	Prop. parts 1	2	3	4	5

Table 22. Napierian logarithms—*Continued*

N	0	1	2	3	4	5	6	7	8	9	Prop. parts 1	2	3	4	5
6.0	1.7918	1.7934	1.7951	1.7967	1.7984	1.8001	1.8017	1.8034	1.8050	1.8066	2	3	5	7	8
6.1	1.8083	1.8099	1.8116	1.8132	1.8148	1.8165	1.8181	1.8197	1.8213	1.8229	2	3	5	7	8
6.2	1.8245	1.8262	1.8278	1.8294	1.8310	1.8326	1.8342	1.8358	1.8374	1.8390	2	3	5	6	8
6.3	1.8405	1.8421	1.8437	1.8453	1.8469	1.8485	1.8500	1.8516	1.8532	1.8547	2	3	5	6	8
6.4	1.8563	1.8579	1.8594	1.8610	1.8625	1.8641	1.8656	1.8672	1.8687	1.8703	2	3	5	6	8
6.5	1.8718	1.8733	1.8749	1.8764	1.8779	1.8795	1.8810	1.8825	1.8840	1.8856	2	3	5	6	8
6.6	1.8871	1.8886	1.8901	1.8916	1.8931	1.8946	1.8961	1.8976	1.8991	1.9006	2	3	5	6	8
6.7	1.9021	1.9036	1.9051	1.9066	1.9081	1.9095	1.9110	1.9125	1.9140	1.9155	1	3	4	6	7
6.8	1.9169	1.9184	1.9199	1.9213	1.9228	1.9242	1.9257	1.9272	1.9286	1.9301	1	3	4	6	7
6.9	1.9315	1.9330	1.9344	1.9359	1.9373	1.9387	1.9402	1.9416	1.9430	1.9445	1	3	4	6	7
7.0	1.9459	1.9473	1.9488	1.9502	1.9516	1.9530	1.9544	1.9559	1.9573	1.9587	1	3	4	6	7
7.1	1.9601	1.9615	1.9629	1.9643	1.9657	1.9671	1.9685	1.9699	1.9713	1.9727	1	3	4	6	7
7.2	1.9741	1.9755	1.9769	1.9782	1.9796	1.9810	1.9824	1.9838	1.9851	1.9865	1	3	4	6	7
7.3	1.9879	1.9892	1.9906	1.9920	1.9933	1.9947	1.9961	1.9974	1.9988	2.0001	1	3	4	5	7
7.4	2.0015	2.0028	2.0042	2.0055	2.0069	2.0082	2.0096	2.0109	2.0122	2.0136	1	3	4	5	7
7.5	2.0149	2.0162	2.0176	2.0189	2.0202	2.0215	2.0229	2.0242	2.0255	2.0268	1	3	4	5	7
7.6	2.0281	2.0295	2.0308	2.0321	2.0334	2.0347	2.0360	2.0373	2.0386	2.0399	1	3	4	5	7
7.7	2.0412	2.0425	2.0438	2.0451	2.0464	2.0477	2.0490	2.0503	2.0516	2.0528	1	3	4	5	6
7.8	2.0541	2.0554	2.0567	2.0580	2.0592	2.0605	2.0618	2.0631	2.0643	2.0656	1	3	4	5	6
7.9	2.0669	2.0681	2.0694	2.0707	2.0719	2.0732	2.0744	2.0757	2.0769	2.0782	1	3	4	5	6
8.0	2.0794	2.0807	2.0819	2.0832	2.0844	2.0857	2.0869	2.0882	2.0894	2.0906	1	2	4	5	6
8.1	2.0919	2.0931	2.0943	2.0956	2.0968	2.0980	2.0992	2.1005	2.1017	2.1029	1	2	4	5	6
8.2	2.1041	2.1054	2.1066	2.1078	2.1090	2.1102	2.1114	2.1126	2.1138	2.1150	1	2	4	5	6
8.3	2.1163	2.1175	2.1187	2.1199	2.1211	2.1223	2.1235	2.1247	2.1258	2.1270	1	2	4	5	6
8.4	2.1282	2.1294	2.1306	2.1318	2.1330	2.1342	2.1353	2.1365	2.1377	2.1389	1	2	4	5	6
8.5	2.1401	2.1412	2.1424	2.1436	2.1448	2.1459	2.1471	2.1483	2.1494	2.1506	1	2	4	5	6
8.6	2.1518	2.1529	2.1541	2.1552	2.1564	2.1576	2.1587	2.1599	2.1610	2.1622	1	2	3	5	6
8.7	2.1633	2.1645	2.1656	2.1668	2.1679	2.1691	2.1702	2.1713	2.1725	2.1736	1	2	3	5	6
8.8	2.1748	2.1759	2.1770	2.1782	2.1793	2.1804	2.1815	2.1827	2.1838	2.1849	1	2	3	5	6
8.9	2.1861	2.1872	2.1883	2.1894	2.1905	2.1917	2.1928	2.1939	2.1950	2.1961	1	2	3	4	6
9.0	2.1972	2.1983	2.1994	2.2006	2.2017	2.2028	2.2039	2.2050	2.2061	2.2072	1	2	3	4	6
9.1	2.2083	2.2094	2.2105	2.2116	2.2127	2.2138	2.2148	2.2159	2.2170	2.2181	1	2	3	4	5
9.2	2.2192	2.2203	2.2214	2.2225	2.2235	2.2246	2.2257	2.2268	2.2279	2.2289	1	2	3	4	5
9.3	2.2300	2.2311	2.2322	2.2332	2.2343	2.2354	2.2364	2.2375	2.2386	2.2396	1	2	3	4	5
9.4	2.2407	2.2418	2.2428	2.2439	2.2450	2.2460	2.2471	2.2481	2.2492	2.2502	1	2	3	4	5
9.5	2.2513	2.2523	2.2534	2.2544	2.2555	2.2565	2.2576	2.2586	2.2597	2.2607	1	2	3	4	5
9.6	2.2618	2.2628	2.2638	2.2649	2.2659	2.2670	2.2680	2.2690	2.2701	2.2711	1	2	3	4	5
9.7	2.2721	2.2732	2.2742	2.2752	2.2762	2.2773	2.2783	2.2793	2.2803	2.2814	1	2	3	4	5
9.8	2.2824	2.2834	2.2844	2.2854	2.2865	2.2875	2.2885	2.2895	2.2905	2.2915	1	2	3	4	5
9.9	2.2925	2.2935	2.2946	2.2956	2.2966	2.2976	2.2986	2.2996	2.3006	2.3016	1	2	3	4	5

Table 23. Napierian logarithms of powers of 10

u	Nap. log. 10^u	u	Nap. log. 10^u	u	Nap. log. 10^u	u	Nap. log. 10^u
0	0.000 000	2.5	5.756 463	5.0	11.512 925	7.5	17.269 388
0.5	1.151 293	3.0	6.907 755	5.5	12.664 218	8.0	18.420 681
1.0	2.302 585	3.5	8.059 048	6.0	13.815 511	8.5	19.571 973
1.5	3.453 878	4.0	9.210 340	6.5	14.966 803	9.0	20.723 266
2.0	4.605 170	4.5	10.361 633	7.0	16.118 096	9.5	21.874 558

Table 24. Three halves powers, numbers 1.00-4.49

N	0	1	2	3	4	5	6	7	8	9	Prop. parts 1	2	3	4	5
1.0	1.000	1.015	1.030	1.045	1.061	1.076	1.091	1.107	1.122	1.138	2	3	5	6	8
1.1	1.154	1.169	1.185	1.201	1.217	1.233	1.249	1.266	1.282	1.298	2	3	5	6	8
1.2	1.315	1.331	1.348	1.364	1.381	1.398	1.414	1.431	1.448	1.465	2	3	5	7	8
1.3	1.482	1.499	1.517	1.534	1.551	1.569	1.586	1.604	1.621	1.639	2	3	5	7	9
1.4	1.657	1.674	1.692	1.710	1.728	1.746	1.764	1.782	1.800	1.819	2	4	5	7	9
1.5	1.837	1.856	1.874	1.893	1.911	1.930	1.948	1.967	1.986	2.005	2	4	6	7	9
1.6	2.024	2.043	2.062	2.081	2.100	2.119	2.139	2.158	2.178	2.197	2	4	6	8	10
1.7	2.217	2.236	2.256	2.275	2.295	2.315	2.335	2.355	2.375	2.395	2	4	6	8	10
1.8	2.415	2.435	2.455	2.476	2.496	2.516	2.537	2.557	2.578	2.598	2	4	6	8	10
1.9	2.619	2.640	2.660	2.681	2.702	2.723	2.744	2.765	2.786	2.807	2	4	6	8	10
2.0	2.828	2.850	2.871	2.892	2.914	2.935	2.957	2.978	3.000	3.021	2	4	6	9	11
2.1	3.043	3.065	3.087	3.109	3.131	3.153	3.175	3.197	3.219	3.241	2	4	7	9	11
2.2	3.263	3.285	3.308	3.330	3.353	3.375	3.398	3.420	3.443	3.465	2	4	7	9	11
2.3	3.488	3.511	3.534	3.557	3.580	3.602	3.626	3.649	3.672	3.695	2	5	7	9	11
2.4	3.718	3.741	3.765	3.788	3.811	3.835	3.858	3.882	3.906	3.929	2	5	7	9	12
2.5	3.953	3.977	4.000	4.024	4.048	4.072	4.096	4.120	4.144	4.168	2	5	7	10	12
2.6	4.192	4.217	4.241	4.265	4.289	4.314	4.338	4.363	4.387	4.412	2	5	7	10	12
2.7	4.437	4.461	4.486	4.511	4.536	4.560	4.585	4.610	4.635	4.660	2	5	7	10	12
2.8	4.685	4.710	4.736	4.761	4.786	4.811	4.837	4.862	4.888	4.913	3	5	8	10	13
2.9	4.939	4.964	4.990	5.015	5.041	5.067	5.093	5.118	5.144	5.170	3	5	8	10	13
3.0	5.196	5.222	5.248	5.274	5.300	5.327	5.353	5.379	5.405	5.432	3	5	8	11	13
3.1	5.458	5.485	5.511	5.538	5.564	5.591	5.617	5.644	5.671	5.698	3	5	8	11	13
3.2	5.724	5.751	5.778	5.805	5.832	5.859	5.886	5.913	5.940	5.968	3	5	8	11	14
3.3	5.995	6.022	6.049	6.077	6.104	6.132	6.159	6.186	6.214	6.242	3	5	8	11	14
3.4	6.269	6.297	6.325	6.352	6.380	6.408	6.436	6.464	6.492	6.520	3	6	8	11	14
3.5	6.548	6.576	6.604	6.632	6.660	6.689	6.717	6.745	6.774	6.802	3	6	8	11	14
3.6	6.831	6.859	6.888	6.916	6.945	6.973	7.002	7.031	7.059	7.088	3	6	9	11	14
3.7	7.117	7.146	7.175	7.204	7.233	7.262	7.291	7.320	7.349	7.378	3	6	9	12	15
3.8	7.408	7.437	7.466	7.495	7.525	7.554	7.584	7.613	7.643	7.672	3	6	9	12	15
3.9	7.702	7.732	7.761	7.791	7.821	7.850	7.880	7.910	7.940	7.970	3	6	9	12	15
4.0	8.000	8.030	8.060	8.090	8.120	8.150	8.181	8.211	8.241	8.272	3	6	9	12	15
4.1	8.302	8.332	8.363	8.393	8.424	8.454	8.485	8.515	8.546	8.577	3	6	9	12	15
4.2	8.607	8.638	8.669	8.700	8.731	8.762	8.793	8.824	8.855	8.886	3	6	9	12	15
4.3	8.917	8.948	8.979	9.010	9.041	9.073	9.104	9.135	9.167	9.198	3	6	9	12	16
4.4	9.230	9.261	9.293	9.324	9.356	9.387	9.419	9.451	9.482	9.514	3	6	9	13	16

Three halves powers, numbers 0-99

N	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.
....	0.000	1.000	2.828	5.196	8.000	11.18	14.70	18.52	22.63	27.00
1	31.62	36.48	41.57	46.87	52.38	58.09	64.00	70.09	76.37	82.82
2	89.44	96.23	103.2	110.3	117.6	125.0	132.6	140.3	148.2	156.2
3	164.3	172.6	181.0	189.6	198.3	207.1	216.0	225.1	234.2	243.6
4	253.0	262.5	272.2	282.0	291.9	301.9	312.0	322.2	332.6	343.0
5	353.6	364.2	375.0	385.8	396.8	407.9	419.1	430.3	441.7	453.2
6	464.8	476.4	488.2	500.0	512.0	524.0	536.2	548.4	560.7	573.2
7	585.7	598.3	610.9	623.7	636.6	649.5	662.6	675.7	688.9	702.2
8	715.5	729.0	742.5	756.2	769.9	783.7	797.5	811.5	825.5	839.6
9	853.8	868.1	882.4	896.9	911.4	925.9	940.6	955.3	970.2	985.0

Table 25. Fifth powers, numbers 1.0-9.9

N	0	1	2	3	4	5	6	7	8	9
1.	1.0000	1.6105	2.4883	3.7129	5.3782	7.5938	10.486	14.199	18.896	24.761
2.	32.000	40.841	51.536	64.363	79.626	97.656	118.81	143.49	172.10	205.11
3.	243.00	286.29	335.54	391.35	454.35	525.22	604.66	693.44	792.35	902.24
4.	1024.0	1158.6	1306.9	1470.1	1649.2	1845.3	2059.6	2293.5	2548.0	2824.8
5.	3125.0	3450.3	3802.0	4182.0	4591.7	5032.8	5507.3	6016.9	6563.6	7149.2
6.	7776.0	8446.0	9161.3	9924.4	10737	11603	12523	13501	14539	15640
7.	16807	18042	19349	20731	22190	23730	25355	27068	28872	30771
8.	32768	34868	37074	39390	41821	44371	47043	49842	52773	55841
9.	59049	62403	65908	69569	73390	77378	81537	85873	90392	95099

Table 26. Five halves powers, numbers 1-99

N	0	1	2	3	4	5	6	7	8	9
1.0	1.000	1.269	1.577	1.927	2.319	2.756	3.238	3.768	4.347	4.976
2.0	5.657	6.391	7.179	8.023	8.923	9.882	10.90	11.98	13.12	14.32
3.0	15.59	16.92	18.32	19.78	21.32	22.92	24.59	26.33	28.15	30.04
4.0	32.00	34.04	36.15	38.34	40.61	42.96	45.38	47.89	50.48	53.15
5.0	55.90	58.74	61.66	64.67	67.76	70.94	74.21	77.57	81.02	84.55
6.0	88.18	91.90	95.71	96.62	103.6	107.7	111.9	116.2	120.6	125.1
7.0	129.6	134.3	139.1	144.0	149.0	154.0	159.2	164.5	169.9	175.4
8.0	181.0	186.7	192.5	198.5	204.5	210.6	216.9	223.3	229.7	236.3
9.0	243.0	249.8	256.7	263.8	270.9	278.2	285.5	293.0	300.7	308.4
1	316.2	401.3	498.8	609.3	733.4	871.4	1024	1192	1375	1574
2	1789	2021	2270	2537	2822	3125	3447	3788	4149	4529
3	4930	5351	5793	6256	6741	7247	7776	8327	8901	9499
4	10119	10764	11432	12125	12842	13584	14351	15144	15963	16807
5	17678	18575	19499	20450	21428	22434	23468	24529	25619	26738
6	27885	29062	30268	31503	32768	34063	35388	36744	38130	39548
7	40996	42476	43988	45531	47106	48714	50354	52027	53732	55471
8	57243	59049	60888	62762	64669	66611	68588	70599	72645	74727
9	76843	78996	81184	83408	85668	87965	90298	92668	95075	97519

Table 27. Fifth roots and two-fifths powers

n	$n^{1/5}$	$n^{2/5}$	n	$n^{1/5}$	$n^{2/5}$	n	$n^{1/5}$	$n^{2/5}$	n	$n^{1/5}$	$n^{2/5}$
0.01	.3981	.1585	0.65	.9175	.8417	7.5	1.496	2.239	85	2.432	5.912
.02	.4573	.2091	.70	.9311	.8670	8.0	1.516	2.297	90	2.460	6.049
.03	.4959	.2460	.75	.9441	.8913	8.5	1.534	2.354	95	2.486	6.181
.04	.5253	.2759	.80	.9564	.9146	9.0	1.552	2.408	100	2.512	6.310
.05	.5493	.3017	.85	.9680	.9371	9.5	1.569	2.461	150	2.724	7.421
.06	.5697	.3245	.90	.9791	.9587	10	1.585	2.512	200	2.885	8.326
.07	.5857	.3452	.95	.9898	.9797	15	1.719	2.954	250	3.017	9.103
.08	.6034	.3641	1.0	1.000	1.000	20	1.821	3.314	300	3.129	9.791
.09	.6178	.3817	1.5	1.085	1.176	25	1.904	3.624	350	3.227	10.41
0.10	.6310	.3981	2.0	1.149	1.320	30	1.974	3.898	400	3.314	10.99
.15	.6843	.4682	2.5	1.201	1.443	35	2.036	4.146	450	3.393	11.52
.20	.7248	.5253	3.0	1.246	1.552	40	2.091	4.373	500	3.466	12.01
.25	.7579	.5743	3.5	1.285	1.651	45	2.141	4.584	550	3.532	12.48
.30	.7860	.6178	4.0	1.320	1.741	50	2.187	4.782	600	3.594	12.92
.35	.8106	.6571	4.5	1.351	1.825	55	2.229	4.968	650	3.652	13.34
.40	.8326	.6931	5.0	1.380	1.904	60	2.268	5.144	700	3.707	13.74
.45	.8524	.7266	5.5	1.406	1.978	65	2.305	5.311	750	3.758	14.13
.50	.8706	.7579	6.0	1.431	2.048	70	2.339	5.471	800	3.807	14.50
.55	.8873	.7873	6.5	1.454	2.114	75	2.371	5.624	850	3.854	14.85
.60	.9029	.8152	7.0	1.476	2.178	80	2.402	5.771	900	3.898	15.19

Table 28. Equivalent parts of feet, inches and millimeters

Inches		Feet		Mm.
Fraction	Decimal	Fraction	Decimal	
1/16	0.0625	......	0.0052	1.588
1/8	0.1250	......	0.0104	3.175
3/16	0.1875	1/64	0.0156	4.763
1/4	0.2500	......	0.0208	6.350
5/16	0.3125	......	0.0260	7.938
3/8	0.3750	1/32	0.0312	9.525
7/16	0.4375	......	0.0365	11.113
1/2	0.5000	......	0.0417	12.700
9/16	0.5625	3/64	0.0469	14.288
5/8	0.6250	......	0.0521	15.875
11/16	0.6875	......	0.0573	17.463
3/4	0.7500	1/16	0.0625	19.050
13/16	0.8125	......	0.0677	20.638
7/8	0.8750	......	0.0729	22.225
15/16	0.9375	5/64	0.0781	23.813
1	1.0000	......	0.0833	25.400
1 1/16	1.0625	......	0.0885	26.988
1 1/8	1.1250	3/32	0.0937	28.575
1 3/16	1.1875	......	0.0990	30.163
1 1/4	1.2500	......	0.1042	31.750
1 5/16	1.3125	7/64	0.1094	33.338
1 3/8	1.3750	......	0.1146	34.925
1 7/16	1.4375	......	0.1198	36.513
1 1/2	1.5000	1/8	0.1250	38.100
1 9/16	1.5625	......	0.1302	39.688
1 5/8	1.6250	......	0.1354	41.275
1 11/16	1.6875	9/64	0.1406	42.863
1 3/4	1.7500	......	0.1458	44.450
1 13/16	1.8125	......	0.1510	46.038
1 7/8	1.8750	5/32	0.1562	46.625
1 15/16	1.9375	......	0.1615	49.213
2	2.0000	......	0.1667	50.800
2 1/16	2.0625	11/64	0.1719	52.388
2 1/8	2.1250	......	0.1771	53.975
2 3/16	2.1875	......	0.1823	55.563
2 1/4	2.2500	3/16	0.1875	57.150
2 5/16	2.3125	......	0.1927	58.738
2 3/8	2.3750	......	0.1979	60.325
2 7/16	2.4375	13/64	0.2031	61.913
2 1/2	2.5000	......	0.2083	63.500
2 9/16	2.5625	......	0.2135	65.088
2 5/8	2.6250	7/32	0.2187	66.675
2 11/16	2.6875	......	0.2240	68.263
2 3/4	2.7500	......	0.2292	69.850
2 13/16	2.8125	15/64	0.2344	71.438
2 7/8	2.8750	......	0.2396	73.025
2 15/16	2.9375	......	0.2448	74.613
3	3.0000	1/4	0.2500	76.200
3 1/16	3.0625	......	0.2552	77.788
3 1/8	3.1250	......	0.2604	79.375
3 3/16	3.1875	17/64	0.2656	80.963
3 1/4	3.2500	......	0.2708	82.550
3 5/16	3.3125	......	0.2760	84.138
3 3/8	3.3750	9/32	0.2812	85.725

Inches		Feet		Mm.
Fraction	Decimal	Fraction	Decimal	
3 7/16	3.4375	......	0.2865	87.313
3 1/2	3.5000	......	0.2917	88.900
3 9/16	3.5625	19/64	0.2969	90.488
3 5/8	3.6250	......	0.3021	92.075
3 11/16	3.6875	......	0.3073	93.663
3 3/4	3.7500	5/16	0.3125	95.250
3 13/16	3.8125	......	0.3177	96.838
3 7/8	3.8750	......	0.3229	98.425
3 15/16	3.9375	21/64	0.3281	100.013
4	4.0000	......	0.3333	101.600
4 1/16	4.0625	......	0.3385	103.188
4 1/8	4.1250	11/32	0.3437	104.775
4 3/16	4.1875	......	0.3490	106.363
4 1/4	4.2500	......	0.3542	107.950
4 5/16	4.3125	23/64	0.3594	109.538
4 3/8	4.3750	......	0.3646	111.125
4 7/16	4.4375	......	0.3698	112.713
4 1/2	4.5000	3/8	0.3750	114.300
4 9/16	4.5625	......	0.3802	115.888
4 5/8	4.6250	......	0.3854	117.475
4 11/16	4.6875	25/64	0.3906	119.063
4 3/4	4.7500	......	0.3958	120.650
4 13/16	4.8125	......	0.4010	122.238
4 7/8	4.8750	13/32	0.4062	123.825
4 15/16	4.9375	......	0.4115	125.413
5	5.0000	......	0.4167	127.000
5 1/16	5.0625	27/64	0.4219	128.588
5 1/8	5.1250	......	0.4271	130.175
5 3/16	5.1875	......	0.4323	131.763
5 1/4	5.2500	7/16	0.4375	133.350
5 5/16	5.3125	......	0.4427	134.938
5 3/8	5.3750	......	0.4479	136.525
5 7/16	5.4375	29/64	0.4531	138.113
5 1/2	5.5000	......	0.4583	139.700
5 9/16	5.5625	......	0.4635	141.288
5 5/8	5.6250	15/32	0.4687	142.875
5 11/16	5.6875	......	0.4740	144.463
5 3/4	5.7500	......	0.4792	146.050
5 13/16	5.8125	31/64	0.4844	147.638
5 7/8	5.8750	......	0.4896	149.225
5 15/16	5.9375	......	0.4948	150.813
6	6.0000	1/2	0.5000	152.400
6 1/16	6.0625	......	0.5052	153.988
6 1/8	6.1250	......	0.5104	155.575
6 3/16	6.1875	33/64	0.5156	157.163
6 1/4	6.2500	......	0.5208	158.750
6 5/16	6.3125	......	0.5260	160.338
6 3/8	6.3750	17/32	0.5312	161.925
6 7/16	6.4375	......	0.5365	163.513
6 1/2	6.5000	......	0.5417	165.100
6 9/16	6.5625	35/64	0.5469	166.688
6 5/8	6.6250	......	0.5521	168.275
6 11/16	6.8750	......	0.5573	169.863
6 3/4	6.7500	9/16	0.5625	171.450

Table 28. Equivalent parts of feet, inches and millimeters—*Continued*

Inches		Feet		Mm.	Inches		Feet		Mm.
Fraction	Decimal	Fraction	Decimal		Fraction	Decimal	Fraction	Decimal	
6 13/16	6.8125	......	0.5677	173.038	9 7/16	9.4375	......	0.7865	239.713
6 7/8	6.8750	......	0.5729	174.625	9 1/2	9.5000	......	0.7917	241.300
6 15/16	6.9375	37/64	0.5781	176.213	9 9/16	9.5625	51/64	0.7969	242.888
					9 5/8	9.6250	......	0.8021	244.475
7	7.0000	......	0.5833	177.800	9 11/16	9.6875	......	0.8073	246.063
7 1/16	7.0625	......	0.5885	179.388	9 3/4	9.7500	13/16	0.8125	247.650
7 1/8	7.1250	19/32	0.5937	180.975	9 13/16	9.8125	......	0.8177	249.238
7 3/16	7.1875	......	0.5990	182.563	9 7/8	9.8750	......	0.8229	250.825
7 1/4	7.2500	......	0.6042	184.150	9 15/16	9.9375	53/64	0.8281	252.413
7 5/16	7.3125	39/64	0.6094	185.738					
7 3/8	7.3750	......	0.6146	187.325	10	10.0000	......	0.8333	254.000
7 7/16	7.4375	......	0.6198	188.913	10 1/16	10.0625	......	0.8385	255.588
7 1/2	7.5000	5/8	0.6250	190.500	10 1/8	10.1250	27/32	0.8437	257.176
7 9/16	7.5625	......	0.6302	192.088	10 3/16	10.1875	......	0.8490	258.763
7 5/8	7.6250	......	0.6354	193.675	10 1/4	10.2500	......	0.8542	260.351
7 11/16	7.6875	41/64	0.6406	195.263	10 5/16	10.3125	55/64	0.8594	261.938
7 3/4	7.7500	......	0.6458	196.850	10 3/8	10.3750	......	0.8646	263.526
7 13/16	7.8125	......	0.6510	198.438	10 7/16	10.4375	......	0.8698	265.113
7 7/8	7.8750	21/32	0.6562	200.025	10 1/2	10.5000	7/8	0.8750	266.701
7 15/16	7.9375	......	0.6615	201.613	10 9/16	10.5625	......	0.8802	268.288
					10 5/8	10.6250	......	0.8854	269.876
8	8.0000	......	0.6667	203.200	10 11/16	10.6875	57/64	0.8906	271.463
8 1/16	8.0625	43/64	0.6719	204.788	10 3/4	10.7500	......	0.8958	273.051
8 1/8	8.1250	......	0.6771	206.375	10 13/16	10.8125	......	0.9010	274.638
8 3/16	8.1875	......	0.6823	207.963	10 7/8	10.8750	29/32	0.9062	276.226
8 1/4	8.2500	11/16	0.6875	209.550	10 15/16	10.9375	......	0.9115	277.813
8 5/16	8.3125	......	0.6927	211.138					
8 3/8	8.3750	......	0.6979	212.725	11	11.0000	......	0.9167	279.401
8 7/16	8.4375	45/64	0.7031	214.313	11 1/16	11.0625	59/64	0.9219	280.988
8 1/2	8.5000	......	0.7083	215.900	11 1/8	11.1250	......	0.9271	282.576
8 9/16	8.5625	......	0.7135	217.488	11 3/16	11.1875	......	0.9323	284.163
8 5/8	8.6250	23/32	0.7187	219.075	11 1/4	11.2500	15/16	0.9375	285.751
8 11/16	8.6875	......	0.7240	220.663	11 5/16	11.3125	......	0.9427	287.338
8 3/4	8.7500	......	0.7292	222.250	11 3/8	11.3750	......	0.9479	288.926
8 13/16	8.8125	47/64	0.7344	223.838	11 7/16	11.4375	61/64	0.9531	290.513
8 7/8	8.8750	......	0.7396	225.425	11 1/2	11.5000	......	0.9583	292.101
8 15/16	8.9375	......	0.7448	227.013	11 9/16	11.5625	......	0.9635	293.688
					11 5/8	11.6250	31/32	0.9687	295.276
9	9.0000	3/4	0.7500	228.600	11 11/16	11.6875	......	0.9740	296.863
9 1/16	9.0625	......	0.7552	230.188	11 3/4	11.7500	......	0.9792	298.451
9 1/8	9.1250	......	0.7604	231.775	11 13/16	11.8125	63/64	0.9844	300.038
9 3/16	9.1875	49/64	0.7656	233.363	11 7/8	11.8750	......	0.9896	301.626
9 1/4	9.2500	......	0.7708	234.950	11 15/16	11.9375	......	0.9948	303.213
9 5/16	9.3125	......	0.7760	236.538					
9 3/8	9.3750	25/32	0.7812	238.125	12	12.0000	1	1.0000	304.801

Table 29. Compound interest. Amount of one dollar

Yr.	2½%	3%	3½%	4%	4½%	5%	6%	8%	10%
1	1.02500	1.03000	1.03500	1.04000	1.04500	1.05000	1.06000	1.08000	1.10000
2	1.05062	1.06090	1.07122	1.08160	1.09203	1.10250	1.12360	1.16640	1.21000
3	1.07689	1.09273	1.10872	1.12486	1.14117	1.15763	1.19102	1.25971	1.33100
4	1.10381	1.12551	1.14752	1.16986	1.19252	1.21551	1.26248	1.36049	1.46410
5	1.13141	1.15927	1.18769	1.21665	1.24618	1.27628	1.33823	1.46933	1.61051
6	1.15969	1.19405	1.22926	1.26532	1.30226	1.34010	1.41852	1.58687	1.77156
7	1.18869	1.22987	1.27228	1.31593	1.36086	1.40710	1.50363	1.71382	1.94872
8	1.21840	1.26677	1.31681	1.36857	1.42210	1.47746	1.59385	1.85093	2.14359
9	1.24886	1.30477	1.36290	1.42331	1.48610	1.55133	1.68948	1.99900	2.35795
10	1.28008	1.34392	1.41060	1.48024	1.55297	1.62889	1.79085	2.15892	2.59374
11	1.31209	1.38423	1.45997	1.53945	1.62285	1.71034	1.89830	2.33164	2.85312
12	1.34489	1.42576	1.51107	1.60103	1.69588	1.79586	2.01220	2.51817	3.13843
13	1.37851	1.46853	1.56396	1.66507	1.77220	1.88565	2.13293	2.71962	3.45227
14	1.41297	1.51259	1.61869	1.73168	1.85194	1.97993	2.26090	2.93719	3.79750
15	1.44830	1.55797	1.67535	1.80094	1.93528	2.07893	2.39656	3.17217	4.17725
16	1.48451	1.60471	1.73399	1.87298	2.02237	2.18287	2.54035	3.42594	4.59497
17	1.52162	1.65285	1.79467	1.94790	2.11338	2.29202	2.69277	3.70002	5.05447
18	1.55966	1.70243	1.85749	2.02582	2.20848	2.40662	2.85434	3.99602	5.55992
19	1.59865	1.75351	1.92250	2.10685	2.30786	2.52695	3.02560	4.31570	6.11591
20	1.63862	1.80611	1.98979	2.19112	2.41171	2.65330	3.20714	4.66096	6.72750
21	1.67958	1.86029	2.05943	2.27877	2.52024	2.78596	3.39956	5.03383	7.40025
22	1.72157	1.91610	2.13151	2.36992	2.63365	2.92526	3.60354	5.43654	8.14027
23	1.76461	1.97359	2.20611	2.46472	2.75217	3.07152	3.81975	5.87146	8.95430
24	1.80873	2.03279	2.28333	2.56330	2.87601	3.22510	4.04893	6.34118	9.84973
25	1.85394	2.09378	2.36324	2.66584	3.00543	3.38635	4.29187	6.84848	10.8347
26	1.90029	2.15659	2.44596	2.77247	3.14068	3.55567	4.54938	7.39635	11.9182
27	1.94780	2.22129	2.53157	2.88337	3.28201	3.73346	4.82235	7.98806	13.1100
28	1.99650	2.28793	2.62017	2.99870	3.42970	3.92013	5.11169	8.62711	14.4210
29	2.04640	2.35657	2.71188	3.11865	3.58404	4.11614	5.41839	9.31727	15.8631
30	2.09757	2.42726	2.80679	3.24340	3.74532	4.32194	5.74349	10.0627	17.4494
31	2.15000	2.50008	2.90503	3.37313	3.91386	4.53804	6.08810	10.8677	19.1943
32	2.20376	2.57508	3.00671	3.50806	4.08998	4.76494	6.45339	11.7371	21.1138
33	2.25885	2.65234	3.11194	3.64838	4.27403	5.00319	6.84059	12.6701	23.2252
34	2.31532	2.73191	3.22086	3.79432	4.46636	5.25335	7.25103	13.6901	25.5477
35	2.37321	2.81386	3.33359	3.94609	4.66735	5.51602	7.68609	14.7853	28.1024
36	2.43254	2.89828	3.45027	4.10393	4.87738	5.79182	8.14725	15.9682	30.9127
37	2.49335	2.98523	3.57103	4.26809	5.09686	6.08141	8.63609	17.2456	34.0040
38	2.55568	3.07478	3.69601	4.43881	5.32622	6.38548	9.15425	18.6253	37.4043
39	2.61957	3.16703	3.82537	4.61637	5.56590	6.70475	9.70351	20.1153	41.1448
40	2.68506	3.26204	3.95926	4.80102	5.81636	7.03999	10.2857	21.7245	45.2593
41	2.75219	3.35990	4.09783	4.99306	6.07810	7.39199	10.9029	23.4625	49.7852
42	2.82100	3.46070	4.24126	5.19278	6.35161	7.76159	11.5570	25.3395	54.7637
43	2.89152	3.56452	4.38970	5.40050	6.63744	8.14967	12.2505	27.3666	60.2401
44	2.96381	3.67145	4.54334	5.61652	6.93612	8.55715	12.9855	29.5560	66.2641
45	3.03790	3.78160	4.70236	5.84118	7.24825	8.98501	13.7646	31.9205	72.8905
46	3.11385	3.89504	4.86694	6.07482	7.57442	9.43426	14.5905	34.4741	80.1795
47	3.19169	4.01190	5.03728	6.31782	7.91527	9.90597	15.4659	37.2320	88.1975
48	3.27149	4.13225	5.21359	6.57053	8.27145	10.4013	16.3939	40.2106	97.0172
49	3.35328	4.25622	5.39606	6.83335	8.64367	10.9213	17.3775	43.4274	106.719
50	3.43711	4.38391	5.58493	7.10668	9.03264	11.4674	18.4202	46.9016	117.391
51	3.52304	4.51542	5.78040	7.39095	9.43910	12.0408	19.5254	50.6537	129.130
52	3.61111	4.65089	5.98271	7.68659	9.86386	12.6428	20.6969	54.7060	142.043
53	3.70139	4.79041	6.19211	7.99405	10.3077	13.2750	21.9387	59.0825	156.247
54	3.79392	4.93412	6.40883	8.31381	10.7716	13.9387	23.2550	63.8091	171.872
55	3.88877	5.08215	6.63314	8.64637	11.2563	14.6356	24.6503	68.9139	189.059
56	3.98599	5.23461	6.86530	8.99222	11.7628	15.3674	26.1293	74.4270	207.965
57	4.08564	5.39165	7.10559	9.35191	12.2922	16.1358	27.6971	80.3811	228.762
58	4.18778	5.55340	7.35428	9.72599	12.8453	16.9426	29.3589	86.8116	251.638
59	4.29248	5.72000	7.61168	10.1150	13.4234	17.7897	31.1205	93.7565	276.801
60	4.39979	5.89160	7.87809	10.5196	14.0274	18.6792	32.9877	101.257	304.482

Table 30. Compound discount. Present value of one dollar

Yr.	2½%	3%	3½%	4%	4½%	5%	6%	8%	10%
1	.97561	.97087	.96618	.96154	.95694	.95238	.94340	.92593	.90909
2	.95181	.94260	.93351	.92456	.91573	.90703	.89000	.85734	.82645
3	.92860	.91514	.90194	.88900	.87630	.86384	.83962	.79383	.75131
4	.90595	.88849	.87144	.85480	.83856	.82270	.79209	.73503	.68301
5	.88385	.86261	.84197	.82193	.80245	.78353	.74726	.68058	.62092
6	.86230	.83748	.81350	.79031	.76790	.74622	.70496	.63017	.56447
7	.84127	.81309	.78599	.75992	.73483	.71068	.66506	.58349	.51316
8	.82075	.78941	.75941	.73069	.70319	.67684	.62741	.54027	.46651
9	.80073	.76642	.73373	.70259	.67290	.64461	.59190	.50025	.42410
10	.78120	.74409	.70892	.67556	.64393	.61391	.55839	.46319	.38554
11	.76214	.72242	.68495	.64958	.61620	.58468	.52679	.42888	.35049
12	.74356	.70138	.66178	.62460	.58966	.55684	.49697	.39711	.31863
13	.72542	.68095	.63940	.60057	.56427	.53032	.46884	.36770	.28966
14	.70773	.66112	.61778	.57748	.53997	.50507	.44230	.34046	.26333
15	.69047	.64186	.59689	.55526	.51672	.48102	.41727	.31524	.23939
16	.67363	.62317	.57671	.53391	.49447	.45811	.39365	.29189	.21763
17	.65720	.60502	.55720	.51337	.47318	.43630	.37136	.27027	.19784
18	.64117	.58739	.53836	.49363	.45280	.41552	.35034	.25025	.17986
19	.62553	.57029	.52016	.47464	.43330	.39573	.33051	.23171	.16351
20	.61027	.55368	.50257	.45639	.41464	.37689	.31180	.21455	.14864
21	.59539	.53755	.48557	.43883	.39679	.35894	.29416	.19866	.13513
22	.58086	.52189	.46915	.42196	.37970	.34185	.27751	.18394	.12285
23	.56670	.50669	.45329	.40573	.36335	.32557	.26180	.17032	.11168
24	.55288	.49193	.43796	.39012	.34770	.31007	.24698	.15770	.10153
25	.53939	.47761	.42315	.37512	.33273	.29530	.23300	.14602	.09230
26	.52623	.46369	.40884	.36069	.31840	.28124	.21981	.13520	.08391
27	.51340	.45019	.39501	.34682	.30469	.26785	.20737	.12519	.07628
28	.50088	.43708	.38165	.33348	.29157	.25509	.19563	.11591	.06934
29	.48866	.42435	.36875	.32065	.27901	.24295	.18456	.10733	.06304
30	.47674	.41199	.35628	.30832	.26700	.23138	.17411	.09938	.05731
31	.46511	.39999	.34423	.29646	.25550	.22036	.16425	.09202	.05210
32	.45377	.38834	.33259	.28506	.24450	.20987	.15496	.08520	.04736
33	.44270	.37703	.32134	.27409	.23397	.19987	.14619	.07889	.04306
34	.43191	.36604	.31048	.26355	.22390	.19035	.13791	.07305	.03914
35	.42137	.35538	.29998	.25342	.21425	.18129	.13011	.06763	.03558
36	.41109	.34503	.28983	.24367	.20503	.17266	.12274	.06262	.03235
37	.40107	.33498	.28003	.23430	.19620	.16444	.11579	.05799	.02941
38	.39128	.32523	.27056	.22529	.18775	.15661	.10924	.05369	.02673
39	.38174	.31575	.26141	.21662	.17967	.14915	.10306	.04971	.02430
40	.37243	.30656	.25257	.20829	.17193	.14205	.09722	.04603	.02209
41	.36335	.29763	.24403	.20028	.16453	.13528	.09172	.04262	.02009
42	.35448	.28896	.23578	.19257	.15744	.12884	.08653	.03946	.01826
43	.34584	.28054	.22781	.18517	.15066	.12270	.08163	.03654	.01660
44	.33740	.27237	.22010	.17805	.14417	.11686	.07701	.03383	.01509
45	.32917	.26444	.21266	.17120	.13796	.11130	.07265	.03133	.01372
46	.32115	.25674	.20547	.16461	.13202	.10600	.06854	.02901	.01247
47	.31331	.24926	.19852	.15828	.12634	.10095	.06466	.02686	.01134
48	.30567	.24200	.19181	.15219	.12090	.09614	.06100	.02487	.01031
49	.29822	.23495	.18532	.14634	.11569	.09156	.05755	.02303	.00937
50	.29094	.22811	.17905	.14071	.11071	.08720	.05429	.02132	.00852
51	.28385	.22146	.17300	.13530	.10594	.08305	.05122	.01974	.00774
52	.27699	.21501	.16714	.13010	.10138	.07910	.04832	.01828	.00704
53	.27017	.20875	.16150	.12509	.09701	.07533	.04558	.01693	.00640
54	.26358	.20267	.15603	.12028	.09284	.07174	.04300	.01567	.00582
55	.25715	.19677	.15076	.11566	.08884	.06833	.04057	.01451	.00529
56	.25088	.19104	.14566	.11121	.08501	.06507	.03827	.01344	.00481
57	.24476	.18547	.14073	.10693	.08135	.06197	.03610	.01244	.00437
58	.23879	.18007	.13598	.10282	.07785	.05902	.03406	.01152	.00397
59	.23296	.17483	.13138	.09886	.07450	.05621	.03213	.01067	.00361
60	.22728	.16973	.12693	.09506	.07129	.05354	.03031	.00988	.00328

Table 31. Annuities. Amount of one dollar per annum

Yr.	2½%	3%	3½%	4%	4½%	5%	6%	7%	8%
1	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
2	2.02500	2.03000	2.03500	2.04000	2.04500	2.05000	2.06000	2.07000	2.08000
3	3.07562	3.09090	3.10623	3.12160	3.13702	3.15250	3.18360	3.21490	3.24640
4	4.15252	4.18363	4.21494	4.24646	4.27819	4.31013	4.37462	4.43994	4.50611
5	5.25633	5.30914	5.36247	5.41632	5.47071	5.52563	5.63709	5.75074	5.86660
6	6.38774	6.46841	6.55015	6.63298	6.71689	6.80191	6.97532	7.15329	7.33593
7	7.54743	7.66246	7.77941	7.89829	8.01915	8.14201	8.39384	8.65402	8.92280
8	8.73612	8.89234	9.05169	9.21423	9.38001	9.54911	9.89747	10.2598	10.6366
9	9.95452	10.1591	10.3685	10.5828	10.8021	11.0266	11.4913	11.9780	12.4876
10	11.2034	11.4639	11.7314	12.0061	12.2882	12.5779	13.1808	13.8165	14.4866
11	12.4835	12.8078	13.1420	13.4864	13.8412	14.2068	14.9716	15.7836	16.6455
12	13.7956	14.1920	14.6020	15.0258	15.4640	15.9171	16.8699	17.8885	18.9771
13	15.1404	15.6178	16.1130	16.6268	17.1599	17.7130	18.8821	20.1406	21.4953
14	16.5190	17.0863	17.6770	18.2919	18.9321	19.5986	21.0151	22.5505	24.2149
15	17.9319	18.5989	19.2957	20.0236	20.7840	21.5786	23.2760	25.1290	27.1521
16	19.3802	20.1569	20.9710	21.8245	22.7193	23.6575	25.6725	27.8881	30.3243
17	20.8647	21.7616	22.7050	23.6975	24.7417	25.8404	28.2129	30.8402	33.7502
18	22.3864	23.4144	24.4997	25.6454	26.8551	28.1324	30.9057	33.9990	37.4502
19	23.9460	25.1169	26.3572	27.6712	29.0636	30.5390	33.7600	37.3790	41.4463
20	25.5447	26.8704	28.2797	29.7781	31.3714	33.0660	36.7856	40.9955	45.7620
21	27.1833	28.6765	30.2695	31.9692	33.7831	35.7193	39.9927	44.8652	50.4229
22	28.8629	30.5368	32.3289	34.2480	36.3034	38.5052	43.3923	49.0057	55.4568
23	30.5844	32.4529	34.4604	36.6179	38.9370	41.4305	46.9958	53.4361	60.8933
24	32.3490	34.4265	36.6665	39.0826	11.6892	44.5020	50.8156	58.1767	66.7648
25	34.1578	36.4593	38.9499	41.6459	44.5652	47.7271	54.8645	63.2490	73.1059
26	36.0117	38.5530	41.3131	44.3117	47.5706	51.1135	59.1564	68.6765	79.9544
27	37.9120	40.7096	43.7591	47.0842	50.7113	54.6691	63.7058	74.4838	87.3508
28	39.8598	42.9309	46.2906	49.9676	53.9933	58.4026	68.5281	80.6977	95.3388
29	41.8563	45.2189	48.9108	52.9663	57.4230	62.3227	73.6398	87.3465	103.966
30	43.9027	47.5754	51.6227	56.0849	61.0071	66.4389	79.0582	94.4608	113.283
31	46.0003	50.0027	54.4295	59.3283	64.7524	70.7608	84.8017	102.073	123.346
32	48.1503	52.5028	57.3345	62.7015	68.6662	75.2988	90.8898	110.218	134.214
33	50.3540	55.0778	60.3412	66.2095	72.7562	80.0638	97.3432	118.933	145.951
34	52.6129	57.7302	63.4532	69.8579	77.0303	85.0670	104.184	128.259	158.627
35	54.9282	60.4621	66.6740	73.6522	81.4966	90.3203	111.435	138.237	172.317
36	57.3014	63.2759	70.0076	77.5983	86.1640	95.8363	119.121	148.913	187.102
37	59.7340	66.1742	73.4579	81.7023	91.0413	101.628	127.268	160.337	203.070
38	62.2273	69.1595	77.0289	85.9703	96.1382	107.710	135.904	172.561	220.316
39	64.7830	72.2342	80.7249	90.4092	101.464	114.095	145.058	185.640	238.941
40	67.4026	75.4013	84.5503	95.0255	107.030	120.800	154.762	199.635	259.057
41	70.0876	78.6633	88.5095	99.8265	112.847	127.840	165.048	214.610	280.781
42	72.8398	82.0232	92.6074	104.820	118.925	135.232	175.951	230.632	304.244
43	75.6608	85.4839	96.8486	110.012	125.276	142.993	187.508	247.777	329.583
44	78.5523	89.0484	101.238	115.413	131.914	151.143	199.758	266.121	356.950
45	81.5161	92.7199	105.782	121.029	138.850	159.700	212.744	285.749	386.506
46	84.5540	96.5015	110.484	126.871	146.098	168.685	226.508	306.752	418.426
47	87.6679	100.397	115.351	132.945	153.673	178.119	241.099	329.224	452.900
48	90.8596	104.408	120.388	139.263	161.588	188.025	256.565	353.270	490.132
49	94.1311	108.541	125.602	145.834	169.859	198.427	272.958	378.999	530.343
50	97.4844	112.797	130.998	152.667	178.503	209.348	290.336	406.529	573.770
51	100.921	117.181	136.583	159.774	187.536	220.815	308.756	435.986	620.672
52	104.444	121.696	142.363	167.165	196.975	232.856	328.281	467.505	671.326
53	108.056	126.347	148.346	174.851	206.839	245.499	348.978	501.230	726.032
54	111.757	131.137	154.538	182.845	217.146	258.774	370.917	537.316	785.114
55	115.551	136.072	160.947	191.159	227.918	272.713	394.172	575.929	848.923
56	119.440	141.154	167.580	199.806	239.174	287.348	418.822	617.244	917.837
57	123.426	146.388	174.445	208.798	250.937	302.716	444.952	661.451	992.264
58	127.511	151.780	181.551	218.150	263.229	318.851	472.649	708.752	1072.65
59	131.699	157.333	188.905	227.876	276.075	335.794	502.008	759.365	1159.46
60	135.992	163.053	196.517	237.991	289.498	353.584	533.128	813.520	1253.21

Table 32. Annuities. Present value of one dollar per annum

Yr.	2½%	3%	3½%	4%	4½%	5%	6%	7%	8%
1	0.97561	0.97087	0.96618	0.96154	0.95694	0.95238	0.94340	0.93458	0.92593
2	1.92742	1.91347	1.89969	1.88609	1.87267	1.85941	1.83339	1.80802	1.78326
3	2.85602	2.82861	2.80164	2.77509	2.74896	2.72325	2.67301	2.62432	2.57710
4	3.76197	3.71710	3.67308	3.62990	3.58753	3.54595	3.46511	3.38721	3.31213
5	4.64583	4.57971	4.51505	4.45182	4.38998	4.32948	4.21236	4.10020	3.99271
6	5.50812	5.41719	5.32855	5.24214	5.15787	5.07569	4.91732	4.76654	4.62288
7	6.34939	6.23028	6.11454	6.00205	5.89270	5.78637	5.58238	5.38929	5.20637
8	7.17014	7.01969	6.87396	6.73275	6.59589	6.46321	6.20979	5.97130	5.74664
9	7.97087	7.78611	7.60769	7.43533	7.26879	7.10782	6.80169	6.51523	6.24689
10	8.75206	8.53020	8.31661	8.11090	7.91272	7.72173	7.36009	7.02358	6.71008
11	9.51421	9.25262	9.00155	8.76048	8.52892	8.30641	7.88687	7.49867	7.13896
12	10.2578	9.95400	9.66333	9.38507	9.11858	8.86325	8.38384	7.94269	7.53608
13	10.9832	10.6350	10.3027	9.98565	9.68285	9.39357	8.85268	8.35765	7.90378
14	11.6909	11.2961	10.9205	10.5631	10.2228	9.89864	9.29498	8.74547	8.24424
15	12.3814	11.9379	11.5174	11.1184	10.7396	10.3797	9.71225	9.10791	8.55948
16	13.0550	12.5611	12.0941	11.6523	11.2340	10.8378	10.1059	9.44665	8.85137
17	13.7122	13.1661	12.6513	12.1657	11.7072	11.2741	10.4773	9.76322	9.12164
18	14.3534	13.7535	13.1897	12.6593	12.1600	11.6896	10.8276	10.0591	9.37189
19	14.9789	14.3238	13.7098	13.1339	12.5933	12.0853	11.1581	10.3356	9.60360
20	15.5892	14.8775	14.2124	13.5903	13.0079	12.4622	11.4699	10.5940	9.81815
21	16.1846	15.4150	14.6980	14.0292	13.4047	12.8212	11.7641	10.8355	10.0168
22	16.7654	15.9369	15.1671	14.4511	13.7844	13.1630	12.0416	11.0612	10.2007
23	17.3321	16.4436	15.6204	14.8568	14.1478	13.4886	12.3034	11.2722	10.3711
24	17.8850	16.9355	16.0584	15.2470	14.4955	13.7986	12.5504	11.4693	10.5288
25	18.4244	17.4132	16.4815	15.6221	14.8282	14.0939	12.7834	11.6536	10.6748
26	18.9506	17.8768	16.8904	15.9828	15.1466	14.3752	13.0032	11.8258	10.8100
27	19.4640	18.3270	17.2854	16.3296	15.4513	14.6430	13.2105	11.9867	10.9352
28	19.9649	18.7641	17.6670	16.6631	15.7429	14.8981	13.4062	12.1371	11.0511
29	20.4536	19.1885	18.0358	16.9837	16.0219	15.1411	13.5907	12.2777	11.1584
30	20.9303	19.6004	18.3921	17.2920	16.2889	15.3725	13.7648	12.4090	11.2578
31	21.3954	20.0004	18.7363	17.5885	16.5444	15.5928	13.9291	12.5318	11.3498
32	21.8492	20.3888	19.0689	17.8736	16.7889	15.8027	14.0840	12.6466	11.4350
33	22.2919	20.7658	19.3902	18.1477	17.0229	16.0026	14.2302	12.7538	11.5139
34	22.7238	21.1318	19.7007	18.4112	17.2468	16.1929	14.3681	12.8540	11.5869
35	23.1452	21.4872	20.0007	18.6646	17.4610	16.3742	14.4983	12.9477	11.6546
36	23.5563	21.8323	20.2905	18.9083	17.6660	16.5469	14.6210	13.0352	11.7172
37	23.9573	22.1672	20.5705	19.1426	17.8622	16.7113	14.7368	13.1170	11.7752
38	24.3486	22.4925	20.8411	19.3679	18.0500	16.8679	14.8460	13.1935	11.8289
39	24.7303	22.8082	21.1025	19.5845	18.2297	17.0170	14.9491	13.2649	11.8786
40	25.1028	23.1148	21.3551	19.7928	18.4016	17.1591	15.0463	13.3317	11.9246
41	25.4661	23.4124	21.5991	19.9931	18.5661	17.2944	15.1380	13.3941	11.9672
42	25.8206	23.7014	21.8348	20.1856	18.7236	17.4232	15.2245	13.4525	12.0067
43	26.1665	23.9819	22.0627	20.3708	18.8742	17.5459	15.3062	13.5070	12.0432
44	26.5039	24.2543	22.2828	20.5488	19.0184	17.6628	15.3832	13.5579	12.0771
45	26.8330	24.5187	22.4955	20.7200	19.1564	17.7741	15.4558	13.6055	12.1084
46	27.1542	24.7755	22.7009	20.8847	19.2884	17.8801	15.5244	13.6500	12.1374
47	27.4675	25.0247	22.8994	21.0429	19.4147	17.9810	15.5890	13.6916	12.1643
48	27.7732	25.2667	23.0913	21.1951	19.5356	18.0772	15.6500	13.7305	12.1891
49	28.0714	25.5017	23.2766	21.3415	19.6513	18.1687	15.7076	13.7668	12.2122
50	28.3623	25.7298	23.4556	21.4822	19.7620	18.2559	15.7619	13.8008	12.2335
51	28.6462	25.9512	23.6286	21.6175	19.8680	18.3390	15.8131	13.8325	12.2532
52	28.9231	26.1662	23.7958	21.7476	19.9693	18.4181	15.8614	13.8621	12.2715
53	29.1933	26.3750	23.9573	21.8727	20.0663	18.4934	15.9070	13.8898	12.2884
54	29.4568	26.5777	24.1133	21.9930	20.1592	18.5651	15.9500	13.9157	12.3041
55	29.7140	26.7744	24.2641	22.1086	20.2480	18.6335	15.9905	13.9399	12.3186
56	29.9649	26.9655	24.4097	22.2198	20.3330	18.6985	16.0288	13.9626	12.3321
57	30.2096	27.1509	24.5505	22.3268	20.4144	18.7605	16.0649	13.9837	12.3445
58	30.4484	27.3310	24.6864	22.4296	20.4922	18.8195	16.0990	14.0035	12.3560
59	30.6814	27.5058	24.8178	22.5284	20.5667	18.8758	16.1311	14.0219	12.3667
60	30.9087	27.6756	24.9447	22.6235	20.6380	18.9293	16.1614	14.0392	12.3765

Table 33. Sinking fund. Annuity which will amount to one dollar

Yr.	2½%	3%	3½%	4%	4½%	5%	6%	7%	8%
1	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
2	.493827	.492611	.491100	.490196	.488997	.487805	.485437	.483092	.480769
3	.325137	.323530	.321934	.320349	.318773	.317209	.314110	.311052	.308033
4	.240818	.239027	.237251	.235490	.233744	.232012	.228591	.225228	.221921
5	.190247	.188355	.186481	.184627	.182792	.180975	.177396	.173891	.170456
6	.156550	.154598	.152668	.150762	.148878	.147017	.143363	.139796	.136315
7	.132495	.130506	.128544	.126610	.124701	.122820	.119135	.115553	.112072
8	.114467	.112456	.110477	.108528	.106609	.104722	.101036	.097468	.094015
9	.100457	.098434	.096446	.094493	.092575	.090690	.087022	.083486	.080079
10	.089259	.087231	.085241	.083291	.081379	.079505	.075868	.072377	.069029
11	.080106	.078077	.076092	.074149	.072248	.070389	.066793	.063357	.060076
12	.072487	.070462	.068484	.066552	.064666	.062825	.059277	.055902	.052695
13	.066048	.064030	.062062	.060144	.058275	.056456	.052960	.049651	.046522
14	.060536	.058526	.056571	.054669	.052820	.051024	.047585	.044345	.041297
15	.055766	.053767	.051825	.049941	.048114	.046342	.042963	.039795	.036829
16	.051599	.049611	.047685	.045820	.044015	.042270	.038952	.035858	.032977
17	.047928	.045953	.044043	.042199	.040418	.038699	.035445	.032425	.029629
18	.044670	.042709	.040817	.038993	.037237	.035546	.032357	.029413	.026702
19	.041760	.039814	.037940	.036139	.034407	.032745	.029621	.026753	.024128
20	.039147	.037216	.035361	.033582	.031876	.030243	.027185	.024393	.021852
21	.036787	.034872	.033037	.031280	.029601	.027996	.025005	.022289	.019832
22	.034646	.032747	.030932	.029199	.027546	.025971	.023046	.020406	.018032
23	.032696	.030814	.029019	.027309	.025682	.024137	.021278	.018714	.016422
24	.030913	.029047	.027273	.025587	.023987	.022471	.019679	.017189	.014978
25	.029276	.027428	.025674	.024012	.022439	.020952	.018227	.015811	.013679
26	.027768	.025938	.024205	.022567	.021021	.019564	.016904	.014561	.012507
27	.026377	.024564	.022852	.021239	.019719	.018292	.015697	.013426	.011448
28	.025088	.023293	.021603	.020013	.018521	.017123	.014593	.012392	.010489
29	.023891	.022115	.020445	.018880	.017415	.016046	.013580	.011449	.009618
30	.022777	.021019	.019371	.017830	.016392	.015051	.012649	.010586	.008827
31	.021739	.019999	.018372	.016855	.015443	.014132	.011792	.009797	.008107
32	.020768	.019047	.017442	.015949	.014563	.013280	.011002	.009073	.007451
33	.019859	.018156	.016572	.015104	.013745	.012490	.010273	.008408	.006852
34	.019007	.017322	.015760	.014315	.012982	.011755	.009598	.007797	.006304
35	.018205	.016539	.014998	.013577	.012270	.011072	.008974	.007234	.005803
36	.017451	.015804	.014284	.012887	.011606	.010434	.008395	.006715	.005345
37	.016741	.015112	.013613	.012240	.010984	.009840	.007857	.006237	.004924
38	.016070	.014459	.012982	.011632	.010402	.009284	.007358	.005795	.004539
39	.015436	.013844	.012388	.011061	.009856	.008765	.006894	.005387	.004185
40	.014836	.013262	.011827	.010523	.009343	.008278	.006462	.005009	.003860
41	.014268	.012712	.011298	.010017	.008862	.007822	.006059	.004660	.003562
42	.013728	.012192	.010798	.009540	.008409	.007395	.005683	.004336	.003287
43	.013217	.011698	.010325	.009090	.007982	.006993	.005333	.004036	.003034
44	.012730	.011230	.009878	.008665	.007581	.006616	.005006	.003758	.002802
45	.012267	.010785	.009453	.008262	.007202	.006262	.004701	.003499	.002587
46	.011826	.010363	.009051	.007882	.006845	.005928	.004415	.003260	.002390
47	.011407	.009961	.008669	.007522	.006507	.005614	.004148	.003037	.002208
48	.011006	.009578	.008306	.007181	.006189	.005318	.003898	.002831	.002040
49	.010623	.009213	.007962	.006857	.005887	.005040	.003664	.002639	.001886
50	.010258	.008866	.007634	.006550	.005602	.004777	.003444	.002460	.001743
51	.009909	.008534	.007322	.006259	.005332	.004529	.003239	.002294	.001611
52	.009574	.008217	.007024	.005982	.005077	.004295	.003046	.002139	.001490
53	.009254	.007915	.006741	.005719	.004835	.004073	.002866	.001995	.001377
54	.008948	.007626	.006471	.005469	.004605	.003864	.002696	.001861	.001274
55	.008654	.007349	.006213	.005231	.004388	.003667	.002537	.001736	.001178
56	.008373	.007085	.005967	.005005	.004181	.003480	.002388	.001620	.001090
57	.008102	.006831	.005732	.004789	.003985	.003303	.002247	.001512	.001008
58	.007842	.006588	.005508	.004584	.003799	.003136	.002116	.001411	.000932
59	.007593	.006356	.005294	.004388	.003622	.002978	.001992	.001317	.000862
60	.007353	.006133	.005089	.004202	.003454	.002828	.001876	.001229	.000798

Table 34. Equivalents of linear measure

Kilometer (Km.)	Meter (M.)	Centimeter (Cm.)	Millimeter (Mm.)	Inch (In.)	Foot (Ft.)	Yard (Yd.)	Micron (μ)	Millimicron ($\mu\mu$)
1	1000	10^5	10^6	39,370	3280.83	1093.61	10^9	10^{12}
0.001	1	100	1000	39.37	3.28083	1.09361	10^6	10^9
10^{-5}	.01	1	10	0.3937	0.032808	0.0109361	10^4	10^7
10^{-6}	.001	0.1	1	0.03937	0.0032808	0.0010936	10^3	10^6
2.54×10^{-5}	0.02540	2.540	25.40005	1	0.08333	0.02778	25,400	2.54×10^7
3.048×10^{-4}	0.304801	30.480	304.801	12	1	0.33333	304,801	3.048×10^8
9.144×10^{-4}	0.914402	91.440	914.4	36	3	1	914,402	9.144×10^8
10^{-9}	10^{-6}	10^{-4}	10^{-3}	3.937×10^{-5}	3.2808×10^{-6}	1.0936×10^{-6}	1	10^3
10^{-12}	10^{-9}	10^{-7}	10^{-6}	3.937×10^{-8}	3.2808×10^{-9}	1.0936×10^{-9}	10^{-3}	1

Table 35. Equivalents

Kilogram (Kg.)	Gram (Gm.)	Milligram (Mg.)	Grain (Gr.)	Avoirdupois		Troy		
				Pounds (Lb.)	Ounce (Oz.)	Pound (Lb.)	Ounce (Oz.)	Penny-weight (Dwt.)
1	1000	10^6	15,432.4	2.20462	35.2740	2.67923	32.1507	643.015
0.001	1	1000	15.432	2.205×10^{-3}	3.527×10^{-2}	2.679×10^{-3}	3.215×10^{-2}	0.64301
10^{-6}	0.001	1	0.01543	2.205×10^{-6}	3.527×10^{-5}	2.679×10^{-6}	3.215×10^{-5}	6.430×10^{-4}
6.430×10^{-5}	0.06480	64.799	1	1.429×10^{-4}	2.286×10^{-3}	1.736×10^{-4}	2.083×10^{-3}	0.04167
0.45359	453.592	453,592.4	7000	1	16	1.21528	14.583	291.667
0.02335	23.3495	28,349.53	437.5	0.0625	1	0.07595	0.91146	18.229
0.37324	373.242	373,241.8	5760	0.82286	13.166	1	12	240
0.03110	31.103	31,103.5	480	0.06857	1.09714	0.08333	1	20
1.555×10^{-3}	1.55517	1555.17	24	3.429×10^{-3}	0.05486	4.167×10^{-3}	0.05	1
0.03110	31.103	31,103.5	480	0.06857	1.09714	0.08333	1	20
3.888×10^{-3}	3.88794	3887.94	60	8.571×10^{-3}	0.13714	0.01042	0.125	2.5
1.296×10^{-3}	1.29598	1295.98	20	2.857×10^{-3}	0.04571	3.472×10^{-3}	0.04167	0.83333
907.185	9.072×10^5	9.072×10^8	14×10^6	2000	32,000	2,430.56	29,166.67	583,334
1016.05	1.016×10^6	1.016×10^9	1.568×10^7	2240	35,840	2722.22	32,666.7	653,334
1000	10^6	10^9	1.543×10^7	2204.62	35,274	2679.23	32,150.77	643,015

Table 36. Miscellaneous

When unit column is entered as

Units	Cu. ft.	Gallons	Cu. ft.	Liters	Gallons	Liters	Kg. per min.	Tons per 24 hr.
	read							
	Gallons	Cu. ft.	Liters	Cu. ft.	Liters	Gallons	Tons per 24 hr.	Cu. ft. per min. $\times\theta$*
1	7.48	0.1337	28.32	0.0353	3.785	0.264	1.587	1.389
2	14.96	0.2674	56.63	0.0706	7.571	0.528	3.175	2.778
3	22.44	0.4010	84.95	0.1059	11.356	0.793	4.762	4.167
4	29.92	0.5347	113.27	0.1413	15.142	1.057	6.349	5.556
5	37.40	0.6684	141.58	0.1766	18.927	1.321	7.937	6.944
6	44.88	0.8021	169.90	0.2119	22.713	1.585	9.524	8.333
7	52.36	0.9358	198.22	0.2472	26.498	1.849	11.111	9.722
8	59.84	1.0694	226.54	0.2825	30.283	2.113	12.699	11.111
9	67.32	1.2031	254.85	0.3178	34.069	2.378	14.286	12.500

* θ = weight per cubic foot.

of masses and weights

Apothecaries' Ounce (℥)	Apothecaries' Dram (ʒ)	Apothecaries' Scruple (℈)	Short (net) ton	Long (gross) ton	Metric ton
32.1507	257.206	771.618	1.102×10^{-3}	9.842×10^{-4}	0.001
3.215×10^{-2}	0.25721	0.77162	1.102×10^{-6}	9.842×10^{-7}	10^{-6}
3.215×10^{-5}	2.572×10^{-4}	7.716×10^{-4}	1.102×10^{-9}	9.842×10^{-10}	10^{-9}
2.083×10^{-3}	0.01667	0.05	7.143×10^{-8}	6.378×10^{-8}	6.480×10^{-8}
14.583	116.667	350	0.0005	4.464×10^{-4}	4.536×10^{-4}
0.91146	7.29166	21.875	3.125×10^{-5}	2.790×10^{-5}	2.835×10^{-5}
12	96	288	4.114×10^{-4}	3.673×10^{-4}	3.732×10^{-4}
1	8	24	3.429×10^{-5}	3.061×10^{-5}	3.110×10^{-5}
0.05	0.4	1.2	1.714×10^{-6}	1.531×10^{-6}	1.555×10^{-6}
1	8	24	3.429×10^{-5}	3.061×10^{-5}	3.110×10^{-5}
0.125	1	3	4.286×10^{-6}	3.826×10^{-6}	3.888×10^{-6}
0.04167	0.33333	1	1.429×10^{-6}	1.275×10^{-6}	1.296×10^{-6}
29,166 67	233,334	7×10^{5}	1	0.89286	0.90718
32,666.7	261,334	7.84×10^{5}	1.12	1	1.01605
32,150.77	257,206	7.716×10^{5}	1.1023	0.98421	1

equivalents

When unit column is entered as

Units	Gr. per gal.	Gm. per liter	Slope in per cent.	Inches per foot	Slope in per cent.	Degrees	Inches per foot	Degrees
	read							
	Gm. per liter	Gr. per gal.	Inches per foot	Slope in per cent.	Degrees	Slope in per cent.	Degrees	Inches per foot
1	0.017	58.4	0.12	8.33	0° 34′	1.75	4° 46′	0.210
2	0.034	116.8	0.24	16.67	1° 9′	3.49	9° 28′	0.419
3	0.051	175.3	0.36	25.00	1° 43′	5.24	14° 02′	0.629
4	0.068	233.7	0.48	33.33	2° 17′	6.99	18° 26′	0.838
5	0.086	292.1	0.60	41.67	2° 52′	8.75	22° 37′	1.048
6	0.103	350.5	0.72	50.00	3° 26′	10.51	26° 34′	1.257
7	0.120	408.9	0.84	58.33	4° 1′	12.28	30° 15′	1.467
8	0.137	467.3	0.96	66.67	4° 35′	14.05	33° 41′	1.676
9	0.154	525.8	1.08	75.00	5° 10′	15.84	36° 52	1.886

Table 37. Equivalents of surfaces and areas

Square meters (M.^{2})	Square inches (Sq. in.)	Square feet (Sq. ft.)	Square yards (Sq. yd)
1	1550.00	10.7639	1.19599
0.0006452	1	0.006944	0.0007716
0.09290	144	1	0.11111
0.83613	1296	9	1

1 sq. millimeter = 0.01 cm.^{2} = 0.00155 sq. in. = 1217.36 circ. mils.

Table 38. Capacity equivalents

Capacity, dry, English

Bushel (Bu.)	Peck (Pk.)	Quart (Qt.)	Pint (Pt.)
1	4	32	64
0.25	1	8	16
0.03125	0.125	1	2
0.015625	0.0625	0.5	1

Capacity, liquid, English

Pipe	Hogshead (Hhd.)	Tierce *	Barrel (Bbl.)	U. S. gallons † (Gal)	Quart (Qt.)	Pint (Pt.)	Gill
1	2	3	4	126	504	1008	4032
0.5	1	1.5	2	63	252	504	2016
0.25	0.5	0.75	1	31.5	126	252	1008
0.3333	0.6667	1	1.33	42	168	336	1344
0.007935	0.01587	0.023810	0.031746	1	4	8	32
0.001984	0.003968	0.005952	0.007937	0.25	1	2	8
0.000496	0.001984	0.002976	0.003968	0.125	0.5	1	4
0.000248	0.000496	0.000744	0.000992	0.03125	0.125	0.25	1

* The Standard Oil Co. has adopted the tierce, 42 gal., as its barrel, and this practice has been followed by other oil producers and refiners.

† British Imperial gallon = 1.20091 U. S. gal.

Capacity metric. The liter (l.) is the unit and the equivalent of the volume occupied by the mass of 1 kilogram of pure water at maximum density. The smaller units usually employed are the cubic centimeter (cu. cm. or cc.) and the cubic millimeter (cu. mm. or mm.^{3}) which are, for all practical purposes, 0.001 and 0.000001 liter respectively.

Table 39. Equivalent forces or weights per unit of volume

1 dyne per cu. cm. = 0.00101979 gm./cm.^{3} = 0.00118528 poundal/in.^{3}

1 gram per cu. cm. = 980.5966 dynes /cm.^{3} = 1.162283 poundal/in.^{3}

1 poundal per cu. in. = 843.683 dynes/cm.^{3} = 0.860378 gm./cm.^{3} = 0.0310832 lb./in.^{3}

Grams per cu. cm. (Gm./cm.^{3})	Pounds per cu. in. (Lb./in.^{3})	Pounds per cu. ft. (Lb./ft.^{3})	Pounds per cu. yd. (Lb./yd.^{3})	Kilograms per cu. m. (Kg./m.^{3})	Pounds per gal., liquid, U. S.
1	0.03613	62.4283	1685.56	1000	8.34545
27.6797	1	1728	46,656	27,679.7	231
0.01602	5.787×10^{-4}	1	27	16.0184	0.13368
5.933×10^{-4}	2.143×10^{-5}	0.03704	1	0.59327	0.004951
0.001	3.613×10^{-5}	0.06243	1.68556	1	0.008345
0.11983	0.004329	7.48052	201.974	9.30920	1

Table 40. Pressure equivalents

1 dyne per sq. cm. = 0.00101979 gm./cm.^{2} = 0.000466646 poundal/in.^{2}
1 gram per sq. cm. = 980.5966 dynes/cm.^{2} = 0.457592 poundal/in.^{2}
1 poundal per sq. in. = 2142.95 dynes/cm.^{2} = 2.18536 gm./cm.^{2} = 0.0310832 pound/in.^{2}

Kilograms per square centimeter (Kg./cm.^{2})	Pounds per square inch (Lb./in.^{2})	Pounds per square foot (Lb./in.^{2})	Net tons (2000 lb.) per square foot	Atmospheres, standard, 760 mm.	Mercury (Hg = 13.59593 sp. gr.)		Water Maximum density, 4° C.	
					Millimeter	Inches	Meters	Feet
1	14.2234	2048.17	1.02408	0.96778	735.514	28.9572	10	32.8083
0.07031	1	144	0.07200	0.06804	51.7116	2.03588	0.70307	2.30665
0.0004882	0.006944	1	0.00050	0.0004725	0.35911	0.01414	0.004882	0.01602
0.97648	13.8889	2000	1	0.94502	718.216	28.2762	9.76482	32.0367
1.03329	14.6969	2116.35	1.05818	1	760	29.9212	10.3329	33.9006
0.001360	0.01934	2.78468	0.001392	0.001316	1	0.03937	0.01360	0.04461
0.03453	0.49119	70.7310	0.03537	0.03342	25.4001	1	0.34534	1.13299
0.10	1.42234	204.817	0.10241	0.09678	73.5514	2.89572	1	3.28083
0.03048	0.43353	62.4283	0.03121	0.02950	22.4185	0.88262	0.30480	1

Table 41. Equivalents of energy, work and heat

1 dyne-cm. = 1 erg = 0.00101979 gm.-cm. = 7.37612×10^{-8} ft.-lb.
1 gm.-cm. = 980.5966 ergs = 7.233×10^{-5} ft.-lb.
1 ft.-lb. = 13,557,300 ergs = 13,825.5 gm.-cm.

Kilogram-meters (Kg.-m.)	Foot-pounds (Ft.-lb.)	Horsepower-hour		Kilowatt-hours (kw.-hr.)	Joules (10^7 ergs) (j.-sec.)	Thermal units	
		U. S., hp.-hr.	Metric 76 kg.-m.-hr.			Common (B.t.u.)	Metric (Kg.-cal.)
1	7.23300	3.653×10^{-6}	3.704×10^{-6}	2.724×10^{-6}	9.80599	0.009296	0.002342
0.13826	1	5.051×10^{-7}	5.121×10^{-7}	3.766×10^{-7}	1.35573	0.001285	3.239×10^{-4}
273,745	1,980,000	1	1.01387	0.74565	2,684,340	2544.65	641.240
270,000	1,952,910	0.98632	1	0.73545	2,647,610	2509.83	632.467
367,123	2,655,403	1.34111	1.35972	1	3,600,000	3412.66	859.975
0.10198	0.73761	3.725×10^{-7}	3.777×10^{-7}	2.778×10^{-7}	1	9.480×10^{-4}	2.389×10^{-4}
107.577	778.104	3.930×10^{-4}	3.984×10^{-4}	2.930×10^{-4}	1054.90	1	0.25200
426.900	3087.77	0.001559	0.001581	0.001163	4186.17	3.96832	1

Table 42. Equivalents of power, rate of energy and heat

1 erg per sec. = 1 dyne-cm./sec. = 0.00101979 gm.-cm./sec. = 7.37612×10^{-8} ft.-lb./sec.
1 gram-centimeter per sec. = 980.5966 ergs/sec. = 7.238×10^{-5} ft.-lb./sec.
1 ft.-lb./sec. = 13,557,300 ergs/sec. = 13,825.5 gm.-cm./sec.

Kilogram-meters per second (Kg.-m./sec.)	Foot-pounds per second (Ft.-lb./sec.)	Horsepower		Kilowatt (Kw.)	Watt, 10^7 ergs/sec.	Thermal units per second	
		U. S., 550 ft.-lb./sec.	75 Metric, kg.-m./sec.			Common (B.t.u./sec.)	Metric (Kg.-cal./sec.)
1	7.23300	0.01315	0.01333	0.009806	9.80597	0.009296	0.002342
0.13826	1	0.001818	0.001843	0.001356	1.35573	0.001285	3237×10^{-4}
76.0404	550	1	1.01387	0.74565	745.650	0.70685	0.17812
75	542.475	0.98632	1	0.73545	735.448	0.69718	0.17569
101.979	737.612	1.34111	1.35972	1	1000	0.94796	0.23888
0.10198	0.73761	0.001341	0.001360	0.001	1	0.0009480	2.389×10^{-4}
107.577	778.104	1.41474	1.43436	1.05490	1054.90	1	0.25200
426.900	3087.77	5.61412	5.69200	4.18617	4186.17	3.96832	1

SECTION 25

PHYSICS

BY

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1. Density

The density of a substance is the mass per unit volume. When c.g.s. units are employed, the density becomes the SPECIFIC GRAVITY and is, for all practical purposes, the weight in grams per cubic centimeter. For densities of various common substances see Table 1.

Determination of density. For LIQUID, weigh a known volume. For SOLIDS in masses, weigh in air and in a liquid in which they sink and are insoluble. The loss in weight divided by the density of the liquid gives the volume of liquid displaced and hence the volume of the solid. For pulverized solids see Sec. 22, Art. 23.

Hydrometer is a weighted scale which floats in the liquid to be tested, and is weighted in such a way that the depth to which it sinks is proportional to the density of the liquid. Hydrometers are calibrated so that the scale reading cut by the liquid surface is the density of the liquid in the units chosen. Ordinarily specific gravity is read directly, but in some arts the Twaddell scale and in others the Baumé is used. For specific gravity greater than unity: Density = 1 + Twaddell degrees/200. Density at 15° C. = 144.3/(144.3 − Baumé degrees). For specific gravity less than unity; Density = 140/(130 − Baumé degrees).

2. Viscosity

A body that yields continually to forces tending to change its form is called viscous. The rate of shear in a fluid is directly proportional to the shearing stress and inversely to a property of the fluid of the nature of resistance or internal friction which is called the COEFFICIENT OF VISCOSITY.

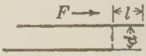

FIG. 1.

Suppose the space between two parallel plates of area A (Fig. 1), and spacing d filled with an adherent fluid. Suppose the force F applied tangentially to one of the plates causes it to slide parallel to the other a distance l in the time t. Then $l/td = v/d$ is the rate of shear where $v = l/t$. The

coefficient of viscosity η is given by the equation $v/d = F/\eta A$. The unit of η is, accordingly 1 gm./1 cm. $\times$ 1 sec. The slowness of the motion v is commonly spoken of as the VISCOSITY of the system. The viscosity of a liquid may be found from the flow through a capillary tube. Let V be the volume which flows in a time, t, L the length of the tube, r its radius and p the pressure due to the head of the liquid, then $\eta = \pi p r^4 t/8LV$.

Table 1. Densities in grams per cc. (For metals, see Sec. 2, Table 1; for minerals, see Sec. 28, Table 2)

1 gm./cc. = 62.4 lb./cu. ft.

Air 0°; 76 cm.	0.001293
Alcohol, ethyl (18°)	0.791
Alcohol, methyl	0.810
Amber	1.06–1.11
Anthracite piled loose	0.84
Asbestos	3.0
Asbestos paper	1.2
Asphalt	1.1–1.5
Basalt	2.4–3.1
Beeswax	0.95–0.96
Benzene	0.899
Brass	8.4–8.7
Brick, fire	1.76
Brick, red	1.92
Bronze	8.7–8.9
Caoutchouc	0.97–0.99
Camphor	1.0
Carbolic acid	0.950–0.965
Carbon dioxide (0°)	0.001977
Celluloid	1.4
Cement, loose	1.15–1.7
Cement, set	2.7–3.0
Chalk	1.9–2.8
Charcoal	0.3 to 0.6
Chloroform	1.480
Clay, dry	1.8–2.6
Coal, bituminous piled loose	0.79
Coke	1.0 to 1.7
Coke piled loose	0.0.45
Concrete, cinder	1.8
Concrete, stone or gravel	2.3–2.5
Ebonite	1.8
Emery	4.0
Ether	0.736
Gas carbon	1.9
Gasolene	0.66–0.69
German silver	8.5–8.9
Glass, crown	2.4–2.6
Glass, flint	2.9–4.5
Glycerine	1.26
Gravel, piled	1.9
Hydrogen	0.00008987
Hydrogen sulphide	0.001523
Ice	0.9168
Leather	0.85–1
Lime	1.03
Lime slaked	1.3–1.4
Masonry	2
Mortar, hard	1.75
Naphtha (petrol. ether)	0.665
Nitrogen	0.001251
Oil, castor	0.969
Oil, cottonseed	0.926
Oil, creosote	1.04–1.10
Oil, lard	0.920
Oil, linseed, boiled	0.942
Oil, mineral lubricating	0.912
Oil, petroleum	0.878
Oil, pine	0.934
Oxygen	0.001429
Paraffin wax	0.87 0.93
Peat	0.84
Phosphor bronze	8.7–8.9
Pitch	1.1
Pumice stone	0.4 0.9
Resin	1.1
Rock salt	2.28–2.41
Rubber, pure	0.91–0.93
Sand, piled	1.68
Sea water	1.025
Silica, fused translucent	2.07
Silica, fused transparent	2.21
Slag	2.0–3.9
Starch	1.53
Sulphur amorphous	1.92–2.07
Sulphur dioxide	0.002927
Snow, fresh fallen	0.136
Snow, compact	0.520
Tallow	0.91–0.97
Tar	±1.02
Turpentine	0.873
Wood, cypress	0.55
Wood, ebony	1.11 1.33
Wood, fir	0.55
Wood, hemlock	0.43
Wood, hickory	0.60 0.93
Wood, lignum vitae	1.17–1.33
Wood, maple	0.62–0.75
Wood, oak	0.60–0.90
Wood, pine, pitch	0.83 0.85
Wood, pine, white	0.35–0.50
Wood, pine, yellow	0.37–0.60
Wood, redwood	0.48
Wood, spruce	0.48–0.70
Wood, teak, African	0.98

Temperature. The coefficient of viscosity decreases rapidly with rising temperature. (See Table 2.)

Relative viscosity or SPECIFIC VISCOSITY is the ratio of the viscosity of the fluid under consideration to that of water at the same temperature.

The apparatus employed for determination varies according to the viscosity of the liquid under investigation. For relatively viscous liquids, some form of the Engler viscosimeter is ordinarily employed; for relatively mobile liquids, the Ostwald apparatus is used.

Table 2. Viscosities

Substance	Temperature, degrees C.	η, $\frac{\text{gm.}}{\text{cm. sec.}}$
Water	0	0.0179
Water	20	0.0101
Water	40	0.00657
Water	60	0.00469
Water	80	0.00356
Mercury	20	0.0156
Ethyl alcohol	20	0.0119
Ether	20	0.00234
Benzene	20	0.00649
Glycerine	10	21.0
Glycerine	20	8.5
Olive oil	15	0.99
Turpentine (d = 0.87)	....	0.0149
Machine oil	19	1.0
Paraffin oil	19	0.02
Sulphuric acid	20	0.22

Engler viscosimeter (Fig. 2), consists of a cup (A), fitted with a pipe orifice (C), a container (B) with a stirring mechanism (E, D), and a graduated flask (G). The cup (A) is provided with an air-insulated cover (c) and a pointed wooden stopper (b).

Take a sample approximately 15 per cent. greater in bulk than required to fill cup (A) to the top of the gage points. Strain the sample through a 100-mesh wire screen to remove foreign matter. Clean cup (A) with solvents such as benzol or alcohol; take particular care that the discharge orifice is thoroughly clean and dry. Soft tissue paper or filter paper is best for cleaning. Assemble the apparatus; place the pointed wooden stopper in the orifice; fill the inner cup to the top of the gage points; place cover (c) in position and insert thermometer (F). Fill chamber (B) with an oil of high boiling point, usually a heavy lubricating oil, and heat slowly, stirring the heating liquid, until the desired temperature is reached. Hold at this temperature until the liquid in cup (A) has reached the same temperature. Place container (G) so that the liquid will flow down the side of the container and thus prevent froth formation. Lift the pointed wood stopper and determine with a stop watch the elapsed time in filling the flask to the graduation mark. Repeat until concordant results are obtained. The time required to discharge an equal volume of distilled water under the same conditions of temperature and head on the discharge orifice should now be determined, after cleaning thoroughly. The ratio of the time of outflow of the oil to that of the water is the specific viscosity of the oil, referred to water at the same temperature, and is called the "Engler degree."

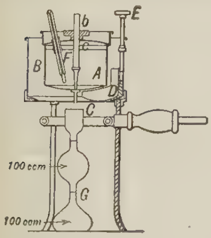

FIG. 2. Engler viscosimeter

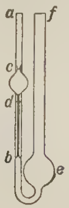

FIG. 3. Ostwald viscosimeter

Ostwald viscosimeter, used for mobile liquids is shown in Fig. 3. It consists of a fine capillary tube (db) about 10 cm. long and 0.4 mm. bore, connected at the upper end to a bulb containing a definite volume between the graduation marks (c) and (d).

Table 3. Relative viscosities of aqueous solutions compared to water

Normal solutions	Temperature, degrees C.	Relative viscosity
Ammonia	25	1.02
Ammonium chloride	17.6	0.98
Calcium chloride	20	1.31
Hydrochloric acid	25	1.07
Potassium chloride	17.6	0.98
Sodium hydrate	25	1.24
Sulphuric acid	25	1.09

The capillary is connected at the lower end by a U-tube to a large bulb (e), which in turn is connected to a tube of convenient size for filling.

The apparatus should be cleaned first with suitable oil solvents such as benzol, acetone, ether, or alcohol and then with warm chromic acid solution, and finally with water, then dried thoroughly by a current of air. Introduce into the leg (f) by means of a calibrated pipette, a definite volume of liquid sufficient to cause the bulb (e) to stand two-thirds full. Heat in a constant-temperature bath from 30 minutes to 1 hour. Next place a rubber tube at (a) and suck oil into this limb to some point above the mark (c). Remove the tube and allow the oil to flow down through the capillary (db) by gravity, taking the elapsed time by stop watch for the meniscus to pass from graduation (c) to graduation (d). Repeat until concordant results are obtained. A variation in elapsed time of 0.1 to 0.5 per cent. is allowable. Repeat with water at the same temperature, using the same volume of water as of oil. The ratio of time of flow of oil multiplied by the specific gravity of the oil, to the time of flow of the water multiplied by the specific gravity of the water gives the specific viscosity of the oil compared with water at the given temperature.

3. Radiation

The energy radiated from a black body depends both in quantity and quality (wave length), upon the temperature.

Table 4. Relative heat-radiating and reflecting powers

	Radiating	Reflecting
Lamp black	100	...
Writing paper	98	2
Ivory, marble	93 to 98	7 to 2
Glass	90	10
Ice	85	15
Gum lac	72	28
Cast iron, polished	25	75
Mercury	23	77
Wrought iron, polished	23	77
Zinc, polished	19	81
Steel, polished	17	83
Platinum, polished	24	76
Tin	15	85
Cast brass, dead polish	11	89
Cast brass, bright polish	7	93
Copper hammered	7	93
Gold plate	5	95
Silver, polished	3	97

Stephan's law. If Q is the quantity of energy in ergs radiated from an area A of a black body in a time t, the rate of loss $R = Q/At$ is $R = KT^4$ where K is a constant having the value 5.3 ergs/cm². sec. (deg.)^4, and T the absolute temperature in degrees C. The radiation from a black body at the temperature T_1 to one at the temperature T_2 will accordingly be $R = K(T_1^4 - T_2^4)$.

The rate at which a hotter body radiates heat and a colder body absorbs heat depends upon the state of the surfaces as well as the temperature. Surfaces that reflect well absorb little and *vice versa*, but radiating power and absorbing power are identical. Consequently radiation and absorption increase with the blackness and roughness of the surface but diminish with the smoothness, polish and whiteness of the surface. It is not possible in the present state of science to give more than relative values for the radiating and reflecting power of surfaces under similar conditions. See Table 4.

Table 5. Heat radiation, B.t.u. per hour per square foot of surface for 1° F. difference of temperature

Copper, polished	.0327
Tin, polished	.0440
Zinc or brass, polished	.0491
Tinned iron, polished	.0858
Sheet iron, polished	.0920
Sheet lead	.1329
Sheet iron	.5662
Glass	.5948
Cast iron	.6480
Iron, rusted	.6868
Wood, stone, brick	.7358

When a surface is not more than 30° F. hotter than the air into which it is radiating and the air is in the vicinity of 50° or 60° F., the rate of radiation may be taken as approximately proportional to the difference in temperature. Newton's law of cooling may be applied thus $Q/At = B(T_1 - T_2)$.

To convert to calories per square centimeter per second for a difference of 1° C. multiply by 1.356 $(10)^{-4}$.

4. Thermometry

Difference of temperature is commonly measured by change in volume of some selected substance, usually mercury in a glass tube.

Fahrenheit scale. The temperature of melting ice is marked 32° F. and that of steam from water boiling under normal atmospheric pressure (76 cm., or 29.92 in. of mercury), is marked 212° F.

Centigrade scale. On this scale the temperatures of the melting and boiling points are called respectively 0° C. and 100° C.

If T_F is a temperature read on a Fahrenheit thermometer and T_C the same temperature on a Centigrade thermometer one reading can be converted into the other by the equation $T_C = 5/9 \times (T_F - 32)$. See also Table 6.

Table 6. Conversion of thermometer readings

Fahrenheit into Centigrade

Fahr.	Cent.	Fahr.	Cent.	Fahr.	Cent.	Fahr.	Cent.	Fahr.	Cent.
0	−17.8	50	10.0	100	37.8	150	65.6	200	93.3
10	−12.2	60	15.6	110	43.3	160	71.1	210	98.9
20	− 6.7	70	21.1	120	48.9	170	76.7	220	104.4
30	− 1.1	80	26.7	130	54.4	180	82.2	230	110.0
40	+ 4.4	90	32.2	140	60.0	190	87.8	240	115.6

Differences for interpolation

Fahrenheit.....	1	2	3	4	5	6	7	8	9	10
Centigrade.....	0.6	1.1	1.7	2.2	2.8	3.3	3.9	4.4	5.0	5.6

Centigrade into Fahrenheit

Cent.	Fahr.	Cent.	Fahr.	Cent.	Fahr.	Cent.	Fahr.	Cent.	Fahr.
−50	−58	0	32	50	122	100	212	150	302
−40	−40	10	50	60	140	110	230	160	320
−30	−22	20	68	70	158	120	248	170	338
−20	− 4	30	86	80	176	130	266	180	356
−10	+14	40	104	90	194	140	284	190	374

Differences for interpolation

Centigrade....	1	2	3	4	5	6	7	8	9	10
Fahrenheit.....	1.8	3.6	5.4	7.2	9.0	10.8	12.6	14.4	16.2	18.0

Examples of use of tables

To convert 164.1° F. to Centigrade.

	Fahr.		Cent.
	160.0°	=	71.1°
Diff.	4.0	=	2.2
Diff.	0.1	=	0.1
	164.1°	=	73.4°

To convert 73.4° C. to Fahrenheit.

	Cent.		Fahr.
	70.0°	=	158.0°
Diff.	3.0	=	5.4
Diff.	0.4	=	0.7
	73.4°	=	164.1°

5. Calorimetry

DEFINITIONS. The quantity of heat required to raise the temperature of 1 gram of water 1° C. is called the CALORIE. As this amount varies slightly with temperature, it is usual to take the average value between 0° and 100° C., which is called the MEAN CALORIE. The quantity of heat required to raise a pound of water 1° F. is called the BRITISH THERMAL UNIT or B.T.U.

THERMAL CAPACITY (c), or water equivalent of a body, is the amount of heat required to raise the temperature of the body one degree. Thus the quantity of heat Q necessary to produce a change of temperature $t_1 - t_2$ in a body whose thermal capacity is c will be $Q = c(t_1 - t_2)$. SPECIFIC HEAT (S) of a substance is the thermal capacity per unit mass. This is numerically the ratio of the quantity of heat required to raise a given mass of the substance one degree to the quantity of heat required to raise an equal mass of water one degree. The quantity of heat Q necessary to produce a change of temperature $(t_1 - t_2)$ in a body whose mass is m and whose specific heat is s is $Q = ms(t_1 - t_2)$. The quantity of heat required to convert a gram of substance from the solid to the liquid state, without change of temperature, is called the LATENT HEAT OF FUSION. The quantity of heat Q necessary to melt m grams of a substance whose latent heat of fusion is L_F is $Q = mL_F$. A similar expression obtains for the change from the liquid to the vapor state and $Q' = m'\ L_V$ where L_V is the LATENT HEAT OF VAPORIZATION. See Table 7.

6. Coefficient of expansion

The fractional increase of a length at zero degrees for a rise of temperature of one degree is called the COEFFICIENT OF LINEAR EXPANSION. If L_t is the length at t degrees, L_0 the length at zero and λ the coefficient of linear expansion, $L_t = L_0(1 + \lambda t)$. For solids the relation $L_{t'} = L_t[1 + \lambda(t' - t)]$ is sufficiently exact. The COEFFICIENT OF CUBICAL EXPANSION is defined by $V_t = V_0(1 + \alpha t)$. For solids the cubical coefficient $\alpha = 3\lambda$. See Table 7.

Cubical expansion of liquids. The volume of a liquid may be exactly expressed as a function of the temperature by the relation: $V_t = V_0(1 + \alpha t + \beta t^2 + \gamma t^3)$. Values of these coefficients are given in the Smithsonian Tables. If, however, the temperature interval is not large, the relation $V_t = V_0(1 + \alpha t)$ may be used. Values of α in Table 8 are mean values for the vicinity of 18° C.

Coefficient of cubical expansion of gases at constant pressure of 76 cm. of mercury, has the practically uniform value of $\alpha = 1/273 = .00367$ per deg. C. WATER VAPOR between 0° and 162° C. has the mean value $\alpha = 0.004189$ per deg. C.

7. Thermal conductivity

The quantity of heat Q that is conducted across a slab of any substance having a facial area A, a thickness d and a difference of temperature $T_1 - T_2$ in a time t is given by the relation $Q = kA(T_1 - T_2)t/d$ where k is a constant defining a property of the material called THERMAL CONDUCTIVITY. See Table 7.

8. Humidity

ABSOLUTE HUMIDITY is the mass of water vapor per cc., *i.e.*, the density of the water vapor actually present. RELATIVE HUMIDITY is the ratio of the amount of water actually present to the amount necessary to saturate the air, the temperature remaining unchanged. The DEW POINT is the temperature at which the aqueous vapor begins to condense.

Table 7. Thermal properties of various substances. (For metals, see Sec. 2, Table 1)

Note—The specific heat is a mean value between 0° C. and 100° C. unless otherwise stated. The boiling points are for 76 cm. of mercury pressure. The specific heats of gases are for constant pressure of 76 cm.

Substance	Melting point, degrees C.	Boiling point, degrees C.	Latent heat, cal. per gm.		Temperature, degrees C.	Specific heat	λ Coefficient of linear expansion $\times 10^{-6}$	Conductivity
			Fusion	Vaporization				
Air, gas					20	.242		.000057
Alcohol, ethyl	−130	78.3		207	0	.547		.00043
Ammonia, gas					20	.520		
Ammonia, liquid				341				
Asbestos						.20		.0006
Basalt						.20–.24		
Benzene		80			10	.340		
Brass	900				Cast	.09	18.8	
					Wire		19.3	
Brick						.22	9.5	.0015
Brine, sp. gr. = 1.2					15	.72		
Cane sugar	160							
Carbon dioxide, gas					20	.202		
Carbon dioxide, liquid		−78.2						
Cement						.27		
Clay						.20		
Coal						.2–.25		
Concrete						.20	10 to 14	.0007
Constantin, 60 Cu, 40 Ni					18	.098	17	.054
Cork								.00013
Cotton								.00055
Cotton wool								.00004
Ether		34.6		88	18	.56		
Felt								.00009
Flannel								.00023
Galena						.047		
Gas carbon						.204	5.4	.010
German silver						.095	18.4	.08
Glass, crown						.16 to .19	8.5	
Glass, flint						.12	9.4	.002
Glass, window								.0025
Glycerine					18 to 50	.58		.00068
Gneiss						.20		
Granite						.19 to .20	8.3	
Graphite						0.19–.20		.012
Gutta percha	100						198	
Hematite						0.16		
Hydrogen gas						3.40		.00033
Hydrogen, liquid	−259	−252.7						
Hydrochloric acid						.60		
Hydrogen sulphide					20	.245		
Ice	0		79.8		−21 to −1	.502	50.7	.005
India rubber						.27 to .48		.00045
Iron, Fe	1530	2450			18	.119		.161
Iron, Fe					225	.137		
Iron, cast gray			23		54	.130	10.2	.114
Iron, cast white			33					
Iron, wrought					18		11.9	.144
Lard	36 to 40							
Limestone						.21		
Magnesium	633	1120				.246	25.4	.376
Magnetite						.16		
Marble						.22	1.4 to 3.5	
Marble, white					18	.22		.0071
Masonry							4 to 7	
Mica						.21		.0018
Nitrogen, liquid	−210.5	−195.7		50				
Nitrogen, gas					0	.235		
Oxygen	−219	−182.9		51	20	.242		
Olive oil	2–6	300			7	.47		
Paper								.0003

Table 7. Thermal properties of various substances. (For metals, see Sec. 2, Table 1)—*Continued*

Substance	Melting point, degrees C	Boiling point, degrees C	Latent heat, cal. per gm. Fusion	Latent heat, cal. per gm. Vaporization	Temperature, degrees C	Specific heat	λ Coefficient of linear expansion, $\times 10^{-6}$	Conductivity
Paraffin oil						.51–.54		.00035
Paraffin wax, soft	38–52	350–390			0–20	.69		
Paraffin wax, hard	52–56	390–430	35				110	.0006
Porcelain						.255	2.8 to 3.4	.0025
Pyrite						.13		
Quartz fused						.20	.518	
Resin	135							
Rubber, hard						.33	64 to 77	.00042
Rubber, Para	120							.00045
Salt, rock						.22		
Sand						.19		.00013
Sandstone						.22	7 to 12	.0055
Sawdust								.00012
Sea water	−2.5				17	.94		
Serpentine						.26		
Silk								.00022
Slate							6 to 10	.0047
Solder 2 Pb, 1 Sn	240						25	
Stearin	72					.114 to .117		
Steel	1300 to 1475				18	0.118	10.5 to 11.6	.108
Steel, 10% Ni							13	
Steel, 20% Ni							19.5	
Steel, 36% Ni¶							0.37 to 0.44	
Sulphur, rhombic	115	444	9	362	30	1.63	70	
Sulphur, monoclinic	119	444	9	362				
Sulphuric acid, $d = 1.87$						.34		
Sulphuric acid, $d = 1.30$						.66		
Tallow, beef	40–45							
Tallow, mutton	44–45							
Turpentine		159		70	18	.42		.0003
Water	0	100	79.8	539	17	1.00§		.0013
Water vapor					100	.465		
Wood, ash							9.5*	
Wood, beech							2.6* 61 †	
Wood, chestnut							6.5* 32 †	
Wood, elm							5.7* 44 †	
Wood, fir						.65		
Wood, mahogany							3.6* 40 †	.0005
Wood, maple							6.4* 48 †	
Wood, oak						.57	4.9* 54 †	.0006
Wood, pine						.47	5.4* 34 †	.0004

* With grain. † Across grain.

§ The specific heat of water varies from 1.00 by only a few parts in a thousand. See Kaye and Laby, *Physical Constants*, page 56.

¶ This alloy, called Invar, has the smallest known coefficient of expansion. An alloy of 42% nickel with iron is called Platinite and has the same coefficient as glass. It is used for incandescent lamp bulbs and armored glass.

Absolute humidity at any temperature, t, may be found from the density of the water vapor, D, at the dew point t_d by the equation

$$\text{A. H.} = D \times \frac{273 + t_d}{273 + t}.$$

Relative humidity may be calculated from the definition or from the ratio

of the vapor pressure at the actual temperature to the vapor pressure at the dew point: R. H. = p_d/p_t.

Table 8. Values of α in the approximate equation for cubical expansion of liquids

Liquid	$\alpha \times 10^5$	Liquid	$\alpha \times 10^5$
Acetic acid	107	Turpentine	94
Alcohol, ethyl	110	Water, see special table	
Benzene	124	Mercury	18.15
Carbon disulphide	121	Solution, 1.6 per cent. NaCl in water	106
Chloroform	126	Solution, 26 per cent. NaCl in water	43.6
Ether, ethyl	163	Sulphuric acid, 100 per cent	57
Glycerine	53	Solution H_2SO_4, 50 per cent., in water	80
Olive oil	70		
Paraffine oil	90		
Petroleum, d = .847	104		

Table 9. Specific volume of water

Temp., deg. C.	Cc. per gm.	Temp., deg. C.	Cc. per gm.
0	1.00013	60	1.01705
4	1.00000	70	1.02270
10	1.00027	80	1.02890
20	1.00177	90	1.03590
30	1.00435	100	1.04343
40	1.00782	110	1.0515
50	1.01207	120	1.0601

Table 10. Freezing mixtures

Parts by weight	Temperature
1 NH_4NO_3 to 1 water	− 15° C.
8 $NaSO_4$ to 5 water	− 17
2 snow to 1 NaCl	− 18
3 snow to 4 $CaCl_2$ crystals	− 48

Example. Given the temperature of the air t = 14.5° C. and the dew point t_d = 9.2° C., required the absolute and the relative humidity.

From Table 11, by transformation of units, density of water vapor in saturated air is $D_{9.2}$ = 8.94 gm. per cu. met., whence A. H. $= 8.94 \cdot \frac{273 + 9.2}{273 + 14.5} = 8.78 \frac{\text{gm.}}{\text{cu. m.}}$.

Again from Table 11, the density for saturation at 14.5° is $D_{14.5}$ = 12.44 gm. per cu. m. whence R. H. $= \frac{\text{A. H.}}{D_{14.5}} = \frac{8.78}{12.44} = 0.706$.

By the tables $p_{9.2}$ = 8.66 mm. Hg and $p_{14.5}$ = 12.22 mm. Hg, hence

$$\text{R. H.} = 8.66/12.22 = 0.708.$$

Table 11. Properties
(p = 14.7 lbs. per

Temperature, degrees	Pressure in pounds per square inch: of vapor component	of air component	Dry air	Weight in pounds per cubic foot: of vapor component	of air component	of saturated mixture	Ratio, vapor/air	Ratio, air/vapor	Specific volume, cubic feet per pound
32	0.089	14.611	.0807	.00031	.0802	.0805	.00379	263.8	3294
35	.100	14.600	.0802	.00034	.0797	.0800	.00427	234	2938
40	.122	14.578	.0794	.00041	.0788	.0792	.00520	193	2438
45	.147	14.553	.0786	.00049	.0778	.0783	.00620	161	2033
50	.178	14.522	.0779	.00059	.0769	.0775	.00769	130.5	1702
55	.214	14.486	.0771	.00070	.0760	.0767	.00921	108.5	1430
60	.254	14.446	.0763	.00082	.0751	.0759	.01092	91.6	1208
65	.304	14.396	.0756	.00097	.0741	.0751	.01310	76.3	1024
70	.360	14.340	.0749	.00114	.0731	.0742	.0156	64.0	871
75	.427	14.276	.0742	.00134	.0721	.0734	.0186	53.8	743
80	.503	14.197	.0735	.00156	.0711	.0726	.0219	45.6	636.8
85	.592	14.108	.0728	.00182	.0700	.0718	.0260	38.5	545.9
90	.693	14.007	.0723	.00212	.0690	.0711	.0306	32.7	469.3
95	.809	13.891	.0715	.00245	.0676	.0701	.0362	27.6	405.0
100	.942	13.758	.0709	.00283	.0662	.0692	.0426	23.4	305.8
105	1.095	13.605	.0702	.00325	.0650	.0683	.0500	20.0	304.7
110	1.267	13.433	.0696	.00373	.0637	.0674	.0586	17.1	265.5
115	1.462	13.238	.0690	.00426	.0622	.0665	.0684	14.6	231.9
120	1.685	13.015	.0684	.00488	.0606	.0655	.0804	12.43	203.1
125	1.932	12.768	.0678	.00544	.0590	.0645	.0939	10.65	178.4
130	2.215	12.485	.0672	.00630	.0572	.0635	.1101	9.08	157.1
135	2.542	12.158	.0667	.00714	.0552	.0624	.1291	7.73	138.7
140	2.879	11.821	.0661	.00806	.0531	.0613	.152	6.58	122.8
145	3.273	11.427	.0655	.00909	.0511	.0601	.178	5.62	109.0
150	3.708	10.992	.0650	.01022	.0487	.0589	.210	4.77	96.9
155	4.193	10.507	.0644	.01145	.0462	.0576	.248	4.03	86.4
160	4.731	9.969	.0639	.01333	.0435	.0568	.305	3.28	77.2
165	5.327	9.373	.0634	.01432	.0405	.0549	.353	2.83	69.1
170	5.985	8.715	.0629	.01602	.0374	.0534	.428	2.33	62.0
175	6.708	7.992	.0624	.01744	.0340	.0518	.521	1.92	55.7
180	7.511	7.198	.0319	.01970	.0304	.0501	.650	1.54	50.15
185	8.375	6.325	.0614	.02181	.0265	.0483	.823	1.215	45.25
190	9.335	5.365	.0609	.02411	.0223	.0464	1.079	.927	40.91
195	10.385	4.315	.0605	.02662	.0178	.0444	1.495	.669	37.04
200	11.526	3.174	.0600	.02933	.0130	.0423	2.25	.443	33.60
205	12.770	1.930	.0596	.03225	.0078	.0401	4.11	.243	30.53
210	14.126	0.574	.0593	.03543	.0023	.0377	15.45	.0647	27.80
212	14.7	0.000	.0592	.0368	.0000	.0368	Infinite	.000	26.79

9. Optics

Image. When light waves proceeding from a point are so modified that they either proceed from another point or appear to proceed from another point, the second point is called the image of the first. In the first case the image is REAL and the second VIRTUAL. In every case the condition of distinct vision is that the light shall proceed definitely from a point, or in other words that the wave front which enters the eye shall be accurately spherical.

Principal focus. When flat wave fronts, that is to say light from a very distant source, fall on a positive lens they are converged to a point (F), (Fig. 4, a), called the PRINCIPAL FOCUS; its distance from the center of the lens is called the focal length of the lens.

of saturated air
square inch = 30 in. Hg)

Heat content in B.t.u. per cubic ft. from from water at 32° F.			Heat in vapor, B.t.u. per pound	Ratio Dry air at 62° F. Saturated mixture		Initial temperature (F.) of dry air required to evaporate 1 lb. of water in saturated mixture, degrees	Latent heat of evaporation at wet-bulb temperature
Vapor	Air	Saturated mixture		By weight	By volume		
.338	.000	.338	1091	258.8	3401	49.7	1073.4
.371	.057	.428	1092	234.4	3080	54.6	1071.7
.448	.150	.598	1094	192.2	2526	63.8	1068.9
.536	.240	.777	1095	158.9	2088	74.0	1066.1
.647	.329	.976	1097	130.4	1714	85.4	1063.3
.768	.415	1.183	1098	108.5	1426	97.6	1060.6
.901	.500	1.401	1100	91.6	1203	110.5	1058.8
1.068	.581	1.649	1101	76.4	1004	125.6	1055.0
1.256	.641	1.897	1103	66.0	868	140.0	1052.3
1.479	.737	2.216	1104	55.0	723	160.0	1049.5
1.725	.811	2.536	1106	45.6	599	182.0	1046.7
2.016	.881	2.897	1107	38.4	505	206.0	1044.0
2.351	.951	3.302	1109	32.5	427	233.0	1041.2
2.721	1.013	3.734	1110	27.6	363	264.0	1038.4
3.147	1.073	4.220	1112	23.5	308	299	1035.6
3.618	1.129	4.747	1113	20.0	263	344	1032.8
4.158	1.181	5.339	1115	17.1	224	385	1030.0
4.764	1.227	5.991	1116	14.6	192	436	1027.2
5.456	1.268	6.724	1118	12.4	163	499	1024.4
6.201	1.304	7.505	1119	10.7	140	567	1021.6
7.062	1.332	8.394	1121	9.10	118	667	1018.8
8.015	1.352	9.367	1123	7.74	102	745	1016.0
9.060	1.367	10.43	1124	6.61	86.8	856	1013.1
10.23	1.372	11.61	1126	5.62	73.8	986	1010.3
11.52	1.366	12.88	1127	4.77	62.6	1,145	1007.4
12.92	1.351	14.27	1129	4.03	53.0	1,332	1004.5
15.06	1.322	16.39	1130	3.26	42.8	1,618	1001.6
16.21	1.282	17.49	1132	2.83	37.1	1,847	998.7
18.15	1.226	19.38	1133	2.33	30.7	2,212	995.8
20.13	1.156	21.29	1135	1.92	25.2	2,665	992.9
22.39	1.068	23.46	1136	1.54	20.3	3,280	989.9
24.82	.964	25.78	1138	1.22	16.0	4,124	986.9
27.47	.838	28.31	1139	0.93	12.2	5,368	983.9
30.37	.690	31.06	1141	0.67	8.8	7,370	980.9
33.52	.519	34.02	1142	0.44	5.8	12,840	977.8
36.89	.323	37.22	1144	0.24	3.2	19,985	974.7
40.58	.098	40.68	1145	0.065	0.7	....	971.6
42.21	.000	42.21	1146	0.000	0.0	....	970.4

Real image. When the light proceeds from a point (O), (Fig. 4, b), farther from the lens than the focal distance, the waves after passing through the lens converge to a point (I)

(a) (b) (c)

FIG. 4.

between the principal focus and infinity. Thus a real image of (O) is formed at (I) and the ratio of the size of the image to the size of the object is the same as the ratio of their respective distances from the lens. Such real images are formed by a photographic lens and by the objective lens of a microscope or a telescope.

Virtual Image. When light proceeds from a point nearer the lens than the focal distance the waves after passing the lens appear to come from a point (I), (Fig. 4, c), behind the object (O), forming a virtual image at (I). The ratio of the size of the object and image is, as before, the same as the ratio of their distances from the lens. This is the way a lens is used as a magnifying glass. For distinct vision (CI) should be about 25 cm. (10 in.), which is taken as the conventional distance of reference. The magnification in this case is approximately 25 cm./(F) cm. since the object is near the principal focus.

Aberrations. Light-wave fronts, after modification by a lens, are (1) neither truly spherical, (2) nor the same for points on the axis and off the axis, (3) nor the same for different colored lights. The assemblage of points in the object is accordingly only imperfectly reproduced in the image formed by a simple lens. Defects (1) and (2) are called spherical aberrations. A sharp point in the object consequently appears hazy and elongated in the image, a plane does not appear flat and the relation of points in it is distorted. Defect (3) is called chromatic aberration and gives rise to overlapping colored images. Spherical aberration may be corrected by using two or more somewhat separated lenses, as in the familiar eyepiece of microscopes and telescopes. A compound lens that has been corrected for spherical aberration is called APLANATIC. Chromatic aberration may be corrected by combining two or more lenses of different kinds of glass. When so corrected the combination is called ACHROMATIC. A lens system that has been corrected for three different wave lengths (colors) is known as an APOCHROMATIC.

Compound microscope. The compound microscope consists essentially of a short focus objective lens (B, Fig. 5) which forms a magnified real image of the object (O), at (I_1). This image is then viewed through a magnifying or ocular lens (C) which forms a virtual image at (I_2). The distance (BI_1) is about the tube length (L) of the microscope. The TOTAL MAGNIFICATION is $25/F \times L/f$ where (F) and (f) are the focal lengths respectively of the ocular and the objective, all in cm.

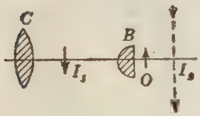

FIG. 5.

In practice an objective may be designated either by its equivalent focal length, f, (in millimeters) or by its magnifying power, 250 mm./f, (Zeiss) or by its initial magnification, L/f (Spencer, Bausch and Lomb) where (L) is conventionally 180 mm. or, sometimes, 160 mm. When the initial magnification is given, the total magnification of the microscope is obtained by multiplying by the power (250 mm./F) of the ocular which is usually stamped upon it, *e.g.*, $5x$ or $10x$.

Depth of focus. With a small aperture the diffusion circle or cross section of the cone of light near the image (I) of a point (Fig. 6, a), does not change much from (a) to (b) so that it is possible to change the focus up or down quite a little without the image losing much in sharpness. With a large aperture (Fig. 6, b), the diffusion circle at (a') or (b') is large, and hence it is not possible to go more than a little way from the true focus without losing distinctness. The depth of focus varies inversely as the magnification and the numerical aperture.

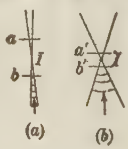

FIG. 6.

Aperture. The resolving power of a microscope or its ability to resolve the details of an object, is determined by the angle of the cone of light proceeding from a point in the object to the objective. This angle, commonly designated by $2u$, is called the angular aperture of the objective. The measure of the optical power is the sine of half this angle. In case an immersion fluid is used the power is increased n times, n being the index of refraction. This expression is called the numerical aperture (N. A.) Thus N. A. = $n \sin u$ where $n = 1$ for air 1.33 for water and 1.52 for oil of cedar (homogeneous immersion).

The measure of the RESOLVING POWER of an instrument is taken as the distance apart of two points which can be seen as separated. This distance at the maximum is $\lambda/2$ N.A., where λ is the wave length of the light used. For white light λ is about $561\mu\mu$ ($1\mu\mu = 10^{-6}$ mm.), for blue light $\lambda = 486\mu\mu$, for ultra-violet $\lambda = 250\mu\mu$. A slightly greater defining power is thus gained by the use of shorter wave lengths but the employment of ultra violet light requires the use of quartz lenses instead of glass.

Illumination. For the study of opaque objects and when employing low powers, light is thrown upon the object from above either by an outside mirror

or a condensing lens, or the microscope may be supplied with a special reflecting device called a vertical illuminator. In order to make full use of the numerical aperture of high power objectives the microscope must be provided with a sub-stage condenser, as carefully corrected for aberrations as the objective and ocular, of sufficient numerical aperture, capable of variation by means of an iris diaphragm. If daylight is not available, nitrogen lamps with special glass filters may be obtained, which give its equivalent.

Dark-field illumination is secured by making the course of the light from the condenser so oblique that although it cannot enter the objective directly, a portion is so scattered by the object as to make parts of its structure appear bright on a dark field.

Ultramicroscope. When dark-field illumination is carried to the extreme limit by using sunlight or the electric arc, the instrument is known as the ultramicroscope. The limit of visibility in the ordinary microscope is about 150 $\mu\mu$ (1 $\mu\mu = 10^{-6}$ mm.) while that of the ultramicroscope is about one hundredth of this. The smallest particle that has been detected is one of gold, estimated to be 1.7 $\mu\mu$.

Stage micrometer is a convenient scale, say 0.1 mm. divided to tenths, ruled on a cover glass and used as a microscope object. To DETERMINE THE MAGNIFICATION OF A MICROSCOPE take two visiting cards and rule a heavy black line across each. Place a stage micrometer under the microscope and focus it. Support one of the cards on a block at the side of the microscope in a plane perpendicular to the axis and at a distance of 25 cm. from the eyepiece. Look through the microscope with one eye and observe the card with the other. Adjust the card so that the line seen with one eye coincides with the image of one of the rulings on the stage seen with the other. Move the head slightly from side to side and observe if the image remains stationary with respect to the card. If it does not the image is not formed in the plane of the card and the focus must be altered until the parallax disappears. Then place the second card by the side of the first and slide it so that the line upon it coincides with another ruling of the image. Measure the distance apart of the card lines with a pair of dividers. The ratio of this distance to the known distance of the rulings on the stage micrometer is the magnification.

Camera lucida. The camera lucida is a reflecting prism attached to the microscope tube so that the image of a card or sheet of paper placed at a distance of 25 cm. appears superposed upon the image of any object seen through the microscope. It is useful for drawing the outlines of objects under investigation. Care must be used to have the field of the microscope and the surface of the paper about equally lighted.

To use the camera lucida for getting magnification, adjust the card so that its image coincides with the image of any two lines upon the stage micrometer. The axis of the microscope may be either horizontal or vertical or inclined, depending upon the type of camera used. With the points of the dividers upon the card adjust their separation so that they appear to coincide with the image of the lines upon the stage micrometer. The ratio of the spread of the dividers to the known distance of the lines is the magnification.

Micrometer eyepiece is a scale ruled upon glass so that its image formed by the eye lens coincides with the image under observation in the microscope. The readings must be calibrated for the exact conditions under which each objective is used. This is readily done by a direct observation of the number of lines on the ocular scale corresponding to one division of a stage micrometer, for that particular objective and position of the draw tube.

Filar micrometer is a micrometer eyepiece having a fixed and a movable cross wire that is carried across the field by means of a fine-pitched screw with a graduated head. The number of complete revolutions may be counted upon a comb which appears in the field, and the value of one of the graduations on the head may be found by counting how much it is necessary to revolve the head to move the cross wire a known distance, say from the image of one of the divisions of the stage micrometer to the next.

Index of refraction. The velocity of light is, in general, different in different media. The ratio of the velocity V_1 in one medium to that in another V_2 is called the relative index of refraction N_{12}, always taken in such sense that

the ratio is greater than unity. Thus $N_{12} = V_2/V_1$. As commonly stated, the index of refraction n of a medium is taken relative to air.

When light passes obliquely from air to another medium the direction changes in a manner given by the relation $n = \sin i/\sin r$ (Fig. 7) where i is called the angle of incidence and r is the angle of refraction, each being measured from the normal PP' to the surface. The index of refraction is also a function of the wave-length, being greater for short than for long waves. Consequently a beam of white light generally appears colored after refraction. This differential effect of refraction is termed DISPERSION. When any translucent object immersed in a liquid is viewed through a microscope the image will appear bounded by a colored fringe unless both the liquid and the substance have the same index of refraction. The refractive index of a substance can be determined by immersing it successively in a series of liquids whose indices of refraction are known until one is found in which the colored fringe disappears. Similarly if a series of transparent solids whose indices of refraction are known be given, the refractive index of an unknown liquid may be ascertained.

FIG. 7. Refraction

Method of displacement of image. When a point (O, Fig. 8) in any medium is viewed normally through a plane boundary AB in air, the apparent depth CO' of the point is less than the actual distance, depending upon the index of refraction. If these distances are measured, for example by focusing first on O then on O', the index of refraction will be given by $n = CO/CO'$.

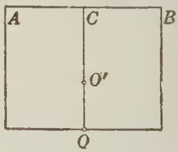

FIG. 8.

Polarization. Light that has been so modified by passing through a crystal that its vibrations are rendered parallel to a plane is called PLANE POLARIZED LIGHT. A beam of such light shows different properties on different sides of its line of propagation, so that it is more copiously reflected in some directions than in others, or it will be transmitted by a transparent crystal section in one position and extinguished by it in another. Any apparatus for thus modifying light is called a POLARIZER. It is usually a divided crystal of calcite called a NICOL PRISM. A similar prism used to test whether the light is polarized is called the ANALYZER. When the planes of polarization of the nicols are parallel, the field is light; when perpendicular (CROSSED), dark.

Micropolariscope. In microscopic work a pair of nicols which may be attached to the instrument is used to test whether a substance is double refracting or not, and thus assist in identification of a specimen. When a double-refracting substance is introduced into the dark field between crossed nicols the light is restored and sometimes colored. In particular circumstances, as with a thin plate of mica or selenite, the color changes as the analyzer is rotated.

Bibliography. For further information upon the microscope, see Gage, Carpenter-Dallinger, Spitta, and A. E. Wright; also the *Substage condenser* by Nelson, and *Elementary chemical microscopy* by Chamot.

10. Electrical phenomena and definitions

Definitions. Electric charges are of two kinds only, POSITIVE, obtained, *e.g.*, on a glass rod rubbed with silk, and NEGATIVE, obtained when a hard-rubber rod is rubbed with wool. The characteristic of these charges is that small bodies charged with like signs repel and with unlike signs attract each other. Any region through which electric forces act is called an ELECTRIC FIELD. A line through the field that has everywhere the direction in which a

positively charged point would be urged is called a LINE OF FORCE. Lines of force always start from positively-charged surfaces and run to negatively-charged surfaces. The magnitude of the force with which a positive-point charge is urged is called the INTENSITY OR STRENGTH OF THE FIELD at any place. The work that would be done by the forces of the field in moving a unit positive charge from one point to another is called the DIFFERENCE OF POTENTIAL between the two points. Since positive charges are urged from places of high to places of low potential, difference of potential, or electromotive force, may be regarded as difference of electrical pressure. The practical unit of potential is called the VOLT. The difference of potential between the copper and the zinc of a Daniell cell (also called the gravity or blue-stone battery) is about one volt.

A body that permits passage of an electric charge is called a CONDUCTOR, *e.g.*, metals and gas carbon; a body that prevents passage of an electric charge is an INSULATOR. Gases, gums and glass-like bodies are examples.

When a conductor is brought into contact with a second and charged body, the first body becomes immediately charged with the same sign as the second. When an insulator is touched to a charged body there is also a transfer of a certain amount of the charge but the electrification of the insulator is confined to the region of contact, except in so far as this charge may slowly creep over the surface, which may be very slightly conducting.

When any body is brought near an electric charge the nearer side of the body becomes electrified with the opposite sign to the original charge, and the farther side with the same sign. This ability of a charged body to influence another through an insulating medium is called ELECTROSTATIC INDUCTION and the medium in this sense is called the DIELECTRIC since lines of force can exist only in a non-conducting medium.

The phenomenon of induction may be differently described as follows: When any body is brought into an electric field it shows a negative charge where the lines of force meet its surface and a positive charge where they leave it. This induced charge is greatest on a conductor and least on an insulator. It also depends upon a constant of the insulating medium surrounding the body, which is called the SPECIFIC INDUCTIVE CAPACITY or DIELECTRIC CONSTANT of the medium. This constant is referred to air, or strictly to a vacuum, as unity. Thus to charge two paralleled metal plates to a difference of potential of one volt will take several times as much charge when they are separated by glass as when they are separated by air. The dielectric constant of ordinary insulators ranges from about 1 to about 10. When a non-conducting body is brought into an electric field the induced surface charge depends upon the dielectric constants inside and outside the body.

If a small metal body is brought near a charged body it will, as a whole, be attracted because the attraction of the nearer induced and unlike charge is greater than that of the farther like charge, the law of the action of two-point charges being inversely as the square of the distance. When the first body touches the second body it will receive a like charge and then be repelled. For a similar reason a small body such as a piece of paper or quartz will first be attracted to a charged body but when it makes contact there will be no new distribution of the charges, if its surface is a very perfect insulator, and it will cling to the charged body. These principles are made use of to separate metallic particles and grains of different dielectric capacity.

Power of points. When a conductor having sharp points is strongly electrified the surface charges are highly concentrated at the points and the field about them is intense. Particles of air are first attracted to contact and then so strongly repelled that they produce a stream of electrified wind in the vicinity, which discharges the conductor and may convey the charge to neighboring bodies. Electrostatic machines are often provided with

a series of points by which electricity is thus transferred from one part to another without actual contact of the parts.

Magnetism. A body that possesses a selective attraction for particles of iron is called a MAGNET. Magnetic phenomena may be conveniently described in terms of surface charges of positive and negative magnetism. An elongated magnetic needle behaves as if these hypothetical charges were concentrated about the ends, which are termed poles. If such a needle be suspended so as to turn freely in a horizontal plane, it will take a position approximately north and south. The end that points north is called the north pole. Positive magnetism is consequently called NORTH MAGNETISM and its opposite, SOUTH MAGNETISM. The behavior of magnetic charges is analogous to electric charges except that there are no conductors of magnetism and consequently it is impossible to magnetize a body entirely with north magnetism or south magnetism. With this exception all the phenomena and definitions given for electricity have their complete analogues in magnets and the magnetic field. The law of attraction or repulsion of two poles, the law of the inverse square of the distance, the passage of lines of force between poles, and the magnetization of a body by induction when placed in a magnetic field, are identically of the same form as in the electrical case. The ability of magnetic lines of force to pass through a body is called the PERMEABILITY of the substance but the permeability of most substances except those of the iron group differs little from air. Compared to the dielectric constant whose maximum value is about 10, the permeability of iron may reach 3000. This taken with the fact that there is no magnetic leakage by conduction readily explains why the total values of magnetic attractions may be made so much greater than electrostatic. Magnetic fields can be produced by the passage of an electric current through a conductor, the lines of force forming closed rings about the conductor. For this reason strong fields, or electromagnets, are easily varied and controlled. The separation of magnetic from non-magnetic mineral grains when placed in a magnetic field is based on these principles.

11. Electrochemical theory

Electrons. Electricity does not appear to be distributed continuously through bodies but in the form of discrete charges. The smallest known quantity of negative electricity has a mass, or is associated with a mass about $\frac{1}{1800}$th of the hydrogen atom. It is the same from whatever source derived and is called an ELECTRON. The electronic charge is $e = 4.7(10)^{-10}$ electrostatic units, and its mass $8.8(10)^{-28}$ gram. The hydrogen atom has a mass of $1.6(10)^{-24}$ gm. Positive electricity on the other hand does not appear to be associated with particles smaller than the atom, and this positive charge is always some multiple of the negative electron.

Atoms. Assume that the neutral atom (of atomic weight roughly $2N$) is a complex structure of N electrons grouped about a positive nucleus having a charge Ne, e being the value of the electronic charge. Assume further that a few of these electrons, called valence electrons, are arranged in the outer shell of the atom so that they may be readily detached. A positively-charged atom is then an ordinary atom which has been deprived of one or more electrons, and a negative atom is one which has acquired one or more free electrons. A MOLECULE is a stable, uncharged group of atoms. Its stability appears to depend upon the grouping of the electrons about the positive nuclei. A charged atom or group of atoms is called an ION.

Valency. The number of electronic charges which an atom carries when it becomes an electrolytic ion may be termed the valence of the ion, but it is not easy to say how many valence electrons an atom possesses, for the atom of an element may sometimes be positive and sometimes negative. The positive charge is the result of the loss of one or more electrons and the negative charge is due to the acquisition of electrons. Thus chlorine has a negative valence 1 in HCl, and a positive valence 7 in Cl_2O_7. Likewise carbon has a negative valence 4 in H_4C and a positive valence 4 in CO_2. In any such grouping the valence electrons can hardly be regarded as particularly belonging to either or any of the atoms which they join.

Electrolysis. When an electric current passes through certain liquids known as ELECTROLYTES, notably aqueous solutions of acids or salts, certain elements of the solution appear, or are set free at the plates (ELECTRODES) by which the current enters or leaves the liquid. The former, or positive electrode, is called the ANODE and the other, or negative, the CATHODE. The amount of any element liberated at one of the electrodes is proportional to the quantity of electricity that has passed, to the atomic weight of the element, and inversely as the valence. This law is simply explained by the view that the liquid contains carriers or IONS such as have been previously described, which are sorted out by the charged electrodes, the latter serving to maintain an electric field in the liquid; the positively charged ion (CATION) is repelled from the anode and attracted to the cathode, the negatively charged ion (ANION) is urged in the opposite direction.

This conclusion is still valid when the ion is not liberated, for here the ion instead of itself being set free enters into combination and releases a chemically equivalent amount of some other substance. Thus in HCl the ions are H^+ and Cl^-; when this is electrolyzed with platinum electrodes, the hydrogen ions migrate to the cathode where, by a rearrangement of electrons, a positive charge is given to the metal and a hydrogen molecule is liberated. In a similar manner Cl is set free at the anode. In a solution of sulphuric acid the ions are HH^{++} and SO_4^{--}. When this solution is electrolyzed there is this difference, that after the ion has given up its charge to the anode a combination of SO_4 with the water is effected forming a new molecule of H_2SO_4 which remains in solution while at the same time O_2 is set free.

Gas pressure. The relation between the pressure, volume, and temperature of a gas is given by $p = R\rho T/M$ where ρ is the density, T the absolute temperature, M the molecular weight (the ratio of the mass of the gaseous molecule to the mass of the hydrogen atom) and R is a UNIVERSAL GAS CONSTANT. If ρ is measured in grams per cubic centimeter, T in degrees C., that is, $T = t_c + 273$ where t_c is the Centigrade temperature, R has the value $8.31(10)^7$ ergs/gm./1° C. for all gases. The pressure p of a gas is regarded as arising from the impact of the molecules against the walls of the containing vessel.

Osmotic pressure. When two gases such as hydrogen and carbon dioxide are separated by a porous plate, *e.g.*, plaster of Paris, the pressure on the CO_2 side rises as a result of the more rapid diffusion of the smaller and more active hydrogen molecules through the pores of the plate. A somewhat analogous experiment may be arranged in the case of liquids. If, for example, an aqueous solution of sugar be separated from pure water by a semi-permeable membrane or partition that allows the water molecules to diffuse through it, but stops the passage of the solute molecule, the level on the side of the solution will rise until a considerable difference of pressure is established between the two sides. This difference is called the OSMOTIC PRESSURE. In dilute solutions this pressure is that which would be produced by the same number of independent particles existing in the gaseous form, *i.e.*, if these independent particles were molecules, the pressure would be given by an equation analogous to the gas law $p = RcT/M$ where c is the concentration in grams per cubic centimeter.

This equation is used to test whether the particles in solution are the same in number as the molecules.

Concentration of solutions. In electrochemical work a mass of a substance equal to its molecular weight is called a GRAM-MOLECULE or briefly 1 MOL. Thus a gram-molecule of H_2SO_4 is $2 + 32.07 + 64 = 98.07$ gm. The concentration of solutions is frequently stated in gram-molecules per liter of solution. One mol. per liter is called a MOLAR solution. Thus a molar solution of sulphuric acid would contain 98.07 grams of H_2SO_4 per liter. A NORMAL solution is usually defined as the molar solution divided by the valence of the highest ion. In the case of sulphuric acid the valence of SO_4 is 2, whence a normal solution would be ½(98.07) grams of H_2SO_4 per liter.

Freezing and boiling points. Another effect of a substance dissolved in water is to produce a change in the vapor pressure, and, in consequence, a lowering of the freezing-point and a raising of the boiling-point. This effect is directly connected with the concentration and the molecular weight and hence may also be used to test the equivalence of the number of the dissolved particles with the number of molecules.

Electrolytic dissociation. When a dilute solution of a non-electrolyte is formed in water, it is found that the resulting osmotic pressure and the changes of the freezing- and boiling-points are in satisfactory agreement with the laws just stated, but that dilute solutions of electrolytes show effects indicating the presence of a number of dissolved particles much greater than that of the solute molecules present, whence the conclusion is to be drawn that the molecules have split up into ions. Of all solvents water is the most effective in producing this dissociation. This has been attributed to the high dielectric constant of water. The law of attraction between two small charged particles is $F = Q(-Q)/Kr^2$, K being the dielectric constant and r the distance between them. Thus as K is increased the attraction between two ions of a molecule would become lessened.

Voltaic electromotive force or potential difference. If two different metals are immersed in an electrolytic solution they become charged and possess a definite difference of potential, which is the electromotive force of the ordinary electric battery. Nernst's explanation is that when a metal is placed in water there is a tendency for the metallic ions to go into solution and as these are always cations, *i.e.*, positively charged, they take this charge with them and leave the metal negatively charged with respect to the solution. The amount of metal actually dissolved is extremely small because the charges on the ions are so large that they soon create an electric field which prevents any more ions leaving the plate. The amount of metal to bring this about, may not, in fact, be sufficient to form a layer one molecule thick. The osmotic pressure exerted by the ions in solution when equilibrium has been attained is considered by Nernst to be a measure of what he terms SOLUTION PRESSURE or the tendency of the ions to go into solution. When on the other hand a metal is placed in a solution already containing a considerable number of ions and exerting an osmotic pressure greater than the solution pressure, this excess of pressure tends to drive some of the ions out of solution, in which process they give up a positive charge to the plate. In this case the plate becomes positively charged with respect to the solution. Thus a copper plate becomes positively charged to +.60 volt when immersed in a normal solution of copper sulphate. Zinc, on the other hand, having a greater solution pressure, becomes negatively charged to −.51 volt when immersed in a normal solution of zinc sulphate.

In a Daniell or gravity cell a plate of Cu and a plate of Zn, each immersed in a solution of its own salt, are combined so that the total electromotive force is the sum of these two differences of potential, *i.e.*, the copper or positive pole is about 1.11 volts higher than the zinc or negative pole. Similar conclusions apply to hydrogen as to metallic ions, since these tend to go into solution as positive ions. When any solution containing hydrogen ions is electrolyzed, the effect of the electric field is to drive them out of the ionic into the gaseous form at conducting surfaces where the force is directed away from the liquid. The theory not only explains the deposition of metals and the evolution of gas at the electrodes but may be applied to the spontaneous decomposition of water by alkali metals. Thus potassium has so great a solution pressure that the resulting electric force or osmotic pressure is sufficient to liberate hydrogen, since there are always a number of hydrogen ions present in water.

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12. Surface tension

Every liquid surface behaves as if it were under tension and in some respects like a stretched elastic membrane. It differs in its behavior from such a membrane in three particulars: (1) It tends to contract indefinitely; (2) the tension is the same in all directions in the surface; (3) the tension is independent of the thickness as long as the latter exceeds a certain small value of the order of $70\mu\mu$ [$1\mu\mu = (10)^{-7}$ cm.]

Surface tension is explicable in terms of the molecular forces that hold a body together; frequently called cohesion or adhesion. Consider a small sphere, whose radius is within the range of molecular forces, to be drawn about a particle in the interior of a liquid. On account of the symmetry of the forces exerted by the other particles of the sphere, the one at the center will be in equilibrium. If, on the other hand, a similar sphere be drawn about a particle at the surface of the liquid, there will be a resultant force on the particle tending to draw it into the interior. This tendency of the surface particles to be drawn inward makes the surface behave as if it were under tension, whence it is convenient to speak as if there were a tensile force acting in the surface. The phenomenon of surface tension is often called CAPILLARITY. Surface tension is a property of an interface or boundary between unlike phases. A thin layer or sheet of liquid having two surfaces is called a FILM.

Measure of surface tension. The force necessary to keep a surface from contracting divided by the width of the surface at right angles to the applied force, is taken as the measure of surface tension T. In the c.g.s. system it is the force in dynes necessary to hold a ribbon of the surface 1 cm. wide from contracting.

Surface energy. Surface phenomena may be considered in terms of surface energy. If two parts of a body originally united are separated against their mutual molecular attractions, a certain amount of work must be done which may be recovered when the surfaces are again allowed to unite. Thus, every square centimeter of a surface may be regarded as the seat of an amount of energy. Whenever the surface of a body, as for example, a liquid, is increased by stretching, a definite amount of work is done, and when this area is contracted, the same amount of work is recovered. This work is easily shown to be numerically equal to the surface tension. Accordingly if A is the area of a surface, the total energy is $W = TA$. Since potential energy tends to a minimum, every liquid surface tends to become as small as the boundary conditions permit, or if the potential energy can diminish by a change in its chemical constitution, or in the concentration of the components of a solution, this change will occur spontaneously.

Angle of contact. When two fluids are in contact with a solid, these surfaces are found experimentally to meet at an angle which is called the angle of contact. In Fig. 9, *a*, let the interfacial tensions be denoted by the letters with subscripts, and the angle of contact of the oil-water surface with the solid by, θ. The condition of equilibrium is $T_{SO} = T_{SW} + T_{OW} \cos \theta$. When the molecular attraction between two substances is large, the intimacy of contact is also great and the interfacial energy or tension is small or may even become negative, in which case the liquid W will tend to spread indefinitely over the solid. In the case illustrated observation shows that water wets the solid, since $T_{SW} < T_{SO}$, or a given area of the solid covered with water represents less energy than the same area covered with oil. If $T_{SO} > T_{SW} + T_{OW}$ the liquid W would tend to cover the solid completely. In Fig. 9, *b*, the oil tends to displace the water. If $T_{SW} > T_{SO} + T_{OW}$ the liquid (O) would tend to cover the solid completely.

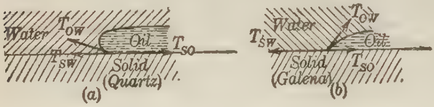

Fig. 9.

Molecular range. The distance at which molecular forces cease to be perceptible is of the order of $50\mu\mu$.

Tension of thin films. No change in the tension of a film occurs until its thickness is reduced to the order of $70\mu\mu$. Tension then first diminishes and afterward increases as the film gets thinner, reaching a stable value at about $12\mu\mu$ which is the thickness of the so-called "black-spot"; below this tension diminishes indefinitely as the thickness gets less. In the case of a thin film of oil on water the surface tension increases slightly as the film is thinned until a thickness of $2\mu\mu$ is reached, which is the thickness required to stop the motion of camphor particles. Between $2\mu\mu$ and $1\mu\mu$ tension increases about 28 per cent. and thereafter but slightly.

Temperature. Diminution of surface tension with increasing temperature, t, is given by the relation $T = T_0 - at$ where T_0 is the tension at 0° C., and a is a constant. For water $a = 0.152$ and for mercury 0.379 dynes per cm. per degree Centigrade.

Surface tension of solutions. Since water has a surface tension which is large compared to that of most ordinary substances, the common effect of a contaminant is to lower the tension, often to a marked degree; this result is intimately bound up with the production of the colloid state. As salts, however, have greater surface tension than water, the surface tension of aqueous salt solutions is generally greater than that of water and increases linearly with the concentration.

Surface concentration. Substances in solution that lower the surface tension of a liquid will concentrate at the surface and substances like salts that raise the tension tend to leave the surface layers. For this reason the presence of a salt does not produce a large increase in surface tension while, on the other hand, substances that reduce surface tension may produce large diminution. Any change in concentration at a surface is called ADSORPTION.

Stability of films. A contaminated surface can adjust so as to effect equilibrium under circumstances where a film of pure liquid would break, *e.g.*, a vertical film of pure liquid would thin down under the influence of its weight until it broke; if, however, the film contained a contaminant which by thinning out increased the surface tension, equilibrium might be regained.

Table 12. Surface tensions of electrolytes

(Solution in water for which T = 75.3 dynes per cm.; temperature, 18° C.; concentration, 1.5 gram-molecules per liter)

HNO_3	74.2	KNO_3	76.9
HCl	74.9	KCl	77.6
H_2SO_4	76.0	K_2CO_3	79.9
NaOH	78.3	$NaNO_3$	77.2
KOH	78.0	NaCl	77.8
NH_3	72.2	NH_4NO_3	77.0

Table 13. Surface tensions

Liquid	Temperature C°	Surface tension dynes per cm.	Liquid	Temperature C°	Surface tension dynes per cm.
Acetic acid	20	23.5	Paraffin oil	25	26.4
Alcohol, ethyl	20	16.5	Sulphuric acid sol., d = 1.14	15	74.4
Alcohol, methyl	20	23.0	Turpentine oil	15	27.3
Ammonia sol., d = 0.96	15	64.7	Pine oil	....	30.0
Benzene	17.5	29.2	Water	15	74.4
Carbon bisulphide	19.4	33.6	Ether, ethyl	20	16.5
Carbon tetrachloride	20	25.7	Mercury	17.5	547
Coke-oven oil	....	28.0	Lead	335	473
Chloroform	15	27.2	Cadmium	365	810
Glycerine	18	65.2	Iron	1200	1000
Kerosene	....	25.2	KI, fused	700	86
Liquid carbon dioxide	15.2	2	KCl, fused	790	100
Liquid chlorine	−72	33.6	KNO_3, fused	338	110
Liquid hydrogen	−252	2	$NaNO_3$, fused	339	106
Liquid oxygen	−183	13.1	HCl solution, d = 1.09	20	74.5
Liquid nitrogen	−196	8.5			
Olive oil	20	32			

Table 14. Interfacial tensions

	Temperature C°	Tension, dynes per cm.
Water-benzene	20	33.6
Water-chloroform	20	29.5
Water-ether	20	12 2
Water-olive oil	20	20.6
Water-paraffin oil	20	48.3
Mercury-water	20	427
Mercury-alcohol	20	399
Mercury-chloroform	20	399
Kerosene-water	....	32.8
Kerosene and pine oil—water	....	11.6
Kerosene and pine oil—0.05% sol. NaOH	....	7.3
Kerosene and pine oil—0.20% sol. NaOH	....	4.5
Kerosene and pine oil—0.20% sol. H_2SO_4	....	13.2
Coke-oven oil—water	....	14.1
Coke-oven oil—0.05% sol. NaOH	....	5.8
Coke-oven oil—0.20% sol. NaOH	....	2.6
Coke-oven oil—0.10% sol. Na_2CO_3	....	6.6
Coke-oven oil—0.20% sol. Na_2CO_3	....	4.4
Coke-oven oil—0.40% sol. H_2SO_4	....	14.4
Coke-oven oil—0.01% sol. saponin	....	9.3
Coke-oven oil—0.01% sol. tannic acid	....	12.7

For this reason durable films must consist of a mixture of substances, *e.g.*, soap and water. High viscosity also aids durability.

Measurement of surface tension. Since the presence of the slightest trace of a contaminant may greatly alter the surface tension, measurements of this quantity are subject to considerable uncertainty and the results may depend on the method employed whether static or kinetic. Even greater uncertainties attach to the angle of contact.

Wilhelmy's method. A thin plate of very clean glass or metal is suspended vertically from one of the arms of a balance and weighed. The lower edge is then immersed in a liquid that wets the plate and the additional mass, m, determined which must be placed in the other pan to balance the pull on the plate partly immersed. Correction must be made, if necessary, for the effect of the fluid displaced. If l is the total length of the blade, θ the angle of contact and g the acceleration of weight, T is found from the equation $2lT \cos \theta = mg$.

Parallel plates. Comparative values of T may be found from the height of rise of liquids between clean parallel glass plates. Two such plates separated at the corners by thin sheet metal and held together at the middle are dipped into the liquids. The surface tensions are proportional to $h/\cos \theta$ where h is the rise and θ is the angle of contact.

Method of falling drops. Let m be the mass of the drop as it falls from a thin-walled tube, r the external radius. The surface tension may be computed by the formula $T = mg/fr$ where f is a constant which may be taken as 3.8 when the radius of tube lies between 0.3 cm. and 1 cm. If very accurate results are desired f must be determined first by experiments on pure water for which T may be taken as 74.4 dynes/cm. at 15° C.

A similar method may be used for determining the interfacial tension when one liquid is allowed to drop out of the tube into another liquid. Let T_{AB} be the interfacial tension, v the total volume of the liquid delivered, D_B the density of one fluid, and D_A the density of the other, f the constant factor and r the external radius of the tube from which n drops are detached. Then $T_{AB} = v(D_B - D_A)g/nrf$. If the effluent fluid does not wet the glass, r is to be taken as the inner radius of the tube. Relative values of the interfacial tensions of different solutions may be thus compared by counting the number of drops in which a given volume of the same fluid flows out into two solutions.

Emulsions. Starting with two liquids, either may be dispersed in the other, but to get a stable emulsion some substance must pass into the separating surface and form a coherent film. This film becomes concave on the side having the higher surface tension, and the liquid on the concave side becomes the internal phase. It is possible to change from an emulsion of oil in water to one of water in oil by changing the nature of the third component.

With sodium soap, oil assumes the internal phase in water while with calcium soap, the water becomes the internal phase in oil. Hydrous ferric oxide may come down as an unquestionable internal phase, or as a jelly, when it is probably the external phase. When water is the external phase, insoluble particles should not affect the surface tension; there should be no osmotic pressure and no diffusion except that due to Brownian movements. If, however, the particles are slightly soluble, all of the effects named may occur in slight degree. If the suspensions are electrified they will move to the cathode or the anode accordingly as they adsorb a positive or a negative ion. Also since this adsorption will in general be selective the nature of the ion will be important as well as the sign of the charge.

Structure. When water becomes the internal phase and the less mobile phase becomes external, the latter will have a honeycomb structure. A distinct change in surface tension and viscosity would be expected but since the less mobile phase is insoluble in the other there would be no appreciable osmotic pressure. As these phenomena are markedly exhibited by gelatine the conclusion is to be drawn that gelatine is the external phase in the so-called aqueous solution of gelatine. Since emulsoid sols are two-phase liquids and since, further, gels may be obtained from them by cooling or evaporation, the conclusion is forced that gels possess a structure similar to emulsoids. We may naturally picture two degrees of solution, the more concentrated one forming the highly viscous walls of the cells which contain the other more dilute solution. Jellies may also have a structure like a sponge when, since each is continuous, there will be neither internal nor external phases, but there is no good reason for assuming that all jellies are of this type.

SECTION 26

THEORETICAL MECHANICS

BY

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STATICS

1. Definitions

Mechanics is the science of motion. It considers the effects of the action of force upon the motion of material bodies. KINEMATICS treats methods of describing motion and involves only concepts of geometry and time. DYNAMICS treats of the effect of forces upon the motion of bodies. KINETICS deals with cases in which the state of motion is changed by action of forces. STATICS deals with cases in which no such change is produced.

These definitions are those used in a large majority of modern texts. A few writers use dynamics as a synonym for mechanics while others use it as a synonym for kinetics.

Force is an action (push or pull) exerted by one body upon a second body. Forces occur always in pairs; the body acted upon exerts an equal and opposite force on the body acting. A force is completely specified by its magnitude, direction and sense, and point of application. In case of rigid bodies, which suffer no change of shape or size, the point of application is not important. The PRINCIPLE OF TRANSMISSIBILITY OF FORCE asserts that a force that acts upon a rigid body may be considered to be applied to any particle of the body that lies in the line of action of the force.

Concentrated and distributed forces. While it is usual to speak of the point of application and action line of a force, these apply strictly to concentrated forces only. An actual force is always a distributed force whose place of application is a surface or a solid. When

a book rests on a table the supporting pressure of the table is applied over the surface of the book in contact with it. When the earth attracts a body the place of application is the three-dimensional space occupied by the body. A CONCENTRATED FORCE is one of finite magnitude whose place of application is a point. This ideal definition can not be fulfilled by an actual force, but the conception is useful and there are forces whose place of application is very small which may be considered as applied at a point. A DISTRIBUTED FORCE is regarded as made up of a large number of small concentrated forces, the combined effect of which, in ordinary problems, is equivalent to a single concentrated force. (See Art. 13.)

Graphical representation. A concentrated force is a vector quantity. It is represented graphically by a straight line the length of which indicates the magnitude of the force, while the direction of the force is indicated by the direction of the line and the sense by an arrow placed on the line. For graphic work two distinct diagrams are employed and a systematic notation is useful. A drawing that represents a body or a structure together with action lines of forces applied to it is a SPACE DIAGRAM. The VECTOR DIAGRAM is one in which vectors are drawn to represent the magnitudes and directions of the forces. In Bow's SYSTEM OF NOTATION forces in a space diagram are designated by lower-case letters placed each side of the line; in the vector diagram by the same capital letters placed at the ends. In Fig. 1 the action line of force W applied to beam at L is indicated by ab, while the magnitude and direction are shown by vector AB.

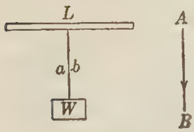


FIG. 1.

Classification of systems of forces is made according to the arrangement of their action lines. If the action lines lie in the same plane the system is COPLANAR, otherwise NON-COPLANAR. If they pass through the same point the system is CONCURRENT, otherwise NON-CONCURRENT. If two or more forces have the same action line they are COLLINEAR. A system of two equal forces, parallel, opposite in sense, and having different action lines is a COUPLE. Two or more forces equivalent to a single force are COMPONENTS of the single force. The single force is the RESULTANT of its components. In general, the resultant of a system of forces is the simplest equivalent system. This may be a single force, a single couple, or a force and a couple. When the resultant is a single force the EQUILIBRANT is a force equal in magnitude, having the same line of action but opposite sense. COMPOSITION is the operation of replacing a system of forces by its resultant. RESOLUTION is the operation of replacing a single force by a system of components.

2 Composition of two concurrent forces

Collinear forces form a special case of concurrent forces. The resultant of two collinear forces having the same sense is a force having the same action line, the same sense, and equal in magnitude to the sum of the magnitudes of the two forces. The resultant of two collinear forces acting in opposite senses is a force whose magnitude is equal to the difference between the magnitudes of the given forces and whose sense is that of the greater. The resultant of any number of collinear forces is a force having the same action line and equal in magnitude and sense to the algebraic sum of the components.

Parallelogram law. If two concurrent forces, P and Q (Fig. 2), whose action lines meet at O are represented in magnitude and direction by OA and OB, their resultant, R, acts along the diagonal, OC, of the parallelogram of which OA and OB are adjacent sides, and its magnitude is represented by the length of OC. This statement does not imply that O is the point of application of the forces. It is necessarily so with a particle (Fig. 2a). But with a rigid body

the point of application of a force may be any point of the body on the action line of the force (Fig. 2*b*). This law applies as well when the point O is outside the body (Fig. 2*c*).

Triangle law. If two concurrent forces, P and Q (Fig. 3), are represented in magnitude and direction by AB and BC, their resultant, R, is represented in magnitude and direction by AC, the third side of the triangle ABC. The action line of R is ac through the point of intersection of the action lines of P and Q. The resultant of two concurrent forces may be computed by trig-

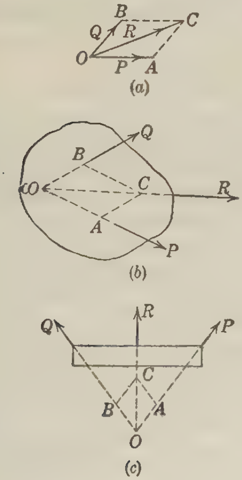


FIG. 2.

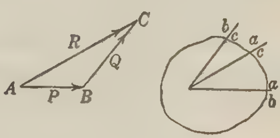


FIG. 3.

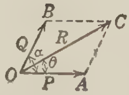


FIG. 4.

onometry. If α (Fig. 4) denotes the angle AOB between forces P and Q, and θ denotes the angle AOC between P and the resultant, R, then

$$R^2 = P^2 + Q^2 + 2PQ \cos \alpha \text{ and } \tan \theta = \frac{Q \sin \alpha}{P + Q \cos \alpha}.$$

Resolution of a force into two components can be made in an infinite number of ways. If a force, F, is represented in magnitude and direction by AB and if C is any point chosen arbitrarily, then AC and CB represent in magnitude and direction the two components whose resultant is the given force, F. The usual case of resolution demands that the two components shall be at right-angles to each other. The RECTANGULAR COMPONENT (or RESOLVED PART) of a force along any line, OX, (Fig. 5) is given by $F_x = F \cos \alpha$. It is understood

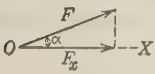


FIG. 5.

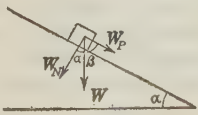


FIG 6.

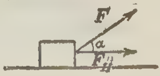


FIG. 7.

that angle α is measured in counterclockwise direction from the positive X-axis to the positive end of the action line of F. The resolved part of a force along any line represents the entire effect of the force in the direction of that line.

Example 1. The weight, W, of a body resting on an inclined plane (Fig. 6), exerts a pressure W_N normal to the plane equal to $W \cos \alpha$ and a force W_P parallel to the plane equal to $W \cos \beta = W \sin \alpha$.

Example 2. A body being dragged along a level plane by a force, F, making an angle, α, with the horizontal (Fig. 7) is urged forward by a force F_H equal to $F \cos \alpha$.

Analytic composition of two forces is accomplished by using rectangular components. If R is the resultant of P and Q and if R_x, P_x, Q_x, are components along the X-axis and R_y, P_y, Q_y are components along the Y-axis then $R_x = P_x + Q_x$ and $R_y = P_y + Q_y$. The resultant is inclined to the X-axis at an angle, θ, given by the formula $\tan \theta = R_y/R_x$. The magnitude of the resultant is $R = \sqrt{R_x^2 + R_y^2}$.

3. Composition of any number of concurrent forces

Coplanar forces. The resultant of any number of coplanar concurrent forces is found graphically by constructing a FORCE (OR MAGNITUDE) POLYGON according to the principle of the force triangle. Suppose the forces to be combined are F_1, F_2, F_3, F_4 whose action lines meet at O (Fig. 8). Beginning at A draw AB to represent F_1 in magnitude and direction, BC to represent F_2, CD to represent F_3, and DE to represent F_4. The resultant is given in magnitude and direction by AE and its line of action passes through O.

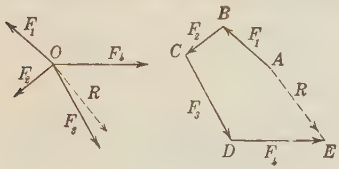


FIG. 8.

The resolved parts of a resultant in the directions of a pair of rectangular axes are obtained by adding the resolved parts of the components. Let R make an angle θ with the X-axis, F_1 the angle α_1, . . ., F_4 the angle α_4, where it is understood that all angles are measured in a counterclockwise direction from the positive X-axis. Then

$$R \cos \theta = R_x = F_1 \cos \alpha_1 + F_2 \cos \alpha_2 + \ldots = \Sigma F_x,$$

$$R \sin \theta = R_y = F_1 \sin \alpha_1 + F_2 \sin \alpha_2 + \ldots = \Sigma F_y,$$

$$R = \sqrt{R_x^2 + R_y^2}, \quad \tan \theta = R_y/R_x.$$

Non-coplanar forces. When the lines of action of a system of concurrent forces do not lie in the same plane the resultant may be found by constructing a space or skew polygon by the same method used for coplanar forces. For analytic work it is customary to use a system of three mutually perpendicular axes. The direction of a force is indicated by three angles α, β, γ (Fig. 9). These are the angles which the action line of the force makes with the X-, Y-, and Z-axes respectively, each angle being less than 180° and measured from the positive co-ordinate axis to the action line of the force. The resolved parts of the force are given by

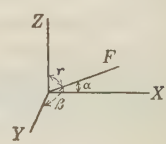


FIG. 9.

$$F_x = F \cos \alpha, \quad F_y = F \cos \beta, \quad F_z = F \cos \gamma.$$

The resultant, R, (direction angles α, β, γ) of a system of n forces, F_1 (direction angles α_1, β_1, γ_1), F_2, . . . F_n is found by the formulas

$$R_x = F_1 \cos \alpha_1 + F_2 \cos \alpha_2 + \ldots + F_n \cos \alpha_n = \Sigma F \cos \alpha,$$

$$R_y = F_1 \cos \beta_1 + F_2 \cos \beta_2 + \ldots + F_n \cos \beta_n = \Sigma F \cos \beta,$$

$$R_z = F_1 \cos \gamma_1 + F_2 \cos \gamma_2 + \ldots + F_n \cos \gamma_n = \Sigma F \cos \gamma,$$

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2},$$

$$\cos \alpha = R_x/R, \quad \cos \beta = R_y/R, \quad \cos \gamma = R_z/R.$$

4. Moments and couples

Moment of a force with respect to (about) a point is the product of its magnitude and the length of the perpendicular from the point to its action line. Thus (Fig. 10) the moment of F about O is Fp. It is equal to twice the area of the triangle OAB. Point O is the CENTER OF MOMENTS and distance p is the MOMENT (OR LEVER) ARM. The moment of a force is the measure of its tendency to produce rotation about an axis passed through O in a direction perpendicular to plane OAB. This measure is expressed in terms of the units of force and length, as ft.-lb., in.-tons. (Some writers use lb.-ft. as the unit of moment of a force to distinguish from ft.-lb. as the unit of work and energy.) Moments are generally considered as positive when they tend to produce rotation in a counterclockwise direction and negative when the direction is clockwise. The moment of a force about a line or axis is found by resolving the force into two rectangular components, one of which is parallel to the line. The product of the perpendicular component and the distance from its action line to the axis is the moment of the given force about the given axis. The value of this moment does not depend on the point of application assumed for the force.

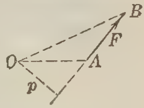


FIG. 10.

Composition of moments is based on Varignon's Theorem: The moment of the resultant of two concurrent forces about any point in their plane is equal to the algebraic sum of the moments of the two forces about the same point. The theorem is valid (*a*) if instead of two there is any number of forces, and (*b*) if the forces are merely coplanar without being concurrent. A similar theorem holds for moments about a line or axis.

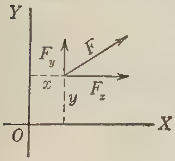


FIG. 11.

For analytic computation the moment of a force (Fig. 11) about the origin (or a line through the origin perpendicular to the XY-plane) is given by the formula:

$$M = xF_y - yF_x,$$

where (x, y) is any point on the action line of the force.

Couples. Two equal and parallel forces of opposite sense constitute a couple. The ARM OF A COUPLE is the perpendicular distance between the action lines. The PLANE OF A COUPLE is the plane determined by the action lines. The MOMENT (OR TORQUE) OF A COUPLE is the product of the magnitude of one of the forces and the arm. It is equal to the algebraic sum of the moments of the two forces of the couple with respect to any point in its plane. The moment is usually called positive if the forces tend to produce counterclockwise rotation, and negative if clockwise. The CENTER OF ROTATION for a couple may be anywhere in its plane. Two couples are equivalent if their planes are parallel and their moments equal. Hence a couple may be turned about in its own plane or moved to a parallel plane or replaced by another

couple (having an arm of any given length but the same moment) without altering its effect on a rigid body.

Resultant of couples. The resultant of any number of couples acting in the same plane or in parallel planes is a couple whose moment is equal to the algebraic sum of their moments. Composition of couples whose planes are not parallel is accomplished by vector addition. A couple is represented by a vector drawn perpendicular to its plane, the length of the vector (to scale) being equal to the moment of the couple. The vector points from the plane toward that side from which the rotation appears counterclockwise. This vector is sometimes called the AXIS OF THE COUPLE. Any number of couples may be combined by adding vectors representing them according to the parallelogram law. (See Art. 2.)

Resultant of single force and couple in the same plane (or parallel planes) is a single force equal and parallel to the original force, at a distance from it equal to the moment of the couple divided by the magnitude of the force; and so situated that the moment of the resultant about the point of application of the original force is of the same sign as the moment of the couple. The couple may be brought into the position shown in Fig. 12. The resultant of P, $-Q$, and Q is R $(=P)$ acting in a line through point C so that $(P - Q) \times AC = Q \times BC$ (See Art. 5). From this it follows that

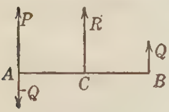


FIG. 12.

$$AC = \frac{Q(AC + BC)}{P} = \frac{\text{moment of couple}}{P}.$$

5. Resultants of parallel forces

Resultant of two parallel forces acting in the same direction is equal in magnitude of the sum of their magnitudes, is parallel to the forces, and divides the line joining their points of application in the inverse ratio of the magnitudes of the given forces. Thus (Fig. 13a) $R = P + Q$ and $P \times AC = Q \times BC$ or $aP = bQ$. Also $aR = cQ$ and $bR = cP$. The resultant of two unequal parallel forces acting in opposite directions is equal in magnitude to difference of their magnitudes, acts in the direction of the larger force, and divides the line joining their points of application externally in the inverse ratio of the magnitudes of the given forces. Thus (Fig. 13b) $R = P - Q$ and $P \times AC = Q \times BC$ or $aP = bQ$. Also $aR = cQ$ and $bR = cP$

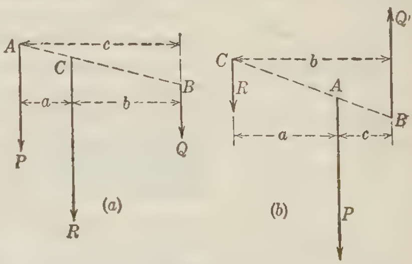


FIG. 13.

Two equal parallel forces acting in opposite directions form a couple (see Art. 4). The equations above show that when P and Q are nearly equal, R is very small and a and b are very large. As the magnitude of Q approaches that of P, R approaches zero and its line of action recedes to an infinite distance. This gives rise to the conception of a couple as equivalent to a force of zero magnitude with an infinite lever arm.

Moment of the resultant of any number of parallel forces in the same plane about any point in their plane is equal to the algebraic sum of the moments of the given forces about that point. In general, the algebraic sum

of the moments of any system of parallel forces about any axis is equal to the moment of the resultant about that axis.

Center of parallel forces is the point of application of the resultant. If the points of application of any system of parallel forces remain the same while the direction of the forces of the system is changed, the resultant will pass always through the center (or CENTROID) of the system (see Art. 13).

Center of two parallel forces may be found by the following construction (Fig. 14). Suppose A and B are the points of application of forces P and Q. From A lay off AA' equal to Q but in the opposite direction to Q. From B lay off BB' equal to P and in the same direction as P. The intersection C of AB and $A'B'$ is the center of P and Q.

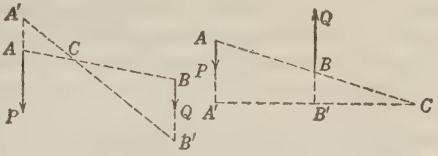


FIG. 14.

The magnitude of the resultant and the center of any system of parallel forces may be found by the following formulas. Using a set of rectangular axes, let the points of application of parallel forces $P_1, P_2, P_3, \ldots$ be (x_1, y_1, z_1), (x_2, y_2, z_2), (x_3, y_3, z_3), . . ., respectively, and let the point of application of the resultant R be $(\bar{x}, \bar{y}, \bar{z})$, then

$$R = P_1 + P_2 + P_3 + \ldots, \qquad \bar{x} = \frac{x_1P_1 + x_2P_2 + x_3P_3 + \ldots}{R},$$

$$\bar{y} = \frac{y_1P_1 + y_2P_2 + y_3P_3 + \ldots}{R}, \qquad \bar{z} = \frac{z_1P_1 + z_2P_2 + z_3P_3 + \ldots}{R}.$$

The sums in these formulas are algebraic sums; proper signs must be given to both coordinates and forces. The formulas are valid for all cases except when $R = \Sigma P = 0$. If the application points all lie in the same plane, this may be taken for the XY-plane and the last formula is unnecessary.

6. Resultant of any system of non-concurrent forces acting on a rigid body

The resultant of any system of forces acting on a rigid body may be (1) a single force, (2) a single couple, (3) a single force and a single couple. A system of coplanar forces can be reduced to either case 1 or case 2. Case 3 can occur only when the action line of the single force is not parallel to the plane of the single couple.

RESULTANT OF ANY SYSTEM OF COPLANAR FORCES

Graphic method. *Case* 1. The resultant is a single force. Suppose the forces to be compounded are ab, bc, cd, de shown in a space diagram (Fig. 15a). The magnitude and direction of the resultant are determined by constructing the force (or magnitude) polygon $ABCDE$ (Fig. 15b), in which AE is the resultant. The action line of the resultant is found by constructing a STRING (or FUNICULAR) POLYGON as follows. Point O (called the POLE) is chosen arbitrarily and rays OA, OB, OC, OD, OE are drawn. Then, using Bow's notation (see Art. 1), oa and ob are drawn, parallel to OA and OB, respectively, from any convenient point on ab; oc is similarly drawn from the point

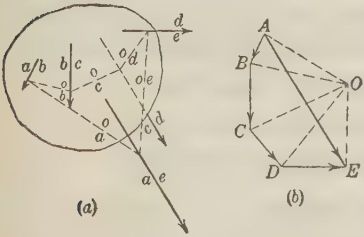


FIG. 15.

where ob meets bc; od is drawn from the point where oc meets cd; and oe is drawn from the point where od meets de. Then the intersection of oa and oe is a point on the action line of the resultant.

It may happen that oa and oe are parallel but, if the force polygon is not closed (see Case 2), this may be avoided by proper choice of point O.

Case 2. The resultant is a single couple. If the force polygon is closed (points A and E coincide) the resultant force is zero. Rays OA and OE coincide and oa and oe must be parallel. If oa and oe do not coincide, the string polygon is open and the resultant of the system is a couple. The moment of the couple is equal to the product of the force represented by OA and the perpendicular distance between oa and oe. If oa and oe coincide, the string polygon is closed and the resultant of the system is zero.

Analytic method. *Case* 1. The resultant is a single force. The magnitude and direction of the resultant may be computed as in the case of concurrent forces (see Art. 3):

$$R_x = \Sigma F_x, \quad R_y = \Sigma F_y, \; R = \sqrt{R_x^2 + R_y^2}, \; \tan\theta = R_y/R_x.$$

The position of the action line of the resultant is determined by the fact that its moment about any point must equal the sum of the moments of the component forces about that point. Thus if $M = \Sigma(xF_y - yF_x)$ denotes the sum of the moments of the given forces about the origin of co-ordinates, the action line of the resultant must be at distance M/R from the origin. The moment arm of R must be laid off in such a way that the sign of the moment of the resultant is the same as the sign of the sum of the moments of the given forces.

Case 2. The resultant is a single couple. In this case $R_x = 0$, $R_y = 0$. The moment of the resultant couple is equal to the sum of the moments of the given forces about any point. The value of this moment will be the same whatever point is chosen as origin.

RESULTANT OF ANY SYSTEM OF NON-COPLANAR FORCES

This is, in general, a single force and a single couple whose plane is not parallel to the action line of the force. The magnitude and direction of the resultant may be computed as in the case of concurrent forces (see Art. 3):

$$R_x = \Sigma F_x, \; R_y = \Sigma F_y, \; R_z = \Sigma F_z, \; R = \sqrt{R_x^2 + R_y^2 + R_z^2},$$

$$\cos\alpha = R_x/R, \; \cos\beta = R_y/R, \; \cos\gamma = R_z/R.$$

The point of application of the resultant force may be chosen arbitrarily. If the origin of co-ordinates is taken at this point, the resultant couple is the vector sum (see Art. 4) of the three component couples M_x, M_y, M_z, where

$$M_x = \Sigma(yF_z - zF_y) = \text{sum of moments about } X\text{-axis},$$

$$M_y = \Sigma(zF_x - xF_z) = \text{sum of moments about } Y\text{-axis},$$

$$M_z = \Sigma(xF_y - yF_x) = \text{sum of moments about } Z\text{-axis}.$$

7. Conditions of equilibrium

Definitions. A system of forces is in equilibrium if their combined action produces no change in motion of the body to which they are applied. There is no change in motion if the body remains at rest or moves in a straight line

with constant speed. A rigid body is in equilibrium if all the external forces acting upon it form a system in equilibrium.

Conditions of equilibrium may be expressed graphically by means of the force and string polygons or analytically by means of algebraic equations. In general, a system is in equilibrium if, and only if, the resultant force is zero and the resultant couple is zero.

System of concurrent forces is shown graphically to be in equilibrium if the force polygon closes. Analytically the sum of the components in two directions must be zero (see Art. 3.)

$$R_x = \Sigma F_x = 0, \quad R_y = \Sigma F_y = 0.$$

System of coplanar forces is shown graphically to be in equilibrium, if (*a*) the force polygon closes (indicating no effect of translation), and (*b*), if a string polygon closes (indicating no effect of rotation). Analytically three and only three independent equations can be written which are necessary and sufficient to insure equilibrium of a system of forces acting in the same plane. A set of three such equations may be written in three different ways. The system will be in equilibrium if (1) the sum of the components is zero for both of any two directions and the sum of moments about any one point is zero; (2) the sum of moments about each of two points is zero and the sum of the components is zero in a direction not perpendicular to the line joining these two points; (3) the sum of the moments is zero about each of three points not in the same straight line. Since only three independent equations can be written, it is possible to solve for only three unknown elements. When more than three elements of a system of coplanar forces acting on a single body are unknown the problem is statically indeterminate.

Non-coplanar forces. Statical problems arising in engineering practice seldom require the use of conditions of equilibrium in three dimensions. It is usually possible to simplify the problem by combining pairs of forces in such a way as to reduce the system to one plane (see Fig. 44). When it is necessary or desirable to use three dimensions, six equations are required to express the conditions of equilibrium. Using a set of rectangular axes these equations are (see Art. 6).

$$R_x = \Sigma F_x = 0; \quad R_y = \Sigma F_y = 0; \quad R_z = \Sigma F_z = 0;$$

$$M_x = \Sigma(yF_z - zF_y) = 0; \quad M_y = \Sigma(zF_x - xF_z) = 0;$$

$$M_z = \Sigma(xF_y - yF_x) = 0.$$

Special case. If three forces are in equilibrium they must be coplanar, and must be concurrent or parallel. If concurrent, each force is proportional to the sine of the angle between the other two; if parallel, each force is proportional to the distance between the other two.

8. Application of conditions of equilibrium

Problems in statics deal with bodies known to be in equilibrium under the action of a system of forces of which some elements are known and some are unknown. The solution of such a problem consists in finding the unknown elements. Success in solving problems in statics depends primarily on the proper choice of body or part of structure to which conditions of equilibrium are to be applied. This fact is emphasized in the examples that follow.

Simple examples. **1.** A body weighing 100 lb. (Fig. 16), suspended by a flexible cord, is pulled horizontally with a force of 40 lb. Determine the direction of the suspending cord and its tension.

The body to which the conditions are to be applied is the weight W in equilibrium under the action of three forces. The action lines of three forces must be concurrent and a moment equation is unnecessary.

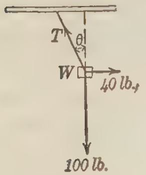


FIG. 16.


FIG. 17.

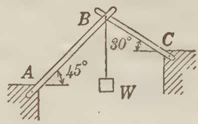


FIG. 18.

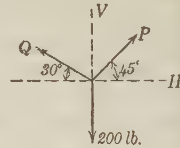


FIG. 19.

First method. Resolution into horizontal and vertical components gives H: $40 - T \sin \theta = 0$, and V: $100 - T \cos \theta = 0$, whence $\tan \theta = 0.4$, $\theta = 21° 48'$, $T = 107.7$ lb.

Second method. Force triangle. Since the force triangle must be closed for equilibrium, θ and T may be calculated from Fig. 17. The graphic method consists in measuring θ and T in the force triangle.

2. In Fig. 18 a horizontal pin B carrying a weight of 200 lb. passes through the members AB and BC inclined respectively at angles of 45° and 30° to the horizontal and held by horizontal pins A and C. Supposing the weights of AB and BC to be small and negligible, find the compression in each member.

The body to which the conditions of equilibrium are to be applied is the pin B which is in equilibrium under the action of three forces (Fig. 19).

First method. Resolution into horizontal and vertical components gives H: $P \cos 45° - Q \cos 30° = 0$, V: $P \sin 45° + Q \sin 30° - 200 = 0$, whence $P = 179.4$ lb. = compression in AB, and $Q = 146.4$ lb. = compression in BC.

Second method. A force triangle is drawn by first laying off (Fig. 20) a vector representing 200 lb. acting vertically downwards, then drawing lines making angles of 45° and 60°, respectively from the extremities of the vector and marking arrows to indicate that the triangle is closed. P and Q are found from the proportion $\frac{200}{\sin 75°} = \frac{P}{\sin 60°} = \frac{Q}{\sin 45°}$.

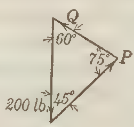


FIG. 20.

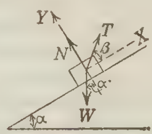


FIG. 21.

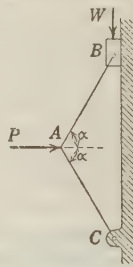


FIG. 22.

The graphic method consists in measuring P and Q in the force triangle.

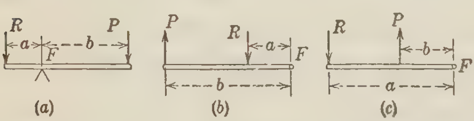


FIG. 23.

3. A body of weight W is held on a smooth plane inclined at angle α to the horizontal, by means of a cord making an angle β with the plane (Fig. 21). Find the tension T in the cord and the pressure on the plane.

The body to which the conditions of equilibrium are to be applied is the weight. N is the normal pressure of the plane on the weight. This is equal and opposite to the pressure of the weight on the plane. Taking an X-axis parallel to plane, the method of resolution gives X: $T \cos \beta - W \sin \alpha = 0$, Y: $T \sin \beta + N - W \cos \alpha = 0$, whence $T = W \sin \alpha \sec \beta$, $N = W(\cos \alpha - \sin \alpha \tan \beta)$.

4. A toggle joint is shown in Fig. 22. A force P is applied at A and a resistance W overcome at B. Find the ratio of P to W. (It is assumed that the weights of arms AB and AC can be neglected.) Let F denote the compression in each arm and consider the pin B. Taking vertical components: $F \sin \alpha = W$. Consider next the pin A. Taking horizontal components: $2F \cos \alpha = P$. Elimination of F between these equations gives $P = 2W \cot \alpha$.

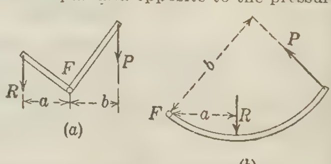


FIG. 24.

5. Levers are classified as follows: First class, fulcrum F between applied force P and resistance R (Fig. 23a); Second class, R between F and P (Fig. 23b); Third class, P between F and R (Fig. 23c). The relation between P and R is found by taking moments about F: $bP = aR$, where a and b represent perpendicular distances from F to the action lines of R and P, respectively. The same relation holds, if the lever is bent (Fig. 24a), or curved (Fig. 24b).

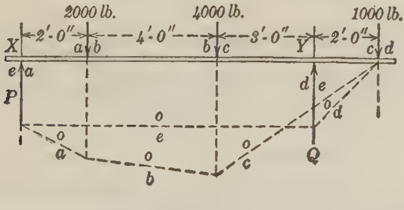


Fig. 25.

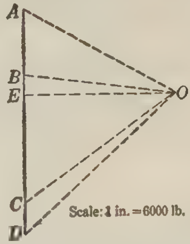


Fig. 26.

6. A beam is loaded as shown in Fig. 25 and supported at X and Y. Find the supporting forces P and Q.

Graphic method (see Art. 6). Since the forces are all parallel, the force polygon consists of segments of a straight line. Segments AB, BC, CD, DE, EA (Fig. 26), represent 2000 lb., 4000 lb., 1000 lb., Q, P, respectively. The location of E is to be determined by a string polygon. Rays OA, OB, OC, OD are drawn. Using Bow's notation (see Art. 1), and starting at point on the action line of P, the polygon oa, ob, oc, od, is drawn with closing line oe. Ray OE parallel to oe locates point E and the unknown forces are $Q = DE = 4330$ lb., and $P = EA = 2670$ lb.

Analytic method. To find P, moments are taken about Y: $2000 \times 7 + 4000 \times 3 - 1000 \times 2 - P \times 9 = 0$, whence $P = 2667$ lb.

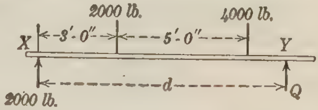


Fig. 27.

To find Q, moments are taken about X: $-2000 \times 2 - 4000 \times 6 - 1000 \times 11 + Q \times 9 = 0$, whence $Q = 4333$ lb.

As a check, the sum of the vertical forces must equal zero. $2667 + 4333 - 2000 - 4000 - 1000 = 0$.

7. A beam is loaded as shown in Fig. 27 and supported at X and Y. The reaction at X is 2000 lb. Find the reaction Q at Y and the distance XY.

Taking vertical components, $2000 - 2000 - 4000 + Q = 0$, whence $Q = 4000$ lb.

Taking moments about X: $-2000 \times 3 - 4000 \times 8 + Qd = 0$, whence $d = 9.5$ ft.

9. Pulleys

In Fig. 28, P denotes the force required to support weight W. If the pulleys were frictionless and if W were moving at a constant speed, this same P would be sufficient to continue the motion at this speed.

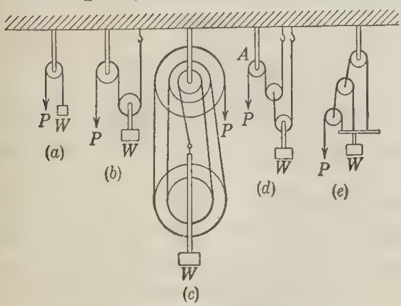


Fig. 28.

(a) Fixed pulley, $P = W$. The only advantage is the change of direction in which P is applied.

(b) Single movable pulley, $P = W/2$.

(c) Block and fall. Two blocks of pulleys of which the upper is fixed while the lower is movable, with a single cord passing alternately around the pulleys in the upper and lower blocks, the portions of cord between successive

pulleys being approximately parallel. If $n (n = 5$ in Fig. 28c) is the number of plies at the lower block, then $P = W/n$.

(d) If A is a fixed pulley and n is the number of movable pulleys, then $P = W/2^n$. In Fig. 28d, $n = 2$, $P = W/4$.

(e) If n is the number of cords attached to a bar supporting W, then $n - 1$ is the number of movable pulleys and $P = W/(2^n - 1)$. In Fig. 28e, $n = 3$, $P = W/7$.

In the relations between P and W given above no account is taken of the weights of the pulleys. In (b) and (c) the weights of the movable pulleys merely form part of the gross weight to be lifted. In (d) the movable pulleys have to be lifted but do not form merely part of the gross weight because only one has to be lifted the same distance as W. In (e) the movable pulleys descend and their weight assists P in lifting W.

10. Trusses

Definitions. A TRUSS is a jointed frame made up of straight bars connected at the ends by pins. Loads are applied only at the joints and, weights of members being neglected, the forces acting on any one member produce tension or compression stresses but no bending. Each member is a two-force piece with equal and opposite forces at each end. To insure rigidity under all conditions of loading the truss must be made up of a series of triangles. A COMPLETE FRAME (Fig. 29a) is made up of just enough bars to preserve the shape under all loads, and must be made up of the minimum number of bars consistent with being composed wholly of triangles. An INCOMPLETE FRAME (Fig. 29b) is not composed wholly of triangles and can carry only symmetrical or other specially arranged loads. A REDUNDANT FRAME has more bars than are necessary to preserve the shape. If both diagonals (Fig. 29c) are pin connected the frame is redundant. If the diagonals are designed to resist tension (or compression) only, the redundancy is only apparent as both diagonals will not be under stress at once. Stress in a redundant frame can not be determined by the laws of elementary statics.

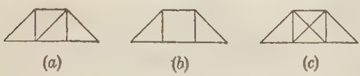


FIG. 29.

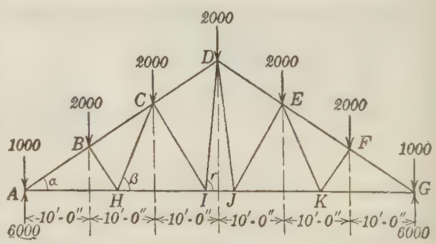
Elementary methods for calculating stresses in truss members are shown in the following examples.

1. *Method of joint resolution.* The roof truss (Fig. 30) has a span of 60 ft. and rise of 15 ft., each inclined upper chord being divided into three equal parts by normal struts. In each set of equations the body considered in equilibrium is the pin at the joint. For writing equations all members are assumed in tension. A plus sign with the answer indicates tension and a minus sign compression.

With loads at the joints of the upper chords as indicated, the supporting forces at A and G are found first and are 6000 lb. each.

Pin A. The horizontal and vertical components are: H: $AB \cos \alpha + AH = 0$, V: $AB \sin \alpha + 6000 - 1000 = 0$, whence $AB = -11{,}200$ lb., $AH = 10{,}000$ lb.

Pin B. The components along and perpendicular to BC are: $BC - AB - 2000 \sin \alpha = 0$, $-BH - 2000 \cos \alpha = 0$, whence $BH = -1800$ lb., $BC = -10{,}300$ lb.



$\sin \alpha = 0.447$, $\sin \beta = 0.800$ $\sin \gamma = 0.949$
$\cos \alpha = 0.894$ $\cos \beta = 0.600$ $\cos \gamma = 0.316$.

FIG. 30.

Pin H. The horizontal and vertical components are: $HI - AH + HC \cos\beta - BH \sin\alpha = 0$, $HC \sin\beta + BH \cos\alpha = 0$, whence $HI = 8000$ lb., $HC = 2000$ lb.

Pin C. The components along and perpendicular to CD are: $CD - BC - HC \cos\alpha - 2000 \sin\alpha = 0$, $-CI - 2000 \cos\alpha - HC \sin\alpha = 0$, whence $CD = -7600$ lb., $CI = -2680$ lb.

Pin I. The horizontal and vertical components are: $IJ - HI + ID \cos\gamma - CI \sin\alpha = 0$, $ID \sin\gamma + CI \cos\alpha = 0$, whence $IJ = 6000$ lb., $ID = 2520$ lb.

Stresses in the remaining members are found by symmetry.

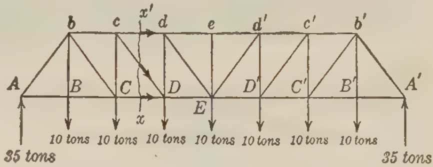


FIG. 31.

It should be observed that, in the preceding method, the order in which pins are considered is important. Before any one pin is considered it is necessary to know all but two of the forces acting on that pin. If this method is employed it is necessary to calculate stresses in all members in order to get stress in ID.

2. *Method of sections.* Fig. 31 represents a PRATT TRUSS of 240 ft. span having 8 equal panels of 30 ft. each and depth 40 ft. With the loading indicated (symmetrical spacing of the loads being assumed) the supporting forces at A and A' are 35 tons. Consider section xx'. The portion of the truss to the left of xx' is a rigid body in equilibrium under the action of three known forces at A, B, and C and three unknown stresses in cd, cD, and CD, which may be found by the conditions of equilibrium as follows: Taking moments about D (assuming the unknown stress as tension): $-40cd + 300 + 600 - 3150 = 0$, whence $cd = -56.25$ tons (compression).

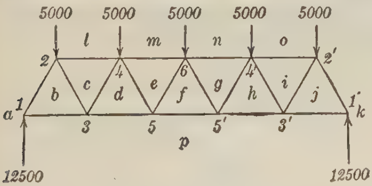


FIG. 32.

Taking moments about c: $40CD + 300 - 2100 = 0$, whence $CD = +45$ tons (tension).

Vertical components give $-0.8cD - 10 - 10 + 35 = 0$, whence $cD = +18.75$ tons (tension).

Stresses in other members may be found by using appropriate sections. The section must not cut more than three members in which stresses are unknown.

3. *Graphic method.* Figure 32 represents a WARREN TRUSS in which each member is 20 ft. long. Figure 33 is the stress diagram. Using Bow's notation the load line $ALMNOK$ is laid off and the reactions KP and PA determined by symmetry. Starting with joint (1) draw AB and BP which determines point B. AB is 14,400 lb. compression (since it acts towards the joint) and BP is 7200 lb. tension. The polygon for joint (2) is AL, LC, CB, BA, the new point being C. LC is 11,700 lb. compression, CB is 8650 lb. tension. The polygon for joint (3) is PB, BC, CD, DP, the new point being D. CD is 8650 lb. compression, DP is 15,800 lb. tension. The polygon for joint (4) is LM, ME, ED, DC, CL, the new point being E. ME is 17,300 lb. compression, ED is 2900 lb. tension. The polygon for joint (5) is PD, DE, EF, FP, the point being F. EF is 2900 lb. compression, FP is 18,750 lb. tension. Stresses in the remaining members are determined by symmetry.

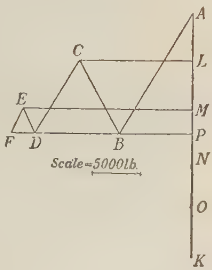


FIG. 33.

11. Cranes and derricks

Simple crane (Fig. 34) consists of a boom AB and tie BC attached to a mast or wall AC, with load supported at B. Stresses in the boom and tie are found by considering the equilibrium of pin B. The force triangle gives the proportion $\dfrac{\text{weight}}{\sin(\alpha+\beta)} = \dfrac{\text{tension in } BC}{\sin\alpha} = \dfrac{\text{compression in } AB}{\sin\beta}$.

Wall crane (Fig. 35) differs from the preceding in that the load is not carried

at B. The boom AD is acted upon by forces at more than two points and stress is no longer simple tension or compression.

Consider the equilibrium of the rigid body AD under the action of four forces: (1) 200 lb. downward at E, (2) 400 lb. downward at D, (3) a force along BC known in direction only, (4) a reaction at A unknown in magnitude and direction. Using two components for the

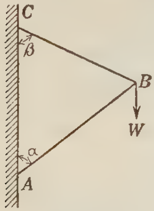


FIG. 34.

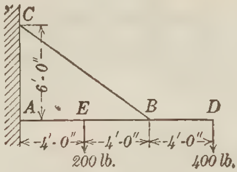


FIG. 35.

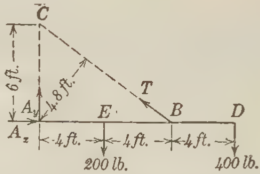


FIG. 36.

unknown forces at A, the problem is to determine the magnitudes of A_x, A_y, T as shown in Fig. 36.

Taking moments about A, $4.8T - 4 \times 200 - 12 \times 400 = 0$, whence $T = 1167$ lb.

Taking moments about B, $-8A_y + 4 \times 200 - 4 \times 400 = 0$, whence $A_y = -100$ lb. (downward).

Taking moments about C, $6A_x - 4 \times 200 - 12 \times 400 = 0$, whence $A_x = 933$ lb.

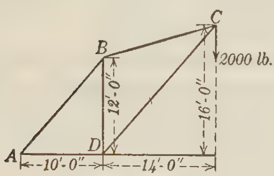


FIG. 37.

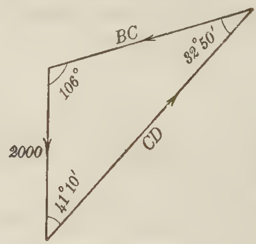


FIG. 38.

Crane with backstay is shown in Fig. 37. The problem is to determine the stresses in the boom CD, tie BC, mast BD, and stay AB. Since all the members are two-force pieces, the stresses are simple tension and compression.

Consider first the pin C for which the force triangle is shown in Fig. 38. The triangle gives $\dfrac{2000}{\sin 32°\,50'} = \dfrac{CD}{\sin 74°} = \dfrac{BC}{\sin 41°\,10'}$, whence $CD = 3550$ lb., $BC = 2430$ lb.

Consider next pin B. From the force triangle $BD = 1280$ lb., $AB = 3040$ lb.

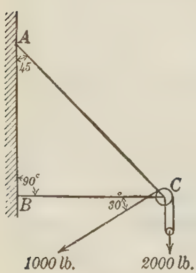


FIG. 39.

Effect of pulleys is illustrated by the crane in Fig. 39. Stresses in the ropes are transferred by means of pulley to pin C, which is in equilibrium under forces shown in Fig. 40.

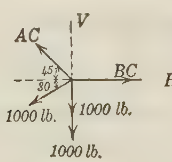


FIG. 40.

Stresses in BC and AC are best determined by taking horizontal and vertical components: H: $BC - 1000 \cos 30° - AC \cos 45° = 0$, V: $AC \sin 45° - 1000 \sin 30° - 2000 = 0$, whence $AC = 3535$ lb., $BC = 3366$ lb.

The frame of Fig. 41 introduces a problem in which the reactions at joints can be determined only by

considering at least two members simultaneously. Complete solution demands the forces on the pins at A, B, C, D, and the supporting force at E (where the frame rests on a smooth floor).

(*a*) Consider the whole frame as a rigid body under the action of (1) 200 lb. downward at F, (2) 300 lb. downward at G, (3) a supporting force acting upward at E, (4) the reaction of the pin at A having components A_x (toward right) and A_y (upward).

Horizontal components show $A_x = 0$.

Taking moments about A: $20E - 200 \times 10 - 300 \times 21 = 0$, whence $E = 415$ lb.

Taking moments about E: $-20A_y + 200 \times 10 - 300 \times 1 = 0$, whence $A_y = 85$ lb.

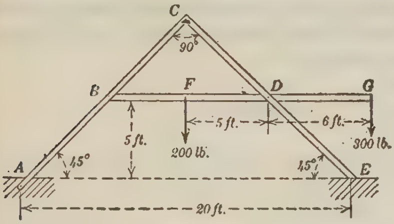


FIG. 41.

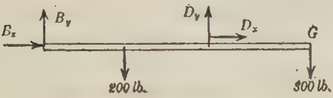


FIG. 42.

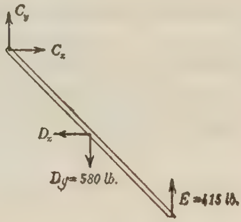


FIG. 43.

(*b*) Consider the member BD in equilibrium under the forces shown in Fig. 42. There are four unknown forces and only three independent equations of equilibrium can be written.

Taking moments about B: $10D_y - 200 \times 5 - 300 \times 16 = 0$, whence $D_y = 580$ lb.

Taking moments about D: $-10B_y + 200 \times 5 - 300 \times 6 = 0$, whence $B_y = -80$ lb. (downward).

Horizontal components give $B_x + D_x = 0$.

(*c*) Consider the member EC in equilibrium under the forces shown in Fig. 43. Observe that the forces exerted by the pin D on member EC are equal and opposite to the forces exerted by the same pin on member BD.

Vertical components give $C_y - 580 + 415 = 0$, whence $C_y = 165$ lb.

Taking moments about C: $-5D_x - 580 \times 5 + 415 \times 10 = 0$, whence $D_x = 250$ lb.

Horizontal components give $C_x - D_x = 0$, whence $C_x = 250$ lb. and, from the equation above, $B_x = -250$ lb. (acts toward the left on BD).

The resulting shear on any pin is found by taking the square root of the sum of the squares of the components. $A = 85$ lb., $B = 262$ lb., $C = 316$ lb., $D = 632$ lb.

Wall crane (Fig. 44) illustrates a problem in space of three dimensions which can be solved by reducing it to problems in two planes.

Plane ABF is perpendicular to wall $ADFE$. The resultant R of the tensions in BE and BD acts along BF. The boom AC is in equilibrium under the action of the forces shown in Fig. 45.

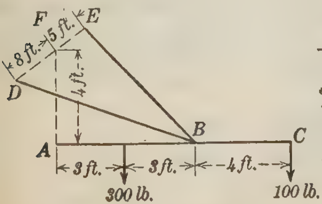


FIG. 44.

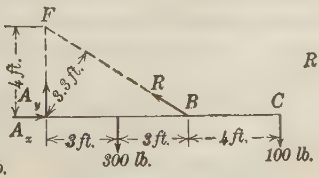


FIG. 45.

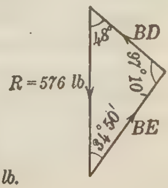


FIG. 46.

Taking moments about A: $3.3R - 300 \times 3 - 100 \times 10 = 0$, whence $R = 576$ lb.
Taking moments about B: $-6A_y + 300 \times 3 - 100 \times 4 = 0$, whence $A_y = 83$ lb.
Taking moments about F: $4A_x - 300 \times 3 - 100 \times 10 = 0$, whence $A_x = 475$ lb.

To find the tensions in BE and BD consider the equilibrium of pin B acted upon by BE, BD, and the boom which exerts a force equal and opposite to R. The force triangle is shown in Fig. 46, from which $BE = 430$ lb., $BD = 335$ lb.

12. Suspended cables

Catenary (Fig. 47) is the curve assumed by a heavy, uniform, flexible, inextensible cord suspended from two points. Let w be the weight per unit length of cord, H be the tension at the lowest point, and $c = H/w$. With axes as shown in Fig. 47 the equation of the catenary is

$$y = \frac{c}{2}(e^{x/c} + e^{-x/c}) = c \cosh x/c.$$

The tension at any point P is given by $T = yw$.

$$\text{Length } LP = \frac{c}{2}(e^{x/c} - e^{-x/c}) = c \sinh x/c.$$

$$\text{Sag } S = \frac{c}{2}(e^{a/c} + e^{-a/c}) - c = \frac{a^2}{2c} \text{ (approximately).}$$

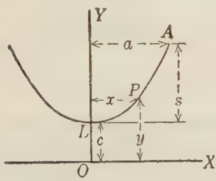


Fig. 47.

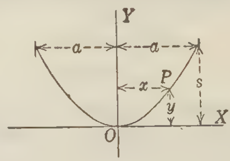


Fig. 48.

Suspension bridge. If the load carried by the cord is distributed uniformly along the horizontal, as in case of a suspension bridge, the curve assumed by the cord is a parabola. Let w be the weight per horizontal unit of length and H be the tension at the lowest point. With axes as shown in Fig. 48 the equation of the curve is $y = wx^2/2H$. If s = sag and $2a$ = span, then $H = wa^2/2s$. The tension at any point P is given by

$$T = \sqrt{H^2 + W^2x^2} = w\sqrt{a^4 + 4s^2x^2}/2s.$$

13. Center of gravity

Definitions. Center of gravity (c. of g.) of a system of particles is the point of application of the resultant of the forces of gravity acting on each particle. For practical engineering purposes these forces of gravity may be regarded as parallel and the center of gravity is given by the formulas of Art. 5. If w_1 is the weight of a particle at (x_1, y_1, z_1), w_2 the weight of a particle at (x_2, y_2, z_2), . . . w_n the weight of a particle at (x_n, y_n, z_n) and c. of g. is at $(\bar{x}, \bar{y}, \bar{z})$, then

$$\bar{x} = \frac{w_1x_1 + w_2x_2 + \ldots + w_nx_n}{w_1 + w_2 + \ldots + w_n} = \frac{\Sigma w_i x_i}{\Sigma w_i},$$

$$\bar{y} = \frac{\Sigma w_i y_i}{\Sigma w_i}, \quad \bar{z} = \frac{\Sigma w_i z_i}{\Sigma w_i}.$$

The center of gravity of any body is the point of application of the resultant of all forces of gravity which act upon every particle of the body. It is given by formulas

$$\bar{x} = \frac{\int x\,dw}{W}, \quad \bar{y} = \frac{\int y\,dw}{W}, \quad \bar{z} = \frac{\int z\,dw}{W},$$

where W = the weight of the body and the limits of the integrals are chosen to cover the space occupied by the body.

If the body is homogeneous, the density D is constant. If V is the volume occupied by the body then $W = DV$ and $dw = D\,dv$ and the formulas become

$$\bar{x} = \frac{\int x\,dv}{V}, \quad \bar{y} = \frac{\int y\,dv}{V}, \quad \bar{z} = \frac{\int z\,dv}{V}.$$

When the body is homogeneous, the center of gravity is called the CENTROID OF THE VOLUME occupied by the body. The CENTROID OF A SURFACE is the limiting position of the center of gravity of a homogeneous thin plate one of whose faces coincides with the surface as its thickness approaches zero. The CENTROID OF A LINE is the limiting position of the center of gravity of a homogeneous thin wire whose axis coincides with the line as the area of its cross-section approaches zero.

Symmetry. The centroid of any geometrical figure lies on any plane or axis of symmetry that the figure may possess.

Table 1. Location of centroids

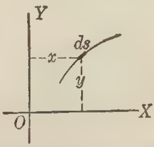
Centroid of	Drawing	Location of centroid
	General	
1. Straight line.		Midpoint.
2. Circle.		Center.
3. Ellipse (see also (25)).		Center.
4. Sphere.		Center.
5. Ellipsoid (see also (36)).		Center.
6. Any regular figure.		Geometrical center.
7. Plane curve.		$\bar{x} = \frac{\int x\,ds}{\int ds}, \quad \bar{y} = \frac{\int y\,ds}{\int ds}.$

Table 1. Location of centroids—*Continued*

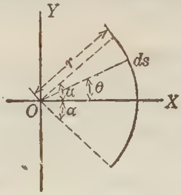
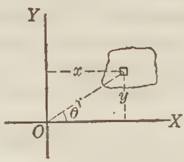
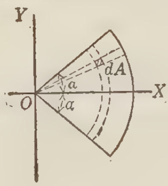
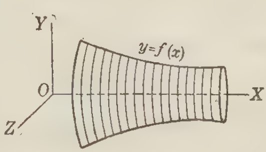
Centroid of	Drawing	Location of centroid
8. Circular arc (see also (16)).		$\bar{x}=\frac{\int x\,ds}{\int ds}=\frac{2\int_0^\alpha r^2\cos\theta\,d\theta}{2\int_0^\alpha r\,d\theta}=\frac{r\sin\alpha}{\alpha}$ $\bar{y}=0.$
9. Any plane area.		$\bar{x}=\frac{\int x\,dA}{\int dA}=\frac{\int\int x\,dx\,dy}{\text{Area}}$ $=\frac{\int\int \rho^2\cos\theta\,d\rho\,d\theta}{\text{Area}}.$ $\bar{y}=\frac{\int y\,dA}{\int dA}=\frac{\int\int y\,dx\,dy}{\text{Area}}$ $=\frac{\int\int \rho^2\sin\theta\,d\rho\,d\theta}{\text{Area}}.$
10. Circular sector (see also (21)).		$\bar{x}=\frac{\int_0^\alpha\int_0^r \rho^2\cos\theta\,d\rho\,d\theta}{\int_0^\alpha\int_0^r \rho\,d\rho\,d\theta}$ $=\frac{\frac{r^3}{3}\int_0^\alpha\cos\theta\,d\theta}{\frac{r^2}{2}\int_0^\alpha d\theta}=\frac{2r\sin\alpha}{3\alpha}.$ $\bar{y}=0.$
11. Surface of revolution. Take X-axis as the axis of revolution and $y=f(x)$ as the generating curve.		$\bar{x}=\frac{\int 2\pi xy\,ds}{\int 2\pi y\,ds}=\frac{\int xy\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx}{\int y\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx}$ $\bar{y}=0,\qquad \bar{z}=0.$
12. Hemispherical surface. Generating curve: $x^2+y^2=r^2$ $\sqrt{1+\left(\frac{dy}{dx}\right)^2}=\frac{r}{y}.$		$\bar{x}=\frac{\int_0^r x\,dx}{\int_0^r dx}=\frac{r}{2}.$ $\bar{y}=0.\qquad \bar{z}=0.$

Table 1. Location of centroids—*Continued*

Centroid of	Drawing	Location of centroid
13. Solid of revolution. When the X-axis is the axis of revolution and the area revolved is that under the curve $y=f(x)$.	For figure, see (11).	$\bar{x}=\dfrac{\int \pi x y^2 dx}{\int \pi y^2 dx}=\dfrac{\pi \int x y^2 dx}{\text{Volume}}$. $\bar{y}=0$. $\bar{}=0$.
14. Solid hemisphere. Generating curve, $x^2+y^2=r^2$.		$\bar{x}=\dfrac{\pi \int_0^r x(r^2-x^2)dx}{\frac{2}{3}\pi r^3}=\frac{3}{8}r$. $\bar{y}=0$. $\bar{z}=0$.

Lines and curves

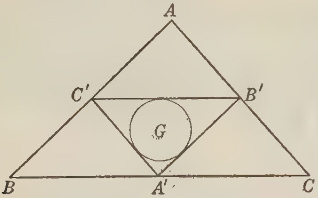
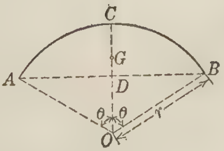
Centroid of	Drawing	Location of centroid
15. Three sides of a triangle ABC.		G = centroid. $A'B'C'$ are midpoints. Distance from G to any side $BC = \dfrac{AB+AC}{AB+AC+BC} \times \dfrac{\text{Altitude from } A}{2}$.
16. Circular arc.		On axis of symmetry. $OG = r \times \dfrac{\text{chord } AB}{\text{arc } AB} = \dfrac{r \sin \theta}{\theta}$ (θ = half of central angle in radians). For a quadrant, $OG=\dfrac{4r}{\pi\sqrt{2}}=0.9003r$. For a semicircle, $OG=\dfrac{2r}{\pi}=0.6336r$. Given rise $h(=DC)$ and chord $k(=AB)$ substitute above, $r=\dfrac{k^2+4h^2}{8h}$; $\sin\theta=\dfrac{4hk}{k^2+4h^2}$. For the distance of the centroid from the middle point of the arc, see Table 2.
17. Four sides of a parallelogram.		Intersection of diagonals.

Table 1. Location of centroids—*Continued*

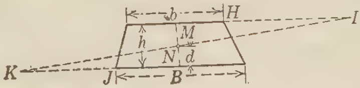
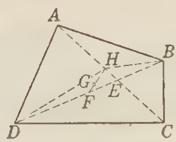
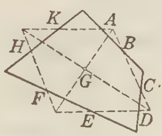
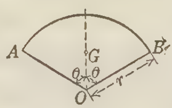
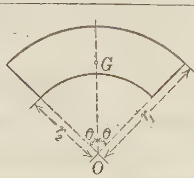
Centroid of	Drawing	Location of centroid
	Surfaces	
18. Triangle.		Intersection of medians.
19. Trapezoid.		On MN joining midpoints of bases. Perpendicular distance from lower base, $d=\frac{h(B+2b)}{3(B+b)}$. To locate graphically, draw $HI=B$ and $JK=b$. Centroid is at intersection of MN and KI.
20. Any quadrilateral $ABDC$		(*a*) Draw diagonal AD. Find centroids of triangles ABD and ACD (18) at E and F. Draw diagonal CB and find centroids of triangles ABC and BCD at G and H. The required centroid is at the intersection of EF and GH.
		(*b*) Draw AC and DB. $F=$ midpoint of DB. Lay off $CH=AE$. The centroid (G) of triangle DHB is the centroid of the quadrilateral and $FG=\frac{1}{3}FH$.
		(*c*) Divide the sides into thirds and construct the parallelogram with sides passing through the third-points as shown. The intersection of the diagonals of this parallelogram is the desired centroid.
21. Sector of a circle.		$OG=\frac{2r\sin\theta}{3\theta}$ $=\frac{2}{3}\text{radius}\times\frac{\text{chord}}{\text{arc}}$ $=\frac{(\text{radius})^2\times\text{chord}}{3\times\text{area}}$ ($\theta=\frac{1}{2}$ angle of sector in radians). See also Table 3.
22. Sector of flat circular ring.		$OG=\frac{2(r_1^3-r_2^3)\sin\theta}{3(r^2{}_1-r_2^2)\theta}$ (θ in radians).

Table 1. Location of centroids—*Continued*

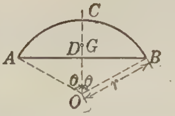
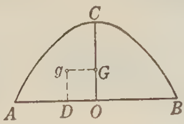
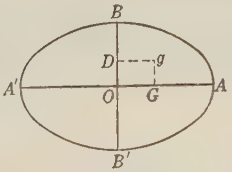
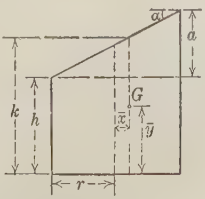
Centroid of	Drawing	Location of centroid
23. Segment of a circle.		$OG=\dfrac{2r\sin^3\theta}{3\theta-\sin\theta\cos\theta}$ (θ in radians). For a semi-circular segment, $OG=\dfrac{4r}{3\pi}=0.4244r$. Given rise h $(=DC)$ and chord $k(=AB)$. $DG=0.4h$ (approximately) when the segment is less than a semi-circle. The exact value lies between $DG=0.4244h$ for a semi-circle ($h=\frac{1}{2}k$) and $DG=0.4000h$ when h is very small.
24. Segment of a parabola.		$OG=\frac{2}{5}OC$. For the half-segment ACO. $Dg=\frac{2}{5}OC$ and $Gg=\frac{3}{8}OA$.
25. Ellipse.		At geometrical center. For semi-ellipse ABB'. $OG=\dfrac{4\,OA}{3\pi}=0.4244\ OA$. For the quarter-ellipse ABO. $Dg=0.4244\ OA$ and $Gg=0.4244\ OB$.
26. Zone of spherical surface.		On axis of zone halfway between bases.
27. Curved surface of cone.		On axis, one third of the distance from the base to the altitude.
28. Slanting surface of a pyramid.		Same as (27).
Solids		
29. Right circular cylinder.		$\bar{x}=\dfrac{ra}{4(2h+a)}=\dfrac{r^2\tan\alpha}{2(2h+a)}$. $\bar{y}=\dfrac{k}{2}=\dfrac{h}{2}+\dfrac{a}{4}+\dfrac{a^2}{16(2h+a)}$.

Table 1. Location of centroids—*Continued*

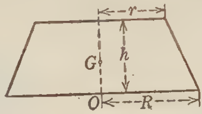
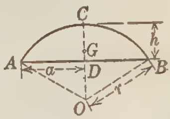
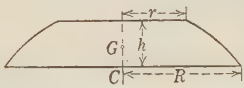
Centroid of	Drawing	Location of centroid
30. Frustum of a right circular cone.		$OG = \frac{h(R^2+2Rr+3r^2)}{4(R^2+Rr+r^2)}$.
31. Cone.		On axis one-fourth of distance from base to apex.
32. Pyramid.		Same as (31).
33. Segment of sphere		$OG = \frac{3(2r-h)^2}{3r-h}$. $DG = \frac{h(2a^2+h^2)}{2(3a^2+h^2)} = \frac{h(4r-h)}{4(3r-h)}$.
34. Zone of sphere.		$CG = \frac{h(2R^2+4r^2+h^2)}{2(3R^2+3r^2+h^2)}$.
35. Paraboloid.		On axis, one-third of the altitude above the base.
36. Ellipsoid. Lengths of semi-axes $= a, b, c$; equation: $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$.		Centroid of one octant is at $\bar{x} = \frac{3}{8}a$; $\bar{y} = \frac{3}{8}b$; $\bar{z} = \frac{3}{8}c$.

Table 2. Distance of centroid of circular arc from middle point of arc

h/k	CG/h	h/k	CG/h
0.01	0.333	0.30	0.347
0.10	0.334	0.35	0.351
0.15	0.337	0.40	0.355
0.20	0.340	0.45	0.359
0.25	0.343	0.50	0.363

Table 3. Location of centroid in circular sectors (Sec. 21, Table 1)

$\theta°$	OG/r	$\theta°$	OG/r	$\theta°$	OG/r	$\theta°$	OG/r
5	0.666	50	0.585	95	0.400	140	0.175
10	0.663	55	0.569	100	0.376	145	0.151
15	0.659	60	0.552	105	0.351	150	0.127
20	0.653	65	0.533	110	0.326	155	0.104
25	0.646	70	0.513	115	0.301	160	0.082
30	0.637	75	0.492	120	0.276	165	0.060
35	0.626	80	0.470	125	0.250	170	0.039
40	0.614	85	0.447	130	0.225	175	0.019
45	0.600	90	0.424	135	0.200	180	0.000

Principle of combination. The centroid of a plane figure composed of two parts lies on the line joining the centroids of the two parts and divides the line in the inverse ratio of the areas of the two parts. A similar statement holds for lines, curved surfaces, and volumes. If the figure is composed of more than two parts, the centroid is found by the formulas (Table 1), in which w_i denotes the area (or length or volume) of one part and (x_i, y_i, z_i) is centroid of this part. The formulas are good for finding the centroid of a figure from which a part has been removed, provided the part removed is given a minus sign.

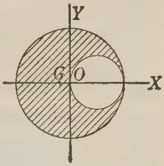


FIG. 49.

Example. Find the centroid of the remainder of a circle of radius r after a circle of radius $r/2$ has been removed (Fig. 49).

From symmetry, $y = 0$. $w_1 = \pi r^2$, $w_2 = -\pi r^2/4$, $x_1 = 0$, $y_1 = 0$, $x_2 = r/2$, $y_2 = 0$.

$$OG = \bar{x} = \frac{\pi r^2 \times 0 - \frac{\pi r^2}{4} \times \frac{r}{2}}{\pi r^2 - \frac{\pi r^2}{4}} = -\frac{r}{6}.$$

14. Moment of inertia

Areas

Moment of inertia of a plane area about an axis is the limit of the sum of the products obtained by multiplying each element of area by the square of its distance from the axis. The moment of inertia is different for different axes. The axis used is indicated by a subscript, thus I_x = moment of inertia about the X-axis.

In Fig. 50,

$$I_x = \int y^2 dA = \int\int y^2 dx\, dy,\ I_y = \int x^2 dA = \int\int x^2 dx\, dy.$$

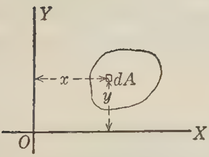


FIG. 50.

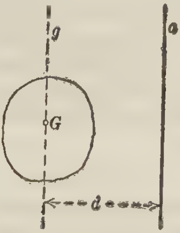


FIG. 51.

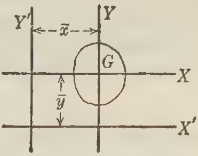


FIG. 52.

The moment of inertia is always positive (never zero). It is called polar if the axis is perpendicular to the plane. I_0 denotes a moment of inertia about an axis through O perpendicular to the XY-plane.

$$I_0 = \int (x^2 + y^2) dA = \int\int (x^2 + y^2) dx\, dy = \int\int \rho^3 d\rho\, d\theta.$$

$$I_0 = I_x + I_y.$$

(Caution. This relation does not hold unless X- and Y-axes are perpendicular.)

Radius of gyration, k is given by $I = Ak^2$, whence $k = \sqrt{I/A}$.

Parallel axes. The moment of inertia of a plane area about any axis is equal to the moment of inertia about the parallel axis through the centroid plus the product of the area and the square of the distance between the axes. Thus (Fig. 51) $I_a = I_g + Ad^2$.

Product of inertia of a plane area with respect to two co-ordinate axes is the limit of the sum of the products obtained by multiplying each element of area by the product of its distances from the co-ordinate axes. Thus $P_{xy} = \int xy\,dA$. The product of inertia may be positive, negative, or zero. It is positive for an area in the first and third quadrants and negative for an area in the second and fourth quadrants.

Parallel axes. The product of inertia of a plane area with respect to a pair of rectangular axes is equal to the product of inertia with respect to a pair of parallel axes through the centroid plus the product of the area and the distances between the axes. Thus (Fig. 52) $P_{x'y'} = P_{xy} + A\bar{x}\bar{y}$.

The product of inertia of an area with respect to a pair of axes one of which is an axis of symmetry is zero.

The moment of inertia of a plane area about different axes lying in the same plane and passing through the same point is calculated by

$$I_l = I_x \cos^2\theta - P_{xy}\sin 2\theta + I_y \sin^2\theta. \quad \text{(Fig. 53.)}$$

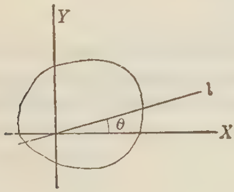


FIG. 53.

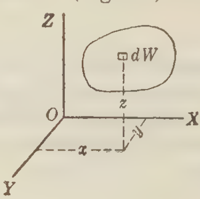


FIG. 54.

Principal axes. Given any plane area and any point in its plane. Consider all axes passing through this point and lying in its plane. Among them there is one about which the moment of inertia of the area is a maximum and one about which it is a minimum. (If the figure is a regular polygon and the point is its center, the moment of inertia is the same for all the axes.) These are called PRINCIPAL AXES. They are perpendicular and the product of inertia with respect to them is zero.

Solid bodies

Moment of inertia of a solid body about an axis is the limit of the sum of the products obtained by multiplying each element of weight by the square of its distance from the axis. (Moment of inertia is also defined by using mass instead of weight.) In Fig. 54

$$I_x = \int (y^2 + z^2)dW, \quad I_y = \int (z^2 + x^2)dW, \quad I_z = \int (x^2 + y^2)dW.$$

Radius of gyration, k, is given by $I = Wk^2$, whence $k = \sqrt{I/W}$.

Parallel axes. The moment of inertia of a solid body about any axis is equal to the moment of inertia about the parallel axis through the center of gravity plus the product of the weight and the square of the distance between the axes.

Product of inertia of a solid body with respect to two co-ordinate planes is the limit of the sum of the products obtained by multiplying each element of weight by the product of its distances from the co-ordinate planes. Thus

$$P_{xy} = \int xy\,dW,\ \ P_{yz} = \int yz\,dW,\ \ P_{zx} = \int zx\,dW.$$

The product of inertia of a solid body with respect to a pair of planes, one of which is a plane of symmetry, is zero.

Principal axes. Given any solid body and any point as origin, among all the axes passing through this point there is one about which the moment of inertia of the body is a maximum and one about which it is a minimum. (For certain regular bodies, if the origin is at the center, the moment of inertia is the same about all axes.) These two axes are perpendicular and, with a third axis perpendicular to each of them, determine a system of co-ordinates for which the three products of inertia vanish. These axes are the PRINCIPAL AXES. If a body has a plane of symmetry, the perpendicular line at any point of the plane is a principal axis for that point. If a body has two planes of symmetry (at right angles to each other) their line of intersection is a principal axis for every point on it. If a body has three planes of symmetry (at right angles to each other) their lines of intersection are the principal axes at the point of intersection.

Table 4. Moments of inertia

a I_O = polar moment of inertia. O is centroid

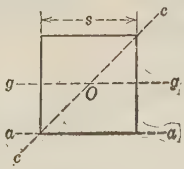
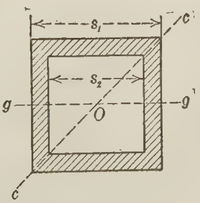
Figure	Drawing	Moment of inertia (*a*)
Plane figures		
1. Solid square.		$I_g = I_c = \frac{s^4}{12}$. $I_a = \frac{s^4}{3}$. $I_O = \frac{s^4}{6}$.
2. Hollow square.		$A = s_1^2 - s_2^2$. $I_g = I_c = \frac{s_1^4 - s_2^4}{12} = \frac{A(s_1^2 + s_2^2)}{12}$. $I_O = \frac{s_1^4 - s_2^4}{6} = \frac{A(s_1^2 + s_2^2)}{6}$.

Table 4. Moments of inertia—*Continued*

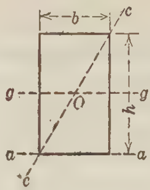
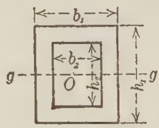
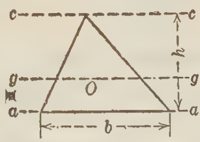
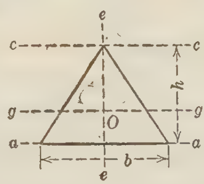
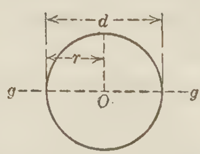
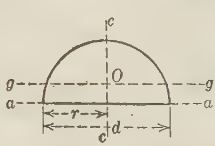
Figure	Drawing	Moment of inertia (a)
3. Solid rectangle.		$I_g=\frac{bh^3}{12}$. $I_a=\frac{bh^3}{3}$. $I_c=\frac{b^3h^3}{6(b^2+h^2)}$. $I_O=\frac{bh(b^2+h^2)}{3}$.
4. Hollow rectangle.		$I_g=\frac{b_1h_1^3-b_2h_2^3}{12}$. $I_O=\frac{b_1h_1(b_1^2+h_1^2)-b_2h_2(b_2^2+h_2^2)}{12}$.
5. Triangle.		$I_g=\frac{bh^3}{36}$. $I_a=\frac{bh^3}{12}$. $I_c=\frac{bh^3}{4}$.
6. Isosceles triangle.		$I_g=\frac{bh^3}{36}$. $I_a=\frac{bh^3}{12}$. $I_c=\frac{bh^3}{4}$. $I_e=\frac{b^3h}{48}$. $I_O=\frac{bh(3b^2+4h^2)}{144}$.
7. Solid circle.		$I_g=\frac{\pi r^4}{4}=\frac{\pi d^4}{64}=0.7854r^4=0.0491d^4$. $I_O=\frac{\pi r^4}{2}=\frac{\pi d^4}{32}=1.5708r^4=0.0982d^4$.
8. Solid semi-circle.		$I_g=\left(\frac{\pi}{8}-\frac{8}{9\pi}\right)r^4$ $=0.1098r^4=0.00686d^4$. $I_c=I_a=\frac{\pi r^4}{8}=\frac{\pi d^4}{128}$ $=0.3927r^4=0.0245d^4$. $I_O=\left(\frac{\pi}{4}-\frac{8}{9\pi}\right)r^4$ $=0.5025r^4=0.0314d^4$.

Table 4. Moments of inertia—*Continued*

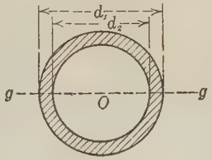
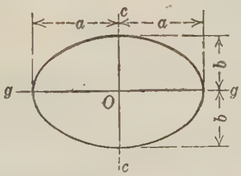
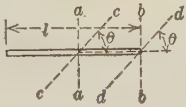
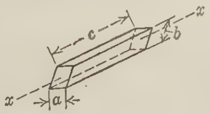
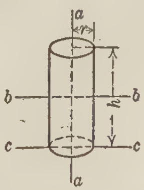
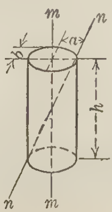
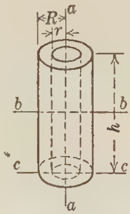
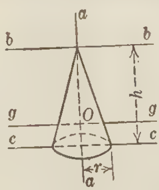
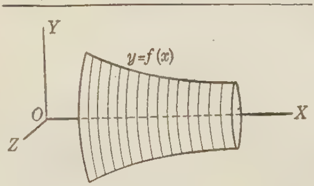
Figure	Drawing	Moment of inertia (a)
9. Hollow circle.		$A=\frac{\pi}{4}(d_1^2-d_2^2)$. $I_g=\frac{\pi}{64}(d_1^4-d_2^4)=\frac{A}{16}(d_1^2+d_2^2)$. $I_O=\frac{\pi}{32}(d_1^4-d_2^4)=\frac{A}{8}(d_1^2+d_2^2)$.
10. Ellipse.		$A=\pi ab$. $I_g=\frac{\pi ab^3}{4}=\frac{Ab^2}{4}$. $I_c=\frac{\pi a^3 b}{4}=\frac{Aa^2}{4}$. $I_O=\frac{A(a^2+b^2)}{4}$.
Homogeneous solid bodies. (W = weight)		
11. Thin straight rod.		$I_a=\frac{Wl^2}{12}$. $I_b=\frac{Wl^2}{3}$. $I_c=\frac{Wl^2\sin^2\theta}{12}$. $I_d=\frac{Wl^2\sin^2\theta}{3}$.
12. Rectangular prism.		$I_x=\frac{W(a^2+b^2)}{12}$.
13. Right circular cylinder.		$I_a=\frac{Wr^2}{12}$. $I_b=\frac{W(3r^2+h^2)}{12}$. $I_c=\frac{W(3r^2+4h^2)}{12}$.
14. Right elliptical cylinder.		$I_m=\frac{W(a^2+b^2)}{4}$. $I_n=\frac{W(3a^2+h^2)}{12}$.

Table 4. Moments of inertia—*Continued*

Figure	Drawing	Moment of inertia (*a*)
15. Hollow circular cylinder.		$I_a = \frac{W(R^2+r^2)}{2}$. $I_b = \frac{W(3R^2+3r^2+h^2)}{12}$. $I_c = \frac{W(3R^2+3r^2+4h^2)}{12}$.
16. Sphere (r = radius).		About diameter. $I = \frac{2Wr^2}{5}$.
17. Hollow sphere, outer radius = R, inner radius = r.		About diameter. $I = \frac{2W}{5}\left(\frac{R^5-r^5}{R^3-r^3}\right)$.
18. Thin hollow sphere, $R = r$.		$I = \frac{2Wr^2}{3}$.
19. Ellipsoid, semi-axes, a, b, c.		About axis $2a$. $I = \frac{W}{5}(b^2+c^2)$.
20. Right circular cone.		$I_a = \frac{3Wr^2}{10}$. $I_b = \frac{3W(r^2+4h^2)}{20}$. $I_g = \frac{3W(4r^2+h^2)}{80}$. $I_c = \frac{W(12r^2+23h^2)}{80}$.
21. Any solid of revolution. Volume = V. Weight = W.		About geometrical axis (X-axis) $I = \frac{\pi W}{2V}\int y^4 dx$. About an axis parallel to the base (Y-axis). $I = \frac{\pi W}{V}\int\left(\frac{y^4}{4}+x^2y^2\right)dx$.

KINETICS

15. Definitions and laws

Velocity and acceleration. VELOCITY is the rate (with respect to time) of change of position of a point. It is a vector quantity (having magnitude and direction). The magnitude of velocity is SPEED. ACCELERATION is the rate (with respect to time) of change of velocity and is a vector quantity. In considering plane motion of a point two methods of taking components of velocity and acceleration are useful in engineering problems.

(1) *Components parallel to fixed axes.* The position of the point is given by co-ordinates (x, y).

$$v_x = dx/dt, \quad v_y = dy/dt, \quad v = \sqrt{v_x^2 + v_y^2}.$$

$$a_x = dv_x/dt = d^2x/dt^2, \quad a_y = dv_y/dt = d^2y/dt^2, \quad a = \sqrt{a_x^2 + a_y^2}.$$

(2) *Components along the tangent and normal to the path.* The position of a point is given by the distance s from a fixed point A on the path (Fig. 55).

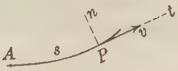


FIG. 55.

$$v_n = 0, \quad v_t = v = ds/dt,$$

$$a_n = v^2/r, \quad a_t = dv/dt = d^2s/dt^2, \quad a = \sqrt{a_n^2 + a_t^2},$$

where r = radius of curvature.

Momentum of a particle is the product of mass and velocity.

Momentum = weight × velocity/g, where g = acceleration due to gravity.

Newton's laws of motion. I. Every body continues in its state of rest, or of uniform motion in a straight line, except in so far as it may be compelled by force to change that state.

II. Change of motion (rate of change of momentum) is proportional to the force applied and takes place in the direction in which the force acts.

III. To every action there is always an equal and contrary reaction; or, the mutual actions of two bodies are always equal and oppositely directed.

16. Motion in a straight line

Uniform motion. Acceleration is zero, velocity, v, is constant. If the position of the moving point is given by its distance, x, from the origin and if $x = x_0$ when $t = t_0$, then

$$x - x_0 = v(t - t_0).$$

Uniformly accelerated motion. Acceleration, a, is constant. If $x = x_0$, $v = v_0$, when $t = t_0$,

$$dv/dt = a, \quad v - v_0 = a(t - t_0), \quad x - x_0 = \tfrac{1}{2}a(t - t_0)^2 + v_0(t - t_0),$$

$$v^2 - v_0^2 = 2a(x - x_0).$$

For bodies moving in a vertical line under the force of gravity, $a = g$. The value of g varies slightly for different positions on the earth's surface. The standard value of g adopted by international agreement is 32.1740 ft./sec.^{2} = 980.665 cm./sec.^{2} The value ordinarily used is g = 32 ft./sec.^{2}

If a body falls from rest

$$v = gt, \quad x = \tfrac{1}{2}gt^2, \quad v^2 = 2gx.$$

If body is projected down with velocity v_0,

$$v = gt + v_0, \qquad x = \tfrac{1}{2}gt^2 + v_0t, \qquad v^2 = 2gx + v_0^2.$$

If body is projected up with velocity v_0,

$$v = v_0 - gt, \qquad x = v_0t - \tfrac{1}{2}gt^2, \qquad v^2 = v_0^2 - 2gx.$$

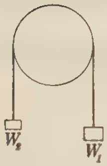


FIG. 56.

Time of ascent $= v_0/g$, height reached $= \tfrac{1}{2}v_0^2/g$.

For motion under gravity on a smooth plane inclined at angle θ to horizontal, $a = g \sin \theta$.

Atwood's* machine consists of two weights attached to the ends of a cord passing over a pulley (Fig. 56). Suppose the inertia of the pulley is neglected (see Art. 19) or that the cord passes over a smooth peg. If $W_1 > W_2$, the acceleration of the weights is given by $a = \dfrac{W_1 - W_2}{W_1 + W_2}g$, and the tension in the cord is $T = \dfrac{2W_1W_2}{W_1 + W_2}$.

General force equation. If a particle of weight W moves along the X-axis under the action of forces whose resultant is F,

$$\frac{W}{g}a = F.$$

This equation is homogeneous in force and acceleration and any system of units may be used. In engineering practice force is measured in pounds and acceleration in feet per second per second. The usual problem in kinetics is one in which F is known and the initial position, x_0, and initial velocity, v_0, are given (at time $t = 0$) and it is desired to determine the subsequent motion. The solution demands integration of a differential equation of the second order for which no general method can be given. Three formulas for acceleration are useful,

$$a = dv/dt = d^2x/dt^2 = v\, dv/dx.$$

Methods of solution are illustrated below.

Example. A 30-ton car moving with a speed of 3 mi./hr. has a spring in the draft rigging requiring a force of 60,000 lb. to compress it 1 in. How much will this spring be compressed when the car strikes a bumping post?

Let $t = 0$ be the instant when the car strikes the post and x = distance moved after $t = 0$; then $x_0 = 0$. $v_0 = 3$ mi./hr. $= 22/5$ ft./sec. The force exerted by the spring is proportional to the compression ($= x$). Hence $F = -720{,}000x$, where F is measured in lb. and x in ft., the minus sign being used because the force on the car is opposite to the direction of motion. The force equation becomes $60{,}000a/32 = -720{,}000x$, whence $v\,dv/dx = -384x$. Integrating, $v^2/2 = -384x^2/2 + C$, where C is a constant to be determined by the initial conditions. $v = 22/5$ when $x = 0$, whence $C = 242/25$. The problem is to determine compression when the car has stopped. Setting $v = 0$, $x = 11\sqrt{96}/480$ ft. $= 2.69$ in.

Resisting medium. The resistance of air depends on the speed of the moving body, being proportional to the square for low speeds. The factor of proportionality depends on the size and shape of the body.

Consider the effect of air resistance on a falling body. The resultant force is $W - k_1v^2$, and the force equation gives $Wa/g = W - k_1v^2$, whence $a = dv/dt = g - kv^2$, where $k = k_1g/W$. To integrate, write $dv/(g - kv^2) = dt$, whence

$$\frac{1}{2\sqrt{g}} \log \left(\frac{\sqrt{g} + \sqrt{k}v}{\sqrt{g} - \sqrt{k}v}\right) = \sqrt{k}\,t + C.$$

The constant of integration, C, is zero if $x = 0$, and $v = 0$ when $t = 0$. Solving for v,

$$v = \frac{dx}{dt} = \frac{\sqrt{g}(e^{\sqrt{kg}\,t} - e^{-\sqrt{kg}\,t})}{\sqrt{k}(e^{\sqrt{kg}\,t} + e^{-\sqrt{kg}\,t})} = \sqrt{\frac{g}{k}}\tanh\sqrt{kg}\,t.$$

Integrating,

$$x = \frac{1}{k}\log\left(\frac{e^{\sqrt{kg}\,t} + e^{-\sqrt{kg}\,t}}{2}\right) = \frac{1}{k}\log\cosh\sqrt{kg}\,t.$$

The expression for acceleration shows that the speed has a limiting value, $v = \sqrt{g/k}$. The relation between v and x is obtained by using $a = v\,dv/dx$ and integrating,

$$\log\left(\frac{g}{g - kv^2}\right) = 2kx \quad \text{or} \quad v^2 = \frac{g}{k}(1 - e^{-2kx}).$$

17. Motion of a particle in a circle

Notation. When a particle moves in a circle of radius r, its position is given by its angular displacement, θ (Fig. 57) from some fixed diameter. Angular velocity, ω, and angular acceleration, α, are given by

$$\omega = \frac{d\theta}{dt}, \quad \alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2} = \omega\frac{d\omega}{d\theta}.$$

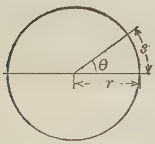


Fig. 57.

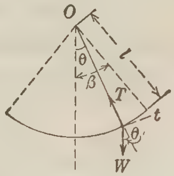


Fig. 58.

Distance, velocity, and acceleration are given by

$$s = r\theta, \quad v = r\omega, \quad a = r\alpha.$$

Simple pendulum consists of a small heavy bob on a light string (Fig. 58). The forces acting on a particle are the weight, W, and tension, T. The resultant force along the tangent $= -W\sin\theta$; the resultant force along the normal $= T - W\cos\theta$. The force equations are $Wa_t/g = -W\sin\theta$, $Wa_n/g = T - W\cos\theta$. Since $a_n = 0$, tension $= W\cos\theta$.

To determine the motion: $a_t/g = l\alpha/g = -\sin\theta$.

The solution of this equation leads to elliptic functions. Approximate solution can be obtained by putting $\sin\theta = \theta$. (Difference between θ and $\sin\theta$ is less than one per cent. if θ is less than 14°.) The differential equation becomes $\omega\,d\omega/d\theta = -g\theta/l$. If the pendulum is at the end of its swing when $t = 0$, then $\theta = \beta$, $\omega = 0$. Integrating, $\omega^2 = g(\beta^2 - \theta^2)/l$; $\omega = \dfrac{d\theta}{dt} = \pm\sqrt{\dfrac{g}{l}}(\beta^2 - \theta^2)$. Integrating, $\theta = \beta\cos\sqrt{\dfrac{g}{l}}t$. Period of oscillation $= 2\pi\sqrt{\dfrac{l}{g}}$.

Conical pendulum (Watt governor) consists of a heavy bob suspended from a fixed point by a light string so that it can be made to rotate about the vertical axis through the fixed point (Fig. 59). If the bob rotates with constant angular velocity, ω, the quantities ϕ, r, h are constants. Since there is no vertical acceleration, $T\cos\phi = W$. The force acting

inward on the bob is $T \sin \phi$. Hence the force equation gives $T \sin \phi = Wa_n/g = Wv^2/gr$, and $\tan \phi = v^2/gr = r\omega^2/g$. Also $h = g/\omega^2$, $T = Wl\omega^2/g$.

Super-elevation of outer rail of a railroad track is determined by the preceding equations where (Fig. 60) r = radius of curvature in ft. and v = speed in ft./sec., $\tan \phi = v^2/gr$. For small angles we may use the sine instead of the tangent and, if h = super-elevation of the outer rail in inches, $h = 56.5v^2/gr$.

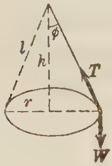


FIG. 59.

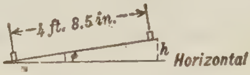


FIG. 60.

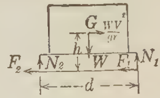


FIG. 61.

Skidding and tipping. Suppose a car (Fig. 61) is taking a curve of radius r ft. at a speed of v ft./sec., G is center of gravity, N_1 is the vertical and F_1 the horizontal pressure on the outer wheel. The problem becomes one of statics by introducing $Wv^2/gr = F_1 + F_2 = F = fW$. If $f < v^2/gr$, the car will skid. Suppose $f > v^2/gr$, then $N_1 = W(\frac{1}{2} + v^2h/dgr)$, $N_2 = W(\frac{1}{2} - v^2h/dgr)$. The critical speed is $v_1 = \sqrt{dgr/2h}$, when the total weight is borne on the outer wheel. If this critical speed is exceeded, the car will tip over.

18. Plane curvilinear motion of a particle

Rectangular force equations. If the co-ordinates of a particle are x, y, components of velocity and acceleration are v_x, v_y, and a_x, a_y (see Art. 15). If the F_x, F_y, are the components of the resultant force on the particle, the force equations are

$$\frac{W}{g}a_x = \frac{W}{g}\frac{d^2x}{dt^2} = \frac{W}{g}\frac{dv_x}{dt} = \frac{W}{g}v_x\frac{dv_x}{dt} = F_x,$$

$$\frac{W}{g}a_y = \frac{W}{g}\frac{d^2y}{dt^2} = \frac{W}{g}\frac{dv_y}{dt} = \frac{W}{g}v_y\frac{dv_y}{dt} = F_y.$$

Normal and tangential force equations. If a particle, P, moves along a curve OP (Fig. 62) and s is the distance along the curve from a fixed point O, then the total velocity is along the tangent and $v_n = 0$. If F_n and F_t are the normal and tangential components of the resultant force on the particle, the force equations are

$$\frac{W}{g}a_n = \frac{Wv^2}{rg} = \frac{W}{rg}\left(\frac{ds}{dt}\right)^2 = F_n, \quad \frac{W}{g}a_t = \frac{W}{g}\frac{d^2s}{dt^2} = \frac{W}{g}\frac{dv}{dt} = \frac{W}{g}v\frac{dv}{ds} = F_t.$$

FIG. 62.

FIG. 63.

Constrained motion occurs when a particle is compelled to follow a given curve. Suppose the particle slides under action of its own weight and friction is neglected. If v_0 = velocity at A (Fig. 63) and v = velocity at B, then $v^2 = v_0^2 + 2gh$. The final speed depends only on the vertical height fallen and not on the shape of the curve or the distance moved. The speed at B' is the same as the speed at B. If P = pressure of the curve on the particle,

Wa_t/g = tangential component of weight, $Wa_n/g = Wv^2/gr = P$ + *normal component of weight;* and $P = Wv^2/gr$ − *normal component af weignt,* where the positive direction of the normal is toward the concave side of the curve.

Example. A particle slides on a smooth vertical circular wire and is just started from rest at the top. Find the pressure on the curve at B and C (Fig. 64).

At B, $v^2 = 2gr$, the normal component of weight = 0, and $P = 2W$. At C, $v^2 = 4gr$, the normal component of weight = − W, and $P = 5W$.

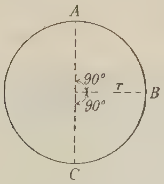


FIG. 64.

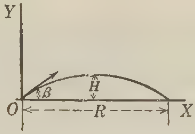


FIG. 65.

Projectile (Fig. 65). If air resistance is neglected, the only force acting (after the initial impulse) is weight. The force equations are $Wa_x/g = 0$, $Wa_y/g = -W$. If, when $t = 0$, $x = 0$, $y = 0$, $v = v_0$, and the angle of the projection with the horizontal $= \beta$, $x = tv_0 \cos \beta$, $y = tv_0 \sin \beta - \frac{1}{2}gt^2$. The trajectory is the parabola $y = x \tan \beta - gx^2/2v_0^2 \cos^2 \beta$. The height, $H = v_0^2 \sin^2 \beta/2g$. The horizontal range $R = v_0^2 \sin 2\beta/g$. The time of flight $T = 2v_0 \sin \beta/g$.

19. Work, energy, power

Work is said to be done on a body by a force when its application point is displaced so that the displacement has a component along the action line of the force. If the force F is constant in magnitude and direction and the displacement in the direction of the force is s, the work done is $F \times s$. The engineering unit of work is the FOOT-POUND (ft.-lb.). If F is constant in direction but variable in magnitude, the *work* $= \int F ds$. If the displacement of the application point is along a curved path, the normal component of force does no work and *work* $= \int F_t ds$. The work done in rotation of a rigid body is $\int G\, d\theta$, where G = the moment of forces (torque) producing the rotation. If G is constant, the work done $= G \times \theta$, where θ is the angle turned through.

Energy is the capacity for doing work which is possessed by a body or system of bodies by virtue of their motion or relative positions. POTENTIAL ENERGY is that due to position. KINETIC ENERGY is that due to motion. For a particle of weight W moving with velocity v,

$$KE = \frac{1}{2}\frac{W}{g}v^2.$$

The fundamental relation between work and kinetic energy is as follows: The work done upon a rigid body by an external system of forces is equal to the change in kinetic energy of the body. If the body is a particle (or body which does not rotate) and if v_0 = initial velocity and v = final velocity, the work done $= W(v^2 - v_0^2)/2g$.

Power is the rate of doing work. The engineering unit is the horsepower (hp.) = 550 ft.-lb./sec. = 33,000 ft.-lb./min.

Work diagram. Let F be the component of force in the direction of motion of the point of application, P, and suppose P moves a distance AD (Fig. 66). Curve BC is obtained by plotting distance as abscissa and force as ordinate. Then the work done by the force is given by

$$\int_{s_1}^{s_2} F\,ds = \text{Area } ABCD.$$

Example is furnished by steam pressure on a piston head. Up to the point of cut-off, K, the pressure is practically constant and equal to that in the boiler, and the work done is represented by the area $ABB'K$ (Fig. 67). After the entering steam **is cut** off, the pressure varies (approximately) inversely as the distance moved from K, and the work done by the expanding steam is represented by the area $KB'CD$. If on the backward stroke the piston head moves against a pressure, the work done by this pressure will be negative and be represented by the area $AHED$, which must be subtracted. The resultant work done on the piston head during one revolution is represented by the area $HBB'CE$. The curve obtained in practice is somewhat irregular (Fig. 68) and the work done is calculated by the measuring **area with a planimeter.**

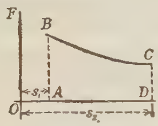


Fig. 66.

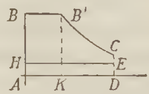


Fig. 67.

Fig. 68.

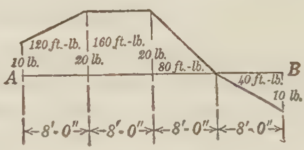


Fig. 69.

Examples of work and energy

1. The work diagram for the resultant of forces moving an 80-lb. weight along a horizontal plane is shown in Fig. 69. If the weight starts from rest at A, find the speed at B.

The work done = 120 + 160 + 80 − 40 = 320 ft.-lb. $KE = \frac{1}{2}\,80v^2/32 = 320$, whence $v = 16$ ft./sec.

2. Water falling from a height of 120 ft. at the rate of 1000 cu. ft./min. drives a turbine directly connected to an electric generator at 120 r.p.m. If the total resisting torque due to friction is 250 lb.-ft., and the water leaves the turbine blades with a velocity of 15 ft./sec find the power developed by the generator.

$1000 \times 120 \times 62.5 = 7{,}500{,}000$ ft.-lb. = work done by force of gravity per min. $.1000 \times 62.5 \times 15 \times 15/2 \times 32 = 219{,}700$ ft.-lb. = KE of water leaving turbine blades per min. $250 \times 120 \times 2\pi = 188{,}500$ ft.-lb. = work used to overcome friction. $7{,}500{,}000 - 219{,}700 - 188{,}500 = 7{,}091{,}800$ ft.-lb./min. = 214 hp. = 160 kw.

Mechanical efficiency of a machine is the ratio of useful work W_u performed by the machine to the work applied W_a to the machine,

$$\text{Efficiency} = \frac{W_u}{W_a}.$$

$W_a - W_u$ = work done in overcoming friction, or loss of energy due to friction.

20. Impulse, momentum, impact

Impulse of a force constant in magnitude and direction is the product of the force and the time during which it acts. If the force is variable in magnitude but constant in direction, impulse $= \int F\,dt$. The unit of impulse is the POUND-SECOND. Impulse is a vector quantity having the same direction as the force, if this direction is constant. If the direction of the force varies, the resultant impulse is the vector sum of the impulses of the components of the force.

Momentum of a particle of weight W moving with velocity v is Wv/g, and is a vector quantity having the same direction as the velocity. The angular momentum of a rigid body rotating about a fixed axis is $I\omega/g$, where I = the moment of inertia about the axis of rotation, and ω = angular velocity. The fundamental relation between impulse and momentum is: The impulse of the resultant force acting upon a particle is equal to the change in momentum of the particle. This is to be understood in the vector sense and may be stated: The component along any line of the impulse of the resultant force acting on a particle is equal to the change in momentum of the particle along that line.

Example. A jet of water strikes a concave vessel with a velocity of 80 ft /sec. and leaves it with a velocity which has the same magnitude but makes an angle of 120° with the original direction. If the diameter of the jet is 1 in. find the force necessary to hold the vessel in position.

The sustaining force F must bisect the acute angle between the lines representing the original and final velocities. Let the line of action of F (Fig. 70) be taken as the X-axis. There is no change in the Y component of momentum. The impulse of the force in the X direction in t sec. = $F \times t$ lb.-sec. The weight of water deflected in t sec. is $W = 80\pi \times 62.5\,t/576$ lb. The component of original momentum in the X direction $= -80\ W \cos 30°/g$ lb.-sec. The component of final momentum in the X direction $= 80\ W \cos 30°/g$ lb.-sec. The change in momentum in the X direction $= 160\ W \cos 30°/g = 5W \cos 30°$ lb.-sec. The fundamental relation gives $F \times t = 5 \times 80\pi \times 62.5 \times \cos 30° \times t/576$, whence $F = 118$ lb. Observe that the sustaining force F does no mechanical work and that the water suffers no loss of kinetic energy.

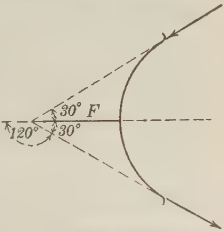


FIG. 70.

Impact occurs when two bodies collide. It is DIRECT when the motion is perpendicular to the striking surfaces; otherwise it is OBLIQUE. It is CENTRAL if the normal to the striking surfaces passes through the centers of gravity of both bodies; otherwise it is ECCENTRIC.

Phenomena of impact. During the impact of two bodies some deformation occurs, beginning at the instant the bodies come in contact and increasing to a maximum. Deformation may be permanent (bodies are inelastic) or the bodies may regain their original shapes (completely, if bodies are perfectly elastic; partially, if bodies are imperfectly elastic). The time of impact of elastic bodies is divided into two parts; first, the period of compression (or deformation) during which the force exerted by one body on the other is the force of compression; second, a period of restitution, with a corresponding force of restitution. There is no period of restitution in the case of inelastic bodies, which will remain in contact with maximum deformation and will move on with a common velocity. The time of impact is very short and it is not possible to determine the force acting at any instant. (But see Example, Art. 16.) The quantity which can be measured is momentum, from which the value of the impulse may be inferred. Ratio, e, of impulse of restitution to

impulse of compression is the COEFFICIENT OF RESTITUTION (or elasticity). For inelastic bodies, $e = 0$; for perfectly elastic bodies, $e = 1$; for imperfectly elastic bodies, $0 < e < 1$. When two bodies collide there is no loss of momentum of the system, but there is loss of kinetic energy except when the bodies are perfectly elastic.

Direct central impact. Let W and W' be the weights of two bodies in collision. Let u = velocity of W before impact, u' = velocity of W' before impact, v = velocity of W after impact, v' = velocity of W' after impact. Then $Wu + W'u' = Wv + W'v'$. If this equation be divided by g we have the statement: The total momentum of the system is the same after as it was before the impact. The coefficient of restitution is, $e = (v' - v)/(u - u')$. The velocities after impact are, $v = \dfrac{Wu + W'u' - W'e(u - u')}{W + W'}$, $v' = \dfrac{Wu + W'u' + We(u - u')}{W + W'}$.

The loss of kinetic energy is expressed by the equation,

$$(\tfrac{1}{2}Wu^2 + \tfrac{1}{2}W'u'^2) - (\tfrac{1}{2}Wv^2 + \tfrac{1}{2}W'v'^2) = \frac{WW'(1 - e^2)(u - u')^2}{2(W + W')}.$$

FIG. 71.

Ballistic pendulum is a device for determining the velocity of a bullet. The bullet is imbedded in soft material so that $e = 0$. Let W (Fig. 71) = weight of bullet, W' = weight of pendulum, k = radius of gyration about the axis of suspension O, u = velocity of bullet to be determined, u' = velocity of pendulum before impact = 0, $v = r\omega$ = velocity of bullet after impact, ω = angular velocity of pendulum after impact, rWu/g = angular momentum of system before impact, $Wr^2\omega/g + W'k^2\omega/g$ = angular momentum of system just after impact. Then $rWu = Wr^2\omega + W'k^2\omega$, and $u = (Wr^2 + W'k^2)\omega/Wr$. If h is the height to which the center of gravity rises, $k^2\omega^2 = 2gh$, and $u = (Wr^2 + W'k^2)\sqrt{2gh}/Wrk$. The quantities on the right-hand side of this equation are easily measured and u is calculated from the equation.

CENTER OF PERCUSSION is the point where the bullet should strike in order that there shall be no horizontal impulse on the axis of rotation. Its distance below P equals $k^2/\bar{r}$.

21. Rotation of a rigid body about a fixed axis

Torque equation. The position of a rigid body rotating about a fixed axis through O (Fig. 72) is given by the angle θ which line OG, fixed in the body, makes with line OX, fixed in space. OG is the line joining O to the center of gravity of the body unless the center of gravity is at O in which case OG is any line fixed in the body.

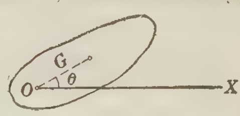


FIG. 72.

ANGULAR VELOCITY, $\omega = d\theta/dt$. ANGULAR ACCELERATION, $\alpha = d\omega/dt = d^2\theta/dt^2 = \omega \; d\omega/d\theta$. TORQUE, G = the sum of the moments of the applied forces about the axis of rotation. I_0 = the moment of inertia of the body about the axis of rotation. The TORQUE EQUATION (equation of angular motion) is, $I_0\alpha/g = G$. If G is constant, α is constant and, if $\theta = \theta_0$, $\omega = \omega_0$, when $t = t_0$, $\omega - \omega_0 = \alpha(t - t_0)$, $\theta - \theta_0 = \tfrac{1}{2}\alpha(t - t_0)^2 + \omega_0(t - t_0)$, $\omega^2 - \omega_0^2 = 2\alpha(\theta - \theta_0)$. If ω is constant, $\alpha = 0$, and $G = 0$. KINETIC ENERGY, $KE = I_0\omega^2/2g$. ANGULAR MOMENTUM, $= I_0\omega/g$. For pressure on axis see Art. 22.

Examples. 1. Atwood's machine (Fig. 73), in the simplest form, consists of two weights W_1 and $W_2 (W_1 > W_2)$ connected by a light cord which passes without slipping over a

pulley of weight W and radius r in the form of a solid cylinder which rotates about a fixed horizontal axis. The problem is to find the acceleration of the system and the tension in the two parts of the cord.

Let a = acceleration of W_1, downward; then a = acceleration of W_2, upward; and $\alpha = a/r$ = angular acceleration of cylinder. For the motion of W_1, $W_1a/g = W_1 - T_1$. For the motion of W_2, $W_2a/g = T_2 - W_2$. For motion of W, $Wr^2\alpha/2g = rT_1 - rT_2$. From these three equations,

$$a = \frac{2(W_1\,W - {}_2)g}{2(W_1 + W_2) + W}, \quad T_1 = \frac{(4W_2 + W)W_1}{2(W_1 + W_2) + W}, \quad T_2 = \frac{(4W_1 + W)W_2}{2(W_1 + W_2) + W}.$$

2. Compound pendulum is any rigid body suspended from a horizontal axis about which it may rotate under the action of its own weight. The forces acting on the body are its weight, acting downward at G (Fig. 74), and the reaction of the axis at O. Let D = distance OG, k_0 = radius of gyration about O. The torque equation gives $Wk_0^2\alpha/g = -WD \sin\theta$, whence $\alpha = -Dg\sin\theta/k_0^2$. This is the equation of a simple pendulum (see Art. 17) of length $l = k_0^2/D$, called the length of the equivalent simple pendulum. The motion of a compound pendulum is the same as the motion of the equivalent simple pendulum.

3. A 2400-lb. wheel with radius of gyration = 4 ft. is making 180 r.p.m. (*a*) How much work must be done to increase the speed to 240 r.p.m? (*b*) If, when making 240 r.p.m., a braking force is applied to the shaft at a distance of 6 in. from the center, find the tangential force if the wheel makes 360 revolutions before stopping.

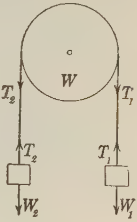


FIG. 73.

180 r.p.m. = 6π rad./sec., 240 r.p.m. = 8π rad./sec.

(*a*) KE (at 180 r.p.m.) = $2400 \times 16 \times 36\pi^2/2 \times 32$ = 213,180 ft.-lb. KE (at 240 r.p.m.) = $2400 \times 16 \times 64\pi^2/2 \times 32$ = 378,990 ft.-lb. Increase in KE = 165,810 ft.-lb. *Ans.*

(*b*) Torque of tangential braking force $F = \frac{1}{2}F$. Work done by torque in 360 revolutions is, $\frac{1}{2}F \times 360 \times 2\pi$ = 378,990, whence F = 335 lb. *Ans.*

4. A punch is required to exert a force of 100,000 lb. through a distance of $\frac{1}{4}$ in. and the work is to be supplied by a flywheel of radius of gyration = 1.5 ft. making 120 r.p.m. Find the weight of the wheel, if the speed is not to be reduced below 100 r.p.m.

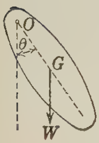


FIG. 74.

ω_1 = 120 r.p.m. = 4π rad./sec., ω_2 = 100 r.p.m. = $\frac{10}{3}\pi$ rad./sec. Work done by punch = 100,000/48 ft.-lb. = reduction in KE of flywheel. Change in $KE = W \times 2.25(\omega_1^2 - \omega_2^2)/64 = W \times 2.25(\omega_1 - \omega_2)(\omega_1 + \omega_2)/64$. Hence

$$\frac{W \times 2.25}{64}(2\pi/3)(22\pi/3) = \frac{100{,}000}{48},$$ whence W = 1230 lb. = minimum weight of flywheel.

22. Plane motion of a rigid body

A rigid body has plane motion if every particle of the body moves in a path that is parallel to a fixed plane. (A book sliding on a table and a rolling wheel are examples. This latter is a case of translation and rotation combined.) The fundamental theorem is that (1) the center of gravity moves as if all the weight were concentrated at that point and all forces applied there; (2) rotation about an axis through the center of gravity takes place as if the center of gravity were fixed in space. The equations for motion of the center of gravity are the equations for motion of a particle. The equation for rotation is the torque equation (see Art. 21). The kinetic energy of a body having plane motion equals the sum of the kinetic energy of translation and the kinetic energy of rotation. If W = weight of body, v = velocity of center of gravity, I = moment of inertia about an axis through the center of gravity, and ω = angular velocity, then

$$KE = Wv^2/2g + I\omega^2/2g.$$

Examples. 1. In Fig. 75 $\tan\beta = \frac{3}{4}$, the wheel weighs 100 lb., diameter = 4 ft., radius of gyration = 1.6 ft. (*a*) Find the acceleration of the center, if the wheel rolls without slipping. (*b*) Find the least coefficient of friction to prevent slipping. (*c*) If the

coefficient of friction = 0.1 find the acceleration of the center and the number of turns made while the center moves 20 ft.

(*a*) The forces acting to move the wheel are $W \sin \beta = 60$ lb. and friction, F. The equation of motion of the center is $100a/32 = 60 - F$. The force acting to turn the wheel is F. The torque equation is $100 \times 1.6 \times 1.6\alpha/32 = 2F$. Since the wheel does not slip, $a = 2\alpha$. Elimination of F gives $a = 11.7$ ft./sec.^{2}

(*b*) Friction = coefficient of friction × normal pressure, or $F = fW \cos \beta = f \times 80$. From the equation above $F = 23.4$ lb., whence $f = 0.29$.

(*c*) The relation between a and α is not known when the wheel slips. $F = 80 \times 0.1 = 8$ lb. The equation of motion of the center is $100a/32 = 60 - 8 = 52$, whence $a = 16.6$ ft./sec.^{2} Distance moved by center, $x = 8.3t^2$. Time to move 20 ft. is given by $t^2 = 20/8.3$. Torque equation is $100 \times 1.6 \times 1.6\alpha/32 = 2 \times 8$, whence $\alpha = 2$ rad./sec.^{2} The angle turned through, $\theta = t^2 = 20/8.3 = 2.41$ rad. $= 0.38$ revolution.

2. In Fig. 76 A is a solid cylinder of weight W about which is wrapped a light cord. B is a smooth peg, C is a weight W' which slides without friction on a horizontal plane. Find the acceleration of A and of C.

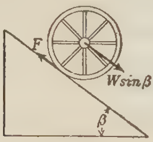


Fig. 75.

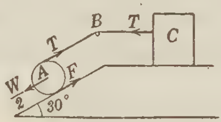


Fig. 76.

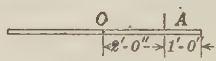


Fig. 77.

As A rolls down the plane, the cord is wrapped about it and the distance s' moved by W' equals twice the distance s moved by the center of W. Then $s' = 2s$ and $a' = 2a$. The equation of motion of the center of A is $Wa/g = W/2 - T - F$. The torque equation is $Wr^2\alpha/2g = r(F - T)$. Assuming the cylinder does not slip, $r\alpha = a$. The equation of motion of C is $W'a'/g = T$. Elimination of T and F gives $a = Wg/(3W + 8W')$.

3. A straight rod, 6 ft. long, of uniform section and weighing 60 lb., revolves in a horizontal plane about a vertical axis through its middle point O at a speed of 500 r.p.m. (*a*) Find the torque necessary to impart the speed of 500 r.p.m. in 20 sec.

(*b*) Find the tension due to centrifugal force at the middle section of the rod and at a section 2 ft. from O. (Fig. 77.)

To find the torque, $I\alpha/g = G$, where $I = \frac{60}{3} \times 9 = 180$; $\alpha = (500 \times 2\pi)/(60 \times 20) = 2.62$, $G = 14.6$ lb.-ft. To find the tension at the mid-section consider the motion of half the rod. Its center of gravity revolves in a circle of radius 1.5 ft. with a speed $v = 500 \times 2\pi \times 1.5/60$ ft. sec. The normal force (tension at mid-section) is given by $T = 30v^2/1.5g = 3850$ lb. To find the tension at a section 2 ft. from O, consider the motion of part A, the weight of which is 10 lb. and whose center of gravity moves in a circle of radius 2.5 ft. with a speed $v = 500 \times 2\pi \times 2.5/60$ ft. sec. The normal force on A is $T = 10v^2/2.5g = 2140$ lb.

Pressure on axis of rotation is found by the method illustrated in problem 4.

4. A straight rod of uniform section and material, 4 ft. long and weighing 40 lb. is suspended from a horizontal axis through a point O, 1 ft. from the upper end (Fig. 78). A force F of 8 lb. is applied perpendicular to the rod. Find the pressure on axis O if F is applied at the lower end; at the upper end. Where must F be applied if there is no pressure on the axis perpendicular to the rod?

Fig. 78.

At the instant F is applied the angular velocity of the rod is zero and the pressure on the axis in the direction of the rod is the weight = 40 lb. When the angular velocity = ω, the pressure on the axis in the direction of the rod is found by the method of the preceding problem.

To find the pressure on the axis perpendicular to the rod, find the angular acceleration α. $I\alpha/g$ = moment of F. $I = \frac{280}{3}$ lb.-ft.^{2}

When F is applied at the lower end, $\alpha = \frac{288}{35} = 8.2$ rad./sec.^{2} The center of gravity moves in a circle of radius, = 1 ft. and $a = 8.2$ ft./sec.^{2} Let P = the pressure of the axis on the rod in the same direction as F. The equation of motion of the center of gravity is $Wa/g = F + P$, whence $P = 2.3$ lb.

When F is applied at upper end its moment $= -8$ lb.-ft. and $\alpha = -\frac{96}{35} = -2.7$ rad./sec^2. $P = -11.4$ lb. (opposite to direction of F).

To find the center of percussion (point where F must be applied to give no pressure on the axis perpendicular to the rod) assume x = distance from O of point of application.

Then $280\alpha/3 \times 32 = 8x$, and $a = \alpha = 96x/35$. $^{40}\!/_{32} \times {}^{96}\!/_{35} \times x = 8 + 0$, whence $x = {}^{7}\!/_{3}$ ft. below 0.

23. Friction

Sliding friction is resistance to motion that is developed when one body slides, or tends to slide, over the surface of another. STATIC FRICTION is that which opposes any tendency to move on the part of two bodies in contact, that are relatively at rest. KINETIC FRICTION occurs when the bodies are in motion relative to each other.

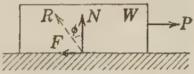


FIG. 79.

Coefficient of sliding friction. Weight W (Fig. 79) is pulled along a horizontal plane by the horizontal force P. The reaction of the plane is R, having a normal (vertical) component N and a tangential (horizontal) component F = friction. The ratio $f = F/N = \tan\phi$ is the COEFFICIENT OF SLIDING FRICTION and ϕ is the ANGLE OF FRICTION.

If P is not large enough to move W the friction is static and $F = P$. As P increases F will increase until W is just about to move. LIMITING FRICTION is the maximum value of static friction. The COEFFICIENT OF STATIC FRICTION is the ratio of the limiting friction to the normal pressure and is usually larger than the coefficient of sliding friction.

Suppose the plane on which W rests is inclined to the horizontal at angle θ. If $\theta < \phi$, W will not slide down; if $\theta > \phi$, W will slide down.

Laws of friction for surfaces which are dry, or nearly so, are

1. Friction between two given bodies is directly proportional to the pressure; the coefficient of friction is constant for all pressures.

2. The coefficient and amount of friction for given pressures is independent of the area of contact.

3. The coefficient of friction is independent of the relative velocity, although static friction is greater than kinetic friction.

The preceding laws are only approximately true. The coefficient of friction is slightly greater for small pressures upon large areas than for great pressures on small areas. The coefficient of friction decreases as the speed increases.

The following table of coefficients of friction (Table 5) is from experiments by Morin about 1830.

Friction of lubricated surfaces does not follow the laws for dry surfaces, but depends on the viscosity and thickness of the lubricant and the form of the surfaces in contact. The LAWS given by Goodman are

1. The coefficient of friction of well-lubricated surfaces is from $1/6$ to $1/10$ that of dry or poorly-lubricated surfaces.

2. The coefficient of friction for moderate pressures and speeds varies approximately inversely as the normal pressure; frictional resistance varies as the area of contact, normal pressure remaining the same.

3. For low speeds the coefficient of friction is abnormally high but as the speed of the rubbing surfaces increases from about 10 to 100 ft. min. the coefficient of friction diminishes and again rises when that speed is exceeded, varying approximately as the square root of the speed.

4. The coefficient of friction varies approximately inversely as the temperature.

Rolling friction is the resistance to motion developed when one body rolls over the surface of another, and depends on the hardness of the surfaces in contact and the radius of the rolling surface. The theory is based on the idea that surfaces are slightly deformed at the place of contact and that the effect of rolling friction is the same as if the surfaces were not deformed and the rolling body

Table 5. Coefficients of friction.

Material	Condition of surface	f	ϕ
Brass on oak	Dry	0.62	31° 48′
Brick on limestone	Dry	0.67	33° 50′
Cast iron on cast iron	Slightly greased	0.16	9° 6′
Cast iron on oak	Wet	0.65	33° 2′
Copper on oak		0.17	9° 38′
Copper on oak	Greased	0.11	6° 17′
Hemp cord on oak	Dry	0.80	38° 40′
Leather on cast iron		0.28	15° 39′
Leather on cast iron	Wet	0.38	20° 49′
Leather on cast iron	Oiled	0.12	6° 51′
Leather on oak	Fibers parallel	0.74	36° 30′
Leather on oak	Fibers crossed	0.47	25° 11′
Oak on oak	Fibers parallel, dry	0.62	31° 48′
Oak on oak	Fibers crossed, dry	0.54	28° 22′
Oak on oak	Fibers parallel, soaped	0.44	23° 45′
Oak on oak	Fibers crossed, wet	0.71	35° 23′
Oak on oak	Fibers end to side, dry	0.43	23° 16′
Oak on oak	Fibers parallel, greased	0.07	4° 6′
Oak on oak	Heavily loaded, greased	0.15	8° 45′
Oak on pine	Fibers parallel	0.67	33° 50′
Oak on limestone	Fibers on end	0.63	32° 15′
Oak on hemp cord	Fibers parallel	0.80	38° 40′
Pine on pine	Fibers parallel	0.56	29° 15′
Pine on oak	Fibers parallel	0.53	27° 56′
Tanned leather on oak	Leather flatwise, dry	0.61	31° 23′
Tanned leather on oak	Leather on edge, dry	0.43	23° 16′
Tanned leather on oak	Leather on edge, wet	0.79	38° 19′
Wrought iron on oak	Wet	0.62	31° 48′
Wrought iron on oak		0.65	33° 2′
Wrought iron on wrought iron		0.28	15° 39′
Wrought iron on cast iron		0.19	10° 46′
Wrought iron on limestone		0.49	26° 7′
Wood on metal	Greased	0.10	6° 0′
Wood on smooth stone	Dry	0.58	30° 7′
Wood on smooth earth	Dry	0.33	18° 16

passed constantly over a small obstruction. Let P (Fig. 80) be the horizontal force required to overcome the small obstruction B. Then $hP = aW$, and, since h is nearly equal to r, $P = aW/r$ (approximately). COEFFICIENT OF ROLLING FRICTION is a. It is a linear distance and is usually given in inches. For iron railroad wheels a varies from 0.0196 to 0.0216 in.

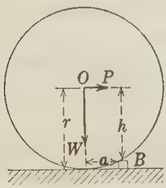


FIG. 80.

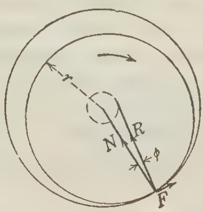


FIG. 81.

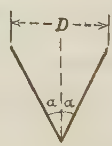


FIG. 82.

Axle friction. If a cylindrical axle fits loosely in a cylindrical bearing, the bearing surface will be a narrow strip or element. If the axle turns in the direction of the arrow (Fig. 81) it will rise in the bearing to some position as

shown. F is friction, N normal pressure, and $R = \sqrt{F^2 + N^2}$ = resultant pressure of bearing on axle. Let f = coefficient and ϕ = angle of friction between the surfaces. $F = N \tan \phi = R \sin \phi$. The FRICTION CIRCLE is the circle drawn about O and tangent to the line of action of R. Its radius is $\rho = r \sin \phi$ and the moment of F is $Fr = R\rho$. Angle ϕ is usually small and $\tan \phi$ is used instead of $\sin \phi$, giving $F = R \tan \phi = fR$, $\rho = r \tan \phi = fr$.

The work done by friction during each revolution of the shaft is $2\pi Fr = 2\pi R\rho$. The power necessary to overcome friction when the axle makes n revolutions per sec. is,

$$Power = 2n\pi Fr = 2n\pi R\rho = 2n\pi rfR \text{ ft.-lb./sec.}$$

Pivot friction occurs when the end of a vertical (or inclined) shaft rests in a bearing. If the end of the shaft is flat and the bearing pressure is constant over the surfaces in contact, the work necessary to overcome friction during one revolution is $2\pi fDT/3$, where f = coefficient of friction, D = diameter of shaft, T = total thrust on bearing. If the end of the shaft is conical (Fig. 82), the work necessary to overcome friction during one revolution = $2\pi fDT/3 \sin \alpha$.

Belt friction. When power is transmitted by a belt over a pulley, the tension T_1 on one side is greater than the tension T_2 on the other side of the pulley. The relation between T_1 and T_2 is $\dfrac{T_1 - Wv^2/g}{T_2 - Wv^2/g} = e^{f\alpha}$, where W = weight of belt per linear ft., v = speed of belt in ft./sec., f = coefficient of friction between belt and pulley, α = angle of contact between belt and pulley, e = base of natural logarithms = 2.718+. For small speeds the centrifugal force (Wv^2/g) may be neglected and $T_1/T_2 = e^{f\alpha}$.

Power transmitted is given by

$$\text{Power} = (T_1 - T_2)v \text{ ft.-lb./sec.} = (T_1 - T_2)v/550 \text{ hp.}$$

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SECTION 27

APPLIED MECHANICS

STRENGTH OF MATERIALS. HYDRAULICS

BY

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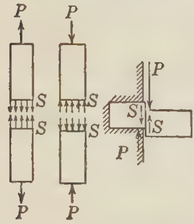
1. Simple stresses. Elasticity. Deformation

Definitions. STRESS is internal force between molecules; it resists change in form and rupture when a body is subject to external forces. Internal stresses hold external forces in equilibrium and in their action are equal and opposite to the action of these forces. UNIT STRESS (intensity) is the total uniform stress in a body divided by the area over which the stress is distributed. It is expressed in lb. per sq. in., tons per sq. ft., kg. per sq. cm., etc. TENSION (Fig. 1, a) is stress tending to keep two adjacent planes in a body from being pulled apart. It resists increase in length of the body in the direction of applied forces. External forces act away from, and internal stresses toward, any given section area normal to the direction of stress. COMPRESSION (Fig. 1, b) is stress resisting decrease in the distance between two adjacent planes in a body. External forces act toward, and stresses away from any given section. SHEAR (Fig. 1, c) is stress tending to keep two adjacent planes in a body from sliding on each other under the action of two equal and opposite external forces which are slightly separated; the action is similar to a pair of shears. Shearing stress is assumed equally distributed over the resisting area unless otherwise noted. Shearing stresses always occur in pairs and are of equal magnitude on all faces of a unit cube (Fig. 2). If vertical stresses S_v resist external shearing forces a couple is necessary to prevent rotation. This is supplied by the horizontal shearing stresses S_v'.



(a) (b) (c)

a, Tension. *b*, Compression. *c*, Shear.

FIG. 1.—Simple stresses.

The two pairs of shearing stresses in any plane may be combined into two equal and opposite tensile stresses tending to elongate one diagonal or into compressive stresses tending to shorten the other diagonal. The resultant action tends to give the unit cube an angular deformation. (Fig. 3.) This diagonal tension is important in design of concrete structures.

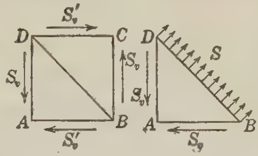


FIG. 2.

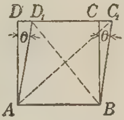


FIG. 3.

Shearing stresses in tension and compression. When a bar is subjected to simple tension or compression, shearing forces occur on diagonal planes. In Fig. 4 consider a plane making angle θ with the axis of a bar whose area at right-angles to P is A. Stress on this plane can be resolved into compressive stress normal to the plane and shearing stress along the plane. Value of shearing stress $S_v = P(\sin 2\theta)/2A$ which has a maximum value of $P/2A$ when $\theta = 45$ deg. Brittle materials under compressive loads fail by shearing along diagonal planes.

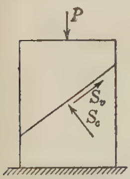


FIG. 4.—Shear due to compression.

Simple-axial stresses occur when resultant forces act along the axis of a body of symmetrical form and the stresses are equally distributed over section area. Axial tension and compression are common in engineering structures. Let P = load or one of a pair of forces producing tension, compression, or shear, A = area over which stress is distributed, and S = unit stress. Then $P = SA$.

Deformation or strain is change in form produced in a body by the action of external forces. Deformations are longitudinal, accompanied by stresses of tension or compression, or angular, produced by shearing forces. Stress resists deformation; the two occur simultaneously under the action of external force.

Poisson's ratio. When longitudinal deformation takes place in body, lateral deformation also occurs. If a block is shortened an amount d under a compressive load, it will increase in width an amount d_1 (Fig. 5). The corresponding unit deformations are $e = d/l$ and $e_1 = d_1/l_1$ and the ratio e_1/e is defined as Poisson's ratio m. Some values of m are: brass, 0.333; cast iron, 0.27; wrought iron, 0.278; steel, 0.303; concrete, 0.10 to 0.25.

Hooke's law states that, when the unit stress in a body does not exceed a certain limit, which varies with the nature of the material, stress bears a constant ratio to deformation or strain.

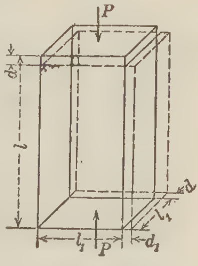


FIG. 5.

Elastic limit. Experiments show that within certain limits a body deformed under load will return to the original form upon removal of the load. Beyond this limit deformation increases more rapidly than the corresponding stress and the body, upon removal of load, will not return to the original form but will have permanent deformation, called SET. The maximum unit stress at which stress and deformation are directly proportional and which will cause no permanent set is called THE ELASTIC LIMIT.

Modulus of elasticity is the ratio of unit stress to unit deformation within the elastic limit. Let E = modulus of elasticity, then $E = S/e = Sl/d = Pl/Ad$. Modulus of elasticity is sometimes called YOUNG'S MODULUS or COEFFICIENT OF ELASTICITY. It is measure of the stiffness of a body. The reciprocal of E is a measure of relative deformation of different materials under the same unit stress.

Example. Average values of E for steel, cast iron, and timber are 30,000,000, 12,000,000 and 1,500,000 lb. per sq. in., respectively. Rods of these materials 10 ft. long under unit tension of 1000 lb. per sq. in. will elongate $d = 1000 \times 120/E = 0.004$, 0.01, and 0.08 in., respectively. Relative deformations are 1, 2.5, 20.

Stress-deformation diagram shows graphically the behavior of material under stress from no-load to rupture.

A bar is placed in a testing machine, load applied, and deformations read with strain gage or extensiometer. Unit deformation is plotted as abscissa and unit stress as ordinate. Figure 6 shows the relative behavior under tensile stress of three materials of construction. The curve is a straight line from the origin to the *elastic limit*, marked (a). The tangent to this curve is $S/e = E$. After passing the elastic limit e increases faster than S until, in the case of medium steel, the curve becomes horizontal at (b), the YIELD POINT. This point is defined as the unit stress at which deformation increases without increase in load and is readily determined by a testing machine as the point where the beam drops. At this point also a hard mill scale begins to peel off the specimen. The yield point is generally known as

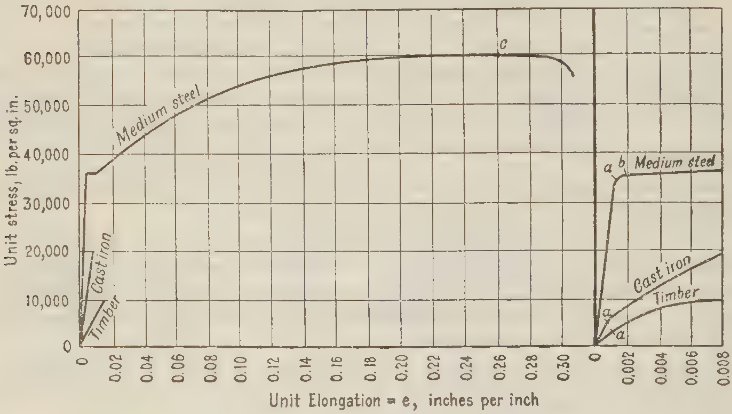


FIG. 6.—Stress-deformation diagram.

the COMMERCIAL ELASTIC LIMIT in commercial testing. The true elastic limit is often several thousand pounds below the yield point and is determined by plotting the stress-deformation diagram and noting the point of departure from a straight line. Many materials have no yield point or clearly defined elastic limit. For these cases Johnson has defined an APPARENT ELASTIC LIMIT as the unit stress at which the slope of the curve is 50 per cent. greater than at the origin.

Shearing modulus of elasticity or COEFFICIENT OF RIGIDITY is $G = S_v/\theta$, in which S_v = unit shearing stress and θ = angle in radians, Fig. 3. Unit of G is lb. per sq. in.

Elongation of a bar due to its own weight is one-half that produced by an equal weight suspended from the lower end.

Uniform bar of weight W lb. and length l in. is suspended vertically and carries a load P lb. at the lower end. Total elongation $d = (P + W/2)l/AE$ in.

Stress due to temperature charge. If a bar is free to expand and contract, its change in length for a temperature change t is $d = ntl$, in which n = the coefficient of linear expansion per degree of temperature. If the bar is rigidly fixed so that change in length is prevented, the resulting unit stress is the same as that required to cause deformation d. Hence $d = Sl/E = ntl$ or $S = ntE$ lb. per sq. in.

Example. Railroad rails 30 ft. long are laid end to end at temperature 50° F. At 90° F. the unit compressive stress is $S = 0.0000065 \times 40 \times 30{,}000{,}000 = 7800$ lb. per sq. in. (For values of coefficient of expansion, see Sec. 25, Art. 6.)

Stresses in compound bodies or bodies of two or more materials are directly proportional to the respective values of E. Let a short block carrying a compressive load P be composed of two materials, the section areas being A_1, A_2 and moduli of elasticity, unit stresses, and loads being E_1, E_2; S_1, S_2; and P_1, P_2, respectively. Then, since deformation of each material is the same, $d = P_1l/A_1E_1 = P_2l/A_2E_2$. Also $P_1 + P_2 = P$. Hence $P = P_1(A_1E_1 + A_2E_2)/A_1E_1 = P_2(A_1E_1 + A_2E_2)/A_2E_2$ and $S_1 = PE_1/(A_1E_1 + A_2E_2)$; $S_2 = PE_2/(A_1E_1 + A_2E_2)$. $S_1/S_2 = E_1/E_2$.

Example. If a block is a 6 × 4-in. wooden stick with two steel plates 6 in. by ¼ in. on the sides, and the total load is 84,000 lb., finding E from Table 1, stresses in wood and steel are 1000 and 20,000 lb. per sq. in., respectively.

Resilience is potential energy stored in a body under stress when the latter does not exceed the elastic limit and equals the work done in producing deformation in the body. For axial stress, if V = volume of the body, *resilience = work done = average force × deformation* $= Pd/2 = \frac{1}{2}P \times Sl/E = VS^2/2E$, in.-lb. per cu. in.

Modulus of resilience, R, is the elastic energy stored in a body per cu. in. at the elastic limit. $R = S_1{}^2/2E$ in which S_1 = elastic limit of material. Values of R for cast iron, structural steel, and tempered spring steel, are 1.04, 15, and 205 in.-lb., respectively. Cast iron is a poor material in which to store elastic energy while spring steel is excellent.

2. Stresses beyond the elastic limit

When stressed beyond the elastic limit, ductile materials such as soft steel become semi-plastic. At the yield point a flow of material occurs, deformation increasing without increase in load, accompanied by peeling off of hard scale. Beyond the yield point stress again increases and the material elongates rapidly, being in semi-plastic, semi-elastic condition. Just preceding rupture the bar necks down as shown by Fig. 7, rupture finally occurring at the contracted section.

Ultimate strength is maximum load divided by the *original* area. It occurs before rupture, point c, Fig. 6.

(*a*) (*b*)
a, Hard material
b, Ductile material.
FIG. 7.—Fractures in tension.

Ultimate elongation is expressed as 100 × *total elongation divided by original length.* It is measure of DUCTILITY or the ability of material to withstand shock and absorb energy without rupture. It varies from approximately zero for hard brittle materials to 30 per cent. or more for rivet steel. It varies with length of specimen hence the latter is always specified, *i.e.*, for rivets and structural steel, minimum unit elongation in 8 in. is specified as 1500 ÷ *ultimate strength.*

Reduction in area is $(A - A_1)/A$ where A = original area and A_1 = area of reduced section or neck. This factor is also a measure of ductility.

Permanent deformation occurs when metals are stressed beyond the elastic limit, Fig. 8. The effect is increase in ultimate strength, increase in elastic limit, increase in hardness, and decrease in ductility. Rolling, drawing, and forging of steel greatly increase its ultimate strength and elastic limit but cause marked decrease in elongation and reduction of area.

Annealing an overstrained bar by heating to 1400° F., and allowing to cool for several days, restores the original properties.

Failure under compression. Direct compressive stresses exist only in short blocks whose length in the direction of stress is less than 10 × *least diameter*. Longer blocks are classified as columns and fail by bending as well as by compression.

Soft ductile materials have no ultimate compressive strength since such material, when stressed beyond the yield point, spreads out indefinitely, Fig. 9. The value is generally assumed the same as that for tension although the term has no real significance. Hard brittle materials fail in compression by shearing along diagonal planes, the more brittle the material, the less the angle the plane makes with the vertical. ULTIMATE STRENGTH is a function of the shearing strength and coefficient of friction. Figure 9 shows failure of common materials of construction under compression.

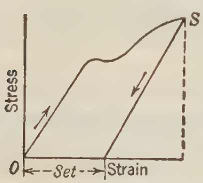
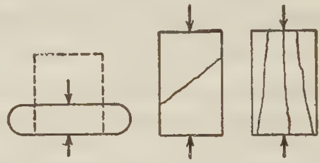


FIG. 8.



Wrought iron. Cast iron. Brick.

FIG. 9.—Fractures in compression.

Repeated stresses may cause rupture at a stress considerably below the elastic limit. The greater the range of stress, the less the stress required for rupture. Bars stressed repeatedly from zero to the elastic limit require an enormous number of applications of load to cause rupture. If stress alternates from tension to compression the bar will rupture after a large number of applications at little more than half the elastic limit.

3. Properties of materials

Average properties of materials are given in Table I. Strength of steel varies with the chemical composition, heat treatment and mechanical working.

Table 1. Average properties of materials

Material	Weight per cubic foot	Elastic limit, pounds per square inch	Ultimate strength, pounds per square inch			Modulus of rupture, pounds per square inch	Modulus of elasticity, ten., comp., shear, 1000 lb. per square inch	
			Tension	Comp.	Shear			
Wrought iron..........	480	28,000	50,000	50,000	40,000	....	28,000	12,800
Cast iron..............	450	6,000*t*	20,000	90,000	18,000	35,000	15,000	6,500
Malleable cast iron.....	....	20,000*c*	38,000	......	......	....	......	......
Steel, soft..............	490	25,000	50,000	50,000	38,000	....	30,000	13,000
Steel, medium..........	490	35,000	60,000	60,000	50,000	....	30,000	13,000
Steel, hard.............	490	40,000	70,000–110,000	......	....	....	30,000	......
Timber................	35	3,000	8,000	8,000	500*a* 3,000*b*	10,000–15,000	1,500	300
Stone.................	165	....	....	6,000	1,500	2,000	5,000	2,700
Brick.................	125	....	300	3,000	1,000	800	2,000	......
Concrete (1 : 2 : 4)......	150	....	125	2,400	400	400	2,500	1,000

a With grain. *b* Across grain. *c* Compression. *t* Tension.

ULTIMATE TENSILE STRENGTH varies from 45,000 to over 200,000 lb. per sq. in. HARDNESS increases and DUCTILITY decreases as tensile strength increases. Timber values are subject to wide variation, depending on the kind of wood, closeness of grain, moisture content, and homogeneity of the specimen. More detailed data are given in Arts. 6, 7, 15, 16.

Working stress is the safe unit stress allowable in material as used in design of members of a structure. It should be well below the elastic limit and is generally fixed by specification.

Working stresses in accordance with building laws of NEW YORK and CHICAGO are given in Tables 2 and 3.

Table 2. Working stresses, tension and compression, in accordance with building laws, New York and Chicago (1913)

Material	New York		Chicago	
	Pounds per square inch			
	Tension	Compression	Tension	Compression
Rolled steel, mild	16,000	16,000	14,000	14,000
Rolled steel, medium	16,000	16,000	14,000	14,000
Wrought iron	12,000	12,000	12,000	10,000
Cast iron	3,000	16,000	....	10,000
Steel pins, rivets (bearing)	....	20,000	....	20,000
Timber:				
Oak, with grain	1,000	900	....	....
Oak, across grain	....	800	....	250 (Boston)
Yellow pine, with grain	1,200	1,000	....	....
Yellow pine, across grain	....	600	....	250 (Boston)
White pine with grain	800	800	....	....
White pine across grain	....	400	....	150 (Boston)
Spruce, with grain	800	800	....	....
Spruce, across grain	....	400	....	150 (Boston)
Hemlock, with grain	800	500	....	....
Hemlock, across grain	....	500	....	41⅔ (Phila.)
Concrete, Portland, 1 : 2 : 4	....	230	....	55 (Chicago)
Concrete, Rosedale 1 : 2 : 4	....	125	....	55 (Chicago)
Brick, flatwise	....	300	....	....
Brickwork, cement mortar	....	250	....	....
Granite (according to test)	....	1,000–2,400	....	....
Limestone (according to test)	....	700–2,300	....	....
Sandstone (according to test)	....	400–1,600	....	....

Factor of safety is the ratio of ultimate strength to working stress or ratio of load to cause rupture to existing load on the structure. A factor of safety is necessary to guard against possible defects in material and to allow for unforeseen increase in stress. It depends for its value on the degree of certainty with which the ultimate strength is known and on similar knowledge in respect to applied loads.

Steel and iron have ultimate strengths that are nearly constant for given composition and treatment while the strength of wood varies widely, hence the factor of safety for steel is lower than that for timber. A steady load permits a lower factor than variable load or shock. Average values of factor of safety, according to *Merriman*, are given in Table 4.

Suddenly-applied loads cause a unit stress and deformation that is twice that caused by a static load of the same value.

Table 3. Working stresses, flexure and shear

Material	New York		Chicago	
	Pounds per square inch			
	Extreme fiber stress, bending	Shear	Extreme fiber stress, bending	Shear
Rolled steel	16,000	9,000	16,000	10,000
Shop rivets and pins	20,000	10,000	22,500	......
Wrought iron	12,000	6,000	12,000	......
Cast iron	16,000*C*	3,000	......	......
	3,000*T*	......	2,500*T*	......
Oak, with grain	1,000	100	1,000	150
Oak, across grain	......	600	......	250
Yellow pine, with grain	1,200	70	1,250	100
Yellow pine, across grain	......	500	......	250
White pine, with grain	800	40	750	80
White pine, across grain	......	250	......	150
Spruce, with grain	......	50	750	80
Spruce, across grain	800	320	......	150
Hemlock, with grain	600	40	......	......
Hemlock, across grain	......	275	......	......
Granite	180	......	......	......
Limestone	150	......	......	......
Sandstone	100	......	......	......
Concrete (Portland) 1 : 2 : 4	30	......	......	......
Concrete (Rosedale) 1 : 2 : 4	16	......	......	......

C, compression. *T*, tension.

Loads applied gradually, so that stress increases gradually from 0 to final value S are equivalent to static loads and $S = P/A$. When the load is suddenly applied so that the full load P acts through the whole deformation d_1 the work done is Pd_1. Had a load P_1 been applied gradually to produce the same deformation d_1, the work done would have been $P_1/2 \times d_1$. Hence $P \times d_1 = P_1/2 \times d_1$ or $P_1 = 2P$ and $d_1 = 2d$.

Table 4. Average values of factor of safety

Material	Steady stress	Variable stress	Shocks
Cast iron	6	10	20
Wrought iron	4	6	10
Structural steel	4	6	10
Hard steel	5	8	15
Timber	8	10	15
Brick and stone	15	25	40

Impact factor or coefficient of impact is the factor introduced to allow for the effect of sudden application of load. In designing structural members, static or dead-load stresses and live-load stresses are computed separately. The latter are found in the ordinary manner and then increased an amount iS in which i is the impact factor and S is stress due to live load treated as static load.

For suddenly applied loads as in hoisting machinery i = unity. For more gradually applied loads as in case of cranes or trains entering a bridge, i is less than unity. For railroad bridges the common value for impact factor is $i = 300/(L + 300)$ in which L is the loaded length in feet that produces maximum stress in the member. For highway bridges American Bridge Co. specifies $i = 0.25$. In many cases in design the impact factor is allowed for in the specified value of the working stress or in the factor of safety.

Stresses due to impact are those caused by a moving load striking a body. Let load P, moving horizontally with velocity V strike the end of a horizontal bar. The kinetic energy of the moving load $= PV^2/2g$ (or Ph, if h is the vertical distance through which the load must fall to acquire the velocity V). This kinetic energy goes into work of deformation of the bar, hence, if $d_1 =$ deformation, and $P_1 =$ static load required to produce this deformation, $Ph = P_1/2 \times d_1$. Also if $d =$ deformation produced by P acting as a static load then $d_1/d = P_1/P$. Solving these equations for P_1 there results $P_1 = P(2h/d)^{1/2}$ and $d_1 = (2hd)^{1/2}$. The corresponding unit stresses are $S_1 = S(2h/d)^{1/2}$.

If the bar be vertical and the moving load fall upon it from height h, the work done is $P(h + d_1)$ and $S_1 = S + S(1 + 2h/d)^{1/2}$; $d_1 = d + d(1 + 2h/d)^{1/2}$ in which $S =$ static stress due to load P and $S_1 =$ impact stress. These values apply only when the elastic limit of the material is not exceeded. Results are a little too large for heavy bars since the inertia of the bar has been neglected. Actually part of the energy is used to overcome inertia of the molecules and is dissipated in heat. For light bars the values are correct.

Example. A 100-lb. weight drops 6 in. and strikes the end of a vertical steel rod 2 in. $\times$ 2 in. $\times$ 10 in. Impact stress $S_1 = 25 + 25(1 + 12 \times 30{,}000{,}000/25 \times 10)^{1/2} = 31{,}275$ lb. per sq. in. Static stress is 25 lb. per sq. in.

Rupture by impact may be caused by light loads moving with considerable velocity. The preceding formulas do not apply. Experiments show that the work necessary to cause rupture is about 30 per cent. greater if it is the result of impact than if done by a static load.

4. Riveted joints

Riveted joints fail by tearing the plate between rivet holes, by shearing rivets or by crushing the plate and rivets where they mutually bear on each other.

Efficiency of a joint is the ratio of its strength to that of a solid unriveted plate.

Let $P =$ load in lb. on each repeating section, $p =$ pitch of rivets or length of repeating section, $t =$ thickness of plate, $d =$ diameter of rivet, $S_t =$ unit tensile stress in plate, $S_c =$ unit compressive stress on rivet, $S_s =$ unit shearing stress on rivet. Let there be n rivets in each repeating section and m rivet sections in shear. The loads to rupture the joint by tension, compression, and shear are respectively $P_t = (p - d)tS_t$; $P_c = ndtS_c$, and $P_s = \frac{m\pi d^2}{4} S_s$. Corresponding efficiencies are found by dividing the above values by the strength of the unriveted plate, $P = ptS_t$. For MAXIMUM EFFICIENCY design the pitch so that the efficiency in tension equals that against shearing or crushing, being equal to the smaller of the two. For VESSELS UNDER PRESSURE the pitch may be determined by the necessity for a tight joint. The above criterion generally satisfies this condition.

Ultimate strengths for rivets and plates of iron or soft steel are $S_t = 55{,}000$, $S_s = 38{,}000$, $S_c = 80{,}000$ lb. per sq. in. Working stresses for structural rivets are given in Art. 14.

General rules for design of joints:

1. Butt joints are preferred to lap joints since the latter introduce dangerous bending stresses.
2. Minimum distance from center line of rivet holes to edge of plate is $1.5d$, preferably $2d$.
3. Diameter of rivet $d = 1.2\sqrt{t}$ to $1.5\sqrt{t}$, being smaller for butt joints than for lap joints.
4. In multiple riveting the distance between rows of rivets is $1.7d$ for staggered riveting and 2 to $2.5d$ for chain riveting.
5. Thickness of straps in butt joints is $0.75t$ to $0.875t$.

Example 1. Design a double-riveted lap joint for $\frac{3}{8}$-in. plate.

There are two rivets per section and each rivet is in single shear hence $n = m = 2$. Let $d = 1.4\sqrt{t} \doteq \frac{7}{8}$ in. Then $P_S = \frac{2\pi d^2}{4} \times 38{,}000 = 45{,}500$ lb., and $P_C = 2dt \times 80{,}000 = 52{,}500$. Equating $P_t = P_S$, there results $(p - \frac{7}{8})\frac{3}{8} \times 55{,}000 = 45{,}500$ and $p = 3\frac{1}{8}$ in. Strength of un-riveted plate is $P_i = pt \times 55{,}000 = 64{,}500$, hence efficiency is $45{,}500 \div 64{,}500 = 70.5$ per cent.

Example 2. Triple-riveted butt joint (Fig. 10), has 5 rivets per section, 4 of which are in double shear, hence $n = 5$, and $m = 9$. Find the pitch for $\frac{1}{2}$-in. plate using 1-in. rivets. $P_S = \frac{9\pi d^2}{4} \times 38{,}000 = 268{,}000$. $P_C = 5dt \times 80{,}000 = 200{,}000$, hence $(p - d)t \times 55{,}000 = 200{,}000$ and $p = 8.25$ in.

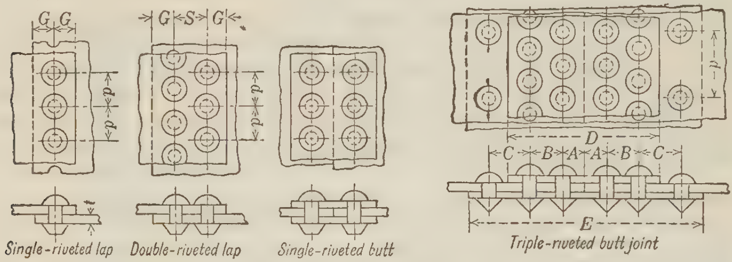


Fig. 10.—Riveted joints.

Forms of riveted joints for boilers and tanks as abstracted from recommendations of Hartford Steam Boiler Inspection and Insurance Co. are given by Fig. 10 and Tables 5 and 6.

Table 5. Dimensions of riveted lap joints

(All dimensions in inches. Letters refer to Fig. 10. e = efficiency in per cent.)

Thickness of plate (steel)	Diameter of rivet (iron)	Single-riveted			Double-riveted				Triple-riveted			
		P	G	e	P	G	S	e	P	G	S	e
$\frac{1}{4}$	$\frac{5}{8}$	$1\frac{5}{8}$	$1\frac{1}{8}$	57	$2\frac{5}{8}$	$1\frac{1}{8}$	$1\frac{7}{8}$	72	$3\frac{1}{2}$	$1\frac{1}{8}$	$2\frac{1}{16}$	80
	$1\frac{1}{16}$	$1\frac{7}{8}$	$1\frac{3}{16}$	60	$2\frac{7}{8}$	$1\frac{3}{16}$	$1\frac{15}{16}$	74	4	$1\frac{3}{16}$	$2\frac{1}{8}$	81
$\frac{3}{8}$	$\frac{3}{4}$	$1\frac{11}{16}$	$1\frac{1}{4}$	52	$2\frac{9}{16}$	$1\frac{1}{4}$	$1\frac{7}{8}$	68	$3\frac{3}{8}$	$1\frac{1}{4}$	$2\frac{3}{16}$	76
	$\frac{7}{8}$	$2\frac{1}{8}$	$1\frac{7}{16}$	55	$3\frac{1}{4}$	$1\frac{7}{16}$	$2\frac{3}{16}$	71	$4\frac{7}{16}$	$1\frac{7}{16}$	$2\frac{1}{2}$	79
$\frac{1}{2}$	$\frac{7}{8}$	$1\frac{13}{16}$	$1\frac{7}{16}$	48	$2\frac{11}{16}$	$1\frac{7}{16}$	2	65	$3\frac{1}{2}$	$1\frac{7}{16}$	$2\frac{1}{4}$	73
	1	$2\frac{3}{16}$	$1\frac{5}{8}$	51	$3\frac{5}{16}$	$1\frac{5}{8}$	$2\frac{1}{4}$	68	$4\frac{7}{16}$	$1\frac{5}{8}$	$2\frac{11}{16}$	76

Table 6. Dimensions of triple-riveted butt joints (steel rivets)

(All dimensions in inches. Letters refer to Fig. 10. e = efficiency in per cent.)

Thickness of plate (steel)	Diameter of rivet hole	e	Long pitch $= p$	Short pitch	A	B	C	D	E	Thickness of straps
$\frac{1}{4}$	$\frac{9}{16}$	87.5	$4\frac{1}{2}$	$2\frac{1}{4}$	$\frac{27}{32}$	$1\frac{3}{8}$	$1\frac{11}{16}$	$6\frac{1}{8}$	$9\frac{1}{2}$	$\frac{3}{16}$
$\frac{3}{8}$	$\frac{13}{16}$	87.5	$6\frac{1}{2}$	$3\frac{1}{4}$	$1\frac{7}{32}$	2	$2\frac{7}{16}$	$8\frac{7}{8}$	$13\frac{3}{4}$	$\frac{5}{16}$
$\frac{1}{2}$	$1\frac{1}{16}$	85.8	$7\frac{1}{2}$	$3\frac{3}{4}$	$1\frac{19}{32}$	$2\frac{1}{4}$	$3\frac{3}{16}$	$10\frac{7}{8}$	$17\frac{1}{4}$	$\frac{3}{8}$
$\frac{5}{8}$	$1\frac{1}{16}$	86.3	$7\frac{3}{4}$	$3\frac{7}{8}$	$1\frac{19}{32}$	$2\frac{5}{16}$	$3\frac{3}{16}$	11	$17\frac{3}{8}$	$\frac{15}{32}$
$\frac{3}{4}$	$1\frac{3}{16}$	84.1	$7\frac{7}{8}$	$3\frac{15}{16}$	$1\frac{25}{32}$	$2\frac{3}{8}$	$3\frac{9}{16}$	$11\frac{7}{8}$	19	$\frac{9}{16}$

5. Cylinders and rollers

Thin cylinders. Pipes and tubes under internal pressure tend to fail in tension along an element. The bursting force is resisted by hoop tension in the shell. If the thickness of the shell is small compared with the diameter of the cylinder, stress is considered uniformly distributed over the resisting area.

Let p = internal pressure, lb. per sq. in.; d = diameter, in.; t = thickness, in.; L = length of cylinder, in.; S = unit tensile stress, lb. per sq. in. (Fig. 11). TOTAL BURSTING FORCE in any direction is pdL and RESISTING TENSION is $2StL$. Hence $S/p = d/2t$. If the shell has a riveted joint with efficiency e, then $t = pd/2Se$. This is the formula used for boilers and pipes.

Hollow spheres under internal pressure have the same stress as the transverse stress in cylindrical boilers due to pressure on the head. BURSTING FORCE = $p\pi d^2/4$; RESISTING TENSION = πdtS, hence $S/p = d/4t$. Transverse stress in a cylindrical shell is one-half the longitudinal stress.

Cylinders under external pressure. The preceding formulas do not apply since the load tends to distort the cylinder into an ellipse and distortion is accompanied by increase in stress. No rational formulas exist. For lap-welded Bessemer-steel tubes, Stewart gives the collapsing pressure P in lb. per sq. in. as follows:

$$P = 1000(1 - \sqrt{1 - 1600(t/d)^2}\,) \qquad (a)$$
$$P = 86{,}670(t/d) - 1386 \qquad (b)$$

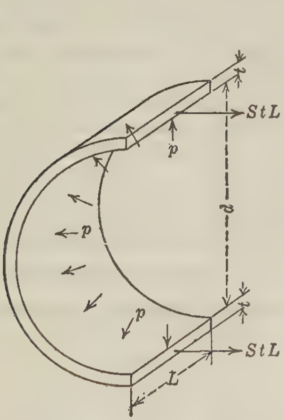


FIG. 11.

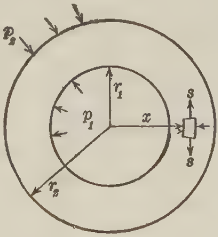


FIG. 12.

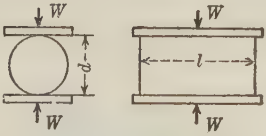


FIG. 13.

Formula (a) applies when P is less than 581 lb. per sq. in. or t/d less than 0.023, d being the outside diameter, in. For greater pressures and thicknesses use (b). Stewart also gives $P = 50{,}210{,}000\ (t/d)^3$ in place of (a).

Thick cylinders. When thickness is large compared with the diameter, tangential stress is not uniformly distributed.

Let p_1 and p_2 be internal and external pressures and r_1 and r_2 be corresponding radii (Fig. 12). Tangential stress at distance x from the center is given by LAME'S FORMULA as $S = \left[r_1^2p_1 - r_2^2p_2 + \dfrac{4r^2r_2^2}{x^2}(p_1 - p_2)\right] \Big/ 3(r_2^2 - r_1^2)$. This formula is based on a value of Poisson's ratio of ⅓. Stress decreases as x increases. If $p_2 = 0$, MAXIMUM STRESS at the inside wall is $S = \frac{1}{3}p_1(r_1^2 + 4r_2^2)/(r_2^2 - r_1^2)$. Radial compressive stress has maximum value p_1 at inside surface and minimum value p_2 at outside surface.

Rollers. Fig. 13 shows a roller of length l and diameter d placed between two plates carrying load W. Assuming the roller only to be deformed S =

$[(9W^2E)/81^2d^2)]^{1/3}$; $ld = (3W/2S)(E/2S)^{1/2}$. LOAD PER UNIT OF LENGTH $w = W/l = \frac{2}{3}dS(2S/E)^{1/2}$. Recent bridge specifications give load per linear inch $w = 600d$.

6. Beams

Notation

P = Concentrated load, lb.
W = Total distributed load, lb.
w = Unit distributed load, lb.
R = Reaction.
V = Vertical shear.
S = Normal unit stress.
S_v = Unit vertical shearing stress.
S_n = Unit horizontal shearing stress.
I = Rectangular moment of inertia.
f = Maximum deflection, in.

Bending or flexure occurs when a bar is subject to such loading that the axis assumes the form of a curve. This occurs in the case of beams, columns, and bars or blocks under eccentric load. Bending induces stresses of tension, compression, and shear.

Simple beam is a horizontal bar supported at each end without constraint, carrying vertical loads. The axis of the beam is deflected convex downward so that the upper fibers are shortened and the lower fibers lengthened. Loads and reactions at the supports are not in the same line, hence shearing stresses exist between adjacent vertical planes. CANTILEVER BEAM is a member with one end rigidly fixed and the other end projecting and free to deflect under vertical loads. CONSTRAINED BEAMS have one or both ends rigidly fixed, *e.g.*, beams with ends built into a brick wall. CONTINUOUS BEAMS have more than two supports. When a beam deflects, fibers on the convex side are in tension and those on concave side in compression. Between the two extremes is a surface of no normal stress called THE NEUTRAL SURFACE; the trace of this surface in any cross-section is called THE NEUTRAL AXIS. This axis passes through the center of gravity of the section and stress in any fiber varies directly as its distance from this axis, provided the elastic limit is not exceeded.

End reactions are found from the laws of equilibrium. Sum of loads = sum of reactions. Moment of loads about any point = moment of reactions.

Example. Figure 14 shows a simple beam with two concentrated loads. $R_1 + R_2 = 6000$. $M_a = 2000 \times 3 + 4000 \times 7 - R_2 \times 10 = 0$. $R_2 = 3400$ lb.; $R_1 = 2600$ lb.

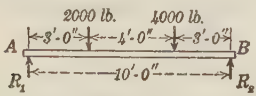


FIG. 14.

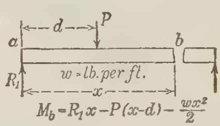


FIG. 15.

Vertical shear. At any section of a beam there is a resultant upward force on one side and an equal downward force on the other side of the section. This produces a shearing force on the section, equal in value to either force. By definition VERTICAL SHEAR on any section is the *sum of all external forces on the left of that section*, upward forces being considered positive and downward forces negative.

For a simple beam, shear at any section at a distance x from the left end is $V = R_1 - wx - P$, in which R_1 = left-end reaction, w = load per unit of length and P = sum of all concentrated loads between left end and given section. For a cantilever beam, the vertical shear at any section is the total load between that section and the free end.

Bending moment at any section of a beam is the *sum of the moments of all*

loads and reactions to the left of the section about an axis through the section. Moments with clockwise rotation are positive, counter-clockwise negative.

For a SIMPLE BEAM, the bending moment $M = R_1x - wx^2/2 - P(x - d)$ (Fig. 15). MAXIMUM MOMENT occurs at the section where vertical shear is zero, known as the DANGER SECTION. For a CANTILEVER BEAM maximum moment occurs at the point of support.

Shear and moment diagrams show graphically the value and variation of vertical shear and bending moment throughout the length of span of a beam. The ordinate under any section gives the shear or moment for that section. Table 7 gives these diagrams together with maximum shears, moments, and deflections for various types of beams and loadings.

Construction is explained in following problem, illustrated by Fig. 16: Assume a simple beam, span = 20 ft.; W = 100 lb. per linear ft.; concentrated loads of 2000 and 1000 lb. 4 ft. and 12 ft. from the left end respectively. The reactions, found by moments, are R_1 = 3000 lb., R_2 = 2000 lb. The SHEAR DIAGRAM is drawn by erecting an ordinate = 3000 at the left end noting that the distributed load causes uniform decrease in value of ordinates of 100 lb. per linear ft. while concentrated loads cause perpendicular drop equal to their value. The danger section is found graphically or, having located its position as between the concentrated loads, from the equation $R_1 - 2000 - 100x = 0$. The shear diagram for concentrated loads is composed of horizontal and vertical straight lines; for uniform load, of a uniformly sloping line. Figure 16 shows a combination. The MOMENT DIAGRAM is drawn by calculating the bending moment under each concentrated load and at the danger section. For concentrated loads this diagram is composed of straight lines intersecting at the loads; for uniform load, it is a parabola of equation $y = wlx/2 - wx^2/2$. Values of moment at points a, b, and c are $M_a = 3000 \times 4 - 50(4)^2 = 11{,}200$ lb.-ft.; $M_b = 3000 \times 10 - 2000 \times 6 - 50(10)^2 = 13{,}000$ lb.-ft. (max.); $M_c = 3000 \times 12 - 2000 \times 8 - 50(12)^2 = 12{,}800$ lb.-ft.

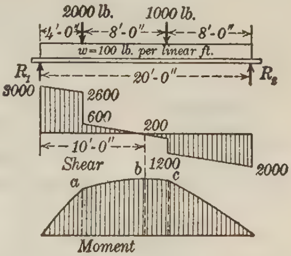


FIG. 16.—Shear and moment diagrams.

Flexure formula. From the laws of equilibrium, vertical shears and external bending moments must be resisted by equal and opposite internal shearing stresses and resisting moments. If V is the vertical shear on any section and A the section area then, assuming uniform distribution, $S = V/A$. For a simple beam maximum shear is equal to the larger reaction; for a cantilever beam it equals the total load. The internal resisting moment consists of a couple formed by the tensile and compressive stresses acting on opposite sides of the neutral axis. If S is maximum unit stress, tension or compression, and c the distance from the neutral axis to the remotest fiber, the RESISTING MOMENT in inch-pounds is given by

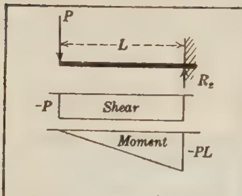
$$M = SI/c; \quad S = M(c/I),$$

in which S is in lb. per sq. in.; I, in.^{4}; c, in.

The term I/c is constant for any section and termed the SECTION MODULUS.

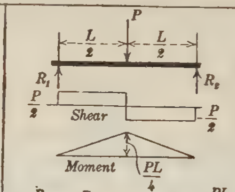
For a uniformly-loaded simple beam, $M = WL/8$, hence $W = \frac{8S}{L}\left(\frac{I}{c}\right)$ and the load varies directly as the working stress and section modulus, and inversely as the span. For a rectangular beam with breadth b and depth d, $I/c = bd^2/6$, hence the strength varies directly as the breadth and as the square of the depth. For simple and cantilever rectangular beams load $= aSbd^2/L$. For a cantilever beam with concentrated load at the end, $a = 1/6$; for distributed load, $a = 1/3$; for a simple beam with concentrated load at the center of the span, $a = 2/3$; for a distributed load, $a = 4/3$, hence the ratio of strengths is 1 : 2 : 4 : 8.

Table 7. Beams of uniform cross-section, transverse loading.



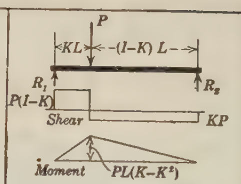
$R_2 = P$ $\quad M_{max} = -PL$

$V_{max} = -P$ $\quad f = \frac{PL^3}{3EI}$



$R_1 = \frac{P}{2}$, $R_2 = \frac{P}{2}$ $\quad M_{max} = \frac{PL}{4}$

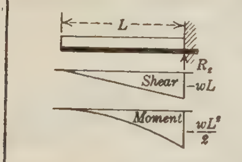
$V_{max} = \pm\frac{P}{2}$ $\quad f = \frac{PL^3}{48EI}$



$R_1 = P(1-K)$, $R_2 = KP$

$V_{max} = +R_1$ or $-R_2$ $\quad f = \frac{W \cdot c^2 c_1^2}{3EIl}$

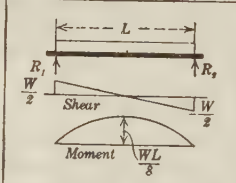
$M_{max} = PL(K-K^2)$



$R_2 = W = wL$

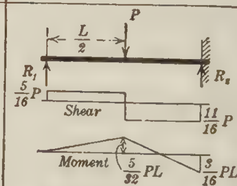
$V_{max} = -W$ $\quad f = \frac{WL^3}{8EI}$

$M_{max} = -\frac{WL}{2}$



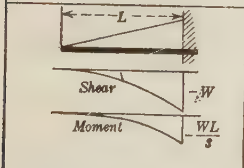
$R_1 = \frac{W}{2}$, $R_2 = \frac{W}{2}$ $\quad M_{max} = \frac{WL}{8}$

$V_{max} = \pm\frac{W}{2}$ $\quad f = \frac{5}{384}\frac{WL^3}{EI}$



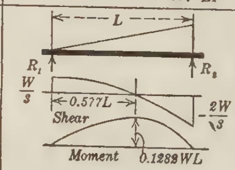
$R_1 = \frac{5}{16}P$, $R_2 = \frac{11}{16}P$ $\quad M_{max} = -\frac{3}{16}PL$

$V_{max} = -\frac{11}{16}P$ $\quad f = \frac{1}{108}\frac{PL^3}{EI}$



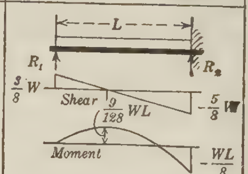
$R_2 = W$ $\quad M_{max} = -\frac{WL}{3}$

$V_{max} = -W$ $\quad f = \frac{WL^3}{15EI}$



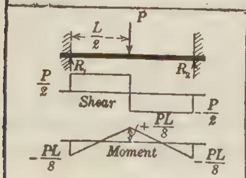
$R_1 = \frac{W}{3}$, $R_2 = \frac{2W}{3}$ $\quad M_{max} = 0.1282WL$

$V_{max} = -\frac{2}{3}W$ $\quad f = \frac{0.01304 WL^3}{EI}$



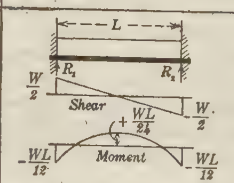
$R_1 = \frac{3}{8}W$, $R_2 = \frac{5}{8}W$ $\quad M_{max} = -\frac{WL}{8}$

$V_{max} = -\frac{5}{8}W$ $\quad f = \frac{1}{185}\frac{WL^3}{EI}$



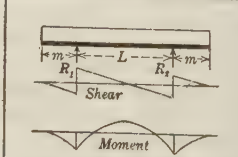
$R_1 = \frac{P}{2}$, $R_2 = \frac{P}{2}$ $\quad M_{max} = \pm\frac{PL}{8}$

$V_{max} = \pm\frac{P}{2}$ $\quad f = \frac{1}{192}\frac{PL^3}{EI}$



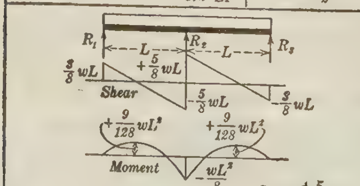
$R_1 = \frac{W}{2}$, $R_2 = \frac{W}{2}$ $\quad M_{max} = -\frac{WL}{12}$

$V_{max} = \pm\frac{W}{2}$ $\quad f = \frac{1}{384}\frac{WL^3}{EI}$



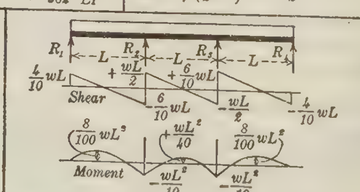
$R_1 = \frac{W}{2}$, $R_2 = \frac{W}{2}$ $\quad V_{max} = \frac{W}{2} - wm$, or $-wm$

$M_{max} = \frac{W}{4}\left(\frac{L}{2} - m\right)$, or $-\frac{wm^2}{2}$



$R_1 = \frac{3}{8}wL$, $R_2 = \frac{10}{8}wL$, $R_3 = \frac{3}{8}wL$ $\quad V_{max} = \pm\frac{5}{8}wL$

$M_{max} = -\frac{wL^2}{8}$



$R_1 = 0.4wL$, $R_2 = 1.1wL$, $R_3 = 1.1wL$, $R_4 = 0.4wL$

$V_{max} = 0.6wL$

$M_{max} = \frac{wL^2}{10}$

Safe loads in pounds uniformly distributed for rectangular wooden beams one inch wide, figured for allowable fiber stress of 1000 lb. per sq. in., are given in Table 8. For any rectangular beam of breadth b and allowable working stress S, multiply the tabular value by $bS/1000$.

Design of beams. For a rectangular beam of any material, type, and loading find the maximum bending moment from Table 7 or by calculation, find the ratio of the moment to $WL/8$ (value for simple beam uniformly loaded) and divide the tabular values, Table 8, by this ratio. The allowable loads as given in the table are multiplied by $bS/1000$ and divided by $M \div WL/8$.

Table 8. Uniformly-distributed loads on rectangular wooden beams one inch wide

Span in feet	Depth of beam in inches											
	6	7	8	9	10	11	12	13	14	15	16	18
5	800	1090	1420	1800	2220	2690	3200	3755	4355	5000	5690	7200
6	667	907	1185	1500	1850	2240	2667	3130	3630	4170	4740	6000
7	570	780	1015	1285	1585	1920	2285	2685	3110	3570	4060	5140
8	500	680	890	1125	1390	1680	2000	2350	2720	3130	3560	4500
9	445	605	790	1000	1230	1490	1780	2090	2420	2780	3160	4000
10	400	540	710	900	1110	1340	1600	1880	2180	2500	2840	3600
11	360	490	650	820	1010	1220	1450	1710	1980	2270	2590	3240
12	330	450	590	750	930	1120	1330	1560	1810	2080	2370	2970
13	310	420	550	690	850	1030	1230	1440	1680	1920	2190	2790
14	290	390	510	640	790	960	1140	1340	1560	1790	2030	2610
15	270	360	470	600	740	900	1070	1250	1450	1670	1900	2430
16	250	340	440	560	690	840	1000	1170	1360	1560	1780	2250
17	230	320	420	530	650	790	940	1100	1280	1470	1670	2070
18	220	300	400	500	620	750	890	1040	1210	1390	1580	1980
19	210	290	380	470	590	710	840	990	1150	1320	1500	1890
20	200	270	360	450	555	670	800	940	1090	1250	1420	1800
21	190	260	340	430	530	640	760	895	1035	1190	1355	1710
22	180	250	320	410	505	610	725	855	990	1135	1290	1635
23	175	240	310	390	480	585	695	815	945	1085	1235	1565
24	166	230	290	375	460	560	665	780	905	1040	1185	1500
25	160	220	285	360	445	540	640	750	870	1000	1140	1440
26	154	210	275	345	425	515	615	720	840	960	1095	1385
27	148	200	265	330	410	500	595	695	810	925	1055	1335
28	143	195	255	320	395	480	570	670	780	895	1015	1285
29	138	190	245	310	385	465	550	650	750	860	980	1240
30	133	180	235	300	370	450	535	625	725	835	950	1200

Example 1. Find the allowable concentrated load at the center of a cast-iron beam 2 in. $\times$ 6 in., span 10 ft., working stress 3000 lb. per sq. in., beam fixed at one end, supported at the other end.

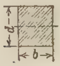
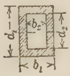
From Table 7, $M = 3PL/16$ hence ratio of moments is $3/2$. Tabular value of load for beam 6 in. deep with 10-ft. span is 400 lb. Hence $P = 400 \times 2/3 \times 2 \times 3000/1000 = 1600$ lb.

Example 2. Find the width of a simple wooden beam 12 in. deep with 16-ft. span to carry 2000 lb. at the center with a working stress of 800 lb. per sq. in.

$M = PL/4$ hence the ratio of moments is 2; of stress $8/10$. Tabular value for 12-in. depth and 16-ft. span is 1000 lb. For the given conditions the load per inch width is $1000 \times 1/2 \times 8/10 = 400$ lb. Required width is $2000 \div 400 = 5$ in.

For beams of various cross-sections, express the bending moment in inch-pounds, then $M/S = I/c$ and the section is determined by the value of the section modulus. Table 9 gives properties of some common sections. For moments of inertia of other sections see Sec. 26, Table 4. For rolled shapes see Tables 21 to 24, incl.

Table 9. Properties of sections of beams

Section of beam	Moment of inertia, I	Section modulus, I/c	Radius of gyration, r
	$bd^3/12$	$bd^2/16$	$d/\sqrt{12} = 0.289d$
	$\dfrac{b_1d_1^3 - b_2d_2^3}{12}$	$\dfrac{b_1d_1^3 - b_2d_2^3}{6d_1}$	$\sqrt{\dfrac{b_1d_1^3 - b_2d_2^3}{12(b_1d_1 - b_2d_2)}}$
	$\pi d^4/64$	$\pi d^3/32$	$d/4$
	$\pi(d_1^4 - d_2^4)/64$	$\pi(d_1^4 - d_2^4)/32d_1$	$\sqrt{d_1^2 + d_2^2}/4$
	$bd^3/36$	$bd^2/24$ (min.)	$d/\sqrt{18} = 0.236d$

Deflection of beams. When a beam is subject to an external bending moment the fibers in tension are elongated and those in compression shortened. This causes the beam to deflect and the neutral surface assumes the form of a curve. The trace of this surface in the plane of bending is called the ELASTIC CURVE and the radius of curvature at any point is $R = EI/M$. Replacing R by an approximate value $R = 1 \div d^2y/dx^2$, $EId^2y/dx^2 = M$. This is integrated by expressing M as function of x and determining constants from the conditions of the problem. Maximum deflection f for various beams and loadings is given in Table 7. In general $f = PL^3/nEI$ in which n is a constant depending on the type of beam and loading. STIFFNESS $= 1/f$. Beams are often designed so that deflection shall not exceed a specified value, usually $L/360$. Since, with increase in span, deflection increases more rapidly than stress, design of long spans may be controlled by the former rather than the latter.

Example. Specifications give a working stress not to exceed 16,000 lb. per sq. in. and deflection not to exceed $L/360$. Find the distance d between adjacent I-beams for a floor to carry a uniform load of 100 lb. per sq. ft. Span = 25 ft., I-beam 10 in. deep weighing 30 lb. per ft.

$W = 100 \times 25 \times d = 2500d$; $I/c = 26.8$ (Table 21). $M = WL/8 = S(I/c)$ hence for stress of 16,000 lb. per sq. in., $2500d \times 25 \times {}^{12}/_{8} = 16{,}000 \times 26.8$, and $d = 4.6$ ft. For deflection: $L/360 = 5WL^3/384EI$. Taking $E = 30{,}000{,}000$ (Table 1). $I = 134.2$ and $W = 2500d$, $d = 3.83$ ft., hence the beams must be spaced 3.83 ft. or less center to center. Had the span been 20 ft. of less, the load to cause a stress of 16,000 would not have caused deflection greater than $L/360$.

Relation between stress and deflection. Bending moment is PL/m where m is constant, hence $PL/M = S(I/c)$ or $S = PLc/mI$. Likewise $f = PL^3/nEI$, hence $f/S = ML^2/nEc$. For any given material and section $f/S = kL^2$ in which k is constant. Hence for given load and section, if the span is doubled S is doubled but f is increased 8 times. Also, if the span is doubled and the load is reduced so that S remains constant, f is increased 4 times.

Constrained beams. The ends are built into a wall in such a manner that the tangent to the elastic curve at the end is fixed in direction. Constrained beams are stronger and stiffer than simple beams. (See Table 7.) Most beams with riveted connections have some constraint but unless the latter is known to be rigid it should be neglected and the beams treated as simple.

Continuous beams are those resting on several supports. Shears and moments are similar to those of constrained beams. Shear at any point equals the sum of all the loads and reactions to the left of the point. The moment at any section B equals the moment at any other section A plus the moment of shear at A about B, plus the moment of all intervening forces.

$$M_b = M_a + V_a x - P(x - d) - wx^2/2.$$

If two adjacent spans of a continuous beam have lengths L_1 and L_2 and unit loads w_1 and w_2, then the moments at the three points of support are connected by the theorem $M_1L_1 + 2M_2(L_1 + L_2) + M_3L_2 = - w_1L_1^3/4 - w_2L_2^3/4$. If $L_1 = L_2$ and $w_1 = w_2$ then $M_1 + 4M_2 + M_3 = - wL^2/2$. From these equations moments, shears, and reactions can be computed. Table 7 shows diagrams for uniformly loaded continuous beams of two and three equal spans. The sign of shear changes on opposite sides of the point of support and the reaction is the sum of the two values.

Impact on beams. See axial impact, Art. 3. If a load W falls through a vertical height h and strikes a horizontal beam, $S_1 = S + S(1 + 2h/d)^{1/2}$; $d_1 = d + d(1 + 2h/d)^{1/2}$, in which S_1 and d_1 are stress and deflection due to impact and S and d the corresponding values for the same load applied as a static load. S is found by the flexure formula and d from the value of f in Table 7.

Example. A cast-iron beam 1 × 6 in. has a span of 10 ft. If a weight of 50 lb. falls 2 ft. and strikes at the center of the span, find the stress due to impact.

$S = M\frac{c}{I} = \frac{50 \times 120 \times 6}{4 \times 36} = 250$ lb. per sq. in. $d = \frac{PL^3}{48EI} = \frac{50 \times (120)^3 \times 12}{48 \times 15{,}000{,}000 \times 6^3} = \frac{1}{150}$. $S_1 = 250 + 250(1 + 24 \times 150)^{1/2} = 15{,}250$ lb. per sq. in.

7. Columns

Columns are compression members whose unsupported length is more than 8 or 10 times their least dimension. Failure occurs in long columns by buckling at a stress below the elastic limit. Short columns, such as those commonly used in structures, fail by a combination of direct compressive stress and stress due to bending. The strength of such columns is a function of both section area and its least moment of inertia. SLENDERNESS RATIO is defined as l/r, in which l is the unsupported length of the column and $r = \sqrt{I/A}$ = LEAST RADIUS OF GYRATION, both generally expressed in inches.

Long columns, l/r greater than 100 or 125, fail by buckling. The load necessary to cause failure depends on the condition of the ends of the columns, those with ends rigidly fixed being stronger and stiffer than those with round ends (Fig. 17). The buckling load for round ends is given by Euler's formula as $P = \pi^2EI/l^2$, or $P/A = \pi^2E(r/l)^2$. For columns with both ends fixed, $P/A = 4\pi^2E(r/l)^2$.

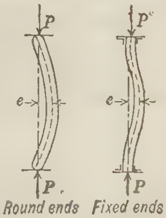


FIG. 17.—Flexure of columns.

Short columns fail by direct stress P/A and bending stress Pec/I in which Pe = bending moment due to deflection e. But $e = kl^2/c$, in which k = constant, hence total stress $S = P/A[1 + k(l/r)^2]$ and $P/A = S/[1 + k(l/r)^2]$ which is Rankine's formula, much used in design for values of l/r = 20 to 150. Values of k

recommended by *Merriman* are given in Table 10. S = allowable compressive stress in material.

Table 10. Values of constant k in Rankine's formula

Material	Both ends fixed	One end fixed, other end round	Both ends round
Timber..............	1/3,000	1.95/3,000	4/3,000
Cast iron............	1/5,000	1.95/5,000	4/5,000
Wrought iron.........	1/36,000	1.95/36,000	4/36,000
Steel................	1/25,000	1.95/25,000	4/25,000

Straight-line column formula, much used in recent specifications, is $P/A = S - c(l/r)$, in which S = working stress in material and c = a constant. The formula represents a straight line tangent to Euler's curve and gives values on the side of safety.

Allowable values of P/A for structural-steel columns

Am. Bridge Co.	A. R. E. Assn.	Gordon's Formula	New York
$19{,}000 - 100\frac{l}{r}$ 13,000 max.	$16{,}000 - 70\frac{l}{r}$ 14,000 max.	$\dfrac{12{,}500}{1 + \dfrac{l^2}{36{,}000r^2}}$	$15{,}200 - 58\frac{l}{r}$

For timber columns Am. R. R. Eng. Assn. gives $P/A = S(1 - l/60d)$ in which S = safe compressive stress along grain, l = length of column, and d = least dimension; l and d are expressed in the same unit. For $l/d < 15$, $S = P/A$. Table 11 is based on this formula with S = 1000 lb. per sq. in. For other values of S solve by proportion.

Table 11. Safe loads for wooden columns, in units of 1000 lb.
(Based on safe end-bearing compression of 1000 lb. per square inch)

Unbraced length, feet	Size of column, inches						
	4×4	6×6	8×8	10×10	12×12	14×14	16×16
4	16.0	......	......	......	......	......	......
6	11.2	36.0	......	......	......	......	......
8	9.6	26.3	......	......	......	......	......
10	8.0	24.1	64.0	......	......	......	......
12	6.4	21.6	44.8	100.0	......	......	......
14	4.8	19.1	41.6	72.0	144.0	......	......
16	......	16.9	38.4	68.0	105.6	196.0	......
18	......	14.4	35.2	63.0	100.8	145.0	256.0
20	......	12.0	32.0	60.0	96.0	140.0	192.0
22	......	9.7	28.8	57.0	91.3	134.0	185.6
24	......	......	25.6	52.0	86.4	128.0	179.2
26	......	......	22.4	48.0	81.6	123.5	172.8
28	......	......	19.2	43.0	76.4	117.6	166.4
30	......	......	......	40.0	72.0	111.8	160.0
32	......	......	......	36.0	67.6	106.5	153.6
34	......	......	......	33.0	62.0	100.8	147.2
36	......	......	......	......	57.6	95.2	140.8
38	......	......	......	......	53.3	89.6	134.4
40	......	......	......	......	48.0	84.0	128.0
42	......	......	......	......	......	78.4	121.6
44	......	......	......	......	......	72.8	115.2
46	......	......	......	......	......	67.2	108.8
48	......	......	......	......	......	......	102.3
50	......	......	......	......	......	......	96.0

Round cast-iron columns have an average ultimate strength, according to Burr, of $P/A = 30{,}500 - 165(l/d)$. This formula may be used if a factor of safety of at least 4 is introduced and l/r made less than 70. Cast iron is not good structural material.

Eccentric loads on a prism. Short blocks carrying loads not in line with their axes have a non-uniform stress distribution that is similar to the case of short columns.

Let the eccentricity of the load be e, then the moment of the load about the c.g. of the section is Pe which produces stress Pec/I. Hence the maximum unit stress is $S_1 = P/A + Pec/I = P/A(1 + ec/r^2)$. Minimum stress $S_2 = P/A(1 - ec/r^2)$. If the section is rectangular with breadth b and length unity, $S_1 = P/A(1 + 6e/b)$. The stress has straight-line distribution (Fig. 18) average value $= P/A$.

If the eccentricity for a rectangular section $= b/6$ then $S_1 = 2P/A$ and $S_2 = 0$. If e is greater than $b/6$, S_2 reverses in sign. Hence the condition to be satisfied in the design of DAMS and RETAINING WALLS, in order that there be no tension in the joint, is that the resultant pressure fall within the middle third of the section.

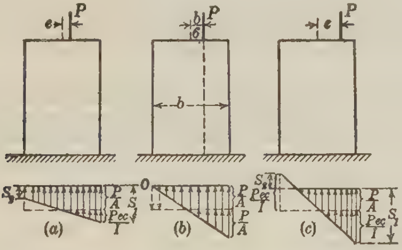


FIG. 18.

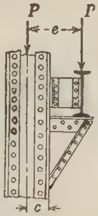


FIG. 19.

Eccentric loads on columns (Fig. 19) produce stresses which are found by combining the formulas of the preceding paragraph with Rankine's formula, hence $P/A = S/(1 + k\frac{l^2}{r^2} + \frac{ec}{r^2})$. The stresses can also be found by use of the straight-line formula. The combined stress due to axial load and bending moment should not exceed the value of P/A as given by the formula. No direct solution is possible but the problem is solved rapidly by trial.

Let P = direct load in lb. P_1 = eccentric load in lb. M = bending moment due to eccentric load in in.-lb. $= Pe$. I = moment of inertia of column in direction of bending. c = extreme fiber distance. S = allowable value of P/A as given by the column formula. A = area of section in sq. in.

The value of S should be equal to or greater than $(P + P_1)/A + Mc/I$, the combined stress due to compression and bending. In design, assume a section in excess of that required for the direct compressive load $P + P_1$ and compute the combined stress. If this is too great make another trial.

Example. Required to select a standard-channel column 20 ft. long to carry an axial load of 40,000 lb. and eccentric load of 20,000 lb., 15 in. from the center on axis 2–2. (Table 22.)

Try 12-in. @ 20.5-lb. channels back to back. $A = 12.06$, $l = 256$, $c = 6$, $r = \sqrt{I/A} = 4.61$. Hence fiber stress $= 60{,}000/12.06 + (20{,}000 \times 15 \times 6)/256 = 11{,}980$ lb. per sq. in. Allowable value of P/A, using A.R.E. formula, is $P/A = 16{,}000 - (70 \times 240)/4.61 = 12{,}360$, hence the column chosen is satisfactory.

8. Torsion of shafts

If a bar is firmly fixed at one end and a twisting couple P_p is applied at the other end (Fig. 20), the bar is twisted through an angle α and the fibers undergo shearing stress. In case of a round shaft, unit shearing stress in any section perpendicular to the axis, varies directly as the distance from the axis.

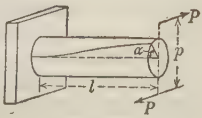


FIG. 20.

Let $M_t = P_p$ = twisting moment. S = unit shearing stress in remotest fiber. c = distance from axis to remotest fiber. J = polar moment of inertia $= I_x + I_y$. G = shearing modulus of elasticity. Then $M_t = SJ/c = aGJ/L$.

For a solid circular shaft $J = \pi d^4/32$ and $c = d/2$, therefore $M_t = \pi d^3 S/16$. If the shaft were designed to deliver H horsepower at N revolutions per minute, then $d = 68.5(H/NS)^{1/3}$. Hollow shafts are stronger than solid shafts of the same weight since the material is removed from the vicinity of the axis where the stress is small. If d_1 and d_2 represent outside and inside diameters respectively, then $J = \pi(d_1^4 - d_2^4)/32$ and $c = d_1/2$, hence $M_t = S\pi(d_1^4 - d_2^4)/16d$ and $S(d_1^4 - d_2^4)/d_1 = 321{,}000H/N$.

The angular distortion of a circular shaft in degrees is: $\alpha = 57.3M_t l/GJ = 3{,}610{,}000HL/NGJ$.

Note.—For steel the value of G varies from 12,000,000 to 15,000,000 lb. per sq. in.

9. Combined stresses

Combined flexure and torsion occurs when a twisted body undergoes bending, *e.g.*, a shaft supported by bearings at each end and carrying a pulley at its center.

Let $S = Mc/I$ = unit bending stress and $S_r = M_t c/J$ = unit shearing stress due to torsion. For circular shafts $S = 8PL/\pi d^3$; $S_r = 16M_t/\pi d^3$. S and S_v combine to produce a normal stress S_m and tangential stress S_t on an interior plane. Maximum values are:

$$S_n = \tfrac{1}{2}S \pm \sqrt{S_v^2 + (s/2)^2}; \quad S_t = \sqrt{S_v^2 + (s/2)^2}.$$

The formulas apply to any case of normal stress combined with shear. Use (+) sign to find maximum tension where S is tension and (−) sign for compression.

Combined flexure and compression occurs when a truss member is under bending as well as direct stress (Fig. 21). Maximum stress is the sum of the direct stress and the bending stresses.

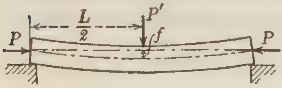


FIG. 21.

Let $S = P/A$ = direct stress and Mc/I = bending stress where M is total moment due to both P and P'. If P is small compared with P', the total stress $S = P/A + M_0c/I$, in which M_0 is the bending moment due to P' only. If compression is large, bending due to P must be considered. Moment $= Pf$, where f = maximum deflection. Total moment is $M_0 + Pf$ and resultant stress is

$$S = \frac{P}{A} + \frac{M_0c/I}{1 - PL^2/CEI},$$

in which L = the span and C is a constant having the value of 10 for hinged-end columns and 32 for fixed ends.

REINFORCED CONCRETE

Concrete is excellent material in compression and has great durability but it is weak in tension. Steel has high tensile strength but lacks durability when exposed to the atmosphere. Concrete-steel structures, in which tensile stresses are carried entirely by the steel, while concrete carries compression and at same time acts as a protective coating for steel, are ideal both as regards strength and permanency. Tension members should be all steel with concrete as a protective coating, massive compression members may be concrete only, but all members where bending is involved should be reinforced by steel bars which carry the total tensile stress. In the latter class are beams, floors, retaining walls, dams, and foundations.

Notation

Rectangular beams. (See Fig. 22.) S_s = tensile unit stress in steel. S_c = compressive unit stress in concrete. E_s = modulus of elasticity for steel. E_c = modulus of elasticity for concrete. $n = E_s/E_c$. M = moment of resistance or bending moment in general. M_s and M_c = resisting moments with respect to steel and concrete respectively. A_s = area of steel. b = breadth of beam. d = depth of beam to center of steel. k = ratio of depth of neutral axis to effective depth d. z = depth of the resultant compressive force below the top of the beam. j = ratio of the lever arm of the resisting couple to depth d. $jd = d - z$ = arm of resisting couple. p = steel ratio (not percentage). V = vertical shear. v = unit shearing stress.

T-beams (in addition to preceding). b = width of flange. b_1 = width of stem. t = thickness of stem.

Columns. A = total net area. A_s = area of longitudinal steel. A_c = area of concrete. P = total safe load.

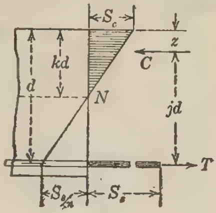


Fig. 22.

10. Reinforced-concrete beams

Steel reinforcement is placed in the tension flange to prevent failure by tension; more rarely in the compression flange to give the beam greater strength; the ends are reinforced by stirrups or by bending up the horizontal bars, or both, in order to prevent failure by diagonal tension produced by a combination of vertical and horizontal shear. Fig. 23 shows lines of maximum

Fig. 23.

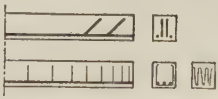


Fig. 24.

tensile stress in beams. If no reinforcement were present, the beam would tend to rupture along planes perpendicular to these lines, producing cracks as shown. Reinforcement must in general be parallel to these lines of stress. Fig. 24 shows methods of reinforcing the web against shear.

Formulas for beams

The assumptions are: perfect adhesion between steel and concrete, straight-line distribution of stress in concrete, tension in concrete neglected.

Rectangular beams. Position of neutral axis; $k = \sqrt{2pn + (pn)^2} - pn$ (**1**). Arm of resisting couple: $jd = d(1 - k/3)$; $j = 1 - k/3$ (**2**). Resisting moments: $M_s = S_s A_s jd = S_s p j b d^2$ (**3**). $M_c = S_c k j b d^2/2$ (**4**). Fiber stress for a given bending moment M is $S_s = M/A_s jd$ (**5**). $S_c = 2M/jkbd^2$ (**6**). $S_c/S_s = 2p/k$ (**7**). Steel ratio for a given stress is

$$p = \frac{1}{\dfrac{2S_s}{S_c}\left(\dfrac{S_s}{nS_c} + 1\right)} \quad (8).$$

For usual beams of 1 : 2 : 4 Portland-cement concrete, average values are, $j = 0.875$, $k = 0.375$, $p = 0.0077$, $S_s = 16{,}000$, $S_c = 650$, $n = 15$. In design, the breadth of beam b should be $0.5 \times d$ to $0.75 \times d$.

Beams that, when tested to failure, collapse by crushing concrete are said to be over-reinforced. If failure by tension occurs in the steel they are under-reinforced. If the beam is equally strong in tension and compres-

sion, that is, if stress is fully developed in both steel and concrete, the beam is said to be BALANCED. Table 12, calculated from formulas (1) to (8), shows that for balanced beams p, k, and j vary with S_s and S_c. A high value of S_c and low value of S_s demands a larger percentage of steel.

Table 12. Constants for balanced beams
(Portland-cement concrete, $n = 15$)

S_s	S_c	p	k	j	M/bd^2, in.-lb.
14,000....	500	0.0062	0.349	0.884	77.0
	600	0.0084	0.391	0.870	103.0
	650	0.0096	0.413	0.862	116.0
16,000....	500	0.0050	0.320	0.893	71.3
	600	0.0067	0.360	0.880	95.0
	650	0.0077	0.379	0.874	107.4
	700	0.0087	0.397	0.868	120.5
18,000....	550	0.0048	0.315	0.895	78.0
	600	0.0055	0.333	0.889	88.8
	650	0.0063	0.350	0.883	100.5
	700	0.0072	0.370	0.877	113.5

Example 1. Design a reinforced-concrete beam with 15-ft. span to carry a total uniform load of 400 lb. per linear ft.

Assume $S_s = 16{,}000$, $S_c = 650$. Hence $k = \frac{3}{8}$, $j = \frac{7}{8}$, $p = 0.0077$ (Table 12), $M = wl^2/8 = 400 \times 15^2 \times 12/8 = 135{,}000$ in.-lb. From Table 12, $M/bd^2 = 107.4$, hence $bd^2 = 135{,}000/107.4 = 1255$. It is now necessary to assume a ratio of b to d. Assume $b = d/2$ for the first approximation. Then $d^3 = 2510$ and $d = 13.6$ in. Making $d = 13.5$ in., $b = 7$ in., and $A_s = pbd = 0.727$ sq. in. Table 13 shows that two ⅝-in.

Table 13. Areas, weights, perimeters of bars

Size, inches	Round bars			Square bars		
	Area	Weight	Perimeter	Area	Weight	Perimeter
¼	0.049	0.167	0.785	0.062	0.213	1.000
⅜	0.110	0.376	1.178	0.141	0.478	1.500
½	0.196	0.668	1.571	0.250	0.850	2.000
⅝	0.307	1.043	1.964	0.391	1.328	2.500
¾	0.442	1.502	2.356	0.562	1.913	3.000
⅞	0.601	2.044	2.749	0.766	2.603	3.500
1	0.785	2.670	3.142	1.000	3.400	4.000
1⅛	0.994	3.380	3.534	1.266	4.304	4.500
1¼	1.227	4.172	3.927	1.562	5.313	5.000
1⅜	1.485	5.049	4.320	1.891	6.428	5.500
1½	1.767	6.008	4.712	2.250	7.650	6.000

(Areas in square inches, weights in pounds per linear foot; perimeter in inches)

square bars will meet this condition. Also four ½-in. round bars might be used. Economy demands a minimum number of bars, but if some bars are to be bent up for web reinforcement, more than two must be used. The quantity d is the depth to center of steel; a further depth, added for fire protection, must be figured to get the total depth. For d less than 10 in., add 1 in.; 10 to 20 in., add 1.5 in.; for d greater than 20 in., add 2 in. For slabs add from 0.75 in. to 1.25 in. The shearing strength of plain concrete is 40 lb. per sq. in. and the area resisting shear is taken as $7bd/8$. For this problem maximum shear is 3000 lb., hence the unit shearing stress $v = 3000/(0.875 \times 7 \times 13.5) = 36.4$ lb. per sq. in. and no web reinforcement is necessary.

Shearing stress and bond. Web stress is due to diagonal tension but the usual method is to consider vertical shear only. The safe unit shearing stress in 1 : 2 : 4 concrete is 40 lb. per sq. in. Stress is found from the formula $v = 8V/7bd$. Unit stress in excess of 40 lb. must be carried by stirrups or by bent up bars, or both; in the latter case the dimensions of the beam may be found from the formula $V = 120bjd$. This applies to short heavily-loaded beams for which shearing stress is the controlling factor in design. (See Example 2.) STRESSES IN STIRRUPS are found from the formula $P = mvsb = 1.14mVs/d$ (**9**), in which P = total load on stirrup, s = horizontal spacing, mV = shear to be carried by stirrups. For bars bent up at 45°, $P = 0.8mVs/d$ (**10**), in which s = horizontal projection of sloping part, and V = average shear in space s. BOND STRESS is given by the formula $u = 8V/7d\Sigma o$ (**11**), in which u = unit bond stress and Σo = the sum of the perimeters of all bars in tension reinforcement. A safe value for u is 80 lb. per sq. in. Maximum bond stress occurs at points where V is maximum; here tension steel should be run into the adjoining wall or section a sufficient distance to develop the required bond, or else bent over in the form of hook to give the required anchorage.

T-beams (Fig. 25). When the neutral axis lies in the flange use the same formulas as for rectangular beams. Removal of concrete below the neutral axis strengthens the beam by decreasing the weight. The area resisting shear is $7b_1d/8$, *i.e.*, the stem only is considered effective in shear. For independent T-beams, $b_1 \geqq b/3$, $t \gtreqless d/3$ (Fig. 25). T-beams are seldom balanced but designed from considerations of shear, headroom, and the like. When the stem joins a slab as part of a floor system, the portion of the slab with a width not exceeding one-fourth the span and not overhanging the stem more than $4t$ is considered to form the flange of a T-beam (Fig. 27).

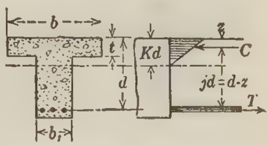


FIG. 25.

Formulas for true T-beams. The neutral axis lies in the stem and compression in the stem between axis and flange is neglected. POSITION OF NEUTRAL AXIS, $kd = (2ndA_s + bt^2)/(2nA_s + 2bt)$ (**12**). POSITION OF RESULTANT COMPRESSION, $z = \dfrac{t(3kd - 2t)}{3(2kd - t)}$ (**13**). ARM OF RESISTING COUPLE, $jd = d - z$. RESISTING MOMENT $M_s = S_sA_sjd$, $M_c = S_cbt(kd - t/2)jd/kd$ (**14**). FIBER STRESS $S_s = M/A_sjd$ (**15**); $S_c = Mkd/bt(kd - t/2)jd$ (**16**); $S_c/S_s = k/n(1 - k)$ (**17**).

Example 2. Design a continuous independent T-beam for a span of 16 ft. and load of 1000 lb. per lin. ft. (Fig. 26)

Since the beam is not homogeneous, positive and negative moments may be chosen with some latitude provided the sum is $WL/8$. The positive moment at the center of span is generally made $WL/12$, hence the negative moment at the supports is $WL/24$. In this case $+M$ = 256,000 in.-lb.; $-M$ = 128,000 in.-lb.; $V_{\max.}$ = 8000 lb. For a first approximation use $V = 120b_1jd$, which assumes bent-up bars and stirrups. This gives $b_1d = 76.2$ which is satisfied by a 6-in. × 13-in. stem. WIDTH OF FLANGE may be found by trial or by using values of Table 12 as approximations. Thus $bd^2 = 256{,}000/107.4 = 2390$, and b = 14 in. If the thickness of the flange is as much as ⅜ of d, the beam may be considered

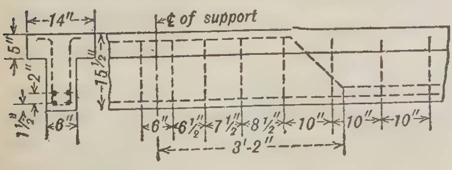


FIG. 26.—Continuous T-beam.

rectangular, hence make $t = 5$ in. The proportions of the beam are now determined except that b_1 may need to be increased to accommodate the reinforcing steel. From equation 5, or from Table 12, $A_s = 1.4$ sq. in. Conditions governing choice of bars are area and perimeter (bond unit stress is not to exceed 80 lb. per sq. in.). From Equation 11 the required perimeter is $8000 \times 8/(7 \times 13 \times 80) = 8.8$ in. These conditions are satisfied by two $\frac{5}{8}$-in. round and two $\frac{5}{8}$-in. square bars, necessitating a double row. (The practice of using both round and square bars is not generally resorted to unless a large number of similar beams or slabs is to be constructed, in which case the close figuring on steel is justified.) Maximum shear = 8000 lb. Concrete carries $6 \times 13 \times 40 = 3120$ lb., hence the shear to be carried by the stirrups = 4880 lb. Using Equation 9 and making the allowable shearing stress 10,000 lb. per sq. in., if $\frac{3}{8}$-in. square bars are used, the spacing at the end should be 6.5 in. Stirrups are extended into the point where unit shear = 40 lb. per sq. in., which in this case = $(3120/8000) \times 8 = 3.12$ ft. from center of span. The negative moment at the supports being half the maximum positive moment, two $\frac{5}{8}$-in. square bars are sufficient. The flange width (6 in.) is less than one-half the top width, but, due to the reinforcing steel, it is sufficient. Bars are generally bent up at the one-fifth points but this point should be checked by moments. Bend the bars up where they are no longer needed for tension reinforcement.

Example 3. To design a T-beam that is part of floor system; $L = 20$ ft., $M = 1{,}200{,}000$ in.-lb., $V = 20{,}000$ lb., $t = 5$ in. (Fig. 27.)

Knowing that the maximum shear is $V = 120 b_1 d \times \frac{7}{8}$, $b_1 d = 1900$. Assume $b_1 = d/2$ for the first trial, $d^2 = 380$; $d = 20$ in.; $b_1 = 10$ in. For trial solution use Table 12; $b = b_1 + 8t = 50$ in., $S_s = 16{,}000$, $S_c = 650$, $j = \frac{7}{8}$, $A_s = M/S_s jd = 4.28$ sq. in. Using this value of A_s in Equations 12 to 15, $k = 0.31$, $z = 1.94$, $j = 0.903$. Re-calculating with these values, $A_s = 4.15$, $k = 0.3$, $z = 1.91$, $j = 0.903$. The steel area may be made up of eight $\frac{3}{4}$-in. square bars giving 4.49 sq. in. Maximum bond stress (Equation 11) is 47.5 lb. per sq. in., which is satisfactory. Since $kd = 6.2$ in., the beam is a true T-beam. Applying Equations 12 to 15, $S_s = 14{,}800$, $S_c = 424$ lb. per sq. in. The steel must be placed in two rows of four bars each. The MINIMUM HORIZONTAL DISTANCE BETWEEN BARS is $3i$ where i is diameter of bar. Since the shear exceeds 40 lb. per sq. in., the bars must be bent up and stirrups used (Fig. 27). The bars are bent up in pairs and each pair is good for 15,600 lb. shear (Equation 10). Since the shear to be carried by the steel is $20{,}000 - (20 \times 10 \times 40 \times \frac{7}{8}) = 13{,}000$ lb., no stirrups are necessary in the space covered by the bent-up bars. They are put in nevertheless, at intervals of $\frac{3}{4}d$. For the remainder of the space, the interval between stirrups is to be calculated as Example 2.

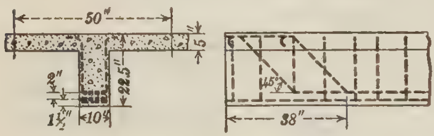


FIG. 27.—T-beam as part of floor system.

11. Reinforced-concrete columns

Reinforced-concrete columns may have longitudinal steel only, or longitudinal steel combined with circular hoops or spirals. The formula for SAFE LOAD is: $P = S_c(A_c + nA_s) = S_cA[1 + (n - 1)p]$.

Safe values are $S_c = 450$, $n = 15$, $p = 0.01$ to 0.06. The column length should not be more than 12 to 15 diameters and the concrete should be rich, 1 : 2 : 4 or better. In figuring the area of the concrete, only that within the steel should be considered; the outside shell or fire-proofing should be neglected. No advantage obtains from increasing the steel percentage above 5 or 6 since the steel can only be stressed to nS_c; with the values above, 6750 lb. per sq. in. Hoop or spiral reinforcement in addition to longitudinal steel adds considerably to the resisting strength of concrete; S_c may be taken as 650 lb. per sq. in. The area of longitudinal steel alone is to be considered in figuring the load.

Columns of rigid structural shapes encased in concrete, in which the cross-section of the steel exceeds 4 per cent., may be considered equal in strength to that of the steel column alone, figured by the usual methods, plus the strength of the concrete within the steel shapes, figured with S as 75 per cent. of normal value. The strength of the outer shell must be neglected.

Example. Design a circular reinforced column 15 ft. long to carry 100,000 lb. Assume $p = 0.02$, then $P = 450A(1 + 0.28)$; $A = 100{,}000/57 = 174$, $d = 14.9$ in. Area of steel $= 0.02 \times 174 = 3.56$. Use eight $\frac{3}{4}$-in. round rods and make the outside diameter 18 in.

BUILDING CONSTRUCTION

12. Foundations

Bearing power of soils varies widely with the nature of the soil and the load. It depends on depth of foundation, which should always extend well below frost line; on lateral confinement or inability of the soil to flow sideways, on moisture content, inclination of strata, and on distribution of pressure. For important work it is advisable to dig a test pit, apply a known load to a given area, and observe settlement. Table 14 is summarized from a number of city building codes and from typical practice not so limited.

Table 14. Safe bearing power of soils

Nature of soil	Safe load, tons per sq. ft.	
	Minimum	Maximum
Hard rock in thick beds in place	200	
Rock, well-consolidated sedimentary	15	30
Rock, soft sedimentary, or badly broken harder rock	5	10
Clay, dry, sandy	4	6
Clay, moderately dry	2	4
Clay, soft	1	2
Gravel and coarse sand, well cemented	8	10
Sand, compact and well cemented	4	6
Sand, clean dry	2	4
Quicksand, alluvial soils, etc	0.5	1

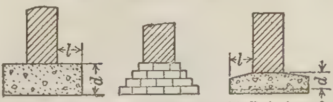
Rock will generally carry any load that can be brought upon it by masonry or concrete footings. It should be leveled off to prevent slip and all soft spots should be filled with concrete.

Clay soils vary greatly in bearing power. Slate or shale will carry any load while soft wet clay will flow or squeeze out under moderate loads. Drainage greatly improves bearing power.

Sand and gravel in thick beds, protected from the action of running water, make good foundation and may carry 6 to 10 tons per sq. ft.

Semi-liquid soils, such as quicksand and alluvium, have little bearing power and should not be relied on, if it is possible to remove them or to drive piles to a more solid sub-stratum. They can, however, support a considerable load due to hydrostatic pressure, which increases with depth.

Soil increases in compactness and bearing power with depth and is generally improved by drainage. Some settlement always occurs, and unit bearing pressures should be low. Structures should be designed for uniform settlement, hence the unit pressure on the base should be constant. This can be accomplished only by having the resultant load pass through the center of area of the base, otherwise eccentric loading occurs with consequent unequal pressures and settlement. Uniformity of pressure is facilitated by making foundation of each wall and pier separate and proportional in area to the load to be carried. This rule is not applied to dams and retaining walls, but if neglected in the case of buildings, uneven settlement may cause dangerous cracks. Compactness of clay soils may be increased by driving small piles 6 in. diameter and 6 ft. long at intervals of two to four ft. This method is not effective in sand and sandy soils since these are incompressible.



(a) Concrete (b) Brick (c) Reinforced concrete

FIG. 28.—Footings.

Footings are designed to spread a concentrated load over sufficient area so that the load per square foot falls within allowable limits. See Fig. 28. Ratio of offset l to depth d is given in Table 15. In general $l = d\sqrt{R/pf}/6$ in which R = modulus of rupture in lb. per sq. in., p = pressure in tons per sq. ft., f = factor of safety.

Timber and reinforced-concrete footings are designed as beams, the offset l being a cantilever beam under uniform load. When l is large, great masses of masonry may be avoided by the use of a grillage of wood or steel beams, or a reinforced-concrete slab with reinforcement in both directions.

Table 15. Safe offset for masonry footing courses

Kind of stone	R	p in tons per sq. ft.		
		0.5	1.0	2.0
Bluestone	5026	5.2	3.7	2.7
Granite	1844	3.2	2.2	1.6
Limestone	1377	2.8	2.0	1.4
Sandstone	1378	2.8	2.0	1.4
Good brick, natural cement mortar, after 60 days	120	0.8	0.6	0.4
Concrete, 1: 2: 4 (Portland)	400	1.6	1.1	0.7
Timber	1000	7.5	5.3	3.7

Pile foundations. Timber piles are used extensively; when below water level they last indefinitely. Piles should be 6 to 8 in. at the small end and preferably not over 14 in. at the butt. If the pile bears on hard rock it should be figured as a column, to obtain the safe load. Ordinary piles depend on friction for bearing power. The ENGINEERING NEWS FORMULAS for safe load are:

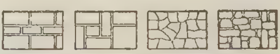
FOR PILE DRIVEN WITH DROP HAMMER, $P = 2wh/(s + 1)$.

FOR PILE DRIVEN WITH STEAM HAMMER, $P = 2wh/(s + 0.1)$ in which P = safe load, lb.; w = weight of hammer, lb.; s = penetration under last blow in inches. Piles should not be closer than 2.5 ft. center to center. The average allowable load is about 12 tons per pile; maximum, 20 tons.

Concrete piles are both cast-and-driven, and cast in place. They cost from four or five times as much as timber piles and carry 25 to 35 tons. They are commonly jet driven.

13. Masonry construction

Stone should be strong, durable and cheap; dense, hard, and uniform, and show a clean, sharp fracture free from loose grains. Stonework can be divided into two general classes, viz.: ashlar and rubble. ASHLAR MASONRY consists of stones of rectangular shape and may be further classified as coursed range and broken range (Fig. 29). Good ashlar is laid with 1 : 2 cement mortar



Coursed ashlar — Broken ashlar — Rubble (Dressed joints) — Rubble (Undressed joints)

FIG. 29.—Ashlar and rubble masonry.

Table 16. Safe working stresses for masonry construction

Kind of stone	Pounds per square inch	Tons per square foot
Granite ashlar	700	50
Limestone, hard	650	47
Marble ashlar	600	43
Soft limestone and sandstone ashlar	500	36
Rubble in Portland cement	250	18
Rubble in lime cement	140	10
Brickwork in Portland cement	350	25
Brickwork in natural cement	250	18
Brickwork in lime mortar	140	10
Concrete, Portland, 1 : 2 : 4	400	29
Concrete, Portland, 1 : 3 : 6	350	25
Concrete, natural, 1 : 2 : 5	150	10

with joints not more than ½ in. wide. The safe bearing values given in Table 16 are based on this assumption. RUBBLE MASONRY is built up of rough stones laid in mortar, the latter constitutes 20 to 50 per cent. of the total mass. Dry rubble is laid without cementing material. RUBBLE CONCRETE consists of concrete in which large stones are imbedded. RIPRAP is loose stone piled around piers and on earth to prevent wash.

The largest stones in walls should be at the bottom, individual stones laid on the broadest face; if stratified, bedding planes should be normal to stress. Porous stone should be wet before being laid. All voids should be filled with cement or mortar.

Brickwork. Strong brickwork is built of hard, well-burned brick and laid up with MORTAR composed of one part Portland cement, one part lime paste, and three parts sand. AVERAGE MORTAR consists of 1 cement, 1 lime, 6 sand. COMMON BRICK is generally $8\frac{1}{4} \times 4 \times 2\frac{1}{4}$ in. and a deep salmon color. When broken it should show fine, uniform texture and give a clear ringing sound when struck. Soft or under-burned brick is light in color and should not be used in heavy structures. Good brickwork may safely carry 20 tons per sq. ft., if laid in 1 : 3 cement mortar, and 8 tons per sq. ft. if laid in lime mortar (1 lime, 6 sand). Isolated piers of brickwork, because of increase of stress due to column action and the possibility of eccentric loading, should have unit loads of only 60 to 75 per cent. of these values. In LAYING BRICK, the structure must be tied together by alternate use of stretchers and headers. Common types of bond are shown in Fig. 30.

FIG. 30.—Bonds used in brickwork.

14. Retaining walls and dams

Retaining walls are designed to resist earth pressure. They are built of rubble masonry, rubble concrete, and reinforced concrete. Gravity walls depend upon their weight to resist sliding and the overturning effect of pressure on their backs, the only forces acting being the earth pressure and the weight of the wall.

If Fig. 31, let W = weight of wall in pounds per lin. ft., E = earth pressure in pounds per lin. ft., and R = resultant force found by combining W and E.

Failure may occur in three ways; (1) by sliding along base (AB) (2) by overturning about the toe at A; (3) by crushing the wall or the foundation at A. Sliding is prevented by friction between the wall and the ground. Gravity walls are generally safe against sliding if conditions (2) and (3) are provided for. Light reinforced-concrete walls demand special investigation and usually have a projection on the base (Fig. 34). The wall will not overturn provided the resultant falls within base AB. Only small walls are in danger of failure by overturning; in large walls, the masonry will crush at the toe before overturning. CRUSHING AT THE TOE is prevented by keeping unit loads within the safe bearing value. Pressure at the base varies uniformly from A to B (Fig. 31), due to eccentric loading.

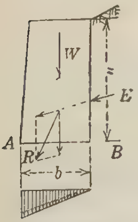


FIG. 31.

MAXIMUM UNIT LOAD (Art. 7) is given by $S = P(1 + 6e/b)/b$ in which P = vertical component of resultant pressure in lb. per lin. ft., b = width of base in ft., e = eccentricity, or distance from center of base to point where R cuts base line. The usual requirement is that the resultant pressure fall within

the middle third of the base. This insures that the maximum unit pressure will not exceed twice the average pressure (Fig. 18*b*) and no tension will exist at the heel.

Earth pressure. Lateral pressure due to the weight of earth varies directly as the depth, as in the case of water pressure. (Art. 20.) The resultant acts one-third up from the base. If the wall is surcharged, the resultant is assumed parallel with the surface (Fig. 32). The general formula is $E = kwh^2/2$ in which E = earth pressure, lb. per lin. ft.; h = height of wall, ft.; w = weight of filling per cu. ft.; k = a constant which is independent of the weight and height but depends on the angle of repose (or friction) of the material and the angle of surcharge. Rankine gives the value of k as

$$k = \cos\theta \frac{\cos\theta - \sqrt{\cos^2\theta - \cos^2\phi}}{\cos\theta + \sqrt{\cos^2\theta - \cos^2\phi}}.$$

in which θ = angle of surcharge and ϕ = angle of repose.

Table 17. Slope of repose and weights for loose earth

Kind of earth	Slope of repose	Angle of repose	Weight, lb. per cu. ft.
Sand, clean........	1.5 to 1	33° 41′	90
Sand and clay......	1.33 to 1	36° 53′	100
Clay, dry..........	1.33 to 1	36° 53′	100
Clay, damp, plastic.	2 to 1	26° 34′	100
Gravel, clean......	1.33 to 1	36° 53′	100
Soil...............	1.33 to 1	36° 53′	100
Rock, soft rotten...	1 to 1	45°	110
Cinders...........	1 to 1	45°	45

Table 18. Values of coefficient k. (*After Rankine*)

Value of θ	Angle of repose, ϕ degrees								
	0	10	15	20	25	30	35	40	45
0	1.00	0.71	0.59	0.49	0.40	0.33	0.27	0.22	0.17
10	....	0.99	0.65	0.53	0.41	0.35	0.28	0.23	0.18
15	....	....	0.97	0.60	0.47	0.38	0.30	0.24	0.19
20	....	....	....	0.94	0.54	0.42	0.32	0.25	0.20
25	....	....	....	....	0.91	0.50	0.38	0.27	0.21
30	....	....	....	....	....	0.87	0.44	0.31	0.23
35	....	....	....	....	....	....	0.82	0.39	0.27
40	....	....	....	....	....	....	....	0.77	0.34
45	....	....	....	....	....	....	....	....	0.71

Design. Small rectangular walls are often built without computation. The base is made $0.4h$ to $0.6h$, where h = height in ft. If frost is likely, the wall should extend at least four feet below the ground surface. Trapezoidal walls are more economical than rectangular. The back of the wall is often built in steps because of greater ease in constructing forms; the face generally has a batter of 1 in 12 to prevent forward settlement due to uneven pressure distribution. The base of trapezoidal walls is made $0.4h$ to $0.6h$ and the top width 1.5 to 3 ft., the larger value for walls with surcharge. Adequate drainage should be provided. The usual method in design is to lay out the wall and

then investigate for stability. Many methods are used, the following example illustrates a common method.

Example. To design a concrete retaining wall with 16 ft. clear height to withstand the pressure of soil weighing 100 lb. per cu. ft. and having an angle of repose of 37°; surcharge angle = 15°.

The wall weighs 150 lb. per cu. ft. Assuming frost likely, the total height will be 20 ft. Width of base is determined by the allowable bearing pressure at the toe, the softer the foundation, the wider the base. Assume $b = 0.45h = 9$ ft. Take top width as 2 ft. Batter of face is 1 : 12. With these data lay out wall as in Fig. 32. Draw vertical line BC. Assume earth pressure to act on this line one-third up from B, as though line BC were the back of the wall. The weight of the triangle of earth BCD is combined with the earth pressure. The resultant of these two forces combined with the weight of the wall gives the final resultant. From Table 18, interpolating for $\phi = 37$ and $\theta = 15$, the value of k is 0.276, hence earth pressure $E = kwh^2/2 = 0.276 \times 100 \times 20^2/2 = 5520$ lb. per linear ft. of wall. The weight of triangle BCD and the position of its center of gravity are found by scaling from Fig. 32; area = 54.3 sq. ft., hence weight = 5430 lb. per linear ft. The resultant of this force and the earth pressure = 8740 lb. The weight of the wall at 150 lb. per cu. ft. = 16,500 lb. per linear ft. See Sec. 26, Art. 13, for center of gravity of wall. Final resultant is 23,800 lb., cutting the base 1.2 ft. from the center; vertical component $P = 22{,}800$. Maximum unit pressure $= P(1 + 6e/b)/b = 22{,}800 \times 1.8/9 = 4560$ lb. per sq. ft. Table 14 shows this to be a conservative value for moderately dry soil. The fact that the resultant falls within the middle third of the base assures absence of tension at the heel. The horizontal component of the resultant is 5500 lb. The coefficient of friction for concrete on dry clay and sand varies from 0.4 to 0.5, hence the factor of safety against sliding is 1.6 to 2.1 which is satisfactory.

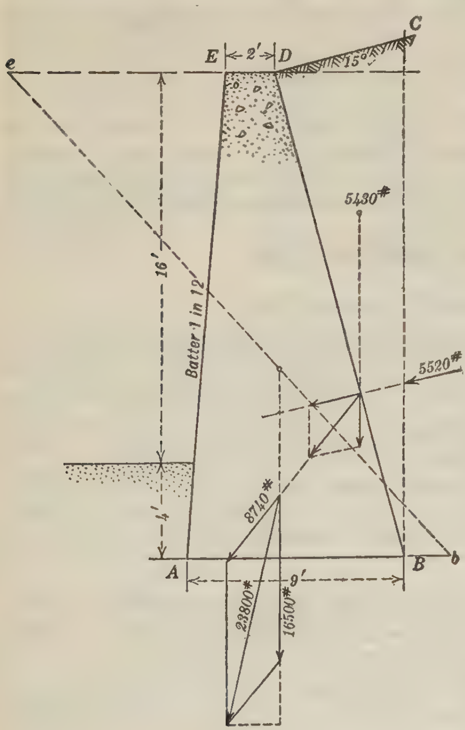


FIG. 32.

FIG. 33.

FIG. 34.

Some designers consider earth pressure to act horizontally instead of parallel to the surface; this increases the overturning moment and hence is on the side of safety. Offsets for foundations are often specified. Fig. 33 shows a conventional type of wall with offset foundation, stepped back, and drainage system.

Masonry dams are designed to meet the same conditions as retaining walls, and in addition the requirement that the foundation must be impermeable. Water pressure in lb. per ft. of run is $P = wh^2/2 = 31.25h^2$, where h is the total head in feet. P acts one-third up from the base. The foundation should be

on rock and impervious; if water seeps under the dam or into any horizontal joint, it will exert hydrostatic pressure upward and reduce the effective weight of the masonry, hence cut-off walls are built at the heel and the foundation is drained. Some specifications provide for hydrostatic pressure by assuming full static head at the heel, diminishing uniformly to zero at the toe, the resultant upward pressure is then one-half the total static pressure and acts one-third the distance from heel to toe. For masonry dams under 30 ft. in height a trapezoidal section is economical. Top width should be 4 to 5 ft., height such as to be three or more feet above high water, back with slight batter, and base such that the resultant force falls within the middle third. (See Retaining walls.) Safe bearing pressure for rubble or concrete dam on rock foundation is 10 to 12 tons per sq. ft. Typical small dam sections are shown in Fig. 35.

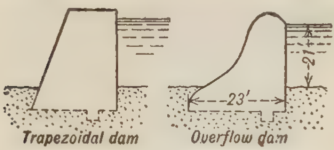


FIG. 35.—Typical dam sections.

STEEL CONSTRUCTION

15. Strength of iron and steel

Cast iron, wrought iron, and steel are alloys of iron and carbon which also contain manganese, silicon, sulphur, and phosphorus. The percentage of carbon has the determining influence on the relative properties of the materials.

Cast iron contains 2 to 4 per cent. CARBON and varying amounts of other elements. Its TENSILE STRENGTH varies from 18,000 to 20,000 lb. per sq. in. and COMPRESSIVE STRENGTH averages 90,000. It has poorly defined ELASTIC LIMIT and no YIELD POINT. MODULUS OF ELASTICITY is 12,000,000 to 18,000,000 lb. per sq. in. Resilience and work to rupture are low hence it has small RESISTANCE TO SHOCK.

As a STRUCTURAL MATERIAL cast iron is unreliable, due to the possibility of hidden defects such as blow holes, cracks, and to internal stresses due to cooling; its use is often prohibited. It is used for a great variety of machine parts, pipe under low pressure, fittings, and short columns under steady load.

Malleable cast iron is made by slow annealing of hard white-iron castings packed in iron oxide. This process de-carbonizes the iron and produces a malleable material with TENSILE STRENGTH of 40,000 lb. per sq. in. and ELONGATION 8 to 10 times that of cast iron.

Malleable iron is used for pipe fittings and small machine parts, but because de-carbonization extends but a short distance below the surface its use is limited to small castings.

Wrought iron is made without fusion; has carbon content 0.05-0.3 per cent.; is tough, ductile, malleable, easily welded but not readily fusible. It shows a fibrous structure on fracture. Its ULTIMATE STRENGTH in tension is 40,000 to 50,000 lb. per sq. in., shear 40,000; MODULUS OF ELASTICITY, 28,000,000.

It is tougher and resists CORROSION better than soft steel but the latter has entirely superseded it for STRUCTURAL WORK due to its lower cost and greater strength.

Steel is an iron-carbon alloy intermediate in composition between cast iron and wrought iron but has higher specific gravity and strength than either. It is made chiefly by three processes, the crucible, open-hearth, and Bessemer;

the quality and relative cost decrease in the order named. Strength is a function chiefly of the carbon content, a rough rule is: $S_t = 45{,}000 + 108{,}000\, C$, in which S_t is tensile strength of un-annealed acid open-hearth steel and C is the carbon content in per cent. A general classification according to carbon content is as follows:

Soft, 0.05–0.20 per cent. C, not temperable, easily welded; medium, 0.15–0.40 per cent. C, poor temper, weldable; hard, 0.30–0.70 per cent. C, good temper, not easily welded; very hard, 0.60–1.00 per cent. C., high temper, not weldable.

Table 19. Average properties of steel

Kind of steel	Elastic limit	Tensile strength	Elongation, per cent	Remarks
Structural-steel rivets. .	30,000	55,000	30	Modulus of elasticity for all grades, 30,000,000; shearing modulus, 10 to 15 million
Structural, rolled shapes	35,000	60,000	27	
Machinery steel.	40,000	75,000	20	
Axle steel.	60,000	100,000	15	
Spring steel.	60,000	125,000	12	
Cable-wire steel.	100,000	200,000	8	

Strength increases and elongation decreases with carbon content. Roughly the percentage of elongation $= 1{,}500{,}000/S_t$.

Nickel steel contains about 3.5 per cent. nickel; it is used to a small extent for structural purposes. Heat-treated nickel steel may have tensile strength in excess of 200,000. Structural nickel steel has an elastic limit 15 per cent. higher and ultimate strength 25 per cent. higher than ordinary structural steel.

16. Steel structures

Steel structures are fabricated of standard rolled-steel shapes. Properties of selected I-beams, channels, and angles, are given in Tables 21 to 24. Safe rules for design follow:

Loads. For structures carrying moving loads add 25 per cent. to the stresses caused by said loads; for wind pressure, allow 20 lb. per sq. ft. on the vertical projection of all surfaces exposed to wind; allow 25 lb. per sq. ft. for snow for slopes under 20°; for other slopes use one pound less for each degree of slope above 20°.

Safe unit stresses for the sum of dead, live, impact, and wind stresses are as follows:

	Lb. per sq. in.
Tension, net section.	16,000
Direct compression.	16,000
Bending on extreme fibers of rolled shapes.	16,000
Shear on pins and shop rivets.	12,000
Shear on bolts and field rivets.	10,000
Shear on rolled sections.	10,000
Bearing pressure on shop rivets and pins.	24,000
Bearing pressure on field rivets and bolts.	20,000

Axial compression allowable on gross section of columns for ratio of l/r up to 120 = $19{,}000 - 100(l/r)$ lb. per sq. in., where l = effective length of member and r is radius of gyration of section, both in inches. For values of l/r in excess of 120 use the following quantities:

Ratio (l/r).	130	140	150	160	170	180	190
Quantity.	6500	6000	5500	5000	4500	4000	3500

Compression members in the main structure should have a length not in excess of 120 × *least radius of gyration;* for secondary members this ratio should not exceed 200.

Beams and girders should not have an unsupported length in excess of 40 × *width of compression flange.*

Riveting. Mininum distance between rivet holes should be 3 diameters. For built-up shapes the maximum pitch (spacing) in line of stress is 6 in. for ⅞-in. rivets, 5 in. for ¾-in., 4½ in. for ⅝-in., and 4 in. for ½-in. rivets.

Standard beam connections are given in Fig. 36 and STANDARD GAUGES FOR ANGLES in Table 20. PLATES for connecting structural members and size

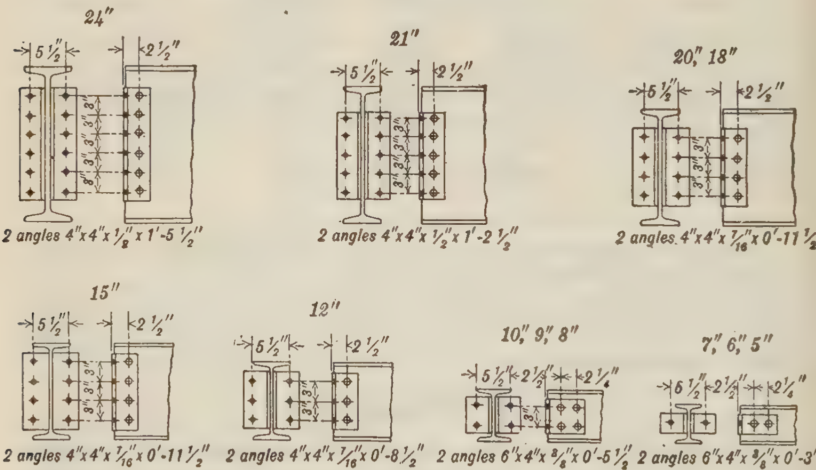
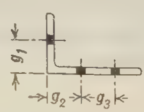


FIG. 36.—Standard beam connections.

Table 20. Gages for angles, inches

Leg	8	7	6	5	4	3½	3	2½	2	1¾	1½	1⅜	1¼	1	¾
g_1	4½	4	3½	3	2½	2	1¾	1⅜	1⅛	1	⅞	⅞	¾	⅝	½
g_2	3	2½	2½	2	...	...	...	...	...	...	...	...	...	...	...
g_3	3	3	2¼	1¾	...	...	...	...	...	...	...	...	...	...	...
Maximum rivet.....	1⅛	1	⅞	⅞	⅞	⅞	⅞	¾	⅝	½	⅜	⅜	⅜	½	½



and number of rivets should be designed for shearing strength of rivets and bearing strength of plates and webs of members to be connected. See Art. 4.

Properties of selected I-beams are given in Table 21.

I is moment of inertia, r is radius of gyration, S is section modulus = (I/c); coefficient of strength is the product of the span in ft. and the safe uniformly-distributed load in lb. For concentrated load at center of span divide coefficient by two. Axis 1–1 is perpendicular to the web at the center, axis 2–2 is coincident with the web.

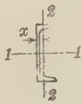
Example. Select an I-beam with a span of 20 ft. to carry a uniformly-distributed load of 10,000 lb.

(a) By calculation, the moment, $M = WL/8 = 10{,}000 \times 20 \times 12/8 = 300{,}000$. Section modulus (S) = moment/stress = 300,000/16,000 = 18.75. From Table 21 a 9-in. beam weighing 21 lb. per ft. is the lightest section that fulfills the requirements.

Table 21. Properties of selected standard I-beams (*Carnegie*)

Depth, inches	Weight per foot, pounds	Area of section, square inches	Width of flange, inches	Thickness of web, inches	Axis 1-1 *I* In.⁴	Axis 1-1 *r* In.	Axis 1-1 *S* In.³	Axis 2-2 *I* In.⁴	Axis 2-2 *r* In.	Axis 2-2 *s* In.³	Coefficient of strength for unit-stress = 16,000 lb. per square inch, axis 1-1
24	110	32.48	7.938	0.688	2883.5	9.42	240.3	81.0	1.58	20.4	2,563,000
	100	29.41	7.254	0.754	2379.6	9.00	198.3	48.6	1.28	13.4	2,114,000
	90	26.47	7.131	0.631	2238.4	9.20	186.5	45.7	1.31	12.8	1,989,000
	80	23.32	7.000	0.500	2087.2	9.46	173.9	42.9	1.36	12.3	1,855,000
20	95	27.94	7.210	0.810	1606.6	7.58	160.7	50.8	1.35	14.1	1,714,000
	85	25.00	7.063	0.663	1508.5	7.77	150.9	47.3	1.37	13.4	1,609,000
	75	22.06	6.399	0.649	1268.8	7.58	126.9	30.3	1.17	9.5	1,353,000
	65	19.08	6.250	0.500	1169.5	7.83	117.0	27.9	1.21	8.9	1,248,000
18	85	25.00	7.163	0.725	1220.7	6.99	135.6	50.0	1.42	14.0	1,446,000
	75	22.05	7.000	0.562	1141.3	7.19	126.8	46.2	1.45	13.2	1,353,000
	65	19.12	6.177	0.637	881.5	6.79	97.9	23.5	1.11	7.6	1,045,000
	55	15.93	6.000	0.460	795.6	7.07	88.4	21.2	1.15	7.1	943,000
15	70	20.59	6.194	0.784	663.7	5.68	88.5	29.0	1.19	9.4	944,000
	60	17.67	6.000	0.590	609.0	5.87	81.2	26.0	1.21	8.7	866,000
	50	14.71	5.648	0.558	483.4	5.73	64.5	16.0	1.04	5.7	687,000
	42	12.48	5.500	0.410	441.8	5.95	58.9	14.6	1.08	5.3	628,000
12	50	14.71	5.489	0.699	303.4	4.54	50.6	16.1	1.05	5.9	539,000
	40	11.84	5.250	0.460	269.0	4.77	44.8	13.8	1.08	5.3	478,000
	35	10.29	5.086	0.436	228.3	4.71	38.0	10.1	0.99	4.0	406,000
	31.5	9.26	5.000	0.350	215.8	4.83	36.0	9.5	1.01	3.8	384,000
10	40	11.76	5.099	0.749	158.7	3.67	31.7	9.5	0.90	3.7	339,000
	35	10.29	4.952	0.602	146.4	3.77	29.3	8.5	0.91	3.4	312,000
	30	8.82	4.805	0.455	134.2	3.90	26.8	7.7	0.93	3.2	286,000
	25	7.37	4,660	0.310	122.1	4.07	24.4	6.9	0.97	3.0	261,000
9	35	10.29	4.772	0.732	111.8	3.29	24.8	7.3	0.84	3.1	265,000
	25	7.35	4.446	0.406	91.9	3.54	20.4	5.7	0.88	2.5	218,000
	21	6.31	4.330	0.290	84.9	3.67	18.9	5.2	0.90	2.4	201,000
8	25	7.50	4.271	0.541	68.4	3.02	17.1	4.8	0.80	2.2	183,000
	20.5	6.03	4.087	0.357	60.6	3.17	15.2	4.1	0.82	2.0	162,000
	18	5.33	4.000	0.270	56.9	3.27	14.2	3.8	0.84	1.9	152,000
7	20	5.88	3.868	0.458	42.2	2.68	12.1	3.2	0.74	1.7	129,000
	17.5	5.15	3.763	0.353	39.2	2.76	11.2	2.9	0.76	1.6	119,000
	15	4.42	3.660	0.250	36.2	2.86	10.4	2.7	0.78	1.5	110,000
6	17.25	5.07	3.575	0.475	26.2	2.27	8.7	2.4	0.68	1.3	93,100
	14.75	4.34	3.452	0.352	24.0	2.35	8.0	2.1	0.69	1.2	85,000
	12.25	3.61	3.330	0.230	21.8	2.46	7.3	1.9	0.72	1.1	78,000
5	14.75	4.34	3.294	0.540	15.2	1.87	6.1	1.7	0.63	1.0	65,000
	9.75	2.87	3.000	0.210	12.1	2.05	4.8	1.2	0.65	0.82	51,300
4	10.5	3.09	2.880	0.410	7.1	1.52	3.6	1.0	0.57	0.70	38,400
	7.5	2.21	2.660	0.190	6.0	1.64	3.0	0.77	0.59	0.58	32,000
3	7.5	2.21	2.521	0.361	2.9	1.15	1.9	0.60	0.52	0.48	20,700
	5.5	1.63	2.230	0.170	2.5	1.23	1.7	0.46	0.53	0.40	17,600

Table 22. Properties of standard channels (*Carnegie*)

Depth of channel, inches	Weight per foot, pounds	Area of section, square inches	Width of flange, inches	Thickness of web, inch	Axis 1-1 			Axis 2-2, r, inch	x, inch	Distance back-to-back for equal radii of gyration	Coefficient of strength for fiber stress of 16,000 pounds per square inch, foot-pounds
					I, inches^4	r, inches	s, inches^3				
	55.0	16.18	3.818	0.818	430.2	5.16	57.4	0.87	0.82	8.53	612,000
	50.0	14.71	3.720	0.720	402.7	5.23	53.7	0.87	0.80	8.72	573,000
15	45.0	13.24	3.622	0.622	375.1	5.32	50.0	0.88	0.79	8.92	533,000
	40.0	11.76	3.524	0.524	347.5	5.43	46.3	0.89	0.78	9.16	494,000
	35.0	10.29	3.426	0.426	319.9	5.58	42.7	0.91	0.79	9.42	455,000
	33.0	9.90	3.400	0.400	312.6	5.62	41.7	0.91	0.79	9.51	445,000
	40.0	11.76	3.418	0.758	196.9	4.09	32.8	0.75	0.72	6.60	350,000
	35.0	10.29	3.296	0.636	179.3	4.17	29.9	0.76	0.69	6.83	319,000
12	30.0	8.82	3.173	0.513	161.7	4.28	26.9	0.77	0.68	7.06	287,000
	25.0	7.35	3.050	0.390	144.0	4.43	24.0	0.79	0.68	7.35	256,000
	20.5	6.03	2.940	0.280	128.1	4.61	21.4	0.81	0.70	7.68	228,000
	35.0	10.29	3.183	0.823	115.5	3.35	23.1	0.67	0.70	5.18	246,000
10	30.0	8.82	3.036	0.676	103.2	3.42	20.7	0.67	0.65	5.41	220,000
	25.0	7.35	2.889	0.529	91.0	3.52	18.2	0.68	0.62	5.66	194,000
	20.0	5.88	2.742	0.382	78.7	3.66	15.7	0.70	0.61	5.96	168,000
	15.0	4.46	2.600	0.240	66.9	3.87	13.4	0.72	0.64	6.33	143,000
	25.0	7.35	2.815	0.615	70.7	3.10	15.7	0.64	0.62	4.83	168,000
9	20.0	5.88	2.652	0.452	60.8	3.21	13.5	0.65	0.59	5.14	144,000
	15.0	4.41	2.488	0.288	50.9	3.40	11.3	0.67	0.59	5.48	121,000
	13.25	3.89	2.430	0.230	47.3	3.49	10.5	0.67	0.61	5.62	112,000
	21.25	6.25	2.622	0.582	47.8	2.77	11.9	0.60	0.59	4.22	127,000
8	18.75	5.51	2.530	0.490	43.8	2.82	11.0	0.60	0.57	4.37	117,000
	16.25	4.78	2.439	0.399	39.9	2.89	10.0	0.61	0.56	4.53	106,000
	13.75	4.04	2.347	0.307	36.0	2.98	9.0	0.62	0.56	4.72	96,000
	11.25	3.35	2.260	0.220	32.3	3.11	8.1	0.63	0.58	4.92	86,000
	19.75	5.81	2.513	0.633	33.2	2.39	9.5	0.56	0.58	3.49	101,000
	17.25	5.07	2.408	0.528	30.2	2.44	8.6	0.57	0.56	3.65	92,000
7	14.75	4.34	2.303	0.423	27.2	2.50	7.8	0.57	0.54	3.82	83,000
	12.25	3.60	2.198	0.318	24.2	2.59	6.9	0.58	0.53	4.00	74,000
	9.75	2.85	2.090	0.210	21.1	2.72	6.0	0.59	0.55	4.21	64,000
	15.5	4.56	2.283	0.563	19.5	2.07	6.5	0.53	0.55	2.90	69,000
6	13.0	3.82	2.160	0.440	17.3	2.13	5.8	0.53	0.52	3.08	62,000
	10.5	3.09	2.038	0.318	15.1	2.21	5.0	0.53	0.50	3.29	54,000
	8.0	2.38	1.920	0.200	13.0	2.34	4.3	0.54	0.52	3.51	46,000
	11.5	3.38	2.037	0.477	10.4	1.75	4.2	0.49	0.51	2.35	44,000
5	9.0	2.65	1.890	0.330	8.9	1.83	3.6	0.49	0.48	2.57	38,000
	6.5	1.95	1.750	0.190	7.4	1.95	3.0	0.50	0.49	2.79	32,000
	7.25	2.13	1.725	0.325	4.6	1.46	2.3	0.46	0.46	1.88	24,000
4	6.25	1.84	1.652	0.252	4.2	1.51	2.1	0.45	0.46	1.96	22,000
	5.25	1.55	1.580	0.180	3.8	1.56	1.9	0.45	0.46	2.08	20,000
	6.0	1.76	1.602	0.362	2.1	1.08	1.4	0.42	0.46	1.10	15,000
3	5.0	1.47	1.504	0.264	1.8	1.12	1.2	0.42	0.44	1.17	13,000
	4.0	1.19	1.410	0.170	1.6	1.17	1.1	0.42	0.44	1.29	12,000

(b) By use of coefficient of strength, multiply the load by the span in ft., 10,000 × 20 = 200,000, then select a 9-in. @ 21-lb. I-beam, coefficient of strength = 201,000.

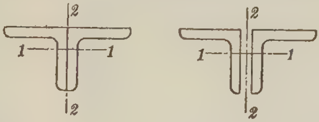
Properties of channels are given in Table 22. The symbols are the same as Table 21 except that x is the distance from the neutral axis to the back face of the web. Two channels back to back and connected with lattice bars is a common type of column. The distance back-to-back of channels should equal or exceed the tabular to develop the full strength of the column.

Example. Find the safe load for a column built of two 12-in. @ 25-lb. channels spaced 7½ in. back-to-back and properly latticed, if the unsupported length is 36 ft.

Since the distance back-to-back exceeds the value in Table 22, the least radius of gyration is about the axis 1–1 and is 4.43. Load $P = A(19{,}000 - 100(l/r)) = 14.70 \times \left(19{,}000 - \frac{100 \times 36 \times 12}{4.43}\right) = 136{,}000$ lb.

Properties of angles. Tables 23 and 24 give the properties of selected standard angles and radii of gyration for two angles separated by varying thicknesses of connecting plates. This latter arrangement is the usual form of member for roof trusses and light bridges; angles with unequal legs give more economical arrangement because the radii of gyration about axes 1–1 and 2–2 are more nearly equal.

Table 23. Properties of equal-leg angles

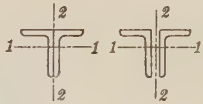


Single angle: Size, inches	Single angle: Weight, pounds per foot	Two angles, area, square inches	Radii of gyration, inches: Axis 1–1	Axis 2–2: In contact	Axis 2–2: ¼ in. apart	Axis 2–2: ⅜ in. apart	Axis 2–2: ½ in. apart
8 × 8 × 1⅛	56.9	33.46	2.42	3.42	3.51	3.55	3.60
½	26.4	15.50	2.50	3.33	3.41	3.45	3.50
6 × 6 × 1	37.4	22.00	1.80	2.59	2.68	2.72	2.77
⅜	14.9	8.72	1.88	2.49	2.58	2.62	2.66
5 × 5 × 1	30.6	18.00	1.48	2.19	2.28	2.33	2.38
⅜	12.3	7.22	1.56	2.09	2.17	2.21	2.26
4 × 4 × 13/16	19.9	11.68	1.18	1.75	1.85	1.89	1.94
¼	6.6	3.88	1.25	1.66	1.75	1.79	1.84
3½ × 3½ × 13/16	17.1	10.06	1.02	1.55	1.65	1.70	1.75
¼	5.8	3.38	1.09	1.46	1.55	1.59	1.64
3 × 3 × ⅝	11.5	6.72	0.88	1.32	1.41	1.46	1.51
¼	4.9	2.88	0.93	1.25	1.34	1.38	1.43
2½ × 2½ × ½	7.7	4.50	0.74	1.09	1.19	1.24	1.29
¼	4.1	2.38	0.77	1.05	1.14	1.19	1.24
2 × 2 × 7/16	5.3	3.12	0.59	0.88	0.98	1.03	1.08
¼	3.19	1.88	0.61	0.85	0.94	0.99	1.04

Example. Design the compression chord of a roof truss for a maximum load of 35,000 lb. and unsupported length of 10 ft.

Assume as the first approximation that $l/r = 100$, hence $P/A = 9000$ and an area of 3.9 sq. in. is required. Try two angles 4 × 3 × 5/16 in. with ⅜-in. connecting plates. The least radius of gyration is 1.27 and area is 4.18. $P = 4.18 \times (19{,}000 - 100 \times {}^{120}/_{27}) =$ 39,800 lb., which is satisfactory. Two 4 × 4 × ¼-in. angles also meet the requirements and are somewhat lighter. If this member is subjected to bending stresses by the purlins, it should be designed by the formula of Art. 7 for combined compression and bending.

Table 24. Properties of unequal-leg angles



Single angle		Two angles, area, square inch	Radii of gyration, inches				
			Axis 1–1	Axis 2–2			
Size, inches	Weight, pounds per foot			In contact	¼ in. apart	⅜ in. apart	½ in. apart
6 ×4 ×⅜	12.3	7.22	1.93	1.50	1.58	1.62	1.67
⅝	20.0	11.72	1.90	1.53	1.62	1.67	1.71
⅞	27.2	15.98	1.86	1.58	1.67	1.71	1.76
5 ×3½×⅜	10.4	6.10	1.60	1.34	1.42	1.46	1.51
⅝	16.8	9.86	1.56	1.37	1.46	1.51	1.56
⅞	22.7	13.36	1.53	1.42	1.51	1.56	1.61
5 ×3 ×5/16	8.2	4.82	1.61	1.09	1.17	1.22	1.26
9/16	14.3	8.38	1.58	1.13	1.22	1.26	1.31
13/16	19.9	11.68	1.55	1.17	1.27	1.32	1.37
4 ×3 ×5/16	7.2	4.18	1.27	1.17	1.25	1.30	1.34
9/16	12.4	7.26	1.24	1.21	1.30	1.34	1.39
13/16	17.1	10.06	1.21	1.25	1.35	1.40	1.45
3½×2½×¼	4.9	2.88	1.12	0.96	1.04	1.09	1.13
½	9.4	5.50	1.09	1.00	1.09	1.14	1.19
11/16	12.5	7.32	1.06	1.03	1.13	1.18	1.23
3 ×2½×¼	4.5	2.64	0.95	1.00	1.09	1.13	1.18
⅜	6.6	3.86	0.93	1.02	1.11	1.16	1.21
9/16	9.5	5.56	0.91	1.05	1.15	1.20	1.25
2½×2 ×3/16	2.8	1.62	0.79	0.79	0.88	0.92	0.97
⅜	5.3	3.10	0.77	0.82	0.91	0.96	1.01
½	6.8	4.00	0.75	0.84	0.94	0.99	1.04

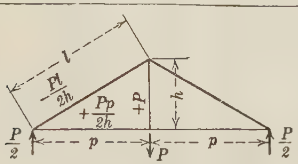


FIG. 37.—Kingpost truss.

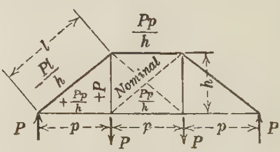


FIG. 38.—Queenpost truss.

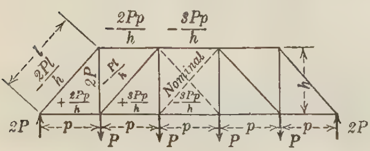


FIG. 39.—Howe truss.

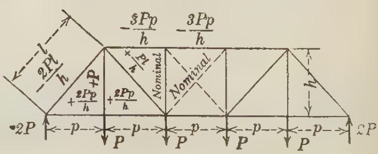


FIG. 40.—Pratt truss.

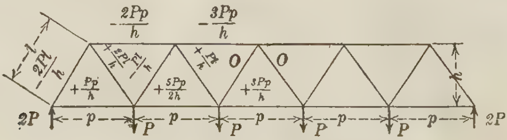


FIG. 41.—Warren truss.

Trusses. See Sec. 26, Art. 10, for methods of solution. Figs. 37–41 show common types of bridge trusses with the stresses expressed in terms of panel load P, panel length p, depth of truss h, and length of diagonal l.

Table 24a. United States standard gage for sheet steel

Gage number	Thickness in decimals of an inch	Weight per square foot in pounds	Gage number	Thickness in decimals of an inch	Weight per square foot in pounds
0000000	0.5	20.40	17	0.05625	2.30
000000	0.46875	19.13	18	0.05	2.04
00000	0.4375	17.85	19	0.04375	1.79
0000	0.40625	16.58	20	0.0375	1.53
000	0.375	15.30	21	0.034375	1.40
00	0.34375	14.03	22	0.03125	1.28
0	0.3125	12.75	23	0.028125	1.15
1	0.28125	11.48	24	0.025	1.02
2	0.265625	10.84	25	0.021875	0.893
3	0.25	10.20	26	0.01875	0.765
4	0.234375	9.56	27	0.0171875	0.701
5	0.21875	8.93	28	0.015625	0.638
6	0.203125	8.29	29	0.0140625	0.574
7	0.1875	7.65	30	0.0125	0.510
8	0.171875	7.01	31	0.0109375	0.446
9	0.15625	6.38	32	0.01015625	0.414
10	0.140625	5.74	33	0.009375	0.383
11	0.125	5.10	34	0.00859375	0.351
12	0.109375	4.46	35	0.0078125	0.319
13	0.09375	3.83	36	0.00703125	0.287
14	0.078125	3.19	37	0.006640625	0.271
15	0.0703125	2.87	38	0.00625	0.255
16	0.0625	2.55			

HYDRAULICS

17. Physical properties of water

Weight of water is usually taken as 62.5 lb. per cu. ft., or 8.355 lb. per U. S. gal. Accurate weights for pure water are given in Table 1. Sea water weighs 64 lb. per cu. ft., Great Salt Lake water, 69 to 76; sewage, 62.4 to 62.7; ice, 57.2 to 57.5 lb. per cu. ft.

Compressibility varies with temperature and pressure. The average value of the BULK MODULUS OF ELASTICITY for low temperature and pressures less than 1000 lb. per sq. in. is 294,000 lb. per sq. in. This value increases with both temperature and pressure. A pressure of 1000 lb. per sq. in. decreases the volume only 0.33 per cent., hence water may be considered incompressible.

18. Atmospheric pressure

MEAN ATMOSPHERIC PRESSURE at sea level is 14.7 lb. per sq. in.; it causes mercury to rise in an exhausted tube to a height of 30 in.; the corresponding height of a water column is 34 ft. Water boils at this pressure at 212° F.; as altitude increases, atmospheric pressure and the boiling point of water decrease. If the air at the top of a barometer tube is only partially exhausted, the height of the mercury column measures the difference in pressure inside and outside. PRESSURES BELOW ATMOSPHERE are often measured in inches of mercury, each inch corresponding to negative or suction pressure of 0.491 lb. per sq. in. Atmospheric pressure has marked effect on the flow of water but where it acts on all parts of a system its effect is neutralized. Pressures below atmosphere are treated as negative pressures and corresponding heads as negative heads.

Table 25. Weight of distilled water

Temperature, degrees F.	Pounds per cubic foot	Temperature, degrees F.	Pounds per cubic foot
32	62.42	130	61.55
39.3 (max.)	62.424	140	61.39
50	62.41	150	61.20
60	62.37	160	61.01
70	62.30	170	60.80
80	62.22	180	60.59
90	62.12	190	60.36
100	62.00	200	60.14
110	61.86	210	59.89
120	61.72	212	59.84

19. Transmission of pressure

Pascal's law. A perfect liquid is unable to withstand tangential stress, hence pressure at any point in a liquid acts with equal intensity in every direction. If water in a closed vessel be subjected to unit pressure p, this pressure is transmitted with undiminished intensity throughout the liquid; the total pressure on any area A will be pA and its direction will be normal to that area. This principle is applied in the hydraulic press, jack, accumulator, and in heavy hydraulic machinery such as punching and riveting machines.

Hydraulic press consists of two cylinders of different diameters, fitted with pistons, and connected by a pipe. If a force P is applied to the small piston it produces pressure on the water which is transmitted to the large piston and enables it to support a load W such that $P/W = d^2/D^2$. This equation is modified by friction of the packing and, if the pistons are not at same elevation, by pressure caused by the difference in head. D/d may be made any desired value, hence it is possible to obtain forces limited only by the ability of the machines to withstand the strain.

20. Pressure due to weight

Unit pressure on a horizontal layer at distance h below the surface in a body of still water is $p = wh$ where w = weight of a cubic unit of water and h = depth below the surface, called HEAD. Pressure per sq. ft. is $p = 62.5h$ and per sq. in. is $p = 0.434h$; pressures are frequently expressed also in feet of head. INTENSITY OF PRESSURE depends only on the density of the liquid and the head; it is independent of the size and shape of the containing vessel and the amount of liquid present.

21. Total pressure and center of pressure

Total normal pressure on any submerged area is $p_n = wAh_0$. In common units, $w = 62.5$, A = total area in sq. ft., and h_0 = the vertical distance from the surface to the center of gravity of the area in ft.

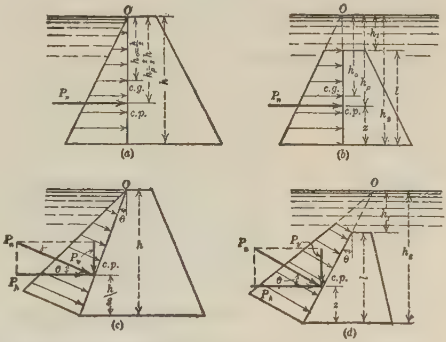
In the case of a VERTICAL DAM of height = h, the area per lin. ft. is h and $h_0 = h/2$. hence $P_n = wh^2/2$.

Pressure in a given direction is $P_1 = P_n \cos\theta = wA_1h_0$, where θ is the angle between the given direction and the normal and A_1 is the projection of the submerged surface on a plane normal to the given direction.

Examples. Horizontal pressure on a DAM is $P_n = wh^2/2$ whether the up-stream face is vertical or inclined. The driving pressure of a PUMP PLUNGER is the same whether the latter is plane or spherical.

Center of pressure of a submerged surface is the point of application of the resultant pressure.

For a RECTANGLE with the top in the liquid surface, the depth of the center of pressure is ⅔h. For submerged rectangles the total pressures and centers of pressure may be found by constructing pressure diagrams (Fig. 42). The area of the diagram represents total pressure and the c.g. of the diagram is opposite the center of pressure. A general rule for location of the center of pressure is: $y_p = I/S$, where I is moment of inertia of the area (see Sec. 26, Art. 14), S is the moment of the area (= area × distance of c.g. from the axis), and y_p is the arm of the center of pressure, all with respect to an axis which is the intersection of the plane of the submerged surface with the water level. A more convenient form is $y_p = y_0 + k_0^2/y_0$, where y_0 is the distance from the axis to the c.g. and k_0 is the radius of gyration of the submerged area about its c.g. The distance k_0^2/y_0 becomes negligible when the head exceeds three or four times the vertical dimensions of the submerged surface.



a. $P_n = P_h = wh^2/2$

b. $P_n = w(h_2^2 - h_1^2)/2 = wl(l + 2h_1)/2$

$$h_p = \frac{2}{3}\left(\frac{h_2^3 - h_1^3}{h_2^2 - h_1^2}\right). \quad z = \frac{l}{3}\left(\frac{l + 3h_1}{l + 2h_1}\right)$$

c. $P_n = wh^2/2 \cos\theta$. $P_h = wh^2/2$. $P_v = (wh^2 \tan\theta)/2$

d. $P_n = wl(l + 2h_1)/2 \cos\theta$

$P_h = wl(l + 2h_1)/2 = P_h \tan\theta$

$z = l(l + 3h_1)/3(l + 2h_1)$

FIG. 42.

Example 1. A square gate, 2 × 2 ft., inclined 60° from the horizontal, with its c.g. 5 ft. below the surface, covers a rectangular opening (Fig. 43). The gate is hinged at the upper edge and it is required to find the moment about the hinge necessary to open the gate.

$P_n = wAh_0 = 62.5 \times 4 \times 5 = 1250$ lb. $y_0 = 5 \sec 60° = 5.77$ ft. $y_p = 5.77 + 0.333/5.77 = 5.828$ ft. Moment about hinge = 1250 × 1.05 = 1320 lb.-ft.

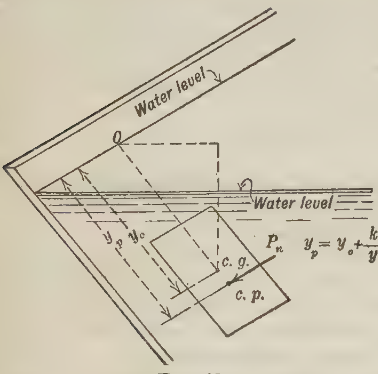


FIG. 43.

Example 2. A vertical wall has 15 ft. of water on one side and 6 ft. on the other. Find the overturning moment on the wall per lin. ft.

$P_1 = 62.5 \times 15^2/2 = 7040$ lb. $P_2 = 62.5 \times 6^2/2 = 1125$ lb. $M = 7040 \times 15/3 - 1125 \times 6/3 = 32{,}950$ lb.-ft.

22. Loss of weight in water. Flotation

Archimedes' principle. A submerged body loses an amount of weight equal to that of a volume of water which is the same as the volume of the submerged body. A body weighing W lb. in air will weigh submerged $W(1 - 62.5/d)$ where d is density of the body in lb. per cu. ft.

Example. If water percolates under a rectangular masonry dam in such a way that full hydrostatic pressure is developed, the upward pressure of water decreases the stability against overturning as though the unit weight of the masonry were decreased 62.5 lb. per cu. ft. Such percolation should be prevented by cut-off walls and drains.

Depth of flotation. A floating body displaces a volume of water that has weight equal to that of the body. To find the depth of flotation y, equate the weight of the body to $62.5 \times$ volume of water expressed in terms of y.

Example. A loaded ship having vertical sides has a waterline enclosing 10,000 sq. ft. What is the change of draft (fresh water) when 500 tons of coal have been burned? Solution, $500 \times 2000 = y \times 10{,}000 \times 62.5$. $y = 1.6$ ft

Stability of flotation. The resultant upward pressure on a floating body acts through the c.g. of the displaced water. This point is called the CENTER OF BUOYANCY. If this resultant also acts through the c.g. of the body the latter is in equilibrium. Otherwise the body will roll under the action of an unbalanced couple.

FIG. 44.

Consider a symmetrical floating body to be displaced from its position of equilibrium through angle θ (Fig. 44). The resultant water pressure through the center of bouyancy B will intersect the axis of symmetry at M, THE METACENTER. The equilibrium of the body is stable, neutral, or unstable according as M is above, coincident with, or below the c.g. of the body.

23. Pressure on gates and tanks

Resultant pressure on a submerged gate is due to the difference of head h on the two sides and is of uniform intensity over the whole of the submerged surface. Pressure $P = wAh$; the intensity is constant and the center of pressure and c.g. coincide.

Unit tensile stress in a pipe or circular tank is given by the formula for HOOP TENSION, $Pd = 2St$, where P is internal pressure in lb. per sq. in., d is diameter in inches, t is thickness of shell, and S is the unit stress in lb. per sq. in. The lower limit of applicability is $d = 30t$, the expression applies to pipes, steel tanks, steel bands for wood stave pipe and steel for reinforced concrete tanks. For thick cylinders see Art. 5.

Example. A standpipe 60 ft. high and 30 ft. diameter is to be built of riveted steel plates, the efficiency of the joints being 70 per cent. and allowable stress 10,000 lb. per sq. in.

Divide the pipe into any number of sections (say five) and design each section for the internal pressure at its lower edge. For the lowest section, $P = 0.434 \times 60 = 26.04$; therefore $26.04 \times 360 = 2 \times 10{,}000 \times t \times 0.70$, and $t = 0.67$ in. or ¾-in. plates. The other sections would be made of ½-, ⅜-, and ¼-in. plates respectively. The same principle of division applies to spacing steel bands on wood-stave tanks and reinforcing bars in concrete tanks.

24. General laws of flow

Torricelli's theorem. The velocity of flow of a jet discharging under a head of h ft. is the same as the velocity acquired by a body falling freely through the height h. Thus $v = \sqrt{2gh}$ = velocity in ft. per sec.

Bernouilli's theorem. For steady frictionless flow the sum of the pressure head and the velocity head equals the hydrostatic head that obtains when there is no flow. For actual conditions involving friction, the sum of the pressure head, velocity head, and elevation head above some assumed datum plane at some station 1 equals the sum of the corresponding heads at any other station 2, plus or minus FRICTION HEAD h_f; plus if 2 is downstream and minus if upstream.

If $h = p/w$ = pressure head, and z = elevation head above the assumed datum plane, $h_1 + z_1 + v_1^2/2g = h_2 + z_2 + v_2^2/2g + h_f$. Multiplying both sides of this equation by the weight of fluid passing per unit of time, the expression states that the total energy per unit

of time available at 1 equals the total energy at 2 plus the energy lost between 1 and 2. Bernouilli's theorem is a form of statement of the law of the conservation of energy and is the basis of most hydraulic formulas.

25. Flow from orifices and mouthpieces

Coefficients. The discharge from a standard orifice (Fig. 45) contracts, at a distance $d/2$ from the plane of the orifice, to an area that is 62 per cent. of that orifice. Pressure head does not become zero and velocity does not attain its value $v = \sqrt{2gh}$ until this contracted section is reached. Head should be measured to the center of pressure but if h is 3 or 4 times d the error involved by reading h to the center of the orifice is small. The ratio of the contracted area to the area of the orifice is C' = the COEFFICIENT OF CONTRACTION. Actual velocity is slightly less than theoretical and is given by $v = C_1\sqrt{2gh}$ where C_1 is COEFFICIENT OF VELOCITY. Discharge in cu. ft. per sec. is the product of velocity and area of the contracted jet and is

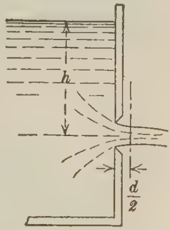


FIG. 45.—Standard orifice.

$$q = C'C_1A\sqrt{2gh} = CA\sqrt{2gh},$$

where C is the COEFFICIENT OF DISCHARGE and A is the area of the orifice in sq. ft.

Standard orifice gives an accurate means of measuring the flow of water. The center of the orifice should be at least $3d$ from the sides and bottom of the tank; the edges should be sharp in order that full contraction may be developed. For SMALL ORIFICES, 1 in. or less in diameter, C may be taken as 0.61 for circular and 0.62 for square orifices. For LARGER ORIFICES and HIGH HEADS the average value of C = 0.597 for circular orifices and 0.604 for square orifices may be used. A precision involving an error of less than 3 per cent. may be expected. If greater accuracy is desired, the orifice should be calibrated or reference had to a table of coefficients in any standard work on Hydraulics. An error less than 1 per cent. may be obtained by calibration.

Large orifices under low heads. Variation of pressure in the place of the orifice is so great that the preceding discharge formula does not hold. To calculate discharge it is necessary to divide the orifice into elementary horizontal strips $x\,dy$ under head y and integrate the discharges through these strips.

For a rectangular orifice of breadth b: $q = \frac{2}{3}C\sqrt{2g}\cdot b(h_2^{3/2} - h_1^{3/2})$ where h_1 and h_2 are the heads on the top and bottom of the orifice respectively. This expression should be used if the head on the center of gravity of the orifice ($h_{c.g.}$) is less than three times the depth of the orifice.

Submerged orifices. Flow is due to the difference in level on the two sides of the orifice and does not depend on the depth of submergence; it is slightly less than for discharge into the atmosphere, the average difference being about 1 per cent. For large orifices the difference is negligible; for orifices 1 in. in diameter or less it may be 2 per cent. The value of C = 0.60 is in general use.

Velocity of approach. If water in a channel approaching an orifice has velocity v_a, the head on the orifice is increased by the velocity head $h_v = v^2_a/2g$ and the EFFECTIVE HEAD is $H = h + v^2_a/2g$. Then $q = CA\sqrt{2gH}$. If the approaching channel and the orifice have areas a and A respectively,

$$v = C_1\sqrt{\frac{2gh}{1 - C^2\left(\frac{A}{a}\right)^2}};\quad q = A\sqrt{\frac{2gh}{\left(\frac{1}{C}\right)^2 - \left(\frac{A}{a}\right)^2}}.$$

For a RECTANGULAR ORIFICE under low heads,

$$q = \tfrac{2}{3}C\sqrt{2g} \cdot b[(h_2 + h_v)^{3/2} - (h_1 + h_v)^{3/2}].$$

Suppression of contraction is effected by an internal projection at the perimeter of the orifice. Suppression increases discharge. For a square orifice with one side suppressed the increase is about 3.5 per cent.; two sides, 7.5 per cent. For rectangles with the lower edge suppressed the increase is 6 to 7 per cent., if $b = 4d$, and 8 to 12 per cent., if $b = 20d$. Avoid suppressed orifices for accurate work.

Discharge under a falling head. If Y is area enclosed by the waterline when the head on an orifice is y, the time for the head to fall from H to h is

$$t = \int_h^H \frac{Yy^{-\frac{1}{2}}}{CA\sqrt{2g}}dy,$$

where Y is, in general, some function of y. If the cross-section is constant and equal to a, the time to empty a vessel is $t = 2a\sqrt{H}/CA\sqrt{2g}$, which is twice that required to discharge a similar amount under a constant head H.

The miner's inch is a unit of flow used in mining and irrigating work; it is rapidly becoming obsolete. It is defined as the discharge from an orifice one inch square under a head on its center of 6.5 in. The value of this unit and its definition vary in different states. The legal equivalent in California is 40 miner's inches = 1 cu. ft. per sec., in Colorado 38.4, while in Arizona, Idaho, Nevada and Utah, the value is 50, by common agreement.

Flow under pressure. When water in a closed vessel is under unit pressure p in addition to its own weight, the velocity and discharge from an orifice are found by computing the equivalent head in feet, $H = h + p/w$. If discharge takes place into a vessel in which the unit pressure is p_0, the equivalent head is $H = h + p/w - p_0/w$.

Example. A 1-in. circular orifice under a head of 10 ft. and an additional pressure of 10 lb. per sq. in. discharges into a 26-in. vacuum and it is desired to find the discharge.

The 26-in. vacuum is equivalent to a negative head of $26 \times 34/30 = 29.5$ ft. which increases the effective head, hence $H = 10 + 10 \times 2.304 + 29.5 = 62.5$ ft. $q = CA\sqrt{2gH}$ $= 0.61 \times 0.00546\sqrt{64.4 \times 62.5} = 0.212$ cu. ft. per sec.

26. Tubes, nozzles, and jets

Short tubes whose lengths are 2 to 3 diameters may be treated in the same manner as orifices. LONG TUBES are classified as pipes. The addition of a short tube to an orifice decreases velocity but INCREASES discharge, although friction losses are also increased. This is due to the fact that the jet after passing the contracted section re-expands to fill the whole cross-section of the tube. Velocity at the contracted section is greater than that at the discharge end of the tube, pressure is correspondingly less (Bernouilli's theorem), and hence below atmospheric, if discharge is into the atmosphere. This negative pressure increases the effective head and discharge is increased. Fig. 46 shows common types of tubes and end connections.

FIG. 46.—Short tubes and mouthpieces.

(a) is the STANDARD SHORT TUBE for which $C' = 1.0$, $C_1 = C = 0.82$, provided that the jet re-expands to fill the tube. For high heads the discharge will jump clear of tube and the device becomes a standard orifice with $C = 0.61$. When the jet re-expands to fill the

tube its appearance becomes turbulent or broomy. (*b*) is a ROUNDED ORIFICE which has a coefficient of discharge varying from 0.61 to 0.99 depending upon the curvature. Even a slight dulling of the edge of a standard orifice increases the discharge several per cent. while if the curvature of the orifice conforms to the direction of the stream lines, the contraction is entirely suppressed and $C_1 = C = 0.96–0.99$. This is an ideal end connection to avoid energy losses. (*c*) is an inward-projecting tube for which $C_1 = C = 0.72$. It is a common type of end connection for a pipe line when no attempt is made to avoid entrance losses. (*d*) is BORDA'S TUBE for which $C = 0.5–0.53$. (*e*) is a conical converging tube, the discharge from which varies with the value of the angle θ: the maximum value of C is 0.94 which occurs when $\theta = 13.5°$. (*f*) is a VENTURI OR COMPOUND TUBE composed of a rounded orifice and an expanding tube with angle of about 10°. Pressure at section AB is less than atmospheric and may be as low as $-$ 24 ft.; the equivalent head is $h + 24$ which gives theoretical values of $C = 8$ or 9 for section AB. Experiments by Francis show C actually to be as high as 2.43. The coefficient for the discharge end is always less than unity.

Loss of head for any pipe, or orifice discharging freely with velocity v under total head h is $h_f = h - v^2/2g$. Loss of head for an orifice or tube whose coefficient of velocity is C_1 is

$$h_f = (1 - C_1^2)h = (1/C_1^2 - 1)v^2/2g.$$

Example. For a standard orifice, $h_f = 0.04h = 0.041v^2/2g = 0.11v_0^2/2g$ where v_0 is the velocity in the plane of the orifice. For a standard short tube (Fig. 46, *a*) $h_f = 0.33h = 0.49v^2/2g$. For an inward-projecting tube (Fig. 46, *c*) $h_f = 0.48h = 0.93v^2/2g$.

Nozzles. Types of nozzle tips are shown in Fig. 47. The SMOOTH NOZZLE is most common. If properly designed there should be no contraction and $C_1 = C = 0.97 - 0.99$. The RING TIP was designed from the mistaken notion that the ring would increase velocity. The coefficient of velocity is the same as for the smooth type while the coefficient of discharge depends on the relative areas of opening and ring. For a SQUARE RING an average value is $C = 0.74$ while an UNDERCUT RING has a value somewhat less. The NEEDLE NOZZLE is designed to regulate flow by moving the needle in and out and has an efficiency of 95 to 98 per cent.; the coefficient of discharge varies from 0.82 to 0.95 being least when the nozzle is nearly closed. Expressions for the velocity and discharge through a nozzle are the same as those for an orifice with velocity of approach, Art. 25.

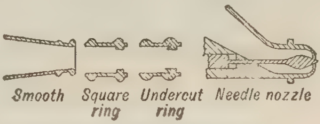


FIG. 47.—Types of nozzles.

Discharge in gallons per minute from a nozzle D in. diameter, attached to a pipe d in. diameter, with the pressure at the base of the tip $= p$ lb. per sq. in. is

$$q = 29.83D^2\sqrt{\frac{p}{(1/C)^2 - (D/d)^4}}.$$

A standard fire stream is one flowing 250 gal. per min. through a $1\frac{1}{8}$-in. smooth nozzle with a pressure at the base of the tip of 45 lb. per sq. in. The hydrant pressure required to throw this stream through 50 ft. of the best rubber-lined hose is 56 lb. per sq. in.; for 200 ft., 77 lb. per sq. in. The pressure drop per 100 ft. of best-quality hose is about 14 lb. per sq. in.; for poor-quality hose it may be double this figure. The best hydrant pressure for fire service is 80 to 100 lb. per sq. in.

Power of a jet discharging W lb. per sec. is $Wv^2/2g = wAv^3/2g$ ft.-lb. per sec., where w = weight of a cu. ft. of fluid, A = cross section of the jet in sq. ft., and v = velocity in ft. per sec. HORSEPOWER $= wAv^3/1100g$. Power of a jet discharging from an orifice or nozzle (coefficient of discharge $= C$ and coefficient of velocity $= C_1$) is $P = CC_1wA\sqrt{2g}\,h^{3/2}$ ft.-lb. per sec. The efficiency of such a jet is $E = C_1^2$.

Example. The respective powers of a standard orifice and standard short tube are $0.58wA\sqrt{2g}\,h^{3/2}$ and $0.55wA\sqrt{2g}\,h^{3/2}$. Corresponding efficiencies are 96 per cent. and 67 per cent. respectively.

Impulse of a jet in a given direction in pounds is $W/g \times$ *change of velocity in that direction*, in which W = lb. of water deflected per sec. If the jet impinges on a flat plate (Fig. 48, a), the impulse is Wv/g lb.; if direction is reversed without loss in friction, the impulse is $2Wv/g$; while for a curved vane, (Fig. 48, c) the impulse in the original direction of jet is $\frac{W}{g}v(1 - \cos\theta)$. If a jet discharging W lb. per sec. (strikes a flat plate moving in the direction of the jet with velocity u, the water that strikes the plate per sec. is $W(v - u)/v$ and the impulse in direction of motion is $W(v - u)^2/vg$. If the jet strikes a series of flat vanes so that all the water discharged strikes some vane, the impulse is $W(v - u)/g$.

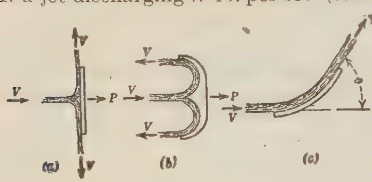
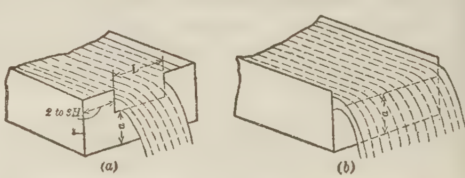


FIG. 48.

Reaction of a jet issuing from the side of a vessel is $R = Wv/g = wAv^2/g = 2wAh$, in which w = weight per cu. ft. of the fluid, A = cross section of the jet in sq. ft. and h = head on orifice in ft. The reaction is equal to the impulse of the jet on a flat plate and is twice the hydrostatic head. If the jet issues from a standard orifice with coefficient of velocity 0.98 and coefficient of contraction = 0.62, the reaction of the jet on the containing vessel is $R = 1.22wAh$.

27. Weirs

A WEIR is a dam or obstruction placed in an open channel, over which water is caused to flow, or a notch in the side of a vessel through which water flows. If the top or CREST of the weir is a thin plate with a sharp edge, flow is analogous to that through a rectangular orifice with head on the upper edge = zero. If the weir is a rectangular notch with ends $3H$ or more from the walls of the approaching channel, full contraction of the stream is developed and the device is known as a CONTRACTED WEIR (Fig. 49, a). If the length of the weir is the same as that of the approaching channel, so that no contraction takes place at the ends, the weir is called a SUPPRESSED WEIR (Fig. 49, b).



Contracted weir. Suppressed weir.

FIG. 49.

Discharge from a weir. The theoretical discharge is $Q = \frac{2}{3}\sqrt{2g}\,LH$, and the actual, $Q = \frac{2}{3}C\sqrt{2g}\,LH^{3/2}$ in which H is the height above the crest of the weir to the level of still water, L is the length of crest over which water is flowing, and C is a constant which varies slightly with the type of weir and head but has an approximate value of 0.62.

Velocity of approach. It is not always possible to measure the head H to the level of still water on account of velocity in the approaching channel. Mean velocity of approach v_a is found by dividing the discharge by the cross-sec-

tion of the approach channel and the corresponding velocity head $h_v = v_a^2/2g$. The observed head H must be increased by some function of the velocity head giving the general form

$$Q = \tfrac{2}{3}C\sqrt{2g}\,L(H + nh_v)^{3/2},$$

in which n varies from 1.00 to 2.00 depending on the ratio of head to height of crest above the bottom and on the ratio of mean velocity of approach to surface velocity.

Francis formulas. J. B. Francis replaced the expression $\tfrac{2}{3}C\sqrt{2g}$ by the constant 3.33. His formulas for SUPPRESSED WEIRS are $Q = 3.33LH^{3/2}$ and $Q = 3.33[(H + h_v)^{3/2} - h^{3/2}]$. Later investigators prefer the form $Q = 3.33L(H + 1.4h)^{3/2}$.

For CONTRACTED WEIRS Francis decreased the length L by $0.1H$ for each end contraction, making the expression for the ordinary rectangular notch

$$Q = 3.33(L - 0.2H)(H + 1.4h)^{3/2}.$$

The Francis formula is widely used and should give results within 1 to 3 per cent. for carefully constructed SUPPRESSED WEIRS with heads from 0.3 to 2.0 ft.; for $H = 0.2$, increase the discharge 3 per cent. and increase 7 per cent. for $H = 0.1$. If the height of crest a exceeds $4H$ the influence of velocity of approach will cause increase in discharge of 1 per cent. or less, which is negligible in view of other uncertainties. For CONTRACTED WEIRS, which are a little less accurate, velocity of approach is negligible unless the ratio of H to a is abnormally high.

Fig. 50 gives the discharge in cu. ft. per sec. per ft. of length by the Francis formula. To use the scale, find first the approximate discharge, neglecting velocity of approach. Divide this by the area of the approaching channel to get v. Replace H by $H + 1.4v^2/2g$, taking the value of the velocity head from Fig. 50. For CONTRACTED WEIRS replace L by $L - 0.2H$.

Example 1. Find the discharge from a SUPPRESSED WEIR 4 ft. long under a head of 0.625 ft.; crest 2 ft. above the bottom of the channel.

From Fig. 50, the discharge per ft. of length is 1.65, hence $Q = 6.60$. The area of the channel is $4 \times 2.625 = 10.5$ sq. ft., hence $v_a = 0.628$ ft. per sec. and $h_v = 0.0061$. Then $H = 0.625 + 1.4 \times 0.0061 = 0.6335$ and $Q = 6.72$ cu. ft. per sec.

Example 2. A CONTRACTED WEIR is to be designed to discharge 4 cu. ft. per sec.

Either the length or the head may be assumed for purposes of trial. Assuming $H = 0.5$, the discharge per foot of width is 1.18 and the necessary width is $4/1.18 = 3.4$. Adding $0.2H$ for the effect of end contractions the width is 3.5 ft., which lies between $4H$ and $8H$ and hence is satisfactory. End contractions should have a minimum value of 1 ft. each, making the width of channel 5.5 ft. The depth of crest should not be less than 1.5 ft., making the area of the approach channel 11 sq. ft.; velocity of approach, 0.364; and velocity head $h_v = 0.0019$, which is negligible.

Bazin formula. $Q = m\sqrt{2g}\,LH^{3/2}$ where $m = (0.405 - 0.00984 - H)\left[1 - 0.55\left(\dfrac{H}{a - H}\right)^2\right]$ in which a is the height of the crest above the bottom of the approaching channel. This formula needs no correction for velocity of approach and is probably the most accurate for a wide range of heads. It may be used safely for heads between 0.2 and 6.0 ft. For extremely low heads it gives high results.

Measurement of flow by weirs. A weir is the standard device for measurement of moderate flows of water. To get correct results, it must be constructed in a standard manner. The crest must be level and have sharp or square edges. The sheet of water or NAPPE must jump free of the downstream face and allow free access of air to the under side. In SUPPRESSED WEIRS the channel walls should extend beyond the crest to prevent lateral expansion of the nappe. If END CONTRACTIONS are used they should be at least $2H$ and preferably $3H$ or more in length to develop full contraction, and the height of crest a (Fig. 49) should be at least $3H$. It is preferable that the length L be between $4H$ and $8H$ although weirs of 20 ft. or more in length are in common use. The velocity of approach should be low. For a CONTRACTED WEIR the area of the approach channel should be $6LH$ or more, which makes the velocity of ap-

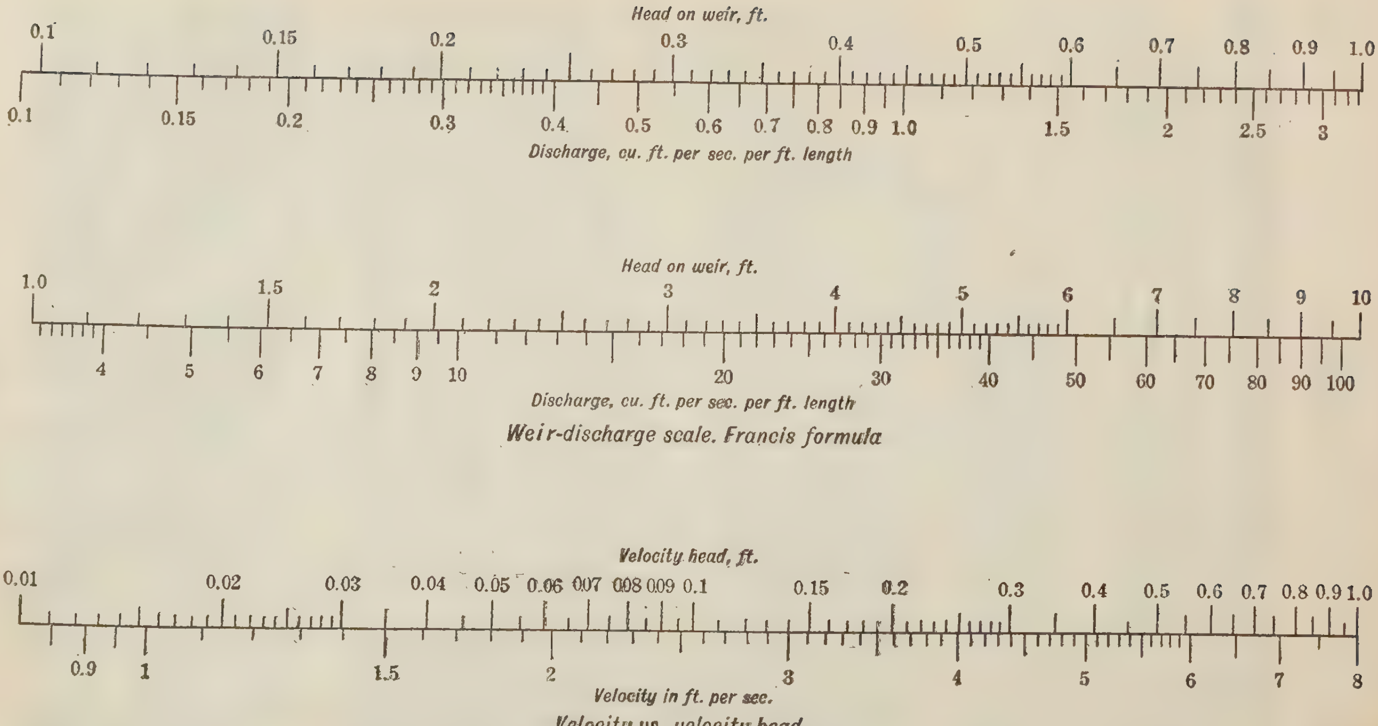


FIG. 50.—Scales for weir calculations.

proach negligible. HEAD should be measured accurately a distance more than $6H$ upstream to eliminate the slope of the surface toward the weir. For approximate work a stake is driven into the channel above the weir with its top level with the crest and head is measured directly by a scale held upon this stake. For more accurate work water is led from the channel to a pail upon which a hook gage is mounted.

Triangular-notch weirs are convenient for measurement of very small to moderate discharges and give good results for heads between 3 and 10 in. DISCHARGE is $Q = \frac{4}{15}c\sqrt{2g}\,LH^{3/2}$ where L is width of notch H ft. above the vertex; $c = 0.60$ for angles at vertex of 90° and 60°. For a 90° weir $L = 2H$ and $Q = 2.53H^{5/2}$ cu. ft. per sec.

Trapezoidal or Cippoletti weir is designed with end slopes of one horizontal to four vertical, which will just compensate for the effect of end contractions. DISCHARGE is then given by the Francis formula $Q = 3.33LH^{3/2}$. Some authorities give $Q = 3.36/LH^{3/2}$.

ROUNDING OF CREST suppresses contraction and increases discharge. INCLINING UPSTREAM FACE with the current increases discharge while sloping the face upstream causes decrease. WIDENING THE CREST so that the nappe no longer jumps free increases friction and reduces discharge.

Dams and spillways may be regarded as weirs; they are by no means accurate as measuring devices.

The coefficient in the Francis formula varies from 2.6 to 4.0 or more depending on the form of dam. For a wide flat-topped dam with width of crest greater than H the coefficient is 2.64 giving discharge 80 per cent. of standard. Rounding the crest to conform to stream lines and inclining the upstream face with the current gives a coefficient of 4.0 or more. For such a dam the discharge varies from 97 to 120 per cent. of standard as the head varies from 0.5 to 4 ft. In designing a waste weir or spillway, the probable flood discharge should be estimated, a large factor of safety introduced, and the dimensions of the weir determined from $Q = MLH^{3/2}$. A common value of M is 3.0.

28. Flow in pipes

When water under pressure flows in a pipe there is a continuous loss of pressure head due to friction between the water and the pipe and internal friction in the water. This condition is shown by Fig. 51, which shows pressure heads necessary in pumping water from (A) to (B). This friction and the corresponding friction head h_f vary widely with the nature and condition of pipe, diameter, and velocity of flow. There is much difficulty in obtaining a satisfactory formula due to the number of variable factors and to the fact that roughness cannot be defined mathematically; also the degree of roughness does not remain constant in service. There are many formulas but all may be placed in one of two classes: (1) friction head is assumed to vary directly as v^2 and inversely as d. $h_f = flv^2/2dg$; the friction factor f is constant for any assumed condition but is varied as v and d vary, l = length in ft., v = velocity in ft. per sec., and d = diameter in ft. (2) $h_f = alv^n/d^r$ in which a is constant and n and r are chosen for average conditions.

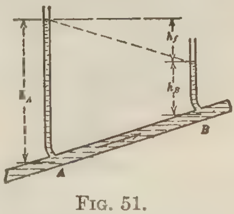


FIG. 51.

Attainable precision. For carefully laid new pipe, a discharge within 10 per cent. of calculated value is all that may be expected. After a few months friction may increase as much as 20 per cent. and in the course of years the diameter may be reduced 50 per cent. by tubercules of iron rust; this deposit forms more rapidly in some waters than others so that each case is a special problem. Friction factors are given for both new and old pipes and in pipe diagrams allowances are made for decrease of capacity with age.

Secondary losses of head are negligible in long pipes but may be a large part of the total loss in pipes of length $50d$ to $500d$. LOSS AT ENTRANCE $= mv^2/2g$ where $m = 1/C_1^2 - 1$,

in which C_1 is the coefficient of velocity for end connection; $m = 1.0$ for inward projecting pipes, 0.5 for square-edged entry with end of pipe flush with the inside of the reservoir wall, and 0 for rounded edge conforming to stream-line flow. Loss DUE TO 90° BENDS is the same as that for straight pipe of length 10 to $20d$, it is smallest when radius $= 3d$; it is greater for large pipes than for small ones, varying approximately as d. Loss DUE TO VALVES is uncertain but equivalent to a length of $6d$ at full gate. Loss DUE TO EXPANSION OF SECTION occurs when water flowing in a small pipe with velocity v_1 suddenly enters a large pipe in which the velocity is v_2; this loss is $h_l = (v_1 - v_2)^2/2g$. It occurs in hydraulic machines, pipes and channels, whenever the section of flowing water is suddenly enlarged; it can be prevented by having the increase gradual as in the Venturi meter (Art. 32). Loss DUE TO SUDDEN CONTRACTION OF SECTION is similar to that at the entrance to square-edged pipe; it depends on the ratio of pipe diameters and has a maximum value of $0.5v^2/2g$, in which v is the velocity in the smaller pipe.

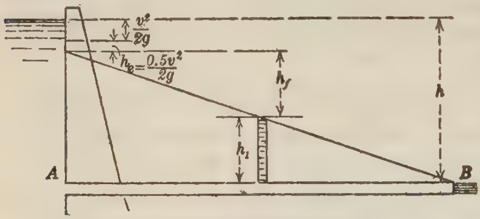


FIG. 52.—Hydraulic gradient, short pipe.

Hydraulic gradient is a line that represents the height of the pressure head at any point in a pipe line. Fig. 52 shows this line for a short horizontal pipe with square-edged entry. The pressure at any point is found from the relation: *pressure head = total head − velocity head − lost head.*

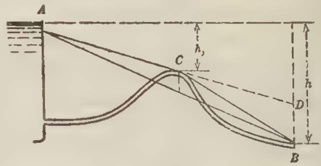
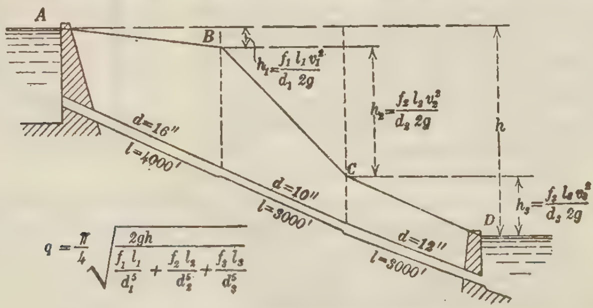


FIG. 53.

When there is no flow, the head at (B) is h. When free discharge occurs at (B), the drop at (A) = the velocity head $v^2/2g$ plus entrance loss, $h_e = 0.5v^2/2g$; the head then drops uniformly to zero at (B). If the pipe were laid on the hydraulic gradient there would be no pressure head and the slope would be just sufficient to give the required flow.

When a pipe rises above the hydraulic gradient (Fig. 53), negative pressure occurs at (C). If this pressure is less than 25 ft., water will syphon over and discharge at (B) under head h. But it is almost impossible to keep the pipe air tight and air dissolved in the water is liberated under the reduced pressure; hence air collects at (C) and breaks the vacuum.



$$q = \frac{\pi}{4}\sqrt{\frac{2gh}{\frac{f_1 l_1}{d_1^5} + \frac{f_2 l_2}{d_2^5} + \frac{f_3 l_3}{d_3^5}}}$$

FIG. 54.—Hydraulic gradient for compound pipe.

The pipe then discharges at (C) under head h_1; section (CB) acts as a conduit running partly full. This may be avoided by closing a valve at (B) until the hydraulic gradient rises above (C), then opening an air valve at (C). Negative pressure at (C) may be avoided by a compound pipe that makes the hydraulic gradient ACB, Fig. 53. Fig. 54 shows the hydraulic gradient for a long compound pipe in which velocity head and secondary losses have been neglected. If the friction factor f is assumed the same for each pipe, the slope of

the gradient varies as v^2/d and since the discharge q is the same for each section, v varies as l/d^2, hence the slope varies inversely as d^5.

Flow in short pipes. If the loss at entrance is $mv^2/2g$ and loss due to bends, valves, and other secondary causes is $m_1v^2/2g$, then $h - v^2/2g = (m + m_1 + fl/d)v^2/2g$ and $v = \sqrt{2gh}/\sqrt{1 + m + m_1 + fl/d}$. If losses due to valves, bends, etc., are allowed for by using an equivalent length of straight pipe, the velocity and discharge for square-edged entry are

$$v = \sqrt{\frac{2gh}{1.5 + f\frac{l}{d}}}; \quad q = 0.7854d^2\sqrt{\frac{2gh}{1.5 + f\frac{l}{d}}}$$

from which the diameter in feet required for discharge q is

$$d = 0.479(1.5d + fl)^{1/5}(q^2/h)^{1/5}.$$

To solve the equation for diameter, use a trial value of $f = 0.02$ and omit the term $1.5d$ for the first approximation. Use a value of f from Table 26 that corresponds with the approximate value of d and solve again. For very short pipes where $l < 50d$, the coefficient m should be replaced by $1/C_1^2 - 1$ and l by $l - 3d$ hence $v = \sqrt{2gh}/\sqrt{1/C_1^2 + f(l - 3d)/d}$.

Table 26. Friction factors (f) for clean iron pipes

Diameter in feet	Velocity, in feet per second						
	1	2	3	4	6	10	15
0.05	0.047	0.041	0.037	0.034	0.031	0.029	0.028
0.1	0.038	0.032	0.030	0.028	0.026	0.024	0.023
0.25	0.032	0.028	0.026	0.025	0.024	0.022	0.021
0.50	0.028	0.026	0.025	0.023	0.022	0.020	0.019
0.75	0.026	0.025	0.024	0.022	0.021	0.019	0.018
1.00	0.025	0.024	0.023	0.022	0.020	0.018	0.017
1.25	0.024	0.023	0.022	0.021	0.019	0.017	0.016
1.50	0.023	0.022	0.021	0.020	0.018	0.016	0.015
1.75	0.022	0.021	0.020	0.018	0.017	0.015	0.014
2.00	0.021	0.020	0.019	0.017	0.016	0.014	0.013
2.5	0.020	0.019	0.018	0.016	0.015	0.013	0.013
3	0.019	0.018	0.016	0.015	0.014	0.013	0.012
3.5	0.018	0.017	0.016	0.014	0.013	0.012	0.012
4	0.017	0.016	0.015	0.013	0.012	0.011	....
5	0.016	0.015	0.014	0.013	0.012	....	....
6	0.015	0.014	0.013	0.012	0.011	....	....

Long pipes. Where length exceeds $4000d$, secondary losses and velocity head are omitted and the formulas become

$$v = \left(\frac{2ghd}{fl}\right)^{1/2}; \quad q = 6.3\left(\frac{hd^5}{fl}\right)^{1/2}; \quad d = 0.479\left(\frac{flq^2}{h}\right)^{1/5}.$$

From Darcy's experiments the friction factor $f = 0.02 + 0.00167/d$, where d is in feet, for new clean cast-iron pipe with well-laid joints, and twice this value for old, foul pipe. This value is too high for large cast-iron pipe and wood-stave pipe and too low for riveted steel pipe. Table 26 gives Merriman's values for f for new well laid cast-iron pipe (*Treatise on Hydraulics*, John Wiley & Sons). For OLD PIPE these values should be multiplied by two for diameters of 3 in. or less and by 1.5 for 36 in. or more. Choice of a multiplier is a

matter of judgment. To determine the diameter required for a given discharge, use a trial value of $f = 0.02$ and solve. Then select the value of f that corresponds to the first result and solve again. Pipe-flow diagrams lessen labor and are sufficiently accurate.

Example. Find the diameter required to deliver 7 cu. ft. per sec. through a pipe 5000 ft. long under a head of 25 ft.

TRIAL SOLUTION: $d = 0.479(0.02 \times 5000 \times 7^2 \div 25) = 1.37$ ft., for which $v = 4.7$. The corresponding value of $f = 0.021$, hence $d = 1.39$ ft. or 16.7 in. The commercial size selected should be 16 or 20 in.

Chezy formula is most generally used for investigation of flow in long pipes, channels and conduits. It states that $v = C\sqrt{Rs}$ in which C is a constant, R is the hydraulic radius in ft., and s is the slope of the hydraulic gradient. The formula applies to everything from smooth pipes to turbulent streams.

HYDRAULIC RADIUS OR HYDRAULIC MEAN DEPTH is the section area of the flowing stream divided by the wetted perimeter and is expressed in feet. Velocity and discharge vary with R and are maximum for a given area when R is maximum. If p = wetted perimeter and A = section area, $R = A/p$, hence for circular conduits full or half full $R = d/4$ and for a rectangular flume b ft. wide and d ft. deep $R = bd/(b + 2d)$. SLOPE OF HYDRAULIC GRADIENT is the friction head per unit of length, hence $s = h_f/l$. It is given as feet per 1000 ft., ft. per mile, and as a ratio. The CHEZY CONSTANT, C, varies with R, s and with the nature of the conduit; it is made to embrace a wide variety of conditions. See Tables 27 and 28.

Table 27. Chezy constant (C) for cast-iron pipe

Diameter of pipe, inches	Velocity, in feet per second							
	New pipes				Old pipes			
	1	3	6	10	1	3	6	10
3	95	98	100	102	63	68	71	73
6	96	101	104	106	69	74	77	79
9	98	105	109	112	73	78	80	84
12	100	108	112	117	77	82	85	88
15	102	110	117	122	81	86	89	91
18	105	112	119	125	86	91	94	97
24	111	120	126	131	92	98	101	104
30	118	126	131	136	98	103	106	109
36	124	131	136	140	103	108	111	114
42	130	136	140	144	105	111	114	117
48	135	141	145	148	106	112	115	118
60	142	147	150	152	....	....	....	....

Table 28. Chezy constant (C) for riveted-steel pipe

Diameter of pipe, inches	Velocity in feet per second			
	1	3	5	10
3	81	86	89	92
11	92	102	107	115
11	93	99	102	105
15	109	112	114	117
38	115	113	113	113
42	102	106	108	111
48	105	105	105	105
72	110	110	111	111
72	93	101	105	110
103	114	109	106	104

Discharge of a pipe running full is given by the Chezy formula as $q = \dfrac{C\pi d^2\sqrt{ds}}{8}$; also $d = \left(\dfrac{8q}{\pi c\sqrt{s}}\right)^{2/5}$. The diameter is best found by trial using a tentative value of $C = 125$.

Relation between friction factor and Chezy constant. The Chezy formula applied to pipes under pressure is in the same form as the friction-factor formula for long pipes, $v = \sqrt{2ghd/fl}$. Replacing s by h/l and R by $d/4$, $v = C\sqrt{dh/4l}$. Equating these two values of v, there results $f = 8g/C^2$.

Pipe-flow diagrams. Fig. 55 (*U. S. Reclamation Service*), shows the relation between discharge, friction head, velocity, and diameter; if any two are known the other two may be determined.

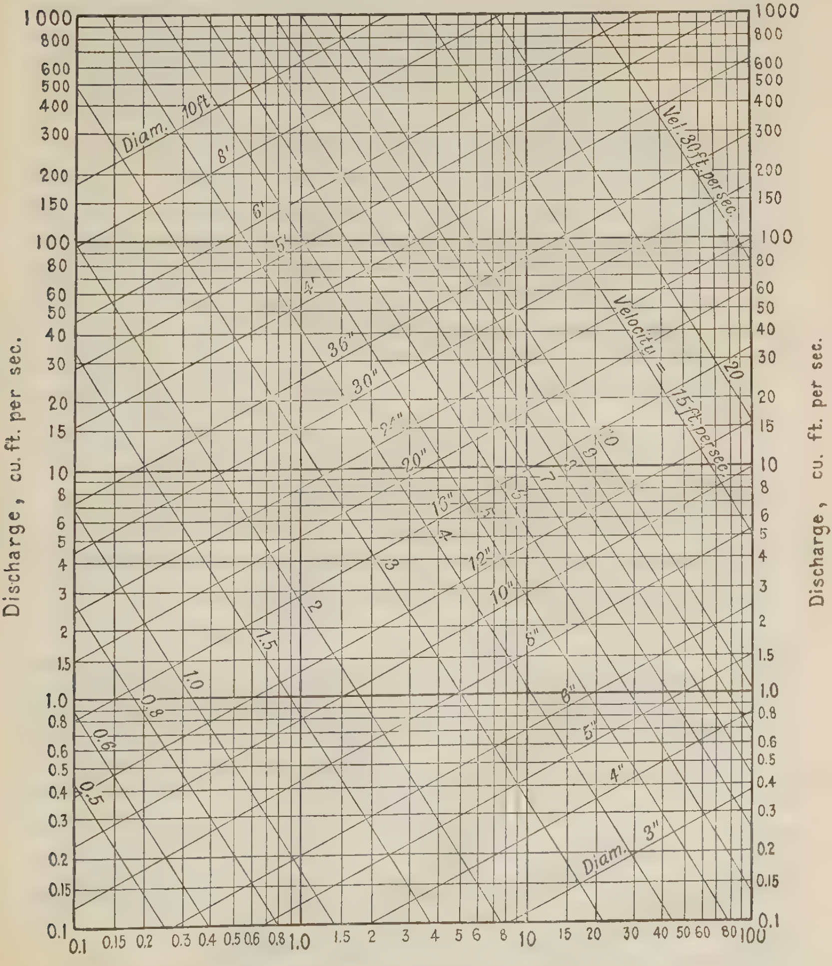


FIG. 55.—Pipe flow, U. S. Reclamation Service formula.

The diagram is based on the formula $h_f = 0.38v^{1.86}/d^{1.25}$ in which h_f is friction head per 1000 ft. of length. The constant and the exponents of v and d are based on average results of a large number of experiments; results agree quite closely with those obtained by use of Tables 27 and 28, and with values found by Kutter's coefficient $n = 0.011$ (Art. 30). Results are for cast-iron and wrought-iron pipe in good condition; for OLD PIPE in service 10 years or more and for riveted-steel pipe, the friction head should be multiplied by 1.45 to 1.63 and discharge divided by 1.20 to 1.28 for velocities of 2 to 5 ft. per sec. For this case the formula is $h_f = 0.50v^2/d^{1.25}$.

Example. New pipe two miles long is to discharge 10 cu. ft. per sec. with drop of head of 40 ft., what diameter is required?

$s = 40/10{,}560 = 0.00379$ or 3.79 ft. per 1000 ft. Enter the diagram on the vertical line representing 3.79 ft. per 1000; at intersection with the horizontal line representing 10 cu. ft.

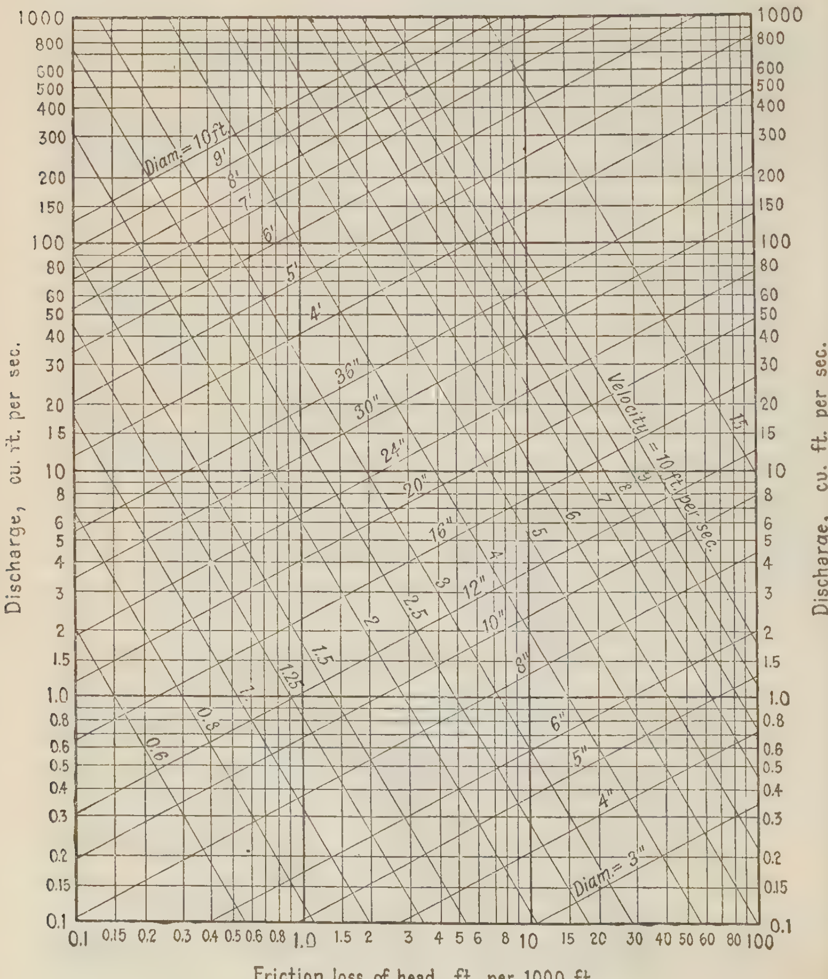


FIG. 56.—Pipe flow, Hazen-Williams formula (*after Bleich*).

per sec. read 20-in. pipe required. If the pipe is to remain in service 10 years or more, divide discharge by 1.25; 20-in. pipe gives $10.8 \div 1.25 = 8.65$ cu. ft. per sec. and 24-in. pipe gives 13.6 cu. ft. per sec.

Hazen-Williams formula, $v = 1.318\, CR^{0.63}s^{0.54}$ is based on average results.

The constant 1.318 is introduced to make the coefficient C approximately the same as in the Chezy formula. Hazen and Williams values for C are: best cast-iron pipe, 140; good new cast-iron pipe, 130; tuberculated pipe, 80 to 110; for DESIGN of cast-iron pipe lines, 100; new riveted-steel pipe, 110, ordinary wrought-iron pipe, 100; lead, brass, tin pipe, 140; smooth wood pipe, 120; vitrified pipe, 110; smooth clean masonry, 140; slime-coated masonry, 130; ordinary masonry, 120; brick sewers, 100. Application is much simplified by use of Fig. 56 (S. D. Bleich). The diagram is constructed for $C = 100$; for any other value of C, multiply velocity and discharge by $c/100$. To SELECT PIPE to give discharge q, multiply q b y $100/C$ giving q^1, and using this value find d from the diagram. To get ACTUAL VELOCITY, multiply the diagram value by $C/100$. For CHANNELS, multiply the hydraulic radius by 4 to get the equivalent diameter.

Example. Find the diameter of riveted-steel pipe to carry 40 cu. ft. per sec. with slope $s = 1.001$. $C = 110$. Multiply 40 by 100/110, then $q^1 = 36.36$ and $d = 46$ in. The corresponding velocity is 3.2, but since the actual discharge is 40, actual velocity is $3.2 \times 110/100 = 3.52$ ft. per sec.

Comparison of U. S. Reclamation Service and Hazen-Williams formulas. F. W. Schoder (*Marks*) states that Fig. 55 gives discharges that are slightly too great for new pipe and hence may be used safely for design when the pipe is to remain in service 5 to 10 years, if the velocity is sufficient to prevent sedimentation and the water is not highly corrosive. But these values correspond to the friction factors for new pipe (Tables 27 and 28) and to values of C in Hazen-Williams formula as follows: 6-in. pipe, 120; 12-in. pipe, 127; 24-in. pipe, 130; 10-ft. pipe, 140. These are coefficients for new cast-iron pipe and pipe in best condition, hence Fig. 56 is more conservative than Fig. 55, but the latter probably gives a closer indication of discharge of a pipe in fairly good condition. Hazen-Williams formula can be used for any class of pipe by choosing the proper value of C.

Compound pipe. A pipe line is to be made of sections of different diameters, the same discharge flowing through all; required to find the discharge.

Solution. $h = \dfrac{f_1 l_1 V_1^2}{2gd_1} + \dfrac{f_2 l_2 V_2^2}{2gd_2} + \dfrac{f_3 l_3 V_3^2}{2gd_3}$, which shows that friction head for each section varies approximately as l/d^5. From this relation friction heads may be found by trial such that each section gives the same discharge. In Fig. 54 let the diameters be 16 in., 10 in., and 12 in., and the lengths 4000 ft., 3000 ft., and 3000 ft., and let the total head be 120 ft. Friction heads for the sections vary as $4000/16^5 : 3000/10^5 : 3000/12^5$, or as 3.8 : 30 : 12. Hence the respective heads are $120 \times 3.8/45.8$, $120 \times 30/45.8$, $120 \times 12/45.8$ or 9.95 ft. 78.6 ft., and 31.4 ft. Fig. 55 shows that each section will discharge 4.4 cu. ft. per sec. under these heads.

Branching pipe. To design branching pipe (Fig. 57) to discharge given amounts at C and D assume a pressure drop from (A) to (B) and design the section $(A - B)$ for a discharge equal to the sum of those required at (C) and (D). Then design sections $(B–C)$ and $(B–D)$ to give the desired discharge under their respective heads. If the diameter of any section is fixed, all pressure drops are fixed and no assumptions are necessary.

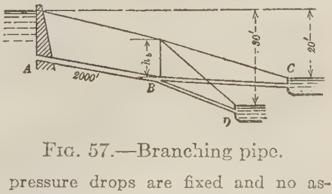


FIG. 57.—Branching pipe.

Example. Find the diameters required to discharge 10 cu. ft. per sec. at (C) and 5 cu. ft. per sec. at (D). Assuming a drop of 10 ft. from (A) to (B), section $(A–B)$ must discharge 15 cu. ft. per sec. under 10 ft. head or 5 ft. per 1000 ft. From Fig. 55 d lies between 20 and 24 in. Assuming the latter, the drop is 6 ft., leaving heads on (C) and (D) of 14 and 24 ft. respectively. $(B–C)$ is then between 16 and 20 in., and $(B–D)$ between 10 and 12 in. It is best to select the larger diameters and control the discharge by gate valves.

To find the discharge from an existing pipe line, make a series of trials assuming different drops from (A) to (B) until one is found that gives a flow through $(A–B)$ that equals the

sum of those through (B–C) and (B–D). If, in Fig. 57, the diameter of (A–B) is 30 in., (B–C) 24 in., and (B–D) 16 in., assuming 10-ft. drop from (A) to (B), the discharges are: (A–B), 33 cu. ft. per sec.; (B–C), 20; and B–D, 14.

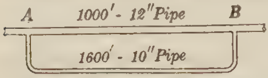


Fig. 58.—Looped pipe.

Looped pipe (Fig. 58). Discharge from each branch is found from the fact that the pressure drop from (A) to (B) is the same in each.

Example. Find the pressure difference from (A) to (B) to give a flow of 5 cu. ft. per sec. and find the flow through each pipe.

For the same friction head, the discharge q varies as $(d^5/l)^{1/2}$, approximately. Hence the ratio of discharges is $\left(\frac{12^5}{1000}\right)^{1/2} : \left(\frac{10^5}{1600}\right) = 15.7 : 7.75$ or nearly $2 : 1$, which gives values of 3.33 and 1.67 cu. ft. per sec. for the two branches. The corresponding lost heads are 6 and 5.8 ft. (Fig. 55). A head of 5.9 ft. will give discharges of 3.2 and 1.8 cu. ft. per sec., as required.

Pump pipe lines. The general problem is selection of pipe for the greatest economy. The pumping head is the sum of the static head and the friction head, the latter varying as d^5 for constant discharge.

Example. Let the pump supply a reservoir with 600 gal. per min. against a static head of 60 ft. through a pipe 2400 ft. long. Valves and bends increase the equivalent length to 2500 ft. Select the pipe.

$q = 1.335$ cu. ft. per sec. The head lost per 1000 ft. for this discharge is approximately 1, 2.5, 7.2, and 30 ft. for diameters of 12, 10, 8 and 6 in. respectively (Fig. 55). Pipes 6-in. or less diameter give too large friction loss. The total losses for 8-, 10- and 12-in. pipes are 17.5, 6.25, and 2.5 ft. to which the static head must be added to find the pump pressure. The most economical pipe will be the one for which the interest on first cost of pipe line plus the value of power lost in friction will be a minimum. Note that a small change in diameter makes a large difference in friction head.

Pumping. Three cases are represented in Fig. 59. (a) The total head is the static head plus friction head and the net power required is $WH/550$ hp., where W = pounds pumped per sec. and H is total head. (b) The pumping head is the difference between the friction and static heads. The negative head at (C) should not be greater than 25 ft. Since the slope of the hydraulic gradient varies inversely as d^5 it is possible to control the pressure at (C) by a proper selection of pipe diameter. (c) The pipe is arranged to eliminate negative head at (C). The static head is the difference in elevation between (A) and (C) and the friction head is the pressure drop from (A) to (C). Pipe (CB) is a conduit designed to carry the given discharge under head h. Its diameter is less than that of the line (AC).

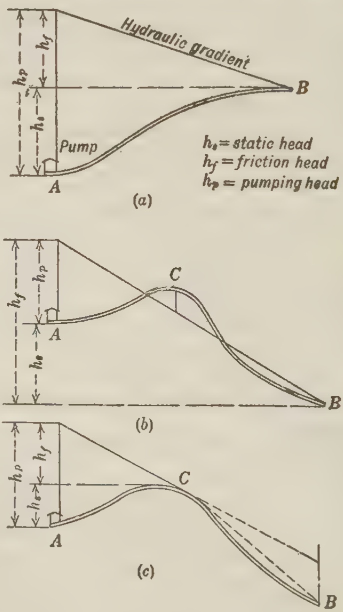


Fig. 59.—Pump pipe lines.

Example. (AC) = 2000 ft., (CB) = 4000 ft.; elevation of (A) = 20 ft., (C) = 50 ft., (B) = 0. Find the power required to pump 4 cu. ft. per sec. through 12-in. pipe. (b) Fig. 55 gives h = 7.9 ft. per 1000 ft. = 47.4 ft. Static head = − 20 ft. hence the pumping head = 27.4 ft. and Hp. = 12.45 net. (c) If no negative pressure at (C) is permissible, the pumping head becomes $30 + (2 \times 7.9) = 45.8$ ft. and (CB) is designed to discharge 4 cu. ft.

per sec. under a head of 50 ft. The diameter lies between 10 and 12 in. hence the 12-in. pipe will run partly full. This can be avoided by partially closing a gate valve at (B). Were the section (AC) made of 16-in. pipe, the friction head per 1000 ft. becomes 1.9 ft., making the pumping head 33.8 ft., a saving of 26.6 per cent.

Power delivered by a pipe is proportional to the product of the discharge and head at the delivery end. In Fig. 60, h_1 is a maximum when $q = 0$ and $h_1 = 0$ when q = maximum. Power varies as $q(h_1 + v^2/2g)$ and has a maximum value when one-third of the static head is consumed in friction. Usually the value of the power is such that the pipe-line is designed to consume but 5 to 10 per cent. of the total head in friction. Selection of a proper pipe diameter is governed by the same factors that control in pump pipe lines. Fig. 61 shows relations between head, power, and discharge for pipe delivering power.

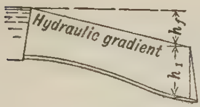


Fig. 60.—Pipe with nozzle delivering power.

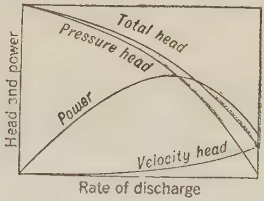


Fig. 61.—Head in pipe and power delivered.

Example. Water under a total head of 1000 ft. is brought to a 1000-kw. Pelton wheel by a pipe 5000 ft. long. The efficiency of the wheel is 85 per cent. Design a pipe so that the friction loss shall be about 10 per cent. of the total head.

Assuming the head on the wheel = 900 ft., $q = 1000 \times 550/(900 \times 62.5 \times 0.746 \times 0.85) = 15.5$ cu. ft. per sec. For cast-iron pipe in good condition, a 120-ft. head is required to discharge this quantity through 16-in. pipe and 40-ft. for 20-in. pipe (Fig. 55). 16-in. pipe will supply 16 cu. ft. per sec. with a loss of 125 ft. or 12.5 per cent., which is the amount required to give 1000 kw. under 875-ft. head, while 20-in. pipe will give the same power with a loss of 3.5 per cent., the discharge being 14.5 cu. ft. per sec. After 10 to 15 years' service the 20-in. pipe will maintain the required power with a loss of 6 per cent. and discharge of 15 cu. ft. per sec. (Fig. 56 with $C = 100$.)

Water hammer or ram is the shock or blow produced by the dynamic action of water when the flow is suddenly checked by the rapid closing of a valve or by other obstruction. The sudden decrease in velocity sets up a pressure wave which produces a maximum excess pressure of $p = 63v$ lb. per sq. in. where v = change in velocity or "extinguished" velocity; the SPEED OF PROPAGATION is about 4500 ft. per sec. If the time of closing the valve exceeds 0.0004281 sec., $p = 0.0271v/t$. If the pressure wave enters a branch pipe with a dead end, the excess pressure may be increased two- or three-fold; a series of branches may build up a dangerous pressure.

When the velocity in a long pipe supplying power to a water wheel is suddenly checked a pressure wave is built up which travels back to the reservoir and, due to the elasticity of water and pipe, forces water back, thus producing a wave of reduced pressure. The result is establishment of an oscillating wave with gradually decreasing amplitude and period depending on the length of pipe. Water hammer may be greatly reduced by the use of air chambers on pumps and of surge tanks on pipe lines supplying water wheels; automatic relief valves employing springs are not as good. Velocity should never be checked suddenly hence the valves are designed to close slowly. Allowance should be made for water hammer in design of pipe.

29. Design of pipe

Determination of diameter usually involves selection of a standard size of pipe that will furnish the required discharge under given conditions. Dia-

grams for selection of diameter give as great accuracy as conditions warrant. The roughness factor is so uncertain that discrepancies of 10 per cent. or more between calculated and actual discharge may be expected.

Cast-iron pipe. Standard sizes are given in Table 29. Bell-and-spigot (Fig. 62) or flanged joints are provided. Flanged pipe is required for pump connections and for conditions where it may be desired to remove a section. Exposed pipe is generally flanged and submerged pipe of bell-and-spigot variety.

Table 29. Standard weights and thicknesses of cast-iron bell-and-spigot pipe
(*U. S. Cast Iron Pipe & Foundry Co. Based on specifications of Am. Soc. for Test. Mat.*)

Nominal inside diam., in.	Approx. laying length, ft.	Class A 100-ft. head, 43 lb. pressure			Class B 200-ft. head, 86 lb. pressure			Class C 300-ft. head, 130 lb. pressure			Class D 400-ft. head, 173 lb. pressure			Approximate pounds of lead per joint 2 in. thick	Approximate pounds of hemp per joint
		Thickness, in.	Outside diam., in.	Weight per foot, pounds.	Thickness, in.	Outside diam., ft.	Weight per foot, pounds	Thickness, in.	Outside diam., ft.	Weight per foot, pounds	Thickness, in.	Outside diam., ft.	Weight per foot, pounds		
3	12	0.38	3.80	13.9	0.42	3.84	16.2	0.45	3.90	17.1	0.48	3.96	18.0	6.0	0.18
4	12	0.42	4.80	20.0	0.45	5.00	21.7	0.48	5.00	23.3	0.52	5.00	25.0	7.5	0.21
6	12	0.44	6.90	30.8	0.48	7.10	33.3	0.51	7.10	35.8	0.55	7.10	38.3	10.3	0.31
8	12	0.46	9.05	42.9	0.51	9.05	47.5	0.56	9.30	52.1	0.60	9.30	55.8	13.3	0.44
10	12	0.50	11.10	57.1	0.57	11.10	63.8	0.62	11.40	70.8	0.68	11.40	76.7	16.0	0.53
12	12	0.54	13.20	72.5	0.62	13.20	82.1	0.68	13.50	91.7	0.75	13.50	100.0	19.0	0.61
14	12	0.57	15.30	89.6	0.66	15.30	103.0	0.74	15.70	117.0	0.82	15.70	129.0	22.0	0.81
16	12	0.60	17.40	108.0	0.70	17.40	125.0	0.80	17.80	144.0	0.89	17.80	158.0	30.0	0.94
18	12	0.64	19.50	129.0	0.75	19.50	150.0	0.87	19.90	175.0	0.96	19.90	192.0	33.8	1.00
20	12	0.67	21.60	150.0	0.80	21.60	175.0	0.92	22.10	208.0	1.03	22.10	229.0	37.0	1.25
24	12	0.76	25.80	204.0	0.89	25.80	233.0	1.04	26.30	279.0	1.16	26.30	307.0	44.0	1.50
30	12	0.88	31.70	292.0	1.03	32.00	333.0	1.20	32.40	400.0	1.37	32.70	450.0	54.3	2.06
36	12	0.99	38.00	392.0	1.15	38.30	454.0	1.36	38.70	546.0	1.58	39.20	625.0	64.8	3.00
42	12	1.10	44.20	513.0	1.28	44.50	592.0	1.54	45.10	717.0	1.78	45.60	825.0	75.3	3.62
48	12	1.26	50.50	667.0	1.42	50.80	750.0	1.71	51.40	908.0	1.96	52.00	1050.0	85.5	4.37
54	12	1.35	56.70	800.0	1.55	57.10	933.0	1.90	57.80	1140.0	2.23	58.40	1340.0	97.6	6.25
60	12	1.39	62.80	917.0	1.67	63.40	1110.0	2.00	64.20	1340.0	2.38	64.80	1580.0	108.0	8.25
72	12	1.62	75.30	1280.0	1.95	76.00	1550.0	2.39	76.90	1900.0	....	.....	......	131.3	12.50
84	12	1.72	87.50	1630.0	2.22	88.50	2100.0	....	.....	......	....	.....	......	152.0	15.00

Fig. 62.

Fig. 63.

The latter is made tight by calking and then pouring in a lead collar which is afterwards calked. Special flexible lead joints (Fig. 63) are made which are capable of motion of several degrees without causing leakage. Joints allow for change in length due to temperature change, hence no temperature stresses are produced.

Thickness of cast-iron pipe is given by Brackett's formula,

$$T = 0.25 + (P + P^1)r/3300,$$

in which T = thickness in inches, P = maximum static pressure in lb. per sq. in. P^1 = allowance for water hammer and r = radius in inches. This formula allows a large factor of safety to cover inequalities in casting and strains other than that due to internal pressure

and gives sufficient thickness to insure the pipe against excessive breakage in handling and shipping. For large pipe the value of $P^1 = 70$ lb. per sq. in.; for smaller pipe Brackett gives the following:

Diameter of pipe, inches.	36	30	24	20	16	12	10 to 3
P^1, pounds per square inch.	75	80	85	90	100	110	120

In ordering pipe for given pressure or head, refer to makers' tables.

Coating. Cast-iron pipe is always given a protective coating by dipping in a solution of coal tar and linseed oil at 300° F.

Pipe is subject to formation of tubercles of RUST, especially if the water contains much CO_2. These deposits may greatly reduce the diameter in the course of years hence suitable allowance must be made. Hazen-Williams formula with $C = 100$ (Art. 28, Fig. 56) allows a considerable factor of safety and should give the correct discharge after 16 years' use, if corrosion is not abnormal. Cast-iron pipe should have a LIFE of 40 to 80 years. Pipe is sold by the net ton; PRICE varies with the price of pig iron. The price for the pre-war period averaged \$25–\$30 per ton with an average of \$60 per ton for special shapes as *Y*'s, *T*'s, curves, etc.

Wrought-iron and steel lap- and butt-welded pipe is used for distribution of water, gas, and steam in sizes ⅛ in. to 15 in. nominal diameter and in three weights, standard full-weight, extra-strong, and double extra-strong. Special pipe for hydraulic machinery capable of withstanding internal pressure of 10,000 lb. per sq. in. is made by boring solid steel forgings. Most lap- and butt-welded pipe now on the market is mild steel but is often sold as wrought-iron. Large lap-welded steel pipe can be furnished in any diameter in thickness from ⅛ to 1¼ in. Forged flanges and connections are furnished in all sizes. Standard sizes are given in Table 30.

Flow through wrought-iron and steel pipe should be calculated for the actual internal diameter, which differs considerably from the nominal diameter. Small pipe is generally employed in short lengths with many elbows, valves, and other fittings, hence secondary losses of head play an important part. Table 31 gives dimensions of small sizes of standard full-weight pipe and the flow that may be expected for given pressures and equivalent lengths. (Williams and Hazen, *Hydraulic Tables*, Wiley.)

Riveted-steel pipe is used for high pressures and for long mains 30 in. or more in diameter; it is not generally used in smaller sizes. It is cheaper than cast-iron for large diameters. The pipe is made of sheets, 7 or 8 ft. wide, which are bent around and riveted with a double-riveted lap joint. (Butt joints with straps on the outside have also been used.) End joints are made by single-riveting in and out courses, alternate rings being larger and smaller. The pipe is also made in taper lengths, one end designed to slip into the large end of the next length. Continuous riveting is generally used on pipe lines, each length being securely riveted to the next. The pipe must then withstand temperature stresses which, under most unfavorable conditions, may be as high as 9000 lb. per sq. in. Strong anchorages must be provided to withstand the resultant thrust due to temperature change and especially designed valves and fittings must be employed to resist this pressure. Temperature stresses may be avoided by the use of expansion joints. Riveted pipe is also furnished in lengths up to 40 ft. with pressed or forged-steel flanges which bolt together as in cast-iron pipe.

Spiral-riveted pipe is furnished in diameters up to 40 in. Each length is made of single sheet wound with overlapping edges which form a single-riveted spiral seam of great strength. Spiral-riveted pipe is stronger than straight riveted pipe of the same weight and thickness and, due to spiral seams, has more resistance to bending or transverse loading. End connections (Fig. 64)

Table 30. Standard full-weight wrought-iron and steel pipe. (*National Tube Co.*)

Diameter, inches			Nominal thickness, inches	Circumference, inches		Transverse areas, square inches			Length of pipe per square foot of		Length of pipe containing 1 cu. ft., feet	Nominal weight per foot, pounds	Number of threads per inch of screw
Nominal internal	Actual external	Approximate internal		External	Internal	External	Internal	Metal	External surface, feet	Internal surface, feet			
⅛	0.405	0.27	0.068	1.27	0.85	0.13	0.06	0.07	9.44	14.15	2513.00	0.24	27
¼	0.540	0.36	0.088	1.70	1.14	0.23	0.10	0.12	7.08	10.49	1383.30	0.42	18
⅜	0.675	0.49	0.091	2.12	1.55	0.36	0.19	0.17	5.66	7.76	751.20	0.57	18
½	0.840	0.62	0.109	2.63	1.95	0.55	0.30	0.25	4.55	6.15	472.40	0.85	14
¾	1.050	0.82	0.113	3.30	2.59	0.87	0.53	0.33	3.64	4.64	270.00	1.13	14
1	1.315	1.05	0.134	4.13	3.29	1.36	0.86	0.50	2.90	3.65	166.90	1.68	11½
1¼	1.660	1.38	0.140	5.22	4.34	2.16	1.50	0.67	2.30	2.77	96.25	2.27	11½
1½	1.900	1.61	0.145	5.97	5.06	2.84	2.04	0.80	2.01	2.37	70.66	2.72	11½
2	2.375	2.07	0.154	7.46	6.49	4.43	3.36	1.07	1.61	1.85	42.91	3.65	11½
2½	2.875	2.47	0.204	9.03	7.75	6.49	4.78	1.71	1.33	1.55	30.10	5.79	8
3	3.500	3.07	0.217	11.00	9.63	9.62	7.39	2.24	1.09	1.25	19.50	7.57	8
3½	4.000	3.55	0.226	12.57	11.15	12.57	9.89	2.68	0.96	1.08	14.57	9.11	8
4	4.500	4.03	0.237	14.14	12.65	15.90	12.73	3.18	0.85	0.95	11.31	10.79	8
4½	5.000	4.51	0.246	15.71	14.16	19.64	15.96	3.68	0.76	0.85	9.02	12.54	8
5	5.536	5.05	0.259	17.48	15.85	24.31	19.99	4.32	0.69	0.76	7.20	14.62	8
6	6.625	6.07	0.280	20.81	19.05	34.47	28.89	5.59	0.58	0.63	4.98	18.97	8
7	7.625	7.02	0.301	23.96	22.06	45.66	38.74	6.92	0.50	0.54	3.72	23.54	8
8	8.625	8.07	0.276	27.10	25.35	58.43	51.15	7.28	0.44	0.47	2.82	24.69	8
8	8.625	7.98	0.322	27.10	25.07	58.43	50.02	8.41	0.44	0.48	2.88	28.55	8
9	9.625	8.94	0.344	30.24	28.08	72.76	62.72	10.04	0.40	0.43	2.29	33.91	8
10	10.750	10.14	0.278	33.77	32.01	90.76	81.55	9.21	0.36	0.37	1.76	31.20	8
10	10.750	10.14	0.306	33.77	31.86	90.76	80.75	10.01	0.36	0.38	1.78	34.24	8
10	10.750	10.02	0.366	33.77	31.47	90.76	78.82	11.94	0.36	0.38	1.82	40.48	8
11	11.750	11.00	0.375	36.91	34.56	108.43	95.03	13.40	0.33	0.35	1.51	45.56	8
12	12.750	12.09	0.328	40.06	37.98	127.68	114.80	12.88	0.30	0.32	1.25	43.77	8
12	12.750	12.00	0.375	40.06	37.70	127.68	113.10	14.59	0.30	0.32	1.27	49.56	8
13	14.000	13.250	0.375	43.96	41.60	153.86	137.81	16.05	0.27	0.29	1.04	54.57	8
14	15.000	14.250	0.375	47.10	44.70	176.62	159.39	17.23	0.25	0.27	0.90	58.57	8
15	16.000	15.250	0.375	54.24	47.90	200.96	182.55	18.41	0.24	0.25	0.75	62.58	8

are of flanged, bolted or slip-joint type. The latter, used for low pressures, consists of a sleeve wrapped in burlap soaked in red lead or tar and driven into the adjoining pipe. Lugs on the pipe are then connected by wire. See also Sec. 23, Table 14.

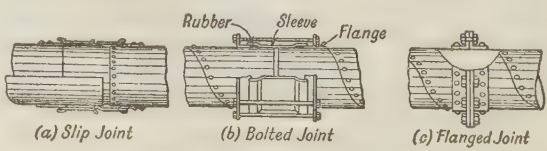


FIG. 64.—Joints for riveted pipe.

Lock-bar pipe (Fig. 65) is made by upsetting the edges of steel plates and inserting them in grooves of a lock bar of H section which is then forced down

Table 31. Flow in small pipes

Wrought iron, inches		Discharge in U. S. gallons		Velocity, feet per second	Loss of head, feet, per 1000 ft.		
Nominal size	Actual diameter	Per minute	Per 24 hr.		Smooth new iron, $C=120$	Ordinary iron, $C=100$	Old iron, $C=80$
⅛	0.270	0.2	288	1.12	44	62	94
		0.4	576	2.24	158	220	335
		0.6	864	3.36	335	470	710
		0.8	1,152	4.48	570	800	1210
		1.0	1,440	5.60	860	1210	1830
¼	0.364	0.5	720	1.54	56	78	118
		1.0	1,440	3.08	200	280	430
		1.5	2,160	4.62	425	600	910
		2.0	2,880	6.16	730	1030	1550
½	0.623	2.0	2,880	2.10	53	74	112
		4.0	5,760	4.21	192	270	410
		6.0	8,640	6.31	410	570	870
		8.0	11,520	8.42	700	980	1480
		10.0	14,400	10.52	1050	1470	2230
¾	0.824	2	2,880	1.20	14	19	29
		4	5,760	2.41	50	70	105
		6	8,640	3.61	105	147	224
		8	11,520	4.81	180	250	380
		10	14,400	6.02	271	380	580
		12	17,280	7.22	380	530	800
		15	21,600	9.02	570	800	1220
		20	28,800	12.03	970	1360	2060
1	1.048	5	7,200	1.86	23.2	32.5	49.1
		10	14,400	3.72	84	117	177
		15	21,600	5.58	178	246	378
		20	28,800	7.44	301	420	640
		25	36,000	9.30	455	640	960
		30	43,200	11.15	640	890	1350
		35	50,400	13.02	850	1190	1800
		40	57,600	14.88	1090	1520	2300
1¼	1.380	5	7,200	1.07	6	8.4	12.7
		10	14,400	2.14	21.8	30.5	46
		20	28,800	4.29	79	111	168
		25	36,000	5.36	119	166	251
		30	43,200	6.43	169	235	358
		35	50,400	7.51	223	312	470
		40	57,600	8.58	285	400	610
		50	72,000	10.72	432	600	920
		60	86,400	12.87	610	850	1290
		70	100,800	15.01	810	1130	1700
		80	115,200	17.16	1030	1450	2200
1½	1.611	10	14,400	1.57	10.2	14.3	21.7
		20	28,800	3.15	37	52	78
		30	43,200	4.72	78	110	166
		40	57,600	6.30	133	188	281
		50	72,000	7.87	202	284	428
		60	86,400	9.44	281	396	600
		70	100,800	11.02	376	530	800
		80	115,200	12.59	480	680	1020
		90	129,600	14.17	600	840	1260
		100	144,000	15.74	730	1020	1540
		110	158,400	17.31	870	1220	1840
		120	172,800	18.90	1020	1430	2170
2	2.0	20	28,800	2.04	12.9	18.2	27.5
		40	57,600	4.08	46.8	66	99
		60	86,400	6.13	99	139	210
		80	115,200	8.17	169	237	258
		100	144,000	10.21	256	358	540
		120	172,800	12.25	360	500	760
		140	201,600	14.30	479	670	1020
		160	230,400	16.34	610	860	1290
		180	259,200	18.38	760	1070	1620
		200	288,000	20.42	920	1290	1960
		220	316,000	22.47	1110	1540	2340

over them by hydraulic pressure. The resulting joint has an efficiency of 100 per cent. in place of 72 per cent. for a double-riveted seam, but the joint is usually figured for 90 per cent. efficiency. Carrying capacity is greater than riveted pipe on account of absence of rivet heads.

Fig. 65.—Lock bar for steel pipe.

Coating. Steel pipe has a shorter life than cast-iron pipe and is liable to rust and pitting. It is therefore dipped in hot asphalt solution or is galvanized, which both protects the pipe and renders the joints tighter after calking.

Carrying capacity of steel pipe. Riveted pipe has a rough interior due to projecting rivet heads; these cause eddy currents and increase pipe friction. It carries 10 to 15 per cent. less water than cast-iron or lock-bar pipe. Hazen-Williams formula with $C = 110$ may be used for new pipe and $C = 95$ for pipe in service several years. Values of Chezy constant for new and old pipe are given in Table 27.

Thickness of steel pipe $= \dfrac{\textit{diam. in inches} \times \textit{pounds pressure.}}{2 \times 16{,}000 \times \textit{efficiency of joint}}$ A working stress of 16,000 lb. per sq. in. gives a factor of safety of 3.5. The efficiency of a single-riveted joint a 55 per cent.; double-riveted lap joint, 72 per cent.; lock-bar, 90 per cent. Due allowsince must be made for water hammer and deterioration.

Wood-stave pipe is much used in localities where lumber is cheap and steel and concrete difficult to obtain. It is usually built with diameter greater than 24 in. and for pressure heads of 20 to 250 ft. The pressure should be sufficient to keep the wood saturated. Continuous wood-stave pipe is built in place, the lower half being assembled in a cradle and the upper half assembled over a pipe ring. Staves break joints and end connections are made water-tight by the use of a steel tongue which fits into saw kerfs in each stave. Staves are held together by steel bands; these are not tightened until the wood is thoroughly saturated. Staves are from $1\frac{1}{4}$ to $2\frac{1}{2}$ in. thick and 6 to 8 in. wide. They are cut with true cylindrical surfaces and radial edges. Bands are $\frac{3}{8}$ to $\frac{3}{4}$ in. diameter and are designed and spaced to carry total hoop tension due to pressure in addition to swelling pressure of the wood. This latter may be assumed at 100 lb. per sq. in. Machine-banded pipe is cut and banded in the shop and pipe lengths are joined with standard cast-iron or special fittings. Staves are made with tongues and grooves and banding is a continuous wire wound on under high tension. Flat bands are also used.

Advantages of wood-stave pipe are ease of transport, easy curves made without special fittings, high carrying capacity, relative cheapness and durability. Kutter, $n = 0.009$ to 0.011; Hazen-Williams, $C = 120$.

Reinforced-concrete pipe comes in diameters of 2 to 10 ft. and lengths of 3 to 5 ft. Large diameters are generally poured in place. The pipe is usually reinforced longitudinally with steel bars and transversely with spiral wire, wire mesh, or steel bands, to resist the total hoop tension. When the pipe is not poured in place, longitudinal reinforcement is designed to provide interlocking of adjacent sections so that when the cement joint is made a continuous pipe results. Reinforced-concrete pipe is usually employed for conduits and for pipe under low heads although reinforcement can be designed for any desired pressure. Thickness of concrete averages one inch per foot of diameter, being slightly greater for small sizes.

30. Flow in open channels

Open channels include flumes, conduits, canals, rivers, and closed pipes when the latter flow partly full. FRICTION HEAD appears as a drop in water surface, not as a loss of pressure; the drop in a given length is just equal to the head required to produce flow through that length. The condition is analogous to that of a pipe line laid on its hydraulic gradient. Flow in existing channels is measured directly when possible; in designing channels and in determining flow where direct measurement is impracticable the Chezy formula $v = C\sqrt{RS}$ is used. Values of C for a wide range of conditions are given in KUTTER'S FORMULA,

$$C = \frac{41.6 + \left(\frac{0.0028}{S}\right) + \left(\frac{1.81}{n}\right)}{1 + \left(41.6 + \frac{0.0028}{S}\right)\left(\frac{n}{\sqrt{R}}\right)},$$

and BAZIN'S FORMULA,

$$C = \frac{158}{1 + N/\sqrt{R}}.$$

Values of C depend in both cases on the roughness of the channel and on the hydraulic radius R; Kutter also introduces the slope S. His formula is most used in this country.

Value of coefficient of roughness in Kutter and Bazin formulas

CHANNELS OF UNIFORM SECTION	Kutter, n	Bazin, N
Well-planed timber, evenly laid	0.009	
Neat cement; best pipe	0.010	0.11
Cement, one-third sand; smooth pipe	0.011	
Unplaned timber; ordinary pipe	0.012	
Ashlar; brick work; new sewer pipe	0.013	0.29
Ordinary brick work and sewers; foul pipe	0.015	
Rubble masonry; rough concrete	0.017	0.83
CHANNELS OF NON-UNIFORM SECTION		
Canals in firm gravel, section nearly uniform	0.02	1.54
Earth canals and rivers free from large stones and weeds	0.025	2.35
Canals and rivers in bad order	0.03–0.04	3.2

Limitations of Kutter's formula. Since the formula was designed to cover a wide range of conditions, considerable error is to be expected. For hydraulic radii greater than 10 ft., for velocities greater than 10 ft. per sec., and for slopes less than 0.0001, the formula should be used with caution. If the slope exceeds 0.001, the value of C for $S = 0.001$ may be used with an error less than the probable error from the formula. Great refinement is unnecessary since an error of 0.001 in the selection of the roughness coefficient n may change C 5 to 17 per cent. The value of the formula depends largely on the proper selection of coefficient n; for smooth flumes and conduits, cleaned periodically, an error of 5 per cent. is to be expected; for canals and rivers in bad order, the formula is but a rough approximation.

Open-channel flow diagrams greatly facilitate the use of Kutter's complicated formula. Tables are also prepared giving values of C for all values of R, S and n found in ordinary practice. One of the best diagrams, prepared by Kennison, is given by Fig. 66.

To find velocity when the hydraulic radius, slope and roughness coefficient are known enter the lower part of the diagram on the horizontal line representing the value of R,

follow this line to the intersection with the given value of n, thence vertically to intersection with the assigned slope line; a horizontal line through this latter intersection determines the velocity.

Example 1. Given $R = 3$ ft., $n = 0.015$, $S = 0.0005$, find velocity. Enter at $R = 3$, proceed to intersection with $n = 0.015$. Project vertically from this point to line $S = 0.0005$ (marked 0.5). This point is on the horizontal line $v = 4.7$, which is the result required.

Example 2. An unplaned timber flume 2 ft. wide and 1 ft. deep is required to supply 10 cu. ft. per sec. What slope is necessary? $v = q/a = 5$ ft. per sec. $R = 2/4 = 0.5$ ft.;

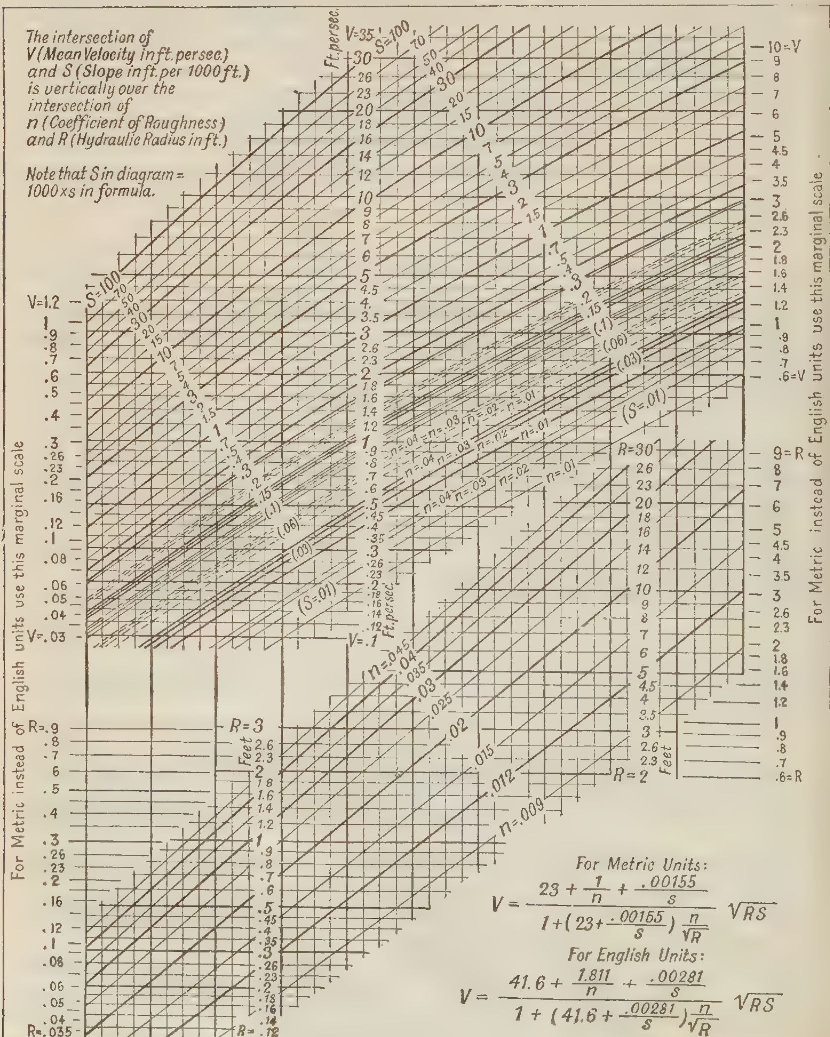


FIG. 66.—Diagram for Kutter formula (*after Kennison*)

n for unplaned timber = 0.012. Vertical from intersection of $R = 0.5$ and $n = 0.012$ intersects line $v = 5$ on line $S = 0.006$ (marked 6) which is the necessary slope. For small values of S, as 0.00001, the position of the line representing S varies with the value of n.

Design of open channels. The Chezy formula shows that velocity increases as R increases. The constant C also increases with R, so that v varies nearly as $R^{2/3}$. Hence for a given discharge, that cross-section should be used which makes R maximum. This condition also makes the perimeter and area of the channel a minimum. A SEMI-CIRCLE meets these requirements hence a semi-circular channel will discharge more water than any other form, for given values of area, slope, and degree of roughness.

Figure 67 shows the channels having the most advantageous elements. The depth of a RECTANGULAR FLUME should be one-half the breadth. This gives the least value of perimeter p hence the least cost for the flume and also the least area for given discharge, hence the least cost for excavation. The best TRAPEZOIDAL SECTION is one-half hexagon. If the side slopes are fixed by the nature of the soil, they should be tangent to a circle of radius d as in Fig. 67. The foregoing sections are not used for UNLINED DITCHES because a shallow ditch is cheaper to dig and maintain. AVERAGE DEPTH is $d = 0.5\sqrt{A}$ where d = depth in ft. and A = area in sq. ft. SIDE SLOPES depend on the material; usual values are 1 : 1, 1.5 : 1, and 2 : 1. For average loam use 1.5 : 1 (Fig. 67).

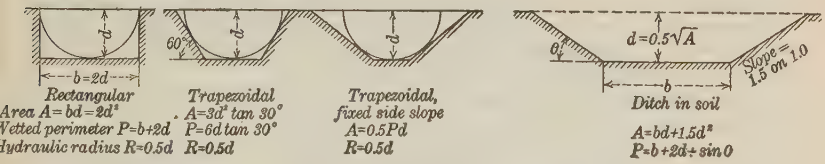


FIG. 67.—Best cross-sections for channels.

Velocity of water should be great enough to prevent deposits of silt and growth of weeds but should not be sufficient to erode the bottom of the channel; 2 ft. per sec. will prevent deposits. The following values of mean velocity are safe against erosion (*Peele*):

Very light loose sand	1.0 to 1.5	Conglomerate, cemented gravel, soft rock	6.0 to 8.0
Average sandy soil	2.0 to 2.5	Hard rock	10.0 to 15.0
Average loam or alluvial soil	2.75 to 3.0	Concrete, water carrying coarse sand	7.0 to 12.0
Stiff clay or ordinary gravel	4.0 to 5.0	Concrete, water carrying fine sand	15.0 to 20.0
Coarse gravel, cobbles	5.0 to 6.0		

The usual velocity for ditches is 2 to 3 ft. per sec., which requires a slope of 3 to 7 ft. per mile. If the available slope is greater than this, the ditch must be lined or else a series of vertical drops introduced. These drops may be wooden or concrete weirs or ramps. The ROUGHNESS COEFFICIENT for unlined earth ditches is usually taken $n = 0.0025$.

Seepage losses in earth ditches may be as high as 25 per cent.; they may be reduced or almost eliminated by linings, which also permit greater velocity and prevent weed growth. LININGS may be puddled clay, road asphalt, or 2 to 4 in. of concrete, placed in position without forms on slopes 1:1 or less. Concrete practically eliminates seepage.

Uniform cross-section is desirable; any change in section causes loss of head by impact and reduces flow. A smooth channel with small changes in section should be regarded as a rough channel in selecting a coefficient of roughness.

Example 1. Design an unplaned timber flume to deliver 20 cu. ft. per sec. with a slope of 0.001.

For the most economic section, $b = 2d$, $A = 2d^2$, $R = 0.5d$, $n = 0.012$. Use Fig. 66 with trial values of d; for values of $d = 1.5$, 1.6, and 1.7 the velocities are 3.3, 3.5, 3.6, giving discharges of 15, 17.8, and 20.8 cu. ft. per sec. Therefore a flume 17 × 34 in. will give the desired flow.

Example 2. Design a ditch in average soil with end slopes 1.5 : 1 to discharge 100 cu. ft. per sec.

Assuming safe velocity = 2.5 ft. per sec., the area must be 40 sq. ft. $d = 0.5\sqrt{A}$, $= 3.2$ ft.; $A = bd + 1.5d^2$, hence $b = 7.7$ ft. Wetted Perimeter $P = b\frac{2d}{\sin\theta} = 19.16$ ft. $R = A/p = 2.09$ ft. Required slope (Fig. 66), is $S = 0.0007$ or 3.7 ft. per mile.

31. Gaging flow in channels

A weir should be constructed for SMALL STREAMS. If a weir or dam does not exist and construction is not feasible, velocity is generally measured by use of FLOATS or by a CURRENT METER. Calculation based on the Chezy formula involves measurement of minute differences of elevation and arbitrary choice of roughness coefficients, hence this method is not recommended unless all others fail.

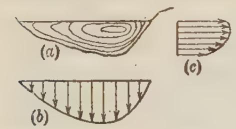


FIG. 68.—Distribution of velocity in stream channel.

Distribution of velocity. Velocity is least at the sides and bottom of the channel and maximum at the deepest part a little below the surface. (Fig. 68.) (*a*) shows contours of equal velocity, (*b*) shows velocity variation in a horizontal plane, (*c*) shows the corresponding curve for a vertical plane passing through the point of maximum velocity. The MEAN VELOCITY, $v = q/A$, is the average of the velocities of all small filaments in the cross-section.

Stream gaging. To measure the discharge directly the velocity must be determined at many points. Plot the transverse profile of the stream bed and divide it into a series of small areas (Fig. 69). Calculate the area and measure the mean velocity for each section. If the areas are $A_1, A_2, A_3, \ldots$, and the corresponding velocities $v_1, v_2, v_3, \ldots$, discharge $q = A_1v_1 + A_2v_2$, etc.

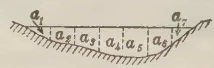


FIG. 69.

SURFACE FLOATS are used for rough field work. The time of passage over a given course is observed and the average velocity taken as 0.8 times the maximum surface velocity. The average velocity in any vertical is 0.86 times its surface velocity. These figures are subject to much variation; the mean velocity may be 90 to 95 per cent. of the surface velocity, hence surface floats are valuable chiefly as a check on other methods and may be 10 to 20 per cent. in error. ROD FLOATS are wooden sticks weighted on one end, or hollow tin cylinders loaded at one end with sand so as to float vertically, nearly touching the bottom and projecting about 6 in. above the surface. Rod floats are very accurate in a channel of uniform cross-section, *e.g.*, a lined conduit. If the float nearly touches bottom its velocity will be average for a given vertical. If the section area is divided into small areas and the velocity of each found by a rod float, the discharge may be found with an error of 5 per cent. or less, but this requires nearly uniform cross-section. Float measurements are laborious and have the added objection that it is often difficult to find any considerable stretch of stream that has approximately uniform section area.

The best method of measurement, aside from a weir, is by CURRENT METER (Art. 32). The U. S. G. S. gives the following four methods of procedure:

(1) Measure velocity at equal intervals of 10 to 20 per cent. of the depth in any vertical, plot the vertical velocity curve, and obtain the average velocity graphically.

(2) Measure velocity at 0.2 and 0.8 times depth and average these results for the mean velocity.

(3) Measure velocity at 0.6 times depth and consider this the mean velocity.

(4) Measure velocity 0.5 to 1 ft. below the surface and multiply by a factor ranging from 0.78 to 0.98, using 0.85 to 0.90 for ordinary stages of flow.

Method (1) should be adopted for precise work. Methods (2) and (3) are based on the assumption that velocity distribution is normal and that the velocity curve is a parabola. Experimental results justify these general assumptions; if, in a given case, the velocity distribution departs widely from the normal, (2) and (3) will be correspondingly in error.

32. Gages and meters

Float gage (Fig. 70) is the best instrument for measuring the elevation of a water surface. The float is a hollow box of non-corrosive metal, either spherical or cylindrical with a conical top and bottom; it is attached to a rigid vertical rod carrying an index mark which is guided along a vertical scale, or else the rod actuates an automatic recording mechanism. For accurate work the float is placed in a vertical pipe or stilling box about 1 in. greater in diameter than the float itself. Lateral motion is prevented by fins. If the pipe is capped on the bottom and small holes are made in the side to allow equalization of level, minor wave and surge action is suppressed. For large changes in level, the float is attached to a light metal graduated tape which winds about a drum provided with a suitable counterweight; the drum may actuate a pointer or a continuous recording mechanism.

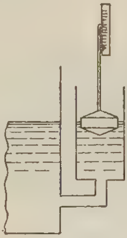


Fig. 70. Float gage.

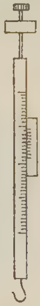
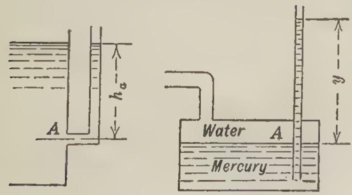


Fig 71. Hook gage.

Hook gage (Fig. 71) is a metal hook on the end of a graduated rod, equipped with vernier scale and slow motion screw. It is operated by submerging the hook and then raising it until a protuberance or pimple appears on the water surface. The hook is then depressed until the pimple just vanishes. The Vernier scale usually reads to 0.001 ft. but precise instruments read to 0.0001 ft. Under favorable conditions readings to 0.0002 ft. may be obtained.

The hook gage is usually employed for measurement of the head on a weir; for this purpose it is placed in a stilling box connected to the channel of approach some distance back of the crest. The connecting pipe is set at right angles to the current or else is perforated to eliminate velocity head. The zero reading is obtained by precise leveling. An approximate method is to read the gage when water just trickles over the weir.

Open liquid column or piezometer tube (Fig. 72) is the simplest device for measuring pressure head. Mercury is used for high pressures; water or light oil for small gas pressures. Readings should be taken tangent to the middle point of the meniscus. Tubes should be ⅜ in. or more in diameter to avoid errors due to capillarity.



$h_a = 13.57y$

Fig. 72.—Piezometer tubes.

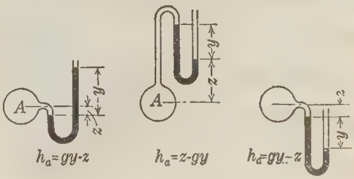


Fig. 73.—U-tubes.

Open U-tubes are used to measure small pressures both above and below atmospheric pressure. (In Fig. 73, g = specific gravity of the liquid in the tube.) Any liquid that does not react chemically with the substance gaged may be used. If applied to gas pressures, $z = 0$.

Hydraulic pressure gages are also available built on the principle of the Bourdon steam gage and of the aneroid barometer.

Differential gages (Fig. 74) measure differences in pressure only, such as the pressure loss in a pipe line due to a valve or obstruction. The mercury gage consists of a U-tube containing mercury with upper ends of the tube connected to vessels under pressure. Difference in pressure at (*A*) and (*B*) is indicated by difference in weight of a column of mercury and a column of water each y ft. in height. $h_a - h_b = 13.57yh - y = 12.57y$. An OIL GAGE is used for small differences in pressure. If the specific gravity of the oil is g, $h_a - h_b = (1 - g)y$.

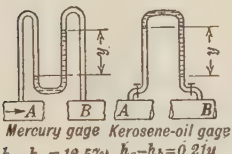


FIG. 74.—Differential gages.

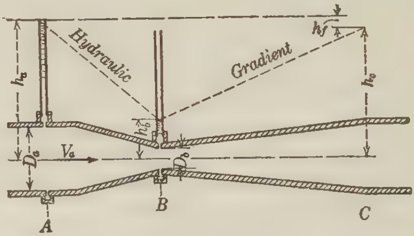


FIG. 75.—Venturi meter.

Venturi meter (Fig. 75) is an accurate and easily constructed device for measuring flow in pipes and closed conduits with any diameter. Two hollow truncated cones having the same base diameter are inserted in the pipe line; piezometer tubes are connected at the entrance and at the contracted section or throat; the pressure difference at these points is an indication of discharge.

By Bernouilli's theorem for frictionless flow in a horizontal tube, the sum of pressure head and velocity head is constant. Hence $h_a + v_a^2/2g = h_b + v_b^2/2g$. The product of velocity and cross-section area is also constant, hence if K be the ratio of large to small diameter $= D_a/D_b$, then $v_b = K^2 v_a$, and $h_a - h_b = (K^4 - 1)v_a^2/2g$ or $v_a = \sqrt{\dfrac{(h_a - h_b)2g}{K^4 - 1}}$. Introducing a constant C to allow for friction losses, discharge $q = C\dfrac{\pi D_a^2}{4} \cdot v_a$ or $q = \dfrac{C\pi D_a^2}{4\sqrt{K^4 - 1}} \cdot \sqrt{2g(h_a - h_b)}$. For any given meter this becomes $q = aH^{1/2}$ where a is a constant and H is the difference in head in feet, obtained usually by the use of a mercury differential gage. The value of C varies from 0.96 to 0.98 and the head lost in passing the meter is from 10 to 15 per cent. of the difference in head $h_a - h_b$. The ratio of pipe to throat diameter varies from 2 to 4, the larger values being used for low velocities, since in this case greater relative increase of velocity at the throat is necessary to give a reading of required sensitiveness. Care must be used that the pressure at the throat does not fall much below atmospheric; preferably it should be positive.

Venturi meters are built in standard-pipe sizes up to diameters of 60-in. They are of cast iron with bronze throat pieces and have interior surfaces highly polished. The expanding cone is about three-times the length of the contracting cone to avoid losses in impact. Piezometer connections are made to annular pressure chambers which surround the throat and entrance and are connected to the interior by a series of holes spaced equally about the circumference. Large meters are built of steel, wood stave, and reinforced concrete. Those of the CATSKILL WATER SYSTEM are of the latter type with bronze throat pieces. Diameters are 210 in. and 93 in. and capacity 650,000,000 gal. per day. Builders Iron Foundry Co. of Providence, R. I., build meters for diameters of 2 to 60 in., equipped with an automatic recording mechanism operated by floats which are supported by the mercury columns of a differential gage.

Example. Find a suitable ratio of diameters for a Venturi meter on a 24-in. pipe line discharging 16 cu. ft. per sec., if the location of the meter is 20 ft. below the hydraulic gradient.

The head at the throat is to remain positive. $v_a = 16/\pi = 5.1$. $h_a - h_b = 20$ ft. $= (K^4 - 1)v_a^2/2g$, hence $K = 2.66$. Adopting the ratio $K = 2.5$ and assuming $C = 0.97$, the discharge becomes $q = 3.95 \times H^{1/2}$.

Pitot tube is an instrument for measurement of velocity by means of the dynamic pressure of a jet. If a curved tube (Fig. 76) is placed in a jet with the plane of its orifice normal to the direction of flow, water will rise in the tube to a height h which varies directly as v^2. Theoretically the impulse of the jet should balance the head $= v^2/2g$ but experiments show that $h = Cv^2/2g$ or $v = C\sqrt{2gh}$ where C is very nearly unity; h is hence a direct measure of velocity head. The Pitot tube is used in pipes under pressure by employing two tubes, one the impact tube, and the other a tube set to record static pressure only. If h is the difference in head, $v = C\sqrt{2gh}$, and if the pressure tube has an orifice flush with the wall of the pipe C is unity.

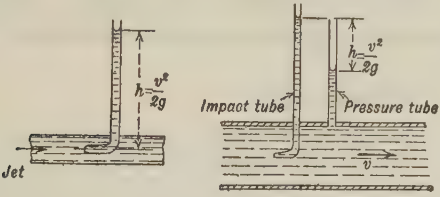


FIG. 76.—Pitot tubes.

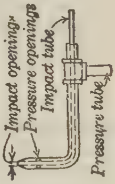


FIG. 77.

The **commercial** form of tube combines impact and pressure tubes in a single brass rod, preferably of stream-line form (Fig. 77). In such a tube the pressure openings do not record full static pressure because of disturbance produced by the presence of the tube, hence C is less than unity and depends on the relative positions of the impact and pressure openings; for Fig. 77 it varies from 0.84 to 0.88. For precise work the tube should be calibrated by moving it through still water at known velocity or by placing in a stream of known velocity.

PITOTMETER is a recent form of instrument which uses two Pitot tubes, one facing up and the other down stream. $v = 0.84\sqrt{2gh}$. Difference of head is measured by a differential gage using a mixture of gasolene and carbon tetrachloride with specific gravity 1.25 to 1.5. The instrument is equipped with a photographic recording device.

Use of Pitot tubes in pipes. Distribution of velocity in pipes is shown by Fig. 78. The velocity curve for normal distribution is a semi-ellipse; the mean velocity is 0.83 × *the center velocity*. If a straight pipe, 50-diameters or more in length, without valves or other obstructions, is obtainable, normal distribution of velocity may be assumed and discharge computed from measurement of the center velocity. For greater precision a complete traverse of the pipe cross-section is necessary. The section is divided into ten concentric circles that divide the area into tenths. The impact opening of the tube is placed on the circumference of odd-numbered circles and ten readings obtained. Since each velocity represents one-tenth area, the mean velocity is the average of the readings. Traverses should be made on two diameters at right-angles.

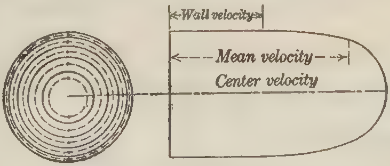


FIG. 78.—Velocity distribution in pipe, 10-point method of sectioning.

Current meter consists of a small wheel-like screw or water wheel which, when placed in a flowing stream, revolves due to the dynamic action of water, its angular speed depending on the velocity in the neighborhood of the point of immersion. The wheel is equipped with a rudder, weights, and rod or guy rope for holding at the desired position in the stream. Revolutions of the wheel are indicated by a make-and-break electric circuit and by revolution counter.

Price current meter, used by the U. S. Geologic Survey, has five conical buckets on a vertical shaft, supported by a frame which carries a rudder. The frame is pivoted to the vertical brass rod, the upper end of which has an eye for attachment of a wire or cable, and the lower end carries a lead weight, also equipped with a rudder, to hold the meter in position. In shallow water the meter is attached to a graduated brass rod without weight or rudder. Meter is equipped with revolution counter and telephone buzzer, the latter being an indication that the wheel is revolving properly. Haskell current meter has a wheel of the screw-propeller type revolving on a horizontal shaft. In other respects it is similar to the Price meter.

Calibration of a current meter. While it is possible to construct a meter for which the revolutions are proportional to the velocity, no such assumption can be made. Every instrument must be calibrated individually and at frequent intervals, especially after tightening or loosening a screw. The meter is calibrated for the desired range either by moving it through still water at the desired velocity or by holding it in a stream whose velocity is known. Rough calibration can be performed by holding the meter just below the stream surface and obtaining the velocity by surface floats. This introduces all the inaccuracies of surface-float measurements. Comparison of meter readings with those obtained from rod floats or a Pitot tube is good.

33. Water supply

Consumption of water in American cities averages about 100 gal. per day per capita; the range is from 30 to 200 gal., depending on supply, percentage metered, and nature of industries. Maximum daily consumption may exceed the mean 40 to 50 per cent.; maximum hourly consumption is determined largely by fire service and may be three times the mean daily average. In mining communities and in isolated plants consumption is determined by mill requirements; in dry or arid regions it is fixed by the amount of water available.

Rainfall and evaporation vary with the locality and vary greatly for the same locality from year to year. U. S. Weather Bureau records give the following values of mean annual rainfall over a period of nearly 30 years: Vicksburg, 53.8; New York, 43.7; Chicago, 33.4; Omaha, 30.8; Helena, 13.3; Yuma, 2.7.

Table 32. Variation in rainfall for the same locality

Locality	Yearly average	Maximum year	Minimum year	Highest 7 consecutive months	Lowest 5 consecutive months	Mean temperature
Boston	45.3	67.7	27.2	47.9	7.9	49
Croton, N. Y.	49.0	63.7	36.9	46.8	10.8	54
Philadelphia	42.6	61.2	29.7	47.7	8.1	54
Atlanta	49.2	65.0	33.0	52.7	8.1	61
Pittsburgh	35.8	50.6	25.3	38.3	6.8	53
Duluth	29.5	45.3	18.1	38.4	2.9	39
Denver	14.3	23.0	8.5	19.1	1.1	51
Phoenix	7.4	19.7	3.8	13.6	0.0	70

Table 32 shows the necessity of having records extending over a period of years.

Evaporation depends on the quantity and distribution of rainfall, temperature, barometric pressure, and nature of vegetation. For New Jersey Vermeule (*U. S. Geol. Surv. of N. J.*, 1894, *vol.* 3, *p.* 76) gives $E = F(15.50 + 0.16R)$ where E = yearly evaporation in in., $F = (0.05T - 1.48)$, T = mean yearly temperature, deg. F; R = yearly rainfall, in. In arid regions evaporation may account for the entire rainfall.

Run-off and yield. Run-off from a water-shed is the amount of water that reaches streams which drain the shed and is the difference between rainfall

and evaporation. YIELD is the collectible portion of rainfall and cannot be accurately computed until the following data are known; (1) catchment area, (2) rainfall, (3) minimum year and a series of years, (4) available storage capacity on streams, losses in evaporation and percolation, (5) measurement of actual discharge of streams. Run-off in New York and New England averages about 45 per cent. of rainfall but the percentage may vary 100 per cent for the same annual rainfall. Average yield is nearly 1,000,000 gal. per day per sq. mile. Small watersheds give a smaller yield per sq. mile than large ones. For the Pacific States Gunsky determined that the run-off may be expressed in identical percentages of precipitation expressed in inches, *i.e.*, if rainfall is 25 in., run-off is 25 per cent. or 6.25 in.; if rainfall is 10 in., run-off is 10 per cent. of 10 or 1 in.

Ground water is that part of precipitation, termed RUN-IN, which soaks into the ground. Sands and gravels permit large run-in, clay is nearly impervious. In Connecticut the run-in is 50 per cent. of the rain-fall, in New South Wales, 2 per cent. Ground water is a source of supply to wells and springs, and to streams except immediately after precipitation. For the same annual rainfall, the ratio between run-in and run-off is a factor of supreme importance in water-supply problems. A high percentage of run-in makes uniform flow in streams and gives abundant supply to wells; low run-in makes arid regions because most of the rainfall finds its way immediately to the water courses and is lost, unless excessive storage capacity is provided. Desert regions are characterized by water courses that are dry except during the period immediately after rain when the water passes off in a flood.

Wells. The horizon below which soil is saturated is called the WATER TABLE. This horizon fluctuates in height with the amount of rainfall and is, in general, parallel to, and a few feet below, the ground surface. Wells sunk below the level of the water-table are a source of supply, the yield depending on the ground water present and ease with which it can flow through the neighboring soil; this depends on the nature of the soil, typography of the country, and geologic features of the rocks. ARTESIAN WELLS are those sunk through the first impervious layer to lower water-bearing strata in which the water is under pressure. FLOW FROM WELLS can be determined only by tests and by records of existing wells in the vicinity. Long records are desirable since flow may decrease rapidly with time.

Stream flow is the usual source of water supply. Stream-flow records covering several years are necessary for dependable estimates of possible yield of a region. When records for short period only are available, rainfall records for the same period should be compared with the run-off record and with the rainfall record for a dry season. When no stream records are available, monthly rainfall records must be used, making allowance for evaporation, deep seepage, and other possible losses. To determine probable fluctuation in flow, rate of precipitation must be considered and ratio of run-in to run-off estimated from topographic features, soil, and vegetation.

Storage is impounding of water during periods of maximum flow for use during dry periods. The usual method is to build a dam in a narrow valley, or a series of such dams on tributaries wherever natural storage sites occur. The storage necessary to assure any assumed continuous draft is determined by study of stream-flow records.

In Fig. 79 the minimum draft can be raised from line A–B to line C–D by supplying storage equal to the larger shaded area representing excess of demand over supply. The height of line C–D increases with the capacity of the reservoir until it equals the mean

annual flow minus evaporation and leakage. EVAPORATION LOSSES from a reservoir are 30 to 100 in. per annum, the high figure applying to arid regions.

Example. If a hydrograph is plotted with a vertical scale of 1 = 1,000,000 gal. per day, and horizontal scale of 1 = 1 month, each square inch of area will represent 30,000,000 gal. The total storage area × 30,000,000 plus the estimated losses gives the REQUIRED STORAGE CAPACITY.

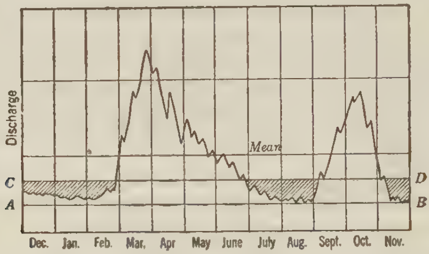


FIG. 79.

Pipe lines should have sufficient capacity to maintain a discharge of 150 per cent. average flow. The size of distribution pipes is determined by fire-service requirements. A standard stream of 250 gal. per min. through 1⅛-in. nozzle requires a pressure of 45 lb. per sq. in. at the base of the tip. Higher pressures are desirable for large buildings. Hydrants should be located so that not more than 300 or 400 ft. of hose is necessary in any one line, the pressure drop being 14 to 30 lb. per sq. in. per 100 ft., depending on the quality of hose. Two or three streams should be available for any one point. Ten standard streams are desirable for a large building.

SECTION 28

APPENDIX

Table 1. The chemical elements (based on oxygen as 16.00) (47 ACS 597 [1925]).

Element	Symbol	Atomic weight	Valence	Element	Symbol	Atomic weight	Valence
Aluminum	Al	26.97	iii	Mercury	Hg	200.61	i, ii
Antimony	Sb	121.77	iii, v	Molybdenum	Mo	96.0	iii, iv, vi
Argon	A	39.91	0	Neodymium	Nd	144.27	iii
Arsenic	As	74.96	iii, v	Neon	Ne	20.2	0
Barium	Ba	137.37	ii	Nickel	Ni	58.69	ii, iii
Beryllium (glucinum)	Be	9.02	ii	Nitrogen	N	14.008	iii, v
Bismuth	Bi	209.0	iii, v	Osmium	Os	190.8	ii, iii, iv, viii
Boron	B	10.82	iii	Oxygen	O	16.00	ii
Bromine	Br	79.92	i	Palladium	Pd	106.7	ii, iv
Cadmium	Cd	112.41	ii	Phosphorus	P	31.027	iii, v
Caesium	Cs	132.81	i	Platinum	Pt	195.23	ii, iv
Calcium	Ca	40.07	ii	Potassium	K	39.096	i
Carbon	C	12.00	ii, iv	Praseodymium	Pr	140.92	iii
Cerium	Ce	140.25	iii, iv	Radium	Ra	225.95	ii
Chlorine	Cl	35.46	i	Radon	Rn	222	0
Chromium	Cr	52.01	ii, iii, vi	Rhodium	Rh	102.09	iii
Cobalt	Co	58.94	ii, iii	Rubidium	Rb	85.44	i
Columbium	Cb	93.1	iii, v	Ruthenium	Ru	101.7	iii, iv, vi, viii
Copper	Cu	63.57	i, ii	Samarium	Sa	150.43	iii
Dysprosium	Dy	162.52	iii	Scandium	Sc	45.10	iii
Erbium	Er	167.7	iii	Selenium	Se	79.2	ii, iv, vi
Europium	Eu	152.0	iii	Silicon	Si	28.06	iv
Fluorine	F	19.00	i	Silver	Ag	107.88	i
Gadolinium	Gd	157.26	iii	Sodium	Na	22.997	i
Gallium	Ga	69.72	iii	Strontium	Sr	87.63	ii
Germanium	Ge	72.60	iv	Sulphur	S	32.064	ii, iv, vi
Gold	Au	197.2	i, iii	Tantalum	Ta	181.5	v
Hafnium	Hf	180.8	iv	Tellurium	Te	127.5	ii, iv, vi
Helium	He	4.0	0	Terbium	Tb	159.2	iii
Holmium	Ho	163.4	iii	Thallium	Tl	204.39	i, iii
Hydrogen	H	1.008	i	Thorium	Th	232.15	iv
Indium	In	114.8	iii	Thulium	Tm	169.4	iii
Iodine	I	126.93	i	Tin	Sn	118.7	ii, iv
Iridium	Ir	193.1	iii, iv	Titanium	Ti	48.1	iii, iv
Iron	Fe	55.84	ii, iii	Tungsten	W	184.0	vi
Krypton	Kr	82.9	0	Uranium	U	238.17	iv, vi
Lanthanum	La	138.9	iii	Vanadium	V	50.96	iii, v
Lead	Pb	207.2	ii, iv	Xenon	Xe	130.2	0
Lithium	Li	6.94	i	Ytterbium	Yb	173.6	iii
Lutecium	Lu	175.00	iii	Yttrium	Yt	88.9	iii
Magnesium	Mg	24.32	ii	Zinc	Zn	65.38	ii
Manganese	Mn	54.93	ii, iv, vi, vii	Zirconium	Zr	91.0	iv

Table 2. Composition and specific gravity of minerals

Mineral	Composition	Specific gravity	Mineral	Composition	Specific gravity
Actinolite	$Ca(Mg \cdot Fe)_3(SiO_3)_4$	3 – 3.2	Hydrozincite	$ZnCO_3 \cdot 2Zn(OH)_2$	3.58 – 3.8
Albite	$NaAl\ Si_3O_8$	2.62 – 2.65	Hypersthene	$(Fe, Mg)SiO_3$	3.4 – 3.5
Almandite	$Fe_3Al_2(SiO_4)_3$	3.9 – 4.2	Ilmenite	$FeTiO_3$	4.5 – 5.0
Amosite	$(Fe, Ca, H_2, Mn)O \cdot SiO_2$	2.2 – 2.3	Iridosmine	Ir and Os (with Rh, Pt, Ru or other metals)	19.3 – 21.12
Amphibole	See actinolite, tremolite, hornblende		Kainite	$MgSO_4 \cdot KCl + 3H_2O$	2.05 – 2.2
Analcite	$NaAlSi_2O_6 + H_2O$	2.22 – 2.29	Kaolinite	$H_4Al_2Si_2O_9$	2.6 – 2.63
Andradite	$Ca_3Fe_2(SiO_4)_3$	3.8 – 3.9	Limonite	$2Fe_2O_3 \cdot 3H_2O$	3.6 – 4.0
Anglesite	$PbSO_4$	6.12 – 6.39	Linnaeite	Co_3S_4 (with Fe or Cu)	4.8 – 5.0
Anorthite	$CaAl_2Si_2O_8$	2.74 – 2.76	Magnesite	$MgCO_3$	3.0 – 3.12
Antimony	Sb (sometimes with As, Fe or Ag)	6.5 – 6.72	Magnetite	Fe_3O_4	5.17 – 5.18
Anthophyllite	$(Mg \cdot Fe)SiO_3$	3.0 – 3.2	Malachite	$CuCO_3 \cdot Cu(OH)_2$	3.9 – 4.03
Apatite	$(Cl \cdot F)Ca_5(PO_4)_3$	3.17 – 3.23	Manganite	$Mn(OH)_3Mn_2O_3$	4.2 – 4.4
Argentite	Ag_2S	7.20 – 7.36	Marcasite	FeS_2	4.85 – 4.90
Arsenopyrite	$FeAsS$	5.9 – 6.2	Melaconite	CuO	±6.0
Atacamite	$Cu_2(OH)_3Cl$	3.75 – 3.77	Millerite	NiS	5.3 – 5.65
Azurite	$2CuCO_3 \cdot Cu(OH)_2$	3.77 – 3.83	Mispickel	See arsenopyrite.	
Barite	$BaSO_4$	4.3 – 4.6	Molybdenite	MoS_2	4.7 – 4.8
Bauxite	$Al_2O_3 \cdot 2H_2O$	2.55	Molybdite	MoO_3	4.49 – 4.5
Bentonite	$SiO_2, Al_2O_3, Fe_2O_3, CaO, MgO$	2.13	Monazite	$(Ce, La)PO_4$	4.9 – 5.3
Beryl	$Be_3Al_2(SiO_3)_6$	2.63 – 2.80	Muscovite	$H_2KAl_3(SiO_4)_3$	2.76 – 3
Biotite	$(H,K)_2(Mg,Fe)_2Al_2(SiO_4)_3$*	2.7 – 3.1	Nephelite	$NaAlSiO_4$	2.55 – 2.65
Bismite	Bi_2O_3	4.36	Niccolite	$NiAs$	7.33 – 7.67
Bismuth	Bi	9.8	Ochre	Pulverized Fe_2O_3 or $Fe_2(OH)_6Fe_2O_3$	About 4
Bismuthinite	Bi_2S_3	6.4 – 6.5	Olivine	$(Mg, Fe)_2SiO_4$	3.3
Bismutite	$(BiO)_2 \cdot CO_3 \cdot H_2O$	6.9 – 7.7	Orpiment	As_2S_3	3.4 – 3.5
Borax	$Na_2B_4O_7 \cdot 10H_2O$	1.69 – 1.72	Orthoclase	$KAlSi_3O_8$	2.47 – 2.62
Bornite	$Cu_2S \cdot CuS \cdot FeS$	4.9 5.4	Pentlandite	$(Fe, Ni)S$	4.60 – 5.0
Bournonite	$PbCuSbS_3$	5.7 – 5.9	Plagioclase	See albite and anorthite.	
Braunite	$3Mn_2O_3 \cdot MnSiO_3$	4.75 – 4.82	Platinum	Pt (alloyed with Fe, Ir, Os, etc.)	14 –19
Brochantite	$CuSO_4 \cdot 3Cu(OH)_2$	3.91	Polybasite	$(AgCu)_{16}Sb_2S_{11}$	6.0 – 6.2
Calamine	$(ZnOH)_2SiO_3$	3.4 – 3.5	Proustite	Ag_3AsS_3	5.57 – 5.64
Calaverite	$(Au \cdot Ag)Te_2$	9.04	Psilomelane	H_4MnO_5 *	3.7 – 4.7
Calcite	$CaCO_3$	2.71	Pyrargyrite	Ag_3SbS_3	5.77 – 5.86
Cassiterite	SnO_2	6.8 – 7.1	Pyrite	FeS_2	4.95 – 5.10
Cerargyrite	$AgCl$	5.55	Pyrolusite	MnO_2	4.82
Cerussite	$PbCO_3$	6.46 – 6.57	Pyromorphite	$Pb_5Cl(PO_4)_3$	6.5 – 7.1
Cervantite	Sb_2O_4	4.08 – 5.28	[illegible]	[illegible]	[illegible]
Chalcanthite	$CuSO_4 \cdot 5H_2O$	2.12 – 2.3			
Chalcedony	SiO_2	2.6 – 2.64			

Chert	SiO_2	2.6 – 2.64	[illegible]	$Ca(Mg,Fe)(SiO_3)_2$	3.2 – 3.38
Chromite	$FeCr_2O_4$	4.32 – 4.57	Pyrrhotite	$Fe_{11}S_{12}$	4.58 – 4.64
Chrysocolla	$CuSiO_3+2H_2O$	2 – 2.24	Quartz	SiO_2	2.65 – 2.66
Chrysolite	$(Mg\cdot Fe)_2SiO_4$	3.27 – 3.37	Realgar	AsS	3.56
Chrysotile	$H_4Mg_3Si_2O_9$	2.22	Rhodochrosite	$MnCO_3$	3.45 – 3.60
Cinnabar	HgS	8.0 – 8.2	Rhodonite	$MnSiO_3$	3.4 – 3.68
Clays	See kaolinite.		Ruby	See corundum	
Coal	C+H+O+N (varies)	1 – 1.8	Rutile	TiO_2	4.18 – 4.25
Cobaltite	CoAsS	6.0 – 6.3	Sapphire	See corundum.	
Colemanite	$Ca_2B_6O_{11}\cdot 5H_2O$	2.42	Scheelite	$CaWO_4$	5.9 – 6.1
Copper	Cu	8.8 – 8.9	Senarmontite	Sb_2O_3	5.3
Corundum	Al_2O_3	3.95 – 4.10	Serpentine	$H_4Mg_3Si_2O_9$	2.5 – 2.65
Covellite	CuS	4.59 – 4.64	Siderite	$FeCO_3$	3.83 – 3.88
Crocidolite	$NaFe(SiO_3)_2\cdot FeSiO_3$	3.2 – 3.3	Silver	Ag (with some Au,Cu and sometimes Pt, Sb, Bi, Hg)	10.1 –11.1
Cryolite	$3NaF\cdot AlF_6$	2.95 – 3.0	Smaltite	$CoAs_2$	6.4 – 6.6
Cuprite	Cu_2O	5.85 – 6.15	Smithsonite	$ZnCO_3$	4.30 – 4.45
Diamond	C	3.516– 3.525	Spessartite	$Mn_3Al_2(SiO_4)_3$	4.0 – 4.3
Dolomite	$Ca,MgCo_{(3)2}$	2.8 – 2.9	Sphalerite	ZnS	3.9 – 4.1
Emery	See corundum.		Spinel	$MgAl_2O_4$	3.5 – 4.1
Enargite	Cu_3AsS_4	4.43 – 4.45	Spodumene	$LiAl(SiO_3)_2$	3.13 – 3.20
Epidote	$Ca_2(AlOH)Al_2(SiO_4)_3$	3.25 – 3.5	Stannite	Cu_2FeSnS_4	4.5 – 4.52
Erythrite	$Co_3As_2O_8\cdot 8H_2O$	2.95	Stephanite	Ag_5SbS_4	6.2 – 6.3
Ferberite	See wolframite.		Stibnite	Sb_2S_3	4.52 – 4.62
Fluorite	CaF_2	3.01 – 3.25	Sulphur	S	2.07 – 2.09
Franklinite	$(Zn,Mn)Fe_2O_4$	5.07 – 5.22	Sylvanite	$(Au,Ag)Te_2$	7.9 – 8.3
Galena	PbS	7.4 – 7.6	Talc	$H_2Mg_3(SiO_3)_4$	2.7 – 2.8
Garnet	See various forms (p. 115).		Tennantite	$Cu_8As_2S_7$	4.37 – 4.49
Garnierite	$H_2(Ni\cdot Mg)SiO_4+H_2O$	2.27 – 2.8	Tenorite	CuO	5.82 – 6.25
Gibbsite	$Al(OH)_3$	2.3 – 2.4	Tetrahedrite	$Cu_8Sb_2S_7$	4.4 – 5.1
Gold	Au (with Ag and sometimes Cu or Fe)	15.6–19.3	Titanite	$CaSiTiO_5$	3.4 – 3.56
Graphite	C	2.09 – 2.23 (2.9, impure)	Topaz	$Al_2F_2SiO_4$	3.4 – 3.6
			Tourmaline	$R_9Al_3B_2(OH)_2Si_4O_{19}$ †	2.98 – 3.20
Grossularite	$Ca_3Al_2(SiO_4)_3$	3.42 – 3.72	Tremolite	$CaMg_3(SiO_3)_4$	2.9 – 3.1
Gypsum	$CaSO_4+2H_2O$	2.31 – 2.33	Uvarovite	$Ca_3Cr_2(SiO_4)_3$	3.41 – 3.52
Halite	NaCl	2.4 – 2.6	Valentinite	Sb_2O_3	5.57
Halloysite	$H_4Al_2O_3\cdot 2SiO_2+H_2O$*	2.0 – 2.2	Wad	See psilomelane	3.0 – 4.26
Hematite	Fe_2O_3	4.9 – 5.3	Willemite	Zn_2SiO_4	3.89 – 4.18
Hessite	Ag_2Te	8.31 – 8.45	Witherite	$BaCO_3$	4.27 – 4.35
Hornblende	$mCa(MgFe)_3(SiO_3)_4 + n(Al,Fe)(F,OH)SiO_3$	±3.2	Wolframite	$(Fe,Mn)WO_4$	7.2 – 7.5
			Wulfenite	$PbMoO_4$	6.7 – 7.0
Hübnerite	$MnWO_4$	6.8	Zincite	ZnO	5.43 – 5.7
			Zircon	$ZrSiO_4$	4.68 – 4.70

* Approximate. † R = Fe, Mg and the alkalies.

INDEX

(Numbers in parenthesis indicate notes or occurrences in flow-sheets)

B

C

G

H

I

L

N

Q

R

U

V

Y

Z